ANALYSIS OF SINGLE-PHASE RECTIFIER CIRCUITS

by

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I. INTRODUCTION

Single-phase rectifier circuits, especially of the condenser-filter type, play an important role in the production of direct currents and voltages from an alternating-current source. They are widely used in battery chargers, radio and television power supplies, detectors in radio receivers, and in many other ways.

Since a voltage drop occurs across the diode tube while current is flowing, a power loss occurs in the tube. In most cases, the alternating-current source is connected to the rectifier through a step-up or a step-down transformer. An effective internal series impedance, therefore, exists in the practical circuits. The analysis of rectifier circuits that are made in several books[1,2,4,5] and a paper[12] is based on the assumption that the transformer is ideal and that the internal impedance of the source and the tube drop are zero. This assumption causes a serious error, especially when the value of load resistance is low and the equivalent tube and source resistance is high.

The purpose of this study is to analyze single-phase rectifier circuits using a high-vacuum thermionic diode and taking into account the tube drop and the resistive loss of the source transformer. Although it is true that practical circuits also involve other undesirable factors than those as mentioned above, for instance, the leakage inductance of transformer,

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For all numbered references, see Bibliography.

imperfect load resistor and shunt capacitor, etc., nevertheless inclusion of these factors increases the complexity of analysis. In order to simplify the analysis in this paper the nonlinear effective resistance of the high-vacuum diode is assumed to be replaced analytically by an equivalent fixed resistance, the leakage inductance of transformer has zero value, and all other circuit elements are perfect. The filter circuits connected to the rectifier include: (1) load shunted by a capacitor; (2) load in series with an inductor; (3) the choke-input type. The equations for the steady state direct output current and voltage in each case have been derived. These equations have been experimentally verified.

II. REVIEW OF LITERATURE

The behavior of the rectifier circuit consisting of a thermionic diode, output load and smoothing condenser taking into account the tube drop has been discussed by many authors[6,9,10,15]. The results of investigation are summarized below.

The equations for power output, power loss in the diode, and effective value of diode current have been derived by Moullin[6], and those for the direct output voltage can be found in either Roberts's or Waidelich's[15] paper. The equations mentioned above were developed under the condition that a linear characteristic and zero source impedance had been assumed.

Waidelich gave the results of the analysis in the form of curves giving the angles at which the tube begins conducting and ceases conducting and also the direct output voltage for various values of the ratio of the load resistance to the tube resistance. Similarly, experimental curves showing the ratio of the direct output voltage to the peak value of the source voltage were presented by Roberts and by Schade[10]. These curves may be helpful in estimating the behavior of the rectifier circuits and in the design of rectifiers.

Before the analysis is made, the idealization of the voltage-current characteristic curve of a high-vacuum thermionic diode is reviewed.

Idealization of The Voltage-Current Characteristic Curve of A high-vacuum Thermionic Diode[5,6,9].

Due to the fact that the voltage-current characteristic curve of a high-vacuum thermionic diode is nonlinear in general, analysis of rectifier circuits is difficult. Because of this inherent difficulty of analysis, simplifying assumptions must be made if useful results are to be obtained for even the most elementary circuits. A simplification that may be made is an idealization of the voltage-current characteristic curve of the thermionic diode. This characteristic curve in the conducting direction may be approximated by:

1) A straight line through the origin, as is shown in Fig. 2-1a.

2) A straight line through one point, which differs from the origin by an amount of E'_o, as is shown in Fig. 2-1b.

The second approximation may be made to fit reasonably well the actual characteristics of the diode. However, the E'_o may be negligible in comparison with E_{sm}, the peak value of an alternating-voltage source, and its presence in the equivalent rectifier circuits would increase the complexity of analysis.

As shown in Fig. 2-1a, the current-voltage characteristic of a high-vacuum thermionic diode is assumed to have a relatively constant slope in the conducting region and hence may be approximately represented as an ideal rectifier in series with a resistance R_o, for the current-voltage curve of such a

combination is identical with the idealized curve of the rectifier if R_o is chosen equal to the reciprocal of the slope of the rectifier current-voltage curve in the conducting region. The ideal rectifier, may be visualized as a switch which is closed and has zero resistance if the current flows through it in one direction, and is opened and conducts no current if, because of a reversed voltage, current tends to pass through it in the opposite direction. Therefore, when analyzing rectifier circuits, it is convenient to replace the diode by an ideal rectifier in series with a resistance.

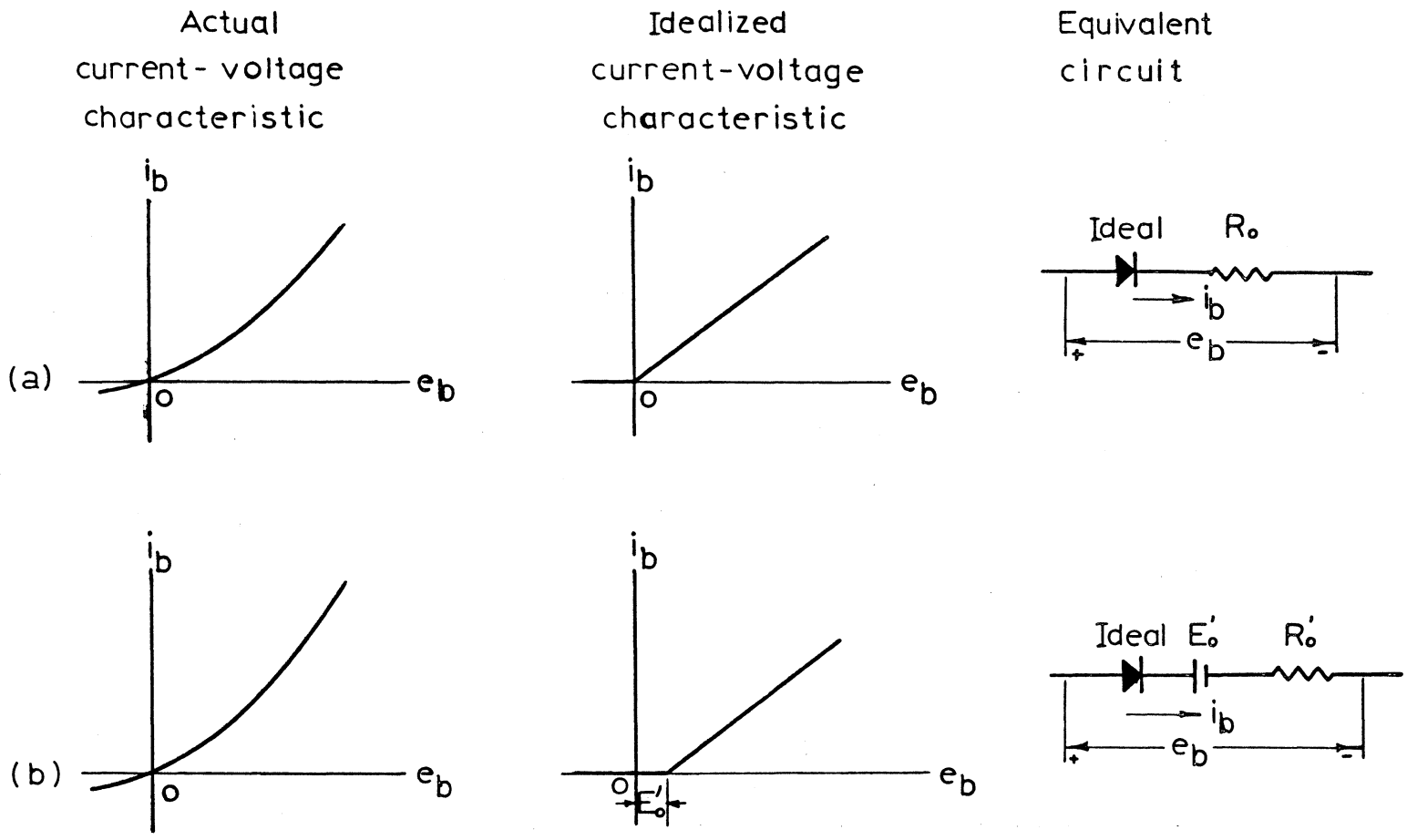


Fig 2-1. Idealized current-voltage characteristic and equivalent circuit of a high-vacuum thermionic diode.

III. ANALYSIS OF RECTIFIER CIRCUITS WITH A SMOOTHING CAPACITOR[6,9,10,15].

The following is the analysis of the rectifier circuits with a smoothing capacitor taking into account the voltage drop across the tube and source transformer, including both half-and full-wave rectifications. The main symbols used in this analysis are listed below.

e_s	sinusoidal source voltage.
e	instantaneous value of the output voltage.
E_{sm}	peak value of the source voltage.
w	angular frequency of the source.
R	load resistance.
R_t	equivalent resistance of tube and transformer.
C	capacitance of the filter condenser.
i	instantaneous current in the load resistance.
i_b	instantaneous plate current of rectifier.
i_c	instantaneous current in the condenser.
I_{dc}	average rectified output load current.
E_{dc}	average or d-c output voltage across the load.
α	ignition angle, at which the tube begins to conduct.
β	extinction angle, at which the tube ceases to conduct.
γ	conduction angle, during which the tube conducts.

For simplification of analysis, the following assumptions are made:

1) Diode tube is replaced by an ideal rectifier in series with a resistance R_o, as shown in Fig. 2-1a.

2) The alternating-voltage applied to the rectifier is assumed to be a pure sine wave.

3) The effective internal series impedance of the source transformer is to be considered as a pure resistance, i.e., the leakage inductance of the transformer is neglected.

4) All other circuit elements are assumed to be perfect.

The analysis is divided into two parts. First, half-wave rectification is discussed. Second, full-wave rectification will be considered.

(a) Half-Wave Rectification

According to the above assumptions, the actual circuit shown in Fig. 3-1a may be idealized as the circuit shown in Fig. 3-1b, where R_t is the total resistance, which involves the equivalent resistance of the tube and the transformer.

Applying Kirchhoff's law to the circuit of Fig. 3-1b yields two differential equations. They are

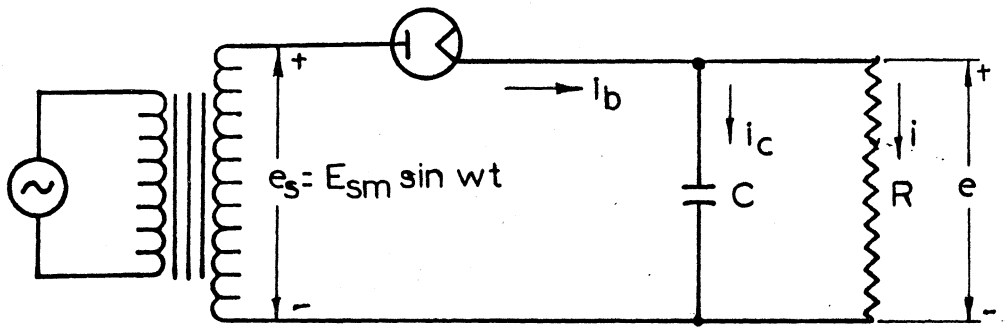
$$E_{sm} \sin wt = R_t i_b + Ri \tag{3-1}$$

$$0 = -\frac{1}{C}\int i_c \, dt + Ri \tag{3-2}$$

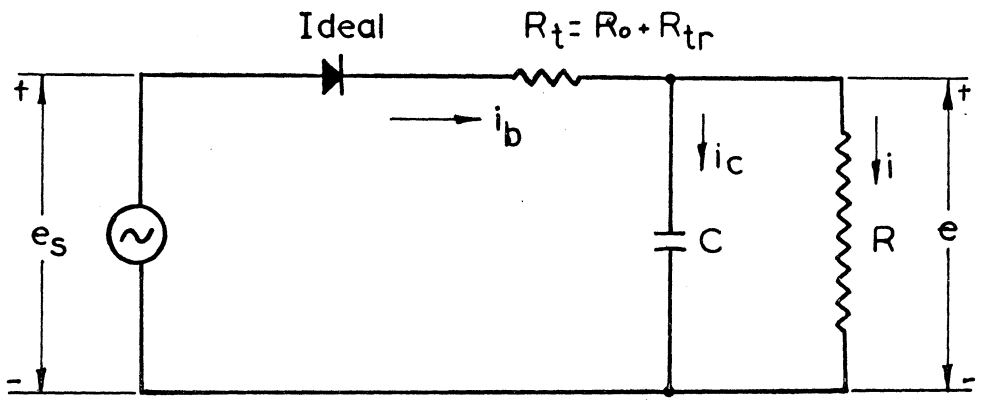
or

$$i_c = RC \frac{di}{dt} \tag{3-3}$$

During the conducting period, the ideal rectifier acts as a short circuit and the tube current i_b passes through the



(a) Actual circuit



(b) Idealized circuit

Fig. 3-1. Half-wave rectifier circuit with a shunt capacitor for smoothing.

resistor R_t and filter load. The relations among the currents are expressed

$$i_b = i_c + i \tag{3-4}$$

Combining Equations (3-1), (3-3), and (3-4), we obtain the expression that involves only the current passing through the load resistor. It is

$$\frac{di}{dt} + \left(\frac{R_t + R}{RR_tC}\right) i = \frac{E_{sm}}{RR_tC} \sin wt \tag{3-5}$$

This is a first order, non-homogeneous linear differential equation. The solution of this equation may be put in the form

$$i = I_m \sin (wt - \theta) + I_{o1} \epsilon^{-wt \cdot \cot \theta} \tag{3-6}$$

where

$$\theta = \tan^{-1}\left(\frac{wRR_tC}{R + R_t}\right) = \tan^{-1}\left(\frac{wRC}{1 + \frac{R}{R_t}}\right)$$

and I_{o1} is an arbitrary constant, which represents the coefficient of the transient term in the expression for load current i during the portion of the first cycle when the tube is conducting, and may be determined by the initial condition of the load current. Examination of Eq. (3-6) reveals that the load current involves two components, namely steady state and transient, and its resultant waveform during the conducting period is, of cource, no longer a sinusoidal wave. To determine the coefficient I_{o1}

of the transient term in Eq. (3-6), it is assumed that the load current passing through the resistor R is I_1 at the instant when wt equals α_1.

Therefore

$$I_{o1} = \left[I_1 - I_m \sin (\alpha_1 - \theta) \right] \cdot \epsilon^{\alpha_1 \cdot \cot \theta} \tag{3-7}$$

I_1 and α_1 may be set to zero if the operation of the rectifier starts at the condition when the capacitor is uncharged.

The current through the capacitor during the first conducting period can be obtained from Eq. (3-3) and expressed by the equation

$$i_c = - wRC \cdot \cot \theta \cdot I_{o1} \epsilon^{-wt \cdot \cot \theta} + wRC \cdot I_m \cdot \cos (wt - \theta) \tag{3-8}$$

Substitution of the expressions of i and i_c, i.e., Equations (3-6) and (3-8), into Eq. (3-4) gives the expression of the tube current i_b during the portion of the first cycle when the tube is conducting. It is

$$i_b = I_m \cdot \left[wRC \cos (wt - \theta) + \sin (wt - \theta) \right] + (1 - wRC \cot \theta) \cdot I_{o1} \epsilon^{-wt \cdot \cot \theta} \tag{3-9}$$

Eq. (3-9) can be simplified to

$$i_b = I_{mb} \sin (wt - \theta + \Phi) - \frac{R}{R_t} I_{o1} \epsilon^{-wt \cdot \cot \theta}$$

where $\Phi = \tan^{-1} wRC$

and

$$I_{mb} = I_m \sqrt{(wRC)^2 + 1}$$

The extinction angle, denoted by β_1, at which the diode acts to disconnect the filter circuit from the source voltage during the first cycle, is easily obtained by letting i_b equal zero at wt= β_1. Thus

$$\frac{R}{R_t} I_{o1} \epsilon^{-\beta_1 \cdot \cot\theta} = I_{mb} \sin(\beta_1 - \theta + \Phi) \qquad (3\text{-}10)$$

Beyong the instant that wt equals to β_1, the ideal rectifier acts as an open circuit, the filter circuit is disconnected from the source, the capacitor discharges through the load, and the rectifier operates in its nonconducting state. During each nonconducting period, the tube ceases to conduct, i.e., i_b has a zero value, and the currents that pass through the load resistor and the shunt capacitor have the same magnitude but opposite directions. The load current during this nonconducting period of the cycle is described by Eq. (3-2), where i_c=-i, which has the solution

$$i = D_1 \epsilon^{-\frac{t}{RC}} \qquad (3\text{-}11)$$

where D_1 is an arbitrary constant and may be determined from the condition that the capacitor voltage, and hence the current through the load resistor, must be continuous from the conducting period to the blocking period at the instant when wt equals to β_1, that is, the currents given by Equations (3-11) and (3-6) must be equal

for wt = β_1. Thus

$$D_1 \epsilon^{-\frac{\beta_1}{wRC}} = I_m \sin(\beta_1 - \theta) + I_{o1} \epsilon^{-\beta_1 \cdot \cot\theta}$$

or

$$D_1 = I_m \sin(\beta_1 - \theta) \cdot \epsilon^{\frac{\beta_1}{wRC}} + I_{o1} \epsilon^{-\frac{\beta_1}{wR_tC}} \qquad (3\text{-}12)$$

Substituting Eq. (3-12) into Eq. (3-11) yields the expression for the load current during the portion of the first cycle when the tube ceases to conduct. Thus

$$i = I_m \sin(\beta_1 - \theta) \cdot \epsilon^{-\left(\frac{wt-\beta_1}{wRC}\right)} + I_{o1} \epsilon^{-\frac{1}{wC}\left(\frac{wt}{R} + \frac{\beta_1}{R_t}\right)} \qquad (3\text{-}13)$$

During the nonconducting period, the terminal voltage falls exponentially until the source voltage exceeds this terminal voltage, and the next conduction period is reached. The ignition angle of the second cycle α_2 may be determined by the condition that the source voltage e_s equals the terminal voltage Ri (i should be the load current in blocking period) at wt = $2\pi + \alpha_2$. Thus

$$E_{sm} \sin(2\pi + \alpha_2) = Ri$$

$$= R \cdot \left[I_m \sin(\beta_1 - \theta) \epsilon^{-\left(\frac{2\pi + \alpha_2 - \beta_1}{wRC}\right)} + I_{o1} \epsilon^{-\frac{1}{wC}\left(\frac{2\pi + \alpha_2}{R} + \frac{\beta_1}{R_t}\right)} \right] \qquad (3\text{-}14)$$

or

$$\sin\alpha_2 = \frac{R}{E_{sm}} \cdot \left[I_m \sin(\beta_1 - \theta) \cdot \epsilon^{-\left(\frac{2\pi + \alpha_2 - \beta_1}{wRC}\right)} + I_{o1} \cdot \epsilon^{-\frac{1}{wC}\left(\frac{2\pi + \alpha_2}{R} + \frac{\beta_1}{R_t}\right)} \right] \tag{3-15}$$

The switching in reactive circuits gives rise to "transients". The transients in this problem are repeated during each cycle and require an iterative solution. The load currents during both the conduction and the nonconduction period, and the ignition and the extinction angles for the second cycle, the third, the fourth, . . . etc., can be found by a process similar to that for the first cycle. The following are the expressions for the load currents and the angles at which the tube begins to conduct and ceases to conduct during the nth cycle.

$$i = \begin{cases} I_m \sin(wt - \theta) - I_{on} \epsilon^{-wt \cdot \cot\theta} \\ \qquad 2(n-1)\pi + \alpha_n \le wt \le 2(n-1)\pi + \beta_n \\ \qquad n = 1, 2, 3, \cdots \end{cases} \tag{3-16}$$

$$\begin{cases} D_n \epsilon^{-\left(\frac{wt}{wRC}\right)} \qquad 2(n-1)\pi + \beta_n \le wt \le 2n\pi + \alpha_{n+1} \\ \qquad n \quad 1, 2, 3, \cdots \end{cases} \tag{3-17}$$

for the load currents, and

$$\frac{R}{R_t} I_{on} \cdot \epsilon^{-\cot\theta \cdot [2(n-1)\pi + \beta_n]} = I_{mb} \sin(\beta_n - \theta + \Phi) \tag{3-18}$$

for the extinction angle. Also

$$\sin \alpha_{n+1} = \frac{R}{E_{sm}} \left\{ I_m \sin(\beta_n - \theta) \epsilon^{-\left[\frac{2\pi + \alpha_{n+1} - \beta_n}{wRC}\right]} + I_{on} \cdot \epsilon^{-\frac{1}{wC}\left[\frac{2n\pi + \alpha_{n+1}}{R} + \frac{2(n-1)\pi + \beta_n}{R_t}\right]} \right\} \quad (3\text{-}19)$$

for the ignition angle, where

$$I_{on} = \left[I_n - I_m \sin(\alpha_n - \theta) \right] \cdot \epsilon^{[2(n-1)\pi + \alpha_n] \cdot \cot\theta}$$

$$D_n = I_m \sin(\beta_n - \theta) \cdot \epsilon^{\frac{2(n-1)\pi + \beta_n}{wRC}} + I_{on} \epsilon^{-\frac{2(n-1)\pi + \beta_n}{wR_t C}}$$

$$I_n = \frac{E_{sm}}{R} \sin \alpha_n \quad (3\text{-}20)$$

The notations in the above equations refer to Fig. 3-2. Equations (3-18) and (3-19) can be simplified by substituting the expressions of I_m, I_{mb}, I_{on} and I_n into them. Then they become

$$Q \cdot \epsilon^{-(\beta_n - \alpha_n) \cdot \cot\theta} = \frac{\sin\theta}{\sin\Phi} \sin(\beta_n - \theta + \Phi) \quad (3\text{-}21)$$

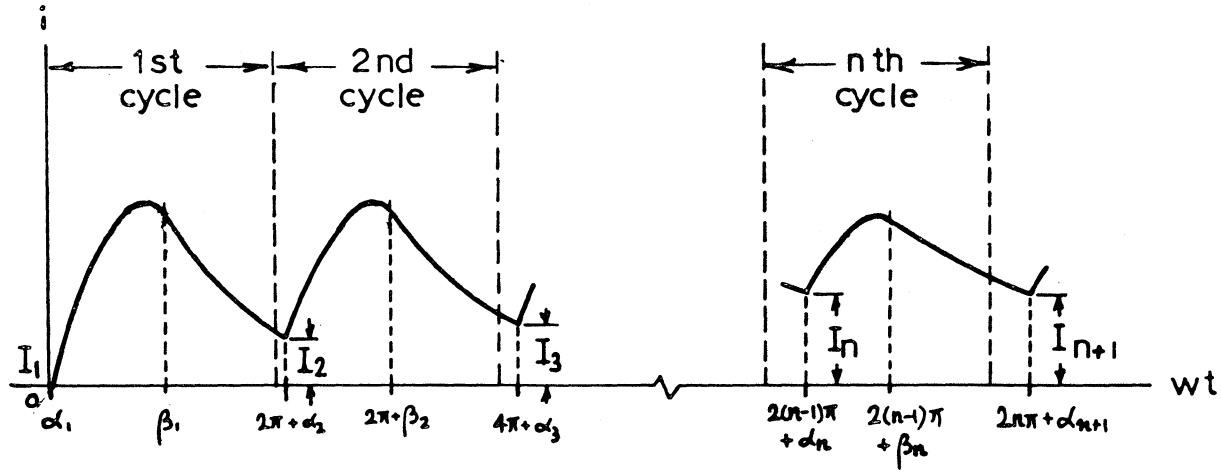


Fig. 3-2. Waveform of output current showing the behavior of the rectifier circuit with a smoothing capacitor.

for β_n, and

$$\sin \alpha_{n+1} = \frac{\frac{R}{R_t}}{\sqrt{(wRC)^2 + \left(1 + \frac{R}{R_t}\right)^2}} \sin(\beta_n - \theta) \cdot \epsilon^{-\frac{2\pi + \alpha_{n+1} - \beta_n}{wRC}}$$

$$+ Q \cdot \epsilon^{-\left[\frac{2\pi + \alpha_{n+1} - \alpha_n + \frac{R}{R_t}(\beta_n - \alpha_n)}{wRC}\right]} \qquad (3\text{-}22)$$

for α_{n+1}, where

$$Q = \sin \alpha_n - \frac{\frac{R}{R_t}}{\sqrt{(wRC)^2 + \left(1 + \frac{R}{R_t}\right)^2}} \sin(\alpha_n - \theta)$$

The circuit operation is called steady state when consecutive cycles are identical. Fig. 3-2 illustrates that when the value of the load current at the instant that the tube begins to conduct after some cycle, namely the (n + 1)st cycle, equals to that of the previous cycle, i.e., $I_{(n+1)} = I_n$, the steady state condition is reached. Now, a question to be answered is "would the waveforms for successive cycles become equal after a time or would they become higher and higher indefinitely?" In order to find out whether these initial values of current will stabilize or not, a comparison among a few values of load current will be made as follows.

