CHAPTER 2
SIMULATION METHOD AND SENSOR MODEL

Throughout this dissertation we will use examples to help explain the concepts and theories that we will develop. In the examples we will use numerical simulation of a structure. In the first half of this chapter we will discuss the structure, modeling, and simulation, and give some insight into the choice of the adaptive identification algorithm. In the second half of the chapter, we will discuss the development of the theory of segmented sensors, the configuration of the sensor, and its principle of operation.

2.1 Model of Variable Host Structure

2.1.1 Description of Structure

The structure we choose should be simple enough to model mathematically, but also allows for variations in mechanical properties. For this purpose, we choose a simply-supported beam with some torsional stiffness at one of its simple supports (Fig. 2.1).

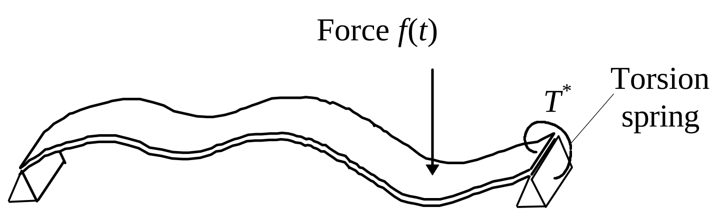

Figure 2.1 Beam with pin and pin-with-torsion-spring boundary conditions.

The modal properties of the structure depends partly on the torsional spring stiffness T^*. The normalized torsion spring constant is

$$T^* = TL/(EI)\,, \tag{2.1}$$

where T^* is the torsion spring constant in m^{-1}, L is the length of the beam in m, E is the modulus of elasticity in Pa, and I is the cross-section area moment of inertia in m^4.

The following paragraphs explain the modeling and simulation of the structure. We assume a linear Euler-Bernoulli beam with internal viscous damping. As in most vibrational analyses of structures, first of all we need to obtain the eigen-properties of the structure. Then we present the equations of motion in the modal coordinates.

2.1.2 Eigen-Properties of Structure

The m^{th} eigenvalue λ_m of the beam is the m^{th} non-trivial solution to the characteristic equation for the undamped case is (Gorman, 1975).

$$\left(\cos\lambda \sinh\lambda - \sin\lambda \cosh\lambda\right)T^{*} - 2\lambda \sin\lambda \sinh\lambda = 0 . \tag{2.2}$$

The m^{th} natural frequency in rad/s, ω_m, is

$$\omega_m = 2\pi f_m = \frac{\lambda_m^{\;2}}{L^2}\sqrt{\frac{EI}{\rho_L}} , \tag{2.3}$$

where f_m is the natural frequency in Hz, λ_m is the m^{th} root of Eq. (2.2), L is the length of the beam, EI is the product of the modulus of elasticity and the area moment of inertia, and ρ_L is the mass per unit length. The mass-normalized mode shape is

$$\phi_m(x) = \frac{1}{c_{mass\ norm}}\left(\sin(\lambda_m x / L) - \frac{\sin\lambda_m}{\sinh\lambda_m}\sinh(\lambda_m x / L)\right), \tag{2.4}$$

where the mass normalization constant $c_{mass\ norm}$ for each mode can be calculated by setting

$$\int_0^L \rho_L \phi_m^{\;2}(x)\,\mathrm{d}x = 1 \tag{2.5}$$

for all m. This equation results in a mass normalization constant for each mode shape.

$$\begin{aligned} c_{massnorm} = \sqrt{L\rho_L}\Bigg\{ 0.5 - \frac{\sin 2\lambda_m}{4\lambda_m} + \frac{\sin^2\lambda_m}{\sinh^2\lambda_m}\left(\frac{\sinh^2\lambda_m}{4\lambda_m} - \frac{\lambda_m}{2}\right) - \\ \frac{\sin\lambda_m}{\lambda_m \sinh\lambda_m}\left(\sin\lambda_m \cosh\lambda_m - \cos\lambda_m \sinh\lambda_m\right)\Bigg\} \end{aligned} \tag{2.6}$$

2.1.3 Equation of Motion and Its State Space Form

The velocity response of the structure to a point force excitation can be expressed in terms of modal expansion as

$$\dot{w}(x,t)=\sum_{m=1}^{M}\phi_m(x)\dot{\eta}_m(t) \tag{2.7}$$

where $\phi_m(x)$ is the m^{th} mass-normalized mode shape of the structure, and $\eta_m(t)$ is the modal coordinate, which is the solution to the equation of motion

$$\ddot{\eta}_m(t)+2\zeta_m\omega_m\dot{\eta}_m(t)+\omega_m^2\eta_m(t)=\phi_m(x_f)f(t) \tag{2.8}$$

where $\eta_m(t)$ denotes modal coordinates. The dot above the variables denote time derivation. The summation limit M in Eq. (2.7) is theoretically infinity, but in practice must be limited to the number of modes included in calculations; ω_m^2 is the eigenvalue; ζ_m denotes the modal damping; $\phi_m(x_f)$ is the value of the mass-normalized mode shape at the forcing point position.

