Optimal Demand Shaping Strategies for Dual-channel Retailers in the Face of Evolving Consumer Behavior

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ABSTRACT

The advent of the Internet has not only enabled traditional brick-and-mortar retailers to open online channels, but also provided a platform that facilitated consumer-to-consumer information exchange on retailers and/or products. As a result, the purchasing decisions of today's consumers are often affected by the purchasing decisions of other consumers. In this dissertation, we adopt an interdisciplinary approach that brings together tools and concepts from operations management, economics, systems dynamics and marketing literatures to create analytical models in order to address a dual-channel retailer's optimal demand shaping strategy, through e-commerce advertisement efforts, store service levels, and pricing, in this new environment. Our findings show that the retailer's optimal demand shaping strategy, in terms of store service levels and e-commerce advertisement effort, critically depends on the product's e-commerce adoption phase. We also show that in the presence of higher operating costs for the store channel compared to the online channels, a channel-tailored pricing policy always dominates a uniform pricing strategy. Our work sheds light on the benefits of channel integration for multi-channel retailers. We show that the retailer can leverage the online channels to provide in-store pricing and inventory availability information in order to enable a more transparent shopping experience for consumers, and this strategy results in a "win-win" situation for all parties.

"Not I, nor anyone else can travel that road for you.
You must travel it by yourself.
It is not far. It is within reach.
Perhaps you have been on it since you were born, and did not know.
Perhaps it is everywhere - on water and land."

-Walt Whitman

To Mom and Dad

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Chapter 1

Introduction

1.1 Motivation and Objectives

Just when retailers had largely mastered replenishing stores, the retail game changed with a seismic shift to omni-channel retailing. The extensive use of the Internet, along with the introduction of smartphones, gave rise to a number of additional sales channels beyond the traditional brick-and-mortar stores. As such, consumers adopted new shopping mechanisms, such as ordering an item online, through a mobile application, by phone using a catalogue, or a combination thereof. The use of technology has moved far beyond the point of purchase, though. Today's consumers use it throughout their entire shopping experience, from awareness and research, to purchasing, taking possession, and obtaining service and support (Toshiba Global Commerce Solutions 2012). For example, a consumer can start her shopping journey online by reading reviews about a specific item, move to the purchasing stage through her mobile phone, and complete her shopping experience at the store, where she chooses to take possession of her order. As channels blend together, the rules of the game are changing and retailers are trying harder than ever to come up with innovative ideas to make consumers' shopping journeys more enjoyable at all stages. In addition to new features that enable consumers to check the in-store product availability and price on the retailer's mobile app and/or websites, new shopping mechanisms such as Order Online/Pick up at the Store or Order at the Store/Ship Home have emerged in the past few years to provide consumers with a more transparent, convenient, and faster shopping experience.

The advent of the Internet in the past decade also provided a platform that facilitated consumer-to-consumer information exchange on retailers and products (Chiang and Chhajed 2005, Toshiba Global Commerce Solutions 2012). As a result, the purchasing decisions of today's consumers are greatly influenced by the purchasing decisions of other consumers (Netessine and Tang 2009). Consumer interactions in the physical and in the virtual world are crucial when it comes to adopting new services and shopping mechanisms such as e-commerce. The information obtained through word of mouth is also important for consumers when they are building estimates about the retailer's service quality (e.g. inventory service levels). However, the ever-growing interactions among consumers create challenges for a dual-channel retailer's demand forecasting and management by leading to dynamically evolving demand patterns across channels over time and by amplifying the effect of poor service levels on demand.

Consumer demand is also impacted by the pricing of the item across various channels (Trade Extensions 2014). Interestingly, research indicates that one of the biggest challenges that dual-channel retailers face is that consumers expect a similar experience through all available channels rather than a channel-specific one. This implies that products must be available at all channels, and the pricing structures must be consistent across the supported channels. That is, "inconsistent messaging, pricing, promotions, or experiences cause customer confusion, leading to low customer satisfaction and reduced spending" (Aberdeen Group 2012). In order to mitigate these challenges, it becomes crucial for the retailer to shape the demand optimally across channels using the decision levers the retailer has.

1.2 Contributions

A main strength and a unique characteristic of this dissertation is adopting an interdisciplinary approach that brings together tools and concepts from the operations management (OM), economics, systems dynamics and marketing literatures. In particular, using these different methodologies, we develop analytical models so as to determine a dual-channel retailer's optimal demand shaping strategy, through three types of levers: e-commerce advertisement efforts, store service levels, and pricing.

In our first study, we develop a novel model that represents both the diffusion of e-commerce within the society and the consumer's channel choice, in the presence of consumer learning, which refers to learning about the product's availability at the store through own experiences and the information shared by other consumers. We characterize the settings in which e-commerce advertisement is profitable for the retailer, and show that the retailer's optimal demand shaping strategy critically depends on the product's e-commerce adoption phase. Interestingly, if the retailer provides the consumers with information on store availability levels, then this results in a "win-win" situation for all parties. In this case, the retailer's optimal service levels do not need to vary over time, even as e-commerce adoption grew.

In our second study, we consider the retailer's optimal demand shaping policy through pricing across channels. Specifically, we shed light on the impact of channel-tailored pricing policies on consumer behavior, and examine some information-sharing strategies the retailer could use to mitigate the aforementioned challenges resulting from those channel-tailored policies. Our study shows that when there is information asymmetry on the store price, i.e. when consumers use the online price as a proxy to estimate the store price, a channel-tailored pricing policy always dominates a uniform pricing policy. In this setting the retailer can further improve its profit by simply making its store price information available on its website, hence, providing consumers with accurate information regarding the actual store price.

The remainder of this dissertation is organized as follows. In Chapter 2, we present recent trends and advances in the retailing industry, and provide a comprehensive review of the relevant literature. In Chapter 3, we present the manuscript of our first study, which is on the retailer's optimal demand shaping strategy using e-commerce advertisement and store service levels under consumer learning. Then, in Chapter 4, we present our second study, which focuses on how a dual-channel retailer should optimally shape its demand through pricing. Lastly, we conclude with future research directions in Chapter 5. To improve the presentation of this dissertation, all proofs are relegated to the Appendix.

Chapter 2

Recent Advances in the Retailing Industry and Related Research

2.1 Recent Advances in the Retailing Industry

The technological revolution in the past decade has transformed the retailing industry tremendously by enabling: 1) manufacturers (e.g. Dell) that would typically sell their products through a brick-and-mortar retailer (e.g. Best-Buy) to open online channels, allowing them to reach consumers directly, and 2) traditional brick-and-mortar retailers (e.g. Best-Buy) to expand their marketing, sales, and distribution efforts to multiple channels such as online and mobile (see Figure 2.1). By taking advantage of the emerging sales mechanisms, retailers have been transitioning to a new supply chain strategy – "omni-channel retailing" – a business model that focuses on multi-channel ordering and delivery methods that enable products to be bought, received, and returned anywhere within an integrated supply chain network in which a uniform consumer experience with the brand across all available channels is essential.

In this increasingly competitive marketplace where information is only milliseconds away, end-to-end consumer experience is the new battleground. Success in retailing today requires companies to deliver a superior, differentiated shopping experience that fits ever-changing consumer needs and preferences (Achabal et al. 2005). Figure 2.2 shows how the retail industry has evolved from single-
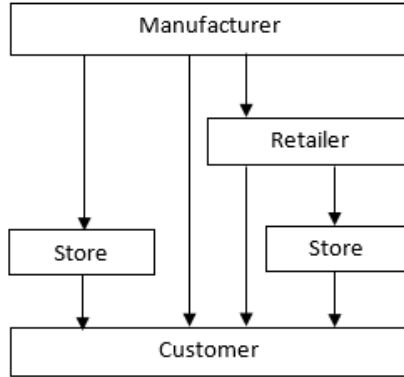
Figure 2.1: Multi-channel supply chain.

channel to omni-channel throughout the years. A single-channel strategy, which allowed consumers to shop mostly at brick-and-mortar stores (an exception includes Sears Roebuck and Company, which began as a mail order catalog company in 1886 and opened retail locations beginning in 1925), has been outdated with the rise of e-commerce in the 1990s. On the other hand, shopping through multiple channels that operate independently of each other is no longer sufficient for consumers. In the new millennium, consumers want to be able to shop through a variety of channels (e.g. store, online, mobile, catalog, and phone) and have the flexibility to switch channels at different stages of their shopping experience. For instance, a shopper might want to do her product research online, order the item through an app on her smartphone, and pick her order up at the store. Most retailers operate in various channels: while this allows for cross-channel interactions with their brands, the channels still remain loosely connected. This retailing model, though, is slowly transforming to a superior one where cross-channel reality is supplemented by technology to deliver a truly seamless shopping experience across multiple channels. With this new business model, consumers can easily switch channels at various phases of their shopping journey by experiencing a uniform view of the brand at all stages (Achabal et al. 2005). According to Tadd Wilson, Senior Managing Consultant at Toshiba Global Commerce Solutions (2013), "Most retailers face a cross-channel reality and have omni-channel aspirations."

Omni-channel retailing can be formally defined as "a set of decision and business processes that enable consumers to have a seamless experience with the brand across all available channels such
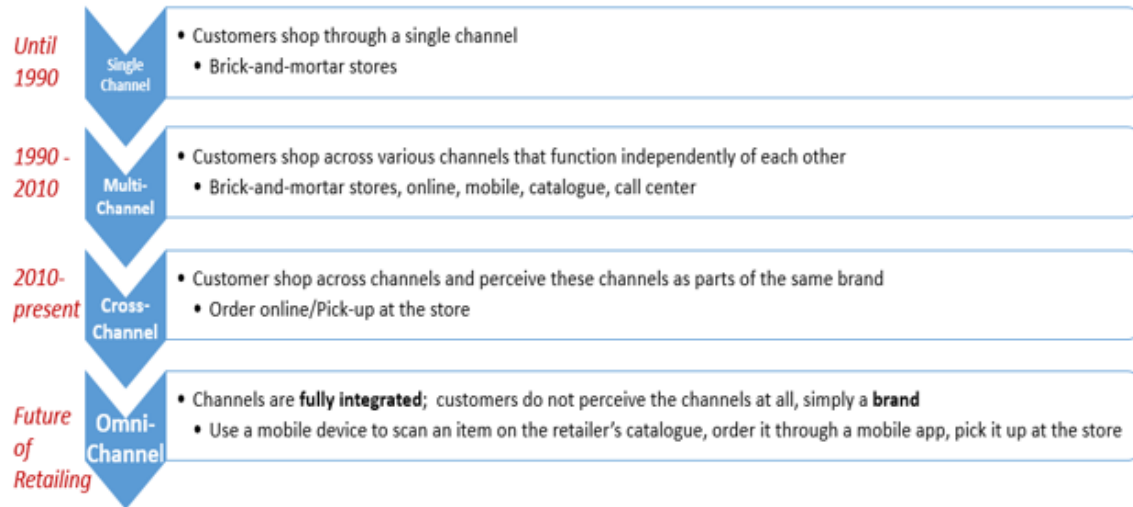
Figure 2.2: The evolution of omni-channel retailing.

as brick-and-mortar stores, the Internet, catalogues and a call center through an integrated supply chain network design, data management, pricing strategy and marketing plan." Omni-channel retailing demands continuous improvement of multi-channel workflow integration within merchandising, order management, marketing and consumer experience through all stages of consumers' shopping experiences from awareness and research to purchasing, taking possession and obtaining service and support (e.g. exchange and return) (Aberdeen Group 2012). Figure 2.3 illustrates that omni-channel retailing provides consumers with a uniform view of the brand, which is sustained through a strong integration among the available channels and through effectively managing inventory, product assortment, pricing, delivery decisions, and consumer loyalty programs and promotions.

In an omni-channel environment, shoppers view a given retailer's sales channels not as separate entities (e.g. a store, a website and a catalog operation), but as a single brand and company that operates in a consistent and harmonious way regardless of the touch point (Achabal et al. 2005). Therefore, consumers demand similar experience through all the outlets regarding pricing, promotions, product offerings etc. (Aberdeen Group 2012). This "anytime, anywhere" retailing places great pressure on organizations to reinvent their infrastructures because "Retailers are serving greater volumes of shoppers through increasing number of channels, but they are doing this

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Figure 2.3: The components of omni-channel retailing.

by using supply chains that were built to serve stores" states RIS News Group Editor-in-Chief Joe Skorupa (4R Systems 2013). In order to overcome the challenge, most companies concentrate on "front-end" activities that consumers directly experience (e.g. promotions, apps, in-store Wi-fi etc.); however, a focus on what goes on behind the scenes (product offerings, pricing, delivery) is much needed to accommodate the changes in the industry. Figure 2.4 presents a framework that shows various decisions that retailers need to make with respect to consumers' channel choices and respective product flows in an integrated supply chain where consumers can easily move from one channel (e.g. online) to another (e.g. store). In addition to providing the consumers with new tools and technologies (e.g. mobile apps, in-store Wi-fi and personalized promotions) to improve omni-channel shopping experience, retailers must optimize various activities in the channel planning and execution stages. Such decisions include strategic decisions such as supply chain network design as well as tactical (e.g. pricing, product assortment and consumer segmentation) and operational decisions (e.g. inventory management, warehouse management and data management). According to Retail TouchPoints Research Group, many challenges revolve around inventory efficiencies across the distribution network due to uncertain demand from various channels (RetailTouchPoints

2012). In order to promote high consumer satisfaction (which leads to high brand loyalty), retailers should serve consumers from various channels efficiently. In that regard, an effective order allocation model, which takes into account various factors including consumer segmentation, product assortment, pricing, etc., is needed to excel in the industry (Achabal et al. 2005).
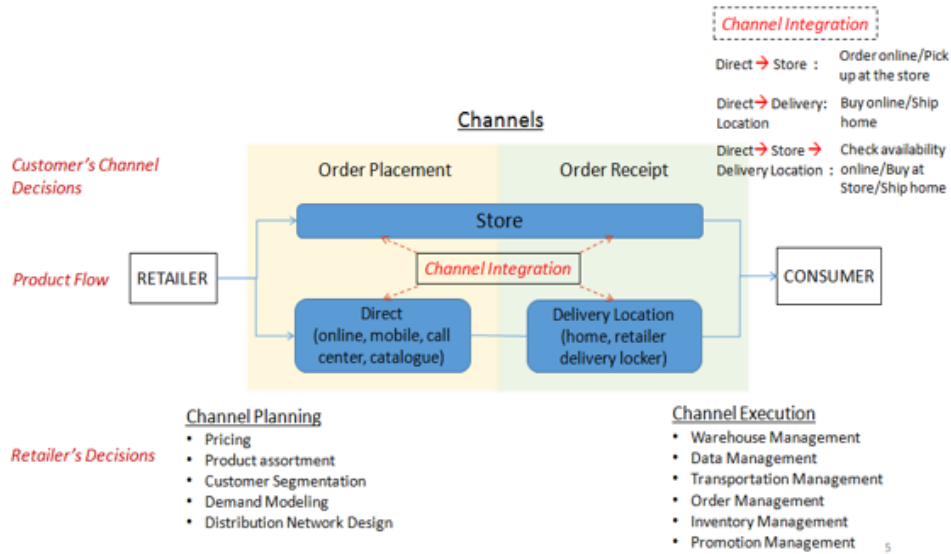
Figure 2.4: Omni-channel supply chain design.

As consumers demand to shop "anytime, anywhere," retailers are trying various approaches to reinvent their traditional store-serving supply chains so that they can better address these emerging issues. For instance, Walmart is investing $430 million to better its e-commerce operations, which includes building new warehouses for online orders, while Macy's is turning its stores into online fulfillment centers (Banjo 2013, Ryan 2013). On the other hand, Amazon, the largest e-tailer, invests in Amazon lockers, which are located at various stores including Seven-Eleven, RadioShack and Staples to reduce distribution costs (Banjo 2013). Figure 2.5 shows alternative inventory locations and delivery paths.

In order to obtain a thorough understanding of omni-channel retailing research, the current state of this new business model, and the challenges that are yet to be addressed, we review a variety of resources in industry and academia in the next two sections. Industry research reveals the current state of omni-channel retailing through surveys and benchmark studies, and provides roadmaps
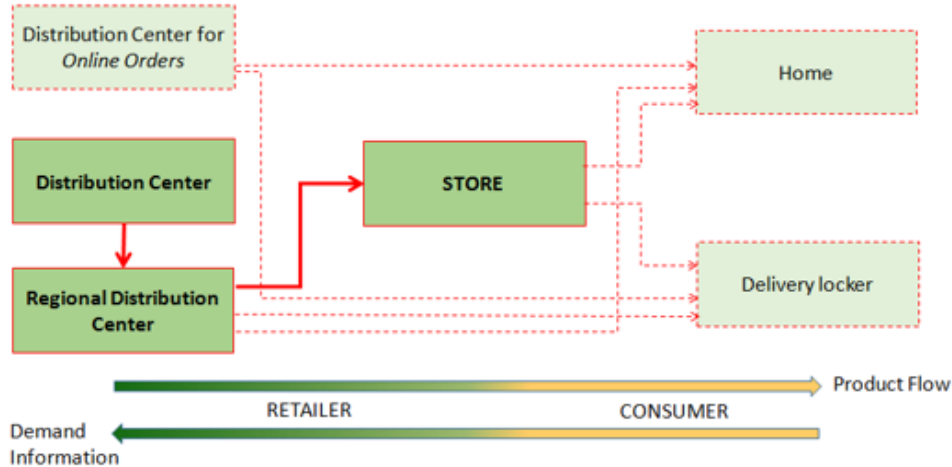
Figure 2.5: Emerging order fulfillment paths in omni-channel retailing.

and tools (e.g. software) for companies to excel in this new business environment. Although qualitative research emphasizes the need for tools which would integrate various aspects of the problem (e.g. pricing, inventory, and order fulfillment), current software only provides modules that deal with these issues separately. Scholarly research, is still scarce. The majority of papers come from the dual-channel literature in which online and store channels work independently with no interaction, and the dynamic change in consumers' adoption of emerging channels is not addressed. Marketing literature provides insights on pricing issues, while operations management literature provides inventory control and distribution models in multi-channel supply chains. Consequently, this literature provides practitioners with a very limited knowledgebase on how to structure omni-channel supply chains and how to make decisions in an omni-channel setting.

2.2 Industry Research

By contrast to single-channel retailing and multi-channel retailing, which have been widely explored by marketing and operational perspectives, industry research on omni-channel retailing is limited. The majority of industry white papers focus on defining the term, and providing roadmaps for a successful transition from a multi-channel to an omni-channel approach to retailing as most companies are still in the process of converting their supply chains to omni-channel supply chain networks which are not mature enough for optimization yet.

A white paper by Toshiba Global Commerce Solutions (Toshiba Global Commerce Solutions 2012) states that although non-store channels only account for 5% of total retail spending globally, retailers view cross-channel shoppers as being more valuable than single-channel shoppers. Consequently, it is crucial that retailers make every effort to gain the loyalty of cross-channel consumers through a seamless shopping experience across all channels and at all stages of their shopping process. Providing a superb shopping experience across all channels, however, requires an integrated supply chain infrastructure with an efficient order fulfillment strategy. Similarly, an Aberdeen Group survey (2012) of 80 organizations reveals that the biggest omni-channel pressure felt by retailers (49% of the retailers responded "Yes" to this question) is "Lost sales opportunity costs," which explains why "Ensuring product availability across all channels" has been selected as the most important strategic action in the survey. Furthermore, survey data shows that the "leaders" are twice as likely as "followers" to have accurate demand data across all channels. Aberdeen Group concludes that accurate demand data across channels not only improves merchandising plans, but also helps establish right product assortment-mix across channels, inventory allocation at stores and distribution centers, and replenishment strategies to prevent stock-out situations that incur high opportunity costs (Aberdeen Group 2012). Likewise, Retail TouchPoints Research emphasizes the profitability of the cross-channel shoppers and concludes that "the crux of many cross-channel challenges revolves around the advanced collection of data, and using the data to create inventory efficiencies" (RetailTouchPoints 2012). 4R Systems suggests that as channels blend more and more through consumers' cross-channel shopping habits, inventory efficiency can no longer be sustained through separate inventory streams for in store and digital commerce (4R Systems 2013). Lastly, IBM Business Consulting and Santa Clara University's Retail Management Institute present a roadmap for cross-channel optimization in omni-channel businesses. The framework consists of four stages:

1. Align fundamentals (ensure basic value propositions such as assortment and pricing are in sync),

2. Achieve proficiency (integrate key consumer facing processes),

3. Leverage across channels (exploit channel-specific capabilities and drive cross-channel collaboration), and

4. Optimize operating model (optimize resource allocation at enterprise level) (Achabal et al. 2005).

Some multi-channel software is available in the market. Such products include WebSphere Commerce (IBM), Celerant (Celerant Command Retail), CoreSense (CoreSense, Inc.), Hybris (hybris), and StoreNet Live (VendorNet), which provide businesses with multi-channel solutions regarding marketing, promotions, social commerce, order management, pricing, cross-channel order processing, employee management and warehouse management. Figure 2.6 provides a comparison chart for various multi-channel software available in the market with respect to the modules they contain. While built-in modules help companies to enhance their front-end activities to provide more enjoyable and personalized shopping experience for the consumers, the existing software fails to provide a comprehensive model that takes into account the interdependencies between price, product assortment, lead time, and demand, and the combined effect of these factors on order fulfillment strategies. By focusing on marketing activities, industry software helps retailers increase their non-store sales, but without firm infrastructure for effective decision making regarding inventory management, pricing, and order fulfillment, higher sales do not always lead to increased profits. Therefore, there is a big research opportunity for a much needed omni-channel decision support system, which handles these issues hand in hand in the face of growing adoption of emerging channels by the consumers over time.

2.3   Academic Research

Several research papers and books such as Agatz et al. (2008) and Simchi-Levi et al. (2004) provide excellent reviews on the impact of the Internet on existing supply chains. Most previous work up to that point, though, considers either online-only supply chains or coordination strategies and inventory and distribution management at a strategic level in multi-echelon supply chain where the higher echelon is the manufacturer and the lower echelon is the retailer. In particular, Agatz et

	Marketing	Promotions	Social Commerce	Order Management	Pricing	Cross-Channel Order Processing	Employee Management	Warehouse Management
IBM	√	√	√	√	√	√		
Celerant	√	√		√	√	√	√	√
CoreSense	√	√	√	√	√	√		√
hybris	√	√	√	√	√	√		√
VendorNet				√		√		√

Figure 2.6: Multi-channel software comparison.

al. (2008) identifies the issues and existing models related to two core areas: 1) sales and delivery planning, and 2) supply management. Sales and delivery planning includes delivery service design, pricing and forecasting, order promising and revenue management, and transportation planning. Supply management, on the other hand, covers areas such as distribution network design, warehouse design, and inventory and capacity management. Alternatively, Simchi-Levi et al. (2004) discusses the specific aspects of online and multi-channel retailing, regarding conflict and coordination in multi-channel distribution systems, coordination of pricing inventory decisions, etc. However, with the growing use of the Internet, additional services such as mobile shopping, in-store Wi-fi, Order online/Pick up at the store, and Order in store/Ship to home are emerging. Hence, new research is being conducted to answer some of the questions related to these emerging services.

In the remainder of this section, we first review a representative set of recent marketing papers on multi-channel retailing. This strand of research primarily focuses on consumer behavior through empirical studies. In particular, the drivers of consumer behavior toward online and offline channels are discussed. Then, we review analytical research recently conducted in the operations management (OM) literature. The first strand of research in this area is focused on the dynamics between a manufacturer and retailer in a decentralized supply chain in the event that the manufacturer opens a direct sales channel, which competes with its already existing retailer channel.

Specifically, coordination strategies for the manufacturer and the retailer which face both vertical and horizontal competition are discussed in this section at a strategic level. The second strand of research in the OM literature is concerned with the optimization of tactical decisions in a multi-channel supply chain. The research in this area is primarily focused on inventory and distribution strategies formulated in a way that is similar to multi-echelon supply chain settings.

2.3.1 Marketing Literature

The marketing literature on multi-channel retailing is primarily focused on consumer behavior within emerging channels in the retail industry. Specifically, authors adopt a common surveying approach complemented with statistical analyses to present their findings in two main areas: 1) drivers of channel choice within the multi-channel framework, and 2) consumers' adoption level of emerging online channels with respect to various product categories.

Understanding the factors that drive consumers to choose various channels within a centralized supply chain is paramount to understanding consumer behavior, which is at the core of designing efficient pricing, product assortment, inventory management, and distribution strategies for the entire supply chain. Schoenbachler and Gordon (2002) provide one of the earliest works that attempt to provide a conceptual model to explain various factors that influence consumers' choice of channels. In particular, the authors identify five main factors that impact the likelihood that a consumer will purchase through a particular channel. These factors, listed here, include not only consumer-related attributes but also retailer- and product-related factors:

1. "Perceived risk" refers to financial, social or physical risk related to the purchase. Perceived risk related to a specific channel stems from the individual's familiarity with the retailer, the brand or the Internet as well as her perception of the security of personal information or purchase in the specific channel.

2. "Past direct marketing experience" refers to when and how the consumer purchased in the past. Since past behavior is a strong indicator of future behavior, a consumer who had a pleasant experience with a channel in the past would be more willing to purchase through the same channel in the future.

3. "Motivation to buy from a channel" refers to consumer demographics such as age, income, gender, number of credit cards and lifestyle factors such as need for convenience and views on shopping for entertainment as well as the purpose of the purchase (e.g. gift vs. self). For example, consumers who view shopping as a form of entertainment would be more likely to shop in store.

4. "Product category" undoubtedly impacts consumers' shopping behavior. For instance, automobiles are most often purchased via traditional retail means, while electronics are more likely to be purchased at the store.

5. "Web site design" refers to the user friendliness of the web page and purchase process as well as the eye appeal of the interface. Consumers would be more willing to purchase through web sites that have better designs.

