

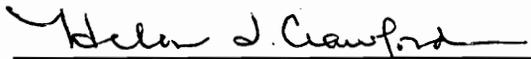
The Relation of Visuospatial and Analytical Skills  
and Span of Short-Term Memory to  
Academic Achievement in High School Geometry

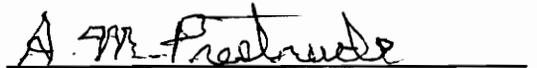
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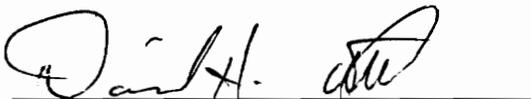
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The Relation of Visuospatial and Analytical Skills  
and Span of Short-Term Memory to  
Academic Achievement in High School Geometry

Martha L. Brown

(ABSTRACT)

The purpose of this research was to investigate hypothesized relations of visuospatial and logical reasoning skills, and span of short-term memory to achievement in geometry. In addition, major subfactors of visuospatial ability (visualization, speeded rotations, spatial orientation, and disembedding) were assessed to determine which were significant predictors of geometry achievement. Vernon's (1965) model of intelligence and Baddeley's model of working memory provided the theoretical framework for these hypotheses.

Subjects ( $N = 110$ ) were students in seven sophomore level geometry classes in two schools in southwest Virginia. Cognitive measures of speeded rotations, visualization, spatial orientation, disembedding, Gestalt closure, logical reasoning, and short-term memory span were administered. Two measures of geometry achievement were used: The standardized New York Regents Geometry Exam, and z-transformations of the classroom final grade.

A model of geometry achievement is proposed and major predictions of the model were supported. Within this sample, regression analysis showed the measures of

visualization, logical reasoning, and short-term memory predicted achievement on the New York Regents Geometry Exam. Separate regression analyses for each gender revealed visualization predicted geometry achievement for the girls, while logical reasoning and short-term memory span predicted geometry achievement for the boys. Gender differences favoring boys were found on measures of speeded rotations, spatial orientation, and Gestalt closure. Girls had significantly higher scores on the measure of short-term memory span and the classroom measure of geometry achievement.

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## INTRODUCTION

". . . the universe . . . stands continually open to our gaze, but it cannot be understood unless one first learns to comprehend the language and interpret the characters in which it is written. It is written in the language of mathematics, and its characters are triangles, circles, and other geometrical figures, without which it is humanly impossible to understand a single word of it; without those, one is wandering about in a dark labyrinth" (Galilei, 1623).

Galileo Galilei, the famous Italian physicist and astronomer, understood the importance of mathematics in comprehending the universe. This view has been expressed by more recent scholars as well. For example, Fox (1981) stated that "in our modern technological society, an understanding and appreciation of mathematics is becoming more and more important in almost every aspect of human endeavor" (p.34). Teachers are encouraged to begin instruction in geometric principles as early as elementary school because geometry is helpful in "representing and

describing the world in which we live" (National Council of Teachers of Mathematics, 1989).

Galileo also assumed the relation of mathematics and spatial concepts is interwoven. His assumption presages investigations of the relationship in the current century. This relation between the two cognitive abilities was also assumed by others as well. McGee (1979) reports Hamley defined mathematical ability as "a compound of general intelligence, visual imagery and ability to perceive number and space configurations and to retain such configurations as mental patterns" (p. 897). Teaching standards for teachers of mathematics at the elementary and secondary levels include the development of "spatial understanding" (National Council of Teachers of Mathematics, 1989, p. 48) because it is considered necessary for development of mathematical understanding.

*Cedric*  
Theories of intelligence have conceptualized mathematical ability and visuospatial ability as related.

While Thurstone (1938) conceived of mathematical and visuospatial skills as two of seven primary mental abilities that have little relationship to one another, Vernon's (1965) hierarchical model of intelligence relates ~~mathematical~~ abilities to both "spatial-practical-mechanical

and verbal-educational abilities" (p. 176). Several researchers (e.g., for reviews see Bishop, 1980; McGee, 1979) have demonstrated a positive relationship between tests of visuospatial skills and ~~mathematical~~ <sup>mathematical</sup> achievement.

Geometry is defined as "the branch of mathematics that deals with the deduction of the properties, measurement, and relationships of points, lines, angles, and figures in space from the defining conditions by means of certain assumed properties of space" (Urdang, 1968). By its definition geometry problem solving involves the application of a deductive reasoning process to spatial information using mathematical operations.

Deductive reasoning is defined by Mayer (1991) as "deriving a conclusion from given premises . . . [which] involves the combining of existing information by following specific mental operations as in addition or subtraction" (p. 116-117). He states that solving <sup>implies</sup> mathematical proofs is a deductive process, <sup>as well as a ~~deductive~~ ~~process~~ ~~on~~ ~~research~~ ~~to~~ ~~develop~~</sup>

The most recent standards established for geometry education include the aforementioned development of spatial understanding and the development of skills in deduction (National Council for Teachers of Mathematics, 1989). Students at the secondary education level are to have

experiences in "visualiz[ing] and represent[ing] geometric figures . . . exploration of the transformation of figures . . . deduc[ing] properties of, and relationships between, figures from given assumptions" (National Council for Teachers of Mathematics, 1989, p. 48).

Given these standards it is logical that skill in processing visuospatial information would enhance achievement in high school geometry. It is also logical that analytical reasoning, or deductive reasoning, would be important in successful geometry problem solving. It is a goal of the research described herein to test if geometry is a branch of mathematics which, as Vernon's model conceptualizes it, is comprised of both visuospatial and analytical reasoning processes.

In addition to the skills of visuospatial and analytical reasoning, the capacity of hold information in short-term memory should also be a significant process for successful geometry problem solving. The importance of short-term memory is congruent with the working memory model of Baddeley (1986, 1992).

In Baddeley's (1986, 1992) model short-term memory is a small element of a larger, more complex working memory system. The model includes a central executive which makes

decisions, supervises and coordinates the activities of subsidiary slave systems, identified as the phonological loop and the visuospatial scratch pad. The phonological loop consists of a phonological store which includes the control processes of articulatory rehearsal. The other subsidiary system, the visuospatial scratch pad, is used for the performance of visuospatial tasks. Baddeley (1986, 1992) argues working memory is important in performance of cognitive operations and experimental support has been provided by several researchers. Yet to be investigated is the relation between mathematical performance and working memory capacity within the context of Baddely's model. This research examined the role of short-term memory span in geometry problem solving.

In addition, the issue of gender is addressed since visuospatial and mathematical reasoning are two cognitive processes where differences favoring boys and men have most reliably been found (Maccoby & Jacklin, 1974). Maccoby and Jacklin note, as have other researchers (e.g., Burnett, 1986; Hyde, Fennema, & Lamon, 1990) the male advantage typically emerges during adolescence and is maintained during the adult years. Smith (1964) proposed gender differences in visuospatial skills might be the factor

contributing to gender differences in mathematics. He suggested "boys [possess] . . . a greater capacity . . . to perceive, recognize, and assimilate patterns within the conceptual structure of mathematics" (p. 123). Given the persistence of gender differences in mathematical and visuospatial skills, gender differences were also investigated in this thesis.

