Demand Management in Evacuation Planning: Models, Algorithms, and Applications

Douglas R. Bish

Dissertation submitted to Faculty of the
Virginia Polytechnic Institute and State University
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy
in
Civil Engineering

Dr. Antoine G. Hobeika, Co-Chair
Dr. Hanif D. Sherali, Co-Chair
Dr. Hojong Baik
Dr. Pushkin Kachroo
Dr. Antonio A. Trani

July 31, 2006
Blacksburg, Virginia

Demand Management in Evacuation Planning: Models, Algorithms, and Applications

Douglas R. Bish

Abstract

Evacuation planning is an important disaster management tool. A large-scale evacuation of a region by automobile is a difficult task, especially as demand is often greater than supply. This is made more difficult as the imbalance of supply and demand actually reduces supply due to congestion. Currently, most of the emphasis in evacuation planning is on supply management. The purpose of this dissertation is to introduce and study sophisticated demand management tools, specifically, staging and routing of evacuees. These tools can be used to produce evacuation strategies that reduce or eliminate congestion. A strategic planning model is introduced that accounts for evacuation dynamics and the non-linearities in travel times associated with congestion, yet is tractable and can be applied to large-scale networks. Objective functions of potential interest in evacuation planning are introduced and studied in the context of this model. Insights into the use of staging and routing in evacuation management are delineated and solution techniques are developed. Two different strategic approaches are studied in the context of this model. The first strategic approach is to control the evacuation at a disaggregate level, where customized staging and routing plans are produced for each individual or family unit. The second strategic approach is to control the evacuation at a more aggregate level, where evacuation plans are developed for a larger group of evacuees, based on pre-defined geographic areas. In both approaches, shelter requirements and preferences can also be considered. Computational experience using these two strategic approaches, and their respective solution techniques, is provided using a real network pertaining to Virginia Beach, Virginia, in order to demonstrate the efficacy of the proposed methodologies.

Acknowledgments

I would like to extend a special thanks to both of my advisors, Dr. Antoine G. Hobeika and Dr. Hanif D. Sherali, for their guidance and support throughout my Ph.D. Working with them was both an honor and a valuable learning experience. I would also like to thank the rest of my committee, Dr. Antonio A. Trani, Dr. Hojong Baik, and Dr. Pushkin Kachroo.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 Motivation and Background

Evacuation is often the only viable option when disasters, such as hurricanes, floods, fires, or nuclear incidents, occur. This is especially true if the disaster cannot easily be averted. Some of these evacuations are large-scale events, affecting potentially millions of people (especially evacuations in response to hurricanes, which can affect large geographic regions including densely populated coastal areas, or when large metropolitan areas are involved). In the United States, a majority of large-scale evacuations are accomplished by automobile, which is often the only transportation option available to a majority of evacuees. Unfortunately, when evacuating a metropolitan or densely populated coastal area by automobile, the number of evacuees can easily overwhelm the available roadways. The imbalance between evacuation supply (roadway capacity) and demand (the population trying to evacuate) leads to traffic congestion and gridlock, which in turn aggravates the imbalance by effectively decreasing supply. Besides being quite inconvenient and increasing the time required to complete an evacuation, gridlock can potentially be very dangerous; it can discourage people from evacuating threatened areas and leave evacuees extremely vulnerable if they are trapped on the roadway when the disaster strikes. This problem is only likely to increase as populations grow while roadway capacities remain fairly static. Currently, most metropolitan areas experience congestion on a daily basis due to imbalances between supply and commuter demand, let alone the higher demands that evacuations can generate. To evacuate metropolitan and densely populated coastal areas in a timely manner, the roadways must be

1

used efficiently, which implies avoiding congestion, i.e., somehow balancing supply and demand. This requires proper strategies for evacuation management. We categorize evacuation management strategies into two distinct classes; 1) supply management strategies, which try to manage/modify the roadway infrastructure to make it more efficient for evacuation, and; 2) demand management strategies, which try to organize/manage the evacuee population to improve the evacuation process. In practice, supply management tools are much more prevalent; demand management tools are far less understood. In this dissertation, we study two demand management tools, namely "staging," which refers to determining when defined groups should start evacuating, and "routing," which refers to determining the specific route each group should use to evacuate.

To provide context, in this dissertation we study hurricane evacuations. Despite this, the work can be applied to any extensive regional evacuation, and to a lesser extent, to the evacuation of large buildings or facilities as well. We note that special considerations might be appropriate for a specific type of disaster, based on characteristics such as advanced warning, risk, and extent that the population to be evacuated has to treated or cared for after/during the evacuation. Important characteristics of hurricane evacuations are their large-scale, the ability to obtain advanced warning, and a fairly defined region where they are likely to strike. Currently, hurricane evacuations are also occurring with an increased frequency. The well-defined prone regions (the Atlantic and Gulf coasts) and increased frequency of hurricanes insures that planning for hurricane evacuations is mandatory and of current interest to policy makers. Next we offer some examples of hurricanes, their impacts, and the success of current evacuation planning strategies.

In 1900, before the technology to predict hurricanes was available, and hence before evacuation was a viable option, a hurricane killed at least 8,000 people in Galveston, Texas, while leveling 12 city blocks, nearly three-quarter of the city at that time. In 1938 a hurricane struck Long Island, destroying more than 8,000 homes and 6,000 boats, while causing 60 deaths and extensive flooding in the New England area. In 1957, a hurricane hit the low-lying areas of coastal Louisiana causing 390 deaths. It was thought that evacuees still had one day left to evacuate the area due to forecasting errors. In 1999 Hurricane Floyd caused hurricane warnings from South Florida to Massachusetts. An estimated two million people were evacuated. In 2004 Hurricane Ivan threatened New Orleans (among many other places). The following is from the New Orleans City Business newspaper, September 27, 2004: "State

officials claim the evacuation process worked during Hurricane Ivan but they admit it needs major improvements. Governor Kathleen Babineaux Blanco ordered a review of state plans for emergency readiness in the wake of massive traffic problems during the evacuation. In many reported instances, it took more than 10 hours to make the 90-minute trip from New Orleans to Baton Rouge [a distance of about 85 miles]." *Contra-flow*, which increases the supply out of the endangered area by reversing the flow on selected roadway segments, was the major component of the evacuation plan for New Orleans. In 2005, some two million plus people were evacuated from the Galveston and Houston areas due to Hurricane Rita. Despite contra-flow measures, this evacuation caused massive gridlock, with traffic jams 100 miles long, which in turn caused fuel shortages leaving many evacuees without fuel on the highway, all in hot, humid weather. Around 60 fatalities were connected with this evacuation.

Although current strategies are effective, as we can see from the above descriptions of recent evacuations, more needs to be done to improve the evacuation process. Current strategies mainly rely on supply management, the most important supply management tool being contra-flow, i.e., the reversal of flow on selected roadway segments to increase roadway capacity out of the endangered area. While contra-flow is effective, it does present challenges, and has limitations. Contra-flow can require extensive modifications of traffic flow, often affecting roadways in multiple jurisdictions, thus it can have a large impact on commerce and traffic in areas not directly impacted by the hurricane. Contra-flow lanes can rarely be used to their full capacity due to loading and unloading issues. Lanes that can be converted by contra-flow are also sometimes limited based on the need to allow emergency equipment/personnel and buses to enter the evacuation area to assist in disaster management and to evacuate that portion of the population without access to vehicles or ability to drive. Other supply management tools, often used in conjunction with contra-flow, are modifying traffic flows at select intersections and removal of tolling apparatus. Currently, evacuation demand management tools are quite rudimentary; they mainly consist of issuing evacuation orders (with a level of severity) to affected areas, while trying to limit unnecessary evacuation from unaffected areas, the designation of evacuation routes, and the identification of evacuation shelters. There is an interest in more advanced demand management strategies, such as staging, which refers to temporally spreading demand by determining when different evacuee groups should start their evacuation, as opposed to a single, blanket order for the whole, at-risk, population. In fact, the evacuation plan for the coastal areas of Virginia

and the New Orleans area incorporate some simple staging strategies, which are based on risk considerations (low-lying and coastal areas first, more inland areas last). These staging schemes are not necessarily scientifically determined, and they might not have a large impact on congestion. The underlying philosophy seems to be that if the disaster strikes before the evacuation is complete, at least the evacuees from higher risk areas have been evacuated. The manner in which staging should be implemented is not well understood. Implemented correctly, we believe a staging strategy for a hurricane evacuation should not need to be based on risk, as it is desirable to complete the evacuation for all at-risk populations before any risk is incurred. Instead the evacuation should be based on an optimal usage of the roadways; the goal is to manage the evacuation demand so as not to overwhelm the roadways, thus avoiding congestion and its unwanted reduction in effective supply, thus improving the evacuation process.

One of the main goals of this research is to produce a body of academic work, using analytical models, that allows for a better understanding of evacuation demand management strategies, specifically those strategies based on staging and routing. In order to accomplish this goal, this dissertation presents a novel modeling framework that combines two areas of research, dynamic traffic assignment (DTA) and evacuation planning. This unique framework has the following features: 1) traffic congestion on a segment of roadway is modeled using a well-founded traffic flow model, which implicitly takes into account the non-linearities in travel time caused by congestion; 2) the network effects of congestion are modeled, i.e., how congestion on one segment of roadway impacts other road segments; 3) the dynamics of how an evacuation unfolds through time are modeled; 4) evacuation shelters both with and without capacity limitations are modeled (a mix of shelter types is common in practice; shelters without limits on capacity often refer to regions outside the affected area, and perhaps not an actual physical shelter; and, 5) evacuee shelter preferences and requirements are considered.

Within the context of this model, we accomplish the following: 1) different objective functions, appropriate for the modeling of evacuations, are studied; 2) the benefits of evacuation strategies based on staging and routing are studied and insights developed; 3) the impact of shelter capacity limits are examined; 4) the benefits of different levels of planning detail are examined; and finally, 5) this methodology is tested on a large-scale, realistic network, based on Virginia Beach, Virginia, an area that is potentially at risk for a hurricane strike,

which actively plans for hurricane evacuations.

Evacuation management is a very complex and rich problem. There are many aspects that are beyond the scope of this dissertation, especially those involving stochasticity. Human behavior and compliance issues are very important aspects of evacuation management, especially when we are considering incorporating demand management strategies. Human behavior also induces a high level of stochasticity usually found in this problem. We believe this is partially due to the lack of demand management; evacuees are usually given fairly open-ended instructions, e.g., evacuate the area as soon as possible. If the area is inhabited by two million people, this leads to many possible scenarios. In this research, we model only limited evacuee behavior, and assume a somewhat idealized response to the evacuation plan. While for the most part, evacuees do behave well during an evacuation, further study will be needed to determine behavior parameters given the more complex plans and higher level of organization that demand management requires. These parameters, in turn, will have to be incorporated into planning models. We also do not study how best to package and disseminate evacuation plans or enforce them.

This dissertation is organized as follows. In the remainder of this chapter we review the relevant literature. In Chapter 2 we define the model notation, present the general constraint set (which later models will utilize) and various objective functions useful for evacuation planning. Then we discuss the modeling of traffic flow and congestion in more detail. In Chapter 3 we study planning at the most disaggregate level, including the impact of the various objective functions considered on solution time and other properties of the evacuation strategy, and include some managerial insights gleaned from this research, along with solution strategies. In Chapter 4 we study more aggregate evacuation strategies, and discuss solution approaches to this difficult combinatorial problem. In Chapter 5 we present a case study using the city of Virginia Beach, Virginia. Finally, Chapter 6 concludes the dissertation and discusses future research into this interesting and complex problem.

1.2 Literature Review

In this section we examine the evacuation literature in detail and the dynamic traffic assignment (DTA) literature briefly. We begin with papers that offer a general understanding of the regional evacuation problem. We follow this with a brief review of papers that discuss

some behavioral aspects of evacuation. Next, we describe related models in the literature beginning with simulation models and followed by analytical models of regional evacuations. Next, we review the literature of a closely related topic, namely the evacuation of buildings and facilities. We concentrate on papers that use network based models to study building and facility evacuation, as this line of research has many similarities with our modeling paradigm. To complete the evacuation literature, we discuss existing work on staging in evacuation, which consists of only two simulation papers. Next, we briefly review the literature on dynamic traffic assignment problems.

The following papers contribute to an understanding of regional evacuations in general. For example, in an article on evacuation issues, Wolshon and Meehan (2003) discuss demand management, but in the context of limiting evacuees from areas where evacuation is not required, thus easing the burden on the transportation system. Urbina and Wolshon (2003) discuss current evacuation management policies, which are mainly concerned with supply management. Petruccelli (2003) discusses evacuation after a seismic incident. The article mentions coordinated (i.e., staged) evacuations to ease demand on the system, but just as an aside. Wolshon, Catarella-Michel, and Lambert (2006) along with Wolshon (2006) discuss the evacuation of Louisiana due to Hurricane Katrina, while Litman (2006) investigates evacuation problems caused by both Hurricanes Katrina and Rita. In these papers, any discussion of staging is minor, or non-existent. Most of the focus is on supply management.

A considerable amount of the literature describes modeling aspects of evacuee behavior, most of which does not directly impact this dissertation. Murray-Tuite and Mahmassani (2003) studied trip-chaining behavior during an evacuation. In a hurricane preparedness study for the city of Virginia Beach, Hobeika, Radwan, and Jamei (1985) use a sigmoidal or S-shaped loading curve to describe aggregate evacuee behavior in respect to evacuee departure time decisions. Several other studies used similar curves, see for instance Lewis, 1985. The US Army Corps of Engineers (http://chps.sam.usace.army.mil/USHESdata/heshome.htm) performs hurricane evacuation assessment studies for various hurricanes (Opal and Hugo, for instance). Most loading curves constructed in their studies are roughly 'S'-shaped. In Fu and Wilmot (2003) a sequential logit model is used to model evacuation demand. While this logit model produces a more complex loading curve, overall it has a rough 'S' shape.

The following is a brief review of papers that use simulation for analysis, or describe simulation systems for evacuation management. Sheffi, Mahmassani, and Powell (1982) discuss

the NETVAC simulation model, which focuses on determining the network clearance time for an evacuation. Evacuees choose routes based on a good knowledge of the network under normal circumstances. This routing can then be modified en route based on the current traffic conditions. Hobeika, Kim, and Beckwith (1994) discuss the TEDSS system, a decision support tool for evacuation planning. Hobeika and Kim (1998) discuss the simulation module of TEDSS, called MASSVAC. An evacuation is simulated using MASSVAC to compare two traffic routing strategies; specifically a user equilibrium traffic assignment and an assignment using Dial's algorithm. Their results suggest that the user equilibrium assignment produces a faster evacuation. Tufekci (1995) presents a conceptual framework for an evacuation decision support system. Pidd, deSilva, and Eglese (1996) review an evacuation planning tool, CEMPS, which integrates a microscopic simulator with a GIS.

The number of analytical papers on regional evacuations is quite small. Sherali, Carter, and Hobeika (1991) examine the combined shelter location and traffic routing problem using a static network model. This type of static representation does not model the dynamics of an evacuation, which is especially important when capacitated shelters are considered, or when the network impact of congestion is significant. Consequently, this analysis was combined with a specialized evacuation simulation to address some of these issues. Church and Cova (2000) present a model for assessing evacuation risk by finding small neighborhoods (i.e., a cluster of nodes) having a high population compared to the exit capacity. Cova and Johnson (2003) present a static network model for determining the shortest paths for all evacuees, while eliminating cross traffic at intersections (a potentially large cause of delay, especially in an urban area) and by constraining the number of traffic merges. Each arc is given a travel time that is unaffected by congestion.

Related to our topic is the modeling of facility and building evacuation using a network approach (for a more complete review see Hamacher and Tjandra, 2001). Chalmet, Francis, and Saunders (1982) model the evacuation of a building using a dynamic transshipment model where each arc has a constant capacity and a travel time. The travel time across the arc is assumed to be independent of the flow on the arc. Jarvis and Ratliff (1982) show that for the dynamic transshipment problem, the flow that maximizes the output for each time interval also minimizes the number of time intervals needed to transship all material (in other words, complete the evacuation), and also minimizes the average time for all flow to arrive at the sink. The arc transit times remain constant. Hamacher and Tufekci (1987)

discuss this class of evacuation models along with multiple cost types for each arc and lexicographic minimum cost flows. This allows the consideration of multiple hierarchical objectives. Choi, Hamacher, and Tufekci (1988) investigate evacuations using a dynamic, time expanded, network flow model with side constraints that limit the capacity of an arc (the number of evacuees the arc can tranship in one time interval), based on the number of evacuees at the arc's entrance node. This models the effect of crowding (i.e., congestion) on flow. While the authors cite research that shows that the flow/density relationship is unimodal for pedestrians, they have modeled it as an upward sloping linear function, which is unrealistic as it does not model the negative effects of congestion. The authors cite a technical report where a piece-wise linear approximation is used to model the flow/density relationship (much as we shall do), but only one room is examined, and the goal is to determine its maximum capacity, given a desired evacuation time. Hoppe and Tardos (2000) examine the transshipment problem on a dynamic network with multiple sources and sinks, where each arc has a capacity and a travel time. The objective is to determine the quickest transshipment scheme. They develop polynomial-time algorithms for solving this problem. Smith (1991) models the evacuation of a facility using a state-dependent network queuing model.

Next, we discuss two papers that explore staging in evacuation. Chen and Zhan (2004) is one of the first studies of staging in evacuation. Using an agent-based simulation, several staged evacuation strategies are compared to a non-staged evacuation on three different network topographies. Routing of evacuees is based on the shortest path under normal circumstances, thus specialized evacuation routing is not considered. They find that some staged evacuation strategies outperform the simultaneous evacuation strategy when population densities are high and a generic grid network is used. This is not the case on a generic ring network, or on a more realistic network based on the city of San Marcos, Texas. On these networks, the simultaneous evacuation strategy outperforms all staging strategies under all population densities. However, as this approach is purely simulation-based, only a limited number of staging strategies are tested, and it is difficult to ascertain if these are particularly good strategies. Sbayti and Mahmassani (2006) also look at staging to improve the evacuation process. In this study a zone, which is a small portion of a larger network, is evacuated. A simultaneous evacuation is compared to a staged evacuation. It is assumed that the traffic flow in the larger network as a whole is unaffected by the evacuation (at least

as far as demand and routing are concerned) except for those travelers going to the affected zone; these travelers cancel their trip, or return to their origins if they have already begun their trip. The evacuees from the zone are sent to a specified shelter. They use an iterative bi-level formulation, where the upper problem determines routes and the lower-problem determines how to load the network. Neither staging paper discusses in great detail what a simultaneous evacuation actually entails. Do all evacuees leave as quickly as they can given the roadway capacities (as implied), which might be fairly unrealistic, or is a loading curve used?

We now briefly discuss some dynamic traffic assignment (DTA) literature. There is a vast extent of DTA literature; here we mention some salient papers that study analytical models based on mathematical programming. For a more detailed review we refer the reader to Peeta and Ziliaskopoulos (2001). One of the first DTA papers was by Merchant and Nemhauser (1978), in which traffic flow is propagated using an exit function for each arc. Ziliaskopoulos (2000) introduces a linear programming cell transmission model. Carey, Ge, and McCartney (2003) and Carey and McCartney (2004) study exit-flow and travel-time models for each arc. These DTA papers, like most, use either a system optimal or user equilibrium based objective, or some variation of these objectives.

Chapter 2

General Modeling Framework

2.1 Introduction

In this chapter, we present the general modeling framework used throughout this dissertation. In Section 2.2 we present some general modeling notation. We note that additional notation is introduced throughout this dissertation, as needed, for specific analysis. Next, in Section 2.3, we introduce the general constraint set for the Regional Evacuation Model (REM). This constraint set represents the rules that govern traffic flow, along with some evacuation-specific constraints, e.g., capacity constraints on shelters. In Section 2.4, we introduce a series of objective functions that are of interest in evacuation planning. In Section 2.5 we introduce and discuss the main traffic flow model used in this work. Finally, in Section 2.6 we introduce a test network that is used throughout this dissertation, along with two evacuation scenarios, each based on the test network, but with varying parameters for several entities such as population density.

2.2 Modeling Notation

Consider a network (N, A), where N and A respectively denote the set of nodes and arcs. Set N is composed of the following disjoint sets: the set of origins $V \subset N$, the set of junctions $W \subset N$, and the set of shelters $Y \subset N$. Furthermore, by appropriately augmenting the network if necessary, we assume, without loss of generality, that each origin $k \in V$ has only outgoing arcs, and each shelter $j \in Y$ has only incoming arcs. Each origin repre-

sents a geographic zone with a known number of evacuees. These evacuees can potentially be classified into different types, represented by the set Z, based on their shelter preferences/requirements. For instance, in a hurricane evacuation, one evacuee type might include those that prefer to leave the evacuation area and find their own shelters, e.g. with families, friends, or in hotels, while another evacuee type might include those that require space in hurricane shelters. Other possible evacuee types include those that require medical care, perhaps representing the movement of patients from one hospital to another, or those having other special needs and requiring correspondingly equipped shelters, be it with medical staff, or a reliable source of electricity (provided by generators) to run medical equipment, or even the ability to handle pets. On this network, we define the following(several groups of these notation shall be used in different models in late sections):

Parameters and Sets:

P	:	set of evacuation paths in the network (N, A)
Z	:	set of evacuee types based on shelter preferences/requirements
T	:	number of intervals, of equal duration (typically one minute), in the planning horizon
F	:	last interval to start an evacuation for any origin and still complete the evacuation
$D_{k,z}$	:	number of evacuees of type z from origin k, $\forall k \in V\ z \in Z$
D	:	total number of evacuees, that is, $D = \sum_{k \in V} \sum_{z \in Z} D_{k,z}$
$h_f^{t,k,z}$	:	evacuees of type z that exit origin k in interval t if that node begins evacuation in interval f, $\forall k \in V,\ t = 1, \cdots, T,\ f = 1, \cdots, F$
$C_{i,z}$	:	capacity of shelter i for evacuees of type z, $\forall i \in Y,\ z \in Z$
$v_{ij}(.)$	:	cost function for arc (i, j), $\forall (i, j) \in A$
q_{ij}^{max}	:	maximum flow on arc (i, j), $\forall (i, j) \in A$
k_{ij}^{max}	:	maximum evacuee density on arc (i, j), $\forall (i, j) \in A$ (also referred to as *jam density*)
u_{ij}^{max}	:	maximum speed on arc (i, j), $\forall (i, j) \in A$ (also referred to as *free-flow speed*)
$P^{k,z}$	:	set of evacuation paths from origin k to shelters having type z capacity, $\forall k \in V,\ z \in Z$
$P^{t,k,z}$	:	set of time indexed paths from set $P^{k,z}$ that reach shelter in interval t,

$$\forall k \in V, \ z \in Z, t = 1, \cdots, F$$

δ_{ij}^l :   binary indicator parameter, which is 1 if arc (i, j) is on path l, and 0 otherwise, $\forall (i, j) \in A, \ k \in V, \ z \in Z, \ l \in P_{k,z}$

Variables:

a_{ij}^t :   number of evacuees that enter arc (i, j) in interval t, $\forall (i, j) \in A, \ t = 1, \cdots, T$

b_{ij}^t :   number of evacuees that exit arc (i, j) in interval t, $\forall (i, j) \in A, \ t = 1, \cdots, T$

x_{ij}^t :   number of evacuees on arc (i, j) at the beginning of interval t,
$\forall (i, j) \in A, \ t = 1, \cdots, T$

$a_{ij}^{t,k,z}$:   number of evacuees of type z from origin k that enter arc (i, j) in interval t,
$\forall (i, j) \in A, \ k \in V, z \in Z, \ t = 1, \cdots, T$

$b_{ij}^{t,k,z}$:   number of evacuees of type z from origin k that exit arc (i, j) in interval t,
$\forall (i, j) \in A, \ k \in V, z \in Z, \ t = 1, \cdots, T$

$x_{ij}^{t,k,z}$:   number of evacuees of type z from origin k on arc (i, j) at the beginning
of interval t, $\forall (i, j) \in A, \ k \in V, z \in Z, \ t = 1, \cdots, T$

$\gamma_f^{k,z}$:   binary indicator variable, which is 1 if evacuees of type z start the evacuation
of origin k in interval f, and 0 otherwise, $\forall k \in V, \ z \in Z, \ f = 1, \cdots, F$

$\lambda_{ij}^{k,z}$:   binary indicator variable, which is 1 if arc (i, j) is part of the evacuation route for
evacuees of type z and origin k, and 0 otherwise, $\forall (i, j) \in A, \ k \in V, \ z \in Z$

ρ^l :   binary indicator variable, which is 1 if evacuation path l is utilized,
and 0 otherwise, $\forall k \in V, \ z \in Z, \ l \in P^{k,z}$

E^t :   binary indicator variable, which is 1 if the evacuation is ongoing in time interval t,
and 0 otherwise, $t = 1, \cdots, T$.

In the next section we introduce our modeling assumptions and constraints, along with some terminology.

2.3   Constraint Set of the Regional Evacuation Model

We now formulate a model for prescribing an evacuation plan, based on the staging and routing of evacuees and the allocation of evacuees to shelters, in order to accomplish various objectives such as minimizing the number of time intervals needed to complete the evacu-

ation, minimizing the average evacuation time, or maximizing the number of evacuees to reach shelter during each interval. While the proposed model is a dynamic network flow model, and as such, does not represent discrete evacuees, it conceptually provides control of when specified numbers of evacuees start evacuating, which shelters they choose, and what routes they travel. The modeling assumptions include the following:

1. The network represents the road network of interest. Each origin node $k \in V$ represents a zonal centroid; hence, some details of the network are subsumed by these nodes. The zones can be determined to allow for various levels of network detail.

2. The term evacuee and vehicle will be used synonymously throughout this work since we are studying the automobile-centric evacuation, i.e., the evacuation of a region using private vehicles. In actuality, the number of evacuees and vehicles will be related by a vehicle occupancy rate.

3. Without loss of generality, we assume that each arc (i, j) can be traversed in a single time interval at the arc's maximum speed, u_{ij}^{max}. Note that any network with arcs having travel times that are integral functions of the length of the time interval used can be transformed into this type of network by introducing dummy nodes.

4. The number of evacuees on arc (i, j) at the beginning of the first time interval, x_{ij}^1, $\forall (i, j) \in A$, is a given parameter. Here we assume that $x_{ij}^1 \equiv 0$, $\forall (i, j) \in A$. This assumption can easily be relaxed with some minor modifications to the formulation.

5. The traffic flow parameters for arc (i, j), q_{ij}^{max}, k_{ij}^{max}, and u_{ij}^{max} are scaled to the arc and time interval length, i.e., q_{ij}^{max} is the maximum flow on each arc in a single interval, k_{ij}^{max} is the maximum number of vehicles each arc can hold, and u_{ij}^{max} is, by Assumption 3, equal to one arc length per time interval.

We now present the mathematical formulation of the constraint set of the REM. This represents the rules that govern the flow of traffic flow, specify the number of evacuees in each geographic zone, and their composition with regards to shelter preferences/requirements, and the shelter capacities. These constraints represent a minimal set of restrictions for the evacuation problem. Further constraints will be added, as needed, to delineate different classes of evacuation strategies.

Model 2.1 Constraint Set of the REM

$$x_{ij}^{t+1,k,z} - x_{ij}^{t,k,z} - a_{ij}^{t,k,z} + b_{ij}^{t,k,z} = 0, \quad \forall (i,j) \in A, \ k \in V, \ z \in Z,$$

$$t = 1, \cdots, T-1 \tag{2.1}$$

$$\sum_{j:(j,i)\in A} b_{ji}^{t,k,z} - \sum_{j:(i,j)\in A} a_{ij}^{t,k,z} = 0, \quad \forall i \in W, \ k \in V, \ z \in Z, \ t = 1, \cdots, T \tag{2.2}$$

$$\sum_{t=1}^{T} \sum_{j:(k,j)\in A} a_{kj}^{t,k,z} - D_{k,z} = 0, \quad \forall k \in V, \ z \in Z \tag{2.3}$$

$$\sum_{k\in V} \sum_{z\in Z} a_{ij}^{t,k,z} \leq q_{ij}^{max}, \quad \forall (i,j) \in A, \ t = 1, \cdots, T \tag{2.4}$$

$$\sum_{k\in V} \sum_{z\in Z} b_{ij}^{t,k,z} \leq \sum_{k\in V} \sum_{z\in Z} x_{ij}^{t,k,z}, \quad \forall (i,j) \in A, \ t = 1, \cdots, T \tag{2.5}$$

$$\sum_{k\in V} \sum_{z\in Z} b_{ij}^{t,k,z} \leq (k_{ij}^{max} - \sum_{k\in V} \sum_{z\in Z} x_{ij}^{t,k,z}) q_{ij}^{max} / (k_{ij}^{max} - q_{ij}^{max}),$$

$$\forall (i,j) \in A, \ t = 1, \cdots, T \tag{2.6}$$

$$b_{ij}^{t,k,z} \leq x_{ij}^{t,k,z}, \quad \forall (i,j) \in A, \ k \in V, \ z \in Z, \ t = 1, \cdots, T \tag{2.7}$$

$$\sum_{t=1}^{T} \sum_{i:(i,j)\in A} \sum_{k\in V} b_{ij}^{t,k,z} - C_{j,z} \leq 0, \quad \forall j \in Y, \ z \in Z \tag{2.8}$$

$$a_{ij}^{t,k,z}, b_{ij}^{t,k,z}, x_{ij}^{t,k,z} \geq 0, \quad \forall (i,j) \in A, \ k \in V, \ z \in Z, \ t = 1, \cdots, T. \tag{2.9}$$

Equation (2.1) defines the state of the system in time interval $t + 1$ based on the state and flows from time interval t. Equation (2.2) represents flow conservation across each junction $i \in W$. Equation (2.3) insures the complete evacuation of all evacuees of type z from each origin $k, \ \forall k \in V, \ z \in Z$. Equation (2.4) limits the flow entering an arc to the arc's maximum flow, q_{ij}^{max}. This is justified because if a flow rate higher than this maximum flow entered the arc, it would quickly be reduced due to a high local density at the entrance of the arc. Equations (2.5), (2.6), and (2.7) represent the Pipes traffic flow relationship (see Section 2.5) that defines each arc's exit-flow limits. The Pipes relationship is a dual-regime model: Equation (2.5) represents uncongested flow, while Equation (2.6) represents the congested regime. Both regimes have a linear flow-density relationship, which implies a constant speed over the uncongested regime, and a speed that decreases in a non-linear fashion (as density increases) in the congested regime. This traffic-flow relationship limits the number of evacuees on arc (i,j) to k_{ij}^{max}, because if $\sum_{k\in V} \sum_{z\in Z} x_{ij}^{t,k,z} > k_{ij}^{max}$ then $\sum_{k\in V} \sum_{z\in Z} b_{ij}^{t,k,z} < 0$, which is prohibited by Equation (2.9), and it also limits the flow, $\sum_{k\in V} \sum_{z\in Z} b_{ij}^{t,k,z}$, from arc

(i, j) to q_{ij}^{max} and insures that $\sum_{k \in V} \sum_{z \in Z} b_{ij}^{t,k,z} \leq \sum_{k \in V} \sum_{z \in Z} x_{ij}^{t,k,z}$ via Equations (2.5) and (2.6). Equation (2.7) also insures that the flow on any arc has the correct composition, with regard to evacuee type and origin. The Pipes relationship is an attractive choice for this type of strategic model, because it provides a good representation of traffic flow while being analytically tractable, as we discuss in more detail in Section 2.5. While Equations (2.4)-(2.7) do not replicate traffic flow with a high fidelity, we believe they are appropriate for a strategic evacuation planning model. Equation (2.8) preserves the capacity of $C_{j,z}$ for each shelter $j \in Y$, for each evacuee type $z \in Z$. Equation (2.9) enforces logical non-negativity restrictions.

