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Munir H. Nayfeh and Ali H. Nayfeh

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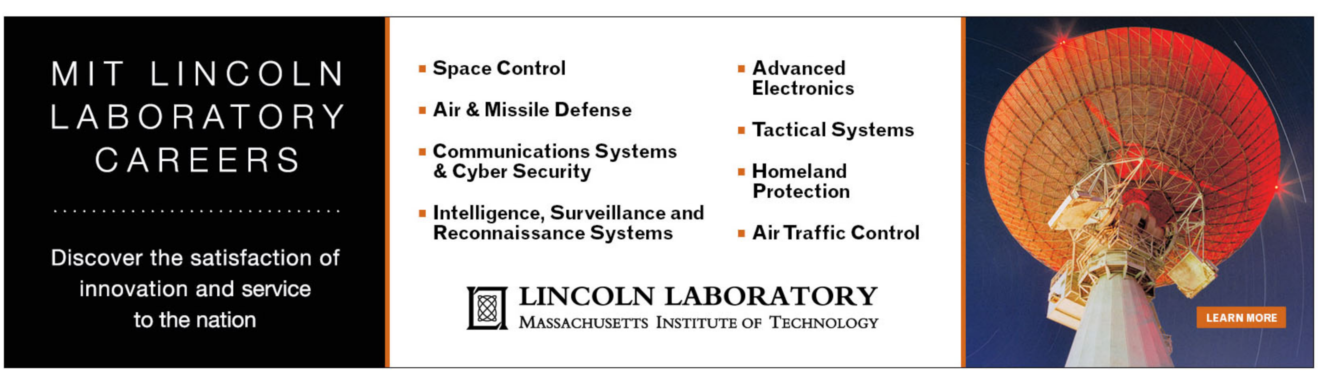

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Population difference of two-level atomic system due to a running pulsed field

Munir H. Nayfeh

Oak Ridge National Lab, Oak Ridge, Tennessee 37830

Ali H. Nayfeh

Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061
(Received 10 February 1975; in final form 21 May 1975)

The method of multiple scales is used to derive an expression for the population difference in an absorbing atomic system due to a running pulsed field. The main contribution to this expression comes from a quasi-steady-state part which has the same functional form as the hole produced by a continuous running field, except that the saturation parameter contains the time dependence of the field. The expression includes also an oscillatory term and a quasisteady term, which decay with a rate that is equal to the inverse of the lifetime of the levels.

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I. INTRODUCTION

We consider the effect of pulsed running waves on the population difference of a two-level atomic system (i.e., modified hole burning model). This study is motivated by recent spectroscopic measurements (saturation spectroscopy with a pulsed dye laser)[1] in which strong and weak running pulsed waves are sent in opposite directions in a resonant medium to determine the shape and wavelength of its resonant lines.

For simplicity, we assume the lifetimes of the levels to be equal in the absence of collisions. Using a matrix density formulation, one can show that the system is governed by the following two equations in a reference frame moving with the atom[2]:

$$\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial z} + \gamma\right)\Delta n = -i\hat{\mu}(\mathcal{E}\rho^* - cc) + \gamma N(v), \tag{1}$$

$$\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial z} + i\omega_0 + \gamma\right)\rho = i\hat{\mu}\,\mathcal{E}\,\Delta n, \tag{2}$$

where $\Delta n = \rho_{aa} - \rho_{bb}$, $\rho = \rho_{ab}$, ρ_{ij} are the elements of the density matrix, t is the time, z is the distance in the direction of the propagation, v is the speed of the atom, γ^{-1} is the lifetime of the levels a and b, $\mathcal{E}$ is the complex excitation, ω_0 is the natural frequency of the transition, $\hat{\mu} = \mu/\hbar$, μ is the dipole moment of the transition, $\hbar$ is Planck's constant, $N(v)$ is the normalized population difference in the absence of any radiation field, and ρ^* is the complex conjugate of ρ.

