A SOLUTION OF THE TWO PARAMETER GAMMA MODEL TO RELATE UNIT HYDROGRAPH FEATURES TO BASIN CHARACTERISTICS

by

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INTRODUCTION

The prediction of flood peaks has historically been the most important problem addressed by the science of hydrology. Peak discharges must be estimated with reasonable accuracy in both the planning and operation stages of project development. The most popular method of accomplishing this objective among practical hydrologists has been through the use of the unit hydrograph. The unit hydrograph can be viewed simply as a means of converting precipitation excess into discharge hydrographs.

Given a situation where stream discharge records are available, the unit hydrograph of a particular catchment is relatively easy to obtain. However, in cases where no discharge data are available, as is the case in the vast majority of localities where small scale projects are contemplated, the derivation of the unit hydrograph is a complex and frequently subjective procedure. There are several such procedures, the most common one being attributed to the work of Snyder. A unit hydrograph derived without the benefit of historical record is termed a synthetic unit hydrograph. Virtually all synthetic procedures rely upon the success of correlating the salient features of the unit hydrograph, i.e. peak discharge and time to peak, with the physical topographic characteristics of the catchment. This necessary correlation has thus far remained elusive. Many investigators have found large degrees of error in all of the synthetic methods presently in use.

In the past few years interest in the unit hydrograph among researchers has waned due to its inherent limitations and to the increasing popularity of other catchment modeling techniques. The unit hydrograph has been relegated to the status of an outmoded relic by many researchers who view empirical approaches with disdain. However, due to its simplicity and its generally reasonable accuracy, the unit hydrograph has continued to be the primary tool of runoff prediction used by practical hydrologists. Thus a dilemma is created. Further research is needed to refine the accuracy of the synthetic unit graph method, but research has been neglected in the recent past.

The objective of this research project was to correlate the constants of the two-parameter gamma hydrograph model proposed by Edson (5) to the pertinent characteristics of the watershed. It is basically a variation of a method outlined by Gray in reference (4) of this report. It is believed that the approach presented in this paper, being much less complex and requiring far less specialized knowledge than the above referenced procedure will be of more use to practical hydrologists who need the methodology but lack the specialized knowledge for its application.

THEORETICAL CONSIDERATIONS

The concept of the unit hydrograph is generally credited to Sherman (23). As proposed by him, the "unit-graph" represented the time-discharge relationship of direct run-off resulting from one inch of effective rainfall distributed uniformly over the basin area at a uniform rate in a specified time period. The specified time period is known as the duration of effective precipitation (t_R). Several limiting assumption are associated with the unit hydrograph theory, the two most important of which are as follows: 1) There is a direct proportionality between the ordinates of the direct-runoff hydrograph and the total amount of runoff constituting that hydrograph and 2) The hydrograph of a given catchment under a specified duration of rainfall is a result of the influence of all the combined physical characteristics of the basin (26). These two principles are known as the principles of superposition and time invariance respectively. Thus, by the first, the ordinates of a hydrograph containing two inches of runoff in a specified time interval will simply be twice the ordinates of the unit hydrograph of the same duration of rain. It also follows from this theory, known as linear hydrograph theory, that the ordinates of unit hydrographs are mutually proportional and therefore can be added or superimposed in proportion to the volume of direct runoff. By the second assumption the unit hydrograph resulting from a given pattern of effective precipitaiton is considered invariable. This reflects the assumption that the physical characteristics of the basin remain constant.

Both of the above assumptions are known to be incorrect at least to a degree. The relationship between effective rainfall or runoff and discharge is not strictly linear, but can be assumed so in many cases with reasonable accuracy. However, care should be taken when applying the unit graph principle to ascertain to the engineer's satisfaction that this assumption holds reasonably well in his case.

The second limiting assumption can never be satisfied. Many natural forces are acting continuously to change the physical character of the catchment. Progressive stream meandering certainly modifies the stream length and slope over the course of a few years. Many investigators have determined the effect of man-made changes in the basin on the resulting unit hydrograph. Eagleson (10), Espey, et. al. (11), Bras (15), and Rao, et. al. (21) all found catchment response drastically altered by the effects of urbanization. The increase in impervious cover, as well as the channel improvements which accompany urban development make the basin much more efficient in discharging excess precipitation received during storm periods. Decreases in time to peak of 40 percent and increases in peak discharge of up to 200 percent were commonly encountered (21).

Carter (18) and Martens (27) and later Andersen (25) and Putnam (24) have made the most extensive studies of urban effects on basin response to date. These studies confirm the results of earlier investigators as to the drastic and sometimes disastrous increases in flood discharge due to increased urban development in the particular catchment.

Other factors, particularly storm characteristics, critically affect the unit hydrograph resulting from a given unit duration of rainfall. Storm characteristics include duration of rainfall and total volume of runoff produced by the storm (28). The limitations implied in the unit-graph definition, i.e., uniform intensity and uniform areal distribution of rainfall, should be adhered to as strictly as possible as criteria for selection of storms from which to derive unit hydrographs. The necessity for maintaining these two criteria in unit graph development has the effect of limiting the size of basins to which the method can be applied. Chow (26) gives a lower limit on drainage area of 4 acres while Linsley, Kohler, and Paulus (28) give an upper limit of approximately 2000 $mi.^2$

Fig. 1 shows the derivation of a typical unit hydrograph for a relatively small drainage basin. The basin lag (t_p in fig. 1) as used herein is defined as the time interval between the centroid of the precipitation excess and the peak of the runoff hydrograph. Unfortunately, lag can also be defined in several other ways. The U.S. Geological Survey defines lag as the time interval between the centroid of precipitation excess and the centroid of the runoff hydrograph. Lag is also sometimes defined as the interval from the time of beginning of runoff-producing rainfall to the centroid of the resulting hydrograph. For the purposes of this thesis, dealing with storms of short duration over small drainage basins (less than 50 mi^2), the differences between these various definitions is considered insignificant.

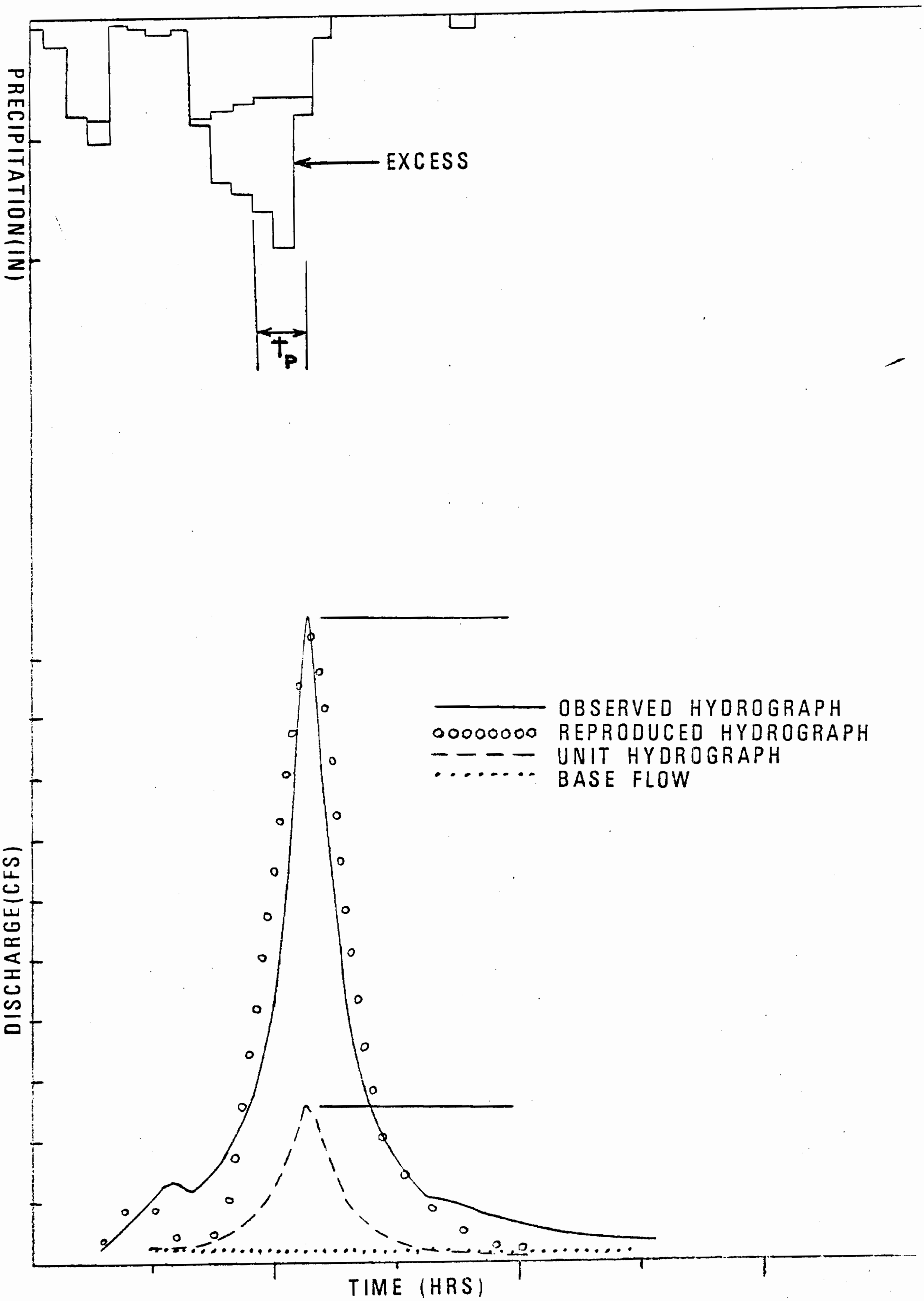

FIG. 1: DERIVATION OF A TYPICAL UNIT HYDROGRAPH (34)

The derivation of unit hydrographs for basins where discharge data was insufficient or non-existent received early attention from McCarthy (29) and Snyder (1). McCarthy attempted to correlate three hydrograph characteristics (peak, lag, and total time base) with three basin parameters (area, average basin slope, and stream density or number of major streams in the basin). He furnished correlation curves from which to estimate the three unit-graph parameters given the three basin characteristics.

The most popular synthetic unit graph method in use today was proposed by Snyder in 1938. Snyder was able to derive an expression for the basin lag as a function of drainage basin characteristics through the use of two parameters assumed to remain constant for the basin. The peak discharge and the effective unit durations are then expressed as a function of the lag. Snyder's equation for the lag is:

$$t_p = c_t(L\ L_{ca})^{0.3} \qquad (1)$$

where: t_p = lag time from the centroid of precipitation excess to the peak of the unit hydrograph.

L = Length of the main stream course from the divide to the station.

L_{ca} = Stream length from the station to the center of gravity of the drainage area.

c_t = Coefficient for lag depending on units and basin characteristics.

Snyder found a variation in C_t from 1.8 to 2.2 with an average value of 2.0 for the Appalachian Highlands in which this study was conducted.

With the lag calculated, Snyder then expressed the peak discharge of the unit graph as:

$$q_p = c_p \frac{640}{t_p} \qquad (2)$$

where: q_p = Peak discharge of unit graph in cfs per sq. mile

c_p = coefficient for peak depending on units and drainage basin characteristics.

Snyder found a variation in C_p from .56 to .69 with an average value of 0.63 for his areas. In his study, Snyder adopted the expression: $t_r = \frac{t_p}{5.5}$ for the standard unit duration of effective rainfall.

Other investigators have found similar relationships in attempting to derive expression for calculating unit hydrograph features from known basin characteristics. Gray (3) proposed the elimination of the term L_{ca} in the expression for lag due to its high degree of correlation with the L term used in the same expression. Taylor and Schwarz (2) introduced the use of the square root of the average channel slope in the expression for lag:

$$c' = .6/\sqrt{s} \qquad (3)$$

where: c' = lag of unit graph

s = slope of a uniform channel having the same length as the longest watercourse and an equal travel time However, Taylor and Schwarz also found the term (LL_{ca}) to be significant in the calculation of an expression for the rate of change of lag with storm duration. Since then, several investigators have proposed expressions for the lag in terms of ($\frac{L}{\sqrt{S}}$) (7,4,18). This term would appear to give an accurate account for the time of travel factor which is of paramount importance in the development of basin lag times.

Eagleson (10), working with urbanized and partly sewered basins of relatively small size, derived the following expression for the basin lag of catchments of a fairly high degree of urbanization:

$$t_p = \frac{\bar{L}}{60V} \tag{4}$$

where: t_p = basin lag

$\bar{L}$ = mean travel distance = area under the area-distance curve divided by the total basin area

and V is calculated from Manning's equation

$$V = \frac{1.5}{n} R^{2/3} S^{1/2}$$

where: V = velocity of fluid flow in storm sewer

n = Manning's roughness coefficient

R = hydraulic radius of channel

S = average slope of channel

Several investigators have proposed expressions for peak discharges without use of the basin lag. Potter (7) proposed a logarithmic correlation between peaks of a given recurrence interval and certain watershed characteristics.

$$\log q = .490 - .299 \log A \qquad (5)$$

where: q = peak of 10 year recurrence interval (cfs/ac)

A = basin area

Later, by adding in a term consisting of a function of $(\frac{L}{\sqrt{S}})$, Potter was able to significantly increase the reliability of his proposed expression.

The peak of the unit hydrograph was correlated to the basin characteristics by Getly and McHughs (8).

$$q_p = \frac{110,860}{A^{.45}(\frac{LL_{ca}}{\sqrt{S}})^{.32}} \qquad (6)$$

where: q_p = peak of the unit hydrograph in (cfs/mi^2) and the other terms are as previously defined.

Rao and Delleur (2) proposed an expression for the peak flow of direct runoff in terms of physiographic basin characteristics and storm parameters:

$$Q_p = C_o A^{C_1} (1+U)^{C_2} P_E^{C_3} T_R^{C_4} \qquad (7)$$

where: Q_p = direct runoff peak in cfs

U = fraction of impervious area

P_E = magnitude of effective precipitation (inches)

T_R = duration of effective precipitation (hours)

C_0, C_1, C_2, C_3, C_4 regression constants

At this point it is necessary to begin a discussion of the problem of duration of effective precipitation. In all of the previously discussed methods, it was necessary to adopt a standard unit duration of effective rainfall. This is the period of time during which runoff is actually occurring. The adopted standard duration depends on basin and storm characteristics. As previously stated, Snyder in his classic paper, used the basin lag time divided by 5.5 as the unit duration and his example has been followed by many subsequent investigators. The relationship between the time of concentration and the unit duration is generally recognized. The Corps of Engineers (13) recommends the use of a standard unit duration of about one half the basin log time for basins of less than 100 mi^2 in size.

However, when unit hydrographs are calcualted from discharge records, the duration of runoff-producing rainfall naturally depends on the storms used in the calculation. Thus it is necessary to convert all the calculated unit graphs to a common unit duration. This is generally accomplished by means of the so called S-curve. This procedure utilizes the first assumption of unit hydrograph theory, i.e. that the ordinates of a unit hydrograph can be numerically superimposed and added together. Thus, to convert a one hour unit hydrograph to a two hour graph it is only necessary to add the ordinates of two one-hour unit graphs lagged one hour and divide each ordinate by two.

The need for shorter durations of effective rainfall, as well as more accurate determinations of the relationship between runoff and basin characteristics led to the development of the instantaneous unit hydrograph (IUH), in which the precipitation excess is distributed in a zero time interval. The concept of the IUH was introduced by Clark (22) in 1945. Clark derived the IUH by routing the time-area concentration curve as inflow through a linear reservoir by the Muskingum method with a x value of zero. The time-area concentration curve is derived by dividing the basin into zones of equal time of travel. The area of each zone is then measured and plotted against its time of travel. Fig. 2 shows the derivation of a typical time-area curve. With this curve as inflow, the outflow hydrograph would be the result of an instantaneous effective rainfall and is therefore an instantaneous unit hydrograph. This hydrograph can be converted into a unit hydrograph of any desired unit duration by merely averaging the proper number of ordinates.