First, choose n equal to 2, 3 and 4 and substitute the corresponding value of sin α_n, which can be obtained from Eq. (3-19), into Eq. (3-20). Thus

$$I_2 = I_m \sin(\beta_1 - \theta) \cdot \epsilon^{-\left(\frac{2\pi + \alpha_2 - \beta_1}{wRC}\right)} + I_{01} \cdot \epsilon^{-\frac{1}{wC}\left(\frac{2\pi + \alpha_2}{R} + \frac{\beta_1}{R_t}\right)}$$

$$I_3 = I_m \sin(\beta_2 - \theta) \cdot \epsilon^{-\left(\frac{2\pi + \alpha_3 - \beta_2}{wRC}\right)} + I_{02} \cdot \epsilon^{-\frac{1}{wC}\left(\frac{4\pi + \alpha_3}{R} + \frac{2\pi + \beta_2}{R_t}\right)}$$

$$I_4 = I_m \sin(\beta_3 - \theta) \cdot \epsilon^{-\left(\frac{2\pi + \alpha_4 - \beta_3}{wRC}\right)} + I_{03} \cdot \epsilon^{-\frac{1}{wC}\left(\frac{6\pi + \alpha_4}{R} + \frac{4\pi + \beta_3}{R_t}\right)}$$

by the proper substitution of I_{o2} and I_{o3} into the expressions of I_3 and I_4 respectively, and the expansion of these equations in terms of I_{o1} and the angles, the expressions for I_3 and I_4 that are shown above become

$$\begin{aligned} I_3 = I_m \Bigg\{ & \sin(\beta_2 - \theta) \cdot \epsilon^{-\left(\frac{2\pi + \alpha_3 - \beta_2}{wRC}\right)} \\ & + \sin(\beta_1 - \theta) \cdot \epsilon^{-\frac{1}{wC}\left(\frac{4\pi + \alpha_3 - \beta_1}{R} + \frac{\beta_2 - \alpha_2}{R_t}\right)} \\ & - \sin(\alpha_2 - \theta) \cdot \epsilon^{-\frac{1}{wC}\left(\frac{2\pi + \alpha_3 - \alpha_2}{R} + \frac{\beta_2 - \alpha_2}{R_t}\right)} \Bigg\} \\ & + I_{01} \cdot \epsilon^{-\frac{1}{wC}\left(\frac{4\pi + \alpha_3}{R} + \frac{\beta_2 + \beta_1 - \alpha_2}{R_t}\right)} \end{aligned} \qquad (3\text{-}23)$$

$$I_4 \quad I_m \Big\{ \sin(\beta_3-\theta)\cdot\epsilon^{-\left(\frac{2\pi+\alpha_4-\beta_3}{wRC}\right)} + \sin(\beta_2-\theta)\cdot\epsilon^{-\frac{1}{wC}\left(\frac{4\pi+\alpha_4-\beta_2}{R}+\frac{\beta_3+\alpha_3}{R_t}\right)}$$

$$+ \sin(\beta_1-\theta)\cdot\epsilon^{-\frac{1}{wC}\left(\frac{6\pi+\alpha_4-\beta_1}{R}+\frac{\beta_3+\beta_2-\alpha_3-\alpha_2}{R_t}\right)}$$

$$- \sin(\alpha_3-\theta)\cdot\epsilon^{-\frac{1}{wC}\left(\frac{2\pi+\alpha_4-\beta_3}{R}+\frac{\beta_3-\alpha_3}{R_t}\right)}$$

$$- \sin(\alpha_2-\theta)\cdot\epsilon^{-\frac{1}{wC}\left(\frac{4\pi+\alpha_4-\beta_2}{R}+\frac{\beta_3+\beta_2-\alpha_3-\alpha_2}{R_t}\right)} \Big\}$$

$$+ I_{01}\cdot\epsilon^{-\frac{1}{wC}\left(\frac{6\pi+\alpha_4}{R}+\frac{\beta_3+\beta_2+\beta_1-\alpha_3-\alpha_2}{R_t}\right)} \qquad (3\text{-}24)$$

Equations (3-23) and (3-24) show that the terms having the coefficient I_{01} decrease gradually and will die out or become negligible as the cycles repeat. Consequently, the number of terms having the coefficient I_m in Eq. (3-24) is more than that of Eq. (3-23). This means that the number of terms having the coefficient I_m in the expression for I_n will increase if the corresponding value of I_{on} is replaced in term of I_{01}. But the sum of these large number of terms has a value which is smaller than that of the other terms as the cycles repeat; therefore, the difference between I_n and $I_{(n+1)}$ will decrease gradually. Finally, the circuit operation becomes repetitive.

A quantitative example is given to show the behavior of the rectifier circuit in the following.

Example 3-1. Calculate the values of α_n, β_n, and load current at the instant that the tube begins to conduct for the rectifier circuit shown in Fig. 3-1 until the steady state is reached. The circuit constants are given by

E_{sm} = 250 volts; f = 60 c. p. s.

R = 5000 ohms; C = 10 μF

R_t = 600 ohms;

Assume the operation starts at wt = 0, with the capacitor uncharged.

The results of the calculation is shown in Table 3-1.

Table 3-1. Computed results of Example 3-1.

n	α_n (deg.)	β_n (deg.)	I_n (ma)
1	0	150.53	0
2	23.37	141.49	19.83
3	29.65	138.57	24.75
4	31.56	137.60	26.17
5	52.18	137.28	26.66
6	32.38	137.28	26.78
7	32.38	137.28	26.78

It is apparent that the answer of the question considered above is "the load current will become stable after several cycles." The behavior of the rectifier circuit can then be described as a steady state consisting of a series of "repeative transients". Equations (3-21) and (3-22) tell us that the ignition and the extinction angles are function of wRC and the ratio R/R_t. It is found that the time required to reach the steady state condition of the circuit depends upon the values of the circuit parameters.

After the steady state is reached, the same load current waveform will be repeated during each cycle. The equations expressing the steady state values of the ignition and the extinction angles, denoted by α and β respectively, can be found by replacing the α_n or α_{n+1} and β_n by α and β respectively in Equations (3-21) and (3-22), since α_n equals to α_{n+1} when the steady state condition is reached. Thus

$$Q \cdot \epsilon^{-(\beta-\alpha)\cdot \cot\theta} = \frac{\sin\theta}{\sin\Phi} \sin(\beta - \theta + \Phi) \tag{3-25}$$

$$\sin\alpha = \frac{\frac{R}{Rt}}{\sqrt{(wRC)^2 + (1+\frac{R}{Rt})^2}} \sin(\beta-\theta)\cdot \epsilon^{-\left(\frac{2\pi+\alpha-\beta}{wRC}\right)} + Q\cdot\epsilon^{-\left[\frac{2\pi+\frac{R}{Rt}(\beta-\alpha)}{wRC}\right]} \tag{3-26}$$

where

$$Q = \sin\alpha - \frac{\frac{R}{Rt}}{\sqrt{(wRC)^2 + (1+\frac{R}{Rt})^2}} \sin(\alpha-\theta)$$

The results of these equations may be given in the form of curves, as appear in Figures 3-3 and 3-4, showing the steady state values of the ignition and the extinction angles respectively as functions of wRC for various values of the ratio of the load resistance to tube and transformer resistance. The data for plotting these curves were obtained by the use of an IBM 7040 computer. The source program is shown in the appendix. Meanwhile, the corresponding curves for the conduction angle, which is defined as the difference between the extinction angle β and the ignition angle α, may be seen in Fig. 3-5. It is found that when other circuit parameters are held constant, an increase in R_t increases the conduction angle or conducting period of the tube.

After the load current waveform becomes stable, the average direct load current can be found by integrating over the interval between $2(n-1)\pi+\alpha$ and $2n\pi+\alpha$ and then dividing the value of the integral by the period of the cycle, 2π. Thus

$$I_{dc}=\frac{1}{2\pi}\left\{\int_{2(n-1)\pi+\alpha}^{2(n-1)\pi+\beta}\left[I_m \sin(wt-\theta)+I_{on}\cdot\epsilon^{-wt\cdot\cot\theta}\right]d(wt)\right.$$

$$\left.+\int_{2(n-1)\pi+\beta}^{2n\pi+\alpha} D_n\cdot\epsilon^{-\frac{wt}{wRC}}\,d(wt)\right\}$$

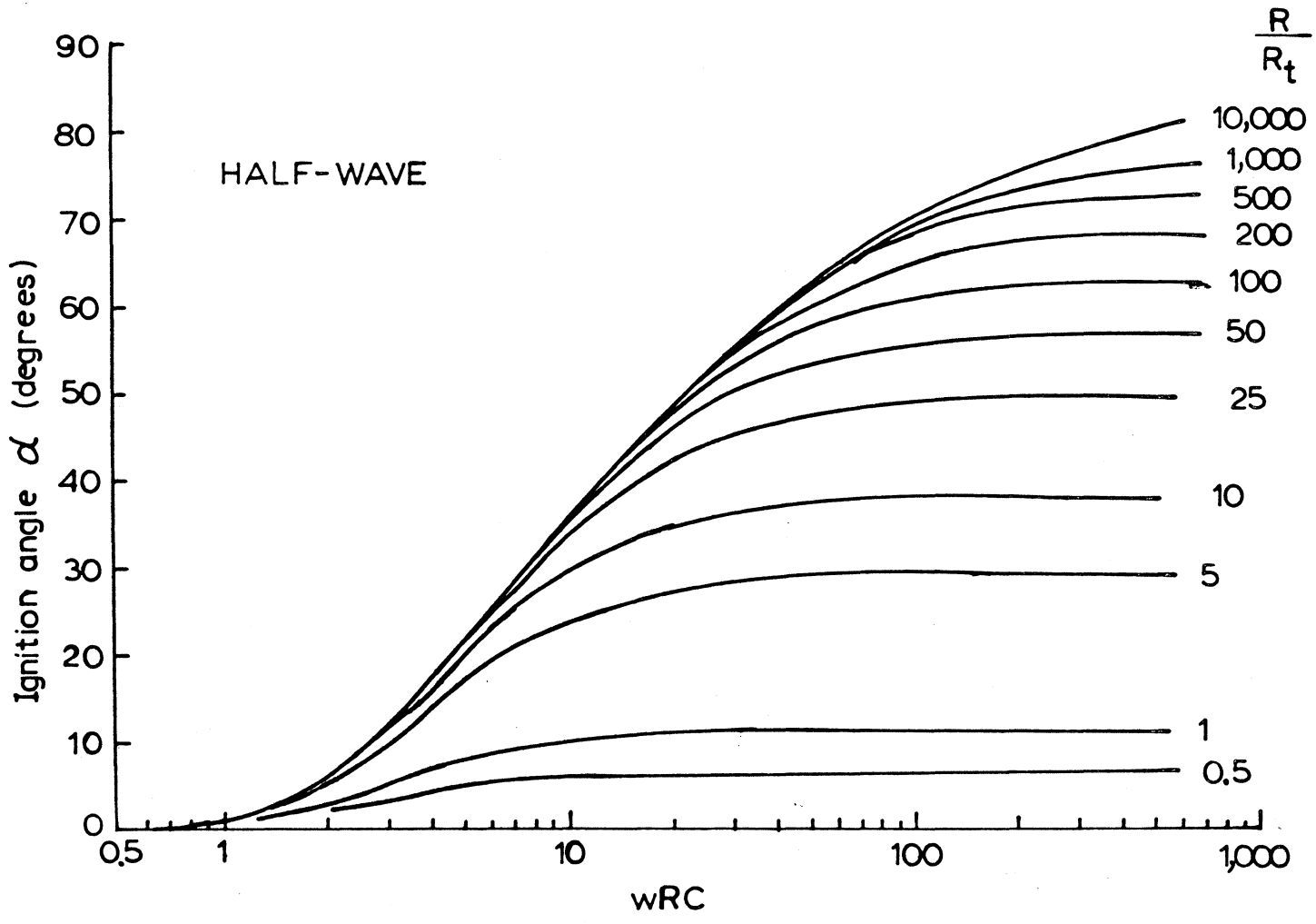


Fig. 3-3. Curves of the ignition angle α versus wRC for various values of R/R_t.

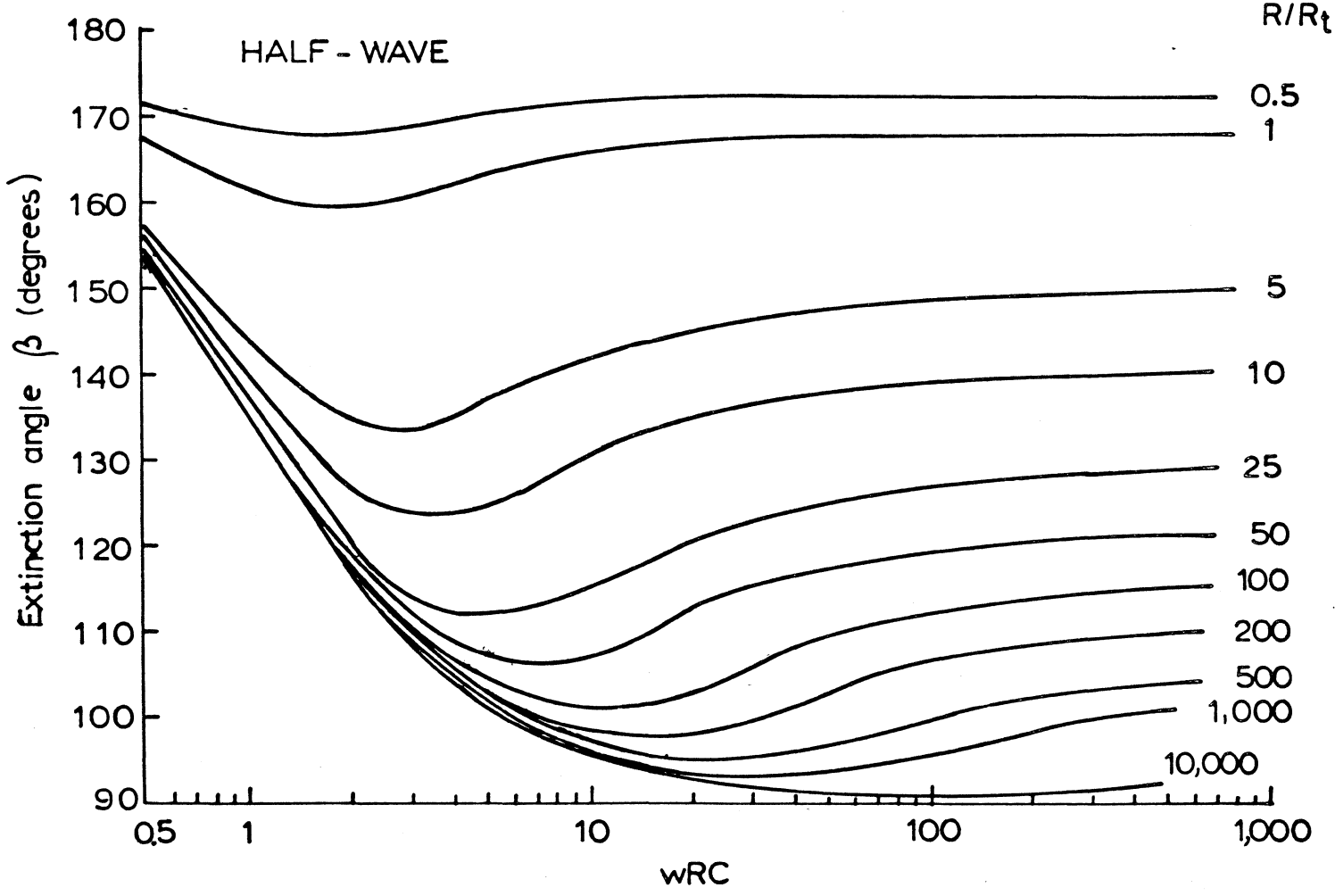


Fig. 3-4. Curves of the extinction angle β versus wRC for various values of R/R_t.

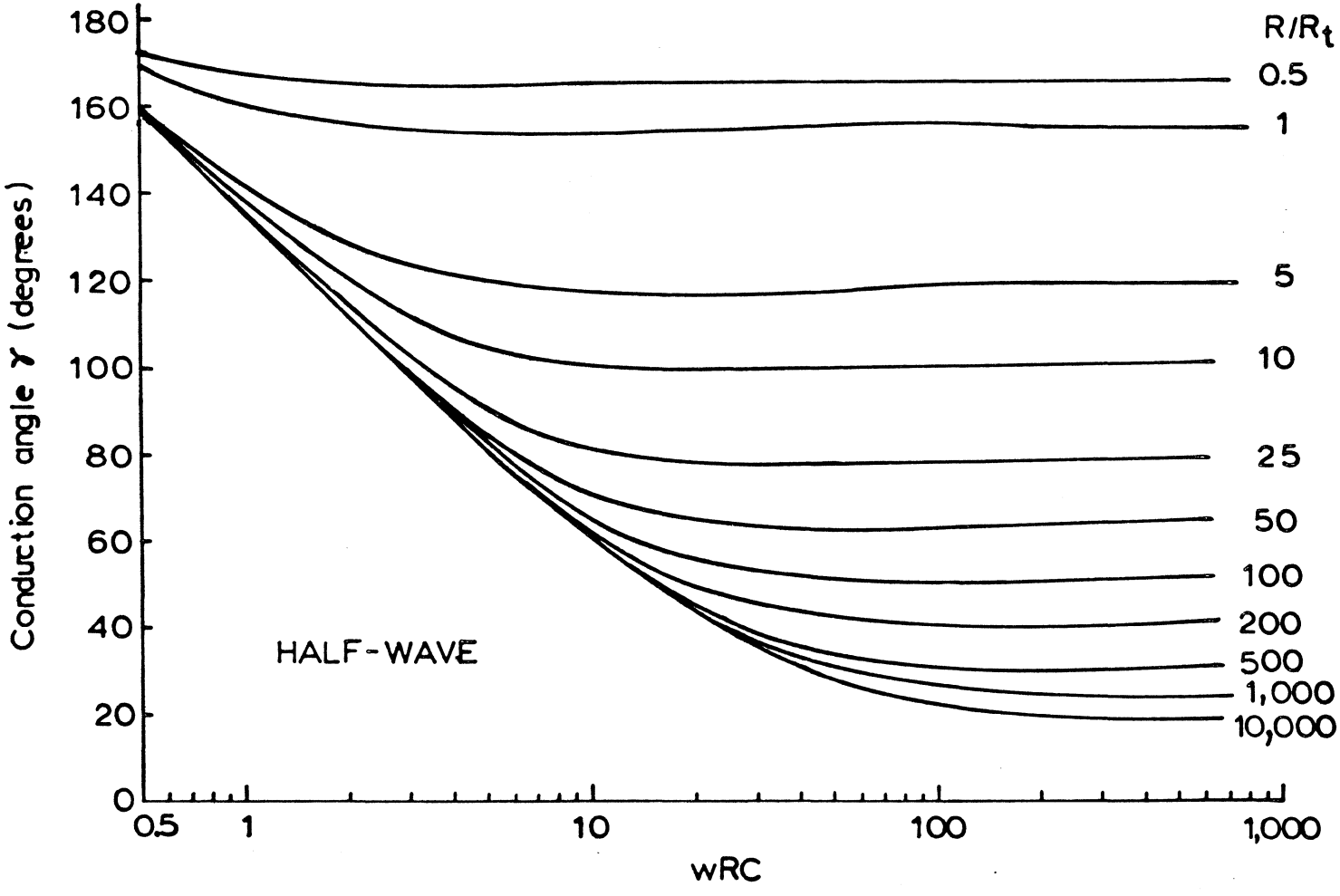


Fig. 3-5. Curves of the conduction angle γ versus wRC for various values of R/R_t.

