

Non- Iterative Technique for Balancing an Air Distribution System

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(Abstract)

Balancing an air distribution system consists primarily of measuring airflow and adjusting volume control devices to get specified airflow. Flow calculation methods are not accurate enough to ensure proper balancing by duct design alone. To assure proper balancing, dampers within an air distribution system must be adjusted until design flows are met throughout the system to within $\pm 10\%$. By properly balancing an air distribution system, operating costs in the system will be reduced, comfort for the occupants in the building will be increased, and the life of the HVAC equipment will be improved.

Existing balancing techniques are iterative methods that require several measurements and damper adjustments. The flows and pressures are first measured, and then the dampers are adjusted to match design airflow. The flows are measured again and the measuring and adjusting process is repeated on a trial-and-error process until design flow is achieved. This iterative process is time consuming and expensive.

The proposed new balancing technique is to use a computer program that, based on a few measurements, determines damper adjustments that will achieve design airflow throughout the system. Each terminal damper is adjusted only once to a specified flowrate that is determined by the computer program, making the balancing process quicker and less expensive. No iteration is required.

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Nomenclature

Symbols:

A	Fan constant: pressure when flowrate is zero (psi)
B	Fan constant: sensitivity (psi/cfm ²)
K	Friction loss coefficient (psi/cfm ²)
n	Number of diffusers
P	Pressure (psi)
Q	Air flowrate (cfm)
u	Uncertainty
V	Velocity (ft/min)
X	Percent of design flow (%)
x	Independent variable
y	Dependent variable
z	Difference between calculated and measured flowrate (cfm)

Greek:

ρ	Density (lbs/ft ³)
--------	--------------------------------

Subscripts:

a	First pass
b	Second Pass
d	Design
f	Final
m	Measured
p	Positioning

Overbars:

(bold caps)	Matrix
(bold)	Vector

Chapter 1

Introduction

1.1 Description of Air Distribution Systems

Air distribution systems are commonly used for ventilating, warm air heating, and central air conditioning. Heating, ventilating, and air-conditioning (HVAC) systems are designed to satisfy the environmental requirements for comfort in a conditioned space and are used in residential, commercial, and industrial facilities.

HVAC systems are designed for cooling during hot months and for heating during cold months. Heating is required in a building when the ambient temperatures are low enough to demand additional warmth to provide comfort. Boilers or furnaces generate the heat; pipes or ducts distribute the heat, and diffusers at the terminal outlets deliver the heat.

Cooling systems utilize cooler outdoor air when available, a refrigeration cycle, or other heat rejection method to supply cool air to occupied spaces. Cooled air is distributed by ducts throughout the building to terminal units (diffusers). These end units deliver the cooling to the desired areas.

Ventilating systems operate to provide fresh outdoor air, to minimize odors, and to reduce unhealthy dust or fumes. In many spaces, simple operable windows satisfy ventilation requirements. Alternatively, ventilation may be provided to a building by exhaust fans or fresh air intakes.

Air distribution HVAC systems are mainly divided into constant volume or variable volume systems. Within these two air distribution groups, there can be dual duct systems, terminal reheat systems, double duct systems, triple duct systems, induction systems and multizone systems. Some of these types of systems will be further described in Section 1.5.1.

1.2 Motivation for Research

Balancing of an HVAC air-supply system can be very challenging. Existing air system balancing techniques are iterative methods that require several measurements and damper adjustments to achieve design flow.

Airflow measurements are used to balance a system's air distribution to the designer's specification, ensuring energy-efficient operation, thermal comfort, and proper ventilation. System balancing is typically an expensive and time-consuming job that makes up an estimated 1 to $1\frac{1}{2}$ man-hours of work per balancing point (Konkel, 1987).

System balancing is done either by a specialized testing, adjusting and balancing (TAB) contractor, or by the HVAC contractor who constructs the system. A TAB process is used to achieve proper operation of heating, ventilating, and air conditioning systems. Calibrated instruments are used to measure temperatures, pressures, and velocity and air qualities for an evaluation of equipment and system performance. Devices such as dampers and valves are adjusted along with fan speeds to achieve maximum system performance and efficiency.

There are numerous methods of balancing an HVAC system, but one method commonly used is a trial and error process. The flows and pressures are first measured and the dampers adjusted to match design airflow. The flows are measured again at each balancing point, and the process is repeated until design airflow is achieved through every outlet, indicating that the system is balanced. This measuring and adjusting process is iterative. It must be repeated several times due to a change in pressure and flow throughout the system when any damper is adjusted. Usually, three or more measurements are made at each point before balancing is complete. Two balancing methods commonly used in industry today will be further described in Section 1.5.4.

The motivation for this project is to find a balancing technique for a system that does not require an iterative process. The proposed new balancing technique is to use a computer program that first determines the system characteristics based on flow and pressure measurements, and then gives information for balancing the system by adjusting each damper only once. This balancing technique starts by making air flow and fan exit-pressure measurements to determine system characteristics (static pressures in junctions, flow loss coefficients in ducts), then calculating damper positions that correspond to design airflow, and finally, adjusting dampers only once at the terminal outlets to a calculated airflow value. Thus, no iteration is required.

1.3 Thesis Objectives

The goals of this project are:

1. To develop a way to determine system characteristics from a small number of measurements.
2. To develop a way to determine design damper positions based on system characteristics and design airflow.
3. To develop a way to adjust all dampers in the system once their ideal position is known.
4. To develop an error analysis that shows uncertainty in measurements.

1.4 Thesis Preview

The chapters of this thesis are organized in the following matter:

1. Chapter 1 introduces the topic of balancing air distribution systems and provides a literature review.
2. Chapter 2 provides an engineering analysis that shows the origin of the equations and theories used in the non-iterative balancing technique.
3. Chapter 3 shows how a computer model is developed from the equations derived in Chapter 2.
4. Chapter 4 presents and discusses the results obtained from the computer model and theories from the previous two chapters.
5. Chapter 5 draws conclusions and summarizes the results from chapter 4 and offers recommendations for additional developments.

1.5 Literature Review

1.5.1 Types of HVAC Air Distribution Systems

There are two main types of air distribution systems: constant volume and variable volume systems.

1.5.1.1 Constant Volume Systems

Constant volume systems maintain the flowrate of the supply air to each conditioned space constant and rely on the control of the temperature of the supply air to meet the load requirements in different conditioned spaces. There are many different ways in which constant volume systems modify the supply air temperature to different rooms. Two of these are through a terminal reheat system, and through a mixed-air or double duct system.

In a terminal reheat system the total supply air is cooled such that its temperature is cold enough to satisfy the worst condition of all conditioned spaces. When full cooling is not required, the cold supply air is heated to a higher temperature by a heating coil in the terminal device that supplies conditioned air to the space to satisfy the space's needs. The primary purpose of the terminal reheating method is to control the temperature and not the humidity in a central air distribution system that serves multiple conditioned spaces. Nevertheless, a side benefit to the terminal reheat system is that the humidity level is more evenly maintained in all conditioned spaces.

In a mixed air or double-duct system, one central air unit provides two air streams at different temperatures. One air stream passes through a cooling coil to provide cool air and the other air stream passes through a heating coil to supply hot air. For each

conditioned space, the temperature is regulated by mixing proper amounts of hot and cold air to achieve the load requirements. Like the terminal reheat system, the mixed air system's primary purpose is to control the temperature and not the humidity.

Constant volume systems have the advantage of being easy to design and reliable to operate. Since these systems maintain the supply air volume to each conditioned space constant, they are used in buildings where constant pressure relationships between conditioned spaces are critical. Constant volume systems also have good efficiency if all areas served have similar loads and the supply temperature is reset to meet the needs of the warmest space.

The major disadvantage of constant volume systems is that they consume too much energy. Constant volume systems have poor efficiency when different spaces have different loads. Since a majority of conditioned spaces do not operate at their peak load conditions for prolonged periods of time, an air-conditioning system almost always operates under partial load conditions. For a constant volume system to satisfy partial load conditions, the system must either reheat a cold air supply, or mix cold and hot air supplies in order to obtain proper air temperatures for conditioned spaces. Whether the system reheats or mixes the air supply depends on the type of constant volume system. Both the reheating and the mixing process waste cooling and heating energy (Sun, 1994).

1.5.1.2 Variable Volume Systems

Variable volume systems operate on the principle of changing the volume of supply air, instead of varying the supply air temperature to each conditioned space. By definition, the temperature in most variable volume systems is held constant. This

implies that a variable volume system can be either a cooling system or a heating system but not both. Variable volume systems are usually designed for cooling loads since most of the load variations in a building are cooling loads, and variable volume systems are more energy efficient than constant volume systems.

The major advantage of a variable volume system is that, compared to a constant volume system, it will not waste thermal energy. Under partial load conditions, which is usually the operating condition, there is no need to reheat a cold air stream or mix hot and cold airstreams to maintain a specific load condition. Therefore, there is no thermal waste. In addition, fan motor horsepower reduces significantly as the volume of air is reduced.

A major disadvantage of variable volume systems is that they can only be used for cooling purposes. Since the supply air temperature is fixed and lower than the temperature maintained in the conditioned space, fixing temperature differences in the system implies that a pure variable volume system has to be a cooling system. As a result, some form of supplementary heating must be used in conjunction with the variable volume system to compensate for the heating load.

Another disadvantage of variable volume systems is that it is more difficult to maintain pressure relationships between adjacent conditioned spaces. If a pressure relationship between two critical areas is needed, complex controls are installed in the system in order to maintain those certain relationship. Variable volume systems require that a greater consideration should be given to maintaining the proper amount of ventilation air throughout the system. A design aimed mostly at conserving energy can

impede the minimum air circulation to maintain the required indoor air quality and comfort conditions in an occupied space (Sun, 1994).

1.5.2 Balancing an Air Distribution System

Balancing an air distribution system consists primarily of measuring airflow and adjusting volume control devices to get design airflow. Flow calculation methods are not accurate enough to ensure proper balancing by duct design alone (See Section 1.5.3: Duct Design). Additionally, a system's description cannot capture all of the details of installation. To assure proper balancing, dampers and fan speed within an air distribution system must be adjusted until design flows are met throughout the system. To minimize fan power, balancing is done with one damper left fully open.

Adjustments are made until the value of the air flowrate at each inlet or outlet device is found to be within $\pm 10\%$ of its design airflow. By properly balancing an air distribution system, operating costs in the system will be reduced, comfort for the occupants in the building will be increased, and the life of the HVAC equipment will be improved.

1.5.3 Duct Design

1.5.3.1 Duct Design and Balancing

Duct design is very important in a system in order to supply the proper amount of air to each space. The overall task of duct design is to size and route the ducts from the fan to each space within constraints of available space and acceptable noise levels.

The reason balancing is needed is that typical methods of duct design do not produce a system that provides the proper amount of air to each branch. Some design methods do not try to produce a balanced system, while those that do try, can only get an approximate balance due to modeling errors and construction variations.

The flowrate in the system is regulated by the use of dampers in main branch ducts or diffusers. The dampers are adjusted to restrict the flow to branches having too much flow, thereby increasing the flow to branches with too little flow. If just adjusting dampers is not enough to increase flow through branches that do not have enough flow, then the fan speed may be increased. The most common practice used in balancing is to dampen the branches receiving too much flow. If some branches have too little flow at full open damper, then the fan speed is increased. Increasing the fan speed increases the static pressure and restricts the flow to all branches except those requiring more flow. Thus, even a very poor duct design can usually be made workable if the fan can be operated to provide a sufficiently high static pressure. It should be noted that increasing the fan speed increases the amount of energy used and increases the noise level in the system.

1.5.3.2 Methods of Duct Design

There are various methods of doing duct design that are commonly used in industry. Four of these methods are: the constant velocity method, the velocity reduction method, the static regain method, and the equal friction method.

The constant velocity method is normally used for exhaust systems carrying materials, and consists of sizing the duct as to maintain a constant flowrate velocity. The

ducts are sized to operate with a minimum velocity component so that the materials being carried stay in progressive order.

The velocity reduction method is similar to the constant velocity method, but involves having different velocities for different duct sizes. This method consists of selecting an appropriate airflow velocity at the fan outlet, and then progressively using duct size changes to reduce the velocity of the airflow at the junctions or branch ducts. The velocity of the air in ducts that are closer to diffusers is lower than the velocity in ducts closer to the fan, and therefore noise effects are reduced.

The equal friction method consists of sizing the ducts in the system so as to cause equal pressure losses per unit length of duct. After the ducts are sized and the pressure losses in the ducts are calculated, the pressure losses for each run are determined. The differences in pressure losses between the duct runs are counteracted by damping the branches that have lower pressure losses. This will cause a higher pressure loss through that branch, and thus higher friction. An additional design iteration of reducing the duct size may be used to achieve equal pressure-loss in the system. However, doing this increases the noise level in the ducts due to a higher airflow velocity through them, and may therefore require noise attenuators for the duct system (Stanford, 1988).

The static regain method consists of selecting velocities so that the static pressure throughout the system is constant. The equation used to calculate static pressure is:

$$P_{stat} = P_{tot} - \frac{\rho V^2}{2}. \quad (1.1)$$

As the total pressure rises due to friction, the velocity is increased to keep the static pressure constant throughout the system.

Each duct sizing method has its advantages and disadvantages, but none of them is accurate enough for achieving design airflow in the system. The constant velocity, velocity reduction, and equal friction method are reasonable for sizing ducts, but they do not try to balance the system. The static regain and equal pressure-loss methods are very time consuming from a design point of view, but they approximately balance the system. There are always inaccuracies in design airflow. The error comes from duct size fabrication differences, and inaccuracies in pressure loss information from elbows and tees. Using standard duct sizes means that one cannot design for exact pressure losses. Thus, balancing procedures are required to counteract inaccuracies.

1.5.4 Balancing Methods

Two methods of balancing commonly used in the HVAC industry are the proportional and the stepwise method. These balancing methods are similar to each other and other balancing methods in that: 1) the value of the airflow throughout the system is measured and found to be within $\pm 10\%$ of design flow, and 2) the terminal outlet with the greatest pressure drop in the system with respect to the fan (usually the branch furthest from the fan) is left completely open.

1.5.4.1 Proportional Balancing Method

The proportional balancing method involves starting at the low airflow point in a system and working back towards the fan. At least one outlet volume damper (the one with the lowest percent of design flow) must remain fully open during the entire balancing procedure, and all of the outlets in a branch must be balanced in proportion to

another. A schematic of an air distribution system is shown in Figure 1.1.

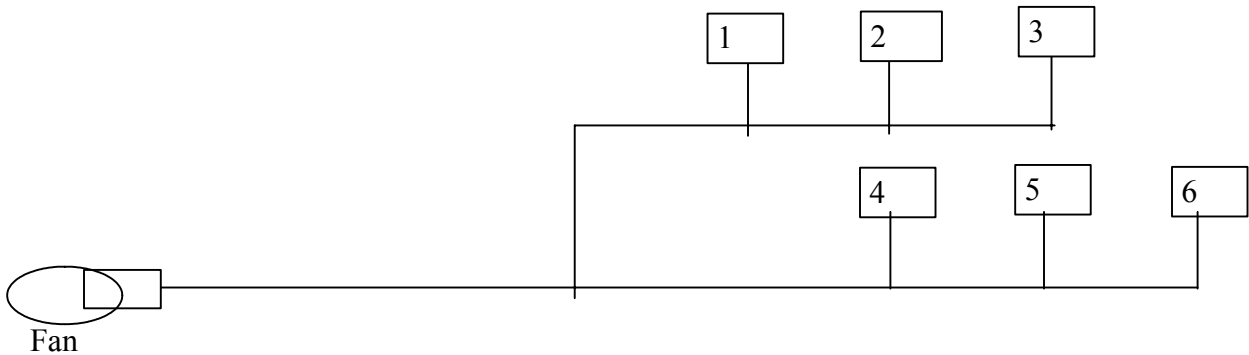


Figure 1.1: Schematic of an Air-Supply System

The proportional balancing procedure is as follows:

1. Because the system must be set and balanced for maximum required airflow, the airflow throughout the system is first measured with all dampers fully opened.
2. The outlets are numbered in order of increasing percentage of design airflow (X%). The outlet with the lowest X% remains open. The percentage of design airflow (X%) for each outlet is calculated by using the following equation:

$$\frac{Q_m}{Q_d} = X\% \quad [1.2]$$

where: Q_m = measured airflow

Q_d = designed airflow

3. The flowrate on the branch with the lowest X% is not adjusted.
4. The flowrate on the branch with the 2nd lowest X% is adjusted so that it has the same X% as the branch with the lowest X%.

5. The flowrate on the branch with the 3rd lowest X% is adjusted so that it has the same X% as the 2nd and 1st branch with the lowest X%.
6. The same procedure is repeated until all branches have the same X%.
7. If needed, fan speed or zone dampers are adjusted to bring the system to within $\pm 10\%$ of design flow.
8. Repeat until further adjustments are not required. (Monger, 1990 and SMANCA, 1993).

1.5.4.2 Stepwise Balancing

In the stepwise balancing method, diffusers are initially adjusted to approximate design flow throughout. The procedure is as follows:

1. Zones are adjusted to approximate the total required airflow.
 - For fans serving single zones with no volume dampers, the fan speed is adjusted.
 - For fans serving multizones with volume dampers, the fan speed and the volume dampers are adjusted.
2. Airflow at all terminal outlets is measured.
3. 1st pass \rightarrow the terminals with highest excess flow are adjusted to approximately 10% below design airflow.
4. 2nd pass \rightarrow the rest of the terminals are adjusted to design airflow.
5. 3rd pass \rightarrow the terminals are again adjusted to design airflow. Three passes are usually enough to get the system to within $\pm 10\%$ of design flow.

6. Since balancing requires that the damper on the terminal with the greatest resistance be wide open, the fan is adjusted if necessary to bring the system to within $\pm 10\%$ of design flow (SMACNA, 1993).

These balancing procedures are necessary to counteract the physical changes that occur to the air flowrate throughout the system when a single damper is adjusted. When a damper is adjusted, the static pressure and the velocity of the airflow change at different points in the system. In order to understand why these changes occur, it is important to understand the main relationships between the static pressures and the air flow rates in an air distribution system.

Chapter 2

Engineering Analysis

2.1 Fluids

2.1.1 Conservation of Mass

To conserve mass in steady state, the incoming rate must equal the departing flowrate. If the inlet is designated as (1) and the outlet as (2), it follows that $\dot{m}_1 = \dot{m}_2$. Therefore conservation of mass requires that

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \quad (2.1)$$

If the density remains constant, then $\rho_1 = \rho_2$, and the above equation becomes the continuity equation for incompressible flow.

$$\boxed{A_1 V_1 = A_2 V_2 \text{ or } Q_1 = Q_2} \quad (2.2)$$

In any air distribution system, this theory applies. Air distribution systems are made of ducts that are divided into branches and separated by junctions. The flowrate coming into each junction is equal to the flowrate going out of that junction. From figure 2.1 it can be deduced that

$$Q_1 = Q_2 + Q_3 \quad (2.3)$$

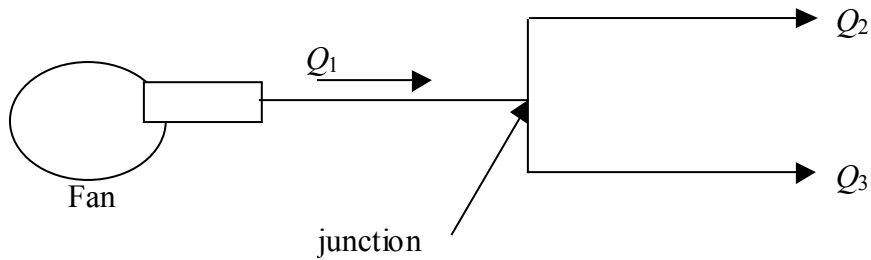


Figure 2.1: Flowrate passing through junction

2.1.2 Pressure Drop through Branches

When balancing an air distribution system, calculating the pressure drop across its branches is necessary. For any element in the system, the pressure drop ΔP can be calculated from the volume flow rate Q and the air density ρ using

$$\Delta P = \left(\frac{K}{A^2} \right) \left(\frac{\rho Q^2}{2} \right) \quad (2.4)$$

Here A is the cross-sectional area and K is a friction loss coefficient. By measuring flow and pressure drop across an element in the system, we can calculate the value of

$\frac{K}{A^2}$. Once this value is known, the pressure drop across an element can be calculated for

any given flowrate. For any element in the system that includes an adjustable damper,

$\frac{K}{A^2}$ varies as the damper is adjusted, but is constant for a single damper position.

