MIXED-INTEGER MATHEMATICAL PROGRAMMING OPTIMIZATION MODELS AND ALGORITHMS FOR AN OIL TANKER ROUTING AND SCHEDULING PROBLEM

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(Abstract)

This dissertation explores mathematical programming optimization models and algorithms for routing and scheduling ships in a maritime transportation system. Literature surveyed on seaborne transportation systems indicates that there is a scarcity of research on ship routing and scheduling problems. The complexity and the overwhelming size of a typical ship routing and scheduling problem are the primary reasons that have resulted in the scarcity of research in this area.

The principal thrust of this research effort is focused at the Kuwait Petroleum Corporation (KPC) Problem. This problem is of great economic significance to the State of Kuwait, whose economy has been traditionally dominated to a large extent by the oil sector. Any enhancement in the existing ad-hoc scheduling procedure has the potential for significant savings.

A mixed-integer programming model for the KPC problem is constructed in this dissertation. The resulting mathematical formulation is rather complex to solve due to (1) the overwhelming problem size for a typical demand contract scenario, (2) the integrality conditions, and (3) the structural diversity in the constraints. Accordingly,

attempting to solve this formulation for a typical demand contract scenario without resorting to any aggregation or partitioning schemes is theoretically complex and computationally intractable.

Motivated by the complexity of the above model, an aggregate model that retains the principal features of the KPC problem is formulated. This model is computationally far more tractable than the initial model, and consequently, it is utilized to construct a good quality heuristic solution for the KPC problem.

The initial formulation is solved using CPLEX 4.0 mixed integer programming capabilities for a number of relatively small-sized test cases, and pertinent results and computational difficulties are reported. The aggregate formulation is solved using CPLEX 4.0 MIP in concert with specialized rolling horizon solution algorithms and related results are reported. The rolling horizon solution algorithms enabled us to handle practical sized problems that could not be handled by directly solving the aggregate problem.

The performance of the rolling horizon algorithms may be enhanced by increasing the physical memory, and consequently, better solutions can be extracted. The potential saving and usefulness of this model in negotiation and planning purposes strongly justifies the acquisition of more computing power to tackle practical sized test problems.

An ad-hoc scheduling procedure that is intended to simulate the current KPC scheduling practice is presented in this dissertation. It is shown that results obtained via the proposed rolling horizon algorithms are at least as good, and often substantially better than, results obtained via this ad-hoc procedure.

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Chapter I

Introduction

1.1 An Overview

Water transportation is one of the major transportation modes in the world. Data compiled by Ronen (1993) from Lloyd's Register (1991) indicates that over one year, the world merchant fleet increased by 12.4 million gross tons. The world fleet is made up of 80,030 ships (each over 100 gross tons) of different types (Ronen, 1993). The world seaborne trade achieved an increase of 16% in 1991 over 1981 and the distance weighted trade grew by 11% in 1990 over 1981 (Ronen, 1993). These facts highlight the reliance of the world economy on seaborne trade and hence emphasize the need for efficient and reliable maritime transportation systems.

Routing and scheduling of ships is the most elaborate and significant level of planning of fleet management in any maritime transportation system. In this process one should properly assign shipments to vessels and decide on the route a vessel should take. Furthermore, one needs to efficiently determine ship size, ship speed, shipment size, fleet size, number of time-chartered vessels, number of spot-chartered vessels, whether it is lucrative to perform a spot-charter for another operator, and so on. Efficient routing and scheduling of ships has the potential of enormous savings in the total fleet operation costs. This is true, especially when taking into account the facts that a typical ship in a merchant fleet usually costs millions of US dollars and the daily operating costs of a ship amounts to tens of thousands of US dollars (Ronen, 1983,1993).

In spite of the vital role of routing and scheduling of ships in any maritime transportation system, ship routing and scheduling problems have not attracted adequate attention of researchers in the past. The complexity of a typical ship routing and scheduling problem and the high degree of uncertainty involved in ship operations have tremendously contributed to the scarcity of research in this area. In contrast, vehicle routing and scheduling problems have been extensively examined in the literature (Bodin et al., 1983). In fact, routing and scheduling of vehicles is deemed to be a major success story, mainly because of efficient modeling and implementation and the close connection between theory and practice (Assad, 1988). Unlike the rail industry, which is seriously looking forward to enhancing its operational efficiency and rationalizing its planning decisions further (Assad, 1980), the ocean shipping industry is conservative and reluctant to adopt new ideas. As a matter of fact, most of the existing quantitative models were developed in vertically integrated organizations wherein ocean shipping is only one aspect of the business (Ronen, 1983).

Water transportation is of special importance to many oil exporting and importing countries in the world, where oil and oil-related products are mainly moved by ships. For the United States, for example, which is the world's largest energy consumer, seaborne petroleum imports are a necessary lifeline to the oil deposits of the Middle East, East Asia, and Latin America (Warf, 1989). Many countries, for instance the Arabian Gulf countries, own fleets of oil tankers. The daily operational cost of a typical oil tanker amounts to tens of thousands of dollars. Therefore, the efficient utilization of such a tanker becomes necessary. This dissertation is concerned with developing and testing routing and scheduling models, along with related algorithms, for Kuwait Petroleum Corporation (KPC) that can enhance the performance of their existing ad-hoc manual scheduling procedure.

1.2 Keywords and Terminology

Routing: The process of assigning a sequence of ports of call to vessels (Ronen, 1983).

Scheduling: The process of specifying time windows for the different activities on a ship's route (Ronen, 1983).

Short Term, Medium Term, and Long Term Scheduling: Short term scheduling is for up to several weeks, medium term scheduling is for up to several months, and long term scheduling is beyond that (Ronen, 1983).

Voyage (Spot) Charter: A vessel is said to be spot chartered when it is chartered for a particular voyage or voyages, not for a period of time. In such a situation all operating costs are undertaken by the vessel owner (Rana and Vickson, 1988).

Time Charters: A vessel is said to be time chartered when it is chartered for a specified period of time (Branch, 1981).

Fleet Deployment: The process of allocating vessels to routes, assigning service frequencies and chartering (time chartered or spot chartered) of vessels, if needed, to supplement the owned fleet in order to meet the transportation requirements (Perakis, and Jaramillo, 1991).

Liner Operation: Liner operators bear many resemblances to domestic common carriers, where vessels travel on regularly scheduled service routes between a group of ports. Vessels in a liner travel according to specified dates, even when the available cargo is not a full shipload (Branch, 1981). A liner company announces the schedule of its transportation services and competes for cargo. Demand for a liner operator is affected by the liner's schedule, where it depends on the frequency of the transportation services provided as well as the ports visited. In a liner operation, a voyage origin and destination can hardly be identified; liners usually operate in closed routes and may load and discharge cargo in each port of call, and may never be empty (Ronen, 1983). Liners range anywhere from ordinary freighters, which are similar to the tramp vessels, to big cargo vessels that are particularly designed for the freight line service, to the biggest passenger-carrying vessels. Some liners specialize in carrying freight exclusively, others a combination of freight and passengers, and yet others carry passengers only (De Kerchove, 1961). The objective of a liner company is to maximize profits per time unit (Ronen, 1993).

Tramp Operation: Tramp is very similar to a taxicab operation. Ships are dispatched according to the availability of cargoes (usually bulk cargoes such as coal, grain, and ores) and not on a fixed sailing schedule (Branch, 1981). Usually, cargoes are full shiploads with a single loading port and one or two discharging ports. The objective of a tramp company is to maximize the profit per time unit (Ronen, 1983). It is worth mentioning that liner companies advertise for their scheduled services between a number of specified ports, whereas tramp operators do not. Furthermore, liners operate faster and more costly ships and undertake a more extensive network of cargo. Liners sometimes adjust their schedules and bid for bulk cargoes in competition with tramp operations (Lawrence, 1972).

Industrial Operation: In an industrial operation, both cargoes and vessels are controlled by one entity (Lawrence, 1972). The main purpose of an industrial operation is usually to assure transportation services for the company's cargo at a minimum expense (Mehrez et al., 1994). It is often the case that the industrial operation fleets do not meet the organization's fluctuating transportation requirements, and thus, the organization must resort to time and spot charters from independent owners, where vessels can be chartered in and out on an international exchange (Ronen, 1983).

KPC: Kuwait Petroleum Corporation .

1.3 KPC Oil Tanker Scheduling Problem

In this section, some information about KPC is presented and then a precise statement of the KPC Oil Tanker Scheduling Problem is given.

KPC is an integrated oil company that enjoys international recognition in the petroleum industry. It is the top company of the Kuwaiti government's operations in the petroleum sector. KPC incorporates a number of companies: Kuwait Oil Company (KOC), Kuwait National Petroleum Company (KNPC), Petrochemical Industries Company (PIC), Kuwait Oil Tanker Company (KOT), and Kuwait Foreign Petroleum Exploration Company (KFPEC). KOC is in charge of producing oil and gas, and KNPC is in charge of refining oil and promoting domestic sales. PIC handles the production of ammonia and urea, and KOT handles the transportation of oil and oil related products from Kuwait to different parts of the world. KOT owns 35 crude oil and refined carriers, making it the largest tanker company in the Organization of Petroleum Exporting Countries (OPEC). KFPEC is an investing company with several concessions in developing countries. KPC mainly markets its products in North America, Europe, and Japan (Background Notes, 1995).

Crude oil and a number of refined oil-related products are to be shipped from three ports in Kuwait to customers located in Europe, North America, and Japan. Deliveries of cargoes are undertaken either by KPC or by other shipping companies, according to contracts agreed upon by KPC and the other shipping companies. A shipment not undertaken by KPC has no impact on the scheduling process of KPC's fleet of vessels, where assuring the delivery according to a specified time frame is the responsibility of the shipping company. Nevertheless, such shipments may impose some restrictions on the loading ports in Kuwait. The decision of whether to use KPC's fleet or to use another shipping company for a given product is made by KPC.

Two classes of vessels are considered: the first is the fleet of vessels controlled by KPC and the second is the vessels that are spot chartered. KPC's controlled fleet consists of the company-owned vessels in addition to a number of leased vessels (time chartered vessels). Each voyage has a single origin (loading port) and a single destination (discharging port). Each cargo is a full shipload and is characterized by its type (oil, refined products, etc.), loading port, loading date, discharging port, and discharging date. Two routes are available for tankers. A tanker may be routed either through the Suez Canal, where canal dues are required, or around the Cape of Good Hope, making a much longer voyage. Loaded tankers cannot pass through the Suez Canal and therefore must go around the Cape of Good Hope. Furthermore, certain tankers are not permitted to enter certain ports or may be unavailable for scheduling during part of the planning horizon.

A penalty is incurred on shipments that are not delivered within the specified time frame. Different customers impose different penalties, where penalties are predetermined according to contracts agreed upon by the KPC and the customers. Since Kuwait is a member of OPEC, a predetermined exporting quota is assigned to it (usually 1.5 - 2.0 million barrels per day). In case this quota changes, KPC has to react accordingly to adjust for this change. If the quota increases, KPC may need to expand the existing controlled fleet or resort to spot chartering. If the quota decreases, KPC may want to reduce the size of the existing fleet.

KPC needs to satisfy demand requirements according to specified time windows. Usually KPC's controlled fleet cannot fulfill demand requirements, hence resorting to spot chartering.

<u>Elements of the problem are presented as follows:</u>

<u>The Operation</u>

- KPC-owned vessels
- KPC-leased vessels (time chartered vessels)
- Spot chartered vessels

- Shipping crude oil and refined products from three ports in Kuwait to Europe, North America, Japan, and some other countries
- Several discharging ports
- No return cargoes
- Shipments on a recurrent basis

The Vessels

- Each vessel is characterized by its size (capacity), speed, and the capability to carry different types of cargo
- No loaded vessels through the Suez Canal
- Dues incurred on vessels passing through the Suez Canal
- A vessel may not be available during part of the planning horizon
- vessels may be diverted at sea

The Ports

- Three loading ports in Kuwait
- Discharging duration for each port
- Ports' dues
- Ports' restrictions

The Shipments

- A shipment may consist of crude oil or refined oil-related products
- Each shipment is a full shipload

- A shipment is characterized by its type (crude oil or other oil related products), revenue, size, origin (loading port), destination (discharging port), loading date, and discharge date
- Certain shipments may be spot chartered

The Costs

- Daily cost of controlled vessels (KPC-owned vessels and time chartered vessels)
- Spot chartering cost
- Bunker fuel
- Auxiliary systems fuel
- Port entry charges and Canal dues
- Demurrage cost (cost of vessel's waiting time)
- Penalties associated with shipments that are not delivered within the specified time frame

The Customers

- Delivery time restrictions
- Customers' priorities

1.4 Objective and Significance of this Research

In general, this research is intended to highlight the importance of routing and scheduling of ships in any maritime transportation system. The principal thrust of the research effort is focused at the KPC Oil Tanker Scheduling Problem. This problem is of great economic significance to the state of Kuwait, which is a small country having a massive oil reserve amounting to about 77 billion barrels of recoverable oil. The amount of reserve Kuwait has is second only to Saudi Arabia, which has the largest reserve in the world.

Kuwait is a major exporting member of OPEC. The Kuwaiti economy has been traditionally dominated to a large extent by the oil sector, which provides well over 80% of Kuwait's national revenues. KOT owns 35 crude oil and refined product carriers, making it the largest tanker company in an OPEC country (Background Notes, 1995 & The World Economic Factbook, 1994/1995). Kuwait markets its products in Europe, the United States, Japan, and other countries in the world. The delivery of oil and oil-related products to customers is mainly undertaken by KOT, which is a branch of KPC. This dissertation is concerned with building a systematic scheduling scheme models and algorithms for KPC that can economically enhance its existing ad-hoc manual scheduling procedure. Certainly, any improvement in the existing scheduling scheme has the potential for significant savings.

1.5 Organization of Dissertation

This dissertation is outlined as follows. In Chapter II, we discuss some of the reasons that have contributed to a noticeable scarcity of research in seaborne transportation systems, and we present a literature review on ship routing , scheduling, and related problems. Operational issues of the KPC problem and a mixed integer programming model for this problem are presented in Chapter III. An aggregate reformulation for the KPC problem is given in Chapter IV. Solution strategies and algorithms, along with pertinent computational results are given in Chapter V. A summary of this research and relevant future research ideas are given in Chapter VI.

Chapter II

Literature Review

This chapter is composed of three sections. It commences with a discussion on some of the reasons that have resulted in a relative scarcity of research and shortcomings of realistic analytical models on ship routing, scheduling, and related problems in the past. In Section 2, a classification scheme for ship routing, scheduling, and related problems is presented. This scheme is very helpful to classify and identify a given ship routing and scheduling problem. It is also helpful to identify existing techniques and algorithms to solve it. Finally, the chapter is concluded with a literature review on ship routing, scheduling, and related problems. Other relevant routing and scheduling models are briefly discussed.

2.1 Scarcity of Research in Seaborne Transportation Systems

Transportation routing and scheduling problems have attracted extensive research in the past. Researchers have tried to investigate many real world problems attempting to build acceptable realistic mathematical models. The bulk of the published work deals with vehicle routing and scheduling problems. A comprehensive review of vehicle routing and scheduling problems and models was presented by Bodin et al. (1983). Other transportation modes such as air, rail, and sea have drawn considerably less attention despite the fact that an enormous capital and significant operating costs are involved in these modes. Ronen (1983) highlighted a noticeable scarcity of published work in designing, planning, and managing seaborne transportation systems and gave a number of impediments associated with ship scheduling problems as follows:

A. Seaborne shipping is a minor transportation mode in the United States, where most cargo is transported by trucks or trains. Numerous companies operate fleets of trucks, yet comparatively few companies operate fleets of vessels. In fact, much of the published work related to ship routing and scheduling problems was initiated in Europe.

B. Ship routing and scheduling problems are fairly complex and involve a large diversity in structure and operating environments.

C. A high degree of uncertainty is associated with vessel operations. A vessel may be delayed or rerouted due to severe weather conditions, mechanical problems, or strikes (both on board and on shore). Such delays or rerouting may incur a tremendous capital loss. Levy et al. (1977) investigated the schedule performance of commercial vessels and concluded that the probability of meeting a quarterly schedule (about three voyages) is only about 30%. Delays in loading and unloading cargoes that result from manpower problems at ports and breakdowns of ports' facilities are other sources of uncertainty (Rana and Vickson, 1991).

D. The long existing traditions of the ocean shipping industry and the fact that ships have been around for centuries make the shipping industry reluctant to adopt new ideas.

Ronen (1993) re-investigated work published during the period 1983-1993 on ship routing and scheduling problems. He concluded that even with the rapid development of computing power that may facilitate optimal solutions to ship routing and scheduling problems, relatively little research has been done on ship scheduling and this situation has not altered significantly since 1983.

2.2 Classification Scheme for Ship Routing and Scheduling Problems

In the sequel, a slightly modified classification scheme to the one proposed by Ronen (1983) is presented. This scheme is quite comprehensive, but by no means exhaustive.

Different problems may involve different twists.

A. Mode of Operation

1. Liner
2. Tramp
3. Industrial
4. Other Modes (Naval, Barge, Coast Guard and Fishing)

B. Loading and Discharging Times

1. Specified (ship scheduling problem)
2. Time windows
3. Open (routing problem)

C. Number of Origins

1. One
2. Multiple

D. Number of unloading Ports

1. One
2. Multiple

E. Number of Loading Ports per Vessel Voyage

1. One
2. Multiple

F. Number of Discharging Ports per Vessel Voyage

1. One

2. Multiple

G. Number of Products to be Shipped

1. One
2. Multiple

H. Sizes of Vessels

1. One
2. Multiple

I. Vessels' compartments Capacities

1. One
2. Multiple

J. Type of Vessels

1. One
2. Multiple

K. Status of Vessels

1. Owned
2. Time Chartered
3. Spot Chartered

L. Demand Structures and Time windows

1. Deterministic
2. Stochastic (continuous , discrete)

M. Cruising Speed as a Decision Variable

1. Yes
2. No

N. Fleet Size and Composition

1. Specified and cannot be changed (short term problem)
2. Can be changed (medium term problem)
3. Constant over a scheduling period
4. Changes permitted over a scheduling period

O. Port Entry Restrictions on Vessels

1. Exist
2. None

P. Sea Route Restrictions on Vessels

1. Exist
2. None

Q. Costs

1. Fixed costs (operating cost and capital cost)
   a. In operation
   b. In lay-up
   c. Change of status cost
2. Variable costs
   a. Steaming costs
   b. Port entry charges
   c. Time in ports

    d. Unit shipping cost
    e. Demurrage (cost of vessel's waiting time)
3. Penalties incurred by late shipments

R. Objectives

1. Minimize costs
2. Maximize profits
3. Maximize utility

S. Cargo Transshipment

1. Allowed
2. Excluded

T. Time between Events

1. Deterministic
2. Stochastic

U. Other Problem-Specific Characteristics

1. Some customers stipulate that shipments of products are to be made based on specified minimum and maximum desired storage levels
2. Some customers specify in advance the times and quantities of shipments of products
3. A customer may not except a shipment of a certain product if the compartments carrying such product have carried another specific product in the previous trip. For example, some customers do not except shipments of Naphtha if the compartments carrying this product have carried crude oil in the previous trip.

2.3 Literature Review

Published work on ship routing, scheduling, and related problems has been fairly sparse. Although a spectrum of ship routing and scheduling problems have been investigated, no specific problem has been the focus of researchers. Researchers in the last decade have utilized a range of OR techniques and the powerful state-of-the-art computers to tackle ship scheduling problems.

In the following, a survey of published work and research on ship routing and scheduling problems is presented. Pertinent articles surveyed in Ronen (1983) and (1993) are also discussed. Relevant research on other modes of transportation (land and air) is briefly touched upon. The survey is divided into three categories: (1) ship routing and scheduling problems, (2) relevant land transportation problems, and (3) relevant air transportation problems. Categories 2 and 3 are briefly touched upon and they are not intended to provide a comprehensive review of land and air routing and scheduling transportation models. They are presented to provide some insights on some of the existing techniques and algorithms employed to approach relevant routing and scheduling problems.

2.3.1 Ship Routing and Scheduling Models

This category is further divided into four ship routing and scheduling sub-categories: (1) liner, (2) tramp, (3) industrial, and (4) other models. The first three modes (liner, tramp, and industrial) of operation are not sharply defined nor mutually exclusive (Lawrence, 1972). The fourth sub-category deals with ship routing and scheduling problems whose operations cannot be clearly classified as liner, tramp or industrial. It is worth mentioning that water transportation routing and scheduling models mainly deal with the transport and delivery of cargoes. Very few models have been developed to tackle ship routing and scheduling problems in the context of passengers. This is a natural consequence of the fact that most cargo in the world is moved by ships, while passengers are mainly moved by air or land.

2.3.1.1 Liner Operations

The literature on modeling techniques and approaches for liner shipping is fairly limited; however, in recent years an increased activity in this area has become evident (Lane et al., 1987). This type of maritime transportation has drawn little attention from researchers, at least in its quantitative aspects. This might be due to the nature of some of the dominant variables and factors that affect the operation of a liner company, such as some minimum required service frequencies, subsidies, and government regulations. Moreover, international liner shipping is barely regulated (Le Gendre, 1991). These factors have discouraged efforts towards a systematic approach to the analysis and optimization of liner transportation systems.

Due to the high degree of uncertainty associated with liner operations, the major modeling techniques utilized to tackle liner problems have been simulation and heuristic decision rules. Uncertainty stems primarily from the relatively large number of ports of call involved in a voyage and from cargo availability. Techniques such as linear, integer, or non-linear programming can be utilized to approach liner fleet deployment and scheduling problems, given that the cargo forecasts are reliable (Perakis and Jaramillo, 1991).

Datz et al. (1964) presented a simulation model for liner operations which produces a schedule based on the available cargo and its profitability. The model took into account the probability that a “promised” cargo could be canceled (Perakis and Jaramillo, 1991). Simulation was also used by Kydland (1969) and Olson et al. (1969) to solve liner problems. Kydland (1969) developed a stochastic simulation model which utilizes linear programming to determine the optimal number of vessels for providing a specified service frequency. Olson et al. (1969) developed a deterministic simulation model to generate medium term regular schedules for a liner company, operating between the US west coast and Hawaii. The model was also used to evaluate scheduling decisions and to investigate the impact of factors such as waiting in port for additional cargo or increasing competition (Perakis and Jaramillo, 1991).

Another simulation model was developed by Gallagher and Meyrick (1984). They presented a cost-based simulation model designed to evaluate the economic characteristics of a liner on a trade route. The information on system performance may then be employed to examine various means of decreasing costs. Similar to the MOPASS model (which was developed by Stott and Douglas (1981)), this approach was evaluative. The model did not incorporate optimization features that could be helpful in recognizing new innovative service patterns (Lane et al., 1987).

Devanney et al. (1972,1975) built a computer-based model to minimize the total cost of a fleet of identical ships to meet the specified shipping demands of a liner operator. The decision variables incorporated the size and design characteristics of the fleet. They used the model results to deduce the efficiencies and inefficiencies of conference rate-making between the eastern United States and the west coast of South America. A number of assumptions were made to simplify the problem. First of all, all cargoes were assumed to go the entire one way route (southbound or northbound); the second assumption was that port time did not depend on the amount of cargo loaded, and finally shipping charges were identical for all cargoes. Though these assumptions relatively eased the problem, they also limited the usefulness of the model (Lane et al., 1987).

Boffey et al. (1979) wrote an interactive computer program and developed an optimization model that was solved heuristically to schedule container vessels over the north Atlantic route. They employed a greedy heuristic to produce schedules; however, managers would rather have the package without the heuristic part. The program was used to provide information on profitability, timing, transit times, and total slack for various inputs of vessel speed and combinations of ports to be called (Perakis and Jarmaillo, 1991).

Datz (1968) developed a simple calculative method for scheduling a liner and suggested some techniques for evaluating the financial results of such a schedule. Almogy et al. (1970) attempted to tackle a simple ship scheduling problem in a rigorous fashion. They developed a stochastic model that enabled them to determine what cargo should be chosen out of the available cargoes in order to maximize profit per time unit. The objective function of their model was linear and separable. Neuhof (1974) presented a procedure for selecting ports of call for a liner operator.

Nemhauser and Yu (1972) studied a model for rail service which can be used for a liner problem. Dynamic programming was used to find the optimal frequency of services that maximizes profit over the planning horizon. Demand for service was a function of two variables, namely service frequency and timing. Lane et al. (1987) presented a dynamic model to determine a cost-efficient fleet that meets the known demand for shipping services on a specified trade route for a given time horizon. The problem was partitioned into a number of manageable components; the emphasis of each component was to minimize the expenses of providing liner services. The model was developed to handle various vessel types, port characteristics, and commodity types. The authors state that this model has been advocated in Australia and Canada.

Bradley et al. (1977) employed linear programming to build a model that could be used for planning the mission and composition of the US merchant marine fleet. This linear programming formulation was a modification of a similar formulation put forth by Everett et al. (1972). The objective of this model was to decide on the number of vessels of various types and the required voyages to meet the annual shipping demands on a specified set of possible routes at minimum cost. The principal use of this model was to determine cost-efficient construction plans to ensure US representation in sea shipping in the future (Lane et al., 1987).

Rana (1985) presented a real-world situation of a container vessel operation. He formulated a mixed integer programming model for the routing of container vessels

(Rana, and Vickson, 1991). Later, Rana and Vickson (1988) developed a deterministic mathematical programming model for optimally routing a chartered container vessel. The formulation involved nonlinearity, which was handled by converting the nonlinear model into a number of mixed integer programs. Bender's decomposition was applied to the mixed-integer programs, wherein the integer network subprograms were solved by a specialized algorithm.

Rana and Vickson (1991) later expanded the work of their 1988 paper, by allowing multiple ships. They formulated a mathematical programming model for a container-ship routing problem that determined the following: (1) the optimal sequence of ports of call for each vessel, (2) the number of trips each vessel makes in the planning horizon, and (3) the amount of cargo delivered between any two ports by each ship. The problem was solved using Lagrangean relaxation which, decomposed it into several sub-problems, one for each vessel. Each sub-problem was further decomposed into a number of mixed-integer linear programs .

Hersh and Ladany (1989) discussed a problem of a company that was leasing a luxury ocean liner during the Christmas holiday season for cruises from southern Florida to the Caribbean. The company needed to decide on the type of cruises to be offered, i.e., the company needed to determine cruise times, routes, and frequency. The decision variables incorporated the routing, the duration**,** the departure dates, and the fare schedules of the cruises. The technique used to solve this problem involved two stages. In the first stage, the demand curve for different itineraries and fares was estimated using multiple nonlinear regression analysis. In the second stage, the authors formulated a discrete dynamic programming model to maximize the net profit for the season by finding the required optimal values of the decision variables. The dynamic programming model used the demand relationships found in the first stage as an input data.

This approach was examined for a case involving cruises to the Bahamas, Jamaica, and Puerto Rico. The authors states that this technique appears to be appropriate for developing optimal operating policies for some general classes of cruise problems such as: (1) cruises from the same point of origin to the same set of possible destinations (ports of call) for various lease periods and at different times of the year, (2) cruises from the same point of origin to any group of possible destinations (ports of call) in the same or nearby region, and (3) cruises from any point of origin to any suitable group of possible destinations (ports of call). For these cases, the demand function established in stage one must be determined for each season. The authors also state that the dynamic programming model can be modified to deal with cases where the company owns a fleet of vessels and needs to determine an optimal set of policies regarding the itineraries, assignment of the vessels to the itineraries, and the fare policies.

Perakis and Jaramillo (1991) developed a linear programming model to minimize the annual operating costs of a fleet of liners. The operating costs included fuel costs, daily running costs, port charges, and canal fees. They also presented independent approaches for fixing both the service frequencies in the various routes and the speed of the vessels. In a subsequent paper, Jarmaillo and Perakis (1991) presented a model for the optimum deployment of a liner fleet that may include both owned and chartered vessels subject to realistic constraints on time, frequency, and other characteristics. The speed of the ships and the frequency of the service in each route were fixed to avoid nonlinearity and hence to formulate the problem as a linear programming problem.

The authors employed sensitivity analysis to provide insights into the impact of the different cost components and constraints on the profitability of the liner operator. In particular, sensitivity analysis indicated that the operating costs were very sensitive to the targeted frequency of service on each route and to the number of owned vessels in the fleet (the more owned ships the higher the operating expenses). The authors presented an example based on the data presented by Flota Mercante Grancolombiana S.A. (FMG), a large liner firm operating in different trade routes between Colombia and Europe, and the US and the Far East. They states that if the deployment strategy presented in this model was used for the above example, then a substantial savings in the operating costs could be obtained. They also state that this model incorporated an appropriate level of detail that makes it realistic and accurate, but not too complicated.

2.3.1.2 Tramp Operations

Very little research has been done on the allocation, routing, and scheduling of tramp shipping. The dearth of research related to tramp shipping may be motivated by the presence of many comparatively small operators in the tramp market. Large shipping firms usually regard the tramp market as a secondary one, principally due to the uncertainty of ship availability and the fact that ships are involved in a tramp operation when such an operation is very lucrative or when ship owners don't have any better occupation for the ships. Research on refrigerated shipping provides the only work on ship routing and scheduling in tramp operations.

Appelgren (1969) discussed a ship scheduling problem obtained from a Swedish shipowning company. In this problem, a shipowning company was involved in world-wide operations of a large number of vessels. A set of cargoes were provided for the planning period, which was 2-4 months. Each cargo had a loading date within the planning period; however, cargo could be delivered after the specified period. Each cargo was characterized by its type, size, earliest loading date, latest loading date, origin (loading port), destination (discharging port), voyage expenses, earnings for optional cargoes, and by the time needed for loading and discharging.

Most of the cargoes were contracted and had to be transported by the company. Occasionally, additional cargoes became available on the market. The scheduling problem considered was to optimally assign a sequence of cargoes to each ship. The author described an algorithm which used the Dantzig-Wolf decomposition method for linear programming. The subprograms were modeled as network flow problems that were solved by dynamic programming. The master program in the decomposition algorithm was modeled as a linear program with only zero-one elements in the matrix and on the right-hand side. The author tested this algorithm with about 40 ships and 50 cargoes. This case was solved in about 2.5 minutes on an IBM 7090 computer. Integer solutions were not assured; however, the solution of a large number of problems showed that the frequency of fractional solutions was about 1-2 percent.

Later, Appelgren (1971) utilized integer programming methods to solve a vessel scheduling problem. The problem was to determine an optimal sequence of cargoes to each vessel in a given fleet during a specified time period. This paper was an attempt to deal with some of the shortcomings associated with the technique used by the author's 1969 paper. Appelgren (1969) used a decomposition algorithm; however, the algorithm gave some non-integer solutions that cannot be interpreted as valid schedules. To avoid fractional solutions, a branch and bound algorithm was developed, where the branching was performed on one of the "essential" non-integer variables and the bounds were computed by the decomposition algorithm.

2.3.1.3 Industrial Operations

As defined in Chapter I, industrial operators have control over both the vessels and the cargoes. This mode of operation is famous in carrying bulk and semi-bulk commodities, such as oil, ore, coal, grain, lumber, pulp, and sugar. Industrial operations have drawn more research than tramp or liner shipping (Perakis and Jaramillo, 1991). The problem being investigated in this dissertation (KPC Oil Tanker Scheduling Problem) belongs to this category of operation.

Dantzig and Fulkerson (1954) studied a tanker scheduling problem of a homogeneous fleet, i.e., carrying capabilities, speeds, and operating expenses were the same for all vessels. Loading and discharging dates were predetermined. The objective was to minimize the number of vessels to meet a specified schedule. One loading and one discharging port per vessel voyage were assumed. The problem was modeled as a transportation problem (Fisher, 1989). Later, Briskin (1966) extended the problem of Dantzig and Fulkerson (1954) by allowing several discharging ports. He described a heuristic clustering procedure in which unloading ports were clustered together and used the transportation method to schedule the tankers. The author utilized dynamic programming to determine the schedule of each ship through its cluster of unloading ports .

Bellmore et al. (1968) presented a modification of the Dantzig and Fulkerson (1954) problem, in which there is an insufficient number of tankers available to satisfy the demand requirements. Delivery "utilities" were presented to determine which deliveries to cancel (Miller, 1987). In another paper, Bellmore et al. (1971) extended the problem of Dantzig and Fulkerson (1954) by allowing various types of vessels and permitting partially loaded vessels. Delivery dates were specified within a certain time interval. The objective was to maximize the total utility of deliveries. The utility of a delivery reflects the desirability of that delivery and the cost of the delivery. Negative utilities were associated with empty legs. The problem was modeled as a mixed integer linear program. The authors suggested employing decomposition with a branch and bound algorithm and network sub-problems. Nevertheless, no application or results have been given.

Laderman et al. (1966) studied a ship routing problem that may be faced either by a tramp shipping company or by an industrial operator. Specified quantities of bulk commodities were to be shipped between specific pairs of ports. The fleet considered was composed of vessels of different characteristics. The objective was to minimize the number of ships needed to meet the shipping requirements. This problem was modeled as a linear program, which assigns vessels to trips, while rounding any resulting fractional number of trips. Rao and Zionts (1968) analyzed a similar problem to the one of Laderman et al. (1966). They considered whether the chartering option was needed. The objective of this problem was to minimize the chartering and operating expenses of vessels. The authors employed a column generation algorithm based on OUT-OF-

KILTER sub-problems to reduce the size of the linear program.

Naslund (1970) studied the problem of delivering wood pulp from many loading ports to numerous discharging ports. This problem is faced by many companies in Northern Europe, where such companies need to determine vessel type, size, and speed. Companies must also efficiently determine suitable ports of call. The objective was to minimize the total transportation expenses from the port of origin to the final inland destination. The author states that this problem has certain characteristics similar to the warehouse location problem. Therefore, a warehouse location algorithm similar to the one proposed by Baumol and Wolf (1958) was employed to solve this problem. The algorithm provides a local minimum; however, it does not assure a global minimum.

Mathis (1972) investigated an oil supply system in which the decision variables were fleet size and mix, sizes of shore tanks, and routes of vessels. Transshipments were permitted at discharging ports. A number of assumptions were made to simplify the problem. Some of these assumptions were (1) only one type of crude oil was considered, (2) a tanker had to stay in its assigned route, (3) sizes of vessels were continuous, and (4) analytical approximations were used for the cost function. The objective of this problem was to optimize (minimize) the total cost of the system. The problem was modeled as a nonlinear integer program and the author exploited the problem's structure to solve it by a branch-and-bound procedure with generalized upper bounds.

Flood (1954) attempted to determine optimum routes and schedules for the US military tanker fleet transporting bulk oil products worldwide. He assumed a given fleet size and minimized the overall distance traveled in ballast (empty) of the vessels by solving a transportation problem. McKay and Hartly (1974) considered the tanker scheduling problem of the US Defense Fuel Supply Center (DFSC) and the Military Sealift Command (MSC) in the worldwide distribution of bulk oil products. They permitted multiple loading ports in a voyage and attempted to minimize the operating expenses of the vessels and the cost of buying the products at the loading ports. The problem was modeled as a mixed-integer linear program and was solved as a linear program with a specialized rounding technique for the discrete variables.

Fisher and Rosenwein (1989) considered the efficient scheduling of a fleet of vessels involved in a pickup and delivery of bulk cargoes. In this model, a menu of candidate schedules for each ship was generated. Selecting an optimal schedule from this menu was modeled as a set packing problem and solved with a dual method. This model was programmed in Pascal on a VAX 8600 and was composed of three modules: (1) the schedule generator, (2) the set packing algorithm, and (3) a color-graphic interface that facilitates display and interactive modification of different schedules. The model was tested based on real-world data obtained from the Military Sealift Command of the US Navy. The resulting solutions indicated a potential for saving up to about $30 million per year over the manual system that was used.

Baker (1981) presented an interactive ship scheduling model for a situation which resembles the vehicle scheduling problem. The model provided schedules for distribution of oil products from a single refinery to several delivery points. A linear programming model and a network model were used to determine voyage selection and quantities of products shipped, respectively.

Brown et al. (1987) presented an elastic set partitioning model of a crude oil tanker scheduling problem. The model incorporated all fleet cost components, including the opportunity cost of vessel time, port and channel tolls, and the costs of demurrage and bunker fuel. The model was used to determine optimal speeds for the vessels, the best routing of ballast (empty) legs, and which shipments to load on controlled vessels and which shipments to spot charter. All feasible schedules along with their costs were computed.

Bausch et al. (1991) extended the work of Brown et al. (1987) to scheduling cargoes of refined petroleum products from various refineries to multiple discharging ports, using a fleet of vessels or barges. A microcomputer system was built with an EXCEL user interface. A detailed cost model was incorporated in the system. The model can be used to determine the optimal cruising speed of the ships, spot chartering requirements, and performing spot charters for other operators.

