# CHAPTER 3

# STRENGTH AND STIFFNESS PREDICTIONS OF COMPOSITE SLABS BY SIMPLE MECHANICAL MODEL

## 3.1. General

One of the purposes of developing simple mechanical based methods for composite slab strength is to provide tools suitable for design purposes. Methods based on this model have been developed worldwide in the past two decades (Stark 1978, Patrick 1990, Stark and Brekelmans 1990, Heagler et al. 1991, Bode & Sauerborn 1992, Easterling and Young 1992, Patrick and Bridge 1994). Despite the complex nature of interactions inside composite slab systems, the methods have demonstrated good performance in predicting the slab strength. In contrast to the so-called m-k method, these methods do not rely heavily on full-scale test results, which becomes the main advantage of the methods.

In this study, two new methods based on simple mechanical model are developed. The methods are based on partial connection theory. Unified formulation for the studded and non-studded slabs and inclusion of shear bond strength at the steel deck-concrete interface offer advancements to the SDI method (Heagler et al. 1991). In comparison to the method developed by Patrick (1990), the remaining strength of the steel deck beyond the shear bond transfer strength is considered. On the other hand, clamping forces at the supports are neglected due to the fact that at the supports, the slab rests on the tip of the supporting beams.

The first of the two new methods is an iterative procedure, in which the slab strength is calculated based on the location of the critical cross section, i.e., the location of the concrete crack that initiates shear bond failure. With this method, the ultimate strength and response history of the slab can be obtained. A computer program is required to perform the iterations. The method is referred to as the *iterative method*.

The second method is one in which simple expressions are used in the formulation. Thus, it is suitable for hand computation. The method is referred to as the *direct method*.

Along with these two new methods, a modified version of the SDI method is presented. This method is referred to as the SDI-M method. The modifications include a corrected yield stress due to concrete casting and omission of the shoring effect to the steel deck yield stress. Modifications were introduced because the SDI method often yields unconservative results if the casting stresses are not introduced and it may give very unconservative results if the shoring stresses are included using the simple approach.

## 3.2. Review of Methods for Prediction of Composite Slab Strength by Means of Semi-Empirical Formulations and Simple Mechanical Models

Although the use of cold-formed steel decks in the U.S. began as early as the 1920's, the standard design procedures for composite steel deck-concrete slabs were not formulated until much later. A landmark research program that led to a design specification for composite slabs was initiated in 1966 at Iowa State University (ISU) under the sponsorship of the American Iron and Steel Institute (Ekberg and Schuster 1968; Porter and Ekberg 1971, 1972). The results of the research led to design recommendations for composite slabs, which later became the basis for an American Society of Civil Engineers design standard for composite slabs (Standard for 1992). These design recommendations were based on two limit states, namely, the flexural and the shear bond limit states. Determination of the slab strength based on shear bond requires a series of full scale-tests.

The flexure limit state is characterized by the achievement of the flexural capacity, $M_u$ (ASCE nomenclature), of the cross section at the maximum positive bending moment location, although slip between the steel deck and concrete may occur anywhere in the slab including at the end of the slab. The shear bond limit state is characterized by the occurrence of slip such that it limits the capability of a section to reach its flexural capacity. Yielding of the steel deck section, however, may occur prior to the failure.

The shear bond limit state was found to be the governing limit state in most composite slab tests conducted at ISU, as well as in other research programs. The formulation of the design method, which is commonly referred to as the m-k method, was chosen to follow the shear equation from the ACI Building Code (Building Code 1995). The expression was developed by Schuster (1970) and refined by Porter and Ekberg (1971). The equation for the limit state is given by:

$$V_u = bd\left(\frac{m\rho d}{L'} + k\sqrt{f_c'}\right) \tag{3-1}$$

where $V_u$ = ultimate shear capacity obtained from experimental test, b = unit width of the slab, d = slab effective depth, measured from the compression fiber to the centroid of the steel deck, $\rho = A_s/bd$, $L'$ = shear span length, $f_c'$ = concrete compressive strength, $A_s$ = steel deck cross sectional area per unit width, m and k are parameters shown in Fig. 3-1, obtained by regression on the values obtained from full scale tests.