Equations (1) and (2) can be easily solved for the case of a continuous-wave excitation (see for example, Refs. 2 and 3). In this paper, we extend these analyses to determine the effect of a long pulsed running wave excitation; that is,

$$\mathcal{E}(z, t) = f(\tau)\exp[i(kz - \omega t)], \tag{3}$$

where ω and k are the frequency and wave number of the excitation, $\tau = \epsilon t$, ϵ is a small dimensionless parameter of the order of ω^{-1}, and f is a real function. Thus, the amplitude of the excitation is a slowly varying function of t with respect to ω^{-1}.

II. APPROXIMATE SOLUTION

We seek a solution of Eqs. (1)—(3) of the form

$$\rho = N(v)[u_1(t) + iu_2(t)]\exp[i(kz - \omega t)], \tag{4}$$

$$\Delta n = N(v)u_3(t), \tag{5}$$

where u_1, u_2, and u_3 are real functions. One can easily show that Δn is real by taking the complex conjugate of Eq. (1). Substituting Eqs. (4) and (5) into Eqs. (1) and (2) and separating the real and imaginary parts yields

$$\left(\frac{d}{dt} + \gamma\right)u_1 - (kv - \Omega)u_2 = 0, \tag{6}$$

$$\left(\frac{d}{dt} + \gamma\right)u_2 + (kv - \Omega)u_1 = \hat{\mu}\,f(\tau)u_3, \tag{7}$$

$$\left(\frac{d}{dt} + \gamma\right)u_3 + 2\hat{\mu}\,f(\tau)u_2 = \gamma, \tag{8}$$

where $\Omega = \omega - \omega_0$.

Since the coefficients in Eqs. (6)—(8) are slowly varying functions of time, we determine an approximate solution to these equations by using the method of multiple scale.[4] We seek a uniform expansion in the form

$$u_1 = F_1(\tau, \epsilon)\exp(\eta) + G_1(\tau, \epsilon), \tag{9}$$

$$u_2 = F_2(\tau, \epsilon)\exp(\eta) + G_2(\tau, \epsilon), \tag{10}$$

$$u_3 = F_3(\tau, \epsilon)\exp(\eta) + G_3(\tau, \epsilon), \tag{11}$$

where

$$\frac{d\eta}{dt} = \xi(\tau). \tag{12}$$

Substituting Eqs. (9)—(12) into Eqs. (6)—(8) and equating the coefficients of $\exp(\eta_n)$ with with $n = 0$ and 1 on both sides, we obtain

$$\epsilon F_1' + (\gamma + \xi)F_1 - (kv - \Omega)F_2 = 0, \tag{13}$$

$$\epsilon F_2' + (\gamma + \xi)F_2 + (kv - \Omega)F_1 - \hat{\mu}f(\tau)F_3 = 0, \tag{14}$$

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$$\epsilon F_3' + (\gamma + \xi) F_3 + 2\hat{\mu} f(\tau) F_2 = 0, \tag{15}$$

$$\epsilon G_1' + \gamma G_1 - (kv - \Omega) G_2 = 0, \tag{16}$$

$$\epsilon G_2' + \gamma G_2 + (kv - \Omega) G_1 - \hat{\mu} f(\tau) G_3 = 0, \tag{17}$$

$$\epsilon G_3' + \gamma G_3 + 2\hat{\mu} f(\tau) G_2 = \gamma, \tag{18}$$

where primes denote differentiation with respect to τ.

Equations (13)–(18) admit expansions of the form

$$F_i = F_{i0}(\tau) + \epsilon F_{i1}(\tau) + \cdots, \tag{19}$$

$$G_i = G_{i0}(\tau) + \epsilon G_{i1}(\tau) + \cdots. \tag{20}$$

Substituting Eqs. (19) and (20) into Eqs. (13)–(18) and equating coefficients of like powers of ϵ, we obtain the following:

order ϵ^0

$$(\gamma + \xi) F_{10} - (kv - \Omega) F_{20} = 0, \tag{21}$$

$$(\gamma + \xi) F_{20} + (kv - \Omega) F_{10} - \hat{\mu} f(\tau) F_{30} = 0, \tag{22}$$