In order for the instantaneous unit hydrograph concept to be useful in correlating hydrograph features to basin characteristics it first must be described mathematically. This description was first accomplished by Edson (5) in 1951. Using the concepts inherent in Clark's derivation of the IUH, Edson was able to derive an expression for the unit hydrograph:

$$q = CY(YT)^X e^{-YT}/\Gamma(X+1)$$

where: Y = recession constant derived from the slope of the recession curve plotted on semi-logarithmic scale

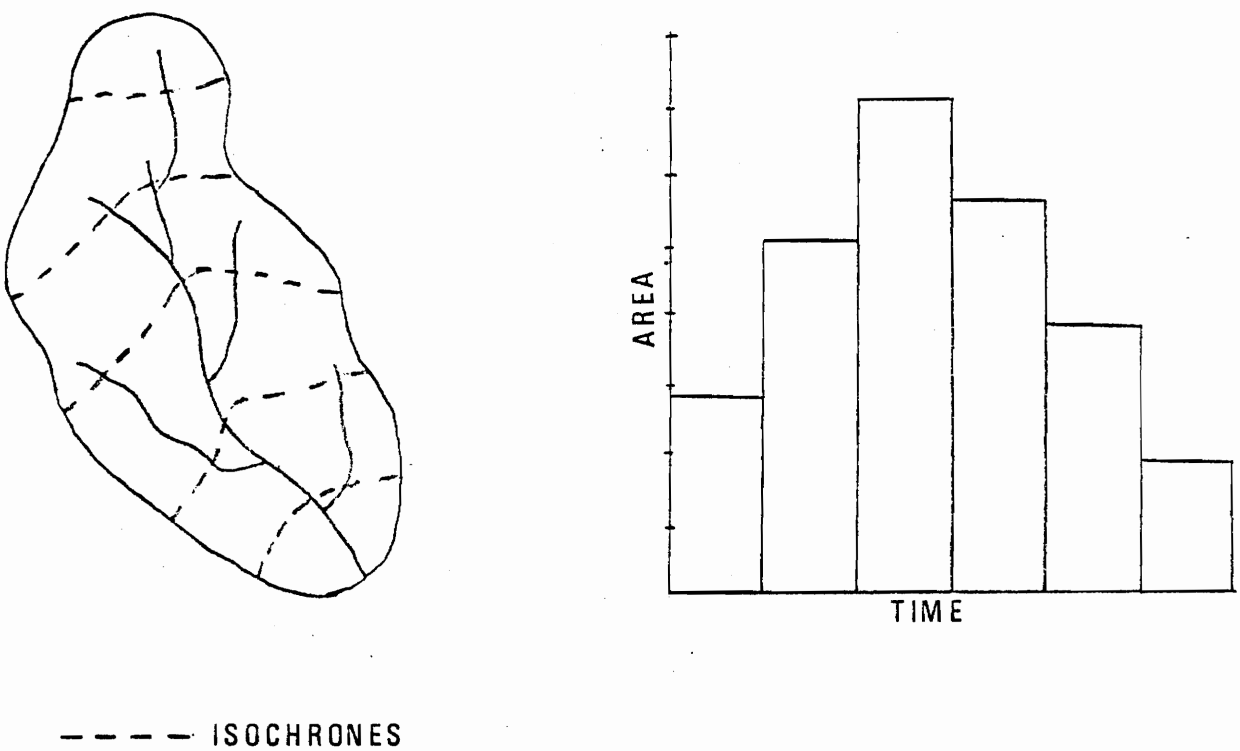

FIG. 2: DERIVATION OF THE TIME-AREA CURVE (28)

X = constant depending on drainage basin characteristics

T = time from beginning of runoff (days)

C = necessary conversion constant

Γ = Gamma function

Later, Nash (19) proposed a similar expression for the instantaneous hydrograph:

$$Q = \frac{Vk^{-n}}{\Gamma(n)} e^{-t/k} t^{n-1}$$

where: Q, t, and Γ are as previously defined

V = volume of surface runoff

n = parameter whose value depends on the storage properties of the basin

k = storage constant

The parameters of the above two expressions are often difficult to evaluate from available data. For determining his parameters, Edsen proposed a graphical procedure which is both cumbersome and inaccurate. Nash (6) suggested the method of moments can be applied to determine k and n, another laborious procedure.

Having noticed the similarity between the above equations and the expression for the two-parameter gamma distribution, Gray (4) proposed a method for estimating the proper parameters with a greater degree of accuracy. The expression for the skew statistical frequency curve is:

$$f(z) = \frac{N(\gamma)^q}{\Gamma(q)} e^{-\gamma x} z^{q-1}$$

In this equation, N is analogous to C in Edsen's expression, γ is analogous to Y, q to X, and z is analogous to t.

By the use of curve-fitting techniques, Gray was able to fit the curve given by the above relationship to the form outlined by the ordinates of several dimensionless unit graphs, thus accomplishing the desired optimization. The dimensionless unit graph is a unit hydrograph whose ordinates are expressed as a percent of the total flow. With the curves fitted and the optimum values of the parameters γ and q obtained, Gray then determined correlations between γ and the pertinent watershed characteristics and between q and γ. Unfortunately, the data appear to break down into lines of very narrow geographical regions. This fact limits the reliability of the estimates because of the lack of sufficient data at any one locality. Thus Gray derived three separate relationships between the parameter γ and the watershed characteristic $(\frac{L}{\sqrt{S}})$ for three separate geographical locations, each with only nine or ten observations available for analysis. This unfortunate reduction of the data also has the effect of severely limiting the effectiveness of the method since the very essence of the synthetic unit graph approach rests on the ability to transpose relationships derived at one locality to the vicinity of the proposed project. Rarely will enough data be available at the desired site to allow the development of the proposed relationships with a high degree of reliability.

There are two other points concerning Gray's analysis which must be discussed. One is the laboriousness and complexity of his procedure. The calculation of this "empirical" unit graph appears to be a cumbersome task and the curve fitting techniques which he employes requires a specialized knowledge far beyond that possessed by most hydrologists. The other point to be covered concerns the questionable accuracy achieved by the process of fitting a pre-conceived curve to the points composing a natural hydrograph. Gray himself admitted the fact that some of his curves appeared to be poorly fitted to the data and he was forced to throw out part of his data because of this fact. This process of curve fitting would seem to add yet another error-producing mechanism to the synthetic unit graph method.

Nevertheless, the use of the two parameter or even three parameter model as proposed by Betsen and Green (16) would appear to give the best indication of the complex relationships which exist between hydrograph peaks and lag times and physiographic basin characteristics. For this reason, it appears to the author that another attempt to derive a less complex and more accurate method using this approach is in order.

One final word is necessary concerning the limitations of the unit hydrograph principle. The danger in acceptance of the lag time as constant for a particular basin has been pointed out by several observers. Barnes (17) has demonstrated the variation in lag times with stage on large mature river systems. The possibility exists that a similar relationship between lag and stage exists for small basins. Certainly, the areal distribution of rainfall is a determining factor in lag development also. Therefore the lag time of a "typical" storm for the basin is difficult to determine and sometimes requires a subjective approach. The importance of this concept of lag time cannot be overemphasized since it is on the determination of lag that most synthetic unit graph methods rest.

In addition, the necessary correlation with basin characteristics is sometimes difficult to ascertain, as evidenced by the varied expressions derived by the several investigators mentioned. The determination of the peak and lag constants necessary for the application of Snyder's method is frequently difficult if not impossible due to scatter in the available data. The reliability of most of the expressions proposed to date, with the exception of the gamma models discussed, is open to question due to the large standard errors generally obtained.

DATA COLLECTION

The data which made this study possible was originally collected for use in two reports published by the U. S. Geological Survey. In his report on the effects of urbanization in the Piedmont region of North Carolina, Putnam (24) collected discharge data from 59 gages in the Piedmont region. Included in these were 20 in the vicinity of Charlotte, N.C., 12 at Durham, 5 at Lenoir, 4 at Morganton, and 18 in and around Winston-Salem. At the most important of these sites tipping-bucket rain gages were also installed.

The survey had also made a careful determination of the pertinent basin characteristics in these areas. The characteristics obtained were drainage area, length of the main water course from the gage to the basin boundary, channel slope between points 10 and 85 percent of the distance upstream of the gage, and the percent of impervious cover contained in each basin. Throughout this report, when reference is made to area, length, slope, and impervious cover, it is the above data which are referenced.

In his report, Putnam also made use of data collected by Anderson in Northern Virginia for the purpose of carrying out a similar investigation (25). Anderson's report contained data on 74 stations in Northern Virginia and Maryland. The collection of data and determination of basin characteristics was carried out in an identical manner to that which was used in North Carolina.

The following criteria were adhered to with respect to the choice of stations to be used in the present investigation from the body of data contained in the two above-referenced works: Preference was given to basins with an area of between 3 and 50 square miles since this is the size of basins generally encountered when planning small scale protection projects. Although this criterion was not strictly adhered to, this report contains only three basins which fall slightly outside of this range. Only basins with both continuous rainfall and discharge data available were chosen, and preference was given to the stations having the longest period of record. Also, basins with varying amounts of urbanization present, ranging from less than one percent to 20 percent of the total basin area, were selected.

Unfortunately, it was discovered that after the publication of Putnam's report much of the data which had been collected to make his investigation possible had been destroyed. Of the stations remaining only five met the above criteria. The U. S. Army Corps of Engineers, Charleston, S. C. District had already computed unit hydrographs on five of the basins whose records had been destroyed. These were generously furnished to the author and used intact. Discharge data were available on six other stations whose records are published in the Geological Survey's water supply papers. Thus, the recorder charts from these gages were available and copies of them, together with the pertinent rating tables were cheerfully furnished by the Survey's Raleigh, N.C. office. The rainfall data for these basins were obtained from the Weather Service hourly precipitation gages at Raleigh-Durham, Greensboro, Yadkinville, and Burlington, all within the State of North Carolina. Thus a total of 16 gages from the North Carolina Piedmont area was used.

All of the desired data from Northern Virginia were available from the Survey's Fairfax office. Of the stations contained in Anderson's report, only 14 met the criteria for inclusion in the study. These data were graciously supplied by the Fairfax office. Therefore, a total of 30 stations were available for inclusion in the investigation at the beginning of the study. Table 1 shows these 30 stations together with their periods of record and measured basin characteristics.

It is important to remember that the Virginia stations were originally intended only to supplement and expand the data obtained from the North Carolina gages. This study is intended to be an investigation into the relationships which exist between unit hydrograph parameters and topographical basin characteristics and urbanization factors in the Piedmont district of North Carolina. No conclusions are drawn concerning any such relationship in Northern Virginia. Due to the eventual reduction of the data into groups by geographical location, the need for the Virginia stations was reduced. It is obvious, however, that the same type of analysis could have been conducted on these basins, probably with somewhat similar results.

TABLE 1

WATERSHEDS UTILIZED IN THE INVESTIGATION

Station #	Location	Period of Record	A (sq mi)	L (mi)	S (ft/mi)	I(%)
1392.0	Bailey Fork nr. Morganton, N.C.	1966-70	7.86	5.90	55.0	1
1396.1	Hunting Creek @ Morganton, N.C.	1966-70	8.26	6.56	28.0	3
1396.5	East Prong nr. Morganton, N.C.	1966-70	8.94	6.06	43.75	2
1411.9	Greasy Creek @ Lenoir, N.C.	1966-70	4.40	3.14	54.94	2
1411.5	Lower Creek @ Mulberry Street, Lenoir, N.C.	1966-70	31.8	8.11	18.0	13
6578.0	Giles Run nr. Woodbridge, Va.	1965-66	4.54	5.5	50.1	3
6459.0	Colvin Run @ Reston, Va.	1961-66	5.09	3.7	49.3	1
6462.0	Scott Run nr. McLean, Va.	1961-66	4.69	4.2	54.0	5
6466.0	Pimmit Run nr. Falls Church, Va.	1961-66	2.87	3.0	59.4	12
6529.1	Backlick Run @ Alexandria, Va.	1960-66	13.4	7.1	28.9	10
6467.0	Pimmit Run @ Arlington, Va.	1961-66	8.12	7.2	38.7	12
1465.0	Little Sugar Creek nr. Charlotte N.C.	1924-70	41.0	11.5	16.2	15
1464.5	Briar Creek @ Sharon Road, Charlotte, N.C.	1962-70	18.5	9.03	14.8	10
1464.7	Little Hope Creek @ Seneca Place, Charlotte, N.C.	1966-70	2.72	2.66	41.0	15
1467.0	McMullen Creek @ Sharon View Road, nr. Charlotte, N.C.	1962-70	6.98	5.06	25.3	6
1466.0	McAlpine Creek @ Sardis Road nr. Charlotte, N.C.	1962-70	38.3	8.75	21.9	2
1463.0	Irwin Creek nr. Charlotte, N.C.	1962-70	30.5	11.4	14.2	11

TABLE 1 (continued)

Station #	Location	Period of Record	A (sq mi)	L (mi)	S (ft/mi)	I(%)
6553.5	Pohick Creek nr. Springfield, Va.	1961-66	15.0	9.0	23.8	1
6550.0	Accotink Creek nr. Accotink Station, Va.	1960-61	37.0	17.1	14.9	5
6526.0	Holmes Run @ Merrifield, Va.	1960-66	2.70	2.8	69.5	10
6539.0	Accotink Creek @ Fairfax, Va.	1961-66	6.80	4.7	35.9	10
6530.0	Cameron Run @ Alexandria, Va.	1955-66	33.7	11.1	30.9	15
6526.9	Holmes Run @ Alexandria, Va.	1960-61	18.9	10.7	31.3	12
1159.0	South Fork Muddy Creek nr. Clemmons, N.C.	1964-70	42.3	12.2	13.1	2
1158.0	Silas Creek @ Clemmons, N.C.	1964-70	11.8	10.6	28.4	6
6526.1	Holmes Run nr. Annandale, Va.	1960-66	7.10	5.8	36.8	12
6525.0	Fourmile Run @ Alexandria, Va.	1961-66	14.4	7.8	42.5	20
0990.0	East Fork Deep River nr. High Point, N.C.	1928-69	14.7	6.36	21.0	2
0955.0	North Buffalo Creek nr. Greensboro, N.C.	1928-68	37.0	15.3	9.92	5
0860.0	Dial Creek nr. Bahama, N.C.	1925-68	4.71	5.06	30.6	1

DESCRIPTION OF AREA

The Piedmont region of North Carolina comprises approximately 40 percent of the land area of the State. It is bounded on the east by the Coastal plain and by the eastern rim of the Appalachian mountains on the west. The average surface slope over the entire area is approximately five feet per mile, beginning at an elevation of 400 feet above sea level at the eastern boundary and rising to an elevation of about 2000 feet in the west.

The major streams in the region generally flow in a southeasterly direction, cutting across the district diagonally from the northwest to the southeast. Channel slopes generally tend to be fairly steep, varying from 15 feet per mile for the major streams to more than 100 feet per mile for some of the minor tributaries.

The average annual precipitation is about 45 inches, generally uniformly distributed through the year. Flooding can occur during any period of the year and is usually caused by one of three types of storms: intense summer thunderstorms, hurricanes, and long duration - low intensity storms resulting from frontal systems.

The Piedmont area contains approximately 70 percent of the urban population of the state. Forty-five percent of that population is concentrated in the 13 counties known as the Piedmont Crescent, extending from Wake County westward to Forsyth and south to Mecklenberg and Gaston Counties. This area contains 15 large urban centers, the largest comprising the City of Charlotte, located in Mecklenberg County.

Of the areas included in this study, two are located in the Piedmont crescent, Charlotte and Winson-Salem-Greensboro. The other area, Morganton-Lenoir, is located in the extreme western part of the Piedmont and boasts a considerably smaller degree of development than the two areas located in the crescent. Figs. 3-6 are maps of these three areas, showing the locations of the stream gaging stations which are contained in each.

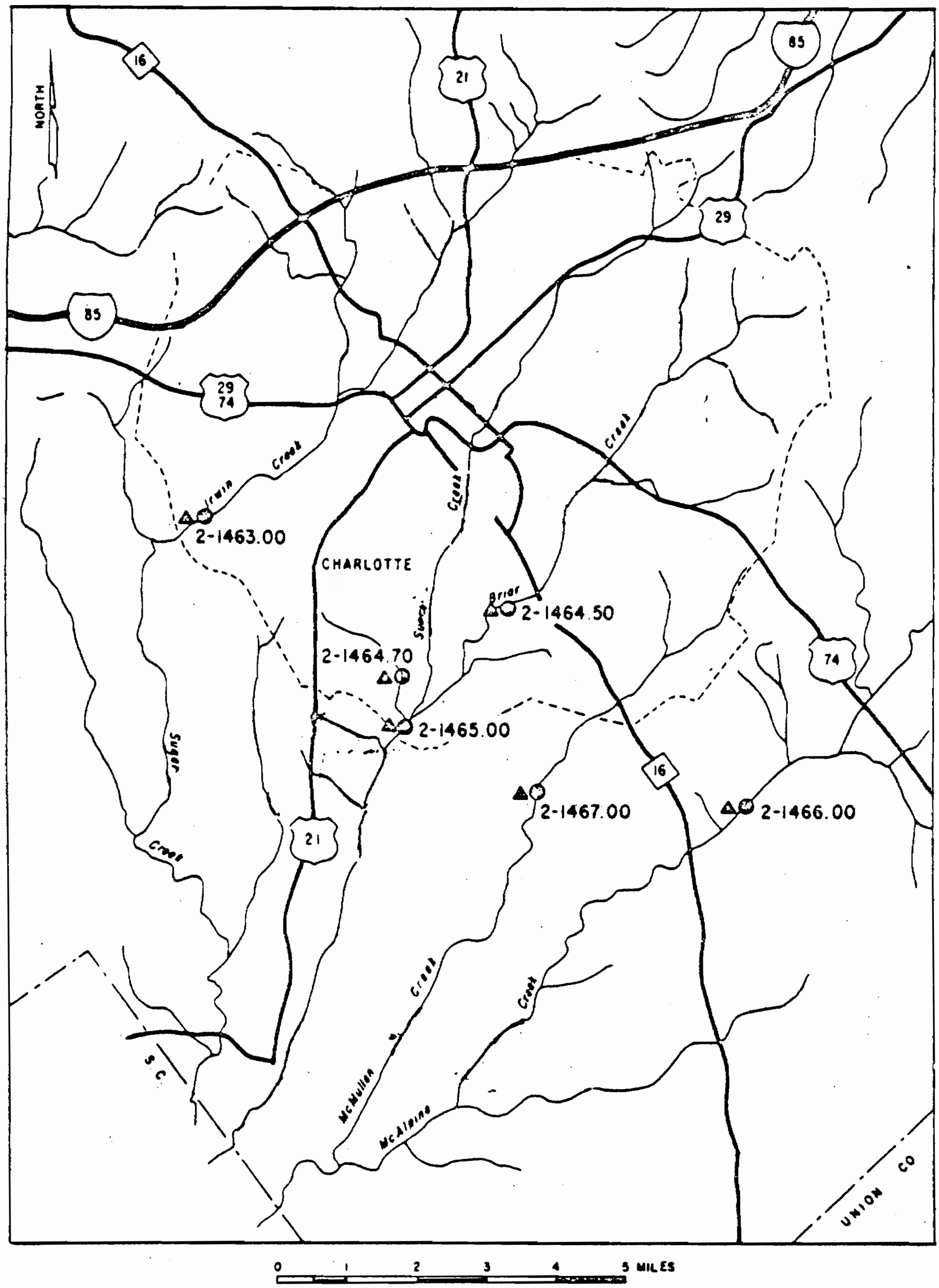

FIG. 3: MAP OF CHARLOTTE NC

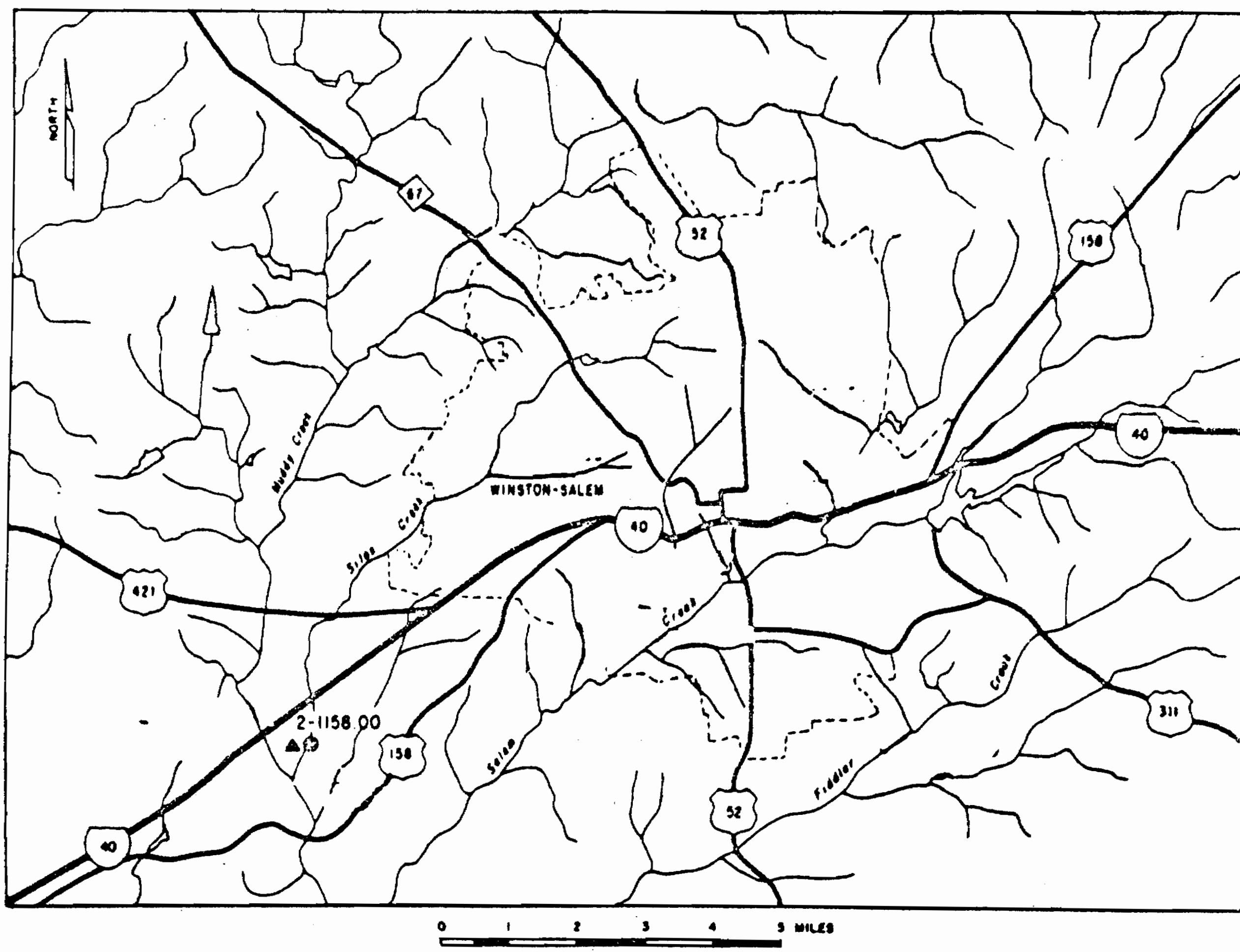

FIG. 4: MAP OF WINSTON SALEM NC

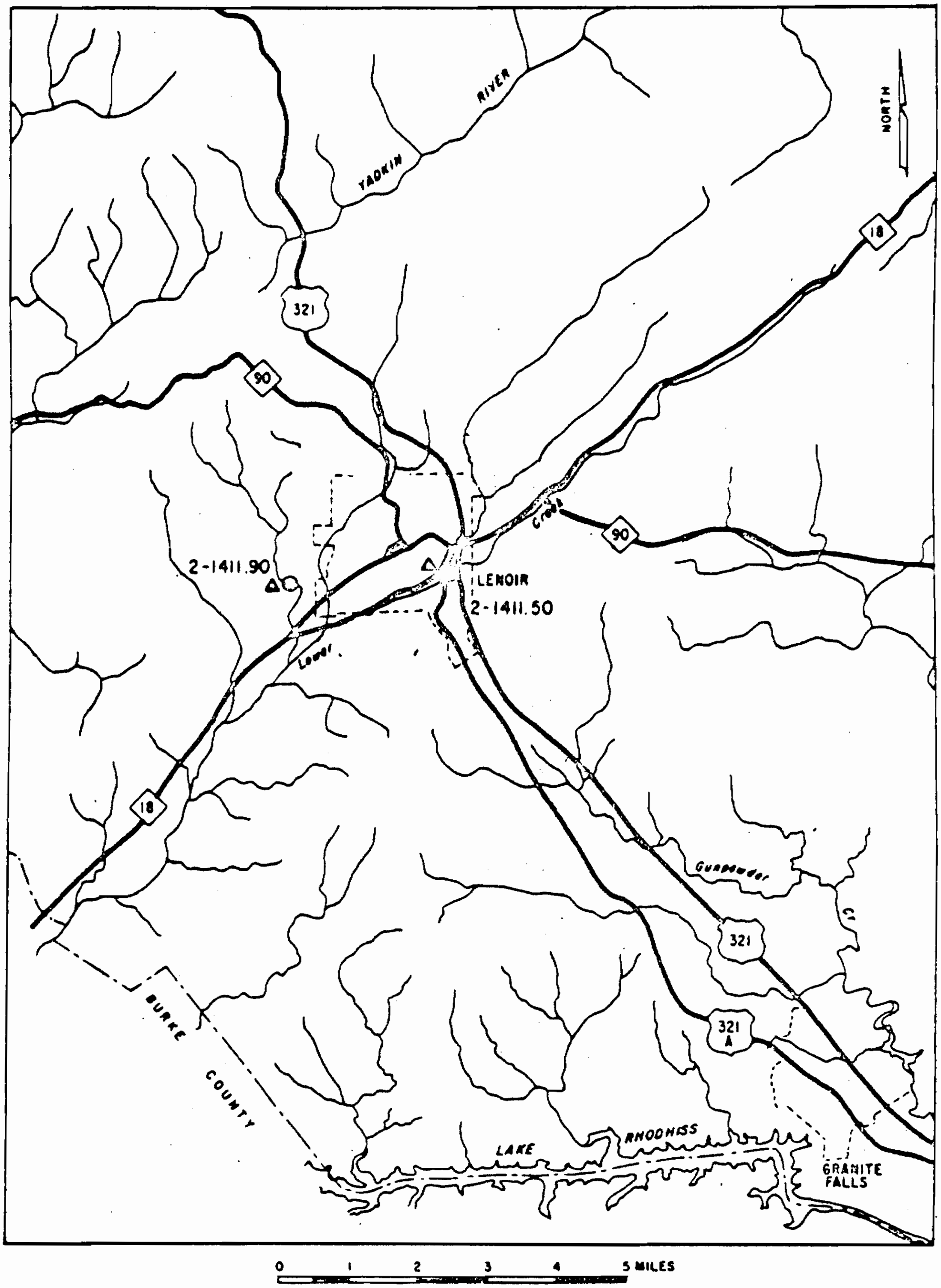

FIG. 5: MAP OF LENOIR NC

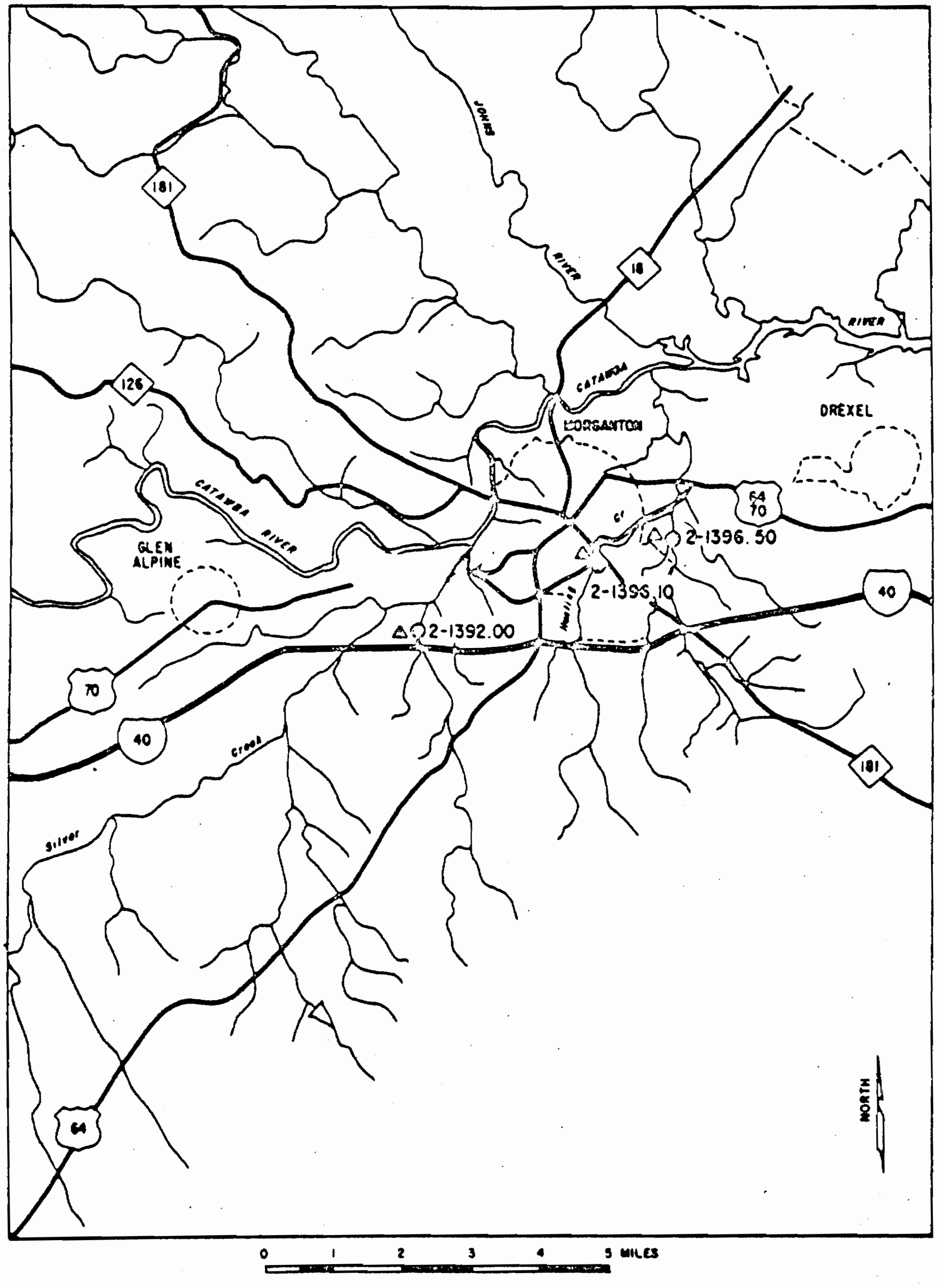

FIG. 6: MAP OF MORGANTON NC

UNIT HYDROGRAPH CALCULATIONS

The computation of the actual unit hydrographs for this study was accomplished by use of the Corps of Engineers standardized computer program 723-X6-L2010, more commonly known as HEC-1. The criteria followed for the selection of storms and other computational procedures is outlined in the standard Corps of Engineers reference on unit hydrograph analysis. (13)

The discharge and precipitation records of each station were first examined for selection of suitable storms from which unit hydrographs could be derived. Preference was given to isolated storms with well-defined single peaks whose rainfall was of fairly uniform intensity over the storm period. In addition, a total volume of 1 inch of precipitation was an initial criteria for storm selection. However, due to the short length of available record at most stations, storms with less than the desired volume were used in several cases. In cases where more than one rainfall gage was used, consideration was given to the distribution of the rainfall over the basin. This was especially true for the six stations in North Carolina taken from the Water Supply Papers. In these cases, as well as for several cases in Virginia where multiple recording precipitation gages were available, the relative weights of each station were determined by the well-known Thiessen polygon method outlined in most standard hydrology references. The storm dates, precipitation gages used, and their relative weights, together with the corresponding basin is presented in table 2.

TABLE 2

STORM DATES AND PRECIPITATION GAGES UTILIZED

Station #	Precipitation Gages Used for All Storms	Weighting (%)	Storm Dates Used
1392.00	Bailey Fork nr. Morganton, N.C.	100	July 1967 Oct. 1968 March 1968
1396.1	Hunting Creek @ Morganton, N.C.	100	July 1967
1396.5	East Prong nr. Morganton, N.C.	100	March 1968 Oct. 1968 May 1968
1411.9	Greasy Creek @ Lenoir, N.C.	100	March 1968 Sept. 1968
1411.5	Lower Creek @ Mulberry Street, Lenoir, N.C.	100	Oct. 1966 June 1968 Oct. 1968
6578.0	Giles Run nr. Woodbridge, Va.	100	March 1965 April 1965 Sept. 1966
6459.0	Colvin Run @ Reston, Va.	100	Feb. 1962 Nov. 1962
6462.0	Scott Run nr. McLean, Va.	100	Aug. 1961 May 1962
6466.0	Pimmit Run nr. Falls Church, Va.	100	Aug. 1961
6529.1	Backlick Run @ Alexandria, Va.	100	Jan. 1960
6467.0	Pimmit Run @ Arlington, Va. Dead Run nr. McLean, Va. Little Pimmit Run nr. Arlington, Va.	40 10 50	April 1961 Aug. 1961
1465.0	Charlotte, N.C. WSO	100	April 1962 March 1968
1464.5	Charlotte, N.C. WSO	100	April 1962 July 1969
1464.7	Charlotte, N.C. WSO	100	Aug. 1967 July 1969
1467.0	Charlotte, N.C. WSO	100	April 1962 March 1966

TABLE 2 (continued)

Station #	Precipitation Gages Used for All Storms	Weighting (%)	Storm Dates Used
1466.0	Charlotte, N.C. WSO	100	April 1962 Oct. 1964 March 1966
6553.5	Rabbit Branch nr. Springfield, Va.	31.1 54.1 14.8	10 April 1961 12 April 1961
6550.0	Accotink Creek nr. Accotink Sta., Va.	100	July 1960 Sept. 1960 Oct. 1960
6526.0	Holmes Run @ Merrifield, Va.	100	Jan. 1960 May 1960
6539.0	Accotink Creek @ Fairfax, Va.	100	14 June 1961 26 June 1961 Aug. 1961
6530.0	Holmes Run @ Alexandria, Va. Tripps Run @ Falls Church, Va. Backlick Run @ Alexandria, Va. Tripps Run nr. Falls Church, Va. Holmes Run nr. Alexandria, Va. Holmes Run nr. Annandale, Va. Turkeycock Run @ Alexandria, Va. Holmes Run @ Merrifield, Va. Backlick Run @ Springfield, Va.	5.6 7.2 8.7 10.6 11.2 12.7 13.4 14.5 15.4	Jan. 1961 April 1961
6526.9	Holmes Run nr. Alexandria, Va. Holmes Run nr. Annandale, Va. Holmes Run @ Merrifield, Va. Tripps Run nr. Falls Church, Va. Tripps Run @ Falls Church, Va.	7.4 19.4 33 16.3 23.9	June 1961 Jan. 1960 Aug. 1961
1159.0	Greensboro, N.C. WSO Yadkinville, N.C. WSO	66 33	Oct. 1964 March 1968
1463.0	Charlotte, N.C. WSO	100	Oct. 1964 March 1968
1158.0	Yadkinville, N.C. WSO	100	Oct. 1964 March 1968
6526.1	Holmes Run @ Merrifield, Va. Tripps Run @ Falls Church, Va. Tripps Run nr. Falls Church, Va. Holmes Run nr. Annandale, Va.	67.5 2.5 .28 29.7	June 1960 July 1960 Nov. 1962

TABLE 2 (continued)

Station #	Precipitation Gages Used for All Storms	Weighting (%)	Storm Dates Used
6525.0	Fourmile Run @ Alexandria, Va.	100	June 1961
0990.0	Greensboro, N.C. WSO	100	Oct. 1964 March 1968
0955.0	Greensboro, N.C. WSO	100	Oct. 1964 March 1968
0860.0	Raleigh-Durham, N.C. WSO	100	March 1968

The HEC-1 package consists of routines which perform various hydrograph computational and routing procedures. The routine utilized in the present instance was the Unit Hydrograph and Loss Rate Optimization program. The objective of the program is to determine the optimum values of a set of continuous variables which will minimize a given function. The procedure used is the univariate gradient technique outlined in "Optimization Techniques in Hydrologic Engineering" by Leo R. Beard.

Input into the program consists of an observed discharge hydrograph together with its precipitation data. From this information, a unit hydrograph is computed along with excess rainfall values. The excess rainfall is then applied to the unit hydrograph ordinates to reproduce the observed hydrograph. The objective function to be minimized in the calculation of the unit graph coefficients is the root-mean-square errors between computed and observed flows. In addition, a volume check is performed which assumes an approximate equality between the volumes of the observed and reproduced hydrographs. Thus, the unit graph is calculated which best reproduces the given observed discharge hydrograph.

The loss rate parameters to be optimized are shown on fig. 7 and defined below:

DLTKR - Initial accumulated rain losses--a function of antecedent soil moisture conditions

STRKR - Starting value of loss coefficient on exponential recession curve (fig. 7)--a function of the soil

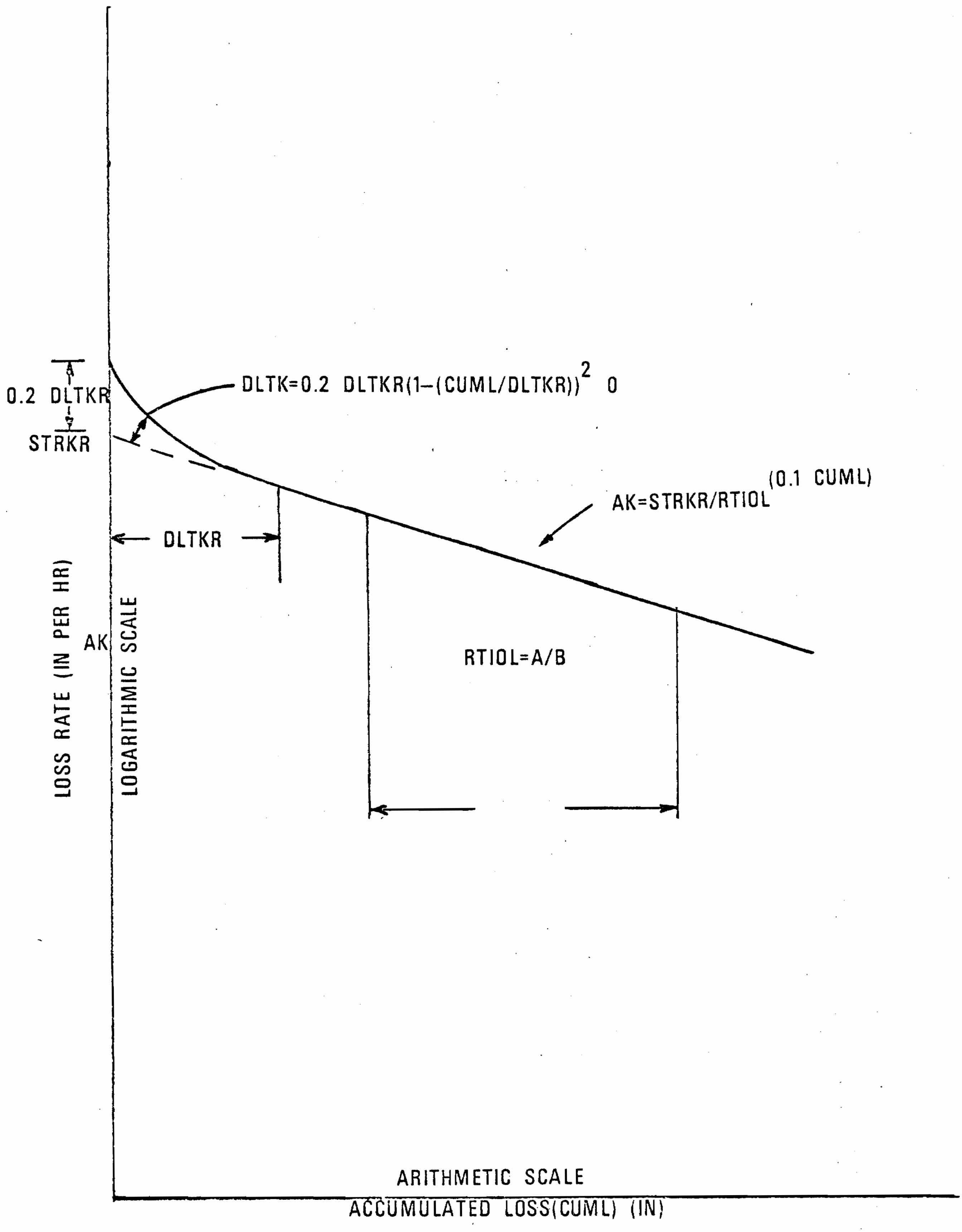

FIG. 7: GENERAL LOSS RATE FUNCTION ON SNOW-FREE GROUND(32)

infiltration capacity

RTIOL - Ratio of loss coefficient on the exponential curve to that corresponding to 10 inches more of accumulated loss

ERAIN - Exponent of precipitation for the exponential function for loss rate

The loss rate by any time interval is calculated from the expression:

$$ALOSS = (AK + DLTK)PRCP^{ERAIN}$$

where: ALOSS = loss rate for the particular time interval

AK = loss rate coefficient at beginning of time interval (from exponential loss curve--fig. 7)

PRCP = rainfall intensity (inches per hour)

DLTK = incremental increase in loss rate coefficient

The unit graph parameters which are optimized are the Clark unit graph time of concentration (TC) and the storage coefficient (R), also from Clark's unit hydrograph procedure. The time of concentration is the time required for water particles to travel from the farthest reaches of the basin to the outflow point. The Clark method consists of routing the time-area curve through a linear reservoir as already described in another section of this paper. For these cases, a program-supplied synthetic time-area curve was used in this procedure. The derivation of this time-area curve, as well as a detailed description of the Clark method can be found in reference (32) of this paper.

The unit graph and loss rate parameters were determined in a successive approximation manner as described in "Optimization

Techniques . . ." by Beard. This procedure is necessary primarily because virtually all of the parameters are interdependent to some degree. The method consists of determining a "regional" value for one of the parameters by analyzing several storms, then fixing that parameter as a constant and repeating the procedure for each of the other variables. The following steps constitute the derivation sequence:

1. Discharge and precipitation data are collected for as many storms as possible for each station.
2. The base flow, or starting value, for each storm is estimated, as well as the slope of the recession limb of the discharge hydrograph and the discharge value at which recession flow begins. These values are input to the program as constants for each particular storm.
3. The six variable unit graph and loss rate parameters are then determined by use of the optimization routine. Thus, the unit hydrographs which best reconstitute each particular flood are calculated.
4. A regional value of ERAIN is chosen based on analysis of the results of step 3.
5. With ERAIN fixed, steps 3 and 4 are repeated for the five other variables until each one is determined on a regional or basin wide basis.

Theoretically, the above procedure produces a single unit hydrograph which best reconstitutes all of the observed hydrographs for each particular basin. However, in applying this procedure in the present investigation several problems were encountered which severely limit the accuracy of the results. Due to the short period of record available at most of the sites, it was necessary to utilize storms of widely varying intensity and total volume at several locations. At a few of these sites, convergence between the various storms used was not obtained. Thus, large differences between the calculated values of unit-graph lags and peaks were encountered in these cases. It was not considered appropriate to take an arithmetic average of the various values of lag and peak discharge in cases where there was a large scatter in the estimated values, therefore the worst of these cases were dropped from the investigation. In most of the cases, where the scatter was not so great, a simple arithmetic average was deemed appropriate. In a few other instances, weighing adjustments were made to bring the results more in line with those obtained by Putnam. Table 3 is a presentation of the storm dates utilized for each basin, as well as the values of lag time, unit graph peak discharge, and the duration of effective rainfall obtained for each storm. An asterisk beside the station number of a particular basin indicates that the scatter was considered to have too large a magnitude and the station was therefore dropped. In the four cases where only one storm of sufficient magnitude to warrant consideration was available for analysis, the resulting unit hydrograph was accepted for the obvious reasons that no better data were available.

TABLE 3

COMPUTED UNIT GRAPH PEAKS AND LAG TIMES

Station #	Storm Date	Tp (hrs)	Qp (cfs)	Tr (hrs)
1392.00	July 1967	3.22	700	1
	Oct. 1968	4.00	947	1
	March 1968	2.61	664	1
1396.1	July 1967	1.68	858	1
1396.5	March 1968	4.19	692	1
	Oct. 1968	3.84	653	1
	May 1968	2.63	1073	1
1411.9	March 1968	2.04	691	1
	Sept. 1968	2.24	657	1
1411.5	Oct. 1966	3.73	1849	1
	June 1968	1.68	2597	1
	Oct. 1968	1.75	1904	1
6578.0	March 1965	2.72	727	1
	April 1965	1.33	890	1
	Sept. 1966	2.71	757	1
6459.0	Feb. 1962	1.52	751	1
	Nov. 1962	2.88	611	1
6462.0	Aug. 1961	1.62	1100	.25
	May 1962	1.11	1621	.25
6466.0*	Aug. 1961	.91	807	.5
6529.1	Jan. 1960	2.06	2684	.5
6467.0	April 1961	3.4	1128	.5
	Aug. 1961	1.53	1376	.5
1465.0	April 1962	1.65	3904	1
	March 1968	1.76	4499	1
1464.5	April 1962	3.36	1602	1
	July 1969	2.80	2609	1
1464.7*	Aug. 1967	.52	1362	.083
	July 1969	.85	1589	.083
1467.0	April 1962	2.42	1373	.5
	March 1966	2.40	972	.5
1466.0	April 1962	5.16	3633	1
	Oct. 1964	5.68	3576	1
	March 1966	6.04	3165	1

TABLE 3 (continued)

Station #	Storm Date	Tp (hrs)	Qp (cfs)	Tr (hrs)
6553.5	10 April 1961	5.15	1415	1
	12 April 1961	5.61	1339	1
6550.0	July 1960	13.95	1418	1
	Sept. 1960	6.98	2504	1
	Oct. 1960	8.22	2258	1
6526.0	Jan. 1960	2.78	369	1
	May 1960	2.76	354	1
6539.0	14 June 1961	2.30	1484	.5
	26 June 1961	2.35	1533	.5
	Aug. 1961	1.92	1370	.5
6530.0	Jan. 1961	3.07	2476	1
	April 1961	3.33	2350	1
6526.9	June 1961	.92	8998	.25
	Jan. 1960	3.21	2640	.5
	Aug. 1961	4.91	1821	.25
1159.0	Oct. 1964	11.84	1446	1
	March 1968	7.03	2351	1
1463.0	Oct. 1964	1.95	3036	1
	March 1968	2.58	2616	1
1158.0	Oct. 1964	9.01	485	1
	March 1968	4.61	914	1
6526.1	June 1960	5.74	339	1
	July 1960	1.77	1015	1
	Nov. 1962	1.54	1050	1
6525.0	June 1961	1.00	6405	.083
0990.0	Oct. 1964	4.79	1634	1
	March 1968	3.90	1728	1
0955.0	Oct. 1964	8.80	2111	1
	March 1968	6.17	3126	1
0860.0	March 1968	6.01	363	1

The final values of basin lag times and unit hydrograph peaks chosen for the analysis are presented in table 4. A standard unit duration of effective precipitation of 1 hour was chosen due to the small size of the basins utilized and the generally short duration of the storms used in the analysis. This choice also appears to be appropriate when consideration is given to the calculated lags. The duration of each computed unit graph was converted to one hour by means of a procedure outlined by Snyder (12). After conversion to a 1 hour effective duration, the adjusted lags and peaks were then averaged to obtain the final values to be used in the rest of the investigation. Table 4 is a listing of the adjusted and averaged values of lag and peak.

TABLE 4

ADJUSTED PEAKS AND LAG TIMES

Station #	Tp (hrs)	qp (cfs/mi^2)
1392.0	2.91	86.77
1396.1	1.68	103.87
1396.5	4.01	75.17
1411.9	2.14	153.18
1411.5	2.74	58.99
6578.0	2.25	174.23
6459.0	2.20	133.79
6462.0	1.55	72.49
6466.0	1.03	281.18
6529.1	2.18	200.30
6467.0	2.50	160.71
1465.0	1.70	102.46
1464.5	3.08	113.78
1467.0	2.53	167.91
1466.0	5.63	90.29
6553.5	5.38	91.80
6550.0	9.72	55.68
6526.0	2.77	133.70
6539.0	2.31	215.00
6530.0	3.20	71.60
6526.9	4.22	70.98
6526.1	3.02	109.58
1159.0	9.44	44.98
1158.0	6.81	59.24
1463.0	2.26	92.65
0990.0	3.90	114.35
0955.0	7.49	70.76
0860.0	6.01	77.07

DERIVATION OF THE MODEL

From an analysis of the time-area diagram shown in Fig. 2, it can be seen that the relationship of cumulative area with time gives the approximate power relationship:

$$A \propto T^X \text{ where } X > 1 \qquad (1)$$

so that it appears that discharge would be given by:

$$Q \propto T^X \text{ for } X > 1 \qquad (2)$$

However, due to the complex interrelationships which are known to exist between the various components of discharge, proportionality (2) is invalidated as a complete description of the discharge process. The resultant delay in time of travel from the various zones has been shown to be the result of valley storage (22). Therefore, the valley of the basin can be compared to a reservoir whose discharge decreases exponentially with time as:

$$A \propto e^{-YT} \text{ with } Y > 0 \qquad (3)$$

Both proportionalities (2) and (3) can be shown to be in effect from the very beginning of runoff so their combined effect is given by the simple proportion

$$Q \propto T^X e^{-YT} \qquad (4)$$

From consideration of the above principles, Edson (5) proposed the following relationship for the discharge at any instantaneous time:

$$Q = BT^{X}e^{-YT} \tag{5}$$

where: Q = discharge in cfs at time T

T = time from beginning of runoff

X = exponent depending on the shape of the time-area curve and therefore on basin characteristics

Y = recession constant determined from a semi-logarithmic plot of the recession curve

e = base of natural logarithms

B = constant of proportionality for the particular hydrograph

If equation (5) is integrated with respect to time and the constants set to correspond to proper units, the result could be used to obtain the relationship of the unit hydrograph. Edson performed this integration by substituting X = (n-1) and Z = YT. Then

$$\int_0^\infty QdT = BY^{-n}\int_0^\infty Z^{n-1}e^{-Z}dz$$

$$= BY^{-n}\Gamma(n) \tag{6}$$

where $\Gamma(n)$ is the gamma function of n. Keeping in mind that all the runoff must be discharged as a unit hydrograph so that: $\int_0^\infty QdT = A$ in-mi^2 must hold, and solving for B and substituting back, then

$$B = CAY^{X+1}/\Gamma(X+1) \tag{7}$$

where: A = drainage area in sq. miles

C = necessary conversion constant

Therefore (5) becomes:

$$Q = CAY(YT)^X e^{-YT}/\Gamma(X+1) \tag{8}$$

Since the unit hydrograph is defined to contain one inch of runoff: C = (24)(26.9) where 26.9 is the number of cfs-days in one inch of runoff, and 24 is, of course, the number of hours in one day. Therefore, time is defined in hours, area in square miles, and discharge in cubic feet per second. So equation (8) can be written:

$$Q = VY(YT)^X e^{-YT}/\Gamma(X+1) \tag{9}$$

where V = CA = volume of runoff in cfs-hrs. By differentiation of (9) and tests for maxima and minima it was shown that (9) is maximized when

$$T_m = X/Y \tag{10}$$

and the maximum discharge is given by:

$$Q_m = VY(X/e)^X/\Gamma(X+1) \tag{11}$$

Thus, expressions for the unit hydrograph peak and time to peak are obtained mathematically.

It is desirable to eliminate the area from the above expressions by substituting q = Q/A into equation (8). Therefore:

$$q = CY(YT)^X e^{-YT}/\Gamma(X+1) \tag{12}$$

and

$$q_m = CY(X/e)^X/\Gamma(X+1) \tag{13}$$

By use of equations (10) and (13) the peak and lag of the unit hydrograph are defined only in terms of the dimensionless parameters X and Y. Given a unit hydrograph, these equations can be solved for X and Y and the parameters can than be related by regression to the basin characteristics. The two major limitations of this procedure are that only one point on the unit graph is utilized in determining the necessary parameters and that the two equations, being highly non-linear, are difficult to solve anyway. However, the only alternative, the curve-fitting procedures utilized by Gray (4) and Betson, et. al. (16), appear to add as much error into the analysis by their approximations to the actual unit graph, as they eliminate by using all of the data points and by-passing the solutions to the equations. It is the opinion of the author that results can be obtained, with an accuracy at least equal to that of the curve-fitting method, by the judicious solution of equations (10) and (13).

SOLUTION OF THE MODEL EQUATIONS

The results presented in the last section have shown that the peak discharge of the instantaneous unit hydrograph is given by the expression:

$$q_m = CY(X/e)^X/\Gamma(X+1) \tag{13}$$

where: q_m = peak discharge in cfs/mi^2 and the other terms are as previously defined

and the time to peak or lag is given by:

$$T_m = X/Y \tag{10}$$

in which T_m = the time in hours from the beginning of runoff to the peak of the UHG.

Given an observed unit hydrograph with known peak and lag, equations (13) and (10) must be solved simultaneously for the correct values of X and Y. The desired solution is best accomplished by numerical techniques. By solving equation (10) for Y in terms of X and substituting the result into equation (13), the following expression is obtained:

$$q_m = C(X/T_m)(X/e)^X/\Gamma(X+1) \tag{14}$$

Subtracting q_m from both sides yields an expression which can then be handled by one of several numerical methods:

$$0 = C(X/T_m)(X/e)^X/\Gamma(X+1)-q_m \tag{15}$$

Figure (10) is a graphical representation of the equation (15) with values of q_m and T_m of 75.17 and 4.01 respectively. It can be seen from the graph that equation (15) represents a smooth curve with no false maxima or minima and having only one root solution. Because of these characteristics the equation is ideally suited to solution by numerical analysis methods. One of the simplest, yet more accurate of these techniques is the method of secants. The principle of the secant method is presented graphically in Figure (11). If a function is known to be continuous between any two points X_0 and X_1, then a secant line can be drawn to the function between those two points. The point on the X-axis where the secant line crosses is then chosen as the next value of X to be inserted into the equation and another secant line is constructed from that point, passing throught the X-axis at a point which is chosen as the next estimate to the root and so on until a desired level of accuracy is attained. The method outlined above sug= gests the following algorithm for the numerical solution of a continuous non-linear function:

$$X_{n+1} = X_n - f(X_n)\,\frac{X_n - X_{n-1}}{f(X_n) - f(X_{n-1})} \tag{16}$$

In the above expression the current estimate to the root is obtained from the previous estimate by adding the correction factor:

$$\frac{-f(X_n)}{[f(X_n) - f(X_{n-1})]/(X_n - X_{n-1})} \tag{17}$$

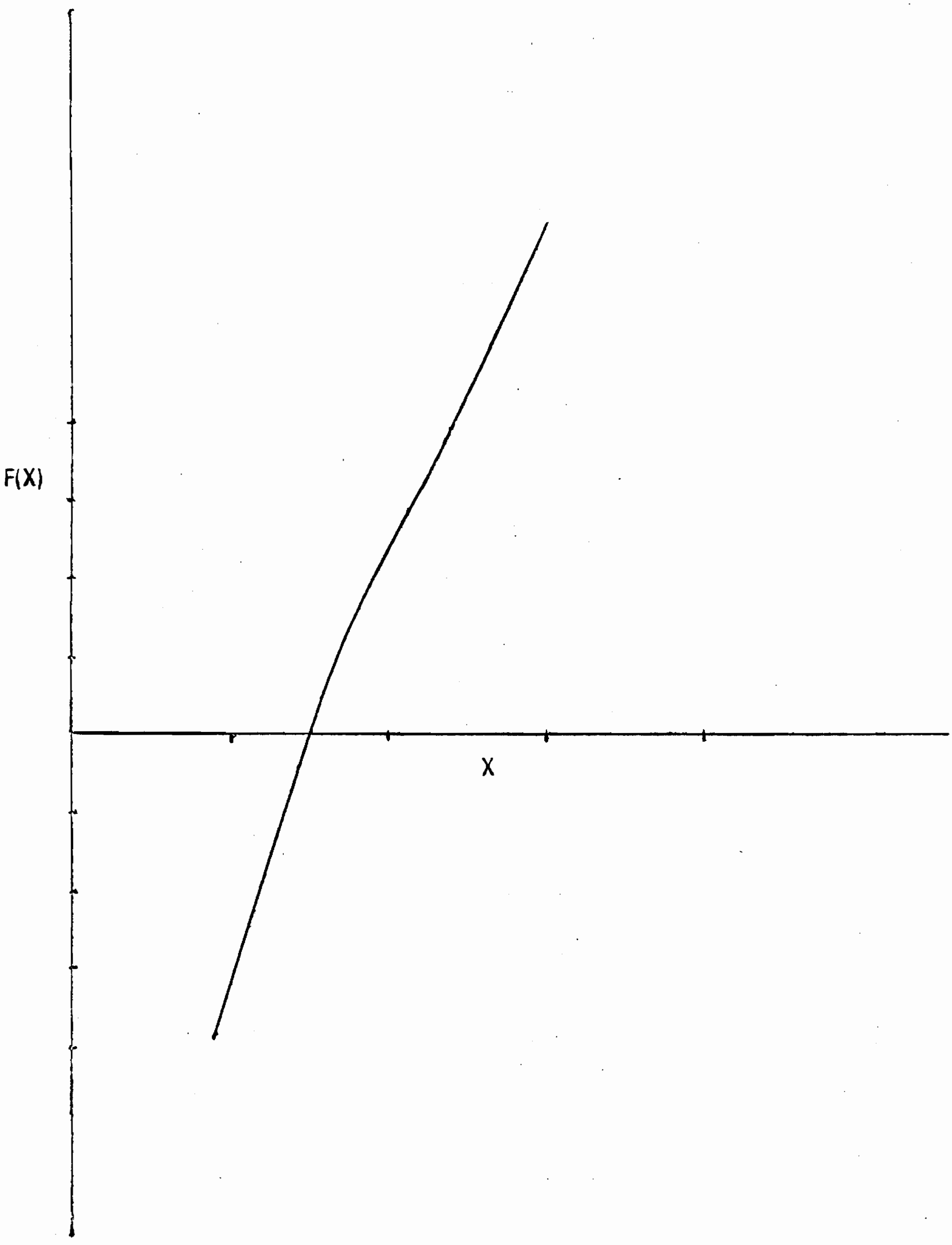

FIG. 8: GRAPHICAL REPRESENTATION OF EQUATION 15

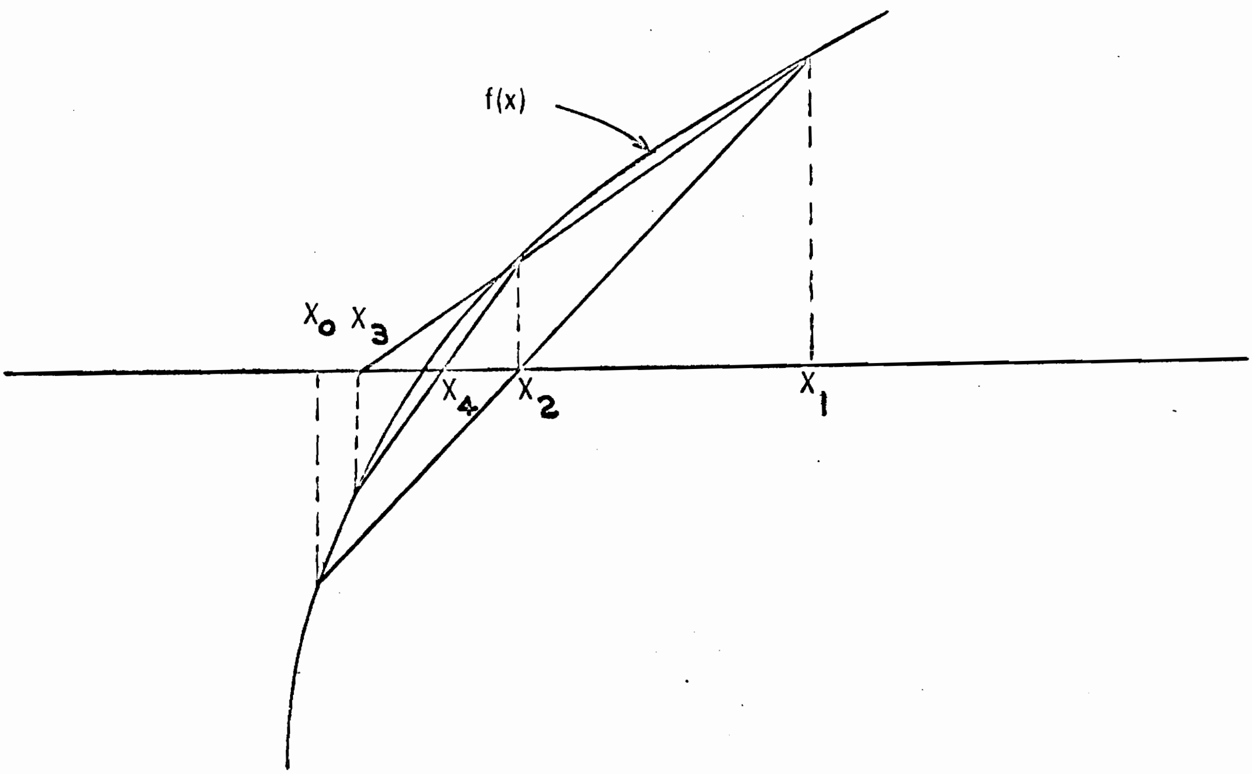

FIG. 9: GRAPHICAL REPRESENTATION OF THE METHOD OF SECANTS

Where $[f(X_n)-f(X_{n-1})]/(X_n-X_{n-1})$ is the slope of the secant to $f(X)$ passing through the points $[X_{n-1},f(X_{n-1})]$ and $[X_n,f(X_n)]$.

The salient feature of the method of secants is represented by the fact that any two values of X can be utilized to start the algorithm. This fact can be used to demonstrate both the major advantage and disadvantage of the procedure. The major advantage is that no specialized values of X are needed to begin the iteration and therefore if it be necessary to solve several such equations in one computer run, the programming is facilitated by the use of the same two points as starting values for every equation. The disadvantage is that the two starting values do not necessarily have to bracket the root and thus the most important indication of the error obtained in each particular iteration is lost. If the starting values do bracket the root, limits on the error of each iteration is obtained by observation of the interval between the current estimate and the previous estimate to the root. The true value of the solution must lie somewhere within that interval. Thus, as the algorithm progresses and the interval becomes increasingly smaller, a useful indication of the deviation of the current estimate from the root is obtained. However, if the starting values do not bracket the root, this indication of the current error is not available. In the present instance, this disadvantage was overcome by the substitution of the calculated values of X and Y into the original equation and a comparison of the calculated peak with the observed values.

The error criteria used in the present investigation for the secant algorithm was the calculation of two successive values of X whose difference was less than or equal to .01. In all cases the algorithm converged successfully after a few iterations with results well within the limits of engineering accuracy when the values of computed and observed peaks were compared.

It will be observed that the mathematically derived expression for lag and the procedure used to calculate the lag in this investigation do not coincide. The lag is calculated by the HEC-1 program according to Snyder's definition, i.e., the time from the centroid of precipitation excess to the peak of the unit hydrograph. Since the lag is calculated mathematically by the maximization of a function in time, it is necessarily measured from the beginning of runoff to the peak of the unit graph. An attempt was made to investigate the magnitude of the error introduced into the analysis by this substitution, but due to the relatively small size of the basins comprising the study and the short lags obtained therein, no significant difference between the lags calculated according to the separate definitions could be ascertained. Furthermore, it was felt that the results of the investigation would not be unduly sensitive to the values of peak discharge and basin lag times used. Therefore, it was determined to accept the values of lag as calculated by Snyder's definition as close approximations of the values as defined by the mathematical definition.

The values of the parameters X and Y obtained from the solution of equation (15) by the method of secants for the basins remaining in the study are presented in Table 5, together with their corresponding stations.

TABLE 5

CALCULATED VALUES OF THE MODEL PARAMETERS

Station #	X	Y
1392.00	1.11	.38
1396.10	.60	.35
1396.50	1.53	.38
1411.9	1.78	.83
1411.5	.53	.19
6578.0	2.48	1.10
6459.0	1.46	.66
6462.0	.30	.20
6466.0	1.42	1.38
6529.1	3.04	1.39
6467.0	2.59	1.04
1465.0	.59	.35
1464.5	2.01	.65
1467.0	2.88	1.14
1466.0	4.06	.72
6526.0	2.23	.80
6539.0	3.88	1.68
6530.0	.94	.29
6526.9	1.51	.36
1159.0	1.02	.15
1463.0	.81	.36
1158.0	2.08	.45
6526.1	1.81	.60
0990.0	2.99	.77
0955.0	2.91	.39
0860.0	3.40	.56

PRESENTATION OF THE RESULTS

As stated in a previous section, the Geological Survey had determined four basin characteristics for each of the basins listed in Table 1. These characteristics were area, length, slope and percent of impervious area contained in each basin. Any or all of these characteristics or any combination thereof could be used in the regression analysis which was the next step of the procedure. To facilitate the selection of the most significant of these characteristics, an analysis was conducted to determine the degree of correlation existing between the characteristics themselves. This procedure, as well as all of the other statistical analyses carried out in this investigation was accomplished by use of the Statistical Analysis System (SAS) program developed at North Carolina State University. The results of the analysis are presented in Table 6. Due to the high degree of correlation found between the area, length and slope factors, it was determined that a term consisting of a combination of these characteristics would be more in order for use in the regression analysis. As previously stated, several other authors have shown the significance of the $L/\sqrt{S})$ term and Gray achieved marked success in his investigation with the use of this combination. It was therefore determined that the factors to be used in the regression analysis would be $(L/\sqrt{S})$ and percent imperviousness.

It is fairly obvious from hydraulic considerations that the magnitude of the storage constant, Y, would vary directly with the stream

TABLE 6

CORRELATION COEFFICIENTS OF BASIN CHARACTERISTICS

	A	L	S	I
A	1.000	.867	-.751	-.0972
L	.867	1.000	-.783	-.126
S	-.751	-.783	1.000	.289
I	-.0972	-.126	.289	1.000

length and inversely with the slope. Therefore, that parameter was chosen for regression analysis with the $(L/\sqrt{S})$ term. Figs. 10-12 show the storage constant Y plotted against $(L/\sqrt{S})$ on semi-logarithmic paper for three regional groupings. This unfortunate reduction of the data into separate curves for the three North Carolina locations was necessary because the data appeared to naturally fall into separate lines according to Geographical location. No significant regression equation could be obtained with the use of all the data in the same analysis. However, the relationships obtained by utilization of the grouping techniques were found to be highly significant in most cases. The three regression equations which were obtained are:

Charlotte:

$$Y = 1.16-1.63(\log(\frac{L}{\sqrt{S}}))$$

Morganton-Lenoir:

$$Y = .40-.93(\log(\frac{L}{\sqrt{S}}))$$

Winston-Salem-Greensboro:

$$Y = .64-.41(\log(\frac{L}{\sqrt{S}}))$$

The coefficient of determination for each group is:

Charlotte:

$$R^2 = 96.6\%$$

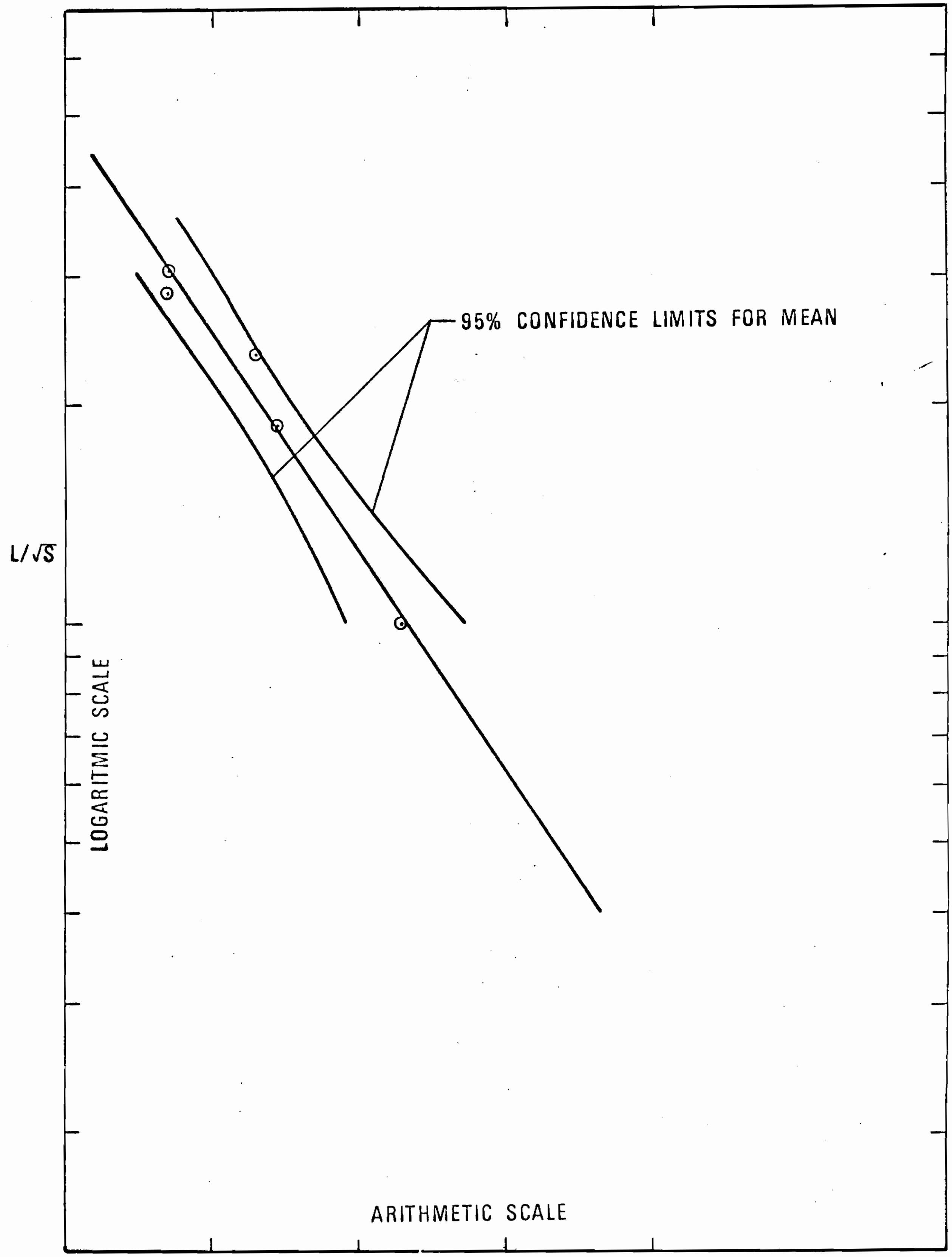

FIG. 10: REGRESSION OF Y VS. $L/\sqrt{S}$ FOR CHARLOTTE, N.C.

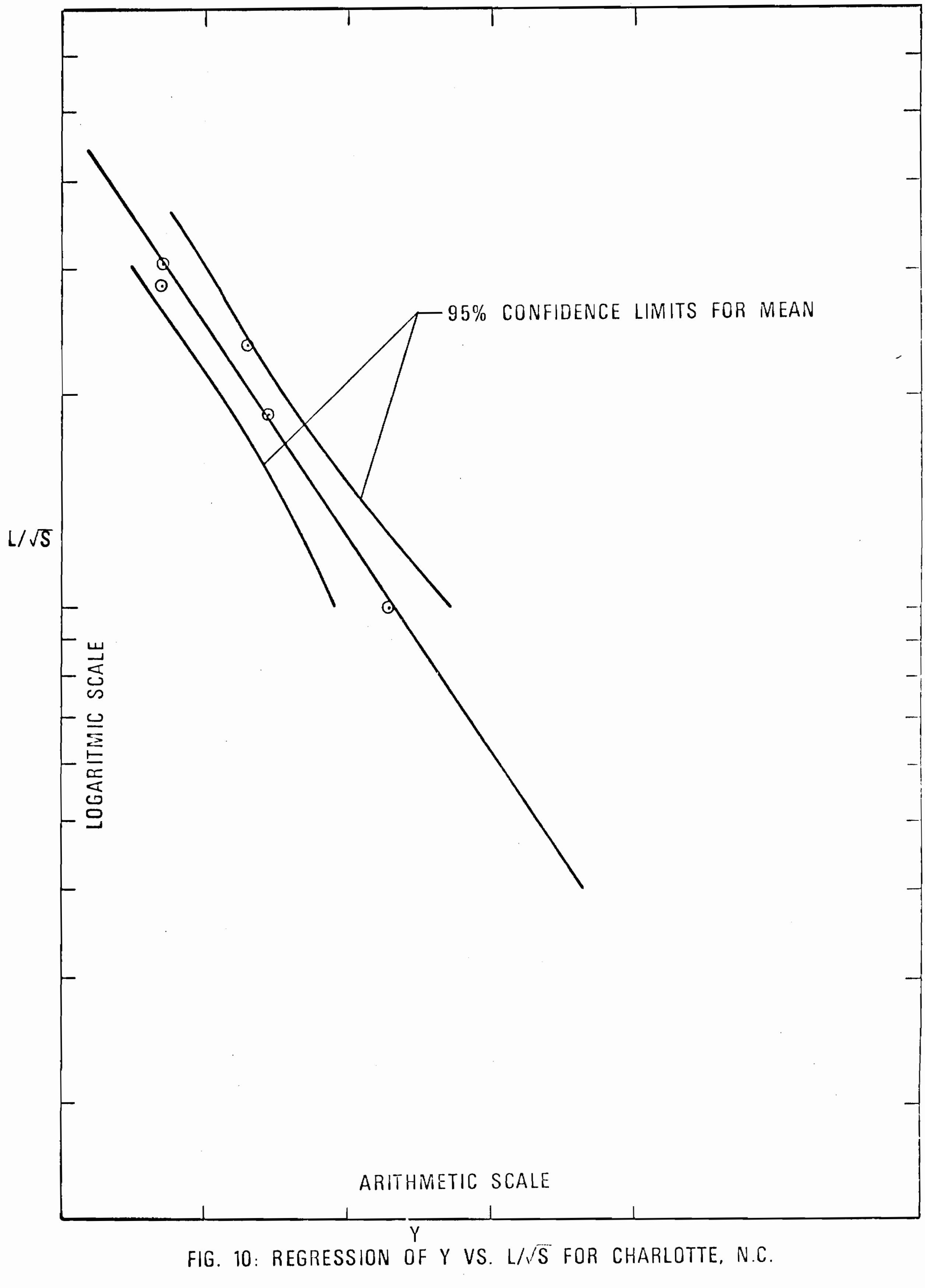

FIG. 10: REGRESSION OF Y VS. L/√S FOR CHARLOTTE, N.C.

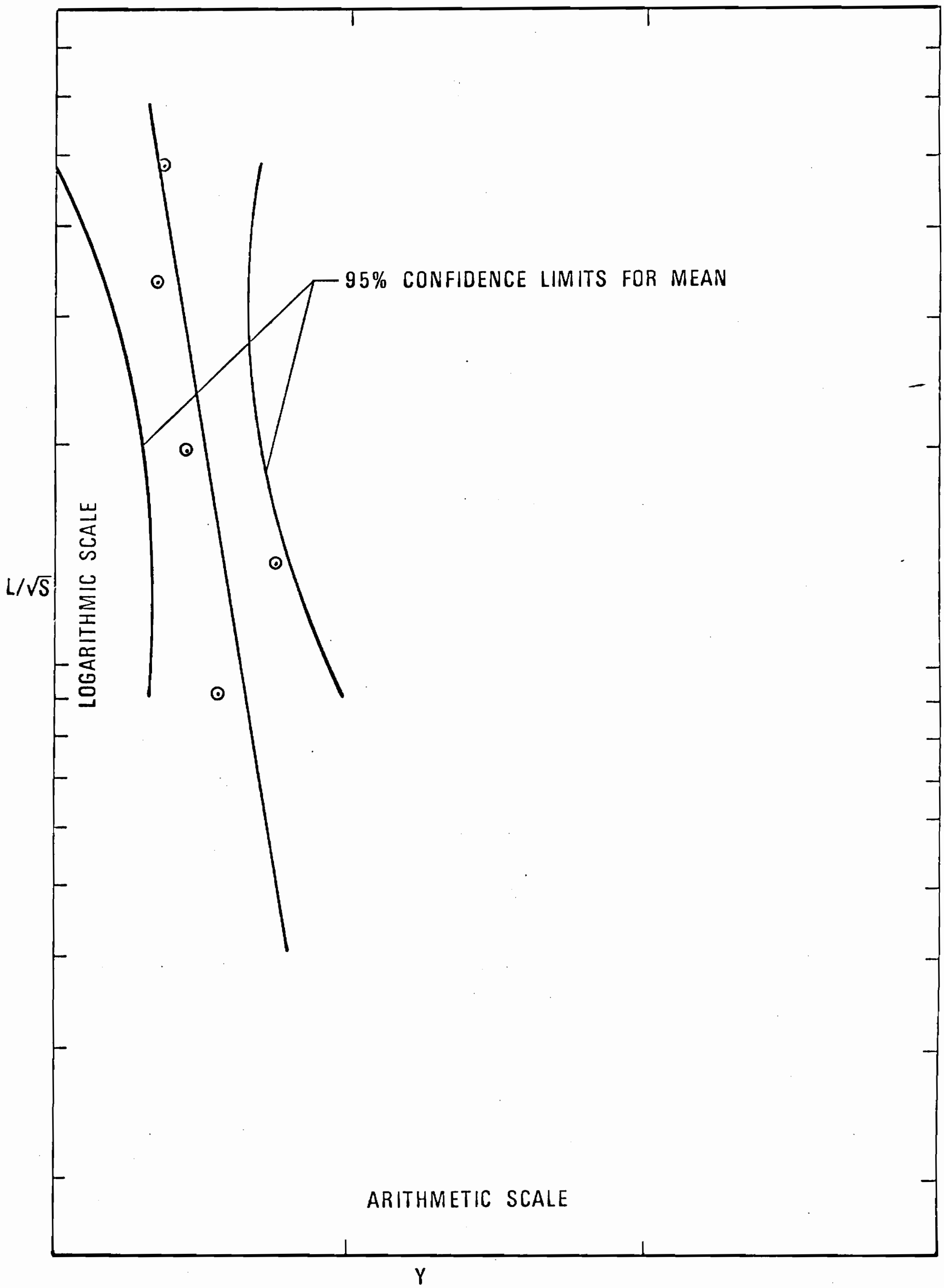

FIG. 12: REGRESSION OF Y VS. $L/\sqrt{S}$ FOR WINSTON–SALEM–GREENSBORO, N.C.

Morganton-Lenoir:

$$R^2 = 88.9\%$$

Winston-Salem-Greensboro:

$$R^2 = 50.8\%$$

In addition, the estimated values of the regression coefficient appear to be significantly different from 0 in all cases. The small size of the data sample for each group somewhat negates the significance of the value of R^2 obtained in each case due to the fact that a fairly large degree of error in any one point has the effect of severly reducing the coefficient of determination for that data set, as in the Winston-Salem-Greensboro case.

The most severe limitation imposed by the lack of sufficient data is the restriction of the confidence inherent to the model, as evidenced by the width of the 95% confidence limits in each case. Due to this lack of confidence in the model, it was not possible to statistically separate the three lines. By use of statistical comparison tests on the slopes and intecepts of the derived lines, it was not possible to reject the Null Hypothesis at any reasonable level of significance. However, due to the apparent natural inclination of the data to maintain the three groupings and the inability to perform any significant statistical tests on the data as a whole, the separation of the data into three groups by geographical location was adhered to in this part of the analysis.

The determination of the relationship between the X parameter and basin characteristics presented a more difficult task. The magnitude of this parameter depends on the shape of the time-area concentration curve for the particular basin. Several factors can have an effect on this curve. It has already been shown that urbanization factors drastically effect the response time of a watershed, so urbanization was chosen as one important characteristic to be considered. The speed with which runoff accrues is certainly dependent upon the slope of the channel as well as the distance traveled. Thus, the time of concentration is seen to be directly related to length and inversely related to the channel slope. Therefore, as in the previous case, the term $(L/\sqrt{S})$ was chosen as the most significant combination of basin topographical characteristics. The following relationships were derived:

Charlotte:

$$X = 4.61 - .70\log(\frac{L}{\sqrt{S}}) - .26I$$

Morganton-Lenoir:

$$X = 1.02 - 2.14\log(\frac{L}{\sqrt{S}}) + .008I$$

Winston Salem-Greensboro:

$$X = 3.31 - .12\log(\frac{L}{\sqrt{S}}) - .16I$$

The magnitude of the coefficient of determination in each case was determined as:

Charlotte:

$$R^2 = 93.8\%$$

Morganton-Lenoir:

$$R^2 = 79.3\%$$

Winston Salem-Greensboro:

$$R^2 = 47.6\%$$

It will be observed from the relatively small magnitude of the coefficient of I for the Morganton-Lenoir area, that the significance of that term is highly questionable in this case. That observation was borne out by statistical significance tests while the coefficients for the urbanization term appeared to be fairly significant in the other cases.

As in the case of the storage factor, the general lack of data severely restricts the validity of the model in each case. Table 7 presents the comparison between the observed and predicted values of the X parameter as well as the calculated confidence limits. Again, the width of the confidence limits in each case prohibits the placement of a great amount of confidence in the model. The significance of the value of the coefficient of determination is again suspect in each case due to the small size of the data sample. Again, as in the case

TABLE 7

PREDICTED VS. OBSERVED VALUES OF X

Obs	Observed Value	Predicted Value	Residual	Lower 95% CL for Mean	Upper 95% CL for Mean
			Charlotte		
1	.59	.33	.25	-1.35	2.02
2	2.01	1.71	.29	.68	2.75
3	2.88	3.03	-.15	.88	5.18
4	4.06	3.89	.16	1.87	5.92
5	.81	1.37	-.56	.02	2.73
			Morganton - Lenoir		
1	1.11	1.23	-.12	.36	2.10
2	.60	.84	-.24	- .20	1.89
3	1.53	1.11	.41	.28	1.94
4	1.78	1.84	-.06	.36	3.31
5	.53	.52	.01	-1.00	2.03
			Winston Salem - Greensboro		
1	2.47	2.92	-.45	1.05	4.79
2	2.00	2.31	-.31	.16	4.45
3	2.99	2.97	.02	1.65	4.29
4	2.91	2.42	.49	.60	4.24
5	3.40	3.15	.25	1.28	5.03

of the Y parameter, the data was grouped according to geographical location because of its natural tendency to fall into such groups and because no significant regression analysis could be conducted on the data as a whole. However, due to the lack of sufficient data in each case, it again was not possible to statistically separate the three curves.

ANALYSIS OF RESULTS

The analysis of the results presented in the last section is necessarily hampered by the lack of sufficient data in each case to make any conclusions which can be rigorously adhered to. However, a few tentative conclusions can be drawn from the results, subject to alteration by later investigations.

The reduction of the data into groups by geographical location is here defended on the basis of the results presented in this thesis. In the cases of both parameters the data appeared to be naturally inclined to fall within such groups. This fact is probably due to both topographical and climatological reasons. Table 8 presents a comparison of the topographical characteristics of the basins comprising the three regions. From an inspection of this table it can be seen that the basins comprising the Morganton-Lenoir area are significantly different from those taken from the other two areas. This difference is most marked in the comparisons of the slopes of the three areas. The average slope of the Morganton-Lenoir basins is 39.9 ft/mile compared to 18.5 and 20.6 for Charlotte and Winston Salem-Greensboro respectively. This fact, combined with the smaller average channel length for these basins leads to a $(L/\sqrt{S})$ ratio significantly smaller for the Morganton-Lenoir area than the corresponding ratios for the other two regions.

It can also be observed from Table 8 that the basins taken from the Charlotte area have a generally milder slope than those

TABLE 8

COMPARISON OF BASIN CHARACTERISTICS

FOR THE THREE GROUPINGS

Sta. #	Area (mi^2)	Length (mi)	Slope (ft/mi)	Impervious Area (%)	$L/\sqrt{S}$
		Morganton - Lenoir			
1392.0	7.86	5.90	55.0	1	.80
1396.1	8.26	6.56	28.0	3	1.24
1396.5	8.94	6.06	43.7	2	.92
1411.9	4.40	3.14	54.9	2	.42
1411.5	31.80	8.11	18.0	13	1.91
Avg.	12.25	5.95	39.9	4.2	1.06
		Charlotte			
1465.0	41.0	11.5	16.2	15	2.86
1464.5	18.5	9.03	14.8	10	2.35
1467.0	6.98	5.06	25.3	6	1.00
1466.0	38.3	8.75	21.9	2	1.87
1463.0	30.5	11.4	14.2	11	3.02
Avg.	27.05	9.15	18.48	8.80	2.20
		Winston-Salem - Greensboro			
1159.0	42.2	12.2	13.1	2	3.37
1158.0	11.8	10.6	28.4	6	1.99
0990.0	14.7	6.36	21.0	2	1.39
0955.0	37.0	15.3	9.92	5	4.86
0860.0	4.71	5.06	30.6	1	.91
Avg.	22.08	9.90	20.60	3.20	2.50

from the other two regions. The Charlotte basins are generally larger and the streams more mature than those contained in the basins from the other two study areas. However, the most marked difference is in the degree of urbanization present in each case. The Charlotte area is a much larger urban center than the Morganton-Lenoir or Winston Salem-Greensboro areas. Thus a far greater amount of urban development is present in the basins taken from Charlotte than in those comprising the other two areas.

An indication of the topographical differences between the basins comprising the various groups might also be obtained by analysis of the final optimum values of the loss rate parameters calculated in the unit hydrograph computations. A comparison for these parameters for the three regional groupings is presented in Table 9. From analysis of the contents of this table it appears that there is no significnat differences in the optimized values of DLTKR, STRKR, or ERAIN for the three areas. However, the basins located in the Winston Salem-Greensboro area appear to have significantly larger values of RTIOL than the basins comprising the other two areas. It will be recalled that RTIOL represents essentially the slope of the exponential loss rate function and thus is the most important of the loss rate paramters. This fact indicates that the infiltration capacity of the soils constituting the basins in this area declines at a much sharper rate than for the basins contained in the other two areas. The extreme variation in the value of RTIOL for the Winston Salem-Greensboro area from that of the other two areas constitutes another supporting argument for separating this

TABLE 9

OPTIMIZED VALUES OF LOSS RATE PARAMETERS

Station #	DLTKR	STRKR	RTIOL	ERAIN
		Charlotte		
1465.0	1.33	.15	2.0	.50
1464.5	1.52	.20	1.5	.50
1464.7	2.0	.50	1.5	.50
1467.0	.75	.13	2.0	.50
1466.0	1.20	.20	2.0	.50
Avg.	1.36	.24	1.80	.50
		Morganton - Lenoir		
1396.5	1.27	.50	1.88	.54
1396.1	1.88	.38	1.59	.50
1392.0	2.19	.58	2.16	.59
1411.5	1.27	.60	1.92	.61
1411.9	2.07	.51	3.86	.55
Avg.	1.73	.51	2.28	.56
		Winston Salem - Greensboro		
0860.0	.20	.20	--	.49
0955.0	.96	.20	3.08	.50
0990.0	1.61	.20	29.18	.47
1158.0	1.77	.25	3.72	.48
1159.0	1.00	.31	2.88	.48
Avg.	1.11	.23	9.71	.48

data set, at least, from the other two.

Climatological factors may also play an important role in the sensitivity of the model to geographical location. The Charlotte area is located in the south-central part of the Piedmont, Winston Salem-Greensboro, in the western highlands and Morganton-Lenoir is in the extreme western portion, almost into the western mountains. Even though they all three fall within a circle whose radius is 80 miles, they are nevertheless subject to different types of storms. Hurricanes that come inland on the Gulf Coast frequently degenerate into tropical storms which follow the mountain chain and affect Morganton and Lenoir severely and Winston Salem and Greensboro to a lesser degree. Charlotte is very rarely affected by this type of storm activity.

The near proximity of the mountains is undoubtedly a factor in storm behavior in the Morganton-Lenoir area while Charlotte and Winston Salem-Greensboro are relatively unaffected by the presence of the mountains. Having performed several hydrologic investigations based within the state of North Carolina, the author is aware of these climatological factors from personal experience.

The degree of sensitivity of the model to geographical location is a matter which is presently open to question. It is believed that the relatively low reliability of the regression equations for the Winston Salem-Greensboro area may be due to an error in combining the data from these two cities. It could be possible that, were sufficient data available, better results could be obtained by separating the data into groups for Winston Salem and Greensboro. Information was not available in the present investigation to adequatley test that hypothesis. However, the distance between the various gages was far greater in this case than for Morganton-Lenoir or Charlotte, where the stations were located in a relatively small area. It can only be stated at the present time, that additional investigations, with far more data than were available in the present case, are needed to make this determination.

The failure to find urbanization to be a significant factor in the determination of the storage constant was an unexpected development. In no case in the present investigation did the addition of percent imperviousness to the analysis for the determination of Y significantly increase the accuracy or reliability of the model. From the present study, it would appear that urbanization factors critically affect the X parameter, and thus the time distribution of runoff up to the peak, but have an insignificant effect on the storage capacity of the basin. It is known that the channel improvements which accompany urban development tend to increase the hydraulic efficiency of the channel and thus should increase the rate of discharge from the basin. Thus, the time required for release of stored water from a particular channel is decreased with increased urban development. This idea may provide one key to the failure in the present investigation. If the information were available, the urbanization data should be broken into groups of various types of channel improvements, i.e., sewered, partly sewered, natural, etc. It is believed that this type of an analysis would lead to better results in determining the affect of urban development on the storage constant.

The same type of an investigation would probably facilitate the determination of urban effects on the time distribution of runoff as well. The success of this determination in the present study may be partly due to the fact that urban development has a more obvious and drastic effect on the basin response time, as adequately shown by previously referenced investigators, than on the storage capacity. Still, better results could be obtained by determining the distribution of urban development throughout the basin, since this distribution would obviously affect the time-area concentration curve and therefore the X parameter. The failure to obtain a significant relationship in the Winston Salem-Greensboro case could well be due to this reason, especially when consideration is given to the relatively large size of two of the basins in that area.

An example of the use of the model in predicting the effects of urban development on basin response is presented in Figure 13. This figure presents unit hydrographs as predicted from the derived relationships and the gamma model for Briar Creek at Sharon Road for various degrees of urbanization. It can be seen from the figure that the first incremental increase in urban development, from 5% impervious area to 10% caused a less significant alteration of basin response than did the next step from 10% to 15% impervious area. This increase in urbanization had the effect of nearly doubling the unit graph peak

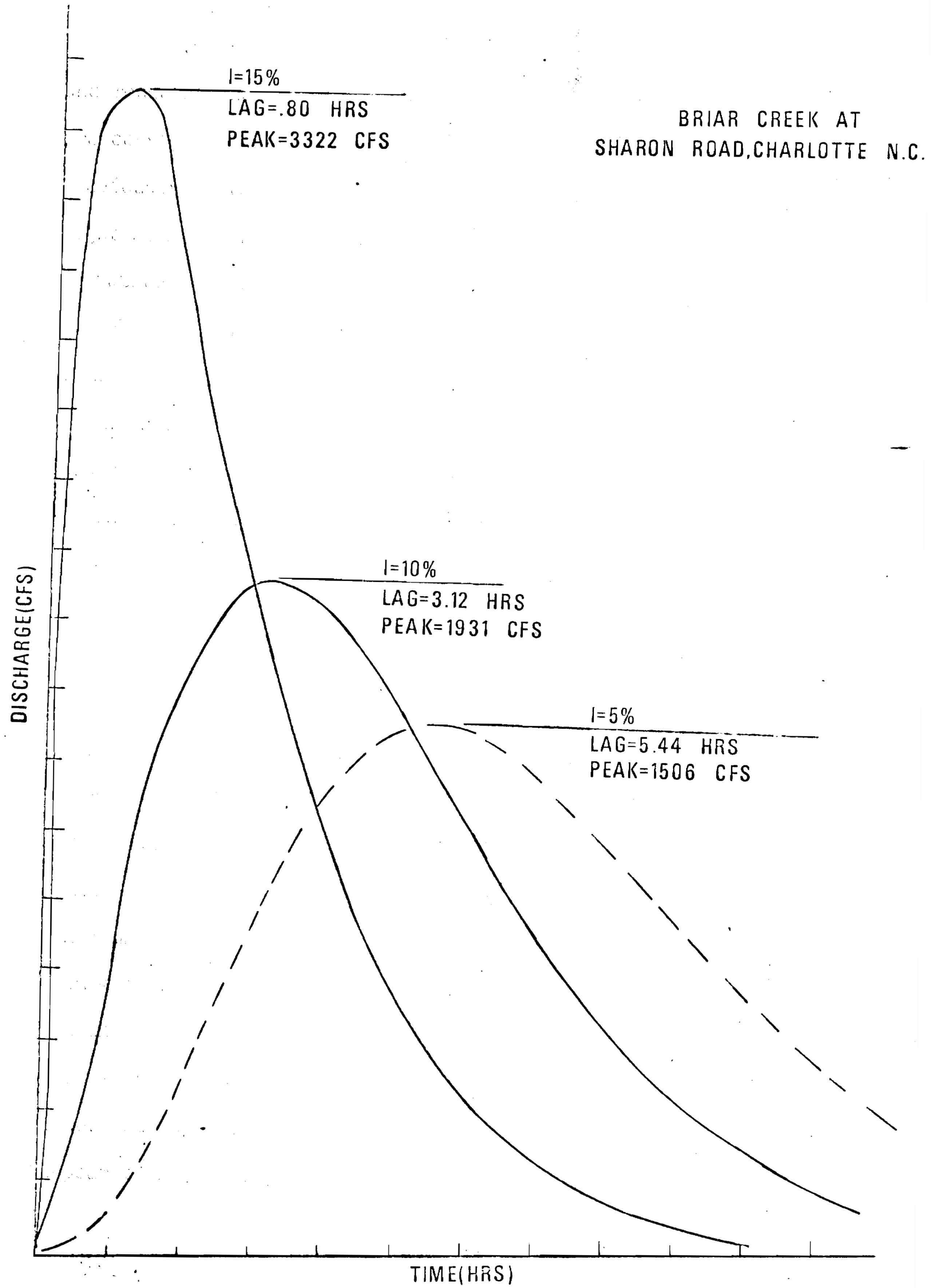

FIG. 13: URBANIZATION EFFECTS ON A TYPICAL BASIN

and reducing the lag time by 75%. The results presented in this figure are consistent with those presented by the other investigators mentioned previously in this report. It is the opinion of the author that much use might be made of the model and the investigative approach outlined above in future urbanization studies.

The lack of sufficient data to obtain regression equations with a high degree of reliability severley limits the confidence placed in any of the relationships derived from that data. Unfortunatly, sufficient data will very rarely be available at any one geographical location to obtain any relationships which have a high degree of reliability. For this reason, an investigation was conducted to determine the sensitivity of calculated peaks and lags to the possible error in values of X and Y derived from the regression equations. For station 1392.0, the values of X can be assumed to lie between .36 and 2.10 and Y to vary from .35 to .63, both with 95% confidence. In other words, 95 out of every 100 values observed will lie within those regions. Now, from this information, the lag would be expected to vary from 1.02 to 3.33 hours and the unit graph peak from 118.28 to 122.48 cfs/sq. mile. By the same analysis it was determined that for station 1159 the lag varied from 6.17 to 7.15 hours and the peak from 40.42 to 65.35 cfs/ sq. mile. For station 1396.5 the figures are: Lag - .93 to 3.46, Peak - 196.86 to 115.16. Considering the large variation between the lower and upper values of X and Y in these cases, the model does not appear to be unduly sensitive to the reliability of the regression equations. With more data available, the accuracy of the model in predicting values of lag and peak could be significantly improved.

The results of a similar investigation, one which compares unit hydrographs derived from the model with the actual unit graphs calculated from rainfall and discharge data are presented in Figures 14 and 15. Figure 14 presents the comparison of actual versus predicted unit graphs for Irwin Creek in Charlotte, N.C. This figure represents results which were about average for the study. It can be seen that the model yields a graph which represents a fair average between the two observed graphs utilized in the study.

On the other hand, Figure 15 presents the results of the comparison for Accotink Creek in Accotink Station, VA., one of the most troublesome basins included in the investigation and one which was eventually dropped from the study. It can be observed from the figure that no accurate convergence could be obtained between the various unit graphs utilized for this basin. For the purpose of continuing with this subsidiary investigation, model values were used which were weighted in favor of the September and October storms of 1960. The figure shows that the model yielded a graph which appears to be a fair weighted average of the three graphs with more weight given to the two storms mentioned above.

From observations of the results presented above, it appears that the model is proficient in reproducing an average unit graph for the basin in which it was applied, at least within the accuracy of the

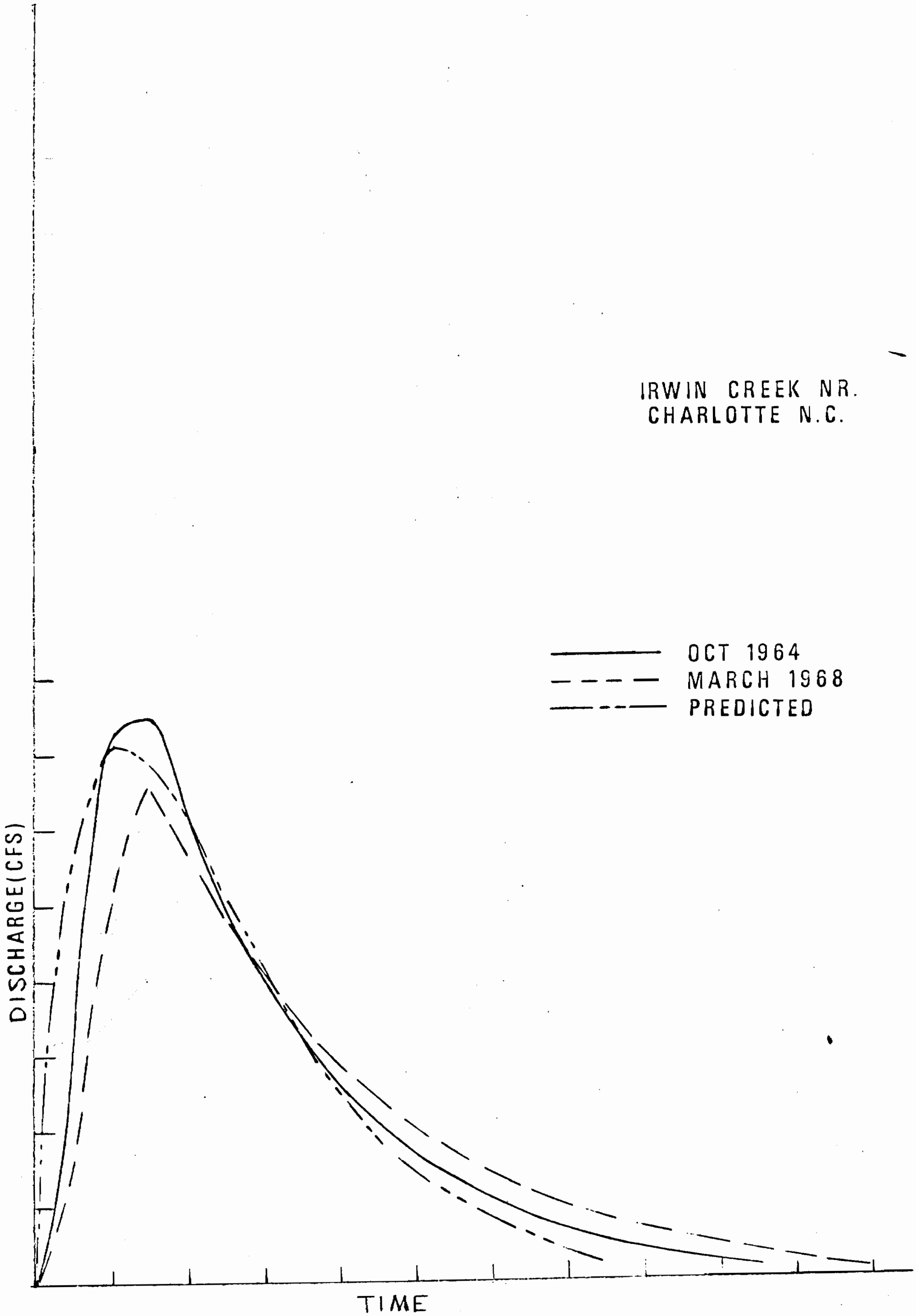

FIG. 14: REPRODUCTION OF DERIVED UNIT GRAPHS

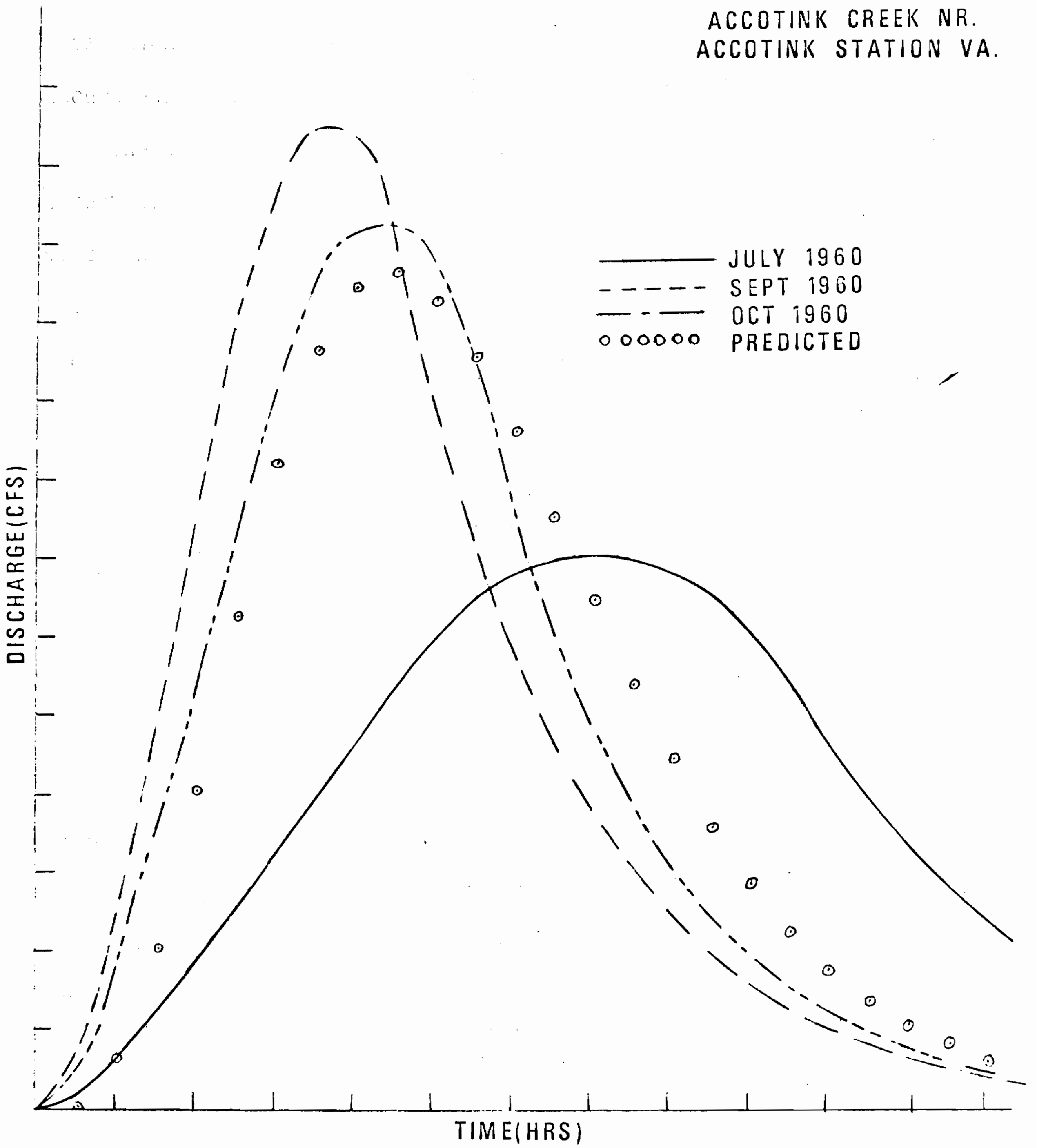

FIG. 15: REPRODUCTION OF DERIVED UNIT GRAPHS

data which was used for its derivation. With more data available, more accurate results could undoubtedly be obtained.

The sensitivity of the model to geographical location has the effect of disqualifying it for use as a synthetic unit graph method since the premise of these methods is the ability to transpose relationships derived at one location to another location. However, much use could be made of this approach in gaining a better determination of which basin characteristics have the most significant effect on the various parts of the runoff hydrograph. From the present investigation, it appears that the $(L/\sqrt{S})$ term adequately accounts for the effects of topographical basin characteristics on both sides of the hydrograph, since a significant regression relationship was derived involving that term in most cases. That is, the relationship was significant within the limits of the data available. No other characteristic or combination of characteristics when used in the analysis gave nearly as good results as that term. It would therefore appear from this study that length, slope, and urbanization are the most important factors present within a given basin which affect the time distribution of runoff.

CONCLUSIONS

From the present investigation the following conclusions are drawn:

1) When use is made of the gamma model for determining the effects of basin characteristics on unit hydrograph parameters, the data tend to reduce to groups according to geographical location. This means that the model is extremely sensitive to small variations in topography - more so than any of the comparable methods presently in use. The effects of this localization of the data are twofold: first, the difficulty in obtaining sufficient data at any one location to derive significant relationships is critical if not insurmountable, and second, the use of the model as a synthetic unit hydrograph method is not encouraging.

2) The model can be extremely useful, given sufficient data in determining the effects of various basin characteristics on the separate parts of the runoff hydrograph. In no other comparable model presently in use is it possible to break the unit hydrograph into its component parts and analyze topographical effects on each part. It is here recognized that the effects of both storage and the speed with which runoff accrues is felt from the beginning of runoff; however the shape

of the time-area concentration curve is most significant to the rising side of the hydrograph and the storage constant is dominant in determining the recession limb.

3) The most significant topographical factors affecting the runoff hydrograph are length of the watercourse and channel slope. A completely adequate regression equation can be obtained by the use of these two characteristics in the ratio $(L/\sqrt{S})$.

4) The possibility exists that urban development has a more significant effect on the speed with which runoff accrues, and thus the lag, than on the storage capacity of the basin. However, this is a tentative conclusion at best, and more study is needed to prove or disprove it.

REFERENCES

1. Snyder, F. F., "Synthetic Unit Graphs," Transactions American Geophysical Union, Part I, 1938. p. 447.

2. Taylor, Arnold B. and Harry E. Schwarz, "Unit Hydrograph Lag and Peak Flow Related to Basin Characteristics," Transactions American Geophysical Union, Vol. 33, No. 2, April 1952.

3. Gray, Donald M., "Interrelationships of Watershed Characteristics," Journal of Geophysical Research, Vol. 66, 1961(a).

4. Gray, Donald M., "Synthetic Unit Hydrographs for Small Watersheds," Proceedings A.S.C.E., Vol. 87, No. HY4, 1961(b).

5. Edson, Charles G., "Parameters for Relating Unit Hydrographs to Watershed Characteristics," Transactions American Geophysical Union, Vol. 32, 1951. p. 591.

6. Nash, J. E., "Systematic Determination of Unit Hydrograph Parameters," Journal of Geophysical Research, Vol. 64, 1959.

7. Potter, W. D., "Rainfall and Topographic Factors that Affect Runoff," Transactions American Geophysical Union, Vol. 34, 1953.

8. Getty, H. C. and J. H. McHughs, "Synthetic Peak Discharges for Design Criteria," Journal of Hydraulics Division, A.S.C.E., Vol. 88, No. HY5, Paper 3255, Sept. 1962.

9. Morgan, P. E. and S. M. Johnson, "Analysis of Synthetic Unit Graph Methods," Journal of Hydraulics Division, A.S.C.E., Vol. 88, No. HY5, Paper 3279, Sept. 1962.

10. Eaglesen, Peter S., "Unit Hydrograph Characteristics for Sewered Areas," Journal of Hydraulics Division, A.S.C.E., Vol. 88, No. HY2, Paper 3069, March 1962.

11. Espey, William H., David E. Winslow and Carl W. Morgan, "Urban Effects on the Unit Hydrograph," Effects of Watershed Changes on Streamflow, W. L. Moore and C. W. Morgan, Editors, 1969.

12. U.S. Army Engineer District, Baltimore District, Unit Hydrographs, Part 1, Principles and Determinations, Civil Works Investigations - Project 152, 1963.

13. U.S. Army Corps of Engineers, EM 1110-2-1405, Flood Hydrograph Analyses and Computations, Aug. 1959.

14. Bensen, M. A., "Spurious Correlation in Hydraulics and Hydrology," Journal of Hydraulics Division, A.S.C.E., Vol. 91, No. HY4, July 1965. p. 35.

15. Bras, Rafael L., "Effects of Urbanization on Catchment Response," Journal of Hydraulics Division, A.S.C.E., Vol. 101, No. HY3, March 1975.

16. Betsen, R. F. and R. F. Green, "Analytically Derived Unit Graphs and Runoff," Journal of Hydraulics Division, A.S.C.E., Vol. 94, No. HY6, Nov. 1968.

17. Barnes, B. S., "Consistancy in Unit Hydrographs," Journal of Hydraulics Division, A.S.C.E., Vol. 85, No. HY8, Aug. 1959. p. 55.

18. Carter, R. W., "Magnitude and Frequency of Floods in Suburban Areas," U.S.G.S. Professional Paper 424-B, pp. 9-11.

19. Nash, J. E., "The Form of the Instantaneous Unit Hydrograph," C. R. et Rapports, Assn. International Hydrologic I.U.G.G., Toronto, 1957, Gentbrugge, 3, 1958.

20. Rao, Ramachandra A., and J. W. Delleur, "Instantaneous Unit Hydrographs, Peak Discharge, and Time Lags in Urban Basins," Bulletin Scientific Hydrology, Vol. 19, No. 2, June 1974.

21. Rao, Ramachandra, A., J. W. Delleur and P. B. S. Sarma, "The Effects of Urbanization on Runoff," Journal of Hydraulics Division, A.S.C.E., Vol. 98, No. HY7.

22. Clark, Charles O., "Storage and the Unit Hydrograph," Transactions of the A.S.C.E., Vol. 110, 1945.

23. Sherman, Leroy K., "Streamflow from Rainfall by the Unit Graph Method," Engineering News Record, Vol. 108, 1932.

24. Putnam, Arthur L., Effect of Urban Development on Floods in the Piedmont Province of North Carolina, U.S. Geological Survey Open-File Report, 1972.

25. Andersen, Daniel G., Effects of Urban Development on Floods in Northern Virginia, U.S. Geological Survey Water-Supply Paper 2001-C, 1970.

26. Chow, V. T., Ed., Handbook of Applied Hydrology, McGraw-Hill Book Company, New York, 1964.

27. Martens, L. A., Flood Inundation and Effects of Urbanization in Metropolitan Charlotte, North Carolina, U.S. Geological Survey Water-Supply Paper 1591-C, 1968.

28. Linsley, R. K., Kohler, M. A. and J. L. Paulhus, Hydrology for Engineers, McGraw-Hill Book Co., Inc., New York, 1958.

29. McCarthy, G. T., "The Unit Hydrograph and Flood Routing," N.A.D., U.S.A.E.D., Providence, Rhode Island, 1939.

30. Conte, S. D. and Carl de Boor, Elementary Numerical Analysis: An Algorithmic Approach, McGraw-Hill Book Company, New York, 1965.

31. Rohlf, F. James and Robert R. Sokal, Statistical Tables, W. H. Freeman and Company, San Francisco, 1969.

32. U.S. Army Corps of Engineers, Hydrologic Engineering Center, HEC-1 Flood Hydrograph Package Users Manual, January 1973.

33. Viessman, Warren, Terence E. Harbaugh and John W. Knapp, Introduction to Hydrology, Intext Educational Publishers, New York, 1972.

34. U.S. Army Corps of Engineers, Charleston District, Reddies River Lake Project, Design Memorandum No. 2, Hydrology and Hydraulics Analysis, December 1974.

35. Barr, Anthony J., James H. Goodnight, John P. Sall, and Jane T. Helwig, A User's Guide to SAS 76, SAS Institute, Inc., Raleigh, North Carolina, 1976.

VITA

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James Cruise

A SOLUTION OF THE TWO-PARAMETER GAMMA MODEL

TO RELATE UNIT HYDROGRAPH FEATURES

TO BASIN CHARACTERISTICS

by

James Franklin Cruise

(ABSTRACT)

The problem of correlating unit hydrograph features to topographic and man-made basin characteristics received attention in this report. The unit graph features considered herein were the peak discharge and the time lag of basin response. In order to facilitate the desired regression analysis, the two-parameter gamma model proposed by Edson was utilized in the investigation. The parameters of the model were obtained by the simultaneous solution of the equations for unit graph peak and lag using observed unit hydrographs for 16 basins in the Piedmont region of North Carolina and 14 basins located in Northern Virginia. In the opinion of many, these parameters are a better measure of the complex relationship which exists between the runoff from a basin and the topographic features of that basin than are the values of the unit graph peak and lag time themselves.

The basin characteristics utilized in the investigation were: basin area, length of the longest streamcourse in the basin, average stream slope between points 10 per cent and 85 per cent downstream of the headwaters, and the per cent of impervious area contained in

the basin. This last factor served as a measure of the amount of urban development present in the watershed.

The investigation was hampered by a regrettable lack of sufficient data to derive regression equations of good reliability. This fact was due to the reduction of the data into groups by narrow geographical ranges. Thus, the number of stations available for analysis in any one group was insufficient for purposes of a reliable regression analysis.

From the investigation, it appears that the most significant basin characteristics affecting runoff are length, slope, and urban development. The strongest regression equations were derived using those three characteristics. It appears that the length and slope factors give better results when combined in the form $(L/\sqrt{S})$.