Mathematics courses have often served as a de facto "filter" in American education, precluding those who do not undertake or successfully complete advanced mathematics from important career options (National Research Council, 1989). Research into the cognitive skills required for successful geometry problem solving is needed to help prevent this "filter" effect and to assist in making geometry serve as a "pump" into careers which emphasize complex cognitive skills (National Research Council, 1989; Sells, 1980).

This thesis presents two studies whose goals were to investigate the relation of visuospatial and analytical skills and short-term memory span to geometry problem solving, and to explore gender differences in these cognitive processes. In order to provide a context for this research and to demonstrate how the research extends earlier work, a review of pertinent literature is provided in the

next section. The first two subsections discuss the relationship of visuospatial skills and mathematical skills to one another and to other cognitive domains, and the third subsection reviews research on the moderating influence of working memory on performance of cognitive tasks. Research investigating gender differences in visuospatial skills and mathematics is discussed in the subsequent two subsections. The last section describes the current research and its hypotheses.

## REVIEW OF THE LITERATURE

Visuospatial Skills: Relationships to Academic Experience

Visuospatial skills have been studied since Galton studied imagery in the late 1800's, and evolved into the testing of these skills with individually administered tests such as Binet-Simon's intelligence test and MacFarlane's wood manipulatives (Wiggly Blocks) in the early 1900's (Eliot & Smith, 1983). World War I provided the impetus for the development of group administered paper and pencil measures of visuospatial skills, and the debate on the relationship between verbal and nonverbal abilities ensued. Since the mid-1920's factor analytic studies of ability measures have consistently demonstrated the separation of verbal and visuospatial factors (Lohman, 1988; McGee, 1979).

Factor analytic studies have revealed a number of visuospatial subfactors (Eliot & Smith, 1983; McGee, 1979). Until recently, there have been different ways of categorizing, describing, and naming these subfactors. In a review of factor analytic studies, Lohman (1988) reported that three spatial factors emerged consistently in review: (1) speeded rotations, solving spatial problems involving mental rotation quickly, such as the Mental Rotations Test (Vandenberg & Kuse, 1978); (2) spatial orientation,

imagining how an arrangement of stimuli would appear from another perspective, measured by such tests as the Card Rotations test, (Ekstrom, French, Harman, & Derman, 1976); and (3) visualization, skill in solving complex two-dimensional problems involving the movement of component parts into an organized whole, measured by tests such as the Surface Development Test (Ekstrom et al., 1976). Factors identified by other researchers include disembedding, skill in identifying a configuration despite background distractors (Crawford, 1989; Stericker & LeVesconte, 1982), such as the Hidden Patterns Test (Ekstrom et al., 1976).

Research has shown that differential course taking of undergraduate university students has had an impact on development of visuospatial skills (e.g., Baenninger & Newcombe, 1989; Burnett & Lane, 1980). This research is reviewed below.

Burnett and Lane (1980) investigated the effect of differential academic experience on the development of spatial visualization skills. Their study was intended to be a replication and extension of an earlier study of Blade and Watson (1955) who found that male engineering undergraduates increased their scores on the Spatial Relations Test, Form Vac-1 of the College Entrance

Examination Board, significantly more than a group of male nonengineering majors after one year of undergraduate experience. Burnett and Lane extended the study by including women and retesting their groups after four semesters rather than two semesters. In addition to examining gender differences, they examined "the contribution of specific course experiences to the improvement in spatial ability" (p. 235).

Burnett and Lane (1980) found that students majoring in science and mathematics improved significantly more on the Guilford Zimmerman Spatial Visualization Test than those majoring in humanities, social sciences, and "undecided." There were no gender differences in the improvement of the scores, though Burnett and Lane noted that women seemed to benefit slightly more than men from their experience in mathematics and sciences courses. The number of mathematics and physical courses taken predicted the improvement in scores, with mathematics courses being the stronger predictor. In addition, when the number of mathematics courses was controlled for, physical science courses no longer contributed significantly to the improvement in spatial visualization. This suggests that mathematics may

be the "critical element" in relevant experience for improvement in spatial visualization.

Burnett and Lane (1980) did not report if there was a significant difference between the different majors on the spatial visualization pretest. They noted in their discussion that there was a correlation between majors and initial scores. Assuming the nonmath/nonscience majors had lower visuospatial scores, the question arises whether this group would similarly benefit from more mathematics and science course experience. Another question is whether gains on the posttest are due to experience on the pretest and if some students differentially benefit from this experience.

A recent meta-analysis by Baenninger and Newcombe (1989) reanalyzed data from studies exploring the role of differential course taking on visuospatial scores and produced results supporting Burnett and Lane (1980). Their analysis showed experiences in physical science, mathematics, and engineering <sup>curricula</sup> compared to liberal arts, humanities, and undecided majors, increased visuospatial test scores over a two to four year period. These studies clearly indicate that course work emphasizing complex problem solving enhances visuospatial skills,

providing further evidence for a relationship between the two cognitive skills.

### Mathematical Skills: Relationships to Other Cognitive Skills

Researchers studying differences in mathematical performance have examined a number of factors contributing to this variability. Cognitive factors such as analytical, verbal, and visuospatial skills are frequently the best predictors of mathematical performance (Briars, 1983). This research is reviewed below, with an emphasis on research examining the relation between visuospatial and mathematical skills.

Battista, Wheatley, and Talsma (1982) examined the effect of mathematics course experience on the development of visuospatial skills. These researchers hypothesized that a geometry course for preservice elementary teachers which included manipulative spatial classroom activities would increase their scores on the Rotations subtest of the Purdue Spatial Visualization Test, which measures the rotation of three dimensional objects. This test would be considered a measure of speeded rotations according to Lohman and Kyllonen (1983). Their hypothesis was confirmed. In addition, there was a positive relationship between speeded

rotations, cognitive level of development, as measured by the Piagetian based Longeot test (Sheehan, 1970), and the final geometry grade, with cognitive development accounting for the larger share of the variance on the final grade. However, their failure to include a control group which did not receive geometry training, and the use of the same test for both pre- and posttests, makes interpretation of the pretest-posttest results problematic. In addition, the subjects were not adequately described, i.e., relative proportions of men and women, year in school, and the number of math classes previously taken. The lack of this information makes interpretation of their results difficult.

Pulos, Stage, and Karplus (1982) included the differential influence of three schools in their study of the relationship of different cognitive variables and performance of eighth graders on mathematical reasoning tasks. The cognitive tasks included the vocabulary tests (postulated to measure crystallized intelligence) and series completion (considered a measure of fluid intelligence), both tests are from the Primary Mental Ability Test (Thurstone, 1962) and the Figural Intersection Task, considered a measure of processing capacity (DeAvila, Havassy, & Pascual-Leone, 1976). They included two tests as

measures of disembedding (a group embedded figures test, the Find a Shape Puzzle; Pulos & Linn, 1979) and the Water Level Task (DeAvila et al., 1976). The authors did not provide descriptions of the tests except to note the skills or abilities they were intended to measure. The mathematical reasoning tests included a proportional reasoning task, a reasoning with unknowns task, and a spatial reasoning task.

Pulos et al.'s results showed that the pattern of correlations and amount of variance in the tasks explained by the cognitive tests varied across the different schools and tasks. The authors hypothesized that this differential pattern may be due to different strategies being taught that have distinct cognitive demands at the different schools, or correlations may be high when subjects are acquiring a skill but later decrease when the skill is automatized. The authors proposed that the differential correlations and variance accounted for by the schools are due to different student attitudes and in the information communicated by the different curriculums. There were no gender differences on any of the mathematical reasoning problems. The only cognitive factor which showed gender differences was the Water level task, with boys outperforming the girls. The authors concluded that a systems approach which takes into

account the many variables influencing students' mathematical performance, sociocultural, educational, and cognitive, is necessary to gain an understanding of differential patterns of mathematics achievement.

Linn and Pulos (1983) extended the Pulos et al. (1982) research on proportional reasoning described above to include experience variables. Their subjects were seventh, ninth, and eleventh graders from three different school districts. They employed the same cognitive tests described above, except they added the Paper Folding test from the Ekstrom et al. (1976) test battery as a measure of general fluid visualization. The cognitive tests were considered their aptitude measures. In addition they included four measures of Piagetian formal reasoning, called Combinations, Permutations, Predicting Displaced Volume, and Controlling Variables. The first two were developed for this research and the last two they had previously developed and used. The proportional reasoning test was the Balance Puzzle (Adi & Pulos, 1980) which was composed of three types of items-- estimation, standard, and complex. Experience in science and mathematics courses was assessed by self-reported number of courses and length of the course.

Results indicated that, overall, performance of both genders and all the grades was quite low. Boys in all grades performed slightly better than girls, and eleventh grade boys performed the best of all the groups. All the cognitive and Piagetian formal reasoning variables correlated significantly for both genders, however a multiple regression analysis showed that none of the cognitive variables or the experience variable accounted for the gender difference in proportional reasoning. The authors found that the factors they called general crystallized (vocabulary) and general fluid visualization (Paper Folding Test) were the strongest predictors of proportional reasoning. The authors suggested that proportional reasoning is closest to what Snow, Lohman, Marshalek, Yalow, and Webb (1977) call general fluid visualization, an aspect of general intelligence. General fluid visualization is defined as "mental manipulation of figural or nonfigural material" (Linn & Pulos, 1983, p. 32). Linn and Pulos speculated that their experience measures, number and length of mathematics and science courses, may not be measuring the variables that influences the gender differences in proportional reasoning.

The question of whether gender differences on a wide range of mathematics problems, including proportional reasoning, was a function of specific spatial skills has also been explored by Pattison and Grieve (1984). Subjects were tenth and twelfth graders from three different high schools, two were single-gender, upper-middle class schools which encouraged mathematics and science achievement. It is assumed the third was a middle-class, coeducational school, though a complete description was not provided. The mathematics test included items which tested arithmetical reasoning, logical problems, geometric reasoning, proportionality and scale, two and three dimensional geometry, and symmetries of the tetrahedron type involving both two- and three-dimensions. The visuospatial tests included Map Planning, Card Rotations, and Surface Development from the Ekstrom et al. cognitive test kit (1976), and the Mental Rotations Test (Vandenberg & Kuse, 1978). Words in Sentences test, a subtest of the Modern Language Aptitude test (Carroll & Sapon, 1958) was included to ascertain if certain language skills, "[apprehension] of structural relationships among words in sentences" (p. 681) might be related to performance on mathematics problems.

Generally, of the spatial tests, Surface Development showed the strongest and clearest positive relationship to the mathematics test, and was a slightly better predictor of performance for the girls. The Words in Sentences test was a better predictor for boys. Canonical correlation analysis of the relationship between mathematics subtests and the spatial and language tests showed that the gender differences in mathematics were not explained by differences in the spatial and language tests. Pattison and Grieve noted in their conclusion that the spatial test that showed the strongest relationship with mathematics was Surface Development which did not demonstrate a gender difference, yet was the better predictor for girls. This finding compromises the hypothesis that boys use more spatial reasoning in solving mathematics problems than do girls. Another complicating finding, that the language test was a better predictor for mathematics performance for boys, is also in opposition to the hypothesis that girls use more verbal strategies in solving math problems.

An examination of the relationship of cognitive variables to solving mathematical story problems presented in three different formats: a drawn version, a conventional verbal version, or a "telegraphic" or abbreviated version

was explored by Threadgill-Sowder, Sowder, Moyer, and Moyer (1985). Subjects were students in grades 3 through 7, with approximately 160 in each grade level from 11 different schools in three cities. The cognitive tests included the following: The Classifications Subtest of the Culture Fair test (Cattell & Cattell, 1960) as a measure of general fluid intelligence; the Spatial Relations Test (Thurstone, 1962) and the Punched Holes Test (a simpler version of the Paper Folding Test from the Ekstrom et al., 1976, test battery) as measures of spatial visualization; Hidden Figures (a modified version of the Hidden Figures Test, Ekstrom et al., 1976) and Find a Shape (described earlier as a test similar to an embedded figures test) as measures of "cognitive restructuring" (or field dependence-independence); and the Test of Reading Comprehension (Brown, Hammill, & Weiderhold, 1978).

Of the cognitive tests the reading test had the highest correlation with the problem solving test ( $r = .50$  to  $.64$ ), and of the spatial and restructuring tests Find a Shape correlated the highest with the problem solving test ( $r = .14$  to  $.47$ ). A two-way analysis of covariance, with the Culture Fair test as the covariate, of two groups formed by dividing them into those in the upper and lower quartiles

revealed that the lower group benefitted the most from the drawn format, while the upper group did not show a clear preference for format. In addition, gender differences were demonstrated on all the cognitive tests (except the Culture Fair test) and math problems, with the girls performing better on the reading comprehension test and boys having a superior performance on the cognitive restructuring, spatial visualization, and problem tests.

Threadgill-Sowder et al. (1985) speculated that the drawn format was more useful for those subjects who score low on Hidden Figures because this format provided additional organization of the information, a goal of the drawings employed in this study, not due to its visual nature. They suggested that drawings that do not provide organization may not be particularly helpful to individuals low in cognitive restructuring skills. The gender differences found in this study are of particular interest since many researchers (e.g., Maccoby & Jacklin, 1974) have reported that these differences are not seen until after puberty.

An investigation into whether individuals discrepant in either visuospatial or verbal skills differ in their ability to solve math problems and their ability to employ

visuospatial and verbal skills to solve those problems was undertaken by Fennema and Tartre (1985) in a three year, longitudinal study of middle school students. The Space Relations subtest of the Differential Aptitude Test (Bennett, Seashore, & Wesman, 1982) was employed as a measure of spatial visualization. The Cognitive Abilities Test, Verbal Battery (a vocabulary test) (Thorndike & Hagen, 1975) was used to measure verbal ability. Sixth grade mathematics achievement was measured by the Mathematics Concepts test (Naslund, Thorpe, & Lefever, 1971) and eighth grade mathematics achievement was measured by the Mathematics Basic Concepts, subtest of the Sequential test of Educational Progress (Educational Testing Service, 1979).

Subjects were tested in the sixth grade and the eighth grade, and chosen for inclusion on the basis of their high and low score profile in the sixth grade. The following groups were formed: High visuospatial/low verbal girls, high visuospatial/low verbal boys, low visuospatial/high verbal girls, low visuospatial/high verbal boys. For each of the three years students were given 10 mathematical problems to solve, and were asked to draw pictures that would facilitate problem solving. Fennema and Tartre found boys in both groups solved significantly more problems than

girls did, and no overall differences between the two spatial groups were demonstrated for solving mathematical problems.