In later chapters, we will need a state-space representation of the structure. The state-space model is

$$\begin{Bmatrix}\dot{\eta}(t)\\ \ddot{\eta}(t)\end{Bmatrix}=\mathbf{A}\begin{Bmatrix}\eta(t)\\ \dot{\eta}(t)\end{Bmatrix}+\mathbf{B}\,f(t)\,, \tag{2.9}$$

where

$$\mathbf{A}=\left[\begin{array}{c|c}\mathbf{0} & \mathbf{I}\\ \hline -\boldsymbol{\omega}_{\mathbf{n}}^{2} & \mathbf{Z}\end{array}\right], \tag{2.10}$$

is the matrix containing the natural frequencies and damping ratios, i.e.,

$$-\boldsymbol{\omega}_{\mathbf{n}}^{2}=\begin{bmatrix}-\omega_1^2 & & \mathbf{0}\\ & \ddots & \\ \mathbf{0} & & -\omega_M^2\end{bmatrix}, \tag{2.11}$$

and

$$\mathbf{Z} = \begin{bmatrix} -2\omega_1\zeta_1 & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & -2\omega_M\zeta_M \end{bmatrix}. \tag{2.12}$$

0 and **I** are M-by-M zero and identity matrices, respectively. In the second term, the input matrix is

$$\mathbf{B} = \left\{ \begin{array}{c} 0 \\ \vdots \\ 0 \\ \hdashline \phi_1(x_f) \\ \vdots \\ \phi_M(x_f) \end{array} \right\}. \tag{2.13}$$

2.2 Simulation of Structure

2.2.1 Sampling and Discrete-Time Model

Because data are normally acquired by sampling and processed by digital signal processing equipment, it is necessary in our analysis to model the input signal, the structure, and the output signal as discrete-time processes.

Figure 2.2 shows a structure excited with a force $f(t)$. At this point we consider the point sensor. The output signal is a velocity $v(t)$ at some point. Because of sampling, the measured force and velocity will be discrete functions of time step or sample number k instead of continuous functions of time. We call the sampled input $f(k)$ and the sampled output $v(k)$. We must represent the frequency response function (FRF) of the structure by a discrete time FRF. This FRF must be such that if the input $f(k)$ is a sampled version of the force $f(t)$, the output $v(k)$ is a sampled version of the velocity $v(t)$.

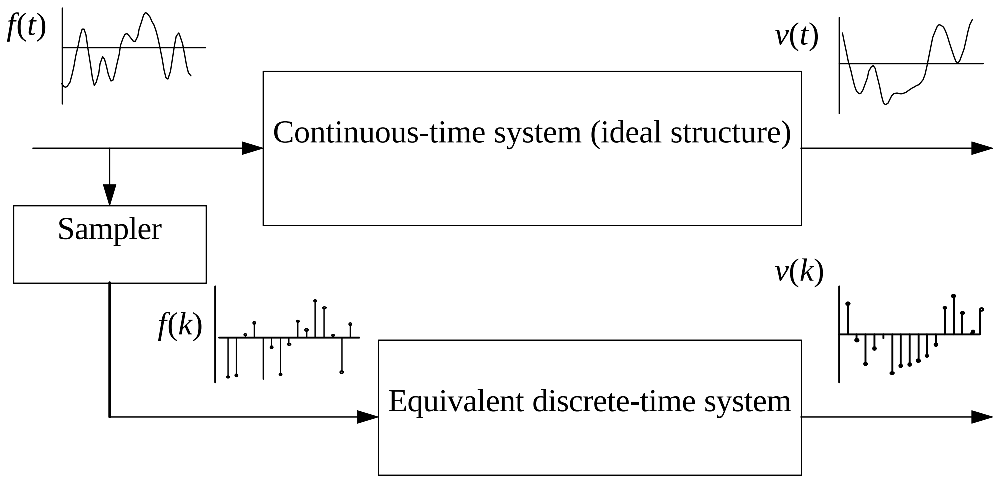

Figure 2.2 Continuous-time and sampled (discrete time) systems.

2.2.2 Discrete-Time Model of One Mode

To give a simple example of the process of representing a structure with a digital filter, first we consider only mode-*m* of the structure. The frequency response of a single mode is like that of a second-order system. According to Eq. (2.8), the continuous-time transfer function from force to a modal coordinate rate $\dot{\eta}$ in the Laplace (s) domain is

$$\frac{\dot{\eta}_m(s)}{f(s)} = \frac{s\eta_m(s)}{f(s)} = \frac{\phi_m(x_f)s}{s^2 + 2\zeta_m\omega_m s + \omega_m{}^2} \tag{2.14}$$