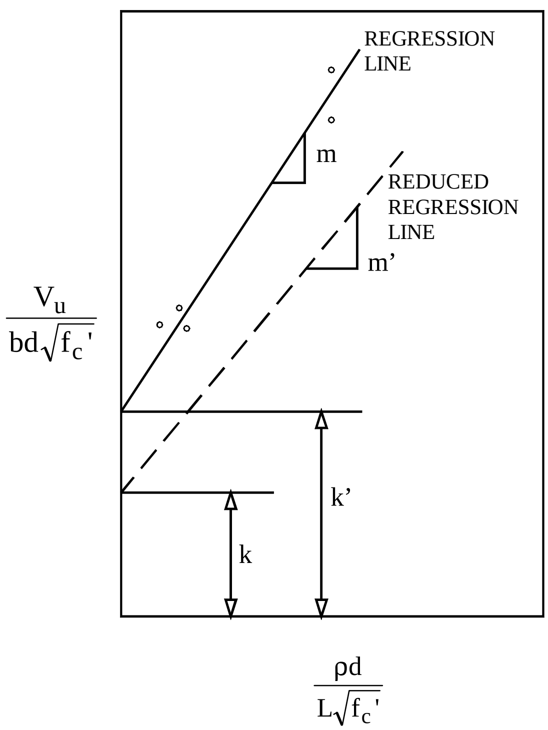


Figure 3-1. m and k shear bond regression line

Because shear bond was found to be the predominant failure mode of composite slabs, the focus of recent research in this area has been to study more closely the behavior of this shear bond action and to improve the performance of this action with or without adding other devices such as end anchorages. Three components were identified in the shear bond action: chemical or adhesion bonding, mechanical interlocking, and surface friction. The afore-mentioned m-k method does not explicitly reflect the action of these components. To substantiate the effects of these actions, tests have been performed and semi-empirical formulations have been developed separately by Schuster and Ling (1980), Luttrell and Prasanan (1984), and Luttrell (1987a, 1987b).

The natures of those design procedures previously described are semi-empirical which rely heavily on full-scale tests. This fact raises some problems as to how to incorporate more parameters without significantly increase the number of full-scale test required and how to cross-examine the design calculations analytically. In 1978, Stark introduced a partial interaction theory similar to that used for composite beam design. The method was developed further by Stark and Brekelmans (1990), in which they view the ultimate bending moment capacity of the slab as built up from two components: (1) the contribution of the normal force of the steel sheet and (2) the contribution of the *reduced* plastic moment $M_p'$ of the deck. The formulation is given by:

$$M_u = N_b . d + M_p' \tag{3-2}$$

$$N_b = k . f_c' . h_b . b \tag{3-3}$$

$$M_p' = 1.25 M_p \left(1 - \frac{N_b}{A_s . f_y}\right) \leq M_p \tag{3-4}$$

where $M_u$ = ultimate bending moment capacity, $M_p$ = steel deck plastic moment capacity, b = slab unit width, $f_y$ = steel deck yield stress, d = repeat definition, k, and $h_b$ are explained in Fig. 3-2. Equation (3-4) is a bi-linear simplification of a nonlinear relation between $M_p'$ and

$N_b$ illustrated in Fig. 3-3.