Model 2.1 represents our general framework. Modifications and additions can be made to this general REM to account for different assumptions on evacuee behavior and the level of control a central evacuation authority can utilize in an evacuation. Specifically, in Chapter **??** we assume a homogeneous population (the set Z only contains one type of evacuee) and disaggregate control of the evacuation by the managing authority. In Chapter **??** we assume the population is, to some degree, heterogeneous (the set Z contains multiple evacuee types), and the central evacuation authority has a more aggregate level of control.

Next, we introduce some terminology that is used throughout this dissertation.

1. **Congestion**: Arc (i, j) experiences *congestion* when the traffic density reaches a point where the excessive density begins to reduce the exit-flow limit (i.e., the bound on b_{ij}^t). Considering the Pipes model, this occurs when the exit-flow limit is defined by Equation (2.6).

2. **Holding-Back**: Arc (i, j) experiences *holding-back* if both the Equations (2.5) and (2.6) hold as strict inequalities, i.e., if the exit flow b_{ij}^t is less than the exit-flow limit, and if this is not caused by the characteristics or state of the desired downstream arc (or arcs) (j, k). For example, if arc (i, j) is congested and we have $q_{jk}^{max} = b_{ij}^t < (k_{ij}^{max} - x_{ij}^t)q_{ij}^{max}/(k_{ij}^{max} - q_{ij}^{max})$ for some $(j, k) \in A$, then this does not constitute holding-back. Carey (1987) provides a good discussion of holding-back, and under what conditions an optimal solution to the particular DTA formulation studied, in that paper, will not have holding-back. In Carey's model, holding-back occurs when flow out of an arc is less than the arc's exit-flow limit. In the real road network this holding-back behavior would not normally occur, i.e., it would require special traffic

control measures, and thus, it usually represents an unrealistic, undesirable property of a solution. In our model, holding-back is somewhat more complex. This is due to an additional constraint, and the use of the Pipes relationships. The constraint, Equation (2.4), limits the flow into an arc to the maximum flow allowed on that arc, i.e., $a_{ij}^t \leq q_{ij}^{max}$, while the Pipes relationship limits the maximum number of evacuees an arc can hold (to the jam density, k_{ij}^{max}). Either of these can cause the exit flow to be less than the exit-flow limit, for the adjacent, upstream arc, but this is not considered holding-back; it is just a consequences of the characteristics or state of the network. This adds realism to the formulation.

3. **Critical Arc-Interval**: Arc (i, j) for time interval t is a *critical arc-interval* if arc (i, j) cannot possibly experience congestion or holding-back in the interval t in an optimal strategy.

4. **Bottleneck Arc-Interval**: Arc (i, j) for time interval t is a *bottleneck arc-interval* if increasing the maximum possible flow, i.e., q_{ij}^{max} (perhaps in conjunction with adjacent arcs), can improve the optimal solution.

The REM (Model 2.1) bears some similarities with mathematical programming formulations of multi-period dynamic traffic assignment (DTA) models, as introduced by Merchant and Nemhauser (1978) and widely studied thereafter (see, for instance, Peeta and Ziliaskopoulos, 2001). Peeta and Ziliaskopoulos (2001) describe two main limitations typically inherent in these mathematical programming formulations: 1) insuring first-in first-out (FIFO) arc flows, and 2) holding-back of traffic flows, which refers to delaying an arc's flow, in favor of another arc's flow, for an unrealistically long time, when they both use a common downstream arc. We discuss these issues as part of our analysis in the subsequent chapters.

Next, we present the objective functions that are of interest for evacuation planning.

2.4   Objective Functions

In this section we present the alternative objective functions considered, and any additional constraints required in conjunction with a particular objective function. This is an important area of research because the objective function plays a critical role in determining the form of the evacuation strategy and, as we shall see, different objective functions can produce

strategies with equivalent results, while some are easier to optimize than others. Most of these objectives do not require a constraint to drive them towards an optimal solution; but a constraint of the type $\sum_{t=1}^{T} \sum_{j \in Y} \sum_{i:(i,j) \in A} b_{ij}^t = \sum_{k \in V} \sum_{z \in Z} D_{k,z}$ can be used to insure that an incomplete evacuation will be infeasible (if the time-window represented by T is not long enough, for instance).

Objective (I): Minimize the number of time intervals (i.e., the total duration) it takes to complete the evacuation, that is,

$$Minimize \quad \sum_{t=1}^{T} E^t,$$

where the following additional constraints are required:

$$E^t \geq (D - \sum_{f=1}^{t-1} \sum_{j \in Y} \sum_{i:(i,j) \in A} b_{ij}^f)/D, \quad t = 1, \cdots, T, \tag{2.10}$$

$$E^t \in \{0, 1\}, \quad t = 1, \cdots, T. \tag{2.11}$$

Equation (2.10) determines if the evacuation is complete, at which point the variable E^t can equal zero; otherwise, this binary variable must equal one, and Equation (2.11) represents the binary restrictions on the E^t-variables. Note that Objective (I) minimizes the maximum evacuation time experienced by any evacuee.

Objective (II): Minimize the average evacuation time, that is,

$$Minimize \quad \sum_{t=1}^{T} \sum_{j \in Y} \sum_{i:(i,j) \in A} t b_{ij}^t / D. \tag{2.12}$$

This is the average time it takes evacuees to reach shelter after the evacuation order is given.

Objective (III): Maximize, lexicographically, the number of evacuees who reach shelter in each time interval, that is,

$$Lexmax\{\sum_{j \in Y} \sum_{i:(i,j) \in A} b_{ij}^t, \quad t = 1, \cdots, T\}. \tag{2.13}$$

Objective (IV): Minimize the total travel cost, that is,

$$Minimize \sum_{t=1}^{T} \sum_{(i,j) \in A} v_{ij}(x_{ij}^t),$$

where the following additional constraint is required:

$$\sum_{t=1}^{T} \sum_{j \in Y} \sum_{i:(i,j) \in A} b_{ij}^t = \sum_{k \in V} \sum_{z \in Z} D_{k,z}. \tag{2.14}$$

This is a common objective in DTA models, generally known as a *system optimal* assignment. For an evacuation, this can be thought of as minimizing the total travel time, in which case the function $v_{ij}(.)$ simply represents the number of evacuees on the arc in time interval t multiplied by the length of the time interval. Equation (2.14) is needed to insure a complete evacuation, if feasible, within the time horizon represented by T.

Objective (V): Minimize an additive risk function, that is,

$$Minimize \sum_{t=1}^{T} [\sum_{k \in V} R_k^t (\sum_{z \in Z} D_{k,z} - \sum_{f=1}^{t-1} \sum_{j:(k,j) \in A} a_{kj}^f) + \sum_{(i,j) \in A} R_{ij}^t x_{ij}^t], \tag{2.15}$$

where $R_k^t \geq 0$ is the risk incurred by evacuees at origin k at the start of time interval t, $\forall k \in V$, $t = 1, \cdots, T$, and $R_{ij}^t \geq 0$ is the risk incurred by evacuees on arc (i,j) at the start of time interval t, $\forall (i,j) \in A$, $t = 1, \cdots, T$.

Two special cases of Objective (V) will also be of interest:

Objective (V.a): Minimize an additive space-constant risk, i.e., the risk is constant across the network during each time interval:

$$Minimize \sum_{t=1}^{T} [\sum_{k \in V} R^t (\sum_{z \in Z} D_{k,z} - \sum_{f=1}^{t-1} \sum_{j:(k,j) \in A} a_{ij}^f) + \sum_{(i,j) \in A} R^t x_{ij}^t], \tag{2.16}$$

where $R^t \geq 0$, $t = 1, \cdots, T$. Thus, we modify Equation (2.15) by setting $R_k^t = R^t$, $\forall k \in V$, $t = 1, \cdots, T$ and $R_{ij}^t = R^t$, $\forall (i,j) \in A$, $t = 1, \cdots, T$.

Objective (V.b): Minimize an additive space- and time-constant risk, that is,

$$Minimize \sum_{t=1}^{T} [\sum_{k \in V} R (\sum_{z \in Z} D_{k,z} - \sum_{f=1}^{t-1} \sum_{j:(k,j) \in A} a_{ij}^f) + \sum_{(i,j) \in A} R x_{ij}^t], \tag{2.17}$$

18

where $R \geq 0$. Thus, we modify Equation (2.15) by setting $R_k^t = R$, $\forall k \in V$, $t = 1, \cdots, T$ and $R_{ij}^t = R$, $\forall (i, j) \in A$, $t = 1, \cdots, T$.

Remark 1 *Objective (V.a) has many alternative forms besides Equation (2.16). One such form is as follows:*

$$Minimize \sum_{t=1}^{T} R^t (D - \sum_{j \in Y} \sum_{i:(i,j) \in A} \sum_{f=1}^{t-1} b_{ij}^f), \qquad (2.18)$$

where each evacuee not in a shelter at the beginning of interval t incurs a risk of R^t. We can also formulate this objective as the following maximization:

$$Maximize \sum_{t=1}^{T} R^t \sum_{f=1}^{t-1} \sum_{j \in Y} \sum_{i:(i,j) \in A} b_{ij}^f, \qquad (2.19)$$

by eliminating the constant term $\sum_{t=1}^{T} R^t D$ in Equation (2.18).

An important characteristic of a hurricane is that it can be tracked and land-fall can be predicted. Because of this advanced warning, regions can start evacuating well in advanced of land-fall. Despite this, authorities are still interested in objectives such as Objective (I), which minimizes the duration of the evacuation, because the accuracy of the forecasted land-fall and intensity increases as the hurricane approaches. Thus, delaying a hurricane evacuation order can be useful to avoid a false alarm and to better demarcate a proper evacuation area. Delaying an evacuation order can also be less disruptive on regional commerce, and the population to be evacuated. We hypothesis that evacuees will respond better to a later evacuation order as they weigh the disruption of an evacuation against the risk. Objectives (II), which minimizes the average time to evacuate, and (III), which maximizes the number of additional evacuees that reach shelter in each interval, are also viable strategic approaches considered in the evacuation literature (perhaps with slightly different formulations; see Chalmet, Francis, and Saunders, 1982 and Jarvis and Ratliff, 1982). Objective (IV) can be used to produce an evacuation plan that reduces the sum of travel times for the evacuees, given a pre-specified overall duration. Objectives (V.a) and (V.b), the two structured risk objectives, do not necessarily model actual risks; instead, they are of interest because of their special properties, which we discuss in Section 3.3. Objective (V), which allows for a less structured risk, is better suited to disasters that occur without much warning, where

risk is present from the start. An example of this is a catastrophic incident at a nuclear power plant. Currently, a typical goal is to evacuate an arbitrary 10-mile radius around the incident in the quickest fashion (see, for instance, Hobeika, Kim, and Beckwith, 1994). An alternative is to evacuate those areas most impacted first, in which case, risk considerations are vital. While it is certainly true that different areas will experience different risks in a hurricane, for example, an area susceptible to a flood surge is far more dangerous than an area situated on higher ground. With advanced warning, the goal should be to evacuate the population before the actual risk manifests, making Objective (V) less relevant in this case. A more detailed study of risk is an interesting area of potential future research.

2.5   Traffic Flow Relationships

The Pipes relationship is used in a number of well-known, high-fidelity, traffic micro-simulators (such as CORSIM and VISSIM, see Rakha and Crowther, 2003). Other traffic flow models have non-linear flow/density relationships, which, if used, would make the REM less tractable. For example, the Greenshields relationship, another well-known traffic flow model, has a linear speed/density relationship, which yields a non-linear flow/density relationship. To calibrate the Pipes relationship for a segment of roadway, three well-known traffic parameters are used: the maximum flow q_{ij}^{max}, the maximum traffic density k_{ij}^{max} (also known as the jam density), and the maximum speed u_{ij}^{max} (also known as the free-flow speed). The relationship between flow, density, and speed are shown graphically in Figure 2.1 for a segment of roadway with unscaled parameters $q^{max} = 1500\ veh/hr$, $k^{max} = 100\ veh/km$, and $u^{max} = 60\ km/hr$.

Equation (2.5), which is represented by the positively sloped line in the flow versus density graph (see Figure 2.1), models the uncongested regime. Equation (2.6), on the other hand, is represented by the negatively sloped line and models the congested regime. As the speed/density graph in Figure 2.1 depicts, the linear flow/density relationship produces a non-linear speed/density relationship. Hence, the time spent on an arc is also non-linear with congestion.

For an evacuation-related example, consider the following link performance function:

$$T_{ij} = (l_{ij}/u_{ij}^{max})[1 + \alpha(x_{ij}/q_{ij}^{max})^{\beta}],$$
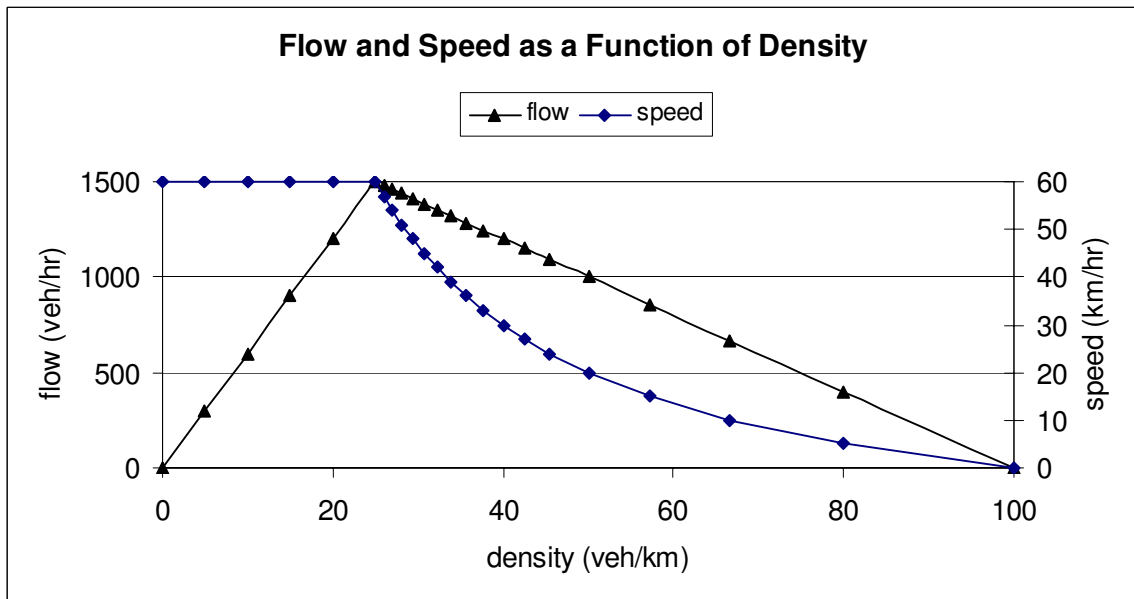
Figure 2.1: The Pipes traffic-flow relationship.
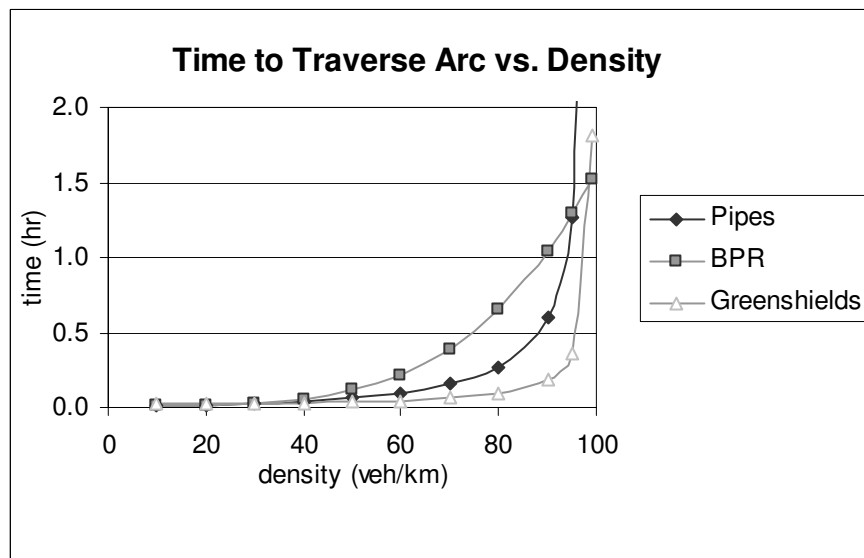
Figure 2.2: Comparison of travel times on an arc using the Pipes relationship and the BPR function.

which was developed by the U.S. Bureau of Public Roads (BPR) to model congestion in a steady-state evacuation model. Here, T_{ij} is the time to traverse arc (i, j), and α and β are non-negative parameters. We have removed the t superscript from the x_{ij}^t variable due to the

steady-state nature of the BPR model. In Figure 2.2, we compare the Pipes model using the same arc characteristics as displayed in Figure 2.1 with the BPR link performance function using typical values of $\alpha = 0.15$ and $\beta = 4.0$, and with the Greenshields relationship, a non-linear, concave function (similar to exit-flow functions often used in the DTA literature; see for instance...). As Figure 2.2 displays, both relationships yield non-linear travel times as traffic density increases. This complicates the solution to the model as the non-linearities are explicit in the mathematical formulation (e.g., see Sherali, Carter, and Hobeika, 1991). On the other hand, in the REM, the non-linearities in Figure 2.2 are a consequence of linear flow/density constraints, making the problem easier to solve. We are therefore able to handle larger sized problems that further consider time dynamic features, which is a great advantage for evacuation planning purposes.

2.6   Test Problems

To better understand the REM, and the related models we examine later, we use a small network, depicted in Figure 2.3, for an initial analysis of the various models. Two scenarios are presented on this network. In both scenarios, evacuees can be categorized based on their shelter preferences (represented by the set Z); those seeking shelter in a local, capacitated, shelter (Y1 and Y2), and those that prefer to leave the area and find their own accommodations (represented by the uncapacitated shelter Y3). In this network, the time horizon is divided into 1 minute time intervals.

Scenario 1: Each arc has a maximum flow of $q_{ij}^{max} = 50$ vehicles per interval and a maximum density of 200 vehicles. Local shelter Y1 has a capacity of 300 and Y2 has a capacity of 500. Each origin has 125 evacuees, 50 of whom prefer a local shelter (Y1 or Y2). Each origin and evacuee type (based on shelter preference) has a five-interval loading curve (when applicable). For this test instance, we used a time horizon of $T = 25$ intervals.

Scenario 2: Each arc has a maximum flow of $q_{ij}^{max} = 100$ vehicles per interval and a maximum density of 400 vehicles. Both local shelters, Y1 and Y2, have capacities of 1000. Each origin has 1000 evacuees, 200 of whom prefer a local shelter (Y1 or Y2). Each origin and evacuee group (based on shelter preference) has a 50-interval loading curve (when applicable). For this test instance, we used a time horizon of $T = 100$ intervals.
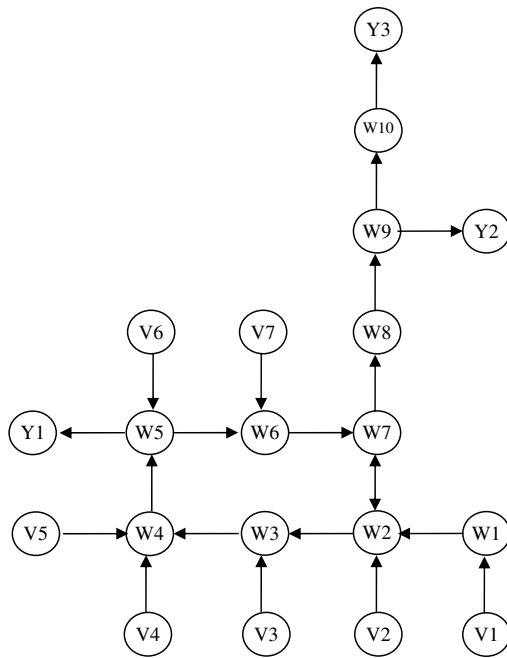
Figure 2.3: Illustrative network.

Chapter 3

Disaggregated Control Strategies

3.1 Introduction

In this chapter we study the Disaggregated Regional Evacuation Model (D-REM), which produces disaggregated control strategies, i.e., strategies that control the evacuation at a disaggregated level. Despite being a dynamic network flow model, which does not represent discrete evacuees, the D-REM conceptually produces strategies that control when individual evacuees start evacuating, which shelter, given their preferences/requirement, they are sent to, and what routes they travel. As such, strategies of this nature would have to be conveyed to each individual, or household, as a customized evacuation plan. While at first this might seem difficult, with the current advances in communications, this scenario could be quite feasible. For example, airlines are developing tools to automatically send messages to passengers when flights are delayed or canceled, using landline phones, cell-phones, e-mail, or text messaging. Internet mapping and routing tools, such as those offered by MapQuest or Google, could be adapted to provide customized evacuation routes. While the dissemination of such customized evacuation plans is not the focus of this research, dissemination, along with the impact of such plans on evacuee behavior, are interesting areas of potential future research. In Section 3.2 we present the constraint set for the D-REM. To delineate a strategy at this disaggregate level, the D-REM only requires one additional constraint beyond those present in the REM. The extra constraint insures evacuee groups evacuate from the proper origin. We also discuss how to greatly simplify the computational effort by producing an optimal flow at a higher level of aggregation, and then recovering the final, disaggregated

evacuation plan using another model. Section 3.2.1 discusses how the constraint set of the D-REM impacts staging. In Section 3.3 we present the analytical results derived for the D-REM, while in Section 3.4 we present some insights derived from our analysis. In Section 3.5 we present solution methodologies for the D-REM with Objective (I), which is a mixed-integer program (MIP), making it more difficult to solve. Finally, in Section 3.6 we provide some preliminary analysis into the D-REM when traffic flow is modeled using a concave, non-linear traffic flow relationship.

3.2 Model Formulation

The constraint set of the D-REM is identical to the REM constraint set with one additional constraint, namely Equation (3.9), which insures that evacuees from each evacuee group originate from the correct origin. We refer to any model using this constraint set as the D-REM with flow-tracking, as we also present a simplified version without flow-tracking.

Model 3.1 Constraint Set of the D-REM with Flow-Tracking

$$x_{ij}^{t+1,k,z} - x_{ij}^{t,k,z} - a_{ij}^{t,k,z} + b_{ij}^{t,k,z} = 0, \quad \forall (i,j) \in A, \ k \in V, \ z \in Z,$$

$$t = 1, \cdots, T-1 \tag{3.1}$$

$$\sum_{j:(j,i)\in A} b_{ji}^{t,k,z} - \sum_{j:(i,j)\in A} a_{ij}^{t,k,z} = 0, \quad \forall i \in W, \ k \in V, \ z \in Z, \ t = 1, \cdots, T \tag{3.2}$$

$$\sum_{t=1}^{T} \sum_{j:(k,j)\in A} a_{kj}^{t,k,z} - D_{k,z} = 0, \quad \forall k \in V, \ z \in Z \tag{3.3}$$

$$\sum_{k\in V} \sum_{z\in Z} a_{ij}^{t,k,z} \leq q_{ij}^{max}, \quad \forall (i,j) \in A, \ t = 1, \cdots, T \tag{3.4}$$

$$\sum_{k\in V} \sum_{z\in Z} b_{ij}^{t,k,z} \leq \sum_{k\in V} \sum_{z\in Z} x_{ij}^{t,k,z}, \quad \forall (i,j) \in A, \ t = 1, \cdots, T \tag{3.5}$$

$$\sum_{k\in V} \sum_{z\in Z} b_{ij}^{t,k,z} \leq (k_{ij}^{max} - \sum_{k\in V} \sum_{z\in Z} x_{ij}^{t,k,z}) q_{ij}^{max} / (k_{ij}^{max} - q_{ij}^{max}),$$

$$\forall (i,j) \in A, \ t = 1, \cdots, T \tag{3.6}$$

$$b_{ij}^{t,k,z} \leq x_{ij}^{t,k,z}, \quad \forall (i,j) \in A, \ k \in V, \ z \in Z, \ t = 1, \cdots, T \tag{3.7}$$

$$\sum_{t=1}^{T} \sum_{i:(i,j)\in A} \sum_{k\in V} b_{ij}^{t,k,z} - C_{j,z} \leq 0, \quad \forall j \in Y, \ z \in Z \tag{3.8}$$

$$a_{ij}^{t,k,z} = 0, \forall i \in V, \ j:(i,j) \in A, \ k \in V, \ k \neq i, \ z \in Z, \ t = 1, \cdots, T \tag{3.9}$$

$$a_{ij}^{t,k,z}, b_{ij}^{t,k,z}, x_{ij}^{t,k,z} \geq 0, \quad \forall (i,j) \in A, \ k \in V, \ z \in Z, \ t = 1, \cdots, T. \tag{3.10}$$

As the model has complete freedom to stage and route the evacuees, as well as to select shelter locations (given the preferences/requirements), we do not actually have to track evacuees by their origin to produce an optimal evacuation flow; we only need to track flows based on shelter preferences/requirements, and hence can produce optimal flows at a higher level of aggregation. Because of this, we are able to simplify the D-REM by removing the origin label from the various variables. The simplified constraint set is as follows:

Model 3.2 Constraint Set of the D-REM without Flow-Tracking

$$x_{ij}^{t+1,z} - x_{ij}^{t,z} - a_{ij}^{t,z} + b_{ij}^{t,z} = 0, \quad \forall (i,j) \in A, \ z \in Z, \ t = 1, \cdots, T-1 \tag{3.11}$$

$$\sum_{j:(j,i)\in A} b_{ji}^{t,z} - \sum_{j:(i,j)\in A} a_{ij}^{t,z} = 0, \quad \forall i \in W, \ z \in Z, \ t = 1, \cdots, T \tag{3.12}$$

$$\sum_{t=1}^{T} \sum_{j:(k,j)\in A} a_{kj}^{t,z} - D_{k,z} = 0, \quad \forall k \in V, \ z \in Z \tag{3.13}$$

$$\sum_{z\in Z} a_{ij}^{t,z} \leq q_{ij}^{max}, \quad \forall (i,j) \in A, \ t = 1, \cdots, T \tag{3.14}$$

$$\sum_{z\in Z} b_{ij}^{t,z} \leq \sum_{z\in Z} x_{ij}^{t,z}, \quad \forall (i,j) \in A, \ t = 1, \cdots, T \tag{3.15}$$

$$\sum_{z\in Z} b_{ij}^{t,z} \leq (k_{ij}^{max} - \sum_{z\in Z} x_{ij}^{t,z})q_{ij}^{max}/(k_{ij}^{max} - q_{ij}^{max}), \quad \forall (i,j) \in A, \ t = 1, \cdots, T \tag{3.16}$$

$$b_{ij}^{t,z} \leq x_{ij}^{t,z}, \quad \forall (i,j) \in A, \ z \in Z, \ t = 1, \cdots, T \tag{3.17}$$

$$\sum_{t=1}^{T} \sum_{i:(i,j)\in A} b_{ij}^{t,z} - C_{j,z} \leq 0, \quad \forall j \in Y, \ z \in Z \tag{3.18}$$

$$a_{ij}^{t,z}, b_{ij}^{t,z}, x_{ij}^{t,z} \geq 0, \quad \forall (i,j) \in A, \ z \in Z, \ t = 1, \cdots, T. \tag{3.19}$$

Equation (3.11) links the time intervals by defining the state of the system in time interval $t + 1$ based on the state and flows from time interval t. Equation (3.12) represents flow conservation across each junction i, $\forall i \in W$. Equation (3.13) insures the complete evacuation of each origin k, $\forall k \in V$. Equation (3.14) limits the flow entering an arc to the arc's maximum flow, q_{ij}^{max}. This is justified because if a flow rate higher than this maximum flow entered the arc, it would quickly be reduced due to a high local density at the entrance of the arc. Equations (3.15), (3.16), and (3.17) define the exit-flow limits for arc (i,j) using the Pipes traffic flow relationship (see Section 2.5). Equation (3.18) defines the capacity limits of shelter j, $\forall j \in Y$. Finally, Equation (3.19) represent the logical non-negativity constraints.