Stott and Douglas (1981) presented a decision support system known as the Marine Operations Planning and Scheduling System (MOPASS) for planning and scheduling the marine transportation of bulk commodities at Bethlehem Steel Corporation's Marine Operations. MOPASS was a collection of models that provided comparisons of voyage costs of various ships and various trades (Lane et al., 1987). Psaraftis et al. (1985) considered a vessel scheduling problem wherein each ship could carry different cargoes at the same time. The authors presented a heuristic technique that decomposed the overall problem by time and employed a network flow algorithm to optimize the sub-problems in conjunction with a utility function of vessel utilization, timely delivery of cargoes, and port congestion (Fisher and Rosenwein, 1989).

Ronen (1986) investigated short term scheduling of ships for shipping bulk or semi-bulk commodities from a single location. The problem was to assign a collection of cargoes (defined by their sizes and destinations) available at a single origin to an available fleet. The objective of the problem was to minimize the fleet operating expenses. The problem was modeled as a mixed-integer nonlinear problem. Various routing algorithms for allocating shipments to ships were compared on a set of real-world situations.

2.3.1.4 Other Ship Routing and Scheduling Models

This category consists of ship routing and scheduling models that do not clearly belong to any of the former categories.

O'Brien and Crane (1959) discussed the problems of scheduling of tugs and barges, which are somewhat different from vessel scheduling because tugs and barges are not one unit. One problem studied was the scheduling of tugs and the determination of the resulting number of barge loads per year that would result. The second problem was to find the suitable balance between tugs and barges for three or four tugs. These problems were solved by simulation. Jaikumar and Solomon (1987) dealt with the problem of determining the minimum tug fleet size required to transport a given number of barges between various ports in a river system. They developed a one-pass algorithm which solved the problem in O(n) time.

Schwartz (1968) considered the problem of determining the routing and timing of movements of barges and towboats to fulfill agreed upon freight movements at minimum fleet expense. The problem was modeled as a linear discrete programming problem. A solution of the model provided the numbers of barges and towboats of each size needed to render the service. Moreover, the solution provided a detailed schedule for the bargeline and specified the location of the barges and determined the status of the barges, boats, and freight at every time unit of the scheduling horizon.

Conley et al. (1968) presented a linear programming formulation for the transport of a homogeneous product from an overseas port through United States ports to over 400 inland destinations. The fleet considered was composed of 50 ships of six types to be assigned to routes between a group of overseas ports and up to seven United States ports. The objective was to minimize the total cost of transporting the product. The expenses of transportation incorporated ocean cost, port cost, cost for unloading and transporting cargoes to inland vehicles, and inland shipping cost. The linear programming solution indicated that fewer ports should be used.

McKenzie (1971) proposed a control system of vessel traffic and scheduling which resembles the air traffic control system. Cheshire (1972) and Hayman (1972) described a commercial medium and a long range Fleet Scheduling Program (FSP). The description was general and did not give any information about mathematical procedures used; nevertheless optimization was claimed. Pruzan and Jackson (1967) considered the routing and scheduling of coastal shipping, in which ships load or discharge in a group of ports along a coast line.

Everett et al. (1972) used linear programming to optimize fleets of large vessels that would carry some 15 percent of the foreign trade of the United States in the major dry and liquid bulk commodities, such as oil, coal, grains, phosphate rock, ores of iron, aluminum, manganese and chromium. This study was conducted between late November 1969 and late March 1970. Part of the decisions to be made were ship size and type. The authors showed, by using sensitivity analysis, that optimum ship characteristics were more sensitive to port depth than to exact trade forecasts.

Zoppoli (1972) considered the problem of determining minimum-time ship routes. He expressed the problem as an N-stage discrete stochastic process and solved it by dynamic programming. Norman (1973) studied the problem of scheduling ships through the Panama Canal. This problem was a very specialized problem which resembled production scheduling problems.

Stochastic aspects of ship scheduling were addressed by Koenigsberg and Lam (1976). In their mode, they studied queuing aspects in a small system of liquid gas tankers operating in closed routes between a small number of terminals. For any particular system, the model could provide the expected number of ships in each stage, the expected number waiting in each stage, and most importantly the expected waiting time in port. Exponential service time distributions were used; however, a series of parallel simulation computations were used to analyze the impact of other distributions.

Later, Koenigsberg and Meyers (1980) extended the work of Koenigsberg and Lam (1976). They developed an analytical model of a system with two independent fleets that share a common loading port. Exponential distributions of service times were used in all queue stages. The authors wrote a simulation program to investigate the behavior of the system when the service time distributions were not exponential and compared the simulation results with the analytical results. The authors states that even the small number of runs showed that the simulation program reproduced the analytical output for exponential distributions.

Schechter (1976) considered a ship routing problem which resembles the vehicle routing problem. The cargoes were collected from various ports to a central transshipment point. The author employed a heuristic that was reverse to that of Clark and Wright (1964), in which the model initially commenced with one ship making all the pickups and added ships until a solution was obtained. The total distance traveled by the ships was minimized.

Levy et al. (1977) built an interactive fleet scheduling model in the USSR for a centralized unit which was in charge of many shipping firms. An integer program was embedded in their model. Johannson (1969) investigated an oil tanker problem in which a tanker had to deliver oil to various ports in Iceland. Demand and shortage capacity of each port were given. The author employed a heuristic based on dynamic programming to find an optimal route. Psaraftis et al. (1990) considered the problem of a single ship loading at a group of ports along a coastline. The objective of this problem was to minimize the maximum loading completion time.

Benford (1981) proposed a simple trial and error procedure to approach a fleet deployment problem. The problem considered was the following. Managers of bulk carriers sometimes find themselves with extra transport capacity. Managers in such situations need to decide on which vessels to use and which to leave idle or probably make available (by sale or charter) to another fleet. Furthermore, while fuel expenses stay high, extra transport capacity provides a potentially lucrative strategy of slow steaming. The objective was to select a fleet deployment strategy that would be most profitable to the owner, while meeting customers' requirements.

To simplify the problem, three assumptions were made. First of all, the manager had only one contract. A quantity of a given commodity needed to be transported between two specific ports according to this contract. Secondly, more than enough ships were available, and some were more efficient than others. The last assumption was that there were no appreciable costs or benefits involved in taking excess ships out of service. The author illustrated his scheme with a very simple example and stated that more complicated cases could be handled with some modification of the proposed scheme.

Perakis (1985) considered the problem of Benford (1981), namely, when managers of bulk carriers find themselves with extra transport capacity. Managers in such situations need to determine which vessels to use and which to leave idle or probably make available (by sale or charter) to another fleet. Furthermore, while fuel prices stay high, extra transport capacity provides a potentially lucrative strategy of slow steaming. The objective was to determine a fleet deployment scheme that would be most profitable to the owner, while meeting customer demand requirements.

The author realized that the solution method proposed by Benford (1981) imposed an artificial constraint on the problem, leading to operating costs at least 15% higher than the minimal achievable. He formulated and solved this problem under assumptions that could be relaxed without serious difficulties. The author employed Lagrange multipliers without the equal unit constraints assumption. Using the example of Benford (1981), results showed an improvement of 15% over what was obtained by Benford (1981).

Ronen (1982) developed basic models for the determination of the optimal speed of one ship for three kinds of legs: income generating leg, positioning (empty) leg, and a leg for which income is related to the speed. The results of this model were applicable to tramp and industrial operators. Boykin and Levary (1985) developed a simulation based interactive decision support system that could be used for scheduling one chemical tanker. This system was used to evaluate different voyage itineraries, including various steaming speeds.

Miller (1987) investigated the problem of fleet scheduling and inventory resupply encountered by an international chemical company. The company had a fleet of small ocean-going tankers to transport bulk fluid to warehouses worldwide. The author developed an interactive computer model, which was successfully employed to deal with daily scheduling concerns as well as longer range planning problems. A network flow model and a mixed-integer programming model were used to analyze the underlying decision problem. A similar problem was discussed by Agin and Cullen (1977). The decision problem was how to generate a collection of routes for each vessel in the fleet and to decide on the amount of each type of cargo to be transported by the vessels at each stop in its assigned route(s). The resulting schedule had to transport cargo to each of many demand locations within specified calendar dates and at a least expense (Miller, 1987).

Perakis and Papadakis (1987,[1]) studied the problem of minimum-cost operation of a fleet of vessels that had to transport a specific amount of cargo between two ports in a given time period for a fixed contract price. Sensitivity analysis was used to investigate the impacts of small or large changes in one or more cost components of the overall cost. The solution generated the full-load and ballast speed, and the vessels (if any) that would have to be laid-up during the given time horizon. The authors presented analytical expressions for the cost components that were a function of ship full-load and/or ballast speeds.

This work was extended by the authors, Perakis and Papadakis (1987,[2]), where the assumption that cost components are constant throughout the time horizon was relaxed. Later, Papadakis and Perakis (1989) extended their initial contract for affreightment problems to consider the case of many origin ports and many destination ports. Nonlinear programming methods were used to determine both ship allocation to particular routes (origin destination pairs) and full and ballast cruising speed which minimize the overall fleet operating expenses.

Perakis and Papadakis (1989) considered the two dimensional minimal time routing problem for a ship traveling from a single origin to a number of ordered destination points. They emphasized that knowing the departure time beforehand could ease the problem tremendously. They derived an optimal bound for the optimal state evolution which significantly reduced the dimentionality of the problem. Finally, they presented numerical examples to validate their methodologies.

Lo and McCord (1991) attempted to evaluate the strategic routing value of ocean currents on trans-Pacific and trans-Atlantic routes for thirty origin-destination port pairs. They accomplished this by calculating the fuel consumption on routes strategically chosen to take advantage of ocean currents relative to that of routes ignoring the current data. Their results indicated that exploiting currents in strategic routing had the potential of reducing the annual fuel cost of the US and the world commercial fleet by $10 million and $7 million, respectively. McCord (1993) emphasized the value of ocean currents for strategic routing, especially, when considering the high quality oceanographic data that can be obtained from the TOPEX/Poseidon satellite, which was jointly built by the US and French space agencies and launched into orbit on Aug. 10, 1992.

Larson et al. (1988) considered a garbage disposal problem (moving garbage to landfills by barges). A simulation model which incorporated an optimization based heuristics was developed to determine the cheapest fleet size and other system characteristics. Millar and Gunn (1991) developed a heuristic method for dispatching a fleet of fishing trawlers. Each trawler was assigned a group of fishing locations and fish processing plants. The objective was to meet the demand for different species at the processing plants at least expense.

Cline et al. (1992) considered the problem of routing and scheduling of buoy maintenance by the United States Cost Guard. They used a best-schedule heuristic for solving a large class of real-world routing and scheduling problems to approach the buoy routing and scheduling problem. The technique used for solving a large class of real-world routing and scheduling problems was presented as a part of a consulting study for the US Coast Guard Research and Development Center. The best schedule technique reduces a routing and scheduling problem to a traveling salesman problem.

For the traveling salesman problem (TSP), the objective is to find the shortest route which permits a salesman to visit each city in a given collection of N cities. For a routing and scheduling problem, the objective is to minimize the cost of the distance traveled as well as the cost of being either early or late at each destination. This model was used to determine the optimum arrival times for the locations (the best schedule) for any given set of locations to be traversed (that is, for any route), and then it employed this information to locate an optimum route.

Few models on the routing and scheduling of warships were discussed in the literature. Williams (1992) considered the problem of replenishing warships at sea while the ships perform their assignments. The author developed a heuristic algorithm to solve this problem. Brown et al. (1990) considered the problem of annual scheduling of naval combat ships. They proposed a method for assigning a fleet of ships to a group of planned activities to satisfy all event and ship-type requirements. They optimally solved a problem with 111 ships and 19 activities by a generalized set partitioning method in a few minutes on a mainframe.

Nulty and Ratliff (1991) discussed the problem of scheduling the United States Navy's Atlantic fleet to meet overseas strategic needs. Ships with certain characteristics were to be deployed to various locations over some time period (e.g., a destroyer with helicopter capability was needed in the Mediterranean sea from July 1 to December 15). An integer programming formulation was presented and an interactive optimization approach was used. Schardy and Wadsworth (1991) developed a computerized system to evaluate (among other things) fuel consumption of naval combat ships. Good estimates of fuel consumption can be utilized to obtain a proper scheduling of resupply activities. This computerized system had been examined and implemented in US naval fleet exercises.

2.3.2 Relevant Land Routing and Scheduling Models

The following is a brief survey on vehicle routing and scheduling problems, and railroad routing and scheduling problems. This survey is intended to provide insights into some of the existing techniques and algorithms employed to tackle these problems.

2.3.2.1 Vehicle Routing and Scheduling Models

This class of problems differ from ship routing and scheduling problems in many aspects. We point out some of these major differences as discussed by Ronen (1983).

- Uncertainty is strongly present in ship scheduling problems, whereas uncertainty in vehicle routing and scheduling problems exists to a lesser extent.
- For the vessel scheduling problems, the scheduling environment relies to a large degree on the mode of operation of the vessels, i.e., either liner, tramp or industrial.
- Vessels do not necessarily return to an origin point.
- Vehicles are mostly operated during the day (except over the road vehicles such as moving trucks), whereas vessels are operated around the clock.
- Destination of vessels may be altered at sea due to, for example, severe weather conditions.
- Ships are usually involved in much longer voyages than vehicles.

We now present a brief discussion on vehicle routing and scheduling problems. A comprehensive survey on vehicle routing and scheduling models is given by Boden et al. (1983).

White (1972) studied the class of dynamic transshipment problems. These are transportation problems that are characterized by the movement of vehicles and goods from location to location over time. Such movements can be represented by a network. The author states that if no directed cycles exist in this network, then an inductive algorithm can be used to optimize the flow of a homogeneous commodity for a linear cost function. The inductive algorithm employs dynamic programming within an out-of-kilter framework. This algorithm can be modified to handle networks in which there are directed cycles.

Desrosiers et al. (1988) studied a vehicle routing problem with full load and maximum length constraints. The problem was formulated as an asymmetric traveling salesman problem. Chen and Kallsen (1988) considered a school bus routing and scheduling problem. The routing aspect of the problem is concerned with the determination of a stop-to-stop route to be traversed to each school by each bus. The scheduling aspect is concerned with the determination of times at all bus stops for each bus. The objective is to minimize the number of buses required in operation, fleet travel time, and to balance the bus loads. The authors developed an expert system approach which was programmed in Turbo PROLOG for use on an IBM/XT and was applied to rural county school system in Alabama.

Yan (1988) presented a heuristic method for scheduling of trucks from many warehouses to many delivery points subject to constraints on truck capacity, traveling time, and loading and unloading time. He considered the truck scheduling problem faced by STARLINK, a warehousing and distribution company based in Hong Kong. This heuristic method was used to build a complete schedule. In each step of the method, two things can happen, a delivery point may be inserted into the set of partial routes, or a delivery point may be moved in the partial solution to another position in the set of routes. The operation "Move" has a higher priority than "Insertion". If there is a gain in repositioning a delivery point within the partial routes, then the action with the most gain is chosen, otherwise the “insertion” action with the least cost is chosen. This process continues until all points have been inserted into the routes and repositioning any single point among the routes is unfavorable. The author reported the success of this method for the STARLINK company with 8.8% average cost improvement over the previously used manual method.

Ferland and Fortin (1989) investigated the problem of scheduling vehicles with sliding time windows. They used a heuristic approach to tackle this problem. The approach was based on the identification of pairs of tasks offering good opportunity costs for reducing the overall cost, and searching for ways to modify the starting times in order to permit them to be linked. This method was first developed for the vehicle scheduling with time windows problem, and then modified to deal with the sliding time windows. The resulting algorithm was implemented in Fortran on the CYBER 173 of the University of Montreal. The authors report the efficiency of this approach based on the performance of this method on a number of real world test problems.

Balakrishnan (1993) described three heuristics for designing an efficient (cost effective) route for the vehicle routing problem with soft time windows. Appropriate penalties are incurred for violated time windows. Upper limits are imposed on the penalty and the waiting time permitted at any customer location. A number of assumptions were considered: 1) the fleet considered is homogenous and stationed at a single depot and 2) the penalty is assumed to be a linear function of the amount of time window violation. The author concluded that the results obtained from a number of benchmark problems showed that by permitting violations of certain customer time window constraints, it could be possible to considerably reduce both the number of vehicles required and/or the total route distances while controlling both customer penalties and waiting times.

2.3.2.2 Railroad Scheduling Models

Railroad is an essential mode of transportation for both cargo and passengers. In the United States, for instance, railroads account for 35.6% of total intercity freight transport and enjoy about 30% of total revenues of all carriers. However, because of the high competition from other modes of transportation, the rail industry is seriously looking forward to enhancing its operational efficiency and to further rationalize its planning decisions. The routing and scheduling aspects of any rail transportation system need to be carefully studied and investigated to achieve any genuine headway in such a system. The overall train routing and scheduling decisions constitute a cumbersome problem that is further complicated by a number of other constraints such as track, crew, and engine availability (Assad, 1980).

As mentioned earlier, water transportation routing and scheduling models mainly deal with the transport and delivery of cargoes. Very few models have been developed to tackle ship scheduling and routing problems in the context of passengers. In contrast, most of the scheduling research for rail has been developed in the context of train passengers and commuter systems (Assad, 1980). This distinction between cargo and passenger models might be instrumental in comprehending the various activities involved in each one. For the passenger systems, stops at different stations do not involve major activities as in classification yards, or the regrouping of traffic in the case of the freight systems (Assad, 1988). The scheduling aspect of the passenger system is somewhat similar to the liner operation, where trains are expected to comply with published timetables.

In the sequel, we give summaries for few railroad scheduling models. For a detailed survey on railroad scheduling models, refer to Assad (1980).

Morlok (1973) developed a model for a suburban railways system. The resulting schedule of the model involves the number of carriers required in the fleet, total carrier miles, and the crew requirements. The model was examined on a Chicago commuter railroad, where 250 trains operate every weekday over a line of 30 miles in length, (Assad, 1980).

Sherali and Tuncbilek (1995) dealt with the design of static and dynamic fleet sizing models for the multi-level rail-car fleet management problem faced by RELOAD, a branch of the Association of American RailRoads (AAR). The static model was motivated by some relevant studies done by Warfield (1992 a,b). Sherali and Tuncbilek point out the static model does not recognize the actual time-varying demand pattern over the year, or the impact of the transit times in the actual re-routing decisions. It is based on a static time-independent, typical month of the year data. This tends to underestimate the actual required fleet size. The dynamic model is more representative and it can be utilized to establish useful guidelines for the storage and retrieval of cars. Furthermore, it can be utilized in calibrating the static model, enabling it to produce more reliable results. An effective heuristic was utilized to solve the underlying large-scale network problem in the model.

2.3.3 Relevant Airlines Scheduling Models

Airline fleet sizing problems are different from railroad fleet sizing problems in the sense that railroad deadheading is inevitable as loads do not initiate in the immediate vicinity of all ramp locations, and moreover, a varying number of cars are involved in each loaded movement (Sherali and Tuncbilek 1995).

We now give a summary on an airline scheduling model. For a detailed survey on airline scheduling models, refer to Deodoric (1988).

Gu et al. (1994) investigated some properties of the fleet assignment problem. They considered the problem of determining the type of planes assigned to each flight segment based on a given flight schedule and fleets of different types of planes. The problem was modeled as a multicommodity flow problem on a time-space directed network. The complexity of the problem and the behavior of the solution as a function of the number of fleets were discussed in this paper. In particular, the authors have proved that the complexity of the feasibility problem for two fleets is unknown and for three fleets is NP-complete. They have also shown that the ratio of the minimum number of planes required by K fleets to the minimum number of planes required by one fleet can be at least as large as 3/8 log.

In the next chapter, we present the operational issues of the KPC problem, and then formulate a mixed integer programming model for this problem.

Chapter III
Model Construction

3.1 Introduction

An integer programming formulation of the KPC Oil Tanker Scheduling Problem is developed in this chapter (Section 3.2). The demand structure, time windows, penalties, notation, assumptions, and variables are also presented. This chapter commences with an overview of research pertinent to the KPC problem. Other related models are discussed to obtain some insights into the modeling and solution techniques employed to tackle this problem. A discussion on variable initialization is presented in Section 3.3. The stochastic aspects of the problem are investigated in Section 3.4.

The KPC Oil Tanker Scheduling Problem addressed in this dissertation has similarities to the problem studied by Brown et al. (1987). They investigated a crude oil tanker scheduling problem faced by a major oil company to ship crude oil from the Middle East to Europe and North America. A voyage usually had one or two (adjacent) loading ports and one or two (adjacent) discharging ports, and each shipment was a full shipload. Empty tankers could be routed through the Suez Canal where canal dues are required, or around the Cape of Good Hope, a much longer voyage. Loaded tankers cannot go through the Suez Canal and must be routed around the Cape of Good Hope. They generated the complete set of feasible schedules (in a column generation framework), along with the schedules' costs, and modeled the problem as an elastic set partitioning problem.

The KPC model developed in this dissertation differs from the model of Brown et al. (1987) in that it incorporates the processes of constructing and selecting feasible schedules within the model itself, rather than separately generating the schedules. This is accomplished by selecting feasible legs that can satisfy demands within the specified time horizon. Furthermore, the model of Brown et al. (1987) does not take into account various ship sizes, nor does it consider different products. The KPC model presented in this chapter deals with various ship sizes and capacities, varying designs of compartments within a ship, and different products. In fact, the principal thrust of the KPC model is to assign products to compartments on selected legs.

3.2 Problem Formulation

In this section, an integer programming formulation of the KPC problem is developed. The demand structure, time windows, penalties, notation, assumptions, and variables of the problem are also presented. The subsequent chapters deal with solution techniques, algorithms, implementation issues, and computational results.

3.2.1 Demand Structure and Time Windows

The demand time windows can be classified into two categories based on the customer information and requirements.

Type I demand time windows: For this type of demand structure, customers impose the time windows in advance. The customer time windows divide the total demand of a product into smaller quantities (partitions) and specify the feasible delivery dates of each quantity. The partitions are usually specified by the minimum and maximum allowable quantities of the product to be shipped to the destination, and the delivery dates of a given quantity lie in a continuous time interval. Some customers impose penalties on shipments that are not delivered according to the specified feasible delivery dates.

Type II demand time windows: In this case, the demand time windows are determined externally (by KPC) based on information obtained from customers. The customers' information includes the following: (1) the total demand of the product at the destination during the time horizon, (2) the storage capacity at the destination (this might include the minimum and maximum allowable levels of the product at the destinations' storage), (3) the rates of consumption of the product at the destination, and (4) other customer-specific requirements such as specifying certain days for deliveries or not permitting deliveries of certain products on specific days. These restrictions on the deliveries of products to destinations define time windows for the demand during the time horizon. For such demand time windows, the storage level of a product should not go below or exceed the minimum and maximum allowable levels, respectively.

Type II demand time windows are not often used by KPC and its customers. Mostly, shipments from Kuwait to customers are determined based on Type I demand time windows. Observe that Type II demand time windows can be employed to determine Type I demand time windows. We will consider both types of demand time windows in our theoretical developments of the mathematical formulations. However, due to the difficulty of the resulting mathematical formulations, we only consider Type I demand time windows in our solution strategies and computational results.

3.2.2 Problem Notation and Assumptions

Let h = 1,..,H denote the days of the time horizon, where H is the number of days in the time horizon. Since contracts between KPC and customers are signed on an annual basis, H should be assigned a value to reflect this fact. Let t = 1,..,T denote the types of ships in the company's fleet, where T is the number of available types. Note that all ships of the same type are assumed to have identical characteristics, i.e., identical capacities, number of compartments, capabilities to carry different products, speeds, loading times, unloading times, etc. However, two ships of the same type need not have identical operating costs. Therefore, two ships of the same type may be chartered for different prices. Capacities of different compartments of a ship are also not necessarily identical.

For a ship-type t, let s = 1,..,M_t denote the ships of type t, where M_t is the number of available ships of type t. Let O_t denote the number of company-owned ships of type t and $CT_t = M_t - O_t$ denote the number of available ships of type t that can be possibly chartered. The company-owned vessels are represented by s = 1,..,O_t while the chartered vessels are represented by s = O_t+1,..,$O_t + CT_t = M_t$. The number O_t is known *a priori* whereas CT_t is assigned a value reflecting the maximum number of available ships of this type that can be possibly chartered. The model determines how many chartered ships of each type are needed. Let $\$_{t,s}$ be the amount in US dollars required to charter a vessel s of type t, where $s = O_t+1,..,CT_t+ O_t$.

Let d = 1,..,D denote the destinations (ports), where D is the total number of destinations (ports) to be visited, and let p = 1,..,P denote the products, where P is the total number of products (crude oil (dirty) and refined products (clean) such as naphtha and natural gas) to be shipped from Kuwait to different ports in the world. Let P+1 denote a dummy product. Throughout this research, a product p refers to an actual product ($p \in \{1,..,P\}$), unless otherwise stated. The dummy product P+1 may be assigned to a compartment of a ship on a selected leg. This compartment can be then interpreted as an empty compartment.

A product may be delivered to a storage facility or a refinery. Some customers prefer to use their own vessels to transport the shipments to their storage facilities or refineries. In this case, customers and KPC decide on locations where KPC's tankers unload their shipments into customers vessels. These locations are usually unchanged throughout the time horizon and can be considered as ports. For a given destinations d, let r = 1,..,5 denote the r^{th} route from Kuwait to destination d. The routes are defined as follows:

r = 1 represents the route from Kuwait, passing through the Suez Canal to destination d, and returning to Kuwait passing through the Suez Canal.
(Kuwait ---> Suez Canal ---> destination d ---> Suez Canal ---> Kuwait)

r = 2 represents the route from Kuwait, passing through the Suez Canal to destination d, and returning to Kuwait by going around the Cape of Good Hope.
(Kuwait ---> Suez Canal ---> destination d ---> Cape of Good Hope ---> Kuwait)

r = 3 represents the route from Kuwait, going around the Cape of Good Hope to destination d, and returning to Kuwait passing through the Suez Canal.
(Kuwait ---> Cape of Good Hope ---> destination d ---> Suez Canal ---> Kuwait)

r = 4 represents the route from Kuwait, going around the Cape of Good Hope to destination d, and returning to Kuwait by going around the Cape of Good Hope.
(Kuwait ---> Cape of Good Hope ---> destination d ---> Cape of Good Hope ---> Kuwait)

For certain destinations, there is only one route available that coincides with none of the above four routes. Examples of such situations are going from Kuwait to one of the ports in Iran or the Far East. Let r = 5 represent such a route. For a given destination involving the first four routes, KPC decides on which route should be selected. In the process of selecting a route, there is a tradeoff between travel time (when a ship travels around the Cape of Good Hope making a longer voyage) and canal dues (when a ship passes through the Suez Canal making a shorter voyage). Nevertheless, some ships cannot take certain routes. For example, super tankers cannot pass through the Suez Canal.

Let $c = 1,..,C_t$ denote the compartments of a ship of type t, where C_t is the total number of compartments in a ship of type t (usually C_t is an even number). A shipment refers to a compartment content, rather than the entire shipload and an infeasible shipment refers to a shipment not delivered on any feasible delivery date. Let $\hat{e}_{t,c}$ denote the capacity of compartment c of a ship of type t. Recall that compartments of a ship of type t may not have identical capacities. Let $D_{d,p}$ denote the demand of product p at destination d. More specifically, let $D_{I,d,p}$ and $D_{II,d,p}$ designate that the demand of product p at destination d must be satisfied according to a Type I demand time window or a Type II demand time window, respectively. For a given product p and a destination d, $D_{d,p}$ is either of type $D_{I,d,p}$ or $D_{II,d,p}$. A particular demand may be satisfied by more than one ship of different types. Most customers are flexible to accept a shortage or an excess of up to a specified percentage of the total agreed upon demand for the time horizon. Let $100\nu_{d,p}$ denote the allowable shortage and excess percentage of the total demand of product p at destination d.

Type I demand time windows specify the quantities (partitions of the demand $D_{I,d,p}$) required to be shipped as well as their feasible delivery dates. Let $N_{d,p}$ denote the number of partitions of the demand $D_{I,d,p}$ and let $D_{i,d,p}$ denote the i^{th} quantity (partition) of product p to be shipped to destination d, for i=1,.., $N_{d,p}$. Let $D_{1(i,d,p)}$ and $D_{2(i,d,p)}$ denote the earliest and latest permitted delivery dates for the i^{th} demand of product p to destination d, respectively. The size of the i^{th} demand $D_{i,d,p}$ is usually specified by the minimum and maximum allowable quantities of product p, denoted by $f_{i,d,p}$ and $F_{i,d,p}$, respectively. The i^{th} demand $D_{i,d,p}$ can be satisfied by more than one vessel. Let $\alpha_{1(i,d,p)}$ and $\alpha_{2(i,d,p)}$ represent the maximum days a shipment can be delivered before and after $D_{1(i,d,p)}$ and $D_{2(i,d,p)}$, respectively. Let $a_{1(i,d,p)} = D_{1(i,d,p)} - \alpha_{1(i,d,p)}$ and $a_{2(i,d,p)} = D_{2(i,d,p)} + \alpha_{2\ (i,d,p)}$. The per barrel penalty incurred on infeasible shipments is denoted by $\sigma_{i,d,p}$.

Type II demand time windows specify the following factors: (1) the total demand, (2) the storage capacity, (3) the rates of consumption, and (4) other customer-specific requirements. The rate of consumption of a product might not be fixed for the entire period of the time horizon. For example, the consumption of certain products during the winter time may exceed the consumption during the summer time. Let $R_{j,d,p}$ denote the rate of consumption of product p at destination d on day j, for $j \in [1,..,H]$. Usually, at most two distinct rates of consumption exist for a given product at a given destination. Let $TC_{(h,d,p)}$ be the total consumption of product p at destination d during the interval of time given by [1,..,h], i.e., $TC_{(h,d,p)} = \sum_{j=1,..,h} R_{j,d,p}$.

Let $\omega_{d,p}$ denote the storage level of product p at destination d at the beginning of the time horizon, i.e., on the first day of the time horizon, and let $SL_{1,d,p}$ and $SL_{2,d,p}$ denote the minimum and maximum desired levels, respectively, of product p at the storage facility of destination d. The storage level of product p at destination d on day h is denoted by $S_{h,d,p}$. Some customers may allow the storage level to go below or exceed $SL_{1,d,p}$ and $SL_{2,d,p}$, respectively; however, with a penalty based on the shortage or excess quantities. Let $\pi_{d,p}$ denote the daily penalty for each shortage or excess barrel of product p at destination d. The permitted shortage and excess quantities of product p at destination d

with respect to the desired levels $SL_{1,d,p}$ and $SL_{2,d,p}$, are given by $A_{1,d,p}$ and $A_{2,d,p}$, respectively. Let $b_{1,d,p} = SL_{1,d,p} - A_{1,d,p}$ and $b_{2,d,p} = SL_{2,d,p} + A_{2,d,p}$.

Let $L_{h,t,s,r,d}$ represent a leg for ship s of type t which travels to destination d and returns to Kuwait by taking route r leaving on day h. Let $G_{h,t,s,r,d,c,p}$ denote a shipment of product p carried in compartment c on leg $L_{h,t,s,r,d}$. The cost associated with leg $L_{h,t,s,r,d}$ is denoted by $C_{h,t,s,r,d}$. Let $T_{t,r,d}$ denote the time required to complete leg $L_{h,t,s,r,d}$. $T_{t,r,d}$ includes the time required to load any ship s of type t in Kuwait, the time this ship takes to travel to destination d, the unloading time at destination d, and finally the time it takes to return to Kuwait, where this voyage follows route r. Let $T_{t,r,d} = T_{1,t,r,d} + T_{2,t,r,d}$, where $T_{1,t,r,d}$ is the time required to load a ship of type t in Kuwait plus the travel time to destination d, and $T_{2,t,r,d}$ is the time required to unload in destination d plus the travel time from destination d to Kuwait. Ships of the same type are assumed to have equal values of $T_{t,r,d}$, and $T_{t,r,d}$ is also assumed to be independent of h; i.e., $T_{t,r,d}$ is independent of the day the leg starts (weather effects are neglected).

3.2.3 Demands and Penalties

The objective of the model is to satisfy the demand requirements within the time horizon according to some feasible delivery schedules at a minimum total cost. This will be accomplished by maximally utilizing the company-owned vessels and then resorting to chartered vessels as needed. As discussed in Section 3.2.1, information relevant to customers and demand requirements is utilized to determine the demand time windows that define the feasible delivery schedules of products to destinations. A shipment that is not delivered via any feasible delivery schedule receives a penalty agreed upon by customers and KPC. The rest of this section deals with different types of penalties associated with Type I demand time windows and Type II demand time windows.

Penalties may be imposed by customers on shipments that are not delivered on any feasible delivery schedules. An example of such an event is when a shipment of a product is delivered to a destination where the storage capacity cannot handle the entire shipment. Penalty functions for Type I demand time windows and Type II demand time windows are defined later in this chapter and will be embedded in the model to indicate the undesirability of not delivering a shipment according to any feasible delivery date (Type I demand time windows) or violating the daily storage required levels (Type II demand time windows). For Type I demand time windows, most customers impose a fixed amount of penalty cost on each early or late day. Furthermore, customers may accept infeasible shipments up to certain dates before the first and after the last feasible delivery days. For Type II demand time windows, penalties are computed based on the shortage or excess quantities.

Penalties Associated with Type I Demand Time Windows

Let $D_{I,d,p}$ be the demand of product p at destination d whose time window structure is specified in advance by the customer at destination d. This type of time windows defines the quantities to be shipped as well as their feasible delivery dates. Consider the i^{th} demand of product p at destination d given by $D_{i,d,p}$. Associated with this demand, we have the first and last feasible delivery dates given by $D_{1(i,d,p)}$ and $D_{2\ (i,d,p)}$. A shipment $G_{h,t,s,r,d,c,p}$ is feasible with respect to the demand $D_{i,d,p}$ if this shipment is delivered during the interval of days given by $[D_{1(i,d,p)}\ ,\ D_{2(i,d,p)}]$. The feasibility of shipment $G_{h,t,s,r,d,c,p}$ with respect to $D_{i,d,p}$ is determined by the starting day of the leg $L_{h,t,s,r,d}$ given by h. In other words, h is a feasible starting day with respect to the $_i{}^{th}$ demand $D_{i,d,p}$, if $(h + T_{1,t,r,d} - 1) \in [D_{1(i,d,p)}\ ,\ D_{2\ (i,d,p)}]$, where $T_{1,t,r,d}$ is the time required to load a ship of type t, plus the travel time to destination d.

For example, let $[D_{1(i,d,p)}, D_{2(i,d,p)}] = [30, 35]$ and $T_{1,t,r,d} = 20$. Then, h is a feasible starting date with respect to this shipment if the vessel arrives at destination d within the interval of days given by [30 , 35]. Now, $h_1 + T_{1,t,r,d} - 1 = D_{1(i,d,p)} \Rightarrow h_1 + 20 - 1 = 30 \Rightarrow h_1 = 11$. Likewise, $h_2 + T_{1,t,r,d} - 1 = D_{2(i,d,p)} \Rightarrow h_2 + 20 - 1 = 35 \Rightarrow h_2 = 16$. Hence, the feasible starting dates are given by [11 , 16]. The size of the i^{th} demand $D_{i,d,p}$ is usually specified by the minimum and maximum quantities of product p that can be shipped to destination d within the time interval $[D_{1(i,d,p)}, D_{2(i,d,p)}]$.

A customer may impose penalties on shipments that are not delivered according to any feasible delivery date. Penalties are usually computed on a daily basis, i.e., the company is penalized a fixed amount for each early or late day. However, customers may accept infeasible shipments up to certain dates before the first and after the last feasible delivery dates. For example, suppose that the feasible delivery dates for the i^{th} demand $D_{i,d,p}$ of product p at destination d are given by the interval [a , b], where $a \leq b$, and the customer accepts infeasible shipments delivered α_1 days before day a and α_2 days after day b. If the customer imposes a penalty of μ dollars for each early or late day and the shipment is delivered in the interval $[a-\alpha_1, a)$ or $(b, b+\alpha_2]$, then the penalty is computed based on how many early or late days the shipment is delivered before or after the earliest or latest feasible delivery dates, respectively.

For example, suppose that the shipment is delivered on day $\gamma \in [a-\alpha_1, a)$. Then the penalty is given by $\mu(a-\gamma)$ dollars. Likewise, if the shipment is delivered on day $\eta \in (b, b+\alpha_2]$, then the penalty is given by $\mu(\eta-b)$ dollars. If the shipment is delivered on day $\iota \notin [a-\alpha_1, b+\alpha_2]$, then the penalty should be sufficiently large to reflect the undesirability of such a shipment. A customer may not accept any infeasible shipments, in which case, a sufficiently large penalty is incurred to indicate the undesirability of such an event. The penalties imposed by customers on infeasible shipments may represent expenses for leasing extra storage if the customer storage cannot handle the entire shipment.

For each $D_{i,d,p}$, define a piecewise linear function representing penalties associated with infeasible shipments with respect to $D_{i,d,p}$ as follows (see Figure 3.1 and Figure 3.2).

