METHODS FOR DETERMINING DISH ANTENNA POINTING ANGLES

by

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1. INTRODUCTION

1.1 Overview of Dish Antenna Pointing

1.1.1 Pointing at Satellites

Before the invention of the communication satellite most terrestrial communication was via ground based communication lines. Beginning with the launch of the first commercial satellite, Early Bird, on April 6, 1965, global communication methods have changed greatly. All international telephone traffic, all international and almost all domestic long-distance television program distribution, and many domestic data and voice communications are via satellite links instead of terrestrial links [1, pp. 1].

To communicate with a satellite, ground based reflector (dish) antennas are utilized. Reflector antennas can focus most of the transmitted power into a narrow region of the sky. This allows for the establishment of communication links over long distances minimizing transmitted power requirements. However, because the power is concentrated in a narrow region of the sky the antenna must be accurately pointed at the received source. The problem of pointing an antenna can be simplified or complicated depending on the orbit of the satellite to be communicated with. Most communication satellites orbit the earth using a geosynchronous orbit. A geosynchronous orbit is an orbit where the satellite's orbital period is 24 hours. A geosynchronous orbit that is in the equatorial plane is called a geostationary orbit. In this orbit the satellite will remain above a fixed location on the earth's equator. This greatly simplifies pointing at the satellite from a ground based antenna.

Many satellites are not in geosationary orbits but in other orbits depending on their use. To communicate with some areas of the globe, satellites are placed in orbits other than geosynchronous. For example, a ground based antenna located in the extreme northern or southern hemisphere cannot communicate with a geostationary satellite due to the curvature of the earth preventing the antenna from viewing the satellite. In other words, the communication satellite will be below the local horizon. Communication satellites used to communicate with these regions of the earth are often placed in non-geosynchronous highly elliptical orbits. Therefore, the satellite does not remain fixed with respect to the earth but rotates around the earth in an orbit inclined with respect to the equator. To communicate with the satellite, a ground based antenna must be able to track the satellite. This greatly complicates the problem of pointing the antenna.

1.1.2 Pointing At Celestial Radio Sources [2, pp. intro 1-6]

A side from communication with satellites, dish antennas are utilized in radio astronomy for pointing at celestial radio sources. From the beginning of man until the 1930's observations of the heavens were made only optically. In the 1930's it was realized that interference with terrestrial radio communication links were being caused by extraterrestrial sources. This discovery has developed into the science of radio astronomy.

Celestial radio sources emit electromagnetic radiation not only in the visible frequency spectrum but across all frequencies. Radio astronomy is concerned with the frequency spectrum from approximately 30 MHz to 30 GHz. Large dish antennas are used to collect this energy for scientific studies. The antennas must be able to accurately point at and track celestial radio sources. This requires compensating for the rotation of the earth and other physical effects.

1.1.3 Overview of Reflector Antennas and Antenna Mounts

A reflector antenna is comprised of a large dish which focuses the transmitted or received energy. A smaller receive antenna known as "the feed" is located at the focus of the dish. The shape of the dish is such that the distance from the feed to all points on the surface of the dish is equidistant. The most common shape which achieves this is a paraboloid. Figure 1-1 illustrates the configuration of a typical parabolic dish antenna.
[1, pp. 80-81]

The antennas 'pattern' is a plot of signal strength versus angle from boresight. The 3-dB beamwidth is twice the angle from boresight at which the transmitted power falls to half power. The 3-dB beamwidth describes how narrowly focused an antenna is. Large dish antennas will have a 3-dB beamwidth of less than 0.1 degrees. Figure 1-2 depicts a typical antenna pattern for a large dish antenna. The 3-dB beamwidth

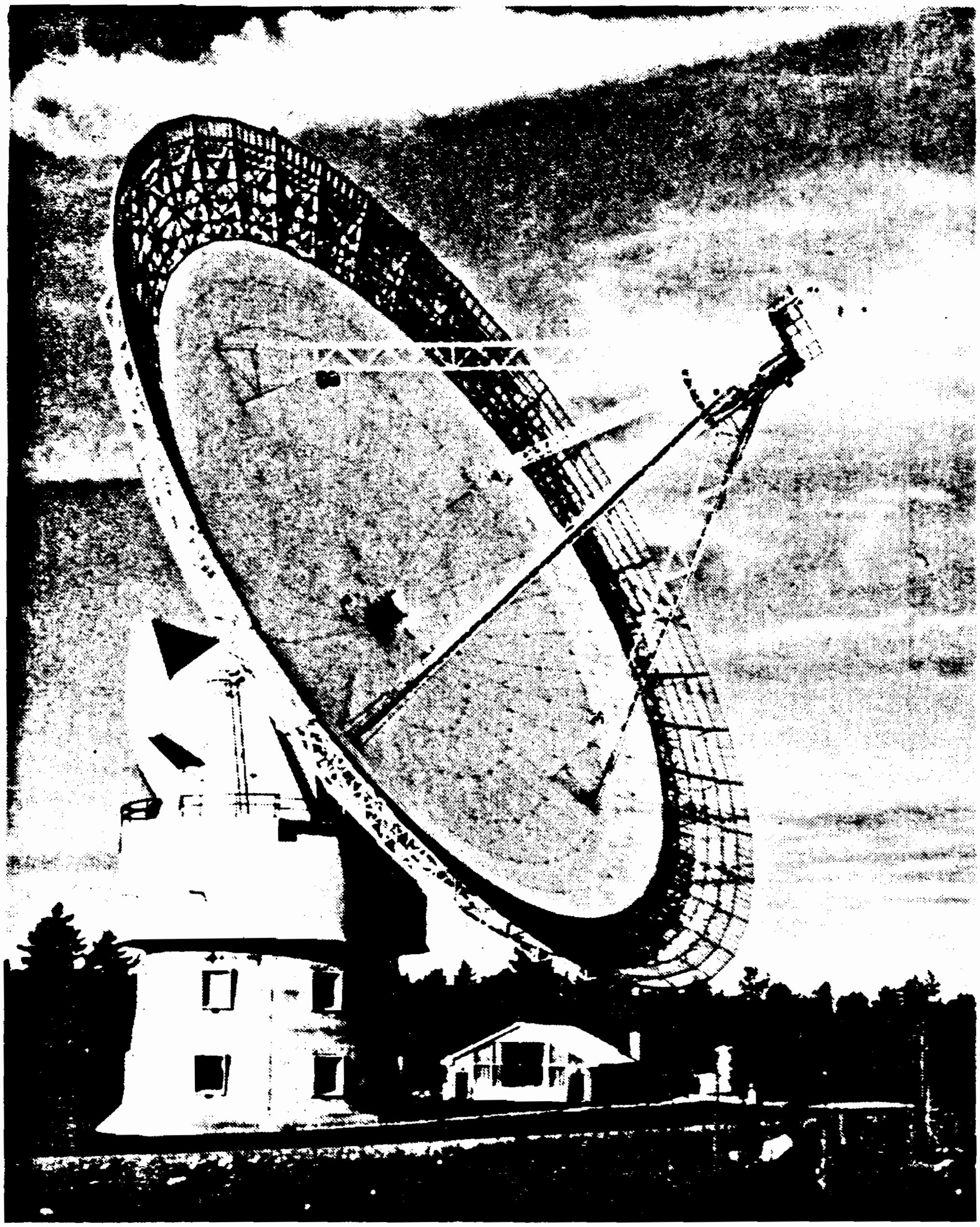


FIGURE 1-1. CONFIGURATION OF A LARGE PARABOLIC DISH ANTENNA [2, pp. 6-70]

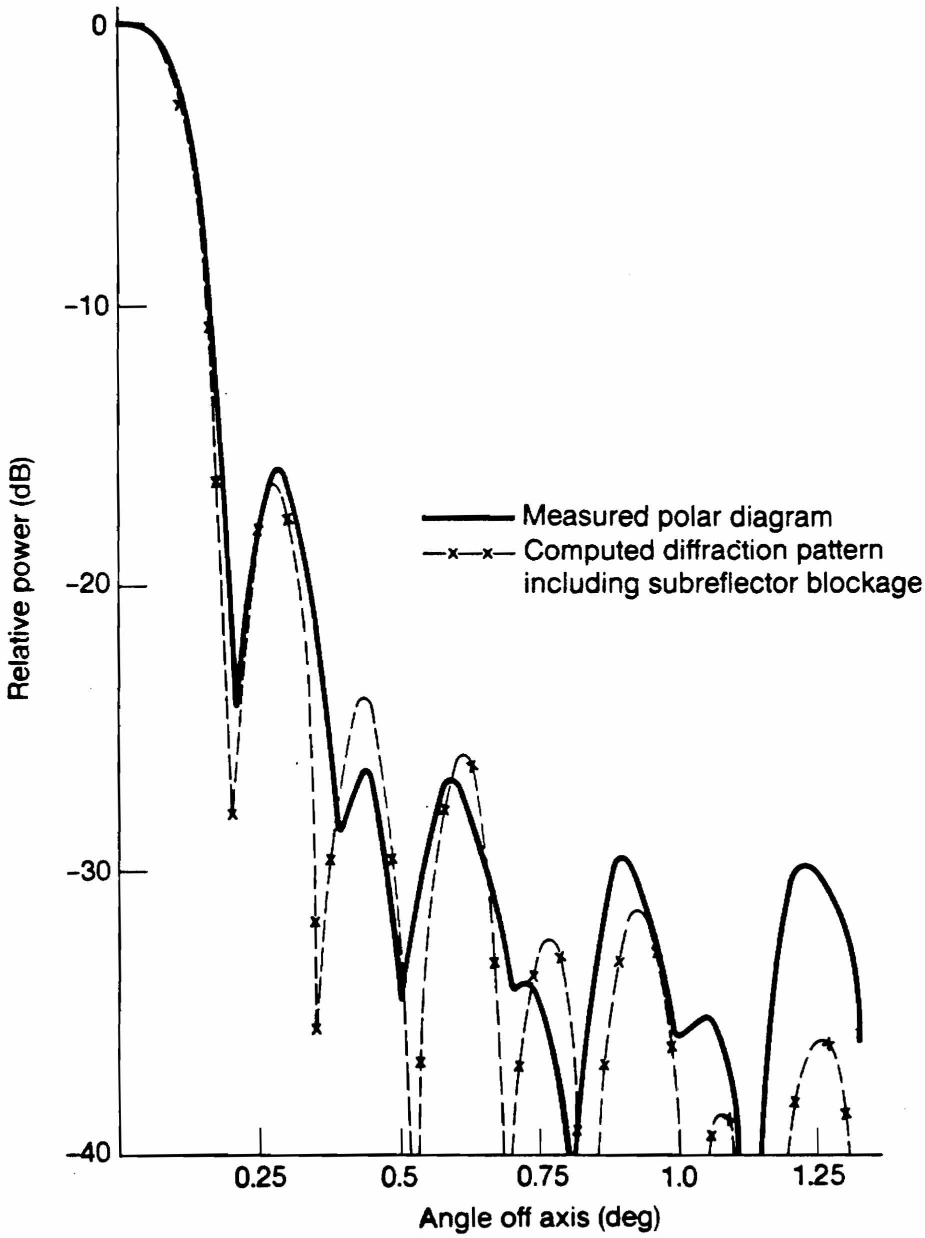


FIGURE 1-2. ANTENNA PATTERN FOR LARGER REFLECTOR ANTENNA [1, pp. 375]

determines how accurately an antenna must be pointed. If the beamwidth is 0.1 degrees then the antenna should point at a source within 0.05 degrees. [1, pp. 78]

To accurately point an antenna an appropriate mount must be used. The most common mount is the alt-azimuth mount. The base of the mount is placed level to the local horizontal of the earth surface (perpendicular to the gravity vector). The pointing direction is measured in azimuth and elevation angles. Azimuth is measured clockwise from true north to where the antenna boresight projects on the plane of the horizon. Elevation is angle between the local horizon and the boresight.

Another common mount is the polar mount. The base of the mount is placed such that the primary axis is parallel to the rotation axis of the earth. The pointing direction is measured in right ascension and declination. Right ascension is measured eastward from the vernal equinox direction (see section 1.5.1.1 for a detailed explanation). Declination is the angle between the celestial equator and the line of sight to the source. [3, pp. 56] Polar mounts are commonly used for astronomical telescopes. Figure 1-3 illustrates the layout of an alt-azimuth mount and the polar mount.

1.2 Study Purpose

The purpose of this study is to gain an overall understanding of determining the angles for pointing a ground based dish antenna at a satellite or celestial object (look angles). The study concentrates on

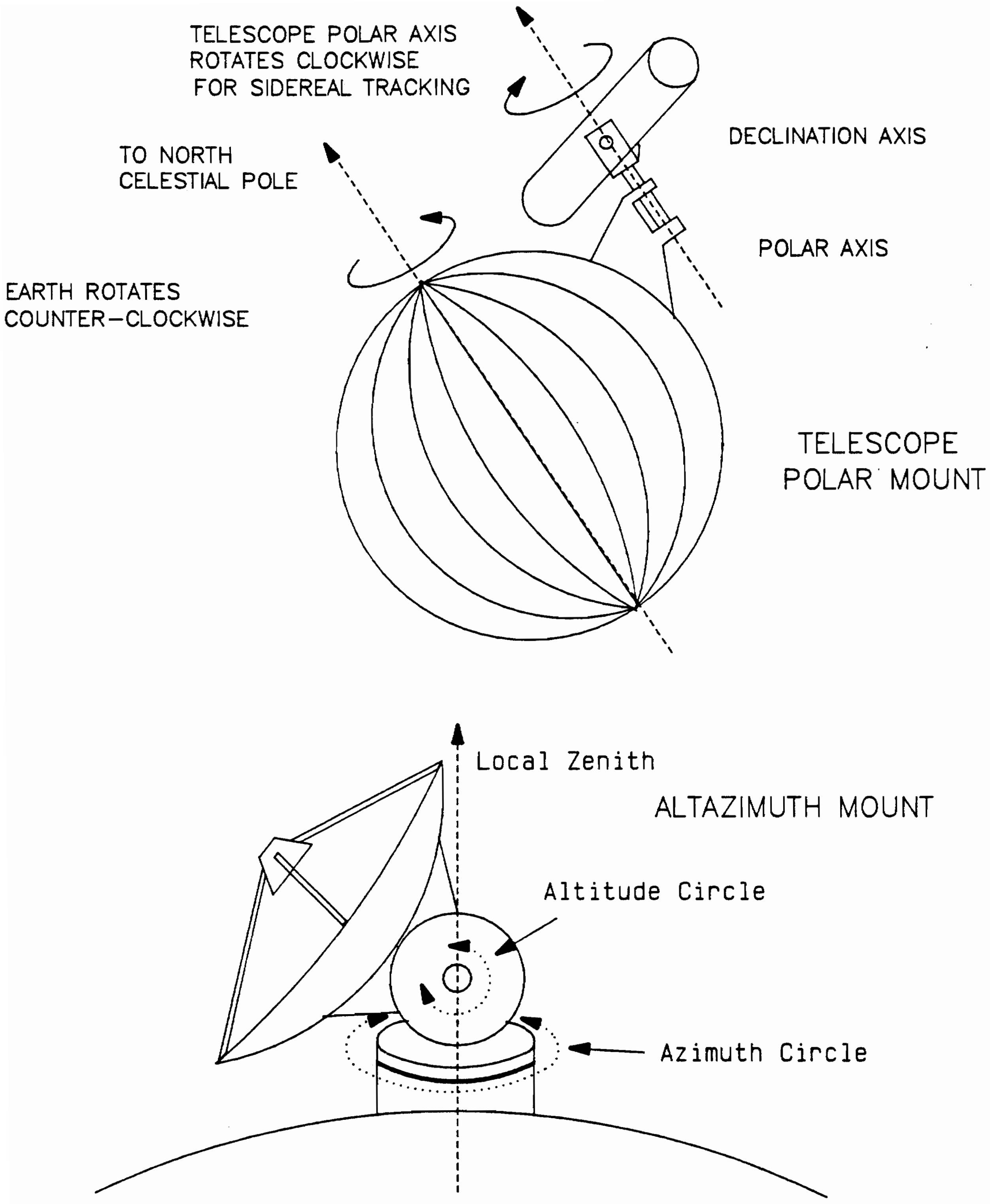


FIGURE 1-3. CONFIGURATION OF ALTAZIMUTH AND POLAR MOUNTS [4, pp. 68,69]

the development of the software required to calculate look angles. All software is original and was developed using Fortran code.

1.3 Study Approach

The study can be divided into three tasks with each task becoming progressively more difficult. The first task is the development of a software routine to calculate the pointing angles from an earth based antenna to a satellite in a geostationary orbit, assuming a spherical earth. In the second task a program was developed to calculate the pointing angles from a ground based antenna to a satellite in a perfect geostationary orbit using geodetic coordinates based on an ellipsoid of revolution. The final task is the development of a program to calculate the pointing angles to an extraterrestrial radio source.

1.4 Document Organization

This document is segmented into chapters to aid in understanding the approach and results of the study. Chapter 2 details the approach for calculating look angles to a geostationary satellite. The angles are calculated using two methods: 1) assuming a spherical earth and 2) assuming an ellipsoidal earth. Chapter 3 explains the software developed for pointing at celestial radio sources. The program corrects for precession, nutation, and polar motion. Chapter 4 is the conclusion highlighting what was learned from the study.

1.5 Definitions

1.5.1 Coordinate Systems

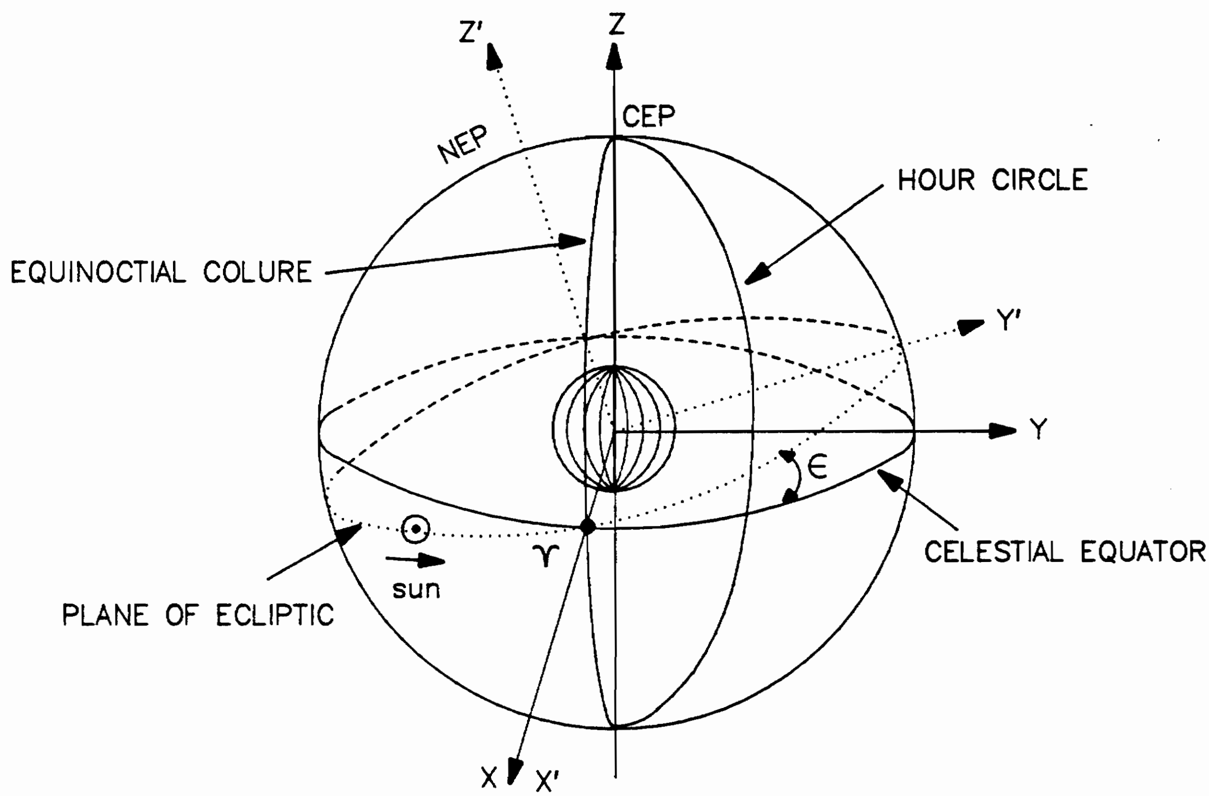
Four types of coordinate systems are used in this study, equatorial, terrestrial, horizon, and geodetic.

1.5.1.1 Equatorial Coordinates

All celestial objects can be considered as lying on an imaginary sphere of an arbitrary radius surrounding the earth, known as the celestial sphere. The earth is the center of the sphere and the equatorial plane of the earth coincides with the equatorial plane of the celestial sphere. Consequently, the polar axis of the earth passes through the polar axis of the celestial sphere. Due to the rotation of the earth, the apparent effect is that the celestial sphere rotates once a day with respect to the earth. [4, pp. 39]

On the celestial sphere great circles containing the poles are called hour circles. The hour circle passing through the poles and the equinoxes (the intersection of the ecliptic and equator) is called the equinoctial colure. The vernal equinox is the point of intersection where the sun crosses the celestial equator from south to north each spring. The plane of the ecliptic contains the path that the sun traces across the celestial sphere every year. Figure 1.4 illustrates the celestial sphere. [5, pp. 29-32]

The primary reference plane for equatorial coordinates is the celestial equator. [5, pp. 34] Spherical angles are given in right



♈ = vernal equinox
ϵ = obliquity of the ecliptic ≐ 23.5 degrees

FIGURE 1–4. THE CELESTIAL SPHERE

ascension and declination. Right ascension, α, is the angle on the plane of the equator between the hour circle of the object of interest and the vernal equinox. Declination, δ, is the angle in the plane of the hour circle between the celestial equator and a line from the center of the earth to the object. The coordinate frame is right handed with the Z axis through Celestial Ephemeris Pole (CEP) and the X axis through the vernal equinox. Figure 1-5 illustrates the equatorial coordinate frame.

1.5.1.2 Conventional Terrestrial Reference Frame

The Conventional Terrestrial Reference System (CTRS) is an earth centered, earth fixed reference frame. This coordinate frame is righthanded with the z axis through the north pole (Conventional Terrestrial Pole, CTP) and the x axis through zero longitude (approximately the Greenwhich Meridian). Directions are given in longitude, λ, and latitude, ϕ. Longitude is measured in degrees east from the Greenwhich Meridian to the meridian of the object. Latitude is measured from the equator to the object, degrees north of the equator being positive. Figure 1-6 shows the terrestrial coordinate system with the difference between the CTP and CEP exaggerated.

1.5.1.3 Horizon Coordinates

The reference plane for the horizon system is the observer's horizon. An object's position is given in azimuth, A, and altitude, a.

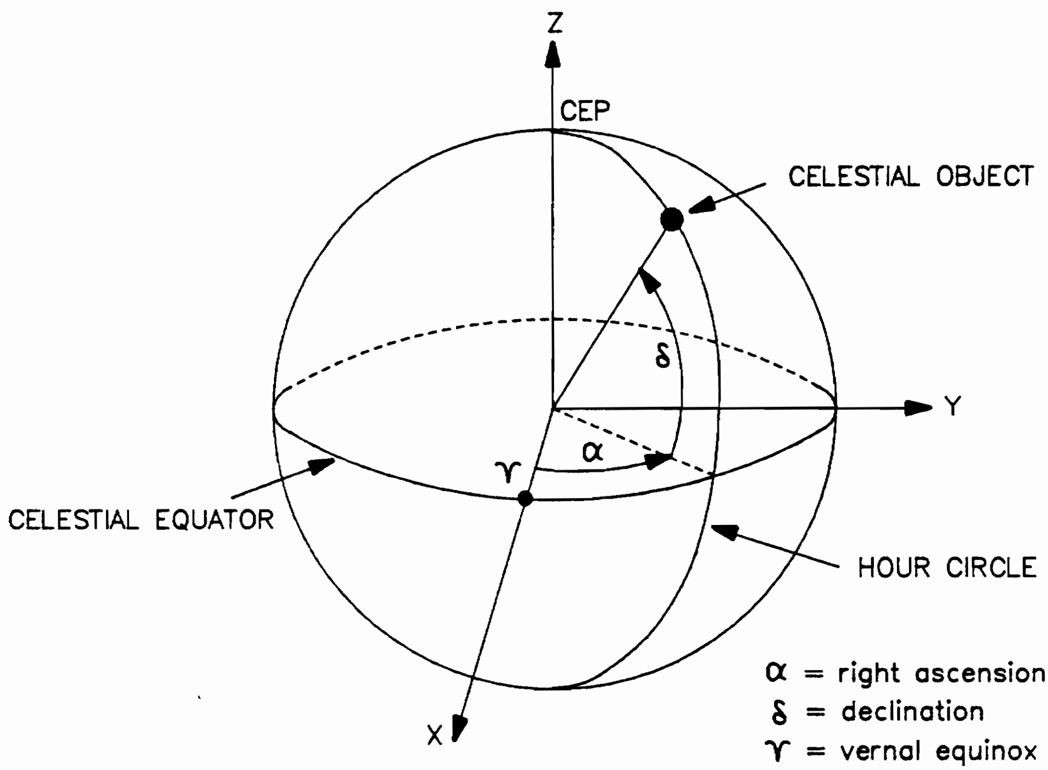


FIGURE 1–5. THE EQUATORIAL COORDINATE SYSTEM

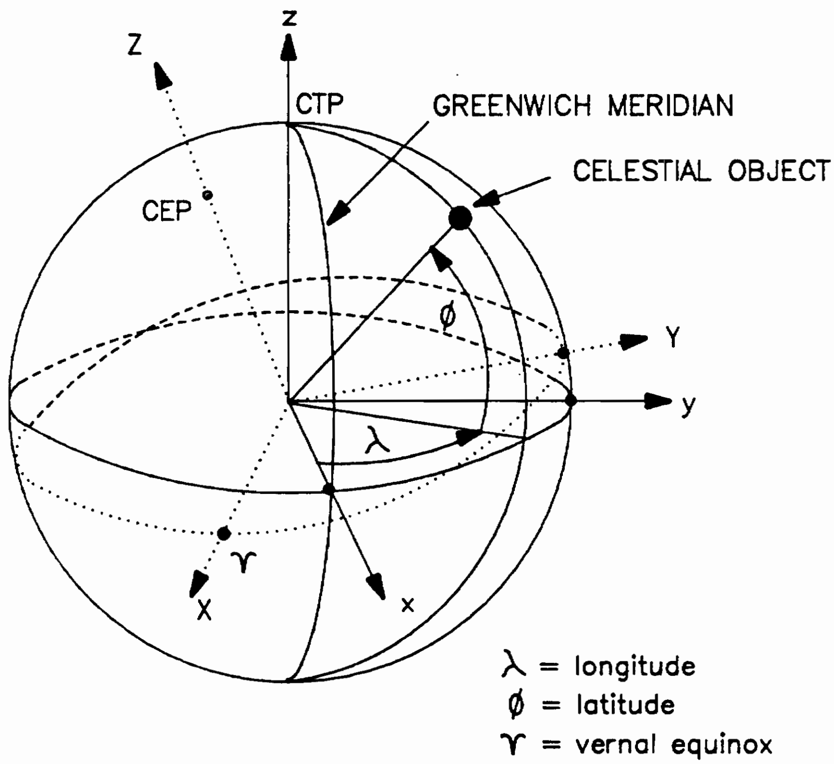


FIGURE 1-6. THE CONVENTIONAL TERRESTRIAL REFERENCE SYSTEM

Altitude is the angle of the object above the horizon measured along the vertical circle. The Azimuth angle is measured from true north clockwise around the horizon to where a vertical circle containing the celestial object intersects the horizon. The horizon coordinate system is illustrated in figure 1-7. The terms altitude and elevation will be used interchangeably.