By not tracking the flow of each evacuee group, and only labeling flows based on shelter preferences/requirements, we drastically simplify the formulation and the time required by the solver to find an optimal solution. To illustrate this we use the test scenarios from Section 2.6. Table 3.1 presents the results from the two test scenarios using the D-REM with flow-tracking, while Table 3.2 presents the same, but using Model 3.2, the D-REM without flow-tracking. The results in both tables are from solving each model with respect

to Objective (II), which minimizes the average evacuation time. We use this objective, instead of Objective (I), to simplify the analysis; Objective (I) transforms each model into a MIP, due to the binary E^t-variables, which is more difficult to solve, as we discuss in later sections.

Table 3.1: Results for the D-REM with flow-tracking

Scenario	Obj. Value	Evac. Duration	Run Time (sec.)	Cont. Vars.	Binary Vars.	Constraints	Dual Simplex Iters	B/B
1	9.886	19	3.375	12,258	0	12,145	4,653	0
2	32.300	67	19,451.9	55,309	0	54,670	710,440	0

Table 3.2: Results for the simplified D-REM without flow-tracking.

Scenario	Obj. Value	Evac. Duration	Run Time (sec.)	Cont. Vars.	Binary Vars.	Constraints	Dual Simplex Iters	B/B
1	9.886	19	0.438	2,861	0	3,790	1,105	0
2	32.300	67	60.2	12,161	0	16,164	30,128	0

Note that the AMPL pre-processor eliminates some variables and constraints from the formulation, the numbers reported in these (and all subsequent) tables are from AMPL after preprocessing. The large differences in run times between Tables 3.1 and 3.2 illustrate that by eliminating the tracking of flow by origin, we greatly reduce the time needed to produce an optimal solution. Of course, without flow-tracking, Model 3.2 does not produce an evacuation plan that specifies paths for each evacuee. To recover an evacuation plan from the more aggregated flows produced by the D-REM, Model 3.2, we can use the following model.

Model 3.3 Evacuation Plan Recovery Model

$$Minimize \quad \sum_{t=1}^{T}\sum_{k\in V}\sum_{z\in Z}\sum_{(i,j)\in A} a_{ij}^{t,k,z} \tag{3.20}$$

subject to

$$x_{ij}^{t+1,k,z} - x_{ij}^{t,k,z} - a_{ij}^{t,k,z} + b_{ij}^{t,k,z} = 0, \quad \forall (i,j)\in A, \ k\in V, \ z\in Z, \ t=1,\cdots,T-1 \tag{3.21}$$

$$\sum_{j:(j,i)\in A} b_{ji}^{t,k,z} - \sum_{j:(i,j)\in A} a_{ij}^{t,k,z} = 0, \quad \forall i\in W, \ k\in V, \ z\in Z, \ t=1,\cdots,T \tag{3.22}$$

$$\sum_{k\in V} a_{ij}^{t,k,z} = a_{ij}^{t,z}, \quad \forall (i,j)\in A, \ z\in Z, \ t=1,\cdots,T \tag{3.23}$$

$$\sum_{k\in V} b_{ij}^{t,k,z} = b_{ij}^{t,z}, \quad \forall (i,j)\in A, \ z\in Z, \ t=1,\cdots,T \tag{3.24}$$

$$\sum_{k\in V} x_{ij}^{t,k,z} = x_{ij}^{t,z}, \quad \forall (i,j)\in A, \ z\in Z, \ t=1,\cdots,T \tag{3.25}$$

$$a_{ij}^{t,k,z} = 0, \forall i\in V, \ j:(i,j)\in A, \ k\in V, \ k\neq i, \ z\in Z, \ t=1,\cdots,T \tag{3.26}$$

$$a_{ij}^{t,k,z}, b_{ij}^{t,k,z}, x_{ij}^{t,k,z} \geq 0, \quad \forall (i,j)\in A, \ k\in V, \ z\in Z, \ t=1,\cdots,T. \tag{3.27}$$

Model 3.3 has an objective function that minimizes the summation of the $a_{ij}^{t,k,z}$ variables. In this model, the three flow variables from Model 3.2, $a_{ij}^{t,z}$, $b_{ij}^{t,z}$, and $x_{ij}^{t,z}$, are parameters. Thus, because of Equation (3.23), the objective is pre-determined, and is just used to produce an evacuation plan from the output of Model 3.2. In this dissertation we do not study in any depth the possible interactions between this model and the D-REM, Model 3.2.

Equation (3.21) defines the state of the system in time interval $t+1$ based on the state and flows from time interval t. Equation (3.22) represents flow conservation across each junction $i \in W$. These constraints insure the same conservation of flow as in Model 3.1. Equations (3.23), (3.24), and (3.25) insure that the flow for the individual evacuee groups conforms to the more aggregate flows produced by Model 3.2. As such, $a_{ij}^{t,z}$, $b_{ij}^{t,z}$, and $x_{ij}^{t,z}$, while variables in Model 3.2, are parameters in this model. Equation (3.26) insures that evacuees originate from the correct origin, based on their evacuee group. Equation (3.27) represents the logical non-negativity constraints.

As Table 3.3 illustrates, Model 3.3 can be solved quite quickly. By dividing the problem, represented by Model 3.1 (the D-REM with flow-tracking), into two sub-problems we can dramatically decrease the required solution time. To illustrate, consider Scenario 2. To produce an evacuation plan, the solver required 19,451 seconds, or about 5.4 hours, with

Model 3.1. In contrast, the solver required only 60.2 seconds to produce the optimal flows at a more aggregate level with Model 3.2, and then less than a second for Model 3.3 to recover the more detailed evacuation plan from these aggregate flows.

Table 3.3: Results for the Evacuation Plan Recovery Model, Model 3.3.

Scenario	Obj. Value	Run Time (sec.)	Cont. Vars.	Binary Vars.	Constraints	Dual Simplex	B/B
1	5,225	0.078	1,086	0	945	194	0
2	51,200	0.203	5,519	0	4,865	969	0

In the remainder of this chapter, when we refer to the D-REM, we refer to Model 3.2. Next, we discuss staging and the D-REM.

3.2.1 Staging

After an evacuation order is given, evacuees depart based on a complex decision process that is not well understood. When viewed at an aggregate level, this behavior can be described by a network loading function; a function that specifies the cumulative number of evacuees that start their evacuation on or before a specified time interval. While this function emerges from the decisions of many different individuals, it tends to have a characteristic shape, often described as an 'S-shaped' curve (see the literature review in Section 1.2). This aggregate behavior is important; if all the evacuees left at once, directly after the evacuation order, the congestion and gridlock would be much worse. Despite rational evacuee behavior (see, for instance, Quarantelli, 1980), and a characteristic loading curve that temporally spreads demand, without staging, the number of evacuees can easily overwhelm the available supply causing congestion and gridlock (as demonstrated in numerous real evacuations). This is partly due to the evacuee's lack of information, as well as the lack of coordination between evacuees. An evacuation plan with staging modifies this characteristic loading function, supplying the needed coordination between evacuees.

The D-REM automatically accommodates staging because it does not enforce any particular form of *a priori* network loading. All the D-REM requires is the complete evacuation

of each origin k, $\forall k \in V$, through Equation (3.13), and thus the model is free to determine when evacuees depart. Each of the objective functions discussed earlier in Section 2.4 will impose staging as a natural part of the optimization process. This is because the D-REM allows congestion, which can severely impact the performance of many of the objectives, and staging can reduce or completely eliminate congestion, depending on the objective considered and the characteristics of the system.

When staging is used in conjunction with routing, it affects how the model routes evacuees. If the D-REM were modified to include a constraint that requires network loading based on a pre-defined characteristic network loading function, the model would often produce solutions that, in an attempt to optimize the various objective functions, have unacceptable, circuitous routes and holding-back of traffic flows. The model would produce this undesirable routing plan to minimize the congestion on the bottleneck arcs. Thus, staging allows us to have a less constrained routing environment without the negative, unrealistic impacts. Without staging, the routing would have to be more constrained, perhaps permitting only pre-specified routes. Holding-back would also have to be eliminated somehow. These problems also occur in DTA models, but they are usually more pronounced in an evacuation setting, as more vehicles are being routed to fewer destinations, creating more potential for congestion.

In the next section we will analyze the D-REM in conjunction with the various objective functions discussed in Section 2.4.

3.3   Analysis

In the following we analyze Model 3.2 with the various objective functions introduced in Section 2.4. Specifically, we first define some terminology and additional notation that is useful in our study. We then characterize various properties of the model and the objective functions, including a comparison of the Objectives (I), (II), (III), (V.a), and (V.b).

3.3.1   Terminology and Additional Notation

To assist in our study of the evacuation problem, we first give an alternate definition for a bottleneck arc-interval that applies to the D-REM:

Bottleneck Arc-Interval: Arc (i, j) for time interval t is a *bottleneck arc-interval* if inflow must be at its maximum possible value, i.e., $a_{ij}^t = q_{ij}^{max}$, in every optimal strategy.

We note that this definition does not invalidate the earlier definition; the D-REM just allows for a more rigorous definition. Next we will introduce the additional notation used in our analysis.

Additional Notation:

s^t	:	number of evacuees reaching shelter in interval t in a feasible solution, where $s^t = \sum_{j \in Y} \sum_{i:(i,j) \in A} b_{ij}^t,\ t = 1, \cdots, T$
n	:	the last interval of a feasible evacuation, i.e., the duration of the evacuation
$L^{t,k,z}$	:	number of evacuees of type z, from origin k, using paths in the set $P^{t,k,z}$, reaching shelter in interval t, in a feasible solution, $\forall k \in V,\ z \in Z,\ t = 1, \cdots, T$.

In addition, when we refer to an optimal solution to Objective (l), $l \in \{$I, II, III, IV, V, V.a, V.b$\}$, we append l to the subscript, i.e., s_l^t, n_l, $L_l^{t,k,z}$, etc.

3.3.2   Results

In this section we study the various objective functions in more detail and discuss how and when they are related, and their impact on the evacuation strategy. First, we introduce a model related to the D-REM, where the arc exit-flows are bounded by a constant equal to the maximum possible flow, q_{ij}^{max}.

Model 3.4 Constraint Set of the Evacuation Model with Constant Exit-Flow Limits

$$a_{ij}^{t,z} - b_{ij}^{t+1,z} = 0, \quad \forall (i,j) \in A, \ z \in Z, \ t = 1, \cdots, T-1 \tag{3.28}$$

$$\sum_{j:(j,i) \in A} b_{ji}^{t,z} - \sum_{j:(i,j) \in A} a_{ij}^{t,z} = 0, \quad \forall i \in W, \ z \in Z, \ t = 1, \cdots, T \tag{3.29}$$

$$\sum_{t=1}^{T} \sum_{j:(k,j) \in A} a_{kj}^{t,z} - D_{k,z} = 0, \quad \forall k \in V, \ z \in Z \tag{3.30}$$

$$a_{ij}^{t} \leq q_{ij}^{max}, \quad \forall (i,j) \in A, \ t = 1, \cdots, T \tag{3.31}$$

$$\sum_{t=1}^{T} \sum_{i:(i,j) \in A} b_{ij}^{t,z} - C_{i,z} \leq 0, \quad \forall j \in Y, \ z \in Z \tag{3.32}$$

$$a_{ij}^{t,z}, b_{ij}^{t,z} \geq 0, \quad \forall (i,j) \in A, \ z \in Z, \ t = 1, \cdots, T \tag{3.33}$$

$$a_{ij}^{t} \geq 0, \quad \forall (i,j) \in A, \ t = 1, \cdots, T. \tag{3.34}$$

In this formulation, we have kept both the flow variables $a_{ij}^{t,z}$ and $b_{ij}^{t,z}$ for continuity with Model 3.2, although from Equation (3.28), only one is needed. Furthermore, we have removed the variables $x_{ij}^{t,z}$, which essentially removes the possibility of an arc holding evacuees for more than one time interval. This will not affect the optimal solution to Model 3.4, as the exit-flow limits are constant, not a function of $x_{ij}^{t,z}$.

Proposition 1 *Considering Objectives (I), (II), (III), (IV), (V.a), or (V.b), an optimal solution to Model 3.4, which has constant exit-flow limits on each arc, is also an optimal solution to Model 3.2, which has variable exit-flow limits.*

Proof: First observe that the optimal solution to Model 3.4 considering Objectives (I), (II), (III), (IV), (V.a), or (V.b) provides a feasible solution for the D-REM. This is because the constant exit-flow limit for each arc in Model 3.4 is set to the maximum variable exit-flow limit for the corresponding arc in the D-REM, and evacuees in the D-REM can clear each arc in each interval. [If this were not the case, i.e., if it took more than a single interval to traverse the arc in the uncongested regime, then the flows from Model 3.4 would not be feasible for the D-REM.] The proof then follows because adding a congested regime does not change the location or capacity of the bottleneck arc-intervals, i.e., the min-cut in the network, and thus no better solution for the D-REM can be found. ∎

We note that Proposition 1 is wholly dependent on staging: Without staging, the characteristic network loading would undoubtedly cause congestion, making the solution to the D-REM infeasible for Model 3.4. Proposition 1 does not necessarily hold for Objective (V), as an optimal solution to this objective might require congestion. The following corollaries are direct consequences of Proposition 1.

Corollary 3.3.1 *There exists an optimal solution for Objectives (I), (II), (III), (IV), (V.a), or (V.b) that does not experience congestion or holding-back.*

Of course there can be multiple optimal solutions if there are multiple ways to supply the bottleneck arc-intervals optimally. This allows for optimal solutions that can have congestion and/or holding-back, but only on certain arcs, as defined in the next corollary.

Corollary 3.3.2 *An optimal solution can have congestion and/or holding-back on the arcs upstream of the bottleneck arc-intervals, but not on the bottleneck arc-intervals or the critical arc-intervals (arcs downstream from the bottleneck arcs).*

Corollary 3.3.3 *If $q_{ij}^{max}, \forall (i,j) \in A$, are integral, then there exists an optimal solution for Objectives (I), (II), (III), (IV), (V.a) or (V.b) having integer flows.*

Figure 3.1 compares a small static network with a space-time version for $T = 4$, which only includes the reachable arcs. Each arc has a capacity of 20 vehicles per interval, except arc $(W3, Y1)$, which has a capacity of 30 vehicles per interval. The space-time network does not have an arc from W1 to W2 in Interval 1, as it is not feasible to reach W2 in Interval 1.

The min-cut arcs in Figure 3.1 are bold. In this example, given enough evacuees, the min-cut arcs would all be bottleneck arc-intervals.

Proposition 2 *In an optimal solution to the D-REM with Objectives (I), (II), (III), (IV), (V.a), or (V.b), the bottleneck arc-intervals are always part of some min-cut of the space-time version of the network (N,A) for Model 3.4, which only includes arcs that are reachable by evacuees in a feasible solution.*

Proof: The min-cut limits the number of evacuees that can reach shelter in any interval. By definition, any arcs upstream of some min-cut, that are not themselves part of a min-cut,
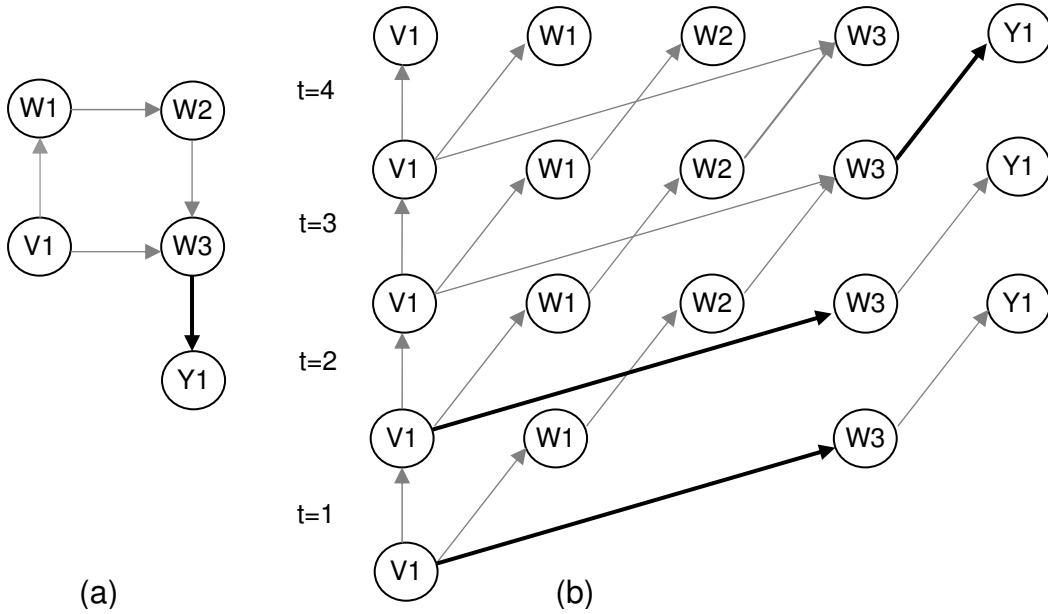
Figure 3.1: A small static network (a) and its space-time equivalent (b).

have higher capacity. The same is true of arcs downstream of a min-cut. By our definition, a bottleneck arc-interval is an arc that must have $a_{ij}^t = q_{ij}^{max}$ in every possible optimal strategy. For Objectives (I), (II), (III), (IV), (V.a), or (V.b), which do not require congestion in an optimal solution, only arcs in a min-cut will necessarily have $a_{ij}^t = q_{ij}^{max}$. While an arc that is not part of a min-cut can have $a_{ij}^t = q_{ij}^{max}$ in an optimal solution, other optimal solutions exist where, for this arc, $a_{ij}^t < q_{ij}^{max}$, as there exists some excess capacity that can be utilized on another arc. ∎

Remark 2 *The min-cut in a space-time network with only reachable arcs does not necessarily correspond with the min-cut of the static network.*

The average evacuation time, i.e., the average time for an evacuee to reach shelter, is also an important objective to consider. Next, we present an equivalence result for this objective.

Proposition 3 *Objective (II) and Objective (V.b) are equivalent with respect to the D-REM; both yield an evacuation strategy that minimizes the average evacuation time.*

Proof: In a feasible evacuation we have $D = \sum_{j \in Y} \sum_{i:(i,j) \in A} \sum_{t=1}^{T} b_{ij}^t$. Substituting for D in Equation (2.18), which is valid for Objective (V.b) if $R^t \equiv R$, $\forall t = 1, \cdots, T$, yields:

$$R \sum_{t=1}^{T} (\sum_{j \in Y} \sum_{i:(i,j) \in A} \sum_{f=1}^{T} b_{ij}^f - \sum_{j \in Y} \sum_{i:(i,j) \in A} \sum_{f=1}^{t-1} b_{ij}^f) = R \sum_{t=1}^{T} \sum_{j \in Y} \sum_{i:(i,j) \in A} \sum_{f=t}^{T} b_{ij}^f,$$

which can be simplified, using $s^t = \sum_{j \in Y} \sum_{i:(i,j) \in A} b_{ij}^t$, as follows:

$$R \sum_{t=1}^{T} \sum_{f=t}^{T} s^f = R \sum_{t=1}^{T} t s^t.$$

Observing that the average evacuation time is given by $\sum_{t=1}^{T} t s^t / D$ and that D and R are constants, the proof follows. ∎

Proposition 4 *Solving Model 3.2 with Objective (I) and without Equation (2.11), i.e., relaxing the binary constraint on the variables E^t, yields an evacuation strategy that minimizes the average evacuation time.*

Proof: We can write, similar to the proof of Proposition 3,

$$D - \sum_{j \in Y} \sum_{i:(i,j) \in A} \sum_{f=1}^{t-1} b_{ij}^f = \sum_{f=t}^{T} s^f, \ t = 1, \cdots, T.$$

Using this relationship in Equation (2.10) yields:

$$E^t \geq \sum_{f=t}^{T} s^f / D, \ t = 1, \cdots, T.$$

As Objective (I) is a minimization, in an optimal solution to Objective (I), without Equation (2.10), we have

$$\sum_{t=1}^{T} E^t = \sum_{t=1}^{T} \sum_{f=t}^{T} s^f / D = \sum_{t=1}^{T} t s^t / D,$$

thus completing the proof. ∎

In this case, E^t can be thought of as the percent of demand that has not reached shelter in time interval t. Thus, Propositions 3 and 4 establish the equivalence between a special case of Objective (I), Objective (II), and Objective (V.b) in minimizing the average evacuation time.

Objectives (I), (II), and (III) each represent an evacuation characteristic that we might want to optimize, namely, minimizing the duration of the evacuation, minimizing the average evacuation time, and maximizing the number of remaining evacuees that reach shelter in each interval, respectively. We next derive relationships between these objectives. First, we compare the duration, n, of an evacuation based on the solution generated for each of these objectives. Our main result on this relationship is given in the following proposition.

Proposition 5 *For all optimal solutions to the D-REM with Objectives (I), (II), and (III), we have $n_I \leq n_{II} \leq n_{III}$.*

In the following, we first discuss some further properties of an optimal solution to Model 3.2 with Objectives (II) and (III) that are useful for the proof of Proposition 5. Then, we provide the proof of Proposition 5.

Proposition 6 *Consider an optimal solution for Objective (II) given by s_{II}^t, $t = 1, \cdots, n_{II}$. In this optimal solution each path in $P^{t,k,z}$, $\forall k \in V$, $z \in Z$, $t = 1, \cdots, n_{II}$, must have a bottleneck arc-interval, unless for origin k we have $\sum_{f=t+1}^{T} L_{II}^{f,k,z} = 0$, i.e., unless origin k has no more evacuees of type z, or $\sum_{f=1}^{n_{II}} \sum_{i:(i,j) \in A} b_{ij,II}^{f,z} = C_{j,z}$, i.e., all the capacity of the shelter j (the destination of a particular path $P^{t,k,z}$) with capacity for type z evacuees is used in the evacuation.*

Proof: The proof follows by contradiction. If a path in $P^{t,k,z}$, $\forall k \in V$, $z \in Z$, $t = 1, \cdots, n_{II}$, does not have a bottleneck arc-interval, then, by definition, it is not fully utilized and can transport more evacuees to a shelter in interval t. If origin k of this path has more evacuees of type z, i.e., $\sum_{f=t+1}^{T} L_{II}^{f,k,z} > 0$, then all the capacity of shelter j for evacuees of type z must be utilized, i.e., $\sum_{f=1}^{n_{II}} \sum_{i:(i,j) \in A} b_{ij,II}^{f,z} = C_{j,z}$. Else, if all the capacity of shelter j is not utilized, then origin k must have no more evacuees. If neither of these conditions holds true, then the solution s_{II}^t cannot be optimal for Objective (II). ∎

Proposition 7 *Consider an optimal solution for Objective (III) given by s_{III}^t, $t = 1, \cdots, n_{III}$. In this optimal solution each path in $P^{t,k,z}$, $\forall k \in V$, $z \in Z$, $t = 1, \cdots, n_{III}$, must have a bottleneck arc-interval, unless $\sum_{f=t+1}^{T} L_{III}^{f,k,z} = 0$, i.e., unless all evacuees of type z from origin k take paths that arrive to shelters in interval t or earlier, or $\sum_{f=1}^{t} \sum_{i:(i,j) \in A} b_{ij,III}^{f,z} = C_{j,z}$, i.e, shelter j (the destination of a particular path $P^{t,k,z}$) with capacity for type z evacuees, has no remaining capacity.*

Proof: The proof is identical to that of Proposition 6. ▌

The following corollary is a direct consequence of Propositions 6 and 7.

Corollary 3.3.4 *Consider an optimal solution for Objective (l), $l \in \{II, III\}$, given by s_l^t, $t = 1, \cdots, n_l$. To increase s_l^b, while maintaining a feasible flow, s_l^a must decrease for some $a < b$.*

In an optimal solution for Objective (III) given by s_{III}^t, $t = 1, \cdots, n_{III}$, s_{III}^t is constrained by the paths in $P^{t,k,z}$, $\forall k \in V$, $z \in Z$. From Proposition 7, the number of evacuees on each path in P_{iz}^t is limited by: 1) bottleneck arcs on the path, i.e., the min-cut of the network, 2) the remaining capacity of shelter j for evacuees of type z, and, 3) the number of remaining evacuees at origin k. Corollary 3.3.4 implies that at least two groups of evacuees must be re-routed from an optimal solution for Objective (III) to reduce the average evacuation time and yield a feasible solution.

Proposition 8 *Consider an optimal solution for Objective (l), $l \in \{II, III\}$. Each evacuee traverses only one arc in each min-cut.*

Proof: Consider a space-time version of the network (N, A), which consists of only those arcs that are reachable in a feasible solution. While this network might contain multiple min-cuts, each evacuee, in an optimal solution for Objective (l), $l \in \{II, III\}$, goes through each min-cut only once. If an evacuee went through a min-cut twice, then the following could also occur; the evacuee could leave the origin later, and only use the arc in the min-cut having a higher time interval number, allowing another evacuee to utilize the other arc in the min-cut, which can only improve the solution. ▌

In the following proposition we show that we cannot reduce the average evacuation time, from an optimal solution for Objective (III), by improving the utilization of the bottleneck arcs, independent of shelter capacity limitations.

Proposition 9 *Consider an optimal solution for Objective (III) in a network where all shelters are uncapacitated. We cannot re-route evacuees to yield a feasible solution having a lower average evacuation time.*

Proof: Consider an optimal solution for Objective (III) given by s_{III}^t, $t = 1, \cdots, n_{III}$. We will show that any re-routing scheme that reduces the average evacuation time is infeasible. As shelters are uncapacitated, a re-routing scheme must decrease the average evacuation time by an improved use of the bottleneck arcs (or min-cut; see Propositions 1, 2, and 8). As a single re-routing cannot reduce the average evacuation time, we will have to examine a scheme consisting of a series of interdependent re-routings, i.e., consecutive re-routings where each successive re-routing makes use of the vacancy in a bottleneck arc produced by the previous re-routing. We can, without loss of generality, assume that each re-routing in the series involves only one evacuee. All independent schemes must begin by re-routing an evacuee to a path, without bottleneck arcs, that reaches a shelter in a later interval. First, we examine a scheme consisting of two re-routings, then we generalize this to all complex re-routing schemes.

We re-route an evacuee from a path $p \in P_{kz}^a$ to a path in P_{kz}^b, where $a < b$, opening the bottleneck arcs on path p. Each of these bottleneck arcs can potentially be used by another evacuee on another path. Consider one such path, path $q \in P_{lz}^c$, where $a < c$. If path q exists, then there also exists a path $q' \in P_{lz}^a$, consisting of the arcs from path q before the bottleneck arc, the arcs from path p after the bottleneck arc, and of course, the shared bottleneck arc. Assume that we re-route one evacuee from a path in P_{lz}^d to path q. Since, this evacuee could also be re-routed to path q', we have that $b \geq d$, else the original optimality for Objective (III) is contradicted. To improve the average evacuation time, we must have $b - a < d - c$, which cannot occur if $b \geq d$.

More complex re-routing schemes are also accommodated by the above analysis. For instance, if another re-routing were possible, using a bottleneck arc vacancy from when we re-routed an evacuee from the path in P_{lz}^d, then the only re-routing of an evacuee to an earlier interval using that bottleneck arc must be from a path in P_{kz}^f, where $d < f < b$, $k \in V$, which cannot decrease the average evacuation time from the original solution for Objective (III). The possibility of using path q' (in the above analysis) imposes a restriction on all subsequent re-routing; they cannot decrease the average evacuation time without contradicting the optimality of the original solution to Objective (III).

Of course, independent re-routing schemes (i.e., not using bottleneck arc capacity made available in another re-routing scheme) can be analyzed separately. ∎

Alternate Proof: Consider an optimal solution for Objective (III) given by s^t_{III}, $t = 1, \cdots, n_{III}$, and an optimal solution for Objective (II) given by s^t_{II}, $t = 1, \cdots, n_{II}$, for a network with uncapacitated shelters. Suppose that the average evacuation time for Objective (III) is more than that for Objective (II). Then for some interval t we must have $s^t_{III} > s^t_{II}$. Let t' be the first such interval, i.e., $s^t_{III} = s^t_{II}, t = 1, \cdots, t' - 1$ and $s^{t'}_{III} > s^{t'}_{II}$. This implies that a bottleneck arc in a path in P^f_{kz}, $\forall k \in V$, $z \in Z$, $f \leq t'$ in the solution for Objective III, is not a bottleneck arc in any path in P^f_{kz}, $\forall k \in V$, $z \in Z$, $f \leq t'$ in the solution for Objective II. From Proposition 6, in the solution for Objective (II), we know that any origin $k \in V$ that has a path containing this arc must be evacuated by interval t', and thus this arc cannot be further utilized after interval t'. The bottleneck arcs in the solution for Objective (III) correspond to the min-cut of the network. If shelters are uncapacitated, the min-cut is the limiting factor in the evacuation, and a solution for Objective (II) that does not fully utilize the min-cut cannot be optimal. We can see this is true because the D-REM with Objective (II) and uncapacitated shelters can be modeled as a minimum cost network flow problem. ∎

Using these results, we can now prove Proposition 5 and establish that the number of intervals required to complete an evacuation for an optimal strategy based on Objective (II), i.e., minimizing the average evacuation time, is less than or equal to that for Objective (III).

Proof of Proposition 5: The first inequality, $n_I \leq n_{II}$, follows from the definition of Objective (I). To prove the second inequality, $n_{II} \leq n_{III}$, we show that an optimal solution for Objective (III) cannot be modified closer to optimality for Objective (II) to produce a feasible solution having a lower average evacuation time and a longer duration, n.

From Proposition 9, modifying the utilization of the bottleneck arcs in an optimal solution for Objective (III) cannot lower the average evacuation time. Thus, we must consider modifying the utilization of the capacitated shelters. As a single re-routing cannot reduce the average evacuation time, we will have to examine a scheme consisting of a series of interdependent re-routings. We can, without loss of generality, assume that each re-routing in the series involves only one evacuee. All schemes must begin by re-routing an evacuee from a full, capacitated shelter, to a different shelter in a later interval. First, we examine a scheme consisting of two re-routings, then we generalize this to all complex re-routing schemes.