Fennema and Tartre's (1985) results revealed different patterns of behavior between the groups. The low visuospatial/high verbal groups provided more detailed descriptions of the relevant information when asked to talk about the problem, and the high visuospatial/low verbal groups drew more complete and detailed pictures if they gave the correct solution. They emphasize that being low in visuospatial skills (and high in verbal) did not hinder the performance of the boys on mathematics achievement or problem solving, whereas the girls with the same skill profile solved the fewest problems and suffered a declination in mathematics achievement. Their method of testing the use of visuospatial skills, and its potential benefit, in solving math problems may not actually be one that assesses its beneficial use. Visuospatial tests such as they employed may actually test short-term memory capacity and speed of processing three-dimensional information. Therefore, drawing pictures to organize the information may not be the appropriate measure of effective use of visuospatial skills. This makes interpretation of

their results and their explanation of their results potentially problematic.

In addition to visuospatial skills, logical reasoning skills are also hypothesized to be important. Battista (1990) investigated the hypothesis that it is a balance between visuospatial and logical skills that contributes best to successful performance in geometry. He argued that if an individual were high in one skill and low in the other, they would use a strategy that emphasizes the skill that was high to the disadvantage of the low skill. To test this hypothesis he administered the following tests to high school geometry students: Guay's (1977) Purdue Spatial Visualization Test, Rotations, modified by the experimenter; a syllogistic reasoning test also developed by the experimenter; the Cooperative Mathematic's Tests, Geometry, Part 1, Form B (Cooperative Mathematics Tests, 1964); and a geometric problem-solving/strategies test developed by the experimenter to test the subjects' ability to solve problems and to identify the strategy used. In addition, he calculated a discrepancy score for each subject by converting the spatial and logical tests to z-scores and subtracting the logical test z-score from the visuospatial test z-score.

Battista's (1990) results showed that visuospatial and syllogistic reasoning tests correlated significantly with geometry achievement and problem solving for both boys and girls. Students who did poorly on the geometry achievement test obtained higher correlations between visuospatial and geometry problem solving. Boys outperformed the girls on the visuospatial test, geometry achievement, and geometry problem solving. There were no gender differences on logical reasoning. Analysis of the discrepancy score revealed that boys, on average, had higher spatial scores than syllogistic reasoning and girls had the opposite pattern. A regression analysis revealed that spatial visualization was the only predictor of performance in geometry for girls and logical reasoning and discrepancy between visuospatial and analytical skills were the only significant predictors for boys. Battista concludes that the two skills, visuospatial and syllogistic reasoning do contribute to geometry performance, but in different ways for the genders. Further research is needed to assess the generality of these findings.

Two visuospatial categories were identified by Tartre (1990). These include: spatial visualization, defined as skills used when the task requires imaginative movement; and

spatial orientation, described as "those tasks that require that the subject mentally readjust her or his perspective to become consistent with a representation of an object presented visually" (Tartre, 1990, p. 217). She further noted that many researchers have speculated about the role visuospatial skills play in mathematics, and most emphasize that visuospatial skills may be instrumental in mathematical problem solving by helping to abstract the relevant features of the problem into a mental representation that can be organized and the "interrelatedness of the components" (p. 218) of the problem made apparent. Tartre asserted that the visuospatial skills that would accomplish the facilitative role just described sound like spatial orientation. The purpose of her study was to investigate whether students who are high or low in spatial orientation solve mathematics problems differently. Tartre employed a qualitative design to achieve this purpose and did not perform a statistical analysis on subjects scores.

Spatial orientation was measured with a gestalt closure task, the Gestalt Completion Test (Ekstrom et al., 1976), which is described by Ekstrom et al. as a measure of speed of closure and is not closely related to previously discussed visuospatial skills (Snow, 1977). She employed a

problem-solving interview where students solved 10 mathematics problems consisting of geometric and nongeometric content. Subjects were tenth grade high school students. She did not find significant differences between high and low gestalt closure groups in total mathematics problems solved, but did find the two groups behaved differently as they solved the problems. Those subjects high in gestalt closure were more likely to estimate correctly the magnitude of a figure, be flexible problem-solvers, not show a rigid mind set, and mentally move or assess the size and shape of part of a figure. While Tartre had some interesting findings, her failure to test statistically for a relationship between mathematics and gestalt closure leaves that significant question unanswered and open to further inquiry.

Table 1 presents a summary of the research reviewed in this section. This research concerning the relationship between the two cognitive processes of visuospatial and mathematical reasoning has illuminated to some degree the nature of the relationship, however, further investigation is still needed to gain a better understanding of the nature of the relationship. Previous research has demonstrated a relation between visuospatial and mathematical processes,

Table 1

Summary of Research on Relations between Mathematics and  
Cognitive Skills

Study	Measure	Skill Measured	Results
Battista et al. (1982)	Purdue Spatial Visualization	speeded rotations	(+) relations between Geo- metry and speeded rotations
	Geometry Course Grades	geometry achievement	
Pulos, et al. (1982)	vocabulary test	crystalized intel.	different pattern of relations for school and test
	series comp- letion	fluid intelli- gence	
	Embedded Figures	disembedding	boys superior on Water Level Test
	Find a Shape	disembedding	
	Water Level Task	cognitive development	
Linn & Pulos (1983)	math problem solving	proportional, spa- tial reasoning, reason w/ unknowns	
	same as above (Pulos, et al.)	see above	vocabulary & series completion strongest predictor of propor- tional reasoning
	Paper Folding	visualization	
	Piagetian Formal Reasoning	cognitive development	
	Balance Puzzle	proportional reasoning	boys superior proportional reasoning
	math & sci- ence courses	experience	

Table 1 continued

Summary of Research on Relations between Mathematics and  
Cognitive Skills

Study	Measure	Skill Measured	Results
Pattison & Grieve (1984)	Map Planning	orientation	Surface Development (+) related to math
	Card Rotations	orientation	
	Surface Development	visualization	differential pattern of relations between meas- ures depend- ing on test and gender
	Mental Rotations Test	speeded rotations	
	Words in Sentences	structural language	
	Math Test	arithmetic, logic, geometry	
Threadgill- Sowder et al. (1985)	Culture Fair Test	general fluid intelligence	Reading Comp. correlated highest with problem solving
	Spatial Relations Test	speeded rotations	
	Punched Holes	visualization	speeded rotations & visualization correlated with problem solving
	Hidden Figures	disembedding	
	Reading Comprehension Test	reading comprehension	girls superior on reading comprehension and boys on visualization and math
	math problems	problem solving	

Table 1 continued

Summary of Research on Relations between Mathematics and  
Cognitive Skills

Study	Measure	Skill Measured	Results
Fennema & Tartre (1985)	Space Relations (DAT)	visualization	boys superior on math problems
	Cognitive Abilities Test	verbal	high and low visuospatial skills do not predict math achievement
	Mathematics Concepts	mathematical achievement	
Battista (1990)	Purdue Spatial	speeded rotations	positive correlation between cognitive skills and geometry
	syllogistic reasoning test	logical reasoning	
	Geometry Test	geometry problem solving	boys superior on geometry and speeded rotations
Tartre (1990)	Gestalt Completion	Gestalt closure	no differences between low and high visuospatial groups in problem solving
	math problems	problem solving	

yet some researchers have failed in this endeavor, or produced conflicting findings. Part of the difficulty is few, if any, of the previous researchers have addressed simultaneously the differential predictive effect of the four major visuospatial factors on performance in mathematics. This review of the literature suggests that the strongest predictor should be visualization.

#### The Role of Working Memory in Performance of Cognitive Tasks

As described in the introduction Baddeley (1986, 1992) proposes a working memory system that includes three components, a central executive which directs and controls attention, supervises and coordinates the activities of the subsidiary slave systems, the phonological loop and the visuospatial scratch pad. The phonological loop consists of a phonological store that is assisted by the process of articulatory rehearsal. The importance of this component has been demonstrated in learning to read, in the comprehension of language, and in the acquisition of vocabulary (Baddeley, 1986, 1992). The visuospatial scratch pad, the second component of the system, is used in the performance of visuospatial tasks, such as in mental arithmetic by abacus experts (Baddeley, 1986), and memory for chess positions by chess masters (Baddeley, 1992).

Working memory is postulated to be involved in the maintenance and manipulation of information, and therefore to be important in learning, reasoning, and comprehending (Baddeley, 1992; Greeno, 1973). Many researchers have demonstrated its importance in many cognitive tasks. This research is reviewed in this section.

The role of working memory in reading comprehension was demonstrated by Daneman and Carpenter (1980) and Oakhill, Yuill, and Parkin (1988). In Daneman and Carpenter's (1980) study subjects with higher "working memory" spans (operationally defined as the ability to recall the last word in a series of sentences) were more fluent readers, better able to draw inferences, and to integrate information.

Similar results were produced by Oakhill, Yuill, and Parkin (1989) with a slightly different measure of working memory, the ability to recall the last digit in a string of digits. Better readers in this study could also recall more digits. Another test of working memory employed by Oakhill, Yuill, and Parkin (1989) was the presentation of apparently inconsistent information presented in sentences, with the pertinent information needed to resolve the inconsistency presented either immediately next to, or two sentences away

from the inconsistency. The subjects' task was to resolve the inconsistency. Poorer readers were less able to resolve the inconsistency when there was a separation of inconsistent and resolving information, yet there was no difference between groups when there was not a separation of resolving information. These researchers concluded that working memory is important for comprehension of text material, and that better readers can comprehend reading material because they can hold more information in working memory while performing a concurrent processing task.

~~Working~~ Working memory has an important role in problem solving as well, as demonstrated by Carpenter, Just, and Shell (1990). These researchers compared subjects' performance on the Raven Progressive Matrices Test with their performance on the Tower of Hanoi puzzle. Results showed a high correlation between errors for both tasks ( $r = .77$ ). Since the Tower puzzle is dependent on goal management and a goal-recursive strategy, performance on both tasks is dependent on the ability to generate and maintain goals in working memory. In addition, a computer simulation program was created in which "goal-management competed with the ability to maintain and apply rules to the extent that goal information was displaced from working memory" (Carpenter et

al., 1990, p. 423). This program's performance on the Ravens test was especially poor on problems requiring more goal management, similar to poorer performers on the Raven test. Carpenter et al. (1990) conclude from these results that one of the processes that distinguishes between individuals on the Ravens test is the ability to manage a large set of problem-solving goals in working memory.

Span of working memory has been demonstrated for other types of problem solving as well. Cooney and Swanson (1990) investigated the role of memory span in the recall of propositional units from algebra story problems. These researchers used a task similar to Daneman and Carpenter's (1980) except the sentences were taken from mathematical story problems. Their results showed high memory span subjects had significantly better recall of relational problems statements and low memory span subjects had better recall of extraneous information. Cooney and Swanson (1990) concluded that memory span is an "important source of individual differences in problem perception, the ability to eliminate extraneous information from the mental representation of mathematical story problems, and the ability to integrate relational information into a problem representation" (p. 570).

Working memory as a source of individual differences between mathematically and verbally precocious (as identified by SAT math and verbal scores) 13 or 14 year old youth was investigated by Dark and Benbow (1991) in two experiments. In their first experiment the working memory span was operationally defined as the number recalled on visually presented lists of four types of stimuli: digits, letters, words, and locations. Their results showed that the mathematically precocious subjects performed significantly better on the digit and location span tests, and the verbally precocious subjects performed significantly better on the word span tests. According to Baddeley's (1986, 1992) model of working memory, information can be maintained by the phonological loop or the visuospatial sketch pad, or general ability. Since the differences between groups was moderated by stimulus type, Dark and Benbow (1991) assert the evidence supports the former hypothesis. Dark and Benbow (1991) also used a paired-associate task (pairing letters with either digits, letters, or locations) as a measure of the ability to manipulate information in working memory. They found that mathematically precocious subjects performed significantly better on the paired-associate task than the verbally

precocious youth. Given these results, Dark and Benbow (1991) concluded that the ability to continually update and manipulate information in working memory distinguishes mathematical skill.

In Dark and Benbow's (1991) second experiment a speed of lexical decision task (recall of word sets comprised of related-prime/unrelated-prime/target triads) was used to determine whether speed of encoding processes distinguished the verbally precocious subjects from the mathematically precocious. Results showed verbally precocious subjects had significantly smaller latencies on the lexical-decision task than the mathematically precocious youth. Dark and Benbow (1991) inferred that speed of activating a word's representation into working memory is a source of individual differences in verbal skill.

Dark and Benbow (1991) concluded that the task, the stimulus, and the individual determines performance on different cognitive measures. The mathematically precocious individual can maintain and manipulate information in working memory, and the verbally precocious shows speed of working-memory encoding. The mathematically precocious individual showed a better working memory representational capacity for digits and locations, and the verbally

precocious individual demonstrated better working memory representational capacity for words. This latter finding may indicate that it is differences in long-term memory representations that is responsible for the results rather than differences in working memory processes.

In summary, differences in working memory has moderated performance on a variety of cognitive tasks, namely reading, Ravens Progressive Matrices Test, algebra story representations, and distinguished between mathematical and verbal precocity. While Dark and Benbow's results indicated that representational differences in long-term memory may account for the differences seen in working memory operations, more research is needed to understand how different information processing skills moderate performance on different cognitive tasks.

#### Gender Differences in Visuospatial Skills

As mentioned earlier, research has shown gender differences in visuospatial skills favoring boys often emerge more strongly during adolescence (Burnett, 1986; Maccoby & Jacklin, 1974). Johnson and Mead (1987) administered a battery of seven visuospatial tests to 1,800 subjects in grades kindergarten through twelfth grade to investigate when this gender difference appears. The tests

they employed were adapted for children from adult visuospatial tests. The tests included: Flags, Hands, and Cubes (Thurstone, 1938); Mental Rotations (Lansman, Donaldson, Hunt & Yantis, 1982); Blocks (King, 1956); Spatial Relations (Thurstone & Thurstone, 1965); and Hidden Figures (Thurstone, 1944). For the upper grades (grades 7-12) the test format and number of items was identical to the adults, the middle grades (grades 4M-6) were administered the tests in three sessions, and the lower grades (1-4L) were administered the tests at a separate session and the items were larger in size and fewer in number. Kindergartners were given tests that were still larger in size and fewer in number. Half the fourth grade was given the middle battery (4M) and the other half was given the lower battery (4L). There was a highly significant male advantage for grades 4H-through 12. At grade 10 there appeared to be a "quantum increase in the male advantage" (p. 735). A limitation is that the number of science and mathematics courses taken by students was not assessed. Therefore, the influence of problem solving experience on visuospatial skills was not determined and may have been a moderating factor as demonstrated by other researchers (eg., Baenninger & Newcombe, 1989; Burnett & Lane, 1980).