Kumar and Venkatesan (2005) extend the prior work by Schoenbachler and Gordon (2002) with a slightly modified conceptual framework that is complemented with an empirical analysis using the consumer database of a high technology hardware and software manufacturer. The authors' conceptual framework consists of "customer-level characteristics" including cross-buying (i.e. the number of different product categories that the consumer buys through the firm), return behavior, consumer tenure, purchase frequency, etc. and "supplier factors" including number and type of contact channels with consumers, the interactions among which can impact multi-channel consumer behavior. The authors evaluate the association of the aforementioned factors on multi-channel shopping by using an ordered logistic regression, and also analyze the performance of multi-channel shoppers compared to single-channel shoppers in terms of several consumer based metrics, including revenues, past consumer value, share of wallet, and predicted propensity to stay in the relationship. Some of the important findings in this work suggest that consumers who buy across multiple product categories and have past experience with the retailer through the online channel have longer tenure and purchase more frequently. The authors also find evidence for a nonlinear relationship between returns and multichannel shopping, and that there is a positive synergy towards multichannel shopping when consumers are contacted through various communication channels. Interestingly,

consumers who shop across multiple transaction channels provide higher revenues, higher share of wallet, have higher past consumer value, and have a higher likelihood of being active than other consumers.

As opposed to the prior work, wherein authors simply define multi-channel shoppers as those who make a purchase in more than one channel in the observed time period, Schröder and Zaharia (2008) define multi-channel shoppers as those who obtain information in one channel and make the purchase in another channel (of the same retailer). They conduct a survey with 525 consumers of a multi-channel retailer that operates five different channels to identify consumers' channel choice with respect to five different factors including recreational orientation, convenience orientation, independence orientation, delivery-related risk aversion, and product- and payment-related risk aversion. Consumers are divided into groups based on the channel in which they obtain information and the channel through which they make a purchase. The survey revealed that consumers who solely shop through store channels care about recreational purposes the most while consumers who shop through online channels only look for convenience and independence. Surprisingly, their findings show that consumers who use multiple channels (i.e. obtain information in the online channel and make a purchase in the store channel) combine the independence feature of online shopping with the reduction of risks associated with buying products in the physical store channel.

Benedicktus et al. (2010) argue that "trust" is one of the greatest factors which could improve the competitiveness of an online-only retailer with multi-channel retailers that have physical presence in the market. The authors first examine the main effects and interactions of consensus information (i.e. the extent of satisfaction agreement among previous consumers), physical store presence, and brand familiarity on trustworthiness and purchase intentions. Then, they extend their work to examine the effects of consensus information and brand familiarity when consumers are actively suspicious and less inclined to trust sellers in general. The authors find that consensus information provides a broad cue that conveys trustworthiness and leads to greater purchase intentions. The authors also find that that consensus information alone is not sufficient to buffer against active, generalized suspicions online. Instead, a combination of high consensus and brand familiarity is necessary for this purpose.

Levin et al. (2005) provide a conceptual model to explain how consumer characteristics such as age, gender, income and computer literacy as well as channel attributes including delivery speed, selection and price for a specific set of products impact online shopping behavior and their model by conducting a survey. In the study, the participants are asked to rate 1) which key attributes are delivered better online than offline, and 2) the importance of each key attribute in both the search stage and the purchasing stage of the product. The findings show that the single best predictor of online/offline shopping preferences is self-reported consumer literacy. Not surprisingly, when consumers demand a fast checkout or a large selection of a product, online shopping is preferred; on the other hand, when attributes such as personal service and ability to see, touch and handle the product are predominant, offline shopping is preferred.

Kacen et al. (2013) uses a similar methodology to test for differences in consumer acceptance of online channels with respect to various product categories. The authors conduct a survey in which the participants rate how well each channel performs in various attributes (such as low price, high quality, availability level, and variety), which have different weights for importance (rated by the consumer). The authors find that all product categories evaluated, including books, shoes, flowers, toothpaste, DVD players, and food items, have lower online acceptance, yet the product category with the highest online acceptance rate is books while the lowest is shoes. Specifically, even though online stores have advantages in brand selection and variety and ease of browsing, those advantages are overcome by shipping and handling charges and exchange-refund policies providing an interesting social or family experience, helpfulness of salespeople, post-purchase service, and uncertainty about getting the right item.

2.3.2 Operations Management Literature

The OM literature on multi-channel retailing is focused on:

1. Strategic decisions with respect to multiple parties in a decentralized supply chain (i.e. a manufacturer and a retailer), and

2. Strategic and tactical decisions in a centralized supply chain (i.e. a single multi-channel retailer).

The first strand of research concentrates on contractual agreements, which are primarily used to coordinate decentralized supply chains in business-to-business settings. The second, on the other hand, focuses on optimal inventory management and distribution strategies for multi-channel retailers in business-to-consumer settings.

Competition and Coordination in Dual-channel Supply Chains

The coordination strategies in decentralized supply chains with a single supplier and single retailer (or multiple competing retailers) have been well studied in the literature (e.g. Chen et al. 2001, Bernstein and Federgruen 2003, and Bernstein and Federgruen 2005). The setting we cover in this review paper, though, is slightly different in that the manufacturer not only is the supplier for the retailer, but also a market competitor. As mentioned before, the common approach in this research area is designing contracts to coordinate the supply chain by using game-theoretic approaches.

Chiang et al. (2003) study the profitability of opening a direct channel that competes with the existing retail channel considering various levels of consumers' acceptance of a direct channel (θ) utilizing a price-setting game. The consumers are heterogeneous in their valuation of the product (v), and purchase the product through the utility maximizing channel given the different pricing scheme (determined by a Stackelberg game where the leader is the manufacturer and the retailer is the follower) across channel and the consumer acceptance level for the product. As the Stackelberg leader, the manufacturer determines the optimal wholesale price and the direct channel price that would maximize its profits anticipating the retailer's pricing decision. Then, given the wholesale price, the retailer determines its optimal selling price. The authors' key finding in this setting is that when the consumer acceptance of the direct channel is high enough, the manufacturer can open a direct channel strategically to alleviate the double-marginalization problem. Double-marginalization occurs when upper and lower echelon parties in a vertical supply chain compete for the same market, which, in turn, results in higher prices, lower demand, and lower profit for both parties. In this setting, the retailer sees the direct channel as such a considerable threat that it cuts the price aggressively, which increases sales in the retailer channel, and, in turn, increases profits of the manufacturer through a higher sales volume.

Similar to Chiang et al (2003), Tsay and Agrawal (2004a) study the optimal channel structure for a manufacturer which can operate two channels-direct and retail. Assuming that the product price is uniform across channels, the authors differentiate between channels by the sales effort in each channel. In that setting, the aggregate deterministic demand in each channel is increasing in the sales effort in both channels. That is, a high sales effort in one channel not only increases demand in that channel, but also in the competing channel. The authors study three different strategies (i.e. direct only, retailer only, and dual-channel) within a game-theoretic framework and, similar to Chiang et al. (2003), find that opening a direct channel is not necessarily detrimental for the retailer. In fact, in order to retain some of the retailer's sales effort, the manufacturer will lower the wholesale price which will create a net system-wide efficiency gain to share because the wholesale price reduction can counteract double marginalization. The authors also find that revisiting the wholesale pricing terms can improve the overall efficiency of a dual-channel system. However, the greatest improvements are realized when the pricing is premised on the reseller's sales effort, which may be difficult or impossible to monitor in practice. Fortunately, certain schemes observed in industry do not have this requirement. These include paying the reseller a commission for diverting all consumers toward the direct channel, or conceding the demand fulfillment function entirely to the reseller.

Boyaci (2005) considers a manufacturer that sells products through a direct channel and an independent retail channel. The author focuses on the stocking levels across both channels in the face of stock-out based channel substitution (horizontal competition) and vertical competition (double-marginalization) due to the hierarchal nature of the supply chain. As opposed to Chiang et al. (2003), the channel structure decision is not considered; that is, it is assumed that the manufacturer utilizes both channels at all times under various prices (uniform across both channels), and wholesale prices which are assumed to be exogenous. The findings show that both channels tend to overstock due to substitution, which gets more pronounced with increasing substitution rates. On the other hand, increasing double-marginalization through a higher wholesale price results in overstocking in the manufacturer's direct channel and understocking in the retailer channel. The author considers various contracts and finds that price-only, buyback, rebate, revenue sharing and

vendor managed inventory contracts do not coordinate the supply chain since they cannot overcome the vertical and horizontal competition inherent in the system. On the other hand, penalty and compensation commission contracts have the capability to coordinate the supply chain although they are hard to implement in practice.

Similarly, Chiang and Monahan (2005) consider a multi-echelon setting and investigate optimal inventory levels for the direct and retail channel channels in the supply chain. Similar to Boyaci (2005), the authors consider parameterized stock-out based channel substitution across channels. As opposed to prior work, the authors use a Markov Model with a Poison demand function as a modeling and solution methodology, and base their conclusions on channel performance on long-run (steady state) average inventory holding and lost sales costs for each channel. The parametric analysis performed by the authors shows that increasing the consumers' willingness to switch channels when a stock-out occurs can possibly increase the total inventory related costs across channels. The authors also examine the performance of two other possible channel distribution strategies–retail-only and direct-only. The numerical study shows that the dual-channel strategy outperforms the other two channel strategies in most cases, and the cost reductions realized by the flexibility of the dual-channel system may be significant under certain circumstances.

Cattani et al. (2006) analyze a similar scenario to that considered in prior work where a manufacturer with a traditional channel partner opens up a direct channel in competition with the traditional channel. The authors study the pricing problem across channels using a game-theoretic approach. Specifically, they consider three scenarios:

1. Employ uniform pricing strategy that keeps wholesale prices as they were before,

2. Keep retail prices as they were before, and

3. Set wholesale and retail prices to maximize the manufacturer's profit.

For each of these strategies, the authors determine how the resulting prices compare to single-channel prices, and determine the resulting profits for the supply chain and its two players. The key findings indicate that the specific equal-pricing strategy that optimizes profits for the manufacturer is also preferred by the retailer and consumers over other equal-pricing strategies. In addition, they

find that equal-pricing strategy is appropriate as long as the Internet channel is significantly less convenient than the traditional channel. If the Internet channel is of comparable convenience to the traditional channel, then the manufacturer has tremendous incentive to abandon the equal-pricing policy, which is detrimental to the traditional retailer.

In addition to the previously mentioned literature that studies conflict and coordination issues at a strategic level in multi-channel supply chains in the existence of a manufacturer and a retailer under both vertical and horizontal competition, another body of literature is focused on multiple aspects of inventory and distribution strategies in centralized multi-channel supply chains. In the next section, we summarize some prominent papers in this literature.

Inventory and Distribution in Dual-channel Supply Chains

One of the earliest works in the inventory and distribution related literature in a multi-channel retailing context is provided by Alptekinoglu and Tang (2005) wherein the authors consider a centralized two-stage supply chain network with multiple depots and sales locations. In particular, the demand at each sales location is stochastic and correlated with other sales locations. In addition, the depots hold no inventory. In this setting, the authors aim to determine the optimal ordering and allocation policies for each depot (i.e. how much stock to order, and how much stock to allocate to each sales location) so that the total expected distribution cost, which consists of transportation, inventory holding, and backordering costs, is minimized. The authors solve this general model (which is applicable to a multi-channel supply chain setting) with a decomposition scheme that breaks down the original multi-channel distribution network into m independent single-depot sub-networks, each of which possesses the structure of a single-depot distribution system. Then they extend the Eppen and Schrage model to analyze these single depot sub-networks. The authors' key findings show that the correlation of demands across sales locations (channels in the multi-channel context) play a significant role in ordering and allocation policies.

A series of papers by Mahar and his colleagues study various aspects of distribution issues in multi-channel retailing. As such, similar to Alptekinoglu and Tang (2005), Bretthauer et al. (2010) study where and how much inventory should be held at each site for a company that operates mul-

tiple sales and distribution channels. As opposed to Alptekinoglu and Tang (2005), who considers a single depot assignment to a sales location, the authors consider multiple depot assignment to a sales location for inventory allocation by using a similar Eppen and Schrage framework. The authors' key findings indicate that 1) the percentage of sales occurring online is the major determinant of the number of sites providing e-fulfillment when all costs (inventory holding, backorder, and shipping) are considered; 2) when holding and backorder costs are the only consideration, the standard deviation of in-store demand is a major factor in determining where online inventory should be located; and 3) an increase in unit shipping costs does not necessarily imply that adding online fulfillment locations will drive the total cost down.

Mahar and Wright (2009) study the order fulfillment postponement strategy within the multi-channel distribution context. Specifically, the author investigate the profitability of postponing the fulfillment of online orders, that is, accumulating the online orders before determining which fulfillment site the orders should be assigned to instead of using the closest fulfillment center to the consumer to fulfill the order right away. The authors develop a "quasi-dynamic" allocation policy that assigns accumulated online sales to fulfillment locations based on expected inventory, shipping, and consumer wait costs, and conclude that accumulating online orders before assigning them to order fulfillment centers is the optimal strategy as this approach leverages more information on the inventory position of each fulfillment site before making the actual fulfillment decisions.

As a follow-up work, Mahar et al. (2012) study a newly available service by the retailers: "Order online/Pick up at the store," wherein the retailer shows the real-time availability of the item at various pick-up locations (i.e. stores) on its website. The paper compares two strategies that can be employed by the retailer: showing all pick-up sites that hold the item vs. showing a subset of pick-up sites with available inventory. The authors develop a dynamic pickup site inclusion policy that utilizes real-time information to determine which of the firm's e-fulfillment locations should be presented at online checkout. The authors' key findings show that it is more profitable for the retailer to show a subset of pickup sites with available inventory, which protects stores with critically low inventory levels and hence reduces backorder costs.

Even though returns by consumers constitute one of the major problems in multi-channel retail-

ing, research that studies this problem is very scarce. Our comprehensive literature search shows that Yao et al. (2005) provides the only analytical work in this field. The author considers a manufacturer and a retailer within a decentralized supply chain and analyzes the impact of information sharing between the manufacturer and the retailer on the manufacturer's optimal return policy within a game-theoretic framework. The author finds that: 1) if the manufacturer and the retailer do not share forecast information about the ratio of the consumer demand across channels, the returns policy becomes more restrictive if the manufacturer overestimates the demand in the direct channel; and 2) the returns policy remains the same regardless of the shift of consumers between two channels if the manufacturer decides to share information with the retailer.

2.4   Conclusions and Future Research Directions

In this chapter, we present some recent advances in the retailing industry, and provide a comprehensive review of some of the most recent papers relevant to omni-channel retailing. Omni-channel retailing is an emerging business model, and even though retailers have been trying to achieve an efficient, seamless supply chain in the past decade, academic research that complements the new developments in the industry is still at its infancy. Hence, our main resource in this work is multi-channel distribution literature. In particular, we provide examples of: 1) descriptive research from the marketing literature that addresses consumer behavior towards multi-channel retailing, and 2) prescriptive research from the OM literature that is focused on two areas: a manufacturer's decision to open an online channel that competes with an already existing retail channel in a decentralized setting, and a multi-channel retailer's optimal inventory and distribution strategy in a centralized setting.

We observe that even though consumer attitudes toward multi-channel retailing is well-studied in the marketing literature, the demand forecasting issue (both short and long term) is completely ignored in a multi-channel setting. It is of utmost importance to the retailer to know the volume of demand across channels for inventory planning purposes, and the current marketing literature does not address this problem. Similarly, the OM literature assumes that demand is deterministic or follows a certain distribution (commonly uniform or normal distribution) in order to study various

decisions such as pricing, inventory, wholesale price or distribution in a single period setting. Hence, the literature lacks predictive models for demand forecasting in a multi-period setting.

In addition, although the conditions under which it is profitable for a manufacturer to open a direct channel that competes with the existing retail channel in a decentralized setting is well-studied in the literature, the retailer's product assortment, pricing, and order fulfillment strategies across multiple channels in a centralized supply chain are not studied well in the OM literature. Specifically, one of the major purposes of opening an online channel is to be able to offer more products as online channels have a larger storage space compared to stores (i.e. warehouses vs. stores). Hence, it is very important for the retailer optimally to leverage the extra space to offer profitable products. Moreover, it is well-acknowledged that the operating costs in online channels are lower due to inventory pooling practices. Hence, uniform pricing across channels, which is a common assumption in the OM literature, might not be the optimal strategy. Although the pricing scheme across channels has been studied in decentralized settings in a game-theoretical framework, the pricing problem in a centralized environment deserves further attention.

Lastly, today's multi-channel retailers are going above and beyond to offer innovative services to provide consumers with a convenient shopping experience. Those new services include in-store Wi-fi, Order online/Pick-up at the store, and Order in store/Ship to home. Even though Order online/Pick-up at the store service is explored by Mahar et al. (2009), research addressing these new services is in its infancy.

In conclusion, the developments in technology lead to many changes in the retailing industry including evolving consumer behavior as well as opportunities for new sales mechanisms and services. The objective in omni-channel retailing is to provide consumers with a seamless experience across all channels through an integrated supply chain. In order to achieve this goal, many processes such as pricing, order fulfillment, and inventory planning have to be executed in an efficient manner. However, the current academic research provides little guidance on how these decisions should be executed. Hence, we believe that omni-channel retailing still encompasses many interesting and challenging problems that are yet to be addressed.

Chapter 3

Optimal E-commerce Advertisement and Store Service Level Decisions under Consumer Learning

3.1 Introduction and Motivation

The advent of the Internet has not only enabled traditional brick-and-mortar retailers to open online channels, but has also provided consumers with a platform that facilitated consumer-to-consumer information exchange on retailers and/or products (Chiang and Chhajed 2005, Toshiba Global Commerce Solutions 2012). As a result, the purchasing decisions of today's consumers are greatly influenced by the purchasing decisions of other consumers (Netessine and Tang 2009). Further, besides organizational factors and pricing, "operational factors related to product availability can have direct impact on consumers' purchasing behavior" (Netessine and Tang 2009). This is because stock-outs are costly due to the time and energy consumers invest in their shopping trips. Hence, "consumers would not patronize a firm without some form of assurance that they can find what they are looking for" (Su and Zhang 2009). Therefore, consumers build estimates about the retailer's product availability through their own prior experiences with the retailer as well as through word of mouth in the physical and in the virtual world (i.e. online reviews and forums), and make their

purchasing decision accordingly (Bernstein and Federgruen 2004).

Consumer interactions are also crucial when it comes to adopting new services and shopping mechanisms such as e-commerce. Data suggest that e-commerce sales in the United States have grown by 14.4% in 2014 (by 13.7%, 15.1%, 17.4% in 2013, 2012, and 2011, respectively) – four times as fast as traditional retail – constituting as much as 34% of the total sales for certain product categories such as electronics (the U.S. Department of Commerce 2014, Egol et al. 2012). Among other factors such as increased spending in e-commerce advertisement and improvements in the speed of online deliveries, consumer interactions play a great role in increasing e-commerce adoption within the society. As OM scholars Debo and Veeraraghavan (2009) state: "Customers do not decide in a 'vacuum' whether to buy a product... Consumers are influenced by what they observe around them. Such customer-to-customer interactions are important determinants that shape a firm's demand or market share, especially [...] for new, innovative products of which some features are unknown." Marketing scholar Avi Goldfarb (2014) complements this statement: "The physical environment shapes online behavior in powerful ways. Neighbors tend to like the same books, music, and cars... So even though we speak of the Internet as a 'place' where users 'visit' websites, this metaphor falls flat when we consider actual behavior. All online behavior has an offline context."

The ever-growing interactions among consumers create challenges for a dual-channel retailer's demand forecasting and management by leading to dynamically evolving demand across channels over time and by amplifying the effect of poor service levels on demand. In order to mitigate these challenges, it becomes crucial for the retailer to shape the demand across channels optimally using the decision levers it has. In our stylized model, we focus on two types of levers: e-commerce advertisement efforts and store service levels. While the former (latter) decision directly impacts the demand in the online (store) channel, it also indirectly impacts the store (online) demand due to consumer migration from one channel to the other. From that perspective, our model brings together the marketing and OM aspects of the problem through explicitly modeling the consumer-to-consumer interactions (e.g. word of mouth, online reviews, and forums), which not only stimulate the e-commerce adoption over time but also impact consumers' predictions of the retailer's store

service levels. Our goal is to gain insight into important research questions including: How should a dual-channel retailer adjust its advertisement efforts and inventory service levels over time to shape demand optimally across channels to mitigate the aforementioned challenges? Do products at different phases of e-commerce adoption and with different profit and sales margins necessitate different strategies? What is the value of providing consumers with accurate information on the retailer's service levels, and how does this impact the retailer's demand shaping strategy?

In order to answer such questions, we develop an analytical model that integrates a *diffusion model* with a *consumer choice model* within a multi-period setting, considering a dual-channel retailer that operates a store and an online channel. In particular, we utilize the consumer choice model to represent the heterogeneity in the consumer base with respect to willingness to wait for online delivery, travel distance to the store, and adoption status of the e-commerce option, where the latter is derived from the diffusion model based on the interactions among e-commerce adopters as well as non-adopters and the retailer's e-commerce advertisement efforts. Consumers' channel choice is also impacted by the product availability at the store, which consumers predict through word of mouth and prior self-experiences with the retailer. Consequently, the retailer's demand across channels is determined *endogenously* as a function of its store service levels and e-commerce advertisement efforts, and is also influenced by how consumers share information with one another. This model allows us to obtain key managerial insight on a dual-channel retailer's multi-period demand shaping strategy in a setting with consumer learning and for products at different phases of e-commerce adoption (i.e. early, established, or mature). In particular, we show that the retailer's optimal target service level in each period follows a threshold policy. This characterization allows us to obtain several important insights. For example, we show that even though e-commerce advertisement improves the retailer's market coverage, it is not necessarily profitable with sufficiently small service radii, i.e. when the majority of the consumers are located in close proximity to the store. With sufficiently large service radii, on the other hand, the retailer's optimal demand shaping policy critically depends on the product's e-commerce adoption level. For example, not surprisingly, if a product is at a mature adoption phase, then the retailer does not need to advertise the online channel as aggressively as in the earlier adoption phases, but invests

more in inventory service levels in the store. Interestingly, we show that if the retailer provides the consumers with accurate information on the store availability levels, then this results in a "win-win" situation for both the retailer and consumers. Further, such information-sharing is most beneficial in settings with slow-learning consumers. We also show that if consumers are provided with accurate information on the retailer's current availability level (so they do not need to seek the information provided by other consumers), then the retailer's optimal service levels do not need to vary over time, even as e-commerce adoption grows. In other words, the retailer's optimal service levels vary over time mainly due to the fact that consumers need to predict the retailer's current availability levels using prior information.

The remainder of this chapter is organized as follows. In Section 3.2, we provide a brief overview of the relevant literature. In Section 3.3, we present the demand model and the consumer learning process, and characterize the retailer's market segmentation as a function of its decision levers. In Section 3.4, we analyze the retailer's optimization problem, and in Section 3.5 we study the value of providing consumers with accurate information on the retailer's product availability at the store, and study how this impacts the retailer's optimal demand shaping strategy. We complement this analysis with a case study in Section 3.6. Finally, we conclude in Section 3.7 with a summary of our main findings and directions for future research. To improve the presentation, all proofs are relegated to the Appendix.

3.2   An Overview of the Related Literature

The related literature spans various streams of research, including research on inventory-driven demand, consumer learning, technology diffusion, and multi-channel retailing. This review is not meant to be comprehensive, but rather indicative of the different models used in each line of research.

The first stream of literature that we draw upon concerns inventory-driven demand; that is, research in which product availability is used as a lever by the firm to stimulate demand. For example, Dana and Petruzzi (2001) consider a single-period newsvendor set-up in which demand is a function of service levels. Chen et al. (2008) study a single-period dual-channel retailer setting in

which consumers' utility from each channel is an increasing function of product availability in that channel. Schwartz (1970) is one of the earliest works that explicitly considers the effect of stock-outs on future demand within a periodic-review infinite-horizon setting for a single firm: the firm's demand in a given period is impacted by the proportion of stock-outs in the long-run. Bernstein and Federgruen (2004) analyze a periodic-review infinite-horizon problem with competing retailers, where each retailer's demand is a function of all the retailers' availability levels and prices. At a high level, the modeling of demand in our work is similar to the aforementioned literature in that demand in each channel is increasing in product availability in that channel. However, as opposed to the above literature, we consider that consumers learn about the retailer's prior performance, in terms of service levels, through repeat purchases and/or social networks, and predict future service levels. This aspect relates our work to the literature on consumer learning in the absence of perfect information, as we explain below.

Adaptive learning models the phenomenon that consumers do not always treat the most recently revealed information as a perfectly reliable reference; rather, they adjust to this new information gradually by merging it with past experience to update their beliefs (e.g. Liu and Van Ryzin 2011). It is common to model this type of learning through an exponential smoothing method in multi-period settings, which we adopt in our work (see, for example, Akerman 1957, Monroe 1973, Sterman 1989, Jacobson and Obermiller 1990, Stidham 1992, Greenleaf 1995, Kopalle et al. 1996, Popescu and Wu 2007, Liu and Van Ryzin 2011, and Wu et al. 2015). In particular, similar to Liu and Van Ryzin (2011), we model consumers' prediction of the retailer's current product availability level as an exponential smoothed average of the product availability level in the previous period and consumers' prior estimate. Further, to study the value of perfect information in the context of product availability, we also study the setting in which consumers can accurately anticipate the retailer's current product availability level through the information provided by the retailer. The fundamental differences between the work by Liu and Van Ryzin (2011) and our research are the impact of availability on demand as well as the problem setting and the research questions that we study. In particular, while high availability stimulates demand in our model by providing consumers with a higher assurance of availability (hence a higher utility), in the setting studied

by Liu and Van Ryzin (2011) the effect is the exact opposite: the expectation of high availability during the sale season discourages consumers from buying during the regular season at full price. In addition, in our setting price does not fluctuate over time; hence, there is no strategic "wait and decide" component (see, for example, Shen and Su 2007); that is, all consumers make their purchasing decision under the uncertainty that the product may be out of stock at the store.

