Aristotle and Frege on Method in the Philosophy of Mathematics

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ACADEMIC ABSTRACT

Aristotle discusses the philosophy of mathematics in the final books of the *Metaphysics* and approaches questions and problems in this subject from a metaphysical perspective. This way of undertaking the philosophy of mathematics is informed by his understanding of the nature and ends of science, according to which metaphysics is the broadest theoretical science concerned with being itself. We will motivate the further use of this approach by considering the problems Frege encounters as a result of his method of investigating the philosophy of mathematics, which begins from a more logico-mathematical perspective. These problems, broadly speaking, concern the degree to which Frege must take on metaphysical commitments despite wanting to show that mathematics deductively follows form logic; at the same time, these problems can also be explained and understood from an Aristotelian point of view which also offers a solution through the adoption of a more metaphysical method. Consequently, the philosophy of mathematics should be studied in a manner closer to Aristotle's and Frege's.

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GENERAL AUDIENCE ABSTRACT

Aristotle discusses the philosophy of mathematics in his work on metaphysics and approaches questions and problems in this subject from a metaphysical perspective. This way of undertaking the philosophy of mathematics is informed by his understanding of science, according to which metaphysics is the broadest science concerned with existence itself. We will motivate the further use of this approach by considering the problems Frege encounters as a result of his method of investigating the philosophy of mathematics, which begins from a more mathematical perspective. These problems, broadly speaking, concern the degree to which Frege must take on metaphysical commitments despite wanting to show that mathematics deductively follows form logic; at the same time, these problems can also be explained and understood from an Aristotelian point of view which also offers a solution through the adoption of a more metaphysical method. Consequently, the philosophy of mathematics should be studied in a manner closer to Aristotle's and Frege's.

Table of Contents

DEDICATION

Ad maiorem Dei gloriam inque hominum salutem

"Mathematical knowledge has always been haunted by ambiguity and is the easiest, among human sciences, to be carried to extremes. Science and art, liberal and practical, inductive and deductive, the work of both the intellect and the imagination, mathematics is akin to logic, physics, and metaphysics and has at various times impersonated each of them. Hovering between physical and metaphysical knowledge, it affects the whole order of human sciences when its object is deformed, and among the lessons of modern thought is the power of mathematics to pass for other sciences and its impotence to reconstruct a single one of them."

-Vincent Edward Smith, *St. Thomas on the Object of Geometry*

Introduction

Ultimately, what we will be considering here is the metaphysical perspective Aristotle brings to mathematics and the philosophy of mathematics. While of course treating mathematics not only as its own science but also as being one of the most theoretical of the sciences, his understanding of mathematics, and specifically of the objects of mathematics, is heavily informed by his metaphysics. Consequently, his method in undertaking the philosophy of mathematics can be accurately described as being a metaphysical one. In order to motivate this consideration we must also consider how Frege must presuppose certain metaphysical views to motivate his theory of number despite lacking the necessary background theories. This lack arises from the fact that his starting point is in logic and mathematics, not metaphysics. In order to adequately argue for his views on numbers he must adopt a method for the philosophy of mathematics that begins with metaphysics as exemplified by that used by Aristotle in his treatment of numbers. It may in fact be that Frege, in trying to understand numbers, is forced to maintain certain metaphysical positions, which means that his line of reasoning only makes sense if those positions are presupposed.

In order to see this point more clearly let us look at how Aristotle begins his discussion of the ontology of number in the *Metaphysics*. He writes, “Where the substance of perceptibles is concerned, we have said what it is, dealing with matter in the methodical inquiry into natural things, and later with substance as activity. But since our investigation is into whether or not beyond the perceptible substances there is any that is immovable and eternal, and, if there is, what it is, we must first get a theoretical grasp on what is said by others.” (Aristotle *Met*. 13.3 1076a6-12) Immediately Aristotle frames the investigation that is to follow within his broader metaphysical inquiries. As a result, the claims he makes about numbers are bolstered and

informed by his background metaphysical commitments. Contrast this with what Frege attempts to do. On the one hand, Frege wants to demonstrate how mathematics is reducible to logic, with numbers being logical objects. On the other hand, this is not enough for a complete understanding of *what* numbers are. Frege himself is aware of this lack. He asks in his letter to Hilbert on the 6th of January, 1900, if, even if one were to demonstrate that omniscience, omnipresence, and omnipotence were not contradictory, that does not thereby show that there exists something which possesses all of those qualities. Likewise, we can ask Frege whether any number of claims about numbers within mathematics are able to prove the existence of numbers as *objects* and to subsequently say as much about numbers as he does. It is here that we can see the value in Aristotle's method. Aristotle does not attempt to leap from mathematics to metaphysics. Rather, he has metaphysics as the foundations of his system, with metaphysics being the standpoint from which all the other sciences are to be understood. While Frege only permits himself to use the tools of mathematics in order to demonstrate the existence of numbers, Aristotle has a much larger toolbox that allows him to get a lot more done. Of course, this does not mean that one must agree with Aristotle on every issue concerning numbers or that one must be an Aristotelian; rather, Aristotle demonstrates an alternative methodology and understanding of the sciences that avoids the problems that Frege's has and fits better into a broader, systematic philosophy, and a consideration of their positions on and treatments of numbers provides an occasion for this difference to be made evident. None of this is to say that Frege's entire philosophy is a mistake; in fact, Frege makes many significant and worthwhile contributions, and this is why he of all people is useful for showing the value and usefulness of Aristotle's method.

We will proceed in the following way. First, we will take a look at why Frege is by necessity committed to some metaphysical positions concerning numbers. In doing this we will

also gain a greater understanding of Frege's particular view on numbers (our first part). This will motivate our investigation into Aristotle's treatment of mathematics and will lead us to consider two of Aristotle's arguments against these positions. We will first consider Aristotle's argument from causation. This will help us highlight Frege's lack of a metaphysical background theory, as the only way for someone maintaining Frege's position to respond to it would be by appealing to other metaphysical theories or by saying that one does not need to be an Aristotelian or to believe that everything that exists has causal powers in order to affirm the existence of numbers or to have a view on their existence. In response to this objection, we will then argue that Frege lacks the metaphysical theory required in order to take a stance on this issue. The second argument we will consider is Aristotle's argument from the nature of science originally made in response to one of Plato's arguments. This will help us further understand how Frege is necessarily limited in what he can do with the tool he provides for himself (all of this constitutes our second part). After this we will examine Aristotle's conception of a science in order to have an outside perspective on Frege's limits and a more positive way for us to consider mathematics (our third part), and this will be followed by a summary and elaboration of what this all means for Frege's position on numbers specifically and for methodology in the philosophy of mathematics generally (our fourth part).

Even if one is not convinced by Aristotle's argument or has metaphysical views which conflict with Aristotelianism, one can still recognize through these arguments that Frege himself makes metaphysical claims without having the proper metaphysical perspective from which he can make them. Consequently, anytime he ventures into metaphysics there is by default a problem. Perhaps more importantly, however, here we will open a door towards an alternative way of approaching the philosophy of mathematics inspired by a more Aristotelian way of

thinking which first makes sense of questions in the philosophy of mathematics within a robust metaphysical framework in an intentional and systematic fashion. While we will not be able to fully articulate everything that there is to say about this method, here we can at least present it with a rough outline and show why it would be beneficial to adopt it.[1]

1) Frege, Plato, and the Nature of Platonism

It perhaps seems unnecessary to even address the similarities between Plato and Frege. In general it is almost treated like common knowledge that Frege is a platonist, with Dummett writing that "[t]he extrusion of thoughts from the mind initiated by Bolzano led to what is often termed 'platonism', as exemplified by Frege's mythology of the 'third realm': for, if thoughts are not contents of the mind, they must be located in a compartment of reality distinct both from the physical world and the inner world of private experience." (Dummett 131) However, if we want to understand how these arguments and positions fit together it will be necessary for us to know what exactly people mean when they say that Frege is a platonist. After all, Aristotle does not criticize Plato for "being a platonist", he criticizes him and his followers for holding specific positions in metaphysics and the philosophy of mathematics. Very easily it could just be the case that this claim is merely an oversimplification meant to make Frege's otherwise difficult-to-articulate metaphysics easy to communicate and understand. If, however, we can

[1] One may have noticed that multiple questions are to be addressed. First, we will be directly criticizing Frege's views on the ontology of number. Next, we will be criticizing the method Frege uses which leads him to these views. Finally, we will consider the nature of scientific knowledge more generally and how Aristotle addresses the relationship between metaphysics and mathematics. Given what we will be arguing, to refrain from investigating these broader subjects would be to contradict our own thesis, though at the same time we are driven to begin with narrower subjects (after all, when one looks at a forest one first sees a tree, and then another, and then another). Of course, what this means is that the trail of ideas we will be following will be less straightforward than if we were to criticize Frege by simply arguing against the truth of one of his premises. However, given the nature of philosophy as it has been understood and practiced over the last two and a half millennia, this is hardly unexpected, though it means that when we engage in our present endeavor we must tread a bit more lightly lest we confuse ourselves too much.

point to specific places where Plato and Frege are in agreement, then this gives credence to the thought that the positions Aristotle is criticizing are comparable to the ones that Frege holds. Alternatively, if it turns out the beliefs of Plato and the platonism of Frege have crucial differences, then the arguments first raised against the former must be adequately adjusted (assuming they can be adjusted in such a way) so as to also be legitimately applicable to the latter. This concern will become especially relevant as we consider the existence of more robust interpretations of Frege's platonism, such as that found in Reck's "Frege on Numbers: Beyond the Platonist Picture." We should additionally clarify that here we will refrain from distinguishing between Plato's various articulations of the theory of Forms, for that which is relevant about Plato's position to our present considerations—the literal existence of unchanging objects that transcend physical objects—persists through these changes.

Now, in the dialogues it is not difficult to find passages affirming the existence of objects beyond the spatial. In the Timaeus it is said that "we must acknowledge that one kind of being is the form which is always the same, uncreated and indestructible, never receiving anything into itself from without, nor itself going out to any other, but invisible and imperceptible by any sense, and of which the contemplation is granted to intelligence only." (Plato *Tim*. 51e-52a) Furthermore, in the *Symposium* it is said about beauty that "[i]t will be neither words, nor knowledge, nor a something that exists in something else, such as a living creature, or the earth, or the heavens, or anything that is—but subsisting of itself and by itself in an eternal oneness." (Plato *Symp*. 211a-b) For Plato there are certain objects that are able to exist independently of physical objects, that are eternal and immutable, and that are not perceptible (this is to be considered independently from the views espoused in the *Meno* and the *Phaedo* concerning recollection). Naturally it must then be asked if numbers are among the objects just described.

Fortunately, we do not need to look far—Plato refers to them in a number of passages, many times referring to them abstractly and not as instantiated in physical objects. For instance, in the *Republic* Socrates says that "the contemplation of the nature of number, by pure thought…it strongly directs the soul upward and compels it to discourse about pure numbers, never acquiescing if anyone proffers to it in the discussion numbers attached to visible and tangible bodies." (Plato *Rep*. 525c-d) Also of significance here is that a certain nobility is also attributed to what is not physical, with Socrates saying in the *Phaedrus* that "[o]f that place beyond the heavens none of our earthly poets has yet sung, and none shall sing worthily…reason alone, the soul's pilot, can behold it." (Plato *Phdr*. 247c) Thus, if something depends on fewer things for its existence and has more things depend on it for their existences, it is higher or worthier, and non-perceptible objects, and in particular numbers, are characterized in this way.

What ultimately motivates these claims is the idea that in order to be able to undertake mathematics in the first place and to reach true mathematical conclusions there must be objects with which mathematical propositions are concerned. As Bostock writes, "Plato's basic argument is very simple: mathematics concerns objects that are perfect or ideal in a way that no perceptible object is; but the statements of mathematics are true; therefore such objects must exist." (Bostock 472) Mathematical truths are clearly true, but in order for them to be this they must be about some true state of the world. This state of the world is not a physical state, and the objects in question are not physical, so there must exist mathematical objects that are not physical. Furthermore, such objects are ideal in the sense previously explained.

It has been thought and assumed (perhaps naively) that Frege is a mathematical platonist in the metaphysical sense. In this line of thinking Frege is imagined to hold metaphysical views quite similar to those of Plato himself. In *Thought* we find the passage "A third realm must be

recognized. Anything belonging to this realm has it in common with ideas that it cannot be perceived by the senses, but has it in common with things that it does not need an owner so as to belong to the contents of his consciousness. Thus for example the thought we have expressed in the Pythagorean theorem is timelessly true, true independently of whether anyone takes it to be true." (Frege 336) These objects neither are perceptible objects nor exist in perceptible objects, and they are not particular to any given mind but are rather able to be accessed by minds. Likewise, Frege also claims in the *Foundations of Arithmetic* that "[w]e are concerned in arithmetic with objects that are not known to us as something foreign from without by the intermediary of the senses, but rather that are directly given to reason, which, as most closely its own, can be completely transparently seen." (Frege §105) If we recall Plato for a moment, this is reminiscent of both the previously seen passage from the *Republic*, in which it is claimed that arithmetic deals with numbers through "pure thought" rather than the physical senses, and the passage from the previously quoted passage from the *Timaeus*. Additionally, this quotation from Frege stresses the proximity numbers have to the intellect; nothing stands in between the mind which grasps the number and the number being grasped. Since there is a more proximate relationship, there is no need to think of numbers as having the same kinds of causal powers as physical objects, which the intellect does not directly access but which rather must first be perceived by the senses that are acted upon. For this reason in the *Grundgesetze der Arithmetik* Frege says, "[I]t is not at all obvious why what persists independently of anyone's making judgements is actual [*wirklich*], that is, must clearly be capable of acting [*wirken*] directly or indirectly on the senses." (Frege 205) Numbers are real, but they do not act on anything. Here the word "*wirklich*" is significant; as Dummett explains in *Frege: Philosophy of Mathematics*, "An object is *wirklich* for him if it is a causal agent." (Dummett 80) Numbers exist, but they are

grasped without there needing to be a causal relation between them and the mind that contemplates them.

However, we must resist the temptation to think that Frege was simply a metaphysical platonist. For starters, Plato and Frege were involved in two very different philosophical projects. Plato was concerned with continuing the Greek philosophical debate on the nature of everything that exists and synthesizing the thoughts of his predecessors such as Heraclitus, Parmenides, and Anaxagoras (among others). Frege, meanwhile, sought to demonstrate that mathematics directly followed from the laws of logic. As a result, it would *prima facie* be odd for them to each be trying to communicate the same ideas when they use similar terms and descriptions as one another. This point aside, one can argue quite convincingly that Frege thought numbers were *logical* objects rather than the kinds of objects which populate a metaphysical third realm. To be a platonist in this sense had precedence by the time of Frege (and here we drift from what Plato himself specifically believed and begin to consider mathematical platonism more generally). Lotze, for instance, contrasts ontological Platonism with a more logical kind, and on this topic Stang writes, "The key role of the theory of Forms, according to Lotze, is to maintain, against the sophists, that the truth about what justice is, is independent of our beliefs, not subject to change, and not sensibly perceptible by us. However, Plato was incorrectly interpreted, including by later members of his own Academy, as asserting that this requires that there be timeless, unchanging, subject-independent 'things' that ground these truths—that is, the Forms." (Stang 147) Reck, meanwhile, provides substantial evidence for thinking that Frege believed numbers to be logical objects, not metaphysical ones. Without dwelling long on his specific arguments, he writes, "Recall once more the three basic aspects of platonism A as just discussed: (i) a certain way of understanding 'objecthood', guided by the example of physical objects; (ii) the explanation of

truth and objectivity as correspondence, based on such "objecthood" and on a related notion of "reference"; (iii) the resulting problem of access, or the possibility of a "fundamental mistake", even if our normal arithmetic is consistent. With respect to all three aspects Frege's position differs from platonism A." (Reck 7)

Let us unpack this. The naive platonist—that is, the one who believes in what Reck calls platonism A—imagines numbers, the forms, and all other objects in the third realm to be like the objects one sees everyday except that they are eternal, unchanging, beyond the physical, etc. One has a conception of physical objects and then from that conception conceives of numbers as objects *similar to* physical objects in some respects but different from them in certain ways. This is what (i) concerns; under the naive view, platonic objects are called objects because they are analogously related to physical objects, but the logical platonist need not affirm this. Concerning (ii), correspondence theories of truth maintain that a proposition is true if it adequately corresponds to the world. Objective facts under this view are those which accurately correspond to what exists in the world. This fits well with the naive, or ontological, platonist view, because if objects in the third realm are comparable to physical objects, then statements about those are true by virtue of accurately referring to those objects. For Frege, however, truth and falsity are denoted by propositions and cannot be further reduced to anything else. Thus, propositions need not denote some external thing that is somewhere else; rather, they simply denote either the true or the false. Finally, (iii) explains why, as we have previously seen, such numbers are directly accessible to the intellect. They are not, as naive platonists often portray it, just in some other realm which we must access analogous to how we access the physical realm through our senses. Instead, numbers follow directly from logic, and since logic is a priori knowledge then mathematical truths are also kinds of a priori knowledge. There is thus (at least according to the

logical platonists) no access problem for the same reason that there is no problem concerning how people have access to logical truths.

Now, calling this alternative kind of platonism "B-Platonism," Stang writes, "The B-Platonist…begins with a conception of objective *judgement*, understood *not* as judgement that corresponds to or matches some metaphysically independent standard (as the A-Platonist does), but as obeying the logical laws internal to judgement itself…to say that numbers exist objectively is just to say that the existence of numbers follows from the laws of logic." (Stang 151) The final part of this passage from Stang is, of course, just logicism, and it is relatively modest when compared to the ontological platonist claim that numbers are a special kind of object to be distinguished from other objects in the world (e.g. physical objects). In general, it seems unlikely that Frege would attempt to reach metaphysical conclusions through purely logical means, especially since he quite clearly advises against doing so in his correspondence with Hilbert; he asks if, should one demonstrate that omnipotence, omniscience, and omnipresence do not contradict one another, one "could infer from this that there was an omnipotent, omnipresent, intelligent being?" immediately following with, "This is not evident to me." (Frege 47) At the same time, if he were to attempt to make blatant metaphysical claims using logical means, it would be difficult for him to do so.

With all this having been said it may seem as though Frege's system is completely devoid of metaphysical commitments. However, as it turns out, a number of metaphysical presuppositions still manage to get in through the backdoor. The reason for this is not that Frege necessarily makes any explicit metaphysical claims; as we have already seen, Frege's position can often be interpreted in an alternative, logical way. Rather, the reason is that what he says and argues only makes complete sense from the perspective of certain metaphysical positions. For

instance, one avenue by which metaphysical presuppositions enter Frege's philosophy is found in the Julius Caesar problem, which we can articulate as, to use Dummett's words, the fact that "[f]rom the criterion of identity between numbers, we cannot determine whether an object not given as a number, such as England or Julius Caesar, is a number at all, and, if so, to what concept it belongs." (Dummett 210-211). Given just the resources of his philosophy which he had articulated up to that point, he is not able to distinguish between everything that is a number and everything that is not a number. Frege himself writes, "[W]e can — to give a crude example — never decide by means of our definitions whether Julius Caesar belongs to a number concept, whether this same well-known conqueror of Gaul is a number or not. We cannot furthermore prove by the aid of our attempted definitions that it must be the case that $a = b$, if the number a belongs to the concept F, and if the number b belongs to the same concept." (Frege §56) Later on in the *Foundations of Arithmetic* Frege speaks of this problem again, saying, "One cannot, for example, decide whether England is the same as the direction of the Earth's axis. Pardon this nonsensical appearing example! Naturally no one is ever going to confuse England with the direction of the Earth's axis; but that is not owing to our example." (Frege §66) The fact that he remarks that naturally people would not confuse the two and that he considers this a problem at all is telling, for it shows that he identifies as obvious the fact that whatever numbers may be, Julius Caesar is certainly not one of them. This presupposes that there is some meaningful difference between numbers and Julius Caesar, and this difference must have some metaphysical import.

Additionally, we should also note that the very idea of numbers being logical objects rather than ontological objects relies on a particular theory of truth. As seen with Reck previously, one way to distinguish Frege's positions from ontological platonism is that the latter

relies on a correspondence theory of truth while the former does not. Frege himself says, “what I have called thoughts stand in the closest connection with truth.” (Frege 342) Frege’s positions on numbers and thoughts generally are tied to his position on the nature of truth, but in order to decide between Frege’s understanding of truth, in which the true and the false cannot be further analyzed, and another one like the correspondence theory of truth, one must provide metaphysical arguments. After all, the question of what theory of truth we should favor does not arise as a question of logic, even if in logic we deal with the truth and falsity of propositions. Consequently, if Frege wants arguments with which he can support his favored theory of truth that are more than just the fact that it fits well with the rest of his system he must adopt theories that are not strictly logical.

If this was not enough, we should also consider what exactly falls within the bounds of logic in the first place. It is not clear what does and does not pertain to logic, and if it turns out that Frege, despite his claims and intentions, uses tools that are outside of pure logic in order to prove the existence of numbers as objects, then it is as we have said, and so he is susceptible to more metaphysical arguments. As it happens, this is a serious concern for Frege. For instance, it is questionable that the existence of extensions and sets is strictly logical; certainly people do not as quickly grasp this idea as they do basic logical truths. Given that they are crucial for Frege’s logicism, this casts doubt on whether Frege really does help himself only to logical tools (this seems to be confirmed by the fact that they are where Frege’s system becomes inconsistent, as Russell’s paradox shows).

There are, of course, other places to which we could point. For instance, he says, “The grasp of a thought presupposes someone who grasps it, who thinks.” (Frege 342) Thus, in discussing thoughts and incorporating them into his system he himself admits to presupposing

the existence of someone with the ability to grasp them. This, however, seems to require some kind of metaphysical commitment, even if that commitment turns out to be a very minimal one. While examples such as this are by themselves not particularly moving, when considered together and in the context of everything we have previously explained they further support the present argument that Frege must hold some metaphysical positions in order to intelligibly argue for his own logicism. The fact that it is possible to plausibly point to passages and sentences by Frege that seem to vaguely discuss metaphysics but which are not explicitly so makes sense in light of the fact that Frege, as we have seen, does not adequately demarcate the proper subjects of logic. The reason for this is that these somewhat metaphysical or adjacently metaphysical claims suggest that Frege is not working with a precise notion of the science of metaphysics or with a precise understanding of its bounds. As we will see, how Frege operates within the sciences is relevant in our latter criticism of his logicism, but it is striking that such considerations already arise.

2) Aristotelian Arguments against the Platonist Thesis

Here we can now consider two of Aristotle's arguments, primarily derived from books M and N of the *Metaphysics*, which can be applied to Frege. Naturally it may be wondered why this text in particular is being cited. As it turns out, there are a number of reasons why Aristotle of all people may be used to further argue against Frege, even if the broader argument concerning the lack of justification for Frege's metaphysical claims does not require us to appeal to any particular positive view of numbers. The first is that Aristotle is responding to Plato's position on the metaphysics of numbers, so his arguments are already tailored for at least some kind of

platonism. The second is that together the arguments we will consider support our overall argument in multiple ways.

We will proceed by first considering Aristotle's views on causality and how they generate an argument against the possibility of the existence of objects outside of any causal chains (which Frege takes numbers to be). Then, we will consider Aristotle's counterargument against the platonist claim that numbers must be objects in the third realm in order for mathematical statements to be able to be true or false. Along the way we will need to take some detours in order to provide important theoretical context and explain some background ideas, but these discursions are permitted for our present goal of adequately presenting these two Aristotelian arguments against the existence of numbers as objects. Besides being two arguments against the first layer of argument concerning the ontological status of numbers, they lead us into more abstract considerations dealing with Frege's methodology and the nature of the sciences.

To begin with the first argument, given that Frege does not think that numbers have causal powers and are instead simply objective, he is susceptible to the argument that his account of numbers cannot be true because everything that exists must have causal powers. Given that numbers exist in some sense, there must be some sense in which they are causes, and so it follows that Frege's account of numbers must be wrong. Broadly this is the structure of the argument, but in order to better understand it we must go into some technical detail. For Aristotle, all things consist of actuality and potency (except for the uncaused cause which is only in actuality, and not in potency), and in order to exist one must be in actuality in some way; nothing that exists is pure potency. What this implies is that everything that exists must have some causal powers, no matter how slight. The reason for this is that causality is simply when one object actualizes the potential in another object, and if something is actual then it can

actualize the right potency in something else. Thus, if something has no causal powers then it cannot be said to exist. If numbers exist in some way then they too must have causal powers, though the way in which they are causes does not need to be the same for everything that does exist. After all, for Aristotle the notion of a cause is broader than what people colloquially mean when they speak of causes (and so for us this means that if numbers are not causes according to Aristotle then a fortiori they are not causes as we tend to conceive of them). However the Forms in general and numbers in particular as characterized by Plato (Aristotle's original targets) are not necessary for explaining causal relations, for particular things are sufficient to explain the causal relations we see in the world; in fact, they seem to complicate matters, as now there are even more objects to consider. Additionally, upon reflection, it is unclear what exactly a number would cause to either exist or to arise in something else; numbers neither move things nor are that towards which things move, and they are neither the materials out of which things are made nor that into which materials are fashioned. For instance, to have two of a certain object does not mean that you possess a single object which is in essence two. Rather, you possess two individual objects, with the "twoness" being contingent upon them. Even if numbers are formal causes (and here let us once again be clear that the word Aristotle uses for cause, αιτια, does not necessarily denote are more narrow understanding of a cause) of *something*, they certainly would not be the formal causes of substances, and even if they *were* the formal causes of substances that does not mean that they themselves are substances too. "Humanness," for instance, is not a substance, even though it is the formal cause of any given human. No one sees or experiences numbers acting on anything else, and to speak as if it were otherwise would be to engage in metaphor. As a result, it is unclear in what way they do exist. As Aristotle says, "But will these magnitudes be Ideas, or what way do they exist? And what do they contribute to beings? In fact,

nothing—just like the objects of mathematics, they contribute nothing." (Aristotle *Met*. 14.3 1090b24-26) While it might be debated that there is one special way in which they do exist or that this is not enough to support the stronger thesis that there is no way in which one can meaningfully and nonmetaphorically say that numbers exist, it is clear that numbers cannot be objects.

At the same time, it is also clear that numbers themselves need to be actualized in such a way that they cannot exist (in any sense) without depending on something else. "But a mathematical magnitude is divisible and quantitative," Aristotle says, "so what is the cause of its being one and holding together?" (Aristotle *Met*. 13.2 1077a23) For Aristotle, things that exist are in some way united. If they were not then it would not be possible for anything to be said about them or for them to have any properties; this is why he says in Book I that "being and the one are the most universally predicated things of all." (Aristotle *Met*. 10.2 1053b20) If something is not naturally united, then there must be something which causes it to be united. Otherwise the fact that they are united is merely a senseless phenomenon without explanation. Since numbers are necessarily divisible, it would follow that there must be something which keeps them together. Consequently, numbers cannot be metaphysically independent substances, and so Aristotle concludes that "it is clear that the objects of mathematics are not separate. For if they were separate their attributes would not be present in bodies." (Aristotle *Met*. 14.3 1090a29-30) In other words, the kinds of things that are predicated of numbers are also predicated of physical objects. One example of this that he gives is the musical scale, since it can be described mathematically; a more contemporary example includes the laws of physics used to model the behavior of physical objects. When we describe the derivative of a parabola on a cartesian plane

and give an equation for it, we are giving an abstracted description of the velocity of a moving object.

Here, however, we should address a possible objection. It comes as no surprise that one does not need to be an Aristotelian to recognize that numbers exist in some way, and the possibility of an Aristotelian position that is to be distinguished from others on this subject is a testament to this fact. One might, for instance, disagree with the claim that anything that exists must have causal powers. Certainly it is not obvious that this is the case, with many holding the contrary view. Consequently, it is not a problem that numbers do not have causal powers, for in that case rather than abandoning the existence of numbers (or the particular kind of way in which numbers exist) we can abandon Aristotle's causal metaphysics should we find the former more believable than the latter.

In response to this it might be argued—and here we begin to look at the second point of Aristotle to be discussed—that it does not matter that we cannot find a place for numbers in the causal order of the world, for we *need* to affirm that numbers exist (in particular, as substances) if we are to coherently speak of them in the manner we do. As we have discussed before, for Plato one motivation for believing in the forms was the mutability of physical objects. Recognizing that Heraclitus had a point when he said that everything is in flux, Plato understood that if this were true then there could be no scientific knowledge of anything; after all, if *everything* is changing, then the moment something is known it also ceases to be. For this reason, he thought, there must be immutable Forms in which all physical objects participate that allow us to have certain, scientific knowledge. However, in response to this Aristotle says that it does not follow from the fact that there are true scientific claims that there are objects independent of the changing perceptible objects. If this were the case, "the objects of astronomy will be beyond the

perceptibles in the same way as the objects geometry is about. But how is it possible for there to be a heaven and its parts [beyond the perceptible one], or anything else with movement?" (Aristotle *Met*. 13.2 1077a2-4) If mathematics is a science that requires there to be mathematical objects beyond perceptible objects in order to exist, then it would seem the same argument would apply to the other sciences, *mutatis mutandis*. If astronomy is a science, for example, it would seem to follow that there are astronomical objects that are distinct from perceptible objects. This, however, is absurd; no one believes the planets to be objects separate from perceptible objects. Consequently, it would also be absurd to think that there are substantial mathematical objects merely because there are true mathematical propositions.

Intuitively this is not the most satisfying argument, and it is easy for us to dismiss it immediately on the basis that Aristotle simply had a false understanding of astronomical objects. If he had a true conception of them, this argument implicitly says, then he would have seen that his argument is hardly an argument at all. This counterargument, however, is itself not much of an argument. Sure, astronomical objects are not the kinds of objects which Aristotle imagined them to be, but this does not change the fact that in studying them we are not studying objects distinct from perceptible objects. We at times deal with idealizations of astronomical objects, but such idealizations are simply abstractions from the original perceptible phenomenon. Ultimately, what Aristotle's argument boils down to is that we can speak intelligibly about astronomical objects without there needing to be special, immutable astronomical objects in the realm of the Forms, and the same thing can be said about all of the other sciences. Thus, the fact that we are able to speak intelligibly about mathematics does not by itself mean that there has to be mathematical objects. The reason this counterargument is convincing in the first place is that there is the intuition that since one cannot see numbers they must be completely different from

astronomical objects, which can be seen. However, in saying that numbers are not separable from physical objects we must understand that we are not saying that they are physical objects; after all, it is of course impossible to touch or see a pure number.

Besides the musical scale and a moving object, a new and slightly different example can help us understand what exactly we mean when we speak of the inseparability of numbers. Aristotle says that "[m]any coincidents, though, belong intrinsically to things insofar as they are, each of them, of a certain sort—for example, insofar as the animal is female and also insofar as it is male, there are attributes special to it (yet there is no male or female separate from the animals)." (Aristotle *Met*. 13.3 1078a5-7) What he is saying here is that the scientist is able to discuss what attributes belong to the males and females of a given species of animal without saying that male and female are substances that exist without existing within particular animals. Similar things can be said about mathematics and numbers. Aristotle continues: "So there are also such attributes of things only insofar as they are lengths or insofar as they are planes. And to the extent, accordingly, that [a science] is concerned with what is prior in account and simpler, to that there is more exactness it has (this is what simplicity does). So there is more exactness without magnitude, certainly, than with magnitude." (Aristotle *Met*. 13.3 1078a8-11) We are able to discuss the attributes an object has by virtue of the fact that it has extension or quantity, and in doing so we can come to discover the truths of mathematics. As with the sex of animals, however, this does not mean that what we are discussing has the same ontological status as the objects which have extension and quantity. Furthermore, the fact that these attributes are under investigation does not, as the use of mathematics in the contemporary sciences might suggest, mean that what is being studied is automatically made more precise and certain. Instead, there is greater simplicity and certainty when we investigate those things that are prior to numbers in the

same way that it is simpler to consider animals insofar as they are simply animals than to consider them insofar as they are male and female.

Here we should address an apparent tension. Frege, as we have already noted, does not maintain the correspondence theory of truth. However, Aristotle's anti-platonic argument seems to assume that the view being criticized understands mathematical claims to be true by virtue of the fact that they accurately correspond to the world of mathematical objects. While of course Frege does not endorse the correspondence theory, one does not need to presuppose this theory in order to advance an argument similar to Plato's, and this is supported by the very fact that Frege does just this. In the *Foundations of Arithmetic* he says, "If in the existing flux of all things there is nothing fixed, eternity persists, then the knowability of the world would end and everything would plummet into confusion." (Frege 15) Similar to how Plato wants to ensure the possibility of true scientific claims, Frege is concerned with ensuring that there can be true propositions so that there can be certainty in mathematics. Frege, like Plato, goes about doing this by arguing that since certain propositions, especially mathematical propositions, remain true or false regardless of the physical circumstances, those propositions must depend on objects that are not physical. Of course, it may be that in reality one must presuppose it in order to make this argument; if this is the case, then this only supports our argument that Frege is making certain metaphysical claims without realizing it.

3) Aristotle's Conception of Science

Let us briefly review the ground we have covered. First, we considered Aristotle's argument on the basis of causal powers. According to Aristotle, if something is to exist it must have causal powers; since numbers do not have causal powers, it follows that they do not exist,

or that one cannot speak of them as existing in any literal and meaningful way. Of course, the Fregean could just deny the claim that something must have causal powers to exist, but in order to take the discussion in this direction the Fregean must commit to certain metaphysical theses beyond what logicism would permit. Next, we considered Aristotle's argument on the basis of the nature of the sciences. It is often argued that mathematical propositions require the existence of substantial mathematical objects beyond physical objects in order to be true or false. Otherwise, the argument goes, the propositions of mathematics can neither be scientific nor meaningful. However, other sciences are able to have true or false propositions without needing to posit substantial objects beyond physical objects. Consequently, prima facie mathematics does not require there to be objects set apart from physical objects in order for it to go about its business. Additionally, what this suggests to us is that we must look deeper into what it means for a science to make meaningful claims about its given subject of study, at the very least because it can help us figure out more precisely we ought to go about filling out our ontological zoo.

At this point we ought to briefly explain what Aristotle means whenever he speaks about a science, for it will clarify the reasoning behind his position. Furthermore, it can serve as a more positive explanation of how mathematics might fail to be enough to justify metaphysical claims. We could, if we wished to keep things simple, say that "science" is just another word for "topic" or "subject." This, however, ignores the fact that for Aristotle "science" is a technical term, and that there are specific requirements for being a science and that there are specific things which sciences seek to do and explain. In the same way that Plato wanted to ensure the possibility of knowledge, so too did Aristotle, and it is in the sciences that we find this possibility. Aristotle says that the different kinds of knowledge are "craft knowledge, scientific knowledge, practical

wisdom, theoretical wisdom, and understanding." (Aristotle *NE* 6.3 1139b15) The original Greek term, *ἐπιστήμη*, is translated into the second of the five, scientific knowledge, and it is what is ultimately gained through scientific investigation. To have it consists of having the ability to make deductive syllogisms which result in certain knowledge, and one can undertake a given science either by reasoning towards the premises of these deductions or by reasoning from those premises towards certain conclusions. Furthermore, this knowledge includes knowledge of the causes of what is being studied. Hence Aristotle says in Book A that "it is clear that theoretical wisdom is scientific knowledge of certain sorts of starting-points and causes," (Aristotle *Met*. 1.1 982a2) and in Book α he says, "It is also correct for philosophy to be called scientific knowledge of the truth…But we do not know the truth without its cause." (Aristotle *Met*. 2.1 993b20-23) Likewise, he says in the *Posterior Analytics*, his primary treatise on the nature of science, "We consider that we have unqualified knowledge of anything (as contrasted with the accidental knowledge of the sophist) when we believe that we know (i) that the cause from which the fact results is the cause of that fact, and (ii) that the fact cannot be otherwise." (Aristotle *APo*. 1.2 71b10-13) Furthermore, we ought to also briefly acknowledge that a science is not merely a collection of facts or theorems; it is a state "in which the soul grasps the truth by assertion and denial." (Aristotle *NE* 6.3 1139b14) To have scientific knowledge of something is to be disposed to think about it in a certain way (namely, in a way that results in necessarily true propositions). The difference between philosophers and sophists is not just that philosophers know the truth; rather, they stand in a particular relationship to the truth and have the virtues necessary for attaining it.

Here it might be wondered how one gets a theoretical grasp on something and achieves this state of the soul. To describe the whole process would be beyond our present scope, though

concerning how one begins to do this Aristotle says, “The best way, though, of getting a theoretical grasp on a given thing would be this—if we posit as separate what is not separate, in just the way the arithmetician and the geometer do. For the human insofar as he is human is one and indivisible. And the arithmetician takes him to be one and indivisible, and then gets a theoretical grasp on what is coincident with a human insofar as he is indivisible.” (Aristotle *Met*. 13.3 1078a20-24) Mathematicians pretend momentarily that numbers are in fact their own objects and exist in a special kind of way (whether it be closer to how Plato imagines it or how Frege imagines it), and it is with this in mind that they are able to proceed. Furthermore, what is also of significance here is that it helps us to explain the psychological plausibility of the claim that numbers are in fact objects beyond the physical and spatial; within the realm of mathematics they practically are. If one is engaged with mathematics or is greatly concerned with its problems in the same way that Frege was, then it is only natural that one would posit the kinds of things that Frege did (this psychological point is, of course, not deductive, and its primary purpose is to bolster the mathematical and metaphysical claim from which it is to be distinguished).

To be clear, this conception of science does not absolutely require one to affirm Aristotle’s views on numbers. Proclus, for instance, maintains a similar hierarchy of the sciences while at the same time believing that numbers and shapes are formal objects (though one could reasonably ask whether the two views paired together are conducive to success in the sciences). “Following Aristotle’s lead,” Glenn R. Morrow writes, “Proclus at times conceives of the mathematical sciences, arithmetic and geometry in particular, each positing a certain genus of subject-matter that it proposes to examine and specific principles that it adopts for handling this subject-matter…Each of them is in this sense bound by the presuppositions from which it starts.” (Morrow 64). Our reason for bringing it up here is that it can help us make more sense of *why*

Frege makes the mistake we have been describing from the Aristotelian perspective we have been using to criticize him. In his commentary on the first book of Euclid's *Elements* Proclus himself says, "For no science demonstrates its own first principles or presents a reason for them; rather each holds them as self-evident, that is, as more evident than their consequences. The science knows them through themselves, and the later propositions through them. This is the way the natural scientist proceeds, positing the existence of motion and producing his ideas from a definite first principle. The same is true of the physician and of the expert in any other science or art." (Proclus 62)

With this in mind, what we see here is an attempt by Frege to undertake metaphysics in order to provide for the needs of mathematics. His linguistic and metaphysical investigations, we should note, were in service to that end (of course, these mathematical inclinations do not diminish the philosophical importance of Frege's work, but at the same time we ought not imagine Frege to have the same concerns as the average metaphysician). Tappenden in "The Riemannian Background to Frege's Philosophy" notes in his discussion of 19th century mathematics that "[r]eduction motivated purely by 'bare philosophical certainty' and ontological economy – whatever its philosophical merits – is in practice of little interest to the working mathematician." (Tappenden 101) Frege himself suggests that he imagines mathematics to have some kind of priority in relation to metaphysics when in the introduction to the *Foundations of Arithmetic* he says, "Thought is in essentials everywhere the same…The differences consist only in the greater or less purity and independence of psychological influences, and on external aids to thought, such as language, number signs, and the like, than, say, on the fineness of the structures of concepts; yet it is precisely in this respect that no science can surpass mathematics, not even philosophy itself," (Frege 12) and later on he says, "Yet, even if these concepts themselves

belong to philosophy, I believe that a decision here cannot be attained without assistance from mathematics." (Frege 19)

At this point one might justifiably demand a reason independent of Aristotle and Frege for putting metaphysics before mathematics or mathematics before metaphysics; after all, it might be argued that all that has been previously described has weight only in a strictly Aristotelian context and that all that has been said is only convincing to those who have already adopted a myriad of other Aristotelian theses. To begin, we must first remember that the very question is a philosophical question, not a mathematical one, and in support of this it should be observed that, despite what some might expect, the arguments explicated here are not mathematical arguments, but are firmly within the sphere of philosophy. What this suggests is that philosophy takes a wider perspective with which it can judge both mathematics and itself. As an aside, we should note that Alexander of Aphrodisias reports that Aristotle himself uses a similar argument in order to argue for why one should study philosophy (Hutchinson and Johnson 197n4). Furthermore, even the phrase "philosophy of mathematics" suggests that there is something beyond mathematics. If mathematics is the final perspective from which one is to undertake analysis, then it would not be possible to appeal to anything else in order to settle debates in mathematics. It follows that metaphysics is more universal than and prior to mathematics, as it gives an account of the starting points and norms of mathematics.

4) The Consequences for Frege and for the Philosophy of Mathematics

To rearticulate our present view, while one may not need to be convinced by Aristotle's particular arguments or to hold the Aristotelian position, one must have some kind of metaphysical argument in order to be able to speak of the metaphysical status of numbers. In

simply showing that mathematics is derived from logic, however, Frege does not offer anything with which one could justify such claims. The reduction of mathematics to logic, as we have noted, does not directly involve metaphysics, so it is not as if Frege has any broader metaphysical theory to which he can appeal. At the same time, however, the way he goes about undertaking this reduction and the particular problems he faces only make sense if particular metaphysical positions are presupposed. Since they are presupposed, they naturally lack any argument in favor of them beyond the plausibility and/or necessity. However, this is ultimately what is being questioned in the first place. In other words, the plausibility of Frege's philosophy is being questioned on the basis of its unfounded metaphysical presuppositions, so a proponent of Frege cannot attempt to support the latter with the certainty of the former.

To return to the example of the rejection of the claim that everything that exists must have causal powers, while we could attribute this to Frege, the maintaining of this position requires a metaphysical theory. It is not just that a mathematical proof would not be able to be made for or against this position; rather, the very idea of causal powers is not something with which mathematics and logic are concerned. Prima facie it seems like it would be difficult to articulate or define causality using only a mathematical vocabulary. The fact that we can make a firm distinction between ontological objects and logical objects only supports this claim, for if the objects of two sciences are so different from one another then it would seem to follow that those sciences are also different. Put in another way, it is unclear how one would be able to intelligibly speak of ontological objects using only terms that apply to logical objects.

Now, for Frege in particular this means that from the beginning there is a problem. He wants to offer a completely logical account of numbers that is necessary and not arbitrary, and he begins the *Foundations of Arithmetic* by considering the question "what the number one is."

(Frege 11) He even says, “if everyone had the right to understand by this name whatever he wants, then the same proposition about one would refer to different things for different people; there would be no common content for such propositions,” (Frege 11) which suggests that he is looking for something objective and common to all. When we speak of numbers we must all be referring to the same thing, and to do otherwise would simply be equivocation. However, as a mathematician the closest he can come to this is to consider what it is that makes mathematical propositions true or false or to show the logical function of numbers in those propositions, and while he may be able to offer a definition that allows us to avoid equivocation, it is neither necessary nor not arbitrary. After all, while the logical validity of a proof or the consistency of a definition may be recognizable to all, it does not follow that these things have some special metaphysical significance or that they constitute the essence of other things. In a letter to Hilbert Frege even says, “I cannot accept such a method of inference from lack of contradiction to truth.” (Frege 48) The mathematical accounts given are not enough to provide for the kind of account desired, and this is supported by the fact that there are alternative ways to define numbers using set theory which analyze them down into things different than Frege. Consequently, Frege does not have the vocabulary to answer the question with which he is most concerned.

One may reply that Frege is not trying to infer the existence of numbers from their possibility or from their lack of contradictions but rather wants to show that mathematics and specifically numbers as objects deductively follow from the laws of logic. This, however, misses the point. As we have already explained, in order to get to the point where he can say that numbers are objects he relies on more than just pure logic. Consequently, even if he gets to this point only through deduction it does not follow that the objecthood of numbers is a law of logic

in the same way that, say, De Morgan's laws are. It follows that numbers are not by necessity strictly logical objects, though in a proposition they may take on that logical role. Of interest here is the multiplicity of possible analyses of numbers, with Zermelo's being one potential alternative. The fact that such analyses are possible suggests that there is not one necessary analysis and that subsequently Frege only demonstrates a possible way to define numbers—a *description* that itself does not express just what exactly numbers are. One may reply that multiple analyses does not undermine the necessity of the conclusion in the same way that the fact that multiple proofs can be undertaken from the same premises to the same conclusion does not mean that the conclusion fails to necessarily follow from those premises. Here, however, the conclusions are each different—since each analysis describes numbers in different ways—while with the different proofs the conclusions are the same. It is similar to the difference between the length of the diameter of a circle and where on the circle it is to be drawn; the former is already certain from the dimensions of the circle, while the latter is a matter of choice and includes an infinite amount of possibilities. At the same time, the mathematicians undertaking these analyses do not and need not all begin with the same premises, and a major part of their projects is to determine the initial axioms from which analysis can begin.

It may also be argued in a Fregean fashion that if numbers are not logical objects then psychologism must be true. The reason for this claim is that if numbers are not directly accessible to the mind through logic then they are accessible to the mind only through psychological processes that are necessary to other understandings of arithmetical epistemology (e.g. abstraction). However, the manner in which we come to know something or come to know *of* something does not define its existence. Once again Frege makes the mistake of putting the cart before the horse; this epistemological claim can only be judged by a metaphysical

framework. To make such statements about the process of abstraction requires knowledge of the faculty of abstraction, and this knowledge comes from knowledge of the mind or soul.