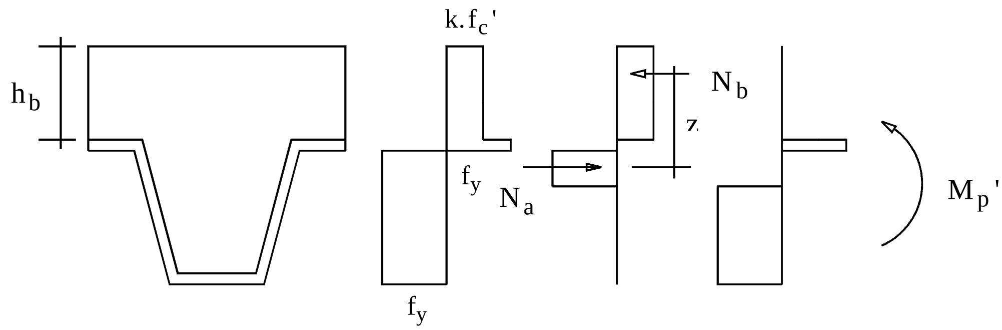


Figure 3-2. Partial interaction theory (Stark and Brekelmans 1990)

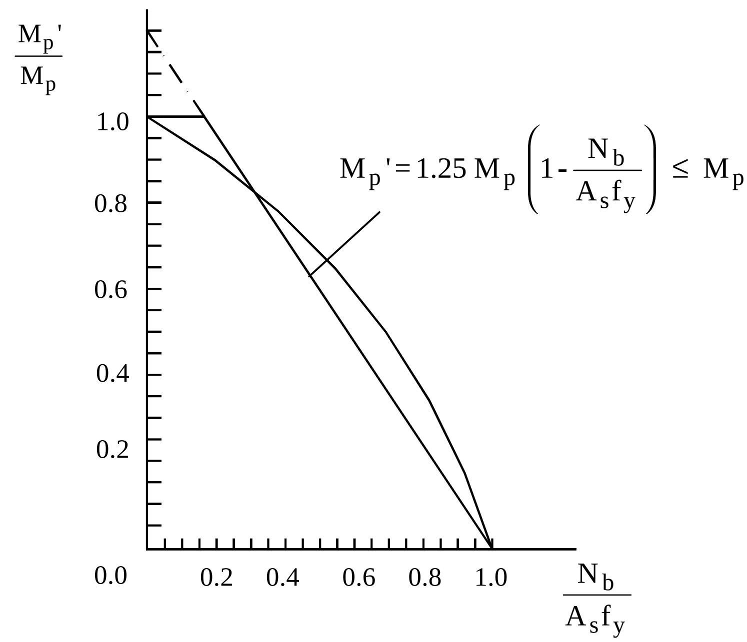


Figure 3-3. Simplified relation between $M_p'$ and $N_b$
(Stark and Brekelmans 1990)

In 1991, the Steel Deck Institute (SDI) launched an alternative formulation to predict the strength of composite slabs for design purposes (Heagler et al. 1991, 1992, 1997, Easterling and Young 1992). These design procedures were based on research conducted at Virginia Polytechnic Institute and State University and West Virginia University sponsored by the SDI.

The advantage of the SDI procedure is that the effect of end anchorages can be taken into account in a simple manner. In the procedure, there is no distinction between ductile and brittle behavior of the slab, however, it recognizes the studded and non-studded slab condition in which generally, the studded shows ductile behavior and the non-studded sometimes has brittle behavior. The nominal moment capacity is calculated based on the expression for a singly reinforced concrete section, given by:

$$M_n = R.A_s.f_y\left(d - \frac{a}{2}\right) \tag{3-5}$$

where

$$a = \frac{A_s f_y}{0.85 f_c^{'} b} \tag{3-6}$$

$$R = \frac{N_r Q_n}{F} \tag{3-7}$$

$$F = f_y\left(A_s - \frac{A_{webs}}{2} - A_{bf}\right) \tag{3-8}$$

with $M_n$ = nominal moment, $A_s$, $f_c'$, $f_y$, b, and d are previously defined, $N_r$ = number of studs per unit width of the slab, $Q_n$ = nominal shear stud strength, $A_{webs}$, $A_{bf}$ = area of the webs and bottom flange of the steel deck, respectively, per unit width of the slab. In the non-studded slabs, the bending capacity of the slabs is predicted by using the moment at first yield, which is given by:

$$M_{et} = \left(T_1 e_1 + T_2 e_2 + T_3 e_3\right) \tag{3-9}$$

where $T_1, T_2, T_3$ are the total forces of the top flange, web and bottom flange of the deck, respectively, and $e_1, e_2, e_3$ are the corresponding moment arms of $T_i$'s to the centroid of the compression side of concrete.