We also benefit from the literature on technology diffusion to model the diffusion of e-commerce adoption within the society. System dynamics models, and, in particular, diffusion models, study the penetration of new technology, innovations, products, and services (e.g. e-commerce) within the society over time (Sterman 2000). In particular, the Bass diffusion model, which is the most commonly used model for this purpose, represents this phenomenon as a function of interactions among current users and non-users, and changes in external factors such as advertisement efforts (e.g. Bass 1969, Mahajan et al. 1995, Dodson 2014). From that perspective, the papers that are most relevant to our research include Lewis and Long (2009), which uses the Bass diffusion model to forecast e-commerce sales of a dual-channel retailer, and Barabba et al. (2002), which utilizes a diffusion model in conjunction with a logit choice model to study the impact of external factors, such as word of mouth, on the number of new subscribers for a new service. However, these studies solely concentrate on the forecasting aspect and do not consider the optimization of the retailer's tactical decisions. In that respect, our main contributions include integrating the Bass diffusion model with a consumer choice model so as to study the retailer's optimal e-commerce advertisement efforts and store service levels, both of which can be used to shape demand optimally across channels.

The last stream of literature relevant to this study is the OM research that focuses on dual-channel retailing (see Cattani et al. 2004, Tsay and Agrawal 2004b, and Agatz et al. 2008 for excellent reviews). A strand of research in this area focuses on two-echelon systems in which a manufacturer and its intermediary are engaged in both vertical and horizontal competition, i.e. a manufacturer with a direct sales channel that competes with the existing retail channel. For example, Chiang et al. (2003), Tsay and Agrawal (2004a), Chiang and Chhajed (2005), Chiang and Monahan (2005), Boyaci (2005), Cattani et al. (2006), and Chen et al. (2008) model price and/or

service interactions between upper and lower echelons to address channel conflict and coordination issues in this setting. From a logistical perspective, Alptekinoğlu and Tang (2005) study optimal distribution strategies in a multi-channel system comprised of multiple depots and sales locations. A series of works by Mahar and colleagues consider various order fulfillment strategies for dual-channel retailers (e.g. Mahar et al. 2009, Mahar and Wright 2009, and Mahar et al. 2012). Some OM scholars examine the impact of recent trends and developments in dual-channel retailing on the retailer's tactical and operational strategies. Balakrishnan et al. (2014) consider consumers' value uncertainty of the product when they buy online, and study the impact of "showrooming" behavior that results in consumers' visiting the store to observe the item before they make their channel-choice decision on the retailer's pricing decision. Gao and Su (2016), on the other hand, study the impact of "buy online and pick up at the store" option on the retailer's store inventory planning. All of these works consider a single-period setting. We contribute to this literature by considering a dual-channel retailer that owns and manages competing channels within a multi-period setting. The multi-period aspect in our setting is especially important, as it enables us to study the retailer's optimal demand shaping strategy as e-commerce adoption evolves.

3.3 The Demand Model

3.3.1 Overview of the Demand Model

Our demand model integrates a diffusion model with a consumer choice model to represent the variation in demand across channels (store and online) over time. In particular, we use the Bass diffusion model to represent, for a certain product, the penetration of the e-commerce option within the consumer base through *imitation* via word of mouth; and *innovation* through the retailer's advertising efforts (Bass 1969, Sterman 2000).

In our context, penetration refers to the adoption of the "idea/option of buying online." However, individual consumers still make rational choices on whether or not to purchase the item and through which channel (store or online) so as to maximize their utility. In order to represent this phenomenon, two processes happen simultaneously in our demand model: 1) adoption of the idea

of acquiring the item through the online channel penetrates within the society over time through advertising efforts, the level of which is determined endogeneously, and word of mouth, which is exogeneous, and 2) individual consumers choose between channels based on their own characteristics and product attributes. We also incorporate availability-based channel substitution in our model (see Figure 3.1).
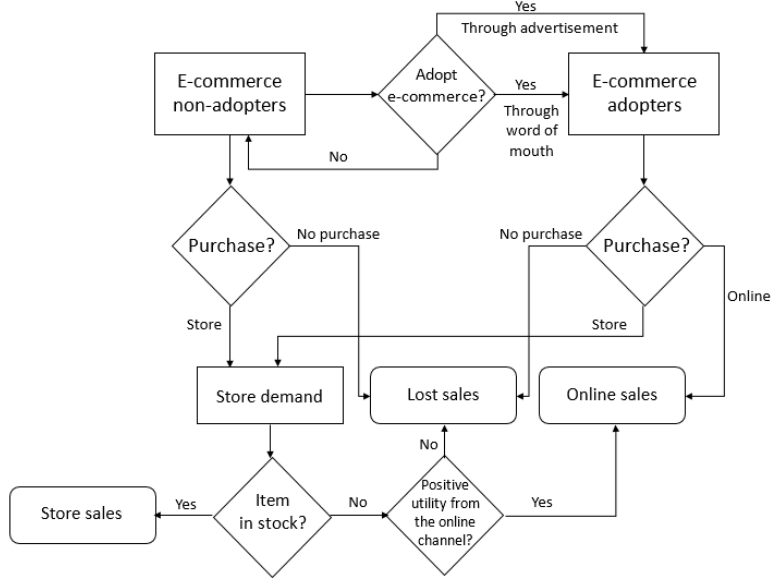
Figure 3.1: Overview of the integrated demand model.

3.3.2 Demand Model and Assumptions

In line with the Bass model, the proportion of the population which will have adopted the e-commerce option by time t is given by the cumulative distribution function (Bass 1969):

$$F_t(\omega) = \frac{1 - e^{-(\omega+a)t}}{1 + \frac{a}{\omega}e^{-(\omega+a)t}}, \tag{3.1}$$

where parameter ω (coefficient of innovation) signifies the effectiveness of the retailer's e-commerce advertisement efforts, and parameter a (coefficient of imitation) can be interpreted as the word of mouth effect on e-commerce adoption. An analogous model applies in discrete-time settings, such as the setting considered in this paper, where the adoption rate can be discretized as $F_t(\omega) = F_{t-1}(\omega) + f_t(\omega)$, for $t \leq T$, with $f_t(\omega)$ representing the proportion of new adopters in time t, i.e. the

probability mass function (Bass 1969). In practice, parameter a depends on the product and the societal structure (Levin et al. 2005, Zhou et al. 2007, Kacen et al. 2013), and can be estimated using maximum likelihood or regression analysis if preliminary data are available (Bass 1969), or by evaluating data from similar products otherwise (e.g. data on mail orders or tele-marketing could be used for e-commerce) (Dodson 2014). On the other hand, the retailer may have some control over the advertisement effectiveness parameter, ω, which we endogenize in our model.

In the discrete choice model, we consider a population of heterogeneous consumers, located uniformly within a given store service radius (k) and differentiated with respect to three dimensions: travel distance to the store (variable $R \sim U[0, k]$), willingness to wait for online delivery (variable $D \sim U[0, 1]$), and adoption status of the online channel at time t (Bernoulli variable X_t). Consumers are brand-sure and interested in buying at most one unit of the item, and make their purchase/no purchase and channel decisions so as to maximize their utility. Our consumer choice model extends those in Chen et al. (2008), Cattani et al. (2006) and Yoo and Lee (2011) by incorporating, into the channel choice, the adoption status of the online channel (variable X_t), which evolves over time. We also model availability-based channel substitution, that is, among the store-bound consumers who face a stock-out at the store, those who also derive a positive utility from the online channel switch to the online channel. We assume, similar to Chen et al. (2008), that the availability level in the online channel is 100%. This assumption is realistic, given the fierce competition and high stock-out costs in the e-commerce arena.

In the remainder of Section 3.3.2, we first characterize the demand split across channels in each time period as a function of the retailer's store service levels and e-commerce advertisement level. Then, using this demand characterization, we study the retailer's optimal decisions in Section 3.4. Throughout, we represent random variables in upper-case letters and their realization in lower-case letters, and represent vectors using arrows. We also use indices o, s, and n to denote the online channel, store channel, and no purchase option, respectively.

Consumer Utilities across Channels

Variables R and D each follow an independent uniform distribution with support $[0,k]$ and $[0,1]$, respectively, similar to Cattani et al. (2006), Chen et al. (2008), and Yoo and Lee (2011). The binary variable X_t, on the other hand, follows a Bernoulli distribution, with $Pr(X_t = 1) = F_t(\omega)$. Consequently, there are two groups of consumers in each time period t: consumers who consider the online option for purchasing the product ($X_t = 1$) and consumers who d o not ($X_t = 0$). Let v denote the value that the consumer places on the item, p the price of the item in both channels, and l the delivery lead time for the online channel. A consumer with index pair (d, r) derives the following utilities from the online and store channels:

$$U_{o,t}(d,r) = \begin{cases} v - p - dl, & \text{if } X_t{=}1 \\ undefined, & \text{otherwise} \end{cases}, \quad \text{and} \quad U_{s,t}(d,r) = \begin{cases} v - p - r, & \text{with probability } \xi_t \\ -r, & \text{otherwise} \end{cases},$$

where ξ_t is the consumers' *predicted* store availability level, which is a function of the retailer's store service levels as detailed in Section 3.3.2. We assume, without loss of generality, that consumers derive zero utility from the no purchase option. Then, given the uncertainty on product availability at the store, the consumer's expected utility for the store channel in period t can be written as:

$$E[U_{s,t}(d,r)] = \xi_t(v - p) - r.$$

Observe that channel demand is driven *endogenously* by both the product availability level at the store, which directly impacts the consumer's predicted availability level ξ_t, and the retailer's e-commerce advertisement level ω, which directly impacts the proportion of e-commerce adopters (i.e. $Pr(X_t = 1) = F_t(\omega)$). Consequently, this demand model allows us to investigate how the interplay between the retailer's store service levels and e-commerce advertisement efforts impacts the retailer's demand shaping capability.

Demand Shaping

We first introduce the following events:

Events:

$B_{i,t}$: Event that a random consumer obtains the highest utility from option $i \in \{s, o, n\}$ in period t, for $t = 1, ..., T$; e.g. $B_{o,t} = \{U_{o,t}(d,r) > \max(E[U_{s,t}(d,r)], 0)\}$.

$B_{c,t}$: Event that a random consumer obtains the highest utility from the store channel and also derives a positive utility from the online channel in period t, for $t = 1, ..., T$; i.e. $B_{c,t} = \{E[U_{s,t}(d,r)] > U_{o,t}(d,r) > 0\}$.

Then, for a random market size of M that is uniformly distributed in $[0, b]$ with probability distribution function $f_M(\cdot)$, we can express the random demand volumes across channels $(Z_{o,t}, Z_{s,t})$, and the number of consumers who choose to not buy the item $(Z_{n,t})$ in period t as follows:

$$Z_{i,t} \equiv M I_{i,t}(\omega, \xi_t), \text{ for } i \in \{s, n\},$$

$$Z_{o,t} \equiv M I_{o,t}^1(\omega, \xi_t) + \frac{I_{o,t}^2(\omega, \xi_t)}{I_{s,t}(\omega, \xi_t)} [Z_{s,t} - q_{s,t}]^+,$$

$$\text{where} \quad I_{i,t}(\omega, \xi_t) \equiv \sum_{x \in \{0,1\}} Pr(B_{i,t}|X_t = x) Pr(X_t = x), \text{ for } i \in \{s, n\},$$

$$I_{o,t}^1(\omega, \xi_t) \equiv Pr(B_{o,t}|X_t = 1) Pr(X_t = 1), \quad I_{o,t}^2(\omega, \xi_t) \equiv Pr(B_{c,t}|X_t = 1) Pr(X_t = 1),$$

and $q_{s,t}$ denotes the retailer's order quantity for the store channel in period t.

(3.2)

For a given set of ω and ξ_t, the random store demand $(Z_{s,t} = M I_{s,t}(\omega, \xi_t))$ consists of consumers (both e-commerce adopters and non-adopters) that obtain the highest utility from the store channel. We assume away certain characteristics of the problem such as inventory carry over and backlogging in order to be able to analyze the retailer's optimal advertisement efforts and service level decisions at a tactical level, while focusing on consumer learning. Then, if the store demand does not exceed the order quantity $(Z_{s,t} \leq q_{s,t})$, the retailer fully satisfies the store demand and ends up with inventory that is discarded with zero salvage value at the end of period t. Otherwise, among those consumers who face a stock-out upon their visit to the store, $[Z_{s,t} - q_{s,t}]^+$, a proportion $\left(\frac{I_{o,t}^2(\omega, \xi_t)}{I_{s,t}(\omega, \xi_t)}\right)$ switches to the online channel, as they also derive a positive utility from the online channel. Therefore, the random demand in the online channel consists of: 1) consumers who derive the highest utility from the online channel $(M I_{o,t}^1(\omega, \xi_t))$, and 2) store-bound consumers who

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switch to the online channel due to availability-based channel substitution $\left(\frac{I_{o,t}^2(\omega,\xi_t)}{I_{s,t}(\omega,\xi_t)}[Z_{s,t}-q_{s,t}]^+\right)$; see Figure 3.1.

Both type 1 and type 2 service levels, defined below for completeness, are relevant to the retailer:

1. Type 1 service level, which we refer to as the "store service level," defined as $\alpha_t \equiv Pr(Z_{s,t} \leq q_{s,t}) = Pr\left(M \leq \frac{q_{s,t}}{I_{s,t}(\omega,\xi_t)}\right)$. Since $M \sim U[0,b]$, the order quantity for the store channel follows:

$$q_{s,t} = \alpha_t\, b I_{s,t}(\omega, \xi_t). \tag{3.3}$$

Similarly, considering 100% type 1 service level for the online channel and incorporating availability-based channel substitution, the retailer's order quantity for the online channel, $q_{o,t}$, follows:

$$q_{o,t} = bI_{o,t}^1(\omega, \xi_t) + (1 - \alpha_t)bI_{o,t}^2(\omega, \xi_t). \tag{3.4}$$

2. Type 2 service level, which we refer to as the "actual store availability level," follows:

$$\beta(\alpha_t) \equiv Pr(\text{a random consumer finds the item at the store})$$

$$= E[Pr(\text{a random consumer finds the item at the store}|Z_{s,t})]$$

$$= \int_0^{b\alpha_t} 1\, f_M(m)\, dm + \int_{b\alpha_t}^b \frac{q_{s,t}}{z_{s,t}} f_M(m)\, dm = \int_0^{b\alpha_t} 1\frac{1}{b}\, dm + \int_{b\alpha_t}^b \frac{b\alpha_t}{m}\frac{1}{b}\, dm$$

$$\Rightarrow \beta(\alpha_t) = \alpha_t(1 - ln(\alpha_t)).$$

In the absence of perfect information on the current store availability level, we model the consumers' prediction process using an exponential smoothing process; that is, at the beginning of each period t, consumers update their predicted store availability level using the actual store availability level in period $t-1$ and their prior estimate:

$$\xi_t = \lambda\beta(\alpha_{t-1}) + (1 - \lambda)\xi_{t-1}, \;\; \text{for } \lambda \in (0,1] \text{ and } t = 2,...,T, \;\; \text{with } \xi_1 = \beta(\alpha_0).$$

Our modeling of the consumers' prediction process is in line with experimental and empirical evidence, which suggests that consumers do not treat the most recent information as perfectly reliable; rather, they adjust to the new information gradually by merging it with past experience (e.g. Liu and Van Ryzin 2011). In that context, λ represents the rate with which consumers adjust to this new information. As λ increases, consumers place a larger weight on the most recently

observed availability level and a smaller weight on their prior estimate, which carries information about availability levels in all the past periods. This adaptive learning model has been widely used in the literature within multi-period settings, see Section 3.2.

Thus, the retailer can use: 1) its store service levels (which impact consumers' predicted availability levels, ξ_t), and 2) e-commerce advertisement (which affects the e-commerce adoption level, $F_t(\omega)$) as levers to shape demand across channels, as we explore in Section 3.4.

3.3.3 Market Segmentation

The retailer's store service radius, k, is a strategic decision, as it strategically places its store to serve a neighborhood of consumers. In addition, the delivery time, l, for the online channel can be highly constrained by the current supply chain infrastructure and/or contracts with third party logistics providers. Hence, we characterize the retailer's business setting, which we consider as exogenous, in terms of parameters k and l. We first provide the market segmentation with respect to the heterogeneous consumer base. Let,

$$
\begin{aligned}
TH_{r,t} &\equiv min\big\{\{r : E[U_{s,t}(d,r)]=0\}, k\big\} = min\Big\{\xi_t(v-p), k\Big\}, \\
TH_{d,t} &\equiv min\big\{\{d : U_{o,t}(d,r)=0\}, 1\big\} = min\Big\{\frac{v-p}{l}, 1\Big\}, \\
TH_{l,t} &\equiv \big\{(d,r) : E[U_{s,t}(d,r)]=U_{o,t}(d,r)\big\} = \Big\{(d,r) : \ d = \frac{(1-\xi_t)(v-p)}{l} + \frac{1}{l}r\Big\}.
\end{aligned}
\tag{3.5}
$$

Then, consumers (both e-commerce adopters and non-adopters) whose travel distance to the store is less than $TH_{r,t}$ (i.e. $r < TH_{r,t}$) derive a positive utility from the store. Similarly, e-commerce adopters whose willingness to wait for the online delivery of an item is less than $TH_{d,t}$ (i.e. $d < TH_{d,t}$) derive a positive utility from the online channel. On the other hand, $TH_{l,t}$ represents the set of e-commerce adopters who are indifferent between the store and online channels.

Figures 3.2(a)-(b) illustrate the market segmentation for e-commerce non-adopters and adopters, respectively, for the case where $TH_{d,t} < 1$ and $TH_{r,t} < k$. In this case, a portion of the consumers choose the store channel, given by events $B_{s,t}|X_t = 0$ (for e-commerce non-adopters) and $B_{s,t}|X_t = 1$ (for e-commerce adopters). Further, for e-commerce adopters, the store-bound con-

sumers who also receive a positive utility from the online channel switch to the online channel if they face a stock-out at the store ($B_{c,t}|X_t = 1$). Consumers who choose the online channel ($B_{o,t}|X_t = 1$) consist of store-bound consumers switching to the online channel when they adopt e-commerce (the gridded region to the left of threshold $TH_{r,t}$) and consumers captured by the online option who would, otherwise, opt not to purchase the item (the gridded region to the right of threshold $TH_{r,t}$). Hence, the online channel expands the retailer's market coverage by reducing the volume of "choice-based lost demand," i.e. consumers who choose not to purchase the item through either channel ($B_{n,t}|X_t = 0 \cup B_{n,t}|X_t = 1$).

Figure 3.2: Market segmentation for (a) e-commerce non-adopters, and (b) e-commerce adopters for the case where $TH_{d,t} < 1$ and $TH_{r,t} < k$.

Lemma 1. *If $TH_{r,t} = k \Leftrightarrow \xi_t > W \equiv \frac{k}{v-p}$, then $(B_{n,t}|X_t = 0 \cup B_{n,t}|X_t = 1) = \emptyset$; that is, there is no choice-based lost demand in period t.*

Lemma 1 suggests that for any product (defined by parameters v and p), there exists a threshold for ξ_t, i.e. the availability level that allows the retailer to capture the entire market demand within the store service radius, k. In particular, the retailer incurs no choice-based lost demand if $\xi_t > W$.

Lemma 2 fully characterizes the demand split across channels and the volume of choice-based lost demand for any given ω and ξ_t; see Eqn. (3.2).

Lemma 2. *$I_{i,t}(\omega, \xi_t)$, $i \in \{s, n\}$, and $I_{o,t}^y(\omega, \xi_t)$, $y \in \{1, 2\}$, are 1) piece-wise continuous in $\xi_t \in (0, 1]$, and 2) continuous and differentiable in $\omega > 0$, and can be fully characterized as in Table 3.1:*

Table 3.1: Proportion of demand across channels as a function of ω and ξ_t.

	$TH_{d,t} = 1 \Leftrightarrow l \in (0, v-p]$	$TH_{d,t} < 1 \Leftrightarrow l \in (v-p, \infty)$
$TH_{r,t} < k \Leftrightarrow \xi_t < W$	$I_{o,t}^1 = \begin{cases} F_t(\omega), & \text{if } \xi_t \leq 1 - \frac{1}{B} \\ \left[1 - \frac{1}{2BW}[1-(1-\xi_t)B]^2\right]F_t(\omega), & \text{otherwise} \end{cases}$ $I_{o,t}^2 = F_t(\omega) - I_{o,t}^1$ $I_{s,t} = F_t(\omega) - I_{o,t}^1 + \frac{\xi_t}{W}(1 - F_t(\omega))$ $I_{n,t} = 1 - I_{o,t}^1 - I_{s,t}$	$I_{o,t}^1 = \left[1 - \frac{\xi_t^2}{2W}\right]BF_t(\omega)$ $I_{o,t}^2 = BF_t(\omega) - I_{o,t}^1$ $I_{s,t} = \frac{\xi_t}{W}\left[1 - \frac{(2-\xi_t)B}{2}F_t(\omega)\right]$ $I_{n,t} = 1 - I_{o,t}^1 - I_{s,t}$
$TH_{r,t} = k \Leftrightarrow \xi_t \geq W$	$I_{o,t}^1 = \begin{cases} F_t(\omega), & \text{if } \xi_t \leq 1 - \frac{1}{B} \\ \left[1 - \frac{1}{2BW}[1-(1-\xi_t)B]^2\right]F_t(\omega), & \text{if } 1 - \frac{1}{B} < \xi_t \leq 1 + W - \frac{1}{B} \\ \left[(1-\xi_t) + \frac{W}{2}\right]BF_t(\omega), & \text{otherwise} \end{cases}$ $I_{o,t}^2 = F_t(\omega) - I_{o,t}^1$ $I_{s,t} = 1 - I_{o,t}^1$ $I_{n,t} = 0$	$I_{o,t}^1 = \left[(1-\xi_t) + \frac{W}{2}\right]BF_t(\omega)$ $I_{o,t}^2 = BF_t(\omega) - I_{o,t}^1$ $I_{s,t} = 1 - I_{o,t}^1$ $I_{n,t} = 0$

where $W \equiv k/(v-p)$ and $B \equiv (v-p)/l$

Not surprisingly, the demand across channels depends on: 1) the retailer's tactical decisions, including store service levels (α_t, through dependence on ξ_t), and e-commerce advertisement level (ω), 2) the retailer's business setting (k and l), and 3) product characteristics (v and p). However, Lemma 2 also indicates that the retailer's demand is piece-wise non-linear, which leads to an optimization problem that is hard to solve, as further discussed in Section 3.4.

3.4 The Retailer's Optimization Problem

In order to study the retailer's optimal demand shaping strategy under dynamically evolving e-commerce adoption with consumer learning, we consider a multi-period setting in which the retailer determines the optimal e-commerce advertisement level, ω^*, and target store service levels over the planning horizon, $\overrightarrow{\alpha}^* = (\alpha_1^*, \alpha_2^*, ..., \alpha_T^*)$, so as to maximize its expected total profit.

Let $CF \equiv \frac{p+s-c}{p+s}$, where p is the unit sales price, c is the retailer's unit purchasing cost, and s is the unit lost sales cost, with $s < c < p$. That is, CF represents the "critical fractile" for

the single-period newsvendor problem with exogenous demand. For analytical tractability, in the remainder of the paper we restrict ourselves to the case where the initial service level equals critical fractile ($\alpha_0 = CF$), and assume that $CF > 0.547$ (under this condition, the first-order optimality conditions become sufficient). This condition is satisfied for a variety of product categories such as fashion items, electronics, and home goods, as demonstrated in the case study of Section 3.6.

The retailer incurs an advertisement cost for its efforts to increase e-commerce adoption. For ease of presentation, we express the cumulative cost of advertisement over the planning horizon by $G(\omega)$, which is convex increasing in the advertisement level, ω. This is in line with the literature, e.g. "as with all inputs used in the production process, the marginal effectiveness of advertising diminishes as more and more is spent on advertising. Thus, advertising is subject to diminishing marginal returns." (Mukherjee 2002). Then, given the retailer's advertising efforts, its expected profit function in period t, $E[\pi_t(\omega, \alpha_t, \xi_t)]$, comprised of the expected sales revenue less the purchasing cost and the lost sales cost, is as follows:

$$E[\pi_t(\omega, \alpha_t, \xi_t)] = p\Big(E[Z_{o,t}] + E[\min(q_{s,t}, Z_{s,t})]\Big) - c(q_{s,t} + q_{o,t})$$
$$- s\left[\left(\frac{I_{s,t}(\omega, \xi_t) - I_{o,t}^2(\omega, \xi_t)}{I_{s,t}(\omega, \xi_t)}\right) E[(Z_{s,t} - q_{s,t})^+] + E[Z_{n,t}]\right]. \tag{3.6}$$

Substituting Eqn.s (3.2)-(3.4) in Eqn. (3.6), the retailer's optimization problem reduces to the following form (see the Appendix for derivations):

$$\begin{aligned} \underset{\substack{\overrightarrow{0} < \overrightarrow{\alpha} \leq \overrightarrow{1} \\ 0 < \omega}}{\text{maximize}} \; E[\Pi(\omega, \overrightarrow{\alpha})] &= -G(\omega) + \sum_{t=1}^{T} E[\pi_t(\omega, \alpha_t, \xi_t)] \\ &= -G(\omega) + b \sum_{t=1}^{T} \left\{ [I_{o,t}^1(\omega, \xi_t) + I_{o,t}^2(\omega, \xi_t)]\left[\frac{p+s}{2} - c\right] \right. \\ &\quad \left. + [I_{s,t}(\omega, \xi_t) - I_{o,t}^2(\omega, \xi_t)]\left[(p+s-c)\alpha_t - (p+s)\frac{\alpha_t^2}{2}\right] - \frac{s}{2} \right\}. \end{aligned} \tag{3.7}$$

The retailer's objective function is non-linear, non-separable in $\overrightarrow{\alpha}$, and not necessarily well-behaved, as we subsequently show in Section 3.4.1. Consequently, in the remainder of this section, we study the structural properties of this problem. In particular, we first derive, in Section 3.4.1, the optimal service level strategy, $\overrightarrow{\alpha}^*(\omega)$, for any given advertisement level ω. Then, in Section 3.4.2, we study how the objective function, $E[\Pi(\omega, \overrightarrow{\alpha}^*(\omega))]$, behaves in ω. In addition to important structural properties, this analysis also leads to an efficient two-stage algorithm that provides the

optimal solution to Eqn. (3.7).