$$(\gamma + \xi) F_{30} + 2\hat{\mu} f(\tau) F_{20} = 0, \tag{23}$$

$$\gamma G_{10} - (kv - \Omega) G_{20} = 0, \tag{24}$$

$$\gamma G_{20} + (kv - \Omega) G_{10} - \hat{\mu} f(\tau) G_{30} = 0, \tag{25}$$

$$\gamma G_{30} + 2\hat{\mu} f(\tau) G_{20} = \gamma; \tag{26}$$

order ϵ^n for $n \geq 1$

$$(\gamma + \xi) F_{1n} - (kv - \Omega) F_{2n} = -F'_{1(n-1)}, \tag{27}$$

$$(\gamma + \xi) F_{2n} + (kv - \Omega) F_{1n} - \hat{\mu} f(\tau) F_{3n} = -F'_{2(n-1)}, \tag{28}$$

$$(\gamma + \xi) F_{3n} + 2\hat{\mu} f(\tau) F_{2n} = -F'_{3(n-1)}, \tag{29}$$

$$\gamma G_{1n} - (kv - \Omega) G_{2n} = -G'_{1(n-1)}, \tag{30}$$

$$\gamma G_{2n} + (kv - \Omega) G_{1n} - \hat{\mu} f(\tau) G_{3n} = -G'_{2(n-1)}, \tag{31}$$

$$\gamma G_{3n} + 2\hat{\mu} f(\tau) G_{2n} = -G'_{3(n-1)}. \tag{32}$$

The solution of Eqs. (24)–(26) is

$$G_{30} = 1 - \frac{2\hat{\mu}^2 f^2(\tau)}{(kv - \Omega)^2 + \gamma^2 + 2\hat{\mu}^2 f^2(\tau)} = 1 - H(\tau), \tag{33}$$

$$G_{20} = \frac{\gamma H(\tau)}{2\hat{\mu} f(\tau)}, \tag{34}$$

$$G_{10} = \frac{(kv - \Omega) H(\tau)}{2\hat{\mu} f(\tau)}. \tag{35}$$

Equations (21)–(23) have a nontrivial solution if, and only if, the determinent of the coefficient matrix vanishes. This condition leads to the following eigenvalues:

$$\xi = -\gamma, \tag{36a}$$

$$\xi = -\gamma \pm i[2\hat{\mu}^2 f^2(\tau) + (kv - \Omega)^2]^{1/2} = -\gamma \pm i\alpha. \tag{36b}$$

Hence, the solution of Eqs. (21)–(23) is

$$F_{30} = A(\tau), \quad F_{20} = \mp \frac{i\alpha}{2\hat{\mu} f(\tau)} A(\tau),$$

$$F_{10} = -\frac{(kv - \Omega)}{2\hat{\mu} f(\tau)} A(\tau), \tag{37a}$$

when $\xi + \gamma = \pm i\alpha$ and

$$F_{30} = B(\tau), \quad F_{20} = 0, \quad F_{10} = \frac{\hat{\mu} f(\tau)}{kv - \Omega} B(\tau), \tag{37b}$$

when $\xi + \gamma = 0$. The functions $A(\tau)$ and $B(\tau)$ are still undetermined at this level of approximation; they are determined by invoking the solvability condition at the next level of approximation.

Since the homogeneous parts of Eqs. (27)–(29) are the same as Eqs. (21)–(23) and since the latter have a nontrivial solution, the inhomogeneous equations (27)–(29) have a solution if, and only if, the inhomogeneous parts are orthogonal to every solution of the adjoint homogeneous problem. This solvability condition provides a relationship among the $F'_{i(n-1)}$. When $n = 1$, this condition yields

$$\left(\frac{2\hat{\mu} f}{kv - \Omega} - 2\hat{\mu}^2 f^2\right) F'_{10} + [1 - \hat{\mu}(kv - \Omega) f] F'_{30} = 0, \tag{38}$$

when $\xi + \gamma = 0$ and

$$(kv - \Omega)^2 F'_{10} \mp i(kv - \Omega)\alpha F'_{20} - \hat{\mu}(kv - \Omega) f F'_{30} = 0, \tag{39}$$

when $\xi + \gamma \neq 0$.