Linear interpolation between the full nominal moment capacity and the first yield moment for slabs that do not have sufficient number of shear studs to provide full anchorage was introduced to the method based on the research by Terry and Easterling (1994). With this interpolation, the studded and non-studded cases can be unified.

Following the development by Stark and Brekelmans (1990), Bode and Sauerborn (1992) developed a method based on the same partial interaction theory that can include the shear bond effect explicitly. To determine the strength of a composite slab, a boundary curve of the slab nominal bending moment resistance vs. the shear bond length for the particular slab for various degree of partial interaction need to be generated (see Fig. 3-4). The expression for the shear bond length is given by:

$$L_s = \frac{N_b}{b.\tau_{\text{shear bond}}} \tag{3-10}$$

where $L_s$ = shear bond length, $N_b$ = normal force developed in the concrete slab (see Fig. 3-4), b = slab unit width, $\tau_{\text{shear bond}}$ = shear bond strength at the interface between the steel deck and concrete. In this case, the shear bond strength is determined from full-scale composite slab tests.

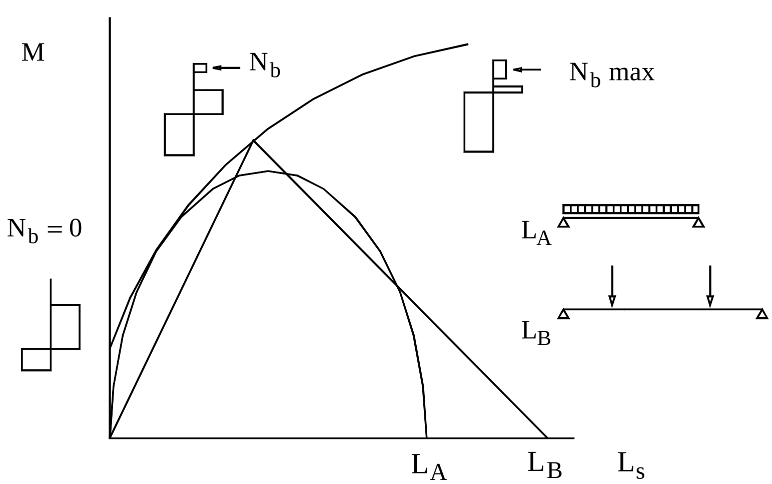

Figure 3-4. Boundary curve based on the partial interaction theory (Bode & Sauerborn 1992)

Patrick and Bridge (1990, 1994) developed a partial shear connection method, which is also based on partial interaction theory. In this method, the effect of the end anchorages and clamping forces over the support as well as the shear bond strength can be taken into account. Similar to the ASCE procedure, the principle of a singly reinforced rectangular concrete section is used to obtain the nominal bending moment, $M_n$. The normal force, T, in the steel deck, which can be viewed as the reinforcing force in a concrete section, can be determined from the free body diagram shown in Fig. 3-5:

$$T = f_s(x + L_c - \gamma D) + \mu\kappa R \leq f_y A_s \tag{3-11}$$

where $f_s$ = shear bond force per unit length, x = distance from the support to the section being investigated, $L_c$ = cantilever length, $\gamma D$ = correction due to diagonal shear cracking, $\mu$ = coefficient of friction between the deck and concrete, R = support reaction, and $\kappa$ = fraction of R that has some contribution in T through a frictional action. With the T value calculated from Eqn. (3-11), the corresponding $M_n$ value can be determined. However, because the shear bond force varies along the slab, then a plot of $M_n$ vs. T (reinforcing force provided by the shear bond, end anchorages, etc.) needs to be generated, as shown in Fig. 3-6, in order to form the boundary curve for the slab load carrying capacity (Fig. 3-7). This concept is very similar to the one introduced by Stark and Brekelmans (1990) (compare Fig. 3-6 to Fig 3-3) and Bode and Sauerborn (1992) (compare Fig. 3-7 to Fig. 3-4). The critical section is then found by matching up the boundary curve to the bending moment diagram due to the applied load, and the first point to intersect with the bending moment capacity diagram is the critical location.

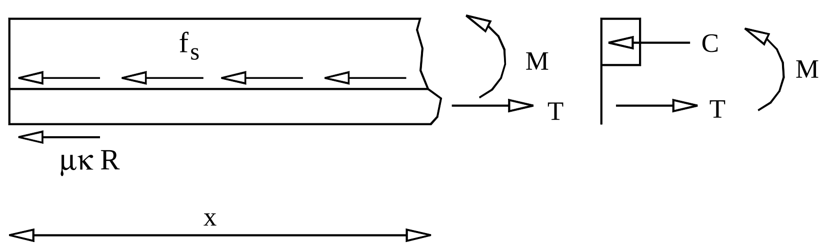


Figure 3-5. Free body diagram of the forces acting in the composite slab section (Patrick 1990, Patrick and Bridge 1994)

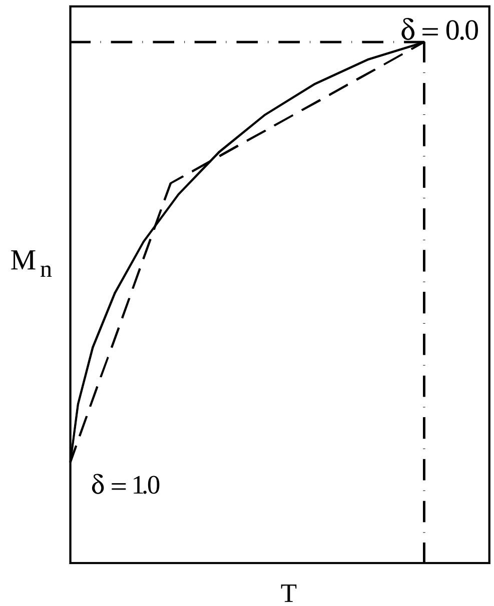


Figure 3-6. Plot of $M_n$ vs. T (Patrick 1990, Patrick and Bridge 1994)

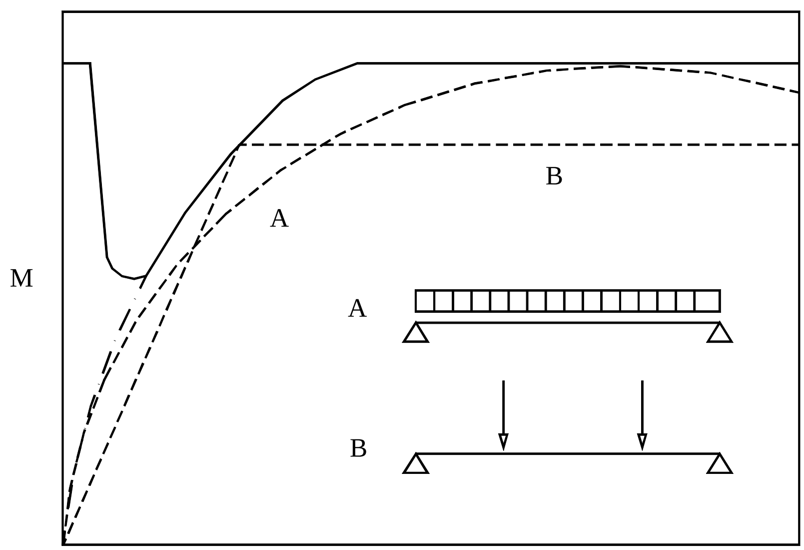


Figure 3-7. Boundary curve for the ultimate bending moment capacity (Patrick 1990, Patrick and Bridge 1994)