(Henceforth, what we denote by 'modal coordinate' actually means the 'time rate of change of the modal coordinate'. This time rate is related to vibration velocity and is often more important than the modal coordinate.) If we know the modal properties of a structure, i.e., the natural frequencies, modal damping, mode shapes, and residues, we can synthesize the digital filter representation using several methods. The digital filter is connected to the analog parts of the system by A/D and D/A converters, usually with a zero-order hold. For this kind of signal conversion, the discrete equivalent of the transfer function in Eq. (2.14) can be computed by (Franklin et al., 1990)

$$H(z) = (1 - z^{-1})\mathcal{Z}\left\{\frac{1}{s}\frac{s\eta(s)}{f(s)}\right\} \tag{2.15}$$

where $Z\{\ \}$ denotes the z-transform operator. It can be shown that substituting Eq. (2.14) into Eq. (2.15) results in the discrete transfer function

$$\frac{\dot{\eta}_m(z)}{f(z)} = \frac{b_m(z^{-1} + z^{-2})}{1 + a_{1m}z^{-1} + a_{2m}z^{-2}} \tag{2.16}$$

where the filter coefficients are

$$b_m = \phi_m(x_f)\frac{\sin(\omega_m\sqrt{1-\zeta_m^2})}{\omega_m\sqrt{1-\zeta_m^2}}\exp(-\zeta_m\omega_m T)\,, \tag{2.17}$$

$$a_{1m} = -2\exp(-\zeta_m\omega_m T)\cos(\omega_m\sqrt{1-\zeta_m^2}) \tag{2.18}$$

$$a_{2m} = \exp(-2\zeta_m\omega_m T) \tag{2.19}$$

In discrete-time domain, the second-order filter representation of the single-mode structure is

$$\dot{\eta}_m(k) = -a_{1m}\dot{\eta}_m(k-1) - a_{2m}\dot{\eta}_m(k-2) + b_m f(k-1) + b_m f(k-2) \tag{2.20}$$

What we have done so far is:
1. Modeled the structure with its modal properties, i.e., ω_m, ζ_m, and $\phi_m(x_0)$
2. Transformed the equations of motion in time domain into transfer functions in the s-domain
3. Transformed the continuous transfer functions in the s-domain into discrete-time transfer functions in the z-domain
4. Transformed the discrete-time transfer functions into digital filter coefficients

2.2.3 Discrete-Time Model of Multi-Mode Structure

Continuous structures, such as the example structure we use in this research work, have an infinite number of modes. In practical calculation and numerical analysis we can only include a limited number of modes. Each mode increases the order of the transfer function by two. In general, an M-mode structure has a $2M^{th}$ order transfer function of the form

$$\frac{\dot{w}(s)}{f(s)} = \frac{\beta_0 + \beta_1 s + \beta_2 s^2 + \ldots + \beta_{2M}s^{2M}}{1 + \alpha_1 s + \alpha_2 s^2 + \ldots + \alpha_{2M}s^{2M}} \tag{2.21}$$

The above transfer function can be represented by a $2M^{\text{th}}$ order digital filter of the form

$$H(z) = \frac{v(z)}{f(z)} = \frac{\mathrm{b}_0 + \mathrm{b}_1 z^{-1} + \mathrm{b}_2 z^{-2} + \ldots + \mathrm{b}_{2M} z^{-2M}}{1 + \mathrm{a}_1 z^{-1} + \mathrm{a}_2 z^{-2} + \ldots + \mathrm{a}_{2M} z^{-2M}} \qquad (2.22)$$

To decompose the multi-mode transfer function into several single-mode transfer functions, we can decompose Eq. (2.22) by partial fractioning into

$$H(z) = \sum_{m=1}^{M} \frac{b_{0m} + b_{1m} z^{-1} + b_{2m} z^{-2}}{1 + a_{1m} z^{-1} + a_{2m} z^{-2}} \qquad (2.23)$$

The last equation can be realized as a bank of second-order infinite-impulse-response (IIR) digital filters connected in parallel with a common input. The outputs of the filters are summed up to produce the total output of the system. This digital-filter representation of the structure is shown in Fig. 2.3.

The representation of the structure as a parallel bank of second order digital filters lends to modal analysis because the transfer function of the parallel filters is exactly the same as the modal expansion of the structure's transfer function. Each second-order filter has three coefficients that are the results of transformations from the natural frequency, modal damping, and modal residues. Even modal analysis could be done adaptively if we find a way to compute the filter coefficients recursively.

2.2.4 Selecting Sensor Configuration

The above conclusion seems very promising because it shows the possibility of doing on-line, adaptive modal analysis. The most difficult part of realizing this concept is the adaptive algorithm that would be necessary to adjust the filter coefficients without requiring a high computational load that precludes the on-line signal processing. There is however another important problem with the parallel IIR filter configuration, namely, stability. Basically, stability requires that

$$\left| -a_{1m} \pm \sqrt{a_{1m}^{\ 2} - 4a_{2m}} \right| < 1\,, \qquad (2.24)$$

