Chapter 2

The Boundary Integral Equation Method

The Boundary Integral Equation Method (BIEM) was developed by Drs. Dunn and Tweed from Old Dominion University and by Dr. Farassat of NASA Langley Research Center. This chapter contains a description of the method when used to compute the sound pressure field generated by a ducted engine fan.

2.1 Introduction

An engine fan surrounded by an infinitesimally thin, finite length cylindrical duct and translating in the axial direction with a uniform speed is considered. The BIEM is based on the equations of linearized acoustics with uniform inflow and predicts the sound scattered by the duct when it is irradiated by the incident sound generated by the fan. A schematic of the model is presented in Figure 2.1. The duct is assumed to have a hard wall exterior and a lined or hard wall interior. The liner has a circumferentially uniform impedance. It can be axially segmented, and positioned anywhere inside the duct. A collection of circumferentially evenly spaced line or point sources, situated on a disc perpendicular to the duct axis, is used to generate the incident sound (the fan noise model is discussed in Chapter 3).

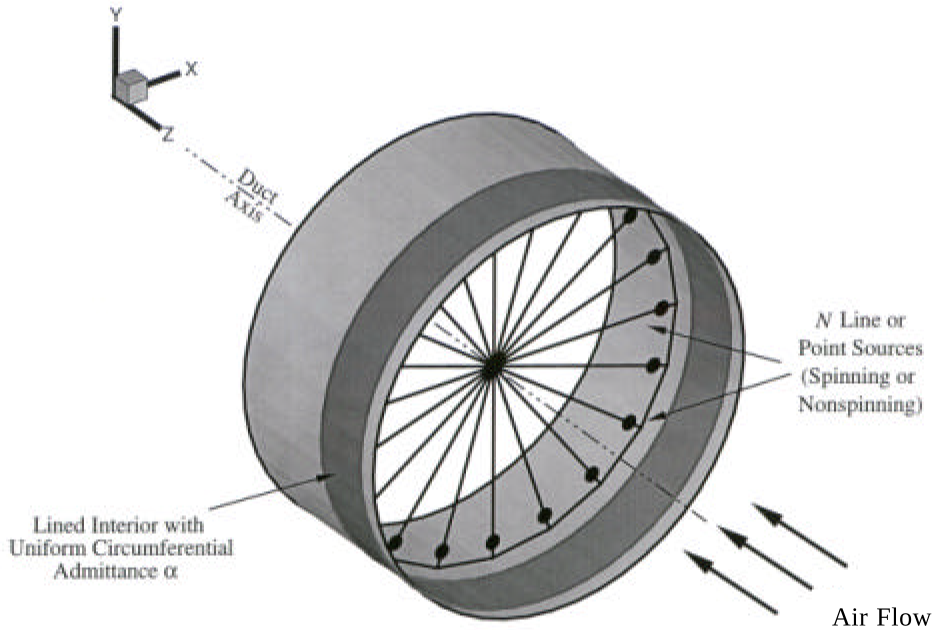

Figure 2.1: BIEM duct geometry (taken from Dunn 1999). Source plane located at z = 0

The application of the BIEM to compute the acoustic pressure field generated by this ducted fan is described next. First, the boundary value problem for the scattered acoustic pressure is derived in accordance with the assumptions that all acoustic processes are linear, generate spinning modes, and occur in a uniform flow field. The boundary value problem is then recast as a system of boundary integral equations for the unknown duct surface quantities (i.e., the values of the scattered pressure and its normal derivative along the duct wall). Only the important steps of the method will be given. A very thorough explanation of the BIEM method can be found in Dunn (1995) and Dunn (1999). The details of the derivations that follow are taken from Dunn (1995).

2.2 Derivation of the boundary value problem

The coordinate system used in this analysis is shown in Figure 2.2. It is assumed that the duct is translating in the axial direction with a uniform velocity V in a quiescent medium and an Eulerian description of the acoustic field is initially considered. At time t=0, the axial coordinates of the fan stage and of the duct inlet and outlet planes are respectively z = 0, z = a and z = b.