3.4.1   The Retailer's Optimal Store Service Levels (Stage 2 Problem)

The following lemma exploits the structure of the retailer's expected profit function for a given advertisement level, ω.

Lemma 3. *For any given $\omega > 0$, $E[\Pi(\omega, \overrightarrow{\alpha})]$ is continuous, and 1) increasing in $\overrightarrow{\alpha}$ for $\overrightarrow{0} < \overrightarrow{\alpha} < CF \cdot \overrightarrow{1}$, and 2) jointly concave in $\overrightarrow{\alpha}$ for $CF \cdot \overrightarrow{1} \leq \overrightarrow{\alpha} \leq \overrightarrow{1}$. Further, for any $\lambda \in (0, 1]$, $\alpha_t^*(\omega) \in [CF, 1)$, $t = 1, ..., T - 1$, and $\alpha_T^* = CF$.*

Remark 1. *For the special case where $\lambda = 0$, $\alpha_t^*(\omega) = \alpha^* = CF, t = 1, ..., T$, for any $\omega > 0$.*

Lemma 3 indicates that consumer learning (which leads to dependence among the periods) forces the optimal store service levels to be at least as high as the critical fractile. This finding is in line with and extends upon the results in the existing literature on inventory-driven demand in the single-channel single-period setting; see Dana and Petruzzi (2001) and the references therein. Specifically, we show that such findings apply in a dynamic multi-channel multi-period setting in which: 1) a high service level in one channel (store) stimulates the channel's own demand, while reducing the demand for the other channel (online), whose adoption level increases over time and is also impacted by the retailer's decisions, 2) consumers switch between channels in case of a stock-out, and 3) consumers learn over time. Indeed, as Remark 1 indicates, when there is no consumer learning (i.e. $\lambda = 0$), there is no dependence among the periods through store service levels and the problem reduces to the single-period newsvendor problem with exogenous demand, whose solution is given by the critical fractile, CF.

In order to answer the research questions posed in Section 3.1, we next explore several structural properties of the retailer's optimal service levels.

Theorem 1. *For a given $\omega > 0$, $\alpha_T^* = CF$, and $(\alpha_1^*(\omega), ..., \alpha_{T-1}^*(\omega))$ has the following structure:*

$$
(\alpha_1^*(\omega), ..., \alpha_{T-1}^*(\omega)) = \begin{cases} (CF, ..., CF), & if \quad W < \beta\,(CF) \\[2mm] (\alpha_1''', ..., \alpha_{T-1}'''), & if \quad \beta\,(CF) \leq W < \min_{t \in [2, ..., T]} \{\xi_t''(\omega)\} \\[2mm] \left\{(\alpha_1, ..., \alpha_{T-1}) : \alpha_t''' \leq \alpha_t \leq \alpha_t''(\omega), \forall\, t = 1, ..., T-1\right\}, & if \quad \min_{t \in [2, ..., T]} \{\xi_t''(\omega)\} \leq W < \max_{t \in [2, ..., T]} \{\xi_t''(\omega)\} \\[2mm] (\alpha_1''(\omega), ..., \alpha_{T-1}''(\omega)), & if \quad W \geq \max_{t \in [2, ..., T]} \{\xi_t''(\omega)\}, \end{cases}
$$

where $W = \frac{k}{v-p}$, $\alpha_1''' = \left\{\alpha : \beta(\alpha) = \frac{W - (1-\lambda)\beta(\alpha_0)}{\lambda}\right\}$ and $\alpha_t''' = \{\alpha : \beta(\alpha) = W\}$, for $t = 2, ..., T$, and $\xi_t''(\omega) \equiv \lambda\beta(\alpha_{t-1}''(\omega)) + (1-\lambda)\xi_{t-1}''(\omega)$, for $\lambda \in (0, 1]$ and $t = 2, ..., T$, with $\xi_1'' = \beta(\alpha_0)$.

Theorem 1 fully characterizes the retailer's optimal service levels for a given advertisement level, ω, and show that they follow a threshold policy. Theorem 1 also underscores that the retailer's service level strategy should be tailored to its business setting (k) and product characteristics $(v$ and $p)$. Specifically, if W is sufficiently small, i.e. $W < \beta(CF)$, then the retailer does not need to increase its store service levels beyond the critical fractile, as this policy enables the retailer to capture the entire market through the store channel due to the sufficiently small service radius; see Lemma 1. Otherwise, the retailer always increases its store service levels beyond the critical fractile. In particular, for relatively small W, i.e. $\beta(CF) \leq W < \min_{t \in [2, ..., T]} \{\xi_t''(\omega)\}$, higher service levels, i.e. $(\alpha_1''', ..., \alpha_{T-1}''')$, allow the retailer to capture the entire market. However, higher service levels come at a cost. Therefore, for larger W, i.e. larger service radius, there is no benefit in increasing the service levels beyond a threshold $(\alpha_1''(\omega), ..., \alpha_{T-1}''(\omega))$, and some choice-based lost demand becomes inevitable. Example 1 demonstrates how the optimal service levels vary with service radius, k, for a given ω.

Example 1. *Let $a = 0.0065$, $\omega = 0.0003$, $\lambda = 0.4$, $v = 29.99$, $p = 24.99$, $c = 10.75$, $s = 0$, $l = 3$, $T = 4$, and $\alpha_0 = CF = 0.57$. Figure 3.3 depicts the optimal service levels as a function of service radius, k, for e-commerce adoption level vector $\overrightarrow{F} = (F_1, F_2, F_3, F_4) = (0.0299, 0.0304, 0.0309, 0.0314)$.*

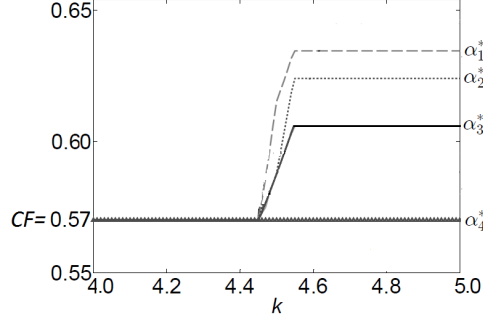Figure 3.3: $\overrightarrow{\alpha}^*$ as a function of k.

It is important to note that although the retailer's optimal service levels for sufficiently small W (i.e. $k \leq 4.45$ in Figure 3.3) are independent of the e-commerce advertisement level (equivalently, e-commerce adoption level) and equal to CF, the expanding e-commerce adoption *does* increase the optimal order quantity for each channel by providing more consumers with a dual-channel option, hence reducing the volume of choice-based lost demand (see Theorem 2 below).

Theorem 2. *For each time period $t = 1, ..., T$, $q_{s,t}^*(\omega)$ is decreasing, $q_{o,t}^*(\omega)$ is increasing, and $q_{s,t}^*(\omega) + q_{o,t}^*(\omega)$, is increasing in ω when $W < \beta(CF)$.*

Next, we study the retailer's optimal advertisement decision in Section 3.4.2.

3.4.2 The Retailer's Optimal Advertisement Level (Stage 1 Problem)

The following lemma characterizes the structure of the retailer's expected profit function with respect to ω.

Lemma 4. $E[\pi_t(\omega, \alpha_t, \xi_t)]$, $t = 1, ..., T$, *is continuous in ω. Further:*

1. *For $\xi_t < W$, $E[\pi_t(\omega, \alpha_t, \xi_t)]$, for $t = 1, ..., T$, is strictly decreasing in ω when α_t satisfies*
   $$\frac{p+s}{2} - c - \frac{\xi_t}{W}\left[(p+s-c)\alpha_t + (p+s)\frac{\alpha_t^2}{2}\right] \leq 0, \text{ and}$$

2. *For $\xi_t \geq W$, $E[\pi_t(\omega, \alpha_t, \xi_t)]$, for $t = 1, ..., T$, is strictly decreasing in ω when α_t satisfies*
   $$\frac{p+s}{2} - c - \left[(p+s-c)\alpha_t + (p+s)\frac{\alpha_t^2}{2}\right] \leq 0.$$

Theorem 1 and Lemma 4 not only allow us to develop an effective two-stage algorithm to determine the retailer's optimal solution, but also indicate the cases where the retailer should not invest in e-commerce advertisement at all.

Theorem 3. *When $W < \beta(CF)$, $E[\Pi_t(\omega, \alpha_t^*(\omega), \xi_t^*(\omega))], t=1, ..., T$, is strictly decreasing in ω.*

Thus, if the retailer's optimal service level strategy allows it to capture the entire market through the store channel (i.e. $W < \beta(CF)$), then it is optimal for its to serve all consumers through the store channel only. Advertising for a *competing* online channel in this setting results in lower profits; hence, if the retailer opens an online channel as a strategic necessity, then it should keep its e-commerce advertisement efforts at a minimal level, i.e. ω^* goes to 0. This policy would allow e-commerce adoption to grow due to the exogenous word of mouth effect only and lower the profit loss due to online shoppers. In other cases, however, the online channel becomes a *complementary* and profitable option, and allows the retailer to shape demand optimally across channels using e-commerce advertisement in conjunction with store service levels.

3.5   The Value of Perfect Information

In this section, we study the value of information sharing strategies that the retailer can adopt in order to improve consumers' experience at the store. In particular, we study the setting in which the retailer shares with the consumers its store service level information in the current period, i.e. $\xi_t = \beta(\alpha_t)$. Then, consumers no longer need to rely on word of mouth and/or prior self-experiences with the retailer to predict the current store availability level. This can be achieved, to some extent, through mechanisms such as providing store service level information on the retailer's website, mobile application, catalogs. This type of information sharing may incur some cost, which depends on the specific setting. We ignore this cost in our analysis, but it can easily be considered in the model. Our goal is to gain insight into the value of this type of information sharing, which helps eliminate the consumers' dependence on the partial information provided by other consumers.

We first present the structure of the retailer's optimal solution for the perfect information setting, which we denote by the superscript P.

Lemma 5. *For a given $\omega > 0$, $E[\Pi^P(\omega, \vec{\alpha})]$ is continuous, and 1) increasing in $\vec{\alpha}$ for $\vec{0} < \vec{\alpha} < CF \cdot \vec{1}$, and 2) jointly concave in $\vec{\alpha}$ for $CF \cdot \vec{1} \leq \vec{\alpha} \leq \vec{1}$. Further, $\alpha_t^{*P}(\omega) = \alpha^{*P} \in [CF, 1)$, for $t = 1, ..., T$.*

Interestingly, the optimal store service levels in the perfect information setting become identical over time, i.e. independent of the consumers' e-commerce adoption level. Hence, the retailer's multi-period service level problem reduces to a single-period problem, and a rise in e-commerce adoption each period leads to a proportional increase (decrease) in online (store) demand. This finding highlights that the retailer's store service levels vary over time mainly due to the fact that consumers do not have information on the current availability level and need to predict it using past information. From a technical perspective, by providing consumers with availability information, the retailer reduces its optimization efforts, as it needs to solve the Stage 2 (service level) optimization problem only once regardless of the length of the planning horizon or e-commerce adoption level of the product.

Theorem 4 shows that the retailer's optimal service level strategy in the perfect information setting continues to follow a threshold policy, but is much easier to implement than the limited information (i.e. adaptive learning) setting.

Theorem 4. *1. For a given ω, the optimal solution, α^{*P}, has the following structure:*

$$
\alpha^{*P} = \begin{cases} CF, & if \quad W < \beta\,(CF) \\ \alpha : \beta(\alpha) = W, & if \quad \beta\,(CF) \leq W < \beta(\alpha') \\ \alpha', & if \quad W \geq \beta(\alpha'). \end{cases}
$$

2. For each time period $t = 1, ..., T$, $q_{s,t}^{*P}(\omega)$ is decreasing, $q_{o,t}^{*P}(\omega)$ is increasing, and $q_{s,t}^{*P}(\omega) + q_{o,t}^{*P}(\omega)$ is increasing in ω;

3. When $W < \beta(CF)$, $E[\pi_t(\omega, \alpha^{*P})], t = 1, ..., T$, is strictly decreasing in ω.

The results is Sections 3.4 and 3.5 enable us to bound the deviation between the optimal service levels in the limited versus perfect information settings. Specifically, Theorem 5 establishes an upper

bound on the absolute difference between α^{*P} and $\overrightarrow{\alpha}^*$ for any problem instance characterized by CF.

Theorem 5. *For any problem instance with a given CF, we have:*

1. $\left|\alpha^{*P}-\alpha_t^*\right| = 0$, *for $t = 1,...,T$, if $W < \beta(CF)$;*

2. $\left|\alpha^{*P}-\alpha_t^*\right| < UB(CF) - CF$, *for $t = 1,...,T$, if $W \geq \beta(CF)$,*

where $UB(CF)$ is an upper bound on $\overrightarrow{\alpha}^*$, for a given CF, and corresponds to the solution of the following equation with $\lambda = 1$ and $F_1 = F_{1+i}$, $i = 1,...,T-1$:

$$UB(CF) \equiv \left\{ \alpha_1 : \left. \frac{\partial E[\Pi(\omega, \overrightarrow{\alpha})]}{\partial \alpha_1} \right|_{\substack{\alpha_j=CF \\ j \neq 1}} = 0 \right\}.$$

Hence, the impact of information sharing on availability levels is especially important with sufficiently large service radii (i.e. $W \geq \beta(CF)$), and for products with small CF for which the absolute difference between α^{*P} and $\overrightarrow{\alpha}^*$ may go up to 8% (for $CF = 0.57$), see Figure 3.4. We use Theorem 5 to quantify the value of information sharing on the retailer's optimal expected profit in Section 3.6.
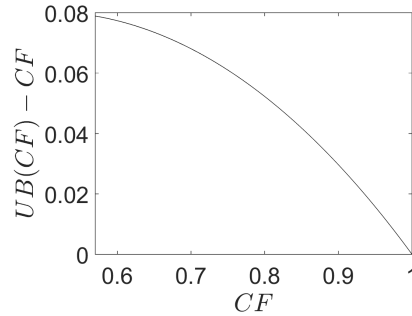

Figure 3.4: Upper bound on the absolute difference between α^{*P} and $\overrightarrow{\alpha}^*$ as a function of CF when $W \geq CF$.

3.6 Case Study

In this section, we present a case study to provide managerial insight on how the retailer's optimal demand shaping strategy is impacted by: 1) consumer-to-consumer interactions, in terms of how

consumers learn (parameter λ) and how they adopt e-commerce (parameter a), and 2) the product's current e-commerce adoption level F_1. Furthermore, we quantify the value of providing consumers with information on the retailer's current store availability level.

3.6.1 Data and Approach

We select three product categories that are currently at different phases of e-commerce adoption, i.e. early, established, and mature adoption phases. According to Egol et al. (2012), a representative set of such product categories includes household items, apparel, and electronics, for which the percentage of e-commerce sales (over all retail sales) are given by 3%, 14%, and 34%, respectively, in 2012, when the overall e-commerce sales constituted 15.1% of the total retail sales. We use these percentages as a proxy for the current adoption rates in the case study. According to Mahajan et al. (1995), the parameter a for the word of mouth effect for monthly e-commerce diffusion typically lies between 0.025 and 0.042. Hence, in our case study, we use $a = 0.025, 0.034, 0.042$ for the household, apparel, and electronics items, respectively, for a planning horizon of four periods ($T = 4$), and with a random market size of $M \sim U[0, 1000]$.

For each product category, we choose a dual-channel retailer (i.e. Garmin Ltd. 2015, Fossil, Inc. 2015, and The Yankee Candle Company, Inc. 2015) and an item that the corresponding retailer offers online (candle, wallet, and auto navigator). We determine the item's online price, p, through the retailer's website, and assume that the item is offered in the store at the same price and that its lost sale cost is zero ($s = 0$). We then use the published gross profit margin, $(p - c)/p$, which equals the critical fractile for the product when $s = 0$, for each retailer (CSIMarket, Inc. 2015, The Wall Street Journal 2013) to estimate the retailer's purchasing cost, c, and assume that $v = p + 5$ for all products (see Table 3.2).

Table 3.2: Financial data for the items considered in the case study.

Item	Category	Profit margin (CF)	List (sales) price (p)	Purchasing cost (c)	Sales margin ($p - c$)
Auto navigator	Electronics	58%	$129.99	$54.60	$75.39
Wallet	Apparel	55%	$50.00	$22.50	$27.50
Candle	Household	57%	$24.99	$10.75	$14.24

3.6.2 Results

We first use the candle example as a representative product to demonstrate how the retailer's optimal decisions and the resulting profit change with various problem parameters when $W \geq \beta(CF)$, as the retailer's optimal solution is trivial otherwise (see Theorems 1 and 3). (The findings are similar for the other products considered.) Then, we compare the optimal decisions for all three products. We consider that the e-commerce advertisement cost function is in the form, $G(\omega) = h \cdot \omega^2$, with $h = 5,000$, and assume $k = 15$ and $l = 10$, unless otherwise noted.

Table 3.3 shows how the retailer's optimal solution and the resulting profit change with λ, i.e. the rate with which consumers adjust to the new information obtained through other consumers and prior self-experiences. Interestingly, for smaller values of λ we observe a monotone pattern over time, i.e. $\alpha_1^* > \alpha_2^* > \alpha_3^* > \alpha_4^*$, while for larger λ, the monotonicity disappears. This is because a smaller λ implies that the consumer places more (less) weight on her earlier (most recent) experience(s). As a result, when λ is small (e.g. $\lambda < 0.96$ for the example in Table 3.3), the optimal service levels in the earlier periods tend to be higher. When λ is large, however, the impact of the earlier periods on the subsequent periods diminishes and a non-monotonic service level pattern becomes possible.

In general, the retailer's optimal service levels increase with λ. With a higher λ, consumers adjust to the new information faster, and this incentivizes the retailer to increase its service levels, which is quickly transformed into higher consumer predictions of store availability. A natural outcome of this strategy is that more consumers are captured and served through the store channel as consumers adjust to the new information faster. Since store service levels and e-commerce advertisement are substitutes, higher service levels also lead to a lower investment in e-commerce advertisement as λ gets higher. Yet, the retailer benefits from faster consumer learning, i.e. its expected profit increases in λ.

The retailer's providing the consumers with accurate information on current store availability levels (i.e. the perfect information setting) is a "win-win" situation, as demonstrated for the candle example below. Table 3.3 shows that when consumers are provided with this information, both the retailer's optimal order quantity (hence, the number of consumers satisfied) and the

optimal expected profit increase compared to the adaptive learning (limited information) setting. Further, as λ increases, the optimal solution for the adaptive learning setting approaches the perfect information setting. Hence, sharing availability information with consumers is most beneficial in settings with slow-learning consumers. Finally, the percent loss in expected profit ($\Delta = (E[\Pi^{*P}] - E[\Pi^*])/E[\Pi^{*P}]$) due to not sharing the availability information with consumers may exceed 2% (when $\lambda = 0$), which can be significant in the retailing industry.

Table 3.3: Retailer's optimal decisions and resulting expected profit for the (a) adaptive learning, and (b) perfect information settings.

(a) Adaptive learning (limited information) setting with various values of λ

λ	ω^*	α_1^*	α_2^*	α_3^*	α_4^*	q_{total}^*	$E[\Pi^*]$	$\Delta(\%)$
0.00	0.0570	0.570	0.570	0.570	0.570	947	4,932	2.14
0.25	0.0563	0.618	0.608	0.593	0.570	980	4,950	1.79
0.50	0.0553	0.638	0.630	0.612	0.570	1,000	4,975	1.29
0.75	0.0546	0.645	0.641	0.629	0.570	1,013	4,996	0.87
1.00	0.0542	0.646	0.644	0.645	0.570	1,021	5,010	0.60

(b) Perfect information setting

ω^{*P}	α_1^{*P}	α_2^{*P}	α_3^{*P}	α_4^{*P}	q_{total}^{*P}	$E[\Pi^{*P}]$
0.0547	0.645	0.645	0.645	0.645	1,054	5,040

Not surprisingly, the product's current e-commerce adoption level (F_1) has a large impact on the retailer's optimal policy. For a given λ ($\lambda = 0.5$), Table 3.4(a) illustrates that as F_1 increases, the retailer's e-commerce advertisement efforts decrease as expected, and the retailer can afford to increase its store service levels. Further, as the dual-channel option becomes available to more consumers, the retailer satisfies a higher demand (due to a decrease in choice-based lost demand), and the retailer's expected profit increases.

Table 3.4: Retailer's optimal decisions and resulting expected profit under (a) various values of F_1, and (b) various values of a.

(a) Under various values of F_1 ($a = 0.025$)

F_1	ω^*	α_1^*	α_2^*	α_3^*	α_4^*	q_{total}^*	$E[\Pi^*]$
0.03	0.0553	0.638	0.630	0.612	0.570	1,000	4,975
0.33	0.0389	0.639	0.630	0.613	0.570	1,402	5,269
0.63	0.0219	0.640	0.631	0.613	0.570	1,822	5,565
0.93	0.0042	0.640	0.631	0.613	0.570	2,265	5,863

(b) Under various values of a ($F_1 = 0.03$)

a	ω^*	α_1^*	α_2^*	α_3^*	α_4^*	q_{total}^*	$E[\Pi^*]$
0.025	0.0553	0.638	0.630	0.612	0.570	1,000	4,975
0.525	0.0597	0.634	0.625	0.610	0.570	1,279	5,145
1.025	0.0583	0.628	0.621	0.608	0.570	1,599	5,355
1.525	0.0445	0.624	0.620	0.610	0.570	1,800	5,515

Interestingly, an increase in the word of mouth effect (a), which stimulates e-commerce adoption (see Eqn. (1)), leads to a non-monotone pattern in the retailer's e-commerce advertisement level, as shown in Table 3.4(b) (for the candle example). In particular, for sufficiently small (large) values of a, the retailer's optimal advertisement level increases (decreases) in a.

Finally, Table 3.5, which reports the optimal strategy for candle, wallet and auto navigator, underscores that the retailer's optimal demand shaping strategy greatly depends on the product's characteristics, including the e-commerce adoption phase (F_1) as well as profit and sales margins . Specifically, while the store service levels highly depend on the product's profit margin (i.e. critical fractile), the optimal advertisement level depends on the sales margin. In summary, a customized strategy highly benefits the retailer.

Table 3.5: Retailer's optimal decisions and resulting expected profit for the three products.

Product	Sales margin	Profit margin	F_1	a	ω^*	α_1^*	α_2^*	α_3^*	α_4^*	q_{total}^*	$E[\Pi^*]$
Candle	$14.24	57%	0.03	0.025	0.0553	0.638	0.630	0.612	0.570	1,000	4,975
Wallet	$27.50	55%	0.14	0.034	0.0249	0.620	0.611	0.593	0.550	1,044	9,051
Auto navigator	$75.39	58%	0.34	0.042	0.1791	0.645	0.638	0.621	0.580	1,711	30,201

3.7 Conclusions and Future Research Directions

As technology is becoming more prevalent in consumers' lives, it is influencing the way they communicate, obtain, and share information. In this research, we study how consumer-to-consumer interactions, empowered by technology, affect a dual-channel retailer's demand shaping decisions. Our results highlight the importance of considering the nature of these consumer-to-consumer interactions (e.g. how consumers adopt e-commerce and how they learn about the retailer's service levels) in the retailer's tactical planning. In particular, when consumers learn fast, both the retailer and the consumers benefit. Yet, the best scenario occurs when the retailer actually provides the consumers with information on its service levels. This strategy is a "win-win" situation for both the retailer and consumers, and also reduces to a strategy that is much easier to implement for the retailer. Our findings also indicate that even though e-commerce advertisement improves the retailer's market coverage, it is not necessarily profitable in small service radii.

Numerous extensions of our model deserve further attention. We assume that product price is exogenous, and that prices and costs are uniform across channels. It is important to expand our model to incorporate several major benefits of operating an online channel, in terms of risk pooling and lower operating costs compared to the store channel. We consider that consumer attributes, such as willingness to wait, travel distance to the store, and adoption probability of the online channel, come from independent distributions. Our extensive simulation study, with correlated distributions, indicates that the retailer's optimal service levels do not differ much (with the highest difference occurring in high service radii settings, but even in this case the differences are low). Nevertheless, it would be interesting to explore the impact of consumer behavior and heterogeneity on the optimal solution in a rigorous manner when consumer attributes are correlated.

Since our main purpose is to study the impact of evolving consumer behavior on a dual-channel retailer's service level decisions at a tactical level, certain characteristics such as inventory carry over and backlogging have been ignored; hence, it would be interesting to study an extension of this problem with the inclusion of these components at an operational level. Lastly, competition among retailers, which has been ignored in our model, can have a significant impact on their optimal decisions. Hence, studying this aspect of the problem would provide valuable insights with respect to dual-channel retailers' advertisement and inventory decisions when multiple parties compete for the market share. Despite these modeling assumptions, we believe our study offers key insight to dual-channel retailers, and sheds light into their optimal service levels in the face of evolving consumer behavior and e-commerce adoption.

Chapter 4

Optimal Pricing Decision under Reference Effects

4.1 Introduction and Motivation

The extensive use of the Internet, along with the introduction of smartphones, enabled traditional brick-and-mortar retailers to open online channels, and changed the way consumers shop. Today's tech-savvy consumers use the Internet not only as a point of purchase, but also as a means for awareness and research about the product attributes such as price and the retailer's service quality regarding availability, delivery, and service, and make their purchasing and channel-choice decisions accordingly (Toshiba Global Commerce Solutions 2012). According to Subramani (2013), today's "shoppers have become increasingly channel agnostic and gravitate towards the channel that offers most value – with pricing becoming a game changer." As consumers become more price-aware and price-sensitive while they are looking for the best deal in-store and online, they expect companies to mirror this ability by actively comparing their prices across all channels in real time (McGovern and Levesanos 2014).

Interestingly, research indicates that a main challenge that multi-channel retailers face is consumers' expectation of a uniform experience through all available channels rather than a channel-specific one. That is, "inconsistent messaging, pricing, promotions, or experiences cause customer

confusion, leading to low customer satisfaction and reduced spending" (Aberdeen Group 2012). However, different channels come with different advantages and disadvantages. For instance, operating costs of online channels are typically lower than store channels due to consolidation of resources such as shop floor personnel, rent for storage space, etc., making it possible for dual-channel retailers to offer lower prices compared to store channels (Schuerman et al. 2013). Further, there is information asymmetry among different channels on pricing. For example, while the online price of an item can be readily observed on the dual-channel retailer's website, mobile app, etc., consumers can only *estimate* the store price using the information provided for the online channel (unless the store price is explicitly shown on the retailer's website, which is not currently a common practice), and the actual store price can only be observed when the consumer physically arrives at the store.