and

$$\begin{aligned} |a_{1m}| &< 2 \\ |a_{2m}| &< 1 \end{aligned} \qquad (2.25)$$

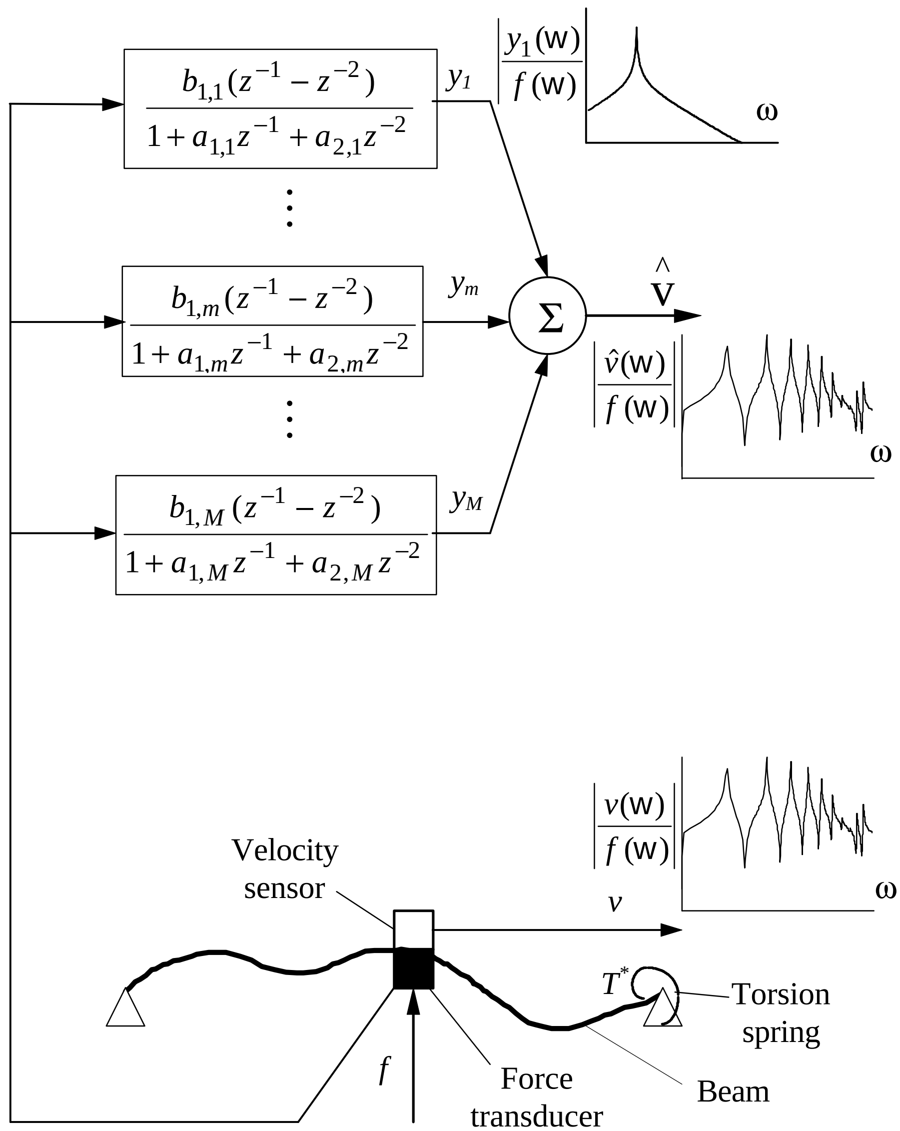

Figure 2.3 Representing a high-order structure with a parallel bank of second-order digital filters.

This issue is discussed separately in appendix D. The conclusions we can draw from the discussion is given below.

If the system to be identified is a high order system such as continuous structures, the order of the acceptable auto-regressive-moving-average (ARMA) model in Eq. (2.22) is very high. With high order models, a slight error or variation from the true values can very easily drift the digital filter poles out of the stability disc. Moreover, the algorithm used to compute the coefficients may easily turn to be unstable itself. The ‘near-instability’ of the structure couples with the instability of the algorithm to make stability problem even more difficult to deal with.

With the second-order filters, it may be easy to add a stability check routine into the adaptive signal processing algorithm to makes sure that the two denominator coefficients for each mode stay within the stable underdamped parabola of Fig. D.1.c in appendix D. However, this ‘brute-force’ approach may create more problems in the development and analysis of the adaptive algorithms. One such problem is discontinuities in the evaluation of the gradients if the gradient-search methods are used, or in the Kalman gains if the recursive-least-squares methods are used.

Because of the stability problem, we will not use the parallel IIR filter configuration in our system identification algorithm. However, we will use this model in *simulating* mechanical structures with known modal properties, and also as an important tool in the development of the on-line modal coordinate sensor. For these two purposes, we know the exact values of the filter coefficients.

2.3 Segmented Sensor Model

Originally, segmentation of strain sensors was invented to avoid the problem of observability. If a sensor is made of one continuous piece of piezoelectric film covering the whole structure, the symmetry of the structure causes the charge resulting from some positive strain to be cancelled out by the charge resulting from some negative strain. The result is that the sensor cannot sense (or observe) the symmetric mode (Cudney, 1992, Tzou and Fu, 1992). Segmentation may solve this observability problem.

This section discusses a sensor system which can sense modal coordinates of vibrating structures in real time. If we used IIR filters, we would need three filter coefficients for each mode. Finite-impulse-response (FIR) filters would require many more coefficients because this type of filters do not reuse results from previous time steps. The lack of ‘computational feedback’ is the reason for the inherent stability of the FIR filters. In many cases, especially in adaptive sensing, this guaranteed stability is worth the extra number of coefficients. One purpose of this research work is to come up with sensor/filter configurations that require minimum number of filter coefficients without tremendous increases in computational loads. A logical approach is to get as much spatial information as possible from the structure. It is for this purpose that we devise the segments of piezoelectric film as distributed sensors on the structures.