We re-route an evacuee from shelter j, which is full, to another shelter such that we decrease s_{III}^a and increase s_{III}^b. This allows another evacuee to be re-routed to shelter j such that we increase s_{III}^c and decrease $s_{III}^{n_{III}}$. To maintain a feasible solution we must have $a < b$ and $a < c$, else the original optimality for Objective (III) is contradicted. To improve the average evacuation time, we must have $b - a < n_{III} - c$. Together, these imply that $b < n_{III}$ and $c < n_{III}$.

Suppose re-routing an evacuee reduces s^t and opens a space in a capacitated shelter. The earliest that this space can be used is interval t, else the original optimality of Objective (III) is contradicted. Thus, if the first re-routing, in which we decrease s_{III}^a and increase s_{III}^b, allows not just one subsequent re-routing, but instead a series of re-routings, each interdependent, using the shelter space vacated by the prior re-routing, the net effect on the average is, at best, $n_{III} - c$, where the last evacuee is re-routed from interval n_{III}. Thus, no matter how complex the scheme, it cannot increase n and decrease the average evacuation time.

Of course, independent re-routing schemes (i.e., not using shelter capacity made available by another re-routing scheme) can be analyzed separately. ∎

The following example shows that strict inequalities in Proposition 5 are possible.
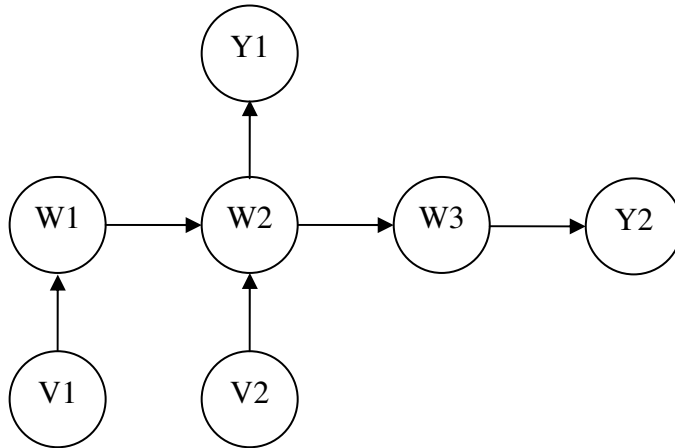

Figure 3.2: A small network with capacitated shelters.

Example 1 Consider the network depicted in Figure 3.2, where nodes V1 and V2 are the origin nodes, each with one evacuee, nodes W1, W2, and W3 are junction nodes, and nodes

Y1 and Y2 are shelters, each with a capacity of one. Each arc can be traversed in one time interval. Each arc and origin node has a risk value $R = 1$.

In this simple example there are two possible strategies. In the first strategy, the evacuee from V1 uses shelter Y1, while the evacuee from V2 uses Y2. In the second strategy, the evacuee from V2 uses shelter Y1 and the evacuee from V1 uses shelter Y2. The first strategy completes the evacuation in time interval four, minimizing Objective (I), but does not maximize the number of evacuees that find shelter in the third interval, a necessary condition of Objective (III)[1]. The second strategy completes the evacuation in time interval five, thus not minimizing Objective (I), but being optimal for Objective (III). Both strategies incur equal risks, and thus both are optimal for Objective (II), i.e., both minimize the average evacuation time for an evacuee.

Next, we will show that for the special case where all shelters are uncapacitated, we have $n_I = n_{II} = n_{III} = n_{V.a} = n_{V.b}$, and furthermore, that the evacuation strategy for Objective (III) will always minimize the average evacuation time. While many shelters are capacitated, the notion of uncapacitated shelters makes sense for certain broadly defined shelters, such as any area outside the region to be evacuated, or an expandable shelter (e.g., with tents).

Proposition 10 *When all shelters are uncapacitated, i.e., $C_{j,z} = \infty$, $\forall j \in Y, z \in Z$, the optimal solutions for Objectives (II), (III), (V.a), and (V.b) are equivalent, and are also optimal for Objective (I). However, the optimal solution for Objective (I) does not necessarily optimize Objectives (II), (III), (V.a), or (V.b).*

Proof: Consider an optimal solution for Objective (III) given by s_{III}^t, $t = 1, \cdots, n_{III}$. As shelter capacity is not a limiting factor on the value of any s_{III}^t, from Proposition 9 we know that the optimal solutions for Objectives (III) and (II), and thus Objective (V.b), are equivalent. The same logic from the proof of Proposition 9 can be applied to show that Objective (V.a) is also equivalent, and that all these solutions are optimal for Objective (I). If, in any of these solutions, some path $p \in P_{kz}^l$, $l \in \{$II, III, V.a, V.b$\}$ does not have a bottleneck arc, then $s_l^{n_l}$ can increase without affecting the optimality of the solution with regard to Objective (I), while rendering the new solution sub-optimal for Objectives (II),

---

[1]Note that we start at interval $t = 1$, not $t = 0$.

(III), (V.a), and (V.b). Thus, the optimal solution for Objective (I) does not necessarily optimize Objectives (II), (III), (V.a), or (V.b). ∎

We note here that Jarvis and Ratliff (1982) produce a result similar to Proposition 10, also for the special case where all shelters are uncapacitated, but for a model that does not allow for the possibility of congestion, i.e., a model with only constant exit-flow limits. In particular, they equate the optimal solutions for the two following objectives to each other, and also show that they are optimal for Objective (I):

$$Minimize \sum_{t=1}^{T} \sum_{j \in Y} \sum_{i:(i,j) \in A} c^t b_{ij}^t, \tag{3.35}$$

where $c^t > 0, t = 1, \cdots, T$, and $c^1 < c^2 < \cdots < c^T$, and

$$Maximize \sum_{f=1}^{t} \sum_{j \in Y} \sum_{i:(i,j) \in A} b_{ij}^f, \quad t = 1, \cdots, T. \tag{3.36}$$

In the following we show the relationship between Equation (3.35) and Objective (V.a).

Proposition 11 *For Equation (3.35), with $c^t > 0$, $t = 1, \cdots, T$, and $c^1 < c^2 < \cdots < c^T$, there exists an equivalent constant risk scheme for Objective (V.a) where $R^t > 0$, $t = 1, \cdots, T$, that is, Equation (3.35) is a special case of Objective (V.a).*

Proof: Substituting $D = \sum_{t=1}^{T} \sum_{j \in Y} \sum_{i:(i,j) \in A} b_{ij}^t$, which holds for any feasible evacuation, in Equation (2.18), we can write Objective (V.a) as follows:

$$\sum_{t=1}^{T} R^t \left(\sum_{f=t}^{T} \sum_{i \in Y} \sum_{j:(j,i) \in A} b_{ji}^f \right) = \sum_{t=1}^{T} \sum_{i \in Y} \sum_{j:(j,i) \in A} b_{ji}^t \left(\sum_{f=1}^{t} R^f \right).$$

Then, setting $R^1 = c^1$ and $R^t = c^t - c^{t-1}$ for $t = 2, \cdots, T$, establishes the result. ∎

As we shall discuss later, the risk approach allows for solutions that can take into account special properties of the network, which Equation (3.35) cannot.

Equation (3.36) is related to Objective (III), but as Example 1 shows, it is not equivalent, that is, an optimal solution for Objective (III) might be mutually exclusive of a solution that optimizes Equation (3.36) for some t (e.g., for $t = 4$ in Example 1). Moreover, Objective (III) is more flexible since there always exists a lexicographic maximum of s^t, $t = 1, \cdots, T$.

Unless all shelters are uncapacitated, an optimal solution for Objective (V.a) is generally not optimal for Objective (I). We can show that a special case of Objective (V.a), with a more restrictive risk structure, will always optimize Objective (I), even with capacitated shelters. This is certain *only if* the risk structure insures that the smallest possible decrease in risk from re-routing evacuees from interval n to an earlier interval is greater than the largest possible increase in risk due to any other re-routing of evacuees required to perform the first re-routing. This observation leads to the following result.

Proposition 12 *An optimal solution for Objective (V.a) with the following risk structure,*

$$\sum_{f=1}^{t-1} R^f < R^t, \quad t = 2, \cdots, T,$$

where $R^t > 0$, $t = 1 \cdots, T$, is also optimal for Objective (I).

Proof: Consider a feasible evacuation plan given by s^t, $t = 1, \cdots, n$, and assume that to decrease s^n at least two re-routings must occur. Observe that re-routing evacuees from s^n to s^{n-1} decreases the risk by R^n per evacuee, which is the smallest possible decrease. The greatest possible increase in risk occurs when evacuees are re-routed from s^1 to s^{n-1}, which will increase the risk by $\sum_{f=2}^{n-1} R^f$ per evacuee. If the decrease in risk is more than the increase, then re-routing will occur. This risk scheme conforms with the concept of minimizing the maximum time any evacuee is in the system, another interpretation of Objective (I). All feasible re-routing schemes can be represented by the simple re-routing scheme discussed here. ∎

Next we will show that another variant of Objective (V.a) can also optimize Objective (III). Once again, we are interested in the more general case where some shelters are capacitated. Here we slightly modify the risks, by allowing the risk in the first interval to be a very small negative number.

Proposition 13 *An optimal solution for Objective (V.a) with the following risk structure,*

$$R^t > \sum_{f=t+1}^{T} R^f, \ t = 2, \cdots, T-1,$$

where $R^1 = 0$ and $R^t > 0$, $t = 2, \cdots, T$, is also optimal for Objective (III).

Proof: Consider a feasible evacuation plan given by s^t, $t = 1, \cdots, n$. Without loss of generality, assume the plan already lexicographically maximizes s^f, $f = 1, \cdots, t-2$, and that to increase s^{t-1} at least two re-routings must occur. Re-routing evacuees from s^t to s^{t-1} yields the smallest decrease in risk, which is R^t per evacuee. Re-routing evacuees from s^t to s^T produces the largest increase in risk, which is $\sum_{f=t+1}^{T} R^f$ per evacuee. If the decrease in risk is more than the increase, then re-routing will occur. All feasible re-routing schemes can be represented by the simple re-routing scheme discussed here. ∎

Both Propositions 12 and 13 use risks to optimally assign evacuees to the capacitated shelters, given the two objective functions.

Remark 3 *An optimal solution for Objective (V) does not necessarily yield an optimal solution for Objectives (I), (II),(III), (IV), (V.a), or (V.b).*

For instance, assigning an extremely high risk to the arcs entering a particular shelter will produce a solution in which this shelter is not used, to the detriment of the other objectives.

Remark 4 *An optimal solution for Objective (IV), a commonly used objective in DTA models, does not necessarily yield an optimal solution for Objectives (I), (II), (III), (V.a), or (V.b).*

With the power to stage and route evacuees, an optimal solution to Objective (IV) would only make use of the least-cost paths between origins and shelters, with no incentive to maximize flow or complete the evacuation in a timely manner.

In summary, Objective (I) minimizes the duration of the evacuation. Objectives (II) and (V.b) minimize the average evacuation time, and each has an evacuation duration that is less than or equal to the duration of Objective (III), which maximizes the number of remaining (unevacuated) evacuees to reach shelter in each interval.

Objective (V.a) is quite versatile: With specialized risk structures, it can minimize the evacuation duration, considering both capacitated and uncapacitated shelters, minimize the average risk, and maximize the number of remaining (unevacuated) evacuees to reach shelter in each interval, i.e., Objective (V.a) can optimize Objectives (I), (II), and (III). Besides this, with certain modifications to the constant risk assumption, it can produce strategies that minimize the duration of an evacuation given certain constraints on the use of the

network. For example, if some arcs are deemed unsafe after a certain time, the risk on these arcs can be set to extremely high values, thus forcing a minimum time evacuation that avoids the unsafe arcs. Similarly, if certain neighborhoods become unsafe earlier than others, these neighborhoods can be given extremely high risk values when unsafe, thus producing a minimum time evacuation strategy, given that certain neighborhoods must be evacuated by certain times.

3.4   Insights

In this section we provide certain additional insights using a series of simple examples. These insights mainly concern how different network topologies can impact strategies and interact with the various objective functions.

Insight 1: Even if shelters are uncapacitated, it is not always optimal for evacuees to use their nearest shelter; as shown by Example 2 below.
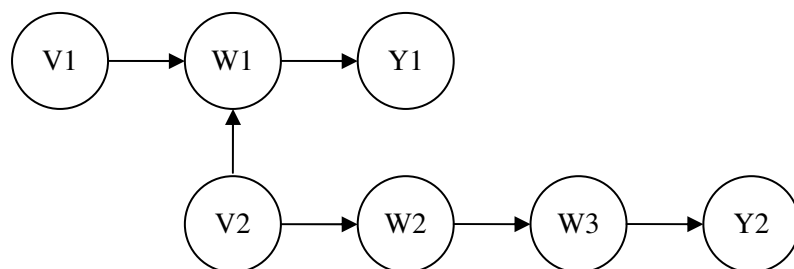
Figure 3.3: A small network with uncapacitated shelters.

Example 2   Consider the network depicted in Figure 3.3, where nodes $V1$ and V2 are origins having 80 evacuees each, nodes W1, W2, and W3 are junctions, and nodes Y1 and Y2 are uncapacitated shelters. Furthermore, each arc has an exit-flow function governed by the Pipes relationship, with a maximum flow of $q_{ij}^{max} = 20$ vehicles per interval. The strategy that will evacuate the network in the fewest number of intervals is to send the evacuees from V1 to shelter Y1 and from V2 to shelter Y2, despite shelter Y1 being the shelter nearest origin node V2.

In Example 2 the bottleneck status of arc (W1, Y1) is the factor that results in sending evacuees to a more distant shelter. Without proper planning and management, if evacuees

46

select the closest shelter this bottleneck will force arcs (V1, W1) and (V2, W1) to become congested, increasing the time required to complete the evacuation.

Insight 2: When shelters share a common bottleneck arc and the shelter closer to the bottleneck arc is capacitated (with less capacity than the number of evacuees), the evacuees sent to the closer shelter might need to leave last, as Example 3 illustrates.
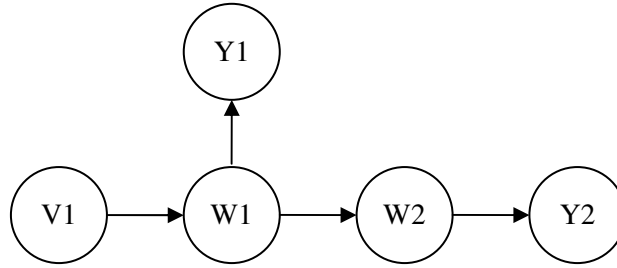
Figure 3.4: A small network with both capacitated and uncapacitated shelters.

Example 3 Consider the network depicted in Figure 3.4, where node V1 is an origin having 60 evacuees, nodes W1 and W2 are junctions, and nodes Y1 and Y2 are shelters, where Y1 has a capacity of 20 and Y2 is uncapacitated. Furthermore, each arc has an exit-flow function governed by the Pipes relationship, with a maximum flow of $q_{ij}^{max} = 20$ vehicles per interval. Sending the last 20 evacuees from V1 to shelter Y1 completes the evacuation in the fewest number of intervals; any other strategy increases the number of intervals required to complete the evacuation.

Insight 3: Shelters should be located such that they do not share a bottleneck arc.

As bottleneck arcs play an important strategic (and operational) role, it is of utmost importance to identify them. Arcs that experienced congestion in a previous, non-staged evacuation might not always identify the bottleneck arcs, which leads to the following insight.

Insight 4: Congestion in a non-staged evacuation does not have to be adjacent to a bottleneck arc, as Example 4 illustrates.

Example 4 Consider the network depicted in Figure 3.5, where nodes V1, V2, V3, and V4 are origins having large numbers of evacuees, nodes W1, W2, and W3 are junctions,
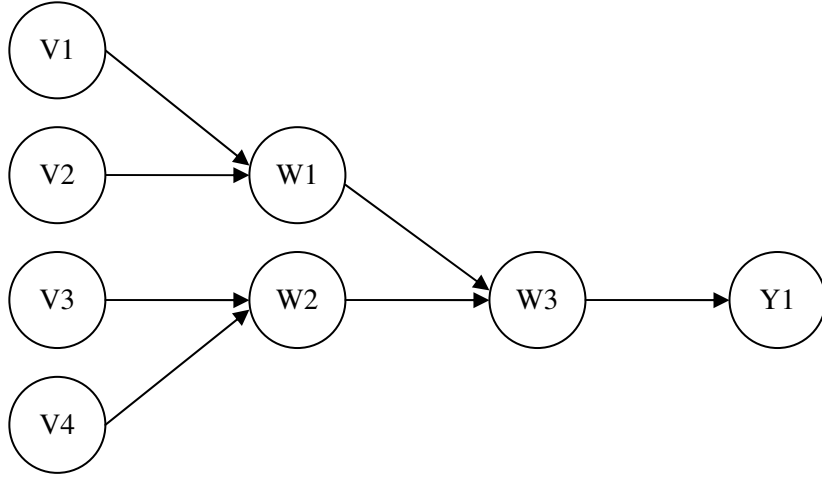
Figure 3.5: A network example of strategy dependent congestion.

and node Y1 is a shelter. Furthermore, each arc has an exit-flow function governed by the Pipes relationship, with a maximum flow of $q_{ij}^{max} = 20$ vehicles per interval. Arc (W3, Y1) represents the min-cut in this network and is a bottleneck arc for all intervals t with flow, when staging is used in this example, i.e., an optimal strategy with staging must insure that this arc maintains a maximum flow of $q_{ij}^{max} = 20$ vehicles per interval, if possible. In an unstaged evacuation, where each origin's characteristic network loading function is such that evacuees leave each origin at a rate of 15 evacuees per interval, congestion will form on arcs (V1, W1), (V2, W1), (V3, W2), and (V4, W2). This congestion is not adjacent to the bottleneck arc, hence, on reviewing the evacuation, one might suppose that arcs (W1, W3) and (W2, W3) are the bottleneck arcs. An optimal staged strategy that controls the release of evacuees will insure that the bottleneck arc (W3, Y1) is fully utilized at its maximum flow limit. The output of Model 3.4 with the Objectives (I), (V.a), (V.b), or (III) would automatically accomplish this goal for this network.

Example 4 shows that congestion is not just a property of the network, and specifically the location of the bottleneck arcs, but is also strategy-dependent. The next two insights deal with Objective (V) and how risk can impact the evacuation strategy.

Insight 5: With Objective (V), risk boundaries and bottleneck arcs interact to govern congestion in an optimal solution, as Example 5 illustrates.
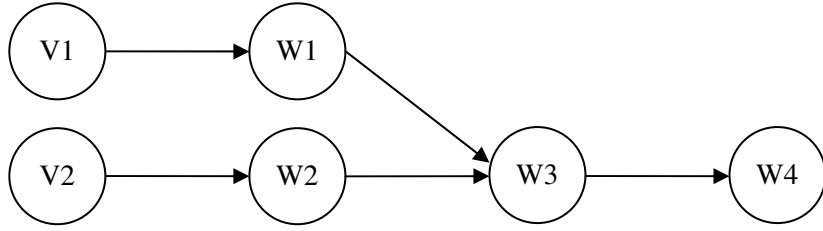
Figure 3.6: Interaction of risk levels and bottleneck arcs.

Example 5 Consider the portion of a network depicted in Figure 3.6, where nodes V1 and V2 are origins in a high risk area with 80 evacuees each, and nodes W1, W2, W3, and W4 are junctions. Furthermore, each arc has an exit-flow function governed by the Pipes relationship, with a maximum flow of $q_{ij}^{max} = 20$ vehicles per interval. If nodes V1 and V2, along with arcs (V1,W1) and (V2, W2), are given high enough risks compared to arcs (W1,W3) and (W2,W3), then congestion will form on arcs (W1,W3) and (W2,W3), while if these arcs are also given a high risk, no congestion will form.

Insight 6: Considering Objective (V), high risk areas do not necessarily evacuate first in an optimal solution.
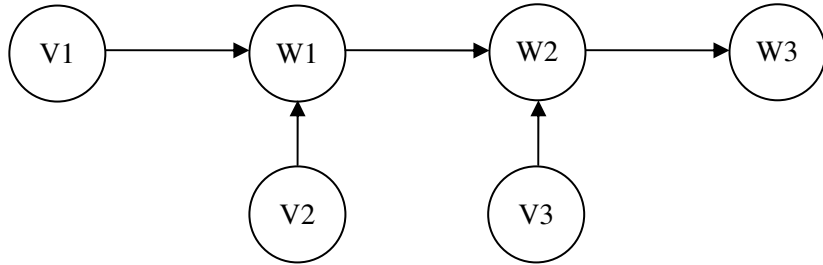
Figure 3.7: Evacuation of high risk areas.

Example 6 Consider the portion of a network depicted in Figure 3.7, where nodes V1, V2, and V3 are origins having 80 evacuees each. Furthermore, V1 and V2 have high risks compared to the other nodes depicted in the network. Each arc has an exit-flow function governed by the Pipes relationship, with a maximum flow of $q_{ij}^{max} = 20$ vehicles per interval, except arc (W2,W3), which has a maximum flow of $q_{ij}^{max} = 30$ vehicles per interval. The optimal solution for Objective (V) will send evacuees from node V3 continuously (despite

its lower risk), until all evacuees are gone, while it will only allow 20 evacuees per interval to escape nodes V1 and V2 combined. Note that an optimal solution would release 20 evacuees from node V3 in the first interval, and 10 evacuees in all subsequent intervals.

3.5   Solution Approaches

As the D-REM is motivated by large-scale regional evacuations, the networks used in practice might be quite large. Furthermore, with Objective (I), the model is a zero-one mixed-integer program (MIP), which is generally difficult to solve to optimality for large networks. Therefore, in this section we first discuss solution techniques for the D-REM, Objective (I). First, we observe that the D-REM is a linear program (LP), except for the binary vector E, which has a special structure at optimality (consecutive ones up to some interval t, followed by zeroes), and for each such hypothesized t value, the resulting feasibility problem is an LP.

To exploit this structure, it is useful to determine a lower bound (denoted LB) on the duration of the evacuation, that is, $LB \leq n_I$. The lower bound in Proposition 14 is determined by dividing the total number of evacuees by the sum of the maximum possible flows into each shelter, and rounded upwards if fractional (denoted $\lceil . \rceil$)

Proposition 14 $LB = \lceil D / \sum_{j \in Y} \sum_{i:(i,j) \in A} q_{ij}^{max} \rceil$.

Proof: The proof follows as $\sum_{j \in Y} \sum_{i:(i,j) \in A} q_{ij}^{max} \geq s^t, t = 1, \cdots, T$, in any feasible solution. ∎

Clearly, the tightness of this lower-bound is highly dependent on the underlying network structure. Next, we present solution techniques for determining an optimal strategy for Model 3.2 with Objective (I).

Solution Technique 1: Solve the D-REM with Objective (I) in its original form as an MIP.

Solution Technique 2: The following algorithm utilizes the special structure of the binary vector E discussed above, and makes use of the lower and upper bounds on n_I developed in Propositions 5 and 14.

1. Determine a lower bound, LB, on n_I using Proposition 14.

2. Determine an upper bound, UB, on n_I by solving the D-REM with Objective (II) (as an LP - see Proposition 5).

3. Perform a bisection search on the discrete values in $[LB, UB]$, solving the feasibility problem at each iteration as an LP.

This problem can be solved in polynomial time having complexity $O(n^3 L \log(UB - LB))$, where n is the number of variables in the problem, L is the number of binary bits required to store the data (see Bazaraa, Jarvis, and Sherali, 2005, for example).

Solution Technique 3: The following algorithm utilizes the special risk structure from Proposition 12.

1. Determine a lower bound, LB, on n_I using Proposition 14.

2. Set $R^t = 0$, $t = 1, \cdots, LB$. Set R^t, $t = LB, \cdots, T$, based on the risk scheme from Proposition 12.

3. Solve Model 3.2 with Objective (V.a) and the given risk structure.

This problem can be solved in polynomial time, using a single LP, having complexity $O(n^3 L)$. In practice, if the lower bound is too far from the optimal solution, appropriate fractional risk values that satisfy Proposition 12 should be selected to control their growth from a computational viewpoint. For example, the choice $R^t = 2^{t-LB}\epsilon$, $t = LB, \cdots, T$, satisfies this relationship, where $0 < \epsilon \le 1$ can be appropriately chosen, depending on T. If the range between the lower bound and T is much larger that 40 intervals, then we can revert to *Solution Technique 2* (possibly, using this in concert with *Solution Technique 3* as suitable).

Each of these solution techniques can be improved using Proposition 1, allowing us to substitute Model 3.4 for the D-REM, which simplifies the underlying LP and improves the form of the optimal solution by eliminating solutions having congestion and holding-back characteristics. The problem based on all the other objectives are easily solvable as they are LPs, and Objectives (II), (III), (V.a),and (V.b) can utilize Model 3.4 to further simplify the problem.

In Chapter 5, we present computational results on applying these solution techniques to a large-scale, realistic network.

3.6 Generic Non-Linear Flow Relationship

Often in the DTA literature, the exit-flow from an arc is modeled as a non-linear function. In this section, we consider an exit-flow function $g_{ij}(x_{ij}^t)$, for arc (i,j), $\forall (i,j) \in A$, that has a non-linear, concave traffic flow/density relationship. We assume that the function $g_{ij}(x_{ij}^t)$ has the following properties:

1. $g_{ij}(x_{ij}^t = 0) = 0$.

2. $g_{ij}(x_{ij}^t = k_{ij}^{max} l_{ij}) = 0$.

3. $g_{ij}(x_{ij}^t) \leq x_{ij}^t$.

4. $g'_{ij}(x_{ij}^t = 0) = 1$.

5. $x_{ij}^{qmax} > q_{ij}^{max}$, where x_{ij}^{qmax} is the number of evacuees on arc (i,j) such that the flow is maximized.

Property 4 stems from the assumption that vehicles can traverse an arc in a single time interval at the free-flow speed (u_{ij}^{max}). Properties 1 and 4, along with the assumption of concavity, imply Property 3, since $g_{ij}(x_{ij}^t) \leq g_{ij}(0) + x_{ij}^t g'_{ij}(0) = x_{ij}^t$. Property 5 insures the uncongested regime is not linear, in which case, the function would act like the Pipes relationship, as discussed in Section 2.5.

An example of a well-known traffic-flow relationship, which adheres to these five properties, is the Greenshields relationship, which is defined as follows:

$$g_{ij}(x_{ij}^t) = u_{ij}^{max} x_{ij}^t / l_{ij} - (u_{ij}^{max}/k_{ij}^{max})(x_{ij}^t/l_{ij})^2. \tag{3.37}$$

Figure 3.8 shows the relationships between flow, density, and speed for the Greenshields relationship.

We see from the Greenshields relationship that $g_{ij}(x_{ij}^t = 0) = 0$ and $g_{ij}(x_{ij}^t = k_{ij}^{max} l_{ij}) = 0$. The derivative of $g_{ij}(x_{ij}^t)$ with respect to x_{ij}^t is:

$$g'_{ij}(x_{ij}^t) = dg_{ij}/dx_{ij}^t = u_{ij}^{max}/l_{ij} - 2u_{ij}^{max} x_{ij}^t/k_{ij}^{max} l_{ij}^2.$$

Note that the maximum flow, q_{ij}^{max}, where the derivative is equal to zero, occurs at a density of $k_{ij}^{max}/2$ and a speed of $u_{ij}^{max}/2$. This represents the transition point from an uncongested to a
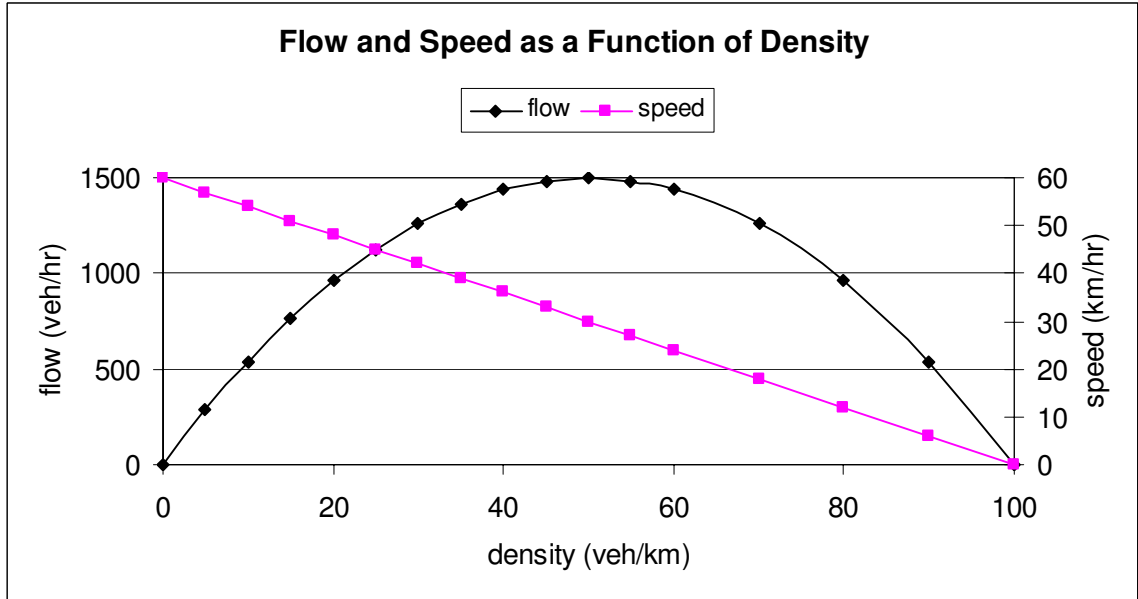
Figure 3.8: The Greenshields traffic flow relationship.

congested regime. The derivative evaluated at $x_{ij}^t = 0$ yields u_{ij}^{max}/l_{ij}. Under the assumption that any arc (i,j) can be traversed in a single time interval, we see that $u_{ij}^{max}/l_{ij} = 1$. As this relationship in concave, the slope of the flow/density curve is decreasing in x_{ij}^t. Thus, with the Greenshields relationship and our assumptions, the flow out of arc (i,j) is always less than or equal to the number of evacuees on the arc, i.e., $g_{ij}(x_{ij}^t) \leq x_{ij}^t$.