Eliminating the F's from Eqs. (37b) and (38) and solving the resulting equation for $B(\tau)$, we obtain

$$B(\tau) = c_1 \alpha^{-1}, \tag{40}$$

where c_1 is an arbitrary constant. Similarly, eliminating the F's from Eqs. (37a) and (39) and solving the resulting equation for $A(\tau)$, we obtain

$$A(\tau) = c_2 \alpha^{-1}(\tau) f(\tau), \tag{41}$$

where c_2 is an arbitrary constant.

Combining Eqs. (5), (9)–(12), (19), (20), (33)–(37), (40), and (41), we obtain the following approximate solution:

$$\Delta n = N(v)\alpha^{-1}(\tau)[c_2 f(\tau)\cos\phi + c_1]\exp(-\gamma t) + N(v)[1 - H(\tau)] + O(\epsilon), \tag{42}$$

$$\rho = N(v)\left[\left(-\frac{(kv - \Omega)c_2}{2\hat{\mu}\alpha(\tau)}\cos\phi + \frac{\hat{\mu} c_1 f(\tau)}{(kv - \Omega)\alpha(\tau)}\right)\exp(-\gamma t) + \frac{(kv - \Omega)H(\tau)}{2\hat{\mu} f(\tau)} + i\left(\frac{c_2}{2\hat{\mu}}\sin\phi\exp(-\gamma t) + \frac{\gamma H(\tau)}{2\hat{\mu} f(\tau)}\right)\right] \times \exp[i(kz - \omega t)] + O(\epsilon), \tag{43}$$

where

$$\phi = \int_0^t \alpha(\epsilon t)\, dt + \phi_0 \tag{44}$$

and ϕ_0 is an arbitrary constant. The constants c_1, c_2, and ϕ_0 can be determined from the initial conditions of the specific problem.

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If the system is initially not polarized, the appropriate initial conditions are

$$\rho(0)=0, \quad \Delta n = N(v). \tag{45}$$

Since the pulsed field is slowly built, $f(0)=0$, $c_1 = c_2 = 0$, and Eqs. (42) and (43) become

$$\Delta n = N(v)[1 - H(\tau)] + O(\epsilon), \tag{46}$$

$$\rho = \frac{H(\tau)}{2\hat{\mu} f(\tau)}(kv - \Omega + i\gamma) N(v) \exp[i(kz - \omega t)] + O(\epsilon). \tag{47}$$

We note that the decaying oscillatory and quasisteady terms in Eq. (42) vanish in this case. However, when the system is initially polarized, c_1 and c_2 do not vanish in general.

III. CONCLUDING REMARKS

The general expression (42) for the population difference of a two-level atomic system due to a pulsed running wave consists of three terms, in contrast with the case of a continuous running wave. One term is oscillatory and has a lifetime equal to those of the atomic levels. The second is quasisteady and decays like the first term. The third term is quasisteady because the saturation parameter $\chi = 2\,\mu^2\gamma^{-2} f^2(\tau)$ is a function of time.

The hole burned in the Maxwell distribution due to the quasisteady term is centered at $kv = \Omega$ with a full width at half-maximum of $2\gamma(1+\chi)^{1/2}$. Thus, the width and height of the hole burned due to the quasisteady term are time dependent because the saturation parameter is time dependent.

The amplitude and frequency of the oscillatory term in the hole burned are time dependent. Moreover, the oscillatory term maximizes at a finite time and it decays as time increases with a decay rate equal to the inverse of the lifetime of the levels. The contribution of this term attains its maximum when $kv = \Omega$. The height and half-width of the hole burned due to this term are time dependent, as are those of the quasisteady term.

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