1.5.1.4 Geodetic Coordinate System

In the science of Geodesy the shape of the earth is often represented as a ellipsoid rather than a sphere. This better represents the actual shape of the earth and the implied gravitational field of the earth. The coordinates (x,y,z) of a particular point are given with respect to a geodetic reference frame. The origin of the coordinate system is the center of the reference ellipsoid. The z-axis coincides with the semi-minor axis of the reference ellipsoid. The x-axis passes through the point where the geodetic latitude and longitude are both equal to zero. The y-axis forms a right hand coordinate system with the x and z axes. [6, pp. 86]

1.5.1.4.1 Curvilinear Geodetic Coordinates

Depending on the problem being analyzed it is often more convenient to employ curvilinear coordinates rather than spatial rectangular coordinates. Curvilinear coordinates are defined in terms of Geodetic longitude, λ, latitude, ϕ, and height, h. Geodetic longitude is the angle between the plane xz and the geodetic meridian

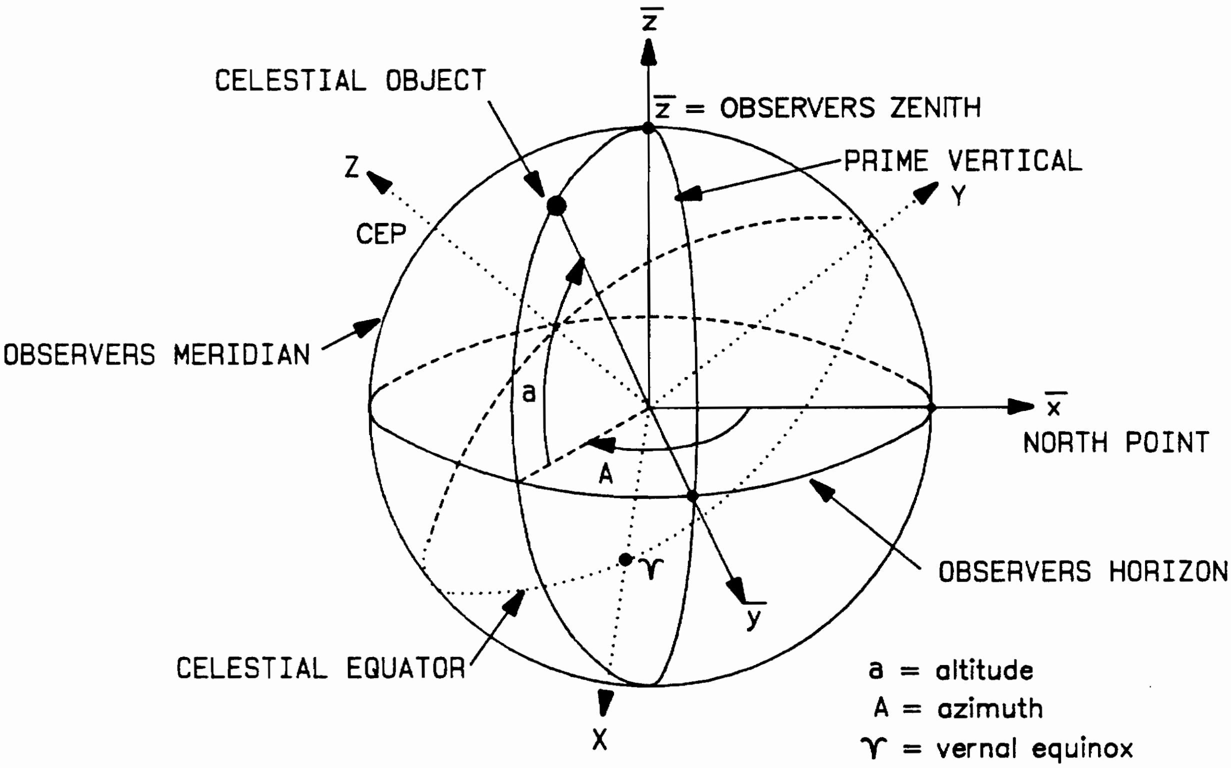


FIGURE 1-7. THE HORIZON SYSTEM

plane of the point of interest, P, measured positive toward the east. Geodetic latitude is the angle between the normal to the ellipsoid at the point of interest and the plane xy. Geodetic height is the "distance along the normal to the reference ellipsoid between P and the surface of the ellipsoid". [6, pp. 89]

1.5.1.4.2 Local Geodetic Frame

The local geodetic frame (e,n,u) is a right handed coordinate system with its origin at the point of interest, P, and is generally not parallel to the x,y,z frame. The *u*-axis ('up' axis), coincides with the normal through P to the reference ellipsoid. The *e*-axis ('east' axis), is normal to the *u*-axis and the plane containing the geodetic meridian. The *n*-axis ('north' axis), forms a right handed coordinate system with the *e* and *u* axes. Figure 1-8 illustrates the relationship between the conventional terrestrial coordinate system and the geodetic system. [6, pp. 90]

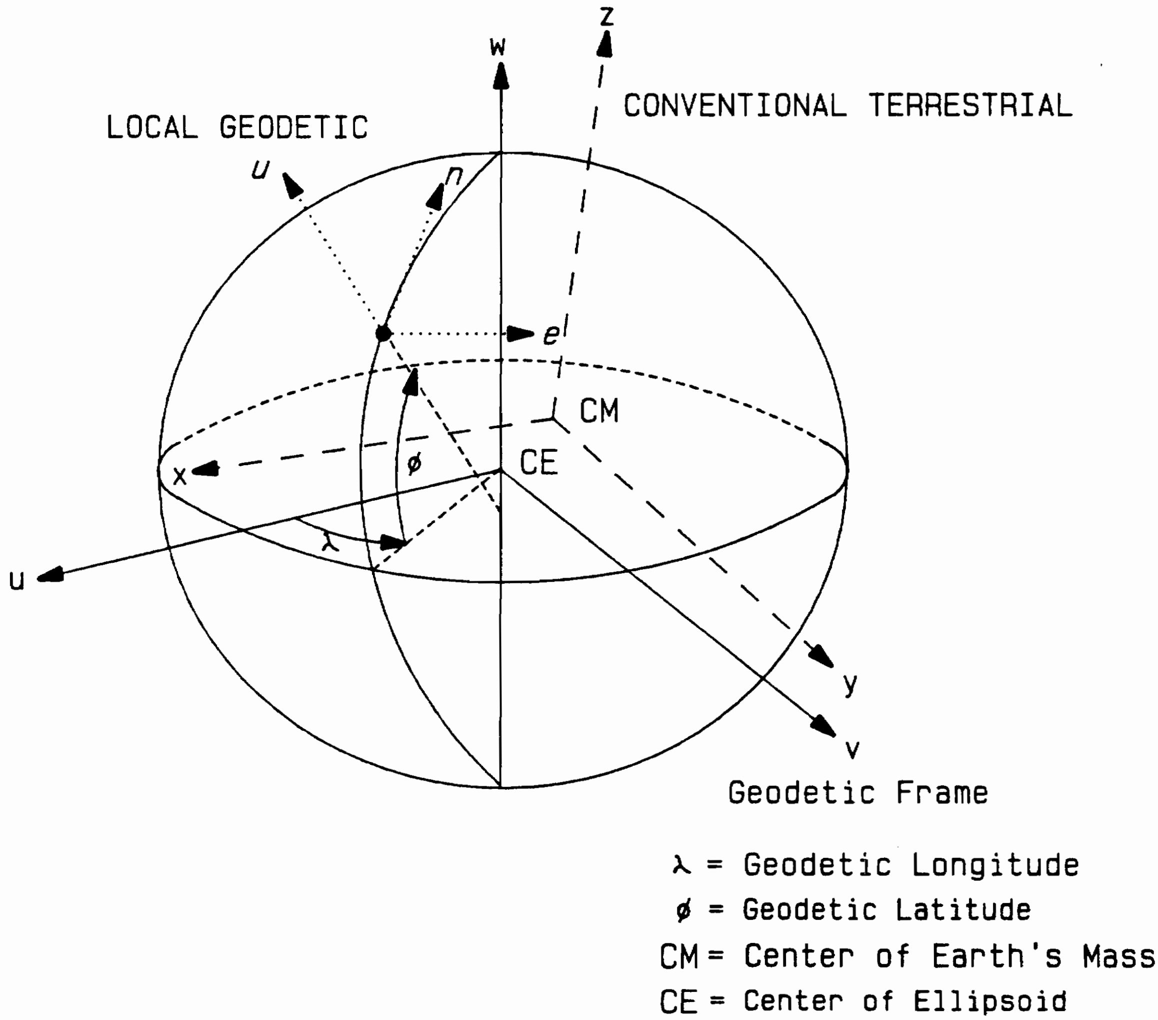


FIGURE 1-8. CONVENTIONAL TERRESTRIAL, GEODETIC, AND LOCAL GEODETIC FRAMES [6, pp. 86]

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CHAPTER 2. POINTING AT FIXED GEOSTATIONARY SATELLITES

2.1 Satellite Look Angles

Satellite look angles are the coordinates or angles, with respect to a predefined coordinate system, to which an earth station antenna must point to communicate with a satellite. Figure 2-1 depicts satellite look angles using the horizon coordinate system. Each satellite has it's own unique set of look angles. The first method developed to calculate a satellite's look angles uses standard plane and spherical trigonometry and assumes a perfectly spherical earth. The second method developed is unique to this paper and is not be found in other sources. The method uses a geodetic reference system which refers to the earth as an ellipsoid instead of a sphere. This second method is a more rigorous approach and readily lends itself to pointing at satellites in any given orbit. Fortran code was developed implementing both methods and Chapter 2 will explain the required calculations.

2.2 Calculating Look Angles Assuming a Spherical Earth [1, pp. 22-29]

In order to calculate pointing angles to a geostationary satellite, general satellite communication textbooks assume a spherical earth. To further simplify the task it is often assumed that the satellite is in a perfect geostationary orbit and not affected by any perturbations (i.e., gravitational, atmospheric, solar radiation, etc.).

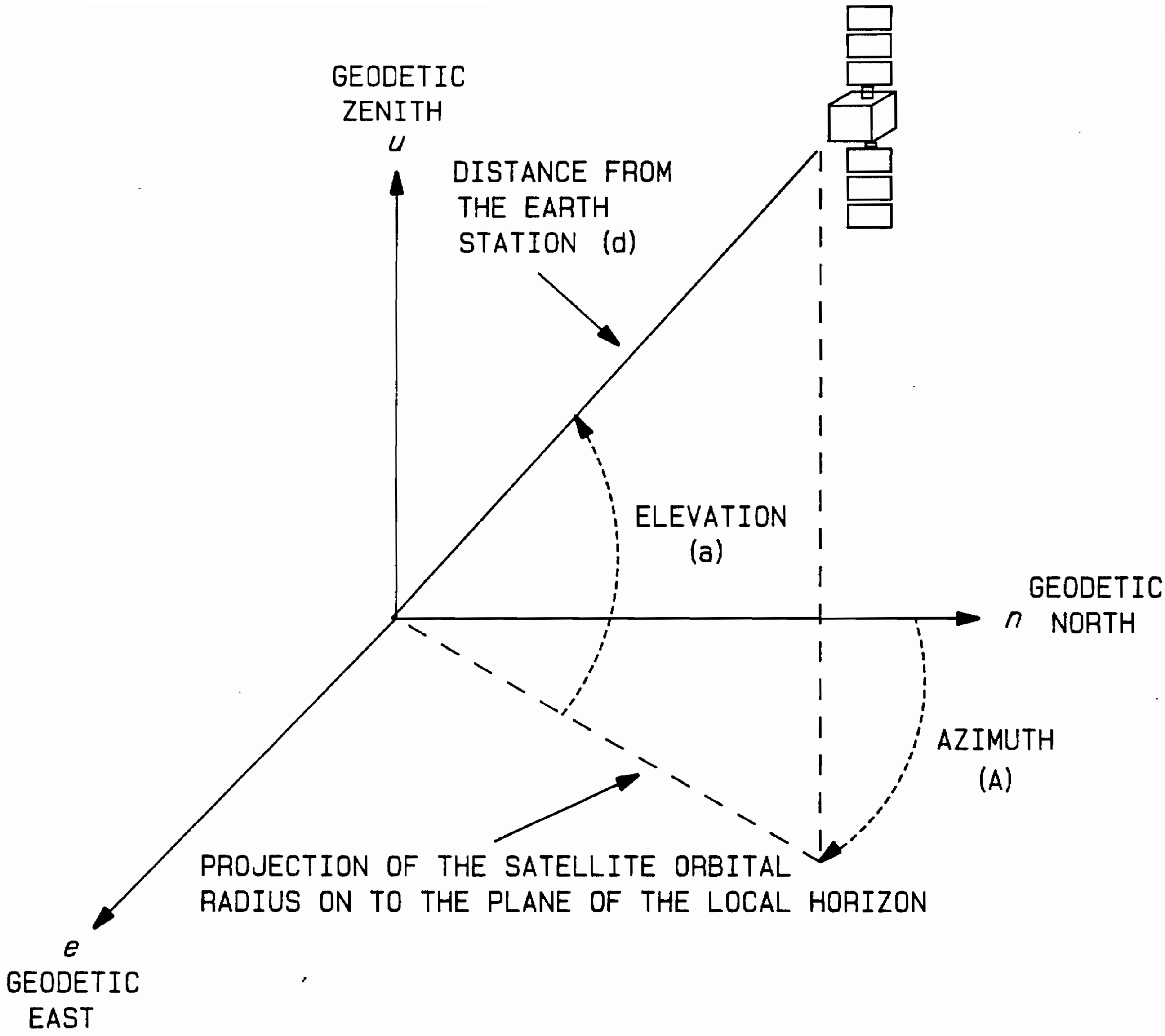


FIGURE 2-1. SATELLITE LOOK ANGLES USING HORIZON COORDINATES [1, pp. 22]

With these assumptions only one parameter is required to determine the location of a satellite, the longitude, λ_s, of the subsatellite point. The subsatellite point is the intersection with the earth's surface of a line drawn from the satellite to the center of the earth. The software program written requires the longitude of the subsatellite point and the location of the observer as operator inputs. The program output is the satellite look angles given in azimuth and elevation. Appendix A contains a listing of the program.

2.2.1 Elevation Look Angle Calculation [1, pp. 24-25]

Figure 2-2 shows the geometry for calculating the elevation look angle of a geostationary satellite. The angle γ, is known as the central angle and is a function of the latitude of the earth station and the longitudinal separation between the earth station and the subsatellite point. Using the right spherical triangle shown in Figure 2-3 the central angle is calculated as follows:

ϕ_o = earth station's latitude

λ_e = earth station's longitude

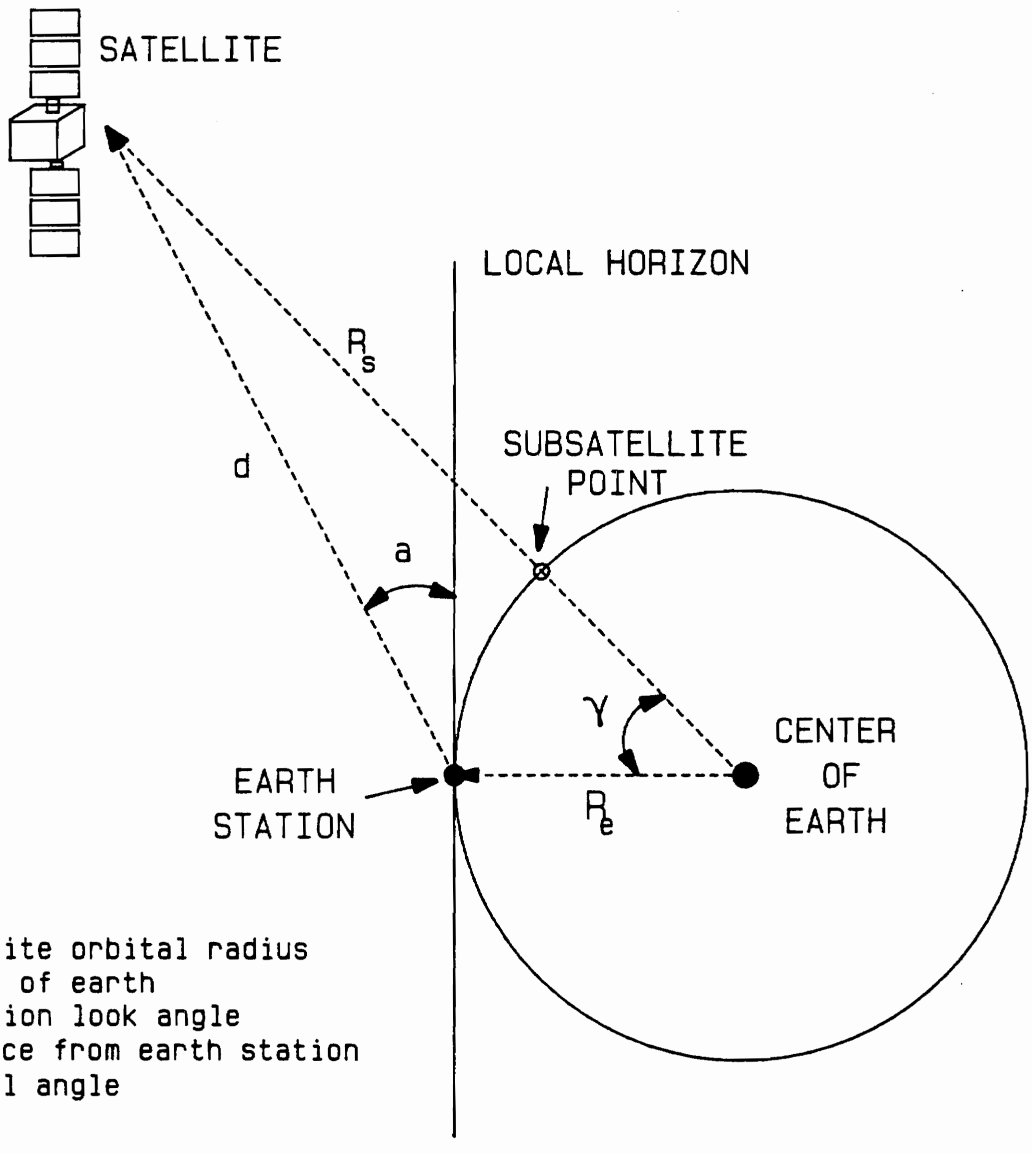
λ_s = longitude of the subsatellite point

γ = central angle

$$\cos\gamma = \sin(90 - \phi_o)\sin[90 - (\lambda_s - \lambda_o)]$$

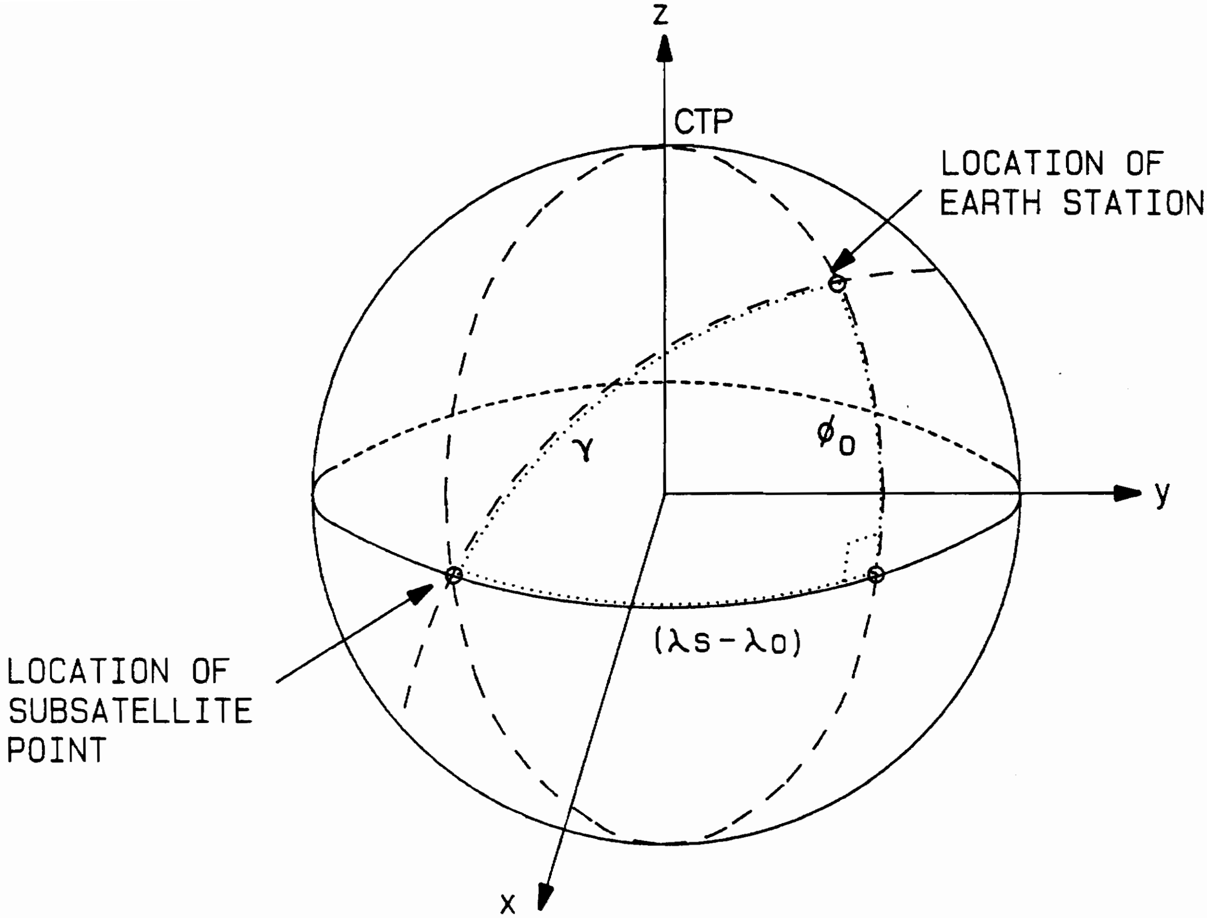
$$= \cos\phi_o\cos(\lambda_s - \lambda_o)$$

Using figure 2-2 it can be seen that the distance from the earth station to the satellite is related to the orbital radius, the radius of the earth, and the central angle. Using the law of cosines the



R_s = satellite orbital radius
R_e = radius of earth
a = elevation look angle
d = distance from earth station
γ = central angle

FIGURE 2-2. THE GEOMETRY FOR CALCULATING THE ELEVATION LOOK ANGLE [1, pp. 22]



$$\cos \gamma = \sin(90 - \phi_0) * \sin[90 - (\lambda s - \lambda o)]$$

ϕ_0 = earth station latitude
λo = earth station longitude
λs = subsatellite point longitude
γ = central angle

FIGURE 2-3. THE SPHERICAL RIGHT TRIANGLE USED FOR CALCULATING THE CENTRAL ANGLE distance is:

d = distance from earth station to the satellite

R_e = radius of the earth

R_s = orbital radius of the satellite

γ = central angle

$$d^2 = R_s{}^2 + R_e{}^2 - 2R_sR_e\cos\gamma$$

$$d = R_s[1 + (R_e/R_s)^2 - 2(R_e/R_s)\cos\gamma]^{1/2}$$

Since the local horizon is perpendicular to a vector from the center of the earth to the ground station, from Figure 2-2 it can also be seen that:

a = elevation

ψ = angle between R_e and d

$$a = \psi - 90^o$$

Using the law of sines the elevation angle can be found:

$$R_s/\sin\psi = d/\sin\gamma$$

$$\sin(a + 90^o) = (R_s\sin\gamma)/d$$

$$\cos a = (R_s\sin\gamma)/d$$

$$a = \cos^{-1}[(R_s\sin\gamma)/d]$$

A common value used for the spherical radius of the earth is R_e = 6370 km. A satellite in a geostationary orbit will have an orbital radius of approximately R_s = 42,242 km. Therefore the above equations for calculating elevation angle can be reduced for the geostationary case to:

$$a = \cos^{-1}[\sin\gamma/(1.02274 - 0.301596 \cos\gamma)^{-1/2}]$$

2.2.2 Azimuth Look Angle Calculation [1, pp. 28-29]

Figure 2-4 illustrates the spherical triangle used for calculating the azimuth look angle for a geostationary satellite. The vertex angle, β, is found through spherical geometry as follows:

ϕ_o = earth station's latitude

β = vertex angle

γ = central angle

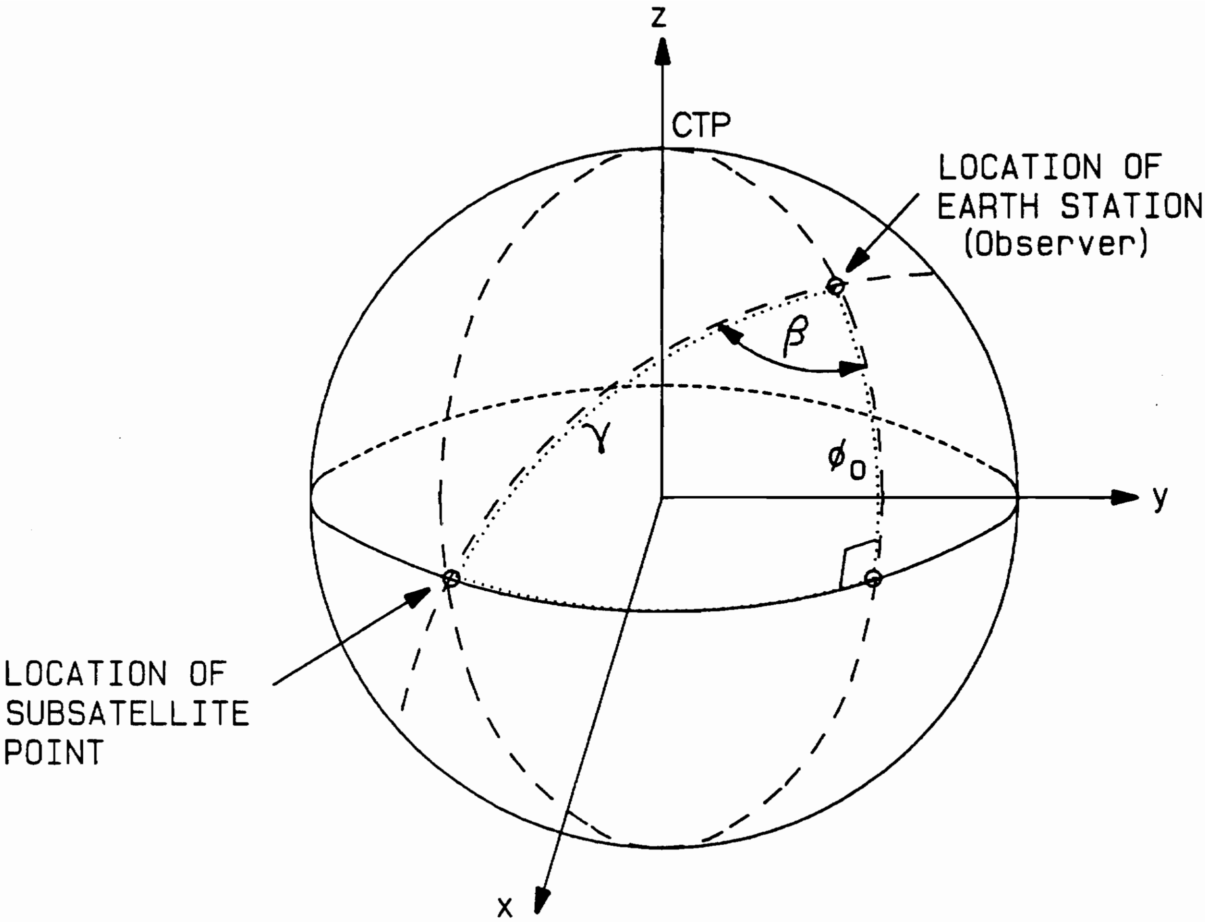
$$\cos\beta = \cot\gamma \cot(90^o - \phi_o)$$

$$\beta = \cos^{-1}[\tan(90^o - \gamma)\tan\phi_o$$

The azimuth look angle is a function of the vertex angle through the relationships stated in Table 2-1.

TABLE 2-1 Calculation of Azimuth(A) Look Angles [1, pp. 29]

Relative Location of Sub-Satellite Point	Equation
1. Southwest of earth station	$A = 180^o + \beta$
2. Southeast of earth station	$A = 180^o - \beta$
3. Northwest of earth station	$A = 360^o - \beta$
4. Northeast of earth station	$A = \beta$



$$\cos \beta = \cot \gamma \cot(90 - \phi_0)$$

β = vertex angle
ϕ_0 = earth station latitude
γ = central angle

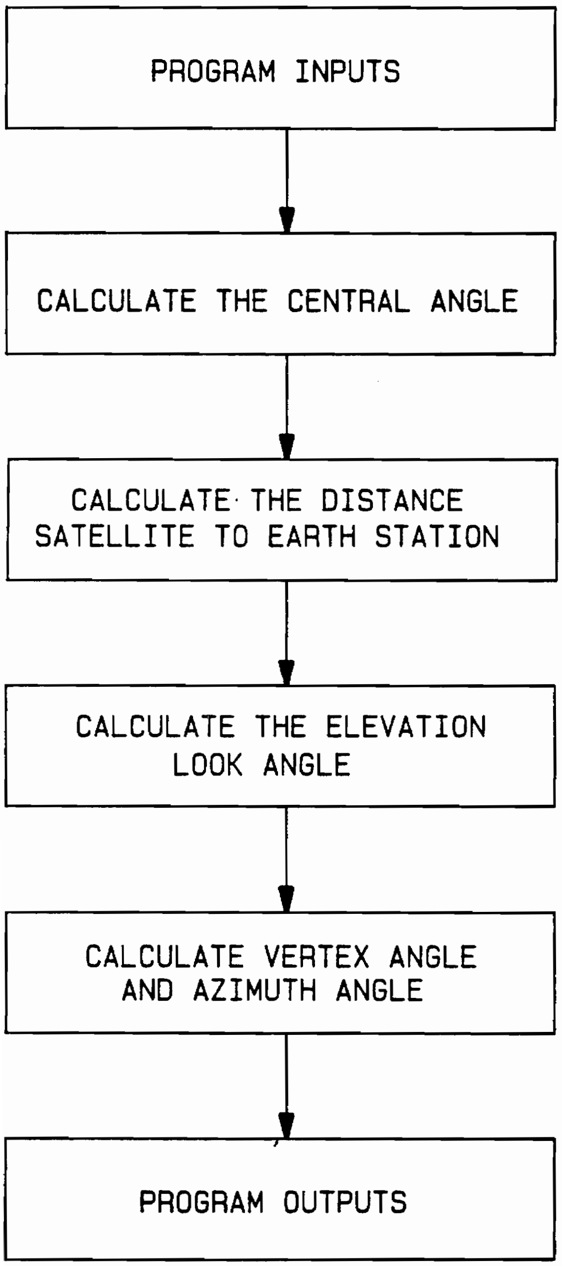
FIGURE 2-4. THE SPHERICAL RIGHT TRIANGLE USED FOR CALCULATING THE VERTEX ANGLE [1, pp. 29]

Figure 2-5 gives a flowchart of the program implementing look angle calculations assuming a spherical earth. Appendix A is a listing of the Fortran code written to perform this calculation.

2.3 Errors Assuming A Spherical Earth

Some problems arise from assuming a spherical earth. The earth is not spherical but in fact resembles a flattened ellipsoid. This oblatness affects the gravity potential of the earth making the geopotential a function of location on the earth. The gravity field is also affected by the distribution of mass inside the earth. The mass of the earth is not homogeneous. A vector which coincides with the local vertical (i.e., the plumbline) may not point to the center of the earth. The direction of the plumbline depends on the local gravity field. The local horizon (astronomic horizon) is perpendicular to the plumbline. The calculations previously developed assumed the local horizon perpendicular to a vector originating from the center of the earth. This assumption is only valid if one assumes a homogeneous spherical earth with an spherical gravity field. The difference can be seen by comparing the geodetic latitude, the astronomic latitude, and the geocentric latitude as illustrated in Figure 2-6. Figure 2-7 shows the difference between geodetic latitude and geocentric latitude. The maximum difference of $0^{o}.1924$ occurs at 45^{o}.

The derivations using spherical trigonometry assume a spherical earth. This will contribute to small errors in pointing. Furthermore,



Observers Latitude ϕ_0
Observers Longitude λ_0
Subsatellite Point Longitude λ_s

$$\gamma = \cos^{-1}[\cos \phi_0 \cos(\lambda_s - \lambda_0)]$$

$$d = 42242 * (1.022 - 0.302 * \cos \gamma)$$

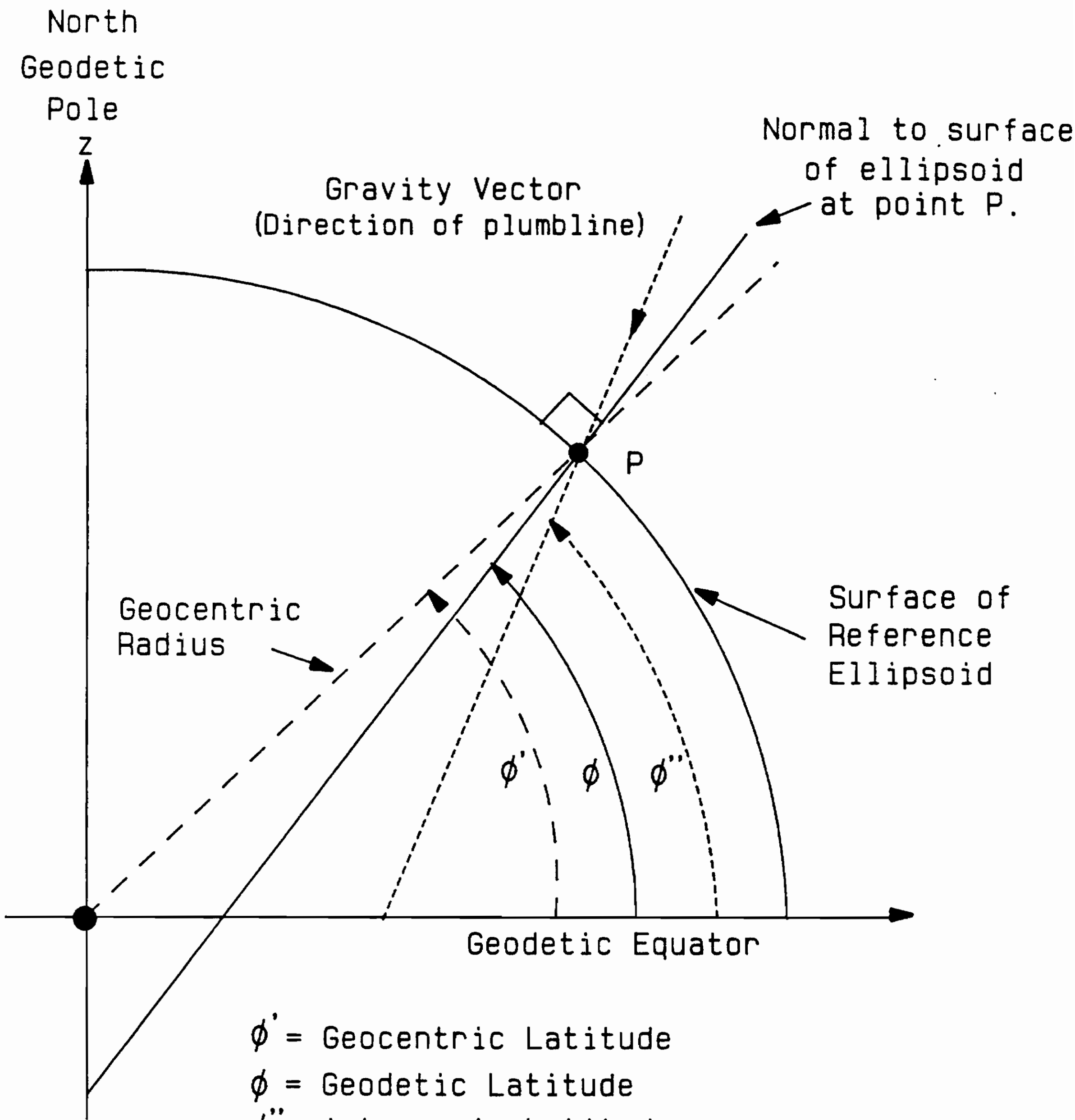
$$a = \cos^{-1}[(\sin \gamma \quad) 42242/d]$$

$$\beta = \cos^{-1}[\tan(\frac{\pi}{2} - \gamma)\tan \phi_0]$$

$$\left.\begin{array}{l} A = 180^0 + \beta \\ A = 180^0 - \beta \\ A = 360^0 - \beta \\ A = \beta \end{array}\right\} \text{Depending on Earth Station Location}$$

Elevation Look Angle
Azimuth Look Angle

FIGURE 2-5. PROGRAM FLOWCHART FOR CALCULATING SATELITTE LOOK ANGLES ASSUMING A SPHERICAL EARTH. [1, pp. 22-30]



ϕ' = Geocentric Latitude

ϕ = Geodetic Latitude

ϕ'' = Astronomic Latitude

$$\phi' = \tan^{-1}\left[\frac{z}{(x^2 + y^2)^{1/2}}\right]$$

FIGURE 2–6. GEODETIC, GEOCENTRIC, AND ASTRONOMIC LATITUDES [4, pp. 15]

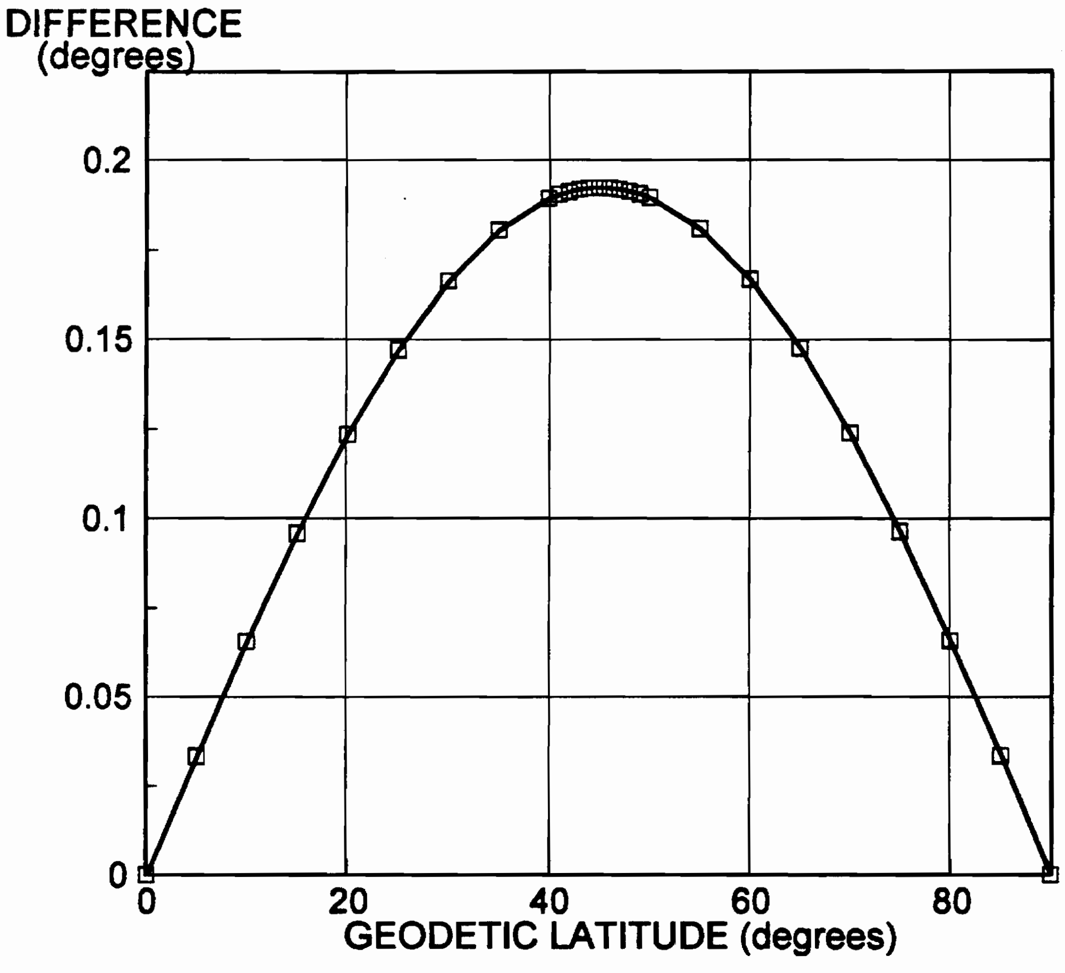


Figure 2-7. Difference Between Geodetic and Geocentric Latitudes.

the calculations assume a constant earth radius of 6370 km. The actual geocentric radius at an earth station is a function of latitude. If highly accurate pointing is required the calculations should not assume that the earth is homogeneous and spherical.

2.4 Calculating Look Angles Using A Geodetic Reference Frame [5, pp. 84-93]

To improve the accuracy of the look angles calculation the variations in the geopotential can be accounted for by using geodetic coordinates based on an ellipsoid of revolution. This approach will also eliminate the errors introduced by the utilization of spherical geometry and errors due to the difference between the geocentric and geodetic latitudes. This method of using a geodetic reference frame for calculating satellite look angles is unique to this paper and to the author's knowledge has never before been published.

2.4.1 An Overview of Geodesy

> Geodesy in the history of the sciences is one of the oldest. It has both scientific and practical missions. The major task of scientific geodesy is the determination of the size, shape, and gravity field of the earth. Using the results obtained, practical geodesy carries out the measurements and computations necessary for the accurate mapping of the earth's surface. The main mission of practical geodesy is the determination of geodetic and astronomic coordinates of fixed terrestrial points. [4, pp. 1]

A physical reference surface called a geoid reflects the distribution of mass in the earth. The surface of the geoid "nearly coincides with the undisturbed mean surface of the oceans." [4, pp. 7] The gravity potential is constant over the surface of the geoid. Since the geoid cannot be expressed by simple mathematical expressions an alternate surface which approximates the geoid and can be described with simple mathematics and is often used to solve geodetic problems.

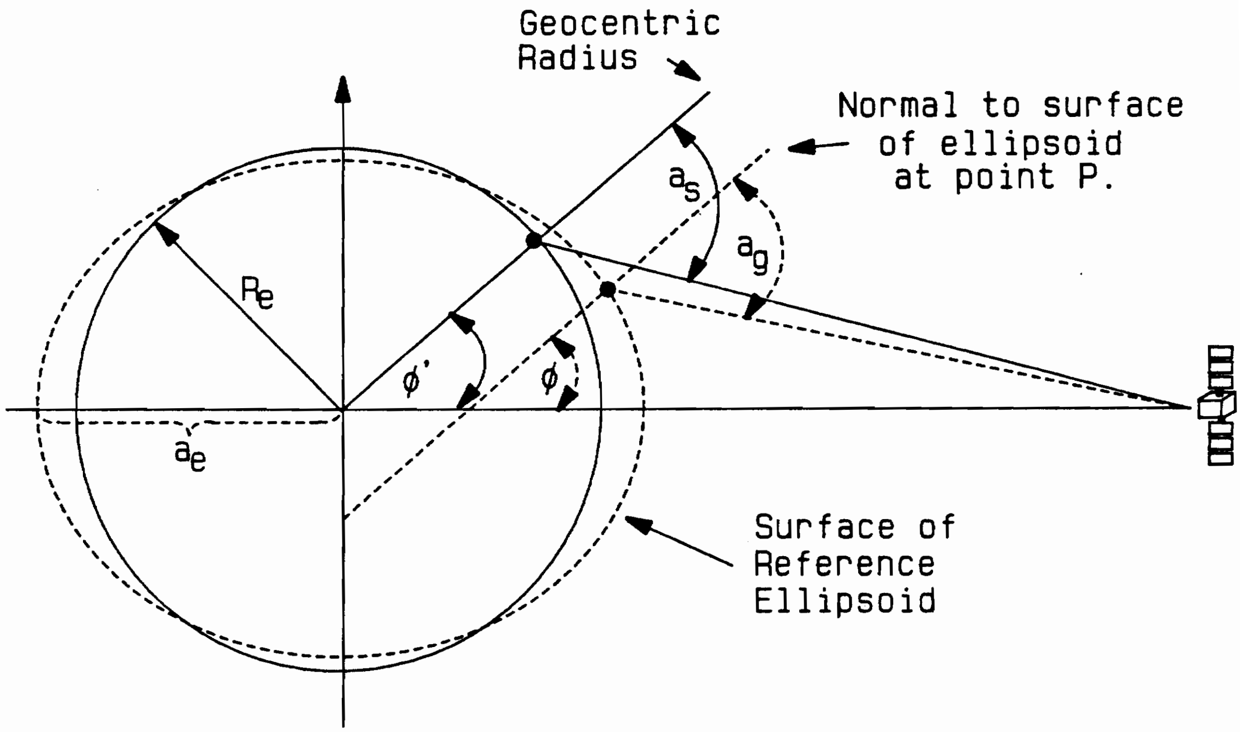
A surface which approximates the geoid well is an ellipsoid of revolution which is created by rotating an ellipse around its minor axis. The ellipsoid can be describe by two parameters, the semimajor axis, a_e, and the flattening, f. Flattening can be described in terms of the semimajor axis and the semiminor axis, b:

$$f = (a_e - b)/a_e$$

There are many different reference ellipsoids used. The ellipsoid adopted for this paper is the GRS 80 (Geodetic Reference System 80) which uses a semimajor axis of ae = 6378137.0 meters and a flattening of 1/f = 298.257222101. Figure 2-8 illustrates the advantage of using an ellipsoidal earth. The value of a_g is more accurate than a_s because the location of the observer, at point A, is closer to the true location, due to the fact that the earth resembles better an ellipsoid than a sphere.

Figure 2-9 shows an ellipsoid within the Cartesian geodetic coordinate frame (x, y, z). The semiminor axis of the ellipsoid coincides with the z-axis of the reference frame. The x-axis is the

FIGURE NOT TO SCALE



ϕ' = Geocentric Latitude
ϕ = Geodetic Latitude } $\phi = \phi'$

a_g = Elevation angle measure using ellipsiodal earth.

a_s = Elevation angle measured using spherical earth.

FIGURE 2-8. IMPROVEMENT IN ACCURACY BY ASSUMING AN ELLIPSOIDAL EARTH

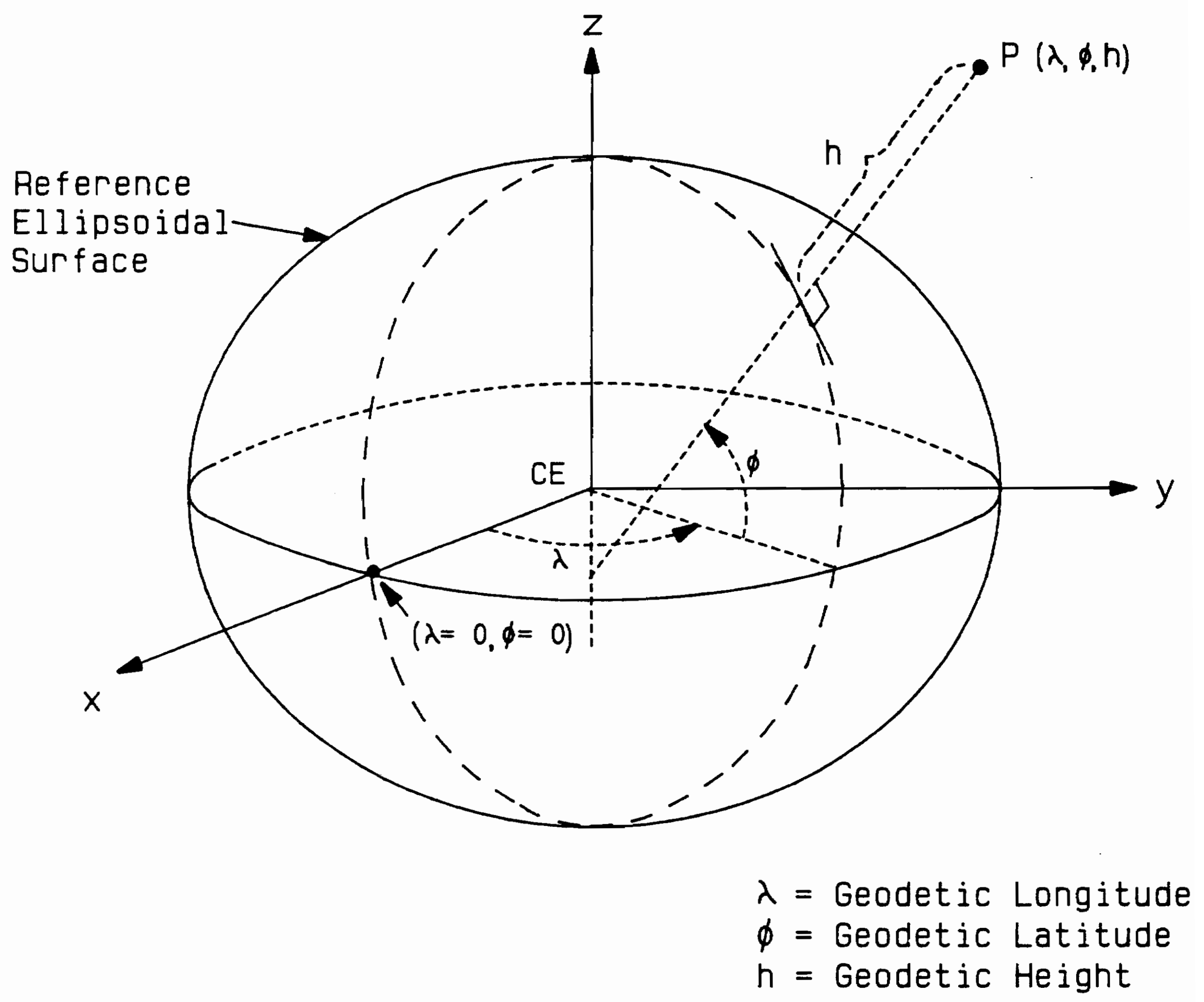


FIGURE 2-9. ELLIPSOIDAL REVOLUTION IN THE GEODETIC CARTESION REFERENCE FRAME WITH POINT 'P' EXPRESSED IN CURVILINEAR COORDINATES (λ, ϕ, h). [3, pp. 184]

ellipsoidal equator and corresponds to the point λ=0, ϕ=0. The length of a line normal to the surface of the ellipsoid from a point, P, to the surface is called the geodetic (ellipsoidal) height, h. The angle between the normal to the ellipsoid and the x-y plane (equatorial plane) is called the geodetic latitude, ϕ. Geodetic longitude, λ, is measured east from the x-axis. The curvilinear geodetic coordinates (λ,ϕ,h), also shown in Figure 2-9, specifies a points location using geodetic latitude, longitude, and ellipsoidal height. [3, pp. 176,271]

Once the location of a point is established with reference to the ellipsoid the relationship between the ellipsoid and the geoid must be known to locate the point with respect to the geoid. This paper assumes that the surface of the ellipsoid and the surface of the geoid are the same. Thus, the height above the ellipsoid, h, and the mean sea level height, H, are the same.