In the remainder of this section, we show some preliminary results for the D-REM, with this type of non-linear exit-flow function. To study this model, we examine the KKT conditions. To do this, we use Objective (II). This is done to avoid the binary variables required by Objective (I). We also eliminate evacuee type from the D-REM to simplify this complex model. The formulation, along with the dual variables, is as follows:

Model 3.5 D-REM, KKT Format

$$Maximize \ \sum_{t=1}^{T}\sum_{j \in Y}\sum_{i:(i,j) \in A} (T-t)b_{ij}^{t} \tag{3.38}$$

subject to

$$v1_{ij}^{t} \ : \ -x_{ij}^{t+1} + x_{ij}^{t} - b_{ij}^{t} + a_{ij}^{t} = 0, \quad \forall (i,j) \in A, \ t = 1, \cdots, T-1 \tag{3.39}$$

$$v2_{i}^{t} \ : \ -\sum_{j:(j,i) \in A} b_{ji}^{t} + \sum_{j:(i,j) \in A} a_{ij}^{t} = 0, \quad \forall i \in W, \ t = 1, \cdots, T \tag{3.40}$$

$$v3_{i} \ : \ -\sum_{t=1}^{T}\sum_{j:(i,j) \in A} a_{ij}^{t} + D_{i} = 0, \quad \forall i \in V \tag{3.41}$$

$$u1_{ij}^{t} \ : \ a_{ij}^{t} - q_{ij}^{max} \leq 0, \quad \forall (i,j) \in A, \ t = 1, \cdots, T \tag{3.42}$$

$$u2_{ij}^{t} \ : \ b_{ij}^{t} - g_{ij}(x_{ij}^{t}) \leq 0, \quad \forall (i,j) \in A, \ t = 1, \cdots, T \tag{3.43}$$

$$u3_{i} \ : \ \sum_{t=1}^{T}\sum_{j:(j,i) \in A} b_{ji}^{t} - C_{i} \leq 0, \quad \forall i \in Y \tag{3.44}$$

$$u4_{ij}^{t} \ : \ -a_{ij}^{t} \leq 0, \quad \forall (i,j) \in A, \ t = 1, \cdots, T \tag{3.45}$$

$$u5_{ij}^{t} \ : \ -b_{ij}^{t} \leq 0, \quad \forall (i,j) \in A, \ t = 1, \cdots, T \tag{3.46}$$

$$u6_{ij}^{t} \ : \ -x_{ij}^{t} \leq 0, \quad \forall (i,j) \in A, \ t = 1, \cdots, T. \tag{3.47}$$

The objective in Equation (3.38) is equivalent to Equation (2.12), which we repeat here,

$$Minimize \ \sum_{t=1}^{T}\sum_{j \in Y}\sum_{i:(i,j) \in A} tb_{ij}^{t}.$$

Proposition 15 *Solving Model 3.4, which has constant exit-flow limits, for Objectives (I), (II), or (III), where the constant exit-flow limits equal the maximum variable exit-flow limits (q_{ij}^{max}) from Model 3.5, yields a lower bound on the solution to Model 3.5 for each of the respective objectives.*

Proof: Based on Property 3 of the exit-flow function, $g_{ij}(x_{ij}^{t})$, and Equation (3.43), we can see that with variable exit-flow limits, the outflow from each arc is always less than or equal to the outflow from the constant exit-flow limit problem. The proof thus follows. ∎

Proposition 16 *Propositions 5-10, which describe the relationship between Objectives (I), (II), and (III) for the D-REM, are also valid for Model 3.5.*

Proof: The proofs to Propositions 5-10 do not depend on the form of the variable exit-flow function, and are thus valid for the convex functions utilized in Model 3.5. ∎

The KKT necessary conditions are as follows:

Model 3.6 KKT Necessary Conditions

$$a_{ij}^t \quad : \quad v1_{ij}^t - v3_i + u1_{ij}^t - u4_{ij}^t = 0, \quad \forall i \in V, \ \forall j : (i,j) \in A, \ t = 1, \cdots, T \tag{3.48}$$

$$a_{ij}^t \quad : \quad v1_{ij}^t + v2_i^t + u1_{ij}^t - u4_{ij}^t = 0, \quad \forall i \in W, \ \forall j : (i,j) \in A, \ t = 1, \cdots, T \tag{3.49}$$

$$b_{ij}^t \quad : \quad -v1_{ij}^t - v2_j^t + u2_{ij}^t - u5_{ij}^t = 0, \quad \forall j \in W, \ \forall i : (i,j) \in A, \ t = 1, \cdots, T \tag{3.50}$$

$$b_{ij}^t \quad : \quad -v1_{ij}^t + u2_{ij}^t + u3_j - u5_{ij}^t = (T-t), \quad \forall j \in Y, \ \forall i : (i,j) \in A, \ t = 1, \cdots, T \tag{3.51}$$

$$x_{ij}^t \quad : \quad -v1_{ij}^{t-1} + v1_{ij}^t - u2_{ij}^t g_{ij}'(x_{ij}^t) - u6_{ij}^t = 0, \quad \forall (i,j) \in A, \ t = 2, \cdots, T \tag{3.52}$$

$$u1_{ij}^t(a_{ij}^t - q_{ij}^{max}) = 0, \quad \forall (i,j) \in A, \ t = 1, \cdots, T \tag{3.53}$$

$$u2_{ij}^t(b_{ij}^t - g_{ij}(x_{ij}^t)) = 0, \quad \forall (i,j) \in A, \ t = 1, \cdots, T \tag{3.54}$$

$$u3_i\Big(\sum_{t=1}^T \sum_{j:(j,i)\in A} b_{ji}^t - C_i\Big) = 0, \quad \forall i \in Y \tag{3.55}$$

$$u4_{ij}^t(-a_{ij}^t) = 0, \quad \forall (i,j) \in A, \ t = 1, \cdots, T \tag{3.56}$$

$$u5_{ij}^t(-b_{ij}^t) = 0, \quad \forall (i,j) \in A, \ t = 1, \cdots, T \tag{3.57}$$

$$u6_{ij}^t(-x_{ij}^t) = 0, \quad \forall (i,j) \in A, \ t = 1, \cdots, T \tag{3.58}$$

$$u1_{ij}^t, u2_{ij}^t, u4_{ij}^t, u5_{ij}^t, u6_{ij}^t \geq 0, \quad \forall (i,j) \in A, \ t = 1, \cdots, T \tag{3.59}$$

$$u3_i \geq 0, \quad \forall i \in Y. \tag{3.60}$$

Note: $x_{ij}^1, \forall (i,j) \in A$, is a parameter, not a decision variable.

Proposition 17 *Consider an arbitrary arc (i,j) which is uncongested in interval t, that is $x_{ij}^t \leq x_{ij}^{qmax}$. Arc (i,j) cannot become congested as long as $b_{ij}^f = g_{ij}(x_{ij}^f), \ \forall f \geq t$, that is, as long as the outflow is unobstructed.*

Proof: If $x_{ij}^t \leq x_{ij}^{qmax}$ and $b_{ij}^t = g_{ij}(x_{ij}^t)$ for arc (i,j) then, from Equation (3.39), we have $x_{ij}^{t+1} \leq x_{ij}^t - g_{ij}(x_{ij}^t) + q_{ij}^{max}$ because $a_{ij}^t \leq q_{ij}^{max}$ (see Equation (3.42)). From the concavity of g_{ij} and Property 4, we have that $q_{ij}^{max} = g_{ij}(x_{ij}^{qmax}) \leq g_{ij}(x_{ij}^t) + g_{ij}'(x_{ij}^t)(x_{ij}^{qmax} - x_{ij}^t) = g_{ij}(x_{ij}^t) + x_{ij}^{qmax} - x_{ij}^t$. Hence, $x_{ij}^{t+1} \leq x_{ij}^t - g_{ij}(x_{ij}^t) + q_{ij}^{max} \leq x_{ij}^{qmax}$. ∎

Corollary 3.6.1 *If $b_{ij}^t = q_{ij}^{max}$ and $b_{ij}^{t-1} = g_{ij}(x_{ij}^{t-1})$ for arc (i,j), then we have $a_{ij}^{t-1} = q_{ij}^{max}$, that is, the flow into arc (i,j) in interval $t-1$ is equal to the maximum possible flow.* ∎

We define arc (i,j) as an *exit arc* if j is a shelter, that is, $j \in Y$. As an exit arc has no downstream obstructions to flow, holding-back is simply defined as $b_{ij}^t < g_{ij}(x_{ij}^t)$. The following propositions discuss properties of exit arcs in an optimal solution.

Proposition 18 *For an exit arc (i,j) in an optimal solution, there is no holding-back, that is, $b_{ij}^t = g_{ij}(x_{ij}^t)$.*

Proof: For Objective (II), the sooner that evacuees on an exit arc enter shelter j, the lower the average evacuation time. This is because holding-back in interval t, i.e., $b_{ij}^t < g_{ij}(x_{ij}^t)$, will not increase total flow into the shelter in intervals t and later, as flow is governed by a concave function. In other words, the slope of $g_{ij}(x_{ij}^t)$ is decreasing as x_{ij}^t increases. ∎

Proposition 19 *For an exit arc (i,j) in an optimal solution, flow will never enter the congested regime, that is, $x_{ij}^t \leq x_{ij}^{qmax}$ (which is equivalent to $x_{ij}^t \leq k_{ij}^{max} l_{ij}/2$ for the Greenshields relationship) and thus $0 \leq g_{ij}'(x_{ij}^t) \leq 1$.*

Proof: This follows from Propositions 17 and 18. ∎

Proposition 20 *For an exit arc (i,j) in an optimal solution, the following relationship holds: $v1_{ij}^{t-1} \leq v1_{ij}^t, t = 2, \cdots, T$.*

Proof: From Equation (3.52) we have, $v1_{ij}^{t-1} = v1_{ij}^t - u2_{ij}^t g_{ij}' - u6_{ij}^t$. As $u2_{ij}^t \geq 0$ and $u6_{ij}^t \geq 0$, and by Proposition 19, $g_{ij}' \geq 0$, the result follows. ∎

Proposition 21 *For an exit arc (i,j) in an optimal solution, if $x_{ij}^t > 0$ then $u2_{ij}^{t-1} > 0$.*

Proof: From Equation (3.51) we have:

$$v1_{ij}^t = -(T-t) + u2_{ij}^t + u3_j - u5_{ij}^t$$

$$v1_{ij}^{t-1} = -(T-(t-1)) + u2_{ij}^{t-1} + u3_j - u5_{ij}^{t-1}.$$

Substituting for $v1_{ij}^t$ and $v1_{ij}^{t-1}$ in Equation (3.52) yields:

$$(T-(t-1)) - u2_{ij}^{t-1} - u3_j + u5_{ij}^{t-1} - (T-t) + u2_{ij}^t + u3_j - u5_{ij}^t - u2_{ij}^t g_{ij}'(x_{ij}^t) - u6_{ij}^t = 0.$$

As $x_{ij}^t > 0$, we get $b_{ij}^t > 0$ by Proposition 18, and thus $u5_{ij}^t$ and $u6_{ij}^t$ are zero from Equations (3.57) and (3.58). Canceling and re-arranging terms yields:

$$u2_{ij}^{t-1} = 1 + u5_{ij}^{t-1} + u2_{ij}^t(1 - g_{ij}'(x_{ij}^t)).$$

As $u5_{ij}^{t-1} \geq 0$, $u2_{ij}^t \geq 0$, and $0 \leq g_{ij}'(x_{ij}^t) \leq 1$, we see that $u2_{ij}^{t-1} > 0$. ∎

Proposition 22 *For an exit arc (i, j) in an optimal solution, whenever shelter capacity is not an issue, i.e., $u3_j = 0$, we have $v1_{ij}^t \leq 0, t = 1, \cdots, T - 1$.*

Proof: For any optimal solution, it is possible to have a last time interval T where $x_{ij}^t = 0$. For time interval T, from Equation (3.51) we have:

$$v1_{ij}^T - u2_{ij}^T = -u5_{ij}^T \leq 0, \tag{3.61}$$

as $u3_j = 0$ and $u5_{ij}^T \geq 0$. From Property 4, $g_{ij}'(x_{ij}^T = 0) = 1$. Combining Equations (3.52) and (3.61) yields $v1_{ij}^{T-1} = v1_{ij}^T - u2_{ij}^T - u6_{ij}^T = -u5_{ij}^T - u6_{ij}^T \leq 0$ as $u6_{ij}^T \geq 0$. Then from Proposition 20 the result follows. ∎

We define l as the first time interval, and m as the last time interval, in a series of intervals where $x_{ij}^t > 0$, $\forall (i, j) \in A$, $t = l, \cdots, m$. We also define m' as the last interval (the first interval being $l-1$), in a series of intervals where $a_{ij}^t > 0$, $\forall (i, j) \in A$, $t = l-1, \cdots, m'$. Unless stated otherwise, these variables are in reference to an exit arc. We know, by definition, that $m' \leq m - 1$. Most of the above propositions are in support of the following result:

Proposition 23 *For an exit arc (i, j) in an optimal solution, $v1_{ij}^{t-2} < v1_{ij}^{t-1}$ for $t = l, \cdots, m$, unless for some $t - 1 \geq l$, the flow into shelter j is equal to the maximum possible flow, i.e., $b_{ij}^{t-1} = g_{ij}(x_{ij}^{t-1}) = q_{ij}^{max}$, in which case $v1_{ij}^{t-2} = v1_{ij}^{t-1}$.*

Proof: From Proposition 21 we have $u2_{ij}^{t-1} > 0$, for $t = l, \cdots, m$. From Equation (3.52) we have $v1_{ij}^{t-2} = v1_{ij}^{t-1} - u2_{ij}^{t-1}g_{ij}'(x_{ij}^{t-1}) - u6_{ij}^{t-1}$. From Equation (3.59) we have $u6_{ij}^{t-1} \geq 0$ and from Proposition 19 we have $g_{ij}'(x_{ij}^{t-1}) \geq 0$. Thus, unless $g_{ij}'(x_{ij}^{t-1}) = 0$, we have that $v1_{ij}^{t-2} < v1_{ij}^{t-1}$. Only when the concave exit-flow function, $g_{ij}(x_{ij}^{t-1})$, is maximized does $g_{ij}'(x_{ij}^{t-1}) = 0$, i.e., when $b_{ij}^{t-1} = g_{ij}(x_{ij}^{t-1} = x_{ij}^{qmax}) = q_{ij}^{max}$. When this occurs we have $u6_{ij}^{t-1} = 0$ because $x_{ij}^{t-1} = x_{ij}^{qmax} > 0$, thus the exception, $v1_{ij}^{t-2} = v1_{ij}^{t-1}$. ∎

Note that for $t = l$ we have $v1_{ij}^{l-2} < v1_{ij}^{l-1}$ since, by definition, $x_{ij}^{l-1} = 0$, and thus we have $g_{ij}'(x_{ij}^{l-1}) = 1$ from Property 4. Also note that by Corollary 3.6.1 and Proposition 18, that for $b_{ij}^{t-1} = g_{ij}(x_{ij}^{t-1} = x_{ij}^{qmax}) = q_{ij}^{max}$ to occur we must have $a_{ij}^{t-2} = q_{ij}^{max}$, and thus the exit arc in interval $t - 2$ is a candidate bottleneck arc-interval. Next, we examine some network properties of an optimal solution.

Proposition 24 *Consider any pair of adjacent arcs (i, j) and (j, k). In an optimal solution if $v1_{jk}^{t-1} < v1_{jk}^{t}$, $a_{jk}^{t} > 0$, $x_{jk}^{t} > 0$, and $b_{ij}^{t} > 0$, then $u2_{ij}^{t-1} > 0$, except possibly when $a_{jk}^{t-1} = q_{jk}^{max}$, i.e., when arc (j, k) is a bottleneck arc in interval $t - 1$.*

Proof: Combining Equations (3.49) and (3.50) for any pair of adjacent arcs (i, j) and (j, k), based on the common $v2_j^t$ variable, yields:

$$v1_{ij}^t = v1_{jk}^t + u1_{jk}^t - u4_{jk}^t + u2_{ij}^t - u5_{ij}^t, \quad \forall t = 1, \cdots, T. \tag{3.62}$$

Substituting for the $v1_{ij}^t$ and $v1_{ij}^{t-1}$ variables in Equation (3.52) for arc (i, j), based on Equation (3.62), and rearranging, yields:

$$u2_{ij}^{t-1} = v1_{jk}^t - v1_{jk}^{t-1} + u2_{ij}^t(1 - g_{ij}'(x_{ij}^t)) + u4_{jk}^{t-1} + u5_{ij}^{t-1} + u1_{jk}^t - u1_{jk}^{t-1}, \tag{3.63}$$

as $u4_{jk}^t = u5_{ij}^t = u6_{ij}^t = 0$ from Equations (3.45)-(3.47) because $a_{jk}^t, b_{ij}^t, x_{ij}^t > 0$.

If, $v1_{jk}^{t-1} < v1_{jk}^t$, as stated in the proposition, then $v1_{jk}^t - v1_{jk}^{t-1} > 0$. Furthermore, from Property 4 and the concavity of the exit-flow function, g_{ij}, we have $u2_{ij}^t(1 - g_{ij}'(x_{ij}^t)) \geq 0$. The remaining variables on the left-hand side of Equation (3.63) are non-negative by definition, with only the last term, $u1_{jk}^{t-1}$, subtracted. Thus, we have $u2_{ij}^{t-1} > 0$, unless $u1_{jk}^{t-1} > 0$, which implies that arc (j, k) is a bottleneck arc in interval $t - 1$, i.e., $a_{jk}^{t-1} = q_{jk}^{max}$ by Equation (3.53). ∎

Proposition 25 *The pattern established for an exit arc (j, k) of $v1_{jk}^{t-1} < v1_{jk}^t$ for intervals $t = l, \cdots, m'$ (see Proposition 23) will propagate upstream to adjacent arc (i, j), following the flow in reverse, i.e., when $b_{ij}^t > 0$, unless the propagation is inhibited by a bottleneck arc-interval.*

Proof: Consider an exit arc (j, k) and adjacent arc (i, j) in intervals $t = l, \cdots, m'$ (referenced to the exit arc). For the intervals considered we have $a_{jk}^t > 0$ and $x_{jk}^t > 0$. From

Proposition 23 we have $v1_{jk}^{t-1} < v1_{jk}^t$, unless $g'_{jk} = 0$, i.e., $g_{jk}(x_{jk}^t) = q_{jk}^{max}$. From Proposition 24 we have $u2_{ij}^{t-1} > 0$ for all intervals $t = l, \cdots, m'$ when $v1_{jk}^{t-1} < v1_{jk}^t$ and $b_{ij}^t > 0$, unless $u1_{jk}^{t-1} > 0$. Both $b_{jk}^t = g_{jk}(x_{jk}^t) = q_{jk}^{max}$ and $u1_{jk}^{t-1} > 0$ imply that $a_{jk}^{t-1} = q_{jk}^{max}$ and hence that arc (j, k) is a bottleneck in interval $t-1$ (see the proof for Proposition 23). When $u2_{ij}^{t-1} > 0$, then $v1_{ij}^{t-2} < v1_{ij}^{t-1}$ unless $g'_{ij} = 0$, i.e., $g_{ij}(x_{ij}^t) = q_{ij}^{max}$. Following the same logic, the propagation continues upstream, following the flow, unless inhibited by a bottleneck arc interval. ∎

Proposition 26 *In an optimal solution, any arc (i, j) that is downstream of a bottleneck arc-interval must have $b_{ij}^t = g_{ij}(x_{ij}^t)$; thus there is no holding-back on an arc unless it is upstream of a bottleneck arc-interval.*

Proof: From Proposition 18 we know that $b_{ij}^t = g_{ij}(x_{ij}^t)$ for an exit arc. The propagation mechanism from Proposition 25 is based on the propagation of non-zero $u2$ variables upstream, against the flow. Thus, $u2_{ij}^t > 0$ (and from Equation (3.54), $b_{ij}^t = g_{ij}(x_{ij}^t)$) for any arc (i, j) upstream of the exit arc, unless a bottleneck arc-interval inhibits the propagation, i.e., any arc (i, j) that is downstream of a bottleneck arc-interval must have $b_{ij}^t = g_{ij}(x_{ij}^t)$. ∎

Proposition 27 *In an optimal solution, any arc (i, j) that is downstream of a bottleneck arc-interval must not be congested, i.e., $x_{ij}^t \leq x_{ij}^{qmax}$.*

Proof: By Proposition 26 any arc (i, j) that is downstream of a bottleneck arc-interval must have $b_{ij}^t = g_{ij}(x_{ij}^t)$. By Proposition 17 these arcs cannot become congested. ∎

Propositions 26 and 27 make intuitive sense; arcs downstream of a bottleneck arc-interval should not delay evacuees due to holding-back or congestion.

Proposition 28 *In an optimal solution, all evacuees must flow through a bottleneck arc-interval, with the possible exception of those evacuees leaving origin i in the last interval having a positive flow from node i.*

Proof: From Proposition 25, if no bottleneck arc-intervals intervene, for each interval t with flow from node i, $i \in V$, onto entrance arc (i, j) in interval t we have $v1_{ij}^{t-1} < v1_{ij}^t$. If arc (i, j) has more than a single interval t with $v1_{ij}^{t-1} < v1_{ij}^t$, then we must use $u1_{ij}^t$ to satisfy Equation (3.48), except for possibly one interval in which $v3_i$ can be used, as by definition

$u4_{ij}^t = 0$, thus $a_{ij}^t = q_{ij}^{max}$, making arc (i,j) a bottleneck in interval t. Furthermore, as $u1_{ij}^t \geq 0$ and is added to $v1_{ij}^t$, we have $v1_{ij}^t \leq 0$. Thus, $v3_i$ can only be used in the last interval t having a positive flow, as this interval will have the smallest $v1_{ij}^t$ value. ∎

Next, we examine a network consisting of sequential arcs, such as the two arc network depicted in Figure 3.9.
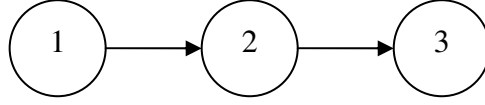
Figure 3.9: A network composed of two sequential arcs.

Proposition 29 *Consider a network composed of F sequential nodes, labeled from 1 to F, where Node 1 is the origin, Node F is the shelter, $q_{ij}^{max} \leq q_{jk}^{max}, i = 1, \cdots, F-2, j = i+1, k = j+1$, and all arcs are initially empty, i.e., $x_{ij}^1 = 0$, $\forall (i,j) \in A$. For this network, the optimal solution takes the following form; for Arc (1,2), we have $a_{12}^t = min[q_{12}^{max}, O_1 - \sum_{f=1}^{t-1} a_{12}^f]$, $t = 1, \cdots, T$, for any pair of adjacent arcs (i,j) and (j,k), we have $a_{jk}^t = b_{ij}^t = g_{ij}(x_{ij}^t)$, and for all arcs $x_{ij}^{t+1} = \sum_{f=1}^t a_{ij}^f - \sum_{f=1}^t b_{ij}^f$, $t = 1, \cdots, T-1$ and $b_{ij}^t = g_{ij}(x_{ij}^t)$, $t = 1, \cdots, T$.*

Proof: From Proposition 18 we know that holding-back will not occur on the exit arc $(F-1, F)$, i.e., $b_{F-1,F}^t = g_{F-1,F}(x_{F-1,F}^t)$, $t = 1, \cdots, T$, and from Proposition 19 the exit arc will not enter the congested regime. For the uncongested regime, the slope of the exit-flow function is non-negative; thus, to maximize flow into the shelter, flow into arc $(F-1, F)$ should be maximized. As $q_{F-2,F-1}^{max} \leq q_{F-1,F}^{max}$, to maximize the flow into arc $(F-1, F)$ we have $a_{F-1,F}^t = b_{F-2,F-1}^t = g_{F-2,F-1}(x_{F-2,F-1}^t)$. As holding-back does not occur on arc $(F-2, F-1)$, by Proposition 17, congestion also does not occur, and as $q_{F-3,F-2}^{max} \leq q_{F-2,F-1}^{max}$, to maximize flow onto arc $(F-2, F-1)$, and thus maximize flow onto arc $(F-1, F)$, we have $a_{F-2,F-1}^t = b_{F-3,F-2}^t = g_{F-3,F-2}(x_{F-3,F-2}^t)$. This same logic continues along all upstream arcs; thus, $b_{ij}^t = g_{ij}(x_{ij}^t), \forall (i,j) \in A$, $t = 1, \cdots, T$. To maximize flow onto the first arc, Arc $(1,2)$, we have $a_{12}^t = min[q_{12}^{max}, O_1 - \sum_{f=1}^{t-1} a_{12}^f]$, $t = 1, \cdots, T$. From Equation (3.52), then, we have $x_{ij}^{t+1} = \sum_{f=1}^t a_{ij}^f - \sum_{f=1}^t b_{ij}^f$, $t = 1, \cdots, T-1$. ∎

Corollary 3.6.2 *For the network defined in Proposition 29, Arc $(1,2)$, the initial arc, is a bottleneck arc until demand is exhausted. Furthermore, the remaining arcs are critical, and by definition will experience no congestion or holding-back in the optimal solution.* ∎

Now we will examine another sequential network where a bottle-neck forms on an arc downstream of the initial arc.

Proposition 30 *Consider a network composed of F sequential nodes, labeled from 1 to F, where Node 1 is the origin, Node F is the shelter, $q_{d,d+1}^{max} < q_{12}^{max} \leq q_{ij}^{max}, i = 2, \cdots, F-1, j = i+1, i \neq d, j \neq d+1$, and all arcs are initially empty, i.e., $x_{ij}^1 = 0, \forall (i,j) \in A$. For this network, there are two possibilities. First, arc $(d, d+1)$ might never reach bottleneck status, in which case, Arc (1,2) is the bottleneck arc-interval and the optimal solution takes the following form; for Arc (1,2), we have $a_{12}^t = min[q_{12}^{max}, O_1 - \sum_{f=1}^{t-1} a_{12}^f], t = 1, \cdots, T$, for any pair of adjacent arcs (i,j) and (j,k), we have $a_{jk}^t = b_{ij}^t = g_{ij}(x_{ij}^t)$, and for all arcs $x_{ij}^{t+1} = \sum_{f=1}^t a_{ij}^f - \sum_{f=1}^t b_{ij}^f, t = 1, \cdots, T-1$ and $b_{ij}^t = g_{ij}(x_{ij}^t), t = 1, \cdots, T$. Another possibility is that arc $(d, d+1)$ is a bottleneck in some interval τ, in which case, we have $a_{12}^t = min[q_{12}^{max}, O_1 - \sum_{f=1}^{t-1} a_{12}^f], t = 1, \cdots, T$.*

Proof: From Proposition 18 we know that holding-back will not occur on the exit arc $(F-1, F)$, i.e., $b_{F-1,F}^t = g_{F-1,F}(x_{F-1,F}^t), t = 1, \cdots, T$, and from Proposition 19 the exit arc will not enter the congested regime. For the uncongested regime, the slope of the exit-flow function is non-negative; thus, to maximize flow into the shelter, flow into arc $(F-1, F)$ should be maximized. As $q_{F-2,F-1}^{max} \leq q_{F-1,F}^{max}$, to maximize the flow into arc $(F-1, F)$ we have $a_{F-1,F}^t = b_{F-2,F-1}^t = g_{F-2,F-1}(x_{F-2,F-1}^t)$. As holding-back does not occur on arc $(F-2, F-1)$, by Proposition 17 congestion also does not occur, and as $q_{F-3,F-2}^{max} \leq q_{F-2,F-1}^{max}$, to maximize flow onto arc $(F-2, F-1)$, and thus maximize flow onto arc $(F-1, F)$, we have $a_{F-2,F-1}^t = b_{F-3,F-2}^t = g_{F-3,F-2}(x_{F-3,F-2}^t)$. This same logic continues along all upstream arcs; thus, $b_{ij}^t = g_{ij}(x_{ij}^t), \forall (i,j) \in A, t = 1, \cdots, T$. To maximize flow onto the first arc, Arc $(1,2)$, we have $a_{12}^t = min[q_{12}^{max}, O_1 - \sum_{f=1}^{t-1} a_{12}^f], t = 1, \cdots, T$. From Equation (3.52), then, we have $x_{ij}^{t+1} = \sum_{f=1}^t a_{ij}^f - \sum_{f=1}^t b_{ij}^f, t = 1, \cdots, T-1$. ∎

Corollary 3.6.3 *For the network defined in Proposition 29, Arc (1,2), the initial arc, is a bottleneck arc until demand is exhausted. Furthermore, the remaining arcs arc critical, and by definition, will experience no congestion or holding-back in an optimal solution.* ∎

Figure 3.10 displays an optimal solution and KKT dual variables for a sequential network composed of two identical arcs. Both arcs have $q_{ij}^{max} = 25$. The first arc, in intervals 1 through 4, is a bottleneck. As Figure 3.10 illustrates, the KKT variables in Figure 3.10 conform to Propositions 17-28 proven in this section.

	arc 1										
prd	a	x	b	g'	v1	v2	u1	u2	u4	u5	u6
1	25.00	0.00	0.00	1.00	-6.95	12.49	2.44	6.47	0.00	0.93	4.05
2	25.00	25.00	18.75	0.50	-5.68	8.23	1.17	2.55	0.00	0.00	0.00
3	25.00	31.25	21.48	0.38	-4.93	6.94	0.41	2.01	0.00	0.00	0.00
4	25.00	34.77	22.68	0.30	-4.51	5.87	0.00	1.36	0.00	0.00	0.00
5	0.00	37.09	23.33	0.26	-4.32	5.05	0.00	0.73	0.19	0.00	0.00
6	0.00	13.75	11.86	0.72	-3.83	4.50	0.00	0.67	0.68	0.00	0.00
7	0.00	1.89	1.86	0.96	-3.00	3.87	0.00	0.87	1.51	0.00	0.00
8	0.00	0.04	0.04	1.00	-2.98	3.00	0.00	0.02	1.53	0.00	0.00
9	0.00	0.00	0.00	1.00	-2.97	2.98	0.00	0.01	1.54	0.00	0.00
10	0.00	0.00	0.00	1.00	-2.97	2.85	0.00	0.00	1.54	0.12	0.00
11	0.00	0.00	0.00	1.00	-2.22	2.76	0.00	0.57	2.29	0.03	0.17
12	0.00	0.00	0.00	1.00	1.88	1.42	0.00	3.30	6.39	0.00	0.80
Total	100.00		100.00		v3	-4.51					

	arc 2									
prd	a	x	b	g'	v1	u1	u2	u4	u5	u6
1	0.00	0.00	0.00	1.00	-11.04	0.00	4.92	1.45	4.97	9.27
2	18.75	0.00	0.00	1.00	-8.23	0.00	2.46	0.00	0.68	0.36
3	21.48	18.75	15.23	0.63	-6.94	0.00	2.06	0.00	0.00	0.00
4	22.68	25.00	18.75	0.50	-5.87	0.00	2.13	0.00	0.00	0.00
5	23.33	28.93	20.56	0.42	-5.05	0.00	1.95	0.00	0.00	0.00
6	11.86	31.70	21.65	0.37	-4.50	0.00	1.50	0.00	0.00	0.00
7	1.86	21.91	17.11	0.56	-3.87	0.00	1.13	0.00	0.00	0.00
8	0.04	6.66	6.21	0.87	-3.00	0.00	1.00	0.00	0.00	0.00
9	0.00	0.48	0.48	0.99	-2.96	0.00	0.04	0.01	0.00	0.00
10	0.00	0.00	0.00	1.00	-2.28	0.00	0.28	0.57	0.57	0.40
11	0.00	0.00	0.00	1.00	-1.61	0.00	0.37	1.15	0.98	0.30
12	0.00	0.00	0.00	1.00	3.09	0.00	3.73	4.51	0.64	0.97
Total	100.00		100.00			u3	0			

Figure 3.10: Solution and KKT dual variables for two arc network.