In this research, we explore how a dual-channel retailer's pricing policy (i.e. uniform vs. channel-tailored) impacts consumers' purchasing behavior, and how this phenomenon, in turn, affects the retailer's profit when operating costs for the store and online channels are different. We also examine whether a dual-channel retailer should leverage the online channels to provide information about the store price, which is otherwise unknown to the consumers, thus, eliminating information asymmetry. In particular, our objective is to answer important questions such as: When should the retailer use channel-tailored vs. uniform pricing across channels? Is it beneficial for the retailer to share the store price information on its website? How does such an information-sharing strategy the store price information on the retailer's website affect the retailer's demand and profit?

In order to answer these questions, we develop an analytical model, considering a dual-channel retailer that operates a store channel and an online channel. In particular, we utilize a consumer choice model to represent the heterogeneity in the consumer base with respect to consumers' hassle costs related to shopping online and in store. Using this model, we study the retailer's pricing problem when consumers lack information on the store price and use the online price as a proxy to estimate it, as they make their purchasing and channel-choice decisions. In this setting, we study whether a channel-tailored pricing policy dominates uniform pricing across channels. Then, we benchmark our model with the case where the retailer provides information about the store price

on its website, and study how the retailer's optimal pricing strategy and profit change.

Using this model, we obtain several important insights. Interestingly, when there is information asymmetry, i.e. when consumers use the online price as a proxy to estimate the store price, a channel-tailored pricing policy, with the store price set higher than the online price, always dominates a uniform pricing policy. Further, when the retailer eliminates this information asymmetry by sharing her store price on her website, this benefits not only the consumers, but also the retailer.

The remainder of this chapter is organized as follows. In Section 4.2, we provide a brief overview of the relevant literature. In Section 4.3, we present our base model and analyze the retailer's optimization problem. In Section 4.4, we study the value of providing consumers with information on the retailer's store price, and study how this strategy impacts the retailer's optimal pricing policy as well as its profit. Finally, we conclude in Section 4.5 with a summary of our main findings and directions for future research. To improve the presentation, all proofs are relegated to the Appendix.

4.2 Literature Review

The related literature spans various streams of research, including research on consumer learning and multi-channel retailing. This review is not meant to be comprehensive, but rather indicative of the different models used in each line of research.

The first stream of literature that we draw upon concerns consumer learning. In multi-period models where price and/or inventory levels are decision variables, adaptive learning models are widely used. In particular, adaptive learning models the phenomenon that consumers do not always treat the most recently revealed information as a perfectly reliable reference; rather, they adjust to this new information gradually by merging it with past experience to update their beliefs (e.g. Liu and Van Ryzin 2011). It is common to model this type of learning through an exponential smoothing method in multi-period settings, which we also adopt in our work (see, for example, Akerman 1957, Monroe 1973, Sterman 1989, Jacobson and Obermiller 1990, Stidham 1992, Greenleaf 1995, Kopalle et al. 1996, Popescu and Wu 2007, Liu and Van Ryzin 2011, and Wu et al. 2015). In single-period models, on the other hand, the rational expectations paradigm is commonly used.

The rational expectations paradigm assumes that consumers can correctly predict the store price. This assumption may hold "in the long run when equilibrium beliefs have formed and stabilized" (Su and Zhang 2009). However, it may not hold in various other settings, i.e. when the consumers have no prior experience with the retailer's store channel, or when the store price fluctuates over time. In such settings, consumers may learn about the retailer's store price through other relevant information. Specifically, we assume that the consumer sets her reference store price as the online price that she observes on the retailer's website in these settings.

The last stream of literature relevant to this study is the OM research that focuses on dual-channel retailing (see Cattani et al. 2004, Tsay and Agrawal 2004b, and Agatz et al. 2008 for excellent reviews). A strand of research in this area focuses on two-echelon systems in which a manufacturer and its intermediary are engaged in both vertical and horizontal competition, i.e. a manufacturer with a direct sales channel that competes with the existing retail channel. For example, Chiang et al. (2003), Tsay and Agrawal (2004a), Chiang and Chhajed (2005), Chiang and Monahan (2005), Boyaci (2005), Cattani et al. (2006), and Chen et al. (2008) model price and/or service interactions between upper and lower echelons to address channel conflict and coordination issues in this setting. Some OM scholars examine the impact of recent trends and developments in dual-channel retailing on the retailer's tactical and operational strategies. For example, Balakrishnan et al. (2014) model the uncertainty in the value consumers place on the item when the item is purchased online, and study how "showrooming" behavior, i.e. the consumers' visiting the store to observe the item, impacts the retailer's pricing decision. Gao and Su (2016), on the other hand, study the impact of "buy online and pick up at the store" option on the retailer's inventory planning. We contribute to this literature by studying how a retailer's channel-tailored strategy can hurt/help it, and how the retailer can leverage the different channels to better integrate its operations.

4.3   Model

We consider a dual-channel retailer that operates a store channel and an online channel, respectively denoted by indices s and o. From the retailer's side, channels are differentiated with respect to

their operating cost, which is higher for the store channel. We represent this difference in terms of the product's inventory purchasing cost, $c_i, i \in \{o, s\}$, with $c_s > c_o$.

Similar to Cattani et al. (2006), Yoo and Lee (2011), and Gao and Su (2016), we denote $v > p_i, i \in \{s, o\}$, as the value placed on the item by the consumers, and assume that consumers are heterogeneous with respect to their hassle costs in the store channel, $h_s \sim U[0, H]$ (due to traveling to the store, looking for the item in the store aisles, etc.) and in the online channel, $h_o \sim U[0, H]$ (due to shipping cost, waiting for the item's delivery, etc.). Consumers make their purchasing and channel decisions so as to maximize their utility surplus.

In the base model, we consider a fixed market size with unit demand, and study the retailer's optimal pricing strategy for both channels when consumers lack accurate information on the store price (p_s), and need to estimate it using the online price (p_o), which is readily available on the retailer's website. Before consumers make their purchasing and channel decisions, they check the retailer's online price (p_o) and *estimate* their utilities under the assumption of uniform pricing across channels (i.e. $p_s^e \equiv p_o$). Therefore, the online utility and the *estimated* store utility for a consumer with a hassle cost vector (h_o, h_s) is given by:

$$u_o(h_o, h_s) = v - p_o - h_o \quad \text{and} \quad u_s^e(h_o, h_s) = v - p_s^e - h_s, \quad \text{with} \quad p_s^e = p_o. \quad (4.1)$$

We assume, without loss of generality, that consumers derive zero utility from the no purchase option, which we denote by index n. Then, based on the estimated utilities, consumers either choose not to purchase the item, or purchase it through their utility-maximizing channel. Consumers who pick the store channel observe the *actual* store price (p_s) after they arrive at the store. Hence, at this point they adjust their *actual* utility and reconsider their purchasing decision. That is, among the store-bound consumers, those with $u_s^a(h_o, h_s) = v - p_s - h_s < 0 \Rightarrow h_s > v - p_s$ decide not to purchase the item and leave the store empty-handed. We define the following events:

Events:

E_o: Event that a random consumer with hassle vector (h_o, h_s) obtains the highest estimated utility, i.e. before observing the actual store price, from the online channel;

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i.e. $E_o = \{u_o(h_o, h_s) \geq \max(u^e_s(h_o, h_s), 0)\}$.

E^i_s: Event that a random consumer with hassle vector (h_o, h_s) obtains the highest estimated utility from the store channel, and the highest actual utility, i.e. after observing the actual store price, from option $i \in \{s, n\}$;

e.g. $E^n_s = \{u^e_s(h_o, h_s) \geq \max(u_o(h_o, h_s), 0)$ and $0 > u^a_s(h_o, h_s)\}$.

Before we introduce the retailer's optimization problem, we derive the market segmentation of the consumers as a function of the retailer's decision variables (p_o, p_s). Figure 4.1 depicts the market segmentation of the consumers for the case with $(v - p_o < H$ and $p_s > p_o) \Leftrightarrow v - p_s < v - p_o < H$. In this case, based on the estimated online and store utilities, a portion of the consumers choose the online channel (event E_o), while another portion chooses the store channel (event $E^s_s \cup E^n_s$). Then, those store-bound consumers who derive a negative actual utility from the store channel leave the store empty-handed (event E^n_s).
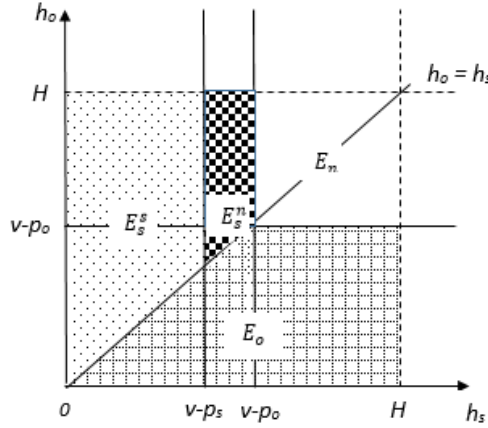

Figure 4.1: Market segmentation for the case where $v - p_s < v - p_o < H$.

Based on the retailer's market segmentation, the following lemma fully characterizes the consumers' choice probabilities, i.e. $Pr(E_o(p_o, p_s))$ and $Pr(E^i_s(p_o, p_s)), i \in \{s, n\}$.

Lemma 6. $Pr(E_o(p_o, p_s))$ and $Pr(E^i_s(p_o, p_s)), i \in \{s, n\}$, are piece-wise continuous and differentiable in p_o and p_s, and can be characterized as in Table 4.2. Further, $Pr(E^s_s(p_o, p_s)) = Pr(E_o(p_o, p_s)) - Pr(E^n_s(p_o, p_s))$.

Table 4.2: Choice probabilities as functions of p_o and p_s.

Region	Choice probabilities
$R_1 = \{(p_o, p_s) : v - H > p_s \geq p_o\}$	$Pr(E_o(p_o, p_s)) = Pr(E_s^s(p_o, p_s)) = 0.5$
$R_2 = \{(p_o, p_s) : v \geq p_s \geq v - H > p_o\}$	$Pr(E_o(p_o, p_s)) = 0.5$ $Pr(E_s^s(p_o, p_s)) = (2H - v + p_s)(v - p_s)/2H^2 \leq 0.5$
$R_3 = \{(p_o, p_s) : v \geq p_s \geq p_o \geq v - H\}$	$Pr(E_o(p_o, p_s)) = (2H - v + p_o)(v - p_o)/2H^2 \leq 0.5$ $Pr(E_s^s(p_o, p_s)) = (2H - v + p_s)(v - p_s)/2H^2 \leq 0.5$
$R_4 = \{(p_o, p_s) : v \geq p_o > v - H \text{ and } p_o > p_s\}$	$Pr(E_o(p_o, p_s)) = (2H - v + p_o)(v - p_o)/2H^2 \leq 0.5$ $Pr(E_s^s(p_o, p_s)) = Pr(E_o(p_o, p_s)) \leq 0.5$
$R_5 = \{(p_o, p_s) : v - H \geq p_o > p_s\}$	$Pr(E_o(p_o, p_s)) = Pr(E_s^s(p_o, p_s)) = 0.5$

Next, we turn our attention to the retailer's optimization problem. The retailer's objective is to shape the demand optimally across channels through its pricing decision so as to maximize its expected profit. Given the consumer behavior described in Lemma 6, the retailer's objective function for the pricing problem (Problem **P**) follows:

Problem **P**:

$$\underset{0 \leq p_o, p_s \leq v}{\text{maximize}} \ \pi(p_o, p_s) = Pr(E_o(p_o, p_s))(p_o - c_o) + Pr(E_s^s(p_o, p_s))(p_s - c_s). \qquad (4.2)$$

The following lemma summarizes the behavior of the retailer's objective function in decision variables (p_o, p_s).

Lemma 7. $\pi(p_o, p_s)$ *is continuous, differentiable, and not concave in* (p_o, p_s).

The following theorem exploits the structural properties of the retailer's profit function and characterizes the retailer's optimal solution.

Theorem 6. *The unique optimal solution to Problem* **P** *has the following structure:*

$$\max\left(v - H, \frac{2c_j + v}{3}\right) < p_j^* < v, \ j \in \{s, o\}, \ \text{and} \ p_o^* < p_s^* < p_o^* - c_o + c_s.$$

Theorem 6 provides important insights on the retailer's optimal pricing strategy. Interestingly, a channel-tailored pricing strategy, with the store price set higher than the online price, is always optimal for the retailer, even though this implies that some store-bound consumers will leave the store empty-handed. That is, the retailer balances between compensating for the higher operating costs and incurring lost sales from store-bound consumers; she achieves this by setting the profit margin for the store channel, $(p_s^* - c_s)$, lower than that of the online channel, $(p_o^* - c_o)$.

At this point, some important questions include: How is the retailer's expected profit affected by the consumers' information asymmetry? Can the retailer improve its expected profit by adopting simple policies such as making the store price available on its website? In order to answer these questions, we next study the scenario in which the retailer makes the store price available on its website.

4.4   The Value of Information Sharing

When the retailer makes the store price available on its website, consumers observe not only the online price, but also the actual store price. Therefore, no store-bound consumer will have to leave the store empty-handed (assuming that the store price does not fluctuate by the time the consumer arrives at the store). Let E_j', $j \in \{s, o, n\}$, represent the event that a random consumer obtains the highest actual utility from option $j \in \{s, o, n\}$, i.e. after observing both the actual store price and the online price on the retailer's website (e.g. $E_o' = \{u_o(h_o, h_s) \geq \max(u_s^a(h_o, h_s), 0)\}$).

In order to demonstrate the impact of such information-sharing on the retailer, we reevaluate the market segmentation depicted in Figure 4.1; see Figure 4.2, which considers the case with $(v - p_o < H$ and $p_s > p_o) \Leftrightarrow v - p_s < v - p_o < H$.
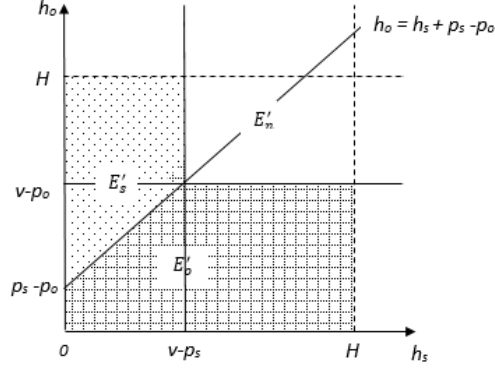
Figure 4.2: Market segmentation for the case where $v - p_s < v - p_o < H$ under information-sharing.

Lemma 8. *When the actual store price information is available on the retailer's website, the retailer's market coverage expands or remains the same, i.e.* $\sum_{j \in \{o,s\}} Pr(E'_j(p_o, p_s)) \geq Pr(E_o(p_o, p_s)) + Pr(E^s_s(p_o, p_s))$.

Interestingly, the retailer expands its market coverage by sharing her store price information with consumers *a priori*. This happens because the reduction in store demand resulting from providing consumers with the actual store price, which is higher than the online price ($p_s > p_o$), is compensated by a greater increase in the online demand (i.e. $Pr(E^s_s(p_o, p_s)) - Pr(E'_s(p_o, p_s)) \leq Pr(E'_o(p_o, p_s)) - Pr(E_o(p_o, p_s))$).

Before we examine the impact of the increase in market coverage on the retailer's prices and study if information-sharing is in fact profitable for the retailer, we first characterize the consumers' choice probabilities $(Pr(E'_j(p_o, p_s)), j \in \{s, o, n\})$ as a function of the decision variables (p_o, p_s) in this new setting.

Lemma 9. $Pr(E'_j(p_o, p_s)), j \in \{s, o\}$, *is piece-wise continuous and non-differentiable in* p_o *and* p_s, *and can be characterized as below. Further,* $Pr(E'_n(p_o, p_s)) = 1 - \sum_{j \in \{s,o\}} Pr(E'_j(p_o, p_s))$.

Table 4.3: Choice probabilities as functions of p_o and p_s under information-sharing.

Region	Choice probabilities
$R_1 = \{(p_o, p_s) : H > p_s - p_o \geq 0 \text{ and } v - H \geq p_o\}$	$Pr(E_o'(p_o, p_s)) = 1 - (H - p_s + p_o)^2/2H^2$ $Pr(E_s'(p_o, p_s)) = 1 - Pr(E_o'(p_o, p_s))$
$R_2 = \{(p_o, p_s) : v \geq p_s \geq p_o \geq v - H\}$	$Pr(E_o'(p_o, p_s)) = (v - p_o)/H - (v - p_s)^2/2H^2$ $Pr(E_s'(p_o, p_s)) = (2H + 2p_o - v - p_s)(v - p_s)/2H^2$
$R_3 = \{(p_o, p_s) : p_s - p_o > H \text{ and } v \geq p_s\}$	$Pr(E_o'(p_o, p_s)) = 1, \qquad Pr(E_s'(p_o, p_s)) = 0$
$R_4 = \{(p_o, p_s) : 0 > p_s - p_o \geq -H \text{ and } v - H \geq p_s \geq 0\}$	$Pr(E_o'(p_o, p_s)) = (H - p_o + p_s)^2/2H^2$ $Pr(E_s'(p_o, p_s)) = 1 - Pr(E_o'(p_o, p_s))$
$R_5 = \{(p_o, p_s) : v \geq p_o > p_s \geq v - H\}$	$Pr(E_o'(p_o, p_s)) = (2H + 2p_s - v - p_o)(v - p_o)/2H^2$ $Pr(E_s'(p_o, p_s)) = (v - p_s)/H - (v - p_o)^2/2H^2$
$R_6 = \{(p_o, p_s) : -H > p_s - p_o \text{ and } v \geq p_o\}$	$Pr(E_o'(p_o, p_s)) = 0, \qquad Pr(E_s'(p_o, p_s)) = 1$

The retailer's optimization problem in this setting is similar in form to Eqn. (4.2):

$$\underset{0 \leq p_o, p_s \leq v}{\text{maximize}} \ \pi'(p_o, p_s) = Pr(E_o'(p_o, p_s))(p_o - c_o) + Pr(E_s'(p_o, p_s))(p_s - c_s). \qquad (4.3)$$

The following lemma summarizes the behavior of the retailer's objective function under information-sharing in decision variables (p_o, p_s).

Lemma 10. $\pi'(p_o, p_s)$ *is continuous, non-differentiable, and not concave in p_o and p_s.*

The following theorem summarizes how information-sharing impacts the retailer's optimal solution $(p_o'^*, p_s'^*)$ and profit.

Theorem 7. *The retailer's optimal solution in the information-sharing setting has the following structure:*

$$\max(v - H, 0) \leq p_o'^* \leq p_s'^* \leq v.$$

Further, $\pi'^* > \pi^*$.

Theorem 7 provides important insights into the benefit of information-sharing. Interestingly, it indicates that the optimal profit increases if the retailer decides to share store price information on her website.

4.5   Conclusions and Future Research Directions

As more and more consumers shop with multi-channel retailers, retailers are under more pressure to offer a uniform experience to consumers across all channels in terms of pricing, inventory availability, service, etc. However, this may not be possible due to complicated channel structures that may lead to different operating costs across channels. In this research, we study the impact of this phenomenon on the retailer's optimal pricing policy. We find that the retailer benefits from a channel-tailored pricing strategy, with higher pricing in the store, even if this results in a number of unsatisfied consumers. More importantly, our findings show that the retailer can leverage the online channels to provide information about the store pricing to enable a more transparent shopping experience for the consumers, which also leads to a higher profit for the retailer. Hence, our study emphasizes the importance of taking advantage of a multi-channel supply chain network in order to enable a more transparent shopping experience for the consumers, leading to a "win-win" situation for all parties.

A number of extensions to our model deserve attention. First, it is important to consider the stochasticity, with respect to item's availability, in either channel. In that respect, examining the interplay between inventory and pricing decisions would be an interesting area to explore from an information-sharing perspective. Price-matching, which allows the retailer to match its store price with the online price in case of higher store pricing, is another important concept to explore. For instance, it would be worthwhile to study which strategy is the best option when the operating costs for the store and online channels are different: price-matching in the store using the limited store inventory or encouraging consumers to purchase through the online channel. The concept of price-matching is especially important when consumers exhibit "showrooming" behavior; that is, consumers visit the store to examine the product but then purchase it online if a better online deal is available (i.e. for fashion items consumers can visit the store to try the item on, but then buy the item online if a better online deal is available). Consumers can find a better deal online on the retailer's website or on a competitor's website. Hence, studying the impact of competition in this setting would provide valuable insights with respect to dual-channel retailers' pricing decisions. Lastly, it is important to study the impact of shipping costs for online purchases on consumers'

purchasing behavior and on dual-channel retailers' profit explicitly. In our model, we assume that the shipping cost is a part of the consumer's hassle cost in the online channel. However, many dual-channel retailers offer free shipping if the consumer's spending exceeds a certain amount. Hence, it would be interesting to examine how free shipping offers impact consumers' purchasing behavior and retailer's pricing strategy.

Chapter 5

Conclusions and Future Research Directions

5.1 Conclusions

Although inventory management and pricing problems are well-studied in various settings in the literature, research that addresses these problems within a multi-channel setting is still in its infancy. Most industry research papers provide merely qualitative frameworks that do not address how these decisions should be made. On the other hand, the academic literature mostly focuses on pricing and distribution problems in dual channel networks, where store and online channels operate independently of each other in a single period setting. Generally, these models do not consider how demand across channels evolves over time due to consumer interactions or how online channels can be leveraged for providing consumers with inventory and/or price information for other available channels in the supply chain to mitigate the consumer's information asymmetry. Cross-channel interactions that allow consumers to obtain information on inventory and/or price information in stores through the retailer's website as well as the diffusion of emerging channels within the consumer base through increasing consumer interaction in the physical and in the virtual world, however, are integral in understanding how demand across channels evolves over time in this new business environment.

In this dissertation, we contribute to the literature by adopting an interdisciplinary approach that brings together tools and concepts from OM, economics, systems dynamics and marketing literatures to create analytical models in order to address a dual-channel retailer's optimal demand shaping strategy, through e-commerce advertisement efforts, store service levels, and pricing. Our findings show that the retailer's optimal demand shaping strategy, in terms of store service levels and e-commerce advertisement effort, critically depends on the product's e-commerce adoption phase. We also show that in the presence of higher operating costs for the store channel compared to the online channels, a channel-tailored pricing policy always dominates a uniform pricing strategy. More importantly, our work also sheds light on the benefits of channel integration for multi-channel retailers. In particular, our findings show that the retailer can leverage the online channels to provide in-store pricing and inventory information in order to enable a more transparent shopping experience for the consumers, which results in a "win-win" situation for all parties.

5.2   Future Research Directions

Numerous extensions of our models deserve further attention. Since our main purpose is to study the impact of evolving consumer behavior on a dual-channel retailer's service level decisions at a tactical level in our first study, certain characteristics such as inventory carry over and backlogging are ignored; hence, it would be interesting to study an extension of this problem with the inclusion of these components at an operational level. In our second study, on the other hand, the pricing decision is studied in a setting where the inventory is assumed to be infinite. In the future, it would be interesting to look at the joint inventory and pricing problem under consumer learning. In addition, examining the interplay between inventory and pricing decisions in this setting can be an exciting problem from an information-sharing perspective. Price-matching, which allows the retailer to match its store price to the online price in case of higher store pricing, is another important concept to explore. For instance, it would be worthwhile to explore which strategy is the best option when the operating costs for the store and online channels are different: price-matching in the store using the limited store inventory or encouraging consumers to purchase through the online channel. The concept of price-matching is especially important when consumers

exhibit "showrooming" behavior; that is, consumers visit the store to examine the product but then purchase it online if a better online deal is available (i.e. for fashion items consumers can visit the store to try the item on, but then buy the item online if a better online deal is available). Consumers can find a better deal online on the retailer's website or on a competitor's website. Hence, studying the impact of competition in this setting would provide valuable insights with respect to dual-channel retailers' pricing decisions.

One of the main disadvantages of online purchases from the consumers' point of view is the shipping cost. Therefore, many dual-channel retailers offer free shipping if a consumer's spending exceeds a certain amount. In order to take advantage of the free shipping opportunity, consumers may shop in baskets. Hence, it would be interesting to examine how free shipping offers impact consumers' purchasing behavior and the retailer's pricing strategy. Also, from retailers' perspective, one of the major advantages of opening an online channel is to be able to offer a greater selection of products as online channels typically have a larger storage space compared to stores. For instance, while multi-channel retailers can only use the storage space in their stores to fulfill in-store purchases, they can use their warehouses as well as their stores to fulfill online orders. Hence, it is very important for multi-channel retailers to take advantage of this setup, and offer products that are tailored for specific channels in the supply chain network.

Lastly, today's multi-channel retailers are going above and beyond to offer innovative services, such as in-store Wi-fi, Order online/Pick-up at the store, and Order in store/Ship to home, to provide consumers with a more convenient shopping experience. However, research addressing these new services is in its infancy. Therefore, it would be interesting to study the impact of such services on multi-channel retailers' pricing and inventory decisions. Fulfillment of online orders is another avenue to explore, as many multi-channel retailers are struggling in determining the optimal order fulfillment strategy for online orders: shipping from a store, shipping from a dedicated warehouse for online orders, shipping from a shared warehouse for store replenishment and online order fulfillment, or a combination thereof.

Bibliography

4R Systems (2013). "Optimizing inventory in the omni-channel supply chain." Date accessed: Sept 11. 2013. URL http://www.4rsystems.com/pdfs/RIS%20News%20May%202012.pdf.

Aberdeen Group (2012). "The 2012 omni-channel retail experience." Date accessed: Nov. 2014. URL http://www.stores.org/assets/whitepapers/EpicorThe2012Omni-ChannelRetailExperience.pdf.