2.4.2 Program Inputs and Outputs

In order to use a geodetic reference frame the program requires the following angles in geodetic curvilinear coordinates:

ϕ_O = Geodetic Latitude of the earth station (in degrees).

λ_O = Geodetic Longitude of the earth station (in degrees).

H = h = Earth stations height above mean sea level (in meters).

ϕ_S = Geodetic Latitude of the subsatellite point (in degrees).

λ_S = Geodetic Longitude of the subsatellite point (in degrees).

Longitudes are measured positive east of Greenwich. Geodetic latitude and longitude are the angles supplied by common maps or atlases. Mean sea level height (H) can be obtained from a topographic map.

The program outputs the pointing angles for the satellite of interest given in the horizon coordinate system:

A = Azimuth (in degrees).

a = Elevation (in degrees).

2.4.3 Transformation From Curvilinear to Cartesian Coordinates

The first step in the calculation is to convert the program inputs, given in geodetic curvilinear coordinates, to cartesian geodetic coordinates. As seen from Figure 2-10 the distance from point P to the z-axis along the geodetic normal is equal to N + h, where N is the maximum radius of curvature at the point and h is the geodetic (ellipsoidal) height. It can be seen from Figure 2-9 that the x and y positions of point P are given as:

$$x = (N + h)\cos\lambda\cos\phi$$

$$y = (N + h)\sin\lambda\cos\phi$$

The distance between z = 0 and where the geodetic normal intersects the z-axis is equal to $Ne^2\sin\phi$, where 'e' is the eccentricity. The z position of point P, as illustrated in Figure 2-10, is then given as:

$$z = (N + h)\sin\phi - Ne^2\sin\phi = [N(1 - e^2) + h]\sin\phi$$

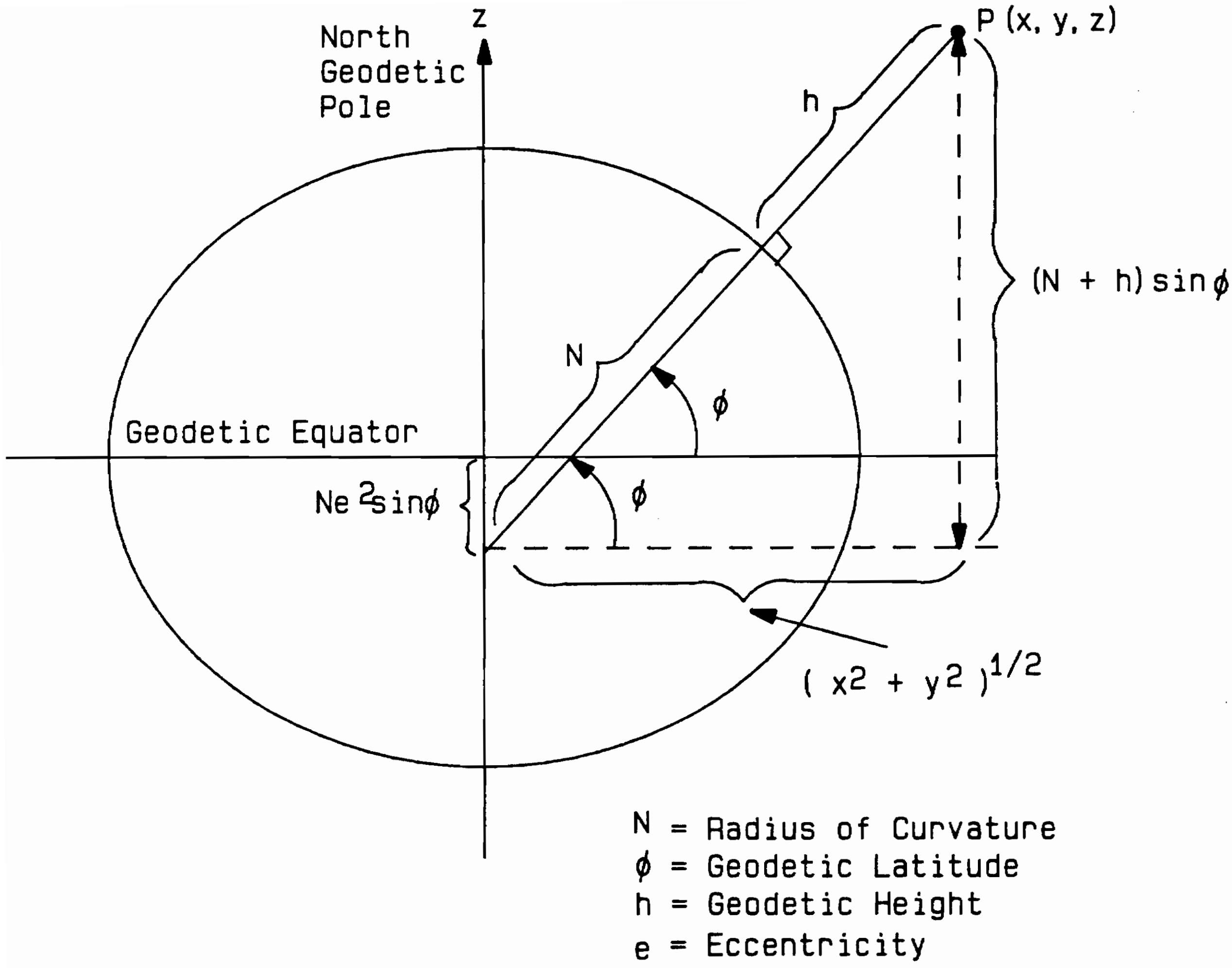


FIGURE 2-10. TRANSFORMING FROM CURVILINEAR GEODETIC TO CARTESIAN GEODETIC COORDINATES

Both the observer's location and the satellite's location are converted to Cartesian coordinates using this method. When calculating the Cartesian coordinates of the satellite the program assumes the satellite is in a perfect geostationary orbit with an orbital radius R_s = 42241558 m which converts to a geodetic height of $R_s - a_e$ = 35863421 m.

2.4.4 Transformation to Local Geodetic Coordinates

In order to transform the satellite's location into the local geodetic reference frame (e,n,u) the satellites position is first transformed to the 'local' (x,y,z) reference frame. The 'local' $(\Delta x, \Delta y, \Delta z)$ coordinates are the difference between the observers location and satellites location:

$$\Delta x = x_s - x_o$$

$$\Delta y = y_s - y_o$$

$$\Delta z = z_s - z_o$$

The subscript, s, indicates the satellites coordinates and the subscript, o, indicates the observers location.

The 'local' $(\Delta x, \Delta y, \Delta z)$ coordinates are then transformed to the local geodetic reference frame by rotating them into the (e,n,u) frame. The required rotation matrix, [R], is:

$$[R] = R_1(\pi/2 - \phi)R_3(\lambda + \pi/2)$$

(See Appendix D for an explanation of rotational matrices.) Then the local geodetic coordinates can be calculated as follows:

$$\begin{bmatrix} e \\ n \\ u \end{bmatrix} = [R] \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

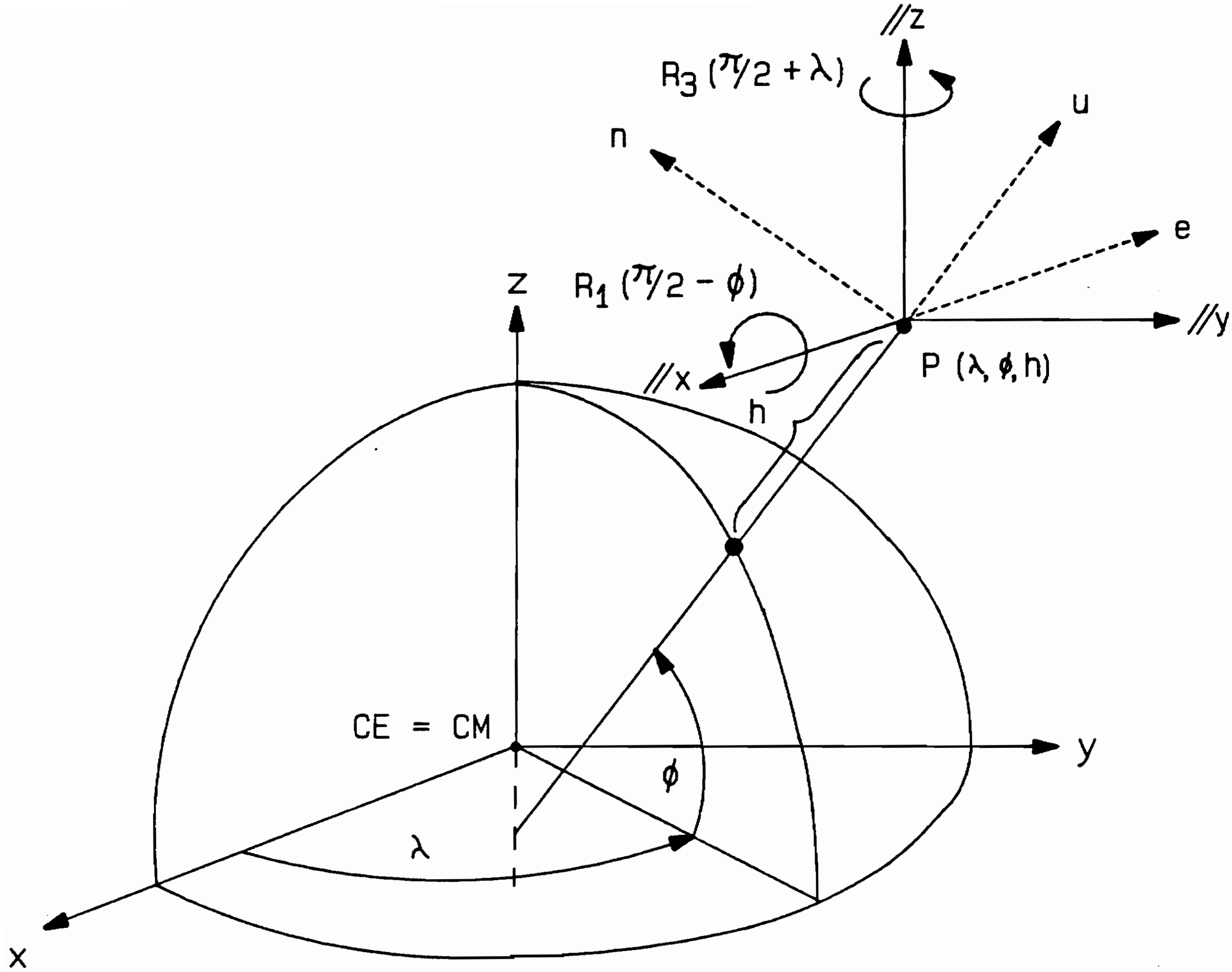
Figure 2-11 illustrates the transformation to the local geodetic reference frame and the rotations involved.

The geodetic azimuth and vertical angles are readily calculated from the (e,n,u) coordinates:

$$\text{Azimuth} = A = \tan^{-1}(e/n)$$

$$\text{Elevation} = a = \tan^{-1}[u/((e^2 + n^2)^{1/2})]$$

This program assumes that the geodetic azimuth and elevation angles are equivalent to the astronomic azimuth and altitude. In reality they differ by the deflection of the vertical components. The geodetic angles are referenced to the local geodetic horizon, which is a plane perpendicular to a vector normal to the surface of the reference ellipsoid at that location. The astronomic angles are referenced to the local astronomic horizon which is a plane perpendicular to the gravity vector at that location. Figure 2-12 shows the flow of the program. Appendix B is a listing of the Fortran code written to calculate satellite look angles assuming an ellipsoidal earth.



λ = Geodetic Longitude
φ = Geodetic Latitude
h = Geodetic Height

FIGURE 2-11. TRANSFORMATION FROM LOCAL GEODETIC (x, y, z) TO LOCAL GEODETIC (e, n, u)

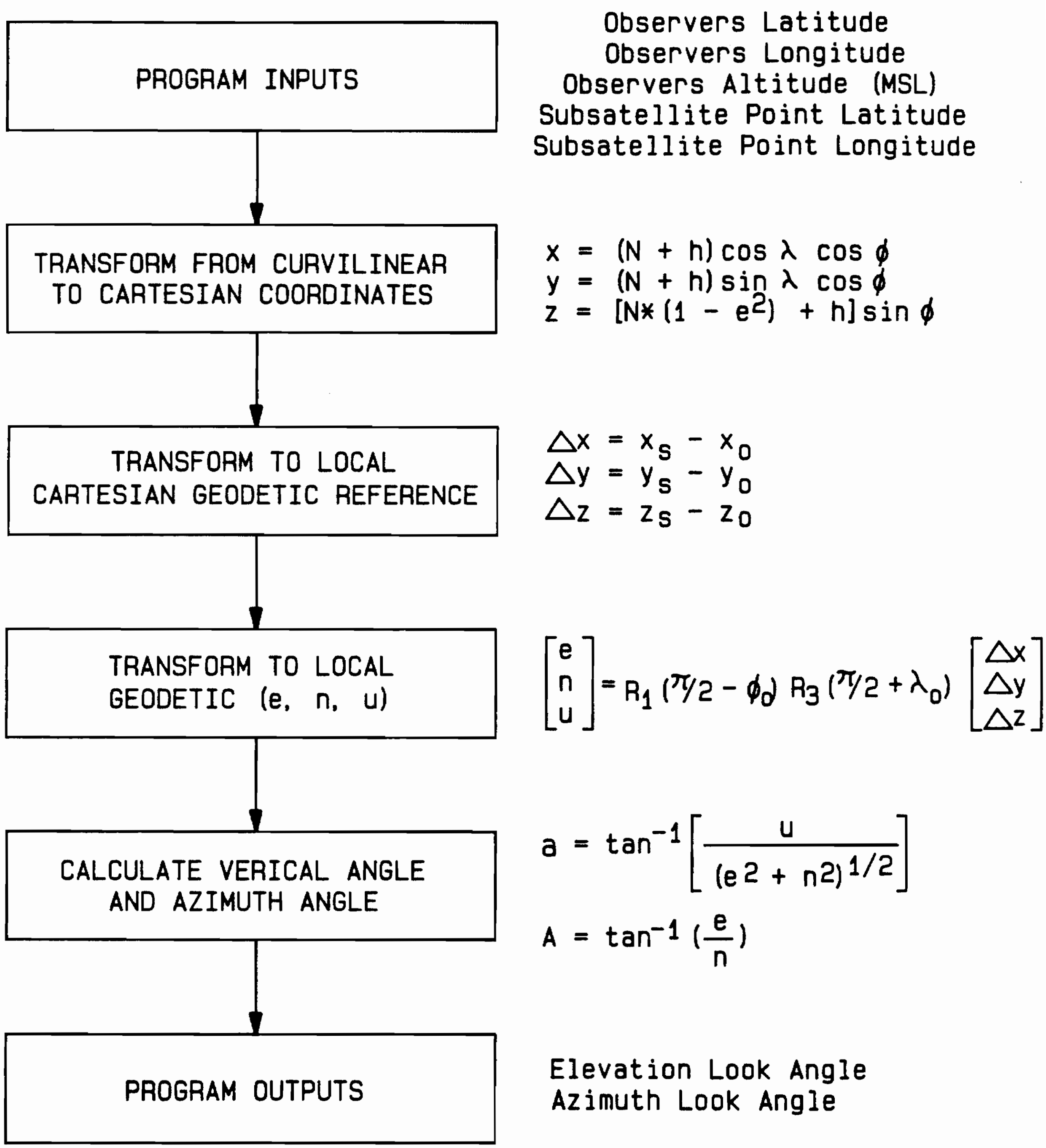


FIGURE 2-12. PROGRAM FLOWCHART FOR CALCULATING SATELITTE LOOK ANGLES USING A GEODETIC REFERENCE FRAME.

2.5 Results

Table 2-2 lists calculated pointing angles using both methods (assuming a spherical earth and using an ellipsoidal earth) with the ground station centrally located in all eight quadrants of the earth and the satellite on the equator at the same longitude as the station. The results show both methods work in any quadrant. The small differences between the two solutions are expected because the ellipsoidal method is more rigorous.

Table 2-3 lists the calculated angles using both methods for northern latitudes between 0^{o} and 80^{o} every 5^{o} and at an arbitrary longitude selected in the table to be 0^{o}. The longitude of the satellite and earth station are identical, isolating the difference in calculated elevation angles. Above 80^{o} the satellite is below the local horizon and is not visible by the earth station. Figure 2-13 is a plot of the difference between the elevation angles versus latitude calculated by the two methods. This plot can also be interpreted as the improvement in accuracy gained by using an ellipsoidal model for the earth.

As shown in Figure 2-13, the worst case error is approximately $0^{o}.023$. Therefore, if the earth station antenna beam width is much greater than $0^{o}.023$, using a geodetic reference frame is not necessary. As the beamwidth approaches this value it becomes increasingly more important to include the oblateness of the earth in the calculation. If the beamwidth is equal to $0^{o}.046$ a pointing error of $0^{o}.023$ would cause

Table 2-2. Calculated Pointing Angles Using Both Methods With the Earth Station in All Eight Quadrants of the Earth (degrees).

EARTH STATION LOCATION		SUBSATELLITE POINT LOCATION		POINTING ANGLES ASSUMING A SPHERICAL EARTH		POINTING ANGLES USING AN ELLIPSOIDAL EARTH	
Longitude	Latitude	Longitude	Latitude	Azimuth	Elevation	Azimuth	Elevation
45	-45	45	0	0	38.194	0	38.216
45	45	45	0	180	38.194	180	38.216
135	-45	135	0	0	38.194	0	38.216
135	45	135	0	180	38.194	180	38.216
225	-45	225	0	0	38.194	0	38.216
225	45	225	0	180	38.194	180	38.216
315	-45	315	0	0	38.194	0	38.216
315	45	315	0	180	38.194	180	38.216

Table 2-3. Calculated Pointing Angles from Different Earth Station Latitudes.

EARTH STATION LOCATION		SUBSATELLITE POINT LOCATION		POINTING ANGLES ASSUMING A SPHERICAL EARTH		POINTING ANGLES ASSUMING AN ELLIPSOIDAL EARTH		DIFFERENCE (ELLIPSOIDAL - SPHERICAL METHOD)	
Longitude	Latitude	Longitude	Latitude	Azimuth	Elevation	Azimuth	Elevation	Azimuth	Elevation
0	0	0	0	-	90.000	180.0005	90.000	0.0005	0.0046
0	5	0	0	180	84.114	180	84.118	0	0.0090
0	10	0	0	180	78.239	180	78.247	0	0.0129
0	15	0	0	180	72.384	180	72.397	0	0.0163
0	20	0	0	180	66.561	180	66.577	0	0.0190
0	25	0	0	180	60.778	180	60.797	0	0.0211
0	30	0	0	180	55.043	180	55.064	0	0.0223
0	35	0	0	180	49.364	180	49.386	0	0.0229
0	40	0	0	180	43.746	180	43.769	0	0.0230
0	45	0	0	180	38.194	180	38.216	0	0.0229
0	50	0	0	180	32.710	180	32.733	0	0.0224
0	55	0	0	180	27.299	180	27.321	0	0.0217
0	60	0	0	180	21.960	180	21.981	0	0.0206
0	65	0	0	180	16.695	180	16.715	0	0.0196
0	70	0	0	180	11.502	180	11.521	0	0.0187
0	75	0	0	180	6.381	180	6.399	0	0.0179
0	80	0	0	180	1.329	180	1.347	0	0.0176
0	85	0	0	*	*	*	*	*	*
0	90	0	0	*	*	*	*	*	*

* Satellite is not visible.

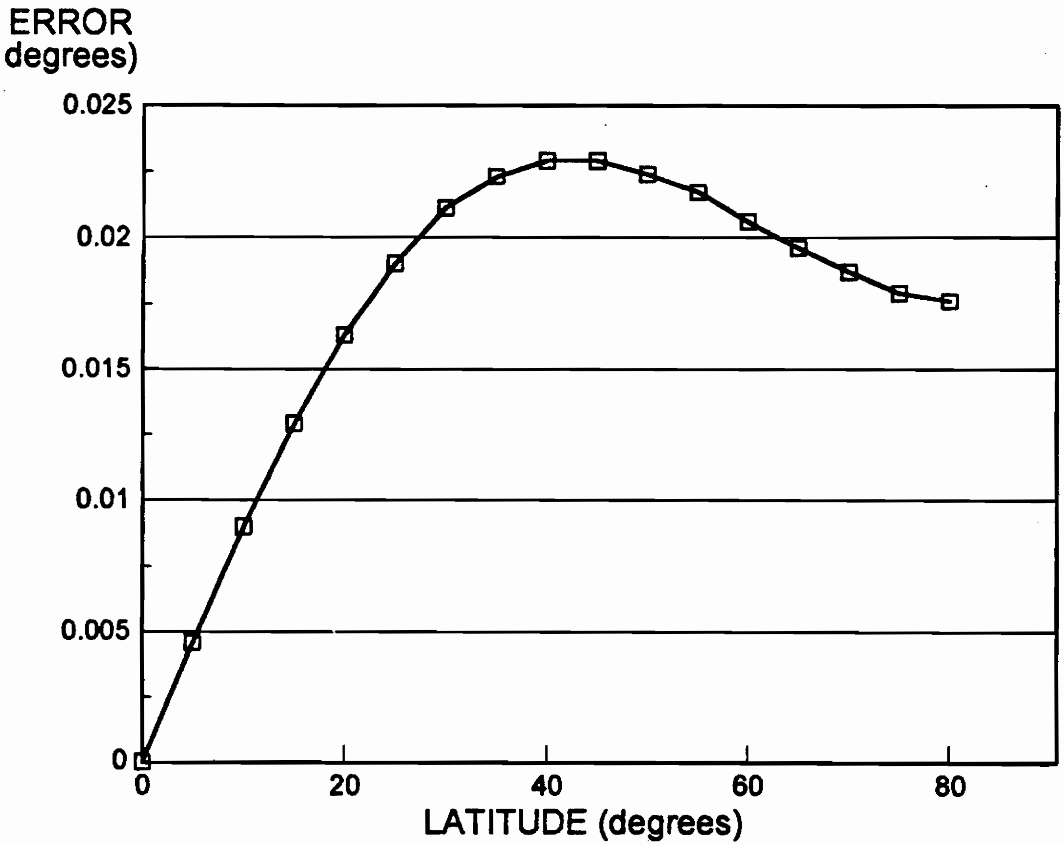


Figure 2-13. Difference in Degrees in Calculated Elevation Pointing Angles Versus Latitude of the Observer (Ellipsoidal Method - Spherical Method)

a 50% decrease in received power. This discrepancy in pointing becomes significant with very high gain antennas and even more crucial when electroptical devices such as lasers are used.

Table 2-4 lists calculated angles for an earth station located at a fixed latitude, 45^{o}, and the satellite at different longitudes. This table shows the difference in azimuth between the two calculations with a peak difference of $0^{o}.0290$.

This chapter has developed and illustrated two methods for calculating pointing angles from an earth station to a satellite. From the results it can be concluded that, where a high degree of pointing accuracy is required, modeling the earth as an ellipsoid is renders greater accuracy.

Table 2-4. Calculated Pointing Angles With the Satellite and Earth Station at Different Longitudes.