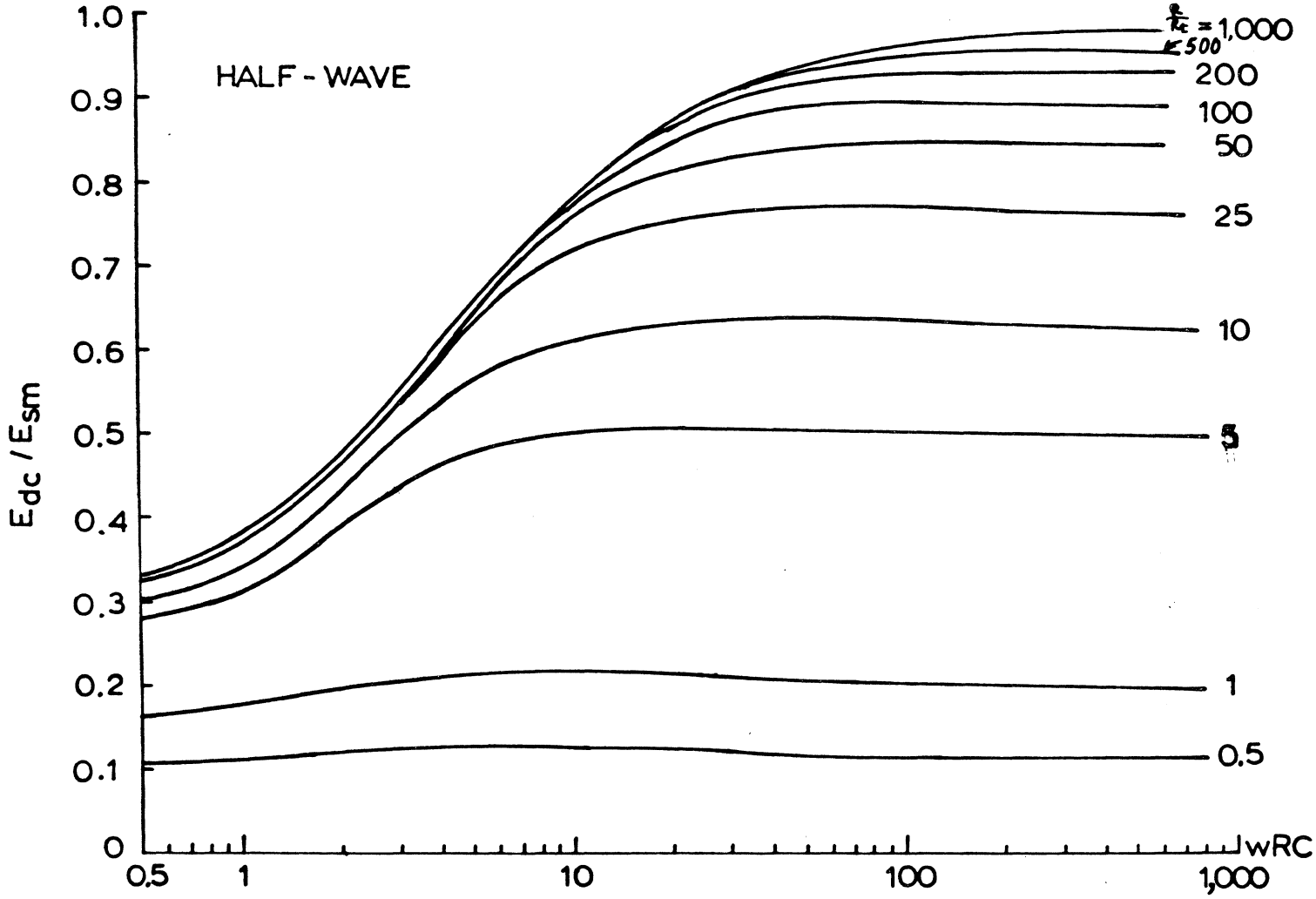


Fig. 3-6. Curves of E_{dc}/E_{sm} versus wRC for various values of R/R_t.

which, after integration, substitution of I_{on}, D_n, and I_m, applying Equations (3-25) and (3-26) and certain trigonometric transformations, may be put in the form

$$I_{dc} = \frac{E_{sm}}{2\pi (R + R_t)} \left\{ \frac{\cos (\alpha + \Phi) - \cos (\beta + \Phi)}{\cos \Phi} \right\}$$

The direct-current output voltage is then expressed by the equation

$$E_{dc} = R \cdot I_{dc}$$

$$= \frac{E_{sm}}{2\pi \left(1 + \frac{R_t}{R}\right)} \left\{ \frac{\cos (\alpha + \Phi) - \cos (\beta + \Phi)}{\cos \Phi} \right\}$$

or

$$\frac{E_{dc}}{E_{sm}} = \frac{\cos (\alpha + \Phi) - \cos (\beta + \Phi)}{2\pi \left(1 + \frac{R_t}{R}\right) \cos \Phi} \qquad (3\text{-}27)$$

The curves showing the variation of the ratio of E_{dc} to E_{sm} versus wRC for different values of the ratio of R to R_t are plotted in Fig. 3-6.

(b) Full-Wave Rectification

The full-wave rectifier connection with a smoothing capacitor may be seen in Fig. 3-7a. A center tap on the transformer secondary is used, and the sinusoidal source voltages e_s and e_s' are therefore equal and 180 degrees out of phase when their positive directions are assigned as indicated; that is

$$e_s = E_{sm} \sin wt$$

$$e_s' = -e_s$$

$$= E_{sm} \sin (wt + \pi)$$

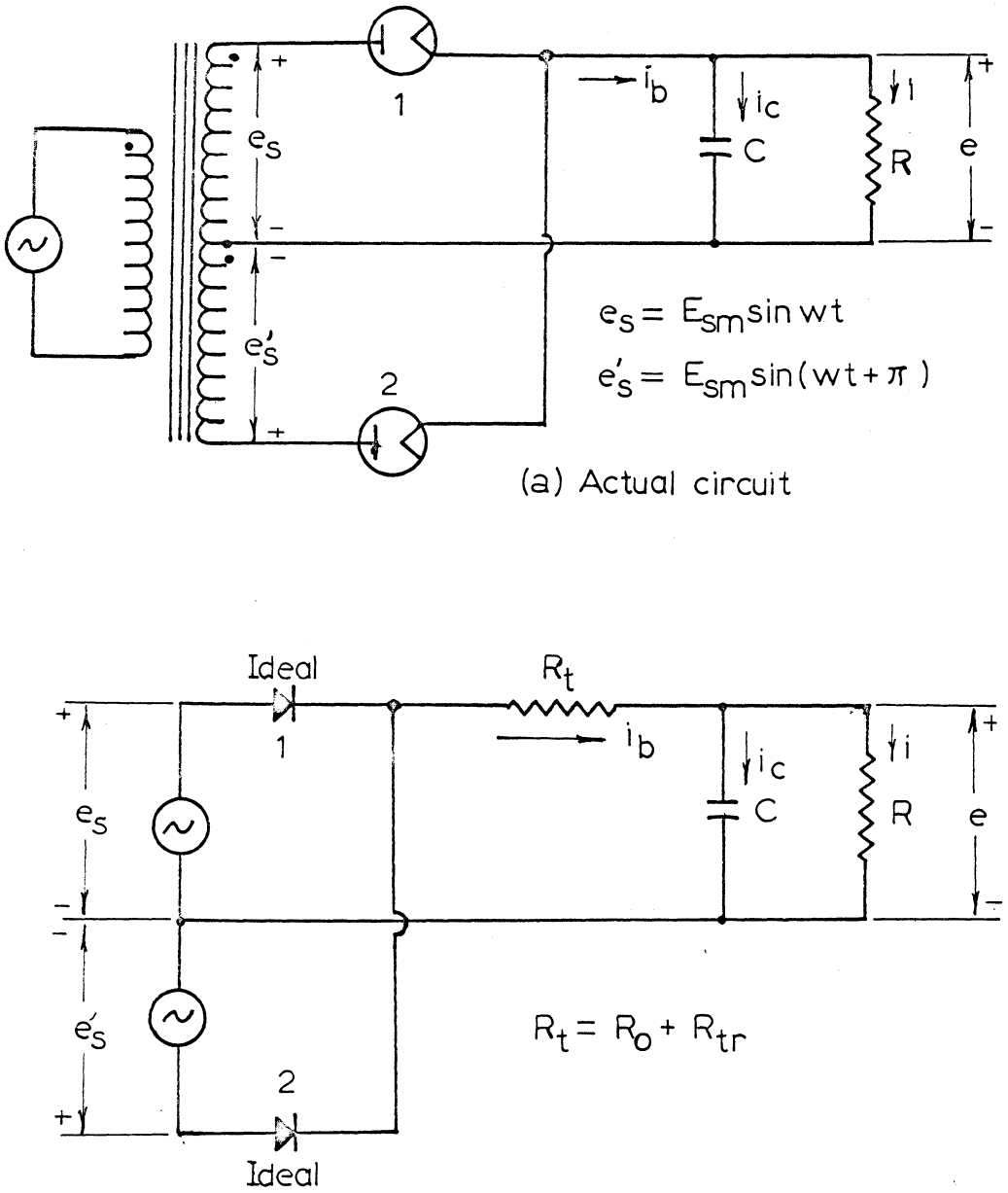


Fig. 3-7. Full-wave rectifier circuit with a shunt capacitor for smoothing.

Under the assumptions at the beginning of this chapter, the rectifier circuit may be idealized as shown in Fig. 3-7b. The two rectifiers are assumed to have identical characteristics. During one half of the cycle, the voltage across rectifier 1 only is in the conduction direction while rectifier 2 is nonconducting, and rectifier 1 passes current when the source voltage e_s is greater than the charged voltage across the capacitor, e. Beyond the crest of the source voltage, e_s, the capacitor discharges and the charged voltage falls exponentially until the other source voltage, e_s', becomes equal to it. During the other half of the cycle, the conditions are reversed. Hence, the operation of the full-wave rectifier circuit with a smoothing capacitor is the same as that of the half-wave circuit, except the capacitor receives the charge twice during each cycle. Thus, the same analysis as used in half-wave rectification with some minor changes may also be applied to the full-wave circuit. After analyzing, it is found that the equations for obtaining the steady state values of the ignition angle α, the extinction angle β and the d-c output voltage E_{dc} are the same as Equations (3-25), (3-26) and (3-27), except 2π must be replaced by π. The curves showing the ignition angle α, the extinction angle β, the conduction angle γ and the ratio of the d-c output voltage to the peak value of the source voltage as functions of wRC for different values of the ratio of the load resistance to the equivalent resistance of the tube and the transformer were plotted as shown in Figures 3-8 through 3-11.

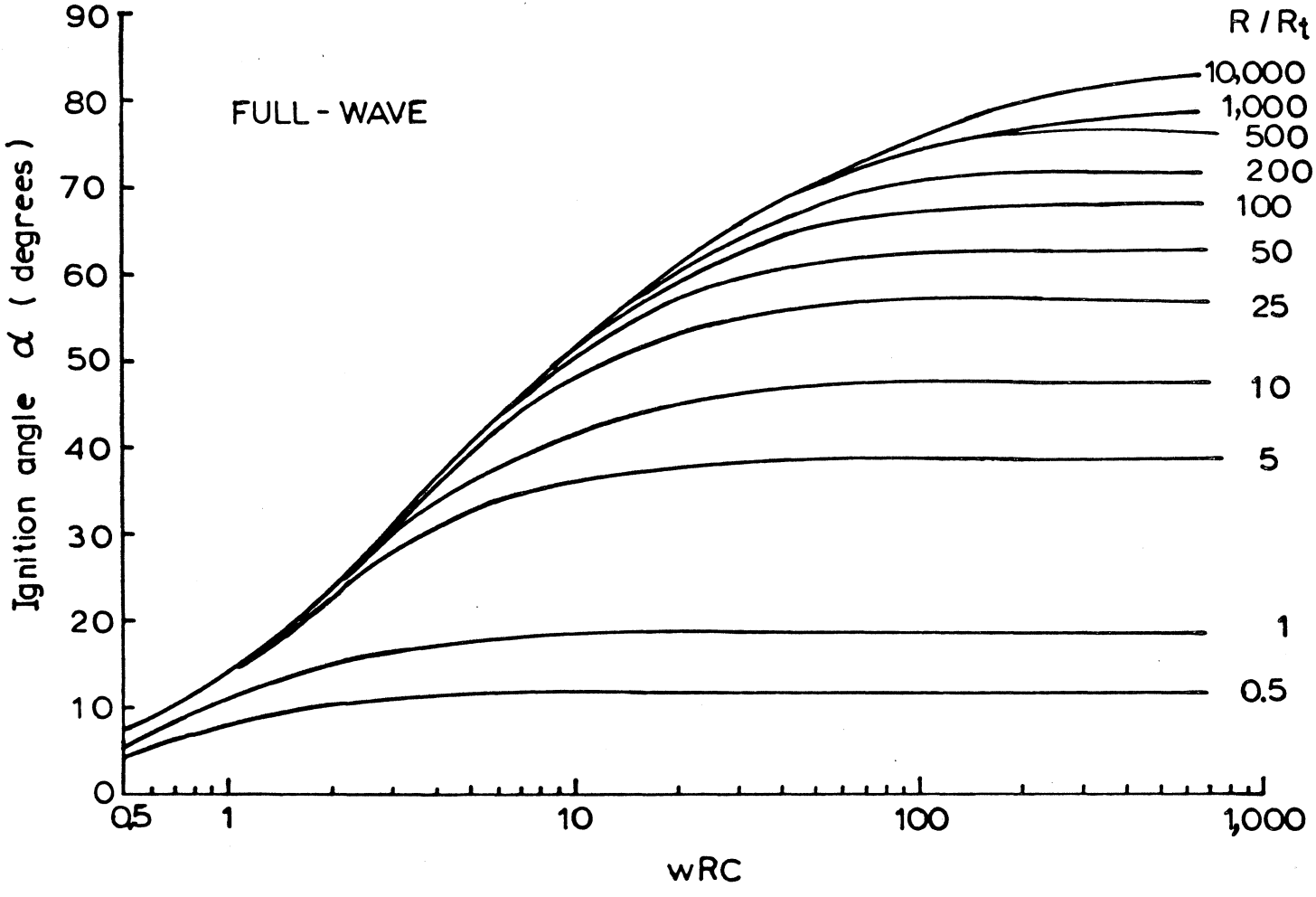


Fig. 3-8. Curves of the ignition angle α versus wRC for various values of R/R_t.

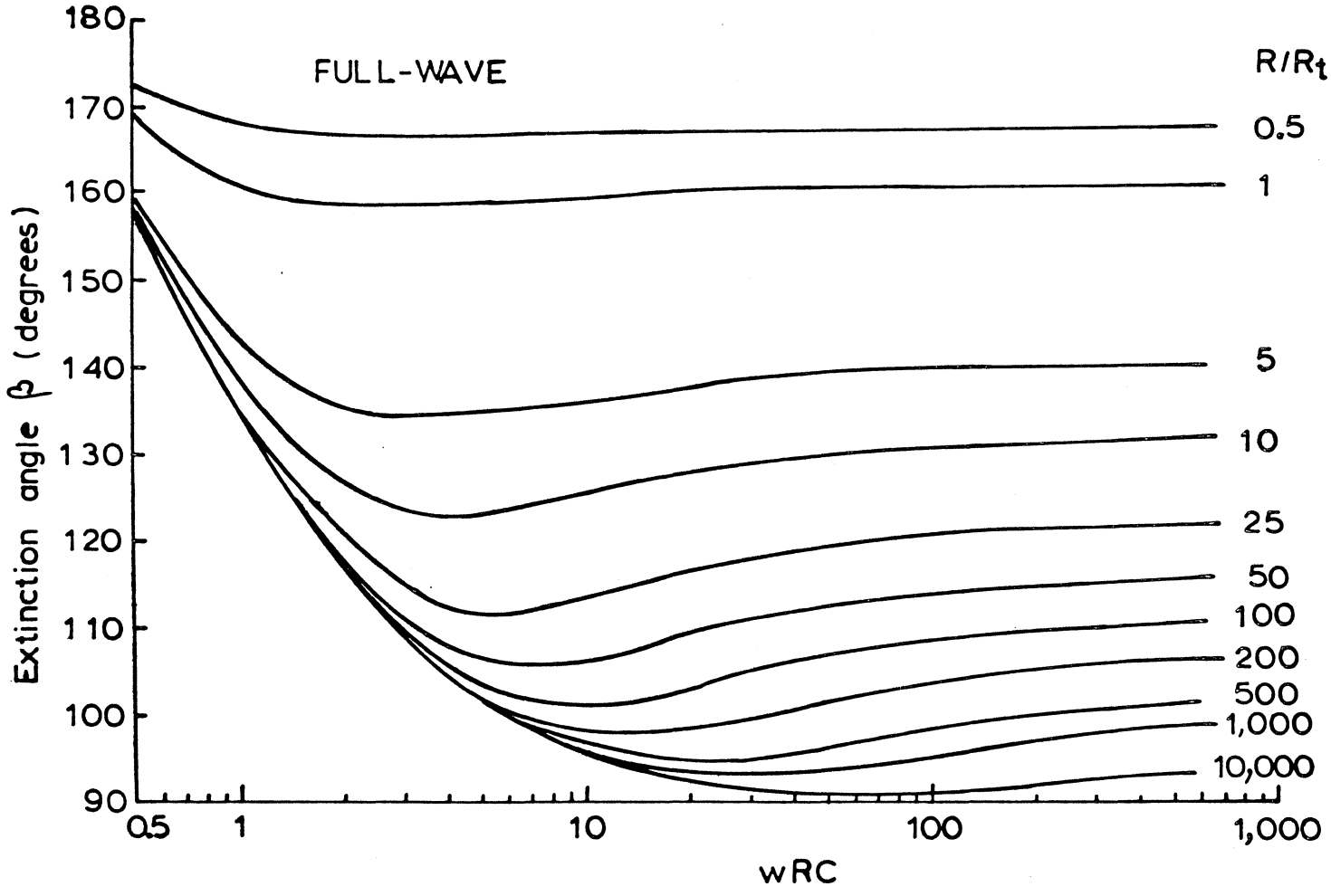


Fig. 3-9. Curves of the extinction angle β versus wRC for various values of R/R_t.

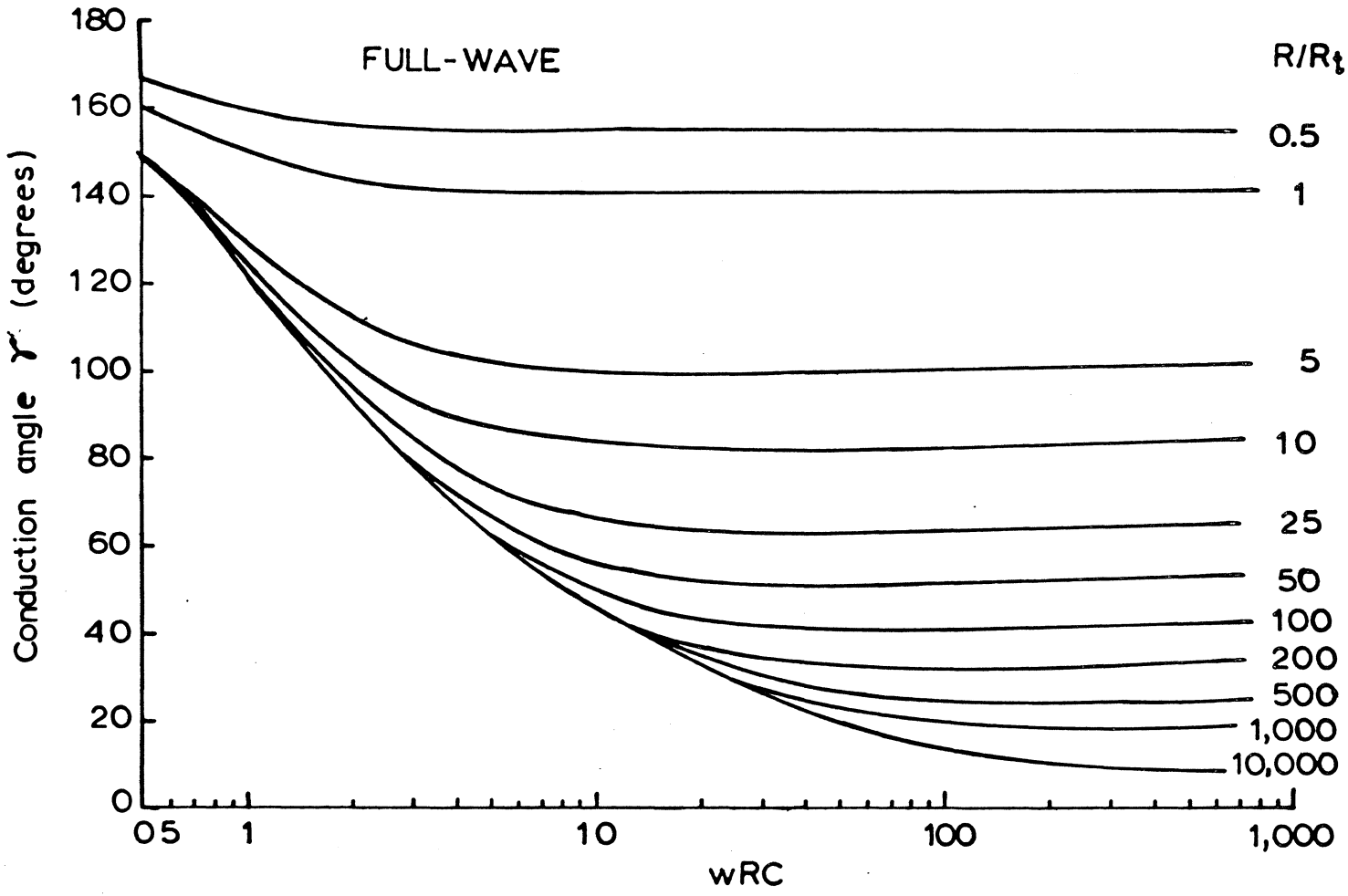


Fig. 3-10. Curves of the conduction angle γ versus wRC for various values of R/R_t.

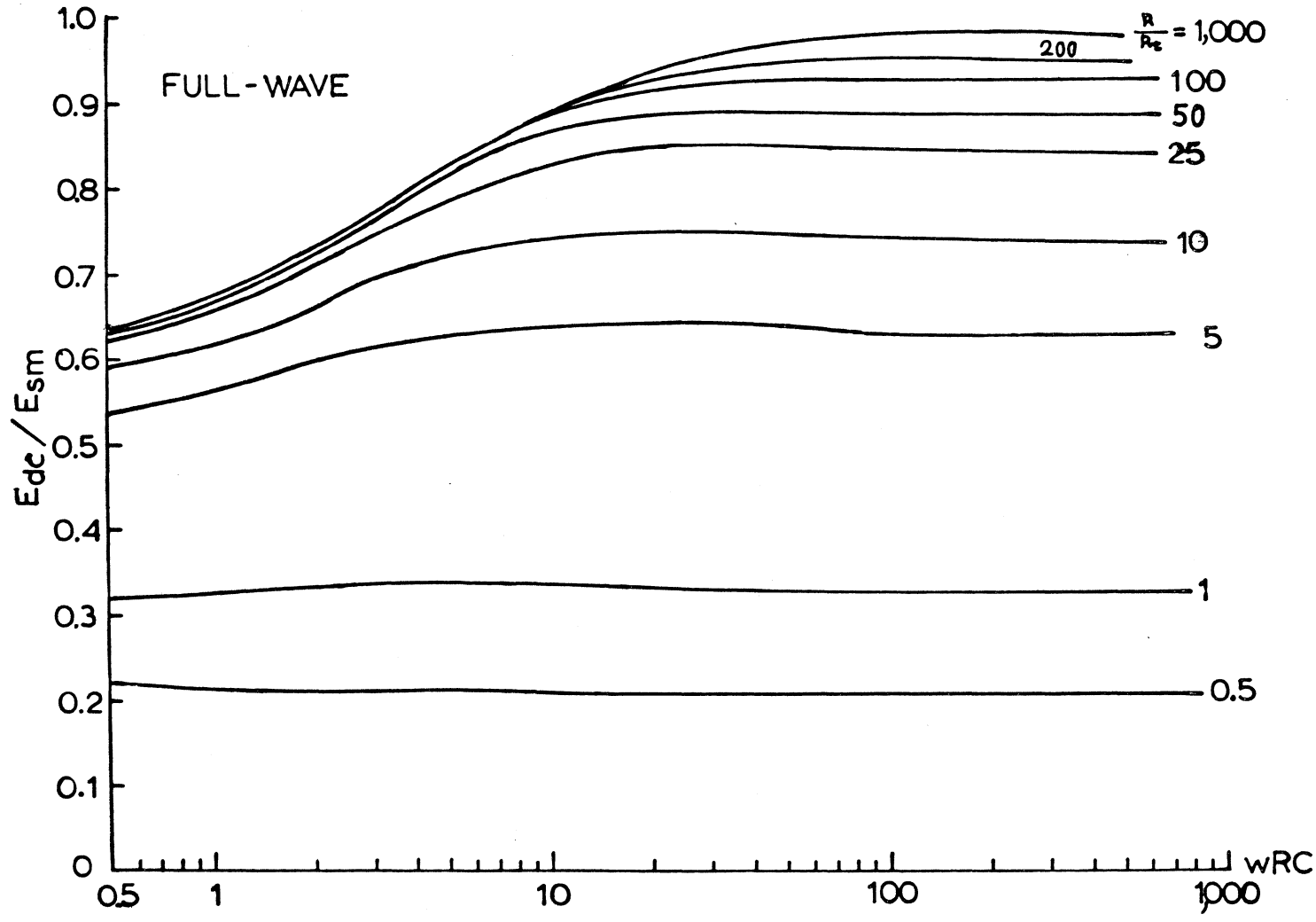
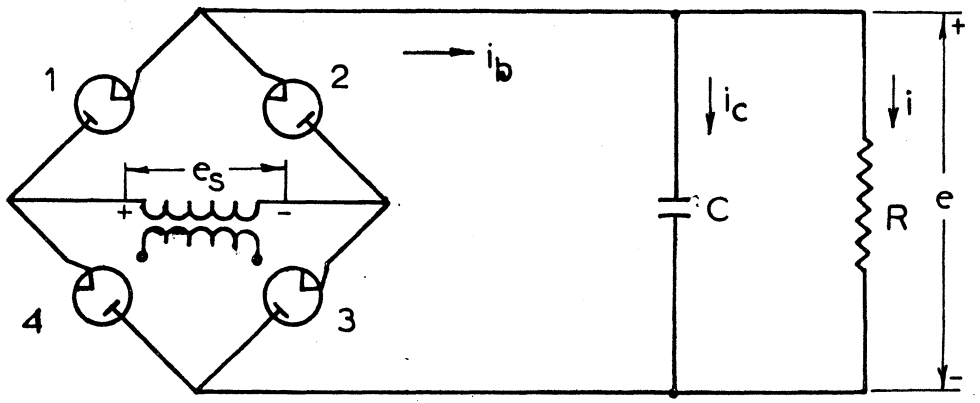
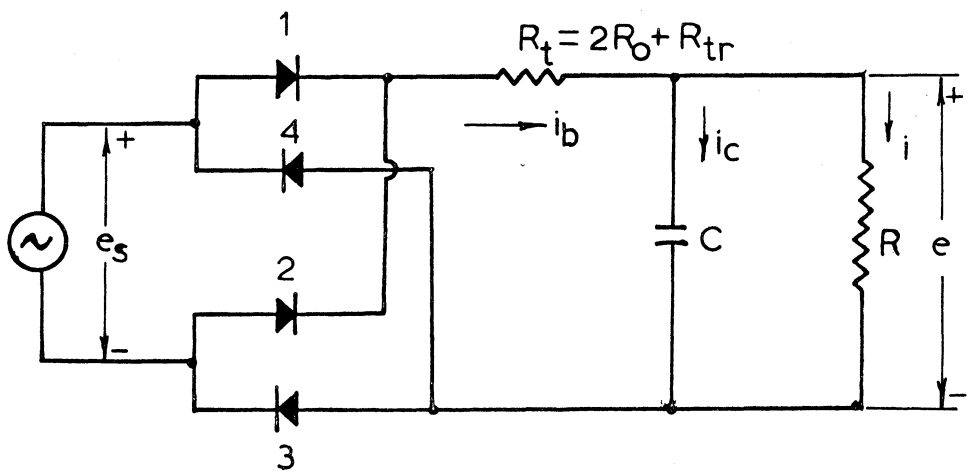


Fig. 3-11. Curves of E_{dc}/E_{sm} versus wRC for various values of R/R_t.



(a) Actual circuit



(b) Idealized circuit

Fig. 3-12 Bridge rectifier circuit with a shunt capacitor for smoothing.

Figure 3-12a represents a bridge rectifier circuit with a smoothing capacitor. This is an arrangement of rectifiers permitting full-wave rectification of the current from a single-phase two-terminal source.[5] To examine the operation of the bridge circuit, one may refer to Fig. 3-12b, the idealized circuit of Fig. 3-12a. It is observed that during the portion of the half-cycle in which the source voltage e_s is positive (relative to the reference direction shown on Fig. 3-12b) and during which the rectifiers 1 and 3 conduct and the rectifier 2 and 4 do not conduct, the capacitor receives a charge. At this time the voltage across the capacitor is built up. When the charged voltage across the capacitor is greater than the source voltage, the filter-load circuit is disconnected from the source, and no tube conducts current. Meanwhile, the capacitor begins to discharge through the load resistor, and the charged voltage falls exponentially until the negative source voltage during the second half of the cycle rises and becomes equal to it. Beyond the instant when the negative source voltage becomes greater than the charged voltage across the capacitor, the rectifiers 2 and 4 conduct current while the rectifiers 1 and 3 act as open circuits. The four rectifiers connected in this circuit are assumed to have identical characteristics. It becomes evident that the operation of the bridge rectifier circuit with a smoothing capacitor is the same as that of the full-wave circuit discussed above. Hence, the results of the analysis in the case for the full-wave circuit of the two-

tube type can also be used for the bridge rectifier circuit. However, care must be taken in choosing the value of the resistance R_t when one calculates the load current and the angles at which the tubes begin conducting and cease conducting for the bridge circuit. Since two tubes are always carrying current during the conduction period, the resistance R_t should be chosen equal to the sum of the transformer resistance and two times the equivalent resistance of each tube, i.e.,

$$R_t = R_{tr} + 2R_o$$

IV. ANALYSIS OF FULL-WAVE RECTIFIER CIRCUIT WITH A SERIES INDUCTOR FOR SMOOTHING.[8,10,16]

An inductor can be connected in series with the rectifier circuits for the purpose of smoothing the current because it stores energy when the voltage across it is positive and releases this stored energy to the load when the voltage is negative. As a matter of fact the current in the inductor for the full-wave rectifier never falls to zero, that is, the current will flow through the inductance and the load resistance at all times. The determination of the current and voltage waves as a series of repeated transients is therefore difficult. For simplifying the analysis, it may be assumed that the current passing through the inductance and the load resistance transfers suddenly from one to the other anode at the instant when one anode becomes negative, and the other one positive[8]. Furthermore, many assumptions are made as follows.

1) diode tube is replaced by an ideal rectifier in series with a fixed resistor R_o.

2) The supply voltage is a pure sine wave.

3) The leakage inductance is neglected.

4) All other circuit elements are perfect.

Under the assumptions outlined above, the rectifier circuit, as shown in part (a) of Fig. 4-1, may be idealized in the form of the circuit given in part (b) of Fig. 4-1. The resistance, denoted by R_t, appearing in the idealized circuit is the sum of the equivalent resistance of tube R_o, the resistance of inductor R_L and

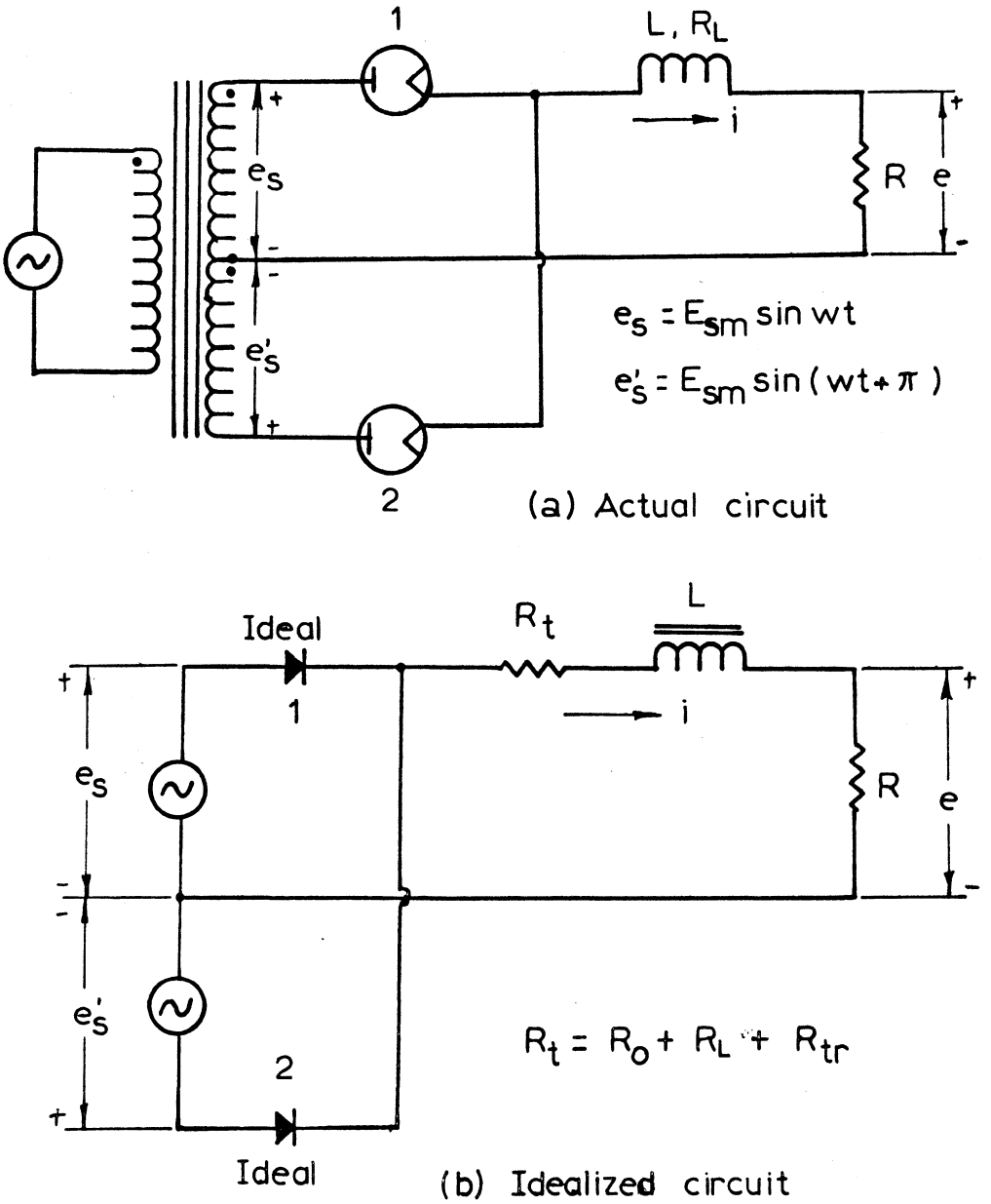


Fig. 4-1. Full-wave rectifier circuit with a series inductor for smoothing.

the resistance of the transformer measuring from the center-tap point to one anode of the tube. Since the rectifiers are assumed to be operated under the condition that when one rectifier ceases to conduct, the other begins, thus only one equivalent resistance of the tube and one half of the total resistance of the secondary transformer are included in the resistance R_t. Hence the resistance R_t may be treated as one part of the filter circuit. The loop voltage equation for the circuit in part (b) of Fig. 4-1 may be written as

$$L\frac{di}{dt} + (R + R_t)i = e_s \qquad (4\text{-}1)$$

during one half of the supply-voltage cycle, and

$$L\frac{di}{dt} + (R + R_t)i = e_s' \qquad (4\text{-}2)$$

during the other half of the cycle. The homogeneous solutions of Equations (4-1) and (4-2) are given respectively by

$$i_h = A \cdot \epsilon^{-(\frac{R+R_t}{wL}) \cdot wt}$$

and

$$i_h = B \cdot \epsilon^{-(\frac{R+R_t}{wL}) \cdot wt}$$

where A and B are the arbitrary constants. Also, the particular solutions of these equations, when solved, are expressed respectively as

$$i_p = \frac{E_{sm}}{\sqrt{(R+R_t)^2 + (wL)^2}} \sin(wt - \lambda)$$

and

$$i_p = \frac{E_{sm}}{\sqrt{(R+R_t)^2 + (wL)^2}} \sin(wt + \pi - \lambda)$$

where

$$\lambda = \tan^{-1}\left(\frac{wL}{R + R_t}\right)$$

Then the complete solution of Eq. (4-1) is of the form

$$\begin{aligned} i &= i_p + i_h \\ &= \frac{E_{sm}}{\sqrt{(R+R_t)^2 + (wL)^2}} \sin(wt - \lambda) + A \cdot \epsilon^{-wt \cdot \cot\lambda} \end{aligned} \tag{4-3}$$

and that of Eq. (4-2) is

$$i = \frac{E_{sm}}{\sqrt{(R+R_t)^2 + (wL)^2}} \sin(wt + \pi - \lambda) + B \cdot \epsilon^{-wt \cdot \cot\lambda} \tag{4-4}$$

The constants A and B may be determined by the initial condition. The initial value of the current at the beginning of each half-cycle is the value of the current at the end of the preceding half-cycle. Hence, the value of the current obtained by Eq. (4-3) at the end of the half-cycle during which the filter circuit is supplied by the source e_s is the initial value of the Eq. (4-4). From Fig. 4-2, the constant A in Eq. (4-3) may be obtained by setting $i = I_n$ when $wt = (n-1)\pi$. Thus

$$I_n = \frac{E_{sm}}{\sqrt{(R+R_t)^2 + (wL)^2}} \sin[(n-1)\pi - \lambda] + A\epsilon^{-(n-1)\pi \cdot \cot\lambda}$$

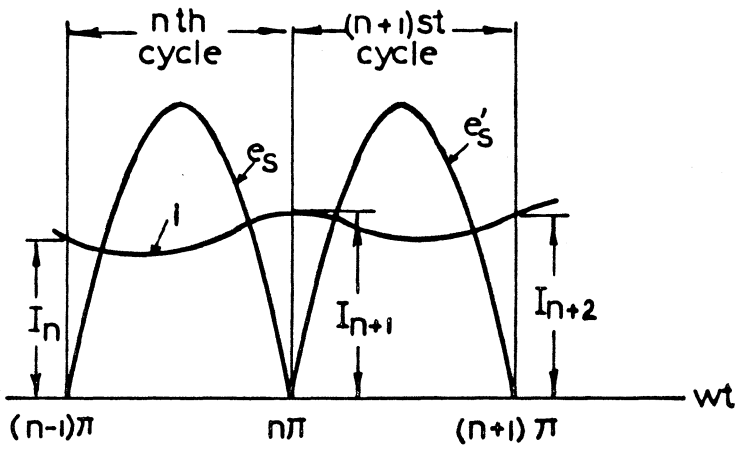


Fig. 4-2. Arbitrary waveform of the load current during one cycle.

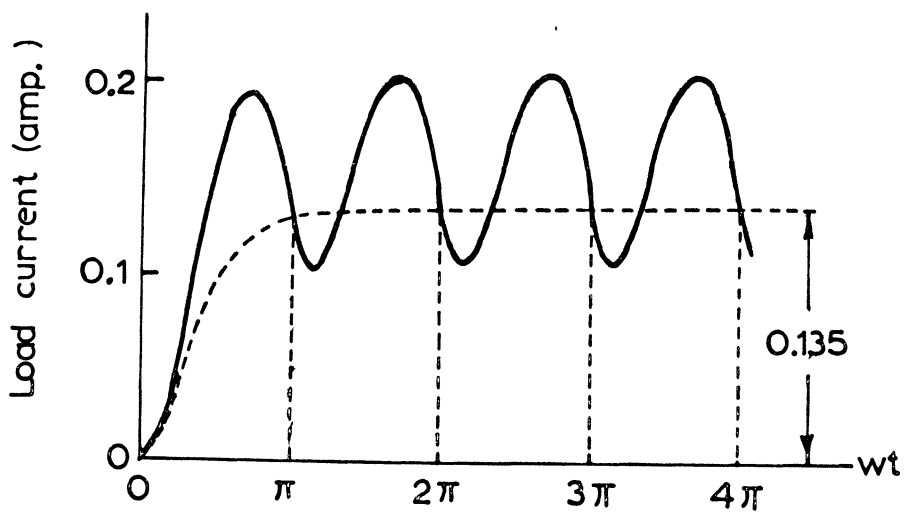


Fig. 4-3. Computed waveform of Example 4-1.

Assuming the operation of the rectifier circuit starts at wt = 0, then n must be an odd integer. Hence

$$A = \left\{ \frac{E_{sm}\, wL}{(R+R_t)^2 + (wL)^2} + I_n \right\} \cdot \epsilon^{(n-1)\pi \cdot \cot\lambda}$$

After substituting the expression of A into Eq. (4-3), the current during the nth half-cycle can be obtained by the equation

$$i = \frac{E_{sm}}{\sqrt{(R+R_t)^2+(wL)^2}} \sin(wt-\lambda) + \left\{ \frac{E_{sm} \cdot wL}{(R+R_t)^2+(wL)^2} + I_n \right\} \cdot \epsilon^{-[wt-(n-1)\pi] \cdot \cot\lambda}$$

$$\text{for} \quad (n-1)\pi \le wt \le n\pi \quad , \quad n = 1, 2, 3 \cdots \tag{4-5}$$

As described above, the initial condition for determining the constant B in Eq. (4-4) is the value of the current at the end of the preceding half-cycle, i.e., the current i in Eq. (4-4) equals $I_{(n+1)}$ when wt = nπ, and from Eq. (4-5)

$$I_{n+1} = \frac{E_{sm}}{\sqrt{(R+R_t)^2+(wL)^2}} \sin(n\pi-\lambda) + \left\{ \frac{E_{sm} \cdot wL}{(R+R_t)^2+(wL)^2} + I_n \right\} \cdot \epsilon^{-[n\pi-(n-1)\pi] \cdot \cot\lambda}$$

or

$$I_{n+1} = \frac{E_{sm} \cdot wL}{(R+R)^2+(wL)^2} \left\{ 1 + \epsilon^{-\pi \cdot \cot\lambda} \right\} + I_n \epsilon^{-\pi \cdot \cot\lambda} \tag{4-6}$$

Therefore the current during the (n+1)st half-cycle is expressed by

$$i = \frac{E_{sm}}{\sqrt{(R+R_t)^2+(wL)^2}} \sin(wt+\pi-\lambda) + \left\{ \frac{E_{sm} \cdot wL}{(R+R_t)^2+(wL)^2} + I_{n+1} \right\} \cdot \epsilon^{-(wt-n\pi) \cdot \cot\lambda}$$

$$\text{for} \quad n\pi \le wt \le (n+1)\pi \quad , \quad n = 1, 2, 3 \cdots \tag{4-7}$$

Examinating of Equations (4-5) and (4-7) shows that the current consists of both a steady state and a transient component. The transients are repeated during each cycle. Hence there is a question whether the current does or does not become stable. Before trying to answer this question, it must be realized that when a steady state is reached, the waveform of the load current must be the same during each cycle. Under this condition, the initial values of the current at the beginning of each cycle must be the same. Therefore the proof may be started by the use of Eq. (4-6).

$$P = \frac{E_{sm} \cdot wL}{(R+R_t)^2 + (wL)^2} \left\{ 1 + \epsilon^{-\pi \cdot \cot \lambda} \right\}$$

and

$$Q = \epsilon^{-\pi \cdot \cot \lambda}$$

It should be noticed that P and Q are constant. Then Eq. (4-6) becomes

$$I_{n+1} = P + Q \cdot I_n \tag{4-8}$$

It may be assumed that the operation of circuit starts at wt = 0 and the inductor stores no energy. Therefore

$$I_1 = 0$$

From Eq. (4-8), it is obtained

$$I_2 = P$$

and

$$I_3 = P + Q \cdot I_2$$
$$= P + P \cdot Q = P(1+Q)$$

Also $$I_4 = P + Q \cdot I_3$$

$$= P + Q \cdot P(1 + Q)$$

$$= P(1 + Q + Q^2)$$

$$\vdots \qquad \vdots$$

$$I_{n+1} = P(1 + Q + Q^2 + \cdots\cdots\cdots + Q^{n-1})$$

As the cycle is repeated infinitively, the series in the parentheses of the above equation approaches a constant. Since Q is always less than unity, except when both the resistances R and R_t have the zero value or when the product of the angular frequency and the inductance, wL, approaches infinity, the series is then convergent to a constant value. Therefore, the initial values of the current at the beginning of each cycle will approach a constant after many cycles. Hence, a steady state condition consisting of repeated transients will be reached and repetitive current waveforms will be obtained. A quantitative example showing the behavior of the full wave rectifier circuit using a series inductor for smoothing is given below.

Example 4-1. Calculate and plot the load current for the rectifier circuit in part (b) of Fig. 4-1 until the steady state is reached. The constants of the circuit are given as:

E_{sm} = 500 volts, w = 377 rad./sec.,

R = 1500 ohms, R_t = 500 ohms,

L = 5 henry.

Assume the operation starts at wt equals to zero with the inductor storing no energy.

The results of Example 4-1 are shown in Fig. 4-3. So far, the behavior of the full-wave rectifier circuit with a smoothing inductor has been proved to be stable. Consequently, the steady state value of the load current can now be determined. Assume the steady state condition of the current in a rectifier circuit is reached at the nth half-cycle. As proved previously, the load current has the same initial values after this cycle. In other words, the value of the current at the beginning of the nth cycle must be equal to that at the end of this cycle. Thus

$$I_{(n+1)} = I_n$$

Substituting this relation into Eq. (4-5) and letting wt equals $n\pi$, yields

$$I_n = \frac{E_{sm}}{\sqrt{(R+R_t)^2+(wL)^2}} \sin(n\pi-\lambda) + \left\{ \frac{E_{sm}\cdot wL}{(R+R_t)^2+(wL)^2} + I_n \right\} \cdot \epsilon^{-\pi\cdot\cot\lambda}$$

or

$$I_n = \frac{wL\cdot E_{sm}\cdot\left(1+\epsilon^{-\pi\cdot\cot\lambda}\right)}{\left[(R+R_t)^2+(wL)^2\right]\cdot\left(1-\epsilon^{-\pi\cdot\cot\lambda}\right)}$$

Then Eq. (4-5) becomes

$$i = \frac{E_{sm}}{\sqrt{(R+R_t)^2+(wL)^2}} \left\{ \sin(wt-\lambda) + \frac{2\sin\lambda}{\left(1-\epsilon^{-\pi\cdot\cot\lambda}\right)} \epsilon^{-[wt-(n-1)\pi]\cdot\cot\lambda} \right\}$$

$$(n-1)\pi \le wt \le n\pi \qquad (4\text{-}9)$$

$$n = 1, 2, 3, \cdots$$

which is the expression for the steady state load current. Now, the d-c load current I_{dc} can be determined by averaging Eq. (4-9) over the interval between wt = (n-1)π and wt = nπ, where n is an odd integer, i. e.,

$$I_{dc} = \frac{1}{\pi}\int_{(n-1)\pi}^{n\pi} i \, d(wt)$$

$$= \frac{E_{sm}}{\pi\sqrt{(R+R_t)^2+(wL)^2}}\left\{\int_{(n-1)\pi}^{n\pi} \sin(wt-\lambda)\, d(wt) + \frac{2\sin\lambda}{(1-\epsilon^{-\pi\cdot\cot\lambda})}\int_{(n-1)\pi}^{n\pi} \epsilon^{-[wt-(n-1)\pi]\cdot\cot\lambda}\, d(wt)\right\}$$

$$= \frac{E_{sm}}{\pi\sqrt{(R+R_t)^2+(wL)^2}}\left\{2\cos\lambda + \frac{2wL\cdot\sin\lambda}{(R+R_t)}\right\}$$

$$I_{dc} = \frac{2E_{sm}}{\pi(R+R_t)} \qquad (4\text{-}10)$$

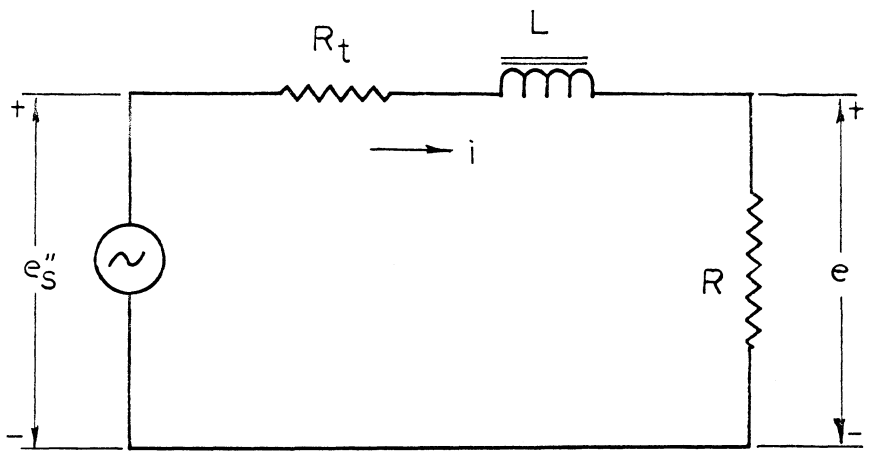
The d-c output voltage can then be determined, that is

$$E_{dc} = R\cdot I_{dc}$$

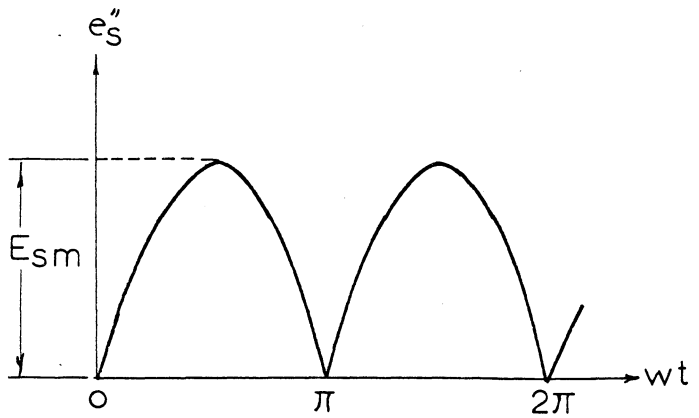
$$= \frac{2E_{sm}}{\pi\left(1+\frac{R_t}{R}\right)} \qquad (4\text{-}11)$$

This result may be checked by the method which is described as follows.