For the array of segments to be modal filters, the outputs must be multiplied by correct sets of gains. If we know the mode shape of the structure, it is easy to construct a sensor that can selectively sense one mode and filter out other modes. Lee (1987) is an excellent reference on how to create on-line ‘modal coordinate analyzers’. Lee’s design method requires one spatially continuous piezoelectric sensor film profile for each mode. The segmented design of modal sensors does not require spatially continuous shaping of the film and does not require a separate layer of sensor film for each mode. The most important advantage of our sensor is that it can be made adaptive.

2.3.1 Voltage Generated by Segment

Within its linear region, piezoelectric film generates electric charge proportional to strain and to the area of the film. Figure 2.4 shows an infinitesimal element of original length dx and width b. The three orthogonal directions for an electrically anisotropic piezoelectric material are shown by the arrows. The electrodes are on the 1-surfaces (top and bottom) of the element. The element is stretched in the 3-direction of the piezoelectric material. The charge generated by this element under strain ε is

$$dq = e_{31}\, dx\, \varepsilon \ , \tag{2.26}$$

where e_{31} is a piezoelectric charge-density-per-strain constant in the 3-direction.

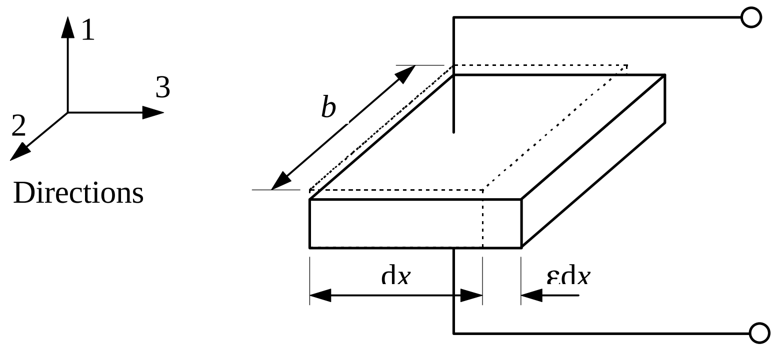

Figure 2.4 An infinitesimal piezoelectric element under strain.

Figure 2.5 shows a segment of a beam with a rectangular piezoelectric film patch attached on the top surface. The left end of the film is at $x = x_1$, and the right end is at $x = x_2$. The beam's deflection as a function of position x is denoted by $w(x)$. The piezoelectric film is so compliant compared to the beam that we can assume that the film does not affect the bending of the beam with any loading. Assuming Euler-Bernoulli beam theory, we can calculate the strain in the film, based on how the film is stretched or compressed as a function of the flexure of the beam.

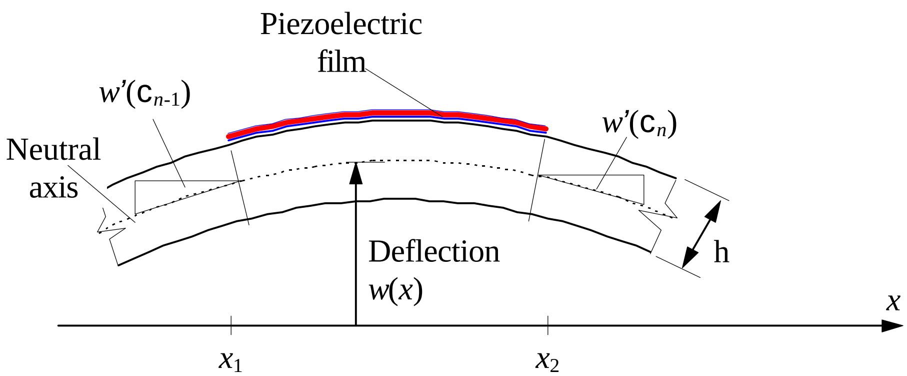

Figure 2.5 Strain in the film as a function of deflection of the beam.

Neglecting the small thickness of the piezoelectric film, the strain on the piezoelectric film can be considered the same as the strain on the surface of the beam, which is

$$\varepsilon(x,t) = -\frac{h}{2}\frac{\partial^2 w(x,t)}{\partial x^2}, \tag{2.27}$$

where h is the thickness of the beam. We obtain the total charge generated in the piezoelectric film by substituting Eq. (2.26) into Eq. (2.27) and integrating over the length of the film to yield

$$q(t) = -e_{31}\, b\, \frac{h}{2}\int_{x_1}^{x_2} \frac{\partial^2 w(x,t)}{\partial x^2}\, dx = -e_{31}\, b\, \frac{h}{2}\left(\left.\frac{\partial w}{\partial x}\right|_{x_2} - \left.\frac{\partial w}{\partial x}\right|_{x_1} \right). \tag{2.28}$$

Thus, the charge generated by the film is proportional to the difference between the slope at the right end and the slope at the left end of the piezoelectric film.