Chapter 4

Aggregated Control Strategies

4.1 Introduction

In this chapter we study the Aggregated Regional Evacuation Model (A-REM), which produces aggregated control strategies, i.e., strategies where staging and routing directions are specified for groups of evacuees. Specifically, the aggregation we consider is based on groups of evacuee types (derived from evacuee shelter requirements or preferences) and origins. We refer to each such aggregated composition as an evacuee group, i.e., every combination of origin $k \in V$ and evacuee type $z \in Z$ constitutes an evacuee group. Thus, evacuee groups represent the level of control, as opposed to individuals as in the D-REM. Each evacuee group is potentially given a unique evacuation start time and route. When a large number of evacuees is given a common evacuation time, it is not expected that they all leave at the same time. Instead a network loading curve is used to describe their aggregate departure behavior. This loading curve represents the statistical effect of many individual evacuees determining their evacuation time after an evacuation order is given (see Section 4.2 for more details).

The rational behind a more aggregate strategy is that it is easier and less expensive to implement and disseminate (a detailed verification of these issues is beyond the scope of this work). In fact, these types of strategies are now being implemented. Consider the state of Virginia. Figure 4.1, from the Virginia Department of Transportation web page (http://www.virginiadot.org/comtravel/hurricane-evac-hro.asp), delineates the evacuation staging zones and strategy. In this case, coastal Virginia is divided into two zones,

termed Phase I (in orange) and Phase II (in yellow). The Phase I evacuation is to occur between 24 to 14 hours prior to the onset of tropical storm force winds, while the Phase II evacuation begins 14 hours prior to the onset of tropical storm force winds. This strategy is based on evacuating those at greater risk first.
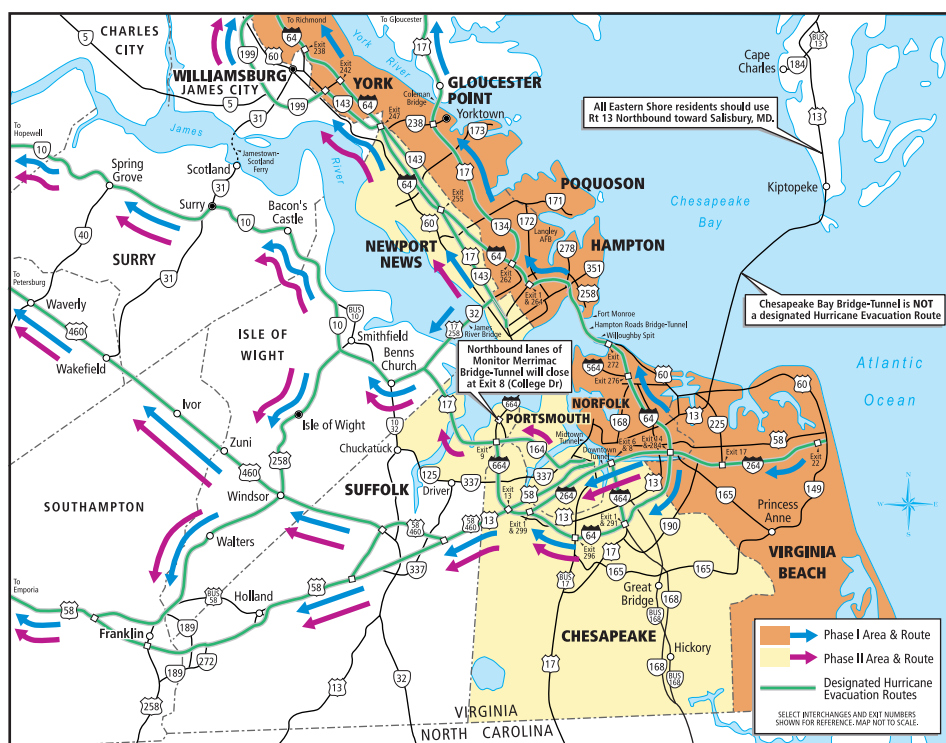
Figure 4.1: Staging zones, Phase I and II, for the evacuation of coastal Virginia, including Virginia Beach.

While not illustrated in Figure 4.1, each phase is further divided into smaller zones (for a total of nine zones, between the two phases), each of which is given a specific evacuation route. In the remainder of this chapter, we discuss a methodology for determining these types of evacuation plans. In the next section, we introduce various formulations to model the A-REM and provide some related results. In Section 4.3 we explore solution methodologies for this problem.

4.2 Model Formulation

Some of the additional assumptions, beyond those specified for the REM (see Section 2.3), inherent in the Aggregated Regional Evacuation Model (A-REM) include:

1. The evacuee loading curve for each evacuee group is known with certainty.

2. If any evacuee group were evacuated in isolation, congestion would not form on any viable path.

3. There is enough specialized capacity for all evacuees of type $z \in Z$.

4. The shape of the loading curve and the total number of evacuees is not a function of evacuation start times for each evacuee group.

The A-REM can be formulated in one of two basic ways: either as an arc-based model, or as a path-based model. We discuss the issues involved with both formulations in later sections. First, we introduce the constraint set for the arc-based formulation, and after discussing this constraint set, we show how it can be modified to produce the path-based formulation. We then introduce a specialized variant of the A-REM, which can also be either arc- or path-based, and discuss some advantages of this formulation. We term this the No-Holding A-REM. All these formulations can be used in conjunction with the various objective functions discussed in Section 2.4. The constraint set of the arc-based A-REM is as follows:

Model 4.1 Constraint Set of the Arc-Based A-REM

$$x_{ij}^{t+1,k,z} - x_{ij}^{t,k,z} - a_{ij}^{t,k,z} + b_{ij}^{t,k,z} = 0, \quad \forall (i,j) \in A, \ k \in V, \ z \in Z, \ t = 1, \cdots, T-1 \tag{4.1}$$

$$\sum_{i:(i,j)\in A} b_{ij}^{t,k,z} - \sum_{i:(j,i)\in A} a_{ji}^{t,k,z} = 0, \quad \forall j \in W, \ k \in V, \ z \in Z, \ t = 1, \cdots, T \tag{4.2}$$

$$\sum_{f=1}^{F} \gamma_f^{k,z} = 1, \quad \forall k \in V, \ z \in Z \tag{4.3}$$

$$\sum_{j:(k,j)\in A} a_{kj}^{t,k,z} = \sum_{f=1}^{F} \gamma_f^{k,z} h_f^{t,k,z}, \quad \forall k \in V, \ z \in Z, \ t = 1, \cdots, T \tag{4.4}$$

$$\sum_{j:(k,j)\in A} \lambda_{kj}^{k,z} = 1, \quad \forall k \in V, \ z \in Z \tag{4.5}$$

$$\sum_{i:(j,i)\in A} \lambda_{ji}^{k,z} = \sum_{i:(i,j)\in A} \lambda_{ij}^{k,z}, \quad \forall j \in W, \ k \in V, \ z \in Z \tag{4.6}$$

$$\sum_{i:(i,j)\in A} \lambda_{ij}^{k,z} \leq 1, \quad \forall j \in W, \ k \in V, \ z \in Z \tag{4.7}$$

$$\sum_{j\in Y} \sum_{i:(i,j)\in A} \lambda_{ij}^{k,z} = 1, \quad \forall \ k \in V, \ z \in Z \tag{4.8}$$

$$a_{ij}^{t,k,z} \leq \lambda_{ij}^{k,z} q_{ij}^{max}, \quad \forall (i,j) \in A, k \in V, \ z \in Z, \ t = 1, \cdots, T \tag{4.9}$$

$$\sum_{k\in V} \sum_{z\in Z} a_{ij}^{t,k,z} \leq q_{ij}^{max}, \quad \forall (i,j) \in A, \ t = 1, \cdots, T \tag{4.10}$$

$$\sum_{k\in V} \sum_{z\in Z} b_{ij}^{t,k,z} \leq x_{ij}^{t}, \quad \forall (i,j) \in A, \ t = 1, \cdots, T \tag{4.11}$$

$$\sum_{k\in V} \sum_{z\in Z} b_{ij}^{t,k,z} \leq (k_{ij}^{max} - \sum_{k\in V} \sum_{z\in Z} x_{ij}^{t,k,z}) q_{ij}^{max}/(k_{ij}^{max} - q_{ij}^{max}),$$
$$\forall (i,j) \in A, \ t = 1, \cdots, T \tag{4.12}$$

$$b_{ij}^{t,k,z} \leq x_{ij}^{t,k,z}, \quad \forall (i,j) \in A, \ k \in V, \ z \in Z, \ t = 1, \cdots, T \tag{4.13}$$

$$\sum_{t=1}^{T} \sum_{i:(i,j)\in A} \sum_{k\in V} b_{ij}^{t,k,z} - C_{j,z} \leq 0, \quad \forall j \in Y, \ z \in Z \tag{4.14}$$

$$a_{ij}^{t,k,z}, b_{ij}^{t,k,z}, x_{ij}^{t,k,z} \geq 0, \quad \forall (i,j) \in A, \ k \in V, \ z \in Z, \ t = 1, \cdots, T \tag{4.15}$$

$$\gamma_f^{k,z} \in \{0,1\}, \quad \forall k \in V, \ z \in Z, \ f = 1, \cdots, F \tag{4.16}$$

$$\lambda_{ij}^{k,z} \in \{0,1\}, \quad \forall (i,j) \in A, \ k \in V, \ z \in Z. \tag{4.17}$$

Equation (4.1) defines the state of the system in time interval $t+1$ based on the state and flows from time interval t. Equation (4.2) represents flow conservation across each junction $j \in W$. Equations (4.3) and (4.4) define the network loading for each origin $k \in V$, i.e.,

the rate of flow of evacuees out of each origin. Equation (4.3) establishes a start interval for the evacuation of each evacuee group, while Equation (4.4) defines the characteristic loading curve for each evacuee group. This curve, which represents the aggregate loading behavior of the evacuee group, is a cumulative curve defining the total number of evacuees that have entered the network, by a given time interval. Research suggests that this curve is usually S-shaped. Given an evacuation start time, f, and an evacuee group defined by indexes k and z, the parameters $h_f^{t,k,z}$ represent a discretized version of the evacuee group's loading curve, that is, they denote how many evacuees of type z leave origin k in interval t given an evacuation start time f. For convenience, we assume that this relationship is described by a parabolic curve, which, when integrated, produces a cumulative loading curve that is S-shaped, as desired. Figure 4.2 displays the S-shaped curve, and the discrete number of evacuees loaded in each interval, for a loading curve (that is used in Scenario 2, see Section 2.6). The parabolic function, $a1t^2 + a2t + a3$, has three parameters: $a1$, $a2$, and $a3$. The first two can be expresses as a function of the loading curve's duration, which we denote as $d^{k,z}$, i.e., $a1 = -6/(d^{k,z})^3$ and $a2 = 6/(d^{k,z})^2$, while the third, $a3$, is always zero. Using this parabolic functional assumption simply provides an easy way to describe the loading curve - we do not make use of any special properties of this curve, beyond that it insures that all evacuees in the evacuee group have left the origin by interval $f + d^{k,z}$. Another functional form is used in Hobeika, Radwan, and Jamei (1985); the loading curve is described using an exponential function, $1/(1 + exp(-\alpha(t - \beta)))$, where α is a parameter that influences the 'steepness' of the curve and β represents the number of time intervals required to load half the evacuees. This exponential function also produces appropriately shaped curves, but the duration of the loading curve is not required to be 2β, as it is influenced by the α parameter. The parameter F, in Equation (4.4), is based on the final interval in which an evacuee group can start evacuating, and potentially finish by interval T, the last interval in the planning horizon.

Equations (4.5)-(4.9) insure that each evacuee group, of type z from origin k, utilizes only one evacuation route to a single shelter. Equations (4.5) and (4.6) insure that the model will choose only one path, and thus only one shelter. Equation (4.7) insures that the evacuation route will contain no loops. The concept of one path and shelter for each evacuee type/origin is to produce a simplified evacuation strategy, which is presumably easier to manage and convey to the populous. This formulation works well if shelters are uncapacitated, but if
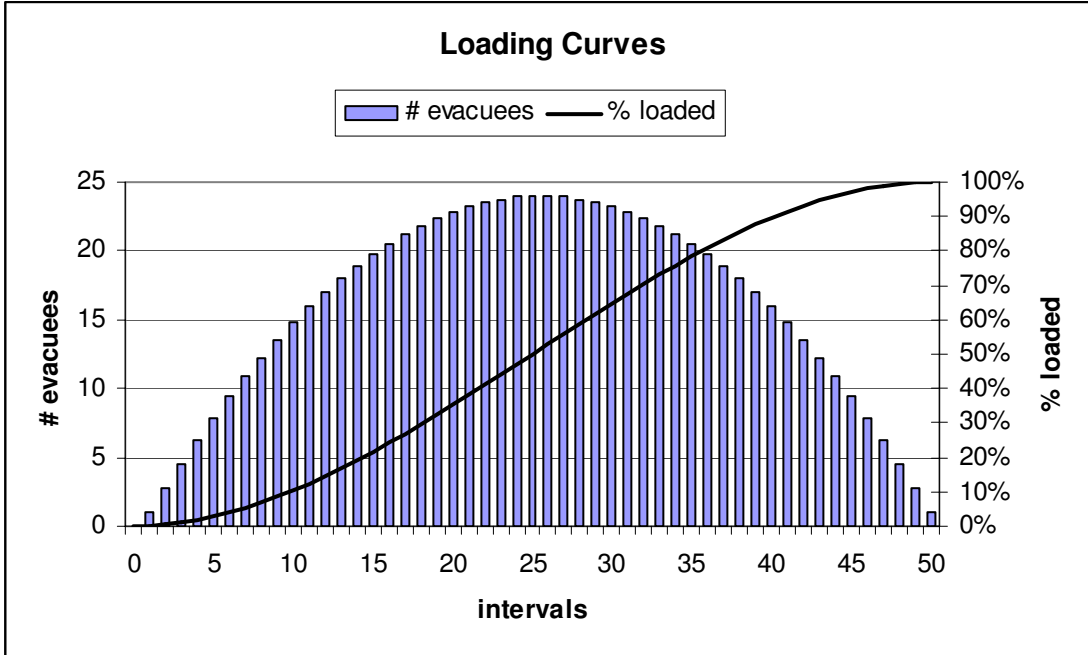
Figure 4.2: A loading curve that requires 50 intervals to completely evacuate an origin with 800 evacuees.

a shelter type is capacitated, a feasible solution might not exist, or the capacity might be poorly utilized with this constraint. Equation (4.8) requires each such path to terminate at exactly one shelter, and Equation (4.9) determines if an arc is used by an evacuee group.

Equation (4.10) limits the total flow entering an arc to the arc's maximum flow, q_{ij}^{max}. This is justified because if a flow rate higher than this maximum flow entered the arc, it would quickly be reduced due to a high local density at the entrance of the arc. Equations (4.11), (4.12), and (4.13) represent the Pipes traffic flow relationship (see Section 2.5) that defines each arc's exit-flow limits. Equation (4.14) sets the capacity of each shelter $j \in Y$, for each evacuee type $z \in Z$, to $C_{j,z}$. Equations (4.15)-(4.17) enforce the logical non-negativity and binary restrictions.

We can re-formulate the arc-based A-REM as a path-based model by replacing Equations (4.5)-(4.9) and (4.17) with the following equations:

$$\sum_{l \in P^{k,z}} \rho^l = 1, \quad \forall k \in V, \; z \in Z \tag{4.18}$$

$$a_{ij}^{t,k,z} \le \sum_{l \in P^{k,z}} \rho^l \delta_{ij}^l q_{ij}^{max}, \quad \forall (i,j) \in A, \; k \in V, z \in Z, \; t = 1, \cdots, T \tag{4.19}$$

$$\rho^l \in \{0,1\}, \quad \forall k \in V, \; z \in Z, \; l \in P^{k,z}. \tag{4.20}$$

There are various ways the path reformulation can be used to determine evacuation strategies, including complete *a priori* enumeration of all possible paths (or possible paths that meet a certain criteria). Of course, this is not always very desirable (or feasible) for a large, complex network. Next, we introduce the No-Holding variation of the A-REM, of which we make extensive use throughout this chapter.

No-Holding Variant of the A-REM

This variant of the A-REM produces strategies where evacuees that enter an arc in interval t always exit the arc in interval $t+1$, i.e., all evacuees must traverse each arc in one time interval, and thus the arcs do not hold evacuees. One advantage of this requirement is that difficulties associated with FIFO and holding-back are non-existent. No-holding strategies of this type are highly dependent on modeling Assumption 2. Thus, the characteristic loading curve should not specify that more evacuees exit an origin than the smallest q_{ij}^{max} on any viable path. This can always be accomplished by sizing the zones (represented by an origin) considering the network structure. In support of this assumption, in most road networks, the capacity tends to increase from local neighborhood streets (most of which are subsumed by the origin nodes) to arterioles, to major arteries, and finally to highways. We also note that based on the Pipes traffic flow model, the No-Holding variant always produces strategies without congestion. The following constraint set, in conjunction with any of the objective functions (see Section 2.4) represents the no-holding strategy of this type.

Model 4.2 Constraint Set of the Arc-Based No-Holding Variant

$$a_{ij}^{t,k,z} - b_{ij}^{t+1,k,z} = 0, \quad \forall (i,j) \in A, \ k \in V, \ z \in Z, \tag{4.21}$$

$$\sum_{i:(i,j)\in A} b_{ij}^{t,k,z} - \sum_{i:(j,i)\in A} a_{ji}^{t,k,z} = 0, \quad \forall j \in W, \ k \in V, \ z \in Z, \ t = 1,\cdots,T \tag{4.22}$$

$$\sum_{f=1}^{F} \gamma_f^{k,z} = 1, \quad \forall k \in V, \ z \in Z \tag{4.23}$$

$$\sum_{j:(k,j)\in A} a_{kj}^{t,k,z} = \sum_{f=1}^{F} \gamma_f^{k,z} h_f^{t,k,z}, \quad \forall k \in V, \ z \in Z, \ t = 1,\cdots,T \tag{4.24}$$

$$\sum_{j:(k,j)\in A} \lambda_{kj}^{k,z} = 1, \quad \forall k \in V, \ z \in Z \tag{4.25}$$

$$\sum_{i:(j,i)\in A} \lambda_{ji}^{k,z} = \sum_{i:(i,j)\in A} \lambda_{ij}^{k,z}, \quad \forall j \in W, \ k \in V, \ z \in Z \tag{4.26}$$

$$\sum_{i:(i,j)\in A} \lambda_{ij}^{k,z} \leq 1, \quad \forall j \in W, \ k \in V, \ z \in Z \tag{4.27}$$

$$\sum_{j\in Y} \sum_{i:(i,j)\in A} \lambda_{ij}^{k,z} = 1, \quad \forall \ k \in V, \ z \in Z \tag{4.28}$$

$$a_{ij}^{t,k,z} \leq \lambda_{ij}^{k,z} q_{ij}^{max}, \quad \forall (i,j) \in A, k \in V, \ z \in Z, \ t = 1,\cdots,T \tag{4.29}$$

$$\sum_{k\in V} \sum_{z\in Z} a_{ij}^{t,k,z} \leq q_{ij}^{max}, \quad \forall (i,j) \in A, \ t = 1,\cdots,T \tag{4.30}$$

$$\sum_{t=1}^{T} \sum_{i:(i,j)\in A} \sum_{k\in V} b_{ij}^{t,k,z} - C_{j,z} \leq 0, \quad \forall j \in Y, \ z \in Z \tag{4.31}$$

$$a_{ij}^{t,k,z}, b_{ij}^{t,k,z} \geq 0, \quad \forall (i,j) \in A, \ k \in V, \ z \in Z, \ t = 1,\cdots,T \tag{4.32}$$

$$\gamma_f^{k,z} \in \{0,1\}, \quad \forall k \in V, \ z \in Z, \ f = 1,\cdots,F \tag{4.33}$$

$$\lambda_{ij}^{k,z} \in \{0,1\}, \quad \forall (i,j) \in A, \ k \in V, \ z \in Z. \tag{4.34}$$

The arc-based A-REM constraint set was modified to produce the No-Holding variant as follows:

1. Replace Equation (4.1) with $a_{ij}^{t,k,z} = b_{ij}^{t+1,k,z}$.

2. Remove Equations (4.11), (4.12), and (4.13).

3. Remove $x_{ij}^{t,k,z}$ from Equation (4.15).

4. Replace the $x_{ij}^{1,k,z} = 0$ assumption with $b_{ij}^{1,k,z} = 0$.

The path-based formulation is as follows:

Model 4.3 Constraint Set of the Path-Based No-Holding Variant

$$a_{ij}^{t,k,z} - b_{ij}^{t+1,k,z} = 0, \quad \forall (i,j) \in A, \ z \in Z, \tag{4.35}$$

$$\sum_{i:(i,j)\in A} b_{ij}^{t,k,z} - \sum_{i:(j,i)\in A} a_{ji}^{t,k,z} = 0, \quad \forall j \in W, \ k \in V, \ z \in Z, \ t = 1, \cdots, T \tag{4.36}$$

$$\sum_{f=1}^{F} \gamma_f^{k,z} = 1, \quad \forall k \in V, \ z \in Z \tag{4.37}$$

$$\sum_{j:(k,j)\in A} a_{kj}^{t,k,z} = \sum_{f=1}^{F} \gamma_f^{k,z} h_f^{t,k,z}, \quad \forall k \in V, \ z \in Z, \ t = 1, \cdots, T \tag{4.38}$$

$$\sum_{l\in P^{k,z}} \rho^l = 1, \quad \forall k \in V, \ z \in Z \tag{4.39}$$

$$a_{ij}^{t,k,z} \le \sum_{l\in P^{k,z}} \rho^l \delta_{ij}^l q_{ij}^{max}, \quad \forall (i,j) \in A, \ k \in V, z \in Z, \ t = 1, \cdots, T \tag{4.40}$$

$$\sum_{k\in V}\sum_{z\in Z} a_{ij}^{t,k,z} \le q_{ij}^{max}, \quad \forall (i,j) \in A, \ t = 1, \cdots, T \tag{4.41}$$

$$\sum_{t=1}^{T}\sum_{i:(i,j)\in A}\sum_{k\in V} b_{ij}^{t,k,z} - C_{j,z} \le 0, \quad \forall j \in Y, \ z \in Z \tag{4.42}$$

$$a_{ij}^{t,k,z}, b_{ij}^{t,k,z} \ge 0, \quad \forall (i,j) \in A, \ k \in V, \ z \in Z, \ t = 1, \cdots, T \tag{4.43}$$

$$\gamma_f^{k,z} \in \{0,1\}, \quad \forall k \in V, \ z \in Z, \ f = 1, \cdots, F \tag{4.44}$$

$$\rho^l \in \{0,1\}, \quad \forall k \in V, \ z \in Z, \ l \in P^{k,z}. \tag{4.45}$$

In the next section, we present some preliminary results.

4.2.1 Results

In this section, we provide some preliminary numerical results comparing the arc-based and path-based formulations of the A-REM and the No-Holding variant. Next, we discuss a problematic issue related with the A-REM formulation, and then we present some results concerning the objective functions (see Section 2.4) and the No-Holding A-REM.

Table 4.1 displays the results for Model 4.1 (the arc-based A-REM) with Objective (II) in conjunction with Scenario 1 presented in Section 2.6. Objective (II) is used to avoid the complications inherent in the formulation using Objective (I), as presented in Section 2.4.

Here, B/B denotes the number of branch-and-bound nodes enumerated. Table 4.1 illustrates that, even for relatively small networks, the run time to optimize the A-REM can be quite large. In fact, for Scenario 2, we had to abort the run due to time considerations.

Table 4.1: Results for Model 4.1: the arc-based A-REM.

Scenario	Obj. Value	Evac. Duration	Run Time (sec.)	Cont. Vars.	Binary Vars.	Constraints	MIP Simplex Iters	B/B
1	10.781	20	2,252.1	15,949	494	20,278	98,918	2

As the network underlying the scenarios described in Section 2.6 is fairly simple, we can easily enumerate all possible paths. The network has 25 possible evacuation routes, excluding paths with loops, and origins V3-V7 each have only one viable route for Type 1 evacuees, i.e., those evacuees going to shelter Y3). Table 4.2 displays the results for the path-based A-REM. We note that the solution times are much lower for the path-based formulation and that the AMPL pre-solver eliminates many more variables and constraints for this formulation.

Table 4.2: Results for the path-based A-REM.

Scenario	Obj. Value	Evac. Duration	Run Time (sec.)	Cont. Vars.	Binary Vars.	Constraints	MIP Simplex Iters.	B/B
1	10.781	20	106.9	8,036	300	11,098	19,476	6
2	39.767	73	28,406.2	36,169	720	49,496	1,169,994	12

As a comparison, we refer to Table 3.1 (see Section 3.2), which displays the results from the D-REM for these scenarios, when flow-tracking and shelter preferences are included. We note that the A-REM requires flow-tracking. The simplification of eliminating flow-tracking (i.e., not tracking evacuees by origin) used in the D-REM to produce more aggregate flows that are still optimal, does not work for the A-REM because of the more restrictive

routing assumption. The more restrictive staging and routing assumption inherent in the A-REM also make it more difficult to optimize because they require binary variables in the formulation of the constraint set. The longer times required to solve the A-REM illustrate the increased difficulty of this model.

Table 4.3 presents the results for Model 4.2 with Objective (II). Once again, we aborted the Scenario 2 run due to time considerations. Table 4.4 shows the results from Model 4.3, the path-based No-Holding variant, with Objective (II).

Table 4.3: Results for Model 4.2: the arc-based No-Holding variant.

Scenario	Obj. Value	Evac. Duration	Run Time (sec.)	Cont. Vars.	Binary Vars.	Constraints	MIP Simplex Iters.	B/B
1	11.543	23	1,014.2	9,423	494	12,562	908,015	39,712

Table 4.4: Results for Model 4.3: the path-based No-Holding variant.

Scenario	Obj. Value	Evac. Duration	Run Time (sec.)	Cont. Vars.	Binary Vars.	Constraints	MIP Simplex Iters.	B/B
1	11.543	23	120.5	5,403	300	7,527	646,901	27,742
2	44.471	95	2,141.9	24,378	720	33,619	1,441,210	53,545

It is interesting to compare the results of the arc-based A-REM (see Table 4.1) and the No-Holding variant of this model (see Table 4.3). We focus on Scenario 1, because a solution was found for all formulations for this scenario. The solver required 2,252 seconds and two branch-and-bound nodes to optimize the arc-based A-REM; in contrast, it required 1,014 seconds and 39,712 branch-and-bound nodes to optimize the No-Holding variant. The A-REM produces a lower objective value than the No-Holding variant. This is to be expected, as the A-REM is less constrained. The interesting statistic we want to focus on is the difference in the number of a branch-and-bound nodes required to find an optimal solution

for each model. The number of branch-and-bound nodes required by the No-Holding variant is considerably greater than that required by the A-REM. This difference is symptomatic of a problem inherent in the A-REM.

The constraint set of the A-REM allows the solver too much latitude in finding an optimal solution; the constraint set can govern the traffic flow in an unrealistic manner, with holding-back and FIFO violations. This is, in fact, a common problem in most analytical DTA models, which we were able to surmount in the D-REM, but not in the A-REM, due to its combinatorial nature. Often, the solver is able to find an optimal solution to the A-REM with very few branch-and-bound nodes by utilizing the flexibility in the constraint set (usually by specifying large amounts of holding-back). Thus, it is often possible for a near-simultaneous evacuation to behave like a more complex staging strategy by forcing unrealistic traffic behavior. The solver only branches once it determines that it cannot optimize the problem by exploiting this flexibility in the constraints. In contrast, the No-Holding variant does not allow this flexibility, thus requiring more branch-and-bound nodes, but yielding traffic flows that are more realistic. To illustrate, we again consider Scenario 1. In the strategy from the A-REM, all evacuee groups, save one, begin their evacuation in the first interval, but many evacuees are held on the initial arc of their evacuation route for multiple intervals - essentially, parked. In contrast, only six evacuee groups begin their evacuation in the first interval with the No-Holding variant, and eight evacuee groups start evacuating in later intervals.