By designating the quantity $\frac{K}{A^2} \left(\frac{\rho}{2} \right)$ as K' , we can write the pressure drop across a

branch as

$$\Delta P = K' Q^2 \quad (2.5)$$

and for group of n flow elements in series, the pressure drop equation becomes

$$\Delta P_{TOTAL} = \sum_{j=1}^n K'_j Q_j^2 \quad (2.6)$$

For a segment consisting of series of elements without branches (no flow entering or leaving), Q_j is constant along the segment, and we get

$$\Delta P_{TOTAL} = Q_j^2 \sum_{j=1}^n K'_j \quad (2.7)$$

The sum of all K'_j for a series of elements with constant flow can thus be determined from a single test.

2.1.3 Fan Performance Curve and System Characteristics

2.1.3.1 Fan Curves and System Matching

Fan performance curves are graphic representations of the performance of a fan from free delivery to no delivery. The static pressure exiting the fan is plotted versus cfm. A performance curve like the one shown in Figure 2.2 will be used to calculate the exit pressure from the fan.

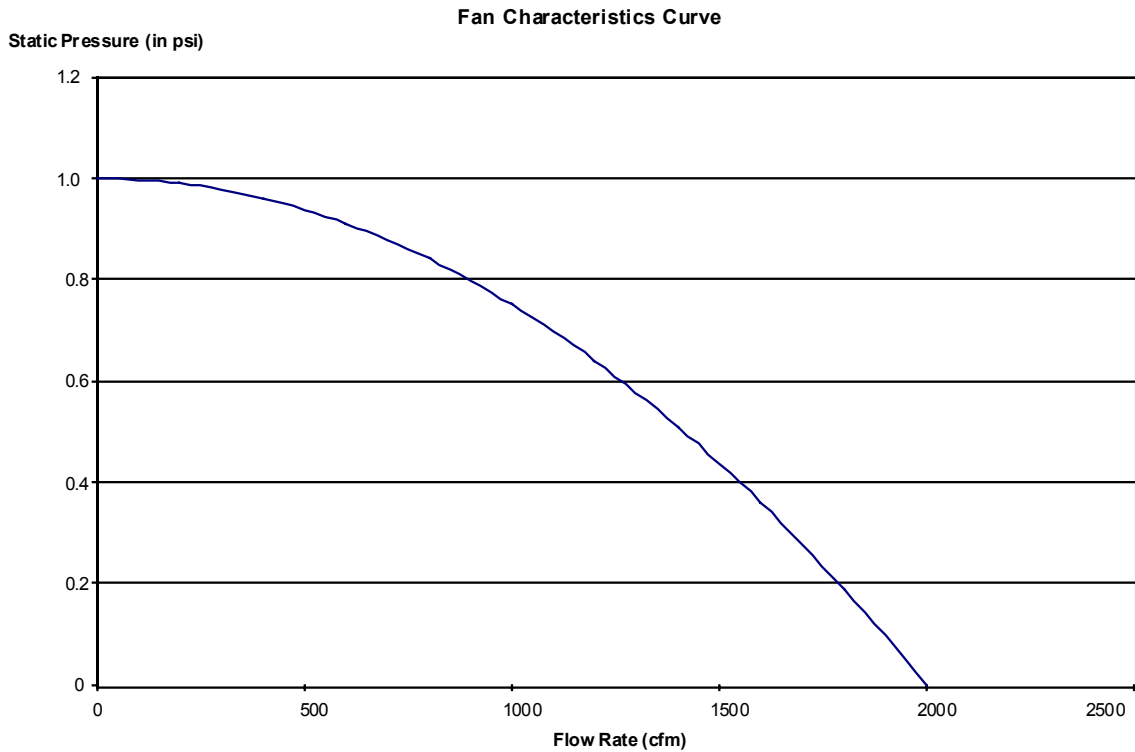


Figure 2.2

Fan performance curves must match system characteristics. The only possible operating points are those where the system characteristics intersect the fan characteristics and occurs at a given fan speed. The pressure developed by the fan exactly matches the system resistance at such points, and the flow through the system equals the fan capacity. For example a system following the curve on Figure 2.3 would run at a static pressure of 0.45 psi and at a flowrate of about 1900 cfm.

Fan Curve and System Curve Matching

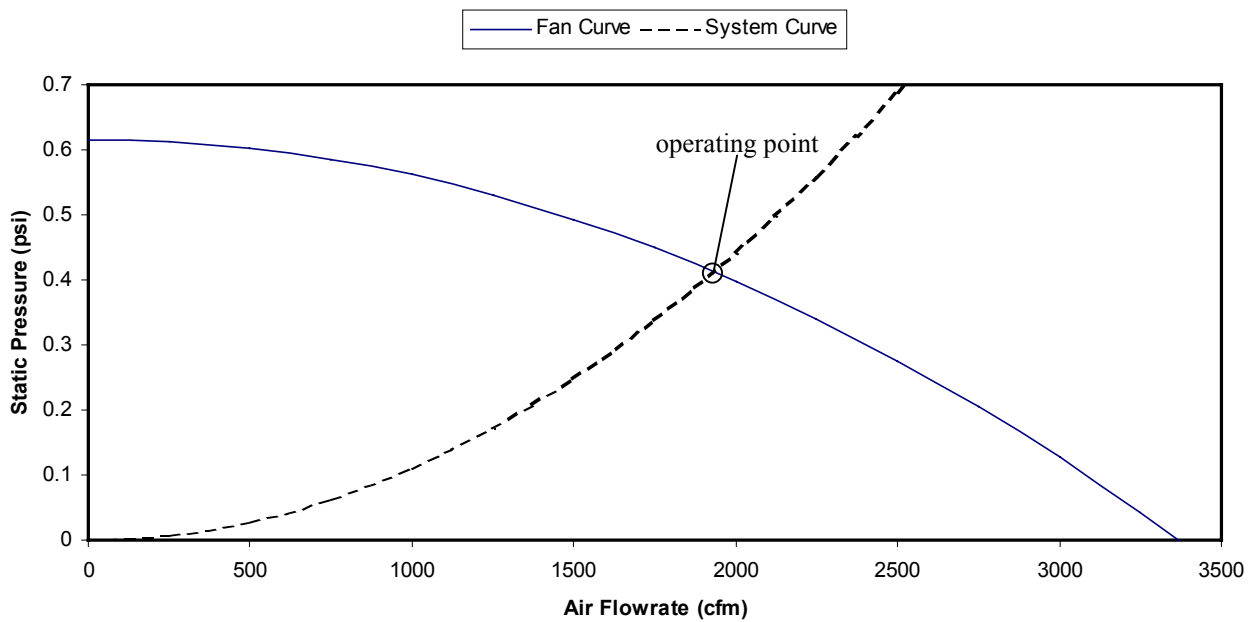


Figure 2.3

If the flowrate through the system does not match design airflow, then either the fan speed or the dampers in the terminal units are adjusted. If either of these two changes takes place, then the operating point changes. If the dampers are adjusted by closing them, then the static pressure exiting the fan and throughout the system will rise, the system curve will change, and the operating point will also change (See Figure 2.4).

Fan Curve and System Curve Matching for a higher fan exit pressure

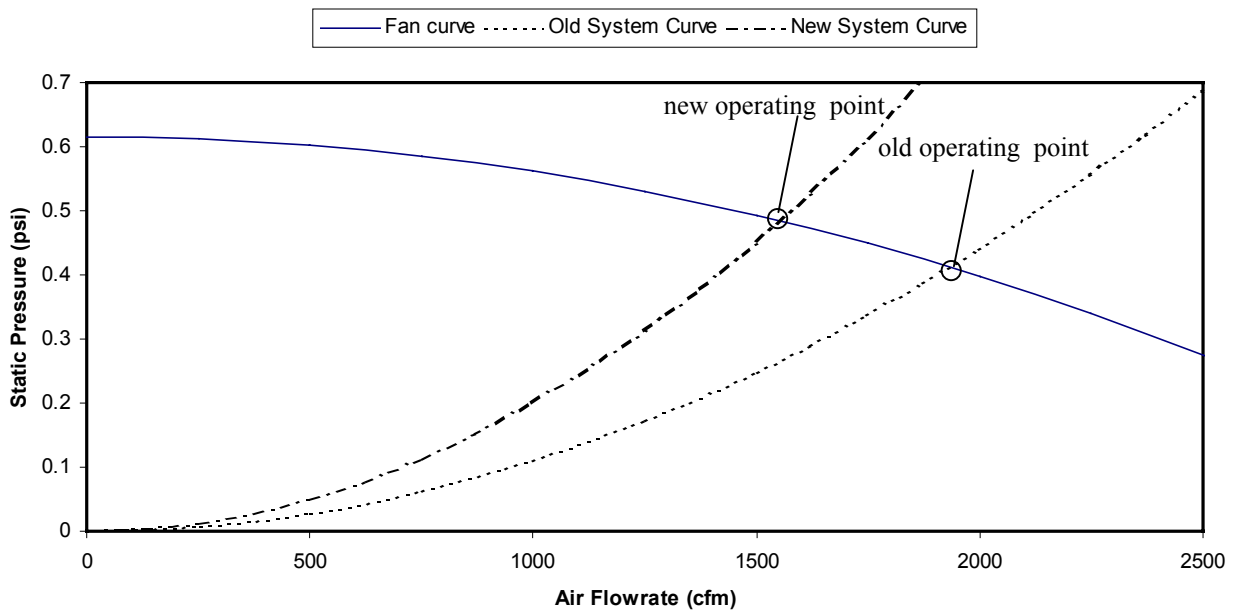


Figure 2.4

If the dampers are adjusted by opening them, then the new system curve will shift below the old system curve. Thus the flowrate increases and the static pressure throughout the system decreases.

If the flowrate through the system is adjusted by lowering the fan speed, then the fan exit static pressure throughout the system decreases and the flowrate decreases (See Figure 2.5).

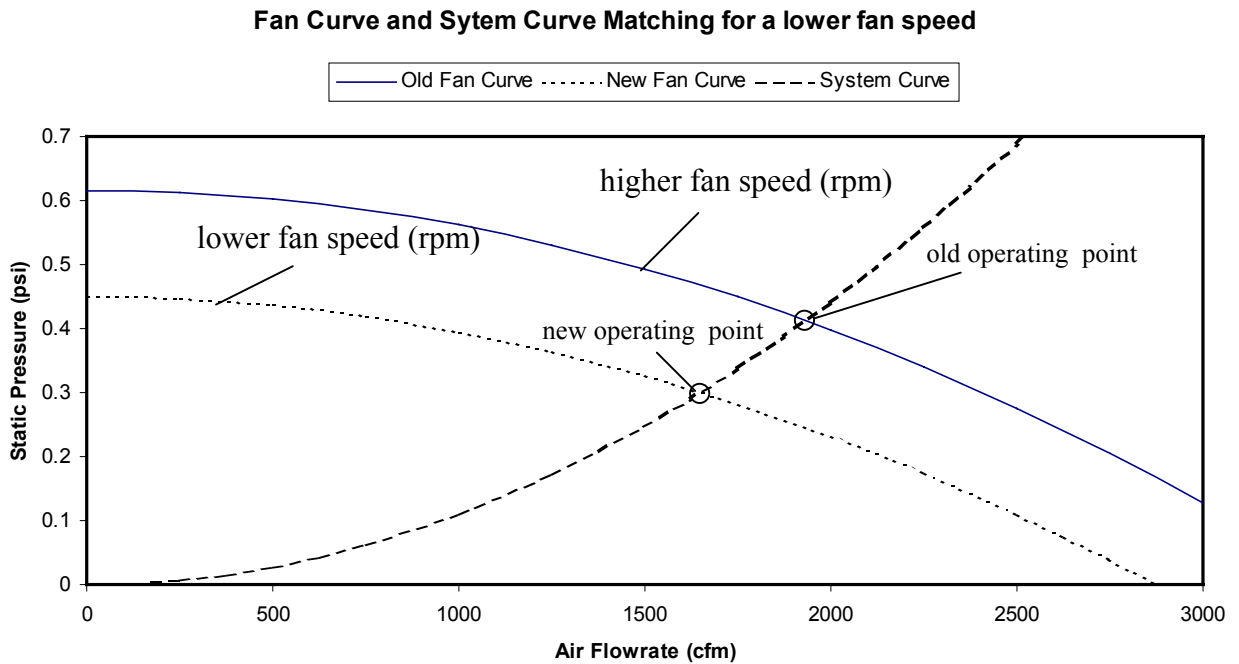


Figure 2.5

2.1.3.2 Fan Laws and System Curve

Fan laws are used to predict fan performance when changing operating conditions or fan size. Some of the basic fan laws are as follows:

1. Air flowrate (Q) varies in direct proportion to fan speed (RPM).

$$\frac{Q_2}{Q_1} = \left(\frac{RPM_2}{RPM_1} \right) \quad (2.8)$$

2. Pressure (P) varies as the square of the fan speed (RPM).

$$\frac{P_2}{P_1} = \left(\frac{RPM_2}{RPM_1} \right)^2 \quad (2.9)$$

3. The system curve shows the static pressure required to overcome the pressure losses in an air distribution system. The air distribution system can only operate along its

system curve, and is represented by the fan law that states that pressure (P) varies as the square of the air flowrate (Q).

$$\frac{P_2}{P_1} = \left(\frac{Q_2}{Q_1} \right)^2 \quad (2.10)$$

2.1.3.3 Approximate Formula

An approximate formula is used to calculate the static pressure exiting the fan for a given air flowrate. Once the fan characteristics are known, the static pressure exiting the fan can be modeled with equation 2.11:

$$P = A - BQ^2 \quad (2.11)$$

This equation is useful when a small section of the fan curve is measured, but poor for modeling the entire curve. A and B are constant parameters for a specific fan operating at a given speed (RPM). Once A and B are determined, the static pressure can be calculated as a function of the air flowrate.

2.1.3.4. Determining Fan Constants A and B at a Given Speed

The parameters A and B can be determined from published performance data. An example of fan data for a size 15 (propeller diameter) vane-axial belt driven fan is shown in Table 2.1.

Table 2.1: Performance Data – Vane axial Fan (Aerovent, Nov. 1988)

Catalog Number			0.250 SP	0.500 SP	0.750 SP
Fan Type	RPM	HP	CFM HP	CFM BHP	CFM BHP
VTBD	1915	$\frac{3}{4}$	2610 0.742	2380 0.753	2097 0.753
VTBD	2108	1	2913 0.983	2714 1.00	2478 1.00
VTBD	2413	1 $\frac{1}{2}$	3384 1.46	3219 1.50	3034 1.51
VTBD	2656	2	3755 1.94	3609 1.98	3449 2.01
VTBD	3041	3	4337 2.89	4213 2.95	4081 2.99
VTBD	3602	5	5178 4.78	5076 4.85	4969 4.91

From such a table, two points are picked at the same RPM. These points represent different static pressures and flowrates on the same fan curve. Equation 2.11 is used to come up with two equations:

$$P_1 = A - BQ_1^2 \rightarrow @ \text{ point 1} \quad (2.12)$$

$$P_2 = A - BQ_2^2 \rightarrow @ \text{ point 2} \quad (2.13)$$

Rearranging equation 2.12 and substituting it into 2.13 we get that:

$$A = P_1 + BQ_1^2 \quad (2.14)$$

and:

$$B = \frac{P_2 - P_1}{Q_1^2 - Q_2^2} \quad (2.15)$$

2.1.3.5. Determining Fan Constants A and B at a Different Speeds

To determine fan constants at any RPM for a given fan size, it is essential to first know the constants A and B for one given RPM . Once the fan constants A and B are known for a given speed, the new fan constants can be determined and a fan curve developed. To start the calculation process, two operating points are picked from a performance data table at different fan speeds (RPM).

$$P_1 = A_1 - B_1 Q_1^2 \rightarrow @ RPM_1 \quad (2.16)$$

$$P_2 = A_2 - B_2 Q_2^2 \rightarrow @ RPM_2 \quad (2.17)$$

We set RPM_2 as the desired fan speed with A_2 and B_2 as the fan coefficients to be solved for. Q_2 is the total flowrate exiting the fan and P_2 is the static pressure exiting then fan.

In order to find the fan constants, we start the analysis by finding the points for which the flowrate is equal to zero. At zero flow, equations 2.16 and 2.17 become:

$$P_1 = A_1 \quad (2.18)$$

$$P_2 = A_2 \quad (2.19)$$

rearranging them, they become:

$$\frac{P_2}{P_1} = \frac{A_2}{A_1} \quad (2.20)$$

and from equation 2.9, equation 2.20 becomes

$$\frac{A_2}{A_1} = \left(\frac{RPM_2}{RPM_1} \right)^2 \quad (2.21)$$

and solving for A_2 ,

$$A_2 = A_1 \left(\frac{RPM_2}{RPM_1} \right)^2 \quad (2.22)$$

The value for A_2 is then determined by using equation 2.22. All the other variables are known so A_2 is easily calculated.

The equation for calculating A_2 was determined by setting the flowrate equal to zero. On the other hand, the equation for calculating B_2 is determined by setting the pressure equal to zero. At zero pressure, equations 2.16 and 2.17 become:

$$Q_1 = \sqrt{\frac{A_1}{B_1}} \quad (2.23)$$

$$Q_2 = \sqrt{\frac{A_2}{B_2}} \quad (2.24)$$

rearranging them, they become

$$\frac{Q_2}{Q_1} = \frac{\sqrt{\frac{A_2}{B_2}}}{\sqrt{\frac{A_1}{B_1}}} \quad (2.25)$$

and from equation 2.8, equations 2.21 becomes

$$\frac{RPM_2}{RPM_1} = \frac{\sqrt{\frac{A_2}{B_2}}}{\sqrt{\frac{A_1}{B_1}}} \quad (2.26)$$

substituting the value for A_2 from equation 2.22 into equation 2.26 results in the RPM 's canceling each other and the A 's canceling each other. This leaves equation 2.26 as

$$B_2 = B_1 \quad (2.27)$$

Having a definition for A_2 and B_2 , the static pressure exiting the fan can be modeled with equation 2.29.

$$P_2 = A_1 \left(\frac{RPM_2}{RPM_1} \right)^2 + B_1 Q_2^2 \quad (2.28)$$

2.2 Balancing Procedure

An air distribution system can be seen as a network of ducts connected by junctions (See Figure 2.6). Airflow starts at the fans, runs through branches, and ends at the terminal units (diffusers) where the adjustable dampers are located. Some systems have balancing dampers in branches mainly to reduce noise, but this will not be addressed in this project. Additionally, some balancing procedures require that the fan speed to be adjusted to either increase or reduce airflow, but this will also not be addressed for this project.

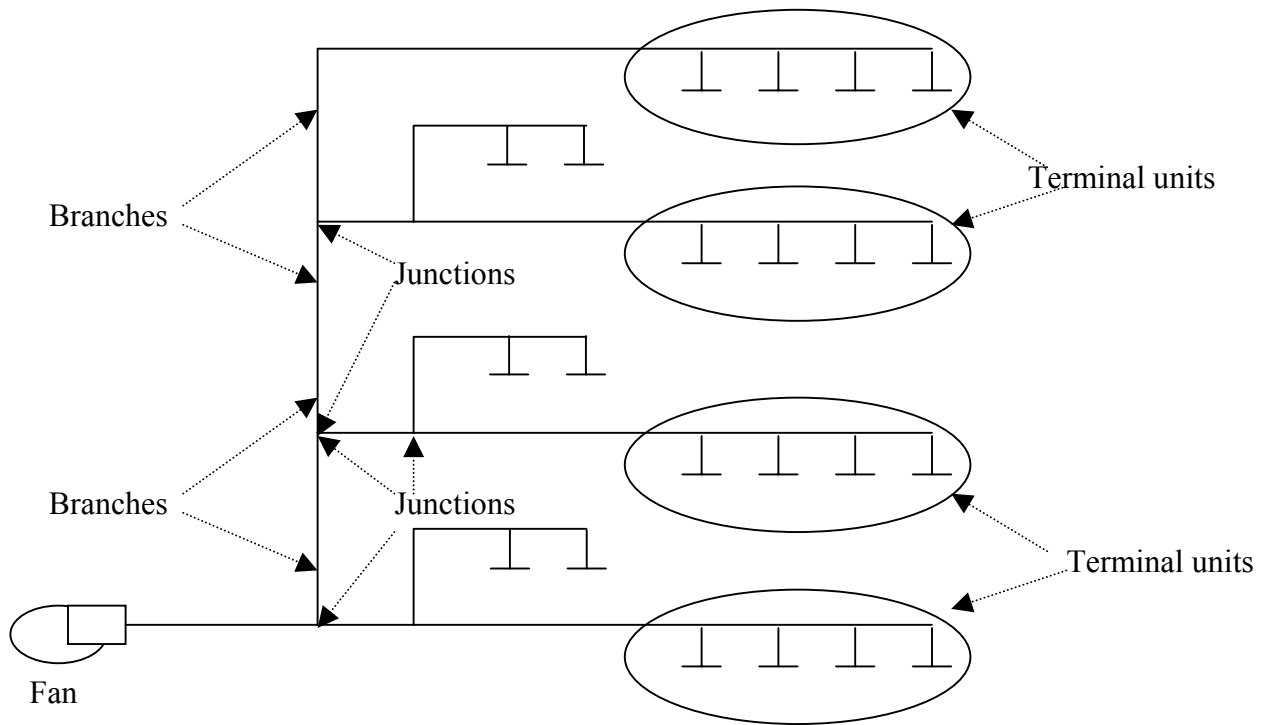


Figure 2.6: Typical Air Distribution Network

For an air distribution system with n diffusers, a system has $2n-1$ branches, and $n-1$ junctions. For example, the system in Figure 2.6 has 22 diffusers, 43 branches, and 21 junctions. This is not valid for air distribution systems with cross-junctions: each cross-junction reduces the number of branches and the number of junctions by 1. The air distribution system for this project will not include cross-junctions.

