A MONTE CARLO CASE STUDY OF RANDOM INPUTS IN THE STOCHASTIC MODEL FOR POLLUTION IN ESTUARIES

by

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I INTRODUCTION

1.1 Estuaries

A special and distinctive environment prevails at and near the mouth of the stream course where the river meets the sea. This zone between the fresh water of the stream and the salt water of the ocean is called an estuary. An estuary has been described [14] as the wide mouth of a river or arm of the sea where the tide meets the river currents. In the interests of sanitary engineering, an estuary can simply be defined as that portion of a river which is affected by tidal action.

Because estuaries present unique advantages in terms of food supply, transportation, and waste disposal, man's historical development is closely linked with estuaries. Of the 110 standard metropolitan areas in the United States, more than 40 are located on estuaries. This constitutes a population total of 55.5 million people [5], or approximately one-fourth of the population of the United States.

Estuarine waters, because they have proved so serviceable to mankind, have consequently become the recipients of a large degree of pollution and contamination, resulting in the devastation of a resource so vital to man's survival.

Fortunately, a growing concern over man's compulsion to poison his environment has led to appropriation of increased funds for rectifying the situation.

The problem of pollution in estuaries is concerned with the temporal and spatial distribution of contaminants introduced into the estuary and with their effect on water quality. Analysis of these effects is important in predicting the degree of treatment required and in ascertaining the quantities of municipal and industrial wastes which can be tolerated by the natural circulatory system of the estuary. Contaminants enter the estuary by means of upland river flow and also through outfall structures with varying amounts of initial dilution through jet entrainment [7].

The control of estuarine pollution is based on the purposes for which this water is used. A minimum quality of water, both in chemical and in physical characteristics, is required for each of the active uses of tidal waters for fisheries, recreation, navigation, or municipal supply. The quality of the water is likely to be depreciated by the passive uses of estuarial waters to convey sewage, industrial wastes, effluent cooling water, and surplus surface water [3].

For the purpose of detecting the strength of organic wastes, the single most useful parameter is the biochemical oxygen demand (BOD). Organic pollution in the presence of dissolved oxygen undergoes an aerobic stabilization process in which organic compounds, containing carbon, hydrogen, oxygen, phosphorus, and nitrogen are decomposed by oxygen consuming bacteria into carbon dioxide, water, nitrates, and phosphates. BOD is, in essence, the amount of oxygen required to achieve this reaction [9].

BOD and dissolved oxygen (DO) are expressed in terms of milligrams per liter (mg./1.) or parts per million (p.p.m.). The difference between the dissolved oxygen saturation level and the amount of dissolved oxygen present is the oxygen deficit (OD).

1.2 Stochastic Simulation

Stochastic simulation, or Monte Carlo methods, comprises that branch of experimental mathematics which is concerned with experiments on random numbers. The greatest successes of this method have been in areas where the basic mathematical problem itself consists of the investigation of some random process, as in problems of neutron physics or in the detection of signals on a phone with random noise [15]. Although still in the early stages of development, Monte Carlo methods have been utilized on occasion in numerous other fields of science, including chemistry, biology, and medicine.

Problems handled by stochastic simulation are of two types called probabilistic or deterministic, according to whether or not they are directly concerned with the behavior and outcome of random processes [6]. In the case of a probabilistic problem, the simplest Monte Carlo approach is to observe random numbers, chosen in such a way that they directly simulate the physical random processes of the original problem, and to infer the desired solution from the behavior of these random numbers. In explanation of the use of Monte Carlo methods for deterministic problems, Hammersley and Handscomb [6] state that one of the main strengths of theoretical mathematics is its concern with abstraction and generality. This strength, however, involves an inherent weakness: the more general and formal its language, the less is theory ready to provide a numerical solution in a particular application. The idea behind the Monte Carlo approach to deterministic problems is to use this strength of theoretical mathematics while avoiding its associated weakness by replacing theory by experiment whenever the theory falters.

Because they arise from raw observational data consisting of random numbers, Monte Carlo answers are uncertain. However, they can serve a useful purpose if the uncertainty can be made fairly negligible, i.e., to make it unlikely that the answers are wrong by much.

According to Hammersley and Handscomb [6], the name and the systematic development of Monte Carlo methods date from about 1944, even though there are a number of isolated and undeveloped instances of their use on earlier occasions. The real use of stochastic simulation as a research tool stems from work on the atomic bomb during the second world war. This work involved a direct simulation of the probabilistic problems concerned with random neutron diffusion in fissile material. The resulting intensive study of Monte Carlo methods in the 1950's, particularly in the United States, discredited the subject, as there was an attempt to solve every problem by Monte Carlo, but not enough attention paid to which of these problems it could solve efficiently, and which it could handle only inefficiently. In the last few years, however, stochastic simulation has regained much of its favor, due to better recognition of those problems in which it is the best, and sometimes the only, available technique. The majority of the latest work done in stochastic simulation has been in the field of statistics. Some examples are studies concerned with empirical Bayes estimators [2,12,13], studies concerned with classical and inverse methods of calibration [10,11], and investigations into the size and power of tests [8].

1.3 Purpose

The determination of the temporal and spatial distribution of contaminants entering estuaries is prerequisite to any program of analysis of and treatment for these substances. Because the ecology varies along different reaches of the estuary, the exact distribution of the pollution at any time is of prime importance in evaluating the problem. Biological and chemical procedures for this end may be difficult, and mathematical models can prove both lengthy and complicated.

This work purports to examine the effect of a random entry of particles of pollution, or BOD, which enter the estuary at a particular point, on the average distribution of the particles after entering the estuary and on the average distribution of the particles of OD which result from disappearing BOD particles. The relationship between BOD and OD will be explained in Section (2.2). Stochastic simulation is employed to quantify these particles and to trace their progress along the estuary, through the use of an IBM-360 computer at the Virginia Polytechnic Institute Computer Center, Blacksburg, Virginia. The number of influent entries is chosen randomly from various distributions and the outcomes are compared with a constant entry and with each other. These entries are considered for 12 estuarial situations, and the resulting average distributions are analyzed for each situation in order to determine the effect of a probability input on the distributions of BOD and OD.

II LITERATURE REVIEW

2.1 Early Models

The first work concerned with the factors affecting the BOD and DO of the stream was presented by H. W. Streeter and E. B. Phelps. These researchers based their study on the pollution decrease by bacterial action, the oxygen decrease by bacterial action, and the oxygen increase by reaeration from the atmosphere.

Later researchers modified the Streeter-Phelps model in order to include factors which affect BOD but not DO concentration. Some of these factors are [1]:

1. Sedimentation and flocculation,
2. Anaerobic destruction of carbon sources as methane, CH_4,
3. Scour, in which BOD particles are released from sludge banks, and
4. Volatilization.

Longitudinal dispersion and local land runoff also affect BOD.

DO also may be affected by additional factors including [16]:

1. Benthal demand,

2. Photosynthesis,

3. Plant respiration, and

4. Longitudinal dispersion.

The assumptions imposed by W. E. Dobbins in his modification of Streeter and Phelps' model are as follows [1]:

1. Flow is steady and uniform.

2. The whole process is steady state; conditions at a point do not change with time.

3. The BOD removals by bacteria and by sedimentation are of the first order.

4. The removal of DO by benthal demand and plant respiration, the addition of DO by photosynthesis, and the addition of BOD from scour and local runoff are all uniform.

5. BOD and DO are uniformly distributed across any cross-section.

Because the models of Streeter and Phelps and Dobbins do not consider the possible variations in BOD and DO for different points in space, Thayer and Krutchkoff developed a stochastic model for BOD and DO in streams [16]. The model used the same assumptions as Dobbins, but considers BOD and DO discretized, i.e., BOD and DO transitions were assumed to be in units of size Δ, a measurable parameter of the individual stream.

In Thayer and Krutchkoff's model [16], a change of one size Δ in the concentration constituted a change of one state. The probability of a change of one state was proportional to a small increment in time. Stochastic equations were set up for the following probabilities [16]:

1. The probability that pollution increases by an amount Δ due to local runoff in a small time increment;
2. The probability that pollution decreases by an amount Δ due to all non-oxygen consuming processes in a small time increment;
3. The probability that pollution decreases and oxygen deficit increases by an amount Δ due to bacterial action in a small time increment;
4. The probability that oxygen deficit decreases by an amount Δ due to benthal demand in a small time increment; and
5. The probability that oxygen deficit decreases by an amount Δ due to reaeration in a small time increment.

Through use of the above probabilities, Thayer and

Krutchkoff were able to obtain the complete BOD and OD distributions in streams.

2.2 Diffusion Model

The model developed by Custer and Krutchkoff [4] and consequently used in the stochastic simulation for this work includes all the factors considered by Dobbins and by Thayer and Krutchkoff and in addition the varying stream velocity inherent in an estuary. The uniform increase of BOD by land runoff and the decrease in DO due to the benthal demand are handled separately and their effects added to the results by the principle of superposition.

Custer and Krutchkoff describe diffusion as the motion of a single particle placed in a medium where it is exposed to a great number of random forces. The general assumption for the diffusion process is that it is certain that the particle will move in any interval of time δt. If δt is small, the movement will be small. The movement of a particle is influenced by the symmetric random forces described as diffusion and the non-symmetric drift or velocity. In the estuary, velocity is a function of time. Also, absorption must be considered in working with a degradable substance such as BOD.

The motion of a single BOD unit is treated as a random walk with decay on the real axis. In each time lapse of length δt, the unit moves forward on the positive axis a distance δx with probability $p(t)$ or backwards the distance δx with probability $q(t)$. This is the only motion possible. However, at each time lapse of length δt, the unit may be absorbed with constant probability r. $p(t)+q(t)+r=1$ for all t.

The following terms are used:

$$\frac{(\delta x)^2}{\delta t} = 2E \quad , \tag{2.2.1}$$

$$p(t)=(1-K_d\delta t)/2 + \frac{U(t)}{2}\frac{\delta t}{\delta x} \quad , \tag{2.2.2}$$

$$q(t)=(1-K_d\delta t)/2 - \frac{U(t)}{2}\frac{\delta t}{\delta x} \quad , \tag{2.2.3}$$

$$r=K_d\delta t \quad , \tag{2.2.4}$$

where

E is the diffusion coefficient,

K_1 is the coefficient of deoxygenation,

K_3 is the coefficient corresponding to all non-oxygen consuming processes,

$K_d = K_1 + K_3$,

and

$U(t)$ is the velocity function.

The parameter δt used in the random walk procedure is the length of time needed for one unit of size Δ to enter the estuary. Thus, δt is a function of the rate at which pollution enters from the source and the cross-sectional area of the estuary. In order to obtain the effect of a continuous source, units of size Δ are released at each time interval δt. The units act independently and their behavior can be studied individually by the random walk mechanism. Therefore the probability of a given concentration at location x after time t is the convolution of the probabilities associated with the individual units. The resulting model is a transient state model, the time parameter being defined as the time lapse since the source of pollution originated.

The diffusing process of a unit of oxygen deficit is handled in a similar manner to that of BOD, with the initial conditions slightly modified. Unlike the BOD units which originate at a uniform rate, a unit of OD is formed simultaneously with the absorption of a unit of BOD. The OD is then absorbed from the system by reaeration in the same manner that BOD is absorbed by the bacteria.

Custer and Krutchkoff [4] give a literal interpretation of the process. One can think of a unit being released at initial time t_o as BOD. The BOD travels in accordance with the random walk mechanism until it is absorbed. The absorption is actually only a "metamorphosis" which changes the identity of BOD to OD.

The probability of a forward step by a unit of OD is represented by $\underset{\sim}{p}(t)$, a negative step by $\underset{\sim}{q}(t)$ and absorption by $\underset{\sim}{r}$. Again $\underset{\sim}{p}(t)+\underset{\sim}{q}(t)+\underset{\sim}{r}=1$ for all t. The transition probabilities are:

$$\underset{\sim}{p}(t)=(1-K_2\delta t)/2 + \frac{U(t)}{2}\frac{\delta t}{\delta x}, \qquad (2.2.5)$$

$$\underset{\sim}{q}(t)=(1-K_2\delta t)/2 - \frac{U(t)}{2}\frac{\delta t}{\delta x}, \qquad (2.2.6)$$

$$\underset{\sim}{r}=K_2\delta t, \qquad (2.2.7)$$

where K_2 is the coefficient of reaeration, and the other parameters are as defined following Equation (2.2.4).

2.3 Objective of the Stochastic Model

In order to maintain a usable quality of water, it has become necessary for local, state, and federal governments to legislate water quality standards. These standards must be rationally and practically determined so that enforcement is possible.

The stochastic model for streams developed by Thayer and Krutchkoff and that for estuaries developed by Custer and Krutchkoff formulate a method for determining a practical level of DO concentration, using the entire DO probability distribution. Since there is a probability of occurrence for each DO concentration, the expected percentage of time that the DO concentration will fall below a given level can be determined from the DO probability distribution. Also, the average DO concentration can be determined from the DO probability distribution. The level of BOD concentration can be handled in the same manner. With this information accessible, water resource managers can set standards on BOD inflow so that the desired water quality can be maintained.

III PROCEDURE AND RESULTS

3.1 Description of the Program

The probability formulas described in Section (2.2) were used to obtain the average distributions of the pollution particles which entered the estuary at a point source and the average distributions of the units of oxygen deficit which resulted from those pollution particles which disappeared.

First the program divided the estuarial reach under consideration into 650 sections which were assumed to be devoid of pollution particles at the initial time. Section 550 was used as the point of pollution inflow. Since the estuary is in continuous motion, the section of entry had to be varied as time was incremented. This does not mean that the particles of pollution were being dumped at different locations, but that the estuary had placed a different section at the point of entry. For example, entry of a unit of pollution into the estuarial system at section 530 meant that the estuary had advanced 20 sections toward the sea since the initial time.

Because there is antagonism between the fresh water river flow and the tidal flow, the movement of the estuary is not a simple one-way procession occurring with the passage of time. The estuarial flow is best approximated by a sinusoidal function. The result is, then, that some of the particles are entered at sections upstream from section 550 and some are entered further toward the sea. The sinusoidal function used in this study was

$$B=550+(5 \sin y - .5t)/.33 \qquad (3.1.1)$$

where

B is the section of BOD input,

y is a function of time,

t is a unit of time.

A sinusoidal function was used also in the probability formulas for the random walk process.

The particle of BOD entered the estuary at the calculated section and a random number was generated, assigning a probability to the pollution particle. The particle then moved backward one section, forward one section, or disappeared, depending on where the random number fell in relation to the probabilities calculated on the basis of the estuarial parameters and the point in time. All the sections were then checked for the presence of BOD particles, and for each particle present, a random number was generated and the particle moved or disappeared according to the appropriate probability.

The same procedure was followed for those pollution particles which disappeared, being transformed into oxygen deficit particles. The unit of time ranged from 0 to 299 and the program was repeated 9 times with each run; the final distributions of the BOD particles and of the OD particles were determined from the average number of particles in each of the 650 sections for the 10 trials.

The concern of this study was to vary the number of particles which entered the estuary with each unit of time and to examine the distributions of the particles resulting from the different random entries. The number of entering particles ranging from 0 to 10 inclusive were selected randomly from a uniform distribution and a binomial distribution and the other random situation had an input of either 0 or 10 particles, each with probability of .5. Each situation had a mean input of 5, so that the outcomes were compared with each other and with the outcome of an entry of exactly 5.

The number of input particles selected randomly from a uniform distribution were entered as shown in Table I; the number selected randomly from a binomial distribution were entered according to Table II; and the number of input particles selected with probability of .5 were entered as seen in Table III.