The electrical circuit model of the piezoelectric film is a charge generator in parallel with the capacitance of the film (Kynar Piezofilm Technical Manual, 1987). The voltage generated by the segment depends also on the signal conditioning circuit connected to the piezoelectric film. The signal conditioning circuit that we will use is an Op-Amp circuit with theoretically zero input impedance. The circuit representation of the film and the signal conditioner is shown in Fig. 2.6.

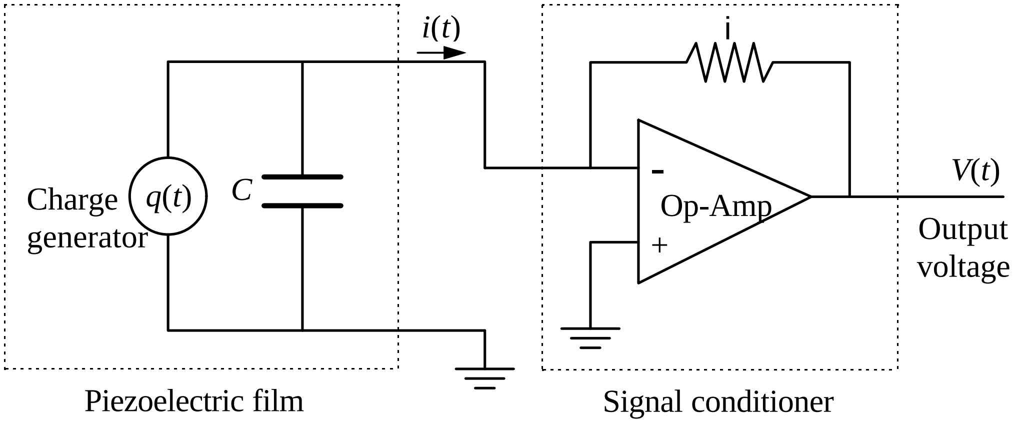

Figure 2.6 Piezoelectric film and zero-impedance signal conditioner.

Because we connect the film to a “zero-impedance” signal conditioner, the output terminals of the film are effectively shorted together. The capacitor *C* does not draw any current and therefore can be ignored. In this case, the output current is simply the time derivative of the charge generated by the film. For the signal conditioning circuit, the output voltage is the product of the current *i*(*t*) and the feedback resistance i, thus,

$$V(t) = -\text{i}\,\frac{dq(t)}{dt} \tag{2.29}$$

Substituting Eq. (2.28) into Eq. (2.29), we obtain the output voltage of the piezoelectric film segment as a function of the deflection of the beam.

$$V(t) = e_{31}\, b\,\frac{h}{2}\,\text{i}\left(\left.\frac{\partial \dot{w}(x,t)}{\partial x}\right|_{x_2} - \left.\frac{\partial \dot{w}(x,t)}{\partial x}\right|_{x_1}\right) \tag{2.30}$$

2.3.2 Array of Piezoelectric Film Segments on Beam

As shown in Fig. 2.7, *N* piezoelectric sensor segments are mounted on the beam. We will construct modal filters, i.e., linear combiners whose output emulates modal coordinates of the vibrating beam. Each modal filter consists of an array of weights (gains) that multiply the outputs of the segments. The mode-*m* filter is an array of gains $\mathbf{W}_{nm}$, where *m* denotes the mode and *n* denotes the segment number to which the gain is connected. The output of each segment is denoted by $\mathbf{V}_n$. The output of the mode-*m* sensor, denoted by $\hat{\eta}_m(t)$, is the weighted sum of the segment outputs,

$$\hat{\dot{\eta}}_{\mathrm{m}}(t) = \sum_{n=1}^{N} \mathbf{W}_{mn} \mathbf{V}_{n}(t)\,. \tag{2.31}$$

If we have *M* modal sensors, we can write the outputs of all of them as

$$\hat{\dot{\eta}} = \mathbf{WV} \tag{2.32}$$

where $\hat{\dot{\eta}}$ is an *M*-vector of sensor output, **W** is an (*M* x *N*) matrix of gains, and **V** is an *N*-vector of segment outputs $[\mathbf{V}_1, \mathbf{V}_2, \ldots, \mathbf{V}_N]^T$. Our task now is to determine the gain matrix **W** so that the modal filter outputs $\hat{\dot{\eta}}$ emulates modal coordinates.

As shown in Fig. 2.8, the left and right end positions of the n^{th} segment are denoted by c_{n-1} and c_n, respectively. We connect each segment to signal conditioning circuit with zero input impedance. For the n^{th} segment, the circuit translates the current into voltage $\mathbf{V}_n$, which can be obtained by Eq. (2.30).

$$\mathbf{V}_n(t) = u\left(\dot{w}'(c_n, t) - \dot{w}'(c_{n-1}, t)\right), \tag{2.33}$$

where

$$u = \frac{h}{2} e_{31} b i\,, \tag{2.34}$$

the dot above the variable *w* denotes time derivative and prime denotes spatial derivative.