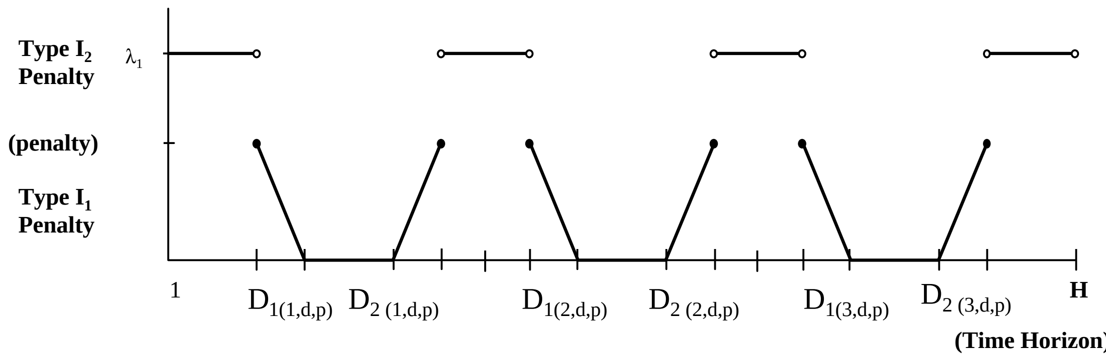

Figure 3.1 (Function for Type I_1 and I_2 Penalties)

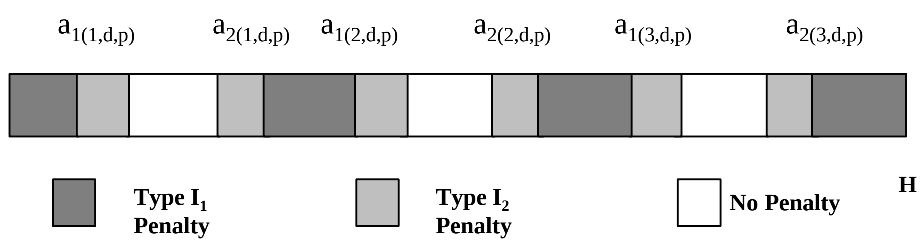

Figure 3.2 (Time Horizon and TypeI_1 &TypeI_2 Penalties)

Let $G_{h,t,s,r,d,c,p}$ denote an arbitrary shipment, then this shipment is penalized if h is not feasible with respect to $D_{i,d,p}$.

Define a penalty function $P_I(h,t,s,r,d,c,p) \rightarrow [0,\infty)$ by

$P_I(h,t,s,r,d,c,p) = \mu_{(i,d,c,p)}$ maximum $[0 , (D_{1(i,d,p)} - h - T_{1,t,r,d}) , (h + T_{1,t,r,d} - D_{2(i,d,p)})]$

if $h + T_{1,t,r,d} \in [D_{1(i,d,p)} - \alpha_{1(i,d,p)} , D_{2(i,d,p)} + \alpha_{2(i,d,p)}]$ and

$P_I(h,t,s,r,d,c,p) = \lambda_1$

if $h + T_{1,t,r,d} \in [\varphi_{1(i,d,p)} , D_{1(i,d,p)} - \alpha_{1(i,d,p)}) \cup (D_{2(i,d,p)} + \alpha_{2(i,d,p)} , \varphi_{2(i,d,p)}]$, where $\mu_{(i,d,c,p)}$, $\varphi_{1(i,d,p)}$ and $\varphi_{2(i,d,p)}$ are defined subsequently and λ_1 should be assigned a sufficiently large value to indicate the undesirability of such an event.

The daily penalty is given by $\mu_{(i,d,c,p)}$ = (number of barrels of product p carried by compartment c) (the penalty per barrel denoted by $\sigma_{i,d,p}$). Observe that the daily penalty is represented by a function of the i^{th} demand, destination d, compartment c, and product p. In other words, we associate a penalty with each $D_{i,d,p}$. Different customers may impose different penalties, and a customer may impose different penalties on different products. Moreover, the customer at destination d may require different penalties for infeasible shipments of product p at different time intervals in the time horizon. Since compartments of a vessel may carry both feasible and infeasible shipments, the penalties are associated with compartments carrying infeasible shipments, rather than with the entire shipload. The penalty associated with leg $L_{h,t,s,r,d}$ is the sum of all penalties incurred on infeasible shipments carried by ship s.

Given a product p and destination d, the penalties associated with shipments delivered to satisfy the i^{th} demand $D_{i,d,p}$ are determined by their delivery dates. If a shipment is delivered on any day that belongs to the interval of days given by $[D_{1(i,d,p)} , D_{2(i,d,p)}]$, then no penalty is incurred. On the other hand, if the shipment is not delivered within the interval $[D_{1(i,d,p)} , D_{2(i,d,p)}]$, then a penalty is incurred based on the delivery date. The penalties associated with infeasible shipments can be categorized based on their delivery dates into two types.

Type I_1 penalty: this penalty is given by

$P_I(h,t,s,r,d,c,p) = \mu_{(i,d,c,p)}$ maximum $[0 , (D_{1(i,d,p)} - h - T_{1,t,r,d}) , (h + T_{1,t,r,d} - D_{2(i,d,p)})]$, for $h + T_{1,t,r,d} \in [D_{1(i,d,p)} - \alpha_{1(i,d,p)} , D_{2 (i,d,p)} + \alpha_{2(i,d,p)}]$.

Type I_2 penalty: this penalty is given by

$P_I(h,t,s,r,d,c,p) = \lambda_1$, for $h + T_{1,t,r,d} \in [\varphi_{1(i,d,p)} , D_{1(i,d,p)} - \alpha_{1(i,d,p)}) \cup (D_{2 (i,d,p)} + \alpha_{2(i,d,p)} , \varphi_{2(i,d,p)}]$.

The interval of time associated with the Type I_1 penalty is given by $[D_{1(i,d,p)} - \alpha_{1(i,d,p)} , D_{2(i,d,p)} + \alpha_{2(i,d,p)}]$, for $i = 1,..,N_{d,p}$. These intervals are assumed to be pairwise disjoint, i.e., $[D_{1(i,d,p)} - \alpha_{1(i,d,p)} , D_{2(i,d,p)} + \alpha_{2(i,d,p)}] \cap [D_{1(j,d,p)} - \alpha_{1(j,d,p)} , D_{2(j,d,p)} + \alpha_{2(j,d,p)}] = \phi$, where $i \neq j$ and ϕ denotes the empty set. A shipment not delivered within $[D_{1(i,d,p)} - \alpha_{1(i,d,p)} , D_{2(i,d,p)} + \alpha_{2(i,d,p)}]$, for $i=1,..,N_{d,p}$ receives a Type I_2 penalty. The interval of time associated with Type I_2 penalty is given by $(D_{2 (i-1,d,p)} + \alpha_{2(i-1,d,p)} , D_{1(i,d,p)} - \alpha_{1(i,d,p)}) \cup (D_{2(i,d,p)} + \alpha_{2(i,d,p)} , D_{1(i+1,d,p)} - \alpha_{1(i+1,d,p)})$ for $i = 1,..,N_{d,p}$. Note that $D_{2 (0,d,p)} + \alpha_{2(0,d,p)} =$ Minimum $[1 , D_{1(1,d,p)} - \alpha_{1(1,d,p)}]$ and $D_{1(Nd,p +1,d,p)} - \alpha_{1(Nd,p +1,d,p)} =$ Maximum $[D_{2(N,d,p)} + \alpha_{2(N,d,p)} , H]$, where H is the last day of the time horizon.

An infeasible shipment delivered in the interval $(D_{2(i-1,d,p)} + \alpha_{2(i-1,d,p)} , D_{1(i,d,p)} - \alpha_{1(i,d,p)})$ or $(D_{2 (i,d,p)} + \alpha_{2(i,d,p)} , D_{1(i+1,d,p)} - \alpha_{1(i+1,d,p)})$, for $i= 2,..,N_{d,p} - 1$ receives a Type I_2 penalty. Now, the interval $(D_{2(i-1,d,p)} + \alpha_{2 (i-1,d,p)} , D_{1(i,d,p)} - \alpha_{1(i,d,p)})$ may be associated with Type II_2 penalty with respect to $D_{i-1,d,p}$ and/or $D_{i,d,p}$. Likewise, the interval $(D_{2(i,d,p)}+ \alpha_{2(i,d,p)} , D_{1(i+1,d,p)} - \alpha_{1(i+1,d,p)})$ may be associated with Type II_2 penalty with respect to $D_{i,d,p}$ and/or $D_{i+1,d,p}$. For the sake of simplification, the interval $(D_{2(i-1,d,p)} + \alpha_{2(i-1,d,p)} , D_{1(i,d,p)} - \alpha_{1(i,d,p)})$ is divided into two equal intervals denoted by $(D_{2(i-1,d,p)} + \alpha_{2(i-1,d,p)} ,\xi)$ and $[\xi ,D_{1(i,d,p)} - \alpha_{1(i,d,p)})$, where ξ is the midpoint of the interval and is arbitrarily included in the first or second interval. Let $\varphi_{2(i-1,d,p)} = \varphi_{1(i,d,p)} = \xi$. Then, the interval $[\varphi_{1(i,d,p)} , \varphi_{2(i,d,p)}) = [\varphi_{1(i,d,p)} , D_{1(i,d,p)} - \alpha_{1(i,d,p)}) \cup [D_{1(i,d,p)} - \alpha_{1(i,d,p)} , D_{1(i,d,p)}) \cup [D_{1(i,d,p)} , D_{2(i,d,p)}] \cup (D_{2(i,d,p)} , D_{2(i,d,p)} + \alpha_{2(i,d,p)}] \cup (D_{2(i,d,p)} +$

$\alpha_{2(i,d,p)}$, $\varphi_{2(i,d,p)}$) and it represents the feasible and infeasible delivery intervals with respect to the demand $D_{i,d,p}$. Hence, the intervals of penalties are defined so that a shipment is either feasible or infeasible with respect to exactly one $D_{i,d,p}$.

The dummy product P+1 may be assigned to a compartment of a ship on a selected leg. This compartment can be deemed as an empty compartment. However allowing empty compartments on a selected leg should be avoided to fully utilize the compartments of the vessels. In order to incorporate the undesirability of empty compartments, a penalty is imposed on all compartments carrying the dummy product. The penalty is given by P_I (h,t,s,r,d,c,P+1) = λ_3 $\forall$ h,t,s,r,d,c, where λ_3 should be assigned an appropriate value to indicate the undesirability of empty compartments.

Penalties Associated with Type II Demand Time Windows

Let $D_{II,d,p}$ be a given demand that has to be satisfied according to Type II demand time windows. The storage level of product p at destination d must be within the minimum and maximum allowable levels on any given day of the time horizon. Some customers impose penalties on KPC when the storage level on a given day falls below the minimum required level or exceeds the maximum permitted level. The maximum permitted level may be the maximum storage capacity, and exceeding this capacity requires extra storage. The penalty associated with a shipment leading to an excess in storage level can be interpreted as the cost of providing this extra storage. Some customers stipulate that the storage level on any given day of the time horizon must lie within the permitted levels and any shipment causing an excess in the storage level will not be accepted.

For Type II demand time windows, the penalty is computed on a daily basis based on the storage level of each day (see Figure 3.3 and Figure 3.4). Let $S_{h,d,p}$ be the storage level of product p at destination d on day h as defined in Section 3.2.2.

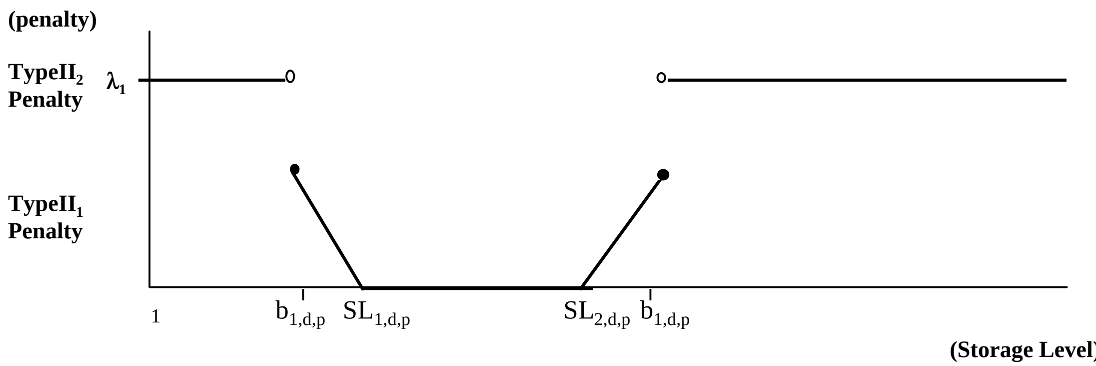

Figure 3.3. Function for Type II$_1$ and Type II$_2$ Penalties

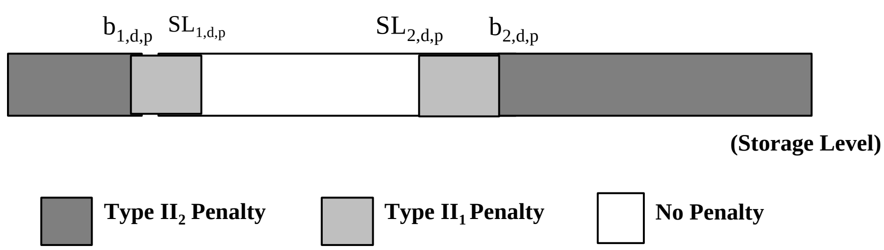

Figure 3.4. Storage Level and Type II$_1$ & Type II$_2$ Penalties

Define $P_{II}(S_{h,d,p}) \rightarrow [0,\infty)$ as

$P_{II}(S_{h,d,p}) = \pi_{d,p}$ Maximum $[0, (SL_{1,d,p} - S_{h,d,p}), (S_{h,d,p} - SL_{2,d,p})]$

if $S_{h,d,p} \in [SL_{1,d,p} - A_{1,d,p}, SL_{2,d,p}\ A_{2,d,p}]$, **(Type II_1 penalty)**

and

$P_{II}(S_{h,d,p}) = \lambda_2$

if $S_{h,d,p} \in [0, SL_{1,d,p} - A_{1,d,p}) \cup (SL_{2,d,p} + A_{2,d,p}, \infty)$, **(Type II_2 penalty)**

where $\pi_{d,p}$ denotes the daily penalty for each shortage or extra barrel of product p at destination d.

The interval of Type II_1 penalty can be expressed as $[SL_{1,d,p} - A_{1,d,p}, SL_{2,d,p} + A_{2,d,p}] = [SL_{1,d,p} - A_{1,d,p}, SL_{1,d,p}) \cup [SL_{1,d,p}, SL_{2,d,p}] \cup (SL_{2,d,p}, SL_{2,d,p} + A_{2,d,p}]$. If $S_{h,d,p} \in [SL_{1,d,p}, SL_{2,d,p}]$, then the storage level lies within the required levels and no penalty is induced, i.e., $P_{II}(S_{h,d,p}) = 0$. If $S_{h,d,p} \in [SL_{1,d,p} - A_{1,d,p}, SL_{1,d,p}) \cup (SL_{2,d,p}, SL_{2,d,p} + A_{2,d,p}]$, then a penalty is induced based on the shortage or excess quantities. On the other hand, if $S_{h,d,p} \in [0, SL_{1,d,p} - A_{1,d,p}) \cup (SL_{2,d,p} + A_{2,d,p}, \infty)$, then a sufficiently large penalty is imposed to indicate the undesirability of such a storage level on any given day.

Alternatively, the Type II_1 and Type II_2 penalties can be represented as follows. Let $S_{h,d,p} = S_{1,h,d,p} - S_{2,h,d,p} - S_{3,h,d,p} + S_{4,h,d,p} + S_{5,h,d,p}$, where $SL_{1,d,p} \leq S_{1,h,d,p} \leq SL_{2,d,p}$, $0 \leq S_{2,h,d,p} \leq A_{1,d,p}$, $0 \leq S_{3,h,d,p} \leq SL_{1,d,p} - A_{1,d,p}$, $0 \leq S_{4,h,d,p} \leq A_{2,d,p}$, and $S_{5,h,d,p} \geq 0$. Now, we verify that the minimization objective formulation which incorporates the term $\overline{P}_{II}(S_{h,d,p})$ will automatically enforce $P_{II}(S_{h,d,p})$. This follows since $0<\pi_{d,p}<\lambda_2$ will assure that for any $S_{h,d,p} \in [0, \infty)$, the corresponding representation of $S_{h,d,p}$ in terms of $S_{1,h,d,p}$, $S_{2,h,d,p}$, $S_{3,h,d,p}$, $S_{4,h,d,p}$, and $S_{5,h,d,p}$ will be determined as follows, where in each case, the remaining (unspecified) variable from this list have zero values.

Let $S_{h,d,p} \in [SL_{1,d,p}, SL_{2,d,p}]$, then $S_{h,d,p} = S_{1,h,d,p}$. If $S_{h,d,p} \in [SL_{1,d,p} - A_{1,d,p}, SL_{1,d,p})$, then $S_{h,d,p} = S_{1,h,d,p} - S_{2,h,d,p}$, where $S_{1,h,d,p} = SL_{1,d,p}$ and $S_{2,h,d,p} = SL_{1,d,p} - S_{h,d,p}$, while if $S_{h,d,p} \in (SL_{2,d,p}, SL_{2,d,p} + A_{2,d,p}]$, then $S_{h,d,p} = S_{1,h,d,p} + S_{4,h,d,p}$, where $S_{1,h,d,p} = SL_{2,d,p}$ and

$S_{4,h,d,p} = S_{h,d,p}$ - $SL_{2,d,p}$. Likewise, if $S_{h,d,p} \in [0\ ,\ SL_{1,d,p} - A_{1,d,p})$, then $S_{h,d,p} = S_{1,h,d,p} - S_{2,h,d,p} - S_{3,h,d,p}$, where $S_{1,h,d,p} = SL_{1,d,p}$, $S_{2,h,d,p} = A_{1,d,p}$, and $S_{3,h,d,p} = SL_{1,d,p} - A_{1,d,p} - S_{h,d,p}$, while if $S_{h,d,p} \in (SL_{2,d,p} + A_{2,d,p}\ ,\ \infty)$, then $S_{h,d,p} = S_{1,h,d,p} + S_{4,h,d,p} + S_{5,h,d,p}$, where $S_{1,h,d,p} = SL_{2,d,p}$, $S_{4,h,d,p} = A_{2,d,p}$, and $S_{5,h,d,p} = S_{h,d,p} - SL_{2,d,p} - A_{2,d,p}$.

Accordingly, we define $\overline{P}_{II} : (S_{h,d,p}) \rightarrow [0,\infty)$ as the linear penalty function $\overline{P}_{II}(S_{h,d,p}) = \pi_{d,p}\ (S_{2,h,d,p} + S_{4,h,d,p}) + \lambda_2\ (S_{3,h,d,p} + S_{5,h,d,p})$. Hence, the function $\overline{P}_{II}(S_{h,d,p})$ renders the required Type II_1 and Type II_2 penalties as described above.

3.2.4 Algorithms for Computing Penalties

For Type I demand time windows, customers provide KPC with the time windows and the penalties associated with infeasible shipments at the time of signing the contract. For Type II demand time windows, detailed information obtained from customers are used to determine the demand time windows and the penalties incurred on shortage or excess storage levels. The information provided by customers includes the following: (1) the total demand of each product at each destination, (2) the rates of consumption of each product at each destination, (3) the storage capacity of each product, (4) any customer-specific requirements, and (5) penalties associated with the shortage or excess levels. The following are algorithms for computing the Type I_1 and Type I_2 penalties associated with Type I demand time windows, and the Type II_1 and Type II_2 penalties associated with Type II demand time windows.

Algorithm for Computing the Type I_1 and Type I_2 Penalties

INPUT

Let $D_{I,d,P}$ be the demand of product p at destination d. For i=1,..,$N_{d,p}$, the following is given.

1. The feasible delivery intervals of $D_{i,d,p}$ given by $[D_{1(i,d,p)}\ ,\ D_{2(i,d,p)}]$.

2. The interval of Type I_1 penalty associated with $D_{i,d,p}$. This interval is given by $[D_{1(i,d,p)} - \alpha_{1(i,d,p)}\ ,\ D_{2(i,d,p)} + \alpha_{2(i,d,p)}\ ]$.

3. The interval of Type I_2 penalty associated with $D_{i,d,p}$. This interval is given by $[\varphi_{1(i,d,p)}, D_{1(i,d,p)} - \alpha_{1(i,d,p)}]$ and $(D_{2(i,d,p)} + \alpha_{2(i,d,p)}, \varphi_{2(i,d,p)}]$.

4. The daily penalty associated with infeasible shipments delivered during the time interval $[D_{1(i,d,p)} - \alpha_{1(i,d,p)}, D_{2(i,d,p)} + \alpha_{2(i,d,p)}]$. This penalty is given by $\mu_{(i,d,c,p)} = \sigma_{i,d,p}$ (number of barrels of product p carried by compartment c).

5. The Type I_2 penalty given by λ_1, which is imposed on infeasible shipments delivered during the time interval $[\varphi_{1(i,d,p)}, D_{1(i,d,p)} - \alpha_{1(i,d,p)}) \cup (D_{2(i,d,p)} + \alpha_{2(i,d,p)}, \varphi_{2(i,d,p)}]$,

i.e., $h + T_{1,t,r,d} \in [\varphi_{1(i,d,p)}, D_{1(i,d,p)} - \alpha_{1(i,d,p)}) \cup (D_{1(i,d,p)} + \alpha_{2(i,d,p)}, \varphi_{2(i,d,p)}]$.

Penalty Computation

Let $L_{h,t,s,r,d}$ be an arbitrary leg. Then,

$P_I(h,t,s,r,d,c,p) = \mu_{(i,d,c,p)}$ maximum $[0, (D_{i,1,d,p} - h - T_{1,t,r,d}), (h + T_{1,t,r,d} - D_{2(i,d,p)})]$

if $h + T_{1,t,r,d} \in [D_{1(i,d,p)} - \alpha_{1(i,d,p)}, D_{2(i,d,p)} + \alpha_{2(i\ d,p)}]$ for some $i \in \{1,..,N_{d,p}\}$,

(Type I_1 penalty)

and $P_I(h,t,s,r,d,c,p,) = \lambda_1$

if $h + T_{1,t,r,d} \in [\varphi_{1(i,d,p)}, D_{1(i,d,p)} - \alpha_{1(i,d,p)})$ or $h + T_{1,t,r,d} \in (D_{2(i,d,p)} + \alpha_{2(i,d,p)}, \varphi_{2(i,d,p)}]$ for some $i \in \{1,..,N_{d,p}\}$, where λ_1 is assigned a sufficiently large value.

(Type I_2 penalty)

Algorithm for Computing the Type II_1 and Type II_2 Penalties

INPUT

1. The demand for product p at destination d, $D_{II,d,P}$.

2. The storage level for product p at destination d on day 1 of the time horizon, $\omega_{d,p}$.

3. The minimum and maximum desired levels of product p at destination d, $SL_{1,d,p}$ and $SL_{2,d,p,}$ respectively.

4. The permitted shortage and excess quantities with respect to the desired levels for product p at destination d, $A_{1,d,p}$ and $A_{2,d,p}$, respectively.

5. The storage level on day h, $S_{h,d,p}$.

6. The daily shortage or excess per barrel penalty, $\pi_{d,p}$ (Type II_1 penalty).

7. Type II_2 penalty given by $\lambda_{2,}$ which is incurred whenever the storage level does not lie within $[SL_{1,d,p} - A_{1,d,p}, SL_{2,d,p} + A_{2,d,p}]$.

Penalty Representation

$\overline{P}_{II}(S_{h,d,p}) = \pi_{d,p}(S_{2,h,d,p} + S_{4,h,d,p}) + \lambda_2 (S_{3,h,d,p} + S_{5,h,d,p})$,

where $S_{h,d,p} = S_{1,h,d,p} - S_{2,h,d,p} - S_{3,h,d,p} + S_{4,h,d,p} + S_{5,h,d,p}$, $SL_{1,d,p} \leq S_{1,h,d,p} \leq SL_{2,d,p}$, $0 \leq S_{2,h,d,p} \leq A_{1,d,p}$, $0 \leq S_{3,h,d,p} \leq SL_{1,d,p} - A_{1,d,p}$, $0 \leq S_{4,h,d,p} \leq A_{2,d,p}$, and $0 \leq S_{5,h,d,p}$.

3.2.5 Operational Issues

The various requirements and characteristics of the KPC problem are investigated in this section along with their mathematical interpretation. The problem variables and the integer formulation are presented in Section 3.2.6.

Demand requirements

A customer demand must be satisfied according to the specified time windows. These time windows are either specified in advance by the customer (Type I demand time windows) or are determined based on the storage capacity, the rates of consumption, and any other customer-specific requirements (Type II demand time windows).

A. Type I Demand Time Windows

In order to satisfy the i^{th} demand $D_{i,d,p}$ of product p at destination d, the total amount of product p carried on selected legs must be within $[f_{i,d,p}, F_{i,d,p}]$. Infeasible shipments with respect to $D_{i,d,p}$ may be selected, but with penalties, either of Type I_1 or Type I_2. Shipments involving Type I_2 penalties are very undesirable due to their large magnitude. If $n_{t,c,p}$ is the number of compartments of type c carrying product p associated with legs $L_{h,t,s,r,d}$ for $h + T_{1,t,r} \in [\varphi_{1(i,d,p)}, \varphi_{2(i,d,p)}]$, then $f_{i,d,p} \leq \sum_t \sum_c n_{t,c,p} \hat{e}_{t,c} \leq F_{i,d,p}$ must hold to satisfy the i^{th} demand $D_{i,d,p}$.

B. Type II Demand Time Windows

For this type of demand time windows, it is very essential to maintain the daily storage level within the specified lower and upper bounds. In other words, the storage level on day $\bar{h} \in [1,..,H]$ given by $S_{\bar{h},d,p}$ must satisfy $SL_{1,d,p} \leq S_{\bar{h},d,p} \leq SL_{2,d,p}$. Now, $S_{\bar{h},d,p}$ is given by $S_{\bar{h},d,p}$ = (the storage level on day 1 of the time horizon) + (the total amount of product p delivered to destination d in the interval $[1,.., \bar{h}]$) - (the total amount of product p consumed in the interval $[1,.., \bar{h}]$). The total amount of product p delivered to destination d in the interval $[1,.., \bar{h}]$ is the quantity that determines the penalty associated with day $\bar{h}$.

C . Total Demands

Most customers are flexible to accept a shortage or an excess of up to a specified percentage (usually 10 percent) of the total agreed upon demand for the time horizon. If $n_{t,c,p}$ is the total number of compartments of type c carrying product p associated with corresponding legs $L_{h,t,s,r,d}$ $\forall$ h,s,r, then the total amount of product p shipped to destination d must satisfy the following:

$D_{d,p}(1 - \nu_{d,p}) \leq \sum_t \sum_c n_{t,c,p}\, \hat{e}_{t,c} \leq D_{d,p}(1+ \nu_{d,p})$. Satisfying the total demand $D_{I,d,p}$ or $D_{II,d,p}$ within the permitted shortage and excess quantities must be enforced to abide by the terms of the contract.

Observe that satisfying the demand $D_{i,d,p}$ does not necessarily imply satisfying the total demand of product p. Suppose, for example, that $D_{I,d,p}$ = 1000,000 barrels of crude oil. Suppose that $N_{d,p}$ = 10, i.e., the total demand is partitioned into 10 smaller demand requirements. Moreover, suppose that $f_{i,d,p}$ = 80,000 and $F_{i,d,\ p}$ = 120,000, for i = 1,..,10. Letting $D_{i,d,p}$ = 80,000 $\forall$ i and $100\nu_{d,p}$ = 10%, then the total amount of product p shipped to destination d is 80,000 x 10 = 800,000, which is less than $D_{I,d,p}(1\text{-}100\nu_{d,p})$ = 1000,0000

(0.9) = 900,000. Likewise for Type II demand time windows, the total demand of a given product at a given destination may not be satisfied even if the daily storage level lies within the desired lower and upper levels throughout the time horizon.

D. Infeasible Shipments (Type I Demand Time Windows)

Infeasible shipments are allowed subject to penalties incurred either of Type I_1 or Type I_2. The selection of an infeasible shipment is determined by the penalty imposed on such a shipment. Shipments with very high penalties would tend to be avoided to minimize the cost. Penalties are computed compartmentwise and the penalty associated with a given leg is the sum of penalties imposed on the infeasible shipments (where a shipment refers to a compartment content). Hence the total operating cost of leg $L_{h,t,s,r,d}$ is given by $C_{h,t,s,r,d}$ + (the total penalty imposed on this leg).

E. The Daily Storage Level (Type II Demand Time Windows)

Violating the daily storage level may be permitted subject to a penalty. A Type II_2 penalty is too costly and should be avoided. As discussed in point B above, the total amount of product p delivered to destination d in the interval [1,.., $\bar{h}$] determines the penalty associated with day $\bar{h}$ of the time horizon. The total penalty imposed by the customer at destination d is the sum of the daily penalties over the time horizon.

Restrictions on Vessels

The usage of a vessel is restricted by a number of factors. Some of these factors are the availability of the vessel during the time horizon, the operating cost of the vessel, the capability of the vessel to go to various destinations and carry different products, the time the vessel takes to travel to a certain destination on a specific route, and whether the vessel is owned by the company or is chartered. Some of these factors are discussed subsequently in this section, while the other factors are discussed in Section 3.5.

A. Selection of Legs

At most one leg can be selected for a ship leaving on a given day. Hence, if $L_{h,t,s,\bar{r},\bar{d}}$ and $L_{h,t,s,r`,d`}$ are two distinct legs associated with a ship s of type t, then at most one of these legs may be selected.

B. Vessel Unavailability During a Selected Leg

If leg $L_{h,t,s,r,d}$ is selected, then the ship involved in this leg must not be assigned to any other leg until this ship completes the current leg. In other words, if ship s of type t is assigned to leg $L_{h,t,s,r,d}$ and $T_{t,r,d} = \rho$ days, then ship s cannot be assigned to any other leg in the time interval given by [h+1,..,h + ρ -1].

C. Vessel Chartering

The company should resort to chartered vessels after maximally utilizing their own vessels. In the current scheduling practice, KPC charters vessel to satisfy about 40-50 percent of the total demand. Moreover, approximately 5-10 percent of the demand is carried by spot chartered vessels. Vessel are chartered for millions of dollars depending on their characteristics. The chartering expenses of a vessel, which is given by $\$_{t,s}$, can be interpreted as a large penalty cost that is incurred when a vessel is chartered.

D. Unavailability of Vessels During Some Intervals of the Time Horizon

A ship may not be available for some intervals of the time horizon. For instance, the ship may require a scheduled maintenance during the time horizon. The unavailability durations of the ship are either known *a priori* or are determined by the company. In the first case, the unavailability time intervals are specified, and therefore the ship should not be involved in any leg during the specified unavailability intervals, whereas in the second case, the company specifies the total number of days the ship can be used within the time horizon.

3.2.6 An Integer Programming Formulation

First, the binary decision variables of the problem are introduced in this section. Then, the integer formulation, and the interpretation of the objective and the constraints of the problem are given.

Let $Y_{h,t,s,r,d}$ = { 1 if leg $L_{h,t,s,r,d}$ is selected
{ 0 otherwise.

Since ships have different compartments of different capacities, we need to explicitly consider the compartments of the vessels in the model. In fact, the principal thrust of the model is to efficiently assign products to compartments. Hence, a binary variable $X_{h,t,s,r,d,c,p}$ is defined as follows:

Let $X_{h,t,s,r,d,c,p}$ = {1 if compartment c carries product p on leg $L_{h,t,s,r,d}$
{0 otherwise.

Recall that the capacity of a selected compartment is given by $\hat{e}_{t,c}$. Moreover, certain compartments may not carry specific products, in which case, the corresponding X-variables are assigned the value of zero.

The binary variables $X_{h,t,s,r,d,c,p}$ and $Y_{h,t,s,r,d}$ for s = 1,..,O_t are associated with the company-owned vessels while $X_{h,t,s,r,d,c,p}$ and $Y_{h,t,s,r,d}$ for s = O_t+1,..,O_t+ CT_t are associated with the available vessels of type t that can be possibly chartered for a specified period of time within the time horizon. Observe that, only time chartering (as defined in Section 1.2 of Chapter I) is considered in the model.

Let $Z_{t,s}$ = { 1 if ship s of type t is selected for chartering for a specified time period within the time horizon
{ 0 otherwise.

The following is a summary of notation, presented here for the sake of convenience.

Ships, Products and Destinations

- $h = 1,..,H$: days of the time horizon.
- $t = 1,..,T$: types of ships in the company's fleet.
- $s = 1,..,M_t$: ships of type t, where M_t is the number of available ships of type t.
- O_t : number of company-owned ships of type t.
- $CT_t = M_t - O_t$: number of available ships of type t that can be possibly chartered.
- $s = 1,..,O_t$: company-owned vessels.
- $s = O_t+1,..,O_t + CT_t = M_t$: chartered vessels.
- $\$_{t,s}$, for $s = O_t+1,..,CT_t+O_t$: chartering expenses.
- $d = 1,..,D$: destinations (ports).
- $p = 1,..,P$: products.
- $P+1$: dummy product.
- $UT_{t,s}$: ship s of type t cannot be used for more than $(H-UT_{t,s})$ days throughout the time horizon.

Routes

- r = 1 : Kuwait ---> Suez Canal ---> destination d ---> Suez Canal ---> Kuwait.
- r = 2 : Kuwait ---> Suez Canal ---> destination d ---> Cape of Good Hope ---> Kuwait.
- r = 3 : Kuwait ---> Cape of Good Hope ---> destination d ---> Suez Canal ---> Kuwait.
- r = 4 : Kuwait --->Cape of Good Hope --->destination d ---> Cape of Good Hope ---> Kuwait.

- $r = 5$: for certain destinations, there is only one route available that coincides with none of the above four routes. Let $r = 5$ represent such a route.

Compartments and Demands

- $c = 1,..,C_t$: compartments of a ship of type t.
- $\hat{e}_{t,c}$: capacity of compartment c of a ship of type t.
- $D_{d,p}$: demand of product p at destination d.
- $D_{I,d,p}$ and $D_{II,d,p}$: demand of product p at destination d that must be satisfied satisfied according to Type I demand time windows and Type II demand time windows, respectively.
- $100\nu_{d,p}$: maximum allowable shortage or excess percentage of the total demand of product p at destination d.

Type I Demand Time Windows

- $(d,p) \in$ Type I : demand for product p at destination d must be satisfied according to Type I demand time windows.
- $N_{d,p}$: number of partitions of the demand $D_{I,d,p}$.
- $D_{i,d,p}$: i^{th} quantity (partition) of product p to be shipped to destination d, for i=1,.., $N_{d,p}$.
- $D_{1(i,d,p)}$ and $D_{2(i,d,p)}$: earliest and latest desired delivery dates, respectively, for the i^{th} demand of product p to destination d.
- $f_{i,d,p}$ and $F_{i,d,p}$: minimum and maximum allowable quantities, respectively, of the i^{th} partition of product p to be shipped to destination d.
- $\alpha_{1(i,d,p)}$ and $\alpha_{2(i,d,p)}$: maximum days a shipment can be delivered before and after $D_{1(i,d,p)}$ and $D_{2\,(i,d,p)}$, respectively.
- $\sigma_{i,d,p}$: the per barrel penalty incurred on infeasible shipments.

- $H_{i,d,p} = [\varphi_{1(i,d,p)}, \varphi_{2(i,d,p)})$, where $(d,p) \in$ Type I demand time windows and $i=1,..,N_{d,p}$.
- $a_{1(i,d,p)} = D_{1(i,d,p)} - \alpha_{1(i,d,p)}$ and $a_{2(i,d,p)} = D_{2(i,d,p)} + \alpha_{2(i,d,p)}$.

Type II Demand Time Windows

- $(d,p) \in$ Type II : demand for product p at destination d must be satisfied according to Type II demand time windows.
- $R_{j,d,p}$: the rate of consumption of product p at destination d on day j, for $j \in [1,..,H]$.
- $TC_{(h,d,p)} = \sum_{j=1}^{h} R_{j,d,p}$: the total consumption of product p at destination d during the interval of time given by [1,..,h].
- $\omega_{d,p}$: storage level of product p at destination d on day 1 of the time horizon.
- $SL_{1,d,p}$ and $SL_{2,d,p}$: minimum and maximum desired levels, respectively, of product p at the storage facility of destination d.
- $A_{1,d,p}$ and $A_{2,d,p}$: maximum permitted shortage and excess quantities, respectively respectively, of product p at destination d with respect to the desired levels given by $SL_{1,d,p}$ and $SL_{2,d,p}$, respectively.
- $S_{h,d,p}$: the storage level of product p at destination d on day h.
- $\pi_{d,p}$: the daily penalty for each shortage or extra barrel of product p at destination d.
- $b_{1,d,p} = SL_{1,d,p} - A_{1,d,p}$ and $b_{2,d,p} = SL_{2,d,p} + A_{2,d,p}$.