The procedure offers a good means that can take into account the shear bond and end anchorage effect in the determination of the bending moment capacity based on the critical cross section.  In the procedure, however, the remaining strength in the steel deck, i.e. the reduced plastic moment of the deck in the method by Stark and Brekelmans (1990), is omitted and on the other hand, as can be noted from Eqn. (3-11), the clamping force at the support due to support reaction is accounted for.  In their research, the shear bond strength used for the procedure was obtained from the slip block test instead of the full-scale tests.

### 3.3.  SDI-M Method

The SDI-M method is a modified version of the SDI design procedure.  All the equations given by Eqns. (3-5) to (3-9) apply.  The modifications are introduced by: (1) replacing $f_y$ (original steel deck yield stress) in Eqns. (3-5) to (3-9) with $f_{yc}$ (corrected steel deck yield stress due to concrete casting), and (2) omission of the construction shoring effect in the $f_{yc}$, thus in this case, the slab is treated as if it were unshored.  Tests on shored composite slabs revealed that unconservative predictions using the SDI method could be resulted when the shoring effect was included in this simple model.

### 3.4.  Iterative Method

The method utilizes a singly reinforced concrete beam section as the basis for the approach.  All effects that help the concrete resists cracking in the positive moment regime are considered as *reinforcement* as indicated in Fig. 3-8.  Such effects come from shear bond action ($f_s$), end anchorages ($F_{st}$), reinforcing bars, etc.

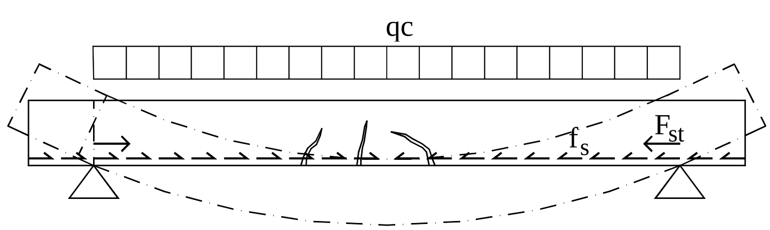


Figure 3-8. Reinforcing effects of some devices

Two phases are considered in the analysis: phase-1, analysis of a composite cross section in which the steel deck acts as a tensile member reinforcing the slab, and phase-2, analysis of the steel deck as a flexural member.  Phase-1 can be regarded as the *composite action* while phase-2 as the *non-composite* action of the system.

In phase-1, analysis is performed exactly in the same manner as one treats a singly reinforced concrete section. Two equilibrium equations are considered: equilibrium of forces and equilibrium of moments on the cross section. Assumptions used in the procedure therefore follow directly from the concrete beam section procedure, with one exception. Because in this procedure one wants to obtain the response of the slab through the entire loading history, the Whitney stress block (equivalent rectangular stress block) for the concrete is replaced by an elasto-plastic model of the stress distribution. This is illustrated in Fig. 3-9 in which, $F_s$ and $F_{st}$ are forces resulting from the effect of shear bond and end anchorages, respectively. Additional effects of welds or pour stop can be added in a way similar to $F_s$ and $F_{st}$.