Each of these three types of entry, as well as a constant entry of 5, was employed for 12 estuarial situations, and for each situation the outcomes for the various types of entry were compared. The 12 estuarial situations used are given in Table IV. δx was .33 miles for all situations considered.

One of the programs used is reproduced in the appendix. Of course, only K_d, K_2, functions of δt, the velocity function, or the number of input particles were varied for different runs of the program.

3.2 Results

The results of the study are pictorially presented in Figures 1-24. The average distributions for the BOD particles for all types of input are plotted on a graph for each estuarial situation, resulting in 12 graphs for BOD with 4 lines on each graph. The 12 graphs for the average distributions of the OD particles are presented in the same manner.

Table I. Uniform Distribution

Range of Random Number x	Number of Particles Entered
$0.0 \le x < .0909$	0
$.0909 \le x < .1818$	1
$.1818 \le x < .2727$	2
$.2727 \le x < .3636$	3
$.3636 \le x < .4545$	4
$.4545 \le x < .5454$	5
$.5454 \le x < .6363$	6
$.6363 \le x < .7272$	7
$.7272 \le x < .8182$	8
$.8182 \le x < .9091$	9
$.9091 \le x \le 1.000$	10

Table II. Binomial Distribution

Range of Random Number x	Number of Particles Entered
$0.0 \leq x < .0010$	0
$.0010 \leq x < .0108$	1
$.0108 \leq x < .0547$	2
$.0547 \leq x < .1719$	3
$.1719 \leq x < .3770$	4
$.3770 \leq x < .6231$	5
$.6231 \leq x < .8282$	6
$.8282 \leq x < .9454$	7
$.9454 \leq x < .9893$	8
$.9893 \leq x < .9993$	9
$.9993 \leq x \leq 1.0000$	10

Table III. Two Point Distribution

Range of Random Number x	Number of Particles Entered
$0.0 \leq x < .5$	0
$.5 \leq x \leq 1.0$	10

Table IV. Values Used in Programs

Situation	K_d 1/Day	K_2 1/Day	δt Day	y Radians	U(t) Mi./Day	E Mi.^{2}/Day
1	.17	.22	1/24	.507t	5 sin y + .5	1.33
2	.33	.40	1/24	.507t	5 sin y + .5	1.33
3	.40	.33	1/24	.507t	5 sin y + .5	1.33
4	.17	.22	1/48	.254t	5 sin y + .5	2.67
5	.33	.40	1/48	.254t	5 sin y + .5	2.67
6	.40	.33	1/48	.254t	5 sin y + .5	2.67
7	.17	.22	1/24	.507t	5 sin y + .23	1.33
8	.33	.40	1/24	.507t	5 sin y + .23	1.33
9	.40	.33	1/24	.507t	5 sin y + .23	1.33
10	.17	.22	1/48	.254t	5 sin y + .23	2.67
11	.33	.40	1/48	.254t	5 sin y + .23	2.67
12	.40	.33	1/48	.254t	5 sin y + .23	2.67