Legs, Processing Times and Costs

- $L_{h,t,s,r,d}$: a leg for ship s of type t.
- $G_{h,t,s,r,d,c,p}$: a shipment of product p carried in compartment c on leg $L_{h,t,s,r,d}$.
- $C_{h,t,s,r,d}$: cost associated with leg $L_{h,t,s,r,d}$.
- $T_{t,r,d}$: processing time associated with leg $L_{h,t,s,r,d}$.

- $T_{t,r,d} = T_{1,t,r,d} + T_{2,t,r,d}$, where $T_{1,t,r,d}$ is the time required to load a ship of type t in Kuwait plus the travel time to destination d and $T_{2,t,r,d}$ is the time required to unload in destination d, plus the travel time from destination d to Kuwait.
- $J_{\bar{h},\bar{t}} = \{(h,r,d)$: if any ship of type $\bar{t}$ departs at day h to destination d by following route r, then this ship would be occupied during day $\bar{h}$, i.e., $\bar{h} \in [h , h + T_{\bar{t},r,d} - 1] \}$.

Penalties

- Type I_1 and Type I_2 penalties : these penalties are associated with Type I demand time windows. Shipments that are delivered within the interval $[D_{1(i,d,p)} - \alpha_{1(i,d,p)}, D_{2(i,d,p)} + \alpha_{2(i,d,p)}]$, for some i, receive the Type I_1 penalty, while shipments that are delivered within the interval $[\varphi_{1(i,d,p)}, D_{1(i,d,p)} - \alpha_{1(i,d,p)}) \cup (D_{2(i,d,p)} + \alpha_{2(i,d,p)}, \varphi_{2(i,d,p)}]$ receive the Type I_2 penalty.
- Type II_1 and Type II_2 penalties : these penalties are associated with Type II demand time windows. A Type II_1 penalty is incurred if the storage level lies within $[SL_{1,d,p} - A_{1,d,p}, SL_{2,d,p} + A_{2,d,p}]$, while a Type II_2 penalty is incurred when the storage level lies within $[0, SL_{1,d,p} - A_{1,d,p})$ or $(SL_{2,d,p} + A_{2,d,p}, \infty)$.
- λ_3 : penalty imposed on empty compartments.

We now present an integer programming formulation for the KPC Oil Tanker Scheduling Problem (KPCP). The objective function and the constraints of the problem are explained subsequently.

KPCP:

Minimize $$\sum_h \sum_t \sum_s \sum_r \sum_d C_{h,t,s,r,d}\, Y_{h,t,s,r,d} + \sum_h \sum_t \sum_s \sum_r \sum_c \sum_d \sum_{\substack{p:\\(d,p)\in \text{Type I}}} P_I(h,t,s,r,d,c,p)\, X_{h,t,s,r,d,c,p}$$

$$+ \sum_{h=1}^{H} \sum_d \sum_{\substack{p:\\(d,p)\in \text{Type II}}} \pi_{d,p}\, [S_{2,h,d,p} + S_{4,h,d,p}] + \sum_{h=1}^{H} \sum_d \sum_{\substack{p:\\(d,p)\in \text{Type II}}} \lambda_2\, [S_{3,h,d,p} + S_{5,h,d,p}]$$

$$+ \sum_t \sum_{s=Ot+1}^{M_t} \$_{t,s}\, Z_{t,s}$$

subject to

(C_1) $$\sum_t \sum_s \sum_r \sum_c \sum_{\substack{h:\\ h+T_{1,t,r,\bar{d}} \in H_{i,\bar{d},\bar{p}}}} \hat{e}_{t,c}\, X_{h,t,s,r,\bar{d},c,\bar{p}} \geq f_{i,\bar{d},\bar{p}} \qquad \forall\ i \text{ and } \forall\ (\bar{d}, \bar{p}) \in \text{Type I}$$

(C_2) $$\sum_t \sum_s \sum_r \sum_c \sum_{\substack{h:\\ h+T_{1,t,r,\bar{d}} \in H_{i,\bar{d},\bar{p}}}} \hat{e}_{t,c}\, X_{h,t,s,r,\bar{d},c,\bar{p}} \leq F_{i,\bar{d},\bar{p}} \qquad \forall\ i \text{ and } \forall\ (\bar{d}, \bar{p}) \in \text{Type I}$$

(C_3) $$S_{\bar{h},d,p} = \omega_{d,p} + \sum_c \sum_r \sum_s \sum_t \sum_{\substack{h:\\ h+T_{1,t,r,d} \in [1,..,\bar{h}]}} \hat{e}_{t,c}\, X_{h,t,s,r,d,c,p} - TC_{\bar{h},d,p} \qquad \forall\ \bar{h} \text{ and } \forall\ (d,p) \in \text{Type II}$$

(C_4) $$S_{h,d,p} = S_{1,h,d,p} - S_{2,h,d,p} - S_{3,h,d,p} + S_{4,h,d,p} + S_{5,h,d,p} \qquad \forall\ h \text{ and } \forall\ (d,p) \in \text{Type II}$$

(C_5) $$\sum_h \sum_t \sum_s \sum_r \sum_c \hat{e}_{t,c}\, X_{h,t,s,r,\bar{d},c,\bar{p}} \geq D_{\bar{d},\bar{p}}(1 - \nu_{\bar{d},\bar{p}}) \qquad \forall\ \bar{d}, \bar{p}$$

(C_6) $$\sum_h \sum_t \sum_s \sum_r \sum_c \hat{e}_{t,c}\, X_{h,t,s,r,\bar{d},c,\bar{p}} \leq D_{\bar{d},\bar{p}}(1 + \nu_{\bar{d},\bar{p}}) \qquad \forall\ \bar{d}, \bar{p}$$

(C_7) $$\sum_{p=1}^{P+1} X_{\bar{h},\bar{t},\bar{s},\bar{r},\bar{d},\bar{c},p} = Y_{\bar{h},\bar{t},\bar{s},\bar{r},\bar{d}} \qquad \forall\ \bar{h}, \bar{t}, \bar{s}, \bar{r}, \bar{d}, \bar{c}$$

(C_8) $$\sum_{(h,r,d)\in J_{\bar{h},\bar{t}}} Y_{h,\bar{t},\bar{s},r,d} \leq Z_{\bar{t},\bar{s}} \qquad \forall\ \bar{h}, \bar{t}, \bar{s}$$

$$(\mathbf{C_9}) \quad \sum_{h} \sum_{r} \sum_{d} Y_{h,\bar{t},\bar{s},r,d}\, T_{\bar{t},d,r} \leq (H - UT_{\bar{t},\bar{s}}) \qquad \forall\ \bar{t},\ \bar{s}$$

$X_{h,t,s,r,d,c,p} \in \{0, 1\} \quad \forall\ h, t, s, r, d, c, p$
$Y_{h,t,s,r,d} \in \{0, 1\} \quad \forall\ h, t, s, r, d$
$Z_{t,s} \in \{0, 1\} \quad \forall\ t, s = O_t+1,..,O_t + CT_t$
$Z_{t,s} = 1 \quad \forall\ t, s = 1,..,O_t$
$S_{h,d,p} \geq 0, \quad SL_{1,d,p} \leq S_{1,h,d,p} \leq SL_{2,d,p} \qquad \forall\ h \text{ and } \forall\ (d,p) \in \text{Type II}$
$0 \leq S_{2,h,d,p} \leq A_{1,d,p}, \quad 0 \leq S_{3,h,d,p} \leq SL_{1,d,p} - A_{2,d,p} \qquad \forall\ h \text{ and } \forall\ (d,p) \in \text{Type II}$
$0 \leq S_{4,h,d,p} \leq A_{2d,p}, \quad S_{5,h,d,p} \geq 0 \qquad \forall\ h \text{ and } \forall\ (d,p) \in \text{Type II.}$

<u>Model Objective and Constraints</u>

The rest this section deals with the interpretation of the objective function and the constraints of problem KPCP.

Objective Function

The objective function of the problem seeks to minimize total cost. Two costs are considered in the operation. The operational cost of selected legs (both for company-owned and chartered vessels), and the penalty costs for violating delivery schedules and demand specifications. Demands cannot be typically satisfied only by the company-owned vessels. In fact, only about half of the demand is usually satisfied by the company-owned vessels and hence, resorting to chartered vessels to satisfy the remaining demands is inevitable. However, in order to minimize the total cost, the company-owned vessels must be fully utilized before resorting to chartered vessels. Indeed, this will be mathematically enforced in the model by associating the chartering expenses with the selected chartered vessels.

The operational cost of a selected leg $L_{h,t,s,r,d}$ is given by $C_{h,t,s,r,d}\, Y_{h,t,s,r,d}$. If $(d,p) \in$ Type I, then the penalty associated with this leg is computed compartmentwise, where the penalty incurred on each compartment is given by $P_I(h,t,s,r,d,c,p)\ X_{h,t,s,r,d,c,p}$. For $(d,p) \in$ Type II, the penalty is computed on a daily basis depending on the storage level, where a Type II_1 penalty is represented by $\pi_{d,p}$ $[S_{2,h,d,p} + S_{4,h,d,p}]$ and a Type II_2 penalty is represented by λ_2 $[S_{3,h,d,p} + S_{5,h,d,p}]$. A ship selected for chartering requires a chartering cost of $\$_{t,s}$ dollars, which is typically high, being of the order of magnitude of about 10 millions.

Type I and Type II Demand Time Windows

Demands of Type I must be satisfied according to feasible delivery schedules. If $(d,p) \in$ Type I, then for each $i \in \{1,..,N_{d,p}\}$, constraints (C_1) and (C_2) assure that the total amount of product p shipped to destination d in the interval $H_{i,d,p}$ lies within $[f_{i,d,p}, F_{i,d,p}]$.
For $(d,p) \in$ Type II, the revisits to destination d are determined by the initial storage level of product p (given by $\omega_{d,p}$), the storage capacity (including minimum and maximum desired levels of product p given by $SL_{1,d,p}$ and $SL_{2,d,p}$), and the rates of consumption R_j. Constraint (C_3) gives the storage level of product p at destination d on day h whereas constraint (C_4) represents $S_{h,d,p}$ in terms of $S_{1,h,d,p}$, $S_{2,h,d,p}$, $S_{3,h,d,p}$, $S_{4,h,d,p}$, and $S_{5,h,d,p}$; the Type II_1 and Type II_2 penalties are incurred in the objective function based on this representation as discussed in Section 3.2.3.

Total Demand

The total demand of Type I and Type II must be satisfied within the time horizon. Observe that constraints (C_1) and (C_2) for Type I demand time windows; or the penalty structure established by (C_3), (C_4), and the objective function for Type II demand time windows, do not necessarily enforce the satisfaction of the total demand of a given product within the time horizon. An example of such an event was given in Section 3.2.5. Constraints (C_5) , (C_6) guarantee that the total demands of Type I and Type II are satisfied within the permitted shortage or excess percentages.

Utilization of Vessels

Constraint (C_7) guarantees that a product $\bar{p}$ can be assigned to compartment $\bar{c}$ of ship $\bar{s}$ of type $\bar{t}$, where this compartment is to be dispatched on day $\bar{h}$, only if the corresponding leg is selected. For example, if leg $L_{\bar{h},\bar{t},\bar{s},\bar{r},\bar{d}}$ is selected, then $Y_{\bar{h},\bar{t},\bar{s},\bar{r},\bar{d}} = 1$, implying by constraint (C_7) that each of the corresponding compartments is assigned a product, i.e., if the corresponding compartment $\bar{c}$ is loaded with product $\bar{p}$, then $X_{\bar{h},\bar{t},\bar{s},\bar{r},\bar{d},\bar{c},\bar{p}} = 1$. On the other hand, if leg $L_{\bar{h},\bar{t},\bar{s},\bar{r},\bar{d}}$ is not selected, then the corresponding compartments cannot be loaded with any product, i.e., $X_{\bar{h},\bar{t},\bar{s},\bar{r},\bar{d},c,p} = 0$, $\forall$ c,p. Observe that, empty compartments are allowed on selected legs by assigning the dummy product P+1 to such compartments, however, with a suitable penalty.

Selection of Legs, Unavailability of a Vessel during a Selected Leg, and Vessel Chartering

Observe that $Z_{t,s} = 0$ implies by constraint (C_8) that $Y_{h,t,s,r,d} = 0$ for all $(h,r,d) \in J_{t,s}$. Suppose that $Z_{t,s} = 1$. If $Y_{\bar{h},\bar{t},\bar{s},\bar{r},\bar{d}} = 1$, then $\bar{h} \in [\bar{h}, \bar{h}+T_{\bar{t},r,d} - 1]$ and $(\bar{h},r,d) \in J_{\bar{t},\bar{s}}$. In this case, constraint (C_8) assures that $Y_{\bar{h},\bar{t},\bar{s},r,d} = 0$, $\forall\ r \neq \bar{r}$ and $\forall\ d \neq \bar{d}$. In other words, constraint (C_8) assures that for a given ship, at most one leg is selected for a given day

If a given leg $L_{\bar{h},\bar{t},\bar{s},\bar{r},\bar{d}}$ is selected, then the ship involved in this leg will not be available during the time interval given by $[\bar{h}+1,.., \bar{h}+T_{\bar{t},\bar{r},\bar{d}} - 1]$. Furthermore, any leg selected prior to $\bar{h}$ for this ship must terminate before day $\bar{h}$. Observe that constraint (C_8) assures this requirement, where (C_8) examines each day (time-slot) $\bar{h}$ for each ship $\bar{s}$ of type $\bar{t}$, and asserts that for any such day, this ship can be occupied by at most one leg. This follows from the definition of the index set $J_{\bar{h},\bar{t}}$.

Constraint (C_8) also examines if a ship $\bar{s}$ of type $\bar{t}$ is selected for chartering. Note that a ship is selected for chartering if this ship is involved in at least one leg during the time horizon. Therefore, if ship $\bar{s}$ of type $\bar{t}$ is assigned to leg $L_{h,\bar{t},\bar{s},r,d}$, then $Y_{h,\bar{t},\bar{s},r,d} = 1$ and hence constraint (C_8) forces $Z_{\bar{t}, \bar{s}}$ to be one, indicating that this ship is selected for chartering.

Vessel Availability During the Time Horizon

The company may require that a ship $\bar{s}$ of type $\bar{t}$ can be used for at most (H-$UT_{\bar{t}, \bar{s}}$) days during the time horizon. This requirement is enforced by constraint (C_9).

3.3 Variable Initialization

Information regarding the status of each vessel must be determined at the beginning of the time horizon. This information includes the location of each vessel at the beginning of the time horizon, the time required for each vessel to return to Kuwait after the beginning of the time horizon (this time equals 0 if the vessel is stationed in Kuwait at the beginning of the time horizon), and its capability to carry different products and visit different destinations. Using the above information, specific indices are defined for each variable to reflect its availability. *Any undefined variable is henceforth assumed to be zero.*

The following are some examples of variables that are assigned either the value of one or zero. Observe that, whenever a Y-variable assumes the value of zero, then constraint (C_7) would force the corresponding X-variables to also have zero values.

(1) $Y_{h, \bar{t}, s, r, \bar{d}} = 0$ if destination $\bar{d}$ does not admit ships of type $\bar{t}$, $\forall$ h, s, r.

(2) $X_{h, \bar{t}, s, r, d, c, \bar{p}} = 0$ if ships of type $\bar{t}$ cannot carry product $\bar{p}$, $\forall$ h, s, r, d, c.

(3) $Y_{h, \bar{t}, s, \bar{r}, \bar{d}} = 0$ if a ship of type $\bar{t}$ cannot go to destination $\bar{d}$ by taking route $\bar{r}$, $\forall$ h, s. An example of such a situation is that loaded super tankers cannot pass through the Suez Canal.

(4) All destinations that do not require passing through the Suez Canal or going around the Cape of Good Hope are assumed to have a unique route represented by r = 5, i.e., $Y_{h,t,s,r,d} = 0$, $\forall$ h,t,s and r = 1,..,4. On the other hand, $Y_{h,t,s,r=5,d} = 0$, for all destinations that require passing through the Suez Canal or going around the Cape of Good Hope.

(5) A ship $\bar{s}$ of type $\bar{t}$ may not be available during the period given by h = h_1 to h_2 of the time horizon. Such a situation occurs if the company requires to perform a scheduled maintenance on a ship during some specific days of the time horizon. This information can be incorporated into the model by letting $Y_{h,\bar{t},\bar{s},r,d} = 0$, $\forall$ $(h,r,d) \in J_{\bar{h},\bar{t}}$ and $\forall$ $h \in \{h_1,..,h_2\}$.

(6) A new demand contract may be signed during a given contract horizon. In this case, the solution of the first demand contract may have variables with values of ones that commit ships over periods concurrent with the second (adjacent) contract horizon. Therefore, proper sets of variables should be defined to be zero to indicate the unavailability of ships during the second contract horizon.

3.4 Discussion

We now conclude this chapter with a discussion on various issues relevant to the KPC model. These issues range from modifying the legs in order to recognize various speed capabilities for the vessels, to discussing the potential usefulness of the model. The stochastic aspects of the problem are also addressed to justify the utilization of a deterministic model. In this regard, note that due to the volatility of the oil market, the oil prices may vary during the time horizon for a given demand contract. The fluctuating price of oil has no impact on the total demand of different products agreed upon between KPC and customers over the time horizon.

The above formulation recognizes only one speed for a given ship s of type t. Nevertheless, a vessel might have 2 to 3 speed capabilities that admit different travel times and costs for a given leg. The formulation can be modified to account for different speeds for a given ship s of type t, by simply incorporating another index level corresponding to this choice. However, since variations in speed en route are frequently used to maintain schedules and to counteract effects of weather delays, we do not incorporate speed decisions in our model. This permits a degree of flexibility in maintaining prescribed schedules.

New demands often emerge during a given time horizon. The company may sign a contract to satisfy customer demands within a certain time horizon, and then the company might enter into another contract during this same time horizon to satisfy a second set of demands. In other words, new demands do not necessarily emerge only at the end of the current time horizon. Contracts are usually signed every 3 to 4 months. Every time a contract is signed, information regarding this contract can be employed to define time windows for the new demands. Since decisions that have not been implemented can be changed, the model can be run in a rolling horizon fashion by modifying it each time the demand structure changes. All committed decisions at such a time can be used to initialize certain variables to specific values as discussed in Section 3.3.

The above formulation assumes a completely deterministic problem, whereas the real operation contains many stochastic aspects. Travel time, delays in loading and unloading, severe weather, breakdowns, and customer demands are some of the stochastic elements of the operation. In the same sprit as the above discussion relating to changes in demands, rescheduling can be employed to deal with the stochastic aspects of the problem. For example, the travel time for leg $L_{h,t,s,r,d}$ is more or less known *a priori* and can be treated as a constant. However, a revamping of decisions might be necessary in the event of severe weather conditions or major breakdowns. In fact, one primary motivation for developing a computerized scheduling capability is to be able to generate new schedules frequently, conveniently, and at a very short notice, as the need arises.

The process of determining the time widows and the intervals of penalties involves a great deal of negotiation between KPC and its customers. It is often the case that customers propose feasible delivery dates of shipments for KPC (either of Type I demand time windows or Type II demand time windows). KPC may accept these delivery dates or negotiate with customers to arrive at mutually acceptable delivery dates. Currently, KPC employs its scheduling experience to negotiate with customers on feasible delivery dates. It is lucrative to decide on an overall cost- effective set of feasible delivery dates, and this is of prime importance to KPC. The proposed model can be used not only for scheduling and planning purposes, but can also serve as a useful tool for gaining an edge in the negotiating process. By running the model in a sensitivity analysis fashion for various possible delivery and penalty options, KPC can pre-assess what effect a given contract can have on its overall operations and costs, and hence on its margin of profit.

Chapter IV
An Aggregate Reformulation
of Problem KPCP

4.1 Introduction

In this chapter, we present an aggregate version of problem KPCP that was formulated in Chapter III. This version is intended to deal with the overwhelming size difficulties associated with problem KPCP. We commence this chapter by examining and exploiting the structure of the constraints of the problem KPCP in order to aggregate some constraints and hence reduce the problem size (Section 4.2). We also give a problem size analysis for problem KPCP in terms of the constraints and variables, along with an example for a typical demand contract scenario (Section 4.3).

An aggregate formulation that retains the essential features of problem KPCP, along with related issues is presented in Section 4.4. This aggregate model, denoted by KPCPxy, is derived from the exact formulation KPCP by using integer variables that represent the number of ships and allocated compartments for each destination in lieu of using the previously discussed binary variables. Implementation issues are addressed in Section 4.5. The subsequent chapters address solution strategies and algorithms, computational results, and sensitivity analysis issues for both problems KPCP and KPCPxy.

It is assumed that for a given destination d, demands of all products are either to be satisfied according to Type I or Type II demand time windows. Accordingly, the destinations can be partitioned based on the demand time windows. Rewriting the constraints of problem KPCP so that all the variables appear on the left-hand side while all the constants appear on the right-hand side, problem KPCP is given as follows.

KPCP:

Minimize $\sum_h \sum_t \sum_s \sum_r \sum_d C_{h,t,s,r,d} Y_{h,t,s,r,d} + \sum_h \sum_t \sum_s \sum_r \sum_c \sum_d \sum_{p \atop (d,p) \in \text{Type I}} P_l(h,t,s,r,d,c,p)\, X_{h,t,s,r,d,c,p}$

$$+ \sum_{h=1}^{H} \sum_d \sum_{p: \atop (d,p) \in \text{Type II}} \pi_{d,p} \left[S_{2,h,d,p} + S_{4,h,d,p}\right] + \sum_{h=1}^{H} \sum_d \sum_{p: \atop (d,p) \in \text{Type II}} \lambda_2 \left[S_{3,h,d,p} + S_{5,h,d,p}\right]$$

$$+ \sum_t \sum_{s=Ot+1}^{M_t} \$_{t,s} Z_{t,s}$$

subject to

(C1) $\sum_t \sum_s \sum_r \sum_c \sum_{h: \atop h+T_{1,t,r,\bar{d}} \in H_{i,\bar{d},\bar{p}}} \hat{e}_{t,c} X_{h,t,s,r,\bar{d},c,\bar{p}} \geq f_{i,\bar{d},\bar{p}}$ $\quad \forall$ i and $\forall (\bar{d}, \bar{p}) \in$ Type I

(C2) $\sum_t \sum_s \sum_r \sum_c \sum_{h: \atop h+T_{1,t,r,\bar{d}} \in H_{i,\bar{d},\bar{p}}} \hat{e}_{t,c} X_{h,t,s,r,\bar{d},c,\bar{p}} \leq F_{i,\bar{d},\bar{p}}$ $\quad \forall$ i and $\forall (\bar{d}, \bar{p}) \in$ Type I

(C3) $\sum_c \sum_r \sum_s \sum_t \sum_{h: \atop h+T_{1,t,r,d} \in [1,..,\bar{h}]} \hat{e}_{t,c} X_{h,t,s,r,d,c,p} - S_{\bar{h},d,p} = TC_{\bar{h},d,p} - \omega_{d,p}$

$\forall \bar{h}$ and $\forall (d,p) \in$ Type II

(C4) $S_{h,d,p} - S_{1,h,d,p} + S_{2,h,d,p} + S_{3,h,d,p} - S_{4,h,d,p} - S_{5,h,d,p} = 0$ $\quad \forall$ h and $\forall (d,p) \in$ Type II

(C5) $\sum_h \sum_t \sum_s \sum_r \sum_c \hat{e}_{t,c} X_{h,t,s,r,\bar{d},c,\bar{p}} \geq D_{\bar{d},\bar{p}}(1 - \nu_{\bar{d},\bar{p}})$ $\quad \forall \bar{d}, \bar{p}$

(C6) $\sum_h \sum_t \sum_s \sum_r \sum_c \hat{e}_{t,c} X_{h,t,s,r,\bar{d},c,\bar{p}} \leq D_{\bar{d},\bar{p}}(1 + \nu_{\bar{d},\bar{p}})$ $\quad \forall \bar{d}, \bar{p}$

(C7) $\sum_{p=1}^{P+1} X_{\bar{h},\bar{t},\bar{s},\bar{r},\bar{d},\bar{c},p} - Y_{\bar{h},\bar{t},\bar{s},\bar{r},\bar{d}} = 0$ $\quad \forall \bar{h}, \bar{t}, \bar{s}, \bar{r}, \bar{d}, \bar{c}$

(C$_8$) $\sum_{(h,r,d) \in J_{\bar{h},\bar{t}}} Y_{h,\bar{t},\bar{s},r,d} - Z_{\bar{t},\bar{s}} \leq 0 \qquad \forall\ \bar{h}, \bar{t}, \bar{s}$

(C$_9$) $\sum_{h} \sum_{r} \sum_{d} Y_{h,\bar{t},\bar{s},r,d}\ T_{\bar{t},d,r} \leq (H - UT_{\bar{t},\bar{s}}) \qquad \forall\ \bar{t}, \bar{s}$

$X_{h,t,s,r,d,c,p} \in \{0, 1\} \quad \forall$ h, t, s, r, d, c, p

$Y_{h,t,s,r,d} \in \{0, 1\} \quad \forall$ h, t, s, r, d

$Z_{t,s} \in \{0, 1\} \quad \forall\ t, s = O_t+1,..,O_t + CT_t$

$Z_{t,s} = 1 \quad \forall\ t, s = 1,..,O_t$

$S_{h,d,p} \geq 0, \quad SL_{1,d,p} \leq S_{1,h,d,p} \leq SL_{2,d,p} \qquad \forall$ h and $\forall$ (d,p) $\in$ Type II

$0 \leq S_{2,h,d,p} \leq A_{1,d,p}, \quad 0 \leq S_{3,h,d,p} \leq SL_{1,d,p} - A_{1,d,p} \qquad \forall$ h and $\forall$ (d,p) $\in$ Type II

$0 \leq S_{4,h,d,p} \leq A_{2d,p}, \quad S_{5,h,d,p} \geq 0 \qquad \forall$ h and $\forall$ (d,p) $\in$ Type II.

Observe that $Z_{t,s} \equiv 1$ for the company owned vessels. For the sake of convenience in presentation, we introduce a list of notation that will be frequently referred to throughout the development of this chapter. We also give variations of problem KPCP in matrix notation. The list of notation is given as follows.

Vectors and Matrices

- $M^{(i,j)}$: $(i,j)^{th}$ entry of matrix M.
- M_i : i^{th} row of matrix M.
- M^j : j^{th} column of matrix M.
- $U^{(i)}$: i^{th} entry of the vector U.
- $M = [M_i]$: The matrix M is composed of the rows (row matrices) M_i.
- M^T : Transpose of the matrix M.
- Rank(M) : Rank of the matrix M.
- det(M) : Determinant of the matrix M.
- adj(M) : Adjoint of the matrix M.

- $\mathbf{1}_n$: The column vector consisting of 1's whose dimension is n.
- $\mathbf{0}_n$: The column vector consisting of 0's whose dimension is n.
- C : The cost vector consisting of the costs $C_{h,t,s,r,d}$.
- P : Penalty vector, where P = $[P_I, \mathbf{0}]$, and where P_I is associated with the Type I demand time windows, being composed of the penalties $P_I(h,t,s,r,d,c,p)$.
- π : Penalty vector associated with Type II demand time windows. This vector is composed of the penalties $\pi_{d,p}$.
- \$: Chartering expense vector which consists of the chartering expenses $\$_{t,s}$.
- X : Shipment vector which consists of the binary variables $X_{h,t,s,r,d,c,p}$.
- Y : Leg vector which consists of the binary variables $Y_{h,t,s,r,d}$.
- Z : Chartering vector which is composed of the binary variables $Z_{t,s}$.
- $S_{k,d,p}$: The vector consisting of the variables $S_{k,h,d,p}$ $\forall$ h, for each d,p and k = 0,..,5, where $S_{0,h,d,p} = S_{h,d,p}$.
- S_k : The vector consisting of the vectors $S_{k,d,p}$ $\forall$ d,p and for each k = 0,..,5.
- $S = [S_0, S_1, S_2, S_3, S_4, S_5]$.
- $f_{d,p}$: The vector consisting of the values $f_{i,d,p}$, for i=1,..,$N_{d,p}$.
- $F_{d,p}$: The vector consisting of the values $F_{i,d,p}$, for i=1,..,$N_{d,p}$.
- b_1 : The vector consisting of the vectors $f_{d,p}$.
- b_2 : The vector consisting of the vectors $F_{d,p}$.
- b_3 : The vector consisting of the values $TC_{h,d,p} - \omega_{d,p}$.
- b_4 : The vector consisting of 0's corresponding to the right-hand side of constraint C_4.
- b_5 : The vector consisting of the values $D_{d,p}(1 - \nu_{d,p})$.
- b_6 : The vector consisting of the values $D_{d,p}(1 + \nu_{d,p})$.
- b_7 : The vector consisting of 0's corresponding to the right-hand side of constraint C_7.
- b_8 : The vector consisting of 0's corresponding to the right-hand side of constraint

$C_{8.}$

- b_9 : The vector consisting of the values (H - $UT_{t,s}$).
- A, B, D, and E : The matrices whose entries correspond to the coefficients of the variables X, Y, Z, and S, respectively, in problem KPCP.
- W = [W_i] : The slack vector associated with the constraint (C_i) in problem KPCP, where $W_i \leq 0$ whenever constraint C_i involves the "≥'' sign, $W_i \geq 0$ whenever constraint C_i involves the "≤'' sign, and $W_i \equiv 0$ whenever constraint C_i involves the "='' sign.
- b = [b_i] : The right-hand side of problem KPCP, where b_i is defined above.
- Λ = {(X,Y,Z,S,W) : where X, Y, Z, S and W are as defined above}

Number of Variables and Constraints

- $N_x(t)$: Number of X-variables for ship-type t.
- N_x : Total number of X-variables, which is given by $N_x = \sum_{t=1}^{T} N_x(t)$.
- $N_y(t)$: Number of Y-variables for ship-type t.
- N_y : Total number of Y-variables, which is given by $N_y = \sum_{t=1}^{T} N_y(t)$.
- $N_z(t)$: Number of chartering variables for ship-type t.
- N_z : Total number of chartering variables, which is given by $N_z = \sum_{t=1}^{T} N_z(t)$.
- N_s : Total number of S-variables.
- $N(C_i)$: Number of constraints in equation (C_i), for i=1,..,9.
- $NC = \sum_{i=1}^{9} N(C_i)$.

Miscellaneous

- $\nu(\mathbb{P})$: Optimal objective value for problem $\mathbb{P}$. If $\mathbb{P}$ is a minimization problem that is infeasible, or a maximization problem that is unbounded, then $\nu(\mathbb{P}) = +\infty$, while if $\mathbb{P}$ is a maximization problem that is infeasible, or a minimization problem that is unbounded, then $\nu(\mathbb{P}) = -\infty$.
- $\overline{\mathbb{P}}$: The linear relaxation of problem $\mathbb{P}$.
- FR(P) : The feasible region of problem P.
- ê$_t$: The capacity of a ship of type t.
- d $\in$ Type I : All products at destination d are to be satisfied according to Type I demand time windows.
- d $\in$ Type II : All products at destination d are to be satisfied according to Type II demand time windows.
- DD_I : Number of destinations of Type I.
- DD_{II} : Number of destinations of Type II.

$|S|$: Number of elements in the set S.

In the remainder of this section, various equivalent representations of problem KPCP are given in matrix notation. These representations are intended to provide some insights into the general structure of the problem. Relevant matrices and vectors are assumed to have conformable dimensions.

Now, problem KPCP can be expressed in matrix notation as follows.

KPCPM1:

$$\textbf{Minimize}\quad PX + CY + \sum_{d}\sum_{\substack{p:\\(d,p)\in \text{Type II}}} \pi_{d,p} \cdot \mathbf{1}_H \cdot [S_{2,d,p} + S_{4,d,p}] + \sum_{d}\sum_{\substack{p:\\(d,p)\in \text{Type II}}} \lambda_2 \cdot \mathbf{1}_H \cdot [S_{3,d,p} + S_{5,d,p}]$$

$$+\ \$\, Z$$

subject to $AX + BY + DZ + ES + W = b.$

$(X, Y, Z, S, W) \in \Lambda$

Let $X = [X_I , X_{II}]$, where X_I and X_{II} correspond to destinations of Type I and Type II, respectively. In a similar fashion, the vector Y is partitioned into Y_I and Y_{II}, and the vector P is partitioned into P_I and the zero vector. Accordingly, the matrix A is partitioned into A_I and A_{II}, and B is partitioned into B_I and B_{II}. Let $A_I = [A_{I,i}]$, for i =1,..,9, where $A_{I,i}$ denotes the matrix whose entries correspond to the coefficients of constraint (C_i). The matrices A_{II}, B_I, B_{II}, D and E, and the vectors W and b are partitioned in a similar fashion. Hence, problem KPCPM1 can be rewritten as follows.

KPCPM2:

Minimize $$P_I X_I + CY + \sum_{d} \sum_{\substack{p: \\ (d,p) \in \text{ Type II}}} \pi_{d,p} \cdot \mathbf{1_H} \cdot [S_{2,d,p} + S_{4,d,p}] + \sum_{d} \sum_{\substack{p: \\ (d,p) \in \text{ Type II}}} \lambda_2 \cdot \mathbf{1_H} \cdot [S_{3,d,p} + S_{5,d,p}] + \$ Z$$

subject to

$$(C_i) \quad A_{I,i} X_I + A_{II,i} X_{II} + B_{I,i} Y_I + B_{II,i} Y_{II} + D_i Z + E_i S + W_i = b_i , \qquad \text{for } i=1,..9$$

$$(X, Y, Z, S, W) \in \Lambda.$$

We further partition some matrices and constraints of problem KPCPM2. Constraint C_1 is partitioned as follows. Let $A_{I,1} = [A_{I,1,(d,p)}]$, where $(d,p) \in$ Type I and $A_{I,1,(d,p)}$ is the matrix whose entries correspond to the variables pertinent to (d,p). The matrices $A_{II,1}$, $B_{I,1}$, $B_{II,1}$, D_1 and E_1, and the vectors W_1 and b_1 are partitioned accordingly. Note that the matrices $A_{II,1}$, $B_{I,1}$, $B_{II,1}$, D_1, and E_1 are zero matrices. Constraint C_2 is partitioned in a similar fashion. Constraint (C_5) is partitioned into ($C_{5.1}$) and ($C_{5.2}$), and constraint (C_6) is partitioned into ($C_{6.1}$) and ($C_{6.2}$), where ($C_{5.1}$) and ($C_{6.1}$) correspond to Type I demand time windows, and ($C_{5.2}$) and ($C_{6.2}$) correspond to Type II demand time windows. Likewise, constraint (C_7) is partitioned into ($C_{7.1}$) and ($C_{7.2}$). The vectors b_5, b_6, and b_7 are partitioned according to their corresponding constraints. Problem KPCPM2 can now be rewritten discarding all zero matrices as follows.

KPCPM3:

Minimize $P_I X_I + CY + \sum_{d} \sum_{\substack{p: \\ (d,p) \in \text{Type II}}} \pi_{d,p} \,.\, \mathbf{1}_H \,.\, [S_{2,d,p} + S_{4,d,p}] + \sum_{d} \sum_{\substack{p: \\ (d,p) \in \text{Type II}}} \lambda_2 \,.\, \mathbf{1}_H \,.\, [S_{3,d,p} + S_{5,d,p}]$

$+ \ \$ Z$

subject to

(C_1)	$A_{I,1,(d,p)} X_I$				$+ W_{1,(d,p)}$	$= f_{d,p}$
(C_2)	$A_{I,2,(d,p)} X_I$				$+ W_{2,(d,p)}$	$= F_{d,p}$
(C_3)	$A_{II,3,(d,p)} X_{II}$				$+ E_3 S$	$= b_3$
(C_4)					$E_4 S$	$= b_4$
($C_{5.1}$)	$A_{I,5.1,(d,p)} X_I$				$+ W_{5.1,(d,p)}$	$= b_{5.1,(d,p)}$
($C_{5.2}$)	$A_{II,5.2,(d,p)} X_{II}$				$+ W_{5.2,(d,p)}$	$= b_{5.2,(d,p)}$
($C_{6.1}$)	$A_{I,6.1,(d,p)} X_I$				$+ W_{6.1,(d,p)}$	$= b_{6.1,(d,p)}$
($C_{6.2}$)	$A_{II,6.2,(d,p)} X_{II}$				$+ W_{6.2,(d,p)}$	$= b_{6.2,(d,p)}$
($C_{7.1}$)	$A_{I,7.1} X_I$	$+ B_{I,7.1} Y_I$				$= b_{7.1}$
($C_{7.2}$)	$A_{II,7.2} X_{II}$		$+ B_{II,7.2} Y_{II}$	+		$= b_{7.2}$
(C_8)		$B_{I,8} Y_I$	$+ B_{II,8} Y_{II}$		$+ W_8$	$= b_8$
(C_9)		$B_{I,9} Y_I$	$+ B_{II,9} Y_{II}$		$+ W_9$	$= b_9$

$(X, Y, Z, S, W) \in \Lambda$.