Achabal, D., Badgett, M., Chu, J., and Kalyanam, K. (2005). "Cross-channel optimization: A strategic roadmap for multichannel retailers." Date accessed: Dec 1. 2013:.
URL http://www-03.ibm.com/industries/ca/en/retail/retail_docs/cross_opt.pdf

Agatz, N.A., Fleischmann, M., and Van Nunen, J.A. (2008). "E-fulfillment and multi-channel distribution–A review." *European Journal of Operational Research*, **187 (2)**, pp. 339–356.

Akerman, G. (1957). "The cobweb theorem: A reconsideration." *The Quarterly Journal of Economics*, **71 (1)**, pp. 151–160.

Alptekinoğlu, A. and Tang, C.S. (2005). "A model for analyzing multi-channel distribution systems." *European Journal of Operational Research*, **163 (3)**, pp. 802–824.

Balakrishnan, A., Sundaresan, S., and Zhang, B. (2014). "Browse-and-Switch: Retail-Online Competition under Value Uncertainty." *Production and Operations Management*, **23 (7)**, pp. 1129–1145.

Banjo, S. (2013). "Wal-Mart's e-stumble with Amazon." Date accessed: June 10. 2013.
URL http://online.wsj.com/article/SB10001424127887323566804578553301017702818.html

Barabba, V., Huber, C., Cooke, F., Pudar, N., Smith, J., and Paich, M. (2002). "A multimethod approach for creating new business models: The General Motors OnStar project." *Interfaces*, **32 (1)**, pp. 20–34.

Bass, F. (1969). "A new product growth for model consumer durables." *Management Science*, **15 (5)**, pp. 215–227.

Benedicktus, R.L., Brady, M.K., Darke, P.R., and Voorhees, C.M. (2010). "Conveying trustworthiness to online consumers: Reactions to consensus, physical store presence, brand familiarity, and generalized suspicion." *Journal of Retailing*, **86 (4)**, pp. 322–335.

Bernstein, F. and Federgruen, A. (2003). "Pricing and replenishment strategies in a distribution system with competing retailers." *Operations Research*, **51 (3)**, pp. 409–426.

Bernstein, F. and Federgruen, A. (2004). "A General Equilibrium Model for Industries with Price and Service Competition." *Operations Research*, **52 (6)**, pp. 868–886.

Bernstein, F. and Federgruen, A. (2005). "Decentralized supply chains with competing retailers under demand uncertainty." *Management Science*, **51 (1)**, pp. 18–29.

Boyaci, T. (2005). "Competitive stocking and coordination in a multiple-channel distribution system." *IIE Transactions*, **37 (5)**, pp. 407–427.

Bretthauer, K.M., Mahar, S., and Venakataramanan, M. (2010). "Inventory and distribution strategies for retail/e-tail organizations." *Computers & Industrial Engineering*, **58 (1)**, pp. 119–132.

Cattani, K., Gilland, W., Heese, H.S., and Swaminathan, J. (2006). "Boiling frogs: Pricing strategies for a manufacturer adding a direct channel that competes with the traditional channel." *Production and Operations Management*, **15 (1)**, pp. 40–56.

Cattani, K.D., Gilland, W.G., and Swaminathan, J.M. (2004). "Coordinating traditional and internet supply chains." In D. Simchi-Levi, S.D. Wu, and Z.J.M. Shen, editors, *Handbook of Quantitative Supply Chain Analysis*. Springer, pp. 643–677.

Chen, F., Federgruen, A., and Zheng, Y.S. (2001). "Near-optimal pricing and replenishment strategies for a retail/distribution system." *Operations Research*, **49 (6)**, pp. 839–853.

Chen, K.Y., Kaya, M., and Özer, Ö. (2008). "Dual sales channel management with service competition." *Manufacturing & Service Operations Management*, **10 (4)**, pp. 654–675.

Chiang, W.K. and Chhajed, D. (2005). "Multi-channel supply chain design in B2C electronic commerce." In J. Geunes and P. Pardalos, editors, *Supply Chain Optimization*. Springer, pp. 145–168.

Chiang, W.K., Chhajed, D., and Hess, J.D. (2003). "Direct marketing, indirect profits: A strategic analysis of dual-channel supply-chain design." *Management Science*, **49 (1)**, pp. 1–20.

Chiang, W.K. and Monahan, G.E. (2005). "Managing inventories in a two-echelon dual-channel supply chain." *European Journal of Operational Research*, **162 (2)**, pp. 325–341.

CSIMarket, Inc. (2015). "Consumer Cyclical." Date accessed: July 2015. URL http://csimarket.com/Industry/Industry_Profitability.php?s=400.

Dana, Jr., J.D. and Petruzzi, N.C. (2001). "Note: The newsvendor model with endogenous demand." *Management Science*, **47 (11)**, pp. 1488–1497.

Debo, L.G. and Veeraraghavan, S.K. (2009). "Models of herding behavior in operations management." In S. Netessine and C.S. Tang, editors, *Consumer-driven Demand and Operations Management Models: A Systematic Study of Information-technology-enabled Sales Mechanisms*. Springer, New York.

Dodson, J. (2014). "New product forecasting: The Bass model." Date accessed: Nov. 2014. URL http://www.learningace.com/doc/531160/fc1252ce3fc46224928578de3c098d60/mktgtool_bass.

Egol, M., Rajagopalan, A., and Sayer, B. (2012). "E-commerce and consumer goods. A strategy for omnichannel success." Date accessed: July. 2015. URL http://www.strategyand.pwc.com/media/file/Strategyand_E-Commerce-and-Consumer-Goods.pdf.

Fossil, Inc. (2015). "Wallets." Date accessed: July 2015. URL http://www.fossil.com/en_US/shop/women/wallets.html?parent_category_rn=357083&departmentCategoryId=331097.

Gao, F. and Su, X. (2016). "Omnichannel Retail Operations with Buy-Online-And-Pickup-In-Store." *Forthcoming in Management Science*.

Garmin Ltd. (2015). "Cars." Date accessed: July 2015. URL https://buy.garmin.com/en-US/US/on-the-road/automotive/cOnTheRoad-cAutomotive-p1.html.

Goldfarb, A. (2014). "The Internet killed distance. Mobile phones brought it back." *MIT Technology Review*, **117 (1)**, pp. 62–63.

Greenleaf, E.A. (1995). "The impact of reference price effects on the profitability of price promotions." *Marketing Science*, **14 (1)**, pp. 82–104.

Jacobson, R. and Obermiller, C. (1990). "The formation of expected future price: A reference price for forward-looking consumers." *Journal of Consumer Research*, pp. 420–432.

Kacen, J.J., Hess, J.D., and Chiang, W.K. (2013). "Bricks or clicks? Consumer attitudes toward traditional stores and online stores." *Global Economics and Management Review*, **18 (1)**, pp. 12–21.

Kopalle, P.K., Rao, A.G., and Assuncao, J.L. (1996). "Asymmetric reference price effects and dynamic pricing policies." *Marketing Science*, **15 (1)**, pp. 60–85.

Kumar, V. and Venkatesan, R. (2005). "Who are the multichannel shoppers and how do they perform?: Correlates of multichannel shopping behavior." *Journal of Interactive Marketing*, **19 (2)**, pp. 44–62.

Levin, A.M., Levin, I.P., and Weller, J.A. (2005). "A multi-attribute analysis of preferences for online and offline shopping: differences across products, consumers, and shopping stages." *Journal of Electronic Commerce Research*, **6 (4)**, pp. 281–290.

Lewis, T. and Long, B. (2009). *Case analysis studies of diffusion models on e-commerce transaction data*. Master's thesis, Massachusetts Institute of Technology, Sloan School of Management.

Liu, Q. and Van Ryzin, G. (2011). "Strategic capacity rationing when customers learn." *Manufacturing & Service Operations Management*, **13 (1)**, pp. 89–107.

Mahajan, V., Muller, E., and Bass, F.M. (1995). "Diffusion of new products: Empirical generalizations and managerial uses." *Marketing Science*, **14 (3-supplement)**, pp. G79–G88.

Mahar, S., Bretthauer, K.M., and Venkataramanan, M. (2009). "The value of virtual pooling in dual sales channel supply chains." *European Journal of Operational Research*, **192 (2)**, pp. 561–575.

Mahar, S., Salzarulo, P.A., and Daniel Wright, P. (2012). "Using online pickup site inclusion policies to manage demand in retail/E-tail organizations." *Computers & Operations Research*, **39 (5)**, pp. 991–999.

Mahar, S. and Wright, P.D. (2009). "The value of postponing online fulfillment decisions in multi-channel retail/E-tail organizations." *Computers & Operations Research*, **36 (11)**, pp. 3061–3072.

McGovern, C. and Levesanos, A. (2014). "Optimizing pricing and promotions in a digital world." Date accessed: Feb 21. 2016. URL https://www.accenture.com/t20150624T211456__w__/us-en/_acnmedia/Accenture/Conversion-Assets/DotCom/Documents/Global/PDF/Industries_14/Accenture-Optimizing-Pricing-Promotions.pdf#zoom=50.

Monroe, K.B. (1973). "Buyers' subjective perceptions of price." *Journal of Marketing Research*, pp. 70–80.

Mukherjee, S. (2002). *Modern Economic Theory*. New Age International.

Netessine, S. and Tang, C.S. (2009). "Introduction." In S. Netessine and C.S. Tang, editors, *Consumer-driven Demand and Operations Management Models: A Systematic Study of Information-technology-enabled Sales Mechanisms*. Springer, New York.

Popescu, I. and Wu, Y. (2007). "Dynamic pricing strategies with reference effects." *Operations Research*, **55 (3)**, pp. 413–429.

RetailTouchPoints (2012). "Completing the cross-channel challenge: Exclusive survey report." Date accessed: Nov. 2014. URL http://f9e7d91e313f8622e557-14a29c251add4cb0f3d45e39c18c202f.r83.cf1.rackcdn.com/RTP_Research_Completing%20The%20Cross-Channel_Challenge.pdf.

Ryan, T. (2013). "Macy's, Others Turn Stores Into Online Fulfillment Centers." Date accessed: Dec 1. 2014. URL http://www.forbes.com/sites/retailwire/2013/04/03/macys-others-turn-stores-into-online-fulfillment-centers/.

Schoenbachler, D.D. and Gordon, G.L. (2002). "Multi-channel shopping: understanding what drives channel choice." *Journal of Consumer Marketing*, **19 (1)**, pp. 42–53.

Schröder, H. and Zaharia, S. (2008). "Linking multi-channel customer behavior with shopping motives: An empirical investigation of a German retailer." *Journal of Retailing and Consumer Services*, **15 (6)**, pp. 452–468.

Schuerman, J., Bellaiche, J.M., and Petzke, A. (2013). "Regaining price control in a multichannel world." Date accessed: Feb. 2013. URL https://www.bcgperspectives.com/content/articles/pricing_sales_channels_regaining_pricing_control_multichannel_world.

Schwartz, B. (1970). "Optimal inventory policies in perturbed demand models." *Management Science*, **16 (8)**, pp. B–509.

Shen, Z.J.M. and Su, X. (2007). "Customer behavior modeling in revenue management and auctions: A review and new research opportunities." *Production and Operations Management*, **16 (6)**, pp. 713–728.

Simchi-Levi, D., Wu, S.D., and Shen, Z.J.M. (2004). *Handbook of quantitative supply chain analysis: modeling in the e-business era*, volume 74. Springer Science & Business Media.

Sterman, J.D. (1989). "Modeling managerial behavior: Misperceptions of feedback in a dynamic decision making experiment." *Management Science*, **35 (3)**, pp. 321–339.

Sterman, J.D. (2000). *Business dynamics: Systems thinking and modeling for a complex world*. Irwin/McGraw-Hill, Boston, 1st edition.

Stidham, S. (1992). "Pricing and capacity decisions for a service facility: Stability and multiple local optima." *Management Science*, **38 (8)**, pp. 1121–1139.

Su, X. and Zhang, F. (2009). "On the value of commitment and availability guarantees when selling to strategic consumers." *Management Science*, **55 (5)**, pp. 713–726.

Subramani, S. (2013). "What's your bet on omni-channel pricing?" Date accessed: Feb. 2016. URL http://www.tcs.com/SiteCollectionDocuments/Newsletters/TCS-Retail-Forum-Issue-2-1013-2.pdf.

The U.S. Department of Commerce (2014). "Monthly and Annual Retail Trade." Date accessed: Nov. 2014. URL http://www.census.gov/retail/.

The Wall Street Journal (2013). "Yankee Candle Agrees to $1.75 Billion Deal." Date accessed: July 2015. URL http://www.wsj.com/articles/SB10001424127887324202304579051044165409288.

The Yankee Candle Company, Inc. (2015). "Jar Candles." Date accessed: July 2015. URL http://www.yankeecandle.com/browse/candles/jar-candles/_/N-8cd.

Toshiba Global Commerce Solutions (2012). "Omni-channel vision, cross-channel reality: Meeting the consumer where they want to buy." Date accessed: Sept. 2013. URL http://www.toshibagcs.com/cross-channel-customer-engagement.

Trade Extensions (2014). "Price is most important factors for consumers, whatever they may say about ethics." Date accessed: Feb. 2016. URL http://www.tradeextensions.com/news/2014/7/30/price-is-most-important-factor-for-consumers-whatever-they-may-say-about-ethics.

Tsay, A.A. and Agrawal, N. (2004a). "Channel conflict and coordination in the e-commerce age." *Production and Operations Management*, **13 (1)**, pp. 93–110.

Tsay, A.A. and Agrawal, N. (2004b). "Modeling conflict and coordination in multi-channel distribution systems: A review." In D. Simchi-Levi, S.D. Wu, and Z.J.M. Shen, editors, *Handbook of Quantitative Supply Chain Analysis*. Springer, pp. 557–606.

Wu, S., Liu, Q., and Zhang, R.Q. (2015). "The Reference Effects on a Retailers Dynamic Pricing and Inventory Strategies with Strategic Consumers." *Operations Research*, **63 (6)**, pp. 1320–1335.

Yao, D.Q., Yue, X., Wang, X., and Liu, J.J. (2005). "The impact of information sharing on a returns policy with the addition of a direct channel." *International Journal of Production Economics*, **97 (2)**, pp. 196–209.

Yoo, W.S. and Lee, E. (2011). "Internet channel entry: A strategic analysis of mixed channel structures." *Marketing Science*, **30 (1)**, pp. 29–41.

Zhou, L., Dai, L., and Zhang, D. (2007). "Online shopping acceptance model-A critical survey of consumer factors in online shopping." *Journal of Electronic Commerce Research*, **8 (1)**, pp. 41–62.

Appendix

Recall that $W \equiv \frac{k}{v-p}$, and $B \equiv \frac{v-p}{l}$. We also define, $Y_t \equiv \begin{cases} \frac{[1-BF_t(\omega)]\xi_t}{W}, & \text{if } \xi_t \leq W \\ [1 - BF_t(\omega)], & \text{otherwise.} \end{cases}$, for $t = 1, ..., T$.

Proof of Lemma 1. In order for no choice-based lost demand to occur (i.e. $(B_{n,t}|X_t = 0 \cup B_{n,t}|X_t = 1) = \emptyset$), both e-commerce adopters and non-adopters must receive positive utility from at least one channel. Since the only available channel for the e-commerce non-adopters is the store channel, it must hold that $TH_{r,t} = k \Leftrightarrow \xi_t > k/(v - p) = W$, i.e. all e-commerce non-adopters derive a positive utility from the store. This condition also implies that all e-commerce adopters derive a positive utility from the store, thus, completing the proof. ∎

Proof of Lemma 2.
Proof of Part 1. The proof proceeds in two parts. First, we derive closed-form expressions for the choice probabilities, $Pr(B_{j,t}|X_t = 1)$, for $j \in \{o, c\}$, and $Pr(B_{i,t}|X_t = x)$, for $i \in \{s, n\}, x \in \{0, 1\}$; and the expressions in Table 2 for $I_{i,t}(\omega, \xi_t)$, for $i \in \{s, n\}$, $I_{o,t}^y(\omega, \xi_t)$, for $y \in \{1, 2\}$, follow directly from Eqn. (3.2). Next, we show that $I_{i,t}(\omega, \xi_t)$, for $i \in \{s, n\}$, $I_{o,t}^y(\omega, \xi_t)$, for $y \in \{1, 2\}$, are piece-wise continuous functions of ξ_t.

In the following, we consider the case where $TH_{d,t} < 1 \Leftrightarrow TH_{d,t} = (v - p)/l < 1$, and $TH_{r,t} < k \Leftrightarrow TH_{r,t} = \xi_t(v - p) < k$ (see Figure 3.2); other cases defined in Lemma 2 can be proven similarly (see Table A.1). We start with the segmentation of e-commerce adopters. Since the y-intercept $(r = 0)$ of the line $TH_{l,t}$ is $d(r=0) = \frac{(1-\xi_t)(v-p)}{l}$, we can write:

$$Pr(B_{o,t}|X_t=1) = \frac{1}{k}\left[k \cdot \frac{v-p}{l} - \frac{\xi_t(v-p)}{2}\left(\frac{v-p}{l} - \frac{(1-\xi_t)(v-p)}{l}\right)\right] = \frac{(v-p)[2k-\xi_t^2(v-p)]}{2kl}. \quad (1)$$

Similarly, it follows that:

$$Pr(B_{s,t}|X_t=1) = \frac{\xi_t(v-p)[2l-2(v-p)+\xi_t(v-p)]}{2kl}, \quad Pr(B_{n,t}|X_t=1) = 1 - \sum_{i \in \{s,o\}} Pr(B_{i,t}|X_t=1),$$

$$\text{(2)}$$

and $Pr(B_{c,t}|X_t = 1) = \frac{v-p}{l} - Pr(B_{o,t}|X_t = 1)$.

Since the online channel is not an option for consumers with $X_t = 0$, consumers with $\{r : r < TH_{r,t}\}$

choose the store; otherwise, they choose to not make a purchase, leading to:

$$Pr(B_{s,t}|X_t = 0) = \frac{\xi_t(v-p)}{k} \text{ and } Pr(B_{n,t}|X_t = 0) = 1 - Pr(B_{s,t}|X_t = 0). \tag{3}$$

By substituting Eqn.s. (1)-(3), along with the equalities, $Pr(X_t = 1) = F_t(\omega)$ and $Pr(X_t = 0) = 1 - F_t(\omega)$, into Eqn. (3.2), we get:

$$I_{o,t}^1(\omega, \xi_t) = \left[1 - \frac{\xi_t^2}{2W}\right]BF_t(\omega), \quad I_{o,t}^2(\omega, \xi_t) = BF_t(\omega) - I_{o,t}^1(\omega, \xi_t), \quad I_{s,t}(\omega, \xi_t) = \frac{\xi_t}{W}\left[1 - \frac{(2-\xi_t)B}{2}F_t(\omega)\right],$$

and $I_{n,t}(\omega, \xi_t) = 1 - \sum_{i \in \{s,o\}} I_{i,t}(\omega, \xi_t)$.

Next, we show that function $I_{o,t}^1(\omega, \xi_t)$ is continuous in ξ_t. (Continuity proofs for $I_{i,t}(\omega, \xi_t)$, for $i \in \{s,n\}$, and $I_{o,t}^2(\omega, \xi_t)$ are similar.) From Table 3.2, we have that:

$$I_{o,t}^1(\omega, \xi_t) = \begin{cases} B\left[1 - \frac{\xi_t^2}{2W}\right]F_t(\omega), & \text{if } \xi_t < \frac{k}{v-p} \\ \left[(1-\xi_t)B + \frac{BW}{2}\right]F_t(\omega), & \text{otherwise.} \end{cases} \tag{4}$$

We have that $\lim\limits_{\xi_t \to \frac{k}{v-p}-} I_{o,t}^1(\omega, \xi_t) = \lim\limits_{\xi_t \to \frac{k}{v-p}+} I_{o,t}^1(\omega, \xi_t) = \left[B - \frac{BW}{2}\right]F_t(\omega)$. Hence, $I_{o,t}(\omega, \xi_t)$ is continuous in ξ_t.

Proof of Part 2. First, notice that we are slightly abusing the notation since the time index t in Eqn. (3.1) stands for the time elapsed after e-commerce for the specific product becomes available to consumers (for a given set of ω and a), while the time index t in Eqn. (3.7) stands for the time period within the planning time horizon. Hence, in order to mimic the behavior of the diffusion curve given in Eqn. (3.1) for a given set of ω and a, we first provide the following transformation. Let $t^*(\omega)$ denote the diffusion time that the retailer's planning time horizon starts; that is, $F_{t^*(\omega)}(\omega) \equiv F_1$, where $\omega > 0$, as $\omega = 0$ implies that $F_t = 0, \forall t \geq 0$. Then, using Eqn. (3.1), the following must hold:

$$F_{t^*(\omega)}(\omega) = \frac{1 - e^{-(\omega+a)t^*(\omega)}}{1 + \frac{a}{\omega}e^{-(\omega+a)t^*(\omega)}} = F_1 \Rightarrow t^*(\omega) = -\frac{1}{\omega+a}\ln\left(\frac{\omega(1-F_1)}{\omega + aF_1}\right). \tag{5}$$

Then, using Eqn. (3.1), the adoption level at time period t within the planning horizon as a function of ω is as follows:

$$F_t(\omega) = \frac{1 - e^{-(\omega+a)(t^*(\omega)+t-1)}}{1 + \frac{a}{\omega}e^{-(\omega+a)(t^*(\omega)+t-1)}}, \quad \omega > 0, \ t = 2, ..., T. \tag{6}$$

Since Eqn. (6) is continuous and differentiable, it trivially follows that $I_{i,t}(\omega, \xi_t)$, $i \in \{s,n\}$, and $I_{o,t}^y(\omega, \xi_t)$, $y \in \{1,2\}$ are continuous and differentiable in ω.

Derivation of the Expected Profit Function. Recall that $M \sim U[0, b]$. Then, using Eqn.s (3.2), (3.3) and (3.4), we derive the following:

Table A.1: Choice probabilities

	$TH_{d,t} = 1 \Leftrightarrow l \in (0, v-p]$	$TH_{d,t} < 1 \Leftrightarrow l \in (v-p, \infty)$
$TH_{r,t} < k \Leftrightarrow$ $\xi_t < W$	$Pr(B_{s,t}\|X_t = 1) = \begin{cases} 0 & \text{if } \xi_t \leq 1 - \frac{l}{v-p} \\ \frac{[l-(1-\xi_t)(v-p)]^2}{2kl} & \text{otherwise} \end{cases}$ $Pr(B_{o,t}\|X_t = 1) = 1 - Pr(B_{s,t}\|X_t = 1)$ $Pr(B_{n,t}\|X_t = 1) = 0$ $Pr(B_{c,t}\|X_t = 1) = 1 - Pr(B_{o,t}\|X_t = 1)$ $Pr(B_{s,t}\|X_t = 0) = \frac{\xi_t(v-p)}{k}$ $Pr(B_{n,t}\|X_t = 0) = 1 - Pr(B_{s,t}\|X_t = 0)$	$Pr(B_{s,t}\|X_t = 1) = \frac{\xi_t(v-p)[2l-2(v-p)+\xi_t(v-p)]}{2kl}$ $Pr(B_{o,t}\|X_t = 1) = \frac{(v-p)[2k-\xi_t^2(v-p)]}{2kl}$ $Pr(B_{n,t}\|X_t = 1) = 1 - \sum_{i \in \{s,o\}} Pr(B_{i,t}\|X_t = 1)$ $Pr(B_{c,t}\|X_t = 1) = \frac{v-p}{l} - Pr(B_{o,t}\|X_t = 1)$ $Pr(B_{s,t}\|X_t = 0) = \frac{\xi_t(v-p)}{k}$ $Pr(B_{n,t}\|X_t = 0) = 1 - Pr(B_{s,t}\|X_t = 0)$
$TH_{r,t} = k \Leftrightarrow$ $\xi_t \geq W$	$Pr(B_{s,t}\|X_t = 1) = \begin{cases} 0 & \text{if } \xi_t \leq 1 - \frac{l}{v-p} \\ \frac{[l-(1-\xi_t)(v-p)]^2}{2kl} & 1 - \frac{l}{v-p} < \xi_t \leq 1 - \frac{l-k}{v-p} \\ 1 - \frac{2(1-\xi_t)(v-p)+k}{2l} & \text{otherwise} \end{cases}$ $Pr(B_{o,t}\|X_t = 1) = 1 - Pr(B_{o,t}\|X_t = 1)$ $Pr(B_{n,t}\|X_t = 1) = 0$ $Pr(B_{c,t}\|X_t = 1) = 1 - Pr(B_{o,t}\|X_t = 1)$ $Pr(B_{s,t}\|X_t = 0) = 1$ $Pr(B_{n,t}\|X_t = 0) = 0$	$Pr(B_{s,t}\|X_t = 1) = 1 - \frac{2(1-\xi_t)(v-p)+k}{2l}$ $Pr(B_{o,t}\|X_t = 1) = 1 - Pr(B_{s,t}\|X_t = 1)$ $Pr(B_{n,t}\|X_t = 1) = 0$ $Pr(B_{c,t}\|X_t = 1) = \frac{v-p}{l} - Pr(B_{o,t}\|X_t = 1)$ $Pr(B_{s,t}\|X_t = 0) = 1$ $Pr(B_{n,t}\|X_t = 0) = 0$

$$
\begin{aligned}
E[min(Z_{s,t}, q_{s,t})] &= \int_0^{q_{s,t}} z_{s,t} \frac{1}{bI_{s,t}(\omega, \xi_t)} d z_{s,t} + \int_{q_{s,t}}^{bI_{s,t}(\omega,\xi_t)} q_{s,t} \frac{1}{bI_{s,t}(\omega, \xi_t)} d z_{s,t} \\
&= \frac{1}{bI_{s,t}(\omega, \xi_t)} \left[\frac{q_{s,t}^2}{2} + q_{s,t} \Big(bI_{s,t}(\omega, \xi_t) - q_{s,t} \Big) \right] = \alpha_t bI_{s,t}(\omega, \xi_t) \left[1 - \frac{\alpha_t}{2} \right], \\
E[(Z_{s,t} - q_{s,t})^+] &= \int_{q_{s,t}}^{bI_{s,t}(\omega,\xi_t)} (z_{s,t} - q_{s,t}) \frac{1}{bI_{s,t}(\omega, \xi_t)} d z_{s,t} \\
&= \frac{1}{bI_{s,t}(\omega, \xi_t)} \left[\frac{b^2 I_{s,t}(\omega, \xi_t)^2 - q_{s,t}^2}{2} - q_{s,t} \Big(bI_{s,t}(\omega, \xi_t) - q_{s,t} \Big) \right] = \frac{bI_{s,t}(\omega, \xi_t)(1 - \alpha_t)^2}{2}, \\
E[Z_{o,t}] &= \frac{bI_{o,t}^1(\omega, \xi_t)}{2} + \frac{I_{o,t}^2(\omega, \xi_t)}{I_{s,t}(\omega, \xi_t)} E[(Z_{s,t} - q_{s,t})^+] = \frac{bI_{o,t}^1(\omega, \xi_t)}{2} + \frac{bI_{o,t}^2(\omega, \xi_t)(1 - \alpha_t)^2}{2}, \\
E[Z_{n,t}] &= \frac{bI_{n,t}(\omega, \xi_t)}{2} = \frac{b(1 - I_{s,t}(\omega, \xi_t) - I_{o,t}^1(\omega, \xi_t))}{2}.
\end{aligned}
\tag{7}
$$

Then, Eqn. (3.7) follows by substituting the terms derived in Eqn. (7), along with Eqn.s (3.2)-(3.4), into Eqn (3.6). This completes the proof. ∎

Proof of Lemma 3. In the following, we consider the case where $TH_{d,t} < 1 \Leftrightarrow TH_{d,t} = (v-p)/l < 1$; the case where $TH_{d,t} = 1 \Leftrightarrow TH_{d,t} = (v-p)/l \geq 1$ can be proven similarly. Before we present the proof for Lemma 3, we derive several properties.