FIGURE 1. BIOCHEMICAL OXYGEN DEMAND - SITUATION 1

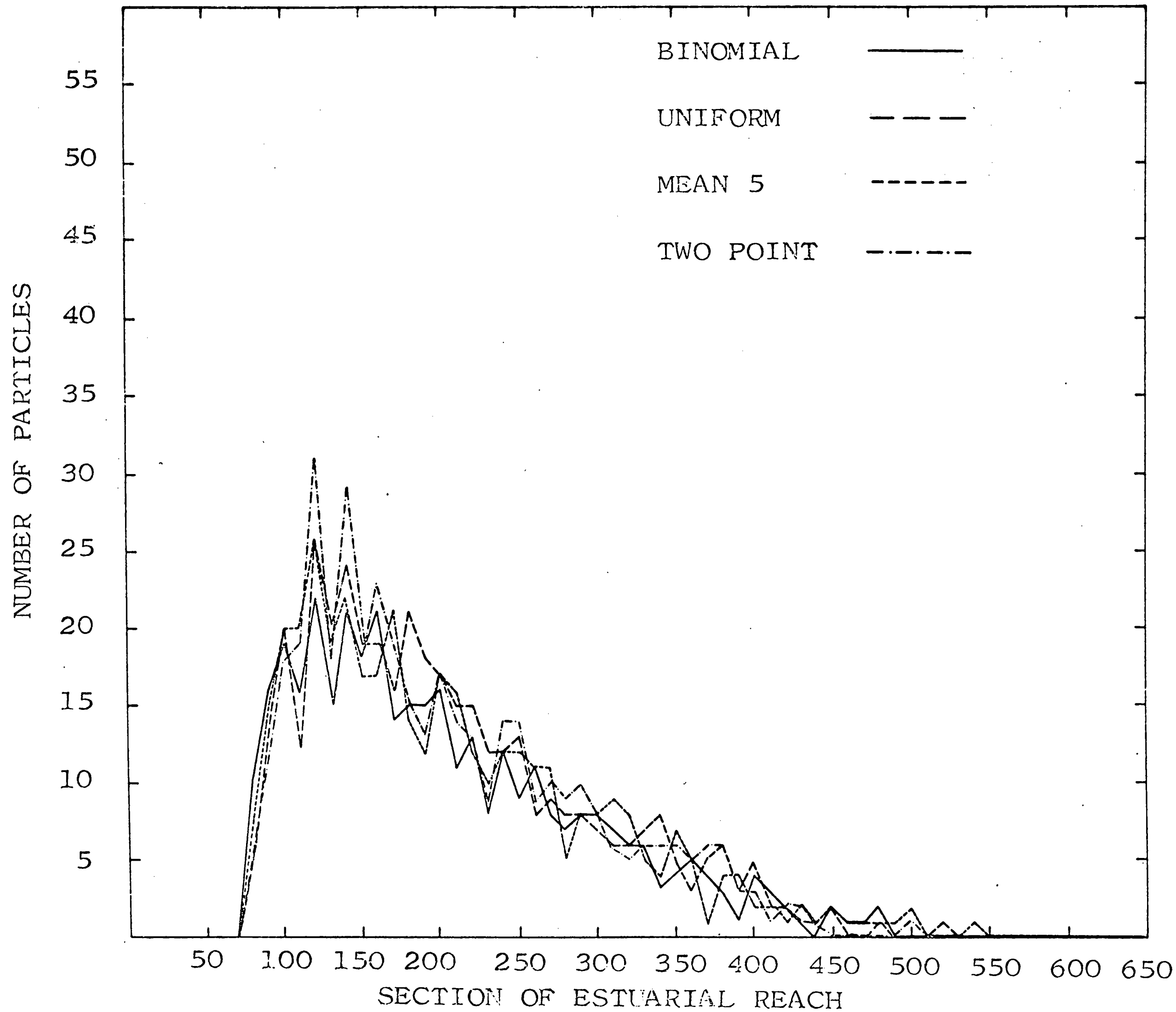


FIGURE 2. OXYGEN DEFICIT - SITUATION 1

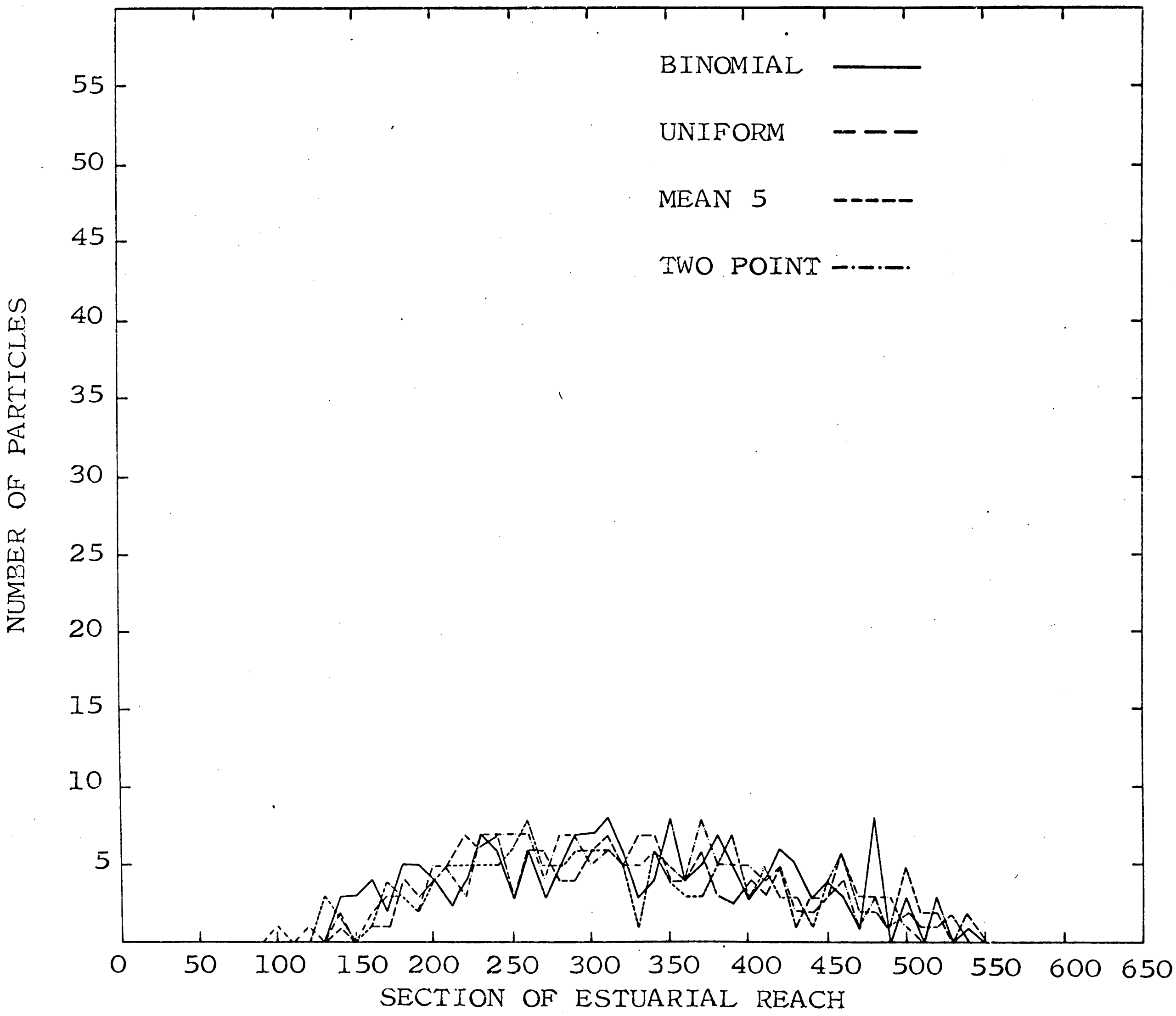


FIGURE 3. BIOCHEMICAL OXYGEN DEMAND - SITUATION 2

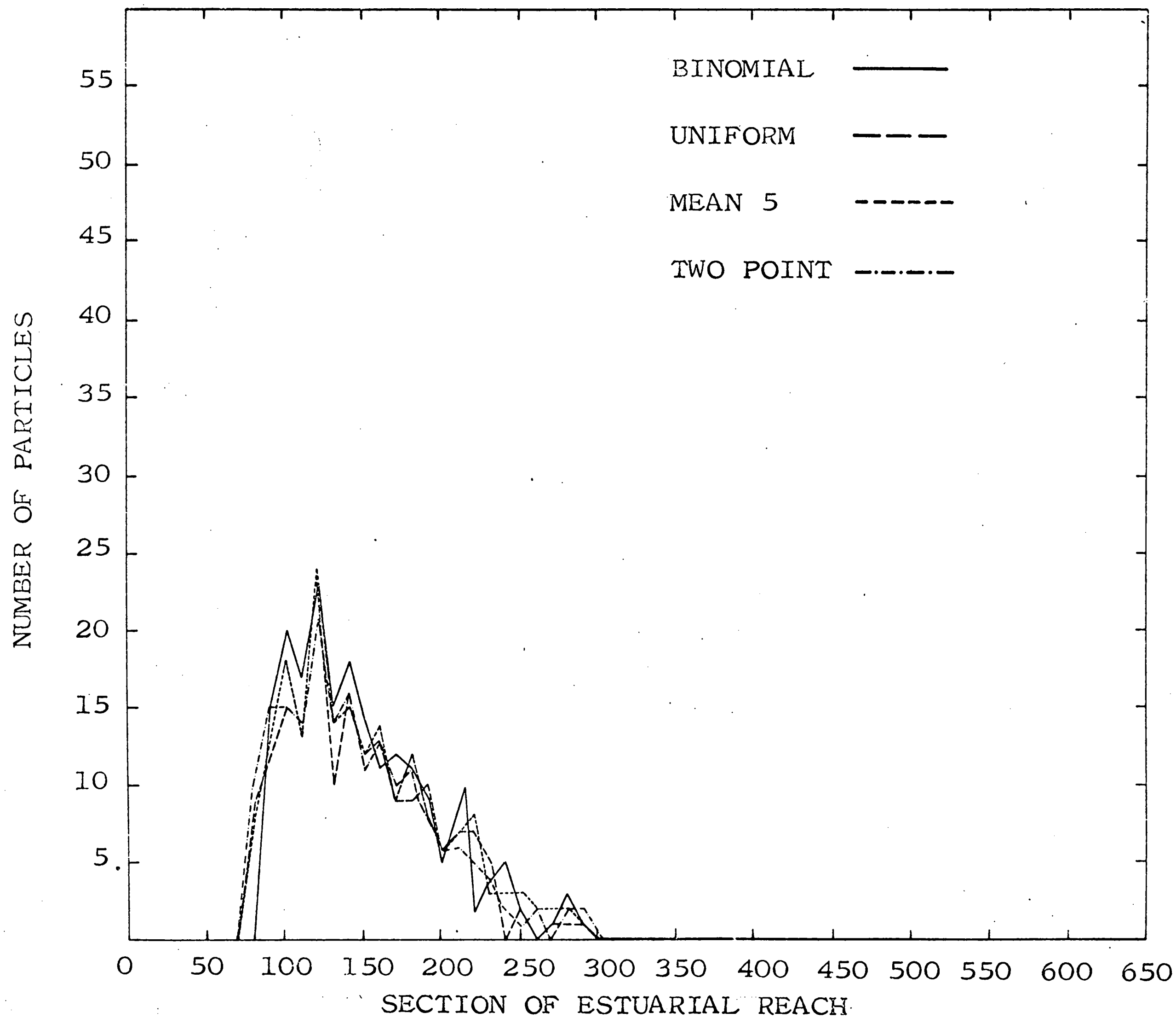


FIGURE 4. OXYGEN DEFICIT - SITUATION 2

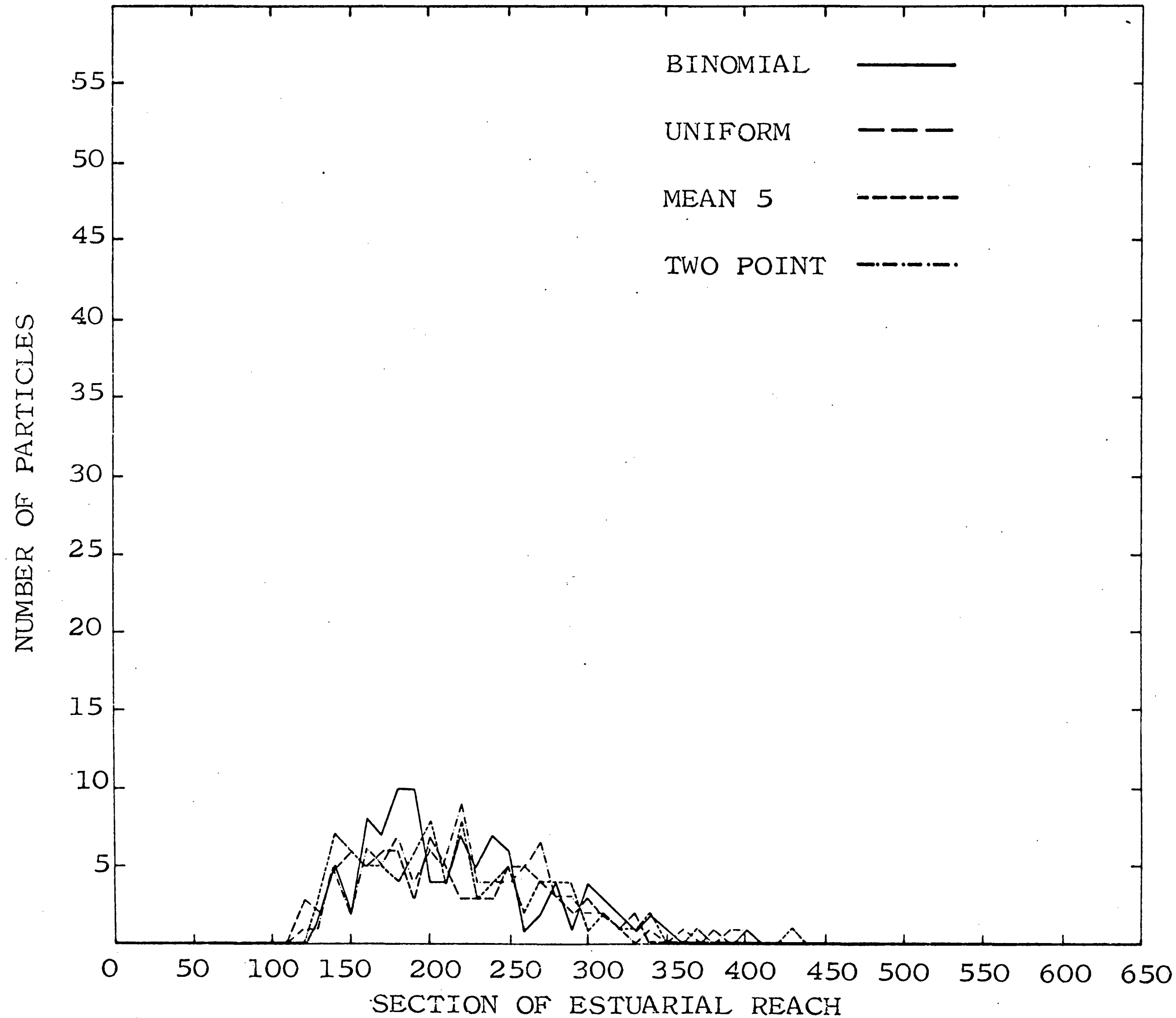


FIGURE 5. BIOCHEMICAL OXYGEN DEMAND - SITUATION 3

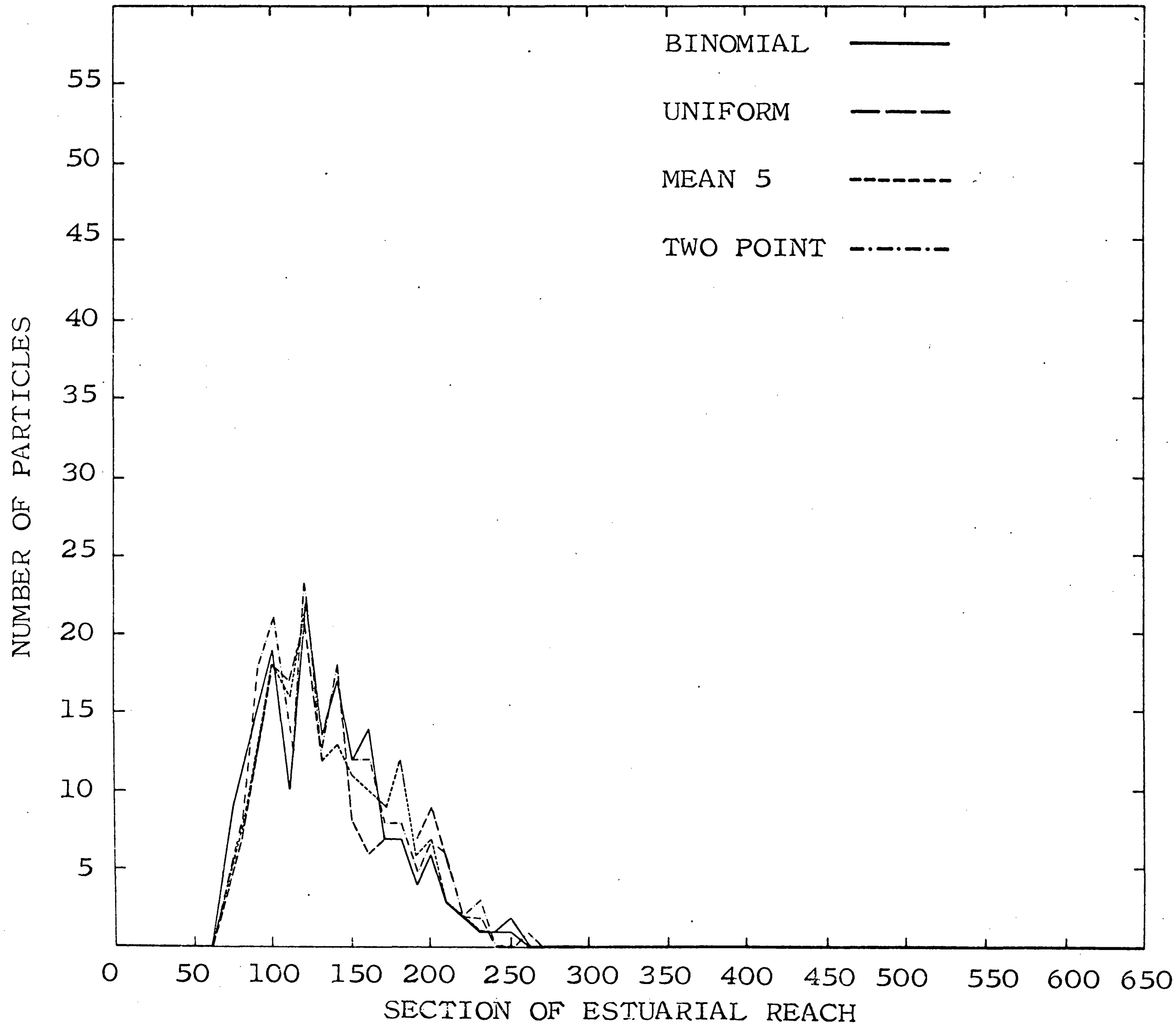


FIGURE 6. OXYGEN DEFICIT - SITUATION 3

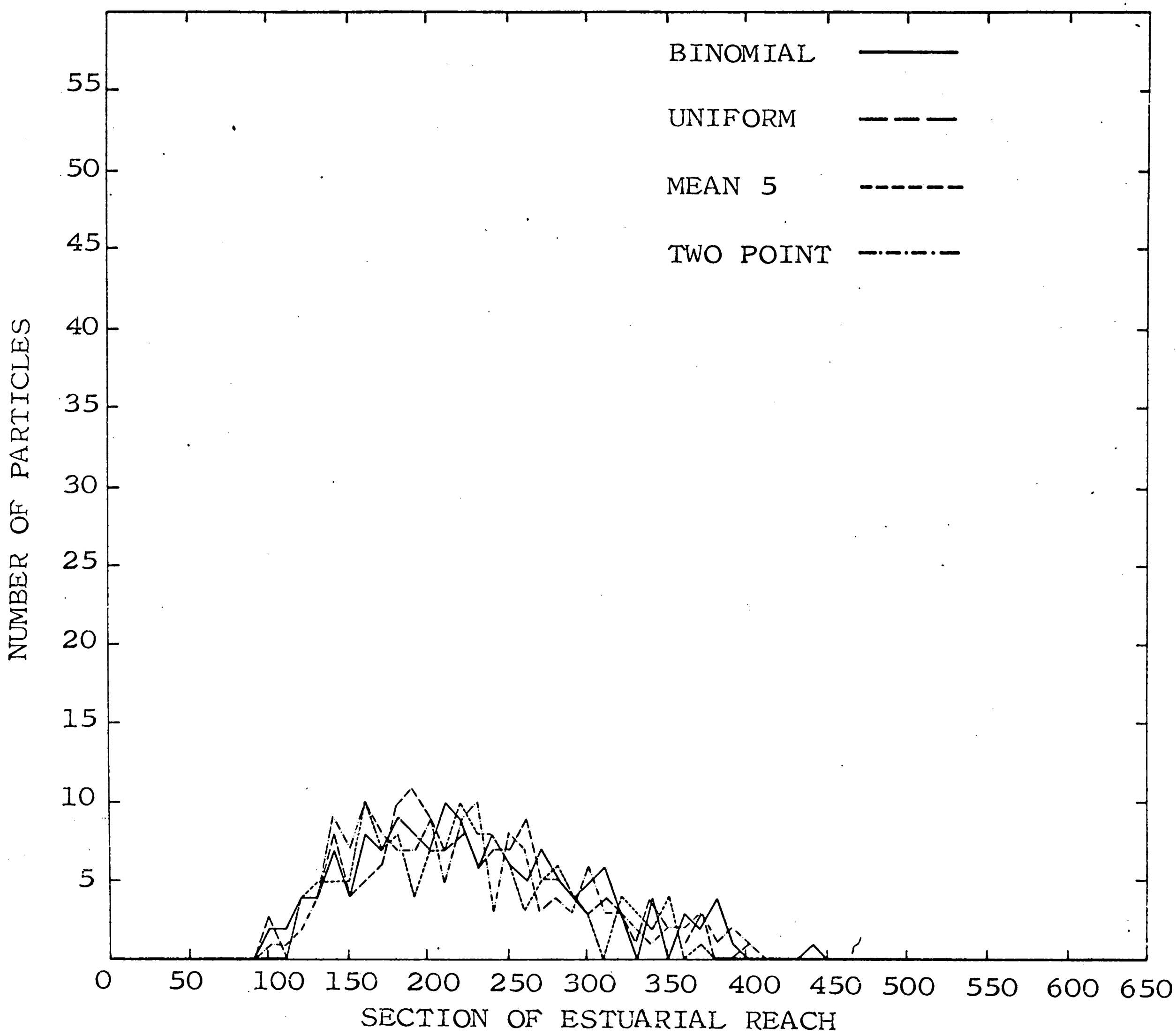


FIGURE 7. BIOCHEMICAL OXYGEN DEMAND - SITUATION 4

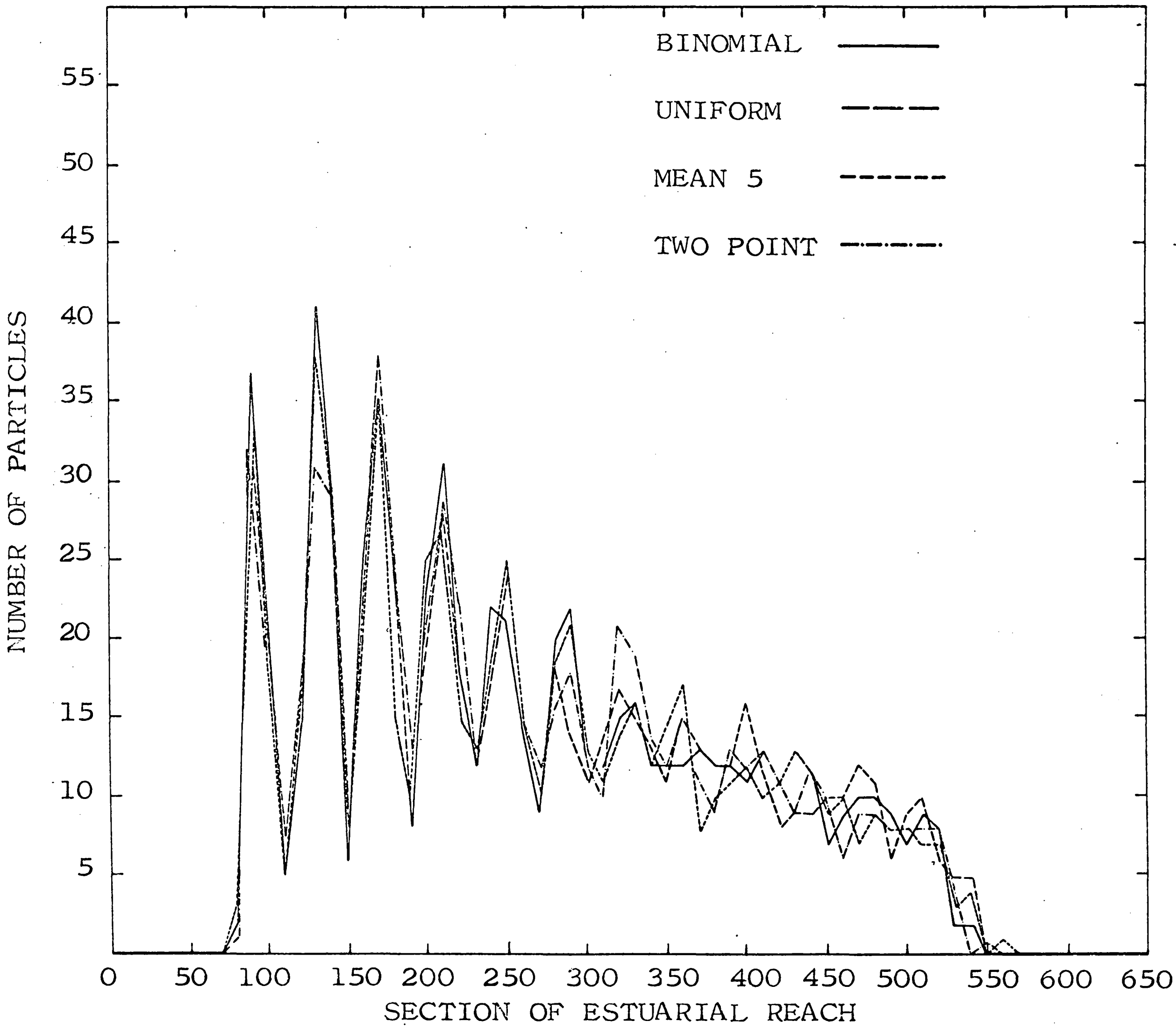


FIGURE 8. OXYGEN DEFICIT - SITUATION 4

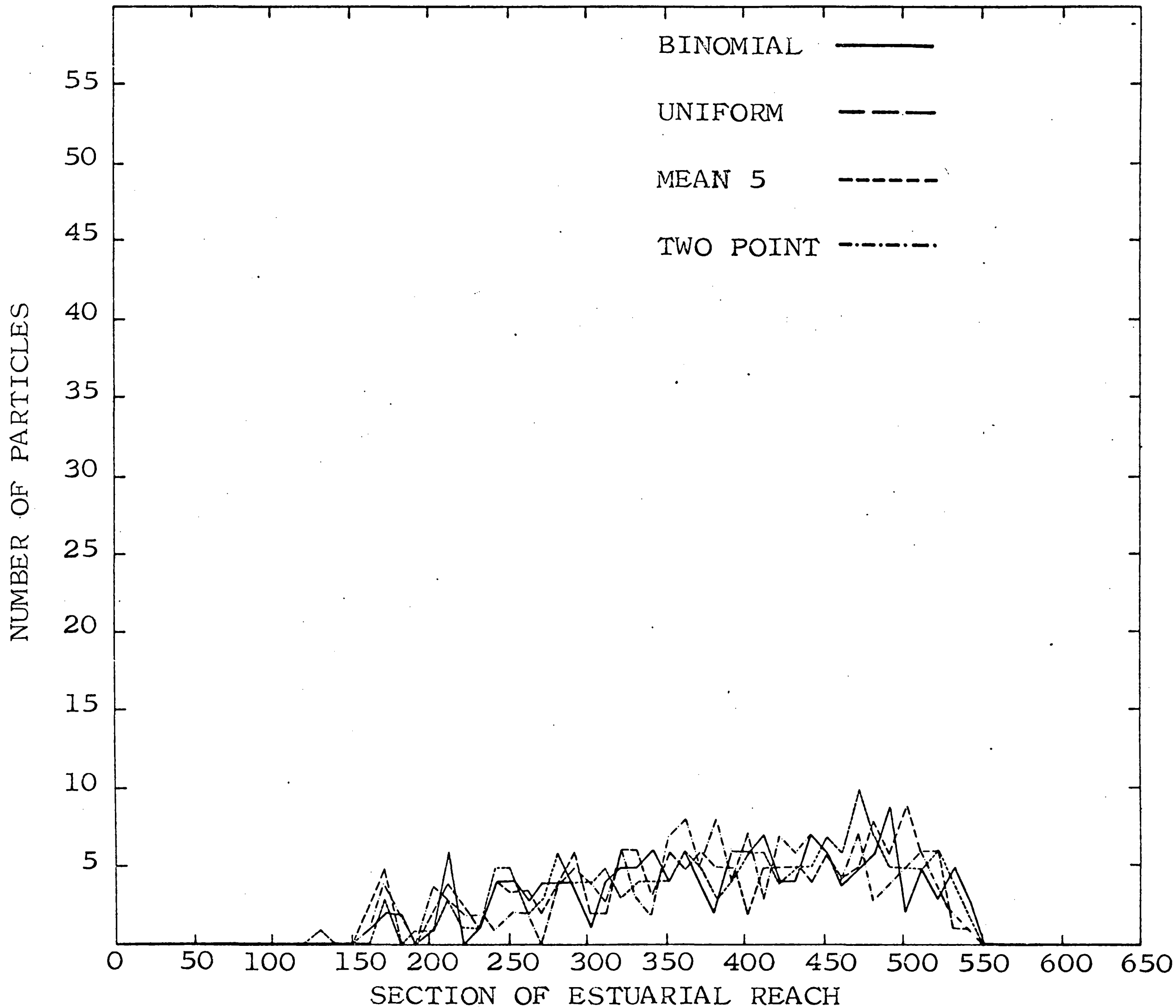


FIGURE 9. BIOCHEMICAL OXYGEN DEMAND - SITUATION 5

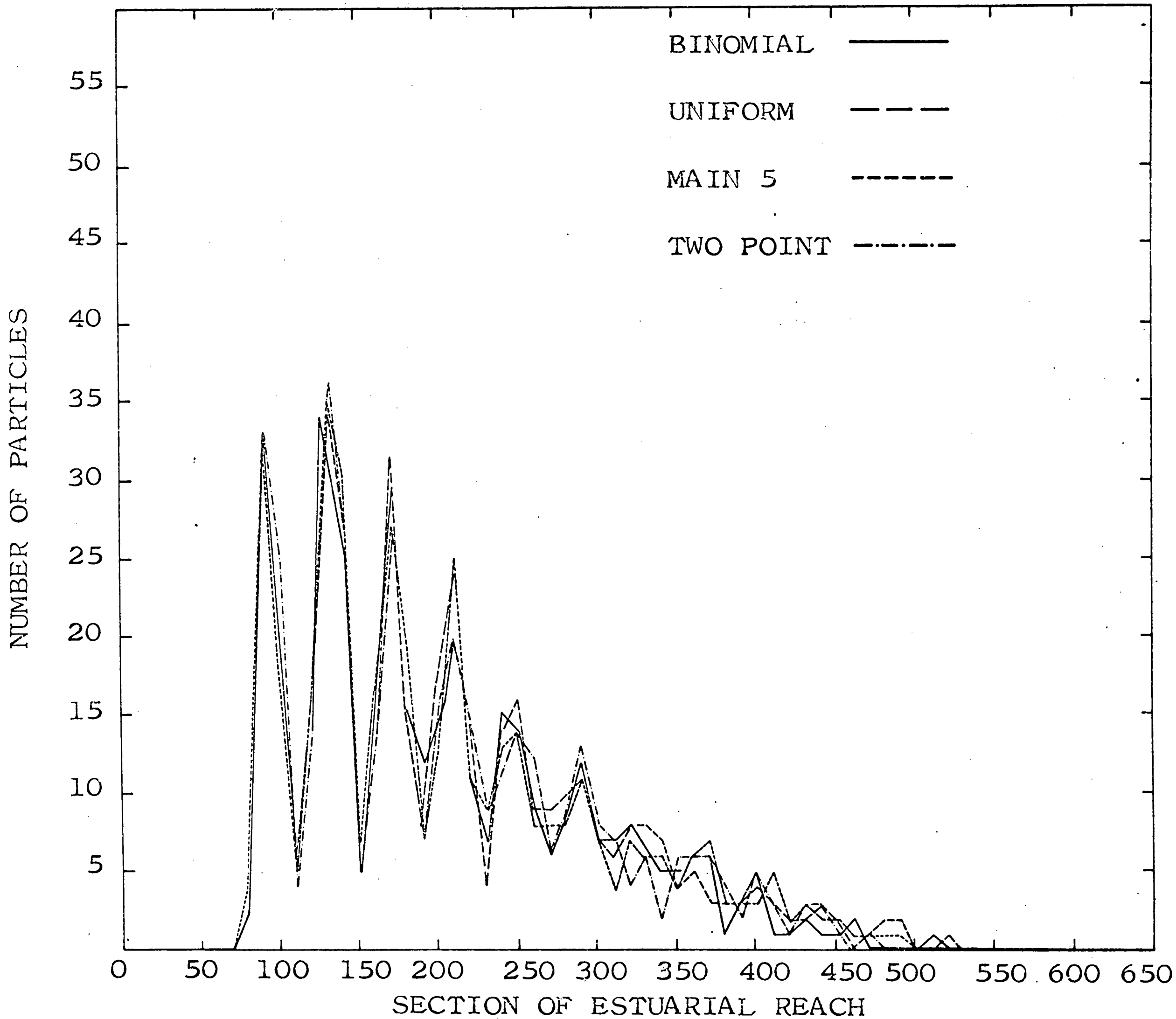


FIGURE 10. OXYGEN DEFICIT - SITUATION 5

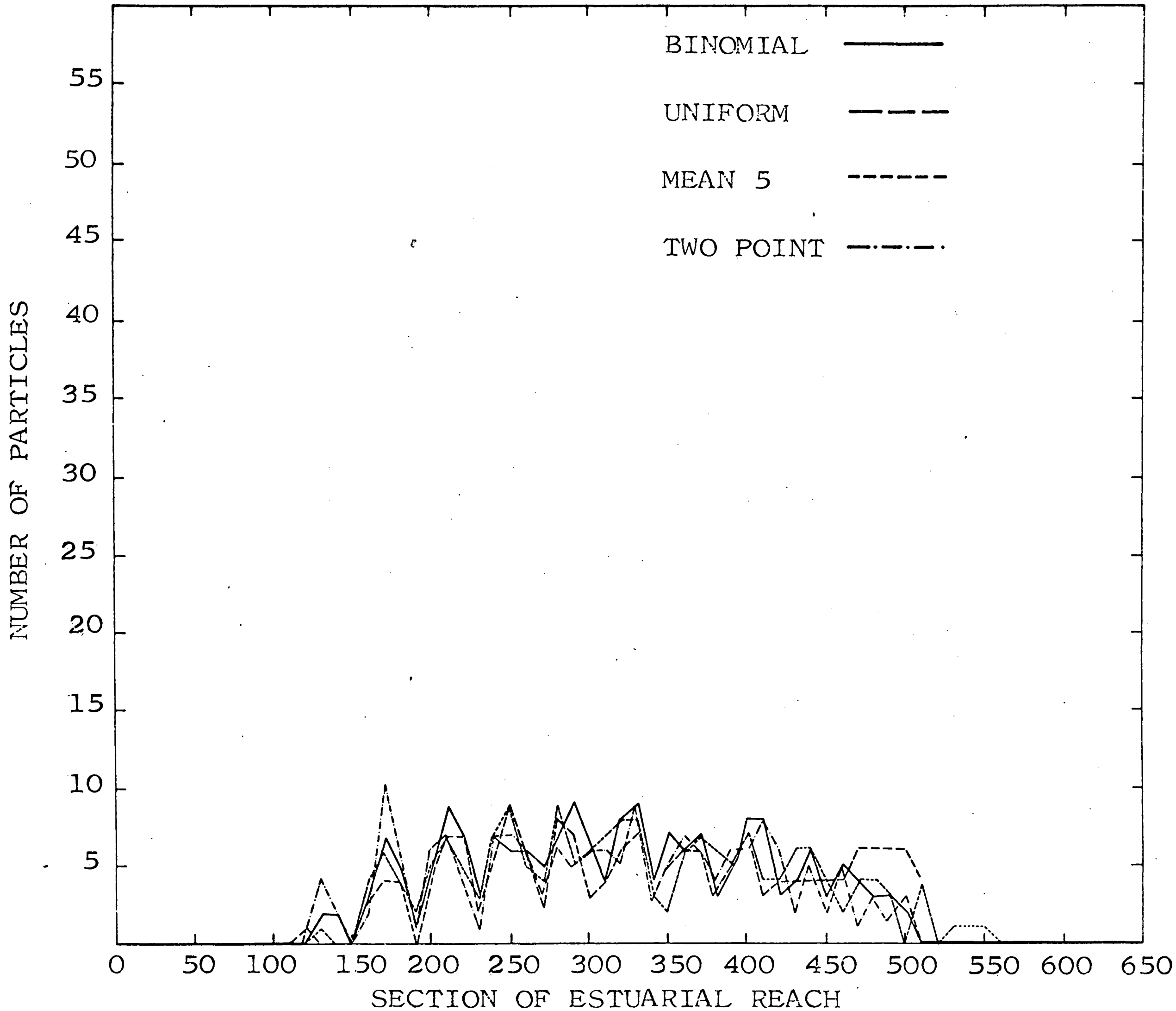


FIGURE 11. BIOCHEMICAL OXYGEN DEMAND - SITUATION 6

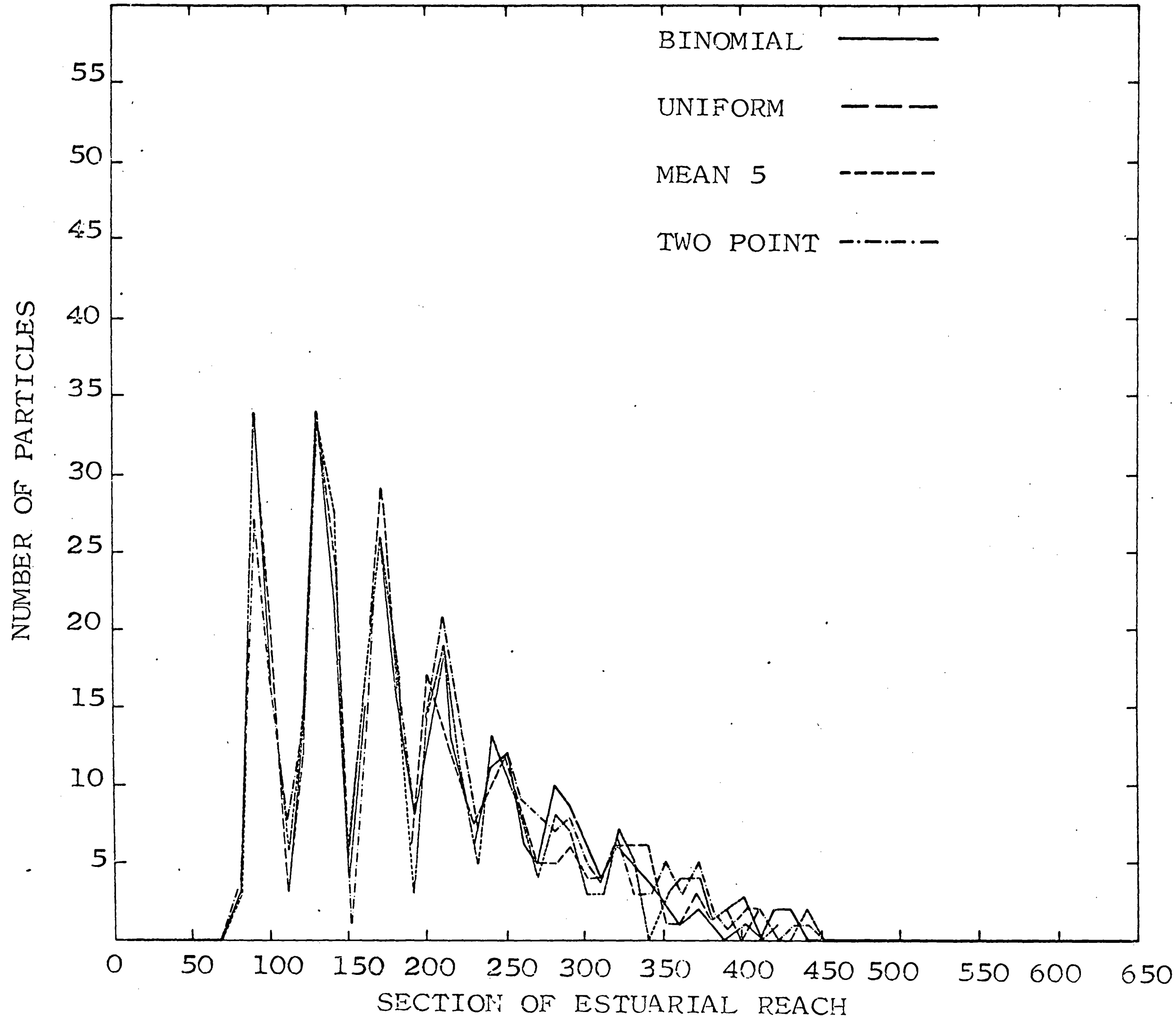


FIGURE 12. OXYGEN DEFICIT - SITUATION 6

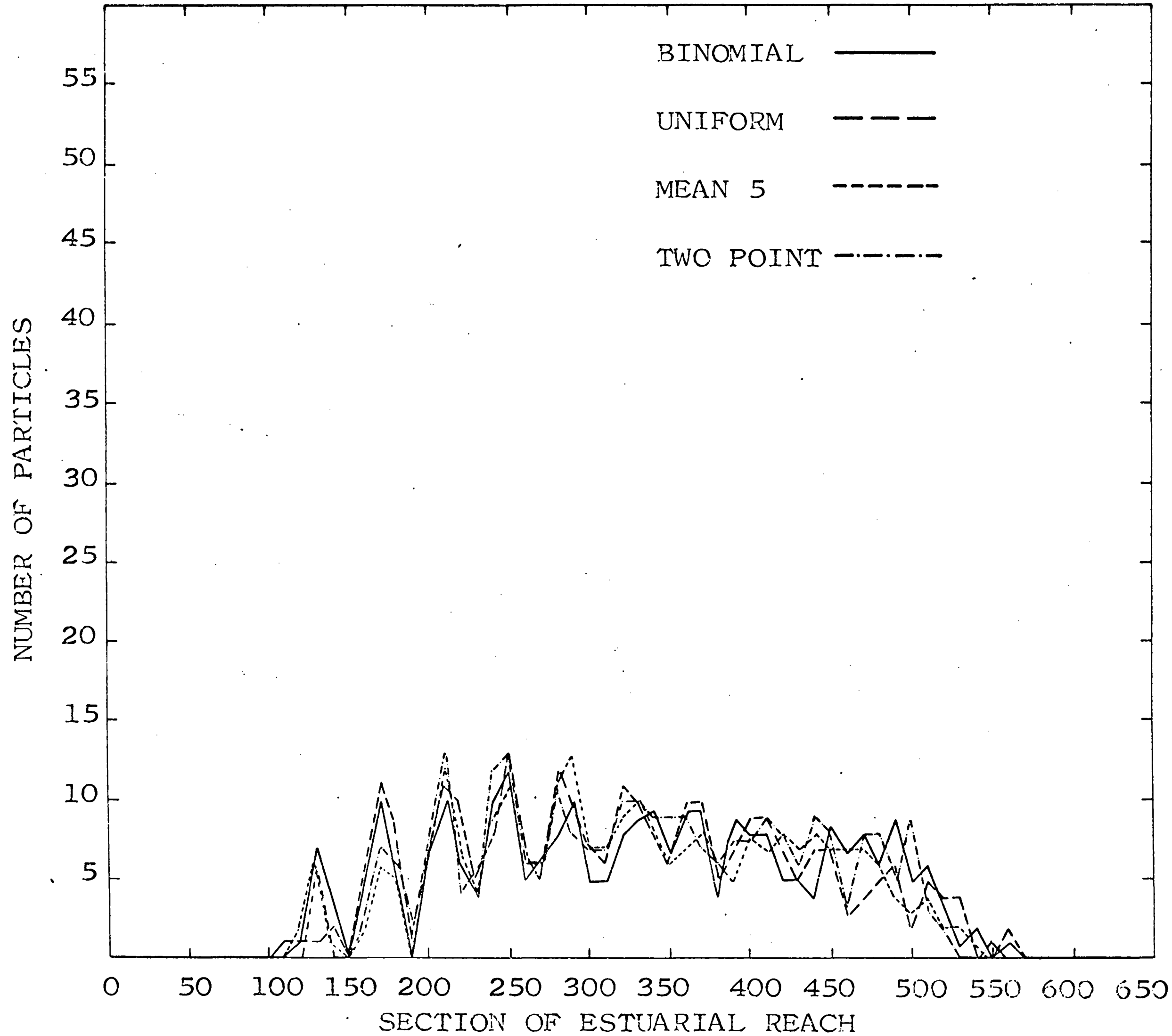


FIGURE 13. BIOCHEMICAL OXYGEN DEMAND - SITUATION 7

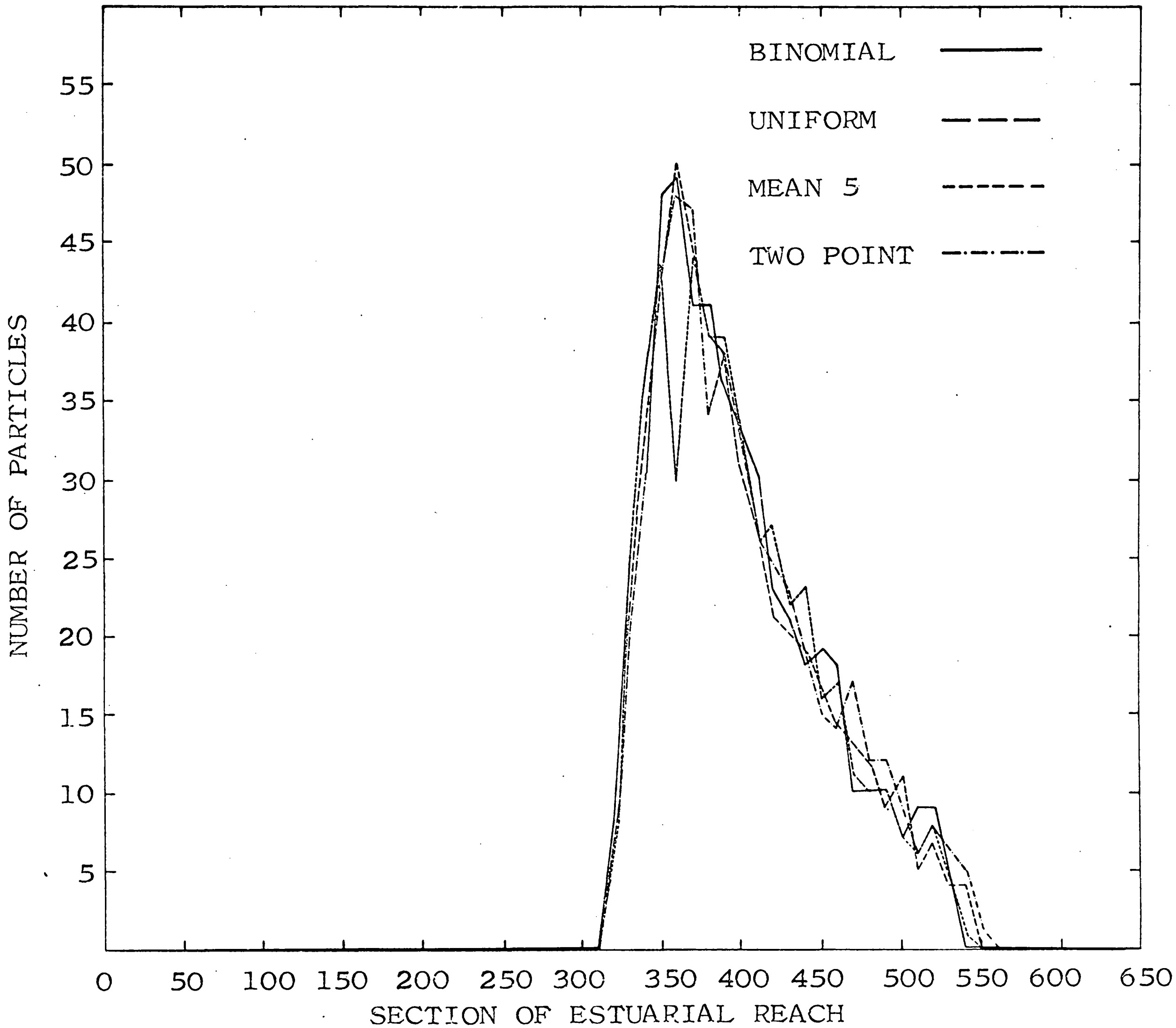


FIGURE 14. OXYGEN DEFICIT - SITUATION 7

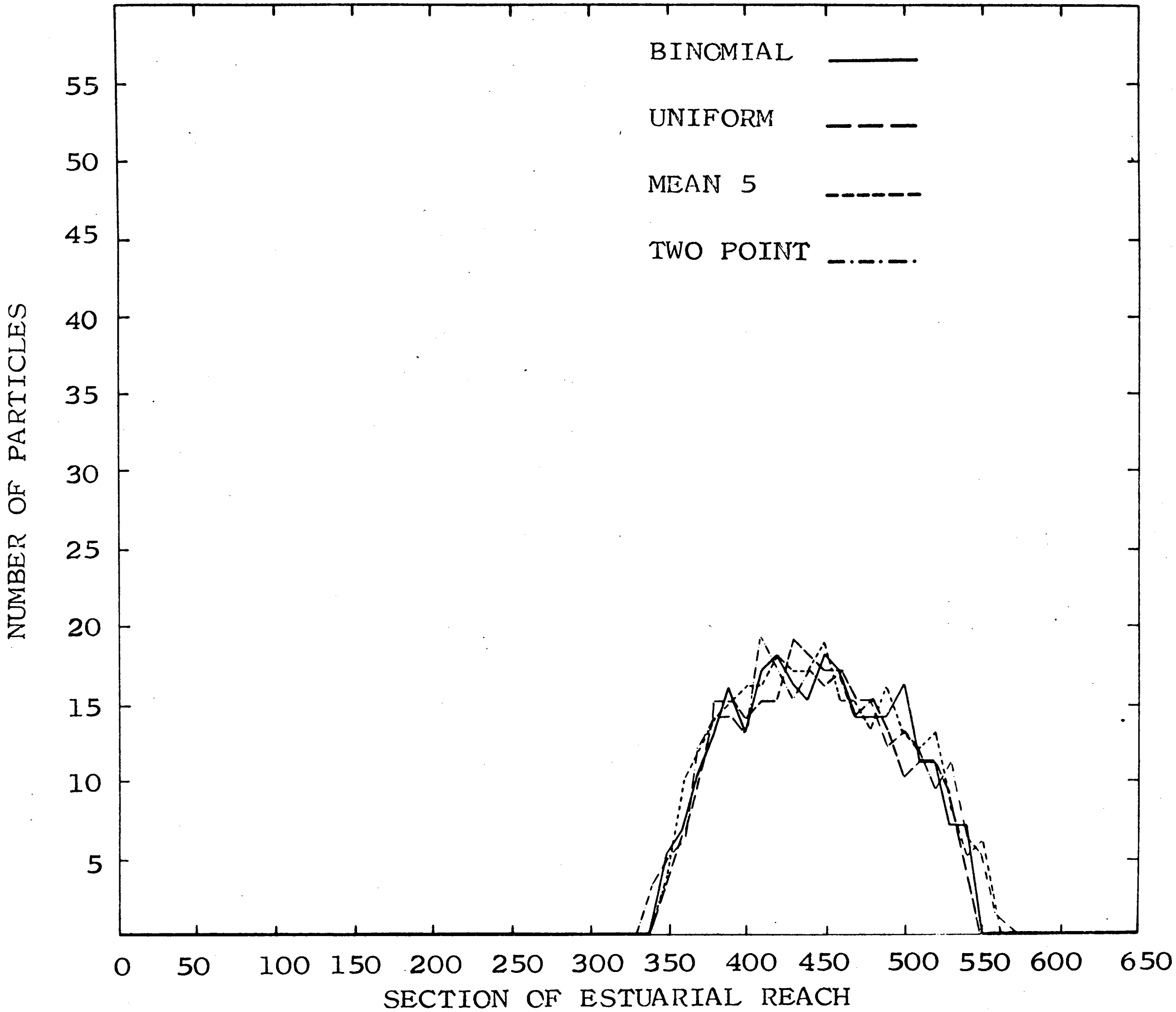