Johnson and Meade (1987) also found the lower grades (1-4L) showed a significant male advantage only after reading scores were used as a covariate. There were no gender differences in kindergarten. The magnitude of the male advantage did not vary with age or grade. A recent study (Lummis & Stevenson, 1990) examining gender differences in reading and mathematics achievement in three cultures (American, Taiwanese, and Japanese) did not find gender differences on a visuospatial test (developed by the researchers, however, no description of the test was included in their report) administered to kindergartners and first graders. Differences on this visuospatial test did emerge in the fifth grade, however. This is noteworthy since it supports Johnson and Mead's (1984) findings that consistent gender differences appeared in their more mature fourth graders. It is also significant that these differences were found in three very different cultures.

Johnson and Mead (1984) note developmental differences in verbal comprehension, attention span, and motivation may mask the expression of spatial skills on tests administered at very young ages. This may be why other researchers have not found gender differences on these skills as early as he found them when he used verbal skills as a covariate.

Johnson and Mead speculate that while girls' verbal precocity may mask a male advantage in testing situations, it may also predispose them to develop verbal strategies for solving problems. However, boys may adopt a spatial strategy since they lack comparable verbal skills.

A recent meta-analysis (Linn & Petersen, 1985) examining gender differences for visualization and speeded rotations (three-dimensional rotations) concluded that gender differences on visualization had declined since 1974 to such an extent as to be considered nonexistent, but the differences between the genders remained for speeded rotations. This conclusion is consistent with findings of Vandenberg and Kuse (1978) who found consistent and large gender differences on the Mental Rotations Test, a measure of speeded rotations. Feingold (1988) also supports this conclusion in his review of the research using the Spatial Relations subtest of the Differential Aptitude Test (DAT) (Bennett et al., 1982), which is a measure of visualization. He analyzed the reported norms for the years 1947, 1962, 1972, and 1980. This analysis revealed by 1980 girls had cut in half the difference demonstrated prior to 1960.

In summary, Linn and Petersen (1985) and Feingold (1988) found in their meta-analyses that the gender

difference in visualization was declining. However, there is agreement across studies that boys and men outperform girls and women on measures of speeded rotations (Johnson and Mead, 1987; Linn & Petersen, 1985; Vandenberg & Kuse, 1978). Gender differences on visuospatial measures emerged as early as first grade in Johnson and Mead's (1984) study when reading was used as a covariate, and in fifth grade in Lummis and Stevenson's cross cultural research. These findings indicate gender differences emerge at a much younger age than prior studies had indicated (e.g., Maccoby & Jacklin, 1974).

Several factors have been explored as contributors to the persistent gender differences in visuospatial skills. These include genetic, neurological, and environmental factors. Some researchers have shown evidence for an X-linked recessive gene hypothesis to explain the disproportionate male advantage in visuospatial skills (Harris, 1979; McGee, 1979; Petersen, 1987). Since the right cerebral hemisphere plays such a crucial role in the performance of visuospatial tasks many researchers have explored differences in cerebral organization and functioning. These researchers have investigated three hypotheses. The first hypothesis is that boys' right

hemisphere is more lateralized earlier and develops faster than girls (e.g., Harris, 1980; McGlone, 1980; Petersen, 1987). The second hypothesis is that there is an "earlier, greater left-hemisphere language lateralization in [girls] and bilateral spatial representation in [boys]" (Harris, 1979, p. 453). This early specialization results in a superiority in language tasks for girls and superiority in visuospatial tasks for boys. The third hypothesis is that there is a "greater bilateral representation for language functions in the female brain from which is drawn a negative outcome for spatial skill" (Harris, 1979, p. 461).

Harris (1979) reported there is more broad based evidence (i.e., from anatomical studies, dichotic listening studies, visual field effects studies, EEG studies, and clinical data) for the third neurological hypothesis than the other two because the emphasis of the first two hypotheses on the early years does not explain the gender differences seen in later years. However, she notes that early development of language skills in girls due to earlier maturity of the left hemisphere may start a course of intellectual development where language skills play a larger role for girls than for boys. Though boys' language skills later catch up with girls, this early deficit may predispose

boys to "encode information nonlinguistically" (Harris, 1979, p. 477). The discussion of the physiological factors, genetic and neurological, which have been explored as contributing to the gender differences in visuospatial skills has had a long and controversial history. A more detailed analysis of these arguments is beyond the scope of this thesis.

Differences in gender role expectations and experiences have been proposed as environmental factors contributing to the gender differences in visuospatial skills (Coser, 1986; Halpern, 1986; Harris, 1979). Halpern (1986) observes that a visit to the toy store reveals "blocks, tinker toys, and models to be assembled--all toys that develop spatial skills" (p. 1015) are in the "boys" section of the store. In the "girls" section, dolls, household appliances, makeup and jewelry are found, toys that emphasize "skills . . . in the more feminine helping professions" (p. 1015).

Halpern (1986) notes there is dramatic increase of women entering into engineering, math and science fields in the United States (Halpern, 1986). She asserts this increase cannot be due to biological reasons, but attributes it to a change in the *Zeitgeist* of the last two decades. If this trend continues, and there is no reason to believe it

will not, perhaps the persistent gender difference in speeded rotations will also decline.

Harris (1979) speculates the genetic-hormonal factors that create males and females "also predispose the operation of modes of both cognitive and physical activity that tend to enlarge and widen initial differences" (p. 487). Boys and girls tend to become involved in experiences that sharpen different skills, and the environment in which they are born further serves to reinforce those tendencies.

In summary, several factors have been explored as contributing the gender differences in visuospatial skills. The research on these factors remains incomplete and inconclusive.

#### Gender Differences in Mathematics

Gender differences in mathematics have been studied intensely for the last 20 years (Fox, 1980; Hyde et al., 1990). Generally, there is agreement that a gender difference exists, with boys and men outperforming girls and women on problems involving complex applications and with girls and women showing an advantage in problems involving less cognitive complexity.

The reason for the superior performance of men in solving mathematical problems was explored by Johnson (1984)

in a series of experiments. He administered a set of problems originally created by Donald Taylor (Johnson, 1984) but revised for use in these experiments with undergraduate students. The first experiment established that men solved significantly more mathematical problems than do women. The other eight experiments, however, showed an even greater male advantage.

Johnson's next two experiments explored the role of previous experience with similar problems. The results demonstrated there were no gender differences associated with familiarity, and Johnson concluded that experience was not responsible for the gender differences in problem solving. The fourth experiment was designed to ascertain if the gender differences extended beyond the solving of word problems and therefore contained verbal analogy problems, Raven Progressive Matrices Test and three computer games. The only tests that showed gender differences were the word problems, with men showing superior performance. The fifth experiment included two problem sets (which he called set A and B) from published collections in the popular press (their name or publisher was not provided) in addition to the original problem set to ascertain if the original set was biased toward men's advantage. The results showed the

male advantage to be of a similar magnitude with the original problem set.

Experiments 6, 7, and 8 examined the roles of spatial and verbal skills in problem solving. In addition to the Raven Progressive Matrices Test and the original problem solving set, the following tests were administered: the Guilford-Zimmerman Aptitude Survey (measuring speeded rotations), the Zaps and Duds problem, and four verbal tests from the Ekstrom et al. (1976) cognitive kit. Interestingly, only the Guilford-Zimmerman Aptitude Survey both correlated significantly with problem solving and showed a gender difference. Though the Raven Progressive Matrices Test correlated significantly with problem solving, there were no gender differences. When the speeded rotations test was used as a covariate the gender differences were reduced to nonsignificance in two of three experiments. Thus, Johnson concluded that visuospatial skills were a significant factor in accounting for gender differences in problem solving.

The relative contributions of aptitude and social learning variable to gender differences in problem solving were investigated in the ninth experiment. A forward stepwise regression analysis revealed that the Scholastic

Aptitude Test (SAT) Math and the visuospatial test were the major contributors of the variance in problem solving. SAT Math was the most significant predictor for both genders. While visuospatial skills entered in second for men, it was third for women. Math importance was second for women, and masculinity was third for men.

Johnson concluded by noting that the gender difference in problem solving was most strongly related to mathematical aptitude (as measured by SAT Math) and visuospatial skills. He opined the gender difference exhibited on these types of problems will not generally impair women using mathematics in solving problems normally encountered in daily adult life. These differences, however, are a symptom of a deficit in overall mathematical ability which can impede women's entry into certain career fields.

A recent meta-analysis of gender differences in mathematics performance by Hyde et al. (1990) supported Johnson's (1984) finding of a male advantage in problem solving. Their meta-analysis also showed the same trend of decline in gender differences in mathematics as was seen in visuospatial skills studies (Linn & Peterson, 1985; Feingold, 1988). Overall, they report women outperformed men by a negligible amount ( $d = -.05$ ). Hyde et al.'s (1990)

analysis of age trends indicates that girls are slightly superior on computation in elementary and middle school years, however this difference declines to insignificance by high school and college. While there were no differences in problem solving in the lower grades, a male superiority emerged in high school and slightly increased in college. The gender difference is even greater for highly selective samples, such as in mathematically precocious youth (Benbow, 1988). This decline in gender differences, except in highly selected groups, has been corroborated by Feingold's (1988) analysis of research employing the Numerical Ability subtest of the DAT (Bennett et al., 1982), the Preliminary Scholastic Aptitude Test (PSAT; Donlon, 1984); and the Scholastic Aptitude Test (SAT; Donlon, 1984) in the years 1947 to 1983.

Linn and Hyde (1989) report that the items on mathematical problem solving tests which reveal the greatest gender differences are those involving science, sports, and complex applications. They note that the magnitude of the difference on problem solving items is similar to the difference between the genders in participation in advanced courses which emphasize complex problem solving applications, such as physics, chemistry, and advanced

mathematics. This is evidence that experience in problem solving contributes to the male performance superiority.

Although Hyde et al. (1990) report the gender difference in problem solving is not found in the lower grades, Lummis and Stevenson's (1990) cross-cultural research into gender differences in academic achievement found a male advantage as early as the first grade in solving word problems, problems involving visual estimation of quantity and distance, and problems requiring the visualization of various transformations of geometrical forms or scenes. These cultures (American, Taiwanese, and Japanese) are quite different, yet the girls in all three samples demonstrated competent and comparable skills in mathematical operations and procedures and the understanding of mathematical concepts, yet did not demonstrate equal skill as demonstrated by boys in using this conceptual and procedural knowledge in solving math problems.

This finding is a "remarkable effect" (Lummis & Stevenson, 1990, p. 262). The strength of the Taiwanese and Japanese mathematics curriculum was effective in developing mathematical skills superior to that of American boys and girls. This same curriculum however, was not instrumental in eliminating the gender differences in mathematical

problem solving skills in Japan and Taiwan. The reason for this gender difference is not clear. Lummis and Stevenson found that the mothers of the children in their study expected their daughters to be better readers than their sons, and although they were less biased in their expectations regarding their childrens' mathematical achievement, they did expect better achievement in mathematics from their sons than their daughters. These researchers speculate that girls may read more since they are expected to do so and will receive positive reinforcement for that activity. Boys will turn to activities other than reading, mathematics for example, for positive encouragement. Lummis and Stevenson propose, although it is not clear the mother's biases were responsible for the gender differences in problem solving, it may be necessary to alter parents' and childrens' beliefs about what subjects children excel in to reduce the gender differences in mathematical problem solving.

#### Summary of the Literature Review

Course work emphasizing complex problem solving has contributed to an enhancement of visuospatial skills. While some researchers have clearly demonstrated a relationship between mathematics and visuospatial skills, others have

not. Span of working memory has been a moderating variable in differences in reading, the Progressive Matrices Test, algebra story problem representations, and distinguishes between mathematical and verbal precocity. The gender differences in visualization are declining, while gender differences in speeded rotations persist, with men outperforming women. Overall, gender differences in mathematics are declining, yet the male advantage in problem solving remains.

Two reviewed studies, Battista's (1990) and Tartre's (1990), were especially interesting for the questions which their research suggests, but are still unanswered. Battista's hypothesis that a balance between logical and visuospatial reasoning is necessary for solving geometry problems is a sound one and worthy of further investigation. Tartre's hypothesis that Gestalt closure (measured by the Gestalt Completion Test) facilitates mathematical problem solving by a process of abstraction is intriguing and had not been investigated before with the type of measurement instrument she employed. Tartre found that Gestalt closure skills were employed by the high group to advantage in nongeometric problems as well as geometric problems. Recall, however, that she did not test for a significant

relationship between Gestalt closure and mathematical problem solving.

## PRESENT INVESTIGATION

Geometry problem solving is an area of mathematics that appears to require skills in the perception and recognition of visuospatial patterns and relationships, as well as skills in abstraction. Another necessary skill is analytical (or deductive) reasoning. This conceptualization of the processes necessary for geometry problem solving is congruent with the pattern of relations in Vernon's model of intelligence. Mathematics is related to both verbal and spatial processes in Vernon's (1965) model. A pilot study was conducted prior to the thesis research. The purpose of both studies was to investigate the hypothesized relation of visuospatial and analytical skills to geometry academic performance.

In addition, the effect of short-term memory capacity on geometry problem solving was investigated in the thesis research. Baddeley's (1990) model of working memory provided the theoretical perspective for the conceptualization of its role in geometry problem solving.

In the pilot study two analytical tests and three visuospatial tests were administered to high school geometry students to investigate which tests predicted achievement in geometry. This study (Brown & Crawford, 1991) was presented

at the annual meeting of the American Psychological Society in Washington, D.C., June, 1991. The thesis research was intended to extend the pilot study with the addition of (a) two visuospatial tests, visuospatial subfactors not tested in Study 1 (disembedding and orientation); (b) a measure of flexibility of closure; (c) a nonspatial and a spatial analytical test; (d) a measure of short-term memory capacity; and (e) a standardized geometry exam.

## THE PILOT RESEARCH

The purpose of the pilot research was to investigate the a priori relation between performance in high school geometry and both visuospatial and logical reasoning skills. Vernon's (1965) model of intelligence provided the conceptual framework for this investigation.

As geometry is an area of mathematics that measures visuospatial patterns and relationships, it was predicted that visuospatial skills would be important for geometry problem solving. Tests which assess visuospatial skills were therefore employed in the pilot study. Since geometry is a division of mathematics in which premises are given from which deductions are to be made, it was predicted the skills of logical reasoning were also critical for successful geometry problem solving. Therefore, tests which measure logical reasoning were also employed in the pilot test.