Now is as good of a time as ever to note that vagueness tends to arise when a science seeks to articulate that which is beyond its limit. The reason for this is that what is beyond its limit is not able to be precisely articulated with the language and concepts at its disposal. This vagueness often manifests itself in such a way that it can often be described as artistic, poetic, or metaphorical. It is for this reason that in the *Tractatus Logico-Philosophicus* Wittgenstein says, "There are, indeed, things that cannot be put into words…They are what is mystical." (Wittgenstein 6.522)[2] Now, Frege openly admits in *Logic* that "[i]t is enough for us that we can grasp thoughts and recognize them to be true; how this takes place is a question in its own right." (Frege 246) Though he has ways by which to gesture to the kind of phenomenon going on, he cannot, despite the rigor he applies to other problems, offer a precise articulation of how this works. Likewise, in the first volume of the *Grundgesetze der Arithmetik* Frege says "The metaphor of grasping is ideally suited to elucidate the matter," (Frege 206) suggesting that rather than directly describing the phenomenon Frege can at best point to it with language not to be taken literally. This, however, is a digression; its primary significance is that it is another weight, however small, on the scale measuring the strength of our argument.

This argument (in case we need to remind ourselves) is that metaphysical problems and distinctions cannot be adequately articulated with the terms of mathematics, and, as we have seen with the previously seen quotation from Proclus, the answers to the questions concerning the foundations of mathematics and number are not discoverable by mathematics but are instead

[2] Incidentally, in this same book Wittgenstein also claims, perhaps not too differently from what we are claiming here, that "[t]he word 'philosophy' must mean something whose place is above or below the natural sciences, not beside them." (Wittgenstein 4.111) Of course, we should not understand Wittgenstein to be endorsing all the metaphysical propositions for which we are presently arguing or for believing science to be what we ourselves take it to be.

already presupposed by them. This, along with the limited view of being which mathematics takes, suggests that what would appear to be the only way for something to exist for a mathematician is merely a kind of existence analogous to that with which a metaphysician is concerned. Hence, we might say that Aristotle would speak of Frege in a manner similar to how Frege himself speaks of the proponents of psychologism in the first volume of the *Grundgesetze der Arithmetik*: "He looks into his psychological peepshow and says to the mathematician: I see nothing at all of what you are defining. And he can only answer: No wonder! For it is not where you are looking." (Frege 207) It follows from this that the existence and intelligibility of mathematical propositions is not enough to determine the ontological status of mathematical objects. At the same time, some arguments derived from metaphysics itself suggest that numbers do not in fact have the properties that Frege attributes to them. In order to counter these claims he would have to tell more of a metaphysical story, but this cannot presently be done with what Frege gives us. It is not unlikely that Frege himself had some understanding of this fact, even if it was not enough to make him completely abandon his project. Speaking of his later philosophy, Dummett writes, "For the non-actual objects of *Grundlagen* and the logical objects of *Grundgesetze*, however, Frege no longer had any philosophical justification; and so they quietly vanished from his ontology." (Dummett 225) Without a more expansive philosophical (and, more specifically, metaphysical) system, it is difficult to argue for the existence of these logical objects.

Conclusion

We first considered the nature of Frege's platonism, drawing the distinction between ontological platonism, which takes numbers to be substantial objects analogous to physical

objects, and logical platonism, which takes numbers to be logical objects primarily understood through the logical role they play. We then considered the ways in which Frege's platonism fails to be strictly logical, requiring certain metaphysical presuppositions to be made in order to defend its assertion that numbers are objects. Subsequently, we took a survey of two arguments derived from books M and N of Aristotle's *Metaphysics* which can be used both to directly push back against Frege's position as well as to reveal the background philosophical problems operating behind the scenes. From this broader perspective we were then able to more robustly offer a methodological critique of Frege's logicism.

In conclusion, Frege is forced to give pronouncements on metaphysical questions without having the means to justify any stance he takes. It is not that his particular metaphysical positions are unfounded; rather, any position he takes (or, more accurately, presupposes) will lack the proper justification. Such follows from the perspective he adopts as the result of his logicism, for one is not able to reach specific metaphysical conclusions just from the laws of logic. As Frege quite explicitly states, consistency and non-contradiction is not enough to make the existence of something a necessary truth. We can contrast this with the method by which Aristotle investigates the philosophy of mathematics. Rather than attempting to have a philosophy of mathematics fully encompassed by logic, Aristotle approaches the subject already having a fleshed-out metaphysical theory. This provides him a broader philosophical context and conceptual framework in which he can bolster his positions within the philosophy of mathematics and justify his presuppositions. Ultimately, Aristotle offers a better method than Frege for undertaking a philosophical analysis of mathematics, and it would be fruitful for us to consider it more intentionally when contemplating numbers and mathematics generally. It does not matter whether one is convinced by Aristotle's particular arguments for his view on the way

in which numbers exist or the extent to which one prefers Frege's position over Aristotle's; regardless of the strengths and weaknesses of each side, by considering them in the way we have here we have gained more insight on another manner in which one can undertake philosophy of mathematics. At the same time, we also have a greater understanding of how one's conception and prioritization of the sciences influences the particular philosophical positions one adopts. There are still avenues by which one can argue in favor of a Fregean metaphysics over an Aristotelian one; what is more difficult to dispute, however, is the impact that one's conception and ordering of the sciences has on how one goes about investigating the question of the metaphysical status of numbers. Additionally, as we have seen, even if there are greater reasons to favor a metaphysics that looks more like Frege's than Aristotle's, Frege does not have the means to get there. Mathematics is not a ladder to metaphysics, and neither is logic. For this reason, regardless of who comes out on top doctrinally, Aristotle approaches metaphysics and the philosophy of mathematics from a more propitious position.

With all of this having been said, it is not as if all is settled and done; rather, this is only the opening up of a realm of investigation. For starters, there is still the unanswered question of what exactly numbers are. Additionally, if a more "Aristotelian" or "metaphysically-minded" methodology and perspective is to be adopted moving forward, there is also the task of updating it to be consistent (or at least more consistent) with the genuine advances of mathematics and philosophical logic. After all, one reason why it was important to address Frege's views here rather than, say, Plato's is that Frege is one of the figures responsible for such advances. Ultimately, however, what is called for by what has been articulated here is a consideration of knowledge and specifically scientific knowledge (or, in other words, the nature and end of philosophy). All of this is, of course, for future projects, and they can only be pointed to or

gestured at in this present moment. Given our current vantage point if we stare at this for too long it will only be dizzying for us, for, after all, it is concerned with the very nature of thought and thinking. We will have succeeded in our task if we have made the possibility of this a possibility.

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