FIGURE 15. BIOCHEMICAL OXYGEN DEMAND - SITUATION 8

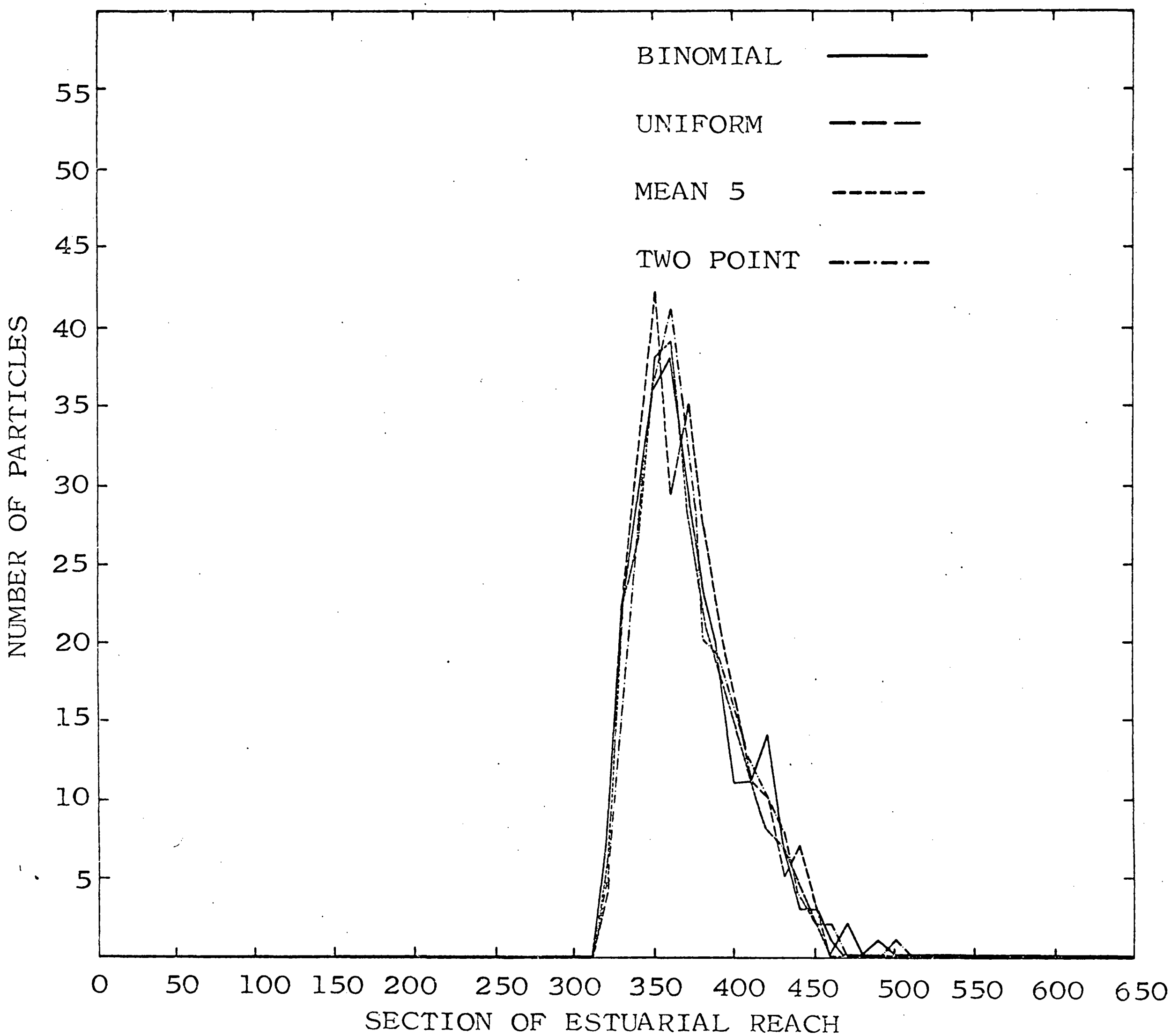


FIGURE 16. OXYGEN DEFICIT - SITUATION 8

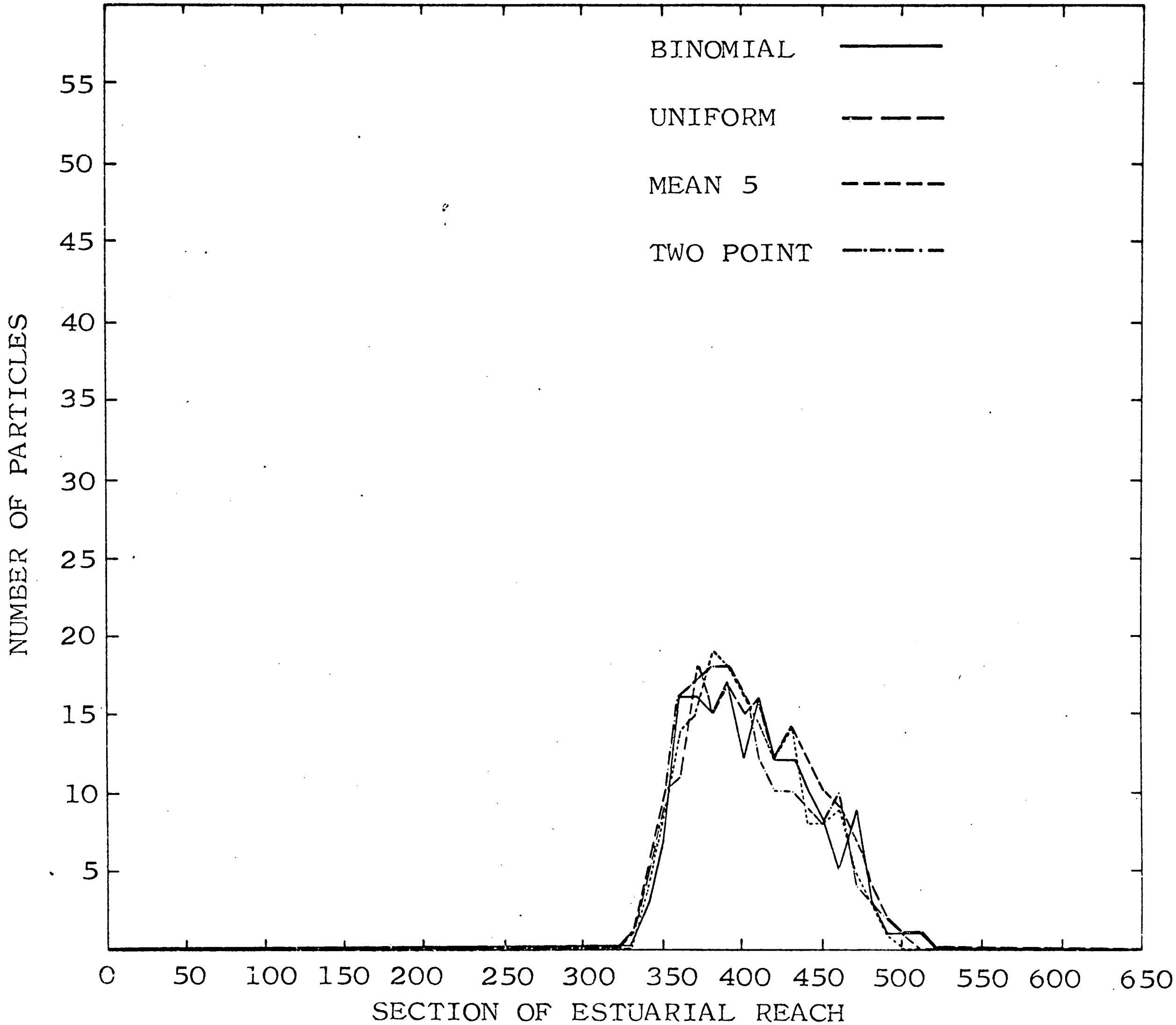


FIGURE 17.   BIOCHEMICAL OXYGEN DEMAND - SITUATION 9

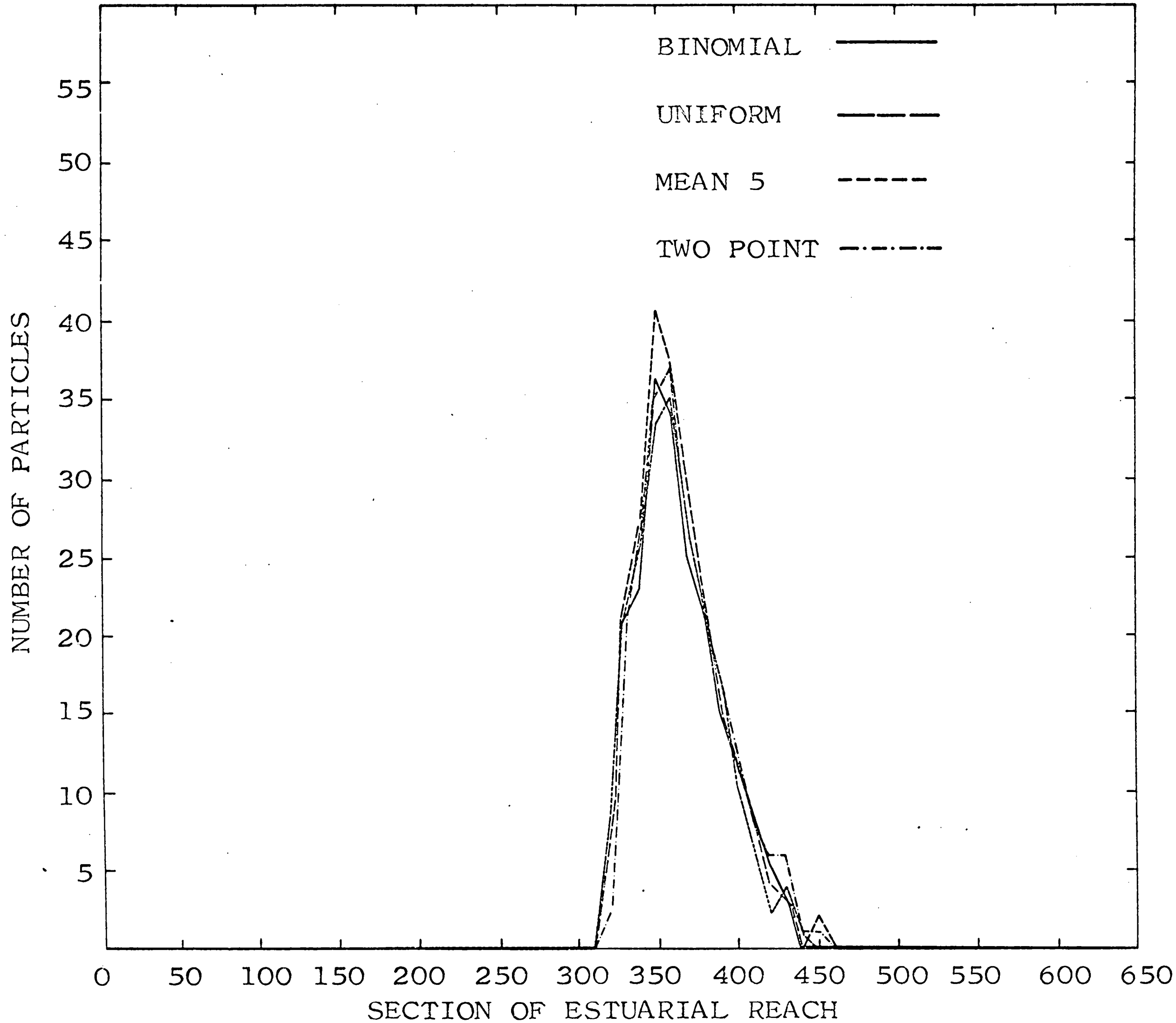


FIGURE 18. OXYGEN DEFICIT - SITUATION 9

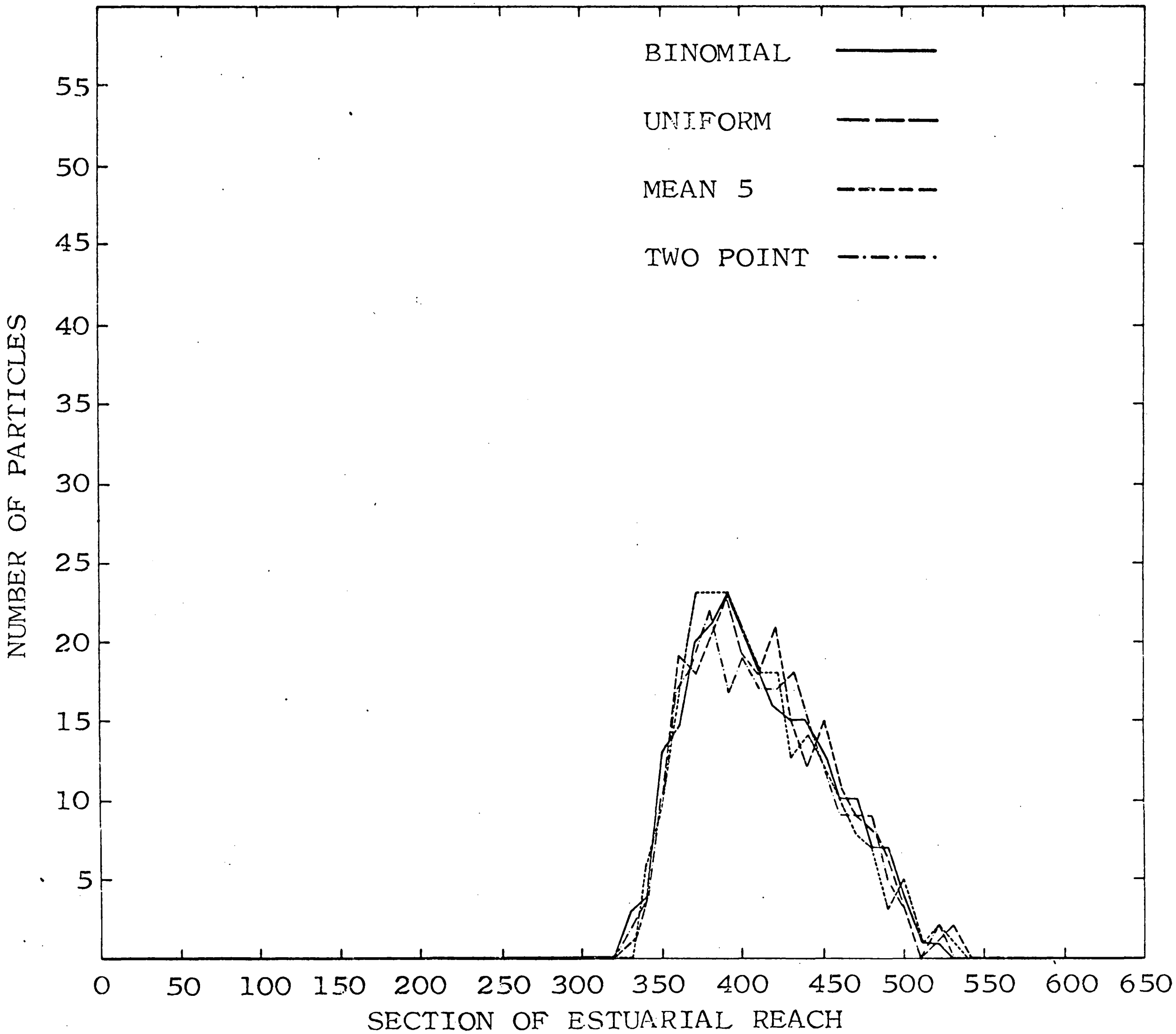


FIGURE 19. BIOCHEMICAL OXYGEN DEMAND - SITUATION 10

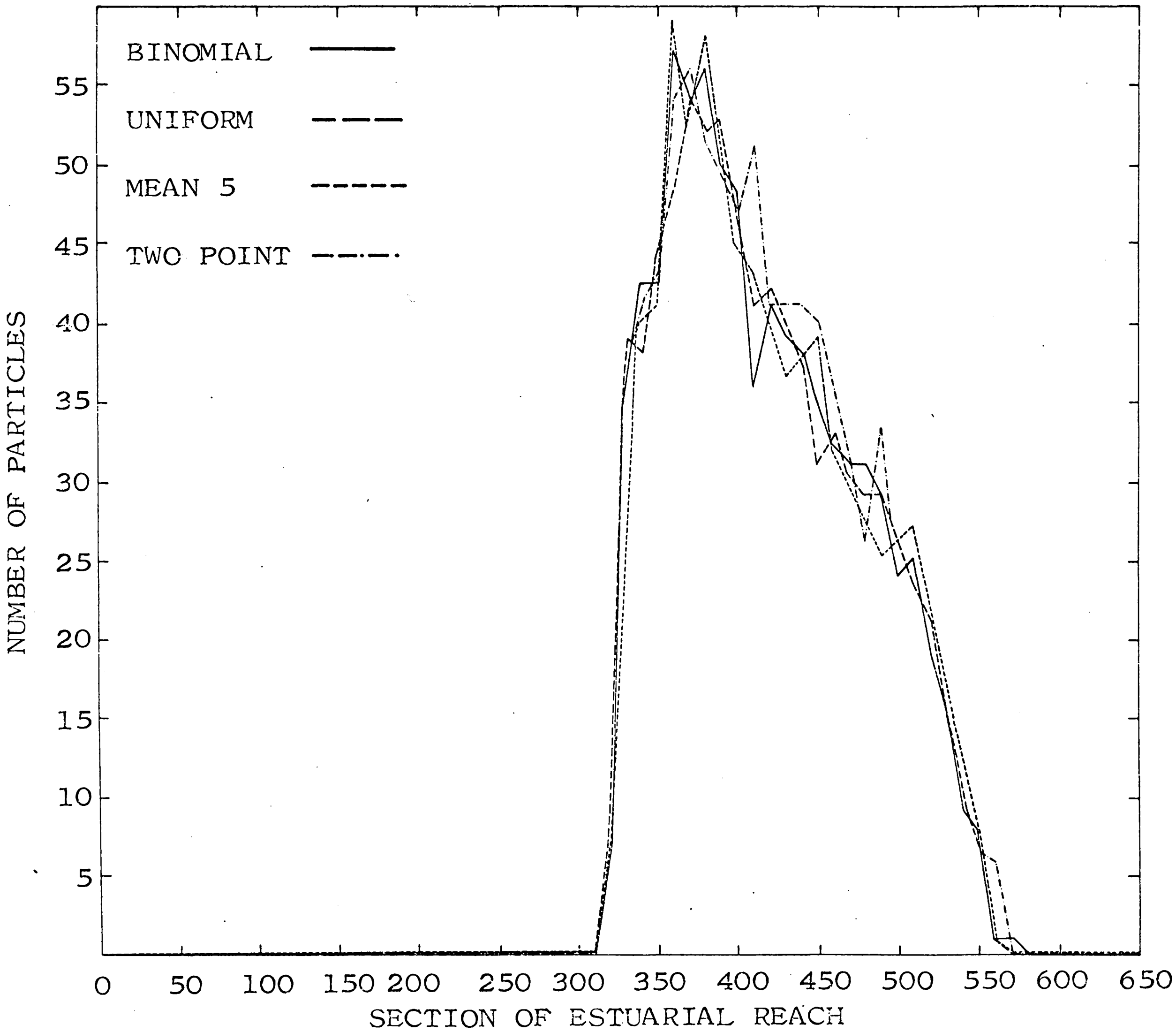


FIGURE 20. OXYGEN DEFICIT - SITUATION 10

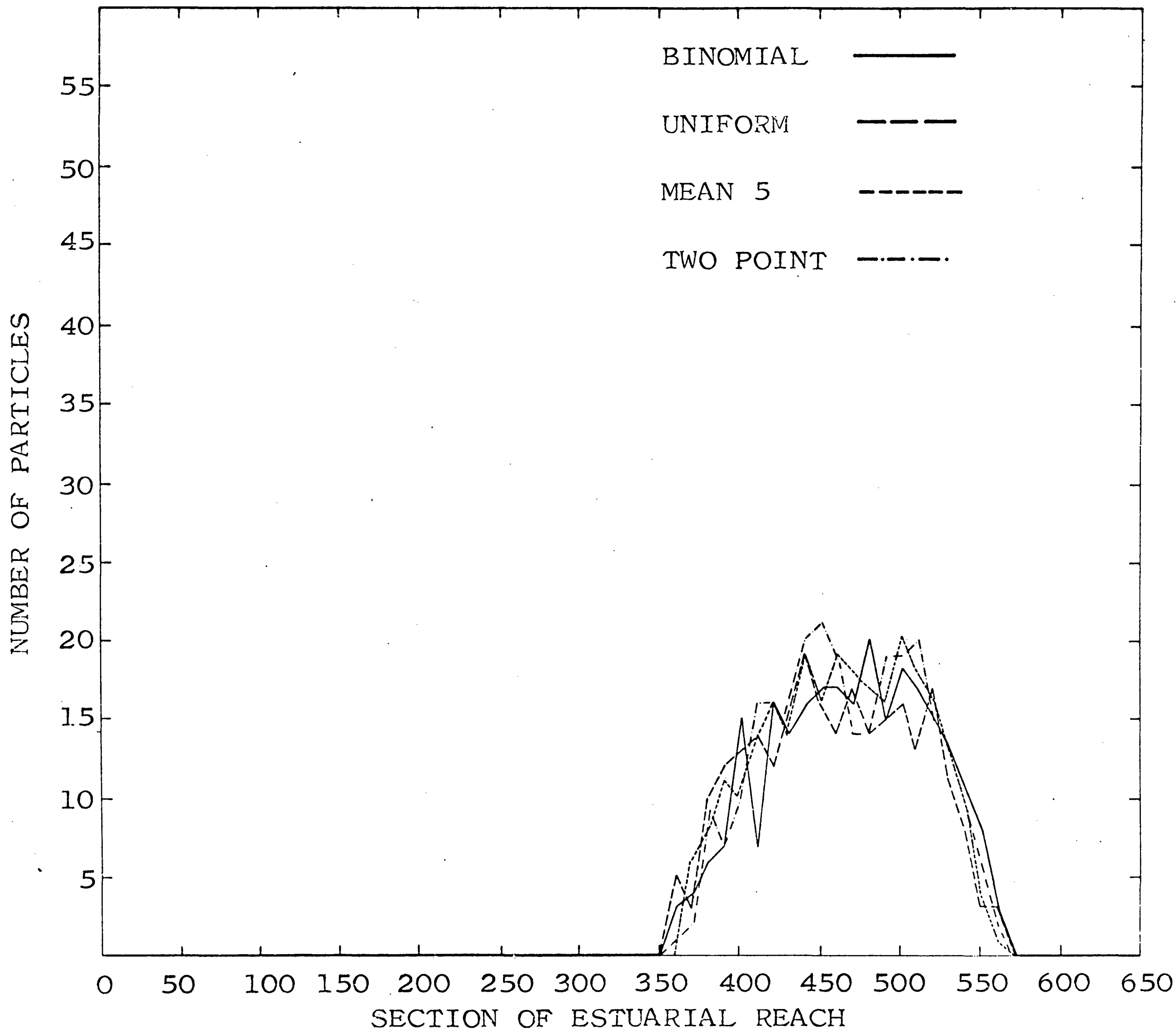


FIGURE 21. BIOCHEMICAL OXYGEN DEMAND - SITUATION 11

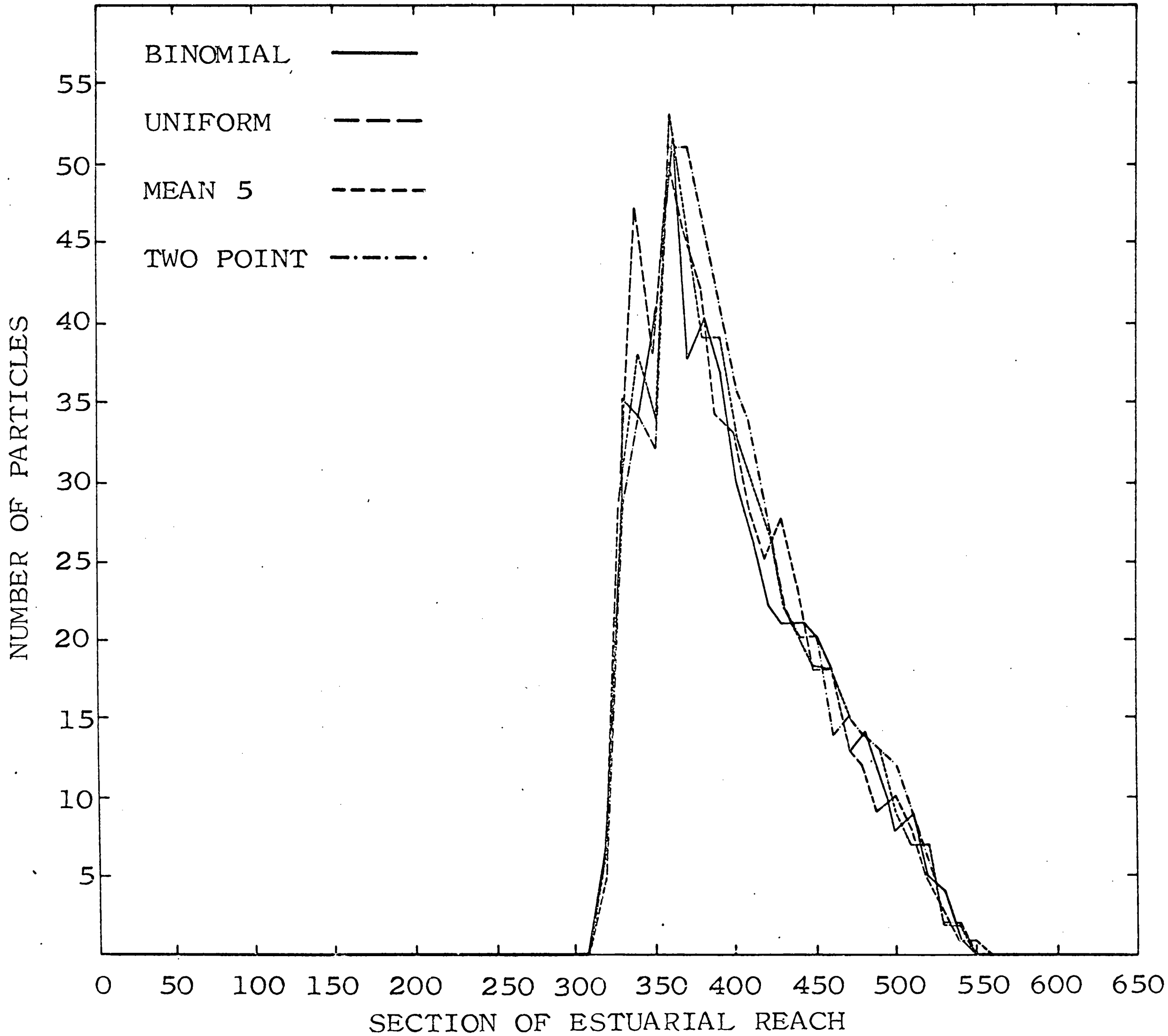


FIGURE 22. OXYGEN DEFICIT - SITUATION 11

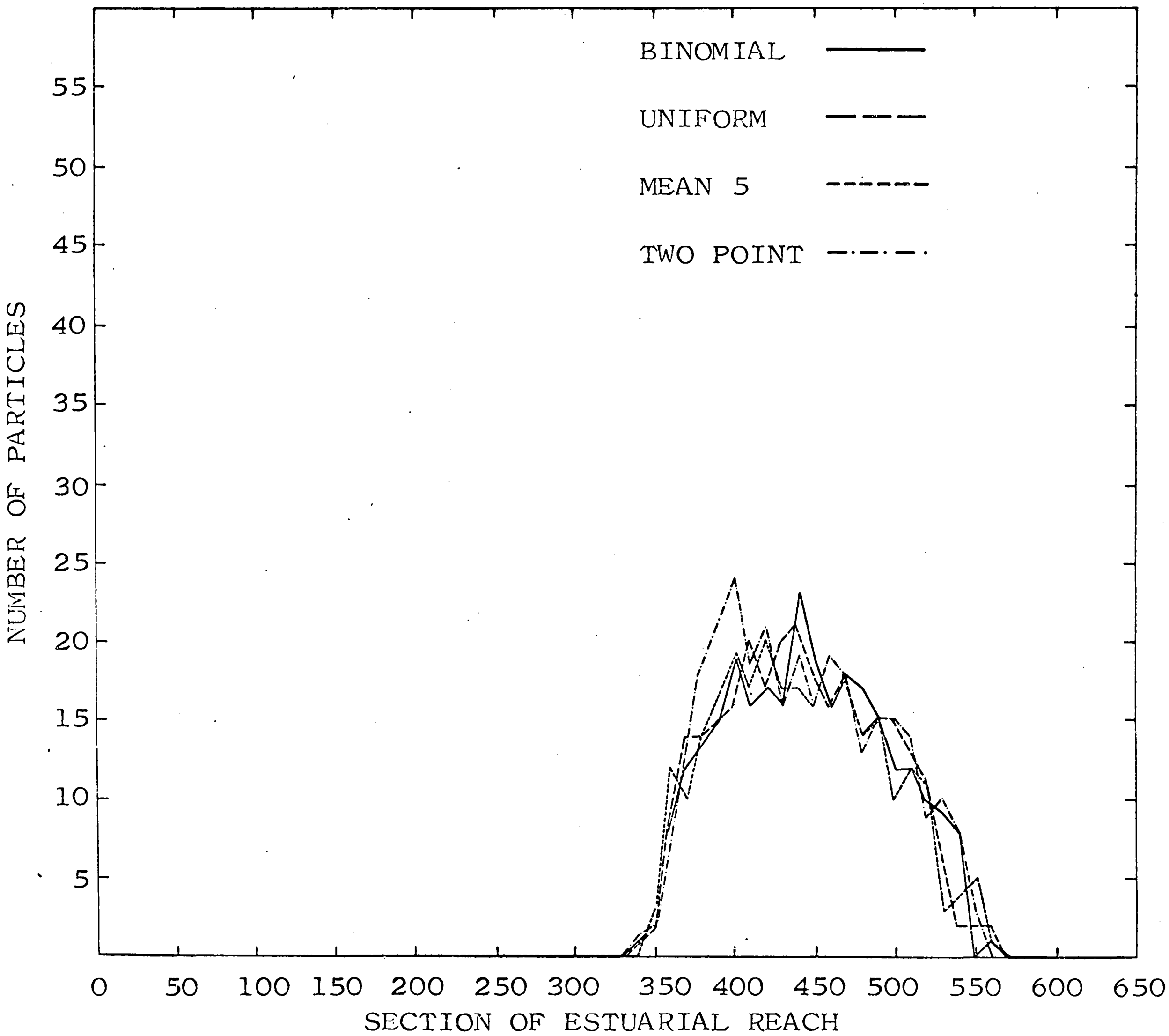


FIGURE 23. BIOCHEMICAL OXYGEN DEMAND - SITUATION 12

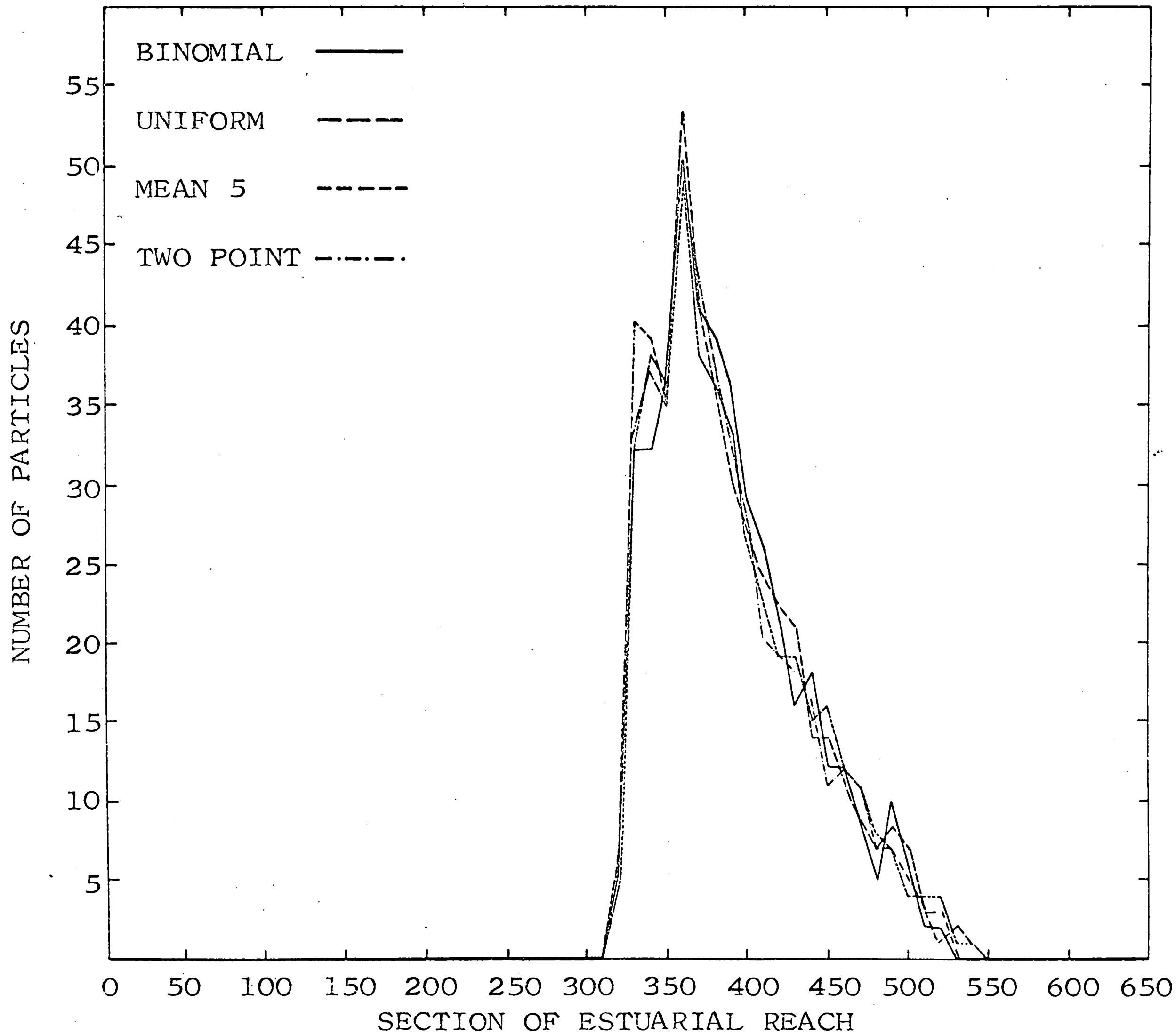
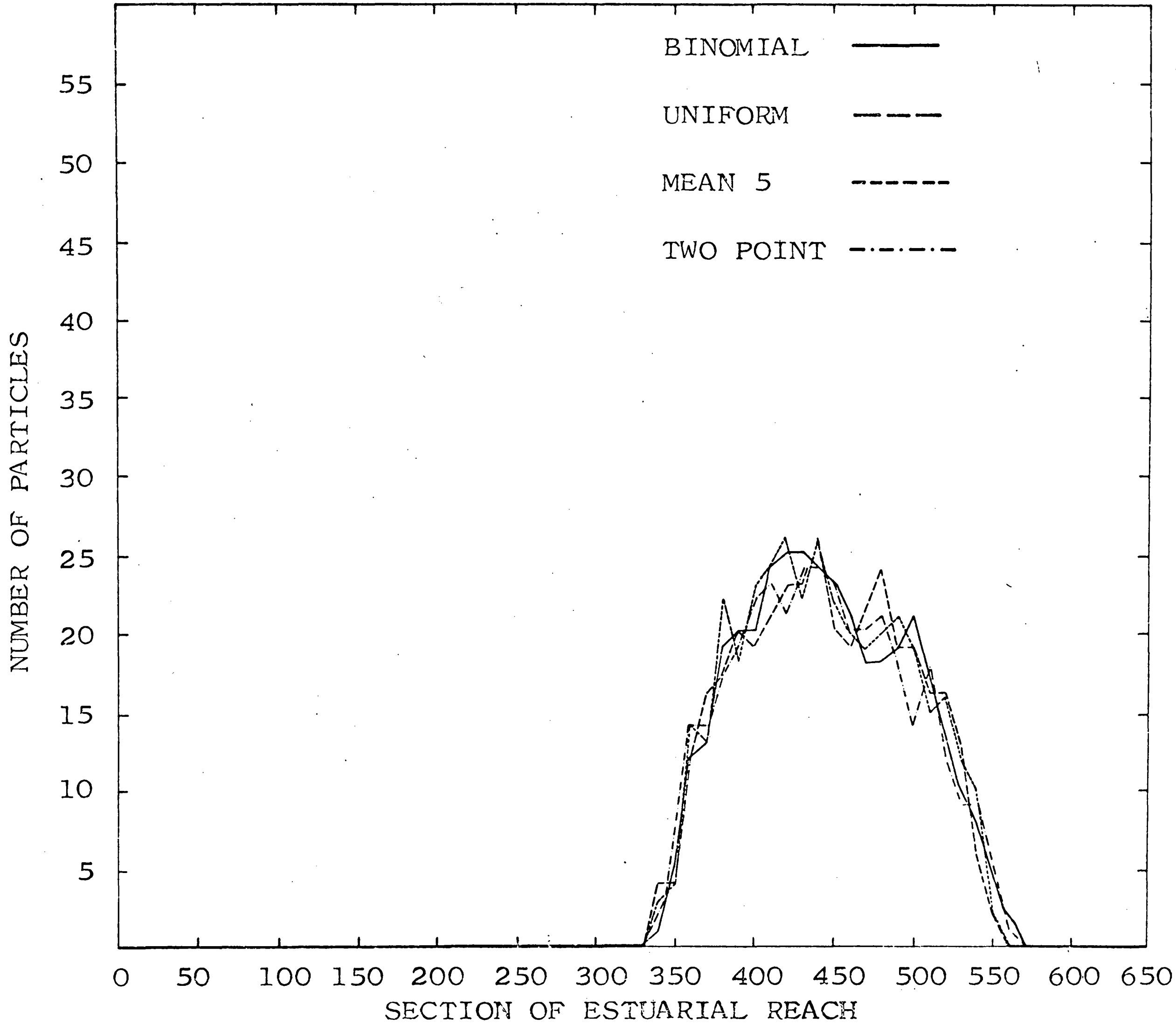


FIGURE 24. OXYGEN DEFICIT - SITUATION 12



IV CONCLUSIONS

This study, using stochastic simulation to determine the average distributions of particles of pollution and the average distributions of particles of oxygen deficit in estuaries resulting from various types of input, showed that the resulting distributions for all the types of input considered were similar. They were so close, in fact, as to make the drafting of the results all but impossible. There was no variation in outcome large enough to be attributed to any factor other than to the randomness of the process itself.

Since the distribution of pollution in estuaries is such an important factor in analyzing the effects of the pollution on the system and in determining the degree and type of treatment needed, it is important to understand exactly what effect the random characteristics of the input units of pollution into the estuary have on the results. This study indicates that the distributional properties of the influent pollution have an insignificant effect on the average distributions of both the pollution after entering the system and the ensuing oxygen deficit.

V BIBLIOGRAPHY

1. Cibulka, John J., Notes from Stream Sanitation 5012; Virginia Polytechnic Institute, Blacksburg, Virginia.

2. Clemmer, B. A., and Krutchkoff, Richard G., "Use of Empirical Bayes Estimators in a Linear Regression Model," Biometrika, Vol. 55, pp. 515-524.

3. Collett, W. F., "The Control of Estuarine Pollution," Chemistry and Industry, No. 11, January 7, 1967, pp. 25-29.

4. Custer, Stephen W., and Krutchkoff, Richard G., Stochastic Models for Biochemical Oxygen Demand and Dissolved Oxygen in Estuaries. Water Resources Research Center, Virginia Polytechnic Institute, Blacksburg, Virginia, February 1969.

5. De Falco, Paul Jr., "The Estuary - Septic Tank of the Megalopolis," Estuaries, George H. Lauff (ed.), Publication No. 83, American Association for the Advancement of Science, Washington, D. C., 1967.

6. Hammersley, J. M., and Handscomb, D. C., Monte Carlo Methods. John Wiley and Sons, Inc., New York, 1964.

7. Harleman, D. R. F., "Pollution in Estuaries," Estuary and Coastline Hydrodynamics, Arthur T. Ippen (ed.), McGraw-Hill Book Co., Inc., New York, 1966.

8. Hudson, J. D., and Krutchkoff, Richard G., "A Monte Carlo Investigation of the Size and Power of Tests Employing Satterthwaite's Synthetic Mean Squares," Biometrika, Vol. 55, pp. 431-433.

9. King, Paul H., Notes for Environmental Engineering 4027; Virginia Polytechnic Institute, Blacksburg, Virginia.

10. Krutchkoff, Richard G., "Classical and Inverse Methods of Calibration," Technometrics, Vol. 9, pp. 425-439.

11. Krutchkoff, Richard G., "Classical and Inverse Regression Methods of Calibration in Extrapolation," Technometrics,

Vol. 11, pp. 605-608.

12. Krutchkoff, Richard G., "Supplementary Sample Nonparametric Empirical Bayes," Biometrika, Vol. 54, pp. 451-458.

13. Martz, H. F., Jr., and Krutchkoff, Richard G., "Empirical Bayes Estimators in a Multiple Linear Regression Model," Biometrika, Vol. 56, pp. 367-374.

14. Reid, George K., Ecology of Inland Waters and Estuaries. Reinholt Publishing Corporation, New York, 1961.

15. Shreĭder, I͡Uliĭ Anatol'evich, The Monte Carlo Method. Pergamon Press, New York, 1966.

16. Thayer, Richard P., and Krutchkoff, Richard G., Stochastics Model for Pollution and Dissolved Oxygen in Streams. Water Resources Research Center, Virginia Polytechnic Institute, Blacksburg, Virginia, August, 1966.

VI APPENDIX-COMPUTER PROGRAM

```
C     INPUT FOR THIS PROGRAM IS UNIFORM
      REAL KD, KR
      INTEGER Z, T, B, C, R, S, U, V, W, G, KS, RS, KP, RP
      DIMENSION K(650),L(650),R(650),S(650),KS(650),RS(650)
     1KP(650),RP(650)
      READ (5,11) IX
   11 FORMAT (I10)
      DO 2101 NN=1,650
      KS(NN)=0
 2101 CONTINUE
      DO 2102 MM=1,650
      RS(MM)=0
 2102 CONTINUE
      DO 9 J=1,650,10
      KP(J)=0
    9 CONTINUE
      DO 8 J=1,650,10
      RP(J)=0
    8 CONTINUE
      KD=.17
      RR=KD*(1./48.0)
      RC=(1-RR)/2.
      KR=.22
      RA=KR*(1./48.0)
      RB=(1-RA)/2.
      DO 2103 LL=1,10
      DO 111 N=1,650
      K(N)=0
  111 CONTINUE
      DO 222 M=1,650
      L(M)=0
  222 CONTINUE
      DO 208 U=1,650
      R(U)=0
  208 CONTINUE
      DO 308 V=1,650
      S(V)=0
  308 CONTINUE
      DO 300 Z=1,150
      T=2*Z-2
      Y=.254*T
      Q=RC-(1./(96.0*.33))*(5.*SIN(Y)+.23)
      QR=Q+RR
      Q1=RB-(1./(96.0*.33))*(5.*SIN(Y)+.23)
      QE=Q1+RA
      B=550+(5.*SIN(Y)-(.23*T))/.33
      CALL SUB(IX,X)
      IF(X.GE.0.0.AND.X.LT..0909)K(B)=K(B)+0
```

```
      IF(X.GE..0909.AND.X.LT..1818)K(B)=K(B)+1
      IF(X.GE..1818.AND.X.LT..2727)K(B)=K(B)+2
      IF(X.GE..2727.AND.X.LT..3636)K(B)=K(B)+3
      IF(X.GE..3636.AND.X.LT..4545)K(B)=K(B)+4
      IF(X.GE..4545.AND.X.LT..5454)K(B)=K(B)+5
      IF(X.GE..5454.AND.X.LT..6363)K(B)=K(B)+6
      IF(X.GE..6363.AND.X.LT..7272)K(B)=K(B)+7
      IF(X.GE..7272.AND.X.LT..8182)K(B)=K(B)+8
      IF(X.GE..8182.AND.X.LT..9091)K(B)=K(B)+9
      IF(X.GE..9091.AND.X.LE.1.000)K(B)=K(B)+10
      DO 55 N=1,650
      IF (K(N) .EQ.0) GO TO 55
      J=K(N)
      DO 66 I=1,J
      CALL SUB (IX,X)
      IF (X.GE.Q.AND.X.LT.QR) GO TO 4001
      IF (N.EQ.1.AND.X.GE.0.0.AND.X.LT.Q) GO TO 4002
      IF (X.GE.0.0.AND.X.LT.Q) GO TO 2001
      IF (N.EQ.650) GO TO 4002
      L(N+1)=L(N+1)+1
      GO TO 66
4001  R(N)=R(N)+1
      GO TO 66
2001  L(N-1)=L(N-1)+1
      GO TO 66
4002  WRITE(6,4003)
4003  FORMAT(' SUBSCRIPT OUT OF RANGE')
 66   CONTINUE
 55   CONTINUE
      DO 44 N=1,650
      K(N)=0
 44   CONTINUE
      DO 707 U=1,650
      IF (R(U).EQ.0) GO TO 707
      W=R(U)
      DO 408 II=1,W
      CALL SUB(IX,X)
      IF(X.GE.Q1.AND.X.LT.QE) GO TO 408
      IF(U.EQ.1.AND.X.GE.0.0.AND.X.LT.Q1) GO TO 4004
      IF(X.GE.0.0.AND.X.LT.Q1) GO TO 2002
      IF (U.EQ.650) GO TO 4004
      S(U+1)=S(U+1)+1
      GO TO 408
2002  S(U-1)=S(U-1)+1
      GO TO 408
4004  WRITE(6,4005)
4005  FORMAT(' SUBSCRIPT OUT OF RANGE')
408   CONTINUE
 707  CONTINUE
```

```
      DO 708 U=1,650
      R(U)=0
  708 CONTINUE
      T=T+1
      Y=.254*T
      Q=RC-(1./(96.0*.33))*(5.*SIN(Y)+.23)
      QR=Q+RR
      Q1=RB-(1./(96.0*.33))*(5.*SIN(Y)+.23)
      QE=Q1+RA
      C=550+(5.*SIN(Y)-(.23*T))/.33
      CALL SUB(IX,X)
      IF(X.GE.0.0.AND.X.LT..0909)L(C)=L(C)+0
      IF(X.GE..0909.AND.X.LT..1818)L(C)=L(C)+1
      IF(X.GE..1818.AND.X.LT..2727)L(C)=L(C)+2
      IF(X.GE..2727.AND.X.LT..3636)L(C)=L(C)+3
      IF(X.GE..3636.AND.X.LT..4545)L(C)=L(C)+4
      IF(X.GE..4545.AND.X.LT..5454)L(C)=L(C)+5
      IF(X.GE..5454.AND.X.LT..6363)L(C)=L(C)+6
      IF(X.GE..6363.AND.X.LT..7272)L(C)=L(C)+7
      IF(X.GE..7272.AND.X.LT..8182)L(C)=L(C)+8
      IF(X.GE..8182.AND.X.LT..9091)L(C)=L(C)+9
      IF(X.GE..9091.AND.X.LE.1.000)L(C)=L(C)+10
      DO 22 M=1,650
      IF (L(M).EQ.0) GO TO 22
      J=L(M)
      DO 33 I=1,J
      CALL SUB(IX,X)
      IF (X.GE.0.AND.X.LT.QR) GO TO 4006
      IF (M.EQ.1.AND.X.GE.0.0.AND.X.LT.Q) GO TO 4007
      IF (X.GE.0.0.AND.X.LT.Q) GO TO 2003
      IF (M.EQ.650) GO TO 4007
      K(M+1)=K(M+1)+1
      GO TO 33
 4006 S(M)=S(M)+1
      GO TO 33
 2003 K(M-1)=K(M-1)+1
      GO TO 33
 4007 WRITE(6,4008)
 4008 FORMAT(' SUBSCRIPT OUT OF RANGE')
   33 CONTINUE
   22 CONTINUE
      DO 77 M=1,650
      L(M)=0
   77 CONTINUE
      DO 808 V=1,650
      IF (S(V).EQ.0) GO TO 808
      G=S(V)
      DO 908 JJ=1,6
      CALL SUB(IX,X)
```

```
      IF(X.GE.Q1.AND.X.LT.QE) GO TO 908
      IF(V.EQ.1.AND.X.GE.0.0.AND.X.LT.Q1) GO TO 4009
      IF(X.GE.0.0.AND.X.LT.Q1) GO TO 2004
      IF (V.EQ.650) GO TO 4009
      R(V+1)=R(V+1)+1
      GO TO 908
 2004 R(V-1)=R(V-1)+1
      GO TO 908
 4009 WRITE(6,4010)
 4010 FORMAT(' SUBSCRIPT OUT OF RANGE')
  908 CONTINUE
  808 CONTINUE
      DO 1208 V=1,650
      S(V)=0
 1208 CONTINUE
 300  CONTINUE
      DO 456 N=1,650
      KS(N)=KS(N)+K(N)
      RS(N)=RS(N)+R(N)
 456  CONTINUE
 2103 CONTINUE
      DO 3001 N=1,650
      KS(N)=KS(N)/10
      RS(N)=RS(N)/10
 3001 CONTINUE
      DO 1 J=1,650,10
      J1=J
      J2=J+9
      DO 2 M=J1,J2
      KP(J)=KP(J)+KS(M)
      RP(J)=RP(J)+RS(M)
   2  CONTINUE
   1  CONTINUE
      DO 1508 U=1,650,10
      WRITE(6,1408) U, KP(U), U, RP(U)
 1408 FORMAT(' KP(',I4,') = ',I5,8X,'RP(',I4,') = ',I5)
 1508 CONTINUE
      STOP
      END
      SUBROUTINE SUB(IX,X)
      IX=IX*65539
      IF (IX .GT. 0) GO TO 1
      IX=IX+2147483647+1
  1   X=IX
      X=X*.4656613E-9
      RETURN
      END
```

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A MONTE CARLO CASE
STUDY OF RANDOM INPUTS IN THE
STOCHASTIC MODEL FOR POLLUTION IN ESTUARIES
by
Sandra Grace Bartley

ABSTRACT

Using Richard Krutchkoff and Stephen Custer's stochastic model for pollution in estuaries, a Stochastic Simulation study of random inputs of pollution particles in estuaries was made. This study investigated four types of input in each of twelve estuarial situations and examined the resulting average distributions of both the biochemical oxygen demand and the oxygen deficit.

For two types of input, the number of input particles ranged from zero to ten and was chosen randomly from a binomial distribution and a uniform distribution. Another type of entry used an input of zero or ten particles, each with probability of .5. These three situations were compared with a constant input of five particles, which was the mean number of input particles for each of the probability-input cases.

Graphs were plotted using all four types of input for each estuarial situation. On the basis of these graphs, the study indicates that the average distributions of both

pollution particles and oxygen deficit particles which result from using random inputs do not differ significantly from each other and from the average distributions obtained by using the mean input.