2.2.1 Pressure Drop Equations and Unknowns in the System

The pressure drop for any section in a duct can be calculated by knowing its flow loss coefficient K' value and the flowrate. For this project, it is assumed every element in the air distribution system has one unique K' that is constant for each branch. For a

branch that includes an adjustable damper, this coefficient varies depending on the positioning of that damper.

To calculate the loss coefficient across a branch, the airflow must be measured. Measuring the airflow leaves the unknowns in the system to be the pressures (P) and the loss coefficients (K'). For any air distribution system, there is one static pressure point at each junction. Since there are $n-1$ junctions, there are $n-1$ P 's in the system. For each branch, there is 1 unique K' , totaling $2n-1$ K' 's. Some K' 's vary and some are constant. The variable K' 's are the damper positioning constants, and the constant K' 's are the duct coefficients. Adding up the K' and P unknowns, there are $3n-2$ unknowns for each set of damper positions.

For every branch or segment in the system, there is one equation for calculating the pressure drop. Since there are $2n-1$ branches, there are $2n-1$ pressure drop equations. Equation 2.5 determines the pressure drop for every branch throughout the system. The static pressure exiting the fan will be measured.

2.2.2 Solving for System Characteristics with Dampers Open

The first step in the balancing procedure is to solve for the system characteristics by measuring the flowrate with all dampers open. For an air distribution system with n junctions, there are of $3n - 2$ unknown K' 's and P 's in the system. Adding the fan constants A and B gives a total of $3n$ unknowns. Adding the fan curve equation (equation 2.11) to the pressure drop equations gives a total of $2n$ equations. Since there are more equations than unknowns, a second pass is required to get enough equations.

The first pass is done with every damper in the system completely open. The second pass will be done with all but one of the dampers open. This closed damper is usually the one furthest from the air supply source (fan). The closed damper in the system will result in no airflow through that branch (i.e. $K' = \text{infinity}$). Since $K' = \text{infinity}$, this will also result in eliminating the pressure drop equation from this branch.

The second pass will bring a total of $2n - 1$ equations to the system. Adding these equations to the $2n$ equations from the first pass results in a total of $4n - 1$ equations. The only unknowns added during the second pass are the P 's because the system characteristics (K 's, A , and B) remain constant. This introduces $n - 1$ unknowns to the system that when added to the $3n$ unknowns from the first pass, the result is $4n - 1$ unknowns. With the second pass, there are enough equations to solve for the $4n - 1$ unknowns (See Table 2.1).

Table 2.1. Equations and Unknowns for an Air Distribution System

Unknowns	Quantity	Equations	Quantity
K'	$2n-1$	$\Delta P = K'Q^2$	$2(2n-1)-1 = 4n-3$
P	$2(n-1) = 2n - 2$	$P = A - BQ^2$	2
A, B	2		
Total	$4n-1$	Total	$4n-1$

2.2.3 Determining Design Damper Positioning Coefficients

The next step in the balancing procedure is to determine the damper positioning coefficients (K_p 's) that will give design airflow. Damper positioning coefficients can now be determined since the fan constants (A, B), and the duct and open-damper loss coefficients in the system are known.

Unlike solving for the system characteristics, determining design damper positioning coefficients does not require any measurement. The reason for this, is that system characteristics are already known from the previous step, and the design airflow (Q_d) through the outlet dampers is given.

The unknowns in this case are the adjustable K_p 's, the P 's at each junction, and the exit pressure from the fan. Since there are n diffusers in the system, there are n dampers that require adjustment, bringing n unknowns in this step. Given that there are $n - 1$ junctions, there are $n-1$ unknown P 's in the system. Additionally the static pressure exiting the fan is also unknown. Adding all the unknowns in this step will result in a total of $2n$ unknowns.

The damper positioning coefficients (K_p 's) and the static pressures (P 's) will be solved for by using both the $2n - 1$ pressure drop equations (2.5) and the fan curve equation (2.11), totaling $2n$ equations.

2.2.4 Determining Flows during Adjustment

Once the K_p 's to achieve design airflow are known, the system can be balanced. The first step is to completely open all the dampers in the system. The value for the open-damper positioning coefficients is already known from the first step in the

balancing procedure (Section 2.2.2: Solving for System Characteristics). The next step is to adjust the damper on the diffuser farthest from the fan to its K_p position, which was determined from the previous step (Section 2.2.4: Determining Damper Positioning Coefficients). When a branch has more than one diffuser, then the diffuser that is the farthest from the fan on that branch, will have its damper adjusted to its K_p position first.

In order to adjust a damper to its K_p value, the flowrate that corresponds to this K_p value must be determined with the computer program. With all other dampers open, the damper on that branch is adjusted until the calculated flowrate (Q_f) reaches such value. By adjusting the airflow on this branch, the airflow will increase through the other diffusers that have their dampers fully open.

After the first damper on that branch is set, the second damper on that branch is set at its K_p value by using the same method described above. And so on with all of the dampers on that branch. After this first branch is balanced, the rest of the branches in the system are set, one by one, in the same way starting from the farthest branch to the closest branch to the fan until the system is balanced. The last damper to be balanced is adjusted to design flow.

An example of how a simple three-diffuser model of an air distribution system works will be explained next.

2.3 Example: Three-Diffuser Model

A simple model with three diffusers as the one shown in Figure 2.7 will be used for testing. This example was taken from Stanford, 1988. The model has 5 branches

with 2 junctions between them. The static pressures are labeled P_1 through P_6 at 6 points in the diagram, and the flowrate values are given at the terminal outlets.

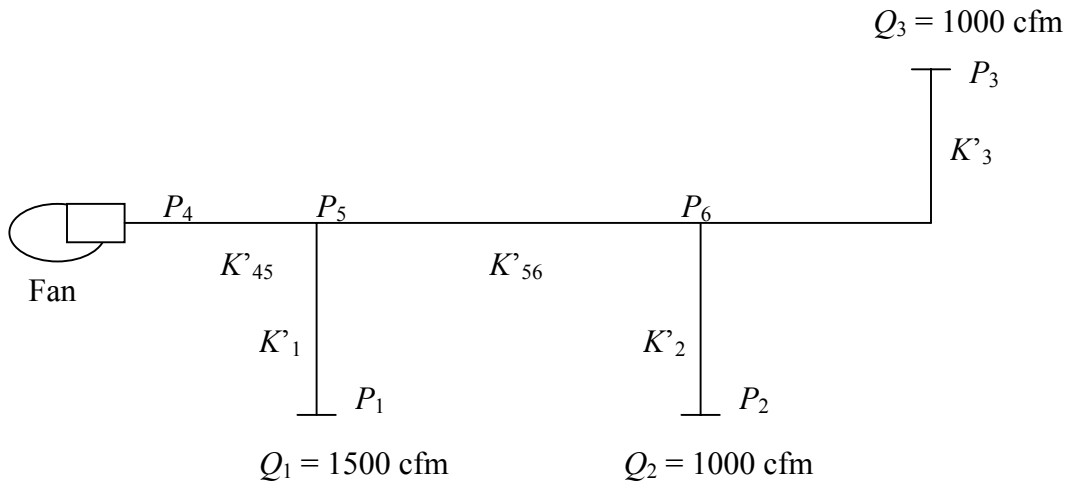


Figure 2.7: Three-Diffuser Model

2.3.1 Determining System Characteristics

To solve for the system characteristics, two passes will be completed at different flowrates and static pressures. For the first pass, all three dampers at the terminal outlets will be left open. Since K'_1 , K'_2 , and K'_3 represent fully open-damper diffusers, they will represent the minimum loss coefficient value.

For the second pass, the damper farthest away from the fan (point 3) is closed and all the other dampers are left open. K'_3 will have value of infinity, and K'_1 and K'_2 will have the same values as those during the first run.

2.3.1.1 Unknowns in the Three-Diffuser System

A three-diffuser system has a total of 11 unknowns. The unknowns are: the system characteristics (flow loss coefficients), the static pressures throughout the system, and the fan constants (A, and B). Looking at figure 2.6, K'_{45} and K'_{56} are the flow loss coefficients pertaining to the main branches in the system. P_4 represents the static pressure exiting the fan, and P_5 and P_6 are the static pressures at the junctions.

Some of the unknown parameters will vary for each pass, and some will remain constant. The exiting flowrates (Q_1 , Q_2 , and Q_3) and the static pressures P_4 , P_{45} , and P_{56} will differ for each run. These parameters will be labeled 'a' and 'b' for the first and second pass, respectively. K'_1 and K'_2 are constant for both passes since they represent flow loss coefficients for branches that have open dampers for both passes. In all, the unknown parameters solved for will be K'_1 , K'_2 , K'_3 , K'_{45} , K'_{56} , P_{45a} , P_{45b} , P_{56a} , P_{56b} , A and B .

2.3.1.2 Equations for the Three-Diffuser System

From running the two passes, the system characteristics in a three-diffuser system are determined by using the following 11 equations:

$$P_{4a} = A - B * Q_{45a}^2$$

$$Q_{45a}^2 * K'_{45} = P_{4a} - P_{5a}$$

$$Q_{56a}^2 * K'_{56} = P_{5a} - P_{6a}$$

$$Q_{1a}^2 * K'_1 = P_{5a} - P_1$$

$$Q_{2a}^2 * K'_2 = P_{6a} - P_2$$

$$Q_{3a}^2 * K'_3 = P_{6a} - P_3$$

$$P_{4b} = A - B * Q_{45b}^2$$

$$Q_{45b}^2 * K'_{45} = P_{4b} - P_{5b}$$

$$Q_{56b}^2 * K'_{56} = P_{5b} - P_{6b}$$

$$Q_{1b}^2 * K'_1 = P_{5b} - P_1$$

$$Q_{2b}^2 * K'_2 = P_{6b} - P_2$$

where: Q = volume flowrate

because of conservation of mass, $Q_{45} = Q_{56} + Q_1$ and $Q_{56} = Q_2 + Q_3$

lumping return duct effects with the fan curve $P_1 = P_2 = P_3 = 0$

The first pass is represented with the subscript 'a' and the second pass with the subscript 'b'. For the second pass, there is no pressure drop equation for the K'_3 branch, and thus a total of 11 equations are used. With these equations, the duct loss coefficients (K'_{45}, K'_{56}) and the fan constants (A and B) are determined.

2.3.2 Determining Design Damper Positioning Coefficients

Since the values for the duct flow coefficients (K'_3, K'_{45}, K'_{56}) and the fan constants (A, B) have been calculated, we can determine the design damper settings ($K'_{1d}, K'_{2d}, K'_{3d}$) by using given design flowrates and the following 6 equations:

$$P_{4d} = A - B * Q_{45d}^2$$

$$Q_{45d}^2 * K'_{45} = P_{4d} - P_{5d}$$

$$Q_{56d}^2 * K'_{56} = P_{5d} - P_{6d}$$

$$Q_{1d}^2 * K'_{1d} = P_{5d} - P_1$$

$$Q_{2d}^2 * K'_{2d} = P_{6d} - P_2$$

$$Q_{3d}^2 * K'_{3d} = P_{6d} - P_3$$

Consequently, the design damper settings are used to adjust the system to a flowrate that will balance it.

2.3.3 Determining Flows during Adjustment

A computer model is used to determine the flowrates that the dampers must be adjusted to balance the system. The procedure of adjusting the flowrates is as follows:

1. The damper on branch 1 is set at its design value (K'_{1d}), and the rest of the dampers are set at their fully open damper loss coefficient (K'_3 and K'_2) values. The computers program then determines the value to set the flowrate on branch number 1. This flowrate value Q_{1f} will be established by adjusting the damper on branch 1.
2. After this damper is set, the damper on branch 2 is set to its design value (K'_{2d}) while leaving the damper on branch 3 at its fully-open damper loss coefficient (K'_3) value. The program will again determine the value for which the flowrate through branch 2 will be set. The flowrate through branch 1 will increase a little.
3. Finally the damper in branch 3 is adjusted to its design valued K'_{3d} value, while the flowrates through branches 1 and 2 increase until they reach their design value, and thus balancing the system.

Chapter 3

Computer Models

This Chapter is divided into 2 major sections: a description of a computational model, and a description of a graphical user interface. The computational model uses MATLAB (Mathworks, 1999) and its solving procedure is based on solving systems of equations. The graphical user interface is written with SIMULINK (Mathworks, 1998), and it uses both MATLAB and SIMULINK to solve the air distribution system model.

3.1 Computational Model

The computational model for this project is solved with a program that calls a linear and a non-linear solver with a computer program known as MATLAB. MATLAB is used to solve for the 3-diffuser example described in Chapter 2. The computational model will use a linear program to solve for the system characteristics and desired damper coefficients, and a non-linear program to solve for the flows that the dampers have to be adjusted to.

3.1.1 Solving for System Characteristics using MATLAB

The first step in the balancing procedure requires that the equations being solved for be put in matrix form, and that the unknowns being solved for be put into a vector, so that a simple matrix multiplication procedure can be used for solving the equations. Since the

model solved for is a 3-diffuser air distribution system, there are 11 $(2n-1)$ equations to solve for 11 unknown system characteristics (branch friction loss coefficients and fan constants). The known variables are the measured flowrates throughout the system and the measured static pressure exiting the fan. For easier reference, a diagram of the three-diffuser system is provided again in Figure 3.1.

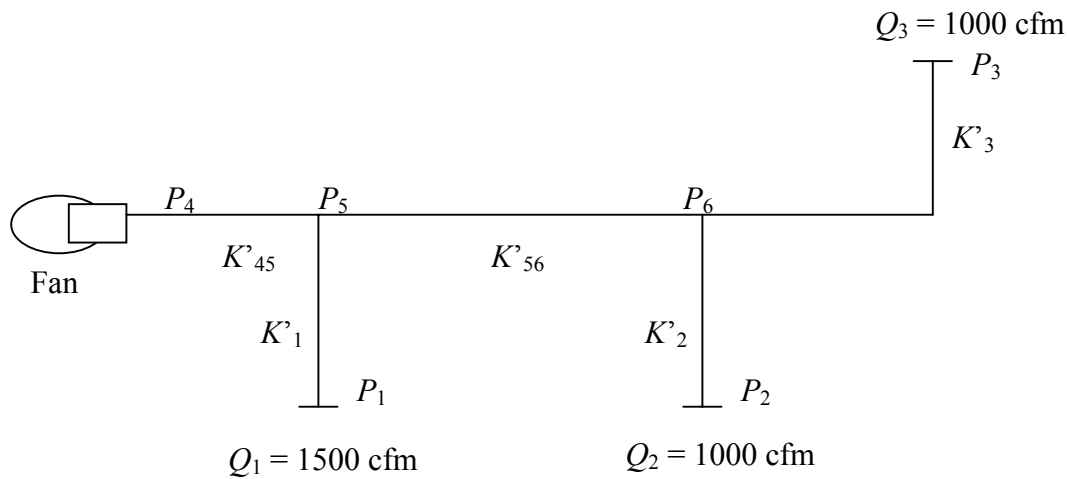


Figure 3.1: Three-Diffuser Air Distribution System

The first step in the computational process is to determine the 11 equations needed to solve for the system characteristics. Section 2.3 (Example: Three-Diffuser Model) shows the equations needed to solve such a system. These equations are used to come up with a linear system of 11 equations and 11 unknowns that are arranged into a matrix, which is then entered into the first MATLAB program. Table 3.1 shows the 11 equations in their basic form with the subscripts 'a' and 'b' to represent the first and second run, respectively.

Table 3.1: Equations to Solve for System Characteristics and Fully-Open Damper Flow Coefficients.

First Run	Second Run
$Q_{45a}^2 * K'_{45a} = P_{4a} - P_{5a}$	$Q_{45b}^2 * K'_{45b} = P_{4b} - P_{5b}$
$Q_{56a}^2 * K'_{56a} = P_{5a} - P_{6a}$	$Q_{56b}^2 * K'_{56b} = P_{5b} - P_{6b}$
$Q_{1a}^2 * K'_1 = P_{5a} - P_1$	$Q_{1b}^2 * K'_1 = P_{5b} - P_1$
$Q_{2a}^2 * K'_2 = P_{6a} - P_2$	$Q_{2b}^2 * K'_2 = P_{6b} - P_2$
$Q_{3a}^2 * K'_3 = P_{6a} - P_3$	$P_{4b} = A - B * Q_{45b}^2$
$P_{4a} = A - B * Q_{45a}^2$	

The purpose of the first MATLAB program is to solve the 11 equations after they are arranged into the matrix form $\mathbf{Ax} = \mathbf{b}$. Arranging the equations in this matrix form allows us to linearly solve for the unknown vector \mathbf{x} by using a matrix solver in MATLAB and setting $\mathbf{x} = \mathbf{A}/\mathbf{b}$. Each one of the elements in the vector \mathbf{x} represents each one of the 11 unknowns in the system, while the matrix \mathbf{A} and the vector \mathbf{b} are composed of the known variables in the system. The 11 equations that are used to solve for the system characteristics and fully open damper coefficients are then rearranged in the following way:

1. The known fan exit static pressures (P_{4a} , and P_{4b}) and the terminal outlet exit pressures (P_1 , P_2 , and P_3) represent the vector \mathbf{b} , and are put on the right side of the equation.

$$P_{5a} + Q_{45a}^2 * K'_{45a} = P_{4a}$$

$$P_{5b} + Q_{45b}^2 * K'_{45b} = P_{4b}$$

$$P_{5a} - Q_{1a}^2 * K'_{1a} = P_1$$

$$P_{5b} - Q_{1b}^2 * K'_{1b} = P_1$$

$$P_{6a} - Q_{56a}^2 * K'_{56a} = P_{5a}$$

$$P_{6b} - Q_{56b}^2 * K'_{56b} = P_{5b}$$

$$P_{6a} - Q_{2a}^2 * K'_{2a} = P_2$$

$$P_{6b} - Q_{2b}^2 * K'_{2b} = P_2$$

$$P_{6a} - Q_{3a}^2 * K'_{3a} = P_3$$

$$A - B * Q_{45b}^2 = P_{4b}$$

$$A - B * Q_{45a}^2 = P_{4a}$$

2. Now that the vector **b** has been determined, the unknown vector **x** is next. This vector consists of the unknown static pressures at the junctions (P_{5a} , P_{5b} , P_{6a} , and P_{6b}), the unknown duct loss coefficients (K_{45} , K_{56a} , and K_{56b}), and the unknown fan constants (A and B). The unknowns are arranged into the vector **x** in such a way, that when multiplied by the matrix **A**, the result will be the vector **b**.
3. The matrix **A** is composed of the measured flowrate through the ducts (Q_{45a} , Q_{45b} , Q_{56a} , and Q_{56b}), and the measured flowrate at the terminal outlets (Q_{1a} , Q_{1b} , Q_{2a} , Q_{2b} , and Q_{3a}). Knowing where all the variables will be placed in the linear system, the final system of the matrix form $\mathbf{Ax} = \mathbf{B}$ will look like this:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & Q_{45a}^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & Q_{45b}^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & -Q_{1a}^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -Q_{1b}^2 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & -Q_{56a}^2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & -Q_{56b}^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -Q_{2a}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -Q_{2b}^2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -Q_{3a}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & Q_{45a}^2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & Q_{45b}^2 \end{bmatrix} * \begin{bmatrix} P_{5a} \\ P_{5b} \\ P_{6a} \\ P_{6b} \\ K_{45} \\ K_1 \\ K_{56} \\ K_2 \\ K_3 \\ A \\ B \end{bmatrix} = \begin{bmatrix} P_{4a} \\ P_{4b} \\ P_1 \\ P_1 \\ 0 \\ 0 \\ P_2 \\ P_2 \\ P_3 \\ P_{4a} \\ P_{4b} \end{bmatrix}$$

Once this matrix is set up, the simple command in MATLAB $\mathbf{x} = \mathbf{A}/\mathbf{b}$ will use Gaussian Elimination to solve for the system friction loss coefficient, the static pressures, and the fan constants. In the same way, any system with n number of equations can be set up as the matrix form $\mathbf{Ax} = \mathbf{b}$, with an $n \times n$ \mathbf{A} matrix that carries known measured flows, and an $n \times 1$ \mathbf{x} vector to solve for the unknowns (see Appendix A for code).