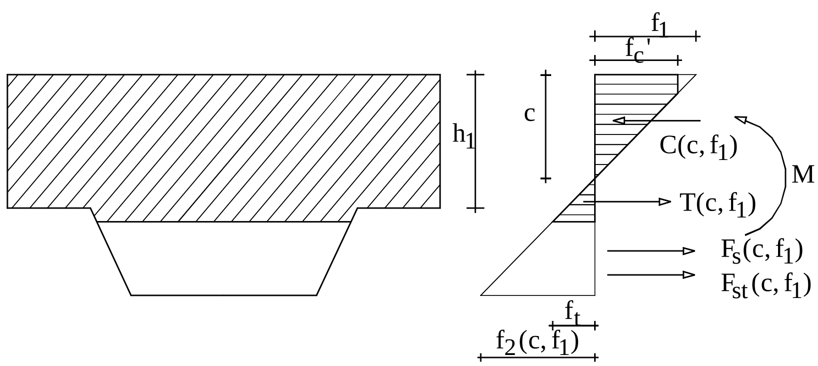


Figure 3-9. Forces acting on the cross section

Two independent variables have to be solved to determine the stress distribution on the cross section. In Fig. 3-9, c and $f_1$ are chosen as the independent variables. They can be solved from the two equilibrium equations on the cross section: equilibrium of forces and equilibrium of moments. The magnitude of $F_s$ and $F_{st}$, however, depends upon the value of slip between the concrete and steel deck which in turn depends on concrete strain at locations where these two forces are acting. The result is a nonlinear relation between $F_s$ or $F_{st}$ and the concrete strain, such that c and $f_1$ are coupled together in a nonlinear system of equations. Therefore, an iterative procedure is needed to solve for c and $f_1$. The iterations are performed for each cross section for a given load level. The greater the number of cross sections considered the more accurate the prediction of the location of the critical section.

The afore-mentioned shear bond force, $F_s$, is computed as follows. Consider the schematic illustration of the shear bond interaction in Fig. 3-10. Figure 3-10a shows a typical relation between shear bond force per-unit length, $f_s$, versus slip at the interface of steel deck-concrete. This relationship is obtained from elemental tests. In general, at a certain load level, the distribution of $f_s$ along the slab is not uniform due to the difference in the amount of slip at different cross sections. This is illustrated by different values of $f_{s,A}$ and $f_{s,B}$ in Fig. 3-10b. The shear bond force, $F_s$, acting on a cross section is the sum of $f_s$ from the end of the slab to the particular cross section (represented by the shaded area in Fig. 3-10b). Figure 3-10c shows the distribution of $F_s$ along the slab. In the case of high strength shear bond, $F_s$ can not be greater than the strength of the steel deck, $f_{yc}A_s$.

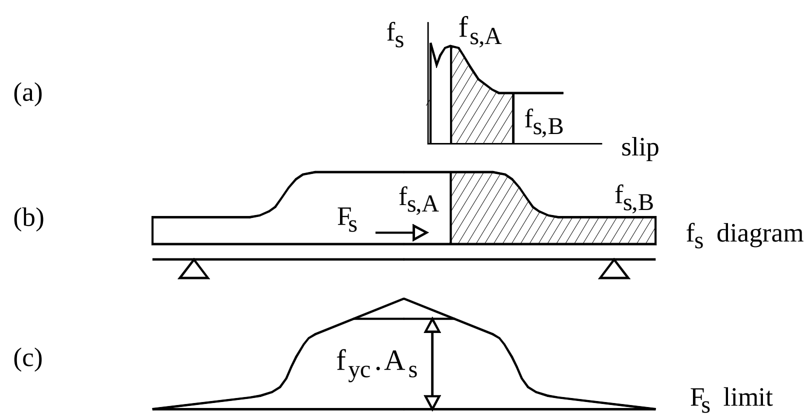


Figure 3-10. Shear bond interaction

Partial interaction between the deck and the concrete is accounted for by limiting the deck contribution to the capacity of the shear bond, such that after a certain phase, the steel deck and concrete no longer have the same amount of strain at the interface. Hence, at any loading point, strength contribution of the deck can not be greater than $F_s$ as shown in Fig. 3-10c, so that, as reinforcement for the concrete, the steel deck strength can be expressed as: