Phase Approximation Model — Approximation Amazingly Accurate

the anisotropy weight be related to the photon absorbing and scattering in the medium, and the optical properties of medium are characterized by the anisotropy weight f along with the absorption and scattering coefficients.

Substitution of Eq. (2) into Eq. (1) yields

$$\mathbf{v}\cdot\nabla L(\mathbf{r},\mathbf{v})+\tilde{\mu}_t L(\mathbf{r},\mathbf{v})=\frac{1}{4\pi}[\tilde{\mu}_s\Phi(\mathbf{r})+S(\mathbf{r})],\quad \mathbf{r}\in\Omega, \tag{3}$$

where $\tilde{\mu}_s=(1-f)\mu_s$, $\tilde{\mu}_t=\tilde{\mu}_s+\mu_a$, and the photon fluence rate $\Phi(\mathbf{r})$ is defined by

$$\Phi(\mathbf{r})=\oint_{S^2}L(\mathbf{r},\mathbf{v})d\mathbf{v}. \tag{4}$$

Equation (3) is a linear, first-order differential equation describing the photon propagation through a heterogeneous medium, and the solution $L(\mathbf{r},\mathbf{v})$ can be formulated as [14]

$$L(\mathbf{r},\mathbf{v})=\frac{1}{4\pi}\int_0^R[\tilde{\mu}_s(\mathbf{r}-\rho\mathbf{v})\Phi(\mathbf{r}-\rho\mathbf{v})+S(\mathbf{r}-\rho\mathbf{v})]\times\exp\left(-\int_0^\rho\tilde{\mu}_t(\mathbf{r}-t\mathbf{v})dt\right)d\rho+L(\mathbf{r}-R\mathbf{v},\mathbf{v})\times\exp\left(-\int_0^R\tilde{\mu}_t(\mathbf{r}-t\mathbf{v})dt\right), \tag{5}$$

where R is a scalar so that $\mathbf{r}-R\mathbf{v}\in\partial\Omega$, representing the distance from point $\mathbf{r}$ to the boundary $\partial\Omega$ along the direction $\mathbf{v}$. Integration of Eq. (5) over all the solid angles results in the following integral formula:

$$\Phi(\mathbf{r})=\frac{1}{4\pi}\oint_{S^2}\int_0^R[\tilde{\mu}_s(\mathbf{r}-\rho\mathbf{v})\Phi(\mathbf{r}-\rho\mathbf{v})+S(\mathbf{r}-\rho\mathbf{v})]$$

Cartesian coordinates $\mathbf{r}'$, an integral equation with respect to photon fluence rate is obtained,

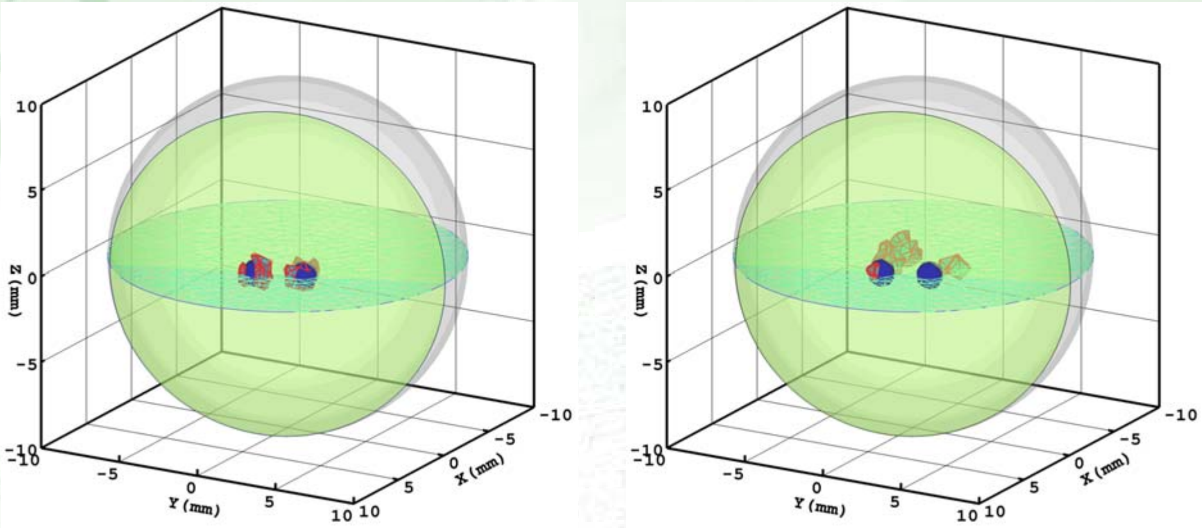
$$\Phi(\mathbf{r})=\frac{1}{4\pi}\int_\Omega[\tilde{\mu}_s(\mathbf{r}')\Phi(\mathbf{r}')+S(\mathbf{r}')]G(\mathbf{r},\mathbf{r}')d\mathbf{r}'-\frac{r_d}{4\pi(1+r_d)}\oint_{\partial\Omega}\Phi(\mathbf{r}')G(\mathbf{r},\mathbf{r}')\boldsymbol{\beta}\cdot\mathbf{n}d\mathbf{r}', \tag{7}$$

where

$$G(\mathbf{r},\mathbf{r}')=\frac{1}{|\mathbf{r}-\mathbf{r}'|^2}\exp\left(-\int_0^{|\mathbf{r}-\mathbf{r}'|}\tilde{\mu}_t(\mathbf{r}-t\boldsymbol{\beta})dt\right),$$

unit vector $\boldsymbol{\beta}=(\mathbf{r}-\mathbf{r}')/|\mathbf{r}-\mathbf{r}'|$, and $\mathbf{n}$ is the outward unit normal at $\mathbf{r}-R\boldsymbol{\beta}$ on the boundary. Equation (7) is a well-posed integral equation of the second kind, and allows an accurate prediction to photon propagation in biological tissues. For simplicity, we call Eq. (7) the phase approximation (PA) equation because it is derived from an approximate phase function. $\tilde{\mu}_s$ and $\tilde{\mu}_t$ (or, equivalently, μ_a, μ_s, and f) in Eq. (7) represent the optical parameters. When the absorption coefficient μ_a and scattering coefficient μ_s in the tissue are unknown, we can directly determine the optical parameters $\tilde{\mu}_s$ and $\tilde{\mu}_t$ using optical tomography techniques based on Eq. (7) [15]. If the absorption coefficient μ_a, scattering coefficient μ_s, and conventional anisotropic factor g in the tissue are known, based on a simple homogenous numerical phantom, such as a spherical or cylindrical object, with a known light source setting, the photon fluence rate on the boundary of the phantom can be generated using Monte Carlo simulation with a appropriate phase function, e.g., the Henyey-Greenstein phase function [16]. Then, a single parameter optimization can be performed to determine the anisotropy weight f by matching the photon fluence rate on the phantom boundary obtained from the phase approximation model to the Monte Carlo simulation results.

The radiative transfer equation (RTE) well describes the photon propagation but its high computational cost is prohibitive for tomographic imaging. The diffusion approximation (DA) to RTE is the most popular but it only works well in weakly absorbing and highly scattering media, and breaks down near sources and across boundaries. ***In 2007, we derived the phase approximation (PA) model from RTE based on the generalized Delta-Eddington phase function.*** *The generalized phase function is a linear combination of isotropic scattering and strongly peaked forward scattering with anisotropy weight as a free coefficient. PA is highly accurate over a broad range of coefficients with a computational complexity comparable to that of DA.*



PA is an enabling model for multi-probe optical molecular tomography that involves a wide spectral range. In the above figure, bioluminescence tomography reconstruction is in a much closer to the truth using PA (Left) than DA (Right).

Papers by Our Team

Cong W, Shen H, Cong A, Wang Y, and Wang G. Modeling photon propagation in biological tissues using generalized Delta-Eddington phase function. Physical Review E 76:051913, 5 pages, 2007

Cong W, Cong A, Shen H, Liu Y, Wang G: Flux vector formulation for photon propagation in the biological tissue. Optics Letters 32:2837-2839, 2007

Cong W, Shen H, Cong A, Wang G: Integral equations of the photon fluence rate and flux based on a generalized Delta-Eddington phase function. Journal of Biomedical Optics, 13:Article ID 024016, 2008