EARTH STATION LOCATION		SUBSATELLITE POINT LOCATION		POINTING ANGLES ASSUMING A SPHERICAL EARTH		POINTING ANGLES ASSUMING AN ELLIPSOIDAL EARTH		DIFFERENCE (ELLIPSOIDAL - SPHERICAL METHOD)	
Longitude	Latitude	Longitude	Latitude	Azimuth	Elevation	Azimuth	Elevation	Azimuth	Elevation
0	45	0	0	180.0000	38.1935	180.0000	38.2164	0	0.0229
0	45	10	0	165.9981	37.2411	165.9883	37.2629	-0.0098	0.0218
0	45	20	0	152.7637	34.5024	152.7459	34.5215	-0.0178	0.0191
0	45	30	0	140.7685	30.2785	140.7453	30.2941	-0.0232	0.0156
0	45	40	0	130.1207	24.9386	130.0943	24.9504	-0.0264	0.0118
0	45	50	0	120.6821	18.8282	120.6540	18.8367	-0.0281	0.0085
0	45	60	0	112.2077	12.2299	112.1789	12.2358	-0.0288	0.0059
0	45	70	0	104.4328	5.3605	104.4038	5.3646	-0.0290	0.0041
0	45	75	0	100.7286	1.8768	100.6996	1.8804	-0.0290	0.0036
0	45	-10	0	194.0019	37.2411	194.0117	37.2629	0.0158	0.0218
0	45	-20	0	207.2363	34.5024	207.2541	34.5215	0.0178	0.0191
0	45	-30	0	219.2315	30.2785	219.2547	30.2941	0.0232	0.0156
0	45	-40	0	229.8792	24.9386	229.9057	24.9504	0.0265	0.0118
0	45	-50	0	239.3179	18.8282	239.3460	18.8367	0.0281	0.0085
0	45	-60	0	247.7923	12.2299	247.8211	12.2358	0.0288	0.0059
0	45	-70	0	255.5672	5.3605	255.5962	5.3646	0.0290	0.0041
0	45	-75	0	259.2714	1.8768	259.3004	1.8804	0.0290	0.0036

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CHAPTER 3. POINTING AT RADIO SOURCES

3.1 Corrections to the Positions of Radio Sources

The positions of galactic or extragalactic radio sources are generally given using equatorial coordinates (section 1.5.1.1). It is often assumed that this position is fixed with respect to the equatorial reference frame and the celestial sphere. However, the actual position of a radio source varies due to a variety of effects:

> "1) The motion of the coordinate systems with respect to inertial space (precession and nutation) and with respect to the solid earth (polar motion).
>
> 2) The apparent displacements in the directions of stars due to physical phenomena" (refraction and, aberration). [1, pp. 59]

Table 3-1 lists the changes due to the motion of the coordinate systems to the corrected position of a radio sources location.

3.1.1 Precession, Nutation, and Polar Motion

Precession and Nutation change the position of the vernal equinox with respect to the equatorial plane. Due to the oblateness of the earth the gravitational pull of sun, moon, and planets produce torques on the earth. The primary torque is caused by the sun and moon. The sun causes the earth to wobble or precess similar to the motion of a simple top.

TABLE 3-1. Spectrum of Changes in the Earth's Rotation*

A. Inerial Orientation of Spin Axis	B. Terrestrail Orientation of Spin Axis (Polar Motion)	C. Instantaneous Spin Rate ω About Axiz
1. Steady precession: amplitude 23°.5; period = 25,700 years. 2. Principal nutation: amplitude 9".20 (obliquity), 6".86 (longitude); period 18.6 years. 3. Other periodic contributions to nutation in obliquity and longitude: amplitudes < 1"; periods 9.3 yeares, annual, semiannual, and fortnightly. 4. Discrepancy in secular decrease in obliquity: 0".1/century (?).	1. Secular motion of pole: irregular, $\doteq$ 0".2 in 70 years. 2. 'Markowitz' wobble: amplitude $\doteq$ 0".02(?); period 24-40 years (?). 3. Chandler wobble: amplitude (variable) = 0".15; period 425-440 days; damping time 10-70 years(?). 4. Seasonal wobbles: annual, amplitude = 0".09; semiannual, amplitude = 0".01. 5. Monthly and fortnightly wobbles; (theoretical) amplitudes = 0".001. 6. Nearly diurnal free wobble: amplitude $\lesssim$ 0".02(?); period(s) within a few minutes of a sidereal day. 7. Oppolzer terms: amplitudes = 0".02; periods as for nutations.	1. Secular acceleration: $\dot{\omega}/\omega \doteq -5 \times 10^{-10}$/yr. 2. Irregular changes: (a) over centuries, $\dot{\omega}/\omega \lesssim \pm 5 \times 10^{-10}$/yr (b) over 1-10 years, $\dot{\omega}/\omega \lesssim \pm 5 \times 10^{-10}$/yr, (c) over a few weeks or months ('abrupt'), $\dot{\omega}/\omega \lesssim \pm 500 \times 10^{-10}$/yr. Short-period variations: (a) biennial, amplitude = 9 msec; (b) annual, amplitude = 20-25 msec; (c) semiannual, amplitude = 9 msec; (d) monthly and fortnightly. amplitudes = 1 msec.

* From [Rochester, 1973]

Since the earth is tilted with respect to the sun at approximately 23.5^{o}, the polar axis, due to precession, will sweep out a conical surface with a semi-vertex angle of 23.5^{o} at a period of approximately 26,000 years.

The moon also perturbs the rotation of the earth producing an additional torque. "The effect of the moon is to superimpose a slight nodding motion called 'nutation', with a period of 18.6 years, on the slow westward precession caused by the sun." [2, pp. 105] This is the principal cause of nutation, however, due to the gravitational pull of the planets there are smaller contributions to nutation. Figure 3-1 depicts the effects of precession and nutation on the rotation of the earth.

Another motion of the equatorial reference frame in relation to radio sources is called polar motion. The Celestial Ephemeris Pole (CEP), which is the Z-axis of the equatorial reference frame, moves with respect to the lithosphere (crust) of the earth. This motion of the pole was first determined by optical observations of the stars.

3.1.1.1 Correcting For Precession [3, pp. 13-16]

As mentioned in section 1.5.1.1 the vernal equinox is the point of intersection where the sun crosses the celestial equator from south to north. The plane which contains the earth's orbit around the sun is called the ecliptic plane. The motion of the vernal equinox due to precession can be described in terms of the motion of the equatorial plane with respect to the ecliptic plane. Figure 1.4 shows the

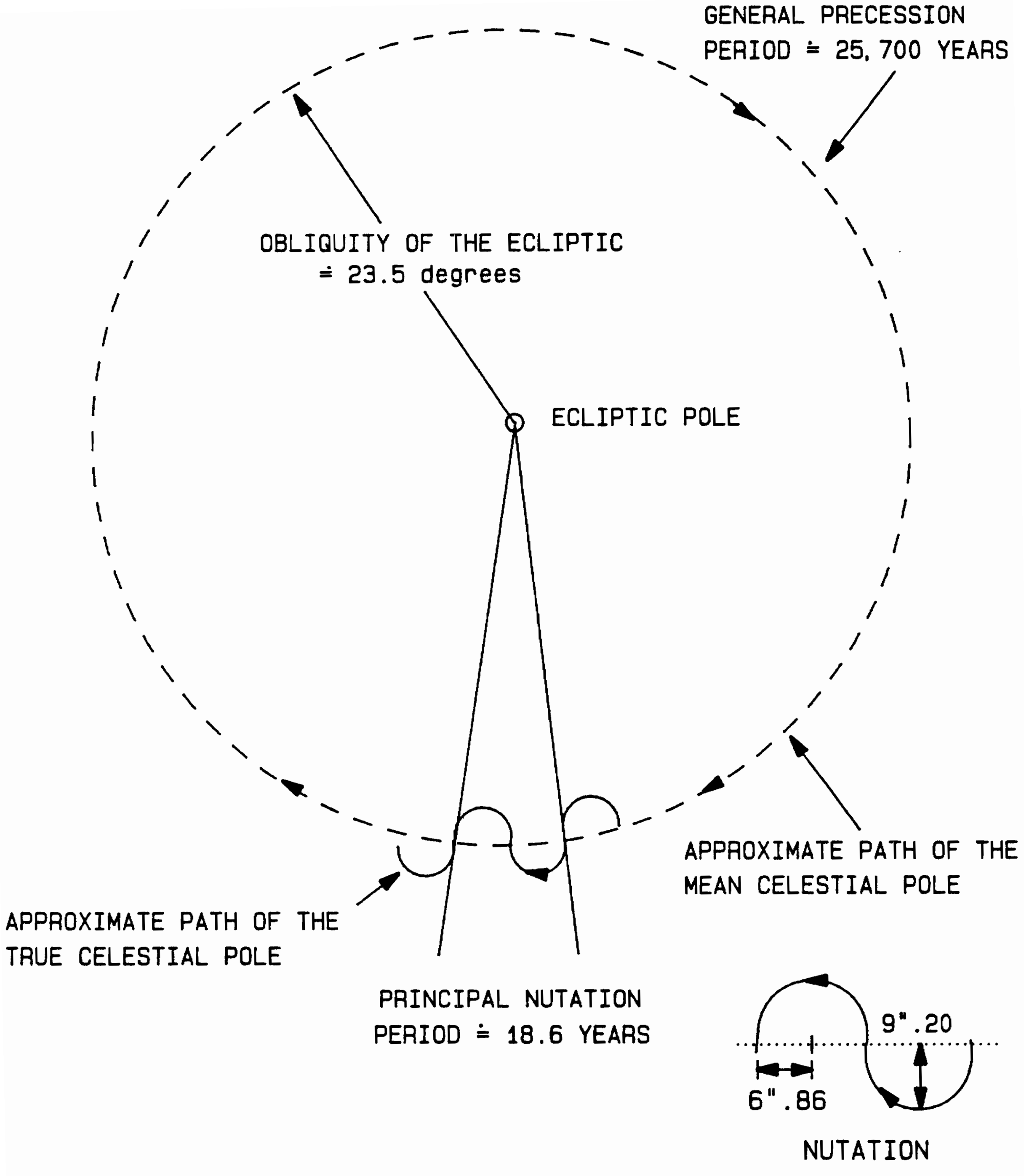


FIGURE 3-1. EFFECTS OF PRECESSION AND NUTATION [3, pp. 70]

relationship between the ecliptic plane and the equatorial plane. The angle between the ecliptic and equatorial plane is termed the obliquity of the ecliptic, ε, and is approximately equal to 23.5^{o}.

Figure 3.2 shows the intersection of the ecliptic and equatorial planes and their motion with respect to each other, due to precession, from some starting epoch, T, to an ending epoch, T_o. *(A Radio Source Catalog gives the positions of source for a particular time (epoch). A source's position must be corrected from the epoch of the catalog (T_o) to the epoch of the observation (T).)* The coordinate system corrected only for "precession is called the 'mean celestial coordinate system' and the respective positions are referred to as the 'mean celestial positions'" (also mean celestial equator, and mean coordinates) [3, pp. 16]. The transformation from epoch T_o to mean epoch T' is given by:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{T'} = [P] \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{T_o}$$

Where [P] is the precession matrix:

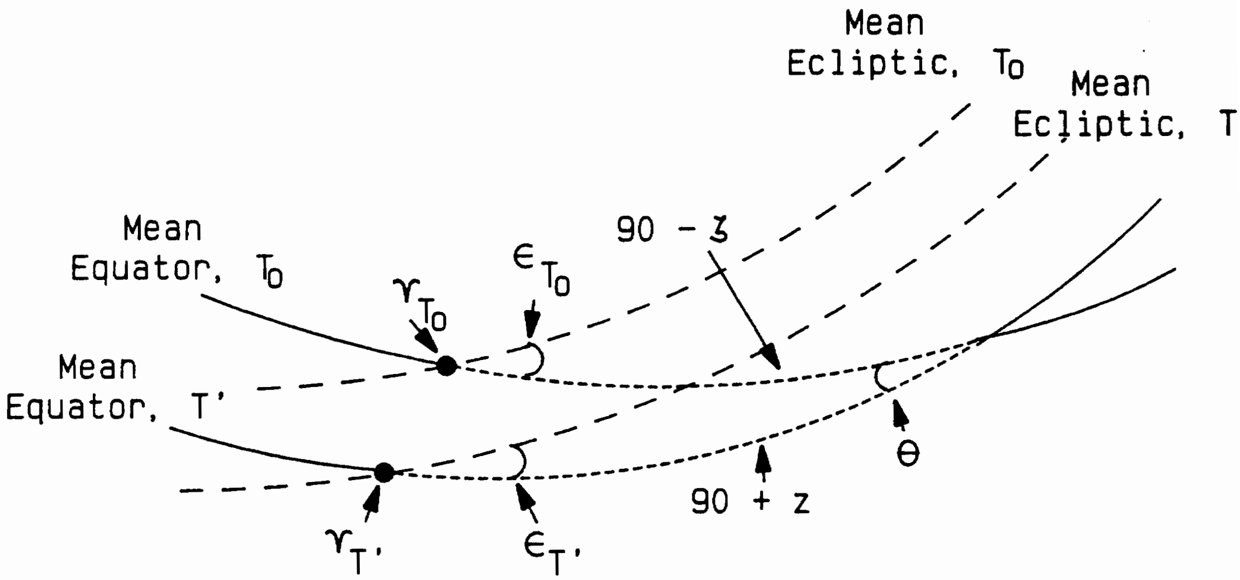
$$[P] = R_3(-\pi/2 - z)R_1(\theta)R_3(\pi/2 - \zeta)$$

Or explicitly:

$$P(1,1) = \cos\zeta\cos\theta\cos z - \sin\zeta\sin z$$

$$P(1,2) = \cos\zeta\cos\theta\sin z - \sin\zeta\cos z$$

$$P(1,3) = \cos\zeta\sin\theta$$



Υ = mean vernal equinox
ϵ = obliquity of the ecliptic
P = precession

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{T'} = P \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{T_0}$$

$$P = R_3(-90 - z) R_1(\theta) R_3(90 - \zeta)$$

FIGURE 3-2. THE MOTION OF THE EQUATORIAL PLANE WITH RESPECT TO THE ECLIPTIC DUE TO PRECESSION. [3, pp. 15]

$P(2,1) = -\sin\zeta\cos\theta\cos z - \cos\zeta\sin z$

$P(2,2) = -\sin\zeta\cos\theta\sin z - \cos\zeta\cos z$

$P(2,3) = -\sin\zeta\sin\theta$

$P(3,1) = -\sin\theta\cos z$

$P(3,2) = -\sin\theta\sin z$

$P(3,3) = \cos\theta$ [4, pp. B18]

The equatorial precession elements, (θ, z, ζ), can be calculated as follows: [4, pp. B18]

$$\zeta = 0^o.6406161*T_o + 0^o.0000839*T_o^2 + 0^o.0000050*T_o^3$$

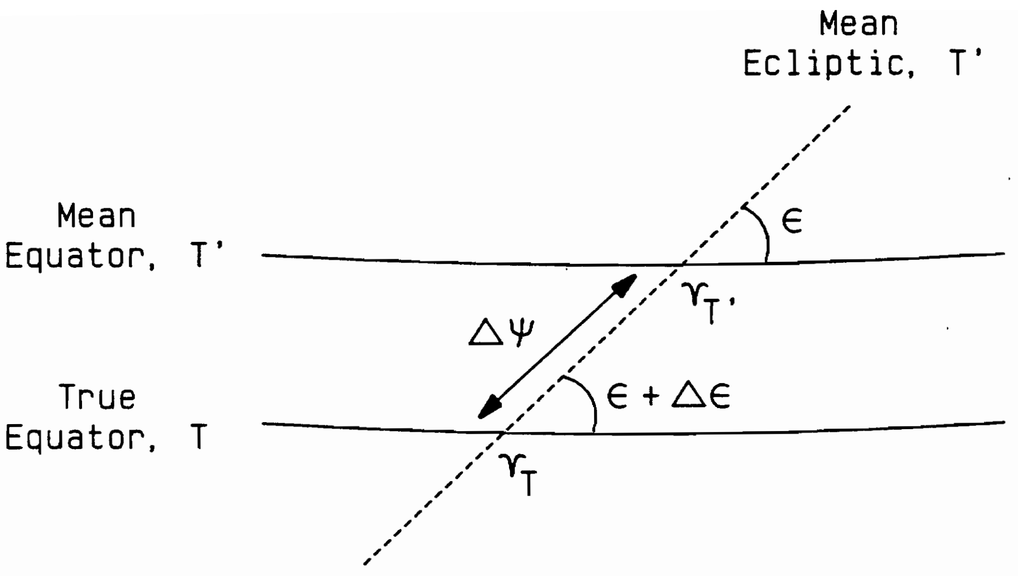
$$z = 0^o.6406161*T_o + 0^o.0003041*T_o^2 + 0^o.0000051*T_o^3$$

$$\theta = 0^o.5567530*T_o + 0^o.0001184*T_o^2 + 0^o.0000116*T_o^3$$

Where $T_o = (JD - 2451545.0)/36525$ and JD is the Julian date of the observation. Appendix E gives a listing of functions written in Fortran including a routine to calculate Julian date.

3.1.1.2 Correction For Nutation [3, pp. 16-18]

A coordinate system corrected for both precession and nutation is referred to as 'true' and the respective positions are referred to as true positions. True celestial positions differ from the mean celestial positions by the nutations calculated for the epoch of the date of observation as illustrated in Figure 3.3. $\Delta\psi$ is the nutation in



Υ = vernal equinox
ϵ = obliquity of the ecliptic
N = nutation

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_T = N \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{T'}$$

$$N = R_1(-\epsilon - \Delta\epsilon)R_3(-\Delta\Psi)R_1(\epsilon)$$

FIGURE 3-3. THE MOTION OF THE EQUATORIAL PLANE WITH RESPECT TO THE ECLIPTIC DUE TO NUTATION. [3, pp. 17]

longitude and $\Delta\varepsilon$ is the nutation is obliquity.

The nutation elements, $\Delta\psi$ and $\Delta\varepsilon$, can be approximated using the 1980 IAU theory of nutation. The IAU theory of nutation represents the nutational elements "by a series expansion of the sines and cosines of linear combinations of five fundamental arguments" [class handout] which "describe the mean positions of the sun and the moon" [3, pp. 18].

$a_1 = l$ = mean anomaly of the moon = $485866''.733 + (1325^r + 715922''.633)T_o + 31''.310T_o^2 + 0''.064T_o^3$

$a_2 = l'$ = mean anomaly of the sun = $1287099''.804 + (99^r + 1291281''.224)T_o - 0''.577T_o^2 - 0''.012T_o^3$

$a_3 = F$ = mean argument of latitude of the moon = $335778''.877 + (1342^r + 295263''.137)T_o - 13''.257T_o^2 + 0''.011T_o^3$

$a_4 = D$ = mean elongation of the moon from the sun = $1072261''.307 + (1236r + 1105601''.328)T_o - 6''.891T_o^2 + 0''.019T_o^3$

$a_5 = \Omega$ = mean longitude of the ascending lunar node = $450160''.280 + (5^r + 482890''.539)T_o + 7''.455T_o^2 + 0''.008T_o^3$

(Note: $1^r = 360^o = 129600''$)

Using these arguments the nutational elements can be calculated as follows:

$$\Delta\psi = S_{(j=1,N)}[(A_{0j} + A_{1j}T)\sin[S_{(i=1,5)}k_{ji}a_i(T)]]$$

$$\Delta\varepsilon = S_{(j=1,N)}[(B_{0j} + B_{1j}T)\cos[S_{(i=1,5)}k_{ji}a_i(T)]]$$

Table 3-2 gives the values for a_i, k_{ji}, A_j, and B_j for the first 13 entries. The complete series uses 106 entries.

Correcting from the mean position (T') to the true position (T) is accomplished using the following transform:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_T = [N] \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{T'}$$

Where [N] is the nutation matrix:

$$[N] = R1(-\varepsilon - D\varepsilon)R_3(-\Delta\psi)R1(\varepsilon)$$

Or explicitly:

$$N(1,1) = \cos\Delta\psi$$

$$N(1,2) = \sin\Delta\psi \cos(\varepsilon + \Delta\varepsilon)$$

$$N(1,3) = \sin\Delta\psi \sin(\varepsilon + \Delta\varepsilon)$$

$$N(2,1) = -\sin\Delta\psi \cos\varepsilon$$

$$N(2,2) = \cos(\varepsilon + \Delta\varepsilon)\cos\Delta\psi \cos\varepsilon + \cos\varepsilon \cos(\varepsilon + \Delta\varepsilon)$$

$$N(2,3) = -\cos(\varepsilon + \Delta\varepsilon)\sin\varepsilon + \sin(\varepsilon + \Delta\varepsilon)\cos\Delta\psi \cos\varepsilon$$

$$N(3,1) = -\sin\Delta\psi \sin e$$

$$N(3,2) = -\sin(\varepsilon + \Delta\varepsilon)\cos\varepsilon + \cos(\varepsilon + \Delta\varepsilon)\cos\Delta\psi \sin\varepsilon$$

$$N(3,3) = \cos(\varepsilon + \Delta\varepsilon)\cos\Delta\psi \sin\varepsilon + \cos(\varepsilon + \Delta\varepsilon)\cos\varepsilon$$

$$\varepsilon = 23.43929111 - 0.0130041666*T_o - 0.000000163889*T_o^2 + 0.0000005036111T_o^3$$

Table 3-2. Partial Listing of 1980 IAU Nutations
[3, pp. 18]

	ARGUMENT					PERIOD	LONGITUDE		OBLIQUITY	
	l	l'	F	D	Ω	(DAYS)	(0".0001)		(0".0001)	
1	0	0	0	0	1	6798.4	-171996	-174.2T	92025	8.9T
2	0	0	0	0	2	3399.2	2062	0.2T	-895	0.5T
3	0	0	2	-2	2	182.6	-13187	-1.6T	5736	-3.1T
4	0	1	0	0	0	365.3	1426	-3.4T	54	-0.1T
5	0	1	2	-2	2	121.7	-517	1.2T	224	-0.6T
6	0	-1	2	-2	2	365.2	217	-0.5T	-95	0.3T
7	0	0	2	-2	1	177.8	129	0.1T	-70	0.0T
8	0	0	2	0	2	13.7	-2274	-0.2T	997	-0.5T
9	1	0	0	0	0	27.6	712	0.1T	-7	0.0T
10	0	0	2	0	1	13.6	-386	-0.4T	200	0.0T
11	1	0	2	0	2	9.1	-301	0.0T	129	-0.1T
12	1	0	0	-2	0	31.8	-158	0.0T	-1	0.0T
13	-1	0	2	0	2	27.1	123	0.0T	-53	0.0T

T_o = (JD - 2451545.0)/36525

Appendix E contains a listing of a routine which calculates the nutational elements.

3.1.1.3 Correcting For Polar Motion

The z-axis of the Conventional Terrestrial Reference System is the Conventional Terrestrial Pole (CTP). The CTP is fixed with respect to the lithosphere of the earth. The position of the CTP was established via optical observations around the year 1900. Polar motion is measured using the coordinates x_p and y_p which describe the displacement of the CEP from the CTP. The CTP is located at x_p = 0 and y_p = 0. Figure 3-4 shows the motion of the CEP with respect to the CTP between the years of 1984 and 1987. Referencing Figure 3-5, a position is corrected for polar motion using the following rotations:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_T = R_2(-x_p)R_1(-y_p)R_3(GAST) \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_T$$

The angle 'GAST' is the angle between the vernal equinox and the Greenwhich Meridian. The rotation R_3(GAST) converts the coordinate system from an inertial celestial fixed system to an earth fixed system. Values for 'GAST' are tabulated in the Astronomical Almanac or can be calculated as follows:

$$GAST = GMST + \Delta\psi\cos(\varepsilon + \Delta\varepsilon)$$

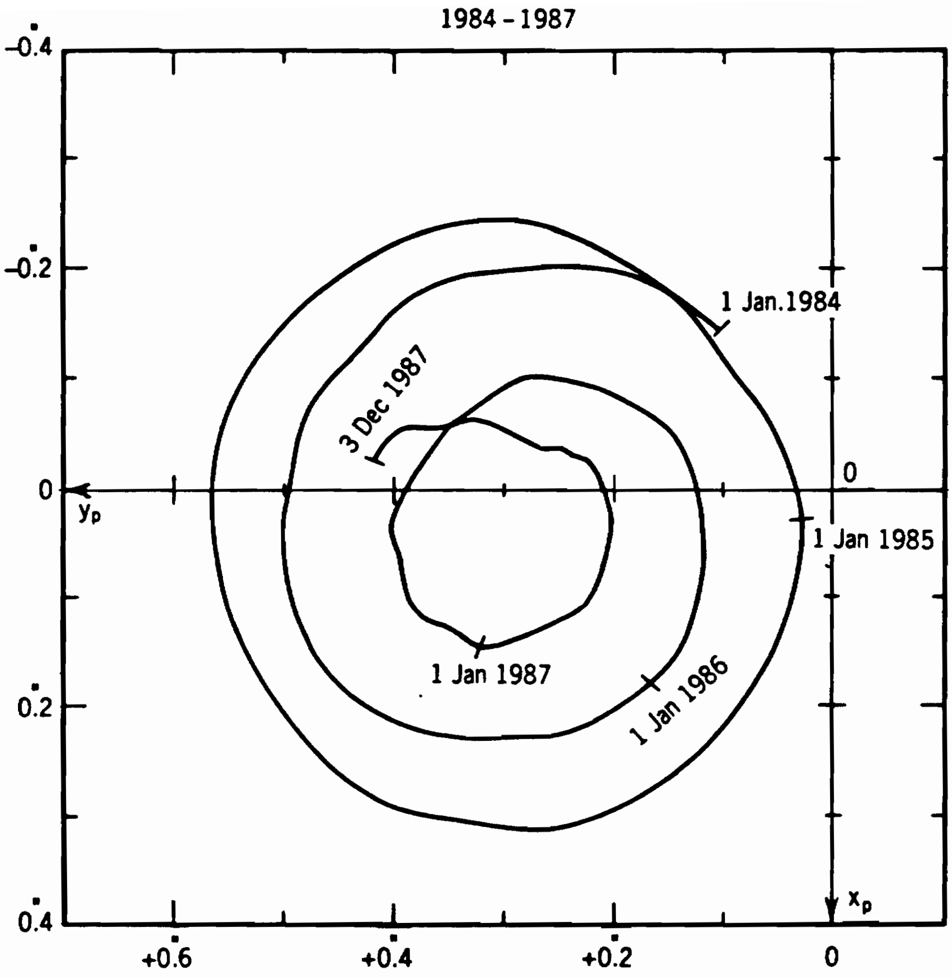
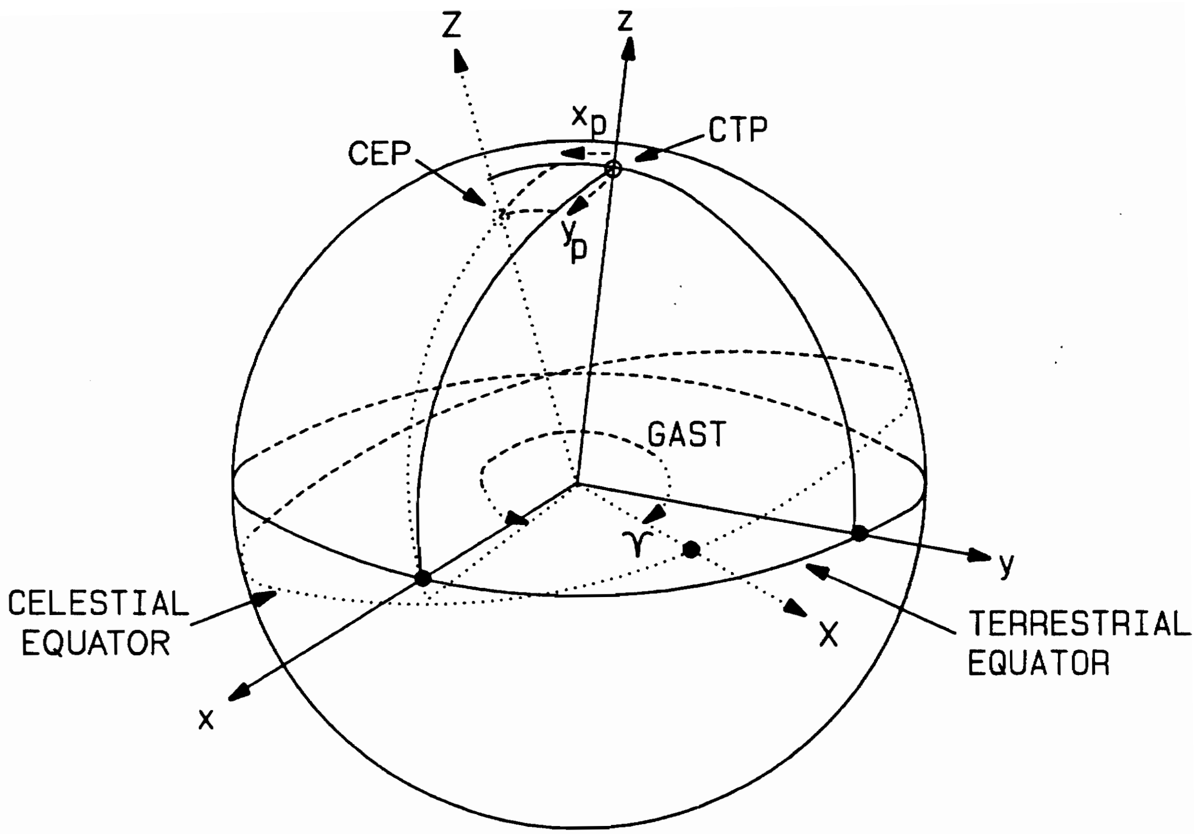


FIGURE 3-4. POLAR MOTION OF THE CEP BETWEEN 1984 AND 1987 [3, pp. 20]



$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_T = M \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_T$$

$$M = R_2(-x_p)\,R_1(-y_p)\,R_3(GAST)$$

FIGURE 3-5. CORRECTION FOR POLAR MOTION AND CONVERSION TO A FIXED TERRESTRIAL COORDINATE SYSTEM.

$$GMST = 24110^s.54841 + 8640184^s.812866T_o + 0^s.093104T^2_o - 6^s.2x10-6T^3_o$$

$$T_o = (JD - 2451545.0)/(36525)$$

The coordinates x_p and y_p are obtained using various "geodetic space techniques, such as Lunar Laser Ranging (LLR), Satellite Laser Ranging (SLR), and Very Long Baseline Interferometry (VLBI)."[3, pp. 20] Publications such as the 'Earth Orientation Bulletin, published by the Subcommission International Radio Interferometric Surveying (IRIS), give values for x_p and y_p.