In fact, the traffic flows should be completely specified for a deterministic model of this type, where we assume no special optimization of the supply, once the staging and routing strategies are determined. This is the case for the No-Holding variant, but at the cost of producing a sub-optimal solution considering all possible realistic traffic flows. Unfortunately, an analytical model that completely specifies the traffic flows in a realistic manner, without the extra constraints inherent in the No-Holding variant, which is yet tractable, has not been developed in the vast DTA literature. A low fidelity simulation could be used to quickly describe the traffic flow, and in fact, this is one strategy currently being studied in the DTA literature and has been implemented in practice, see Peeta and Ziliaskopoulos (2001). This simulation methodology works fairly well for developing system optimal or user equilibrium type flows, which are amendable to various convergent algorithms. Unfortunately, this simulation methodology does not work well for the present evacuation problem because of the

problem's combinatorial nature that limits the evacuee groups to a single path and start-time. We also note that a model such as the No-Holding variant is not useful for DTA type problems because it requires staging, which is not a normal feature in DTA models.

While the No-Holding variant produces sub-optimal strategies, when compared to a hypothetical model that is less constrained (i.e., one that does not have a no-holding requirement), it does insure realistic traffic flows. We believe this sub-optimality is not an important issue in the context of the hurricane evacuation problem. We can consider strategies from the No-Holding variant as being conservative, which might be advisable, given the stochastic nature of this problem.

Unlike the disaggregate case, there exists another model that combines elements of the A-REM and the No-Holding variant; this is a model where holding is allowed, but congestion is not. A model of this type produces strategies where the number of evacuees on an arc never enter the congested regime. This model is produced by modifying the A-REM by replacing Equation (4.12) with $x_{ij}^{t,k,z} \leq q_{ij}^{max}, \quad \forall (i,j) \in A, \ k \in V, \ z \in Z, \ t \in 1, \cdots, T$. Table 4.5 presents the results for this model using Scenario 1. However, the structure of this model still shares the problems inherent in the A-REM.

Table 4.5: Results for the No-Congestion variant of the A-REM.

Scenario	Obj. Value	Evac. Duration	Run Time (sec.)	Cont. Vars.	Binary Vars.	Constraints	MIP Simplex Iters.	B/B
1	10.790	20	571.3	19,863	574	24,560	61,241	11

Due to these issues discussed above, in the remainder of this chapter we study the No-Holding variants of the A-REM. Also, in the remainder of this chapter we study these models in conjunction with Objective (II), to allow continuity with the above results. Nevertheless, we first discuss results pertaining to some of the other objectives. While Objective (I), which is to minimize the evacuation's duration, adds to the complexity of the solution for the A-REM (and the D-REM, see Section 3.5), it is much simplified for the No-Holding A-REM. Consider the following proposition.

Proposition 31 *The number of intervals until an evacuee group is completely evacuated is the number of intervals before the evacuation starts plus the number of intervals in the duration of the loading curve plus the number of arcs in the evacuation path.*

Proof: The number of intervals before the evacuation starts plus the number of intervals in the duration of the loading curve represents the interval in which the last evacuee from the evacuee group leaves the origin, and thus is on the first arc in the evacuation path. As each arc requires one interval to traverse, this last evacuee reaches shelter in as many intervals as there are arcs in the evacuation path. ∎

Corollary 4.2.1 *Given a route, the travel time for each evacuee group, that is the time (number of intervals) from when the first evacuees (from the evacuee group) enter the network until the last evacuee enters shelter, is fixed, and equal to the duration of the loading curve plus the number of arcs in the evacuation path.* ∎

Objective (I), as originally formulated in Section 2.4, utilizes a set of binary variables. We repeat the original formulation and extra constraints, as follows, for reference:

$$Minimize \ \sum_{t=1}^{T} E^t,$$

where the following additional constraints are required:

$$E^t \geq (D - \sum_{f=1}^{t-1} \sum_{j \in Y} \sum_{i:(i,j) \in A} b_{ij}^f)/D, \quad t = 1, \cdots, T,$$

$$E^t \in \{0,1\}, \quad t = 1, \cdots, T.$$

For Model 4.2, the arc-based No-Holding variant, we can greatly simplify Objective (I) as follows:

$$Minimize \ \ L$$

where the following additional constraint is required:

$$L \geq \sum_{f=1}^{F} f \gamma_f^{k,z} + d^{k,z} + \sum_{(i,j) \in A} \lambda_{ij}^{k,z}, \quad \forall k \in V, z \in Z,$$

where $d^{k,z}$ is the duration of the loading curve for evacuee group (k, z). Similarly, for Model 4.3, the path-based No-Holding variant, we can express Objective (I) as follows:

$$Minimize \ \ L$$

where the following additional constraint is required:

$$L \geq \sum_{f=1}^{F} f\gamma_f^{k,z} + d^{k,z} + \sum_{l \in P^{k,z}} \rho^l k^l, \quad \forall k \in V, z \in Z,$$

where k^l is the length of path l, which is a known quantity, and thus a parameter. Both formulations of Objective (I) do not require an extra set of binary variables; thus, this objective is not any more difficult to handle than the other objectives, unlike in the case of the D-REM.

In Section 3.3 we showed that for the D-REM, the Objectives (II) and (III) have identical solutions when shelters do not have capacity limitations, and furthermore, that optimal solutions to these objectives are also optimal with respect to Objective (I) (see Proposition 10). This is not the case with the No-Holding A-REM, as we illustrate with the following example using the network in Figure 4.3.
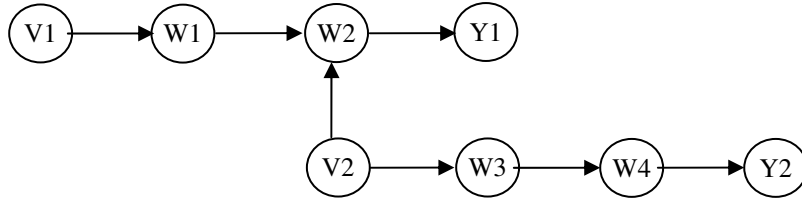
Figure 4.3: An illustrative network where shelters have unlimited capacity.

Example 7 Consider the network depicted in Figure 4.3, where nodes V1 and V2 are the origins, each with one evacuee group having a loading curve duration of three intervals; nodes W1, W2, W3 and W4 are junctions, and nodes Y1 and Y2 are shelters having unlimited capacities. The capacity of each arc and the loading curves are such that only one evacuee group can be on an arc in each time interval.

77

In this example there are three possible strategies. In the first strategy, the evacuees from V1 use shelter Y1, while the evacuees from V2 use Y2. In the second strategy, the evacuees from V1 use shelter Y1, followed by the evacuees from V2, who also use shelter Y1. Finally, in the third strategy, the evacuees from V2 use shelter Y1, followed by the evacuees from V1, who also use shelter Y1. The first strategy completes the evacuation in six time intervals, minimizing Objective (I), but does not maximize the number of evacuees that find shelter in the third interval, a necessary condition of Objective (III)[1]. The first strategy also minimizes the average evacuation time. The third strategy completes the evacuation in eight time intervals; thus it does not minimize Objective (I), but it is optimal for Objective (III), as evacuees reach shelter by interval three. The third strategy is also non-optimal for Objective (II). The second strategy is sub-optimal for all three objectives.

4.3 Solution Methodologies

In Section 4.2.1 we solved, or attempted to solve, the arc-based and path-based No-Holding A-REM for the two test scenarios discussed in Section 2.6. These are all mixed-integer programs of a highly combinatorial nature, and as such, are very difficult to solve. To solve these problems, the solver, CPLEX, uses a sophisticated branch-and-bound cut technique. In this section we discuss various techniques to improve the search for an optimal solution.

Two simple strategic simplifications, that improve solution times by reducing the size of the combinatorial problem, are as follows:

1. *Determine Evacuation Start Times Based on a Larger Time Unit:* In this dissertation a time interval of one minute is commonly used. This is done mainly to model the network and traffic flow in an appropriate manner. We could examine strategies where the evacuation start times are based on some larger unit, e.g., evacuation start times are considered at every 10-interval durations . This will reduce the number of binary variables needed to determine evacuation start times.

2. *Single Origin-Level Evacuation Start-Time:* The A-REM allows each evacuee type $z \in Z$ from each origin $k \in V$ to have an individual evacuation start-time. In the present strategy, the model is reformulated to only allow a single evacuation start-time

---

[1]Note that we start at interval $t = 1$, not $t = 0$.

for each origin $k \in V$, that is, all evacuee types start evacuating at the same time if they share a common origin. In our test case, with two evacuee types, this should reduce the binary variables attributed to start-times by half.

The first strategy certainly seems realistic; making these decisions at minute-intervals is unrealistic and overestimates the accuracy of the model. The second strategy might be appropriate, depending on circumstances.

Next, we examine another method to further reduce the size of the combinatorial problem.

4.3.1 Elimination of Sub-Optimal Staging Strategies

Consider Scenario 1 (see Section 2.6), which has seven origins, two evacuee types (and thus $7(2) = 14$ evacuee groups), and 20 possible starting intervals for each of the 14 evacuee groups. This leads to $20^{14} = 1.6384 \times 10^{18}$ possible staging strategies. In this section we discuss how to reduce the number of possible staging strategies by eliminating cases that we can prove are sub-optimal, thus improving the branch-and-bound search. First, we provide the following proposition.

Proposition 32 *Consider a staging strategy S, represented by the vector $\vec{S}$, which consists of starting times for each evacuee group. Assume that z^* is an optimal solution value for strategy S for Objective (II). Now consider an alternative strategy, represented by the vector $\vec{S'}$ (i.e., strategy S'), where $\vec{S'}$ is obtained by adding a vector of ones, $\vec{1}$, multiplied by positive integer scalar μ to vector $\vec{S}$, that is, $\vec{S'} = \vec{S} + \mu\vec{1}$. Assume that a feasible solution exists for this strategy, let z' be the optimal solution value for strategy S'. For Objectives (II) we always have $z^* < z'$, and thus strategy S' is always sub-optimal. If there is no feasible solution for strategy S, then there is no feasible solution for strategy S'. This also holds for Objective (I).*

Proof: An optimal solution for strategy S' is identical to that for strategy S, except that all evacuation start-times are increased by a constant μ intervals. This is true, because increasing all evacuation start-times by a constant does not change any of the temporal or spatial interactions between evacuee groups, i.e., translating strategy S forward in time, thus forming strategy S', does not allow for any modification of flows to improve strategy S' that could not also be applied to strategy S. Thus, for Objectives (I) or (II), $z^* < z'$, as

79

strategy S' carries the added burden to both objectives of starting later, and thus strategy S' is sub-optimal. Likewise, if S is infeasible, then so is S'. ∎

From Proposition 32, any staging strategy, represented by vector $\vec{S'}$, is a sub-optimal strategy if we can subtract a vector of ones, $\vec{1}$, multiplied by positive integer scalar μ, i.e., $\vec{S'} - \mu\vec{1}$, and obtain a valid strategy.

Corollary 4.3.1 *Any strategy that does not have at least one evacuee group starting to evacuate in interval 1 (i.e., the first interval) is sub-optimal.* ∎

Considering again the Scenario 1 example, of the $20^{14} = 1.6384 \times 10^{18}$ possible strategies, $19^{14} = 7.990 \times 10^{17}$ of these strategies do not have an evacuee group starting their evacuation in the first interval, and are thus sub-optimal. This leaves 8.3939×10^{17} strategies, which represents a 48.8% reduction in the number of strategies to be considered. In general, the number of sub-optimal strategies we can eliminate is $(F - 1)^n$, where F is the number of possible start intervals and n is the number of evacuee groups.

In order to eliminate the sub-optimal strategies, the following constraint, which insures that at least one evacuee group starts evacuating in interval 1, can be added to any of the models:

$$\sum_{k \in V} \sum_{z \in Z} \gamma_1^{k,z} \geq 1. \tag{4.46}$$

We test this constraint using Model 4.2, the arc-based No-Holding variant.

Table 4.6: Results for Model 4.2 (the arc-based No-Holding A-REM) with Constraint (4.46).

Scenario	Obj. Value	Evac. Duration	Run Time (sec.)	Cont. Vars.	Binary Vars.	Constraints	MIP Simplex Iters.	B/B
1	11.543	23	630.7	13,849	546	17,877	556,823	25,517

Comparing Tables 4.6 and 4.3, we see that adding (4.46) to the No-Holding variant of the arc-based A-REM yields a 35.7% reduction in the number of branch-and-bound nodes (B/B) and a 37.9% reduction in the required solution time.

Table 4.7: Results for Model 4.3 (the the path-based No-Holding variant) with Constraint (4.46).

Scenario	Obj. Value	Evac. Duration	Run Time (sec.)	Cont. Vars.	Binary Vars.	Constraints	MIP Simplex Iters.	B/B
1	11.543	23	86.0	5,403	300	7,528	453,580	20,467

Comparing Tables 4.7 and 4.4, we see that adding (4.46) to the path-based No-Holding variant yields a 26.2% reduction in the number of branch-and-bound nodes (B/B) and a 28.6% reduction in the solution time. The reductions in branch-and-bound nodes and solution times for both the arc- and path-based formulations are less than the 48.8% reduction in the number of strategies to be considered because the branch-and-bound algorithm is designed to eliminate many solutions without having to exhaustively enumerate all solutions. Undoubtedly some of the sub-optimal strategies we eliminate using Constraint (4.46) are independently eliminated when a particular branch is eliminated from consideration.

The number of possible strategies eliminated is a function of the number of evacuee groups and start-intervals. The surface in Figure 4.4 shows how the dominant strategies, as a percentage of possible strategies, varies with these parameters. Next, we present another proposition relating staging strategies for the No-Holding A-REM.

Proposition 33 *Consider Objective (I) or (II). Given a fixed routing strategy P that specifies a route for each evacuee group, consider two staging strategies S and S', where strategy S' specifies an evacuation start-time for each evacuee group that is greater than or equal to the corresponding start-times in strategy S. Assume the following: (1) $S \neq S'$; (2) a feasible solution exists for the combined S and P strategy having an objective value of z; and, (3) a feasible solution exists for the combined S' and P strategy having an objective value of z'. Then, we have $z < z'$, and thus strategy S' is sub-optimal, when both strategies are used in conjunction with routing strategy P.*

Proof: The following logic holds for Objectives (I) and (II). Given a fixed route for each evacuee group (specified by a routing strategy P), consider the solution for strategy S. If
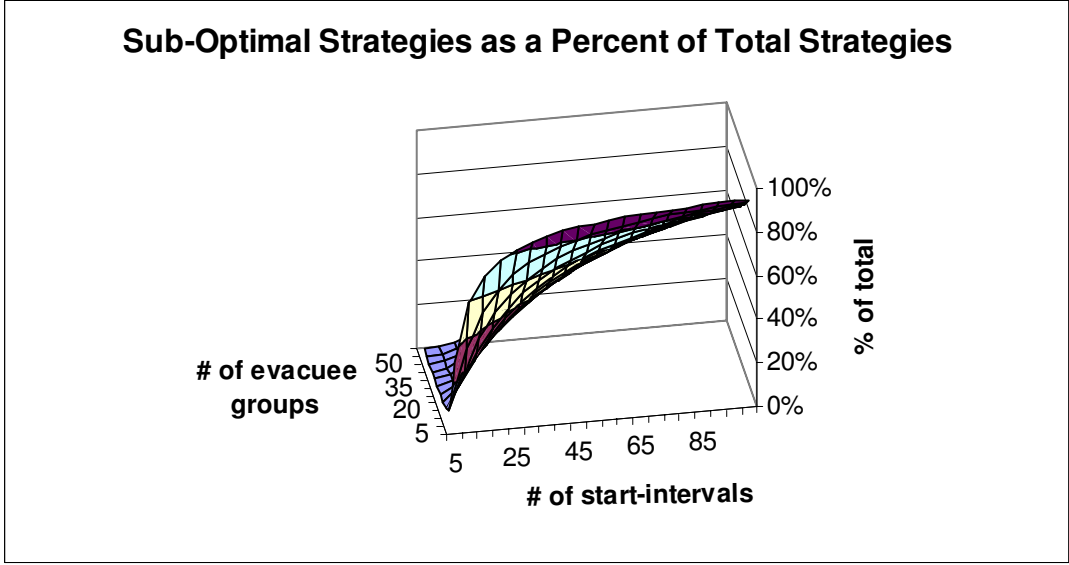
Figure 4.4: Sub-optimal strategies, as a percentage of possible strategies, as a function of the number of evacuee groups and start-intervals.

this solution is feasible, then there are only two possible outcomes of delaying any number of evacuation start-times (thus producing strategy S'): (1) Strategy S' with P is feasible, whereas $z < z'$; or, (2) Strategy S' with P is infeasible. Thus strategy S' is sub-optimal. ∎

Next, we propose a lower bound on the number of evacuee groups that start evacuating in the first interval in an optimal solution. This lower bound is dependent on the characteristics of the network (e.g., bottleneck capacities) and the characteristics of the evacuee groups (e.g., the maximum number of evacuees entering the network in any interval, based on the loading curves).

Proposition 34 *Consider the following process:*

1. *Order the evacuee groups, $k \in V$, $z \in Z$, into a descending list, based on the maximum number of evacuees that can enter the network in any one interval, denoted as $h^{k,z,max}$, where $h^{k,z,max} = max\{h_1^{t,k,z}, \ t = 1, \cdots, T\}$.*

2. *Determine the capacity of the network's static bottlenecks, i.e., those arcs that form a min-cut on the static (not time-expanded) network, denoted as B^{max}.*

82

3. *From the top of the list, sum the $h^{k,z,max}$-values until this summation just exceeds B^{max}. The number of terms in the summation minus one yields a lower bound on the number of evacuee groups starting their evacuation in the first interval.*

Proof: By Proposition 33, any solution can be improved if we can feasibly start the evacuation of a particular evacuee group earlier, using the same path (or a shorter path). Those evacuee groups first using the bottleneck arcs should therefore start evacuation in the first interval. A lower bound on the number of such evacuee groups can be computed by assuming that the groups having the largest single-interval flows are the stated potential groups, and the largest single-interval flows all arrive at the bottleneck at the same interval. ∎

For example, in Scenario 1, the maximum number of evacuees from a single evacuee group that can exit an origin in any interval is 22.1 (based on the network loading curve). With a $q_{ij}^{max} = 50$ vehicles per interval for every arc (i, j) in the network, and the location of the shelters, we can see that at least four evacuee groups should start their evacuation in the first interval in an optimal solution. Modifying Equation (4.46) so that at least four evacuee groups must start in the first interval reduces the number of strategies to be considered by 99.6% (for a total of 6.8374×10^{15} strategies to be considered) of the 1.6384×10^{18} possible strategies.

To determine the number of strategies that we can disregard when we know at least s evacuee groups must start in the first interval, we subtract $\sum_{k=1}^{s-1} \binom{n}{k}(F-1)^{(n-k)}$ from the number of possible strategies. Each term in the summation represents the number of strategies with exactly k evacuee groups starting in the first interval, where $\binom{n}{k}$ is the number of possible combinations of k evacuee groups out of the total n evacuating in the first interval, and $(F-1)^{(n-k)}$ represents the number of possible strategies given each combination.

In the two test scenarios, the duration of the loading curve (how many intervals between the start of the evacuation and the last evacuee leaving the origin) for each evacuee group was identical within the scenario. In this case, we determined F by subtracting the duration from T. In cases where the durations differ, we will need a specific F-value for each evacuee group, based on the duration of each group's loading curve. We can also reduce the F-parameters for each origin group, i.e., define $F^{k,z}$, by determining the shortest path from each group's origin to the closest shelter of the appropriate type. For example, considering Scenario 1 again, Table 4.8 shows the lengths of the shortest paths (in fact for both scenarios, as the

83

underlying network structure is identical). Using these shortest paths, we can produce a set of individual F-parameters, which yields 1.4565×10^{16} possible strategies, or just 0.889% of the original 1.6384×10^{18} possible strategies. This strategy can be more refined in the path-based formulation; in this case, we can determine the appropriate F-parameter for each path based on its actual length.

Table 4.8: Shortest paths to the nearest shelters of the appropriate type.

type/origin	V1	V2	V3	V4	V5	V6	V7
Z1	7	6	9	8	8	2	6
Z2	6	5	4	3	2	2	5

4.3.2 Reformulation Techniques

In this section, various reformulations for the arc-based No-Holding variant are tested to gauge their impact on solution times.

Add Total Flow Constraint: To tighten the formulation, the following extra constraint was added:

$$\sum_{i \in V} \sum_{j:(i,j) \in A} \sum_{t=1}^{T} \sum_{k \in V} \sum_{z \in Z} a_{ij}^{t,k,z} \leq D.$$

Table 4.9: Solution results for case of adding a total flow constraint.

Scenario	Obj. Value	Evac. Duration	Run Time (sec.)	Cont. Vars.	Binary Vars.	Constraints	MIP Simplex Iters.	B/B
1	11.543	23	557.3	9,423	494	12,563	694,549	34,889

This constraint limits the total flow from all origins to the population to be evacuated. Incorporating this constraint reduces the required solution time from 1,014 seconds to 557 seconds; a 45% reduction in solution time. The No-Holding A-REM prohibits evacuees from starting their evacuation from the wrong origin through an interplay of various constraints.

Adding this constraint simplifies this interplay, thus focusing the solver on a smaller solution space. With this in mind, we next study a constraint that does this more explicitly.

Explicitly Forbid Flow from Wrong Origin: To tighten the formulation, we explicitly constrain the system so that only evacuees from origin k can start their evacuation from origin k by adding the following constraint:

$$\lambda_{ij}^{k,z} = 0, \forall i \in V, j : (i,j) \in A, \ k \in V \ , k \neq i, \ z \in Z.$$

Table 4.10: Solution results for case where flow from the wrong origin is explicitly forbidden.

Scenario	Obj. Value	Evac. Duration	Run Time (sec.)	Cont. Vars.	Binary Vars.	Constraints	MIP Simplex Iters.	B/B
1	11.543	23	236.4	6,109	428	8,751	762,473	31,715

Adding this extra constraint reduces the solution time by 77% in Scenario 1, from 1,014 seconds without this constraint to 236 seconds with this constraint. This improved performance is based on a reduction in the number of branch-and-bound nodes required by the solver to obtain a solution, in the original formulation 39,712 branch-and-bound nodes were required, with this additional constraint, only 31,715 were needed.

Use a Single Flow Variable per Arc-Interval: As mentioned when the No-Holding A-REM was introduced, the use of both flow variables $a_{ij}^{t,k,z}$ and $b_{ij}^{t,k,z}$ is just a convention so that the formulation of the No-Holding variant does not differ too much from the A-REM. Here, we eliminate the $b_{ij}^{t,k,z}$ variables, under the premise that fewer variables will improve solution efficiency.

We gain a modest decrease in solution time, from 1,014 seconds to 863 seconds, but inexplicably, an increase in branch-and-bound nodes and simplex iterations.

In the next section, we provide some major re-formulations of this model.

4.3.3 Expedients for the Branch-and-Bound Algorithm

The branch-and-bound algorithm works by solving linear relaxations of the integer program with the integer variables either relaxed, or further constrained (which is much simplified

Table 4.11: Solution results with a single flow variable.

Scenario	Obj. Value	Evac. Duration	Run Time (sec.)	Cont. Vars.	Binary Vars.	Constraints	MIP Simplex Iters.	B/B
1	11.543	22	863.8	4,798	494	7,927	2,049,368	64,659

in our case, as all integer variables are binary, so when branching on a particular integer variable, one branch is fixed at 1 and the other to 0). As discussed in Section 4.3.1, we can simplify this process, and thus improve solution times, if we can eliminate branches *a priori*, which, in effect, potentially reduces the number of linear relaxations that must be solved. Another technique we can use to improve solution times is to reduce the solution time required for each LP relaxation. In the following we discuss how this can be done.

Consider the first linear relaxation that the branch-and-bound algorithm will perform, where no binary variables are fixed. Instead of solving the linear relaxation of the No-Holding A-REM (Model 4.2), we solve the following specialized relaxed formulation:

Model 4.4 Constraint Set of the λ Relaxed No-Holding A-REM

$$a_{ij}^{t,z} - b_{ij}^{t+1,z} = 0, \quad \forall (i,j) \in A, \ z \in Z, \tag{4.47}$$

$$\sum_{i:(i,j)\in A} b_{ij}^{t,z} - \sum_{i:(j,i)\in A} a_{ji}^{t,z} = 0, \quad \forall j \in W, \ z \in Z, \ t = 1,\cdots,T \tag{4.48}$$

$$\sum_{f=1}^{F} \gamma_f^{k,z} = 1, \quad \forall k \in V, \ z \in Z \tag{4.49}$$

$$\sum_{j:(k,j)\in A} a_{kj}^{t,z} = \sum_{f=1}^{F} \gamma_f^{k,z} h_f^{t,k,z}, \quad \forall k \in V, \ z \in Z, \ t = 1,\cdots,T \tag{4.50}$$

$$\sum_{z\in Z} a_{ij}^{t,z} \le q_{ij}^{max}, \quad \forall (i,j) \in A, \ t = 1,\cdots,T \tag{4.51}$$

$$\sum_{t=1}^{T} \sum_{i:(i,j)\in A} b_{ij}^{t,z} - C_{j,z} \le 0, \quad \forall j \in Y, \ z \in Z \tag{4.52}$$

$$a_{ij}^{t,z}, b_{ij}^{t,z} \ge 0, \quad \forall (i,j) \in A, \ z \in Z, \ t = 1,\cdots,T \tag{4.53}$$

$$1 \ge \gamma_f^{k,z} \ge 0, \quad \forall k \in V, \ z \in Z, \ f = 1,\cdots,F. \tag{4.54}$$

In Model 4.4, we have eliminated the path variables, λ, along with all constraints containing λ, as well as the origin superscript, i.e., $k \in V$, from the flow variables, a and b. Note that both the linear relaxations of Models 4.2 and 4.4 differ from the D-REM in that the number of evacuees that leave an origin in each interval is still constrained by the loading curve; evacuees can only evacuate based on convex combinations of the loading curve through time, due to Equations (4.49) and (4.50) and the corresponding equations of Model 4.2. As an example of this difference, if we specify that a certain number of evacuees leave in interval t, we have also made at least a partial decision on the number of evacuees that should leave in interval $t + 1$, which is not the case for the D-REM. Table 4.12 compares the results of Model 4.4 with the linear relaxation of Model 4.2 for the first linear relaxation performed in the branch-and-bound algorithm.

Table 4.12: Results for the initial linear relaxation of Models 4.2 and 4.4.

Scenario	Obj. Value	Evac. Duration	Run Time (sec.)	Cont. Vars.	Binary Vars.	Constraints	Dual Simplex Iters.
1 (Model 4.2)	10.863	21	15.8	9,917	0	12,562	13,813
1 (Model 4.4)	10.857	20	0.141	2,075	0	2,054	439
2 (Model 4.2)	40.931	89	69.0	70,291	0	70,291	26,526
2 (Model 4.4)	40.931	89	4.297	9,120	0	9,725	4,197

As Table 4.12 illustrates, Model 4.4 can be solved much quicker. In Model 4.2 only Equations (4.25)-(4.29) and (4.34) contain the λ-variables, and of these, only Equation (4.29), which we repeat below, directly limits flow.

$$a_{ij}^{t,k,z} \le \lambda_{ij}^{k,z} q_{ij}^{max}, \quad \forall (i,j) \in A, k \in V, \; z \in Z, \; t = 1, \cdots, T.$$

In Model 4.2 this represents a use/no use decision. The limit on flow, represented by q_{ij}^{max} can actually be modified, replacing q_{ij}^{max} with a very large number, M. In this way, we can see that this constraint does not limit flow in the relaxed version; even a very small value of $\lambda_{ij}^{k,z}$ when multiplied by M, will not impede the flow. In Table 4.12, note that the optimal objective values for Scenario 1 differ for the two models. Here we explain why this occurs

(sometimes), and then discuss the ramifications. This difference is able to occur due to the simplicity of the network, and is caused by an interaction between the network loading and the path variables in the formulation of Model 4.2. If an origin and a shelter are connected by two paths, where the second path is simply the first with a loop added, then Model 4.2 is not able to utilize the loop to improve the objective function, while Model 4.4 can. To illustrate this, consider the simple network in Figure 4.5. The linear relaxation of Model 4.2 (as well as the discrete version) only produces strategies that send evacuees from origin V1 on path V1-W1-Y1. Model 4.4 can produce a strategy that sends some evacuees on the following path V1-W1-W2-W1-Y1, which can allow a better utilization of arc (W1, Y1), despite this being an unrealistic strategy from the evacuee point of view. This is because Equation (4.25) insures that $\lambda_{V1,W1}^{V1,z} = 1$, while Equation (4.26) insures that any looping violates Equation (4.27). In a more complicated network, where multiple, independent, paths occur, each path can have a $\lambda < 1$, which will allow the looping to occur. So, in most realistic networks, this difference does not occur.
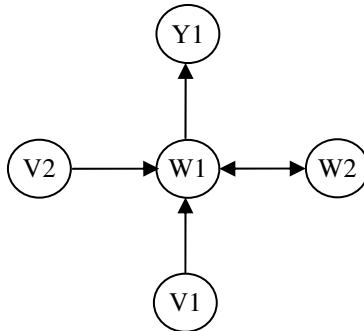

Figure 4.5: A simple network to illustrate potential looping issues.

Whenever we branch on a start-time variable, γ, we can use this specialized *lambda*-relaxation. If we branch on a path variable, λ, we only need the origin superscript for the flow variables for the origin corresponding to the λ-variable; the rest can be untracked (i.e., k-superscripts removed). If we have already branched on the start-time variable for an origin (for all types), then the flow variables become parameters for the branched arcs.