3.1.2 Solving for Design Damper Positioning Coefficients Using MATLAB

The next step in the computational process involves a MATLAB program that solves for the terminal outlet loss coefficients (K_{1d} , K_{2d} , and K_{3d}), when running the system at design flow. These loss coefficients will replace K_1 , K_2 , and K_3 from figure 3.1. Like the previous step (Solving for System Characteristics), this step arranges its equation into the matrix form $\mathbf{Ax} = \mathbf{b}$ in order to solve for the unknowns with a simple algebraic approach.

Since the flow coefficients (K_{45} , and K_{56}) and the fan constants (A , and B) have been calculated, the only unknowns for this step are the static pressures corresponding to design flow (P_4 , P_5 , and P_6), and the modified loss coefficients (K_{1d} , K_{2d} , and K_{3d}). Unlike solving for system characteristics, the static pressure exiting the fan is unknown, since neither the flowrate nor the static pressure exiting the fan is measured during this step.

The process of finding the equations necessary to solve for the design damper positioning coefficients is easier than the process to solve for system characteristics. This process uses the six equations derived from Section 2.3, but with a small modification:

$$Q_{45d}^2 * K_{45} = P_{4d} - P_{5d}$$

$$Q_{56d}^2 * K_{56} = P_{5d} - P_{6d}$$

$$Q_{1d}^2 * K_{1d} = P_{5d} - P_1$$

$$Q_{2d}^2 * K_{2d} = P_{6d} - P_2$$

$$Q_{3d}^2 * K_{3d} = P_{6d} - P_3$$

$$P_{4d} = A - B * Q_{45d}^2$$

The equations have been modified by adding the subscript 'd', which represents variables that have design operating condition values. These equations are then used to come up with a linear system of 6 equations and 6 unknowns, arranged into a matrix, and entered into the program.

The equations are arranged in the form $\mathbf{Ax} = \mathbf{b}$ to allow us to linearly solve for the unknown vector \mathbf{x} , with each one of its elements representing each one of the 6 unknowns to solve for. When rearranged into the matrix form of $\mathbf{Ax} = \mathbf{b}$, the 6 equations will look like this:

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -Q_{1d}^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -Q_{2d}^2 & 0 \\ 0 & 0 & 0 & 1 & 0 & -Q_{3d}^2 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} P_{4d} \\ P_{5d} \\ K_{1d} \\ P_{6d} \\ K_{2d} \\ K_{3d} \end{bmatrix} = \begin{bmatrix} K_{45} * Q_{45d}^2 \\ P_1 \\ K_{56} * Q_{56d}^2 \\ P_2 \\ P_3 \\ A - B * Q_{45d}^2 \end{bmatrix}$$

The reason there are Q 's on the right hand side is to convert every term in the vector \mathbf{b} into a pressure term. Multiplying the Q 's by the loss coefficient K or the fan constant B will change flowrate units into pressure units. Having all the terms of the \mathbf{b} vector with the same unit helps the sensitivity of the matrix. In other words, the matrix is

less prone to calculating errors when the numerical values of all the elements in the vector are of the same magnitude (see Appendix A for code).

The same MATLAB command that was used in the previous step ($\mathbf{x} = \mathbf{A}/\mathbf{b}$) using the method of Gaussian Elimination, is used to solve for damper positioning coefficients and static pressures that correspond to the design flow.

3.1.3 Determining Adjusted Flows at the Outlets Using MATLAB

This final step in the computational process is to write three MATLAB programs that include a non-linear algorithm to solve for the flowrates needed at the terminal outlets, in order to complete the balancing procedure. The first two MATLAB programs are subroutines written to solve for the adjustable flows, and the third program calls on these subroutines to solve them.

3.1.3.1 First Subroutine

The first subroutine in this step determines the flowrate on the first branch with the dampers on branches 2 and 3 fully open, and the damper on branch 1 adjusted to its design value. In calculating this flowrate, the pressure drop equations across branches 2 and 3 will use their open damper friction loss coefficients (K_2 , and K_3), and the pressure drop equation across branch 1 will use its design damper positioning coefficient (K_{1d}).

The equations for the first subroutine are also derived from section 2.3 with minor modifications:

$$Q_{45f}^2 * K'_{45} = P_{4f} - P_{5f}$$

$$Q_{56f}^2 * K'_{56} = P_{5f} - P_{6f}$$

$$Q_{1f}^2 * K'_{1f} = P_{5f} - P_1$$

$$Q_{2f}^2 * K'_2 = P_{6f} - P_2$$

$$Q_{3f}^2 * K'_3 = P_{6f} - P_3$$

$$P_{4f} = A - B * Q_{45f}^2$$

The equations have been modified by having the subscript 'f' indicating variables which values represent the final phase of the balancing procedure, and Q_{1f} is the flowrate for which the damper on branch 1 is adjusted to. If you look at the pressure drop equations across branches 2 and 3, you will notice that the fully-open damper coefficients K'_2 and K'_3 are used, and that across branch 1, the pressure drop equation uses the desired damper positioning coefficient K'_{1d} .

Since the variables solved for are flowrates, and solving them requires taking the square root of them, a non-linear algorithm is necessary to solve the 6 equations. This time, the equations will not be arranged into a matrix. The equations will be implicitly written in the program and will be set equal to zero, since this is the form that MATLAB's non-linear solver requires. After setting them equal to zero, the equations are:

$$Q_{45f}^2 * K'_{45} - P_{4f} + P_{5f} = 0$$

$$Q_{56f}^2 * K'_{56} - P_{5f} + P_{6f} = 0$$

$$Q_{1f}^2 * K'_{1d} - P_{5f} + P_1 = 0$$

$$Q_{2f}^2 * K'_2 - P_{6f} + P_2 = 0$$

$$Q_{3f}^2 * K'_3 - P_{6f} + P_3 = 0$$

$$P_{4f} - A + B * Q_{45f}^2 = 0$$

The equations are solved with an optimization package from MATLAB that includes a command known as *lsqnonlin*. *Lsqnonlin* solves non-linear equations using a least-squares method which solves equations in the form $\mathbf{f}(\mathbf{x}) = 0$, where \mathbf{x} is a vector of unknowns and \mathbf{f} represents six equations in implicit form. The six equations during this phase of the balancing process are then rearranged and entered into MATLAB in the form a vector \mathbf{f} with the vector \mathbf{x} as the unknowns.

$$\mathbf{f} = [x_1 - x_2 - K_{45}*(x_1+x_2+x_3)^2$$

$$x_2 - P_1 - K_{1d}*x_3^2$$

$$x_2 - x_6 - K_{56}*(x_4+x_3)^2$$

$$x_6 - P_2 - K_2*x_4^2$$

$$x_6 - P_3 - K_3*x_5^2$$

$$x_1 - A - B*(x_1+x_2+x_3)^2].$$

Here, x_1 through x_6 represents the six unknowns to solve for in the vector \mathbf{x} . *Lsqnonlin* will find \mathbf{x} such that

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

with the unknowns represented in vector \mathbf{x} being:

$$P_{4f} = x_1$$

$$P_{5f} = x_2$$

$$Q_{1f} = x_3$$

$$Q_{2f} = x_4$$

$$Q_{3f} = x_5$$

$$P_{6f} = x_6$$

After Q_{1fn} is determined, the damper on branch 1 can be adjusted to this flowrate while the other two dampers on the other branches are left wide open.

3.1.3.2 Second Subroutine

The second subroutine is used to determine the adjusted flowrate on the second branch with the damper on branch 1 set at the desired damper position K_{1d} , and the damper on branch 3 completely open. When calculating the flowrate on branch 2, the pressure drop equation across branch 3 uses its open damper friction loss coefficients K_3 , and the pressure drop equation across branch 2 will use its design damper positioning coefficient (K_{2d}).

Like the first subroutine, the equations used on the second subroutine are derived from section 2.3:

$$Q_{45f}^2 * K'_{45} = P_{4f} - P_{5f}$$

$$Q_{56f}^2 * K'_{56} = P_{5f} - P_{6f}$$

$$Q_{1f}^2 * K'_{1d} = P_{5f} - P_1$$

$$Q_{2fn}^2 * K'_{2d} = P_{6f} - P_2$$

$$Q_{3f}^2 * K'_3 = P_{6f} - P_3$$

$$P_{4f} = A - B * Q_{45f}^2$$

Q_{2fn} represents the value that the flowrate on branch 2 will be adjusted to. The pressure drop equations across branch 3 uses its fully-open damper coefficient K'_3 , and the

pressure drop equation across branches 1 and 2 use the desired damper positioning coefficient K'_{1d} and K'_{2d} .

The same command used on the first subroutine (*lsqnonlin*) is used on this subroutine, and the equations are arranged implicitly so they can be called on with the third MATLAB program during the non-linear solving procedure. The equations are arranged in the following form:

$$\mathbf{f} = [x_1 - x_2 - K_{45}*(x_1+x_2+x_3)^2$$

$$x_2 - P_1 - K_{1d}*x_3^2$$

$$x_2 - x_6 - K_{56}*(x_4+x_3)^2$$

$$x_6 - P_2 - K_{2d}*x_4^2$$

$$x_6 - P_3 - K_3*x_5^2$$

$$x_1 - A - B*(x_1+x_2+x_3)^2]$$

and the unknowns are:

$$P_{4f} = x_1$$

$$P_{5f} = x_2$$

$$Q_{1f} = x_3$$

$$Q_{2f} = x_4$$

$$Q_{3f} = x_5$$

$$P_{6f} = x_6$$

After determining the second vector \mathbf{x} , the second branch can be adjusted to the appropriate flowrate Q_{2f} . By adjusting the damper on the second branch to its desired

position, the flowrate through branch 1 will increase so that it will be closer to its design flowrate value on that branch. The flowrate on branch 3 will be adjusted to its design flowrate. Since fan speed is not being adjusted here, the damper on this branch is adjusted to design flow. Thus, the flowrate through branches 1 and 2 will increase until they reach design airflow. This non-iterative balancing procedure is essentially the same for larger systems.

3.1.3.3 Subroutine-Calling Program

The final program in the non-linear procedure is used to call on the two previous-mentioned subroutines with the command *lsqnonlin*, and uses the design values as the initial guesses for the least squares approximation procedure. The final program is also used to write the results to either an output file or the MATLAB command window (see Appendix A for codes).

3.2 Preliminary Graphical User Interface

A graphical user interface of the balancing procedure is developed with a modeling program called SIMULINK. The purpose of a graphical user interface is to show how a potential product would work if it were to be put into production. The plans for the potential future product are:

1. Graphically enter a duct system for an air distribution system.
2. Automatically develop equation sets in subsystems.

3. Have the user enter airflow measurements and fan curve constants.
4. Have the program determine the solution while displaying balancing instructions.

A setback to the graphical user interface procedure is that it is more complicated and more likely to crash than the computational procedure. This is partly due to the fact that the SIMULINK solving process is transient, and the air distribution system that is being modeled does not require a time dependency. The system being solved is assumed to be at steady state.

The method by which the system is solved is through a stochastic process in which the variables are interdependent of each other. A change in pressure will cause a change in airflow and vice-versa.

Three graphical models were developed to solve for a two-diffuser air distribution system using a similar procedure to the three-diffuser model. These graphical models follow a balancing procedure similar to the computational model procedure. The system characteristics are solved for first, next the desired damper positioning coefficients are determined, and then the dampers on the branches are adjusted to match calculated flowrates that will balance the system. The three graphical models are called upon by using a MATLAB program that provides the input to these models and then uses the output for the next models.

3.2.1 Description of Model

SIMULINK has built-in functions and equation solvers that are used to solve for the system characteristics and the other unknowns throughout the balancing process. The two-diffuser air distribution system used is shown in Figure 3.2

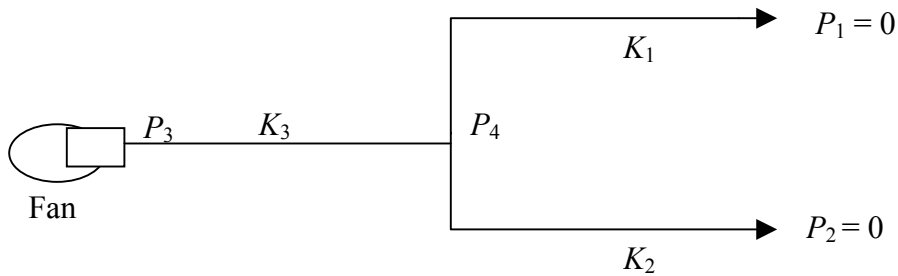


Figure 3.2: Two- Diffuser Air Distribution System

The system characteristics for a two-diffuser model are very similar to that of a three-diffuser model. K'_1 and K'_2 represent the fully open damper flow coefficients, while K'_3 represents the loss coefficient in the main duct. P_3 represents the fan exit static pressure, while P_4 represents the static pressure at the junction connecting the two diffusers with the main duct.

The model will be designed as a system composed of various flow elements with the airflow information going upstream and the pressure information going downstream (see Figure 3.3).



Figure 3.3. Flow Element

With this in mind, the air distribution model for a two-diffuser system is designed like the system in Figure 3.4. The same model used to solve for the system characteristics, is also used to solve for design damper positioning coefficients. The model used to solve for the adjusted flowrates at the terminal outlets is similar model to that on Figure 3.4, but with some modifications (See Figure 3.10). In solving all models, it is assumed that the fan curve for the system is known, so the fan constants are given.

The SIMULINK model is divided into 7 subsystems that are connected to each other through lines of flowrate and pressure. The subsystems are: the Fan subsystem, the Fanduct subsystem, the Tee subsystem, Diffuser 1 and Diffuser 2 subsystems, and Measurement 1 and Measurement 2 subsystems.

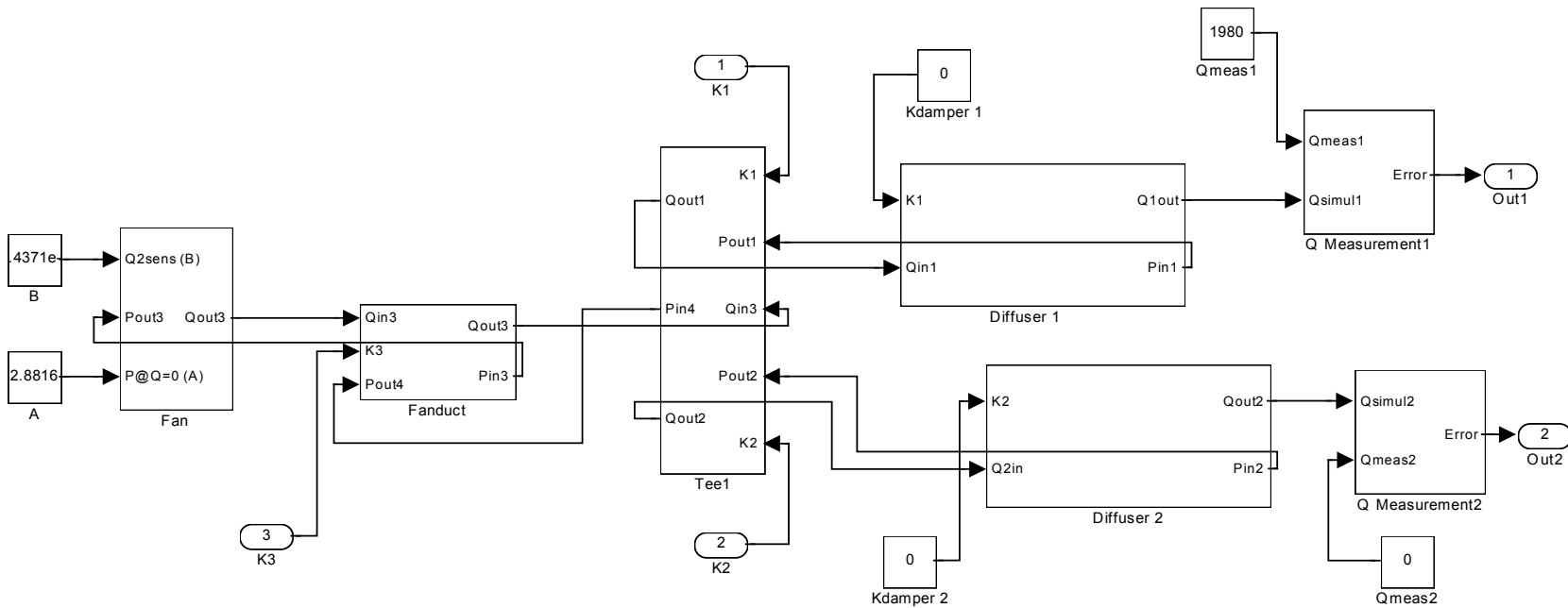


Figure 3.4. Simulink Model of Two-Diffuser Air Distribution System

3.2.1.1 Fan Subsystem

The purpose of the fan subsystem is to calculate the system's operating flowrate for a calculated fan exit static pressure and given fan constants A and B .

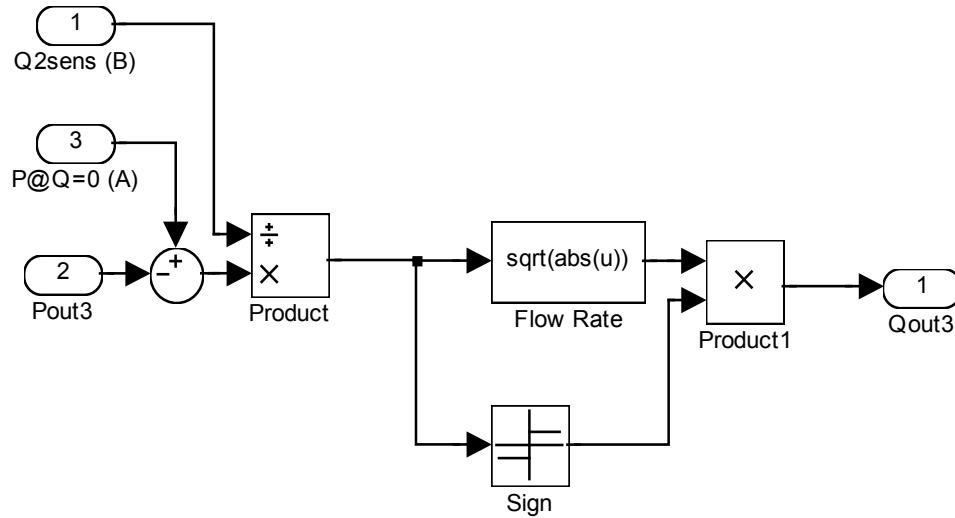


Figure 3.5 Fan Subsystem

The fan subsystem works on the principle of using the equation

$$Q_3 = \pm \sqrt{\frac{A - P_3}{B}}, \quad (3.1)$$

where Q_3 is the flowrate and P_3 is the fan exit pressure. The static pressure entering the fan subsystem is subtracted from the fan constant A , which represents the maximum static pressure for the system when the flowrate is 0. The result is divided by the fan constant B , and the square root of the absolute value is taken to determine the system's

operating flowrate. After the system's flowrate is determined, the sign block ensures that the correct sign is passed along to the next element or subsystem.

3.2.1.2 Fan Duct Subsystem

The fan duct subsystem determines the value for the flow loss coefficient K_3 when determining system characteristics, and then uses this value in the other steps of the balancing process to determine other variables.

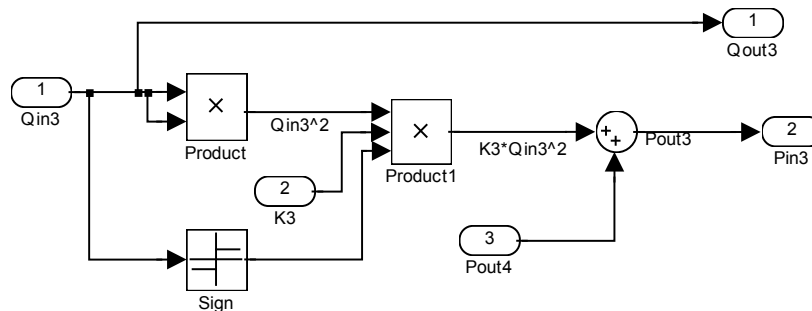


Figure 3.6: Fan Duct Subsystem

The fan duct subsystem uses the airflow Q_3 from the previous element (Fan subsystem) along with the flow loss coefficient K_3 to determine the pressure loss across the fan duct branch. The pressure loss is then added the static pressure at the junction P_4 to determine the fan exit pressure P_3 . This pressure is then fed back upstream to the fan subsystem where it is used to calculate the flowrate Q_3 . The flowrate Q_3 is carried

upstream to the next subsystem.

3.2.1.3 Tee Subsystem

The tee subsystem takes the flowrate Q_3 and splits it into 2 separate flowrates for diffuser 1 and 2 subsystems. This subsystem also determines fully open damper loss coefficients and damper positioning coefficients at design flow.

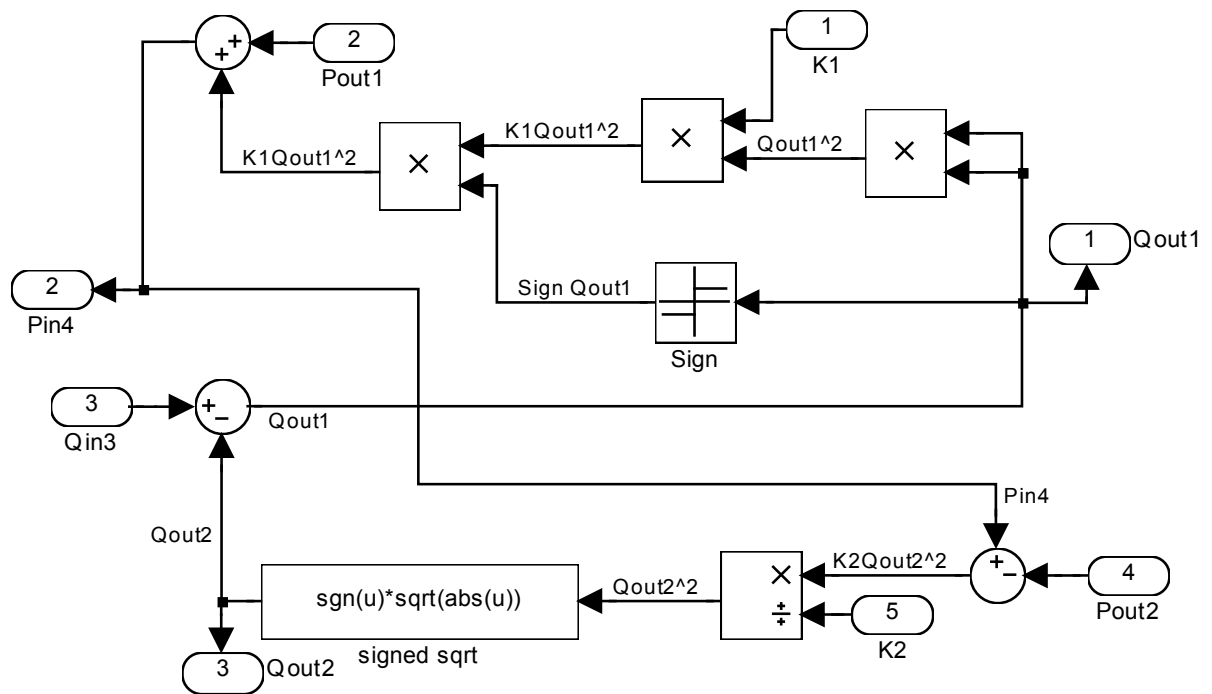


Figure 3.7: Tee Subsystem

The tee subsystem uses the static pressures P_1 and P_2 fed from diffusers 1 and 2, along with the flow loss coefficients from branches 1 and 2 (K_1 , K_2), to determine the

diffuser flowrates Q_1 and Q_2 . From the previous subsystem (Fan Duct), Q_3 enters the Tee subsystem and is used to calculate Q_1 by subtracting Q_2 from Q_3 . From here, Q_1 is squared and then multiplied by K_1 to calculate the pressure drop across branch 1. Again, the sign block is there to ensure that the correct sign is maintained after squaring Q_1 . By using the pressure drop equation

$$P_4 - P_1 = K_1 Q_1^2, \quad (3.2)$$

the pressure drop across diffuser 1 ($K_1 Q_1^2$) is added to the exit pressure from diffuser 1 (P_1), to calculate the static pressure at the junction P_4 . The pressure exiting diffuser 2 (P_2) is subtracted from P_4 to determine the pressure drop through diffuser 2. Knowing the pressure drop across diffuser 2, Q_2 is calculated by dividing the pressure drop by K_2 and taking the square root of the answer:

$$Q_2 = \pm \sqrt{\left| \frac{\Delta P_2}{K_2} \right|}. \quad (3.3)$$

With a sign block assuring that the correct sign is passed, Q_1 and Q_2 are fed upstream to the diffuser 1 and 2 subsystems, respectively, and P_4 is fed downstream to the tee subsystem.

3.2.1.4 Diffuser Subsystem

The purpose of the diffuser subsystem is to determine whether the damper on this diffuser is open or closed.

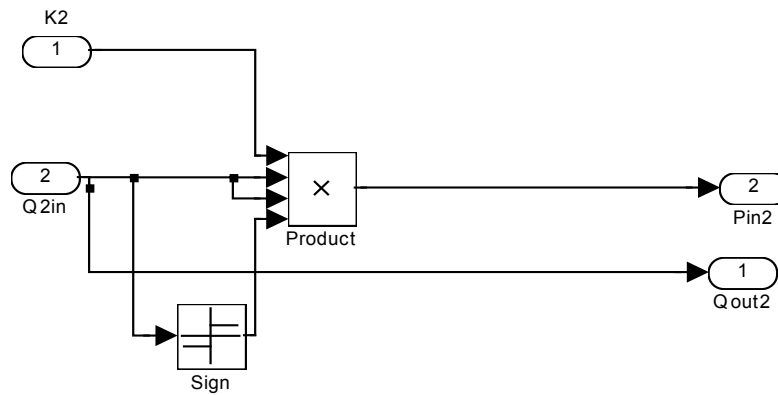


Figure 3.8: Diffuser Subsystem

The diffuser subsystem uses Q_1 or Q_2 and a damper coefficient to determine the diffuser exit pressure P_1 or P_2 . The damper coefficient value K will be either zero when the damper is open, or very high (close to infinity) when the damper is closed. A K value of zero represents the exit pressure for the terminal outlet of an air distribution system, and will always be the case when this damper is not closed. A high K value represents an extremely high pressure for that terminal outlet, or for calculating purposes, a branch that does not exist.

To determine the terminal outlet pressure, K is multiplied by the flowrate squared, and the sign block assures that the correct sign is being passed to the next subsystem.

When determining system characteristics, two SIMULINK models are required to solve for all unknowns: one representing two fully open dampers, and one representing a fully open and a closed damper. When determining design variables and adjusted flows, only one SIMULINK model is needed. Q_1 and Q_2 are fed downstream to measurement 1 and 2 subsystems, and P_1 and P_2 are fed upstream to the tee subsystem.

3.2.1.5 Measurement Subsystem

The object of this subsystem is to calculate the error between the given and measured flowrate, and the flowrate calculated by SIMULINK. For the last phase of the balancing process (calculating adjusted flowrates), this subsystem is not required.

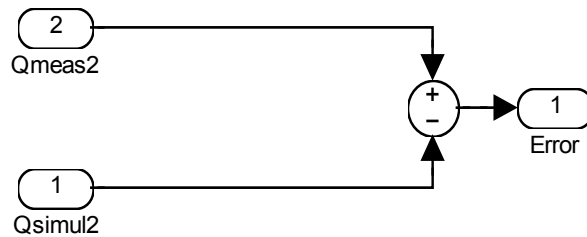


Figure 3.9: Measurement Subsystem

This subsystem uses the flowrates calculated by SIMULINK and subtracts them from flowrates entered into the program by the user, to calculate an error. The error is then output and called upon by a MATLAB function that finds the minimum possible value for the error.

3.2.2 Solving for System Characteristics Using SIMULINK

This model takes a different approach at solving for the system characteristics than the computational model. The model uses an iterative process to find a minimum error between a calculated flowrate by the SIMULINK model and the actual flowrate as measured from the air distribution system.. To find that minimum, guesses are entered for the duct friction loss coefficient K_3 and the open damper flow coefficients K_1 and K_2 . To simplify the computational process, the fan constants (A , B) are given.

The SIMULINK model uses a MATLAB program that first inputs guesses into the SIMULINK file for the flow coefficients K_1 , K_2 , and K_3 , and then uses the command *fminsearch* to call on two models to find the minimum value of the absolute error. The first model represents the first pass, and the second model represents the second pass. The absolute error is determined by adding the absolute value of the differences between the measured and the calculated flowrates. The differences between the measured and the calculated flowrates are individually labeled z_1 through z_3 depending on the errors they represent. The errors z_1 and z_2 represent the difference between measured and calculated flowrates for branches 1 and 2 for the first model. Conversely, the error z_3 represents the flowrate differences for branch number 1 for the second model. The absolute error is calculated with equation 3.4:

$$\text{error}_{\text{abs}} = |z_1| + |z_2| + |z_3|. \quad (3.4)$$

Taking the absolute value of each error gets rid of the possibility of the errors canceling each other due to a negative sign in either of them.

3.2.3 Solving for Design Damper Coefficients Using SIMULINK

The SIMULINK model for determining design damper positioning coefficients works on the principle of entering the main branch friction coefficient K_3 and the design flowrates Q_{1d} and Q_{2d} , to determine the design damper positioning coefficients K_{1d} and K_{2d} .

Solving for desired damper coefficients is simpler than solving for system characteristics. Since K_3 is already known, the iterative error minimizing procedure will be used to solve for K_{1d} and K_{2d} only. Also, there are only four unknowns in the system, so only one SIMULINK model is required to solve for them. The same error minimization procedure is used to determine the design damper positioning coefficients. The only difference is that the absolute error is calculated with:

$$\text{error}_{\text{absd}} = |z_{1d}| + |z_{2d}| \quad (3.5)$$

3.2.4 Solving for Adjusted Flows during Balancing Using SIMULINK

The final SIMULINK model uses the main branch friction loss K_3 , the desired damper coefficient for branch 2 K_{2d} , and the fully open damper coefficient for branch 1 K_1 to determine the flowrate Q_{2f} that the damper on branch number 2 is adjusted to. The flowrate on branch number 1 is adjusted to its design flow.

This is the simplest of all models to solve since the error minimization function is not used. The measurement subsystems are eliminated and only one model is needed to compute the flowrate Q_{2f} (See Figure 3.10). The flow coefficients are entered and SIMULINK computes the flowrate.

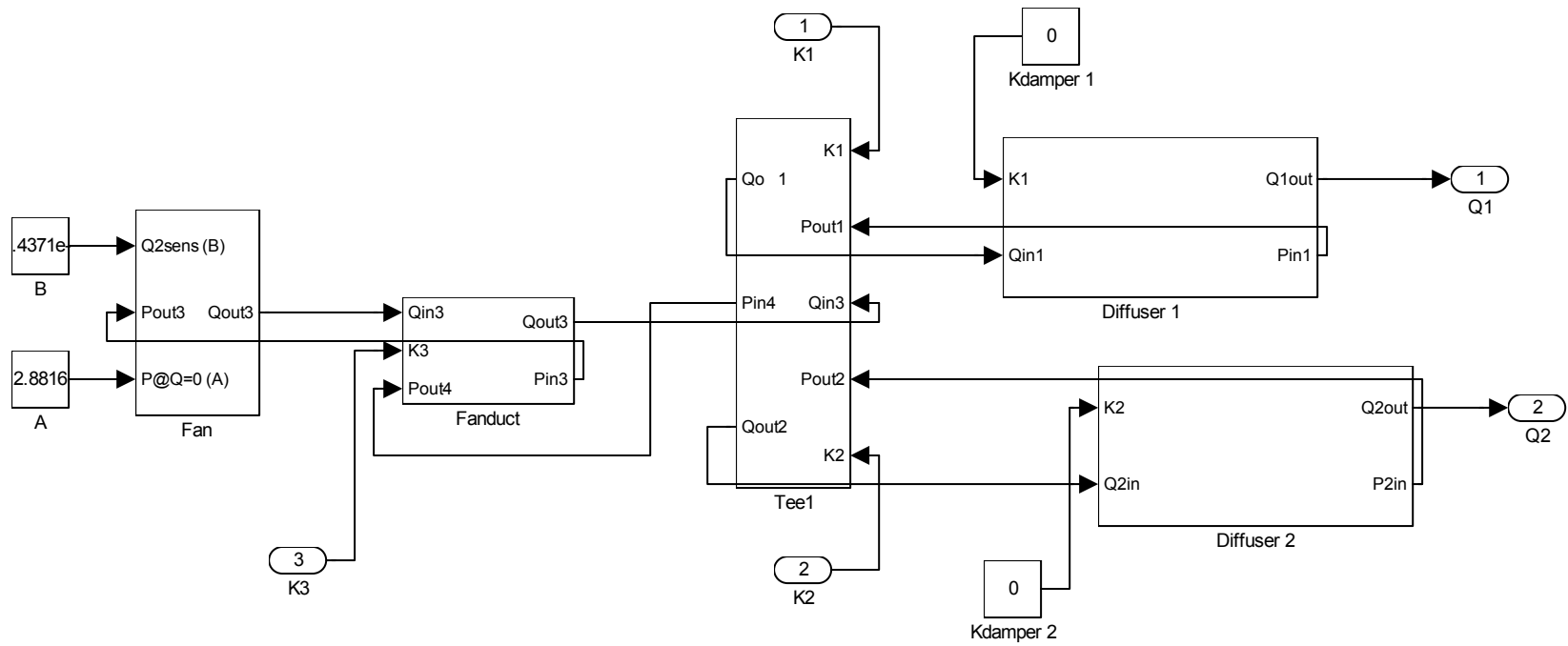


Figure 3.10. Simulink Model for Determining Adjusted Flows During Balancing

3.2.5 MATLAB Programs that run the SIMULINK Models

Three MATLAB programs were written to handle the inputs and outputs from the four SIMULINK models. Two of the programs are subroutines that minimize the absolute error in order to find friction loss coefficients. One program finds the flow loss coefficients for the first step in the balancing procedure and the other program finds the damper positioning coefficients when operating at design flow. The third program is the main program that provides the inputs and outputs for the two other programs and the SIMULINK models. The third program works in the following way:

1. The program calls on the first subroutine by using the command *fminsearch* to find the minimum possible absolute error to determine the main branch friction loss coefficient K_3 and the fully open damper coefficients K_1 and K_2 for the first subroutine.
2. The program inputs K_3 into the second subroutine and uses *fminsearch* to determine the damper positioning coefficients K_{1d} and K_{2d} that correspond to design flow.
3. Next, the program inputs the friction loss coefficient K_3 , the fully-open damper coefficient for branch 1 K_1 , and the damper positioning coefficient at design airflow on branch 2 K_{2d} , to determine the flow Q_{2f} for which to adjust the dampers on branch number 2. The flow Q_{1f} should have the same value as that of design flow Q_{1d} , and is adjusted to this value after Q_{2f} is adjusted (see Appendix A for codes).

The SIMULINK graphical user interface process is more complicated, computationally slower, and more likely to crash than the MATLAB computational process, but it graphically shows how a duct system works and it is the basis for a potential non-iterative balancing program that could be put into production.

Chapter 4

Results and Discussion

This section is divided into two parts: a description of the results of the computational model and a discussion of the results of the graphical user interface.

4.1 Computational Model Results

The values for design flowrate for the model in this research project were taken from an example in a book (Stanford, 1988). The example in the book includes its own flow coefficient values and pressure losses. Since the model being solved in this project has to be solved by completely closing a damper and assuming the flowrate in the system after this damper is closed, the fully open damper coefficient values in the book are not used. Instead, the flow coefficient values from the book were used to determine the flowrate for dampers fully open, and for one damper closed. An educated guess on the fan exit pressure is made to help in determining the flowrates during the first step of the balancing procedure.

The model being solved is a three-diffuser air distribution system. The purpose of this project is to achieve a design flow of $Q_1 = 1500$ cfm, $Q_2 = 1000$ cfm, and $Q_3 = 1000$ cfm (see Figure 4.1) without undergoing an iterative process.

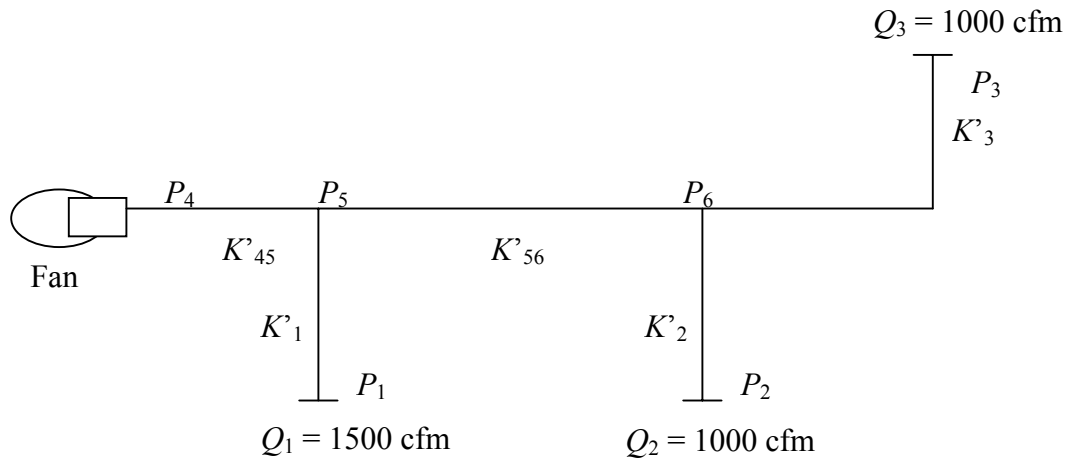


Figure 4.1. Three-Diffuser Air Distribution System

Solving the model requires three steps. During the first step (solving for system characteristics), the measured flowrate and the static pressure exiting the fan will be given. For the second step, the design damper positioning coefficients will be calculated. During the final step, the dampers at the terminal outlets will be adjusted only once to a calculated flowrate that will balance the system.

4.1.1 Solving for System Characteristics

As mentioned earlier, the system characteristics and the fully open damper coefficients are solved for by running two passes that lead to a system of 11 equations and 11 unknowns. Within these 11 equations, the given variables are the measured flowrates and the static pressure exiting the fan. The values for these variables are as follow:

$$Q_{1a} = 1500 \text{ cfm}$$

$$Q_{2a} = 1000 \text{ cfm}$$

$$Q_{3a} = 1307.9 \text{ cfm}$$

$$Q_{1b} = 1846.6 \text{ cfm}$$

$$Q_{2b} = 1610.1 \text{ cfm}$$

$$P_{4a} = 0.28 \text{ psi}$$

$$P_{4b} = 0.35 \text{ psi}$$

where the subscripts 'a' and 'b' represent the first and second pass, respectively. All variables listed have values that represent the air distribution system with the terminal outlets fully open. For the second pass, the branch with the closed damper has its variable (Q_{3b}) eliminated from the system.

A quick analysis can be done on the given variables. Looking at their numerical value, it can be seen that for the second pass, the static pressure exiting the fan (P_4) is higher. This is a result of closing the damper on the third branch. Additionally, closing this damper after it being fully open will decrease the overall flowrate through the system, increase the flowrate through branches 1 and 3, and increase the overall static pressure throughout the system. The total flowrate during the first pass is 3807.9 cfm and for the second pass its 3456.7 cfm.

After entering the given variables and running all 11 equations in MATLAB, the static pressures, system characteristics, and fully open damper coefficients are as follow:

Static Pressures at Junctions 5 and 6	
$P_{5a} = 0.1725 \text{ psi}$	$P_{5b} = 0.2614 \text{ psi}$
$P_{6a} = 0.0843 \text{ psi}$	$P_{6b} = 0.2185 \text{ psi}$
Flow Coefficients Through Main Branches	
$K_{45} = 7.4151 \times 10^{-9} \frac{\text{psi}}{\text{cfm}^2}$	$K_{56} = 1.6561 \times 10^{-8} \frac{\text{psi}}{\text{cfm}^2}$
Fan Constants	
$A = 0.6778 \text{ psi}$	$B = -2.7437 \times 10^{-8} \frac{\text{psi}}{\text{cfm}^2}$
Fully-Open Damper Coefficients	
$K_1 = 7.6658 \times 10^{-8} \frac{\text{psi}}{\text{cfm}^2}$	$K_2 = 8.4271 \times 10^{-8} \frac{\text{psi}}{\text{cfm}^2}$
$K_3 = 4.9264 \times 10^{-8} \frac{\text{psi}}{\text{cfm}^2}$	

Now that we have the system characteristics and fully open damper coefficients, an analysis on the static pressures can be done to check if the results are correct. This is done by using the equation for determining the pressure loss across an element (equation 2.5), and checking if the answer is correct. The pressure losses for each element in the system are:

Table 4.1 Pressure Losses Across Elements

Pressure Losses During First Pass	Pressure Losses During Second Pass
$\Delta P_{45a} = 0.1075 \text{ psi}$	$\Delta P_{45b} = 0.0886 \text{ psi}$
$\Delta P_{56a} = 0.0882 \text{ psi}$	$\Delta P_{56b} = 0.0429 \text{ psi}$
$\Delta P_{51a} = 0.1725 \text{ psi}$	$\Delta P_{51b} = 0.2614 \text{ psi}$
$\Delta P_{62a} = 0.0843 \text{ psi}$	$\Delta P_{52b} = 0.2614 \text{ psi}$
$\Delta P_{63a} = 0.0843 \text{ psi}$	

During the first pass, the first element in the branch closest to the fan has a flow coefficient of $K_{45} = 7.4151 \times 10^{-9} \frac{\text{psi}}{\text{cfm}^2}$ and a flowrate of 3807.9 cfm. Squaring the flowrate, and multiplying it by the flow coefficient we get:

$$(3807.9)^2 \times 7.4151 \times 10^{-9} = \Delta P_{45a} = 0.1075 \text{ psi}$$

Similarly, we can check this for each element in the system:

$(1500)^2 \times 7.6658 \times 10^{-8} = \Delta P_{51a} = 0.1725 \text{ psi}$	$(2307.9)^2 \times 1.6561 \times 10^{-8} = \Delta P_{56a} = 0.0882 \text{ psi}$
$(1000)^2 \times 8.4271 \times 10^{-8} = \Delta P_{62a} = 0.0843 \text{ psi}$	$(1307.9)^2 \times 4.9264 \times 10^{-8} = \Delta P_{63a} = 0.0843 \text{ psi}$

And for the second pass the products are:

$(3456.7)^2 \times 7.4151 \times 10^{-9} = \Delta P_{45b} = 0.0886 \text{ psi}$	$(1846.6)^2 \times 7.6658 \times 10^{-8} = \Delta P_{51b} = 0.2614 \text{ psi}$
$(1610.1)^2 \times (8.4271 \times 10^{-8} + 1.6561 \times 10^{-8}) = \Delta P_{52b} = 0.2614 \text{ psi}$	

For the second pass, branch 5-6 and branch 2 are added up into a single branch 5-2. To determine the pressure loss across this branch (ΔP_{52b}), the flow coefficient K_{56} is added to K_2 to get K_{52} .

4.1.1.1 Fan Curve and System Curve

The results can be understood in terms of the fan curve and system curve. The fan curve and the system curves are determined by using the fan constants (A, B) and the main branch flow coefficient (K_{45}), and producing a graph of pressure versus flowrate. Both the fan and system curve must match at the operating flowrate and pressure of the system. For the first pass, this would be at 3807.9 cfm and 0.28 psi; for the second pass this would be at 3456.7 cfm and 0.35 psi. Since the process of determining system characteristics requires two passes, there are two system curves plotted versus one fan curve. The intersection of the fan curve with the system curves determines the two operation points for the system (See Figure 4.2).

To produce the equations for the fan and system curve, the fan constants and the system flow coefficients are needed. First, to develop the fan curve equation, equation 2.11 from Chapter 2 is taken and substituted with the fan constants A and B to produce:

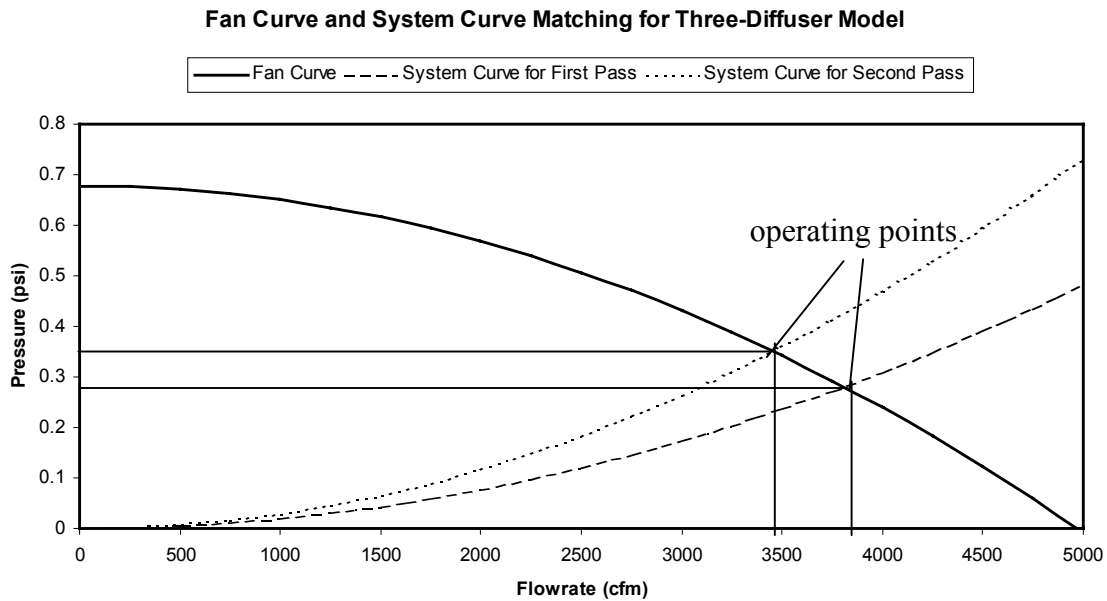
$$P_4 = 0.6778 - (2.7437 \times 10^{-8})Q_{45}^2 \quad (4.1)$$

where P_4 is the fan exit pressure and Q_{45} is the system total flowrate.

Second, to develop the system curve equation, the operating points (P_4, Q_{45}) in the system are taken in conjunction with one of the fan laws (equation 2.11) to produce

$$P_4' = P_4 \left(\frac{Q_{45}'}{Q_{45}} \right)^2 \quad (4.2)$$

where P_4 and P_5 are the constant operating points, and P_4' and Q_{45}' are the varying pressure and flowrates plotted on the graph. Equation 4.2 is used for the system curves of both the first and second passes. As shown in Figure 4.2, the fan curve matches both system curves at their operating points.



As shown in Figure 4.2, the fan curve intersects the system curve for the second pass at a higher pressure and a lower flowrate than the system curve for the first pass. Although obvious, this concept is in accordance with the pressure-flowrate relationship of the fan curve equation (2.11): as the flowrate increases, the pressure decreases. Additionally, closing the damper for the second pass caused an increase in pressure and a decrease in flow.

The system curve equation (equation 4.2) can be analyzed by expressing it in a different way. Rewriting it, it can be generalized as:

$$y = mx^2 \quad (4.3)$$

where the fan exit-pressure (P_4) represents the slope of the curve m . From Figure 4.2, it can be seen that the system curve for the second pass goes up faster than the system curve for the first pass. This is due to the fact that the slope for the second pass ($P_4 = 0.35$ psi) is higher than that of the first pass ($P_4 = 0.28$ psi).

In determining the flow coefficients and fan constants, it was shown that the system behaves in accordance with the fan laws and fan curve equations that govern an air distribution system.

4.1.2 Solving for Design Damper Positioning Coefficients

The main branch flow coefficients (K_{45} , K_{56}) and the fan constants (A , B) are used to solve for the damper positioning coefficients (K_{1d} , K_{2d} , K_{3d}) and the static pressures (P_{4d} , P_{5d} , P_{6d}) corresponding to design flow. To solve for these variables, design airflows are entered in the computer program along with the system characteristics. Design airflow values for branches 1, 2, and 3 are:

$$Q_{1d} = 1500 \text{ cfm}$$

$$Q_{2d} = 1000 \text{ cfm}$$

$$\text{and } Q_{3d} = 1000 \text{ cfm,}$$

for a total design airflow of 3500 cfm. After entering the given parameters and running the 6 equations for design variables (Section 3.1.2) in MATLAB, the design static pressures and damper positioning coefficients are as follow:

Design Static Pressures	Design Positioning Coefficients
$P_{4d} = 0.3417 \text{ psi}$	$K_{1d} = 1.1151 \times 10^{-7} \frac{\text{psi}}{\text{cmf}^2}$
$P_{5d} = 0.2509 \text{ psi}$	$K_{2d} = 1.8466 \times 10^{-7} \frac{\text{psi}}{\text{cmf}^2}$
$P_{6d} = 0.1847 \text{ psi}$	$K_{3d} = 1.8466 \times 10^{-7} \frac{\text{psi}}{\text{cmf}^2}$

Like in the determining system characteristics step (section 4.1.1), a check can now be made to see if these results make sense. Looking at the design damper positioning coefficients (K_{1d} , K_{2d} , and K_{3d}), it can be noted that their value is higher than the fully open damper coefficient values (K_1 , K_2 , and K_3). This is due to the fact that the air flowrate is lower when running at design specifications when the dampers are not completely open, than when running with all dampers fully open. As a general rule, we can conclude that as the flow loss coefficients go up, the overall flowrate goes down.

A second check to be made, is to look at the static pressure exiting the fan (P_{4d}). When determining system characteristics, the system ran at the static pressures of 0.28 psi and 0.35 psi with corresponding flowrates of 3870.9 cfm and 3456.7 cfm for the first and the second pass, respectively. From equation 2.11, the static pressure is inversely proportional to the flowrate squared. Since the overall design flowrate is 3500 cfm, one expects the system to run at design static pressure greater than 0.28 psi, and a little less than 0.35 psi. At a design static pressure of 0.3417 psi, the result seems to be correct.

4.1.2.2 System Curve at Design Flow

A system curve is also made for design variables. Because the fan curve stays constant during the entire balancing procedure, only the system curve is developed for the system running at design flow. The equation for the system curve at design flow is similar to equation 4.2:

$$P_{4d}' = P_{4d} \left(\frac{Q_{45d}'}{Q_{45d}} \right)^2 \quad (4.4)$$

The only difference in the two equations is that design variables are used in this case. Looking at the system curve equation, the system at design flow has an operating point of $P_{4d} = 0.3417$ psi and $Q_{45d} = 3500$ psi with the slope of the curve being the static pressure P_{4d} . The system curve at design flow and the fan curve intersect can be seen in Figure 4.3.

Fan Curve and System Curve at Design Flow Matching

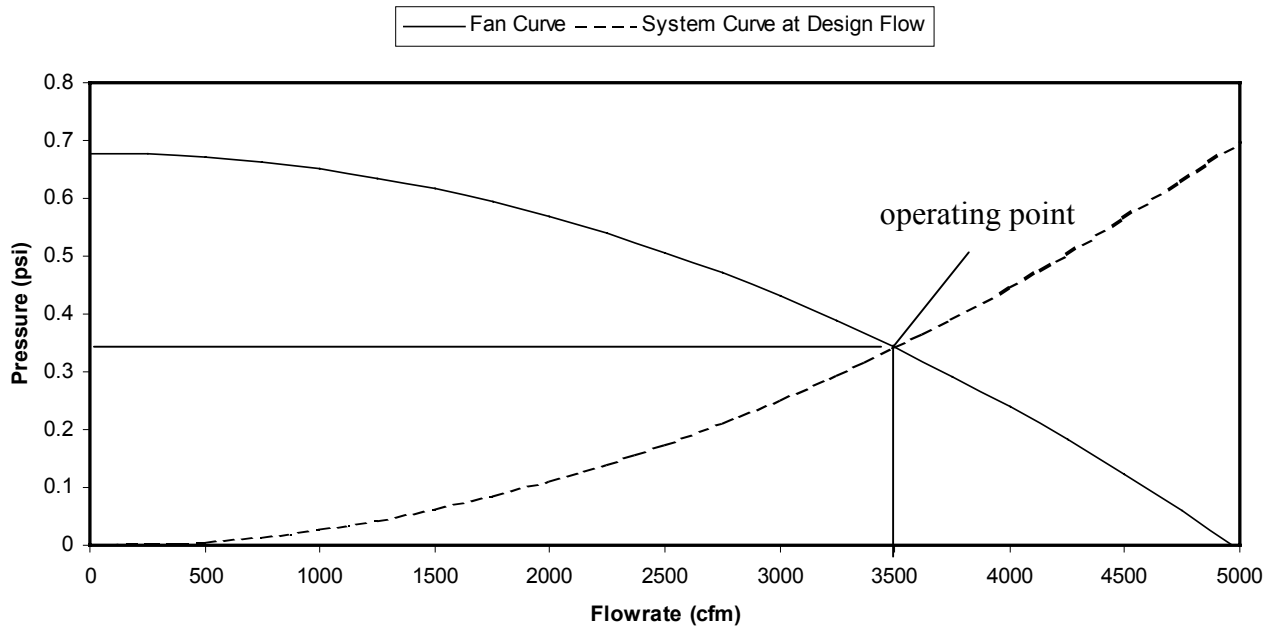


Figure 4.3

4.1.3 Solving for Adjusted Flows at the Terminal Outlets

In solving for adjusted flows at the terminal outlets, the fully open damper coefficients and the damper positioning coefficients are required. The first step in the process, is to adjust the damper in branch number 1 so that the flowrate will reach a calculated value (Q_{1f}). This is done by entering the values for K_{1d} , K_2 , and K_3 along with the fan constants A and B into the MATLAB program. After entering these values and running the 6 equations from section 3.1.3.1, the adjusted flowrate for the first branch is:

$$Q_{1f} = 1320.1 \text{ cfm.}$$

Looking at the numerical value for Q_{1f} , it can be seen that the flowrate through branch number 1 is adjusted to a value lower than its design flow. This makes sense, since after the other two dampers are adjusted (by slightly closing them), the flowrate

through branch 1 will increase. After the second and third step in the flow adjusting process, Q_{1f} should increase to 1500 cfm.

The second step in the process is to adjust the damper on branch number 2 to achieve its calculated flowrate Q_{2f} . Branches 1 and 2 are set at their design positioning coefficients and branch number 3 is set at its fully open flow coefficient. After entering the values for K_{1d} , K_{2d} , and K_3 into the MATLAB program and running the 6 equations from section 3.1.3.2, the adjusted flowrate for the second branch is:

$$Q_{2f} = 806.54 \text{ cfm}$$

Similar to the first branch, the flowrate through branch number 2 is adjusted to a value that is less than its design flow of 1000 cfm. Like on branch number 1, the flowrate through branch 2 will increase when the damper on branch 3 is adjusted.

The third step in the process is to adjust the damper on branch number 3 to its design flowrate of $Q_{3d} = 1000$ cfm. After this flowrate is adjusted, the flows through branches 1 and 2 will revert back to their design flows.

4.2 Uncertainty Analysis

Uncertainty in measurements comes from errors in instruments when flowrates and pressures are measured. Most instruments predict accurate measurements with a tolerance of less than ± 1 % (Kuzara, 1981). In balancing the system, measurements are required to determine the system characteristics. Each of these measurements includes some uncertainty, and these uncertainties add up to create an uncertainty in the final result.

The calculated result y , the dependent variable, is a function of several independent measured variables (x_1, x_2, \dots, x_n). For example, a flow coefficient (y) is a function of pressure (x_1) and flowrate (x_2). Each measured variable has some calculated uncertainty (u_1, u_2, \dots, u_n). These uncertainties lead to a calculated uncertainty in y , which is called u_y . If it is assumed that each uncertainty is small enough, then a first-order Taylor expansion of $y(x_1, x_2, \dots, x_n)$ provides a reasonable approximation:

$$y(x_1 + u_1, x_2 + u_2, \dots, x_n + u_n) \approx y(x_1, x_2, \dots, x_n) + \frac{\partial y}{\partial x_1} u_1 + \frac{\partial y}{\partial x_2} u_2 + \dots + \frac{\partial y}{\partial x_n} u_n. \quad (4.6)$$

Using this approximation, y is a linear function of the independent variables, and the statistical theorem for linear functions (Beckwith et. al, 1993) can be used to come up with the equation

$$u_y = \sqrt{\left(\frac{\partial y}{\partial x_1} u_1\right)^2 + \left(\frac{\partial y}{\partial x_2} u_2\right)^2 + \dots + \left(\frac{\partial y}{\partial x_n} u_n\right)^2} \quad (4.7)$$

For this project, calculating the uncertainty in the dependent variable is a three-step process. A numerical experiment to calculate the percentage change in y for a 1 % change in $x \left(\frac{\partial y}{\partial x}\right)$ is undertaken. First, the independent variables are changed by a small percentage. The changes in dependent variables are calculated for this 1 % change, and their value is recorded. Second, for a dependent variable, $\frac{\partial y}{\partial x}$ is calculated for each change in independent variable. And finally, the total uncertainty in the dependent variable is determined by taking the root mean squared of every $\frac{\partial y}{\partial x}$ calculated.

4.2.1 Uncertainties in System Characteristics

This section analyzes the uncertainty in the results of the system characteristics (A , B , K_{45} , and K_{56}) and the fully open damper flow coefficients (K_1 , K_2 , and K_3) as the flowrates (Q_{1a} , Q_{2a} , Q_{3a} , Q_{1b} , and Q_{2b}) and the fan exit pressures (P_{4a} , and P_{4b}) change by 1%. The independent variables are the flowrates and the fan exit pressures. The dependent variables are the system characteristics and the fully open damper coefficients.

The first step in calculating the uncertainty in y is to record the changes in the dependent variable as the independent variable is changed. This is done by changing one of the independent variables by 1% while all the other independent variables are left constant, and then recording the change in the dependent variables. For example, the measured flowrate through branch number 1 during the first pass (Q_{1a}) is changed from 1500 cfm to 1515 cfm. With all of the other independent variables held constant, the system characteristics and the fully open damper flow coefficients are calculated.

Then, the next independent variable is changed by 1% while all other variables (including the first one) are left constant. The change in dependent variables is then recorded. The same procedure is then repeated with the rest of the independent variables. Tables 4.2, 4.3, 4.4 and 4.5 show the independent variables on the farthest left column with their values before and after they are increased by 1%. The dependent variables are shown on the top row with their numerical values on the columns below them.

Table 4.2. Changes in Fan Constants after Changing Independent Variables by 1%

x	Regular Value	Value 1% high	Old A (psi)	New A (psi)	Old B (psi/cfm ²)	New B (psi/cfm ²)
Q_{1a} (cfm)	1500	1515	0.6678	0.6638	$-2.74e^{-8}$	$-2.63e^{-8}$
Q_{1b} (cfm)	1846.6	1865.066	0.6678	0.6988	$-2.74e^{-8}$	$-2.89e^{-8}$
Q_{2a} (cfm)	1000	1010	0.6678	0.6683	$-2.74e^{-8}$	$-2.66e^{-8}$
Q_{2b} (cfm)	1610.1	1626.201	0.6678	0.6960	$-2.74e^{-8}$	$-2.87e^{-8}$
Q_{3a} (cfm)	1307.9	1320.979	0.6678	0.6614	$-2.74e^{-8}$	$-2.61e^{-8}$
P_{4a} (psi)	0.28	0.2828	0.6678	0.6647	$-2.74e^{-8}$	$-2.63e^{-8}$
P_{4b} (psi)	0.35	0.3535	0.6678	0.6977	$-2.74e^{-8}$	$-2.88e^{-8}$

Table 4.3. Changes in Fully-Open Damper Coefficients after Changing Independent Variables by 1%

x	Regular Value	Value 1% high	Old K_1 (psi/cfm ²)	New K_1 (psi/cfm ²)	Old K_2 (psi/cfm ²)	New K_2 (psi/cfm ²)	Old K_3 (psi/cfm ²)	New K_3 (psi/cfm ²)
Q_{1a} (cfm)	1500	1515	$7.67e^{-8}$	$7.90e^{-8}$	$8.43e^{-8}$	$8.60e^{-8}$	$4.93e^{-8}$	$5.03e^{-8}$
Q_{1b} (cfm)	1846.6	1865.066	$7.67e^{-8}$	$7.28e^{-8}$	$8.43e^{-8}$	$8.24e^{-8}$	$4.93e^{-8}$	$4.82e^{-8}$
Q_{2a} (cfm)	1000	1010	$7.67e^{-8}$	$7.70e^{-8}$	$8.43e^{-8}$	$8.52e^{-8}$	$4.93e^{-8}$	$5.08e^{-8}$
Q_{2b} (cfm)	1610.1	1626.201	$7.67e^{-8}$	$7.61e^{-8}$	$8.43e^{-8}$	$8.13e^{-8}$	$4.93e^{-8}$	$4.75e^{-8}$
Q_{3a} (cfm)	1307.9	1320.979	$7.67e^{-8}$	$7.66e^{-8}$	$8.43e^{-8}$	$8.44e^{-8}$	$4.93e^{-8}$	$4.84e^{-8}$
P_{4a} (psi)	0.28	0.2828	$7.67e^{-8}$	$7.52e^{-8}$	$8.43e^{-8}$	$8.26e^{-8}$	$4.93e^{-8}$	$4.83e^{-8}$
P_{4b} (psi)	0.35	0.3535	$7.67e^{-8}$	$7.89e^{-8}$	$8.43e^{-8}$	$8.67e^{-8}$	$4.93e^{-8}$	$5.07e^{-8}$

Table 4.4 Changes in the System Flow Coefficients after Changing Independent Variables by 1%

<i>x</i>	Regular Value	Value 1% high	Old K_{45} (psi/cfm²)	New K_{45} (psi/cfm²)	Old K_{56} (psi/cfm²)	New K_{56} (psi/cfm²)
Q_{1a} (cfm)	1500	1515	7.42e-9	6.76e ⁻⁹	1.66e ⁻⁸	1.79e ⁻⁸
Q_{1b} (cfm)	1846.6	1865.066	7.42e-9	8.02e ⁻⁹	1.66e ⁻⁸	1.53e ⁻⁸
Q_{2a} (cfm)	1000	1010	7.42e-9	7.33e ⁻⁹	1.66e ⁻⁸	1.61e ⁻⁸
Q_{2b} (cfm)	1610.1	1626.201	7.42e-9	7.50e ⁻⁹	1.66e ⁻⁸	1.69e ⁻⁸
Q_{3a} (cfm)	1307.9	1320.979	7.42e-9	7.44e ⁻⁹	1.66e ⁻⁸	1.63e ⁻⁸
P_{4a} (psi)	0.28	0.2828	7.42e-9	7.84e ⁻⁹	1.66e ⁻⁸	1.62e ⁻⁸
P_{4b} (psi)	0.35	0.3535	7.42e-9	7.07e ⁻⁹	1.66e ⁻⁸	1.70e ⁻⁸

Table 4.5: Change in Junction Static Pressures after Changing Independent Variables by 1%

<i>x</i>	Regular Value	Value 1% high	Old P_{5a} (psi)	New P_{5a} (psi)	Old P_{5b} (psi)	New P_{5b} (psi)	Old P_{6a} (psi)	New P_{6a} (psi)	Old P_{6b} (psi)	New P_{6b} (psi)
Q_{1a} (cfm)	1500	1515	0.1725	0.1812	0.2614	0.2692	0.0843	0.086	0.2185	0.2229
Q_{1b} (cfm)	1846.6	1865.066	0.1725	0.1638	0.2614	0.2532	0.0843	0.0824	0.2185	0.2136
Q_{2a} (cfm)	1000	1010	0.1725	0.1731	0.2614	0.2624	0.0843	0.0869	0.2185	0.2208
Q_{2b} (cfm)	1610.1	1626.201	0.1725	0.1713	0.2614	0.2596	0.0843	0.0813	0.2185	0.2149
Q_{3a} (cfm)	1307.9	1320.979	0.1725	0.1723	0.2614	0.2611	0.0843	0.0844	0.2185	0.2188
P_{4a} (psi)	0.28	0.2828	0.1725	0.1691	0.2614	0.2563	0.0843	0.0826	0.2185	0.2142
P_{4b} (psi)	0.35	0.3535	0.1725	0.1775	0.2614	0.2691	0.0843	0.0867	0.2185	0.2249

Now that the change in the dependent variables for every independent variable is known, the next step is to calculate the fractional change in y for a 1% change in x . The equation is as follows:

$$\frac{\Delta y}{y} = \frac{y_{new} - y_{old}}{y_{new}} \quad (4.8)$$

where y_{new} = dependent variable value after 1% change,

and y_{old} = dependent variable value before 1% change.

The fractional change in y is calculated for each of the dependent variables and their absolute values are recorded. The root mean squared is taken and shown below each dependent variable (See Tables 4.6 and 4.7).

**Table 4.6 Fractional Change in Fan Constants and Fully-Open Damper
Coefficients for a 1% Change in x**

x	$\frac{\Delta A}{A}$	$\frac{\Delta B}{B}$	$\frac{\Delta K_1}{K_1}$	$\frac{\Delta K_2}{K_2}$	$\frac{\Delta K_3}{K_3}$
Q_{1a}	0.0060	0.0429	0.0300	0.0202	0.0202
Q_{1b}	0.0464	0.0528	0.0504	0.0223	0.0223
Q_{2a}	0.0007	0.0290	0.0039	0.0106	0.0309
Q_{2b}	0.0422	0.0457	0.0070	0.0358	0.0358
Q_{3a}	0.0096	0.0500	0.0011	0.0016	0.0181
P_{4a}	0.0046	0.0400	0.0193	0.0193	0.0193
P_{4b}	0.0448	0.0500	0.0294	0.0293	0.0294
RMS %	7.81	11.90	6.89	5.95	6.86

**Table 4.7 Fractional Change in Flow Coefficients and Static Pressures for a 1%
Change in x**

x	$\frac{\Delta K_{45}}{K_{45}}$	$\frac{\Delta K_{56}}{K_{56}}$	$\frac{\Delta P_{5a}}{P_{5a}}$	$\frac{\Delta P_{5b}}{P_{5b}}$	$\frac{\Delta P_{6a}}{P_{6a}}$	$\frac{\Delta P_{6b}}{P_{6b}}$
Q_{1a}	0.0886	0.0798	0.0504	0.0298	0.0202	0.0201
Q_{1b}	0.0809	0.0773	0.0504	0.0314	0.0225	0.0224
Q_{2a}	0.0114	0.0304	0.0035	0.0038	0.0308	0.0105
Q_{2b}	0.0112	0.0205	0.0070	0.0069	0.0356	0.0165
Q_{3a}	0.0032	0.0149	0.0012	0.0011	0.0012	0.0014
P_{4a}	0.0571	0.0194	0.0197	0.0195	0.0202	0.0197
P_{4b}	0.0471	0.0293	0.0290	0.0295	0.0285	0.0293
RMS %	14.19	12.31	7.99	5.64	6.60	5.04

To determine the uncertainty in the dependent variable as a percentage of the actual numerical value, the root mean squared (RMS) of the fractional changes $\frac{\Delta Y}{Y}$ is calculated. This will determine the percentage of change in the output is for a 1% change in the input. In order to do this, it is assumed that the errors caused by the change in independent variables are independent of each other. The RMS % is calculated with the following equation:

$$RMS\% = \sqrt{\left(\frac{\Delta Y_1}{Y_1}\right)^2 + \left(\frac{\Delta Y_2}{Y_2}\right)^2 + \dots + \left(\frac{\Delta Y_n}{Y_n}\right)^2} \times 100 \quad (4.9)$$

where n represents the number of independent variables. After calculating the RMS % for each of the dependent variables, a graph is made to analyze these errors. Figure 4.4 shows a graph of the RMS % uncertainty in dependent variable for 1% change in measurements.

RMS % Uncertainty in Dependent Variable for a 1% Uncertainty in Measurements

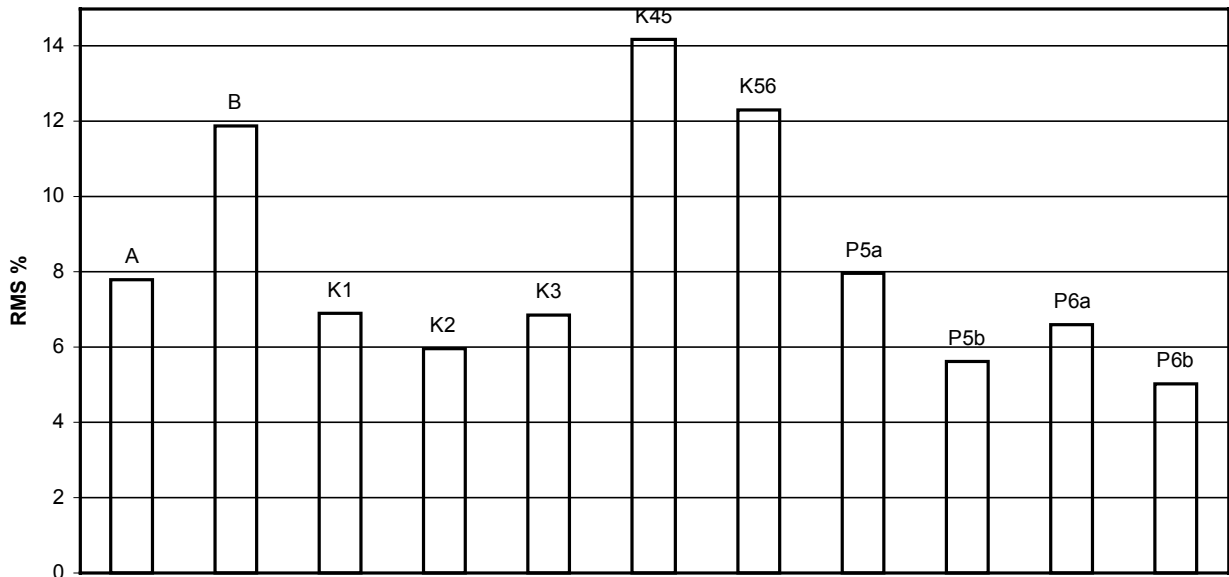


Figure 4.4

Analyzing Figure 4.4, it can be seen that the fully open damper coefficients are less affected by measurement errors in the independent variables than the main ducts flow loss coefficients. Moreover, the flow coefficient closest to the fan (K_{45}) has a higher RMS % than the flow coefficient farthest from the fan. Furthermore, the static pressures closest to the fan (P_{5a} , P_{5b}) have a greater uncertainty error in their results than the other static pressures (P_{5a} , P_{5b}). From these results, it can be concluded that variables that are closer to the fan have a greater uncertainty in their results from measurement errors.

4.2.2 Uncertainties in Design Airflow Variables

This section analyzes the uncertainty in dependent variables as the independent variables change by 1% for design flow variables. The dependent variables are the

damper positioning coefficients (K_{1d} , K_{2d} , and K_{3d}) and the design static pressures (P_{4d} , P_{5d} , and P_{6d}), and the independent variables remain unchanged since the initial flowrates and the static pressure exiting the fan are the only variables being measured.

Uncertainties in design variables are calculated with the same equations (equations 4.8 and 4.9) as the uncertainties in the system characteristics and fully open damper coefficients. The first step, is to determine the change in design variables as the independent variables change by 1% (see Tables 4.8 and 4.9).

Table 4.8 Change in Design Damper Positioning Coefficients after Changing Independent Variables by 1%

x	Regular Value	Value 1% high	Old K_{1d} (psi/cfm ²)	New K_{1d} (psi/cfm ²)	Old K_{2d} (psi/cfm ²)	New K_{2d} (psi/cfm ²)	Old K_{3d} (psi/cfm ²)	New K_{3d} (psi/cfm ²)
Q_{1a} (cfm)	1500	1515	$1.12e^{-7}$	$1.15e^{-7}$	$1.85e^{-7}$	$1.88e^{-7}$	$1.85e^{-7}$	$1.88e^{-7}$
Q_{1b} (cfm)	1846.6	1865.066	$1.12e^{-7}$	$1.10e^{-7}$	$1.85e^{-7}$	$1.86e^{-7}$	$1.85e^{-7}$	$1.86e^{-7}$
Q_{2a} (cfm)	1000	1010	$1.12e^{-7}$	$1.12e^{-7}$	$1.85e^{-7}$	$1.88e^{-7}$	$1.85e^{-7}$	$1.88e^{-7}$
Q_{2b} (cfm)	1610.1	1626.201	$1.12e^{-7}$	$1.12e^{-7}$	$1.85e^{-7}$	$1.85e^{-7}$	$1.85e^{-7}$	$1.85e^{-7}$
Q_{3a} (cfm)	1307.9	1320.979	$1.12e^{-7}$	$1.12e^{-7}$	$1.85e^{-7}$	$1.87e^{-7}$	$1.85e^{-7}$	$1.87e^{-7}$
P_{4a} (psi)	0.28	0.2828	$1.12e^{-7}$	$1.09e^{-7}$	$1.85e^{-7}$	$1.81e^{-7}$	$1.85e^{-7}$	$1.81e^{-7}$
P_{4b} (psi)	0.35	0.3535	$1.12e^{-7}$	$1.15e^{-7}$	$1.85e^{-7}$	$1.90e^{-7}$	$1.85e^{-7}$	$1.90e^{-7}$

Table 4.9 Change in Design Static Pressures after Changing Independent Variables by 1%

x	Regular Value	Value 1% high	Old P_{4d} (psi)	New P_{4d} (psi)	Old P_{5d} (psi)	New P_{5d} (psi)	Old P_{6d} (psi)	New P_{6d} (psi)
Q_{1a} (cfm)	1500	1515	0.3417	0.3421	0.2509	0.2593	0.1847	0.1878
Q_{1b} (cfm)	1846.6	1865.066	0.3417	0.3450	0.2509	0.2468	0.1847	0.1857
Q_{2a} (cfm)	1000	1010	0.3417	0.3420	0.2509	0.2522	0.1847	0.1879
Q_{2b} (cfm)	1610.1	1626.201	0.3417	0.3446	0.2509	0.2527	0.1847	0.1851
Q_{3a} (cfm)	1307.9	1320.979	0.3417	0.3420	0.2509	0.2526	0.1847	0.1869
P_{4a} (psi)	0.28	0.2828	0.3417	0.3421	0.2509	0.2460	0.1847	0.1811
P_{4b} (psi)	0.35	0.3535	0.3417	0.3448	0.2509	0.2583	0.1847	0.1901

Next, fractional change in design variables is calculated using equation 4.8 and their absolute value is recorded. The RMS % is calculated with equation 4.9 and shown in Table 4.10.

Table 4.10. Fractional Change in Damper Positioning Coefficients and Static Pressures at Design Flow for a 1% Change in x

x	$\frac{\Delta K_{1d}}{K_{1d}}$	$\frac{\Delta K_{2d}}{K_{2d}}$	$\frac{\Delta K_{3d}}{K_{3d}}$	$\frac{\Delta P_{4d}}{P_{4d}}$	$\frac{\Delta P_{5d}}{P_{5d}}$	$\frac{\Delta P_{6d}}{P_{6d}}$
Q_{1a}	0.0334	0.0165	0.0165	0.0012	0.0335	0.0168
Q_{1b}	0.0163	0.0056	0.0056	0.0097	0.0163	0.0054
Q_{2a}	0.0051	0.0178	0.0178	0.0009	0.0052	0.0173
Q_{2b}	0.0072	0.0024	0.0024	0.0085	0.0072	0.0022
Q_{3a}	0.0066	0.0121	0.0121	0.0009	0.0068	0.0119
P_{4a}	0.0194	0.0193	0.0193	0.0012	0.0195	0.0195
P_{4b}	0.0293	0.0294	0.0294	0.0091	0.0295	0.0292
RMS%	5.24	4.48	4.48	1.59	4.37	4.46

Finally, the RMS % for design variables is plotted in Figure 4.5.

RMS % Uncertainty in Design Variables for a 1% Uncertainty in Measurements

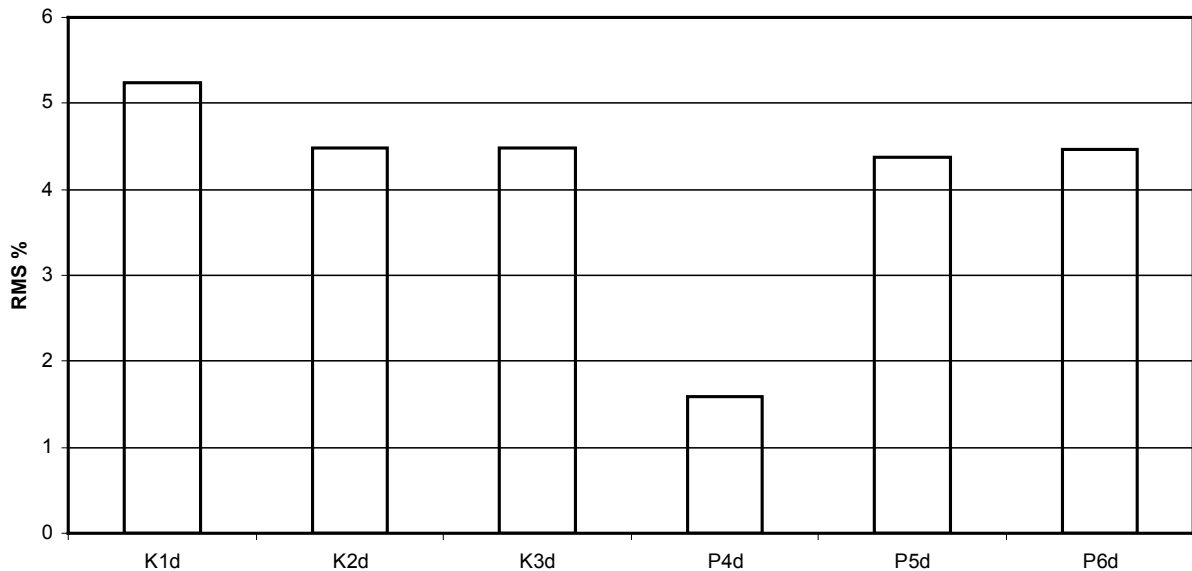


Figure 4.5

From Figure 4.5 it can be seen that expect for the static pressure exiting the fan (P_{4d}), all design variable have approximately the same RMS %. One possible explanation, is that P_{4d} is the only variable in the design system matrix that can be determined without relying on the other dependent variables. If you look at the equation for calculating P_{4d} :

$$P_{4d} = A - BQ_{45d}^2,$$

you will notice that all the other variables are already known. Thus, P_{4d} is easily determined and is less prone to calculating errors.

In addition, it can also be seen that the RMS % in design variables is less than the RMS % for the first set of dependent variables (system characteristics, fully open damper coefficients, and static pressures). A reason for this, is that the matrix used to

solve for system characteristics and damper coefficients is larger than the design system matrix. This could mean, that the first matrix has a larger condition number, which is the ratio of the largest singular value in the matrix to the smallest. For a system of linear equations, the condition number of a matrix measures the sensitivity of the solution to numerical errors from variables. The condition number for the first matrix is 1.3837×10^8 and the condition number for the second matrix is 6.75×10^5 . Thus, one would expect the first matrix to be more prone to error than the second one. Furthermore, some of the errors from the first step could be canceling each other out when they are passed on to the next step.

4.2.3 Uncertainties in Adjusted Flows

This section covers the uncertainty in adjusted flows after changing the independent variables by 1%. The independent variables are again the measured flowrates and the measured static pressures exiting the fan. The dependent variables are the adjusted flowrates (Q_{1f} , Q_{2f}) at branches 1 and 2. The flowrate at branch 3 is not a dependent variable since it is not a calculated value. Its value is design flow (1000 cfm).

The procedure for calculating the uncertainty is the same as the previous two procedures. Equations 4.8 and 4.9 are used to calculate a fractional change by which an RMS % of error can be calculated for the dependent variable. The first step, is to find the change in dependent variables for a 1% change in independent variables (See Table 4.11).

Table 4.11 Change in Adjusted Flowrates at Balancing Points after Changing Independent Variables by 1%

x	Correct Value	Value 1% high	Old Q_{1f} (cfm)	New Q_{1f} (cfm)	Old Q_{2f} (cfm)	New Q_{2f} (cfm)
Q_{1a} (cfm)	1500	1515	1320.1	1329.4	806.5355	808.0818
Q_{1b} (cfm)	1846.6	1865.066	1320.1	1362.1	806.5355	800.5053
Q_{2a} (cfm)	1000	1010	1320.1	1320.0	806.5355	812.1499
Q_{2b} (cfm)	1610.1	1626.201	1320.1	1303.1	806.5355	796.2659
Q_{3a} (cfm)	1307.9	1320.979	1320.1	1308.3	806.5355	804.2747
P_{4a} (psi)	0.28	0.2828	1320.1	1331.1	806.5355	808.6045
P_{4b} (psi)	0.35	0.3535	1320.1	1317.5	806.5355	804.8686

After determining the change, the fractional change and the RMS % for each dependent variable is calculated with equations 4.8 and 4.9, respectively. The absolute values are taken and recorded (See Table 4.12).

Table 4.12 Fractional Change in Adjusted Flows for a 1% Change in x

x	Uncertainty in Q_{1f}	Uncertainty in Q_{2f}
Q_{1a}	0.007045	0.001917
Q_{1b}	0.031816	0.007477
Q_{2a}	$7.58e^{-5}$	0.006961
Q_{2b}	0.012878	0.012733
Q_{3a}	0.008939	0.002803
P_{4a}	0.008333	0.002565
P_{4b}	0.001970	0.002067
RMS %	2.05	1.54

Finally the RMS % in the dependent variable is plotted on a graph (see Figure 4.6).

RMS % Uncertainty in Required Flowrates for a 1% Uncertainty in Measurements

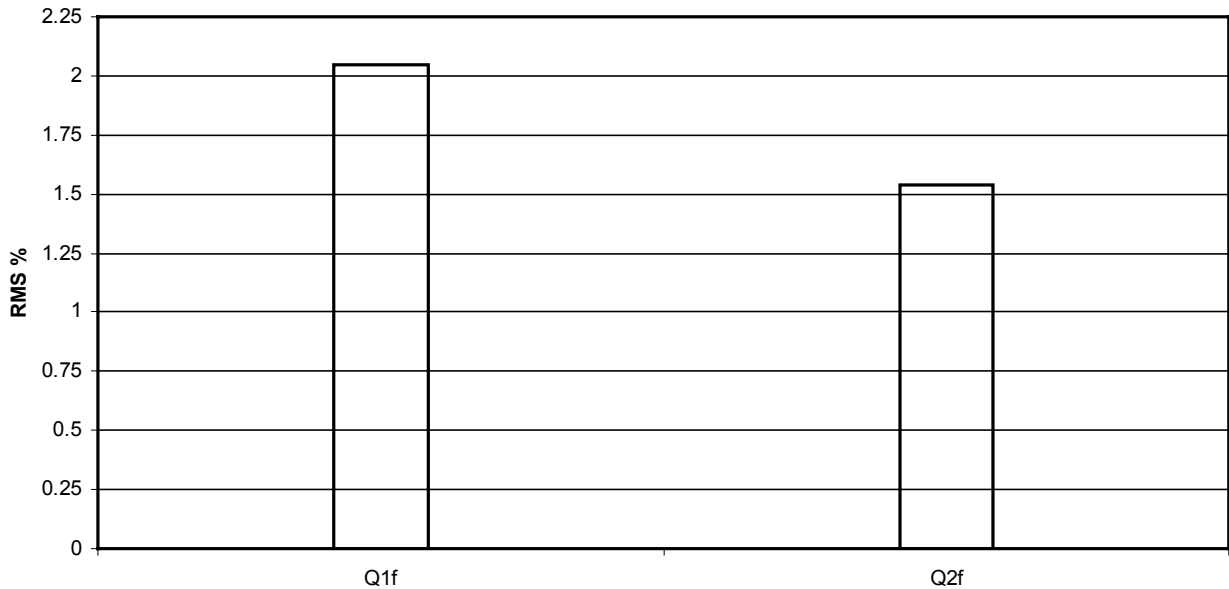


Figure 4.6

From Figure 4.6 it can be seen that the RMS % for the adjusted flows are less than the RMS % for design variables or for the first set of dependent variables. When calculating the adjusted flows at the balancing points, a non-linear system of equations is solved for by using a least square method. Since the equation being solved is a vector of several unknowns and not a matrix, the solution is less prone to errors. Additionally, some of the errors from the previous step could have canceled each other even more when passed on to this step of the calculations.

By looking at Figure 4.6 it can be concluded that a 1% uncertainty in measurements produces a 2% change in the adjusted flows. This percentage of error in the flowrate is well below the balancing cushion of $\pm 10\%$ of design air that is specified in the literature (SMACNA, 1993). Another point to notice is that when these flows are

adjusted, there could be a possible additional 1% error in measurement from either human error or instrument error, but it is not very significant since the adjusted flowrate would still be within $\pm 10\%$ of design airflow.

4.2.3 Final Measurement Error

The final measurement error comes from uncertainty in measuring devices and human error when the dampers are adjusted in the last step of the balancing procedure. The effects of this error will be first taken into account by assuming that all other errors are zero. With this assumption, an RMS % of the errors in the final adjusted flowrates can be evaluated for a 1% change in Q_{1f} , Q_{2f} , and Q_{3f} .

Table 4.13 shows the percentage change in the adjusted flows for a 1% change in adjusted flows. The RMS % is shown on the right hand side.

Table 4.13 Percentage Change in Adjusted Flows for 1% Change in Adjusted Flows

	Q_{1f} 1% high	Q_{2f} 1% high	Q_{3f} 1% high	RMS %
Error in Q_{1f} (%)	1.00	2.37	1.05	2.78
Error in Q_{2f} (%)	0.34	1.00	1.32	1.69
Error in Q_{3f} (%)	0.98	0.85	1.00	1.64

It can be seen from Table 4.11 that the final measurement errors by themselves are relatively small compared to the allowable airflow error of $\pm 10\%$. As a worst case scenario, if the errors were to be added to the uncertainties in the adjusted flow determined in section 4.2.3, the final adjusted flowrate errors would be:

Q_{1f} error	Q_{2f} error	Q_{3f} error
4.83 %	3.23 %	1.64 %

4.3 Other Sources of Error

Besides the uncertainty in measurements, there are other sources of errors that were not addressed in this thesis, but suggest avenues for further research. One other source of error is airflow leakage in ducts and in closed dampers. This error affects the flowrate measurements through ducts or terminal outlets and can aid in giving inaccurate results. Another source of error for this analysis is the use of the fan curve equation (2.11). Since this equation is just an approximation and is not exact, the fan constants A and B are used to determine an approximate fan exit pressure, and thus bring about an error in the final answer.

4.4 Graphical User Interface Results

The SIMULINK model results are very sensitive to measurement error because of the SIMULINK solving process. To solve for the system characteristics and damper positioning coefficients, the minimization program requires a guess that starts close to the answer.

The model being solved, is a two-diffuser air distribution system like the one shown here in Figure 4.7.

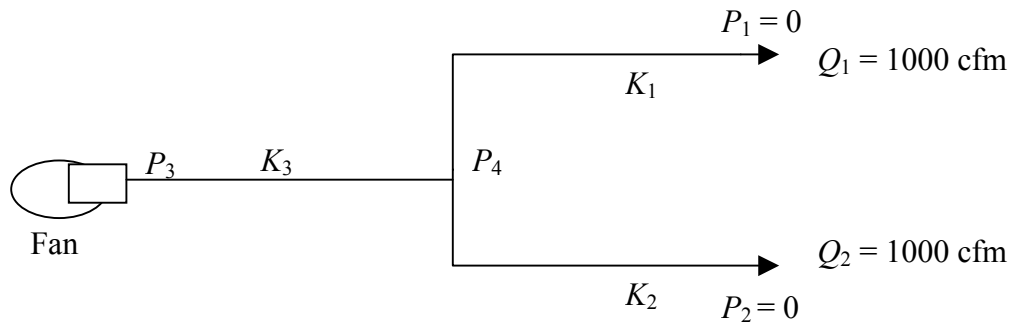


Figure 4.7 Two- Diffuser Air Distribution System

The purpose of the model is to achieve a design airflow of $Q_1 = 1000$ cfm and $Q_2 = 1000$ cfm.

4.4.1 System Characteristics Results

In solving system for the flow coefficient (K_3) and the fully open damper coefficients (K_1, K_2), the variables entered into SIMULINK are: the measured flowrates ($Q_{1a}, Q_{1b}, Q_{2a},$ and Q_{2b}) and the fan constants. The fan constants A and B were determined by running this example in a MATLAB program with the fan exit pressure given, and solving for all system characteristics with a procedure similar to the three-diffuser system. For other cases, the fan constants can be determined from published performance data on the fan. The system is assumed to be running with both dampers open for the first pass, and with one damper closed for the second pass. The given variables are as follow:

Measured Flowrates	
$Q_{1a} = 1100 \text{ cfm}$	$Q_{1b} = 1980 \text{ cfm}$
$Q_{2a} = 1100 \text{ cfm}$	$Q_{2b} = 0 \text{ cfm}$
Fan Constants	
$A = 2.8816 \text{ psi}$	$B = -5.4371 \times 10^{-7} \frac{\text{psi}}{\text{cfm}^2}$

After entering the given variables in the SIMULINK model, a minimization routine in MATLAB is run to determine the K_1 , K_2 , and K_3 values that will give the smallest error. To do this, the MATLAB *fminsearch* command is used, and the results are as follow:

$$K_1 = 1.861 \times 10^{-7} \frac{\text{psi}}{\text{cfm}^2}$$

$$K_2 = 1.832 \times 10^{-7} \frac{\text{psi}}{\text{cfm}^2}$$

$$K_3 = 5.1 \times 10^{-9} \frac{\text{psi}}{\text{cfm}^2},$$

and the minimum value for the error is 0.0763. To get a more accurate answer, the error should be closer to zero. The maximum number of iterations allowed by this computer was used to come up with this error. If a ‘super computer’ is used, it is possible to get the error closer to zero, but for this project’s purpose, the answer above is close enough.

4.4.2 Damper Positioning Coefficients Results

The damper positioning coefficients are determined by entering the K_3 value along with design flowrates for branches 1 and 2 into the SIMULINK model, and using MATLAB to perform an error minimization routine. The values for these variables are:

$$K_3 = 5.1 \times 10^{-9} \frac{psi}{cfm^2}$$

$$Q_{1d} = 1000 \text{ cfm}$$

$$Q_{2d} = 1000 \text{ cfm.}$$

After entering these constants and running the simulation, the results are:

$$K_{1d} = 6.871 \times 10^{-7} \frac{psi}{cfm^2}$$

$$K_{2d} = 6.871 \times 10^{-7} \frac{psi}{cfm^2},$$

with an error of 9.354×10^{-7} . The error is much closer to zero than in the step above. One reason why this iterative procedure was more successful, is that only two damper coefficients are solved for instead of three, making the minimization process easier. Another reason, is that MATLAB is doing the minimization for only one model instead of two, making the iteration procedure quicker. Therefore, this step gives an answer less susceptible to error than the previous step.

4.4.3 Adjusted Flowrates Results

The adjusted flowrate on branch 2 is determined by entering the fully-open damper coefficient value for branch 1 K_1 , the design positioning coefficient value for branch 2 K_{2d} , and the flow coefficient value K_3 into the SIMULINK model, and using SIMULINK

to determine the flowrate. After the flowrate on branch 2 is adjusted, the flowrate on branch 1 is adjusted to design value. This is a direct calculation and no error minimization routine is needed. After entering these constants into the SIMULINK model, the result is:

$$Q_{2f} = 732.2 \text{ cfm.}$$

This value is lower than its design value of 1000 cfm. After the flowrate on branch 1 is adjusted to its design value, the flowrate through branch 2 will increase to 1000 cfm. Since this step does not require an iterative procedure, it is less prone to errors than either of the previous two.

Chapter 5

Conclusion and Recommendations

The primary goal of this research project was to determine a way to balance an air distribution system with only a few measurements and no iterations. The non-iterative balancing procedure was done by using three steps:

1. Determining system characteristics from airflow and fan exit-pressure measurements.
2. Determining design damper positioning coefficients from computer model.
3. Determining the flowrate to which each terminal outlet should be adjusted, taking into account the fact that some outlets had not yet been adjusted.

5.1 Conclusions

Based on the analysis of this thesis, the following conclusions can be drawn from non-iteratively balancing an air distribution system:

1. The air distribution system behaves in accordance with fan laws, fan curves, and system curves.
2. Uncertainties in flowrate and pressure measurements closest to the fan are more important than uncertainties in measurements further from the fan.
3. The equations used for the entire balancing procedure are not highly sensitive to measurement errors.

4. An air distribution system can be balanced non-iteratively by making a few pressure and airflow measurements.

5.2 Recommendations

The results displayed in this thesis suggest several paths for future research:

1. An analysis of balancing an air distribution system with cross-junctions should be developed.
2. A graphical user interface model should be developed with a program that does not solve an assumed steady state system in a transient fashion.
3. A study of the effects of adjusting the airflow by either adjusting a volume damper inside a main branch or by adjusting fan speed should be undertaken.
4. An uncertainty analysis with an example of a system with a significantly larger number of terminal outlets should be developed.
5. A method of solving for system characteristics should be examined where more than one outlet damper is closed during a second pass, for a system with a large number of terminal outlets.
6. A study of other sources of errors such as leakage in ducts and terminal outlets should be performed.

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Appendix A

Matlab Codes

A.1 Solving for System Characteristics

```
function[P5a,P5b,P6a,P6b,K45,K1,K56,K2,K3,Af,B] =
```

```
linearsystem3(Q1a,Q1b,Q2a,Q2b,P1,P2,P3a,P3b)
```

```
%This function computes the constants K1,K2a,K2b,K3, the pressure P4 during
```

```
%both runs and the fan constants A and B
```

```
%Parameters given. P1 and P2 are negligible
```

```
Q1a = 1500;
```

```
Q2a = 1000;
```

```
Q3a = 1307.9;
```

```
Q1b = 1846.6;
```

```
Q2b = 1610.6;
```

```
Q3b=0;
```

```
P4a = 0.28;
```

```
P4b = 0.35;
```

```
P1=0;
```

```
P2=0;
```

```
P3=0;
```

$$Q56a=Q2a+Q3a;$$

$$Q45a=Q1a+Q56a;$$

$$Q56b=Q2b+Q3b;$$

$$Q45b=Q1b+Q56b;$$

%11 equations to solve for the 5 constants K1,K2,K3,A,B and pressures P4a and P4b.

%The 11 equations are arranged into an 11x11 matrix A.

$$A=[0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ -(Q3a)^2\ 0\ 0;$$

$$1\ 0\ -1\ 0\ 0\ 0\ 0\ -(Q56a)^2\ 0\ 0\ 0\ 0;$$

$$0\ 1\ 0\ -1\ 0\ 0\ 0\ -(Q56b)^2\ 0\ 0\ 0\ 0;$$

$$0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ -(Q2a)^2\ 0\ 0\ 0;$$

$$0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ -(Q2b)^2\ 0\ 0\ 0;$$

$$1\ 0\ 0\ 0\ 0\ 0\ -(Q1a)^2\ 0\ 0\ 0\ 0\ 0;$$

$$0\ 1\ 0\ 0\ 0\ 0\ 0\ -(Q1b)^2\ 0\ 0\ 0\ 0\ 0;$$

$$1\ 0\ 0\ 0\ 0\ (Q45a)^2\ 0\ 0\ 0\ 0\ 0\ 0;$$

$$0\ 1\ 0\ 0\ 0\ 0\ (Q45b)^2\ 0\ 0\ 0\ 0\ 0\ 0;$$

$$0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ (Q45a)^2;$$

$$0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ (Q45b)^2;];$$

$$\text{condition1}=\text{cond}(A,\text{inf});$$

$$b=[P3\ 0\ 0\ P2\ P2\ P1\ P1\ P4a\ P4b\ P4a\ P4b]';$$

$$x = A \setminus b;$$

P5a=x(1)

P5b=x(2)

P6a=x(3)

P6b=x(4)

K45=x(5)

K1=x(6)

K56=x(7)

K2=x(8)

K3=x(9)

Af=x(10)

B=x(11)

A.2 Solving for Design Damper Positioning Coefficients

```
function [P4d,P5d,K1d,P6d,K2d,K3d] =
```

```
desired3(P5a,P5b,P6a,P6b,K45,K1,K56,K2,K3,Af,B)
```

```
%This function calculates the desired constants K1d and K2d for the given
```

```
%desired flowrates Q1d and Q2d. It also calculates the pressures P4d and P3d at
```

```
%these given flowrates.
```

```
[P5a,P5b,P6a,P6b,K45,K1,K56,K2,K3,Af,B]=feval('linearsystem3');
```

```
%A=1.7466;
```

```
%B=-1.2444e-7;
```

```
%K45=1.6604e-8;
```

```
%K56=1.979297e-8;
```

```
%Given Parameters
```

```
Q1d = 1500;
```

```
Q2d = 1000;
```

```
Q3d = 1000;
```

```
Q56d = Q2d + Q3d;
```

```
Q45d = Q56d + Q1d;
```

```
P1=0;
```

```
P2=0;
```

```
P3=0;
```

```
%A 6x6 matrix is built to linearly solve for K1d,K2d,P3d,and P4d
```

```
A=[1 -1 0 0 0 0;
```

```
0 1 -Q1d^2 0 0 0;
```

```
0 1 0 -1 0 0;
```

```
0 0 0 1 -Q2d^2 0;
```

```
0 0 0 1 0 -Q3d^2;
```

```
1 0 0 0 0 0];
```

```
condition2=cond(A,inf)
```

```
b=[K45*Q45d^2 P1 K56*Q56d^2 P2 P3 Af+B*Q45d^2]';
```

```
xd=A\b;
```

```
P4d=xd(1);
```

```
P5d=xd(2);
```

```
K1d=xd(3);
```

```
P6d=xd(4);
```

```
K2d=xd(5);
```

```
K3d=xd(6);
```

A.3 Solving for Adjusted Flowrates

```
function F=nonlinear3(x)
```

```
%Solving for the flowrate Q1f with dampers 2 and 3 fully open
```

```
[P5a,P5b,P6a,P6b,K45,K1,K56,K2,K3,Af,B]=feval('linearsystem3');
```

```
[P4d,P5d,K1d,P6d,K2d,K3d]=feval('desired3');
```

```
P1=0;
```


P2=0;

P3=0;

F=[x(1) - x(2) - K45*(x(3)+x(4)+x(5))^2;

x(2) - P1 - K1d*x(3)^2;

x(2) - x(6) - K56*(x(4)+x(5))^2;

x(6) - P2 - K2*x(4)^2;

x(6) - P3 - K3*x(5)^2;

x(1) - Af - B*(x(3)+x(4)+x(5))^2;];

`function F2=nonlinear32(x)`

`%Solving for the flowrate Q2f with branch 1 at K1d and branch 3 fully open`

`[P5a,P5b,P6a,P6b,K45,K1,K56,K2,K3,Af,B]=feval('linearsystem3');`

`[P4d,P5d,K1d,P6d,K2d,K3d]=feval('desired3');`

`P1=0;`

`P2=0;`

`P3=0;`

`F2=[x(1) - x(2) - K45*(x(3)+x(4)+x(5))^2;`

```

x(2) - P1 - K1d*x(3)^2;
x(2) - x(6) - K56*(x(4)+x(5))^2;
x(6) - P2 - K2d*x(4)^2;
x(6) - P3 - K3*x(5)^2;
x(1) - Af - B*(x(3)+x(4)+x(5))^2;];

```

```

%This program calles on the subroutines nonlinear3 and nonlinear32 to solve
%for the flowrates Q1f and Q2f

```

```

% The presssures P4d,P5d,P6d are will be used for the intial guess

```

```

[P4d,P5d,K1d,P6d,K2d,K3d]=feval('desired3');

```

```

Q1d=1500;

```

```

Q2d=1000;

```

```

Q3d=1000;

```

```

x0=[P4d;P5d;Q1d;Q2d;Q3d;P6d];

```

```

options=optimset('Tolfun',1e-6,'MaxFunEvals',10000,'Maxiter',10000,'Tolx',1e-6);

```

```

xt = lsqnonlin('nonlinear3',x0,[],[],options);

```

```

P4f = xt(1);

```

```

P5f = xt(2);

```

```

Q1fn = xt(3)

```

Q2f = xt(4)

Q3f = xt(5)

P6f = xt(6);

xt2 = lsqnonlin('nonlinear32',x0,[],[],options);

P4f2=xt2(1);

P5f2=xt2(2);

Q1fn2=xt2(3)

Q2fn2=xt2(4)

Q3f2=xt2(5)

P6f2=xt2(6);

A.4 Matlab Codes for Simulink

%code to determine K1, K2, and K3

function error=test(a)

global k3d

x=linspace(0,1,2);

u1=linspace(1,1,2).*a(1);

u2=linspace(1,1,2).*a(2);

u3=linspace(1,1,2).*a(3);

```
UT1=[x' u1'];
```

```
UT2=[x' u2'];
```

```
UT3=[x' u3'];
```

```
[t,xstate,z1,z2]=sim('model3old3',[],[],UT1,UT2,UT3);
```

```
[t,xstate,z3,z4]=sim('model3old31',[],[],UT1,UT2,UT3);
```

```
error = abs(z1(3)) + abs(z2(3)) + abs(z3(3));% + abs(z4(3));
```

```
%code to determine K1d and K2d
```

```
function error = testdesired(a)
```

```
global k3d
```

```
x=linspace(0,1,2);
```

```
u1=linspace(1,1,2).*a(1);
```

```
u2=linspace(1,1,2).*a(2);
```

```
UT1=[x' u1'];
```

```
UT2=[x' u2'];
```

```
u3d=linspace(1,1,2).*k3d;
```

```
UT3=[x' u3d'];
```

```
[t,xstate,z1,z2]=sim('desiredmodel3old3',[0,2],[[],[]],UT1,UT2,UT3);
```

```
error = abs(z1(3)) + abs(z2(3));
```

```
%code to determine Q1f and Q2f
```

```
%final model input
```

```
u3f=linspace(1,1,2).*k(3);
```

```
u4f=linspace(1,1,2).*k(1);
```

```
u5f=linspace(1,1,2).*kd(2);
```

```
UT3f=[x' u3f'];
```

```
UT4f=[x' u4f'];
```

```
UT5f=[x' u5f'];
```

```
%final model output
```

```
[tf,xstatef,Q1f,Q2f,P3f,P4f]=sim('finalmod',[0,1],[[],[]],UT3f,UT4f,UT5f);
```

```
Q1f
```

```
Q2f
```

Vita

Mauro Small was born in Panama City, Panama, on July 23 1976. He attended Colegio Episcopal High School in Panama where he graduated in the class of December 1994. In August 1995, he enrolled into Virginia Polytechnic Institute and State University in Blacksburg, Virginia. In May 1999 he graduated with a Bachelor of Science degree in Mechanical Engineering. He then pursued a Masters degree in the same institution, and helped develop a non-iterative technique of balancing an air distribution system