Property 1. *1. $[I_{o,t}^1(\omega, \xi_t) + I_{o,t}^2(\omega, \xi_t)] = BF_t(\omega) \geq 0$ which implies that $\frac{\partial[I_{o,t}^1(\omega,\xi_t)+I_{o,t}^2(\omega,\xi_t)]}{\partial \xi_t} = 0$;*

2. $[I_{s,t}(\omega, \xi_t) - I_{o,t}^2(\omega, \xi_t)] = Y_t > 0$, and $\frac{\partial Y_t}{\partial \xi_t} \geq 0$;

3. Given that $\xi_t = \lambda\beta(\alpha_{t-1}) + \lambda\xi_{t-1}$, where $\beta(\alpha_t) = \alpha_t(1 - ln(\alpha_t))$, it holds that $\frac{\partial \xi_t}{\partial \alpha_{t-i}} =$

$$-\lambda(1-\lambda)^{i-1}\ln\alpha_{t-i}, \text{ for } t\leq T, \text{ and } i\leq t-1.$$

Proof of Property 1. Throughout the proof, we consider the range $\alpha_t \in (0,1]$, for $t = 1, 2, ..., T$, as $\ln\alpha_t$ is undefined at $\alpha_t = 0$.

Proof of Parts 1 and 2. The results directly follow from Table 3.1.

Proof of Part 3. The result directly follows from the definition of ξ_t.

This completes the proof. ∎

Proof of Lemma 3. Using Property 1, for $\lambda \in [0,1]$ it follows that:

$$E[\pi_t(\omega,\alpha_t,\xi_t)] = -G(\omega) + bBF_t(\omega)\left[\frac{p+s}{2}-c\right] + bY_t\left[(p+s-c)\alpha_t - (p+s)\frac{\alpha_t^2}{2}\right] - \frac{bs}{2} \tag{8}$$

$$\Rightarrow \frac{\partial E[\pi_t(\omega,\alpha_t,\xi_t)]}{\partial \alpha_{t-i}} = -b\frac{\partial Y_t}{\partial \xi_t}\lambda(1-\lambda)^{i-1}\ln\alpha_{t-i}\left[(p+s-c)\alpha_t - (p+s)\frac{\alpha_t^2}{2}\right], \text{ for } i < t \tag{9}$$

$$\frac{\partial E[\pi_t(\omega,\alpha_t,\xi_t)]}{\partial \alpha_t} = bY_t\left[p+s-c-(p+s)\alpha_t\right] \tag{10}$$

$$\Rightarrow \frac{\partial^2 E[\pi_t(\omega,\alpha_t,\xi_t)]}{\partial \alpha_{t-i}^2} = -b\frac{\partial Y_t}{\partial \xi_t}\lambda(1-\lambda)^{i-1}\frac{1}{\alpha_{t-i}}\left[(p+s-c)\alpha_t - (p+s)\frac{\alpha_t^2}{2}\right], \text{ for } i < t \tag{11}$$

$$\frac{\partial^2 E[\pi_t(\omega,\alpha_t,\xi_t)]}{\partial \alpha_{t-i}\alpha_{t-j}} = 0, \text{ for } i,j < t, i \neq j \tag{12}$$

$$\frac{\partial^2 E[\pi_t(\omega,\alpha,\xi_t)]}{\partial \alpha_{t-i}\alpha_t} = -b\frac{\partial Y_t}{\partial \xi_t}\lambda(1-\lambda)^{i-1}\ln\alpha_{t-i}\left[p+s-c-(p+s)\alpha_t\right], \text{ for } i < t \tag{13}$$

$$\frac{\partial^2 E[\pi_t(\omega,\alpha_t,\xi_t)]}{\partial \alpha_t^2} = -bY_t(p+s) \tag{14}$$

Using the definition of Y_t, Property 1 and Eqn. (8), it follows that $E[\pi_t(\omega,\alpha_t,\xi_t)]$ is continuous in $\alpha_j \in (0,1]$, for $j \leq t$. Also, Eqn.s (9) and (10) imply that $E[\pi_t(\omega,\alpha_t,\xi_t)]$ is strictly increasing in $\alpha_j \in (0, CF)$, for $j \leq t$. Let $A^- \equiv \{(\alpha_1, \alpha_2, ..., \alpha_{t-1}) : CF\cdot\vec{1} \leq (\alpha_1, \alpha_2, ..., \alpha_{t-1}) \leq \vec{1}, \xi_t < W\}$ and $A^+ \equiv \{(\alpha_1, \alpha_2, ..., \alpha_{t-1}) : CF\cdot\vec{1} \leq (\alpha_1, \alpha_2, ..., \alpha_{t-1}) \leq \vec{1}, \xi_t \geq W\}$, where $\xi_t = \lambda\beta(\alpha_{t-1}) + (1-\lambda)\xi_{t-1}$. Letting $T \equiv b\lambda\left[(p+s-c)\alpha_t - (p+s)\frac{\alpha_t^2}{2}\right]$, and $V \equiv b\lambda\left[p+s-c-(p+s)\alpha_t\right]$, the Hessian matrix for $-E[\pi_t(\omega,\alpha_t,\xi_t)]$ for A^- in this case, where the entries are in the order of $\alpha_1, \alpha_2, ..., \alpha_t$, is as follows.

$$H^{A^-} = \frac{[1-BF_t(\omega)]}{W}\begin{bmatrix} \frac{T(1-\lambda)^{t-2}}{\alpha_1} & 0 & \cdots & 0 & 0 & V(1-\lambda)^{t-2}\ln\alpha_1 \\ 0 & \frac{T(1-\lambda)^{t-3}}{\alpha_2} & \cdots & 0 & 0 & V(1-\lambda)^{t-3}\ln\alpha_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \frac{T(1-\lambda)^1}{\alpha_{t-2}} & 0 & V(1-\lambda)^1\ln\alpha_{t-2} \\ 0 & 0 & \cdots & 0 & \frac{T(1-\lambda)^0}{\alpha_{t-1}} & V(1-\lambda)^0\ln\alpha_{t-1} \\ V(1-\lambda)^{t-2}\ln\alpha_1 & V(1-\lambda)^{t-3}\ln\alpha_2 & \cdots & V(1-\lambda)^1\ln\alpha_{t-2} & V(1-\lambda)^0\ln\alpha_{t-1} & \xi_t(p+s) \end{bmatrix}$$

Using the definitions of T and V, it follows that $T > 0$ and $V \leq 0$, for $\alpha_t \in [CF, 1]$. In order to prove that $E[\pi_t(\omega,\alpha_t,\xi_t)]$ is jointly concave in $\alpha_t \in [CF, 1]$ and $(\alpha_1, \alpha_2, ..., \alpha_{t-1}) \in A^-$, it is sufficient to show that H^{A^-} is positive semi-definite (PSD) in $\alpha_j \in [CF, 1]$, for $j \leq t$. For that, we need to show that all principal minors of H^{A^-}, which we denote by D_i, are greater than or equal

to zero for $\alpha_j \in [CF, 1]$, for $j \le t$. Clearly, $D_1 = \xi_t(p + s) \ge 0$. In order to show that H^{A^-} is PSD, we only need to show that $D_2 \ge 0$, then $D_j \ge 0$ for $j > 2$ can be proven by induction. In order to show that $D_2 \ge 0$ for $\alpha_j \in [CF, 1]$, for $j \le t$, we need to show that:

$$D_2 = \frac{T(1-\lambda)^0}{\alpha_{t-1}} \xi_t(p+s) - V^2(\ln \alpha_{t-1})^2 \ge 0. \tag{15}$$

It follows that D_2 is decreasing for $\alpha_t \in [CF, 1]$; hence, for any given α_{t-1}, D_2 is minimized at $\alpha_t = 1$. Therefore, in order for D_2 to be positive at $\alpha_t = 1$, it must hold that:

$$\lambda \left[\frac{p+s}{2} - c \right] \frac{\xi_t(p+s)}{\alpha_{t-1}} - \lambda^2 c^2 (\ln \alpha_{t-1})^2 \ge 0 \Rightarrow \left[\frac{p+s}{2} - c \right] \frac{\xi_t(p+s)}{\alpha_{t-1}} - c^2 (\ln \alpha_{t-1})^2 \ge 0$$

$$\Rightarrow f_1 = \left[\frac{p+s}{2} - c \right] \xi_t(p+s) - c^2 (\ln \alpha_{t-1})^2 \alpha_{t-1} \ge 0 \tag{16}$$

f_1 is increasing for $\alpha_{t-i} \in [CF, 1]$, for $i < t$. Thus, it must hold that:

$$\left[\frac{p+s}{2} - c \right] CF(1 - \ln CF) - c^2 (\ln CF)^2 CF \ge 0 \Rightarrow \left[\frac{p+s}{2} - c \right] (1 - \ln CF) - c^2 (\ln CF)^2 \ge 0 \tag{17}$$

Recall that $CF = \frac{p+s-c}{p+s}$, letting $\eta \equiv \frac{p+s}{c}$ and simplifying, we can rewrite Eqn. (17) as:

$$f_2 = \eta \left[\frac{\eta - 2}{2} \right] \left(1 - \ln \frac{\eta - 1}{\eta} \right) - \left(\ln \frac{\eta - 1}{\eta} \right)^2 \ge 0 \tag{18}$$

f_2 is increasing in η, which implies that any η that is greater than the root of f_2 satisfies Eqn. (18), which assures that $D_2 \ge 0$. By numerically solving, we get: $f_2 = 0$ at $\eta = 2.206$. Thus, we prove that $D_2 \ge 0$ for $\eta \ge 2.206$, which implies that H^{A^-} is PSD for $\eta \ge 2.206$. Thus, $E[\pi_t(\omega, \alpha_t, \xi_t)]$ is concave in A^-. Next, the Hessian matrix for $-E[\pi_t(\omega, \xi_t)]$ for A^+ is:

$$H^{A^+} = [1 - BF_t(\omega)] \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & (p+s) \end{bmatrix}$$

which is PSD for $\forall \alpha_j \in (0, 1], j \le t$. Then, $E[\pi_t(\omega, \alpha_t, \xi_t)]$ is jointly concave in $\alpha_j \in [CF, 1]$, for $j \le t$ since $E[\pi_t(\omega, \alpha_t, \xi_t)]$ is: 1) jointly concave in $\forall \alpha_j, j \le t$ in both regions A^- and A^+, 2) continuous and differentiable in α_t, and 3) $\frac{\partial E[\pi_t(\omega, \alpha_t, \xi_t)]}{\partial \alpha_{t-i}} \ge 0, i < t$, for $(\alpha_1, \alpha_2, ..., \alpha_{t-1}) \in A^-$, and $\frac{\partial E[\pi_t(\omega, \alpha_t, \xi_t)]}{\partial \alpha_{t-i}} = 0, i < t$, for $(\alpha_1, \alpha_2, ..., \alpha_{t-1}) \in A^+$ (using Eqn. (9)).

Proof of Remark 1. Using Property 1, it follows that $\xi_t = \beta(\alpha_0)$ for $\lambda = 0$. Hence, $E[\pi_t(\omega, \alpha_t, \xi_t)]$ is independent of α_j, for $\forall j \ne t$. Then, Eqn.s (10) and (14) imply that the FOC for $E[\pi_t(\omega, \alpha_t, \xi_t)]$, $\forall t$, for $\lambda = 0$, is satisfied at $\alpha_t = CF$ for $t \le T$.

Proof of Theorem 1. In the following, we consider the case where $TH_{d,t} < 1 \Leftrightarrow TH_{d,t} = (v - p)/l < 1$; the case where $TH_{d,t} = 1 \Leftrightarrow TH_{d,t} = (v - p)/l \ge 1$ can be proven similarly. For a

given $\omega > 0$, since ξ_t is a function of all $\alpha_i, i < t$, it follows that:

$$E[\Pi(\omega, \overrightarrow{\alpha})] = -G(\omega) + \sum_{t=1}^{T} E[\pi_t(\omega, \alpha_t, \xi_t)] \Rightarrow \frac{\partial E[\Pi(\omega, \overrightarrow{\alpha})]}{\partial \alpha_t} = \sum_{i=0}^{T-t} \frac{\partial E[\pi_{t+i}(\omega, \alpha_{t+i}, \xi_{t+i})]}{\partial \alpha_t}, \quad \text{for } t \leq T.$$

$$\Rightarrow \frac{\partial E[\Pi(\omega, \overrightarrow{\alpha})]}{\partial \alpha_t} = Y_t[p+s-c-(p+s)\alpha_t] - \lambda \ln \alpha_t \sum_{i=1}^{T-t} \frac{\partial Y_{t+i}}{\partial \xi_{t+i}} (1-\lambda)^{i-1} \left[(p+s-c)\alpha_{t+i} - (p+s)\frac{\alpha_{t+i}^2}{2} \right]. \quad (19)$$

Then using Eqn. (19), $\alpha_T^* = CF$. Let ξ_t^*, for $t \leq T$, denote ξ_t at optimality. Then, using Eqn.s (9) and (10), it follows that the FOC for α_t, for $t \leq T$, is satisfied at $\alpha_t^* = CF$ if $\xi_t^* = \beta(CF) > W$; otherwise, there exists a vector, say, $\overrightarrow{\alpha}'' = (\alpha_1'', \alpha_2'', ..., \alpha_{T-1}'')$, that satisfies the FOC if $\xi_t''(\omega) \leq W, \forall t \leq T$, where $\xi_t''(\omega) = \lambda\beta(\alpha_{t-1}'') + (1 - \lambda)\xi_{t-1}$, with $\xi_1'' = \beta(\alpha_0)$; thus, $(\alpha_1^*, \alpha_2^*, ..., \alpha_{T-1}^*) = \overrightarrow{\alpha}''$. If $\beta(CF) \leq W < \min_{1<t\leq T}\{\xi_t''(\omega)\}$, then $E[\Pi(\omega, \overrightarrow{\alpha})]$ is increasing in all directions in $(\alpha_!, \alpha_2, ..., \alpha_{T-1})$ when $\xi_t \leq W, \forall t \in [2, T]$; therefore, it follows that $(\alpha_1^*, \alpha_2^*, ..., \alpha_{T-1}^*) = \overrightarrow{\alpha}'''_{prior} \equiv \{(\alpha_1, \alpha_2, ..., \alpha_{T-1}) : \xi_t = W, \forall t \in [2, T]\}$. Consequently, if $\min_{1<t\leq T}\{\xi_t''(\omega)\} \leq W < \max_{1<t\leq T}\{\xi_t''(\omega)\}$, then $\alpha_t''' \leq \alpha_t^* \leq \alpha_t''$ since $E[\Pi(\omega, \overrightarrow{\alpha})]$ is decreasing at $\overrightarrow{\alpha}''$ in some directions while maximized in others. This completes the proof. ∎

Proof of Theorem 2. In the following, we consider the case where $TH_{d,t} < 1 \Leftrightarrow TH_{d,t} = (v-p)/l < 1$, and $TH_{r,t} < k \Leftrightarrow TH_{r,t} = \xi_t(v-p) < k$ (see Figure 3.2); other cases can be proven similarly. Before we present the proof, we derive the following property.

Property 2. $\frac{\partial F_t(\omega)}{\partial \omega} < 0$ for $\omega > 0, t = 2, ..., T$.

Proof of Property 2. Using Eqns. (5) and 6), it follows that:

$$F_t(\omega) = \frac{\omega + aF_1 - \omega(1-F_1)e^{-(\omega+a)(t-1)}}{\omega + aF_1 + a(1-F_1)e^{-(\omega+a)(t-1)}} = 1 - \frac{(a+\omega)(1-F_1)e^{-(\omega+a)(t-1)}}{\omega + aF_1 + a(1-F_1)e^{-(\omega+a)(t-1)}}$$

$$= 1 - \frac{(a+\omega)(1-F_1)}{(\omega+aF_1)e^{(\omega+a)(t-1)} + a(1-F_1)}$$

$$\Rightarrow \frac{\partial F_t(\omega)}{\partial \omega} = (1-F_1)\frac{(\omega+aF_1)e^{(\omega+a)(t-1)} + a(1-F_1) - (a+\omega)e^{(\omega+a)(t-1)}(1+(\omega+aF_1)(t-1))}{\left((\omega+aF_1)e^{(\omega+a)(t-1)} + a(1-F_1)\right)^2}$$

$$= (1-F_1)e^{(\omega+a)(t-1)}\frac{(\omega+aF_1) + a(1-F_1)e^{-(\omega+a)(t-1)} - (a+\omega)(1+(\omega+aF_1)(t-1))}{\left((\omega+aF_1)e^{(\omega+a)(t-1)} + a(1-F_1)\right)^2}$$

$$= (1-F_1)e^{(\omega+a)(t-1)}\frac{a(1-F_1)(e^{-(\omega+a)(t-1)} - 1) - (a+\omega)(\omega+a)(\omega+aF_1)(t-1)}{\left((\omega+aF_1)e^{(\omega+a)(t-1)} + a(1-F_1)\right)^2} < 0.$$

Proof of Theorem 2. Using Eqn.s (3.3), (3.4), Property (2), and Theorem 3, when $\beta(CF) < W$, it holds that:

$$q_{s,t}^*(\omega) = CF\,b\,I_{s,t}(\omega, \beta(CF)) \Rightarrow \frac{\partial q_{s,t}^*(\omega)}{\partial \omega} = -b\,CF\frac{(2-\beta(CF))\beta(CF)\,B}{2W}\frac{\partial F_t(\omega)}{\partial \omega} < 0$$

$$q_{o,t}^*(\omega) = b\,I_{o,t}^1(\omega, \beta(CF)) + (1-CF)\,b\,[BF_t(\omega) - I_{o,t}^1(\omega, \beta(CF))]$$

$$\Rightarrow \frac{\partial q_{o,t}^*(\omega)}{\partial \omega} = b\,CF\,B\left[1 - \frac{\beta(CF)^2}{2W}\right]\frac{\partial F_t(\omega)}{\partial \omega} + b\,(1-CF)B\frac{\partial F_t(\omega)}{\partial \omega} > 0$$

$$\Rightarrow \frac{\partial q_{s,t}^*(\omega)}{\partial \omega} + \frac{\partial q_{o,t}^*(\omega)}{\partial \omega} = bB(1 - CF\beta(CF)) > 0.$$

This completes the proof. ∎

Proof of Lemma 4. In the following, we consider the case where $TH_{d,t} < 1 \Leftrightarrow TH_{d,t} = (v-p)/l < 1$; the case where $TH_{d,t} = 1 \Leftrightarrow TH_{d,t} = (v-p)/l \geq 1$ can be proven similarly. From Eqn. (8), it follows that:

$$\frac{\partial E[\pi_t(\omega, \alpha_t, \xi_t)]}{\partial \omega} = -\frac{\partial G(\omega)}{\partial \omega} + bB\frac{\partial F_t(\omega)}{\partial \omega}\left[\frac{p+s}{2} - c\right] - b\frac{\partial Y_t}{\partial \omega}\left[(p+s-c)\alpha_t - (p+s)\frac{\alpha_t^2}{2}\right] \quad (20)$$

By assumption, $G(\omega)$ is increasing in ω. Then, using the definition of Y_t and Property 2, the proof follows. ∎

Proof of Theorem 3. From Theorem 1, if $W < \beta(CF)$, then $\overrightarrow{\alpha}^* = CF \cdot \overrightarrow{1} \Rightarrow \xi_t^* = \beta(CF) > W, \forall t = 1, ..., T$. Since, $\frac{p+s}{2} - c\left[(p+s-c)\alpha_t^* - (p+s)\frac{(\alpha_t^*)^2}{2}\right] \leq 0$ at $\alpha_t^* = CF$, and $\partial\alpha_t^*/\partial\omega = 0$, using Lemma 4, the proof follows. ∎

Proof of Lemma 5. Given that $\xi_t^P = \beta(\alpha_t) \Rightarrow \frac{\partial\xi_t^P}{\partial\alpha_t} = -\ln\alpha_t$, and using Property 1, it follows that:

$$E[\pi_t^P(\omega, \alpha_t, \xi_t)] = bBF_t(\omega)\left[\frac{p+s}{2} - c\right] + bY_t\left[(p+s-c)\alpha_t - (p+s)\frac{\alpha_t^2}{2}\right] - \frac{bs}{2} \quad (21)$$

$$\Rightarrow \frac{\partial E[\pi_t^P(\omega, \alpha_t, \xi_t)]}{\partial\alpha_t} = -b\frac{\partial Y_t}{\partial\xi_t}\ln\alpha_t\left[(p+s-c)\alpha_t - (p+s)\frac{\alpha_t^2}{2}\right] + bY_t[p+s-c-(p+s)\alpha_t] \quad (22)$$

$$\Rightarrow \frac{\partial^2 E[\pi_t^P(\omega, \alpha_t, \xi_t)]}{\partial\alpha_t^2} = -b\frac{\partial Y_t}{\partial\xi_t}\frac{1}{\alpha_t}\left[(p+s-c)\alpha_t - (p+s)\frac{\alpha_t^2}{2}\right] - 2b\frac{\partial Y_t}{\partial\xi_t}\ln\alpha_t[p+s-c-(p+s)\alpha_t] - bY_t(p+s) \quad (23)$$

Using the definition of Y_t, Property 1 and Eqn. (21), it follows that $E[\pi_t^P(\omega, \alpha_t, \xi_t)]$ is continuous in $\alpha_t \in (0, 1]$, for a given $\omega > 0$. Let $\alpha_{TH} \equiv \{\alpha_t : \xi_t = W\}$, where $\xi_t = \beta(\alpha_t)$. Then, there are three cases to consider: *Case 1.* If $0 < \alpha_{TH} \leq CF \Leftrightarrow 0 < W \leq \beta(CF)$, then $E[\pi_t(\omega, \alpha_t, \xi_t)]$ is strictly increasing in $\alpha_t \in (0, CF)$, and strictly concave and differentiable in $\alpha_t \in [CF, 1]$ with a maximizer at $\alpha_t^{*P} = CF$. *Case 2.* If $CF < \alpha_{TH} \leq 1 \Leftrightarrow \beta(CF) < W \leq 1$, then $E[\pi_t^P(\omega, \alpha_t, \xi_t)]$ is strictly increasing in $\alpha_t \in (0, CF)$, strictly concave in $\alpha_t \in (CF, \alpha_{TH})$, and strictly concave decreasing in $\alpha_t \in (\alpha_{TH}, 1]$ with $\lim_{\alpha_t \to \alpha_{TH}-} \partial E[\pi_t^P(\omega, \alpha_t, \xi_t)]/\partial\alpha_t > \lim_{\alpha_t \to \alpha_{TH}+} \partial E[\pi_t^P(\omega, \alpha_t, \xi_t)]/\partial\alpha_t$ by using Eqn. (22), which implies that $E[\pi_t^P(\omega, \alpha_t, \xi_t)]$ is strictly concave in $\alpha_t \in [CF, 1]$ with a maximizer at $\alpha_t^{*P} \in (CF, \alpha_{TH}]$. *Case 3.* If $\alpha_{TH} > 1 \Leftrightarrow W > 1$, then $E[\pi_t^P(\omega, \alpha_t, \xi_t)]$ is strictly increasing in $\alpha_t \in (0, CF)$ and strictly concave in $\alpha_t \in [CF, 1]$ with a maximizer at $\alpha_t^{*P} \in (CF, 1)$. Also, using Eqn. (22), it follows that the first derivative of $E[\pi_t^P(\omega, \alpha_t, \xi_t)]$ is independent of the adoption level $F_t(\omega)$; therefore, $\alpha_t^{*P} = \alpha^{*P}$ for $t \leq T$. This completes the proof. ∎

Proof of Theorem 4.
Proof of Part 1. It directly follows from Lemma 5 that, for a given $\omega > 0$, if $0 < \alpha_{TH} \leq CF \Leftrightarrow 0 < W \leq \beta(CF)$, then $\alpha^{*P} = CF$. If, on the other hand, $CF < \alpha_{TH} \leq 1 \Leftrightarrow \beta(CF) < W \leq 1$, then $\alpha^{*P} \in (CF, \alpha_{TH}]$; that is, the maximizer of $E[\pi_t^P(\omega, \alpha_t, \xi_t)]$ is also the maximizer of $E[\pi_t^P(\omega, \alpha_t, \xi_t)]$ when $\xi_t \leq W$ (equivalently, $\alpha_t < \alpha_{TH}$).