As assumed previously, the current flowing through the inductance and the load resistance transfers abruptly from one to



(a) Equivalent circuit



(b) Waveform of the source voltage

Fig. 4-4. Equivalent circuit of Fig. 4-1.

the other anode of the tubes. It is easily to see that the circuit conditions are then exactly the same as if we had a single source of voltage furnishing nothing but positive half-waves of voltage to the load resistance and the inductance, with the rectifiers removed completely from the circuit[8]. The equivalent rectifier circuit and the waveform of such a special source appear in part (a) and part (b) of Fig. 4-4. The Fourier series for the source voltage e_s'' is given by

$$e_s'' = \frac{E_{sm}}{\pi}\left(2 - \frac{4}{3}\cos 2wt - \frac{4}{15}\cos 4wt - \cdots\cdots\right)$$

In accordance with the principle of superposition and a-c circuit theory, the load current i can be found as

$$i = \frac{2E_{sm}}{\pi(R+R_t)} - \frac{4E_{sm}\cos(2wt-\lambda_1)}{3\pi\sqrt{(R+R_t)^2+(2wL)^2}} - \frac{4E_{sm}\cos(4wt-\lambda_2)}{15\pi\sqrt{(R+R_t)^2+(4wL)^2}} - \cdots\cdots \qquad (4\text{-}12)$$

where

$$\lambda_1 = \tan^{-1}\left(\frac{2wL}{R+R_t}\right)$$

and

$$\lambda_2 = \tan^{-1}\left(\frac{4wL}{R+R_t}\right)$$

When the integration over a half-cycle is taken for determining the average value of the load current, it will be found that the integration of all the terms on the right hand side of Eq. (4-12), except the first term, are zero. Hence, the d-c output current obtained by this method has the same result as that expressed in Eq. (5-10).

Either of the above methods may apply in the analysis of bridge rectifier circuit using a series inductor for smoothing. The results are the same as a full-wave rectifier, except the resistance R_t includes two times the tube resistance instead of one as that for full-wave, i. e.,

$$R_t = 2R_o + R_L + R_{tr}$$

since two tubes carry current during the conduction period.

The load current waves obtained by the rectifier circuit with a series inductor or a shunt capacitor have a ripple. Although this ripple may theoretically be reduced by increasing the size of the inductor in one circuit or increasing the size of the capacitor in the other, it would be impractical as well as uneconomical to make the inductance too large in an actual circuit. Also, too large a value of capacitance may cause a high peak tube current which would damage the rectifier device. For the purpose of reducing the ripple voltage, a full-wave rectifier circuit with a filter circuit consisting of an inductor in series with a parallel capacitor and a load resistor will be introduced in the following section. Such a filter circuit may be called a "choke-input filter".

V. ANALYSIS OF FULL-WAVE RECTIFIER CIRCUIT WITH CHOKE-INPUT FILTER [2,5,8,11].

Part (a) of Fig. 5-1 shows a full-wave rectifier circuit using a filter consisting of an inductor in series with a parallel capacitor and resistor. Under the same assumption made at the beginning of the preceding chapter, an idealized circuit may be drawn as shown in part (b) of Fig. 5-1. The circuit diagrams of Figures 4-1 and 5-1 show that the rectifier circuit with a choke-input filter is similar to that with a series inductor, except a shunt capacitor is connected across the load resistor. When this capacitor appears in the filter circuit, it receives charge from the source. Then, two conditions of operation may occur in the rectifier circuit. (1) The charged voltage across the capacitor may exceed the source voltage plus the voltage drop across the inductor during each cycle. Thus, the filter circuit will be disconnected from the source. Therefore the current passing through the inductance will fall to zero during the cycle, (2) The current flows continuously through the inductor throughout the entire cycle and the tube never ceases conducting during the conduction cycle.

Although it is possible to analyze the rectifier circuit of this type when it is operated under the first condition mentioned above by the method used in the third chapter of this paper, the procedure is relatively difficult. Also, such a condition seldom occurs in practice, since L is usually large enough to prevent it. Therefore, the analysis that will be made in this section covers

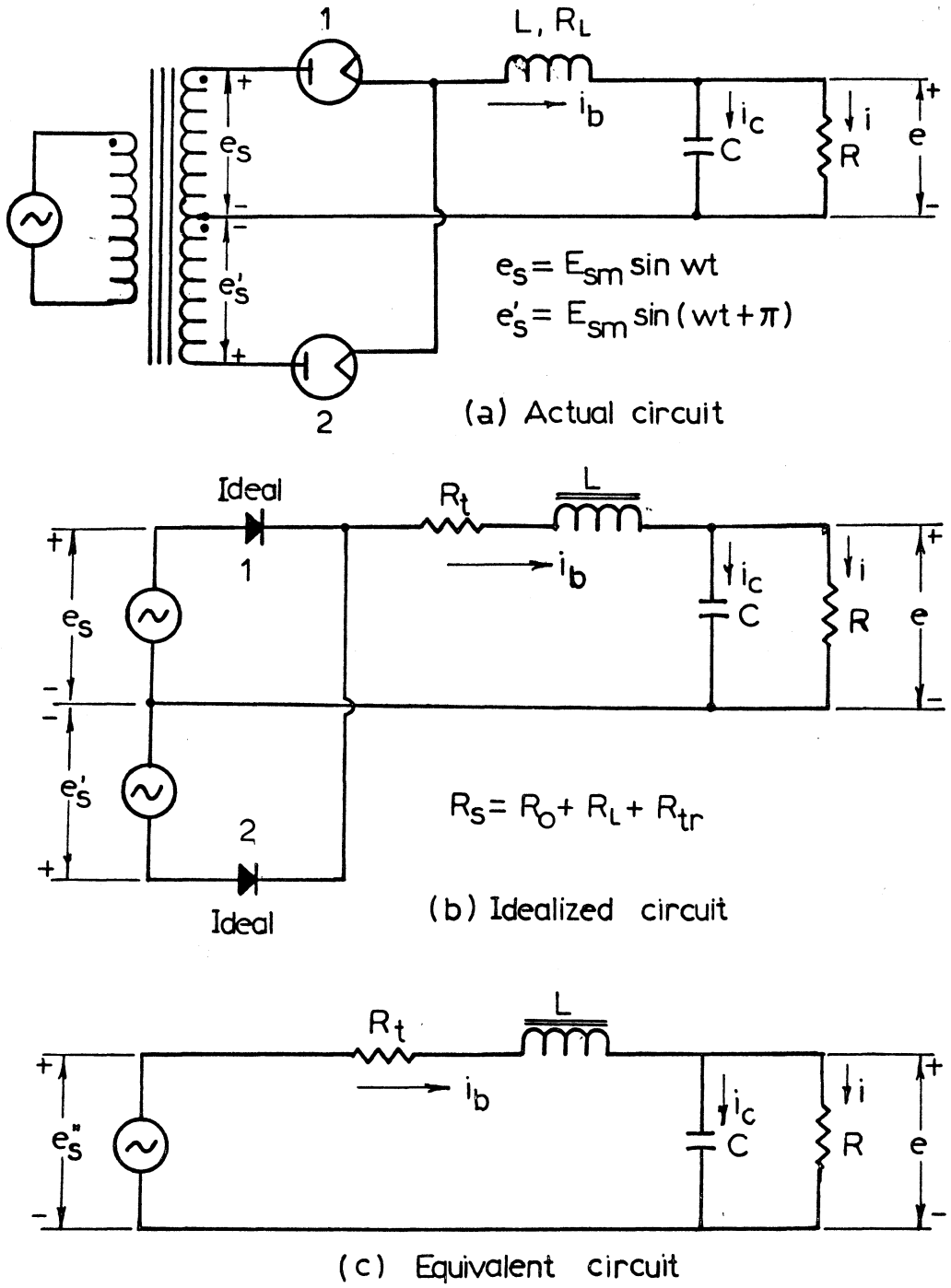


Fig. 5-1. Full-wave rectifier circuit with a choke-input filter.

only the second condition of operation, i. e., under the condition of operation that the current in the inductor is never zero.

To analyze the full-wave rectifier with a choke-input filter under this condition, two methods may be used. They are: (1) Set up the loop voltage equations for the rectifier circuit. The equation for the load current must be a second order nonhomogeneous differential equation, since the circuit consists of both the inductance and the capacitance. Thus, the solution of this equation involves two arbitrary constants, which may be determined by the initial value and the first derivative of the load current at the beginning of each cycle. If the steady state condition could be reached, then the average output current and voltage can be obtained, (2) Remove the ideal rectifiers from the circuit, as is shown in the part (c) of Fig. 5-1, and replace the sources by a special source which furnishes the positive half waves only. Consequently, represent this special source by the Fourier series and apply a-c theory and the principle of superposition to the circuit for obtaining the load current.

Investigation proves that the latter method is simpler. Since in the most usual conditions of operation, the current in the inductor never falls to zero, neither the current in the inductor nor the voltage across the capacitor at the beginning of each cycle is a known condition[5]. The first method of analysis is therefore more difficult than the second one. Consequently, by the use of the second method that was outlined above, the rectifier circuit may be replaced by the equivalent circuit in part (c) of Fig. 5-1. The

special source e_s'' has the waveshape shown in part (b) of Fig. 4-4. The Fourier series representation of e_s'' is

$$e_s'' = E_{sm}\left(\frac{2}{\pi} - \frac{4}{3\pi}\cos 2wt - \frac{4}{15\pi}\cos 4wt - \cdots\right) \tag{5-1}$$

where w is the angular frequency of the supply voltage.

For the steady state continuous-conduction condition, the d-c output current is

$$I_{dc} = \frac{2E_{sm}}{\pi(R + R_t)} \tag{5-2}$$

and the d-c output voltage is

$$E_{dc} = \frac{2E_{sm}}{\pi\left(1 + \frac{R_t}{R}\right)} \tag{5-3}$$

When the second-harmonic component of the source voltage, i. e., the second term on the right hand side of Eq. (5-1), is applied to the filter circuit, the second-harmonic component of the inductor current, denoted by i_{bf2}, is given by

$$I_{bf2} = \frac{4E_{sm}\angle 180^\circ}{3\sqrt{2}\,\pi \cdot Z_{f2}} \tag{5-4}$$

This is an expression in the frequency domain, and the phase is referred to e_s''. The Z_{f2} appearing in the above equation is the impedance of the filter circuit for the frequency of the second-harmonic and is expressed as

$$Z_{f2} = R_t + j2wL + \frac{1}{\frac{1}{R} + j2wC} \tag{5-5}$$

For the purpose of eliminating all of the alternating component of the load current, the following condition is specified

$$X_L >> X_C << R \tag{5-6}$$

Under this condition, the filter capacitor by-passes all of the alternating component of the current and makes the ripple voltage appear across the inductor rather than across the load. Then Eq. (5-5) simplifies to

$$Z_{f2} \approx R_t + j2wL$$

Substituting this relation into Eq. (5-4) yields

$$I_{bf2} \approx \frac{4 E_{sm} \angle (180^\circ - \delta_1)}{3\sqrt{2}\pi\sqrt{R_t^2 + (2wL)^2}} \tag{5-7}$$

where $$\delta_1 = \tan^{-1}\frac{2wL}{R_t}$$

and the voltage across the load may be written as

$$E_{f2} \approx \frac{I_{bf2}}{j2wC} = \frac{2 E_{sm} \angle (90^\circ - \delta_1)}{3\sqrt{2}\pi wC\sqrt{R_t^2 + (2wL)^2}} \tag{5-8}$$

If the relation expressed in (5-6) is true for the second-harmonic, it is an even better approximation for the fourth. Consequently,

the fourth-harmonic component of the inductor current and of the voltage across the load can be found by a process similar to that for the second. Such a process yields

$$I_{bf4} \approx \frac{4 E_{sm} \angle (180^\circ - \delta_2)}{15\sqrt{2}\,\pi \cdot \sqrt{R_t^2 + (4wL)^2}} \qquad (5\text{-}9)$$

where

$$\delta_2 = \tan^{-1} \frac{4wL}{R_t}$$

and

$$E_{f4} \approx \frac{E_{sm} \angle (90^\circ - \delta_2)}{15\sqrt{2}\,\pi \cdot wC \cdot \sqrt{R_t^2 + (4wL)^2}} \qquad (5\text{-}10)$$

From the Equations (5-7) and (5-9), the ratio of the magnitude of the fourth-harmonic current in the inductor to that of the second-harmonic is given approximately by

$$\frac{I_{bf4}}{I_{bf2}} \approx \frac{1}{10} = 10 \quad \text{percent}$$

and the similar ratio for voltage across the load can be obtained from the Equations (5-8) and (5-10). It is

$$\frac{E_{f4}}{E_{f2}} \approx \frac{1}{20} = 5 \quad \text{percent}$$

It is apparent that the fourth-harmonic current and voltage are very small compared with those of the second-harmonic. A similar process may be used to find the higher harmonic components of currents and voltages than those have been found thus far. However this is not

necessary, because not only the amplitude of the high harmonic components of the source voltage e_s'' is smaller but also the harmonic frequency is higher than those of the second harmonic. This will, of course, cause the ratios of the high harmonic components of current and voltage to those of the second-harmonic to be very small values. Thus the ripple components of the current in the inductor and the voltage across the capacitor are predominantly second harmonic. Hence, the expression of the current passing through the inductor may be written approximately in time-domain as

$$\begin{aligned} i_b &= I_{dc} + i_{bf2} \\ &= \frac{2E_{sm}}{\pi(R+R_t)} - \frac{4E_{sm}}{3\pi\sqrt{R_t^2+(2wL)^2}}\cos(2wt-\delta_1) \end{aligned} \tag{5-11}$$

Eq. (5-11) shows that the current in the inductor consists of a d-c component and a a-c component. If the negative peak of the a-c component of the current at any time reaches a value in excess of the d-c component, then there will be a reversal of the current in the tube. However the tube prevents such a reversal current from flowing, the rectifier is therefore disconnected from the load and the current in the inductor happens to be zero during one part of the cycle. As mentioned previously, this condition of operation does not appear in our analysis. In order to fulfill the condition of operation that the current in the inductor possesses throughout the entire cycle, there must be some restriction imposed on the

problem, namely, the condition that the inductance is large enough to prevent the a-c component of the current having an amplitude in excess of the d-c current, since a large value of inductance gives a small amplitude of the a-c component of the current. Under the assumption that the capacitor across the load has a reactance very much smaller than the inductive reactance of the choke coil and if the above condition is fulfilled, the maximum value of the a-c component of the current must be smaller than or equal to the d-c component. In other words, the amplitude of the second term must be smaller than or equal to the first term on the right hand side of Eq. (5-11). Thus

$$\frac{4E_{sm}}{3\pi\sqrt{R_t^2 + (2wL)^2}} \leq \frac{2E_{sm}}{\pi(R+R_t)}$$

If the critical value of the inductance, i. e., the value causing the a-c component to be equal to the d-c component of the current, is caled L_c, the value is found to be

$$L_c = \frac{\sqrt{4R^2 + 8RR_t - 5R_t^2}}{6w} \tag{5-12}$$

If the frequency of the supply voltage is 60 cycles per second, Eq. (5-12) gives

$$L_c = \frac{\sqrt{4R^2 + 8RR_t - 5R_t^2}}{2,262} \tag{5-13}$$

This value of L_c would just let the current drop to zero and therefore represents the minimum value of the inductance in the choke-input filter to enable the current to pass through the rectifier throughout the entire cycle. If the inductor is chosen to have a large Q and the load resistance is large, so that 2wl and R are large compared with R_t, then, Eq. (5-13) may be approximated as

$$L_c = \frac{R}{1,131}$$

For the design purpose, it is desirable to make L_c somewhat large. Thus, L_c is usually taken as

$$L_c = \frac{R}{1,000}$$

The waveforms showing the currents in the inductor when the value of the inductance in the choke coil is less than, equal to and greater than the critical value of inductance are given in the part (a), (b) and (c) of Fig. 5-2 respectively. These curves are reproduced from an oscillogram taken on the circuit, as is shown in the part (a) of Fig. 5-1. Consequently, the output voltage in time domain is given approximately by

$$\begin{aligned} e &= E_{dc} + e_{f2} \\ &= \frac{2E_{sm}}{\pi\left(1+\frac{R_t}{R}\right)} - \frac{2E_{sm}}{3\pi wC\sqrt{R_t^2+(2wL)^2}}\sin(2wt-\delta_1) \end{aligned} \tag{5-15}$$

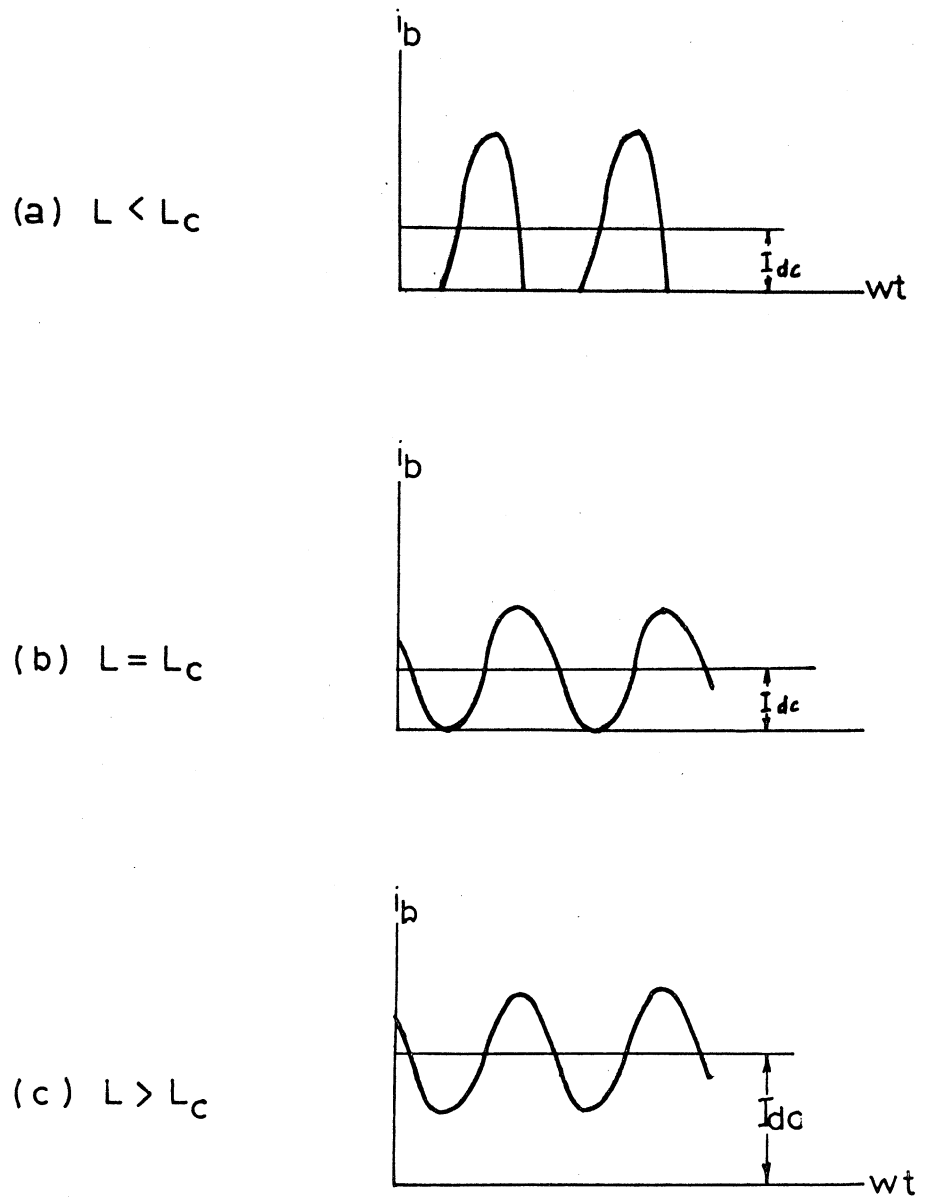


Fig. 5-2. Waveforms showing the tube current in a rectifier circuit using a choke-input filter.

VI. EXPERIMENTAL VERIFICATION

For the purpose of checking the correctness of the equations derived for the rectifier with three different types of filter circuits, three experiments have been made. In these experiments a 5U4-G diode tube was used. Also, a center-tapped transformer was connected for the full-wave circuits. The direct-current output voltages, E_{dc}, were obtained by a direct-current voltmeter. The maximum and the minimum values of the current passing through the tube for each case, and the ignition angle α and the extinction angle β for the capacitor filter circuit were read by means of an oscilloscope. Accurate reading of the angles α and β are difficult to obtain. Some photographs showing the waveforms of the rectified voltage and the tube current have been taken. Each set of the circuit parameters taken in these experiments has been substituted into the corresponding equations for calculating the theoretical results. In an attempt to find out what equivalent value of the tube resistance would make the calculated results close to the experimental results, the values of R_o were chosen in three ways. They were chosen equal to the reciprocal of the slope of a straight line drawn: (1) through the points on the current-voltage curve of the 5U4-G diode corresponding to the maximum and the minimum values of the tube current, (2) tangentially to a point on the current-voltage curve corresponding to the maximum value of the tube current, (3) through the points on the

current-voltage curve corresponding to the maximum and the average value of the tube current. These methods for determining the approximate value of the tube resistance are illustrated respectively by the diagrams as shown in Fig. 6-1.

The experiments taken for three different types of rectifier filter circuit are described separately as follows:

(a) Half-Wave Rectifier With A Smoothing Capacitor.

In this experiment a 124 volt (R. M. S.) source voltage with frequency of 60 cycles per second was used. For the purpose of reducing the effect of the internal impedance of the transformer on the circuit, no transformer was used. Data and results are listed in Table 6-1. The same circuit constants have been substituted into Equations (3-25), (3-26) and (3-27) for calculating the theoretical results, where the values of R_o included in R_t are chosen respectively by the three methods outlined above. The calculated results when R_o is chosen by the first method are shown in Table 6-2 while those using the second and third method are shown in Tables 6-3 and Table 6-4 respectively. Since the diode tube is disconnected from the source when the voltage across the capacitor is greater than the source voltage, the minimum value of tube current is zero. Thus, the value of R_o is obtained by the first method is essentially equal to the reciprocal of the slope of the straight line which is drawn from the origin to a point on the current-voltage curve of 5U4-G diode corresponding to the peak value of the

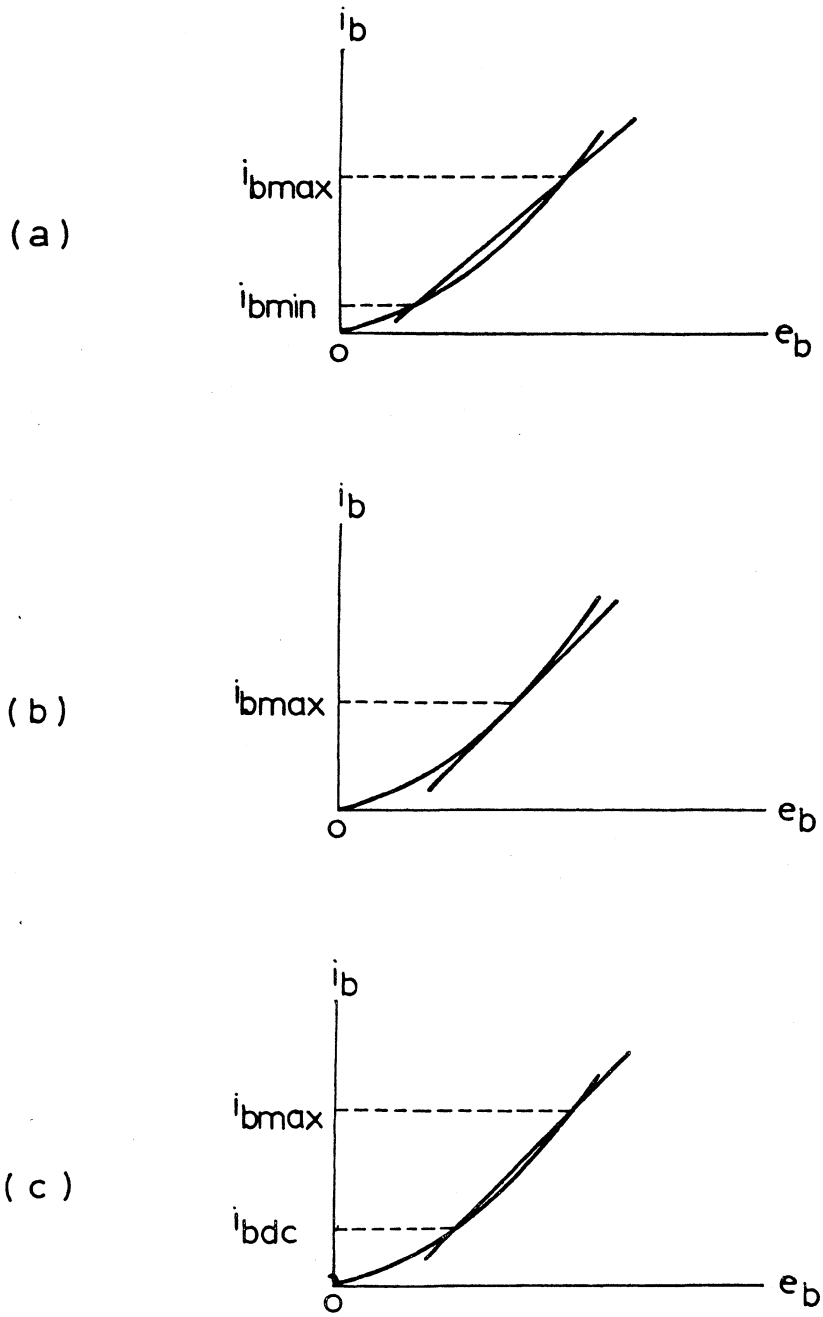


Fig. 6-1. Curves showing the determination of the equivalent tube resistance.

tube current. After examination of the three tables showing the calculated results, it is found that when the values of the tube current are less than 50 ma, the calculated results when R_o was chosen by three different ways didn't make too much difference. However it is indicated that when the tube current has values in the range from 50 ma to 150 ma, the corresponding results listed in Table 6-2 show good agreement, i. e., the first method for determining the values of the tube resistance is the best among the three methods when the tube current of the 5U4-G diode is operated in this range. In contrast, the results shown in Table 6-4 check closely with the results shown in Table 6-1 when the value of the tube current are of the order above 150 ma. This means that the third method for determining the R_o is best when the current passing through the 5U4-G diode is greater than 150 ma. Hence, when the d-c output voltage and the angles α and β are calculated from Equations (3-25) through (3-27) for a half-wave rectifier circuit with a capacitor for smoothing, the values of the tube resistance may be chosen equal to the reciprocal of the straight line which is drawn (1) from the original to the point on the current-voltage curve of the diode corresponding to the peak value of the tube current when the current is low, (2) through the points on the curve corresponding to the maximum and the average values of the tube current when the current is high.

The oscillograms showing the waveforms of the tube current and the output voltages appear in Fig. 6-2. Fig. 6-3 illustrates

the corresponding waveforms and the reading of the angles α and β of Fig. 6-2. The curves of Fig. 6-4 show the computed waveforms of the output voltage and the tube current corresponding to the photographs of Fig. 6-2. Table 6-5 shows the calculated results without consideration of the tube and the transformer voltage drop, and the equations from which these results were calculated are shown in the appendix. The comparisons made between Table 6-1 and Table 6-5 show that the calculation neglecting the voltage drop across the tube and the transformer for the half-wave rectifier circuits with a smoothing capacitor results in a serious error.

(b) Full-Wave Rectifier With A Smoothing Inductor.

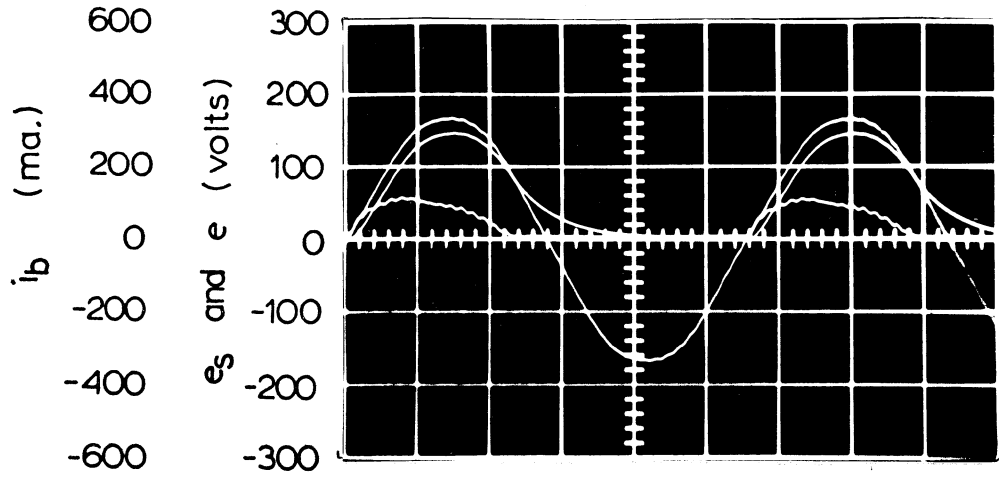
The measurements taken across one section of the center-tapped transformer secondary show that the voltage supplied to the rectifier circuit is 360 volts (R. M. S.) with a frequency of 60 cycles per second and an internal resistance is 150 ohms. Table 6-6 shows the data obtained by experiment. Three methods for determining the value of R_o have been applied in calculating of the theoretical results by Eq. (4-11). It is found that the first method for determining R_o is the best among those. In other words, when Eq. (4-11) is used for calculating the d-c output voltage of the full-wave rectifier circuit with a smoothing inductor, the equivalent resistance of the tube may be chosen equal to the reciprocal of the straight line which is drawn through the points on the characteristic curve of a diode corresponding to the maximum and minimum values of

the tube current. The results calculated by the first method are shown in Table 6-7 but those by the other methods were not included in this paper. Figure 6-5 and Fig. 6-6 show the experimental and the computed waveforms of the output voltage respectively. They are also erroneous if the voltage drop across the tube and the transformer are neglected in the calculation of the d-c output voltage for this type of rectifier filter circuit when the load resistance has a low value.

(c) Full-Wave Rectifier With The Choke-Input Filter.

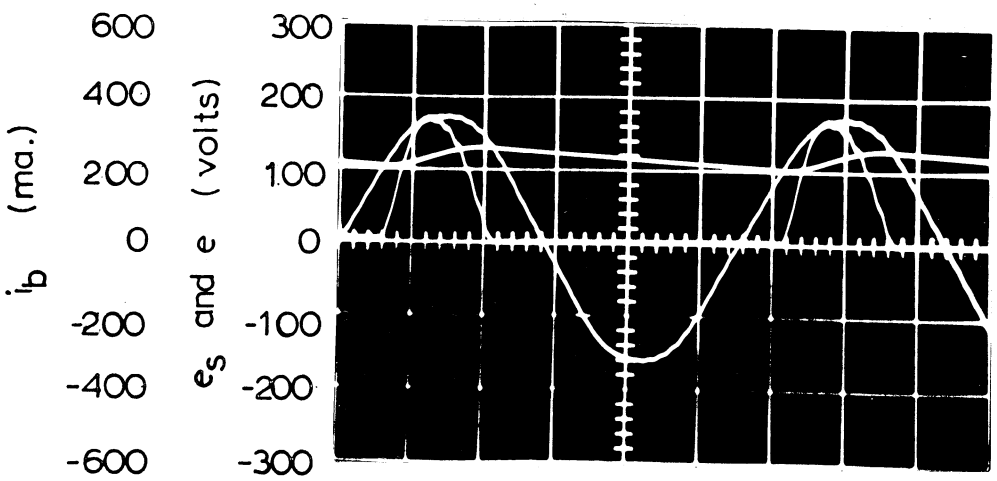
Since the analysis of this type of rectifier circuit in this paper are made under the condition that the current passing through the inductor never falls to zero, the values of the load resistance were chosen below the value that would make the tube current become zero during the conduction cycle. Because the capacitive reactance is assumed to be very small compared with the inductive reactance and the load resistance, a capacitor having a high capacitance was used in this experiment. The supplied voltage and the transformer are the same as that used in section (b). Experimental data and results are shown in Table 6-8. Three methods for determining the R_o have been tried in the calculation of the theoretical results. It is also found that the calculation using the first method to find the value of R_o gives results that are closer to the experimental results. The calculated results are listed in Table 6-9. The oscillograms showing the output voltage and the current in the inductor appear in

Fig. 6-7. Fig. 6-8 shows the computed waves corresponding to the photographs of Fig. 6-7. Also the experimental curve showing the direct output voltage versus the direct output current of the 5U4-G diode for the choke-input filter with L = 33 henry and C = 20 μF is shown in Fig. 6-9.



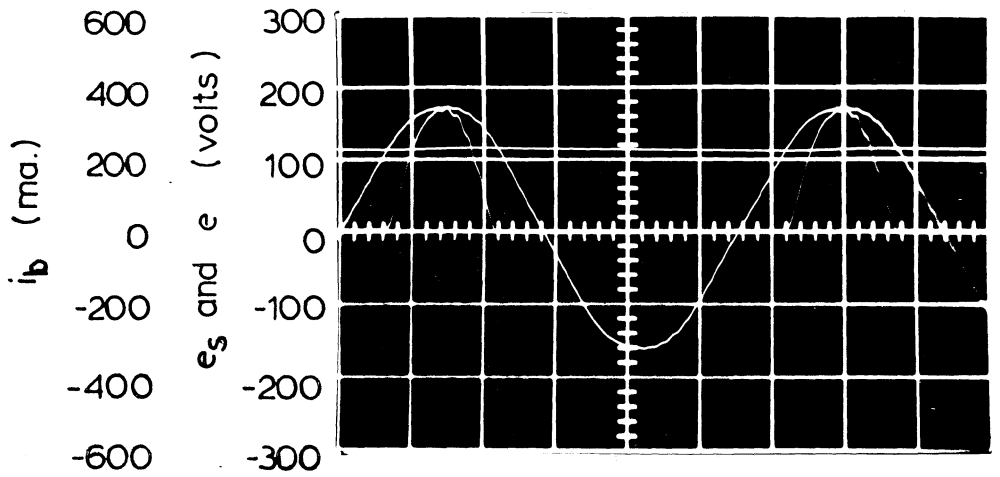
R = 2 KΩ ; C = 1 μF
Sensitivities:
100 volts/div. and 200 ma./div.

Fig. 6 - 2a.



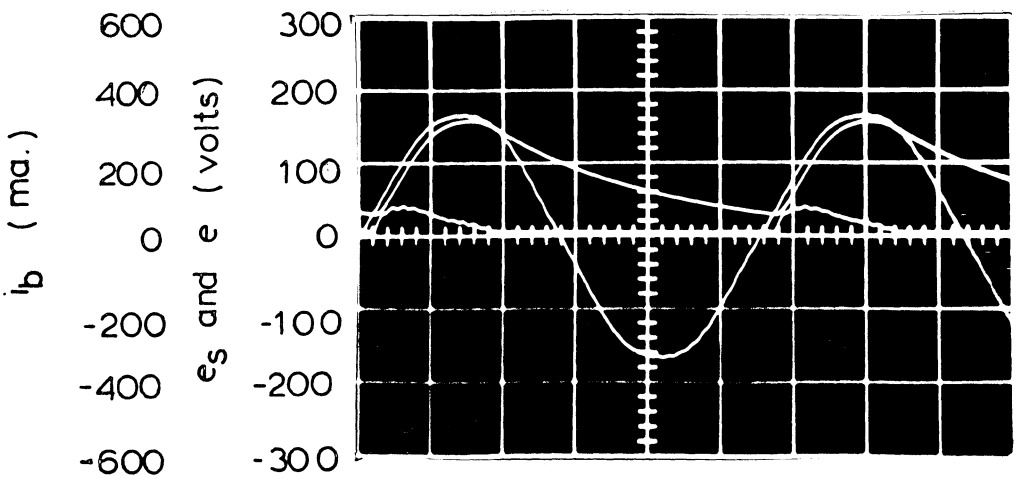
R = 2 KΩ ; C = 20 μF
Sensitivities:
100 volts/div. and 200 ma./div.

Fig. 6 - 2b.



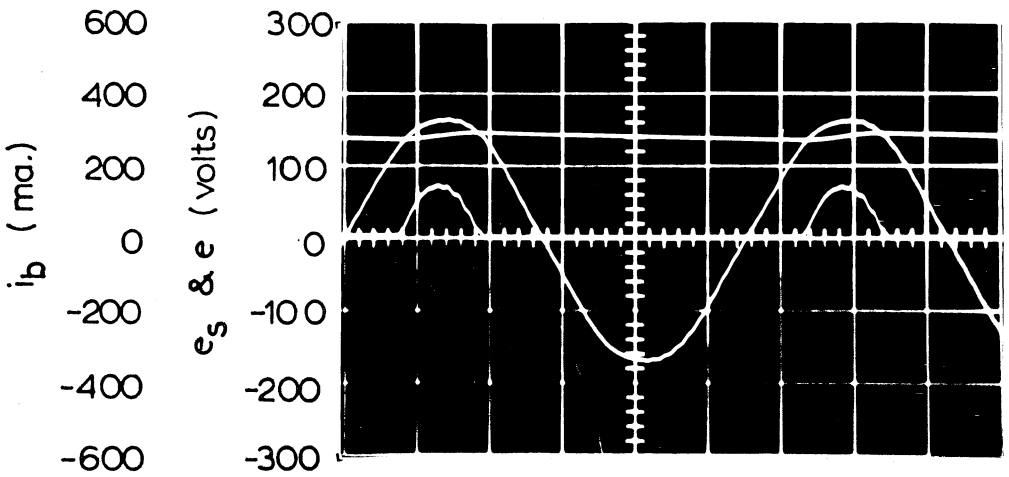
R = 2 KΩ ; C = 200 μF
Sensitivities:
100 volts/div. and 200 ma./div.

Fig. 6 - 2c.



R = 10 KΩ ; C = 1 μF
Sensitivities:
100 volts/div. and 200 ma./div.

Fig. 6 - 2d.



R = 10 KΩ ; C = 20 μF
Sensitivities :
100 volts/div. & 200 ma/div.

Fig. 6-2e.

Fig. 6-2. Oscillograms of output voltage and tube current referring to source voltage. (Capacitive filtering case).

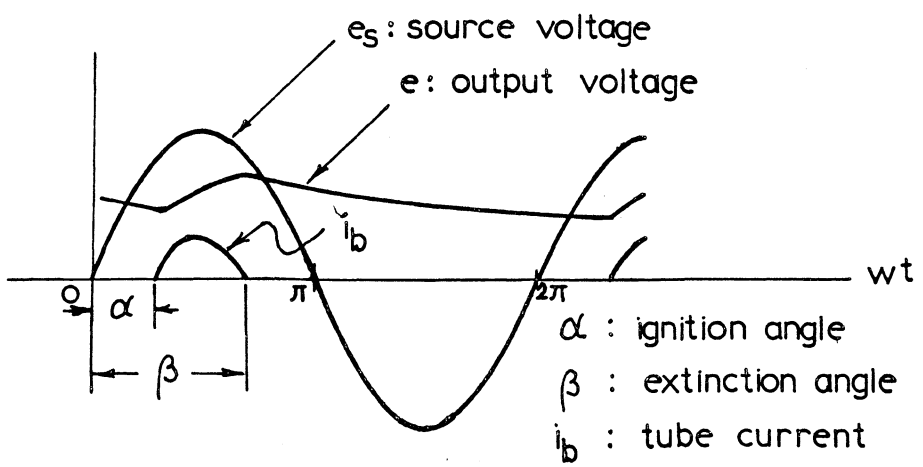


Fig. 6-3. Illustration of waveforms corresponding to Fig. 6-2.

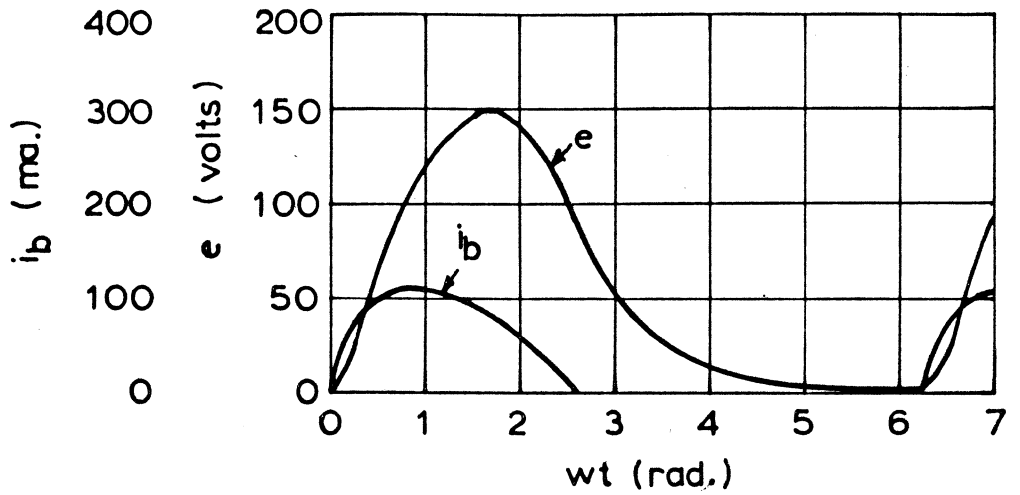


Fig. 6-4a.

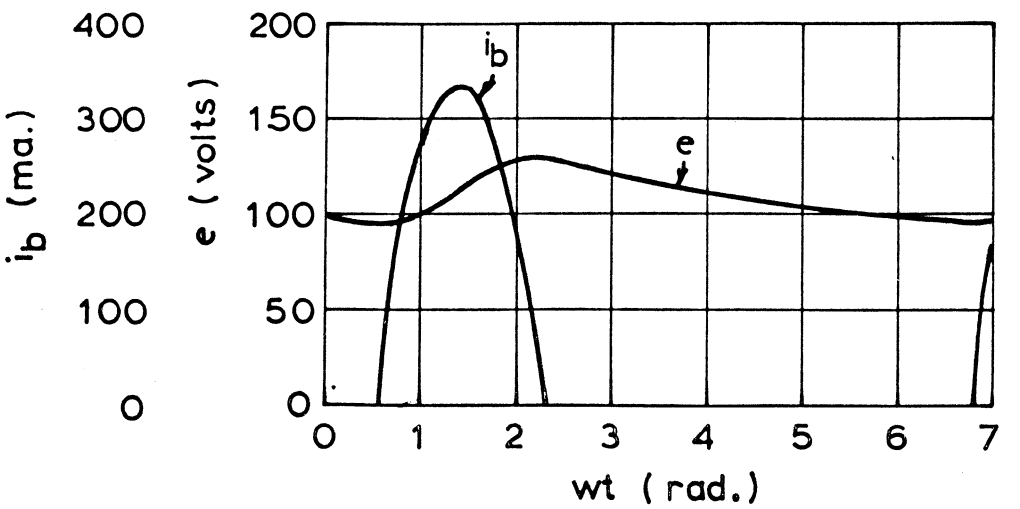


Fig. 6-4b.

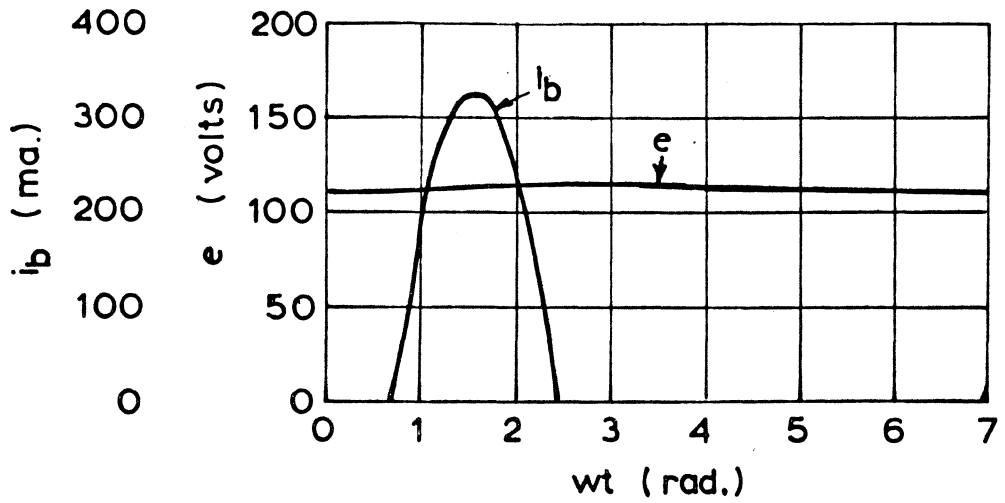


Fig. 6-4c.

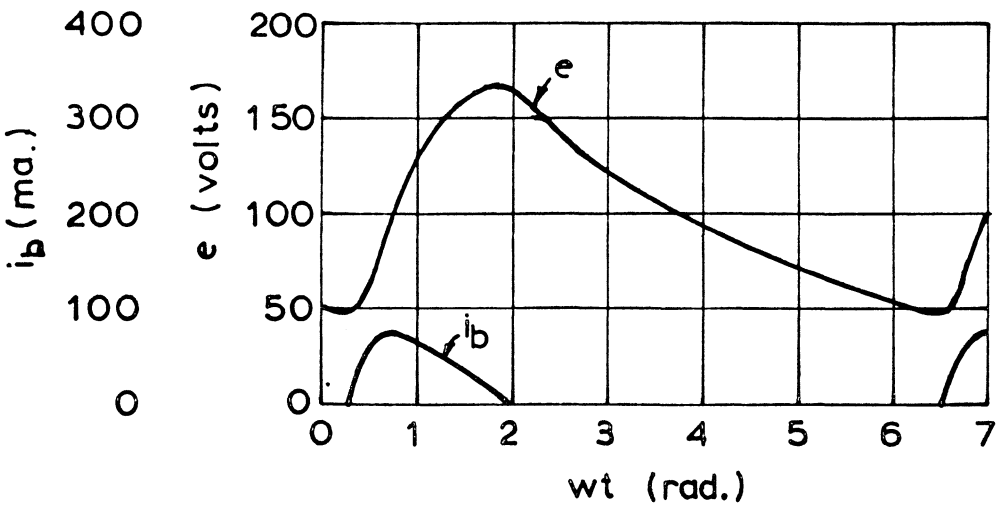


Fig. 6-4d.

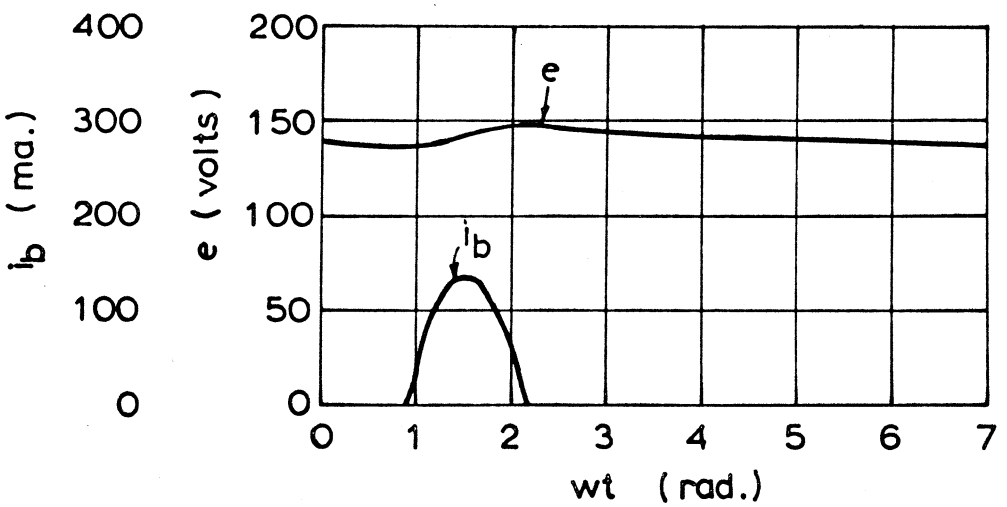
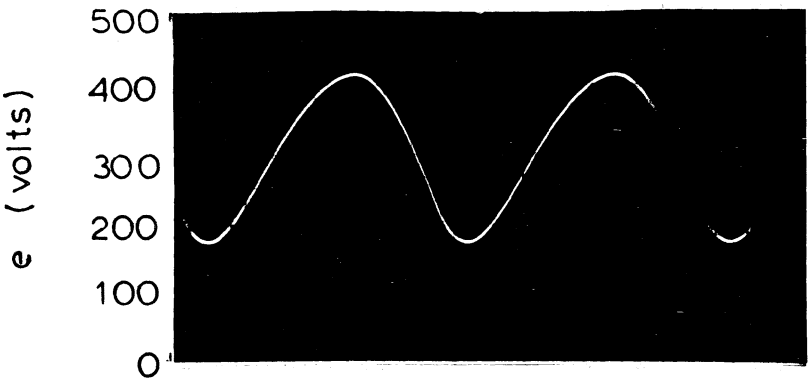


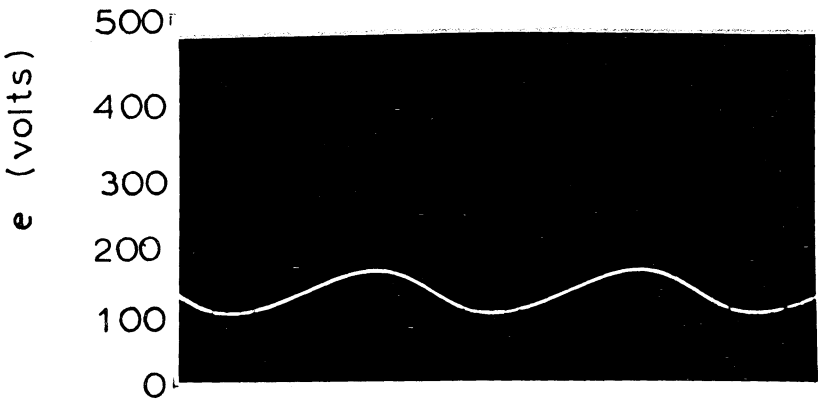
Fig. 6-4e.

Fig. 6-4. Computed waveforms corresponding to the photographs of Fig. 6-2.



L = 8 h ; R = 10 KΩ
Sensitivity : 100 volts/div.

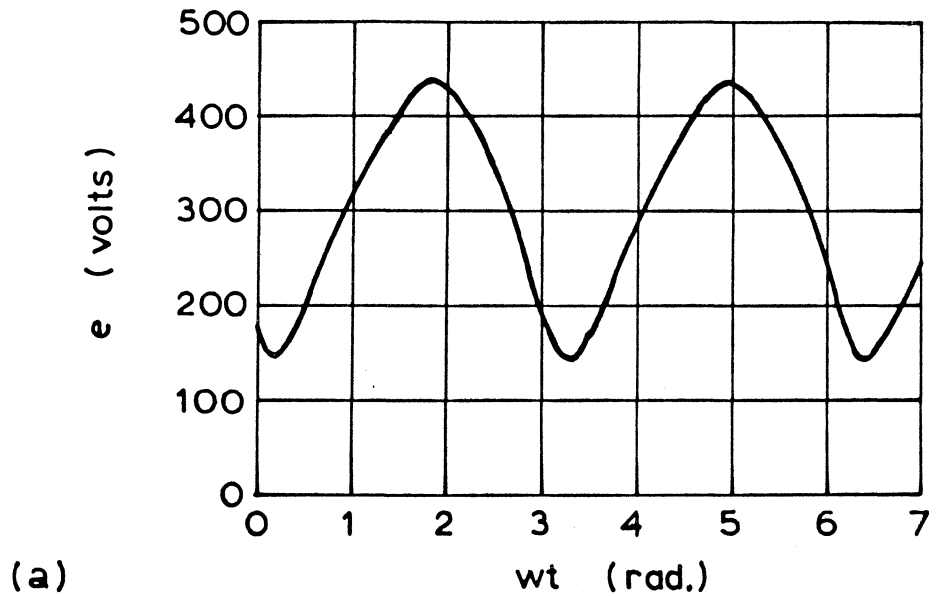
(a)



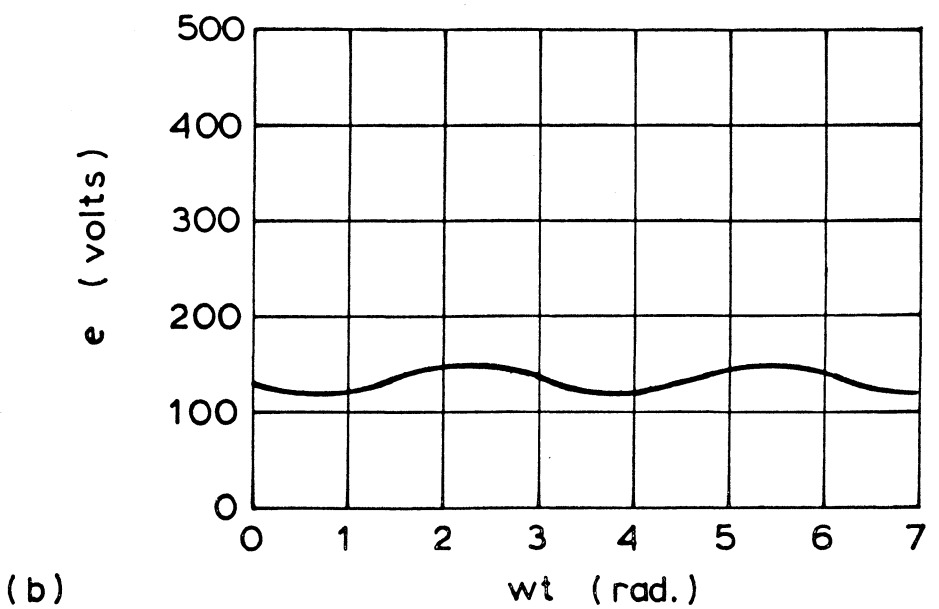
L = 8 h ; R = 400 Ω
Sensitivity : 100 volts/div.

(b)

Fig. 6-5. Oscillograms showing the output voltage of the rectifier circuit with a series inductor for smoothing.

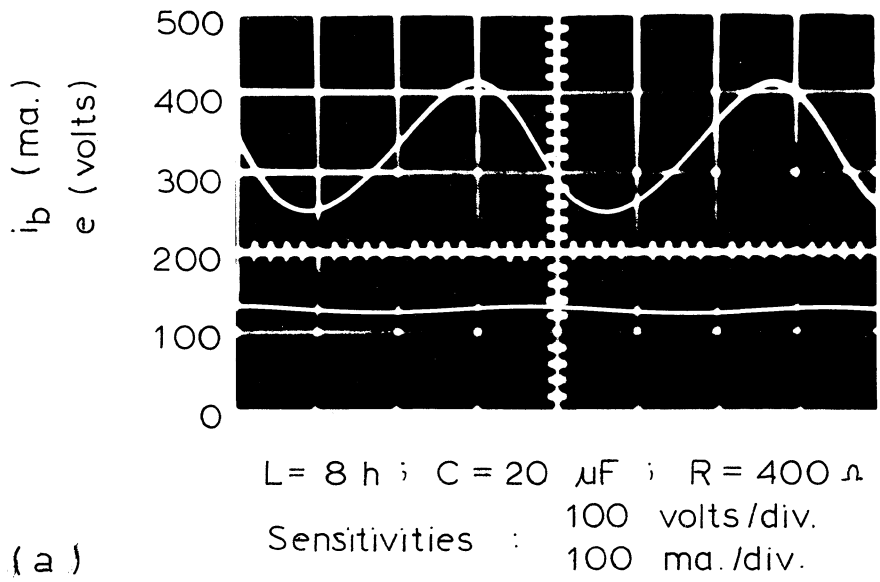


(a)

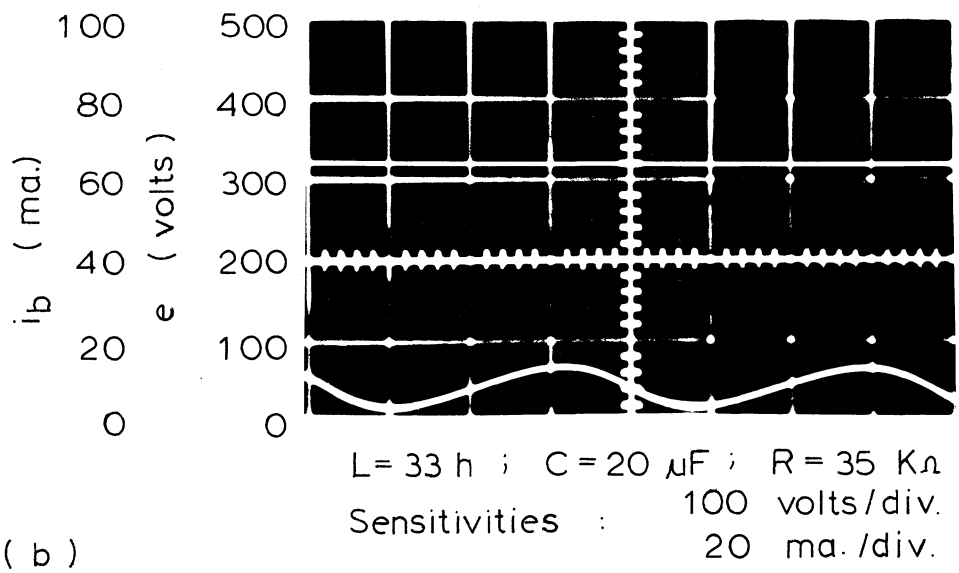


(b)

Fig. 6-6. Computed waveforms corresponding to the photographs of Fig. 6-5.

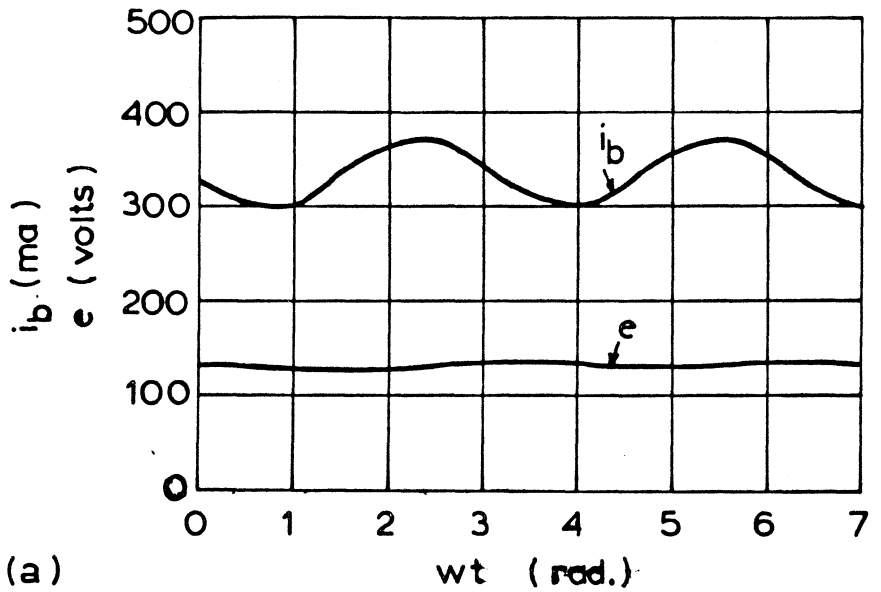


(a)



(b)

Fig. 6-7. Oscillograms showing the output voltage and the inductor current of the rectifier circuit with a choke-input filter.



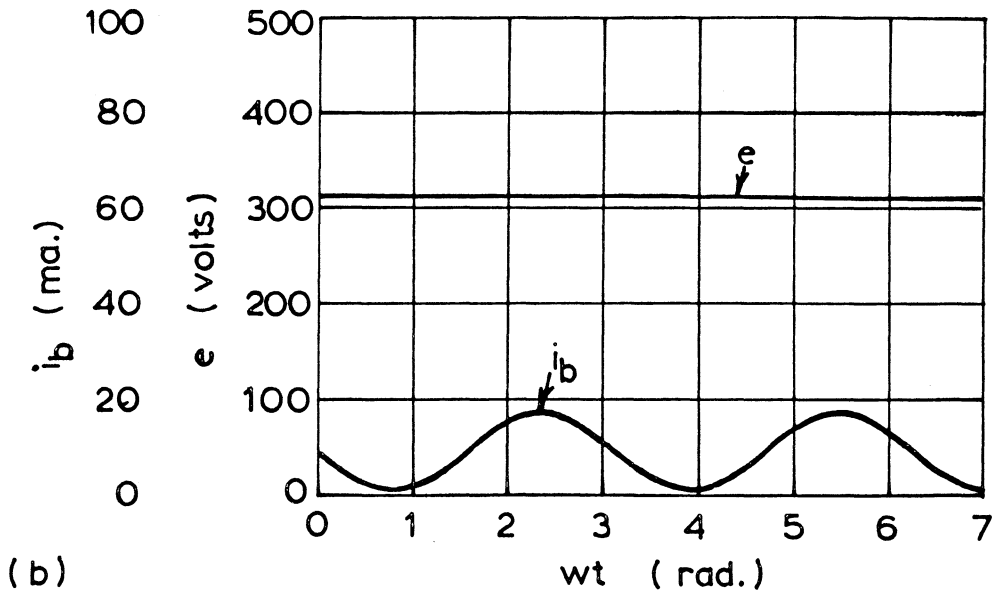


Fig. 6-8. Computed waveforms corresponding to the photographs of Fig. 6-7.

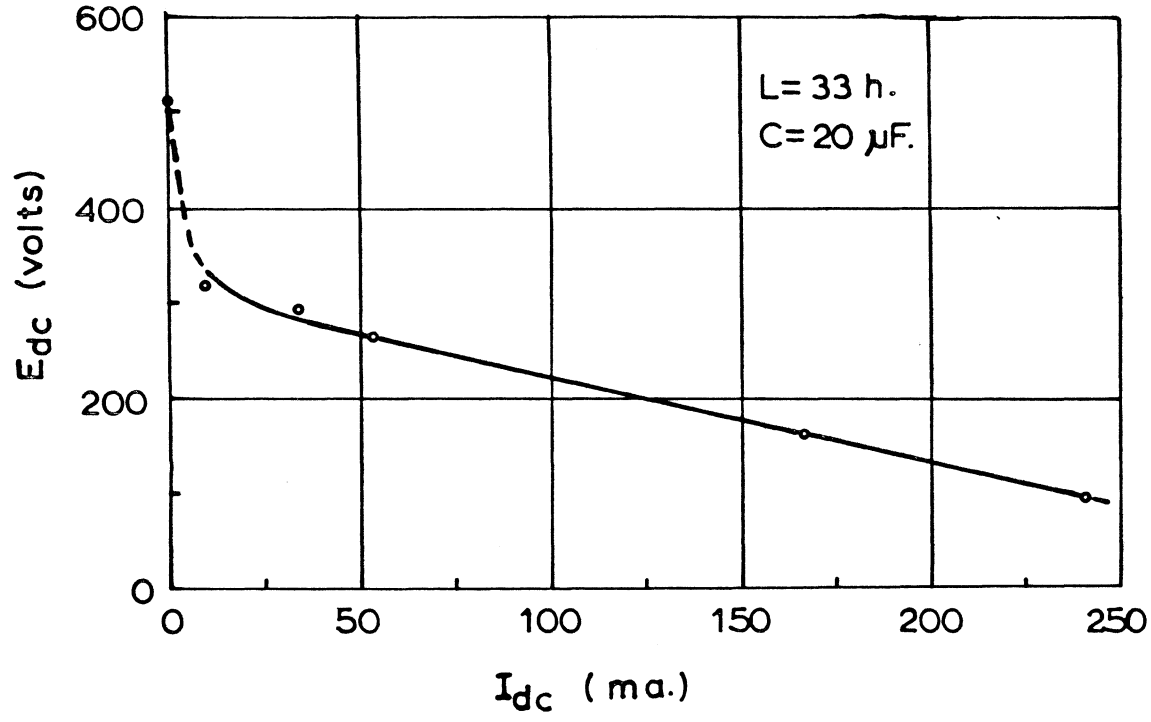


Fig. 6-9. Curve of d-c output voltage versus d-c output current for the rectifier circuit with a choke-input filter.

Table 6-1. Experimental data and results.

No.	R (kΩ)	C (μF)	wRC	i_b max. (ma.)	α (deg.)	β (deg.)	E_{dc} (v)	$\frac{E_{dc}}{E_{sm}}$
1	0.3	1	0.113	400	0	176	32	0.183
2	0.5	1	0.188	270	0	172.5	38	0.217
3	5	0.1	0.138	36	0	168.5	50	0.286
4	8	0.1	0.302	22	0	162	52	0.297
5	0.8	1	0.302	200	0	167	42	0.24
6	10	0.1	0.377	20	0	158.2	52	0.297
7	1	1	0.377	170	0	164.5	44	0.252
8	20	0.1	0.754	13	0	141.5	60	0.343
9*	2	1	0.754	115	0	148	52	0.297
10	40	0.1	1.508	11	2.6	123.8	74	0.425
11	5	1	1.885	80	6.4	128.5	75	0.429
12	60	0.1	2.262	8	6.4	116	86	0.492
13	80	0.1	3.016	8	12.8	107	96	0.55
14	100	0.1	3.77	8	16.7	107	104	0.595
15*	10	1	3.77	75	12.8	115	99	0.565
16	0.7	20	5.276	620	20.5	144	80	0.457
17	1	20	7.54	520	25.7	139	92	0.526
18	30	1	11.31	60	36	105.5	134	0.765
19*	2	20	15.08	340	35.9	134	111	0.635
20	50	1	18.35	50	45.2	102.5	145	0.828

Table 6-1. (continued)

No.	R (kΩ)	C (μF)	wRC	i_b max. (ma)	α (deg.)	β (deg.)	E_{dc} (v)	$\frac{E_{dc}}{E_{sm}}$
21	3	20	22.62	270	39.8	130	121	0.69
22	80	1	30.16	30	52.6	102.5	155	0.885
23	100	1	37.7	25	57.7	102.5	159	0.908
24	6	20	45.24	175	46.2	126	135	0.77
25	0.8	200	60.32	620	31.8	147	90	0.515
26	1	200	75.4	560	34.6	144	10	0.556
27*	10	20	75.4	140	50.1	123.5	142	0.811
28*	2	200	150.8	340	38.5	139	117	0.67
29	30	20	226.2	56	60.5	118	160	0.914
30	50	20	377	40	64.2	115.5	158	0.903
31	5	200	377	215	51	134	142	0.811
32	80	20	603.2	30	66.9	115.5	160	0.914
33	100	20	754	27	69.4	110.5	162	0.925
34	10	200	754	140	54.8	125	141	0.805

* The oscillograms of the output voltage and the tube current are shown in Fig. 6-2.

Table 6-2. Calculated results.

No.	wRC	R_t (Ω)	$\frac{R}{R_t}$	α (deg.)	β (deg.)	$\frac{E_{dc}}{E_{sm}}$
1	0.113	213	1.4	0	176.2	0.186
2	0.188	243	2.03	0	172.8	0.215
3	0.188	450	11.1	0	170.2	0.295
4	0.302	480	16.7	0	164.1	0.307
5	0.302	266	3	0	167.5	0.243
6	0.377	500	20	0	160.3	0.313
7	0.377	288	3.48	0	164.1	0.255
8	0.754	550	36.4	0.2	144.1	0.348
9	0.754	320	6.25	0.2	148.9	0.307
10	1.508	570	70	3	124.7	0.428
11	1.885	360	13.9	5.1	125.1	0.437
12	2.262	600	100	7.4	115.1	0.5
13	3.016	600	133	11.8	109.6	0.558
14	3.77	600	167	15.9	106.1	0.604
15	3.77	367	27.2	15.8	112.4	0.583
16	5.278	180	3.9	16.3	142.2	0.442
17	7.54	200	5	21.4	140.3	0.498
18	11.31	390	77	36.4	105.1	0.775
19	15.08	225	8.9	31.7	135	0.614

Table 6-2 (continued)

No.	wRC	R_t (Ω)	$\frac{R}{R_t}$	α (deg.)	β (deg.)	$\frac{E_{dc}}{E_{sm}}$
20	18.85	420	119	45.5	101.9	0.837
21	22.6	243	12.3	37.2	132.1	0.672
22	30.16	455	176	52.3	115.2	0.86
23	37.7	470	217	58.1	102.1	0.918
24	45.24	280	21	45.5	127.3	0.754
25	60.32	180	4.5	28.1	149.7	0.49
26	75.4	186	5.4	30.6	147.3	0.527
27	75.4	300	33.2	51.5	123.2	0.81
28	150.8	225	8.9	37.3	141.4	0.618
29	226.2	415	72	60.23	117.2	0.913
30	377	438	114	64.3	113.8	0.908
31	377	250	20	47.1	132	0.741
32	603.2	455	176	67.7	111	0.929
33	754	462	217	79.1	109.8	0.937
34	754	300	33.3	52.3	127	0.808

Table 6-3 Calculated results.

No.	wRC	R_t (Ω)	$\frac{R}{R_t}$	α (deg.)	β (deg.)	$\frac{E_{dc}}{E_{sm}}$
1	0.113	133	2.2	0	175.6	0.217
2	0.188	157	3.2	0	171.8	0.244
3	0.188	325	15.4	0	169.9	0.301
4	0.302	420	19	0	164.0	0.309
5	0.302	180	4.5	0	166.4	0.265
6	0.377	400	25	0	160.2	0.316
7	0.377	194	5.2	0	162.7	0.275
8	0.754	480	41.7	0.2	143.9	0.349
9	0.754	225	8.9	0.2	147.3	0.321
10	1.508	510	78.5	3	124.6	0.428
11	1.885	328	21	5.1	122.8	0.448
12	2.262	525	114	7.4	114.9	0.5
13	3.016	525	153	11.8	109.4	0.559
14	3.77	525	190	15.9	105.9	0.605
15	3.77	240	41.7	15.9	109.9	0.592
16	5.278	120	5.8	18.4	134.8	0.505
17	7.54	127	7.9	24.3	132.1	0.574
18	11.31	280	107	37.1	101.3	0.79
19	15.08	150	13.3	35	128.4	0.68

Table 6-3. (continued)

No.	wRC	R_t (Ω)	$\frac{R}{R_t}$	α (deg.)	β (deg.)	$\frac{E_{dc}}{E_{sm}}$
20	18.85	300	167	46.1	99.5	0.851
21	22.62	157	19.1	41.1	125.5	0.734
22	30.16	340	235	53.1	95.8	0.905
23	37.7	410	244	58.3	98.2	0.919
24	45.24	190	31.6	49.3	122	0.802
25	60.32	120	6.7	33.2	144	0.569
26	75.4	125	8	35.6	141.8	0.602
27	75.4	213	47	64.5	119.1	0.855
28	150.8	150	13.3	42.2	135.9	0.719
29	226.2	290	103.5	63.1	113.9	0.903
30	377	320	156	66.5	111.3	0.970
31	377	185	27	50.6	128.3	0.781
32	603.2	340	236	69.5	108.9	0.941
33	754	405	247	69.8	108.8	0.942
34	754	213	47	56.1	123.2	0.836

Table 6-4. Calculated results.

No.	wRC	R_t (Ω)	$\frac{R}{R_t}$	α (deg.)	β (deg.)	$\frac{E_{dc}}{E_{sm}}$
1	0.113	175	1.7	0	175.9	0.201
2	0.188	200	2.5	0	172.4	0.229
3	0.188	400	12.5	0	170.1	0.297
4	0.302	440	18.2	0	164	0.308
5	0.302	220	3.7	0	166.9	0.255
6	0.377	440	22.7	0	160.2	0.315
7	0.377	233	4.3	0	163.3	0.266
8	0.754	520	38.3	0.2	144	0.348
9	0.754	266	7.6	0.2	148	0.315
10	1.508	535	75	3	124.6	0.428
11	1.885	300	16.7	5.1	124	0.443
12	2.262	560	107	7.4	115	0.5
13	3.016	560	143	11.8	109.5	0.559
14	3.77	560	179	15.9	106	0.604
15	3.77	325	30.8	15.8	111.5	0.586
16	5.278	156	4.5	17.2	139.6	0.466
17	7.54	160	6.25	22.9	136.3	0.536
18	11.31	368	81.5	36.5	104.6	0.78
19	15.08	194	10.3	32.9	132.6	0.638

Table 6-4. (continued)

No.	wRC	R_t (Ω)	$\frac{R}{R_t}$	α (deg.)	β (deg.)	$\frac{E_{dc}}{E_{sm}}$
20	18.85	400	125	45.7	102.1	0.842
21	22.62	200	15	39.1	129	0.702
22	30.16	420	190	52.9	101.3	0.895
23	37.7	450	222	58.1	102.3	0.92
24	45.24	250	24	46.9	125.5	0.771
25	60.32	156	5.2	29.8	147.6	0.532
26	75.4	150	6.7	33.3	144.4	0.568
27	75.4	266	37.6	52.6	121.7	0.822
28	150.8	188	10.6	39.4	138.4	0.682
29	226.2	468	81.5	61.2	116	0.922
30	377	433	116	64.5	113.7	0.909
31	377	288	22	48.2	130.8	0.831
32	603.2	420	191	68.2	110.4	0.933
33	754	445	225	69.2	109.5	0.939
34	754	266	37.6	53.7	125.7	0.812

Table 6-5. Calculated results.

No.	wRC	α (deg.)	β (deg.)	$\frac{E_{dc}}{E_{sm}}$
1	0.113	0	173.5	0.318
2	0.188	0	169.5	0.318
3	0.188	0	169.5	0.318
4	0.302	0	163.2	0.318
5	0.302	0	163.2	0.318
6	0.377	0	159.6	0.32
7	0.377	0	159.6	0.32
8	0.754	0	143	0.34
9	0.754	0	143	0.34
10	1.508	3.8	123.5	0.425
11	1.885	6	118	0.465
12	2.262	8	114.9	0.49
13	3.016	15.2	105.8	0.585
14	3.77	16.5	104.8	0.6
15	3.77	16.5	104.8	0.6
16	5.278	21.8	101.4	0.665
17	7.54	31	97.6	0.73
18	11.31	40	95.1	0.8
19	15.08	45	93.8	0.83

Table 6-5. (continued)

No.	wRC	α (deg.)	β (deg.)	$\frac{E_{dc}}{E_{sm}}$
20	18.85	49	93.1	0.86
21	22.62	52	92.5	0.885
22	30.16	55	91.9	0.9
23	37.7	60	91.5	0.93
24	45.24	62	91.25	0.94
25	60.32	65	90.95	0.95
26	75.4	68	90.75	0.96
27	75.4	68	90.75	0.96
28	150.8	80	90.39	0.97
29	226.2	86	90.25	0.98
30	377	87	90.15	0.99
31	377	87	90.15	0.99
32	603.2	87	90.1	0.999
33	754	89	90.1	0.999
34	754	89	90.1	0.999

Table 6-6. Experimental data and results.

No.	L (h)	R_L (Ω)	R (kΩ)	i_{bmax} (ma.)	i_{bmin} (ma.)	E_{dc} (v)
1	8	250	100	5.2	0	325
2	8	250	50	10	0	315
3*	8	250	10	42	17	297
4	8	250	5	72	42	290
5*	8	250	0.4	400	285	130
6	8	250	0.2	500	275	80
7	15	200	100	5.2	0	320
8	15	200	50	10.2	0	319
9	15	200	10	51.5	7	300
10	15	200	5	72	36	282
11	15	200	30	16.5	5.3	315
12	15	200	0.4	500	125	135
13	15	200	0.2	630	150	90
14	33	650	100	5	0	325
15	33	650	50	9.5	0	310
16	33	650	10	34	23	290
17	33	650	5	60	43	260
18	33	650	0.5	250	210	92
19	33	650	0.2	275	245	52

* The oscillograms of the output voltage are shown in Fig. 6-5.

Table 6-7. Calculated results.

No.	L (h)	R (kΩ)	R_o (Ω)	R_t (Ω)	E_{dc} (v)
1	8	100	1,000	1,400	321
2	8	50	580	980	318
3	8	10	342	742	302
4	8	5	300	700	286
5	8	0.4	155	555	135
6	8	0.2	147	547	86
7	15	100	1,000	1,350	321
8	15	50	580	930	319
9	15	10	450	800	301
10	15	5	270	620	289
11	15	30	465	815	317
12	15	0.4	157	507	142
13	15	0.2	150	500	100
14	33	100	1,000	1,800	320
15	33	50	590	1,390	311
16	33	10	330	1,130	288
17	33	5	320	1,120	258
18	33	0.4	167	967	92.5
19	33	0.2	163	963	55.5

Table 6-8. Experimental results.

No.	L (h)	R_L (Ω)	C (μF)	R (kΩ)	i_{bmax} (ma)	i_{bmin} (ma)	E_{dc} (v)
1	8	250	200	8	65	28	288
2	8	250	200	5	85	45	278
3	8	250	20	5	80	40	280
4	8	250	200	1	250	170	200
5	8	250	20	1	250	170	197
6	8	250	200	0.4	440	260	122
7*	8	250	20	0.4	420	250	120
8	8	250	200	0.2	520	310	75
9	8	250	20	0.2	520	310	75
10*	33	650	20	35	14	4	320
11	33	650	200	10	46	34	290
12	33	650	20	10	38	26	296
13	33	650	200	5	66	54	270
14	33	650	20	5	64	52	267
15	33	650	200	1	162	158	164
16	33	650	20	1	160	160	166
17	33	650	200	0.4	265	235	90
18	33	650	20	0.4	265	235	90

* The oscillograms of the output voltage and the tube current are shown in Fig. 6-7.

Table 6-9. Calculated results.

No.	L (h)	C (μF)	R (kΩ)	R_o (Ω)	R_t (Ω)	E_{dc} (V)
1	8	200	8	320	720	298
2	8	200	5	285	685	285
3	8	20	5	280	680	285
4	8	200	1	180	580	206
5	8	20	1	180	580	206
6	8	200	0.4	200	600	130
7	8	20	0.4	200	600	130
8	8	200	0.2	150	550	85
9	8	20	0.2	150	550	85
10	33	20	35	500	1,300	312
11	33	200	10	270	1,070	294
12	33	20	10	280	1,080	293
13	33	200	5	300	1,100	266.5
14	33	20	5	310	1,110	266
15	33	200	1	200	1,000	163
16	33	20	1	200	1,000	163
17	33	200	0.4	180	980	94
18	33	20	0.4	180	980	94

VII. DISCUSSION

Examination of Figures 3-3 through 3-6 and 3-8 through 3-9 shows that the ignition angle α, the extinction angle β, the conduction angle γ and the ratio of d-c output voltage to peak value of the source voltage developed by the high-vacuum diode rectifier operating with a shunt-capacitor filter system not only depend upon the load resistance R and the shunt capacitor C, but also depend upon the equivalent resistance of the tube and the transformer, R_t. Figures 3-5, 3-6, 3-10 and 3-11 indicate that when all the parameters, except the load resistance, for the rectifier circuit with a shunt-capacitor filter are held constant, an increase in the load resistance, R, increases the d-c output voltage and decreases the conduction period of the tube, i. e., the conduction angle, thereby reducing the output ripple voltage and smoothing the wave. Similarly, the change in waveform and direct output voltage are also affected by an increase in the capacitance C; it is found that the higher the capacitance C the smaller the output ripple voltage and the greater the direct output voltage. The effects of the R_t on the behavior of the rectifier circuits are also illustrated by the calculated curves. It is observed that when E_{sm} and wRC are held constant, the conduction angle γ and the direct output voltage decrease as the value of R_t increases. When the value of R_t is too large to make the ratio of the load resistance to the resistance of the tube and

transformer approach infinity, i. e., the ideal rectifier case, the ratio of the d-c output voltage to the peak source voltage has a value less than unity and conduction angle γ does not become zero, even though the value of wRC would have been made infinite. It is apparent that the higher the tube and transformer resistance, the greater the output ripple voltage. Therefore, when a high output voltage with good regulation is desired, the tube and transformer resistance should be made as low as possible[14]. On the other hand, it is a fact that the higher the source resistance, the smaller the peak tube current, so a high source resistance is desirable when it is important that the peak current be kept down[14]. However, the smoothing of the output voltage would be somewhat sacrificed.

Examination of Eq. (4-12) shows that an increase in inductance L or load resistance R decreases the amplitudes of the a-c components of the output current; therefore the ripple voltage is small. The tube and transformer resistance also affect the behavior of the rectifier circuit using a choke-input filter or a series inductor for smoothing. The presence of R_t in Eq. (4-11) and (5-3) indicates that the direct output voltage varies as the ratio of R_t to R varies. When the load resistance is decreases, the ratio of R_t to R becomes appreciably large; therefore the direct output voltage is decreased. Since a decrease in the load resistance gives a high value of the direct output current, the direct output voltage decreases as the direct output current increases. This phenomenon has been proved by the experimental curve shown in Fig. 6-9. The portion of curve

plotted by solid line is obtained experimentally and that shown by dotted line was plotted in accordance with the result of Reference 5.

So far, it could be concluded that the voltage drop across the tube and source transformer is one of the important characteristics of a high-vacuum thermionic diode rectifier circuit. Neglecting this factor can introduce an appreciable error.

After the experiments were made for checking the correctness of the equations derived in this paper, the comparisons made between the calculated results and experimental results showed good agreements if the tube resistance is chosen properly. Also, the calculated curves of the ratio of the d-c output voltage to the peak value of the source voltage for both half- and full-wave rectifier circuit with the shunt capacitor filter type as given in Figures 3-6 and 3-11 respectively compare very well with the experimentally obtained curves as given by Roberts[9] and Schade[10]. Also, curves in Figures 3-3, 3-4, 3-8 and 3-9 have been checked closely with those curves presented in Waidelich's[15] paper. These calculated curves permit the solution of the inverse problem to determine the magnitude of the source voltage necessary to give a certain average voltage output for a given load.

Three different types of filter circuit have been considered in this study. Each of them has relative advantages and disadvantages. With the capacitor-input circuit, the direct voltage across the load may reach a value near the peak value of the source voltage, while with the series inductor circuit, it is only approximately

63 per cent of this value[8]. In addition, the tube current is almost smooth in the series inductor circuit, while it has the form of large pulses of current of short duration in the capacitor filter circuit[8]. To avoid the rectifier device being damaged by a high peak value of the current, a high-vacuum thermionic tube is desirable to use in this circuit, since the high-vacuum tube will limit the peak current to a reasonable value. Otherwise series resistance must be added. Compared with the series inductor filter, the choke-input filter circuit has a smaller ripple in the output voltage if the shunt capacitor is large, since a large capacitor may by-pass the harmonic components of the load current.

VIII. SUMMARY AND CONCLUSIONS

The single-phase rectifier filter circuits, including the half- and full-wave rectifiers with a capacitor and full-wave rectification with either a series inductor or a choke-input filter, have been analyzed taking into account the voltage drop across the tube and the transformer. In these analyses the high-vacuum thermionic diode was considered. The voltage-current characteristic curve of a diode was assumed linear, and the equivalent resistance of the tube was chosen equal to the reciprocal of the slope of the diode characteristic curve in the conducting region.

For the capacitive filtering case, the equations expressing the ignition angle, the extinction angle, and the ratio of the d-c output voltage to the peak value of the applied voltage when the steady state condition is reached have been derived. It was found that these equations not only depend upon the product of the angular frequency of the applied voltage and the time constant of load circuit, but also depend on the ratio of the load resistance to the equivalent resistance of the tube and the source transformer. Most of these equations appear to be of a transcendental form, the solution of which requires either graphical or trial-and-error method. Both of these require tedious work and are time consuming with hand computations. However it is easily accomplished today by the use of an IBM 7040 computer. The calculated curves showing the variation of the ignition angle, the extinction angle, and the

ratio of the d-c output voltage to the peak value of the supply-voltage for half- and full-wave rectifications were plotted with wRC as abcissa for several different values of the ratio of the load resistance to the equivalent resistance of tube and transformer.

The equations derived for the direct output voltage of the rectifier circuit with either the series inductor or the choke-input filters show that they depend only on the ratio of the load resistance to the total resistance of the tube, the transformer, and the inductor.

All the equations mentioned above had been experimentally verified. It is found that the calculated results checked closely with the experimental data if the equivalent resistances were properly chosen.

The effect of circuit parameters on the behavior of rectifier circuits operated with three types of filters have been discussed. It becomes evident that the tube and the transformer resistance of practical circuits appears as an additional parameter which cannot be neglected in the analysis of the single-phase rectifier. However, inclusion of this resistance greatly complicates the analysis.

The method of analysis presented in this paper may be extended to other types of rectifier circuits. Although the analysis made in this paper did not take into account all of the possible factors which may be involved in the practical circuits, nevertheless the results were presented in convenient form for practical use.

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XII. APPENDICES

Appendix A

The following is the results of the analysis of the rectifier circuit with capacitive filtering, where the voltage drop across the tube and the transformer are neglected. The equations for the angles α and β and the d-c output voltage that are outlined below are copied from the MIT EE Staff book[5]. They are given by

$$\beta = \tan^{-1}(-wRC) \tag{1}$$

$$\sin\alpha = \sin(\tan^{-1} wRC)\cdot\epsilon^{-\frac{2\pi+\alpha-\tan^{-1}(-wRC)}{wRC}} \tag{2}$$

$$E_{dc} = \frac{E_{sm}}{2\pi}\sqrt{1+(wRC)^2}\cdot(1-\cos\gamma) \tag{3}$$

where

$$\gamma = \beta - \alpha$$

Appendix B

Fortran Program For Analyzing Half Wave Rectifier With A Smoothing Capacitor

This Fortran program is applied for obtaining the ignition angle α, the extinction angle β, and the ratio of the d-c output voltage E_{dc} to the peak value of the source voltage E_{sm}. Since both the equations for angles α and β appear to be of the transcendental form, to solve these equations the trial-and-error method is required. This method may be performed by applying Newton's formula, which is given by

$$X_{K+1} = X_K - \frac{f(X_K)}{f'(X_K)}$$

where $f(X_k)$ represents the transcendental equation to be solved, and X_k is an estimate of the root of the equation. When the transcendental equation is solved, one may guess a root, X_k, and substitute it into the above formula to obtain the new root X_{k+1}. If X_{k+1} does not equal to X_k or if the difference between them is larger than a small percentage of value that is expected, let X_k equal to X_{k+1} and repreat the calculation until an equality between X_k and X_{k+1} is obtained.

The Fortran program of our problem is written in such a manner that first, an arbitrary value of α is guessed (it may be chosen equal to zero), and substituted it into Eq. (3-25).

Newton's method is then applied for calculating the angle β . After the angle β is obtained, then it is substituted into Eq. (3-22) to obtain the angle α of the next cycle. If the new value of α does not equal the old one or the difference between them is larger than a small percentage of the value expected, substitute this new value of α into Eq. (3-25) again to calculate another new value of β . This iterative calculation is applied repeatedly until the value of α equals or differs from the old one by a small amount. When the angle α remains constant during successive cycles, the steady state condition is reached. The ratio of E_{dc} to E_{sm} can then be determined by Eq. (3-27). The constants read in this program include R, R_t, C and w.

```
C       PROGRAM FOR HALF WAVE RC CIRCUIT
        READ (5,1) R,RT,C,W
      1 FORMAT (4F10.0)
        PI=6.28318
        ALPHA1=0.
        ALPHA=0.
        BETA=3.
        A=R/RT
        B=W*R*C
        T=ATAN(B/(1.+A))
```

```
      S=ATAN(B)
      D=1./SQRT(B*B+(1.+A)*(1.+A))
      TS=SIN(T)/SIN(S)
C     DETERMINATION OF BETA
    2 Q=SIN(ALPHA1)-D*A*SIN(ALPHA1-S)
    5 Q1=Q*EXP(-(BETA-ALPHA1)*COS(T)/SIN(T))
      H=(TS*SIN(BETA-T+S)-Q1)/(TS*COS(BETA-T+S)
     1  +Q1*COS(T)/SIN(T))
      IF (ABS(H)-0.002) 3,3,4
    4 BETA=BETA-H
      GO TO 5
C     DETERMINATION OF ALPHA
    3 U=-(PI+ALPHA-ALPHA1+A*(BETA-ALPHA1))
      X=D*A*SIN(BETA-T)*EXP(U)+Q*EXP(V)
      G=(SIN(ALPHA)-X)/(COS(ALPHA+X/B)
      IF (ABS(G)-0.002) 6,6,7
    7 ALPHA=ALPHA-G
      GO TO 3
    6 E=ALPHA-ALPHA1
      IF (ABS(E)-0.002) 8,8,9
    9 ALPHA1=ALPHA
      GO TO 2
```

```
C       CALCULATION OF EDC/ESM
      8 EDCESM=(COS(ALPHA+S)-COS(BETA+S))/(PI*COS(S)*(1.+A))
        WRITE (6,10) ALPHA,BETA,EDCESM
     10 FORMAT (1H,7H ALPHA=,F10.5,5X,6H BETA=,F10.5,5X,
     1       9H EDC/ESM=,F10.5)
        STOP
        END
```

ABSTRACT

The purpose of this study is to analyze single-phase rectifier circuits taking into account the tube drop and the resistive loss of the source transformer. Included are the analyses of rectifier circuits using a shunt capacitor, a series inductor, and a choke-input filter for smoothing. In these analyses a high-vacuum thermionic diode is considered and a linear characteristic of a diode is assumed. The equations for the direct output current and voltage of each type of circuit are derived. Also, for the capacitor filtering case, the results of the analysis in the form of curves showing the angles at which the tube begins to conduct and ceases to conduct and the ratio of the direct output voltage to the peak value of the source voltage as functions of the product of the angular frequency of the source and the time constant of the circuit for various values of the load to tube and transformer resistance are given. Experiments are performed for checking the correctness of the equations derived.

It is found that the tube and the transformer resistance of practical circuits appears as an additional parameter which cannot be neglected in the analysis of the single-phase rectifier.