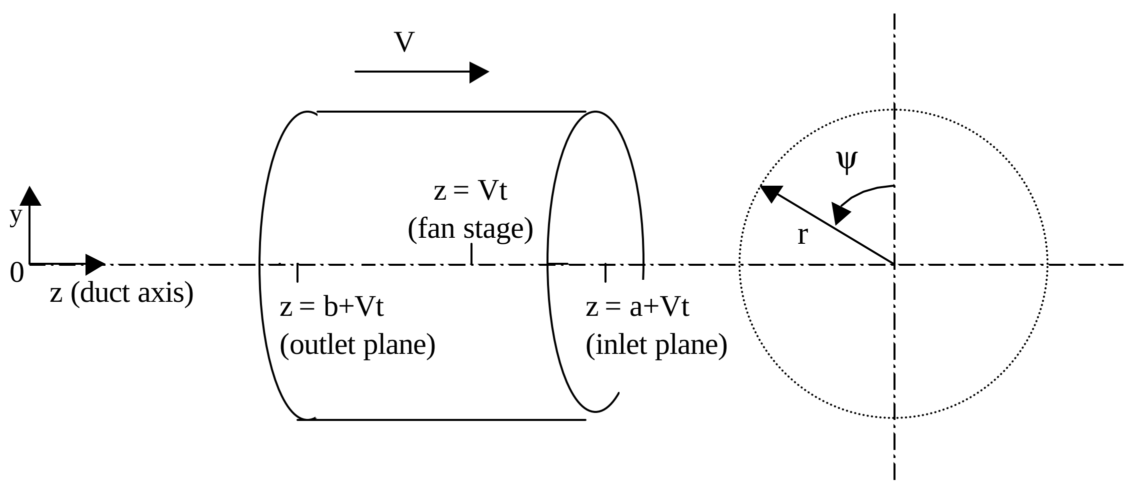

Figure 2.2: The cylindrical coordinate system

The very first step of the method consists of formulating the problem as a scattering problem in which the acoustic pressure field is split into known incident and unknown scattered components: i.e.,

$$\tilde{p}_t(\tilde{r},\psi,\tilde{z},\tilde{t}) = \tilde{p}_i(\tilde{r},\psi,\tilde{z},\tilde{t}) + \tilde{p}_s(\tilde{r},\psi,\tilde{z},\tilde{t}), \tag{2.1}$$

where a tilde over a variable denote a dimensional quantity, (r,ψ, z) are the cylindrical coordinates, t is the time and p_t, p_i and p_s are respectively the total, incident and scattered pressures. The incident pressure field corresponds to the pressure field generated by the fan as if it were in free space (see Chapter 3). The scattered field represents the changes to the free space field due to the presence of the duct. The incident field is computed first, independently of the duct shape and is used as input to the BIEM procedure.

Next, the scattered pressure is computed. It is obtained by expressing its field equations in terms of the unknown surface pressure and normal derivative of the surface pressure along the duct. The field equations are:

(i) The Ffowcs Williams-Hawkings equation (Farassat 1977)

$$\left[\frac{1}{\tilde{c}^2}\frac{\partial^2}{\partial \tilde{t}^2} - \nabla^2\right]\tilde{p}_s(\tilde{r},\psi,\tilde{z},\tilde{t}) =$$

$$-\nabla[\Delta\tilde{p}(\psi,\tilde{z},\tilde{t})\,\delta(\tilde{r}-\tilde{r}_d)\,\hat{i}_r] - \Delta\tilde{p}_{\bar{n}}(\psi,\tilde{z},\tilde{t})\,\delta(\tilde{r}-\tilde{r}_d), \tag{2.2}$$

where c is the ambient speed of sound, r_d is the duct radius, δ is the Dirac Delta function, ∇ and ∇^2 are respectively the divergence and Laplacian operators, and $\Delta\tilde{p}$ and $\Delta\tilde{p}_{\bar{n}}$ are respectively the pressure jump across the duct surface (from the inside to the outside) and the normal derivative of the pressure jump across the duct surface. They are defined as

$$\Delta\tilde{p} = \lim_{\tilde{r}\to\tilde{r}_d^+}\tilde{p}_s(\tilde{r},\psi,\tilde{z},\tilde{t}) - \lim_{\tilde{r}\to\tilde{r}_d^-}\tilde{p}_s(\tilde{r},\psi,\tilde{z},\tilde{t}) \tag{2.3}$$

and

$$\Delta\tilde{p}_{\bar{n}} = \lim_{\tilde{r}\to\tilde{r}_d^+}\frac{\partial\tilde{p}_s}{\partial\tilde{r}} - \lim_{\tilde{r}\to\tilde{r}_d^+}\frac{\partial\tilde{p}_s}{\partial\tilde{r}}. \tag{2.4}$$