Constraints (C_1), (C_2), ($C_{5.1}$), and ($C_{6.1}$) are written for all $(d,p) \in$ Type I, and constraints ($C_{5.2}$) and ($C_{6.2}$) are written for all $(d,p) \in$ Type II.

The above problem is fairly sparse and moreover, the problem possesses special structures that will be exploited in the subsequent sections in order to reduce the problem size and to motivate the formulation of an aggregate model.

4.2 Aggregation of Constraints

In this section, the structure of the constraints is examined to identify redundant constraints and to aggregate some constraints in order to reduce the size of the problem.

Remark 1: Constraints (C_1) and (C_2) enforce that the demand of a given product at destination d $\in$ Type I is satisfied according to a specified time window. Since $A_{I,1,(d,p)} = A_{I,2,(d,p)}$, this requirement can be alternatively enforced using one constraint, given by ($C_{1,2}$): $A_{I,(1,2),(d,p)} X_I - W_{(1,2),(d,p)} = 0 \;\; \forall$ (d,p) $\in$ Type I, where $A_{I,(1,2),(d,p)} = A_{I,1,(d,p)} = A_{I,2,(d,p)}$, and $f_{d,p} \leq W_{(1,2),(d,p)} \leq F_{d,p}$. Note that $W^{(i)}_{(1,2),(d,p)}$, for i=1,..,$N_{d,p}$, represents the total amount of product p delivered to destination d to satisfy the i^{th} demand. Constraint ($C_{1,2}$) implies that $f_{d,p} \leq A_{I,(1,2),(d,p)} X_I \leq F_{d,p}$ as specified by the corresponding time window.

Remark 2: Constraints (C_3) and (C_4) can be combined by replacing $S_{\bar{h},d,p}$ in (C_3) with $S_{1,\bar{h},d,p} - S_{2,\bar{h},d,p} - S_{3,\bar{h},d,p} + S_{4,\bar{h},d,p} + S_{5,\bar{h},d,p}$ from (C_4).

Remark 3: In the same spirit as in Remark 1, constraints ($C_{5.1}$) and ($C_{6.1}$) can be consolidated into one constraint. Since $A_{I,5.1,(d,p)} = A_{I,6.1,(d,p)}$, constraints ($C_{5.1}$) and ($C_{6.1}$) can be alternatively enforced using one constraint, given by ($C_{5.1,6.1}$): $A_{I,(5.1,6.1),(d,p)} X_I - W_{(5.1,6.1),(d,p)} = 0 \;\; \forall$ (d,p) $\in$ Type I, where $A_{I,(5.1,6.1),(d,p)} = A_{I,5.1,(d,p)} = A_{I,6.1,(d,p)}$ and $b_{5.1,(d,p)} \leq W_{(5.1,6.1),(d,p)} \leq b_{6.1,(d,p)}$. Constraint ($C_{5.1,6.1}$) implies that $b_{5.1,(d,p)} \leq A_{I,(5.1,6.1),(d,p)} X_I \leq b_{6.1,(d,p)}$, as specified by the demand contract. Likewise, constraints ($C_{5.2}$) and ($C_{6.2}$) can be consolidated into one constraint given by ($C_{5.2,6.2}$):
$A_{I,(5.2,6.2),(d,p)} X_I - W_{(5.2,6.2),(d,p)} = 0 \;\; \forall$ (d,p) $\in$ Type II, where $A_{I,(5.2,6.2),(d,p)} = A_{I,5.2,(d,p)} = A_{I,6.2,(d,p)}$ and $b_{5.2,(d,p)} \leq W_{(5.2,6.2),(d,p)} \leq b_{6.2,(d,p)}$.

Remark 4: Let (d,p)∈Type I and let $X_{I,(d,p)}$ be the subvector of X_I that corresponds to (d,p). Then, the rows of constraint ($C_{1,2}$) that correspond to (d,p) can be rewritten as follows:

$$A^1_{I,(1,2),(d,p)} X_{I,(d,p)} + \sum_{(\bar{d},\bar{p}) \neq (d,p)} A^2_{I,(1,2),(\bar{d},\bar{p})} X_{I,(\bar{d},\bar{p})} - W_{(1,2),(d,p)} = 0,$$

where $A^1_{I,(1,2),(d,p)}$ is the submatrix of $A_{I,(1,2),(d,p)}$ whose entries correspond to (d,p) and $A^2_{I,(1,2),(\bar{d},\bar{p})}$ is the submatrix of $A_{I,(1,2),(d,p)}$ whose entries correspond to $(\bar{d},\bar{p}) \neq (d,p)$. Likewise, constraint ($C_{5.1,6.1}$) can now be rewritten as follows:

$$A^1_{I,(5.1,6.1),(d,p)} X_{I,(d,p)} + \sum_{(\bar{d},\bar{p}) \neq (d,p)} A^2_{I,(5.1,6.1),(\bar{d},\bar{p})} X_{I,(\bar{d},\bar{p})} - W_{(5.1,6.1),(d,p)} = 0,$$

where $A^1_{I,(5.1,6.1),(d,p)}$ is the submatrix of $A_{I,(5.1,6.1),(d,p)}$ whose entries correspond to (d,p) and $A^2_{I,(5.1,6.1),(\bar{d},\bar{p})}$ is the submatrix of $A_{I,(5.1,6.1),(d,p)}$ whose entries correspond to $(\bar{d},\bar{p}) \neq$ (d,p). Note that for $(\bar{d},\bar{p}) \neq$ (d,p), the matrices $A^2_{I,(1,2),(\bar{d},\bar{p})}$ and $A^2_{I,(5.1,6.1),(\bar{d},\bar{p})}$ are zero matrices.

The matrix $A^1_{I,(5.1,6.1),(d,p)}$ is composed of exactly one row, whose possible nonzero entries correspond to $X_{I,(d,p)}$. The matrix $A_{I,(1,2),(d,p)}$ is composed of $N_{d,p}$ rows, each of which corresponds to a partition $D_{i,d,p}$. Likewise, the vector $W_{(1,2),(d,p)}$ is composed of $N_{d,p}$ entries. Adding the rows of the matrix $A^1_{I,(1,2),(d,p)}$ and noticing constraint ($C_{1,2}$), we obtain the following equation: $B_{\mathbf{(d,p)}} . X_{I,(d,p)} - \mathbf{1}_{N_{d,p}} . W_{(1,2),(d,p)} = 0$, where $B_{(d,p)}$ is the row matrix whose i^{th} entry is obtained by adding the entries of the i^{th} column of the matrix $A^1_{I,(1,2),(d,p)}$, and $\mathbf{1}_{N_{d,p}}$ is the one vector whose dimension is $N_{d,p}$. Note that $B_{\mathbf{(d,p)}} = A^1_{I,(5.1,6.1),(d,p)}$ and hence $A^1_{I,(5.1,6.1),(d,p)} X_{I,(d,p)} - \mathbf{1}_{N_{d,p}} . W_{(1,2),(d,p)} = 0$. Since $A^1_{I,(5.1,6.1),(d,p)} X_{I,(d,p)} - W_{(5.1,6.1),(d,p)} = 0$, we obtain the following equation ($\bar{C}_{5.1,6.1}$) :

$W_{(5.1,6.1),(d,p)} - \mathbf{1}_{N_{d,p}} . W_{(1,2),(d,p)} = 0$.

Accordingly, constraint ($C_{5.1,6.1}$) can be replaced with constraint ($\bar{C}_{5.1,6.1}$). Constraint ($\bar{C}_{5.1,6.1}$) may be considered advantageous over constraint ($C_{5.1,6.1}$) since it involves no binary variables.

4.3 Problem Size Analysis

Before proceeding to address our solution strategy for problem KPCP, we discuss the problem size in terms of the variables and constraints. We also present an example for a typical demand contract scenario. This example will motivate the formulation of problem KPCPxy from problem KPCP by using integer variables representing the number of ships and allocated compartments for each destination in lieu of using binary variables (Section 4.4).

In the following, Table 4.1 and Table 4.2 give the number of variables of problem KPCP, and Table 4.3 gives the number of constraints of problem KPCP. It is assumed that the number of ship-types is T = 5, which reflects both the actual number of available types owned by KPC and those available in the market for possible chartering. Observe that M_t, for t = 1,..,5 denote the total number of ships of type t, CT_t, for t = 1,..,5 denote the number of charter ships of type t, and C_t, for t = 1,..,5 denote the number of compartments on a ship of type t. The number of routes is assumed to be R = 4.

TABLE 4.1. Number of the (X, Y, Z) Variables in Problem KPCP

Variable (X,Y,Z)	Ship-Type $t\in\{1,..,5\}$	Number of Variables for Ship-Type t, ($N_x(t)$, $N_y(t)$, $N_z(t)$)	Total Number of (X,Y,Z) Variables for all Types of Ships
X	1	$N_x(1) = H \,.\, M_1 \,.\, 4 \,.\, D \,.\, C_1 \,.\, P$	
	2	$N_x(2) = H \,.\, M_2 \,.\, 4 \,.\, D \,.\, C_2 \,.\, P$	
	3	$N_x(3) = H \,.\, M_3 \,.\, 4 \,.\, D \,.\, C_3 \,.\, P$	
	4	$N_x(4) = H \,.\, M_4 \,.\, 4 \,.\, D \,.\, C_4 \,.\, P$	
	5	$N_x(5) = H \,.\, M_5 \,.\, 4 \,.\, D \,.\, C_5 \,.\, P$	
			$N_x = \sum_{t=1}^{5} N_x(t)$
Y	1	$N_y(1) = H \,.\, M_1 \,.\, 4 \,.\, D$	
	2	$N_y(2) = H \,.\, M_2 \,.\, 4 \,.\, D$	
	3	$N_y(3) = H \,.\, M_3 \,.\, 4 \,.\, D$	
	4	$N_y(4) = H \,.\, M_4 \,.\, 4 \,.\, D$	
	5	$N_y(5) = H \,.\, M_5 \,.\, 4 \,.\, D$	
			$N_y = \sum_{t=1}^{5} N_y(t)$
Z	1	$N_z(1) = CT_1$	
	2	$N_z(2) = CT_2$	
	3	$N_z(3) = CT_3$	
	4	$N_z(4) = CT_4$	
	5	$N_z(5) = CT_5$	
			$N_z = \sum_{t=1}^{5} N_z(t)$
Total number of (X,Y,Z) variables =			$N_x + N_y + N_z$

TABLE 4.2. Number of the ($S_{h,d,p}$, $S_{1,h,d,p}$, $S_{2,h,d,p}$, $S_{3,h,d,p}$, $S_{4,,h,d,p}$, $S_{5,h,d,p}$) Variables in Problem KPCP

Variable	Number
$S_{h,d,p}$, $S_{1,h,d,p}$, $S_{2,h,d,p}$, $S_{3,h,d,p}$, $S_{4,h,d,p}$, or $S_{5,h,d,p}$	$H \,.\, DD_{II} \,.\, P$
Total number of ($S_{h,d,p}$, $S_{1,h,d,p}$, $S_{2,h,d,p}$, $S_{3,h,d,p}$, $S_{4,h,d,p}$, $S_{5,h,d,p}$) variables	$N_s = 6.\ H.DD_{II} \,.\, P$

TABLE 4.3. Number of Constraints in Problem KPCP

Type of Constraint	Number of Constraints of Type C_i
$C_{1,2}$	$N(C_{1,2}) = \sum_{(d,p) \in \text{Type I}} N_{d,p}$
$C_{3,4}$	$N(C_{1,2}) = H \,.\, D_{II} \,.\, P$
$C_{5.1,6.1}$	$N(C_{5.1,6.1}) = D_I \,.\, P$
$C_{5.2,6.2}$	$N(C_{5.2,6.2}) = D_{II} \,.\, P$
C_7	$N(C_7) = N_x / P$
C_8	$N(C_8) = \sum_{t=1}^{T} H\, M_t$
C_9	$N(C_9) = \sum_{t=1}^{T} M_t$
Total number of constraints = $N(C_{1,2})+ N(C_{3,4})+ N(C_{5.1,6.1})+ N(C_{5.2,6.2}) + N(C_7) + N(C_8) + N(C_9)$	

Example 4.1 (An Example for a Typical Demand Contract Scenario)

A typical demand contract scenario for problem KPCP is presented in TABLE 4.4 and TABLE 4.5 below. Since computational results of Chapter V deals with the Type I demand time windows, we only consider this type of demand time windows in the following example.

TABLE 4.4. Data for a Typical Demand Contract Scenario for Problem KPCP

Time Horizon (H)	H = 300
Number of Available Ship-Types (T)	T = 5
Number of Routes (R)	R = 4
Number of Destinations of Type I (DD_I)	DD_I = 10
Number of Products (P)	P = 5
Number of partitions $N_{d,p}$	$N_{d,p}$ = 5, for all $(d,p) \in$ Type I

TABLE 4.5. Number of Ships and Compartments for Each Ship-Type

Ship-Type (t=1,..,5)	Total Number of Ships of Type t, $M_t = O_t + CT_t$	O_t (Owned Vessels)	CT_t (Chartered Vessels)	Number of Compartments for Ships of Type t
1	15	7	8	6
2	15	10	5	6
3	10	5	5	8
4	10	4	6	8
5	10	5	5	8

Tables 4.1, 4.2, and 4.3 are now reproduced based on the information given in Tables 4.4 and 4.5.

TABLE 4.6. Number of the (X, Y, Z) Variables for Example 4.1

Variable (X,Y,Z)	Ship-Type $t \in \{1,..,5\}$	Number of Variables for Ship-Type t, ($N_x(t)$, $N_y(t)$, $N_z(t)$)	Total Number of (X,Y,Z) Variables for all Ship-Types
X	1	$N_x(1)$ = 300 . 15 . 4 . 10 . 6 . 5 = 5,400,000	
	2	$N_x(2)$ = 300 . 15 . 4 . 10 . 6 . 5 = 5,400,000	
	3	$N_x(3)$ = 300 . 10 . 4 . 10 . 8 . 5 = 4,800,000	
	4	$N_x(4)$ = 300 . 10 . 4 . 10 . 8 . 5 = 4,800,000	
	5	$N_x(5)$ = 300 . 10 . 4 . 10 . 8 . 5 = 4,800,000	
			N_x = 25,200,000
Y	1	$N_y(1)$ = 300 . 15 . 4 . 10 = 180,000	
	2	$N_y(2)$ = 300 . 15 . 4 . 10 = 180,000	
	3	$N_y(3)$ = 300 . 10 . 4 . 10 = 120,000	
	4	$N_y(4)$ = 300 . 10 . 4 . 10 = 120,000	
	5	$N_y(5)$ = 300 . 10 . 4 . 10 = 120,000	
			N_y = 720,000

Z	1	$N_z(1) = 8$	
	2	$N_z(2) = 5$	
	3	$N_z(3) = 5$	
	4	$N_z(4) = 6$	
	5	$N_z(5) = 5$	
			$N_z = 29$
Total number of (X,Y,Z) variables =			25,920,029

TABLE 4.7. Types and Number of Constraints for Example 4.1

Type of Constraint	Number of Constraints of Type C_i
$C_{1,2}$	$N(C_{1,2}) = 5 . 10 . 5 = 250$
$C_{5.1,6.1}$	$N(C_{5.1,6.1}) = 5 . 10 = 50$
C_7	$N(C_7) = (25{,}200{,}000/ 5) = 5{,}040{,}000$
C_8	$N(C_8) = 300 . 60 = 18{,}000$
C_9	$N(C_9) = 60$
Total number of constraints = 5,058,360	

Example 1 shows that the size of the problem KPCP for a typical demand contract scenario is indeed immense. In fact, even attempting to solve the linear relaxation of problem KPCP for a moderate size test problem will most likely lead to memory problems (see Table 5.3 of Section 5.4 of Chapter V). This observation motivates our solution strategy in Section 4.4 to utilize an aggregate version of problem KPCP.

4.4 Formulation of an Aggregate Model for Problem KPCP and Related Issues

4.4.1 Introduction

The formulation of the problem KPCP that was presented in Chapter III explicitly takes into account the different vessel sizes, the various products, the various sizes of compartments, the two types of demand time windows, etc. In the process of formulating this problem, we attempted to simulate the actual operation as closely as possible. Consequently, the resulting formulation of Section 3.3 is rather complex to solve. The complexity of the formulation stems from the following factors: (1) the immensely large number of variables and constraints for a typical demand contract scenario (see example 4.1 for an illustration of problem size), (2) the integrality conditions, and (3) the structural diversity in the constraints. As a result, attempting to solve the problem without any aggregation and partitioning schemes is theoretically complex and computationally intractable, as will be illustrated in Chapter IV via a number of examples. The proposed aggregate problem, denoted by KPCPxy, is designed to deal with the immensely overwhelming size of problem KPCP for a typical demand contract scenario. Problem KPCPxy retains the essential operational features of problem KPCP, and moreover, it is far more computationally tractable than problem KPCP.

This section is organized as follows. A formulation of problem KPCPxy is presented in Section 4.4.2. The size of problem KPCPxy in terms of the variables and constraints is analyzed in Section 4.4.3. A discussion on some of the essential features of problems KPCP and KPCPxy, and the usefulness of the problem KPCPxy in obtaining a good quality feasible solution for problem KPCP is presented in Section 4.5.

We now present the following two propositions which will be referred to when discussing some of the structural properties of problems KPCPxy.

Proposition 1: Suppose that A is an mxn matrix with Rank (A) = m. Furthermore, suppose that every column of A is composed of either one nonzero entry, namely a "+1" or a "-1", or two nonzero entries, namely a "+1'' and a "-1''. If b is an mx1 integer valued vector and M is an mxm invertible submatrix of A, then M^{-1} b is also an integer valued vector.

Proof: See, for example, Bazaraa et al. 1990 or Ahuja et al. 1993.

Proposition 2: Let $\Phi = \{x : Ax = b,\ L \leq x \leq U\}$ be a nonempty set. Suppose that the matrix A is unimodular and the vectors b, L, and U are integer valued vectors. Then, every extreme point of the region Φ is an integer valued vector.

Proof: See, for example, Bazaraa et al. 1990 or Ahuja et al. 1993.

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4.4.2 Formulation of Problem KPCPxy

Let $e = 1,..,e_{max}$ denote the various expected types of trips. Each $e \in \{1,.., e_{max}\}$ is specified by a route and a destination. Thus, e may be represented as an ordered pair as follows: e = (r,d), where $r \in \{1,.., R\}$ and $d \in \{1,..,D\}$. For $d \in \{1,..,D\}$, let $R_d = \{e = (r,d) : \text{where } r \in \{1,..,R\}\}$ be the trips corresponding to destination d. Let h = 1,.., H denote the days of the time horizon. Let q_{t1} denote the number of ships of type t that are initially available. This number includes the company owned vessels and the vessels available for chartering. Thus, q_{t1} might be assigned some large (tight) upper bound.

Let Ω_c, for $c = 1,..,c_{max}$ denote the distinct compartment capacities over all ship types. Let $\Gamma_{t,c}$ denote the number of compartments of size Ω_c on a ship of type t. Let $L_{h,t,e}$ represent a leg for a ship of type t that is consigned during day h to traverse trip e. The number of days that trip e takes using a ship of type t is denoted by $T_{t,e}$. Thus, $T_{t,e}$ is independent of the day during which trip e begins. The cost associated with leg $L_{h,t,e}$ is denoted by $C_{t,e}$. It is assumed that all ships of the same type have the same operational costs corresponding to a given leg $L_{h,t,e}$. If this is not the case, then we can let $C_{t,e}$ be the average of the operational costs associated with leg $L_{h,t,e}$ over all ships of type t. Similarly, let $\$_t$ denote the average of the chartering expenses of all ships of type t that are available for chartering.

Now, denote by $y_{h,t,e}$ the integer variable representing the number of ships of type t that are consigned during day h to traverse trip e. Let $y_{h,t,(e_{max+h})}$ be the slack variable representing the number of unused ships of type t during day h. If $(h + T_{t,e} -1) > H$ for any $e \in \{1,.., e_{max}\}$, then we force the variable $y_{h,t,e}$ to have a zero value. Thus, we are only interested in consigning ships that will arrive during the specified time horizon. Let $E = \{(h,t,e) : (h + T_{t,e} -1) > H, \text{ where } e \in \{1,.., e_{max}\}\}$.

Define $x_{h,c,d,p}$ as the integer variable that represents the number of compartments of size Ω_c that are designated for product p that will arrive at destination d during day h. Let z_t be the integer variable denoting the number of ships of type t that are selected for chartering.

Problem KPCPxy is derived from problem KPCP by using the above integer variables that represent the number of assigned ships and the allocated compartments for the various destinations over the time horizon, in lieu of using binary variables. We now present constraints for problem KPCPxy and discuss some of their special structures. Problem KPCPxy involves two types of constraints given as follows: (A) ship-use constraints and (B) demand constraints. We now give a summary of notation and then define the ship-use and the demand constraints.

The following is a summary of the notation, presented here for the sake of convenience.

- $e = 1,..,e_{max}$: expected types of trips.
- $e = (r,d)$: e specifies a route $r \in \{1,.., R\}$ and a destination $d \in \{1,..,D\}$.
- H : number of days in the time horizon.
- $h = 1,..,H$: days of the time horizon.

- q_{t1} : number of ships of type t that are initially available. This number includes the company owned vessels and the vessels available for chartering.
- q_{th} : number of ships of type t that are known available for the first time on day h in the model horizon.
- Ω_c, for c = 1,..,c_{max} : distinct compartment capacities over all ship types.
- For $d \in \{1,..,D\}$, $R_d = \{e = (r,d) : \text{where } r \in \{1,..,R\}\}$.
- $\Gamma_{t,c}$: number of compartments of size Ω_c on a ship of type t.
- $L_{h,t,e}$: a leg for any ship of type t that is consigned during day h to traverse trip e.
- $T_{t,e}$: number of days that trip e takes using a ship of type t.
- $T_{t,e} = T_{1,t,e} + T_{2,t,e}$, where $T_{1,t,e}$ is the time required to load a ship of type t in Kuwait plus the travel time to destination d, and $T_{2,t,e}$ is the time required to unload in destination d, plus the travel time from destination d to Kuwait.
- $C_{t,e}$: operational cost associated with leg $L_{h,t,e}$.
- $R_{h,d,p}$: rate of consumption of product p at destination d during day h.
- $TC_{h,d,p}$: total consumption of product p at destination d during the interval of time given by [1,..,h].
- $SL_{1,d,p}$ and $SL_{2,d,p}$: minimum and maximum desired storage levels during each day, respectively, of product p at destination d.
- $A_{1,d,p}$ and $A_{2,d,p}$: maximum permitted shortage and excess quantities during each day, respectively, of product p at destination d with respect to the desired levels given by $SL_{1,d,p}$ and $SL_{2,d,p}$, respectively.
- $\$_t$: average of the chartering expenses of all ships of type t available for chartering.
- $E = \{(h,t,e) : (h + T_{t,e} - 1) > H, \text{ where } e \in \{1,.., e_{max}\}\}$.
- $y_{h,t,e}$, for e=1,..,e_{max} : integer variable representing the number of ships of type t that are consigned during day h to traverse trip e.
- $y_{h,t,(emax+h)}$: slack variable representing the number of unused ships of type t during day h.

- $x_{h,c,d,p}$: integer variable representing the number of compartments of size Ω_c that are designated for product p and will arrive at destination d during day h.
- z_t : integer variable denoting the number of ships of type t that are selected for chartering.
- $f_{i,d,p}$ and $F_{i,d,p}$: minimum and maximum allowable quantities, respectively, of the i^{th} partition of product p to be shipped to destination d.
- $H_{i,d,p} = [\varphi_{1(i,d,p)}, \varphi_{2(i,d,p)})$, where (d,p) ∈ Type I demand time windows and i=1,..,$N_{d,p}$.

(A) Ship-Use Constraints of Problem KPCPxy

The ship-use constraints of problem KPCPxy are obtained from the ship-use constraints of problem KPCP by using integer variables representing the number of ships of each type consigned during each day. These new ship-use constraints capture, in essence, constraint (C_8) of problem KPCP, and are given as follows.

$$(RC_1) \quad \sum_{e=1}^{e_{max}} y_{1,t,e} + y_{1,t,(e_{max}+1)} = q_{t1} \qquad \forall\, t$$

$$(RC_2) \quad \sum_{e=1}^{e_{max}} y_{h,t,e} + y_{h,t,(e_{max}+h)} = y_{(h-1),t,(e_{max}+h-1)} + \sum_{e=1}^{e_{max}} y_{(h-T_{t,e}),t,e} + q_{th} \qquad \forall\, t \text{ and } \forall\, h \geq 2$$

$$(RC_3) \quad z_t \geq \sum_{e=1}^{e_{max}} \sum_{i=1}^{T_{t,e}} y_{(h-T_{t,e}+i),t,e} - O_t \qquad \forall\, t, h$$

$y_{h,t,e} \geq 0$ and integer $\forall$ h,t,e with $y_{h,t,e} = 0$ whenever (h,t,e) ∈ E

$z_t \geq 0 \quad \forall$ t.

Constraint (RC_1) states that the total number of ships of type t that are initially available equals the total number of ships of type t that are consigned to various destinations on day 1 plus the unused ships of type t on this day. Constraint (RC_2) asserts that the number of ships of type t that are consigned during day h (h ≥ 2) plus the unused ships of type t during this day equals the total number of ships that are available in this day, either as unused ships from the previous day or ships returning after service from prior days. Thus, these constraints enforce that a ship of type t may be assigned to at most one trip in any day, similar to constraint (C_8) of problem KPCP.

Note that one primary objective of the model is to minimize the number of chartered vessels. In other words, we seek to minimize the sum of $\$_t z_t$ over all ship types. The value of the variable z_t in constraint (RC_3) gives the number of chartered ships of type t that are needed at any day during the time horizon.

Rewriting the system of equations given by (RC_1) and (RC_2) so that all the variables appear on the left-hand side while all the constants appear on the right-hand side, (RC_1) and (RC_2) are given as follows.

$$(RC_1) \quad \sum_{e=1}^{e_{max}} y_{1,t,e} + y_{1,t,(e_{max}+1)} = q_{t1} \quad \forall\, t$$

$$(RC_2) \quad \sum_{e=1}^{e_{max}} y_{h,t,e} + y_{h,t,(e_{max}+h)} - y_{(h-1),t,(e_{max}+h-1)} - \sum_{e=1}^{e_{max}} y_{(h-T_{t,e}),t,e} = q_{t1} \quad \forall\, t \text{ and } \forall\, h \geq 2.$$

We now give constraints (RC_1) and (RC_2) for a simple example. We then show that these constraints describe network-flow constraints. A generalization of this result is presented in Proposition 3.

Example 4.2

For t = 1, let $q_{11} = 3$, $q_{12} = 0$, $q_{13} = 0$, H =3, $e_{max} = 3$, $T_{t,=1,e=1} = 1$, $T_{t,=1,e=2} = 2$, and $T_{t,=1,e=3} = 3$. Then, for t=1, constraints (RC_1) and (RC_2) are given as follows.

$$(RC_1) \quad y_{1,t=1,e=1} + y_{1,t=1,e=2} + y_{1,t=1,e=3} + y_{1,t=1,4} = 3$$

$$(RC_2) \quad y_{2,t=1,e=1} + y_{2,t=1,e=2} + y_{2,t=1,e=3} + y_{2,t=1,e=5} - y_{1,t=1,e=1} - y_{1,t=1,e=4} = 0$$

$$y_{3,t=1,e=1} + y_{3,t=1,e=2} + y_{3,t=1,e=3} + y_{3,t=1,e=6} - y_{1,t=1,e=2} - y_{2,t=1,e=1} - y_{2,t=1,e=5} = 0$$

where $y_{2,t=1,e=3} = y_{3,t=1,e=2} = y_{3,t=1,e=3} = 0$.

Since $y_{2,t=1,e=3} = y_{3,t=1,e=2} = y_{3,t=1,e=3} = 0$, these variables can be discarded from the above system of equations. Let M denote the matrix of coefficients associated with the constraint set given by (RC_1) and (RC_2). Then, each column of M is composed of either a single nonzero entry (namely a "+1'') or two nonzero entries (namely a "+1'' and a "-1''). Thus, (RC_1) and (RC_2) describe network-flow constraints. Letting $y_{i,t=1,j} = y_{i,j}$, the matrix M is given as follows.

$$M = \begin{array}{c} \\ \\ \\ \\ \end{array} \begin{array}{ccccccccc} y_{1,1} & y_{1,2} & y_{1,3} & y_{1,4} & y_{2,1} & y_{2,2} & y_{2,5} & y_{3,1} & y_{3,6} \\ \end{array}$$

$$M = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 & -1 & 1 & 1 \end{bmatrix} \begin{array}{l} h=1 \\ h=2 \\ h=3 \end{array}$$

Let G_1 (N,A) be the directed graph corresponding to (RC_1) and (RC_2), where N is the node-set of the graph and A is the arc-set of the graph. The node-set N is given by N = {h=1, h=2, h=3}. Hence, the node-set is composed of the days of the time horizon. The arc-set A is given by A = {$y_{1,1}$, $y_{1,2}$, $y_{1,3}$, $y_{1,4}$, $y_{2,1}$, $y_{2,2}$, $y_{2,5}$, $y_{3,1}$, $y_{3,6}$} and is based on the trips initiating and ending at various suitable days. Here, $y_{1,1}$ represents the arc going from h=1 to h=2, $y_{1,2}$ represents an arc going from h=1 to h=3, $y_{1,3}$ represents the arc initiating from h=3, etc. The initial and final nodes of each arc are determined based on the positions of the "+1" and the "-1" in the column corresponding to this arc in the matrix M.

Rewrite $y_{i,j}$ as $a_{k,m,n}$, where k and m respectively denote the initial and final nodes of the arc $y_{i,j}$, and n is an integer that uniquely identifies this arc if there is more than one arc going from the node k to the node m. If the column corresponding to an arc $y_{i,j}$ is composed of a single nonzero entry (namely a "+1''), then we let m be some dummy node, say m = h = 4. Hence, we may rewrite the arcs of the above example as follows: $a_{1,2,1} = y_{1,1}$, $a_{1,2,2} = y_{1,4}$, $a_{1,3,1} = y_{1,2}$, $a_{1,4,1} = y_{1,3}$, $a_{2,3,1} = y_{2,1}$, $a_{2,3,2} = y_{2,5}$, $a_{2,4,1} = y_{2,2}$, $a_{3,4,1} = y_{3,1}$, $a_{3,4,2} = y_{3,6}$. Accordingly, M is given as follows.

$$M = \begin{array}{c} \\ \\ \\ \end{array} \begin{array}{ccccccccc} a_{1,2,1} & a_{1,3,1} & a_{1,4,1} & a_{1,2,2} & a_{2,3,1} & a_{2,4,1} & a_{2,3,2} & a_{3,4,1} & a_{3,4,2} \\ \end{array}$$

$$M = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 & -1 & 1 & 1 \end{bmatrix} \begin{array}{l} h=1 \\ h=2 \\ h=3 \end{array}$$

The graph $G_1(N,A)$ is illustrated in Figure 4.1.

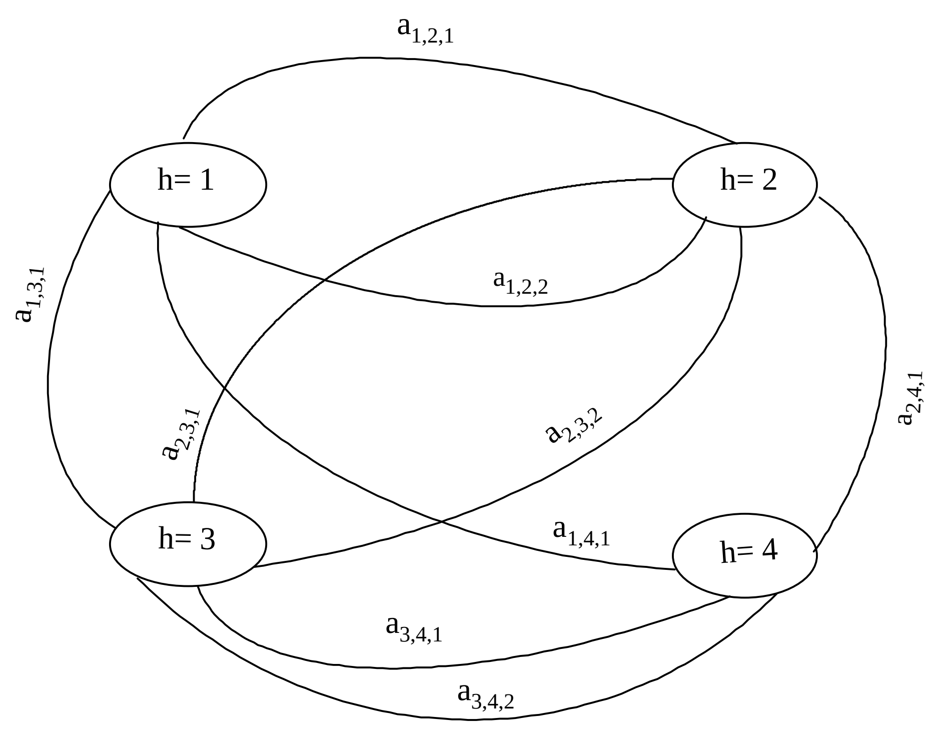

Figure 4.1. Network Graph for Example 4.2.

Proposition 3: For each ship-type t, (RC_1) and (RC_2) yield network-flow constraints.

Proof: Since $y_{h,t,e} = 0$ for all $(h,t,e) \in E$, we may discard these variables from constraints (RC_1) and (RC_2). Let M denote the matrix of coefficients associated with the constraint set given by (RC_1) and (RC_2). We prove that (RC_1) and (RC_2) yield network-flow constraints by showing that each column of M is composed of either a single nonzero entry (namely a "+1'') or two nonzero entries (namely a "+1'' and a "-1''). For $t \in \{1,..,T\}$, the slack variables associated with (RC_1) and (RC_2) are given by $y_{1,t,(emax+1)}$, $y_{2,t,(emax+2)},..,y_{H,t,(emax+H)}$. For $h \in \{1,.., H\}$, $y_{h,t,(emax+h)}$ gives the number of unused ships of type t during day h. Noticing the structure of (RC_1) and (RC_2), we see that $y_{h,t,(emax+h)}$, for $h \in \{1,.., H -1\}$, appears with a positive sign in the row corresponding to h and it appears with a negative sign in the row corresponding to (h+1). For h = H, $y_{H,t,(emax+H)}$ appears with a positive sign only in the row corresponding to H. For a given $y_{h,t,e}$, let $HH = h+T_{t,e}-1$. Then, $y_{h,t,e}$ appears with a positive sign in the row corresponding to h and it appears with a negative sign in the row corresponding to HH +1 only if $HH < H$.
Thus, each column of M is composed of either a single nonzero entry (namely a "+1'') or two nonzero entries (namely a "+1'' and a "-1''). þ

(B) Demand and Availability Constraints of Problem KPCPxy

The demand and availability constraints of problem KPCPxy are intended to approximate the demand and availability constraints of problem KPCP. These constraints are derived from the demand and availability constraints of problem KPCP by using integer variables representing the number of compartments carrying product p that will arrive at destination d during day h. Recall that the demand and availability constraints of problem KPCP are given by (C_1) - (C_7). Sufficient conditions under which some of the demand and availability constraints of problem KPCPxy yield network-flow constraints are presented in Proposition 4.