3.1.2 Aberration, and Astronomic Refraction [5, pp. 44-45]

The actual position of a radio source may appear to be displaced due to a variety of physical effects including annual and diurnal aberration, and astronomic refraction. Annual aberration is the result of the earth orbiting the sun. The velocity of light received from an object combined with the earth's orbital velocity, 30 km/sec., causes an apparent displacement of the objects true position. Over the period of a year a celestial object will appear to move in an ellipse around it's true position. The maximum amount of displacement due to annual aberration is approximately 20".5.

An object's apparent position is also effected in a similar manner by the rotation of the earth about it's axis. This is know as diurnal aberration. The maximum amount of displacement due to diurnal aberration is 0".319 in declination and $0^s.0213$ in right ascension.

"Astronomic refraction is the apparent displacement of the celestial object outside the atmosphere that results from radio waves being bent in passing through the atmosphere. This results in all objects appearing higher above the horizon than they would appear if there were no refraction." [1, pp. 89-90]

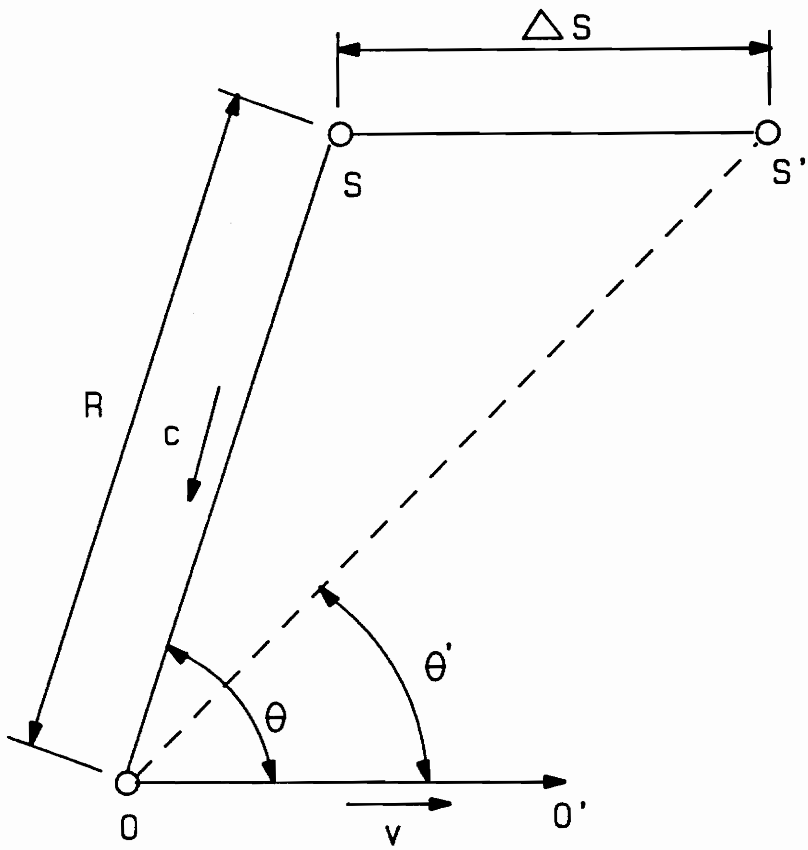
3.1.2.1 Correcting For Aberration [1, pp. 91-99]

Figure 3-6 illustrates the general law of aberration. The law can be stated as follows:

> "The apparent direction of an object seen by an observer in motion is displaced from the true direction in which it would be seen if the observer were motionless by an amount equal in linear measure to the observer's motion at a constant speed during the time interval the light propagated from the object to the observer. The direction of the displacement is that of the observer's motion at the moment of the observation [Newcomb, 1906]" [1, pp. 91]

The linear displacement of the object, Ds, is:

$$\Delta S = v*R/c = \eta R$$



$\Delta S = vR/c = \eta R$

$\eta = \Delta S/R = v/c$

R = distance to object
O = observer
S = celestial object
c = speed of light
v = observer's velocity
θ = angular displacement
η = constant of aberation

FIGURE 3-6. THE GENERAL LAW OF ABERRATION
[1, pp. 91]

Where:

$$\eta = \Delta S/R = v/c$$

The variable 'v' is the observer's velocity, 'R' is the distance from the observer to the object, and 'c' is the speed of light.

As stated earlier, annual aberration is due to earth's orbital velocity around the sun. Figure 3-7 illustrates the geometry used in calculating the effects of annual aberration on the true position of an object. The displacement in declination, $\Delta\delta_a$, is calculate as follows:

$$\Delta\delta_a = -\eta[\cos\lambda_s \cos\varepsilon \, (\tan\varepsilon \cos\delta - \sin\alpha \sin\delta) + \cos\alpha \sin\delta \sin\lambda_s]$$

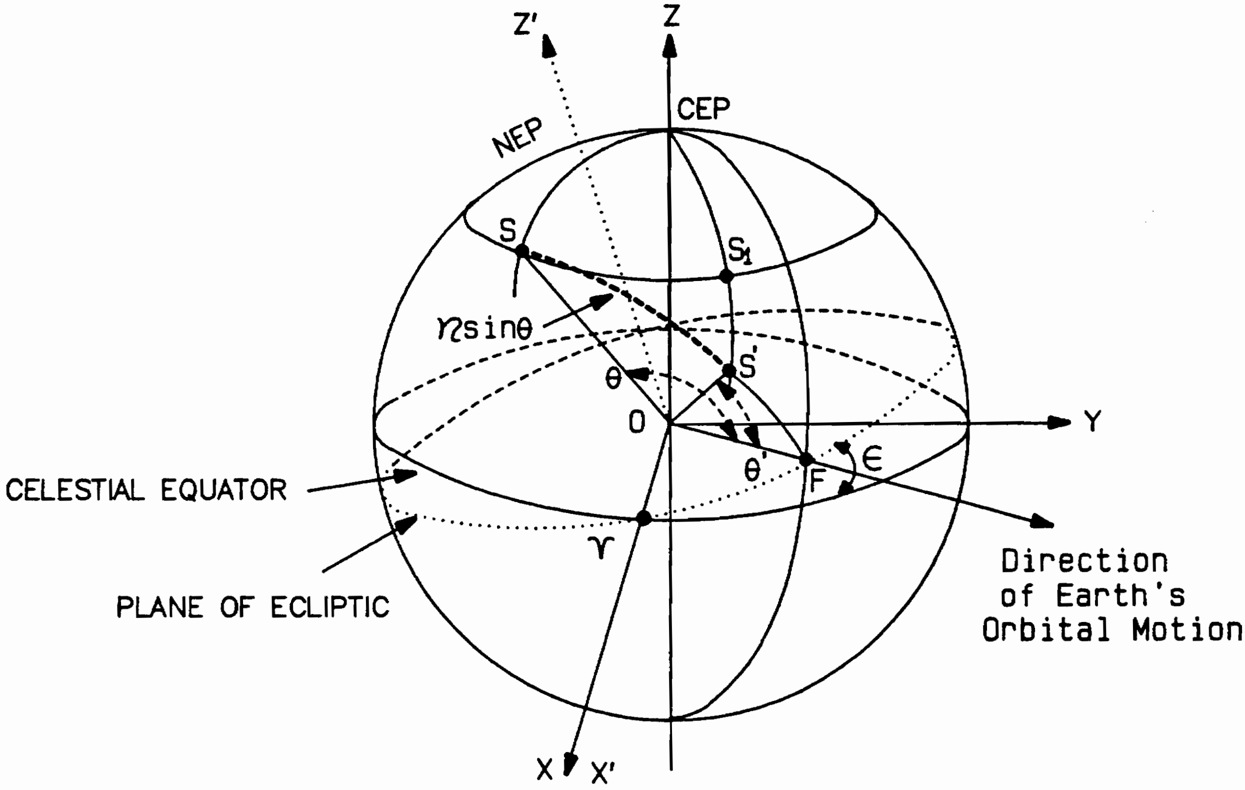
The displacement in right ascension, $\Delta\alpha_a$, is:

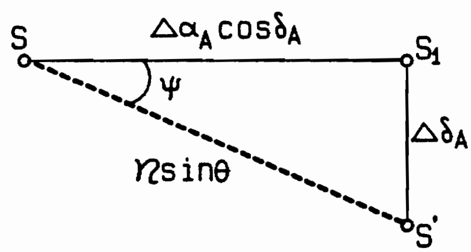
$$\Delta\alpha_a = -\eta \sec\delta[\cos\alpha \cos\lambda_s \cos\varepsilon + \sin\lambda_s]$$

The angle 'λ_s' is the ecliptic longitude of the star and the angle 'ε' is the obliquity of the ecliptic. These calculations assume the earth's orbit around the sun to be circular. Since the actual orbit is eliptical there is a small error associated with these equations.

An objects apparent position is also effected by diurnal aberration as illustrate in Figure 3-8. The correction for diurnal aberration can be expressed using the observer's latitude, ϕ, the hour angle of the star, h, and the radius of the earth expressed in units of equatorial radius, r. The correction in declination, $\Delta\delta_d$, is:

$$\Delta\delta_d = 0''.3198 \, r \cos\phi \sin h \sin\delta$$





O = observer
S = celestial object
θ = angular displacement
$\varkappa$ = constant of aberration

FIGURE 3-7. CALCULATING ABERRATION
[1, pp. 92]

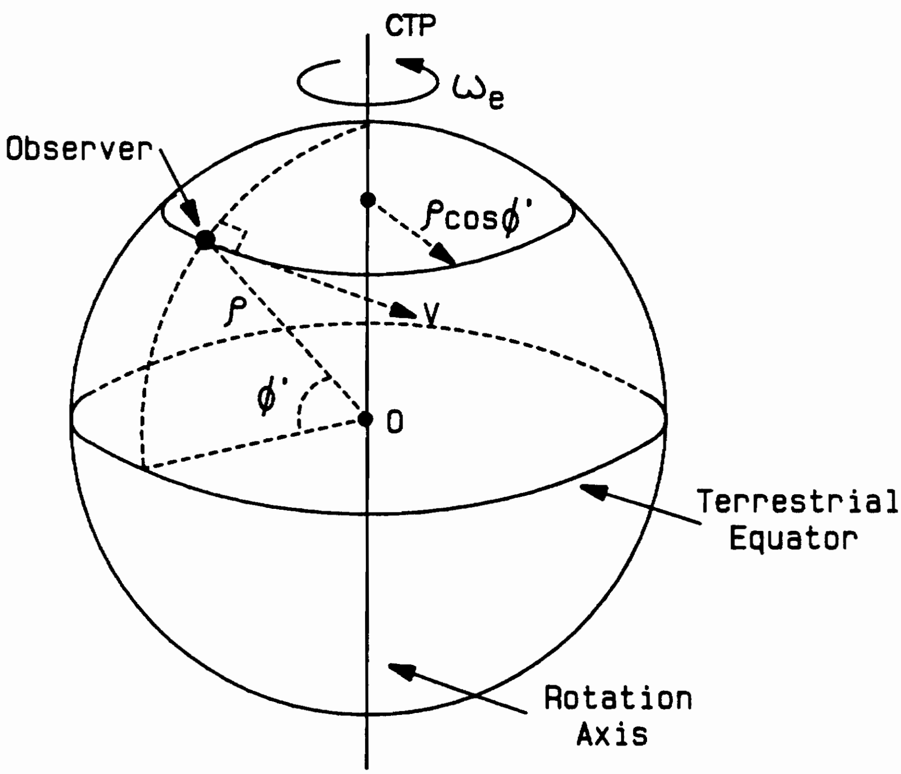


FIGURE 3-8. CALCULATING DIURNAL ABERRATION
[1, pp. 97]

The correction in right ascension, $\Delta\alpha_d$, is:

$$\Delta\alpha_d = 0^s.02132\ r\ \cos\phi \cos h\ \sec\delta$$

3.1.2.1 Correcting For Astronomic Refraction

As electromagnetic radiation from a radio source enters the earth's atmosphere it's direction of propagation is changed due to refraction. The amount of refraction depends on the density and composition of the air the signal is passing through. The amount of refraction a radio wave experiences depends on the frequency of the source. At microwave frequencies the dipole component of the water vapor in the atmosphere is readily excitable contributing significantly to the overall refractive index. The models used to estimate the atmospheric effects on radio signals are very complex and will not be pursued in this study. The program developed does not correct for astronomic refraction, however, refraction is mentioned because in some systems such as VLBI it is a source of error which must be accounted for.

3.3 Program Inputs and Outputs

To calculate the pointing angles to a radio source the program requires the following inputs:

ϕ_o = Observers Latitude (in degrees).

λ_o = Observers Longitude (in degrees).

R/A = Radio Source Right Ascension J2000 (in hours,mins,secs).

DEC = Radio Source Declination J2000 (in degrees, mins, secs).

x_p = Polar Motion in the x-axis (in arcseconds).

y_p = Polar Motion in the y-axis (in arcseconds).

D = Besselian Day Number (in arcseconds).

C = Besselian Day Number (in arcseconds).

date = Date of Observation (Year, Month, Day)

Time = Time of Observation (Hour, Minutes, Seconds)

The program outputs the pointing angles for the radio source of interest given in the horizon coordinate system:

A = Azimuth (in degrees)

a = Elevation (in degrees)

Figure 3-9 is a flow chart of radio source pointing program.

3.4 Results

To verify that the program calculates the correct pointing angles, program output values were compared with values published in the 1992 Astronomical Almanac. This comparison is shown in Table 3-3. As seen the program calculates the correct nutation-precession matrix (NP) to an accuracy greater than the eighth decimal place. Also shown is the calculation for Greenwhich Apparent Sidereal Time (GAST). The value of GAST is identical to the sixth decimal place.

An additional verification was made by entering the position of selected radio sources from a J2000 catalog and verifying that the program calculates the corrected position published in the Astronomical Almanac for the Julian Date 1992.5 . As seen from Table 3-4 the

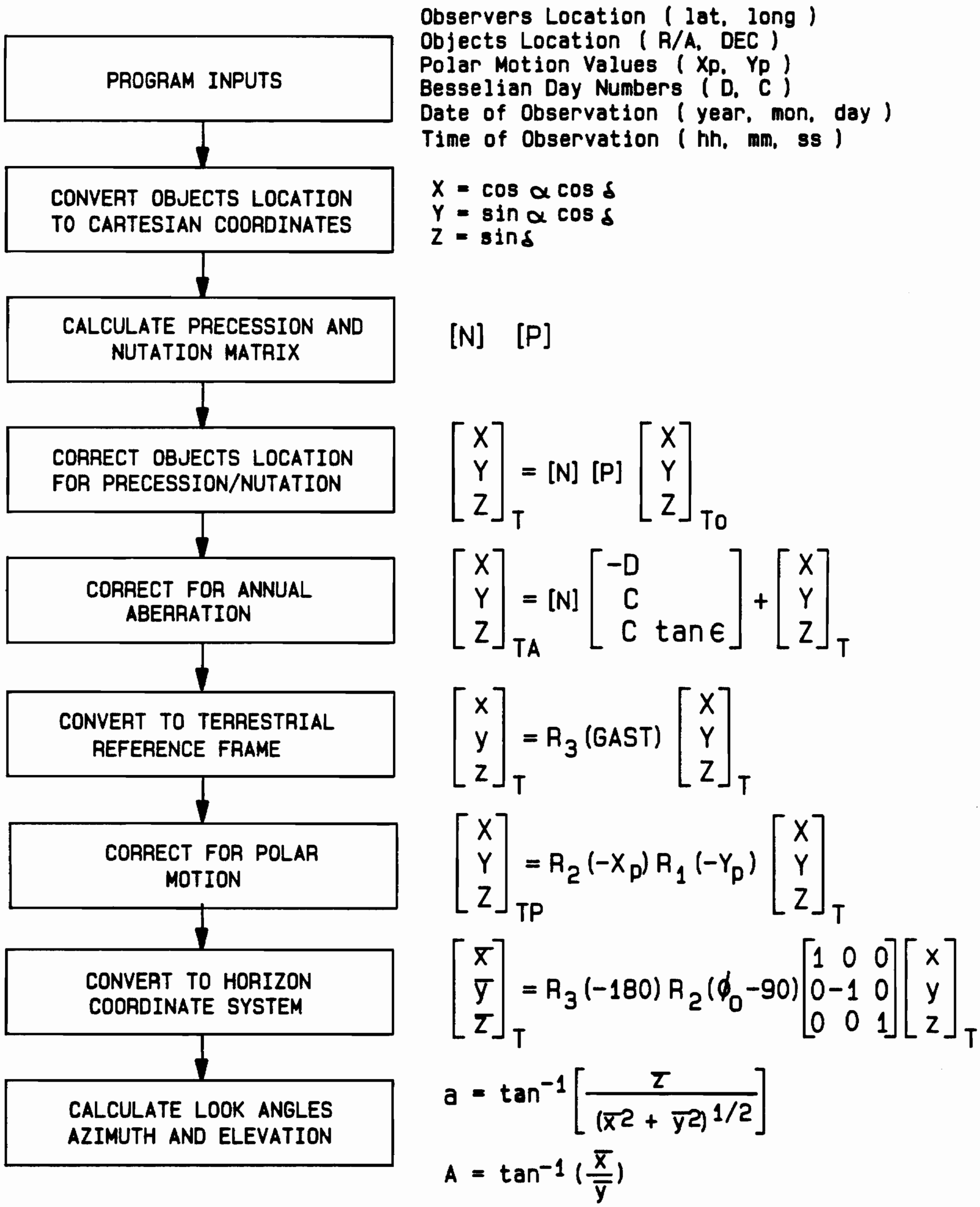


FIGURE 3-9. PROGRAM FLOWCHART FOR POINTING AT RADIO SOURCES

corrected position in right ascension is accurate to the second. In declination there is a difference of one arcsecond (0.00028^o). Also shown in Table 3-4 is a comparison between values calculated by the program and the values given in the Astronomical Almanac for the precession matrix of Julian Date 1992.5 . The values are the same to the published accuracy. Table 3-5 shows additional comparisons for corrected positions of quasars. The discrepancies in corrected positions between those calculated by the program and those given in the almanac can be attributed to the accuracy's of the original position measurements. The position data given in the almanac for the quasars was measured using optical techniques rather than radio telescopes. This will cause errors in the order of several arcseconds.

Table 3-3. Comparison between Outputs from the Radio Source Pointing Program and the "1992 Astronomical Almanac" for the Precession-Nutation Matrix.

Example: Radio Source OX 057

Position: (J2000) R/A 324.160775 degrees

DEC 0.698392 degrees

Observation Date: November 17, 1992

Observation Time: 00:00:00

Observers Location: 278 °east, 38 °north

Program Outputs

Julian Day = 2448943.5

GAST = 56.303066

Nutation-Precession Matrix:

$$\begin{bmatrix} 0.99999862 & 0.00152166 & 0.00066133 \\ -0.00152167 & 0.99999884 & 0.00000543 \\ -0.00066132 & -0.00000644 & 0.99999978 \end{bmatrix}$$

Azimuth = 196.574033 Elevation = 51.50011

Astronomical Almanac

Julian Day = 2448943.5

GAST = 56.303066

Nutation-Precession Matrix:

$$\begin{bmatrix} 0.99999862 & 0.00152166 & 0.00066133 \\ -0.00152167 & 0.99999884 & 0.00000543 \\ -0.00066132 & -0.00000644 & 0.99999978 \end{bmatrix}$$

Table 3-4. Comparison between Outputs from the Radio Source Pointing Program and the "1992 Astronomical Almanac" Comparing the Precession Matrix and the Corrected Positions.

Example: Radio Source NRAO 530

Position: (J2000) R/A $17^h 33^m 2^s.7$

DEC $-13^o 4' 49''.6$

Observation Date: July 2, 1992

Observation Time: 03:00:00 GMT

Program Outputs

Julian Day = 2448805.625

Precession Matrix:

$$\begin{bmatrix} 0.99999833 & 0.00167709 & 0.00072880 \\ -0.00167709 & 0.99999859 & -0.00000061 \\ -0.00072880 & -0.00000061 & 0.99999973 \end{bmatrix}$$

Corrected Position: R/A $17^h 32^m 37^s.2$

DEC $-13^o 4' 31''$

Astronomical Almanac

Julian Day = 2448805.625

Precession Matrix:

$$\begin{bmatrix} 0.99999833 & 0.00167709 & 0.00072880 \\ -0.00167709 & 0.99999859 & -0.00000061 \\ -0.00072880 & -0.00000061 & 0.99999973 \end{bmatrix}$$

Corrected Position: R/A $17^h 32^m 37^s.2$

DEC $-13^o 4' 32''$

Table 3-5. Comparison between Outputs from the Radio Source Pointing Program and the "1992 Astronomical Almanac" Comparing the Corrected Positions of Selected Quasars.

Observation Date: July 2, 1992

Observation Time: 03:00:00 GMT

Julian Day = 2448805.625

Radio Source	Position J2000	Corrected Position: Program Outputs	Astronomical Almanac
NRAO 530	R/A $17^h 32^m 37^s.2$ DEC $-13^\circ 4' 31''$	R/A $17^h 32^m 37^s.2$ DEC $-13^\circ 4' 31''$	R/A $17^h 32^m 37^s.2$ DEC $-13^\circ 4' 32''$
PKS 2145 + 067	R/A $21^h 48^m 05^s.5$ DEC $6^\circ 57' 38''.6$	R/A $21^h 47^m 43^s.6$ DEC $6^\circ 55' 42''.5$	R/A $21^h 47^m 43^s.1$ DEC $6^\circ 55' 33''$
PKS 0537 - 441	R/A $05^h 38^m 50^s.4$ DEC $-44^\circ 5' 8''.9$	R/A $05^h 38^m 35^s.8$ DEC $-44^\circ 5' 18''$	R/A $05^h 38^m 36^s.9$ DEC $-44^\circ 5' 23''$
B2 0552 + 398	R/A $05^h 55^m 30^s.8$ DEC $39^\circ 48' 49''.2$	R/A $05^h 54^m 59^s.2$ DEC $39^\circ 48' 44''.45$	R/A $05^h 54^m 59^s.4$ DEC $39^\circ 48' 46''$
3C 379	R/A $12^h 56^m 11^s.2$ DEC $-5^\circ 47' 21''.5$	R/A $12^h 55^m 48^s.4$ DEC $-5^\circ 44' 59''$	R/A $12^h 55^m 47^s.8$ DEC $-5^\circ 44' 56''$

REFERENCES

1. Ivan I. Mueller and Heinrich Eichhorn, *Spherical and Practical Astronomy*, Fredrick Ungar Publishing Co., New York, 1969.