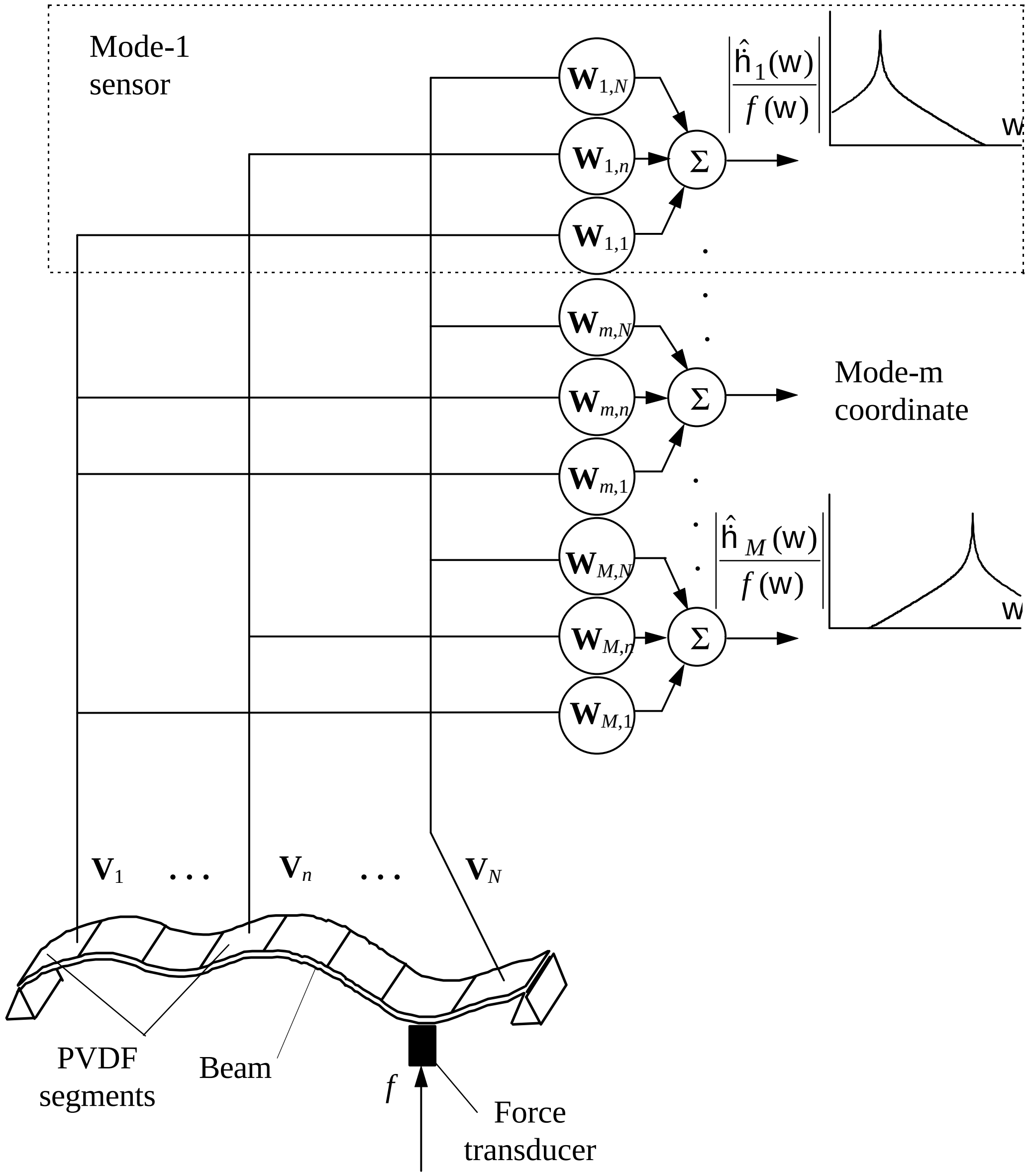

Figure 2.7 Spatial filters as modal sensors

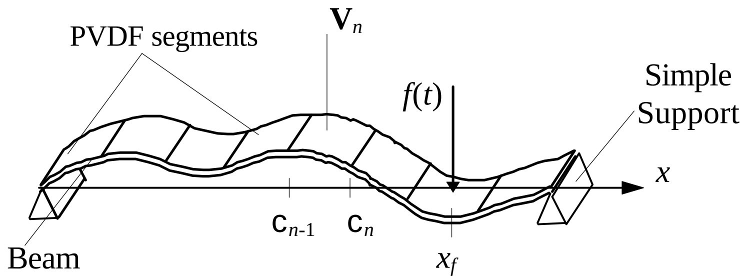

Figure 2.8 Segment positions on beam.

2.3.3 Gain Matrix for Segmented Modal Sensor

The time rate of change of a deflection slope can be expanded into its modal components according to Eq. (1.1),

$$\dot{w}'(x,t) = \sum_{m=1}^{M} \phi'_m(x)\dot{\eta}_m(t)\,, \tag{2.35}$$

where $\phi'_m(x)$ is the slope of the m^{th} mode shape, and $\eta_m(t)$ is the m^{th} modal coordinate.