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Let α' denote the point that sets the FOC of $E[\pi_t^P(\omega, \alpha_t, \xi_t)]$ when $\xi_t \leq W$ equal to zero. It follows from Eqn. (22) that $\alpha' \in (0, 1)$. Then, there are two cases to consider: *Case 1.* If $\alpha_{TH} < \alpha'$ (equivalently, $W < \beta(\alpha')$) then $\alpha^{*P} = \alpha_{TH}$, and *Case 2.* If $\alpha_{TH} \geq \alpha'$ (equivalently, $W \geq \beta(\alpha')$) then $\alpha^{*P} = \alpha'$.

Proof of Part 2. Proof follows similar to the proof of Theorem 2.

Proof of Part 3. Proof follows similar to the proof of Theorem 3.

This completes the proof. ∎

Proof of Theorem 5. Before we present the proof for Theorem 5, we derive the following property.

Property 3. $\partial^2 E[\Pi(\omega, \overrightarrow{\alpha})]/\partial\alpha_t\partial\alpha_j \leq 0$, *for* $\forall \alpha_t, \alpha_j \in (CF, 1), j, t \leq T$.

Proof of Property 3. Taking the derivative of Eqn. (19) with respect to α_j, for $j \leq T$, it follows that $\partial^2 E[\Pi(\omega, \alpha_t, \xi_t)]/\partial\alpha_t\partial\alpha_j \leq 0$ for $j, \forall t \leq T$. This completes the proof. ∎

Proof of Theorem 5.

Proof of Part 1. The proof follows from Theorems 1 and 4.

Proof of Part 2. The proof proceeds in three parts. First, we derive the expression for the upper bound $UB_{\alpha_t^*}(\lambda, CF)$ on $\alpha_t^*, t = 1, 2, ..., T - 1$, for a given pair of (λ, CF) and for any $\omega > 0$. Next, we show that $UB_{\alpha_t^*}(\lambda, CF)$ is decreasing in t and increasing λ, and hence $UB(CF) \equiv UB_{\alpha_1^*}(1, CF)$ is an upper bound on $\overrightarrow{\alpha}^*$ for a given CF. Finally, we complete the proof by showing that $\alpha^{*P} \leq UB_{\alpha^{*P}} \equiv \alpha' < UB(CF)$.

Factoring out $p + s$ in Eqn. (19), we get:

$$\frac{\partial E[\Pi(\omega, \overrightarrow{\alpha})]}{\partial\alpha_t} = (p+s)Y_t[CF - \alpha_t] - (p+s)\lambda \ln\alpha_t \sum_{i=1}^{T-t} \frac{\partial Y_{t+i}}{\partial\xi_{t+i}}(1-\lambda)^{i-1}\left[CF\alpha_{t+i} - \frac{\alpha_{t+i}^2}{2}\right] \quad (24)$$

For a given ω, when $\xi_t^* \leq W, \forall t = 2, ..., T$; that is $\overrightarrow{\alpha}^*(\omega) = \overrightarrow{\alpha}''(\omega)$, it follows that $\partial E[\Pi(\omega, \overrightarrow{\alpha})]/\partial\alpha_t$ is decreasing in α_j, for $j \leq T$ (using Property 3), and increasing in $\frac{[1-BF_{t+i}(\omega)]}{[1-BF_t(\omega)]}$, for $i \in [1, T-t]$ (from Eqn. (24)). Then, $UB_{\alpha_t''}(\lambda, CF)$ sets Eqn (24) equal to zero when $\alpha_j = CF, \forall j \leq T, j \neq t$, i.e. at its lower bound, and $\frac{[1-BF_{t+i}]}{[1-BF_t]} = 1$, i.e. at its maximum, which is obtained by setting $F_t(\omega) = F_{t+i}(\omega)$, for $i \in [1, T-t]$ (Hence, $UB_{\alpha_t^*}(\lambda, CF)$ is independent of B, ω and a.).

When $\alpha_j = CF$, for $\forall j \leq T, j \neq t$, and $F_t = F_{t+i}$, for $i \in [1, T-t]$, the first term in Eqn. (24) is the same for all $\partial E[\Pi(\omega, \overrightarrow{\alpha})]/\partial\alpha_t$; however, the second term is a summation over all $\alpha_{t+i}, i \in [1, T-t]$. Thus, for a larger t, the second term in Eqn. (24) is less positive; hence, $UB_{\alpha_t''}(\lambda, CF)$ is decresing in t. Also, when $\alpha_j = CF$, for $\forall j \leq T, j \neq t$, Eqn. (24) is decreasing in α_t and increasing in λ. Thus, $UB_{\alpha_t''}(\lambda, CF), t < T$, is increasing in λ. Hence, $UB(CF) \equiv UB_{\alpha_1^*}(1, CF)$ is an upper bound on $\overrightarrow{\alpha}^*$.

Setting $\lambda = 1$, $\alpha_j = CF, \forall j \leq T, j \neq 1$, and $F_1 = F_{1+i}$, for $i = 1, ..., T - 1$, in Eqn. (24), the FOC for α_1 follows:

$$\beta(CF)\left[CF - UB(CF)\right] - \ln\left(UB(CF)\right)\left[\frac{CF^2}{2}\right] = 0. \quad (25)$$

When $\beta(\alpha') \leq W$; that is $\alpha^{*P} = \alpha'$, at optimality it must hold that (using Eqn. 22):

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$$\beta(\alpha')[CF - \alpha'] - \ln \alpha' \left[CF\alpha' - \frac{\alpha'^2}{2} \right] = 0. \tag{26}$$

That is, using Eqn.s. (25) and (26),

$$\beta(CF)\left[CF - UB(CF)\right] - \ln\left(UB(CF)\right)\left[\frac{CF^2}{2}\right] = \beta(\alpha')[CF - \alpha'] - \ln \alpha'\left[CF\alpha' - \frac{\alpha'^2}{2}\right] \tag{27}$$

We prove that $\alpha^{*P} \leq UB_{\alpha^{*P}} \equiv \alpha' < UB(CF)$ by contradiction. Assume that $\alpha' \geq UB(CF) > \beta(CF)$, then using Eqn. (27), it must hold that:

$$\beta(CF)\left[CF - UB(CF)\right] - \beta(\alpha')[CF - \alpha'] = \ln\left(UB(CF)\right)\left[\frac{CF^2}{2}\right] - \ln \alpha'\left[CF\alpha' - \frac{\alpha'^2}{2}\right] > 0. \tag{28}$$

In addition, $\ln\left(UB(CF)\right) < \ln \alpha' < 0$ and $\frac{CF^2}{2} > CF\alpha' - \frac{\alpha'^2}{2} > 0 \Rightarrow \ln\left(UB(CF)\right)\left[\frac{CF^2}{2}\right] - \ln \alpha'\left[CF\alpha' - \frac{\alpha'^2}{2}\right] < 0$, which poses a contradiction with Eqn. (28). Hence, it must hold that $\alpha^{*P} \leq UB_{\alpha^{*P}} \equiv \alpha' < UB(CF)$, which implies that $|\alpha^{*P} - \alpha_t^*| < UB(CF) - CF$. This completes the proof. ■

Proof of Lemma 6.. In the following, we assume that $v - H \geq 0$, and then provide the proof for $Pr(E_o(p_o, p_s))$ and $Pr(E_s^i(p_o, p_s)), i \in \{s, n\}$, for the case where $v - p_o < H$ and $p_s > p_o \Rightarrow v - p_s < v - p_o < H$, i.e. $(p_o, p_s) \in R_3$, see Figure 4.1 and Table 4.2. (Other cases can be proven similarly.) Next, we show that $Pr(E_o(p_o, p_s))$ and $Pr(E_s^i(p_o, p_s)), i \in \{s, n\}$, are piece-wise continuous and non-differentiable. Note that if $v - H < 0$, only Regions R_2, R_3, and R_4 would be possible.

As depicted in Figure 4.1, the gridded region shows the proportion of consumers who choose the online channel, i.e. $Pr(E_o(p_o, p_s)) = (2H - v + p_o)(v - p_o)/2H^2 \leq 0.5$. Since consumers choose between the store and the online channel assuming that $p_s^e = p_o$, and they either purchase the item through the store channel or leave the store empty-handed after observing the actual price, it holds that $Pr(E_o(p_o, p_s)) = \sum_{i \in \{s,n\}} Pr(E_s^i(p_o, p_s))$, where $Pr(E_s^s(p_o, p_s)) = (2H - v + p_s)(v - p_s)/2H^2 \leq 0.5$.

In the following, we consider the continuity and differentiability of $Pr(E_o(p_o, p_s))$ in Regions R_2 and R_3; other cases can be proven similarly. From Table 4.2, we have $\lim_{p_o \to v - H-} Pr(E_o(p_o, p_s)) = \lim_{p_o \to v - H+} Pr(E_o(p_o, p_s)) = 0.5$ and $\lim_{p_o \to v - H-} \partial Pr(E_o(p_o, p_s))/\partial p_o = \lim_{p_o \to v - H+} \partial Pr(E_o(p_o, p_s))/\partial p_o = 0$. Hence, $Pr(E_o(p_o, p_s))$ is piece-wise and continuous and differentiable in p_o and p_s. This completes the proof. ■

Proof of Lemma 7. From Eqn. (4.2), it follows that:

$$\frac{\partial \pi(p_o, p_s)}{\partial p_o} = \frac{\partial Pr(E_o(p_o, p_s))}{\partial p_o}(p_o - c_o) + \frac{\partial Pr(E_s^s(p_o, p_s))}{\partial p_o}(p_s - c_s) + Pr(E_o(p_o, p_s)) \tag{29}$$

$$\Rightarrow \frac{\partial^2 \pi(p_o, p_s)}{\partial p_o^2} = \frac{\partial^2 Pr(E_o(p_o, p_s))}{\partial p_o^2}(p_o - c_o) + \frac{\partial^2 Pr(E_s^s(p_o, p_s))}{\partial p_o^2}(p_s - c_s) + 2\frac{\partial Pr(E_o(p_o, p_s))}{\partial p_o} \tag{30}$$

$$\frac{\partial \pi(p_o, p_s)}{\partial p_s} = \frac{\partial Pr(E_s^s(p_o, p_s))}{\partial p_s}(p_s - c_s) + Pr(E_s^s(p_o, p_s)) \tag{31}$$

$$\Rightarrow \frac{\partial^2 \pi(p_o, p_s)}{\partial p_s^2} = \frac{\partial^2 Pr(E_s^s(p_o, p_s))}{\partial p_s^2}(p_s - c_s) + 2\frac{\partial Pr(E_s^s(p_o, p_s))}{\partial p_s} \tag{32}$$

$$\frac{\partial^2 \pi(p_o, p_s)}{\partial p_o \partial p_s} = \frac{\partial Pr(E_s^s(p_o, p_s))}{\partial p_o} \tag{33}$$

Using Lemma 6, Property 4, and Eqns. (29)-(33) it follows that $\pi(p_o, p_s)$ is: 1) continuous and differentiable in p_o and p_s, and 2) strictly jointly concave in p_o and p_s for Regions R_1, R_2, R_3, R_5, but not for R_4. This completes the proof. ∎

Proof of Theorem 6. In this proof, we consider the case when $0 < H < v$, other cases can be proven similarly. The proof follows in two parts. First, we prove that $\max\left(v - H, \frac{2c_j + v}{3}\right) < p_j^* < v, j \in \{s, o\}$, and then we show that $p_o^* < p_s^* < p_o^* - c_o + c_s$. Before we present the proof, we derive the following property:

Property 4.

1. $\frac{\partial Pr(E_o(p_o, p_s))}{\partial p_o} \leq 0$, $\frac{\partial Pr(E_o(p_o, p_s))}{\partial p_s} = 0$, $\frac{\partial Pr(E_s^s(p_o, p_s))}{\partial p_o} \leq 0$, and $\frac{\partial Pr(E_s^s(p_o, p_s))}{\partial p_s} \leq 0$, and

2. $\frac{\partial^2 Pr(E_o(p_o, p_s))}{\partial p_o^2} \leq 0$, $\frac{\partial^2 Pr(E_o(p_o, p_s))}{\partial p_s^2} = 0$, $\frac{\partial^2 Pr(E_s^s(p_o, p_s))}{\partial p_o^2} \leq 0$, $\frac{\partial^2 Pr(E_s^s(p_o, p_s))}{\partial p_s^2} \leq 0$, and $\frac{\partial^2 Pr(E_s^s(p_o, p_s))}{\partial p_o \partial p_s} = \frac{\partial^2 Pr(E_o(p_o, p_s))}{\partial p_o \partial p_s} = 0$.

Proof of Property 4. The proof trivially follows from Lemma 6. ∎

Proof of Theorem 6. We first prove that $p_o^* \leq p_s^*$ by contradiction. Assume that $p' = p_o^* > p_s^* = p''$. Then, from Eqn. (4.2), the retailer's optimal profit equals $\pi(p_o^*, p_s^*) = \pi'(p', p'') = Pr(E_o(p', p''))(p' - c_o) + Pr(E_s^s(p', p''))(p'' - c_s)$, with $Pr(E_o(p', p'')) = Pr(E_s^s(p', p''))$. Now assume that the retailer sets $p' = p_o = p_s$, then the retailer's profit equals $\pi''(p', p') = Pr(E_o(p', p'))(p' - c_o) + Pr(E_s^s(p', p'))(p' - c_s) > \pi'(p_o, p_s)$ since $Pr(E_o(p', p'')) = Pr(E_o(p', p')) = Pr(E_s^s(p', p'')) = Pr(E_s^s(p', p'))$ and $(p' - c_s) > (p'' - c_s)$. Hence, it must hold that $p_o^* \leq p_s^*$.

Now we further show that $\max(0, v - H) \leq p_o^* \leq p_s^* \leq v$. Assume that $v - H \geq 0$. Then, (p_o^*, p_s^*) must belong to Regions R_1, R_2, or R_3. Since $Pr(E_o(p_o, p_s)) = Pr(E_s^s(p_o, p_s)) = 0.5$ for any $(p_o, p_s) \in R_1$, the retailer can simply increase its profit by setting p_o and p_s at their upper bound in this region, i.e. $(p_o, p_s) = (v - H, v - H)$. Further, for any $(p_o, p_s) \in R_2$, it holds that $Pr(E_o(p_o, p_s)) = 0.5$. Hence, for any given p_s in this region, the retailer can increase its profit by setting p_o at its upper limit, i.e. $p_o = v - H$. Hence, it must hold that $\max(0, v - H) \leq p_o^* \leq p_s^* \leq v$, i.e. $(p_o^*, p_s^*) \in R_3$. In addition, using Eqns. (29) and (31), it trivially follows that $\pi(p_o, p_s)$ is increasing when $p_o = v - H$ and $p_i = (2c_i + v)/3, i \in \{s, o\}$, and it is decreasing when $p_s = v$; hence, since $\pi(p_o, p_s)$ is jointly concave in (p_o, p_s) for $\max(0, v - H) \leq p_o^* \leq p_s^* \leq v$, it must hold that $\max((2c_i + v)/3, v - H) < p_i^* < v, i \in \{s, o\}$ and $p_o^* \leq p_s^*$.

Since $\pi(p_o, p_s)$ is strictly jointly concave in (p_o, p_s) for $\max(0, v - H) < p_o^* \leq p_s^* < v$ (using Eqn. (30), (32) and (33)), the First Order Condition (FOC) is necessary and sufficient for optimality. Assume that $p_o^* = p'''$ and $p_s^* = p_o^* - c_o + c_s = p''' - c_o + c_s$. Then, using Eqns. (29) and (31), it

must hold that:

$$\frac{\partial \pi(p_o, p_s)}{\partial p_o}\bigg|_{\substack{p_s = p''' - c_o + c_s \\ p_o = p'''}} = (-H + v - p')(p' - c_o)/H^2 + (2H - v + p')(v - p')/2H^2 = 0 \tag{34}$$

$$\frac{\partial \pi(p_o, p_s)}{\partial p_s}\bigg|_{\substack{p_s = p''' - c_o + c_s \\ p_o = p'''}} = (-H + v - p' + c_o - c_s)(p' - c_o)/H^2 + (2H - v + p' - c_o + c_s)(v - p' + c_o - c_s)/2H^2 \tag{35}$$

$$= \frac{\partial \pi(p_o, p_s)}{\partial p_o}\bigg|_{\substack{p_s = p''' - c_o + c_s \\ p_o = = p''' - c_o + c_s}} = 0 \tag{36}$$

Since $\pi(p_o, p_s)$ is strictly jointly concave in (p_o, p_s) for $\max(0, v - H) < p_o^* \le p_s^* < v$, and by assumption Eqn. (34) holds true, Eqn. (36) poses a contradiction. Hence, it must be the case that $p_s^* < p_o^* - c_o + c_s$. Using a similar argument, it can be easily shown that $p_o^* < p_s^*$. Hence, the optimal solution must satisfy: $p_o^* < p_s^* < p_o^* - c_o + c_s$. This completes the proof. ∎

Proof of Lemma 8. In the following, we consider the case $\max(v - H, 0) \le p_o \le p_s \le v$, i.e. Region R_3 and R_2 for the settings with and without information-sharing, respectively. The proof follows by contradiction. Assume that $\sum_{j \in \{o,s\}} Pr(E_j'(p_o, p_s)) < Pr(E_o(p_o, p_s)) + Pr(E_s^s(p_o, p_s))$. Then, using Tables 4.2 and 4.3, it must hold that:

$$2H(v - p_o) - (v - p_s)^2 + (2H + 2p_o - v - p_s)(v - p_s) < (2H - v + p_o)(v - p_o) + (2H - v + p_s)(v - p_s)$$
$$\Rightarrow (2p_o - p_s)(v - p_s) - (v - p_s)^2 < -(v - p_o)^2 + p_s(v - p_s)$$
$$\Rightarrow 0 < (v - p_s)^2 - (v - p_o)^2 + 2(p_s - p_o)(v - p_s)$$
$$\Rightarrow 0 < -(2v - p_s - p_o)(p_s - p_o) + 2(p_s - p_o)(v - p_s)$$
$$\Rightarrow 0 < -(p_s - p_o)^2$$

Since the above result poses a contradiction, it must hold that $\sum_{j \in \{o,s\}} Pr(E_j'(p_o, p_s)) \ge Pr(E_o(p_o, p_s)) + Pr(E_s^s(p_o, p_s))$. This completes the proof. ∎

Proof of Lemma 9. The proof follows similar to the proof of Lemma 6. ∎

Proof of Lemma 10. From Eqn. (4.3), it follows that:

$$\frac{\partial \pi'(p_o, p_s)}{\partial p_o} = \frac{\partial Pr(E_o'(p_o, p_s))}{\partial p_o}(p_o - c_o) + \frac{\partial Pr(E_s'(p_o, p_s))}{\partial p_o}(p_s - c_s) + Pr(E_o'(p_o, p_s)) \tag{37}$$

$$\Rightarrow \frac{\partial^2 \pi'(p_o, p_s)}{\partial p_o^2} = \frac{\partial^2 Pr(E_o'(p_o, p_s))}{\partial p_o^2}(p_o - c_o) + \frac{\partial^2 Pr(E_s'(p_o, p_s))}{\partial p_o^2}(p_s - c_s) + 2\frac{\partial Pr(E_o'(p_o, p_s))}{\partial p_o} \tag{38}$$

$$\frac{\partial \pi'(p_o, p_s)}{\partial p_s} = \frac{\partial Pr(E_o'(p_o, p_s))}{\partial p_s}(p_o - c_o) + \frac{\partial Pr(E_s'(p_o, p_s))}{\partial p_s}(p_s - c_s) + Pr(E_s'(p_o, p_s)) \tag{39}$$

$$\Rightarrow \frac{\partial^2 \pi'(p_o, p_s)}{\partial p_s^2} = \frac{\partial^2 Pr(E_o'(p_o, p_s))}{\partial p_s^2}(p_o - c_o) + \frac{\partial^2 Pr(E_s'(p_o, p_s))}{\partial p_s^2}(p_s - c_s) + 2\frac{\partial Pr(E_s'(p_o, p_s))}{\partial p_s} \tag{40}$$

Since $Pr(E_i'(p_o, p_s)), i \in \{o, s\}$, is continuous and non-differentiable in p_o and p_s from Lemma 9, it follows that $\pi'(p_o, p_s)$ is continuous and non-differentiable in p_o and p_s using Eqns. (37)-(40). This completes the proof. ∎

Proof of Theorem 7. In this proof, we consider the case when $0 < H < v$, other cases can be

proven similarly. Before we present the proof, we derive the following property:

Property 5. *Assume that $p_o = p^\circ > p_s = p''$. Then, $Pr(E'_o(p^\circ, p'')) = Pr(E'_s(p'', p^\circ))$ and $Pr(E'_o(p^\circ, p'')) = Pr(E'_o(p'', p^\circ))$.*

Proof of Property 5. Assume that $(p_o, p_s) = (p^\circ, p'') \in R_6$, then, from Lemma 9, it must hold that $(p'', p^\circ) \in R_3$. This implies that $Pr(E'_o(p^\circ, p'')) = Pr(E'_s(p'', p^\circ))$ and $Pr(E'_s(p^\circ, p'')) = Pr(E'_o(p'', p^\circ))$. The proofs for other regions follow similarly. ∎

Proof of Theorem 7. We prove that $p'^*_o \leq p'^*_s$ by contradiction. Assume that $p^\circ = p'^*_o \geq p'^*_s = p'' \Rightarrow Pr(E'_o(p^\circ, p'')) \leq Pr(E'_s(p^\circ, p''))$. Then, from Eqn. (4.3), the retailer's optimal profit equals $\pi(p'^*_o, p'^*_s) = \pi'_1(p^\circ, p'') = Pr(E'_o(p^\circ, p''))(p^\circ - c_o) + Pr(E'_s(p^\circ, p''))(p'' - c_s)$. Now assume that the retailer sets $p'' = p_o$ and $p_s = p^\circ$, then, using Property 5, the retailer's profit equals $\pi'_2(p'', p^\circ) = Pr(E'_o(p'', p^\circ))(p'' - c_o) + Pr(E'_s(p'', p^\circ))(p^\circ - c_s) = Pr(E'_s(p^\circ, p''))(p'' - c_o) + Pr(E'_o(p^\circ, p''))(p^\circ - c_s)$. Then, it follows:

$$
\begin{aligned}
\pi'_2(p'', p^\circ) - \pi'_1(p'', p^\circ) &= Pr(E'_s(p^\circ, p''))(p'' - c_o) + Pr(E'_o(p^\circ, p''))(p^\circ - c_s) \\
&\quad - Pr(E'_o(p^\circ, p''))(p^\circ - c_o) - Pr(E'_s(p^\circ, p''))(p'' - c_s) \\
&= Pr(E'_s(p^\circ, p''))(c_s - c_o) - Pr(E'_o(p^\circ, p''))(c_s - c_o) \\
&= [Pr(E'_s(p^\circ, p'')) - Pr(E'_o(p^\circ, p''))](c_s - c_o) \geq 0.
\end{aligned}
$$

Hence, it must hold that $p'^*_o \leq p'^*_s$, i.e. (p'^*_o, p'^*_s) must belong to R_1, R_2 or R_3. Now we further show that $\max(0, v - H) \leq p'^*_o \leq p'^*_s \leq v$. Since $Pr(E'_o(p_o, p_s)) = 1$ and $Pr(E'_s(p_o, p_s)) = 0$ for any $(p_s, p_s) \in R_3$, the retailer can simply increase its profit by setting p_o and p_s at their upper bound in this region, i.e. $(p_o, p_s) = (v - H, v)$. Further, assume that $(p_o, p_s) = (p''', p^4) \in R_1$. Then, for any price vector $\{(p_o, p_s) : (p_o, p_s) \in R_1 \text{ and } p''' - p^4 = p^5 - p^6\}$, it holds that $Pr(E'_o(p''', p^4)) = Pr(E_o(p^5, p^6))$ and $Pr(E'_s(p''', p^4)) = Pr(E'_s(p^5, p^6))$ using Lemma 9. Hence, the retailer can simply increase its profit by setting p_o and p_s at their upper bound in this region, i.e. $(p_o, p_s) = (v - H, v)$. Hence, it must hold that $\max(0, v - H) \leq p'^*_o \leq p'^*_s \leq v$, i.e. $(p'^*_o, p'^*_s) \in R_2$.

Lastly, we prove that $\pi'(p'^*_o, p'^*_s) > \pi(p^*_o, p^*_s)$. First, using Lemma 9, it holds that $Pr(E'_o(p_o, p_s)) - Pr(E_o(p_o, p_s)) = Pr(E^s_s(p_o, p_s)) - Pr(E'_s(p_o, p_s)) + (p_s - p_o)^2/2H^2 > 0 \Rightarrow Pr(E'_o(p^*_o, p^*_s)) - Pr(E_o(p^*_o, p^*_s)) = Pr(E^s_s(p^*_o, p^*_s)) - Pr(E'_s(p^*_o, p^*_s)) + (p^*_s - p^*_o)^2/2H^2 > 0$. Then, it follows:

$$
\begin{aligned}
\pi'(p^*_o, p^*_s) - \pi(p^*_o, p^*_s) &= [Pr(E'_s(p^*_o, p^*_s)) - Pr(E^s_s(p^*_o, p^*_s))](p^*_s - c_s) + [Pr(E'_o(p_o, p_s)) - Pr(E_o(p_o, p_s))](p^*_o - c_o) \\
&= \left[Pr(E_o(p_o, p_s)) - Pr(E'_o(p_o, p_s)) + \frac{(p_s - p_o)^2}{2H^2}\right](p^*_s - c_s) \\
&\quad + [Pr(E'_o(p_o, p_s)) - Pr(E_o(p_o, p_s))](p^*_o - c_o) \\
&= [Pr(E_o(p_o, p_s)) - Pr(E'_o(p_o, p_s))](p^*_s - c_s - p^*_o + c_o) + \frac{(p_s - p_o)^2}{2H^2}(p^*_s - c_s) > 0
\end{aligned}
$$

$\Rightarrow \pi'(p'^*_o, p'^*_s) \geq \pi'(p^*_o, p^*_s) > \pi(p^*_o, p^*_s) \Rightarrow \pi'(p'^*_o, p'^*_s) > \pi(p^*_o, p^*_s)$. This completes the proof. ∎