We now introduce demand and availability constraints for problem KPCPxy.

$$(RC_4)\quad \sum_{p} x_{h,c,d,p} \leq \sum_{e \in R_d} \sum_{t} \Gamma_{t,c}\; y_{(h - T_{1,t,e}+1),t,e} \qquad \forall\ h,c,d$$

$$(RC_5)\quad \sum_{c} \sum_{h \in H_{i,d,p}} \Omega_c\; x_{h,c,d,p} \geq f_{i,d,p} \qquad \forall\ i, p, d \in \text{Type I}$$

$$(RC_6)\quad \sum_{c} \sum_{h \in H_{i,d,p}} \Omega_c\; x_{h,c,d,p} \leq F_{i,d,p} \qquad \forall\ i, p, d \in \text{Type I}$$

$$(RC_7)\quad \sum_{c} \Omega_c\; x_{h,c,d,p} = W_{h,d,p} \qquad \forall\ h, p, d \in \text{Type II}$$

$$(RC_8)\quad \omega_{d,p} + \sum_{h \leq \bar{h}} W_{\bar{h},d,p} - TC_{h,d,p} = S_{1,h,d,p} - S_{2,h,d,p} - S_{3,h,d,p} + S_{4,h,d,p}$$

$$+\ S_{5,h,d,p} \qquad \forall\ \bar{h} \text{ and } \forall\ (d,p) \in \text{Type II}$$

$$(RC_9)\quad \sum_{c} \sum_{h} \Omega_c\; x_{h,c,d,p} \geq D_{d,p}(1 - \nu_{d,p}) \qquad \forall\ p, d \in \text{Type I}$$

$$(RC_{10})\quad \sum_{c} \sum_{h} \Omega_c\; x_{h,c,d,p} \leq D_{d,p}(1 + \nu_{d,p}) \qquad \forall\ p, d \in \text{Type I}$$

$$(RC_{11})\quad \sum_{c} \sum_{h} \Omega_c\; x_{h,c,d,p} \geq D_{d,p}(1 - \nu_{d,p}) \qquad \forall\ p, d \in \text{Type II}$$

$$(RC_{12})\quad \sum_{c} \sum_{h} \Omega_c\; x_{h,c,d,p} \leq D_{d,p}(1 + \nu_{d,p}) \qquad \forall\ p, d \in \text{Type II}$$

$x_{h,c,d,p} \geq 0$ and integer $\qquad \forall\ h,c,d,p$

$W_{h,d,p} \geq 0, \qquad \forall\ h$, and $(d, p) \in$ Type II

$S_{h,d,p} \geq 0, \quad SL_{1,d,p} \leq S_{1,h,d,p} \leq SL_{2,d,p} \qquad \forall\ h$ and $\forall\ (d,p) \in$ Type II

$0 \leq S_{2,h,d,p} \leq A_{1,d,p},\ 0 \leq S_{3,h,d,p} \leq SL_{1,d,p} - A_{1,d,p}$

$\forall\ h$ and $\forall\ (d,p) \in$ Type II

$0 \leq S_{4,h,d,p} \leq A_{2d,p}\ ,\ S_{5,h,d,p} \geq 0 \qquad \forall\ h$ and $\forall\ (d,p) \in$ Type II.

Let $\phi_1(h,c,d) = \sum_{e \in R_d} \sum_t \Gamma_{t,c} \; y_{(h - T_{1,t,e} + 1),t,e}$ and let $\phi_2(t,h) = \sum_{e=1}^{e_{max}} \sum_{i=1}^{T_{t,e}} y_{(h - T_{t,e} + i),t,e}$.

$\phi_1(h,c,d)$ gives the number of compartments of size Ω_c that will arrive at destination d during day h and $\phi_2(t,h)$ gives the number of ships of type t that are used during day h, as in (RC_3). Constraint (RC_4) assures that the total number of compartments of size Ω_c designated for various products and which will arrive at destination d during day h does not exceed the availability $\phi_1(h,c,d)$.

Constraints (RC_5) and (RC_6) assure that the total amount of product p shipped to destination $d \in$ Type I in the interval $H_{i,d,p}$ lies within $[f_{i,d,p}, F_{i,d,p}]$. For $(d,p) \in$ Type II, the revisits to destination d are determined by the initial storage level of product p (given by $\omega_{d,p}$), the storage capacity (including minimum and maximum desired levels of product p given by $SL_{1,d,p}$ and $SL_{2,d,p}$), and the rates of consumption R_j. The continuous variable $W_{h,d,p}$ in constraint (RC_7) gives the total amount of product p delivered to destination d during day h. Accordingly, constraint (RC_7) gives the storage level of product p at destination d on day h. Constraint (RC_8) gives a representation of the storage level during each day of the time horizon in terms of the variables $S_{1,h,d,p}$, $S_{2,h,d,p}$, $S_{3,h,d,p}$, $S_{4,h,d,p}$, and $S_{5,h,d,p}$.

Constraints (RC_9) and (RC_{10}) enforce the satisfaction of the overall demands at destinations of Type I, and Constraints (RC_{11}) and (RC_{12}) enforce the satisfaction of the overall demands at destinations of Type II.

Sufficient conditions under which constraints (RC_4), (RC_5), (RC_6), (RC_7), (RC_8), (RC_9), and (RC_{10}) describe network-flow constraints are presented next.

Proposition 4: Suppose that y is given. Then (RC_4), (RC_5), (RC_6), (RC_7), (RC_8), (RC_9), and (RC_{10}) describe a *generalized* network-flow structure. These constraints describe a network-flow structure if Ω_c is constant for all c (let this assumption be denoted by A_4). Let $A\zeta = \alpha$ denote the system of equations given by (RC_4), (RC_5), (RC_6), (RC_7), (RC_8),

(RC_9), and (RC_{10}). Let $\Phi_1 = \{\zeta: A\,\zeta = \alpha, L \le \zeta \le U\}$, where L and U respectively denote the lower and upper bound vectors of ζ. Note that all integrality conditions are discarded in Φ_1 . If in addition to assumption (A_4) we have the following assumption (denoted by AA_4):

$(f_{i,d,p} / \Omega_c)$, $(F_{i,d,p} / \Omega_c)$, $(D_{d,p}\ (1-\nu_{d,p}) / \Omega_c)$, and $(D_{d,p}\ (1+\nu_{d,p}) / \Omega_c)$ for $(d,p)\in$ Type I, and $(TC_{h,d,p} / \Omega_c)$, $(\omega_{d,p} / \Omega_c)$, $(SL_{1,d,p} / \Omega_c)$, $(SL_{2,d,p} / \Omega_c)$, $(A_{1,d,p} / \Omega_c)$, and $(A_{1,d,p} / \Omega_c)$ for $(d,p)\in$ Type II, are all integer numbers, then any extreme point of Φ_1 is integer valued.

Proof: Constraints (RC_5) and (RC_6) can be consolidated into one constraint given as follows: $(RC_{5,6})$ $\sum_c \sum_{h \in H_{i,d,p}} \Omega_c\ x_{h,c,d,p} - U_{i,d,p} = 0 \quad \forall$ i and $(d,p) \in$ Type I

Where $f_{i,d,p} \le U_{i,d,p} \le F_{i,d,p}$.

Likewise, Constraints (RC_9) and (RC_{10}) can be consolidated into one constraint given as follows: $(RC_{9,10})$ $\sum_i U_{i,d,p} - V_{d,p} = 0 \quad \forall\ (d,p) \in$ Type I

Where $D_{d,p}(1 - \nu_{d,p}) \le V_{d,p} \le D_{d,p}(1 + \nu_{d,p})$, (refer to Remark 4 of Section 4.2 for a proof).

The rows of equation (RC_8) can be equivalently expressed so that each row contains only one $W_{h,d,p}$. This is accomplished as follows. Let $(d,p) \in$ Type II and let $(RC_{8,(d,p,h)})$ denote the constraint corresponding to the parameter (d,p,h) in (RC_8). Let $(\overline{R}C_{8,(d,p,1)}) = (RC_{8,(d,p,1)})$. For $h \in \{2,.., H\}$, let $(\overline{R}C_{8,(d,p,h)}) = (RC_{8,(d,p,h)}) - (RC_{8,(d,p,h-1)})$.

Observe that for $h \in \{2,.., H\}$, constraint $(RC_{8,(d,p,h)})$ is given by

$$(RC_{8,(d,p,h)}) = \sum_{i=1}^{h} (\overline{R}C_{8,(d,p,i)}).$$

Thus, the set of constraints given by $(RC_{8,(d,p,1)})$, $(RC_{8,(d,p,2)})$,.., $(RC_{8,(d,p,H)})$ is equivalent to the set of constraints given by $(\overline{R}C_{8,(d,p,1)})$, $(\overline{R}C_{8,(d,p,2)})$,..,$(\overline{R}C_{8,(d,p,H)})$. Rewriting (RC_4), $(RC_{5,6})$, (RC_7), $(\overline{R}C_8)$, and $(RC_{9,10})$, in standard form, we get the following.

$$(RC_4) \quad \sum_p x_{h,c,d,p} + \eta_{h,c,d} = \phi_1(h,c,d) \qquad \forall\ h,c,d$$

$$(RC_{5,6}) \quad -\sum_c \sum_{h \in H_{i,d,p}} \Omega_c\ x_{h,c,d,p} + U_{i,d,p} = 0 \qquad \forall\ i, \text{ and } (d,p) \in \text{Type I}$$

$$(RC_7) \quad -\sum_c \Omega_c\ x_{h,c,d,p} + W_{h,d,p} = 0 \qquad \forall\ h, \text{ and } (d,p) \in \text{Type II}$$

$$(\overline{R}C_8) \quad -W_{1,d,p} + S_{1,1,d,p} - S_{2,1,d,p} - S_{3,1,d,p} + S_{4,1,d,p} + S_{5,1,d,p} = \omega_{d,p} - TC_{1,d,p}$$

$$-W_{\bar{h},d,p} + S_{1,h,d,p} - S_{2,h,d,p} - S_{3,h,d,p} + S_{4,h,d,p} + S_{5,h,d,p}$$

$$- S_{1,(h-1),d,p} + S_{2,(h-1),d,p} + S_{3,(h-1),d,p} - S_{4,(h-1),d,p} - S_{5,(h-1),d,p} =$$

$$(TC_{h-1,d,p} - TC_{h,d,p}) \qquad \forall\ \bar{h} \text{ and } \forall\ (d,p) \in \text{Type II}$$

$$(RC_{9,10}) \quad -\sum_i U_{i,d,p} + V_{d,p} = 0 \qquad \forall\ (d,p) \in \text{Type I}$$

where $\eta_{h,c,d} \geq 0$,

Let M be the coefficient matrix associated with the constraint set given by (RC_4), ($RC_{5,6}$), (RC_7), ($\overline{R}C_8$), and ($RC_{9,10}$). Examining the columns of the matrix M, we see that each column is composed of either one nonzero entry (namely a "+1'' or a "-1'') or two nonzero entries (namely a "+1'' and either a "-1'' or a "$-\Omega_c$'' for some c). Thus, (RC_4), (RC_5), (RC_6), (RC_7), ($\overline{R}C_8$), (RC_9), and (RC_{10}) describe *generalized* network-flow constraints.

Now, assume that Ω_c is constant for all c. Let $\overline{W}_{h,d,p} = (W_{h,d,p}/\Omega_c)$, $\overline{S}_{j,h,d,p} = (S_{j,h,d,p}/\Omega_c)$, for j=1,..,5, $\overline{f}_{i,d,p} = (f_{i,d,p}/\Omega_c)$, $\overline{F}_{i,d,p} = (F_{i,d,p}/\Omega_c)$, $\overline{\alpha}_{d,p} = (D_{d,p}(1-\nu_{d,p})/\Omega_c)$, $\overline{\beta}_{d,p} = (D_{d,p}(1+\nu_{d,p})/\Omega_c)$, $\overline{TC}_{h,d,p} = (TC_{h,d,p}/\Omega_c)$, $\overline{\omega}_{d,p} = (\omega_{d,p}/\Omega_c)$, $\overline{SL}_{1,d,p} = (SL_{1,d,p}/\Omega_c)$, $\overline{SL}_{2,d,p} = (SL_{2,d,p}/\Omega_c)$, $\overline{A}_{1,d,p} = (A_{1,d,p}/\Omega_c)$, and $\overline{A}_{2,d,p} = (A_{2,d,p}/\Omega_c)$. Dividing constraints ($RC_{5,6}$), ($RC_7$), ($\overline{R}C_8$), and ($RC_{9,10}$) by Ω_c, we get the following.

$$(RC_4) \quad \sum_{p} x_{h,c,d,p} + \eta_{h,c,d} = \phi_1(h,c,d) \qquad \forall\, h,c,d$$

$$(RC_{5,6}) \quad -\sum_{c} \sum_{h \in H_{i,d,p}} x_{h,c,d,p} + \overline{U}_{i,d,p} = 0 \qquad \forall\, i, \text{ and } (d,p) \in \text{Type I}$$

$$(RC_7) \quad -\sum_{c} x_{h,c,d,p} + \overline{W}_{h,d,p} = 0 \qquad \forall\, h, \text{ and } (d,p) \in \text{Type II}$$

$$(\overline{R}C_8) \quad -\overline{W}_{1,d,p} + \overline{S}_{1,1,d,p} - \overline{S}_{2,1,d,p} - \overline{S}_{3,1,d,p} + \overline{S}_{4,1,d,p} + \overline{S}_{5,1,d,p} = \overline{\omega}_{d,p} - \overline{T}C_{1,d,p}$$

$$-\overline{W}_{\bar{h},d,p} + \overline{S}_{1,h,d,p} - \overline{S}_{2,h,d,p} - \overline{S}_{3,h,d,p} + \overline{S}_{4,h,d,p} + \overline{S}_{5,h,d,p}$$

$$-\overline{S}_{1,(h-1),d,p} + \overline{S}_{2,(h-1),d,p} + \overline{S}_{3,(h-1),d,p} - \overline{S}_{4,(h-1),d,p} - \overline{S}_{5,(h-1),d,p} =$$

$$(\overline{T}C_{h-1,d,p} - \overline{T}C_{h,d,p}) \qquad \forall\, \bar{h} \text{ and } \forall\, (d,p) \in \text{Type II}$$

$$(RC_{9,10}) \quad -\sum_{i} \overline{U}_{i,d,p} + \overline{V}_{d,p} = 0 \qquad \forall\, (d,p) \in \text{Type I}$$

where $\eta_{h,c,d} \geq 0,$

$$\overline{f}_{i,d,p} \leq \overline{U}_{i,d,p} \leq \overline{F}_{i,d,p} \qquad \forall\, i \text{ and } (d,p) \in \text{Type I}$$

$$\overline{\alpha}_{d,p} \leq \overline{V}_{d,p} \leq \overline{\beta}_{d,p} \qquad \forall\, (d,p) \in \text{Type I}$$

$$\overline{W}_{h,d,p} \geq 0 \qquad \forall\, h \text{ and } (d,p) \in \text{Type II}$$

$$\overline{S}_{h,d,p} \geq 0, \quad \overline{S}L_{1,d,p} \leq \overline{S}_{1,h,d,p} \leq \overline{S}L_{2,d,p}$$

$$\forall h \text{ and } \forall\, (d,p) \in \text{Type II}$$

$$0 \leq \overline{S}_{2,h,d,p} \leq \overline{A}_{1,d,p}, \quad 0 \leq \overline{S}_{3,h,d,p} \leq \overline{S}L_{1,d,p} - \overline{A}_{1,d,p}$$

$$\forall\, h \text{ and } \forall\, (d,p) \in \text{Type II}$$

$$0 \leq \overline{S}_{4,h,d,p} \leq \overline{A}_{2d,p}, \quad \overline{S}_{5,h,d,p} \geq 0 \qquad \forall\, h \text{ and } \forall\, (d,p) \in \text{Type II.}$$

Let M be the coefficient matrix associated with the constraint set given by (RC_4), $(RC_{5,6})$, (RC_7), $(\overline{R}C_8)$, and $(RC_{9,10})$. Examining the columns of the matrix M, we see that each column is composed of either one nonzero entry (namely a “+1’’ or “-1’’) or two nonzero entries (namely a “+1” and a “-1’’). Thus, (RC_4), $(RC_{5,6})$, (RC_7), $(\overline{R}C_8)$, and

($RC_{9,10}$) describe network-flow constraints. If assumptions (A_4) and (AA_4) of the proposition hold, then by Proposition 2 all extreme points of Φ_1 are integer valued. þ

We now restate problem KPCPxy in its complete form.

KPCPxy:

$$\textbf{Minimize} \quad \sum_{e=1}^{e_{max}} \sum_{t} \sum_{h} C_{t,e}\, y_{h,t,e} + \sum_{t} \$_t\, z_t$$

$$+ \sum_{h} \sum_{d} \sum_{\substack{p:\\(d,p)\in \text{Type II}}} \pi_{d,p}\, [S_{2,h,d,p} + S_{4,h,d,p}] + \sum_{h} \sum_{d} \sum_{\substack{p:\\(d,p)\in \text{Type II}}} \lambda_2\, [S_{3,h,d,p} + S_{5,h,d,p}]$$

subject to

$$(RC_1) \quad \sum_{e=1}^{e_{max}} y_{1,t,e} + y_{1,t,(e_{max}+1)} = q_{t1} \qquad \forall\, t$$

$$(RC_2) \quad \sum_{e=1}^{e_{max}} y_{h,t,e} + y_{h,t,(e_{max}+h)} = y_{(h-1),t,(e_{max}+h-1)} + \sum_{e=1}^{e_{max}} y_{(h-T_{t,e}),t,e} + q_{th} \qquad \forall\, t \text{ and } \forall\, h \geq 2$$

$$(RC_3) \quad z_t \geq \sum_{e=1}^{e_{max}} \sum_{i=1}^{T_{t,e}} y_{(h-T_{t,e}+i),t,e} - O_t \qquad \forall\, t, h$$

$$(RC_4) \quad \sum_{p} x_{h,c,d,p} \leq \sum_{e \in R_d} \sum_{t} \Gamma_{t,c}\, y_{(h-T_{1,t,e}+1),t,e} \qquad \forall\, h, c, d$$

$$(RC_5) \quad \sum_{c} \sum_{h \in H_{i,d,p}} \Omega_c\, x_{h,c,d,p} \geq f_{i,d,p} \qquad \forall\, i \text{ and } (d,p) \in \text{Type I}$$

$$(RC_6) \quad \sum_{c} \sum_{h \in H_{i,d,p}} \Omega_c\, x_{h,c,d,p} \leq F_{i,d,p} \qquad \forall\, i \text{ and } (d,p) \in \text{Type I}$$

$$(RC_7) \quad \sum_{c} \Omega_c\, x_{h,c,d,p} = W_{h,d,p} \qquad \forall\, h \;\; (d,p) \in \text{Type II}$$

$$(RC_8) \quad \omega_{d,p} + \sum_{h \le \bar{h}} W_{\bar{h},d,p} - TC_{h,d,p} = S_{1,h,d,p} - S_{2,h,d,p} - S_{3,h,d,p} + S_{4,h,d,p} + S_{5,h,d,p}$$

$$\forall \ \bar{h} \text{ and } \forall \ (d,p) \in \text{ Type II}$$

$$(RC_9) \quad \sum_c \sum_h \Omega_c \ x_{h,c,d,p} \geq D_{d,p}(1 - \nu_{d,p}) \qquad \forall \ (d,p) \in \text{ Type I}$$

$$(RC_{10}) \quad \sum_c \sum_h \Omega_c \ x_{h,c,d,p} \leq D_{d,p}(1 + \nu_{d,p}) \qquad \forall \ (d,p) \in \text{ Type I}$$

$$(RC_{11}) \quad \sum_c \sum_h \Omega_c \ x_{h,c,d,p} \geq D_{d,p}(1 - \nu_{d,p}) \qquad \forall \ (d,p) \in \text{ Type II}$$

$$(RC_{12}) \quad \sum_c \sum_h \Omega_c \ x_{h,c,d,p} \leq D_{d,p}(1 + \nu_{d,p}) \qquad \forall \ (d,p) \in \text{ Type II}$$

$x_{h,c,d,p} \geq 0$ and integer $\qquad \forall \ h,c,d,p$

$Y_{h,t,e} \geq 0$ and integer $\qquad \forall \ h,t,e$ with $Y_{h,t,e} = 0$ whenever $(h,t,e) \in E$

$z_t \geq 0 \qquad \forall \ t$

$W_{h,d,p} \geq 0, \qquad \forall \ h$, and $(d, p) \in$ Type II

$S_{h,d,p} \geq 0, \quad SL_{1,d,p} \leq S_{1,h,d,p} \leq SL_{2,d,p} \qquad \forall \ h$ and $\forall \ (d,p) \in$ Type II

$0 \leq S_{2,h,d,p} \leq A_{1,d,p}, \ 0 \leq S_{3,h,d,p} \leq SL_{1,d,p} - A_{1,d,p}$

$\forall \ h$ and $\forall \ (d,p) \in$ Type II

$0 \leq S_{4,h,d,p} \leq A_{2d,p}, \ S_{5,h,d,p} \geq 0 \qquad \forall \ h$ and $\forall \ (d,p) \in$ Type II.

Objective Function of Problem KPCPxy

The objective function of problem KPCPxy is composed of four terms. The first two terms give the overall operational costs of both the company owned and the chartered vessels, and the chartering costs associated with the chartered vessels. The last two terms give the sum of the Type II_1 and Type II_2 penalties.

Structural Properties of Problem KPCPxy

The following is a summary of some of the structural properties of problem KPCPxy.

- If the integrality condition of the variable y is explicitly enforced, then any solution of problem KPCPxy would yield integer values for the variables z_t. This can be easily seen by examining the structure of constraint (RC_3).
- Constraints (RC_1) and (RC_2) describe a network-flow structure.
- Constraints (RC_4), ($RC_{5,6}$), (RC_7), ($\overline{R}C_8$), and ($RC_{9,10}$), describe a *generalized* network-flow structure.
- If assumption (A_4) of Proposition 4 holds, then constraints (RC_4), ($RC_{5,6}$), (RC_7), ($\overline{R}C_8$), and ($RC_{9,10}$) describe a network-flow structure.
- If assumptions (A_4) and (AA_4) of Proposition 4 hold, then any extreme point of the region given by Φ_1 as defined in Proposition 4 is integer valued.
- For a fixed value of y, problem KPCPxy can be decomposed into subproblems, one for each destination.

4.4.3 Problem Size Analysis for Problem KPCPxy

An analysis of the size for problem KPCPxy is presented in this section. We give the number of variables and constraints for problem KPCPxy.

In the following, Table 4.8 gives the number of variables in problem KPCPxy, and Table 4.9 gives the number of constraints for problem KPCPxy.

TABLE 4.8. Number of Variables in Problem KPCPxy

Variable	Number of Variables
$x_{h,c,d,p}$	H . c_{max} . D . P
$y_{h,t,e}$	H . T . (e_{max}+1)
z_t	T
$W_{h,d,p}$	H . D_{II} . P

TABLE 4.9. Number of Constraints in Problem KPCPxy

Number of Constraints of Type RC_i
$N(RC_1) = T$
$N(RC_2) = (H-1) . T$
$N(RC_3) = H . T$
$N(RC_4) = H . c_{max} . D$
$N(RC_5) = \sum_{(d,p) \in \text{Type I}} N_{d,p}$
$N(RC_6) = \sum_{(d,p) \in \text{Type I}} N_{d,p}$
$N(RC_7) = H . DD_{II} . P$
$N(RC_8) = H . DD_{II} . P$
$N(RC_9) = DD_I . P$
$N(RC_{10}) = DD_I . P$
$N(RC_{11}) = DD_{II} . P$
$N(RC_{12}) = DD_{II} . P$
Total number of constraints = $\sum_{i=1}^{12} N(RC_i)$

Example 4.3

In this example, we give the number of variables and constraints of problem KPCPxy based on the data provided in Example 4.1 and assuming that the number of distinct compartments is 5.

TABLE 4.10. Number of Variables in Problem KPCPxy for Example 4.3

Variable	Number of Variables
$x_{h,c,d,p}$	300 . 5 . 10 . 5 = 75,000
$y_{h,t,e}$	300 . 5 . 41 = 61,500
z_t	T = 5
Total number of variables = 136,505	

TABLE 4.11. Number of Constraints in Problem KPCPxy for Example 4.3

Number of Constraints of Type RC_i
$N(RC_1)$ = T = 5
$N(RC_2)$ = 299 . (T=5) = 1,495
$N(RC_3)$ = 300 . (T=5) = 1,500
$N(RC_4)$ = 300 . 5 . 10 = 15,000
$N(RC_5)$ = 10 . 5 . 5 = 250
$N(RC_6)$ = 10 . 5 . 5 = 250
$N(RC_7)$ = 0
$N(RC_8)$ = 0
$N(RC_9)$ = 10 . 5 = 50
$N(RC_{10})$ = 10 . 5 = 50
$N(RC_{11})$ = 0
$N(RC_{12})$ = 0
Total number of constraints = 18,600

The number of variables and constraints in problem KPCP for the data provided in Example 4.1 are respectively given by 25,920,029 and 5,058,860, while the number of variables and constraints in problem KPCPxy for the same data are respectively given by 136,505 and 18,600. Consequently, problem KPCPxy is computationally far more tractable than problem KPCP, as will be illustrated in Chapter V via a number of examples.

4.5 Utilization of Problem KPCPxy and Implementation Issues

We conclude this chapter by presenting a discussion on the potential utilization of problem KPCPxy in obtaining a good quality feasible solution for problem KPCP. This is accomplished by addressing the essential differences between problems KPCP and KPCPxy, and discussing pertinent implementation issues as follows.

1. Problem KPCP designate a binary variable ($X_{h,t,s,r,d,c,p}$) for each compartment and a binary variable ($Y_{h,t,s,r,d}$) for each vessel, while problem KPCPxy designate an integer variables ($y_{h,t,e}$) representing the number of ships of type t that are consigned during day h to traverse trip e and an integer variables ($x_{h,c,d,p}$) representing the number of compartments of size Ω_c that are designated for product p and will arrive at destination d during day h. Once the values of the integer variables $x_{h,c,d,p}$ and $y_{h,t,e}$ of problem KPCPxy are determined, we can readily extract the number of vessels of each type required to satisfy overall demand and assign products to the selected compartments.

2. Problem KPCP designate a binary variable ($Z_{t,s}$) for each vessel that is available for chartering, while problem KPCPxy designate an integer variable (z_t) for the number of vessels of type t that are available for chartering. Obviously, once z_t is determined, we can easily obtain the number of vessels of each type needed for chartering.

3. Operational costs of two vessels of the same type may not be identical. Likewise, chartering expenses of two vessels of the same type may not identical. Problem KPCP accounts for the costs and chartering expenses of each vessel. For problem

4. KPCPxy, averages of operational costs of vessels are considered, since only one variable is designated for all ships of the same type. Likewise, averages of chartering expenses of vessels are considered, since only one variable is designated for all ships of the same type that are available for chartering. To enhance the representation of problem KPCPxy, we can partition a given ship type into subtypes, each of which is composed of vessels with identical operational costs of legs and identical chartering expenses.

5. For a given typical demand contract scenario, the number of variables in problem KPCP is tremendously larger the number of variables in problem KPCPxy. Similarly, the number of constraints in problem KPCP is tremendously larger the number of constraints in problem KPCPxy. These observations strongly motivate the usefulness of problem KPCPxy in obtaining a good quality feasible solution for problem KPCP.

6. Problem KPCPxy does not account for the availability constraints (C_9) of problem KPCP. Recall that the availability constraints of problem KPCP are needed to deal with maintenance requirements of vessels. In general, the time duration for maintenance for a given vessel is far less that the time durations of legs to the various destination. We can slightly increase legs durations of vessels and then heuristically decide on the maintenance periods.

Chapter V
Solution Algorithms and Computational Results

5.1 Introduction

Solution strategies and algorithms, along with pertinent computational results for the initial formulation (KPCP) and the aggregate formulation (KPCPxy) are presented in this chapter. As mentioned earlier, only the Type I demand time window is considered in our solution algorithms and computational results.

This chapter is organized as follows. In Section 5.2, we present a realistic set of test problems that represent diverse operational scenarios. Preprocessing techniques and lower bounding schemes that will be instrumental in reducing optimality gaps in problems KPCP and KPCPxy are discussed in Section 5.3. Attempts are made to solve the initial formulation (KPCP) using CPLEX 4.0 MIP capabilities for the test problems of Section 5.2. Computational difficulties experienced are reported along with related results in Section 5.4 for the purpose of further comparisons and analysis. Likewise, attempts are made to solve problem KPCPxy using CPLEX 4.0 MIP for the test problems of Section 5.2, and similar discussions on the computational experience are reported in Section 5.5. Two rolling horizon solution algorithms that facilitate the derivation of good quality feasible solutions are presented in Section 5.6 along with related computational results. These algorithms are based on a judicious sequential fixing of integer variables until a feasible solution is obtained. An ad-hoc routing and scheduling procedure that is employed to simulate the current KPC scheduling practice is discussed in Section 5.7. Results obtained from this ad-hoc scheduling procedure are compared with results obtained from the rolling horizon algorithms (based on the test problems of Section 5.2) in order to acquire insights into the usefulness and efficiency of the proposed rolling horizon algorithms.

The rolling horizon solution algorithms are programmed in C in an interactive fashion using the CPLEX 4.0 callable library routines. The ad-hoc scheduling procedure is also programmed in C. Specifications of the computer used in obtaining our computational statistics are given as follows. Machine type: Sun Workstation, Ultra-1, Sun4u; physical memory: 128 megabytes; and virtual memory (swap): 225 megabytes.

The following list of notation is given here for the sake of convenience in presenting our test problems, solution statistics, solution algorithms, and the ad-hoc scheduling procedure.

- LP : stands for a linear program.
- MIP : stands for a mixed integer program, (a problem that contains both continuous and integer variables).
- (X, Y, Z) : binary variables of the problem KPCP as defined in Section 3.2.6.
- (x, y, z) : integer variables of the aggregated problem KPCPxy as defined in Section 4.4.2.
- H : time horizon.
- T : number of ship-types.
- O : total number of company-owned ships of all ship-types, i.e., $O = \sum_{t=1}^{T} O_t$, where O_t is the number of company-owned ships of type t as defined in Section 3.2.6.
- CT : total number of ships that are available for chartering, i.e., $CT = \sum_{t=1}^{T} CT_t$, where CT_t is the number of ships of type t that are available for chartering as defined in Section 3.2.6.
- R : number of routes.
- P_d : number of products that will be shipped to destination d.
- D_I : number of destinations of Type I.
- N : total number of partitions (shipments) for all products at destinations of

Type I. Note that $N = \sum_{(d,p) \in \text{Type I}} N_{d,p}$, where $N_{d,p}$ is the number of partitions of the demand $D_{I,d,p}$ as defined in Section 3.2.6.

- $f_{1(i,d,p)}$: minimum permitted amount of product p for shipment i at destination d.
- $F_{1(i,d,p)}$: maximum permitted amount of product p for shipment i at destination d.
- $a_{1(i,d,p)}$: first allowable delivery day for the i^{th} shipment (partition) of product p at destination d.
- $a_{2(i,d,p)}$: last allowable delivery day for the i^{th} shipment (partition) of product p at destination d.
- opt-gap : this parameter corresponds to the CPLEX MIPGAP parameter. The parameter MIPGAP is used as a relative tolerance on the gap between the best integer objective value found (let α denote this number) and the least lower bound among the active nodes (let β denote this number). If the value of $(\alpha - \beta) / (1.0 + \text{abs}(\beta))$ falls below the value of MIPGAP parameter setting, then the mixed integer optimization is terminated. The default value for MIPGAP in CPLEX is $(1 . e^{-4})$ and the possible values for this parameter are given by the range $[(1 . e^{-9}) , 1.0]$.
- RT : run time in minutes.
- OP : optimality status, where
  - * : indicates that the obtained solution is optimal,
  - ** : indicates that the obtained solution is within some given optimality gap; however, the optimality of this solution is not verified,
  - ⊗ : indicates that an optimal solution (within some known optimality gap) was not obtained due to out-of-memory problems. In this case, a best feasible solution is reported, if found.
- $NC_i = \begin{cases} 1 & \text{if the compartment having capacity } C_i \text{ is considered in a given problem} \\ 0 & \text{otherwise.} \end{cases}$

- $\overline{\text{KPCP}}$: linear relaxation of problem KPCP.
- $\overline{\text{KPCP}}_Z$: linear relaxation of problem KPCP, but while enforcing the integrality of its chartering variables (i.e., the Z-variables).
- $\overline{\text{KPCPxy}}$: linear relaxation of problem KPCPxy.
- $\overline{\text{KPCPxy}}_z$: linear relaxation of problem KPCPxy, but while enforcing the integrality of its chartering variables (i.e., the z-variables).
- $\overline{\text{KPCPxy}}_z(H_1,\text{opt-gap})$: relaxation of problem KPCPxy for which integrality is enforced on the z-variables and also on the x and y variables that correspond to day 1 through day H_1 of the time horizon. If an optimal solution for this problem exists, then this solution is required to be found within opt-gap of optimality.
- $\nu(P)$: objective function value of the problem P.
- $\nu_{UB}(P)$: best solution (upper bound) found for problem P.
- $\lceil a \rceil$: the ceiling of a real number a, which is the smallest integer number greater than or equal to a.
- $\lfloor a \rfloor$: the floor of a real number a, which is the largest integer number less than or equal to a.
- I_i : test problem instance i.

5.2 Test Problems

The following table contains a set of 24 test problems that represent realistic diverse operational scenarios. In particular, the diversity of these test problems is instrumental in examining the various solution algorithms and in obtaining insights into the sensitivity of the models with respect to various elements of the problem. We assume that there are 6 distinct compartment capacities given as follows: $[C_1, C_2, C_3, C_4, C_5, C_6] = [5,000, 8,000, 10,000, 15,000, 30,000, 40,000]$ barrels. Note that I_j denotes the test problem instance j.

Table 5.1. Test Problems

Test prob	H	(T,O,CT)	D_I	R	N	[NC_1,.,NC_6]
I_1	6	(1,1,1)	1	2	2	[1, 0, 0, 0, 0, 0]
I_2	6	(1,1,2)	1	2	3	[1, 0, 0, 0, 0, 0]
I_3	12	(1,1,2)	2	2	3	[1, 0, 1, 0, 0, 0]
I_4	12	(1,2,4)	2	2	7	[1, 0, 1, 0, 0, 0]
I_5	20	(1,2,3)	1	2	5	[1, 0, 1, 0, 0, 0]
I_6	20	(1,4,4)	1	2	11	[1, 0, 1, 0, 0, 0]
I_7	30	(1,4,4)	1	2	8	[1, 0, 1, 0, 0, 0]
I_8	30	(1,4,4)	1	2	8	[1, 0, 1, 1, 0, 0]
I_9	60	(1,3,4)	2	2	15	[1, 1, 0, 0, 0, 0]
I_{10}	60	(2,6,6)	3	2	24	[1, 1, 1, 1, 0, 0]
I_{11}	90	(2,7,7)	3	2	16	[1, 1, 1, 1, 0, 0]
I_{12}	90	(3,6,10)	4	3	20	[0, 0, 0, 1, 1, 1]
I_{13}	120	(3,7,11)	4	3	36	[0, 1, 1, 1, 1, 0]
I_{14}	120	(3,4,12)	5	3	30	[0, 0, 0, 1, 1, 1]
I_{15}	150	(3,4,12)	5	3	32	[0, 0, 0, 1, 1, 1]
I_{16}	150	(3,4,15)	5	3	32	[0, 0, 0, 1, 1, 1]
I_{17}	180	(3,4,15)	5	3	39	[0, 0, 0, 1, 1, 1]
I_{18}	180	(3,4,15)	6	3	46	[0, 0, 0, 1, 1, 1]
I_{19}	210	(3,4,15)	6	3	49	[0, 0, 0, 1, 1, 1]
I_{20}	210	(3,5,15)	7	3	56	[0, 0, 0, 1, 1, 1]
I_{21}	240	(3,5,15)	8	3	69	[0, 0, 0, 1, 1, 1]
I_{22}	240	(3,5,15)	8	4	69	[0, 0, 0, 1, 1, 1]
I_{23}	300	(3,5,15)	9	4	91	[0, 0, 0, 1, 1, 1]
I_{24}	300	(4,7,18)	10	4	95	[0, 0, 0, 1, 1, 1]

5.3 Preprocessing Techniques and Lower Bounding Schemes

Preprocessing

Four coefficient analysis techniques are employed to generate valid cuts (inequalities) that render some fractional solutions infeasible in both problems KPCP and KPCPxy. This process is called preprocessing and it is utilized to tighten integer or mixed integer problems, and consequently to reduce solution time and effort (see for example, Nemhauser and Wolsey, 1988 or Parker and Rardin, 1988). For each valid cut, we give two examples, illustrating how these cuts render some fractional solutions infeasible in both problems KPCP and KPCPxy. Below, we present four classes of valid cuts, which are respectively denoted by $\mathbf{CUT_I}$, $\mathbf{CUT_{II}}$, $\mathbf{CUT_{III}}$, and $\mathbf{CUT_{IV}}$.

Consider the following constraints:

$(\mathbf{C_I})$ $\Omega_1 X_1 + \Omega_2 X_2 + \Omega_3 X_3 + \ldots\ldots + \Omega_n X_n \geq f$

$(\mathbf{C_{II}})$ $\Omega_1 X_1 + \Omega_2 X_2 + \Omega_3 X_3 + \ldots\ldots + \Omega_n X_n \leq F$

where X_i, for $i = 1,..,n$ is a binary (or an integer) variable.

$\mathbf{CUT_I}$:

Let $\Omega_{max} = \max \{\Omega_1, \Omega_2, \Omega_3, \ldots,\Omega_n\}$. Dividing both sides of $(\mathbf{C_I})$ by Ω_{max}, we get the following inequalities.

$X_1 + X_2 + X_3 + \ldots + X_n \geq$

$(\Omega_1 / \Omega_{max}) X_1 + (\Omega_2 / \Omega_{max}) X_2 + (\Omega_3 / \Omega_{max}) X_3 + \ldots\ldots + (\Omega_n / \Omega_{max}) X_n \geq$

(f / Ω_{max}).

Since X_i, for $i = 1,..,n$ are integer variables, the sum $(X_1 + X_2 + X_3 + \ldots + X_n)$ must also be integer. Accordingly, we obtain the following valid cut (inequality).

$\mathbf{CUT_I}$: $X_1 + X_2 + X_3 + \ldots + X_n \geq \lceil f / \Omega_{max} \rceil$.

Example 5.1.

Consider the constraint:

(1) $2 X_1 + 3 X_2 + 4 X_3 + 5 X_4 \geq 11$, where X_i, i = 1,2,3,4 are binary (or integer) variables.

This constraint is of type C_I above and **CUT_I** for this constraint is given as follows:

(2) $X_1 + X_2 + X_3 + X_4 \geq 3$.

Observe that $X_{(1)} = (X_1, X_2, X_3, X_4) = (0, 2/3, 1, 1)$ and $X_{(2)} = (X_1, X_2, X_3, X_4) = (1/2, 0, 0, 2)$ satisfy (1) above; however, $X_{(1)}$ and $X_{(2)}$ do not satisfy (2). Hence, adding this valid cut (inequality) would render $X_{(1)}$ and $X_{(2)}$, for example, infeasible.

CUT_{II}:

Let $\Omega_{min} = \min \{\Omega_1, \Omega_2, \Omega_3, ..., \Omega_n\}$. Dividing both sides of **(C_{II})** by Ω_{min}, we get the following inequalities.

$X_1 + X_2 + X_3 + ... + X_n \leq$

$(\Omega_1 / \Omega_{min}) X_1 + (\Omega_2 / \Omega_{min}) X_2 + (\Omega_3 / \Omega_{min}) X_3 + ... + (\Omega_n / \Omega_{min}) X_n \leq$

(F/ Ω_{min}).

Since X_i, for i = 1,..,n are integer variables, the sum $(X_1 + X_2 + X_3 + ... + X_n)$ must also be integer. Accordingly, we obtain the following valid cut (inequality).

CUT_{II}: $X_1 + X_2 + X_3 + ... + X_n \leq \lfloor F / \Omega_{min} \rfloor$.

Example 5.2.

Consider the constraint:

(3) $4 X_1 + 4 X_2 + 4 X_3 + 4 X_4 + 5 X_5 + 5 X_6 \leq 17$, where X_i, i = 1,..,6 are binary (or integer) variables.

This constraint is of type C_{II} above and **CUT_{II}** for this constraint is given as follows:

(4) $X_1 + X_2 + X_3 + X_4 + X_5 + X_6 \leq 4$.

Observe that $X_{(3)} = (X_1, X_2, X_3, X_4, X_5, X_6) = (1, 1, 1, 1, 1/5, 0)$ and $X_{(4)} = (X_1, X_2, X_3, X_4, X_5, X_6) = (2, 2, 0, 0, 1/5, 0)$ satisfy (3) above; however $X_{(3)}$ and $X_{(4)}$ do not satisfy (4). Hence, adding this valid cut (inequality) would, for example, render $X_{(3)}$ and $X_{(4)}$ infeasible.

CUT_{III}:

Let Ω_{min} = min $\{\Omega_1, \Omega_2, \Omega_3, \ldots, \Omega_n\}$. Dividing both sides of **(C_I)** by Ω_{min}, we get the following inequalities.

$\lceil \Omega_1 / \Omega_{min} \rceil X_1 + \lceil \Omega_2 / \Omega_{min} \rceil X_2 + \lceil \Omega_3 / \Omega_{min} \rceil X_3 + \ldots\ldots + \lceil \Omega_n / \Omega_{min} \rceil X_n \geq$

$(\Omega_1 / \Omega_{min}) X_1 + (\Omega_2 / \Omega_{min}) X_2 + (\Omega_3 / \Omega_{min}) X_3 + \ldots\ldots + (\Omega_n / \Omega_{min}) X_n \geq$

(f / Ω_{min}).

Since for i = 1,..,n, X_i and $\lceil \Omega_i / \Omega_{min} \rceil$ are integer quantities , the sum

$\lceil \Omega_1 / \Omega_{min} \rceil X_1 + \lceil \Omega_2 / \Omega_{min} \rceil X_2 + \lceil \Omega_3 / \Omega_{min} \rceil X_3 + \ldots\ldots + \lceil \Omega_n / \Omega_{min} \rceil X_n$ is also an integer quantity. Accordingly, we obtain the following valid cut (inequality).

CUT_{III}:

$\lceil \Omega_1 / \Omega_{min} \rceil X_1 + \lceil \Omega_2 / \Omega_{min} \rceil X_2 + \lceil \Omega_3 / \Omega_{min} \rceil X_3 + \ldots\ldots + \lceil \Omega_n / \Omega_{min} \rceil X_n \geq \lceil f / \Omega_{min} \rceil$.

Example 5.3.

Consider the constraint:

(5) $3 X_1 + 3 X_2 + 3 X_3 + 3 X_4 + 4 X_5 + 4 X_6 \geq 11$, where X_i, i = 1,..,6 are binary (or integer) variables.

This constraint is of type C_I above and **CUT_{III}** for this constraint is given as follows:

(6) $X_1 + X_2 + X_3 + X_4 + 2 X_5 + 2 X_6 \geq 4$

Observe that $X_{(5)} = (X_1, X_2, X_3, X_4, X_5, X_6) = (1, 1, 1, 2/3, 0, 0)$ and $X_{(6)} = (X_1, X_2, X_3, X_4, X_5, X_6) = (2, 1, 0, 2/3, 0, 0)$ both satisfy (5) above as well as the CUT_I derived for this constraint which is given as: $X_1 + X_2 + X_3 + X_4 + X_5 + X_6 \geq 3$.

However, $X_{(5)}$ and $X_{(6)}$ do not satisfy (6). Hence, adding this valid cut (inequality) would, for example, render $X_{(5)}$ and $X_{(6)}$ infeasible.

CUT_{VI}:

Let Ω_{ave} = $[(\Omega_1 + \Omega_2 + \Omega_3 + ...+ \Omega_n) / n]$. Dividing both sides of **(C_I)** by Ω_{ave}, we get the following inequalities.

$\lceil \Omega_1 / \Omega_{ave} \rceil X_1 + \lceil \Omega_2 / \Omega_{ave} \rceil X_2 + \lceil \Omega_3 / \Omega_{ave} \rceil X_3 + ... + \lceil \Omega_n / \Omega_{ave} \rceil X_n \geq$

$(\Omega_1 / \Omega_{ave}) X_1 + (\Omega_2 / \Omega_{ave}) X_2 + (\Omega_3 / \Omega_{ave}) X_3 + ... + (\Omega_n / \Omega_{ave}) X_n \geq$

(f / Ω_{ave}).

Since for i = 1,..,n, X_i and $\lceil \Omega_i / \Omega_{ave} \rceil$ are integer quantities , the sum

$\lceil \Omega_1 / \Omega_{ave} \rceil X_1 + \lceil \Omega_2 / \Omega_{ave} \rceil X_2 + \lceil \Omega_3 / \Omega_{ave} \rceil X_3 + ... + \lceil \Omega_n / \Omega_{ave} \rceil X_n$ is also an integer quantity. Accordingly, we obtain the following valid cut (inequality).

CUT_{IV}:

$\lceil \Omega_1 / \Omega_{ave} \rceil X_1 + \lceil \Omega_2 / \Omega_{ave} \rceil X_2 + \lceil \Omega_3 / \Omega_{ave} \rceil X_3 + ... + \lceil \Omega_n / \Omega_{ave} \rceil X_n \geq$

$\lceil f / \Omega_{ave} \rceil$.

Example 5.4.

Consider the constraint:

(7) $2 X_1 + 3 X_2 + 3 X_3 + 3 X_4 + 3 X_5 + 4 X_6 \geq 13$, where X_i, i = 1,..,6 are binary (or integer) variables.

This constraint is of type C_I above and **CUT_{VI}** for this constraint is given as follows:

(8) $X_1 + X_2 + X_3 + X_4 + X_5 + 2 X_6 \geq 5$

Observe that $X_{(7)} = (X_1, X_2, X_3, X_4, X_5, X_6) = (1/2, 1, 1, 1, 1, 0)$ and $X_{(8)} = (X_1, X_2, X_3, X_4, X_5, X_6) = (1/2, 2, 2, 0, 0, 0)$ both satisfy (7), as well as the cuts CUT_I and CUT_{III} derived for this constraint which are respectively given as:

$X_1 + X_2 + X_3 + X_4 + X_5 + X_6 \geq 4$ and

$X_1 + 2 X_2 + 2 X_3 + 2 X_4 + 2 X_5 + 2 X_6 \geq 7$.

However, $X_{(7)}$ and $X_{(8)}$ do not satisfy (8). Hence, adding this valid cut (inequality) would, for example, render $X_{(7)}$ and $X_{(8)}$ infeasible.

Note that if (f / Ω_{max}) is an integer number, then the constraint given by $\mathbf{CUT_I}$ is a redundant constraint. This follows from the fact that (f / Ω_{max}) being integer implies that $(f / \Omega_{max}) = \lceil f / \Omega_{max} \rceil$. Likewise, if (F / Ω_{min}) is an integer number, then the constraint given by $\mathbf{CUT_{II}}$ is a redundant constraint. This follows from the fact that (F/ Ω_{min}) being integer implies that $(F / \Omega_{min}) = \lfloor F / \Omega_{min} \rfloor$. Based on the same line of reasoning, the constraint given by $\mathbf{CUT_{III}}$ is redundant if (f / Ω_{min}) is an integer number and the constraint given by $\mathbf{CUT_{VI}}$ is redundant if (f / Ω_{ave}) is an integer number.

Constraints (C_1) and (C_5) of problem KPCP, and constraints (RC_5) and (RC_9) of problem KPCPxy are of the type (C_I) given above. Constraints (C_2) and (C_6) of problem KPCP, and constraints (RC_6) and (RC_{10}) of problem KPCPxy are of the type (C_{II}) given above. Accordingly, valid cuts of the types $\mathbf{CUT_I}$, $\mathbf{CUT_{III}}$, and $\mathbf{CUT_{VI}}$ are generated corresponding to constraints (C_1) and (C_5) in problem KPCP, and constraints (RC_5) and (RC_9) in problem KPCPxy, while valid cuts of the type $\mathbf{CUT_{II}}$ are generated corresponding to constraints (C_2) and (C_6) of problem KPCP, and constraints (RC_6) and (RC_{10}) of problem KPCPxy.

To illustrate the usefulness of the above cuts in tightening the LP relaxations of problem KPCPxy, we give few examples in which the lower bounds obtained via solving the LP relaxations after generating the above cuts provide tighter lower bounds for problem KPCPxy than those obtained via solving the LP relaxations before generating the above cuts. These examples are based on selected test problems of Section 5.2 and are given in the following table. Note that $v_1(\overline{KPCPxy})$ denotes the objective function value of the LP before generating the above cuts, and $v_2(\overline{KPCPxy})$ denotes the objective function value of the LP after generating the above cuts.

TABLE 5.2. Examples Illustrating the Utilization of the Above Cuts in Tightening the LP Relaxations of Problem KPCPxy

Test Prob I_i	CPLEX(LP) $v_1(\overline{KPCPxy})$	CPLEX(LP) $v_2(\overline{KPCPxy})$	$v_2(\overline{KPCPxy}) - v_1(\overline{KPCPxy})$	% of Improvement
I_{12}	1,203,000	1,365,196	162,196	11.90
I_{13}	874,565	912,892	38,327	4.20
I_{14}	1,515,087	1,698,083	182,996	10.80
I_{15}	746,756	782,312	35,556	4.50
I_{16}	1,353,470	1,377,309	23,839	1.70
I_{17}	920,095	927,930	7,835	0.84
I_{18}	1,352,730	1,368,525	15,795	1.54
I_{19}	2,232,925	2,252,621	19,696	0.87
I_{20}	3,010,304	3,043,622	33,318	1.10
I_{21}	3,043,891	3,098,643	54,752	1.77
I_{22}	2,778,314	2,841,699	63,385	2.23
I_{23}	5,528,632	5,595,769	67,137	1.2

Lower Bounding Schemes

The following four lower bounding schemes are considered in our analysis.

1. Usual lower bounds obtained via solving the linear relaxations $\overline{KPCP}$ and $\overline{KPCPxy}$. Let LB_1 denote the lower bound obtained via this scheme.

2. Lower bounds obtained via solving the linear relaxations $\overline{KPCP}$ and $\overline{KPCPxy}$, but while enforcing the integrality of the chartering (Z and z) variables (i.e., via solving

problems $\overline{\text{KPCP}}_Z$ and $\overline{\text{KPCPxy}}_z$). Let LB_2 denote the lower bound obtained via this scheme.

3. Lower bounds obtained via solving problems KPCP and KPCPxy until either optimality is obtained (in this case, the optimal objective value, or upper bound, coincides with the lower bound) or out-of-memory difficulties emerge (in which case, no lower bound is found although the least global lower bound at this stage could be used as a lower bound). This lower bound may be obtained from the last iteration of the branch-and-bound procedure before it ran into out-of-memory difficulties. Let LB_3 denote the lower bound obtained via this scheme.

4. Lower bounds obtained via solving problem $\overline{\text{KPCPxy}}_z(H_1,\text{opt-gap})$ for the largest possible H_1 and the smallest possible opt-gap before running into out-of-memory problems. Let LB_4 denote the lower bound obtained via this scheme.

In the process of determining the optimality percentage of solutions obtained by the rolling horizon solution algorithms discussed below, we consider the largest of the foregoing lower bounds that is available. Based on running a number of test problems, it was observed that the lower bounds obtained via solving problem $\overline{\text{KPCP}}_Z$ (or $\overline{\text{KPCPxy}}_z$) provide reasonably tight lower bounds. It was also observed that if a given solution of problem $\overline{\text{KPCP}}$ (or $\overline{\text{KPCPxy}}$) contains fractional Z-variables (or z-variables), then the objective function value of the linear relaxation given by $\nu(\overline{\text{KPCP}})$ (or $\nu(\overline{\text{KPCPxy}})$) usually gives a loose lower bound for the underlying problem KPCP (or KPCPxy). The relatively large gap between the LP and MIP solutions is created by the fractionality of the chartering variables, due to the large magnitudes (chartering expenses) associated with the chartering variables. This motivates the consideration of lower bounds obtained via solving the linear relaxations while enforcing the integrality of these chartering variables.

As an example, suppose that the chartering expenses for a ship of type t is given by $1,000,000. Moreover, suppose that in the solution of the corresponding LP, the value obtained for the associated chartering variable is given by 0.5. If in the solution of the related MIP, the value assigned to the corresponding chartering variable is given by 1, then this difference of $500,000 due to the fractionality of this chartering variable would induce a large gap between the LP and MIP solutions.

5.4 Computational Experience in Solving Problem KPCP

Tables 5.3 and 5.4 present some computational results related to solving problem KPCP. Computational statistics are reported corresponding to solving problems $\overline{KPCP}$, $\overline{KPCP}_Z$, and KPCP itself for the set of test problems given in Section 5.2 using CPLEX 4.0. Note that opt-gap is set to the value 0.05. Optimality percentages with respect to the objectives obtained by solving problems $\overline{KPCP}$ and $\overline{KPCP}_Z$ are reported in Table 5.4.

Table 5.3. Computational Statistics for Problems $\overline{KPCP}$ and $\overline{KPCP}_Z$

I_i	# Rows	# Cols	CPLEX(LP) $\nu(\overline{KPCP})$	RT	CPLEX(MIP) $\nu(\overline{KPCP}_Z)$	RT
I_1	104	163	10,800	0.0006	10,800	0.0005
I_2	159	353	14,450	0.0013	21,650	0.0020
I_3	501	974	30,400	0.0043	42,800	0.0046
I_4	1,440	4,108	67,550	0.0385	67,550	0.0400
I_5	807	2,148	45,166	0.0090	45,166	0.0086
I_6	1,314	5,548	90,166	0.0308	90,166	0.0303
I_7	1,914	6,804	81,333	0.0410	101,333	0.0880
I_8	1,914	6,804	87,942	0.0456	100,800	0.063
I_9	11,747	40,716	65,714	0.8560	82,400	1.0440
I_{10}	21,372	78,438	143,756	2.8770	156,060	10.5535
I_{11}	38,402	117,161	1,086,103	6.0100	1,470,075	13.4112
I_{12}	51,848	163,238	1,365,196	5.7221	⊗	N/A
I_{13}	90,310	325,179	⊗	N/A	⊗	N/A

Table 5.4. Computational Statistics for Problem KPCP

I_i	CPLEX(MIP)			% of opt w.r.t	% of opt w.r.t
	ν_{UB}(KPCP)	RT	OP	$\overline{\nu}$(KPCP)	$\overline{\nu}$($KPCP_Z$)
I_1	10,800	0.0006	*	100	100
I_2	21,900	0.4976	**	66.00	98.86
I_3	45,000	0.5000	**	67.60	95.11
I_4	68,100	0.1355	**	99.20	99.20
I_5	45,500	0.0190	**	99.30	99.30
I_6	90,500	0.1342	**	99.63	99.63
I_7	102,400	5.6200	**	79.43	98.96
I_8	102,400	1.3008	**	85.90	98.44
I_9	120,000	439.5826	⊗	54.76	68.67
I_{10}	307,300	N/A	⊗	46.80	50.80
I_{11}	4,548,000	345.84	⊗	23.90	32.33
I_{12}	N/A	N/A	⊗	N/A	N/A

The test problem given by I_{12} was the largest test problem for which the LP solution was obtained. LP solutions for the test problems given by I_i, for i=13,..,24 were not available due to out-of-memory problems. The largest test problem for which a meaningful MIP solution was obtained is problem I_{11}. The overwhelming problem size for a moderate size test problem (such as test problem I_{13}, which is comprised of 90,310 constraints and 325,179 variables) would most likely lead to the exhaustion of memory before obtaining a meaningful LP solution.

5.5 Computational Experiments with Problem KPCPxy

Tables 5.5 and 5.6 present some computational results related to solving problem KPCPxy. Computational statistics are reported corresponding to solving problems $\overline{\text{KPCPxy}}$, $\overline{\text{KPCPxy}}_z$, and KPCPxy itself for the set of test problems given in Section 5.2 using CPLEX 4.0. Note that opt-gap is set to the value 0.05. Hence, for a given test problem, a solution within this optimality gap is reported, or the best solution (if found) before out-of-memory problems emerge is reported. Optimality percentages with respect to the lower bound obtained from the last iteration of the branch-and-bound procedure (i.e., lower bound of the type LB_3 as defined above), and with respect the objectives obtained by solving problems $\overline{\text{KPCP}}$ and $\overline{\text{KPCP}}_Z$ are reported in Table 5.6.

Table 5.5. Computational Statistics for Problems $\overline{\text{KPCPxy}}$ and $\overline{\text{KPCPxy}}_z$

I_i	# Rows	# Cols	CPLEX(LP)		CPLEX(MIP)	
			$\nu(\overline{\text{KPCPxy}})$	RT	$\nu(\overline{\text{KPCPxy}}_z)$	RT
I_1	33	28	10,800	0.0002	10,800	0.0002
I_2	43	36	14,450	0.0006	21,650	0.0002
I_3	97	114	30,400	0.0005	42,800	0.0005
I_4	127	168	67,550	0.0006	67,550	0.0008
I_5	115	145	45,166	0.0006	45,166	0.0006
I_6	155	236	90,166	0.0008	90,166	0.0006
I_7	175	282	81,333.3	0.0006	101,333.33	0.0010
I_8	205	372	87,942	0.0015	100,800	0.0020
I_9	580	1,221	65,714	0.0070	82,400	0.0072
I_{10}	1,120	2,974	143,756	0.0306	156,055	0.0440
I_{11}	1,550	3,444	1,086,103	0.0400	1,470,075	0.0400
I_{12}	1,760	5,701	1,365,196	0.0840	1,508,530	0.1240
I_{13}	2,390	8,329	912,892	0.095	912,892	0.080
I_{14}	2,725	9,764	1,698,083	0.2700	2,014,966	0.5700
I_{15}	3,365	12,196	782,312	0.3053	782,312	0.4150
I_{16}	3,365	12,196	1,377,309	0.4207	1,675,452	0.6095
I_{17}	4,030	14,633	927,930	0.6125	975,595	0.0200
I_{18}	4,615	17,342	1,368,525	1.4555	1,807,533	2.3650
I_{19}	5,350	20,225	2,252,621	1.3670	2,278,109	2.0800
I_{20}	6,025	23,384	3,043,622	2.3265	3,450,726	3.0840
I_{21}	7,630	30,329	3,098,643	8.33	3,574,378	8.87
I_{22}	7,630	36,089	2,841,699	23.10	3,255,607	28.74
I_{23}	10,450	50,513	5,595,769	23.50	5,595,769	23.20
I_{24}	11,980	68,220	5,074,530	11.85	5,094,270	12.85

Table 5.6. Computational Statistics for Problem KPCPxy

I_i	CPLEX(MIP) $\nu_{UB}(KPCP_{xy})$	RT	OPT	LB_3	% of Opt w.r.t ______ v(KPCPxy)	% of Opt w.r.t ______ $v(KPCPxy_z)$	% of Opt w.r.t LB_3
I_1	10,800	0.0002	*	10,800	100	100	100
I_2	21,900	0.0005	*	21,900	66.00	98.86	100
I_3	45,000	0.0023	*	45,000	67.60	95.11	100
I_4	68,100	0.0022	*	68,100	99.20	99.20	100
I_5	45,500	0.0015	**	45,333	99.30	99.30	99.63
I_6	90,500	0.0012	**	90,466	99.63	99.63	99.96
I_7	103,000	0.4745	**	101,733	78.96	98.38	98.77
I_8	102,400	0.6750	**	101,228	85.90	98.44	98.85
I_9	119,600	14.9260	⊗	93,200	54.94	68.90	79.93
I_{10}	282,500	65.6000	⊗	146,481	50.90	55.24	51.85
I_{11}	2,557,000	10.52	⊗	1,127,621	42.5	57.50	44.10
I_{12}	2,426,000	87.82	⊗	1,475,570	53.60	62.20	60.82
I_{13}	1,304,000	90.14	⊗	970,155	70.00	70.00	74.40
I_{14}	4,256,000	281.90	⊗	2,071,288	39.90	47.35	48.67
I_{15}	3,574,000	239.33	⊗	954,594	21.90	21.90	26.77
I_{16}	3,540,000	380.33	⊗	1,648,778	38.91	47.83	46.58
I_{17}	3,968,000	294.25	⊗	1,060,256	23.39	24.59	26.72
I_{18}	4,921,000	71.53	⊗	1,463,444	27.81	36.73	29.74
I_{19}	10,382,000	37.02	⊗	2,308,796	21.70	21.94	22.24
I_{20}	16,842,000	26.22	⊗	3,081,723	10.07	20.50	18.30
I_{21}	18,739,000	547.2	⊗	3,113,399	16.54	19.07	16.62
I_{22}	21,836,000	N/A	⊗	2,928,789	13.02	14.91	13.42
I_{23}	N/A	N/A	⊗	N/A	N/A	N/A	N/A

Solutions for problems $\overline{KPCPxy}$ and $\overline{KPCPxy_z}$ were readily obtained for all test cases. The largest test problem for which a meaningful MIP solution was obtained is test problem I_{22}. Observe that $\nu(\overline{KPCP}) = \nu(\overline{KPCPxy})$ and $\nu(\overline{KPCP_Z}) = \nu(\overline{KPCPxy_z})$ for the test problems $I_1,..,I_{11}$ (i.e., for the test problems in which objective function values for problems $\overline{KPCP}$ and $\overline{KPCP_z}$ were available). Likewise, ν(KPCP) $\geq$ ν(KPCPxy) for the test problems $I_1,..,I_{11}$ (these were all the test problems for which objective function values for problems KPCP and KPCPxy were available). This provides some validation for the efficiency of the aggregation employed in formulating problem KPCPxy.

For a given test problem, the run time needed to solve problem KPCP is usually far more than the run time needed to solve problem KPCPxy. For example, the run times for problems $\overline{KPCP_Z}$ and $\overline{KPCPxy_z}$ based on test problem I_{10} are respectively given as 10.5535 minutes and 0.0440 minutes. This huge difference in run times is anticipated since the number of rows and variables in problem KPCP for test problem I_{10} are respectively given by 21,372 and 78,438, while the number of rows and variables of problem KPCPxy for test problem I_{10} are respectively given by 1,120 and 2,974.

5.6 Rolling Horizon Solution Algorithms for Problem KPCPxy

It was illustrated in the previous section that attempting to solve problem KPCPxy for a problem of practical size (such as test problem I_{23}) will most likely lead into out-of-memory problems before reaching a good quality feasible solution. To alleviate this difficulty, we adopt two rolling horizon solution algorithms, respectively denoted by RHA_1 and RHA_2, that will enable us to construct good quality feasible solutions. The algorithm RHA_2 is a modified version of the algorithm RHA_1, and it is intended to deal with large sized problems. These algorithms are based on the sequential fixing of integer variables. We present the rolling horizon solution algorithm RHA_1 in Section 5.6.1, and report on our computational results for this algorithm based on the test problems of Section 5.2. The rolling horizon solution algorithm RHA_2 is presented in Section 5.6.2, and similar computational results are reported.

5.6.1 Rolling Horizon Solution Algorithm (RHA_1) for Problem KPCPxy

Recall that x and y are integer variables in problem KPCPxy. Let $x = (x_1, x_2, x_3, \ldots, x_H)$, where x_i denotes the vector of x variables associated with the i^{th} day of the time horizon. Likewise, let $y = (y_1, y_2, y_3, \ldots, y_H)$. Let H_1 be the horizon for which the corresponding variables are restricted to be integer valued, and let H_2 be a subset of H_1 for which the determined decisions are fixed in a rolling horizon framework. Let $KK = \lceil ((H - H_1) / H_2) + 1 \rceil$ and let KPCPxy(H_1, H_2, opt-gap, k), for k = 1 ,..,KK, denote problem KPCPxy with the following characteristics.

(1) integrality of all the z-variables is enforced.

(2) x_i and y_i are enforced as integer variables for $i \leq (H_1 + (k - 1)H_2)$.

(3) x_i and y_i for $i \leq ((k - 1)H_2)$ are fixed at the values found from the solution to problem KPCPxy(H_1, H_2, opt-gap, (k-1)).

(4) Optimality gap is given by opt-gap, which is a parameter that corresponds to the MIPGAP CPLEX parameter as defined in Section 5.1.

The following figure illustrates the status of the integer variables at iteration k=4 of the algorithm RHA_1.

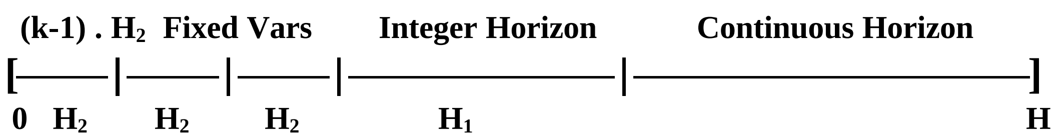

Figure 5.1. Status of the Integer Variables at Iteration k = 4 of the Rolling Horizon Algorithm RHA_1.

The rolling horizon solution algorithm (RHA_1) then proceeds as follows.

Initialization: Let H_1 be some integer number less than or equal to H, $H_2 \leq (H_1 / 2)$, and k=1. Let opt-gap be some given optimality gap criterion. Solve problem KPCPxy(H_1, H_2, opt-gap, 1).

Main Step: If **(**k+1) > KK, then terminate the algorithm; the proposed solution is that obtained from solving problem KPCPxy(H_1, H_2, opt-gap, KK). Otherwise, increase k by one and solve problem KPCPxy(H_1, H_2, opt-gap, k). Repeat the Main Step.

Let BLB(I_j) denote the largest lower bound available for KPCPxy for the test problem I_j. This lower bound is obtained by one of the lower bounding schemes addressed in Section 5.3. The results reported in Table 5.7 were obtained by using some fixed judicious values of H_1, H_2 and opt-gap. The choice of these parameters was based on our computational experimentation with the algorithm RHA_1. Note that ν_{RH1} gives the solution value obtained by the rolling horizon algorithm.

Table 5.7. Computational Statistics for the Algorithm RHA_1

I_i	Algorithm RHA_1 $H_1 = 6, H_2 = 3$, opt-gap = 0.12		$BLB(I_j)$	% of Opt of RHA_1 w.r.t $BLB(I_j)$	% of Opt of KPCPxy w.r.t $BLB(I_j)$
	ν_{RH1}	RT			
I_1	10,800	0.00003	10,800	100	100
I_2	21,900	0.0005	21,900	100	100
I_3	45,000	0.0035	45,000	100	100
I_4	68,100	0.0200	68,100	100	100
I_5	45,500	0.0022	45,333	99.63	99.63
I_6	90,500	0.0035	90,466	99.96	99.96
I_7	102,400	0.0060	101,733	99.35	98.77
I_8	102,600	0.0200	101,228	98.66	98.85
I_9	122,800	0.0930	93,200	75.90	79.93
I_{10}	202,700	0.3665	162,925	80.40	55.24
I_{11}	2,204,000	4.4090	1,470,075	66.70	57.50
I_{12}	2,438,000	110.7000	1,508,530	61.90	62.20
I_{13}	1,493,000	1.4250	970,155	64.98	74.40
I_{14}	4,587,000	49.5670	2,071,288	45.20	48.67
I_{15}	4,566,000	27.0700	956,737	20.95	26.77
I_{16}	3,644,000	12.8360	1,675,452	45.98	47.83
I_{17}	3,897,000	17.7200	1,060,256	27.21	26.72
I_{18}	5,164,000	32.6130	1,807,533	35.00	36.73
I_{19}	6,286,000	52.3300	2,308,796	36.72	22.24
I_{20}	9,046,000	64.9500	3,450,726	38.15	20.50
I_{21}	10,339,000	140.75	3,574,378	34.57	19.08
I_{22}	N/A	N/A	3,255,607	N/A	14.91

The largest test problem for which a meaningful MIP solution was obtained via using the rolling horizon algorithm RHA_1 was problem I_{21}. MIP solutions (upper bounds) for the test problems given by I_i, for i=22,..,24 were not available due to out-of-memory problems. The quality of the solution obtained using Algorithm RHA_1 depends to a large extent on the parameters H_1, H_2, and opt-gap. A given solution may be enhanced by increasing H_1 and decreasing opt-gap. Based on extensive computational experimentation with the algorithm RHA_1, we determined that the best strategy was to gradually increase H_1 and decrease opt-gap until out-of-memory problems emerge. Note that even a slight increase in H_1 or a slight decrease in opt-gap can lead to an out-of-memory indication. For example, if (H_1, H_2, opt-gap) are some given parameters for which a feasible solution can be constructed using the above algorithm, then the parameters ($H_1 + 1$, H_2, opt-gap) might induce out-of-memory problems before reaching a meaningful solution. Table 5.8 presents some examples that illustrate the potential improvement by using more stringent parameter values.

Table 5.8. Examples Illustrating the Sensitivity of the Algorithm RHA_1 with Respect to the Parameters (H_1, H_2, opt-gap)

Test Prob	Algorithm RHA_1 $H_1 = 6, H_2 = 3$, opt-gap = 0.12 ν_{RH1}	RT	$(\bar{H}_1, \bar{H}_2, \overline{\text{opt-gap}})$	Algorithm RHA_1 Using $\bar{H}_1$, $\bar{H}_2$, and $\overline{\text{opt-gap}}$ ν_{RH1}	RT	% of Improvement Using ($\bar{H}_1$, $\bar{H}_2$, $\overline{\text{opt-gap}}$)
I_8	102,600	0.0200	(20, 10, 0.05)	102,400	0.0195	0.20
I_9	122,800	0.0930	(20, 10, 0.05)	110,400	0.0560	10.10
I_{11}	2,204,000	4.4090	(20, 5, 0.05)	1,684,000	2.3815	23.60
I_{12}	2,438,000	110.70	(10, 5, 0.10)	1,910,000	25.3605	21.66

Analyzing the results of Table 5.8, we deduce that using more stringent parameter values can produce better results. In test problem I_{11}, for example, the parameters (H_1, H_2, opt-gap) = (20, 5, 0.05) induced an improvement of 23.60 percent over the parameters (H_1, H_2, opt-gap) = (6, 3, 0.12). However, attempting to solve test problem I_{21} for the parameters (H_1, H_2, opt-gap) = (20, 5, 0.05) have led to out-of-memory indication before reaching a meaningful solution. This justifies our choice of the parameters (H_1, H_2, opt-gap) = (6, 3, 0.12), which have produced meaningful MIP solutions for the test problems I_1,..,I_{21}. For the test problems I_{22},..,I_{24}, we will utilize the rolling horizon solution algorithm RHA_2 presented in the next section. Observe that for problems of practical size, such as test problems I_{20} and I_{21}, the algorithm RHA_1 has produced far better results then the results obtained by directly solving problem KPCPxy (see Table 5.7).

5.6.2 Rolling Horizon Solution Algorithm (RHA_2) for Problem KPCPxy

This rolling horizon algorithm is a modified version of the algorithm RHA_1, that is intended to deal with large sized problems, such as test problems I_{22},..,I_{24}. Let H_1 and H_2 be as defined in the algorithm RHA_1. Let $H_3 = H_1 + \overline{H}$, where $\overline{H}$ is the horizon of continuos variables in a rolling horizon framework (see Figure 5.2). Let KK = $\lceil ((H - H_1) / H_2) + 1 \rceil$ and let KPCPxy(H_1, H_2, H_3, opt-gap, k), for k = 1 ,..,KK, denote problem KPCPxy with the following characteristics.

(1) At iteration k of the algorithm RHA_2, the problem horizon is given by
min {H_3 , H - (k-1)H_2}, and accordingly, the partition of the problem related to the interval of time ((k-1)H_2+ H_3 , H - (k-1)H_2 + H_3] is discarded at this iteration of the algorithm. Note that in the algorithm RHA_1, H_3 is given by H_3 = H - (k-1)H_2.

(2) integrality of all the z-variables is enforced.

(3) x_i and y_i are enforced as integer variables for $i \leq (H_1 + (k - 1)H_2)$.

(4) x_i and y_i for $i \leq ((k - 1)H_2)$ are fixed at the values found from the solution to problem KPCPxy(H_1, H_2, opt-gap, (k-1)).

(5) Optimality gap is given by opt-gap, which is a parameter that corresponds to the MIPGAP CPLEX parameter as defined in Section 5.1.

The following figure illustrates the status of the integer variables and the problem horizon at iteration k=3 of the algorithm RHA_2.

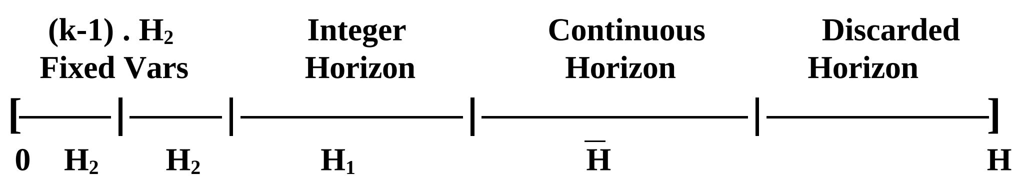

Figure 5.2. Status of the Integer Variables at Iteration k = 3 of the Rolling Horizon Algorithm RHA_2.

The rolling horizon solution algorithm (RHA_2) then proceeds as follows.

Initialization: Let H_1 be some integer number less than or equal to H, $H_2 \leq (H_1 / 2)$, H_3 some reasonable problem horizon, and k=1. Let opt-gap be some given optimality gap criterion. Solve problem KPCPxy(H_1, H_2, H_3, opt-gap, 1).

Main Step: If **(**k+1) > KK, then terminate the algorithm; the proposed solution is that obtained from solving problem KPCPxy(H_1, H_2, H_3, opt-gap, KK). Otherwise, increase k by one and solve problem KPCPxy(H_1, H_2, H_3, opt-gap, k). Repeat the Main Step.

The results reported in Table 5.9 were obtained by using some fixed judicious values of H_1, H_2, H_3 and opt-gap. The choice of these parameters was based on our computational experimentation with the algorithm RHA_2. Note that ν_{RH2} gives the solution value obtained by the rolling horizon algorithm RHA_2. Note also that results for test problems I_1,..,I_{12} are not reported because for the given parameters both of the algorithms RHA_1 and RHA_2 produce similar results.