For Model 4.4, as with all the different variants of the No-Holding A-REM, we can eliminate the b-variables; Equation (4.47) indicates that once we determine the a-variables, the b-variables are also known. Instead of examining this minor change in Model 4.4 (we

explored this for the No-Holding A-REM in Section 4.3.2), we now examine an arc-based model having only binary variables (except for those variables associated with a particular objective function).

Model 4.5 Constraint Set of Arc-Based Binary No-Holding A-REM

$$\sum_{(k,j)\in A}\sum_{f=1}^{F}\lambda_{kj,f}^{k,z} = 1, \quad \forall k \in V, \ z \in Z \tag{4.55}$$

$$\sum_{f=1}^{F}\lambda_{ij,f}^{k,z} \leq 1, \quad \forall(i,j) \in A, \ k \in V, \ z \in Z \tag{4.56}$$

$$\sum_{i:(i,j)\in A}\lambda_{ij,f}^{k,z} = \sum_{i:(j,i)\in A}\lambda_{ji,f+1}^{k,z}, \quad \forall j \in W, \ f = 1, \cdots, F-1, \ k \in V, \ z \in Z \tag{4.57}$$

$$\sum_{j\in Y}\sum_{(i,j)\in A}\sum_{f=1}^{F}\lambda_{ij,f}^{k,z} = 1, \quad \forall k \in V, \ z \in Z \tag{4.58}$$

$$\sum_{k\in V}\sum_{z\in Z}\sum_{f=1}^{F}\lambda_{ij,f}^{k,z}h_f^{t,k,z} \leq q_{ij}^{max}, \quad \forall(i,j) \in A, \ t = 1, \cdots, T \tag{4.59}$$

$$\sum_{t=1}^{T}\sum_{i:(i,j)\in A}\sum_{k\in V}\sum_{f=1}^{F}\lambda_{ij,f}^{k,z}h_f^{t,k,z} \leq C_{j,z}, \quad \forall j \in Y, \ z \in Z \tag{4.60}$$

$$\lambda_{ij,f}^{k,z} \in \{0,1\}, \quad \forall(i,j) \in A, \ f = 1, \cdots, F, \ k \in V, \ z \in Z. \tag{4.61}$$

In Model 4.5 we have eliminated the γ-binary variables (which determine start-times) and have integrated the evacuation start-time decision into the λ-variables. With this change, we have a potential start-time for each arc, for each evacuee group. Thus, $\lambda_{ij,f}^{k,z}$ is one if arc (i,j) is used by evacuees of type z from origin k for the first time in interval f, and zero otherwise. We assume $\lambda_{ij,f}^{k,z} = 0, \forall i \in V, j : (i,j) \in A, f = 1, \cdots, F, \ k \in V, z \in Z, i \neq k$ and that $\lambda_{ij,1}^{k,z} = 0, \quad \forall i \in W, \ (i,j) \in A, \ k \in V, \ z \in Z$. Equations (4.55)-(4.58) specify the path-building rules, and insure that each evacuee group has one evacuation path and evacuation start-time. Equation (4.59) limits the number of evacuees on an arc in each time interval, based on the characteristics of the arc, while Equation (4.60) specifies the capacity of each shelter for each evacuee type. Finally, Equation (4.61) is the binary constraint. Next, we examine the differences between Models 4.4 and 4.5, within the context of the branch-and-bound algorithm.

It is more difficult to implement a branch-and-bound search using Model 4.4 than Model 4.5, as some specialized interaction is required between Model 4.4 and the branch-and-bound

search algorithm; Model 4.4 must be reformulated if branching is to occur on a path (λ-) variable. There is a considerable difference in these two formulations with regards to the variables required. Model 4.4 (and the No-Holding A-REM) requires fewer binary variables than Model 4.5, but many more continuous variables. For Scenario 1, Model 4.4 requires only 560 binary variables, 280 ($n|A|$) arc (λ-) variables and 280 (nF) evacuation start-time (γ-) variables, while Model 4.5 requires 5,600 binary variables ($nF|A|$), where n, the number of evacuee groups, is 14, F, the number of possible start intervals, is 20, and $|A|$, the number of arcs, is 20. Despite the difference in the number of binary variables, the binary variables from both models specify the same number of strategies.

When solving a linear relaxation, Model 4.4 has the least number of variables, followed by Model 4.5, and finally the No-Holding A-REM. Thus, in general, using the dual simplex option, we might expect the linear relaxation of Model 4.4 to take less time to solve than that of Model 4.5, which in turn should require less time than the No-Holding A-REM. Comparing the results for the required solution times in Tables 4.13 and 4.12 illustrates this is so for the two test problems considered.

Table 4.13: Results for the initial linear relaxation of Model 4.5.

Scenario	Obj. Value	Evac. Duration	Run Time (sec.)	Cont. Vars.	Binary Vars.	Constraints	Dual Simplex Iters.	B/B
1	11.166	22	0.172	2,894	0	2,640	297	0
2	44.024	94	17.3	8,069	0	7,705	5,313	0

Despite requiring more time to solve each linear relaxation, Model 4.5 has the considerable advantage of providing a tighter relaxation. Comparing the results from Table 4.13 with Table 4.12 for Scenario 1, we see that the initial linear relaxation of Model 4.5 has an objective value of 11.166, compared to 10.857 for Model 4.4 (the optimal solution for this scenario is 11.543). A tighter relaxation can lead to a significant decrease in the number of branch-and-bound nodes required to reach an optimal solution. Once again examining Scenario 1, (see Table 4.14) Model 4.5 requires 2,906 branch-and-bound (B/B) nodes, while Model 4.2 (see Table 4.3) requires 39,712 branch-and-bound nodes (note, Model 4.4 should

require roughly the same number of branch-and-bound nodes as Model 4.2). The decrease in the number of branch-and-bound nodes should give Model 4.5 a speed advantage over Model 4.4.

Table 4.14: Results for Model 4.5, the all binary reformulation.

Scenario	Obj. Value	Evac. Duration	Run Time (sec.)	Cont. Vars.	Binary Vars.	Constraints	MIP Simplex Iters.	B/B
1	11.543	23	28.4	22	2,646	2,458	88,910	2,906
2	44.471	96	364.5	98	8,086	7,808	149,928	3,358

Model 4.5 produces a tighter linear relaxation due to its more constrained solution space. Equation (4.52) of Model 4.4 offers much more leeway to route evacuees, once they have left their origin, than Equation (4.52) of Model 4.5. In Model 4.5 a given value for a λ-variable constrains the flow, which must be some fraction (determined by the relaxed λ-value) of the total flow. Furthermore, once this fraction is determined, it is constant through time, and through the network, although various flows can diverge and converge. As Table 4.14 illustrates, with this new formulation, we are easily able to find solutions to both test scenarios.

Path-Based No-Holding A-REM: The path-based No-Holding A-REM can also be reformulated to eliminate all non-binary variables, thus greatly simplifying each linear relaxation within the branch-and-bound algorithm. This model is as follows:

Model 4.6 Constraint Set of Path-Based Binary No-Holding A-REM

$$\sum_{l \in P^{k,z}} \sum_{f=1}^{F^l} \rho_f^l = 1, \quad \forall k \in V, \ z \in Z \tag{4.62}$$

$$\sum_{k \in V} \sum_{z \in Z} \sum_{l \in P^{k,z}} \sum_{f=1}^{F^l} \delta_{ij}^l \rho_f^l h_f^{t-\beta_{ij}^l,k,z} \leq q_{ij}^{max}, \quad \forall (i,j) \in A, \ t = 1, \cdots, T \tag{4.63}$$

$$\sum_{t=1}^{T} \sum_{i:(i,j) \in A} \sum_{k \in V} \sum_{l \in P^{k,z}} \sum_{f=1}^{F^l} \delta_{ij}^l \rho_f^l h_f^{t-\beta_{ij}^l,k,z} \leq C_{j,z}, \quad \forall j \in Y, \ z \in Z \tag{4.64}$$

$$\rho_f^l \in \{0,1\}, \quad \forall l \in P^{k,z}, \ f = 1, \cdots, F^l, \tag{4.65}$$

91

where all undefined h-values are assumed to be zero. Equation (4.62) limits each evacuee group (i.e., origin and evacuee type) to a single evacuation path and evacuation start-time, as specified by the binary variable ρ. Here $\rho_f^l = 1$ if path $l \in P^{k,z}$ is used by evacuee group k, z starting in interval $f = 1, \cdots, F^l$. Equation (4.63) limits the number of evacuees on any arc-interval to the maximum that arc can transport in a single interval (the no-holding assumption). The binary parameter δ_{ij}^l simply indicates if arc (i, j) is on path l, while the parameter β_{ij}^l indicates the order of the arcs on path l. If arc (i, j) is the first arc on path l, or is not on path l, then $\beta_{ij}^l = 0$. If arc (i, j) is the second arc on path l, then $\beta_{ij}^l = 1$, and in general if arc (i, j) is the nth arc on path l then $\beta_{ij}^l = n - 1$. In the No-Holding variant, the evacuees that enter the first arc of a path in interval t will enter the second arc in interval $t + 1$ and so on. Thus the β_{ij}^l-parameter determines the correct $h_f^{t,k,z}$ that defines the number of evacuees, of the specified evacuee group, that are on arc (i, j) based on its order in path l and the evacuation start-time. Thus, the variable that represents flow into an arc in Model 4.3, $a_{ij}^{t,k,z}$, can be represented as follows:

$$a_{ij}^{t,k,z} = \sum_{l \in P^{k,z}} \sum_{f=1}^{F} \delta_{ij}^l \rho_f^l h_f^{t-\beta_{ij}^l,k,z}.$$

This, with Equation (4.41), leads to Equation (4.63). Equation (4.64) enforces shelter capacity limitations, while Equation (4.65) enforces the binary restrictions.

Table 4.15: Results for Model 4.6.

Scenario	Obj. Value	Evac. Duration	Run Time (sec.)	Cont. Vars.	Binary Vars.	Constraints	MIP Simplex Iters.	B/B
1	11.543	23	3.5	23	500	506	19,845	1,653
2	44.471	89	85.9	98	1085	1787	35,945	4,977

Table 4.15 illustrates the solution speed advantage of this formulation. The solver, with the original arc-based formulation of the No-Holding variant, requires 1,014 seconds to solve Scenario 1. Compare this with to 3.5 for Model 4.6. We can see that the number of branch-and-bound nodes has also been drastically reduced, from 39,712 to 1,653. For Scenario 2,

the solver requires 2,141 seconds with the original path-based No-Holding A-REM, while the solver only requires 85.9 seconds with this formulation.

In the next chapter we explore using the D-REM and A-REM on a large, realistic network.

Chapter 5

Computational Experience Using a Real Network

In this chapter we study the D-REM and the No-Holding A-REM in a realistic setting. To accomplish this, we utilize a much more realistic and complicated test scenario based on Virginia Beach, Virginia. The goal of this chapter is to test the computational effort required on a large, complex network. Our goal here is not to develop realistic evacuation strategies; this would require much more effort, and include extensive testing of various assumptions, along with detailed simulation studies.

The static Virginia Beach network (i.e., not time-expanded), as modeled in this chapter, consists of 840 arcs, 83 origins, which are defined using the 2000 census tract data, 288 junctions, and nine shelters, three of which are uncapacitated (they represent safe, inland regions), and the remaining six shelters being local municipal buildings (mainly public schools) that can shelter 1000 evacuees each. Figure 5.1 displays a map of Virginia Beach, including major routes and shelter locations. As stated earlier, evacuees and vehicles are synonymous. In this study there are approximately 215,000 vehicles, which is based on a vehicle occupancy rate of two passengers per vehicle. Here we do not include evacuee shelter preferences. All computational results were obtained using CPLEX Version 9.0 on an Intel 2.4GHz Xeon workstation with 1.5 GB RAM.

Disaggregated Regional Evacuation Model (D-REM)

Here we study the D-REM (Model 3.2) with Objective (I). We focus on this objective as it is the most difficult to optimize; it is the only objective that is formulated as an MIP. For
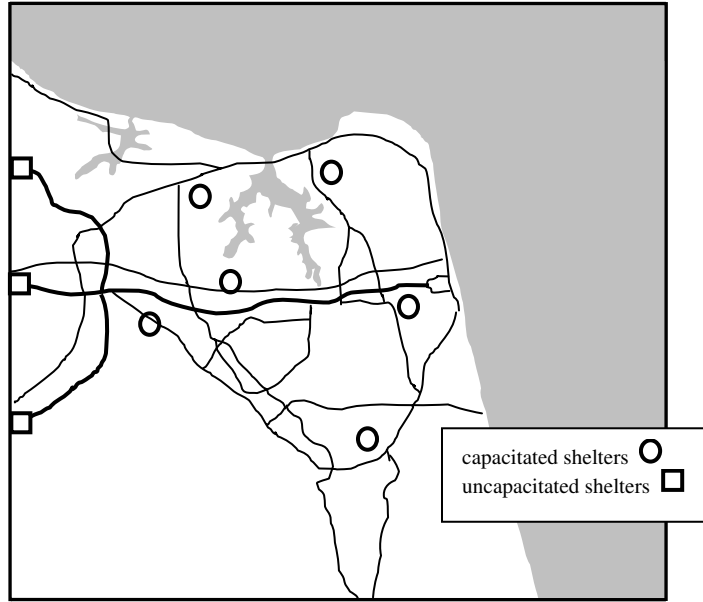
Figure 5.1: A map of Virginia Beach, including major routes and shelters.

the Virginia Beach data set, the overall size of the problem is around 1,500,000 constraints, 1,000,000 continuous variables and non-negativity constraints, and 400 binary variables. Using Proposition 14 we found a lower bound for the staged evacuation to be 345 minutes. Solving the D-REM with Objective (I), the minimum duration required to complete this evacuation turns out to be 351 intervals (i.e., minutes). Figure 5.1 displays the solution times for the various solution techniques delineated in Section 3.5.

Table 5.1: Time required to solve the D-REM with Objective (I) using various solution techniques.

	# LPs	Solution Time
Solution Technique 1	N/A	no solution found due to memory limitations
Solution Technique 2	4	100 h
Solution Technique 3	1	25 h

It is interesting to note that for this particular network configuration, Objective (I) and Objective (II) yield strategies having identical evacuation durations (recall that the problem was solved with Objective (II) to compute an upper bound on the solution for Objective (I)).

95

This indicates that the six capacitated shelters only influence the evacuation time due to their capacities, not their location. Locating the shelters in such a way that Objectives (I) and (II) yield different evacuation durations is possible. However, note that Virginia Beach is fairly compact (see Figure 5.1), with the arcs leading to the three uncapacitated shelters acting as bottlenecks (see Figure 5.1) , and the capacitated shelters, in this example, accommodate relatively few evacuees. Almost all evacuees originate in the upper portion of the Virginia Beach area. Thus, the time to get any evacuee to a shelter is controlled more by the bottlenecks to the uncapacitated shelters than the distances that need to be traveled. In other words, no particular origin forces the evacuation to take 351 minutes; rather it is the consequence of the volume. Figure 5.2 displays the percent of evacuees sheltered by the end of each interval. For the first four intervals, no evacuee reaches a shelter; then the number of sheltered evacuees increases fairly quickly until all shelters are filled, establishing a fairly high exit rate. Once the capacitated shelters reach their capacity limits, the curve settles on a new, slightly lower, exit rate until the evacuation is complete. If there were more capacitated shelters, or if the network had a different, less compact shape with a lower population density (such as the Outer Banks in North Carolina or the Keys in Florida), the location of the capacitated shelters might have had more impact.
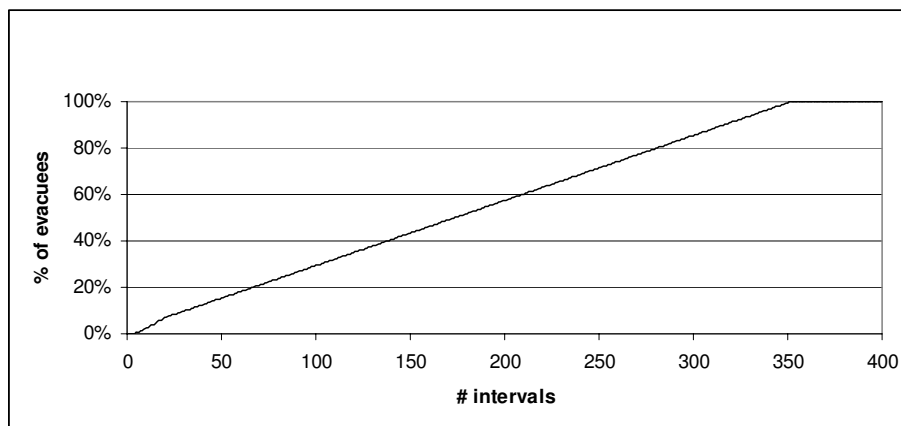

Figure 5.2: The percent of evacuees sheltered by the end of each interval.

Whereas it would be useful to compare these results with non-staged strategies, this is not easy to accomplish. We would have to model the characteristic loading of the network and enforce a sensible routing scheme as well as control the holding-back of traffic. While it might be argued that the staging and routing in this model is too idealized, it does represent

an optimal strategy against which other staged evacuation strategies can be measured.

Aggregated Regional Evacuation Model (A-REM)

Here we study issues related to the A-REM on a large-scale network. This is an extremely difficult problem due to its combinatorial nature. There are also many issues regarding parameter selection, which can have a large impact on the problem's complexity.

In the following we describe some of the issues encountered while trying to determine an optimal evacuation strategy for the Virginia Beach network. First, due to the size of the network, and the large time envelope considered, we determined that the arc-based model (Model 4.5) could not be solved in a reasonable amount of time. Because of this, we concentrate on the path-based model (Model 4.6). A complete enumeration of the possible paths would be quite difficult, and would also produce such a large scale problem that a solution could not be found in a reasonable amount of time. Besides, only a certain subset of possible paths should be considered, i.e., those that are 'reasonable' from an evacuee's point of view. With this in mind, we generate an initial set of paths. This set is composed of the shortest paths between each origin and each of the three uncapacitated shelters. This yields three paths for each origin, for a total of 249 paths. We then determined a duration for each loading curve, based on the minimum duration necessary to insure that all paths are viable. This yielded an average duration of approximately 125 intervals. With these paths and loading curves defined, we produced a rough upper bound of 900 intervals (based on bottleneck-capacity estimates and maximum flows for each evacuee group). We then attempted a first run; unfortunately, the problem was too large, the solver could not even begin optimization, having failed in the pre-processor. Consequently, we lowered the number of path from each origin to one, while balancing the number of paths going to each of the three shelters. This run also experienced memory problems. To circumvent these memory problems, we used a few different techniques:

1. Eliminate arcs that are not on paths currently considered; these arcs need not be considered.

2. Eliminate arcs that are part of only one path. These arcs do not constraint the solution by Assumption 2 (see Section 4.2). Note, this does not affect the duration required to evacuate any evacuee group, as we know the order of the arcs on each path, and thus compensate for any missing arcs.

3. Solve the problem using smaller time intervals, codifying the solution from earlier intervals, and modifying the problem accordingly, e.g., modifying arc-interval capacities.

Using these strategies, we were able to produce a feasible solution that required 847 intervals to complete the evacuation. This is by no means an optimal solution. Overcoming this memory issue required a great deal of customized manipulation of the data and the formulation. It is also a problem that is a function of the computer hardware available for this research; computers with more memory are readily available in the market. Despite this, for the current problem, the processing power is the real issue. Consider Table 5.2, which displays the solution time for the Virginia Beach network, as discussed above, for various planning horizons (T-values). Note that all these solutions are for incomplete evacuations. As Table 5.2 illustrates, the time required to solve the problem increases exponentially, despite using various strategies discussed in Section 4.3. As 37,222 seconds, or 10.3 hours, were required to solve the problem with a planning horizon of 220 minutes, we can easily see that a planning horizon of 850 is prohibitive.

Table 5.2: Timing results for Model 4.6 with various planning horizons.

T-Value	Run Time (sec.)
160	1.047
180	268.7
200	718.7
220	37,222.9
240	memory limitations

More work is required to produce an optimal solution for problems of this size, and more aggregation (i.e., larger groups) might need to be considered. However, from a practical point of view, this might not be required. The evacuation time of 847 intervals, or 14.1 hours, might be reduced in an optimal solution, but the difference might well be within the range of error for a problem of this type. The current solution, though sub-optimal, represents a very reasonable evacuation strategy. In Hobeika, Radwan, and Jamei (1985), a more detailed study of the evacuation of Virginia Beach, evacuation times were estimated to be between 13 and 55 hours, depending on prevalent conditions, all of which were based on

much fewer vehicles than we use in this study (this is mainly a function of the population growth from 1985 to 2000).

Chapter 6

Conclusions and Future Research

In this dissertation we have studied the strategic evacuation planning problem for large-scale, regional evacuations. The proposed evacuation strategies are based on crucial demand management tools, namely, staging and routing. Staging is not much studied in the literature or used in practice, and as such, is an important area of research. Where routing has been researched more, when it is used in conjunction with staging, it must be re-examined because staging is the dominant tool and has the greatest impact on the evacuation. Currently, evacuation planners mostly depend on supply management tools, but as evident from recent hurricane evacuations, which were plagued with gridlock and congestion, these supply management tools are often not enough. The situation is likely to worsen, because, even if managed well, the supply is limited, and not likely to increase significantly. On the other hand, demand is likely to increase, as populations grow and continue to shift to urban and coastal areas.

We have studied two unique models that combine aspects of evacuation and dynamic traffic assignment models. Within this framework, we have explored various objective functions that represent characteristics of interest to an evacuation planner, and have presented properties of these objectives and shown how they are related. These objective functions represent various characteristics of an evacuation that are desirable to optimize, such as the duration of the evacuation, the average evacuation time, or the risk involved.

The timely evacuation of a major urban region is a highly complex undertaking and requires more research to develop appropriate strategies. In the following, we suggest several possibilities for future research.

1. Study risk in more detail, perhaps in the context of various types of spontaneous disasters.

2. Explore the following issues: What is the impact of evacuee behavior on the evacuation strategy? How closely do evacuees follow the plan? Do families use one vehicle, or all available vehicles? How many evacuees want to make use of a designated shelter, and how many just want to leave the area (staying with friends, family or in a hotel)?

3. Study how stochastic elements can impact the evacuation strategy, and ways to produce more robust strategies.

4. Study combining staging and routing with supply management strategies such as contra-flow.

5. Examine strategies when not all evacuees have vehicles. For instance, if a certain percentage of the population must be evacuated by bus, what is the best evacuation strategy?

6. Research how staging impacts loading curves and evacuee decisions, along with the impact of education. Econometric models used to study other travel behavior might be appropriate to model this decision process. With a solid modeling foundation of this behavior, we could study how to influence evacuees to depart in a staged manner.

7. Test strategies using a more detailed simulation approach. This can help determine the impact of assumptions made in the modeling process.

8. Explore more effective solution strategies for the A-REM, possibly including decomposition procedures, column generation, and heuristic procedures.

This is an important area of research, which is largely unstudied. As our preliminary results seem to indicate, staging can have a large impact on the quality and success of an evacuation.

References

Carey, M., Ge, Y.E. and McCartney, M. (2003), "A Whole-Link Travel-Time Model with Desirable Properties," *Transportation Science*, **37**(1), 83–96.

Carey, M. and McCartney, M. (2004), "An Exit-Flow Model Used in Dynamic Traffic Assignment," *Computers & Operations Research*, **31**, 1583–1602.

Chalmet, L.G., Francis, R.L. and Saunders, P.B. (1982), "Network Models for Building Evacuation," *Management Science*, **28**(1), 86–105.

Chen, X. and Zhan F.B. (2004), "Agent-Based Modeling and Simulation of Urban Evacuation: Relative Effectiveness of Simultaneous and Staged Evacuation Strategies," *TRB 2004 Annual Meeting*.

Choi, W., Hamacher, H.W. and Tufekci, S. (1988), "Modeling of Building Evacuation Problems by Network Flows with Side Constraints," *European Journal of Operational Research*, **34**, 98–110.

Church, R.L. and Cova, T.J. (2000), "Mapping Evacuation Risk on Transportation Networks Using a Spatial Optimization Model," *Transportation Research, Part C* **8**, 321–336.

Cova, T.J and Johnson, J.P. (2003), "A Network Flow Model for Lane-Based Evacuation Routing," *Transportation Research, Part A*, **37**, 579–604.

Fu, H. and Wilmont, C.G., (2004), "A Sequential Logit Dynamic Travel Demand Model for Hurricane Evacuation," *TRB 2004 Annual Meeting*.

Garber, N.J. and Hoel, L.E. (2002), Traffic & Highway Engineering, Third Edition, Brooks/Cole, Pacific Grove, CA.

Hamacher, H.W. and Tjandra, S.A. (2001), "Mathematical Modeling of Evacuation Problems: A State of the Art," *Pedestrian and Evacuation Dynamics*, Schreckenberg, M. and Sharma, S.D. Ed, Springer, 227–266.

Hamacher, H.W. and Tufekci, S. (1987), "On the Use of Lexicographic Min-Cost Flows in

Evacuation Modeling," *Naval Research Logistics*, **34**, 487–503.

Hobeika, A.G. and Kim, C. (1998), "Comparison of Traffic Assignments in Evacuation Modeling," *IEEE Transactions on Engineering Management*, **45**(2), 192–198.

Hobeika, A.G., Kim, S. and Beckwith, R.E. (1994), "A Decision Support System for Developing Evacuation Plans Around Nuclear Power Stations," *Interfaces*, **24**(5), 22–35.

Hobeika, A.G., Radwan, A.E., and Jamei, B. (1985), "Virginia Beach Hurricane/Flood Transportation Evacuation Study," Technical Report, the Office of Emergency Services, Virginia Beach, VPI&SU, Blacksburg, VA.

Hoppe, B. and Tardos, E. (2000), "The Quickest Transshipment Problem," *Mathematics of Operations Research*, **25**(1), 36–62.

Jarvis, J.R. and Ratliff, D.H. (1982), "Some Equivalent Objectives for Dynamic Network Flow Problems," *Management Science*, **28**, 106–109.

Litman, T (2006), "Lessons from Katrina and Rita: What Major Disasters Can Teach Transportation Planners," *Journal of Transportation Engineering*, **132**(1), 11–18.

Merchant, D.K. and Nemhauser, G.L. (1978), "A Model and an Algorithm for the Dynamic Traffic Assignment Problem," *Transportation Science*, **12**, 183–199.

Murray-Tuite, P.M. and Mahmassani, H.S. (2003), "Model of Household Trip-Chaining Sequencing in Emergency Evacuation," *Transportation Research Record*, **1831**, 21–20.

Peeta, S. and Ziliaskopoulos, A.K. (2001), "Foundations of Dynamic Traffic Assignment: The Past, the Present, and the Future," *Networks and Spatial Economics*, **1**, 233–265.

Petruccelli, U. (2003), "Urban Evacuation in Seismic Emergency Conditions," *Institute of Transportation Engineers. ITE Journal*, **73**(8), 34–38.

Pidd, M., deSilva, F.N. and Eglese, R.W. (1996), "A Simulation Model for Emergency Evacuation," *European Journal of Operational Research*, **90**, 413–419.

Rakha, H. and Crowther, B. (2003), "Comparison and Calibration of FRESIM and INTEGRATION Steady-state Car-following Behavior," *Transportation Research, Part A*, **37**, 1–27.

Sbayti, H. and Mahmassani, H.S. (2006), "Optimal Scheduling of Evacuation Operations," *TRB 2006 Annual Meeting*.

Sheffi, Y., Mahmassani, H. and Powell, W.B. (1982), "A Transportation Network Evacuation Model," *Transportation Research, Part A*, **16**(3), 209–218.

Sherali, H.D., Carter, T.B. and Hobeika, A.G. (1991), "A Location-Allocation Model and

Algorithm for Evacuation Planning under Hurricane/Flood Conditions," *Transportation Research, Part B*, **25**(6), 439–452.

Smith, J.M. (1991), "State-Dependent Queuing Models in Emergency Evacuation Networks," *Transportation Research, Part B*, **25**(6), 373–389.

Tufekci, S. (1995), "An Integrated Emergency Management Decision Support System for Hurricane Emergencies," *Safety Science*, **20**, 39–48.

Urbina E. and Wolshon, B. (2003), "National Review of Hurricane Evacuation Plans and Policies: A Comparison and Contrast of State Practices," *Transportation Research, Part A*, **37**, 257–275.

Wolshon, B. (2006), "Evacuation Planning and Engineering for Hurricane Katrina," *The Bridge*, **36**(1), 27–34.

Wolshon, B., Catarella-Michel, A. and Lambert, L. (2006), "Louisiana Highway Evacuation Plan for Hurricane Katrina: Proactive Management of a Regional Evacuation," *Journal of Transportation Engineering*, **132**(1), 1–9.

Wolshon, B. and Meehan, B.H. (2003), "Emergency Evacuation - Ensuring Safe and Efficient Transportation out of Endangered Areas," *TR News*, **224**, 3–10.

Ziliaskopoulos, A.K., (2000), "A Linear Programming Model for the Single Destination System Optimal Dynamic Traffic Assignment Problem," *Transportation Science*, **34**(1), 37–49.

Vita

Douglas R. Bish has earned the following degrees:

- B.S. in Industrial Engineering from the California Polytechnic State University (Cal Poly), San Luis Obispo, CA,

- M.S. in Biomedical Engineering from Northwestern University, Evanston, Il,

- M.S. in Industrial Engineering from Northwestern University, Evanston, Il,

- Ph.D. in Civil and Environmental engineering from Virginia Polytechnic Institute and State University (Virginia Tech), Blacksburg, VA.

Doug has over seven years industrial experience at United Airlines working in the area of applied operations research. He is currently starting as an Assistant Professor at Virginia Tech, in the Grado Department of Industrial and Systems Engineering.