2. Rodger R. Bate, Donald D. Mueller, Jerry D. White, *Fundamentals Of Astrodynamics*, Dover Publications Inc., New York, 1971.

3. Alfred Leick, *GPS Satellite Surveying*, John Wiley and Sons, New York, 1990.

4. *The Astronomical Almanac For The Year 1992*, United States Printing Office, Science and Engineering Research Council, 1991.

5. Ian Ridpath, *Norton's 2000.0 Star Atlas and Reference Handbook*, John Wiley and Sons, New York, 1989.

CHAPTER 4. CONCLUSIONS

4.1 Study Conclusions

This study has developed a method , unique to this paper, for applying a Geodetic reference frame (ellipsoidal earth) to satellite look angle calculations. The result is a more rigorous and accurate approach for calculating these angles. Even though the method was developed specifically for pointing at satellites in geostationary orbits, it is applicable to angle calculations for satellites in any given orbit. This method was compared with the standard method for calculating satellite look angles which assumes a spherical earth.

The study also developed a routine which calculates the pointing angles to radio sources correcting for precession, nutation, polar motion, and annual aberration. Calculations for correcting a radio source's position due to the effects of diurnal aberration and astronomic refraction were discussed but not implemented by the program. The program outputs were verified by comparison with data published in the Astronomical Almanac.

4.2 Recommendations for Future Work

The knowledge gained in this study can readily be applied to the calculation of pointing angles for a satellite in any given orbit. The orbital elements of a satellite could be converted into Cartesian coordinates and then corrected to the date of observation using the routines developed in Chapter 3 for precession, nutation, and polar motion. The coordinates could then be referenced to an ellipsoidal earth and converted into pointing angles using the approach detailed in section 2.4.

Continued development could add the capability of tracking a satellite open loop. A satellite's position changes continuously with time. For open loop tracking a routine could update the pointing angles at a rate sufficiently fast enough to keep the satellite in the antenna beamwidth.

The ultimate test would be to implement the routines on an actual system. The pointing method could then be verified and compared with existing methods. Figure 4.1 is a block diagram which depicts the dish antenna control system for a portable satellite communication system. The system is referenced to the Global Positioning System (GPS) satellite network to provide accurate time and position. This will allow for accurate alignment of the antenna mount to inertial space. This also provides accurate time and position used by the pointing algorithms. The system would provide accurate open loop tracking of a satellite based on the a satellite's orbital elements.

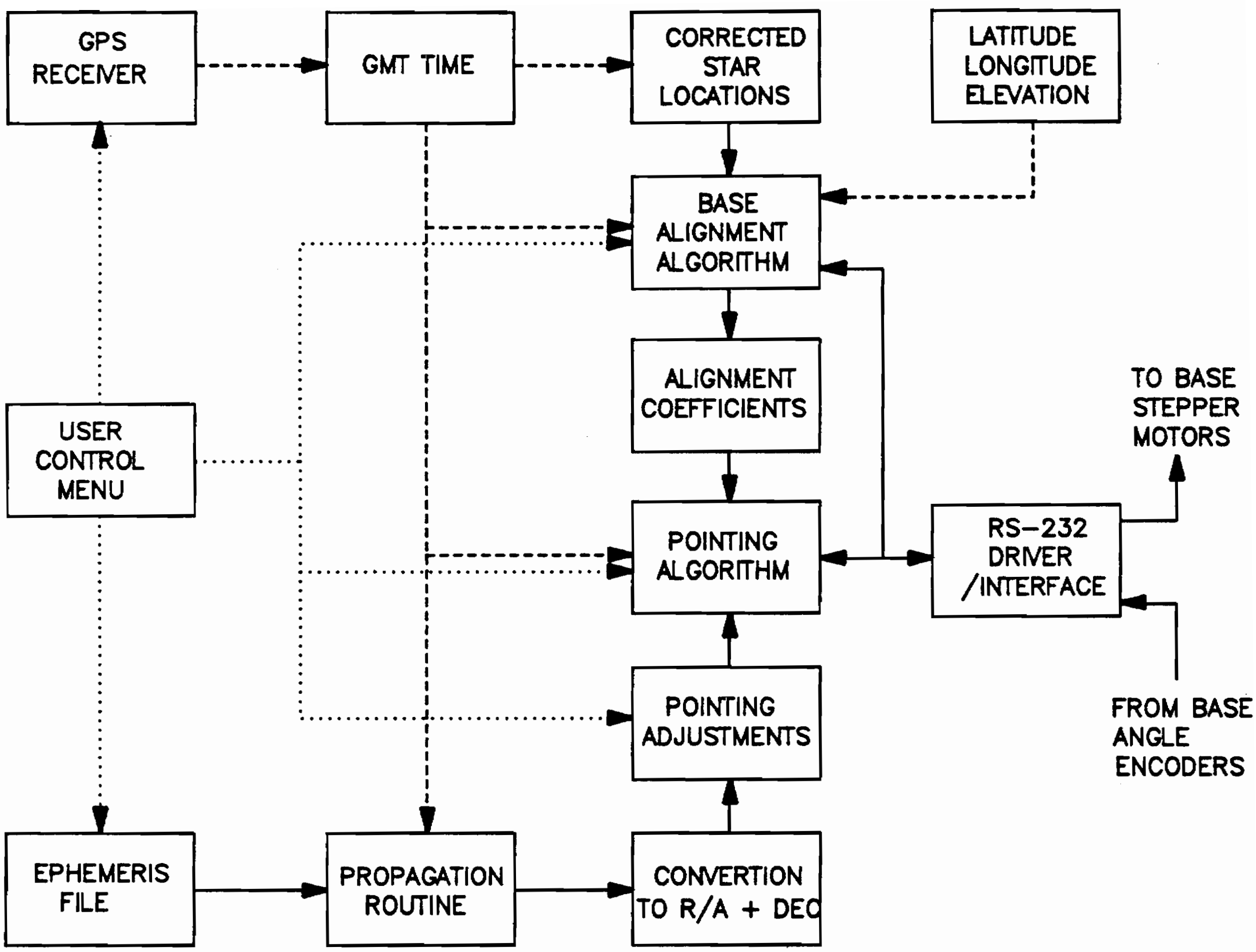


FIGURE 4_1. MOUNT CONTROL SYSTEM ALGORITHM FOR A PORTABLE SATELLITE DISH ANTENNA

FORTRAN LISTING - CALCULATING POINTING ANGLES ASSUMING A SPHERICAL EARTH.

```
CC  GEOPOINT.FOR
CC      GIVEN AN OBSERVERS LOCATION (LATITUDE, LONGITUDE) THIS
CC      PROGRAM WILL CLACULATE THE LOOK ANGLES TO A GEO-LOCATED
CC      SATELLITE LOCATED AT A PARTICULAR LONGITUDE.  THE ANGLES
CC      WILL BE GIVEN IN HORIZON COODINATES (AZIMUTH, ELEVATION).
CC      IT IS ASSUMED THAT THE SATELLITE IS IN A PERFECT GEO-
CC      STATIONARY ORBIT.  THE LONGITUDES ARE GIVEN IN DEGREES
CC      EAST OF GREENWHICH.
CC

      REAL*8    DegRad,RadDeg
      REAL*8    loclat,loclon,geolon,loclatr,loclonr,geolonr
      REAL*8    cenang,dist,elev,maxang,cenmag
      REAL*8    verang,temp1,temp2,temp3,s,azimuth

      DOUBLE PRECISION pi / 3.141592653589 /

      WRITE (*,9000)
      READ  (*,*) loclat
      WRITE (*,9100)
      READ  (*,*) loclon
      WRITE (*,9200)
      READ  (*,*) geolon

      IF (loclon .LT. 0) THEN
       loclon = 360.d0 + loclon
      END IF
      IF (geolon .LT. 0) THEN
       geolon = 360.d0 + geolon
      END IF

C     CONVERT DEGREES TO RADIANS (26)

      loclatr = DegRad(loclat)
      loclonr = DegRad(loclon)
      geolonr = DegRad(geolon)

C     CALCULATE THE CENTRAL ANGLE
```

```
      cenang = dacos(dcos(loclatr)*dcos(geolonr-loclonr))

C     VERIFY VISIBILITY

      maxang = dacos(6370/42242)
      cenmag = ABS(cenang)
      IF ((cenmag) .GT. (maxang)) GOTO 2000

C     CALCULATE THE DISTANCE FROM THE EARTH STATION TO THE
C     SATELLITE IN KILOMETERS                (49)

      dist = 42242*sqrt(1.02274-0.301596*dcos(cenang))

C     CALCULATE THE ELEVATION ANGLE

      elev = dacos(dsin(cenang)*42242/dist)

C     AZIMUTH IS KNOWN IF THE LONGITUDE OF THE SUBSATTELLITE POINT
C     IS EQUAL TO THE LONGITUDE OF THE OBSERVER.

      IF (loclonr .EQ. geolonr) THEN
       IF (loclatr .GT. 0) THEN
       azimuth = pi
       END IF
       IF (loclatr .LT. 0) THEN
       azimuth = 0
       END IF
       IF (loclatr .EQ. 0) THEN
       WRITE (*,9500)
       GOTO 9980
       END IF
      GOTO 1500
      END IF

C     TO CALCULATE THE AZIMUTH YOU FIRST NEED TO CALCULATE THE
C     VERTEX ANGLE. CALCULATE THE VERTEX USING THE RIGHT
C     SPHERICAL TRIANGLE WITH SIDES:  "a" IS THE LONGITUDINAL
C     SEPERATION, "c" IS THE LATUDINAL SEPERATION,  AND "b" IS THE
C     CENTRAL ANGLE.  THE RIGHT ANGLE IS BETWEEN THE SIDES  "c" AND
C     "a".  THE VERTEX ANGLE IS BETWEEN SIDES "c" AND "b".
C                 COS(verang) = COT(b)COT(90-c)
C                 b = cenang              (62)
C                 c = loclatr
```

```
      verang = DACOS(DTAN(pi/2 - cenang) * DTAN(loclatr))

C    ALTERNATE METHOD OF CALCULATING VERTEX ANGLE

C     s = 0.5*(ABS(geolonr - loclonr) + ABS(loclatr) + cenang)
C     temp1 = DSIN(s - cenang) * DSIN(s - ABS(loclatr))
C     temp2 = DSIN(s) * DSIN(s - ABS(loclonr - geolonr))
C     temp3 = SQRT(temp1/temp2)
C     verang = 2*DATAN(temp3)

C     AZIMUTH CALCULATION DEPENDS ON THE QUADRANT OF
C    SUBSATTELLITE POINT  RELATIVE TO THE SATELLITE.  AZIMUTH IS
C    MEASURED CLOCKWISE  FROM NORTH.

      IF (loclatr .GT. 0) THEN
       IF (loclonr .EQ. geolonr) THEN
       azimuth = pi
       GOTO 1500
       END IF
       IF ((geolonr - loclonr) .GT. 0) THEN
          IF ((geolonr .GT. (3*pi/2)) .AND. (loclonr .LT. (pi/2))) THEN
          azimuth = (pi + verang)
          GOTO 1500
          END IF
          azimuth = (pi - verang)
          GOTO 1500
       ELSE
          IF ((geolonr .LT. (pi/2)) .AND. (loclonr .GT. (3*pi/2))) THEN
          azimuth = (pi - verang)
          GOTO 1500
          END IF
          azimuth = (pi + verang)
          GOTO 1500
       END IF
      END IF
      IF (loclatr .LT. 0) THEN
       IF (loclonr .EQ. geolonr) THEN
       azimuth = 0
       GOTO 1500
       END IF
       IF ((geolonr - loclonr) .GT. 0) THEN
          IF ((geolonr .GT. (3*pi/2)) .AND. (loclonr .LT. (pi/2))) THEN
```

```
        azimuth = ((2*pi) - verang)
        GOTO 1500
        END IF
        azimuth = (pi - verang)
        GOTO 1500
     ELSE
        IF (( geolonr .LT. (pi/2)) .AND. (loclonr .GT. (3*pi/2))) THEN
        azimuth = (verang)
        GOTO 1500
        END IF
        azimuth = (pi + verang)
        GOTO 1500
     END IF
    END IF

C     Calculate Azimuth if observers latitude is zero.

    IF (loclatr .EQ. 0) THEN
     IF ((geolonr .LT. (pi/2)) .AND. (loclonr .GT. (3*pi/2))) THEN
        azimuth = pi/2
        GOTO 1500
     END IF
     IF ((geolonr .GT. (3*pi/2)) .AND. (loclonr .GT. (pi/2))) THEN
        azimuth = 3*pi/2
        GOTO 1500
     END IF
     IF (geolonr .LT. loclonr) THEN
        azimuth = 3*pi/2
     ELSE
        azimuth = pi/2
     END IF
    END IF
    GOTO 1500

 1500 azimuth = RadDeg (azimuth)
    elev = RadDeg (elev)
    WRITE (*,1505)
 1505 FORMAT (' ')
    WRITE (*,1510) azimuth,elev
 1510 FORMAT (1X, 'Azimuth = ', F8.3, 1x, 'degrees', 4X, 'Elevation = '
   +     , F8.3, 1X, 'degrees')

    GOTO 9980
```

```
2000 WRITE (*,9300)

9000 FORMAT( ' Latitude of the observers location (degrees): '\ )
9100 FORMAT( ' Longitude of the obsevers location (deg east): '\ )
9200 FORMAT( ' Longitude of the subsatellite point (deg east): '\ )
9300 FORMAT( ' Visibility angles have been exceeded.  The '/
  +        '   longitudinal seperation between the earth station '/
  +        '   and the satellite is to great.: ' )
9500 FORMAT( ' Elevation = 90.0,  Azimuth is undefined.')

9980 END

   REAL*8 FUNCTION DegRad (degree)
   DOUBLE PRECISION pi / 3.141592653589 /
   REAL*8 degree
   DegRad = (degree * 2 * pi)/360
   END

   REAL*8 FUNCTION RadDeg (radians)
   DOUBLE PRECISION pi /3.141592653589 /
   REAL*8 radians
   RadDeg = (radians * 360)/(2*pi)
   END
```

FORTRAN LISTING - CALCULATING POINTING ANGLES USING A GEODETIC REFERENCE FRAME.

```
CC SKYPOINT.FOR
CC    GIVEN AN OBSERVERS LOCATION (LATITUDE, LONGITUDE) THIS
CC    PROGRAM WILL CLACULATE THE LOOK ANGLES TO A GEO-LOCATED
CC    SATELLITE LOCATED AT A PARTICULAR LONGITUDE.  THE ANGLES
CC    WILL BE GIVEN IN HORIZON COODINATES (AZIMUTH, ELEVATION).
CC    IT IS ASSUMED THAT THE SATELLITE IS IN A PERFECT GEO-
CC    STATIONARY ORBIT.  THE LONGITUDES ARE GIVEN IN DEGREES
CC    EAST OF GREENWHICH.
CC

      REAL*8    DegRad,RadDeg,Mul3x3,Arctan
      REAL*8    xs,ys,zs,xp,yp,zp
      REAL*8    a,f,ec,hsat,Wsat,Nsat,Nobs,hobs,Wobs
      REAL*8    loclat,loclon,satlon,satlat,loclatr,loclonr,satlonr,satlatr
      REAL*8    R(3,3),Del(3,3),Penu(3,3)
      REAL*8    e,n,u,alpha,new,k

      WRITE (*,9000)
      READ  (*,*) loclat
      WRITE (*,9100)
      READ  (*,*) loclon
      WRITE (*,9150)
      READ  (*,*) hobs
      WRITE (*,9200)
      READ  (*,*) satlon
      WRITE (*,9225)
      READ  (*,*) satlat
C     WRITE (*,9250)
C     READ  (*,*) hsat

C   CONVERT DEGREES TO RADIANS (28)

      loclatr = DegRad(loclat)
      loclonr = DegRad(loclon)
      satlonr = DegRad(satlon)
      satlatr = DegRad(satlat)
```

```
C  Transform from Curvilinear Geodetic to Cartesian Geodetic:
C  ==========================================================

C  First transform satellites coordinates:

C  Definitions:
C  N = principle radius of curvature in the prime vertical plane
C  a = semimajor axis (GRS 80 is used) (meters)
   a = 6378137
C  hsat = geodetic hieght of satellite
C  ec = eccentricity
C  f = flattening (GRS 80 is used)
   f= 1/298.257222101

C  Calculate e
   ec = DSQRT(2*f - f*f)

C  Calculate N, for satellites latitude
   Wsat = DSQRT(1 - ((ec*ec) * (DSIN(satlatr)*DSIN(satlatr))))
   Nsat = a/Wsat

C  Calculate hsat -- for geosatellites radius = 42241558 meters
   hsat =  (42241558 - Nsat)

C  Calculate satellites geodetic cartesian coordinates
   xs = ((Nsat + hsat)*(DCOS(satlatr))*(DCOS(satlonr)))
   ys = ((Nsat + hsat)*(DCOS(satlatr))*(DSIN(satlonr)))
   zs = ((Nsat*(1 - ec*ec) + hsat)*(DSIN(satlatr)))
   WRITE (*,*) xs,ys,zs

C  Calcutlate N, for observer latitude
   Wobs = DSQRT(1 - ((ec*ec) * (DSIN(loclatr)*DSIN(loclatr))))
   Nobs = a/Wobs
C    WRITE (*,*) Nobs, Wobs,ec

C  Calculate observers geodetic cartesian coordinates  (70)
   xp = ((Nobs + hobs)*(DCOS(loclatr))*(DCOS(loclonr)))
   yp = ((Nobs + hobs)*(DCOS(loclatr))*(DSIN(loclonr)))
   zp = ((Nobs*(1 - ec*ec) + hobs)*(DSIN(loclatr)))
C    WRITE (*,*) xp,yp,zp
```

```
C  Calculate the differential delta's between the satellite
C  and the observer.

   Del(1,1) = (xs - xp)
   Del(1,2) = (ys - yp)
   Del(1,3) = (zs - zp)
C    WRITE (*,*) Del(1,1),Del(1,2),Del(1,3)

C  Transform from local (x,y,z) to local geodetic (e,n,u):
C  ==========================================================

C  Calculate the rotation matric "R".

   R(1,1) = -DSIN(loclonr)
   R(1,2) = -DSIN(loclatr)*DCOS(loclonr)
   R(1,3) = DCOS(loclatr)*DCOS(loclonr)
   R(2,1) = DCOS(loclonr)
   R(2,2) = -DSIN(loclatr)*DSIN(loclonr)
   R(2,3) = DCOS(loclatr)*DSIN(loclonr)
   R(3,1) = 0
   R(3,2) = DCOS(loclatr)
   R(3,3) = DSIN(loclatr)

C  Calculate (e,n,u):                                    (99)
   Penu = Mul3x3(R,DEL,Penu)
   e = Penu(1,1)
   n = Penu(1,2)
   u = Penu(1,3)

C  Calculate geodetic azimuth and vertical angle:
C    WRITE (*,*) e,n,u
   alpha = Arctan(e,n)
   alpha = RadDeg(alpha)
   k = DSQRT(e*e + n*n)
   new = Arctan(u,k)
   new = RadDeg(new)
   WRITE (*,*) alpha,new

   GOTO 9980
2000 WRITE (*,9300)
```

```
9000 FORMAT( ' Enter observers latitude (degrees):  '\ )
9100 FORMAT( ' Enter observers longitude (deg east):  '\ )
9150 FORMAT( ' Enter observers geodetic hieght (meters):  '\ )
9200 FORMAT( ' Enter satellites longitude (deg east):  '\ )
9225 FORMAT( ' Enter satellites latitude (degrees):  '\ )
9250 FORMAT( ' Enter satellites hieght (meters):  '\ )
9300 FORMAT( ' The satellite is below the observers horizon. ')
9500 FORMAT( ' Elevation = 90.0,  Azimuth is undefined.')

9980 END
```

```
CC STARAZEL.FOR
CC    GIVEN AN OBSERVERS LOCATION (LATITUDE, LONGITUDE) THIS
CC    PROGRAM WILL CALCULATE THE POINTING ANGLES TO A CELESTIAL
CC    OBJECT.     THE POINTING ANGLES WILL BE GIVEN IN HORIZON
CC    COODINATES (AZIMUTH, ELEVATION).  THE COORDINATES FOR THE
CC    CELESTIAL OBJECT MUST BE GIVEN IN RIGHT ASCENSION(degrees)
CC    AND DECLINATION(degrees) J2000.
CC

CC ======================== Variables ==============================

      INTEGER  yr,mn,dy,hr,min,date
      REAL*8   jd,sec,time,GmsTim
      REAL*8   DegRad,RadDeg,Mul3x3,Arctan,JulDate,NutRig
      REAL*8   X,Y,Z,XYZTo(3,3),TXYZ(3,3),params(2)
      REAL*8   T,gama,theta,zeta,Prec(3,3),XYZ_T(3,3)
      REAL*8   epsln,epslt,delsi,deleps,Nuta(3,3),NP(3,3)
      REAL*8   GMST,GAST,AST,Xp,Yp,R1(3,3),R2(3,3),R3(3,3)
      REAL*8   PolMot(3,3),xyzT(3,3),xs,ys,zs,xo,yo,zo
      REAL*8   loclat,loclon,loclatr,loclonr,starra,stardec
      REAL*8   stardecr,starrar,ROT2(3,3),ROT3(3,3)
      REAL*8   AZEL(3,3),R3R2(3,3),az,el,Purm2(3,3)
      REAL*8   hourang(3,3)
      REAL*8   aber(3,3),D,C,abcor(3,3),XYZ_A(3,3)
      REAL*8   X_out,Y_out,Z_out,n_out
      REAL*8   DegHMS,scs,DegDMS,scs_d,scs_i,scs_t
      REAL*8   k,l,m,n,ra,dec,ra_o,dec_o,ra_o_d,dec_o_d

      INTEGER  hrs,mns,mns_d,deg,hrs_i,mns_i,deg_t,mns_t

CC======================== Operator Inputs ==========================
      WRITE (*,9000)
      READ  (*,*) loclat
      WRITE (*,9100)
      READ  (*,*) loclon
      WRITE (*,9200)
      READ  (*,*) starra
      WRITE (*,9225)
      READ  (*,*) stardec
      WRITE (*,9226)
      READ  (*,*) Xp
      WRITE (*,9227)
      READ  (*,*) Yp
```

```
      WRITE (*,9228)
      READ  (*,*) D
      WRITE (*,9229)
      READ  (*,*) C
      WRITE (*,9250)
      READ  (*,*) date
      WRITE (*,9275)
      READ  (*,*) time

      DO 1450 I=1,30
       WRITE(*,*)
 1450 CONTINUE

C     CONVERT HMS TO DEGREES

      hrs_i = INT(starra/10000)
      mns_i = INT((starra - hrs_i*10000)/100)
      scs_i = ((starra - hrs_i*10000.d0)/100.d0 - mns_i)*100.d0
      starra = hrs_i*15.d0 + mns_i*0.25d0 + scs_i*0.0041666666666d0

      deg_t = INT(stardec/10000)
      mns_t = INT((stardec - deg_t*10000)/100)
      scs_t = ((stardec - deg_t*10000.d0)/100.d0 - mns_t)*100.d0
      stardec = deg_t + mns_t/60.d0 + scs_t/3600.d0

      WRITE (*,1500) hrs_i,mns_i,scs_i
 1500 FORMAT(1X,'Uncorrected Object R/A =',1X,I2,'h',1X,I2,'m',
     + 1X,F7.2,'s')
      WRITE (*,1510) deg_t,ABS(mns_t),ABS(scs_t)
 1510 FORMAT(1X,'Uncorrected Object DEC =',1X,I2,'d',1X,I2,'m'
     + 1X,F7.2,'s')

C     CONVERT DEGREES TO RADIANS (43)

      loclatr = DegRad(loclat)
      loclonr = DegRad(loclon)
      starrar = DegRad(starra)
      stardecr = DegRad(stardec)

C     CONVERT DATE TO YEAR,MONTH,DAY

      yr = int(date/10000)
      mn = int((date - yr*10000)/100)
```

```
      dy = int(date - yr*10000 - mn*100)

C     CONVERT TIME TO HOUR,MINUTE,SECONDS  (53)

      hr = int(time/10000)
      min = int((time - hr*10000)/100)
      sec = time - hr*10000.d0 - min*100.d0

C     CALCULTATE CELESTIAL OBJECTS POSITION TERRESTRIAL (x,y,z) AT
TIME, t
C ==================================================================

C     CONVERT R/A AND DEC TO (X,Y,Z) J2000

      X = DCOS(starrar)*DCOS(stardecr)
      Y = DSIN(starrar)*DCOS(stardecr)
      Z = DSIN(stardecr)
      XYZTo(1,1) = X
      XYZTo(1,2) = Y
      XYZTo(1,3) = Z

C     Calculate Julian Date

      jd = JulDate(yr,mn,dy,hr,min,sec)

      WRITE (*,1900) jd
 1900 FORMAT(1X,'Julian Day =',1X,F12.3)

C ================ Calculate the precession matrix: ================

C     gama, zeta, theta are angles that serve to specify the position of
C     the mean equinox and equator of date with respect to mean equinox
C     and equator of the initial epoch.

C     From Astronomical Almanac

      T = (jd - 2451545.d0)/36525.d0

      gama = (0.640616139d0*T) + (0.00008385555d0*T*T)
     + + (0.000004999444444d0*T*T*T)
      gama = DegRad(gama)
      zeta = (0.640616139d0*T) + (0.0003040777778d0*T*T)
     + + (0.000005056388889d0*T*T*T)
```

```
      zeta = DegRad(zeta)
      theta = (0.556753028d0*T) - (0.000118513889d0*T*T)
     + - (0.0000116202778d0*T*T*T)
      theta = DegRad(theta)

      Prec(1,1)=DCOS(gama)*DCOS(theta)*DCOS(zeta)-DSIN(gama)*DSIN(zeta)
      Prec(1,2)=DCOS(gama)*DCOS(theta)*DSIN(zeta)+DSIN(gama)*DCOS(zeta)
      Prec(1,3)=DCOS(gama)*DSIN(theta)
      Prec(2,1)=-DSIN(gama)*DCOS(theta)*DCOS(zeta)-DCOS(gama)*DSIN(zeta)
      Prec(2,2)=-DSIN(gama)*DCOS(theta)*DSIN(zeta)+DCOS(gama)*DCOS(zeta)
      Prec(2,3)=-DSIN(gama)*DSIN(theta)
      Prec(3,1)=-DSIN(theta)*DCOS(zeta)
      Prec(3,2)=-DSIN(theta)*DSIN(zeta)
      Prec(3,3)=DCOS(theta)

C ==================== Calculate the nutation matrix: ====================

C     epsln = mean obliquity of the ecliptic
C     epslt = true obliquity of the ecliptic
C     delsi = nutation in longitude
C     deleps = nutation in obliquity

C     From GPS Satellite Surveying

      epsln = 23.439291111d0-0.0130041666d0*T
     + -0.000000163889d0*T*T + 0.000000503611d0*T*T*T

C     Calculate Nutation in longitude and obliquity
      params = NutRig(jd,params)
      deleps = params(2)
      delsi = params(1)
      epslt  = epsln + deleps
      epsln  = DegRad(epsln)
      epslt  = DegRad(epslt)
      deleps = DegRad(deleps)
      delsi  = DegRad(delsi)

C     From Homework 2-10-92 page 3

      Nuta(1,1) = DCOS(delsi)
      Nuta(1,2) = DSIN(delsi) * DCOS(epslt)
      Nuta(1,3) = DSIN(delsi) * DSIN(epslt)
      Nuta(2,1) = -DSIN(delsi) * DCOS(epsln)
```

```
      Nuta(2,2) = DCOS(epslt) * DCOS(delsi) * DCOS(epsln)
     +            + DSIN(epsln) * DSIN(epslt)
      Nuta(2,3) = -DCOS(epslt) * DSIN(epsln) + DSIN(epslt)
     +            * DCOS(delsi) * DCOS(epsln)
      Nuta(3,1) = -DSIN(delsi) * DSIN(epsln)
      Nuta(3,2) = -DSIN(epslt) * DCOS(epsln) + DCOS(epslt)
     +            * DCOS(delsi) * DSIN(epsln)
      Nuta(3,3) = DSIN(epslt) * DCOS(delsi) * DSIN(epsln)
     +            + DCOS(epslt) * DCOS(epsln)

C ==================== Calculate the NP matrix ====================

      NP = Mul3x3(Nuta,Prec,NP)
C     WRITE (*,*) NP
C ================== Correct for Nutation-Precession ==================

      XYZ_T = Mul3x3(NP,XYZTo,XYZ_T)
C     WRITE(*,*) XYZ_T

C ==================== Calculate R/A and DEC at T ====================
      k = XYZ_T(1,1)
      l = XYZ_T(1,2)
      m = XYZ_T(1,3)
      ra_o = arctan(l,k)
      ra_o_d = RadDeg(ra_o)
      n = DSQRT(k*k + l*l)
      dec_o = arctan(m,n)
      dec_o_d = RadDeg(dec_o)
      IF (dec_o_d .GT. 270) THEN
      dec_o_d = dec_o_d - 360
      END IF
C      WRITE(*,*) ra_o_d, dec_o_d

C ======================== Annual  Aberration ========================

      D = D/3600.0d0
      C = C/3600.0d0
      D = DegRad(D)
      C = DegRad(C)

C Above R/A's of 180 degrees D must be made negative.

      IF (hrs_i .GE. 12) THEN
```

```
    D = -D
    END IF

    aber(1,1) = -D
    aber(1,2) = C
    aber(1,3) = C*DTAN(epsln)

C     aber(1,3) = C*DTAN(epsln)/(DSIN(epsln - ABS(stardecr)))

    Abcor = Mul3x3(Nuta,aber,Abcor)
C     WRITE(*,*) Nuta,aber
    XYZ_A(1,1) = abcor(1,1) + XYZ_T(1,1)
    XYZ_A(1,2) = abcor(1,2) + XYZ_T(1,2)
    XYZ_A(1,3) = abcor(1,3) + XYZ_T(1,3)

    X_out = XYZ_A(1,1)
    Y_out = XYZ_A(1,2)
    Z_out = XYZ_A(1,3)

    ra = arctan(Y_out,X_out)
    n_out = DSQRT(X_out*X_out + Y_out*Y_out)
    dec = arctan(Z_out,n_out)
    ra = RadDeg(ra)
    dec = RadDeg(dec)

    IF (dec .GT. 270) THEN
    dec = dec - 360
    END IF

C     WRITE(*,*) ra, dec
    hrs = DegHMS(ra,hrs,mns,scs)
    WRITE(*,2500) hrs,mns,scs
 2500 FORMAT(1X,'Corrected R/A =',1X,I2,'h',1X,I2,'m',1X,F7.2,'s')
    deg = DegDMS(dec,deg,mns_d,scs_d)
    WRITE(*,2510) deg,mns_d,scs_d
 2510 FORMAT(1X,'Corrected DEC =',1X,I2,'d',1X,I2,'m',1X,F7.2,'s')

C    Calculate the terrestrial coordinates at time of observation.
C ==================================================================

C ---------------- Calculate Greenwhich Apparent Sidereal Time ----------------

C   GAST = Greenwhich apparent sidereal time
```

```
C  GMST = Greenwhich mean sidereal time
C  GAST = GMST + equation of equinoxes
C  equation of equinoxes = delsi * cos (epslt)

   GMST = GmsTim(T,hr,min,sec)
   GAST = DegRad(GMST) + delsi * DCOS(epslt)
   AST = GAST + loclonr

   WRITE (*,2851) RadDeg(GAST)
2851 FORMAT (1X,'Greenwhich Apparent Sidereal Time =',F10.6)

C ------------------------ Convert to terrestrial ------------------------------

   DATA R3 / 0.d0,0.d0,0.d0, 0.d0,0.d0,0.d0, 0.d0,0.d0,1.d0/
   R3(1,1) = DCOS(AST)
   R3(1,2) = -DSIN(AST)
   R3(2,1) = DSIN(AST)
   R3(2,2) = DCOS(AST)

   xyzT = Mul3x3(R3,XYZ_A,xyzT)

C ------------------------ Calculate Polar Motion ----------------------------
   Xp = Xp/3600.d0
   Yp = Yp/3600.d0
   Xp = DegRad(Xp)
   Yp = DegRad(Yp)

   DATA R1 /1.d0,0.d0,0.d0, 0.d0,0.d0,0.d0, 0.d0,0.d0,0.d0/
   R1(2,2) = DCOS(-Yp)
   R1(2,3) = -DSIN(-Yp)
   R1(3,2) = DSIN(-Yp)
   R1(3,3) = DCOS(-Yp)

   DATA R2 /0.d0,0.d0,0.d0, 0.d0,1.d0,0.d0, 0.d0,0.d0,0.d0/
   R2(1,1) = DCOS(-Xp)
   R2(1,3) = DSIN(-Xp)
   R2(3,1) = -DSIN(-Xp)
   R2(3,3) = DCOS(-Xp)

   PolMot = Mul3x3(R2,R1,PolMot)
   Txyz = Mul3x3(PolMot,xyzT,Txyz)

C ------------------------ Convert right to left hand ---------------------------
```

```
C From Mueller

     DATA Purm2 / 1.d0,0.d0,0.d0, 0.d0,-1.d0,0.d0, 0.d0,0.d0,1.d0/

     hourang = Mul3x3(Purm2,xyzT,Hourang)

     xs = hourang(1,1)
     ys = hourang(1,2)
     zs = hourang(1,3)

C ==================== Convert to Horizon Coordinates ====================
C     AZEL = R3(-180) * R2(latitude - 90) * xyzT
C From Mueller

     theta = DegRad(90.d0) - loclatr
     DATA ROT2 / 0.d0,0.d0,0.d0, 0.d0,1.d0,0.d0, 0.d0,0.d0,0.d0/
     ROT2(1,1) = DCOS(theta)
     ROT2(1,3) = DSIN(theta)
     ROT2(3,1) = -DSIN(theta)
     ROT2(3,3) = DCOS(theta)

     DATA ROT3 / -1.d0,0.d0,0.d0, 0.d0,-1.d0,0.d0, 0.d0,0.d0,1.d0/

     R3R2 = Mul3x3(ROT3,ROT2,R3R2)
     AZEL = Mul3x3(R3R2,hourang,AZEL)

     xo = AZEL(1,1)
     yo = AZEL(1,2)
     zo = AZEL(1,3)

     az = ArcTan(yo,xo)
     theta = SQRT(xo*xo + yo*yo)
     el = ArcTan(zo,theta)
     az = RadDeg(az)
     el = RadDeg(el)

     WRITE (*,3000) az
3000 FORMAT(1X,'Azimuth Look Angle =',1X,F8.3,1X,'degrees')
     WRITE (*,3010) el
3010 FORMAT(1X,'Elevation Look Angle =',1X,F8.3,1X,'degrees')

     GOTO 9980
4000 WRITE (*,9300)
```

```
9000 FORMAT( ' Enter observers latitude (degrees):  '\ )
9100 FORMAT( ' Enter observers longitude (deg east):  '\ )
9150 FORMAT( ' Enter observers geodetic hieght (meters):  '\ )
9200 FORMAT( ' Enter objects R/A J2000 (hhmmss.ss):  '\ )
9225 FORMAT( ' Enter objects DEC J2000 (ddmmss.ss):  '\ )
9226 FORMAT( ' Enter objects polar motion Xp (arcsec): '\ )
9227 FORMAT( ' Enter objects polar motion Yp (arcsec): '\ )
9228 FORMAT( ' Enter Besselian Day number D (arcsec): '\ )
9229 FORMAT( ' Enter Besselian Day number C (arcsec): '\ )
9250 FORMAT( ' Enter Observation date (ex: YYYYMMDD):  '\ )
9275 FORMAT( ' Enter Observation time, UT1 (ex: HHMMSS.SS):  '\ )
9300 FORMAT( ' The satellite is below the observers horizon. ')
9500 FORMAT( ' Elevation = 90.0,  Azimuth is undefined.')

9980 END
```

TRANSFORMATION BY MATRICES

A convenient way to transform from one coordinate system to another is via rotational matrices. A rotation is denoted by [R] where are R a rotation about one or more axis. A rotation around the X-axis by an angle q is denoted by $R_1(q)$, the Y-axis by $R_2(q)$ and the Z-axis by $R_3(q)$. The rotation matrices are given by:

$$R_1(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$$

$$R_2(\theta) = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$R_3(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Figure 2-11 illustrates the transformation from the local geodetic (x,y,z) to the local geodetic (e,n,u). The transformation is accomplished by rotation about the x-axis of 90 - f and then a rotation about the z-axis of 90 + l. This is denoted by:

$$[R] = R_1(90 - f)R_3(90 + l)$$

Where the rotations R_1 and R_2 are described by the above matrices.

```
CC ================== Degrees to Hour,Min,Sec ==================
   REAL*8 FUNCTION DegHMS (degrees,hrs,mns,scs)

   REAL*8 degrees,scs
   INTEGER hrs,mns
   hrs = INT(degrees/15.d0)
   mns = INT((degrees/15.d0 - hrs)*60)
   scs = ((degrees/15.d0 - hrs)*60 - mns)*60
   DegHMS = hrs
   END

CC ================== DEGREES TO DEG,MIN,SEC ==================

   REAL*8 FUNCTION DegDMS (degrees,deg,mns,scs)

   REAL*8 degrees,scs
   INTEGER deg,mns

   deg = INT(degrees)
   mns = INT((degrees - deg)*60)
   scs = ABS(((degrees - deg)*60 - mns)*60 )
   mns = ABS(mns)
   DegDMS = deg
   END

CC ================== DEGREES TO RADIANS ==================

   REAL*8 FUNCTION DegRad (degree)
   DOUBLE PRECISION pi / 3.141592653589793d0 /
   REAL*8 degree
   DegRad = (degree * 2.d0 * pi)/360.d0
   END

CC ================== RADIANS TO DEGREES ==================

   REAL*8 FUNCTION RadDeg (radians)
   DOUBLE PRECISION pi /3.141592653589793d0 /
   REAL*8 radians
   RadDeg = (radians * 360.d0)/(2.d0*pi)
   END

CC ================== MATRIX MULTIPLICATION ==================
   REAL*8 FUNCTION Mul33 (m1,m2,m3)
```

```
      REAL*8 m1(3,3),m2(3,3),m3(3,3)

      m3(1,1) = m1(1,1)*m2(1,1)+m1(2,1)*m2(1,2)+m1(3,1)*m2(1,3)
      m3(2,1) = m1(1,1)*m2(2,1)+m1(2,1)*m2(2,2)+m1(3,1)*m2(2,3)
      m3(3,1) = m1(1,1)*m2(3,1)+m1(2,1)*m2(3,2)+m1(3,1)*m2(3,3)

      m3(1,2) = m1(1,2)*m2(1,1)+m1(2,2)*m2(1,2)+m1(3,2)*m2(1,3)
      m3(2,2) = m1(1,2)*m2(2,1)+m1(2,2)*m2(2,2)+m1(3,2)*m2(2,3)
      m3(3,2) = m1(1,2)*m2(3,1)+m1(2,2)*m2(3,2)+m1(3,2)*m2(3,3)

      m3(1,3) = m1(1,3)*m2(1,1)+m1(2,3)*m2(1,2)+m1(3,3)*m2(1,3)
      m3(2,3) = m1(1,3)*m2(2,1)+m1(2,3)*m2(2,2)+m1(3,3)*m2(2,3)
      m3(3,3) = m1(1,3)*m2(3,1)+m1(2,3)*m2(3,2)+m1(3,3)*m2(3,3)
      Mul33 = m3(3,3)
      END


      REAL*8 FUNCTION Mul3x3 (m1,m2,m3)
* Multiplies to matrices returns answer in m3
      REAL*8 m1(3,3),m2(3,3),m3(3,3),temp

      DO 100 i=1,3
      DO 95 k=1,3
      temp = 0
      DO 90 j=1,3

      temp = temp + m1(j,i)*m2(k,j)

   90 CONTINUE
      m3(k,i) = temp
   95 CONTINUE
  100 CONTINUE

      Mul3x3 = m3(3,3)
      END

CC ============== FOUR QUANDRANT ARC TANGENT ==============

      REAL*8 FUNCTION Arctan (num,den)
      DOUBLE PRECISION pi /3.141592653589793d0/
      REAL*8 num,den,temp
      temp = DATAN(num/den)
      IF ((num .LT. 0) .AND. (den .LT. 0)) THEN
```

```
      temp = temp + pi
      END IF
      IF ((num .LT. 0) .AND. (den .GE. 0)) THEN
       temp = temp + 2.d0*pi
      END IF

      IF ((num .GE. 0) .AND. (den .LT. 0)) THEN
       temp = temp + pi
      END IF
      Arctan = temp
      END

CC ======================== JULIAN DATE ========================

      REAL*8 FUNCTION JulDate(yr,mn,dy,hr,min,sec)
      REAL*8 sec,temp
      INTEGER yr,mn,dy,hr,min

      temp = dy - 32075 + 1461*(yr + 4800 + (mn -14)/12)/4
     + +367*(mn - 2 - (mn - 14)/12*12)/12 - 3*((yr + 4900 +
     + (mn - 14)/12)/100)/4 - 0.5

      JulDate = temp + ((sec/60 + min)/60 + hr)/24
      END

CC ============== GREENWHICH MEAN SIDEREAL TIME ==============

      REAL*8 FUNCTION GmsTim(T,hr,min,sec)
      REAL*8 T, sec, rasun, UT1, GMST
      INTEGER hr, min

C   GMST = Greenwhich mean sidereal time
C   UT1 = Universal time of the Greenwhich Meridian
C   rasun = right ascension of the mean sun
C
C   GMST = UT1 + rasun - 12 hours

      rasun = 280.460618375d0 + 36000.77005360834d0*T
     +     + 0.0003879333333333d0*T*T - 2.583333333333333E-004*T*T*T

      UT1 = hr*15.d0 + min*15.d0/60.d0 + sec*15.d0/(60.d0*60.d0)

      GMST = UT1 + rasun - 12.d0*15.d0
```

```
840 IF (GMST .GE. 0) GOTO 850
   GMST = GMST + 360.d0
   GOTO 840

850 IF (GMST .LT. 360) GOTO 860
   GMST = GMST - 360.d0
   GOTO 850

860 GmsTim = GMST

      END

CC ========== NUTATION IN LONGITUDE AND OBLIQUITY ==========

CC  This program was written to calculate Nutation in Longitude and
CC  Nutation in Obliquity using the 1980 IAU Theory of Nutation.


CC ==================== Variables ====================
   REAL*8 FUNCTION NutRig(jd,params)
   REAL*8     jd,params(2)
   REAL*8     T,rs
   INTEGER    k1(106),k2(106),k3(106),k4(106),k5(106)
   REAL       A0(106),A1(106),B0(106),B1(106)
   REAL*8     alpha(5),sum,delsi,deleps,DegRad
CC ==================== Nutation Terms ====================

   DATA  k1 /0,0,-2,2,-2,1,0,2,0,0,0,0,0,2,0,0,0,0,0,-2,0,2,0,1,
  + 2,0,0,0,-1,0,0,1,0,1,1,-1,0,1,-1,-1,1,0,2,1,2,0,-1,-1,1,-1,1,
  + 0,0,1,1,2,0,0,1,0,1,2,0,1,0,1,1,1,-1,-2,3,0,1,-1,2,1,3,0,
  + -1,1,-2,-1,2,1,1,-2,-1,1,2,2,1,0,3,1,0,-1,0,0,0,1,0,1,1,2,0,0 /

   DATA k2 /0,0,0,0,0,-1,-2,0,0,1,1,-1,0,0,0,2,1,2,-1,0,-1,0,1,0,
  + 1,0,1,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,-1,0,
  + 0,0,0,0,0,0,-1,0,1,0,0,1,0,-1,-1,0,0,-1,1,0,0,0,0,0,0,0,0,0,0,
  + 0,1,0,0,0,-1,0,0,0,0,0,0,1,-1,0,0,1,0,-1,1,0,0,0,1 /

   DATA k3 /0,0,2,-2,2,0,2,-2,2,0,2,2,2,0,2,0,0,2,0,0,2,0,2,0,0,
  + -2,-2,0,0,2,2,0,2,2,0,2,0,0,0,2,2,2,0,2,2,2,2,0,0,2,0,2,2,2,
  + 0,2,0,2,2,0,0,2,0,-2,0,0,2,2,2,0,2,2,2,2,0,0,0,2,0,0,2,2,0,2,
  + 2,2,4,0,2,2,0,4,2,2,2,0,-2,2,0,-2,2,0,-2,0,2,0 /
```

```
      DATA k4 /0,0,0,0,0,-1,-2,0,-2,0,-2,-2,-2,-2,-2,0,0,-2,0,2,-2,
     + -2,-2,-1,-2,2,2,0,1,-2,0,0,0,0,-2,0,2,0,0,2,0,2,0,-2,0,0,0,2,
     + -2,2,-2,0,0,2,2,-2,2,2,-2,-2,0,0,-2,0,1,0,0,0,2,0,0,2,0,-2,0,
     + 0,0,1,0,-4,2,4,-4,-2,2,4,0,-2,-2,2,2,-2,-2,-2,0,2,0,-1,2,-2,
     + 0,-2,2,2,4,1 /

      DATA k5 /1,2,1,0,2,0,1,1,2,0,2,2,1,0,0,0,1,2,1,1,1,1,1,0,
     + 0,1,0,2,1,0,2,0,1,2,0,2,0,1,1,2,1,2,0,2,2,0,1,1,1,1,0,2,2,2,0,
     + 2,1,1,1,1,0,1,0,0,0,0,0,2,2,1,2,2,2,1,1,2,0,2,2,0,2,2,0,2,1,2,
     + 2,0,1,2,1,2,2,0,1,1,1,2,0,0,1,1,0,0,2,0 /

      DATA A0 /-171996,2062,46,11,-3,-3,-2,1,-13187,1426,-517,217,129,
     + 48,-22,17,-15,-16,-12,-6,-5,4,4,-4,1,1,-1,1,1,-1,-2274,712,-386,
     + -301,-158,123,63,63,-58,-59,-51,-38,29,29,-31,26,21,16,-13,-10,
     + -7,7,-7,-8,6,6,-6,-7,6,-5,5,-5,-4,4,-4,-3,3,-3,-3,-2,-3,-3,2,-2,
     + 2,-2,2,2,1,-1,1,-2,-1,1,-1,-1,1,1,1,-1,-1,1,1,-1,1,1,-1,-1,-1,
     + -1,-1,-1,-1,1,-1,1 /

      DATA A1 /-174.2,0.2,0,0,0,0,0,0,-1.6,-3.4,1.2,-0.5,0.1,0,0,-0.1,
     + 0,0.1,0,0,0,0,0,0,0,0,0,0,0,0,0,-0.2,0.1,-0.4,0,0,0,0,0,0.1,
     + -0.1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
     + 0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
     + 0,0,0,0,0,0,0 /

      DATA B0 /92025,-895,-24,0,1,0,1,0,5736,54,224,-95,-70,1,0,0,9,7,6,
     + 3,3,-2,-2,0,0,0,0,0,0,0,977,-7,200,129,-1,-53,-2,-33,32,26,27,16,
     + -1,-12,13,-1,-10,-8,7,5,0,-3,3,3,0,-3,3,3,-3,3,0,3,0,0,0,0,0,1,1,
     + 1,1,1,-1,1,-1,1,0,-1,-1,0,-1,1,0,-1,1,1,0,0,-1,0,0,0,0,0,0,0,0,0,
     + 0,0,0,0,0,0,0,0 /

      DATA B1 /8.9,0.5,0,0,0,0,0,0,-3.1,-0.1,-0.6,0.3,0,0,0,0,0,0,0,0,0,
     + 0,0,0,0,0,0,0,0,-0.5,0,0,-0.1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
     + 0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
     + 0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 /

C============= Calculate time in julian centries since J2000.0 ==============

      T = (jd - 2451545.d0)/36525.d0

C ============ Calculate Nutation in longitude and obliquity ============

C   delsi = nutation in longitude
C   deleps = nutation in obliquity
```

```
C  From Kaplan 1981, Cannon, 1981 (paper handed out in class)
C   alpha(1) = mean anomaly of the moon      100
C   alpha(2) = mean anomaly of the sun
C   alpha(3) = mean arqument of latitude of the moon
C   alpha(4) = mean elongation of the moon from the sun
C   alpha(5) = mean longitude of the ascending lunar node
C         rs = 1296000 seconds equals 360 degrees
          rs = 1296000.d0

    alpha(5) = 450160.280d0 - (5.d0*rs + 482890.539d0)*T
   + + 7.455d0*T*T + 0.008d0*T*T*T

    alpha(4) = 1072261.307d0 + (1236.d0*rs + 1105601.328d0)*T
   + -6.891d0*T*T + 0.019d0*T*T*T

    alpha(3) = 335778.877d0 + (1342.d0*rs + 295263.137d0)*T
   + - 13.257d0*T*T + 0.011d0*T*T*T

    alpha(2) = 1287099.804d0 + (99.d0*rs + 1292581.224d0)*T
   + -0.577d0*T*T - 0.012d0*T*T*T

    alpha(1) = 485866.733d0 + (1325.d0*rs + 715922.633d0)*T
   + + 31.31d0*T*T + 0.064d0*T*T*T

    DO 100 i=1,5
    alpha(i) = alpha(i) - INT(alpha(i)/1296000.d0) * 1296000.d0
    alpha(i) = alpha(i)/3600.d0
    alpha(i) = DegRad(alpha(i))
  100 CONTINUE
C ==================== Calculate Nutation in Longitude ====================
    delsi = 0.d0
    DO 110 j=1,106
       sum = k1(j)*alpha(1) + k2(j)*alpha(2) + k3(j)*alpha(3)
   +       + k4(j)*alpha(4) + k5(j)*alpha(5)
       delsi = delsi + (A0(j)/10000.d0 + (A1(j)/10000.d0)*T)
   +          *DSIN(sum)
  110 CONTINUE
C ==================== Calculate Nutation in Obliquity ====================
    deleps = 0.d0
    DO 120 j=1,106
       sum = k1(j)*alpha(1) + k2(j)*alpha(2) + k3(j)*alpha(3)
   +       + k4(j)*alpha(4) + k5(j)*alpha(5)
```

```
      deleps = deleps + (B0(j)/10000.d0 + (B1(j)/10000.d0)*T)
   +        *DCOS(sum)
 120 CONTINUE

C     deleps = -0.888d0/3600.d0
C     delsi = 15.570d0/3600.d0
      delsi = delsi/3600.d0
      params(1) = delsi
      deleps = deleps/3600.d0
      params(2) = deleps
      NutRig = params(2)
      END
```

Methods For Determining Dish Antenna Pointing Angles

by

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(ABSTRACT)

Satellite Look Angles are the coordinates to which an earth station antenna must point to communicate with a satellite. Each satellite has it's own unique set of look angles. The first method, developed to calculate a satellite's look angles, uses standard plane and spherical trigonometry and assumes a perfectly spherical earth. The second method developed is unique to this paper and will not be found anywhere, including general satellite communication textbooks. This method uses a geodetic reference system which refers to the earth as an ellipsoid rather than a sphere. This second method is a more rigorous approach to determining look angles and readily lends itself to pointing at satellites in any given orbit. Fortran code was written implementing both methods and it is concluded that employing a geodetic reference frame is viable where high degrees of accuracy are required. Fortran code was also developed to calculate the pointing angles for Radio Sources such as pulsars and quasars. This code corrects for precession, nutation, annual aberration, and polar motion of the earth.