Table 5.9. Computational Statistics for the Algorithm RHA_2

Test Prob	Algorithm RHA_2 $H_1 = 6$, $H_2 = 3$, $H_3 = 90$ opt-gap = 0.12		$BLB(I_j)$	% of Opt	% of Improvement of RHA_2 Over RHA_1
	ν_{RH2}	RT			
I_{13}	3,675,000	8.8909	970,155	26.40	-59.37
I_{14}	5,985,000	8.9834	2,071,288	34.61	-23.36
I_{15}	3,809,000	9.9264	956,737	25.12	16.58
I_{16}	4,054,000	10.7453	1,675,452	41.33	-10.11
I_{17}	4,381,000	11.8265	1,060,256	24.20	-11.05
I_{18}	6,276,000	32.3790	1,807,533	28.80	-17.72
I_{19}	6,593,125	36.6667	2,308,796	35.20	-4.66
I_{20}	10,067,000	38.3300	3,450,726	34.30	-10.14
I_{21}	11,065,000	101.5400	3,574,378	32.30	-6.56
I_{22}	13,527,614	119.6460	3,255,607	24.07	N/A
I_{23}	14,564,987	256.8970	5,595,769	38.42	N/A
I_{24}	12,255,000	298.0981	5,094,270	41.57	N/A

The algorithm RHA_2 enabled us to construct meaningful MIP solutions for large sized problems (such as test problems I_{23} and I_{24}) that we could not tackle by attempting to directly solve the problem KPCPxy or by using the algorithm RHA_1. Although the proven optimality percentage for some problems is quite low, these algorithms (RHA_1 and RHA_2) produce results that are at least as good as the results obtained from the ad-hoc procedure as will be illustrated in Section 5.7, where the importance becomes substantial as problem size increases.

It is worth mentioning that out-of-memory difficulties emerge due to the large size of the branch-and-bound tree created by the CPLEX. Observe also that at the end of each iteration of the algorithms RHA_1 and RHA_2 all allocated memory for this iteration, in particular memory allocated for the CPLEX data structures, is freed. Recall that the rolling horizon algorithms were run on a Sun Workstation with 128 megabytes of physical memory. The performance of the rolling horizon algorithms may be enhanced by increasing the physical memory, and consequently, better solutions for the problem KPCPxy can be extracted. The potential saving and the usefulness of the model KPCPxy in negotiation and planning purposes strongly justifies the acquisition of more computing power (in particular more physical memory) to tackle practical sized test problems.

5.7 An Ad-hoc Routing and Scheduling Procedure

KPC is currently employing an ad-hoc procedure for the routing and scheduling of vessels from Kuwait to the various destinations in the world. An ad-hoc procedure that is employed to simulate the existing manual procedure is presented in this section. Since chartering expenses are of large magnitudes relative to costs of legs and penalties imposed on early or late shipments, the procedure attempts to fully utilize company owned vessels before resorting to chartered vessels. This ad-hoc procedure is programmed in C and is evaluated based on the test problems of Section 5.2. The resulting overall costs obtained from this procedure and the overall costs obtained via the rolling horizon solution algorithm of Section 5.6 are given and analyzed in this section.

We now introduce some notation and then present the ad-hoc scheduling procedure.

- $d = 1,..,D_I$: destinations of Type I. If $d_1 < d_2$ then the total demand of all products at destination d_1 is greater than or equal to the total demand of all products at destination d_2.
- P_d : the number of products to be shipped to destination d.
- $i = 1,..,N_{d,p}$: subdemands of product p at destination d, where $N_{d,p}$ is the number of

shipments of the demand $D_{I,d,p}$ as defined in Section 3.2.6.

- r = 1,..,R : available routes.
- s = 1,..,O : the company-owned ships. If $s_1 < s_2$ then the total capacity of ship s_1 is greater than or equal to the total capacity of ship s_2.
- s = O + 1,..,O + CT : the ships available for chartering. Similar to the company-owned ships, if ($s_1 < s_2$) then the total capacity of ship s_1 is greater than or equal the total capacity of ship s_2.
- total-cost : the overall cost. This cost includes cost of legs, penalties incurred on early or late shipments, and the chartering cost of vessels.
- $\text{chart-cost}_{(s)}$ = { 0 if s is a company-owned ship
  { chartering cost for ship s otherwise.
- $A_{(s)}$[ ] : denote an array whose dimension is given by H. This array reflects the availability of ship s during the time horizon, where $A_s[h] = 0$ indicates that ship s is available on day h of the time horizon, and $A_s[h] = 1$ indicates that ship s is not available on day h of the time horizon.
- extra-del-days[ ] and extra-comp-caps [ ] : these arrays will be utilized to store information about compartments that have not been assigned any products on ships that have been assigned to a given destination. In particular, each entry of the array extra-del-days[ ] contains a delivery day of some given ship, while the corresponding entry in the array extra-comp-caps [ ] contains the capacity of an unassigned compartment on this ship.
- extra-comp : number of unassigned compartments on ships assigned to a given destination. This number is initially given by 0, indicating the unavailability of any compartment.
- $\text{num-comp}_{(s)}$: number of compartments on ship s.
- $\text{comp-caps}_{(s)}$ [ ] : an array whose dimension is given by $\text{num-comp}_{(s)}$. This array consists of the capacities of the compartments of ship s.

- left-over$_{(i,d,p)}$: minimum amount of product p that is still needed to satisfy the ith subdemand at destination d, which is initially given by left-over$_{(i,d,p)}$ = $f_{i,d,p}$. Note that whenever left-over$_{(i,d,p)}$ falls below zero, then this indicates that this subdemand has been satisfied.
- $T_{t,r,d}$: processing time associated with leg $L_{h,t,s,r,d}$ as defined in Section 3.2.6, where s is a ship of type t.
- $T_{t,r,d} = T_{1,t,r,d} + T_{2,t,r,d}$, where $T_{1,t,r,d}$ is the time required to load a ship of type t in Kuwait plus the travel time to destination d and $T_{2,t,r,d}$ is the time required to unload in destination d, plus the travel time from destination d to Kuwait.
- $C_{s,r,d}$: operational cost for ship s when this ship travels to destination d following route r.
- penalty (i,d,p,c) : penalty imposed on the compartment c based on the delivery day of this compartment. Recall that this penalty corresponds to the Type I penalty as defined in Section 3.2.6.
- del-day : denotes a feasible delivery day. Hence, for some given shipment i, product p, and destination d, del-day lies within the time interval [$a_{1(i,d,p)}$, $a_{2(i,d,p)}$].
- dep-day : denotes a departure day, where for some given delivery day, route r, and ship of type t, dep-day is given as dep-day = del-day - $T_{1,t,r,d}$ + 1.
- dem-status : a binary variable reflecting whether all subdemands have been satisfied.

In particular, dem-status = { 1 if all subdemands have been satisfied
{ 0 otherwise.

Ad-hoc Routing and Scheduling Procedure (AHA)

In general, this procedure proceeds as follows. For a given subdemand (shipment) of product p at destination d, we have (1) the feasible delivery dates and (2) minimum and maximum allowable quantities of this product to be shipped to this destination within the specified feasible delivery dates given in (1). We now examine the feasibility of using the company-owned vessels in order to satisfy this subdemand. This process involves exploring various feasible departure and delivery dates based on the various available routes. If this subdemand is not yet satisfied after examining all the company-owned vessels, we resort to chartered vessels to satisfy this demand. If at any stage of the ad-hoc procedure, we have left-over$_{(i,d,p)} > 0$, then if left-over$_{(i,d,p)}$ plus any compartment capacity exceed the quantity $F_{i,d,p}$, then we need to consider only a portion of this compartment.

The proposed ad-hoc routing and scheduling procedure is given as follows.

Step 1: (Initialization)

dem-status = 0.

total-cost = 0.

For h = 1,..,H, $A_{(s)}$[h] = 0 or 1 based on the availability of ship s on day h.

For s = 1,..,O + CT, initialize the array comp-caps$_{(s)}$[ ] based on the capacities of compartments on ship s.

d = 1, p = 1, i = 0, option = next-part.

Step 2: (Update Destinations, Products, Partitions, or Ships)

```
if (option = next-dest)
begin
    d = d +1
    if (d > D_I)
    begin
        status = 1
        goto Step 8 (Termination Step)
    end
    extra-comp = 0
    p = 1, i = 0, option = next-part
    Repeat Step 2
end

else if (option = next-prod)
begin
    p = p + 1
    if (p > P_d )
    begin
        option = next-dest
        Repeat Step 2
    end
    i = 0
    option = next-part
    Repeat Step 2
end
```

```
else if (option = next-part)
begin
    i = i +1
    if (i > N_{d,p})
    begin
        option = next-prod
        Repeat Step 2
    end
    s = 1
    total-cost = total-cost + chart-cost(s)
    chart-cost(s) = 0
    left-over(i,d,p) = f_{i,d,p}
    del-day = a1(i,d,p) - 1
end

else if (option = next-ship)
begin
    s = s + 1;
    if (s > O + CT)
    begin
        goto Step 8 (Termination Step)
    end
    total-cost = total-cost + chart-cost(s)
    chart-cost(s) = 0
end
```

Step 3: (Check Availability and Delivery Days of Unassigned Compartments on Ships that Have Been Already Assigned to Destination d)

for j = 1 to extra-comp

begin

if (extra-del-days[j] $\in$ [$a_{1(i,d,p)}$, $a_{2(i,d,p)}$]

begin

left-over$_{(i,d,p)}$ = left-over$_{(i,d,p)}$ - extra-comp-caps [j]

total-cost = total-cost + penalty (i,d,p, extra-comp-caps [j])

extra-del-day [j] = -1

extra-comp-caps [j] = 0

if (left-over $\leq$ 0)

begin

option = next-part

goto Step 2

end

end

end

Step 4: (Examine Next Feasible Delivery Day)

r = 1

del-day = del-day + 1

if (del-day > $a_{2(i,d,p)}$)

begin

option = next-ship;

goto Step 2

end

<u>Step 5: (Examine Departure Days)</u>

dep-day = del-day - $T_{1,t, r,d}$ + 1

if (dep-day $\leq$ 0)

begin

set r = r + 1

if (r > R) **goto Step 4**

else **Repeat Step 5**

end

<u>Step 6: (Examine Availability of Vessels)</u>

$$\text{ship-status} = \sum_{h\,=\,\text{dep-day}}^{\text{del-day}} A_{(s)}[h].$$

if (ship-status > 0) **goto Step 4**

For h = dep-day to del-day, $A_{(s)}[h] = 1$

total-cost = total-cost + $C_{s,r,d}$

c = 0

Step 7: (Assign Products to Compartments)

```
 c = c + 1
if (c > num-comp(s))     goto Step 4
left-over(i,d,p) =  left-over(i,d,p) - comp-caps(s)[c ]
total-cost = total-cost  +  penalty (i,d,p,comp-caps [ c ])
if (left-over ≤ 0)
begin
   for cc = (c+1)  to  num-comp(s)
   begin
      extra-comp = extra-comp + 1
      extra-del-days[extra-comp]  =  del-day
      extra-comp-caps [extra-comp] = comp-caps_s[cc ]
    end
   option = next-part
   goto Step 2
end
Repeat Step 7
```

Step 8: (Termination)

if (status = 1) **All subdemands (shipments) have been satisfied**

else **Demands have not been satisfied, need more vessels.**

Tables 5.10 presents some computational statistics for the ad-hoc procedure. Note that ν_{AH} gives the total cost obtained by the ad-hoc procedure, and ν_{RH} gives the minimum of ν_{RH1} and ν_{RH2}. Accordingly, for a given test problem, RHA denotes the rolling horizon algorithm that gives the minimum overall cost. The last column of the Table 5.10 indicates if the ad-hoc procedure requires more ships to satisfy the overall demand than those available for the problem. Note that if more ships are needed to satisfy the overall demand, then we may consider more chartered vessels. If no chartering vessels are available and the overall demand is not satisfied, then we terminate the algorithm, deducing that the overall demand cannot be satisfied.

Table 5.10. Computational Statistics for the Procedure AHA

Test Prob	ν_{RH} Total Cost	ν_{AH} Total Cost	**% of Improvement for RHA over AHA**	**More Ships**
I_1	10,800	10,800	0	No
I_2	21,900	22,100	0.90	No
I_3	45,000	64,400	30.12	Yes
I_4	68,100	85,400	20.26	No
I_5	45,500	47,400	4.01	No
I_6	90,500	92,900	2.63	No
I_7	102,400	102,600	0.20	No
I_8	102,600	102,600	0	No
I_9	122,800	125,600	2.23	No
I_{10}	202,700	317,700	36.20	No
I_{11}	2,204,000	5,652,000	61.00	No
I_{12}	2,438,000	4,129,000	40.95	No
I_{13}	1,493,000	6,635,000	77.50	Yes
I_{14}	4,587,000	6,367,000	27.96	No
I_{15}	3,809,000	4,588,000	16.98	No
I_{16}	3,644,000	4,588,000	20.60	No
I_{17}	3,897,000	4,932,000	21.00	No
I_{18}	5,164,000	6,092,000	15.23	No
I_{19}	6,286,000	8,521,000	26.23	No
I_{20}	9,046,000	13,491,000	32.95	No
I_{21}	10,339,000	17,156,000	39.74	No
I_{22}	13,527,614	21,116,000	35.94	No
I_{23}	14,564,987	17,760,000	17.99	No
I_{24}	12,255,000	13,769,000	10.99	No

Observe that for each of the test problems of Section 5.2, the overall cost obtained via the algorithm RHA is at least as good, and often substantially better than, the overall cost obtained via algorithm AHA. In fact, for some cases such as for test problem I_{13}, the gap between the overall cost obtained via RHA and the overall cost obtained via AHA is quite large, given by 40.06 percent ($4,812,000). Accordingly, the utilization of the proposed methodology in the routing and scheduling of ships from Kuwait to the various destinations in the world has the potential for enormous savings in the overall fleet operational cost.

Chapter VI
Summary, Conclusions, Recommendations, and Future Research

Summary, Conclusions, and Recommendations

Water transportation is one of the major transportation modes in the world. Data compiled by Ronen (1993) highlights the reliance of the world economy on seaborne trade (See Section 1.1) and hence emphasizes the need for efficient and reliable maritime transportation systems. Routing and scheduling of ships is the most elaborate and significant level of planning of fleet management in any maritime transportation system. In this process, one should properly assign shipments to vessels and decide on the route a vessel should take. Furthermore, one needs to efficiently determine ship size, ship speed, shipment size, fleet size, number of time-chartered vessels, number of spot-chartered vessels, whether it is lucrative to perform a spot-charter for another operator, and so on.

Water transportation is of special importance to many oil exporting and importing countries in the world, where oil and oil related products are mainly moved by ships. For the United States, for example, which is the world's largest energy consumer, seaborne petroleum imports are a necessary lifeline to the oil deposits of the Middle East, East Asia, and Latin America. Efficient routing and scheduling of ships has the potential of enormous savings in the total fleet operation costs. This is true, especially when taking into account the facts that a typical ship in a merchant fleet usually costs millions of US dollars and the daily operating costs of a ship amounts to tens of thousands of US dollars.

The literature surveyed on seaborne transportation systems indicates that there is a scarcity of research on ship routing and scheduling problems. The complexity of a typical ship routing and scheduling problem has contributed to this scarcity of research. The complexity of these problems may be attributed to the following factors: (1) ship routing and scheduling problems involve a large diversity in structural and operational conditions, (2) since ship routing and scheduling problems involve the selection of routes, schedules and the number of ships needed to fulfill overall stated demand, formulations of these problems involve “mixed’’ integer programming models which are notorious for their difficulty, and (3) ship routing and scheduling problems involve a high degree of uncertainty, as for example, due to severe weather conditions and mechanical problems.

The principal thrust of this research effort is focused at the Kuwait Petroleum Corporation (KPC) Problem. This problem is of great economic significance to the state of Kuwait, whose economy has been traditionally dominated to a large extent by the oil sector, and hence, any enhancement in the existing ad-hoc scheduling procedure has the potential for significant savings. A mixed integer programming model (KPCP) for the KPC problem was formulated in this dissertation. This model takes into account the different vessel sizes, the various products, the various sizes of compartments, the two types of demand time windows, etc. In the process of formulating the problem KPC, we attempted to simulate the actual operation as closely as possible in order to produce a realistic model that can be utilized for scheduling and negotiation purposes in actual operation.

The resulting formulation of the KPC problem confirms the forgoing observations regarding the complexity of ship routing and scheduling problems. In particular, this formulation involves an overwhelming number of integer variables and constraints for a typical demand contract scenario, as was illustrated via an example in Chapter IV. In fact, even attempting to solve the linear relaxation of problem KPCP for a moderate sized test problem (such as test problem I_{13} of Chapter V) will most likely lead to out-of-memory problems. Consequently, attempting to solve the problem KPCP without any aggregation or partitioning scheme is theoretically complex and computationally intractable.

Motivated by the complexity of problem KPCP, an aggregate model (KPCPxy) that retains the essential operational features of problem KPC was formulated. This model is computationally far more tractable than the initial model, and consequently it could be utilized to construct a good quality feasible solution for the KPC problem. Unlike problem KPCP, in which a solution for even its linear relaxation for a moderately sized test problem may not be available due to out-of-memory problems, solutions for the linear relaxation as well as the linear relaxation while enforcing the integrality of the chartering variables of problem KPCPxy (i.e., problems $\overline{K}$PCPxy and $\overline{K}$PCPxy$_z$), were readily obtained for all the test problems

Computational results presented in Chapter V indicate that optimal solutions (within some given optimality gap) for problem KPCPxy can be obtained for moderate sized and some large sized test problems. For the large sized test problems I_{23} and $I_{24,}$ no meaningful MIP solution was obtained due to out-of-memory indications. To alleviate this difficulty, we adopted two rolling horizon solution algorithms (RHA_1 and RHA_2) that are based on the sequential fixing of integer variables until a good quality feasible solution of problem KPCPxy is obtained. These algorithms enabled us to enhance the quality of the solutions obtained by directly solving problem KPCPxy and to handle larger test problems than those that could be handled by directly solving problem KPCPxy. For example, we were able to obtain solutions for problem KPCPxy for the test problems I_{23},..,I_{24} using the algorithm RHA_2.

Recall that the rolling horizon algorithms RHA_1 and RHA_2 were run on a Sun Workstation with 128 megabyte of physical memory. The performance of these algorithm may be enhanced by increasing the physical memory and consequently, good solutions of problem KPCPxy for larger instances can become feasible. The potential savings and the usefulness of the model KPCPxy in negotiation and planning purposes strongly justifies the acquisition of more computing power (in particular more physical memory) to tackle practical sized test problems.

This proposed methodology can serve as a useful tool for gaining an edge in the negotiation process. By running the model in a sensitivity analysis fashion for various possible delivery and penalty options, KPC can pre-assess the effect of a given contract on its overall operations and net cost. Furthermore, the availability of sufficient computing power can facilitate the generation of new schedules frequently, conveniently, and at a very short notice, as need arises.

Future Research

Recall that Type II demand time window (demand based on minimum and maximum storage levels) was considered only in the theoretical development of the exact and aggregate formulations. The solution algorithms and computational results of Chapter V were presented for Type I demand time windows. Even though the Type II demand time window is not widely used in the scheduling process, it could be utilized in the negotiation process to determine cost effective delivery dates and penalty intervals for Type I demands. Thus, a potential future research work is to investigate solution algorithms for problems KPCP and KPCPxy in the context of Type II demand time windows and for both Type I and II demand time windows.

Other relevant research ideas are the investigation of port scheduling problems and refinery scheduling problems. Port scheduling problems and refinery scheduling problems are closely related to ship scheduling problems in the sense that loading and unloading of ships are determined to some extent by the availability of berths and refined products at the port.

Bibliography

Agin, N. I., and Cullen, D. E. (1977), "An Algorithm for Transportation routing and vehicle loading", Logistics (A special issue of Management Science, October 1977).

Ahuja, R. K., Magnanti, T. L., and Orlin, J. B. (1993), "Network Flows", Prentice-Hall, Inc.

Almogy, Y., and Levin, O. (1970), "Parametric Analysis of a Multistage Stochastic Shipping Problem", IFORS, OR 69: Proc. Fifth International Conference on OR (Tavistock, London) 359-370.

Appelgren, L. H. (1969), "A Column Generation Algorithm for a Ship Scheduling Problem", Transportation Science, 3 : 53-68.

Appelgren, L. H. (1971), "Integer Programming Methods for a Vessel Scheduling Problem", Transportation Science, 5 : 64-78.

Assad, A. A. (1980), "Models for Rail Transportation", Transportation Research, A, 14A : 205-220.

Assad, A. A. (1988), "Modeling and Implementation Issues in Vehicle Routing", College of Business and Management, University of Maryland, College Park, MD 20742.

Background Notes (1995), Prepared by the United States Department of State, Bureau of Public Affairs, 3, no. 13.

Baker, T. E. (1981), "Interactive Vessel Scheduling at Exxon", Presented at CORS/TIMS/ORSA Joint National Meeting, Toronto.

Balakrishnan, N. (1993), "Simple Heuristics for the Vehicle Routing Problem with Soft Time Windows", Journal of the Operation Research Society, 44, no. 3 : 279-287.

Baumol, W. J., and Wolf, P. (1958), "A Warehouse-Location Problem", Operations Research, 6 : 252-263.

Bausch, D. O., Brown, G. G., and Ronen, D. (1991), "Elastic Set partitioning - A Powerful Tool for Scheduling Transportation of Oil and Gas", in: M. Breton and G. Zaccour (eds.), Advances in Operations Research in the Oil and Gas Industry, Editions Technip, Paris, 151-162.

Bazaraa, M., Jarvis, J., and Sherali, H. (1990), "Linear Programming and Network Flows", Second Edition, John Wiley & Sons.

Beaujon, G. J., and Turnquist, M. A. (1991), "A Model for Fleet Sizing and Vehicle Allocation", Transportation science, 25, no. 1 : 19-45.

Bellmore, M., Bennington, G., and Lubore, S. (1968), "A Maximum Utility Solution to a Vehicle Constrained Tanker Scheduling Problem", Naval Research Logistics Quarterly, 15 : 403-411.

Bellmore, M., Bennington, G., and Lubore, S. (1971), "A Multi-Vehicle Tanker Scheduling Problem", Transportation Science, 5 : 36-47.

Benford, H. (1981), "A Simple Approach to Fleet Deployment", Maritime Policy and Management, 8, no. 4 : 223-228.

Bodin, L., Golden, B., Assad, A., and Ball, M. (1983), "The State of the Art in the Routing and Scheduling of Vehicles and Crews", Computers and Operations Research, 10, no. 2 : 63-211.

Boffey, T. B., Edmond, E. D., Hinxman, A. I., and Pursglove, C. J. (1979), "Two Approaches to Scheduling Container Ships with an Application to the North Atlantic Route", Journal of the Operational Research Society, 30, no. 5 : 413-425.

Boykin, R. F., and Levary, R. R. (1985), "An Interactive Decision support system for Analyzing Ship Voyage Alternatives", Interfaces, 15 : 81-84.

Bradley, S. P., Hax, A. C., and Magnanti, T. L. (1977), "Applied Mathematical Programming", (Addison-Wesley, Reading, Mass), CH. 7, 291-301.

Branch, A. E. (1981), "Elements of shipping ", Fifth Edition Chapman and Hall.

Briskin, L. E. (1966), "Selecting Delivery Dates in the Tanker Scheduling Problem", Management Science, 12 B : 224-233.

Brown, G. G., Graves, G. W., and Ronen, D. (1987), "Scheduling Ocean Transportation of Crude Oil", Management Science, 33, no. 3 : 335-346.

Brown, G. G., Goodman, C. E., and Wood, R. K. (1990), "Annual Scheduling of the Atlantic Fleet Naval Combatants", Operations Research, 38, no. 2 : 249-258.

Chen, D., and Kallsen, H. A. (1988), "School Bus Routing and Scheduling: an Expert System Approach", Computers and Industrial Engineering, 15 : 179-183.

Cheshire, I. M. (1972), "Most Profitable Scheduling of Bulk Cargo Fleets", Norwegian Shipping News, 11C : 40-41.

Clark, G., and Wright, J. W. (1964), "Scheduling of Vehicles from a Central Depot to a Number of Delivery Points", Operations Research, 8 : 568-581.

Cline, A. K., King, D. H., and Meyering, J. M. (1992), "Routing and Scheduling Coast Guard Buoy Tenders", Interfaces, 22, no. 3 : 56-72.

Conley, J. H., Farnsworth, R. S., Koenigsberg, E., and Wiersema, U. (1968), "A Linear Programming Approach to the Total Movement of a Homogeneous Product", Transportation Science, 2 : 289-302.

Dantzig, G. B., and Fulkerson, D. R. (1954), "Minimizing the Number of Tankers to Meet a Fixed Schedule", Naval Research Logistics Quarterly, 1 : 217-222.

Datz, I. M., Fixman, C. M., Friedberg, A. W., and Lewinson, V. A. (1964), "A Description of the Maritime Administration Mathematical Simulation of Ship Operations", Trans. SNAME, 493-523.

Datz, I. M. (1968), "Planning Tools for Ocean Transportation Ship Scheduling", Norwegian Shipping News, 1064-1069.

De Kerchove, R. (1961), "International Maritime Dictionary", Second Edition Published by Van Nostrand Reinhold Company.

Deodovic, D., "Airline Operations Research", Gordon & Breach Science Publishers, 1988.

Desrosiers, J., Laporte, G., and Soumis, F. (1985), "Vehicle Routing with Full Loads", Computers and OR, 15 : 219-226.

Devanney, J. W. III., Livanos, V. M., and Stewart, R. J. (1972), "Conference Ratemaking and the West Coast of South America", MIT Commodity Transportation and Economic Development Laboratory, Report no. 72-1.

Devanney, J. W. III, Livanos, V. M., and Stewart, R. J. (1975), "Conference Ratemaking and the West Coast of South America", Journal of Transport Economics and Policy.

Everett, J. L., Hax, A. C., Lewinson, V. A., and Nutts, D. (1972), "Optimization of a Fleet of Large Tankers and Bulkers: A Linear Programming Approach", Marine Technology, October : 430-438.

Fisher, M. L., and Rosenwein, M. B. (1989), "An Interactive Optimization System for Bulk-Cargo Ship Scheduling", Naval Research Logistics, 36 : 27-42.

Flood, M. M. (1954), "Application of Transportation Theory to Scheduling a Military Tanker Fleet", Operations Research, 2 : 150-162.

Freland, J. A. and Fortin, L. (1989), "Vehicle Scheduling with Sliding Time Windows", European Journal of Operational Research, 38 : 213-226.

Gallagher, F. D., and Meyrick, S. J. (1984), "ASEAN-Australia Liner Shipping: a Cost-Based Simulation Analysis", ASEAN-Australia Joint Research Project (Kuala Lumpur and Canberra), ASEA-Australia Economic Papers No. 12.

Gu, Z., Johnson, E. L., Nemhauser, G. L., and Wang, Y. (1994), "Some Properties of the Fleet Assignment Problems", Operations Research Letters, 15 : 59-70.

Hayman, C. (1972), "Fleet Scheduling - Best by Computer", Seatrade, (April), 65-66.

Hersh, M., and Ladany, S. P. (1989), "Optimal Scheduling for Ocean Cruises", INFOR (Canadian Journal of Operational Research), 27, no. 1 : 48-57.

Jaikumar, R., and Solomon, M. M. (1987), "The Tug Fleet Size Problem for Barge Line Operations: A Polynomial Algorithm", Transportation Science, 21, no. 4 : 264-272.

Jarmaillo, D. I., and Perakis, A. N.(1991), "Fleet Deployment Optimization for Liner Shipping Part 2: Implementation and Results", Maritime Policy and Management, 18, no. 4 : 235-262.

Johannson, K. (1969), "A Heuristic Approach to the Multi-Discharge Routing Problem", Unpublished Ph.D. dissertation, Illinois Institute of Technology.

Koenigsberg, E., and Lam, R. C. (1976), "Cyclic Queue Models of Fleet Operations", Operations Research, 24, no. 3 : 516-529.

Koenigsberg, E., and Meyers, D. A. (1980), "An Interacting Cyclic Queue Model of Fleet Operations", The Logistics and Transportation Review, 16 : 59-71.

Kydland, F. (1969), "Simulation of Liner Operations", Institute for Shipping Research, Bergen.

Laderman, J., Gleiberman, L., and Egan, J. F. (1966), "Vessel Allocation by Linear Programming", Naval Research Logistics Quarterly, 12 : 315-320.

Lane, D. E., Heaver, T. D., and Uyeno, D. (1987), "Planning and Scheduling for Efficiency in Liner Shipping", Maritime Policy and Management., 14, no. 2, : 109-125.

Larson, R. C., Minkoff, A., and Gregory, P. (1988), "Fleet Sizing and Dispatching for the Marine Division of the New York City Department of Sanitation", in: B.L. Golden and Assad (eds.), Vehicle Routing: Methods and Studies, North-Holland, Amsterdam, 395-423.

Lawrence, S. A. (1972), "International Sea Transport: The Years Ahead" (Lexington Books, Lexington, MA).

Le Gendre, Louis (1992), "Liner Shipping", World Wide Shipping: WWW, 54 : p 24.

Levy, V. D., Lvov, S. P., and Lovetsky, S. E. (1977), "Man-Machine Systems for Merchant Fleet Operation Scheduling", Proc. 7th International Symposium on Transportation and Traffic Theory, Kyoto, (Institute for System Science Research) 823-836.

Lo, H. K., and McCord, M. R. (1991), "Value of Ocean Current Information for Strategic Routing", European Journal of Operational Research, 55 : 124-135.

Mathis, S. J., Jr. (1972), "Synthesis of a Crude Oil Supply System: A nonconvex Network Problem", Unpublished Ph.D. dissertation, Case Western Reserve University.

McCord, M. R. (1993), "New Ocean Views may Cut Shipping Costs", New Ocean News, January 30 : p. 79.

Mckay, M. D., and Hartley, H. O. (1974), "Computerized Scheduling of Seagoing Tankers", Naval Research Logistics Quarterly, 21: 255-264.

McKenzie, J. S. (1971), "The Routing of Ships", J. Transportation Econom. Policy (May), 201-215.

Millar, H. H., and Gunn, E. A. (1991), "Dispatching a Fishing Trawler Fleet in the Canadian Atlantic Groundfish Industry", European Journal of Operational Research, 55 : 148-164.

Miller, D. M. (1987), "An Interactive Computer-Aided Ship Scheduling System", European Journal of Operational Research, 32 : 363-379.

Morlok E. K., Pierskalla W. P., and Vandersypen H. L. (1973), "Schedule Planning and Timetable Construction for Suburban Railways", Proc. Int. Conf. on Trans. Res., PP. 757-767. Transportation Research Forum, Chicago.

Murty, K. G. (1983), "Linear and Combinatorial Programming", John Wiley and Sons, New York.

Naslund, B. (1970), "Combined Sea and Land Transportation", Operational Research Quarterly, 21, no. 1 : 47-59.

Nemhauser, G. L., and Wolsey, L. A. (1988), "Integer and Combinatorial Optimization", Wiley, New York.

Nemhauser, G. L., and Yu, P. L. (1972), "A Problem in Bulk Service Scheduling", Operations Research, 20 : 813-819.

Neuhof, B. (1974), "The Ship Line Decision Problem on Ports of Call", Hansa, 23 : 2013-2017 (in German).

Norman, R. J. (1973), "An Algorithm for the Scheduling of Vessels through the Panama Canal", Unpublished M.S. thesis, Naval Postgraduate School, Monterey, CA.

Nutly, W. G., and Ratliff, H. D. (1991), "Interactive Optimization Methodology for Fleet Scheduling", Naval Research Logistics, 38 : 669-677.

O'Brien, G. G., and Crane, R. R. (1959), "The Scheduling of a Barge Line", Operations Research, 7 : 561-570.

Olson, C. A., Sorenson, E. E., and Sullivan, W. J. (1969), "Medium-Range Scheduling for a Freighter Fleet", Operations Research, 17 : 565-582.

Padberg, M. W. (1976), "A Note on the Total Unimodularity of Matrices", Discrete Mathematics, 14, 273-278.

Papadakis, N. A., and Perakis, A. N. (1989), "A nonlinear approach to the Multiorigin, Multidestination Fleet Deployment problem", Naval Research Logistics, 36, no. 5 : 515-528.

Parker, R. G., and Rardin, R. L. (1988), "Discrete Optimization", Academic Press, Inc.

Perakis, A. N. (1985), "A Second Look at Fleet Deployment", Maritime Policy and Management, 12, no. 3 : 209-214.

Perakis, A. N., and Papadakis, N. A. (1987,[1]), "Fleet Deployment Optimization Model. Part 1", Maritime Policy and Management, 14, no. 2 : 127-144.

Perakis, A. N., and Papadakis, N. A. (1987,[2]), "Fleet Deployment Optimization Models. Part 2", Maritime Policy and Management, 14, no. 2 : 145-155.

Perakis, A. N., and Jarmaillo, D. I. (1991), "Fleet Deployment Optimization for Liner Shipping Part 1. Background, Problem Formulation and Solution Approaches", Maritime Policy and Management, 18, no. 3 : 183-200.

Pruzan, P. M., and Jackson, J. T. Ross (1967), "The Many Product Cargo Loading problem", Naval Research Logistics Quarterly, 14 : 381-390.

Psaraftis, H. N., Orlin, J. B., Bienstock, D., and Thompson, P. M. (1985), "Analysis and Solution Algorithms of Sealift Routing and Scheduling Problems: Final Report", Sloan School of Management Paper no. 1700-85.

Psaraftis, H. N., Solomon, M. M., Magnanti, T. L., and Kim, T. U. (1990), "Routing and Scheduling on a Shoreline with Release Times", Management Science, 36 : 212-223.

Rana, K (1985) , " Routing and Scheduling Container Ships Using Lagrangian Relaxation and Decomposition", Unpublished Ph.D. dissertation, University of Waterloo, Canada.

Rana, K., and Vickson, R. G. (1988), "A Model and Solution Algorithm for Optimal Routing of a Time-Chartered Containership", Transportation Science, 22, no. 2 : 83-95.

Rana, K., and Vickson, R. G. (1991), "Routing Container Ships Using Lagrangian Relaxation and Decomposition", Transportation Science, 25, no. 3 : 201-214.

Rao, M. R., and Zionts, S. (1968), "Allocation of Transportation Units to Alternative Trips, a Column Generation Scheme with Out-of-Kilter Subproblems", Operations Research, 16 : 52-63.

Ronen, D. (1982), "The Effect of Oil Price on the Optimal Speed of Ships", Journal of the Operational Research Society, 33 : 1035-1040.

Ronen, D. (1983), "Cargo Ships Routing and Scheduling: Survey of Models and Problems", European Journal of Operational Research, 12 : 119-126.

Ronen, D. (1986), "Short-Term Scheduling of Vessels for Shipping Bulk or Semi-Bulk Commodities Originating in a Single Area", Operations Research, 34, no. 1 : 164-173.

Ronen, D. (1993), "Ship Scheduling: The Last Decade", European Journal of Operational Research, 71 : 325-333.

Schardy, D., and Wadsworth, D. (1991), "Naval Combat Logistics Support System", Journal of the Operational Research Society, 42 : 941-948.

Schechter, J. (1976), "A Minimum Cost Vessel Routing Model Applied to the US Trust Territory of the Pacific Islands", Maritime Studies and Management, 3 : 195-204.

Schrijver, A. (1986), "Theory of Linear and Integer Programming", John Wiley & Sons, New York.

Schwartz, N. L. (1968), "Discrete Programs for Moving Known Cargoes from Origins to Destinations on Time at Minimum Bargeline Fleet Cost", Transportation Science, 2 : 134-145.

Sherali, H. D., and Tuncbilek, C. H. (1995), "Static and Dynamic Time-Space Strategic Models and Algorithms for Multi-Level Rail-Car Fleet Management", Management Science, to appear.

Stott, Jr. K. L., and Douglas, B. W. (1981), "A Model-Based Decision Support System for Planning and Scheduling Ocean-Borne Transportation", Interfaces, 11, no. 4 : 1-10.
The World Economic Factbook (1994/1995).

Taha, H. A. (1975), "Integer Programming, Theory, Applications, and Computations", Academic Press, New York.

Turnquist, M. A., and Jordan, W. C. (1986), "Fleet Sizing Under Production Cycles and Uncertain Travel Times", Transportation Science, 20, no. 4 : 227-236.

Warf, B. (1989), "US Imports of Petroleum and Related Products in the 1980s", Maritime Policy and Management, 16, no. 4 : 265-275.

Warfield, T. P. (1992a), "Estimated Fleet and Empty Mileage Saving of Universal Trilevel and Bilevel Multilevel Fleets", Association of American Railroads, Freight Equipment Management Research-Demonstration Program, 50 F. Street, N.W., 7th floor, Washington, D.C..

Warfield, T. P. (1992b), "Estimated Impact on Loaded and Empty Miles of Using Bilevel Only Cars for General Motors and Toyota Automobile Loadings", Association of American Railroads, Freight Equipment Management Research-Demonstration Program, 50 F. Street, N.W., 7th floor, Washington, D.C..

White, W. W. (1972), "Dynamic Transshipment Networks: An Algorithm and its Application to the Distribution of Empty Containers", Networks, 2 : 211-236.

Williams, T. M. (1992), "Heuristic Scheduling of Ship Replenishment at Sea", Journal of the Operational Research Society, 43, no. 1 : 11-18.

Yau, C. (1988), "A Heuristic Method for Scheduling of Trucks from Many Warehouses to Many Delivery points", Computers in Industry, 11 : 175-180.

Zoppoli, R. (1972), "Minimum Time Routing as an N-Stage Process", J. Appl. Metorol., 11 : 249-435.

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