Substituting Eq. (2.35) into Eq. (2.33), we obtain the segment outputs $\mathbf{V}_n$, n = 1, ..., N, as

$$\mathbf{V} = \mathbf{C_S}\dot{\boldsymbol{\eta}}\,. \tag{2.36}$$

where the (n,m) component of the *segment output matrix* $\mathbf{C_S}$ is defined as

$$\mathbf{C_S}(n,m) = \psi\big(\phi'_m(\xi_n) - \phi'_m(\xi_{n-1})\big);\;\; n = 1,\ldots,N;\; m = 1,\ldots,M\,. \tag{2.37}$$

and $\dot{\boldsymbol{\eta}}$ is the modal coordinate vector

$$\dot{\boldsymbol{\eta}} = \begin{Bmatrix} \dot{\eta}_1(t) \\ \vdots \\ \dot{\eta}_M(t) \end{Bmatrix}. \tag{2.38}$$

The segment output voltages in Eq. (2.36) are input to the linear combiners (i.e., multiplied by the gain matrix $\mathbf{W}$) to produce the sensor outputs in Eq. (2.32). The sensor output vector $\hat{\dot{\boldsymbol{\eta}}}$ and the gain matrix $\mathbf{W}$ contain as many rows as there are linear combiners. One linear combiner emulates one modal filter.

Substituting Eq. (2.36) into Eq.(2.32), we can express the outputs of the linear combiners as functions of the gain matrix, the segment output matrix, and the modal coordinates,

$$\hat{\dot{\boldsymbol{\eta}}} = \mathbf{W}\mathbf{C_S}\dot{\boldsymbol{\eta}}. \tag{2.39}$$

For the segment array to behave as modal coordinate sensors, the segment outputs **V** must be multiplied by correct gains. The gain matrix **W** is the matrix that ideally will transform the segment outputs **V** into the modal coordinates $\dot{\boldsymbol{\eta}}$. Equation (2.39) shows that the estimated modal coordinates $\hat{\dot{\boldsymbol{\eta}}}$ will be equal to the ideal modal coordinates $\dot{\boldsymbol{\eta}}$ only if the gain matrix is

$$\mathbf{W} = \mathbf{C_S}^{-1} \tag{2.40}$$

We will use the above equation to obtain the ideal gain matrices of linear combiners.

If we substitute the above weight matrix into Eq. (2.39), at the first glance it seems that the estimated modal coordinates are equal to the ideal modal coordinates. However, this is not exactly the case. Note that the gain matrix **W** and the segment output matrix $\mathbf{C_S}$ must be square. From Eq. (2.40), it is clear that to create a sensor to sense M modes we need at least $N = M$ segments. Theoretically, the structure has an infinite number of modes, therefore, the size of the segment output matrix $\mathbf{C_S}$ should be infinity. In practice, we can only use a finite number of segments. Therefore, the segment output matrix $\mathbf{C_S}$ is actually a truncated version of the ideal, and the weight matrix obtained by Eq. (2.40) is only an approximation of the ideal weights for modal filters. This approximation will result in a non-ideal estimate for the modal coordinates, as will be shown in a later chapter.

2.4 Chapter Summary

In this chapter we reviewed and developed some concepts that are required to create a sensor-and-signal-processor system that functions as a modal coordinate sensor. This system will receive its input from the mechanical vibration of a structure and produce outputs that are proportional to the modal coordinates of the structural vibration. As a transducer to convert mechanical vibration into electrical signal, we choose an array of segments of piezoelectric film. The signal-processing part of the system consists of linear combiners that act as modal filters. Each modal filter shall be sensitive to only one mode, and filter out any other modes. This chapter reviewed and developed some concepts necessary to design the modal filters.

To facilitate understanding of the concepts, we developed a structural model to use as examples in analytical development and numerical simulation. The model we have chosen is a simply supported beam with an additional torsion spring at one end. The excitation is a point force. We derived the natural frequencies, mode shape, and equation of motion for the structure.

We developed a discrete-time model for the structure. Mechanical structures can be modeled by equivalent electrical circuits. When the signals are sampled, both can in turn be modeled with digital filters. The modal-expansion form of the response of the mechanical structure is a summation of second-order-type responses. Therefore, we derived a method to model a mechanical structure with a filter composed of parallel second-order digital filters. This method transforms the modal properties of the structure (natural frequency, damping, and residue) into filter coefficients. The resulting discrete-time model is not only useful in simulating the structure with a digital computer, but also useful in further analysis of the characteristics of the structure.

The relationship between the filter coefficients and the modal properties is known precisely. Therefore, if we can adaptively adjust the filter to match the structure, we shall be able to extract the modal properties of the structure from the filter coefficients. Thus, the parallel-second-order-filter model of the structure could be used as a basis for developing an adaptive algorithm that can be used in on-line modal parameter estimation. A problem with this approach is stability. Many metal structures have very low damping that cause the z-plane poles of the structures to be very close to the edge of the unit circle. Adaptive algorithms for identifying these poles can easily come up with an unstable pole in the adaptation process, throwing the whole adaptive identification process into instability. Stability problems force us to consider other filter configuration for system identification. We chose a simple linear combiner configuration to do the filtering in space rather than time.