(ii) The radial component of the momentum equation

$$\frac{\partial \tilde{u}_r(\tilde{r},\psi,\tilde{z},\tilde{t})}{\partial \tilde{t}} + \frac{1}{\tilde{\rho}_0}\frac{\partial \tilde{p}_t(\tilde{r},\psi,\tilde{z},\tilde{t})}{\partial \tilde{r}} = 0, \quad (2.5)$$

where $\tilde{u}_r$ is the radial component of the acoustic velocity and $\tilde{\rho}_0$ is the density of the undisturbed medium.

(iii) The hard wall boundary condition

$$\lim_{\tilde{r}\to\tilde{r}_d^+} \tilde{u}_r(\tilde{r},\psi,\tilde{z},\tilde{t}) = 0 \quad \text{for} \quad \tilde{z} \in [\tilde{a}+\tilde{V}\tilde{t}, \tilde{b}+\tilde{V}\tilde{t}], \quad (2.6)$$

where $(\tilde{a}+\tilde{V}\tilde{t})$ and $(\tilde{b}+\tilde{V}\tilde{t})$ are respectively the duct inlet and outlet planes axial coordinates in the fixed reference frame, and

(iv) The lined wall boundary condition (Eversman 1991)

$$\lim_{\tilde{r}\to\tilde{r}_d^-} [\tilde{u}_r(\tilde{r},\psi,\tilde{z},\tilde{t}) - \frac{1}{\tilde{Z}_{imp}}(1 - i\frac{M}{\tilde{k}}\frac{\partial}{\partial \tilde{z}})\, \tilde{p}_t(\tilde{r},\psi,\tilde{z},\tilde{t})] = 0 \ \text{for} \ \tilde{z} \in [\tilde{a}+\tilde{V}\tilde{t}, \tilde{b}+\tilde{V}\tilde{t}] \quad (2.7)$$

where $\tilde{Z}_{imp}$ is the acoustic impedance of the duct inner wall, $i = \sqrt{-1}$, $M = \frac{\tilde{V}}{\tilde{c}}$ and $\tilde{k}$ is the wavenumber. The hard wall boundary condition is applied on the outer surface of the duct, and the lined or hard wall boundary conditions are applied on the inner surface of the duct.

These governing equations are then nondimensionalized and expressed in a stretched and moving reference frame, where the axial coordinate z is replaced by

$$Z = \frac{1}{\boldsymbol{b}}(z - Vt) \quad (2.8)$$

with

$$\boldsymbol{b} = \sqrt{1 - M^2}, \quad (2.9)$$

and the axial coordinates of the leading and trailing edges of the duct (i.e., the inlet and outlet planes) are replaced by

$$A = \frac{a}{\beta} \tag{2.10}$$

and

$$B = \frac{b}{\beta}. \tag{2.11}$$

For the nondimensionalization procedure, length is divided by $\tilde{r}_d$, mass by $\tilde{\rho}_0\,\tilde{r}_d^3$, and time by $\tilde{\Omega}^{-1}$, where $\tilde{\Omega}$ is the shaft speed of the fan.

In addition, in the frame of reference moving with the duct, the symmetry of the problem is such that all acoustic variables can be expressed as a linear superposition of time harmonic circumferential spinning modes. Thus, in that reference frame and after nondimensionalization, the scattered acoustic pressure (and similarly the incident and total acoustic pressures) can be written as

$$p_s(r, \psi, Z, t) = \sum_{h=-\infty}^{\infty} P_s^h(r, Z)\; e^{ihN(t-\psi)}, \tag{2.12}$$

and the radial component of the acoustic velocity can be written as

$$u_r(r, \psi, Z, t) = \sum_{h=-\infty}^{\infty} U_r^h(r, Z)\; e^{ihN(t-\psi)}, \tag{2.13}$$

where h is the harmonic number, N is the number of fan blades and $t = \tilde{t}\,\tilde{\Omega}$ is the nondimensional time. Similarly, the pressure jump and the normal derivative of the pressure jump across the duct surface can respectively be expressed as

$$\Delta p(\psi, Z, t) = \sum_{h=-\infty}^{\infty} \Delta P^h(Z)\; e^{ihN(t-\psi)}, \tag{2.14}$$

and as

$$\Delta p_{\bar{n}}(\psi, Z, t) = \sum_{h=-\infty}^{\infty} \Delta P_{\bar{n}}^h(Z)\; e^{ihN(t-\psi)}. \tag{2.15}$$

Applying Eq. (2.8) through (2.15), the nondimensionalized field equations for the scattered acoustic pressure take the following forms, when expressed in the stretched moving reference frame:

$$\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)-\frac{hN}{r^2}+\frac{\partial^2}{\partial Z^2}+\kappa^2\right)\left[e^{i\kappa MZ}\,P_s(r,Z)\right]=$$

$$e^{i\kappa MZ}\left[\Delta P(Z)\left(1+\frac{\partial}{\partial r}\right)+\Delta P_{\bar{n}}(Z)\right]\delta(r-1) \tag{2.16}$$

for the Ffowcs Williams- Hawkings equation, where

$$\kappa=\frac{k}{\beta}=\frac{1}{\beta}\frac{hN}{c}=\frac{1}{\beta}\,\frac{hN\tilde{\Omega}}{\tilde{c}}\,\tilde{r}_d\,, \tag{2.17}$$

k being the nondimensional wave number,

$$\left(ik-\frac{M}{\beta}\frac{\partial}{\partial Z}\right)U_r(r,Z)+\frac{1}{c}\frac{\partial P_t(r,Z)}{\partial r}=0 \tag{2.18}$$

for the radial component of the momentum equation, and

$$\lim_{r\to 1^+} U_r(r,Z)=0 \quad \text{for} \quad Z\in[A,B] \tag{2.19}$$

and

$$\lim_{r\to 1^-}\left[-e^{-i\frac{\kappa}{M}Z}U_r(r,Z)+\frac{iM}{\kappa Z_{imp}}\frac{\partial}{\partial Z}\left(e^{-i\frac{\kappa}{M}Z}P_t(r,Z)\right)\right]=0 \quad \text{for} \quad Z\in[A,B] \tag{2.20}$$

for the hard wall and lined wall boundary conditions, respectively. Note that $Z_{imp}=\dfrac{\tilde{Z}_{imp}}{\tilde{\rho}_0\,\tilde{c}}$ is the specific acoustic impedance.

In order to simplify these equations and the rest of the derivations, new variables are introduced. These variables are Q, Q_s, Q_i, and V_r, and are defined by the following relations

$$Q_t(r,Z)=P_t(r,Z)\,e^{i\kappa MZ}\,, \tag{2.21}$$

$$Q_s(r,Z)=P_s(r,Z)\,e^{i\kappa MZ}\,, \tag{2.22}$$

$$Q_i(r,Z) = P_i(r,Z)\, e^{i\kappa MZ}\,, \tag{2.23}$$

and

$$V_r(r,Z) = U_r(r,Z)\, e^{i\kappa MZ}\,, \tag{2.24}$$

where Z is the axial coordinate in the stretched and moving reference frame as defined earlier in Eq. (2.8). Similarly we define

$$\Delta Q(r,Z) = \Delta P(r,Z)\, e^{i\kappa MZ}\,, \tag{2.25}$$

and

$$\Delta Q_{\overline{n}}(r,Z) = \Delta P_{\overline{n}}(r,Z)\, e^{i\kappa MZ}\,. \tag{2.26}$$

When expressed in term of these new variables, the Ffowcs Williams – Hawkings equation becomes

$$\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) - \frac{m}{r^2} + \frac{\partial^2}{\partial Z^2} + \kappa^2\right) Q_s(r,Z) \;= \left[\Delta Q(Z)\left(1 + \frac{\partial}{\partial r}\right) + \Delta Q_{\overline{n}}(Z)\right]\delta(r-1) \tag{2.27}$$

which is the inhomogeneous Helmhotz equation for Q_s. Note that the circumferential mode order m defined by m=hN is introduced. Similarly, applying Eq. (2.21) through (2.24) to the radial component of the momentum equation yields

$$\left(i\kappa - \frac{M}{\beta}\frac{\partial}{\partial Z}\right) V_r(r,Z) + \frac{1}{c}\frac{\partial Q_t(r,Z)}{\partial r} = 0 \tag{2.28}$$

or, after integration with respect to Z:

$$V_r(r,Z) = -\frac{\beta}{cM}\, e^{i\frac{\kappa}{M}Z} \int_{-\infty}^{Z} e^{-i\frac{\kappa}{M}\zeta}\, \frac{\partial\, Q_t(r,\zeta)}{\partial\, r}\; d\zeta\,. \tag{2.29}$$

Finally, application of Eq. (2.21) through (2.24) to the boundary conditions gives

$$\lim_{r\to 1^+} V_r(r,Z) = 0 \quad \text{for} \quad Z \in [A,B] \tag{2.30}$$

and

$$\lim_{r\to 1^-}\left[- e^{-i\frac{\kappa}{M}Z} V_r(r,Z) + \frac{iM}{\beta^2 \kappa Z_{imp}}\frac{\partial}{\partial Z}\left(e^{-i\frac{\kappa}{M}Z} Q_t(r,Z)\right)\right] = 0 \quad \text{for} \quad Z \in [A,B]\,. \tag{2.31}$$

In order to ensure the uniqueness of the solution of the boundary value problem defined by equations (2.27) - (2.31), the behavior of the acoustic pressure is constrained in certain regions of the field. Thus, the acoustic pressure is required to satisfy the Sommerfeld radiation condition in the far field (Pierce 1989), and the Kutta condition at the trailing edge of the duct (Bertin and Smith 1989). This means that the acoustic field should vanish at large distance from the sound source and only outgoing waves are possible (Sommerfeld radiation condition) and that the flows from the outer surface and the inner surface of the duct should join smoothly at the duct trailing edge (Kutta condition).

Using traditional numerical methods (finite differences or finite element methods) to solve this system of equations would lead, as it was stated earlier, to heavy computational time and storage. Here instead, potential theory is used to transform the boundary value problem into a system of one dimensional boundary integral equations. This procedure will allow the calculation of the scattered pressure (and also the total acoustic pressure) at any location in the field by evaluating integrals along the surface of the duct.

2.3 Formulation of the system of boundary integral equations

The first step of the reformulation of the boundary value problem into a system of boundary integral equations consists of developing an analytical solution of Eq. (2.27), the Helmhotz equation. A solution of Eq. (2.27) can be found by using the Green's function technique. The Green's function for the Helmhotz equation in an unbounded space and in cylindrical coordinates (Goldstein 1976) is

$$G(r, r', Z - Z') = \frac{1}{2\pi} \int \cos m\psi \frac{e^{-i\kappa R}}{R} \, d\psi \tag{2.32}$$

where

$$R = \sqrt{r^2 + r'^2 - 2r\, r' \cos\psi + (Z - Z')^2} \; . \tag{2.33}$$

Using Eq. (2.32), the solution of Eq. (2.27) can be written as

$$Q_s(r, Z) = \iint_{E^2} \left[\Delta Q(Z') \left(1 + \frac{\partial}{\partial r'} \right) + \Delta Q_{\bar{n}}(Z') \right] \delta(r' - 1)\ G(r, r', Z - Z')\ r' dr' dZ' . \quad (2.34)$$

After integration of the delta functions the above equation simplifies to

$$Q_s(r, Z) = \int_A^B [\Delta Q_{\bar{n}}(Z')\ G(r, 1, Z - Z') - \Delta Q(Z') \frac{\partial G}{\partial \bar{n}'}(r, Z - Z')]\ dZ' \qquad (2.35)$$

where

$$\frac{\partial G}{\partial \bar{n}'}(r, Z - Z') = \lim_{r' \to 1} \frac{\partial}{\partial r'} G(r, r', Z - Z') . \qquad (2.36)$$

Introducing the following operators

$$s[f](r, Z) = \int_A^B f(Z')\ G(r, 1, Z - Z')\ dZ' \qquad (2.37)$$

$$d[f](r, Z) = \int_A^B f(Z') \frac{\partial G}{\partial \bar{n}}(r, Z - Z')\ dZ' \qquad (2.39)$$

where f(Z) has the smoothness properties imposed by potential theory (cf. Smirnov 1964, and Kleinman 1974), Eq. (2.35) can be rewritten as

$$Q_s(r, Z) = s[\Delta Q_{\bar{n}}](Z) - d[\Delta Q](Z) . \qquad (2.39)$$

Eq. (2.39) is the analytical solution of the Helmholtz equation for Q_s.

Using Eq.(2.39), the total field and its radial derivative can therefore be written as

$$Q_t(r, Z) = s[\Delta Q_{\bar{n}}](Z) - d[\Delta Q](Z) + Q_i(r, Z) \qquad (2.40)$$

and

$$\frac{\partial Q_t}{\partial r}(r, Z) = \frac{\partial}{\partial r} s[\Delta Q_{\bar{n}}](Z) - \frac{\partial}{\partial r} d[\Delta Q](Z) + \frac{\partial}{\partial r} Q_i(r, Z) . \qquad (2.41)$$

This solution for the scattered (and total) field is expressed in terms of $\Delta Q(Z)$ and $\Delta Q_{\bar{n}}(Z)$ which, as defined earlier, are functions relating to the jumps in acoustic pressure and its normal derivative across the duct surface. These functions are the unknowns that need to be determined in order to compute the scattered acoustic field. Therefore, integral equations for these unknown functions need to be developed. This will be done next and will complete the formulation of the problem in terms of boundary integral equations.

First, the following notation is defined

$$f^{\pm}(Z) = \lim_{r \to 1\pm} f(r, Z) \qquad Z \in (A, B) \tag{2.42}$$

where f(r,Z) is an arbitrary field function, and + and – signs refer, respectively, to the duct exterior and interior surfaces. By applying this notation, the radial component of the momentum equation, Eq. (2.29), can be written as

$$V_r^{\pm}(Z) = -\frac{\beta}{M} e^{i\frac{\kappa}{M}Z} \int_{-\infty}^{Z} e^{-i\frac{\kappa}{M}\zeta} \frac{\partial Q_t^{\pm}(\zeta)}{\partial r} \, d\zeta \qquad Z \in (A, B) \tag{2.43}$$

when evaluated in the limit as the field point approaches the duct surface.

Similarly, applying Eq. (2.42) and combining Eq.(2.43) with the boundary conditions (Eq. (2.30) and (2.31)) yields

$$\int_{-\infty}^{Z} e^{-i\frac{\kappa}{M}\zeta} \frac{\partial Q_t^{\pm}(\zeta)}{\partial r} \, d\zeta = 0 \qquad Z \in (A, B) \tag{2.44}$$

and

$$\int_{-\infty}^{Z} e^{-i\frac{\kappa}{M}\zeta} \frac{\partial Q_t^{\pm}(\zeta)}{\partial r} \, d\zeta + \frac{iM^2}{\beta^3 \kappa Z_{imp}} \frac{d}{dZ}\left(e^{-i\frac{\kappa}{M}Z} Q_t^{-}(Z) \right) = 0 \qquad Z \in (A, B). \tag{2.45}$$

Adding Eq. (2.44) to Eq. (2.45) give

$$-\int_{-\infty}^{Z} e^{-i\frac{\kappa}{M}\zeta} \Delta Q_{\bar{n}}(\zeta) \, d\zeta + \frac{iM^2}{\beta^3 \kappa Z_{imp}} \frac{d}{dZ}\left(e^{-i\frac{\kappa}{M}Z} Q_t^{-}(Z) \right) = 0 \qquad Z \in (A, B). \tag{2.46}$$

Finally, application of the boundary conditions, Eq. (2.44) and Eq. (2.46), to the governing equations, Eq. (2.40) and Eq. (2.41), yields

$$\int_{-\infty}^{Z} e^{-i\frac{\kappa}{M}\zeta}\left\{D_{\bar{n}}[\Delta Q](\zeta)-\left(\frac{1}{2}I+S_{\bar{n}}\right)[\Delta Q_{\bar{n}}](\zeta)\right\}d\zeta = \int_{-\infty}^{Z} e^{-i\frac{\kappa}{M}\zeta}\,\frac{\partial Q_i}{\partial r}(\zeta)\,d\zeta \quad Z\in(A,B) \quad (2.47)$$

and

$$-\int_{-\infty}^{Z} e^{-i\frac{\kappa}{M}\zeta}\Delta Q_{\bar{n}}(\zeta)\,d\zeta+\frac{iM^2}{\beta^3\kappa Z_{imp}}\frac{d}{dZ}\left(e^{-i\frac{\kappa}{M}Z}\left\{-\left(\frac{1}{2}I+D\right)[\Delta Q](Z)+S[\Delta Q_{\bar{n}}](Z)\right\}\right)=$$

$$-\frac{iM^2}{\beta^3\kappa Z_{imp}}\frac{d}{dZ}\left(e^{-i\frac{\kappa}{M}Z}Q_i(1,Z)\right) \qquad Z\in(A,B) \qquad (2.48.a)$$

which is a system of one-dimensional integral equations for the unknown surface quantities of the scattered pressure $\Delta Q(Z)$ and $\Delta Q_n(Z)$. Note that in the rigid walled case Eq. (2.48.a) reduces to

$$\int_{-\infty}^{Z} e^{-i\frac{\kappa}{M}\zeta}\Delta Q_{\bar{n}}(\zeta)\,d\zeta = 0 \quad Z\in[A,B]. \qquad (2.48.b)$$

The operators S, D, S_n and D_n that appear in Eq. (2.47) and Eq. (2.48.a) (cf. Smirnov 1964, and Kleinman 1974) are defined as

$$S[f](Z) = s[f](1,\ Z) \quad Z\in[A,B] \qquad (2.49)$$

$$D[f](Z) = d[f](1,\ Z) \quad Z\in[A,B] \qquad (2.50)$$

$$S_{\bar{n}}[f](Z) = \int_A^B f(Z')\lim_{r\to 1}\frac{\partial}{\partial r}\left[G(r,1,Z-Z')\right]dZ' \quad Z\in[A,B] \qquad (2.51)$$

$$D_{\bar{n}}[f](Z) = \int_A^B f(Z')\lim_{r,r'\to 1}\frac{\partial^2}{\partial r\partial r'}\left[G(r,1,Z-Z')\right]dZ' \quad Z\in[A,B] \qquad (2.52)$$

The evaluation of the integrals in Eq. (2.47) and Eq. (2.48) can be simplified by differentiating these equations (Eq. (2.47) and (2.48)) with respect to Z. This operation leads to

$$D_{\bar{n}}[\Delta Q](Z)-\left(\frac{1}{2}I+S_{\bar{n}}\right)[\Delta Q_{\bar{n}}](Z)=\frac{\partial Q_i}{\partial r}(Z) \qquad Z\in(A,B) \qquad (2.53)$$

and

$$
-e^{-i\frac{\kappa}{M}\zeta}\Delta Q_{\bar{n}}(\zeta)+\frac{iM^2}{\beta^3\kappa Z_{imp}}\frac{d^2}{dZ^2}\left(e^{-i\frac{\kappa}{M}Z}\left\{-\left(\frac{1}{2}I+D\right)[\Delta Q](Z)+S[\Delta Q_{\bar{n}}](Z)\right\}\right)=
$$

$$
-\frac{iM^2}{\beta^3\kappa Z_{imp}}\frac{d^2}{dZ^2}\left(e^{-i\frac{\kappa}{M}Z}Q_i(1,Z)\right)\qquad Z\in(A,B)\,. \tag{2.54}
$$

Eq. (2.53) and Eq. (2.54), unlike Eq. (2.47) and (2.48), do not have a unique solution due to the differentiation. To recover the information lost during the differentiation procedure, two auxiliary conditions are added. These auxiliary conditions are obtained by evaluating Eq. (2.47) and (2.48) at the trailing edge of the duct (i.e., at Z= A).

Eq. (2.53) and Eq. (2.54) together with the two auxiliary conditions and the edge conditions form the complete system of boundary integral equations that is required to compute the scattered field. The source terms of the resulting system of integral equations are expressed in terms of the incident pressure field generated by the fan in an unbounded space. This incident pressure field is an input to the BIEM procedure and its derivation is described next, in Chapter 3.