It was also predicted knowledge of arithmetic operations used to solve mathematical problems is important for successful geometry problem solving. An instrument to measure this knowledge was included in the pilot study. The test employed, Necessary Arithmetic Operations Test is described by Ekstrom et al. (1976) as a test of general

reasoning and has a central position in nonmetric scaling studies examining the interrelations among cognitive tests (Snow, Kyllonen, & Marshalek, 1984). Snow et al. found in a helix analysis this test loads near to the Raven Progressive Matrices Test, and other complex reasoning tests, which are at the center of their solution and indicative of general intelligence. Tests measuring more specialized processes (e.g., Gestalt closure measures) are at the periphery. The centrality of the Necessary Arithmetic Operations Test indicates it is a measure of general intelligence. Use of this test with more specialized tests helps determine what specific skills, beyond general ability, are important for effective geometry problem solving.

The hypotheses for this investigation include the following:

(1) Subjects who are both high in visuospatial and logical reasoning skills will demonstrate higher achievement in geometry than those who are low in both visuospatial and logical reasoning (Battista, 1990).

(2) Subjects who are low in either visuospatial or logical reasoning skills will demonstrate lower achievement in geometry than subjects who are high in both visuospatial and logical reasoning (Battista, 1990).

(3) Gender differences favoring boys will be found on tests of speeded rotations, but not on the measures of visualization (Feingold, 1988; Linn & Petersen, 1985).

(4) There will be no gender differences on measures of logical reasoning (Battista, 1990).

(5) There will be gender differences favoring boys on measures of achievement in geometry problem solving (Hyde et al., 1990).

#### Method

##### Subjects

Subjects were 84 students, 41 boys and 43 girls, in four sophomore level geometry classes at a moderately sized high school which drew students from both a university town and the surrounding rural county in Southwestern Virginia. Subjects ranged in age from 14 to 19 (mean = 15.68, S.D. = 1.34). Subjects were students in ninth through twelfth grade. Two of the tenth grade classes were regular college preparatory classes and the third was a lower-performing group of students. The fourth class was a ninth grade class of advanced students.

Permission to participate was obtained from the parents and students, as well as the participating school. Please see Appendix 1 for a copy of the consent form. In addition,

this research was approved by the Human Subjects Committee of the Psychology Department and the Institutional Review Board of Virginia Tech.

### Instruments and Procedure

Subjects were administered the following tests from the Revised Kit of Factor-Referenced Cognitive Tests (Ekstrom et al., 1976): A logical reasoning test, Diagramming Relationships Test, and Necessary Arithmetic Operations Test, and three visuospatial tests (Paper Folding, Cube Comparisons, and Surface Development). These tests have demonstrated a respectable level of reliability as reported in the manual for the Ekstrom et al., 1976 test battery: Diagramming Relations Test,  $r = .79$ ; Necessary Arithmetic Operations Test,  $r = .73$ ; Paper Folding Test,  $r = .84$ ; Cube Comparisons Test,  $r = .84$ ; Surface Development Test,  $r = .92$ .

A test of speeded rotations, the Mental Rotations Test (Vandenberg & Kuse, 1978) was also administered. This test has shown respectable reliability in the .83-.88 range (Vandenberg & Kuse, 1978).

These tests were administered by the teachers under the guidance of the researchers over a period of several weeks near the end of the school year. The teachers administered

the tests during the last fifteen minutes of their class period on various days.

Geometry test grades were used as measures of geometry performance. Z-score transformations were performed on the final grade in order to compare grades between teachers and classes since there were two teachers, and students were grouped into classes according to performance level. These z-scores were used in the analyses.

### Results

Table 2 presents the means and standard deviations for the measures of cognitive skills and geometry achievement for the whole sample, and each class.

T-tests did not reveal the anticipated gender differences. No gender differences were found on any of the measures of cognitive skills or achievement in geometry.

Table 3 presents the correlations between the measures of cognitive skills and geometry achievement. As can be seen from the table, with the exception of the Mental Rotations Test, all of the cognitive tests correlated with the final grade at the .01 level of significance.

A stepwise multiple linear regression analysis was performed using the z-score transformations of the final grade as the dependent variable, and the visuospatial and

Table 2  
Means and Standard Deviations of the Cognitive and Geometry Measures for the Total Sample, and Each Class

Measure	Total Sample		Class 1		Class 2		Class 3		Class 4	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Diagramming Relations	16.11	6.15	9.56	3.93	20.16	5.77	14.72	5.34	16.94	4.37
Necessary Arithmetic	6.56	2.24	5.15	1.80	8.45	1.81	7.75	5.11	5.11	2.09
Surface Development	19.61	6.21	14.03	7.81	7.64	3.65	20.06	6.01	18.70	5.10
Paper Folding	11.87	3.68	9.50	4.11	13.39	3.94	11.53	2.34	12.00	3.37
Mental Rotations	19.66	8.03	17.75	12.99	20.07	7.27	20.59	6.64	19.40	7.13
Cube Comparisons	16.11	6.15	9.56	3.93	20.16	5.77	14.72	4.37	16.94	4.37
Final Grade	77.48	14.33	69.13	12.08	86.68	14.44	77.82	15.15	72.85	14.67

Table 3  
Correlations between Measures of Cognitive Skills and Geometry Achievement

	Paper Folding	Surface Development	Diagramming Relations	Arithmetic Operations	Cube Comparisons	Final Grade
Mental Rotations	.53**	.55**	.05	.17	.54**	.06
Paper Folding		.58**	.37**	.46**	.40	.37**
Surface Development			.36**	.53**	.59**	.29**
Diagramming Relations				.46**	.18	.37**
Arithmetic Operations					.37**	.47**
Cube Comparisons						.25*

\*p < .05

\*\*p < .01

Table 4

Results of Multiple Linear Regression Analysis with the Final Exam as Criterion and Cognitive Skills as Predictors

Variable	B	SE B	Beta	t	p=
Total Sample (N = 84)					
R Squared = .28					
Necessary Arithmetic	1.13	.321	.410	3.52	.0008
Diagramming Relations	1.64	.647	.339	2.54	.0140
Boys (N = 41)					
R Squared = .36					
Necessary Arithmetic	.109	.024	.603	4.53	.0001
Diagramming Relations	.048	.024	.360	2.11	.0534
Surface Development	.026	.011	.377	2.34	.0255
Girls (N = 43)					
R Squared = .19					
Necessary Arithmetic	.083	.036	.345	2.30	.0207
Diagramming Relations	.089	.033	.438	2.72	.0107
Cube Comparisons	.062	.023	.429	2.69	.0113

analytical tests as the predictors. The analysis indicated that there was a significant relationship between the final grade and the Necessary Arithmetic Operations,  $F(1,61) = 12.37$ ,  $p = .008$ , and Diagramming Relations Test,  $F(1,50) = 6.48$ ,  $p = .01$ . The combined R squared was .28 for these tests indicating a moderate relationship between these measures and achievement in geometry. A summary of the regression analyses for the total sample, and boys and girls separately, are presented in Table 4.

Separate multiple linear regression analyses were performed for boys and girls. As can be seen from Table 4, there was a different pattern of predictors for boys and girls and the R squared was higher for boys.

#### Discussion

Within this sample, for both boys and girls, knowledge of arithmetic operations, as measured by the Necessary Arithmetic Operations Test, and logical reasoning, as measured by the Diagramming Relations Test, are important predictors for achievement in geometry as measured by the students' final grade.

Different visuospatial factors emerged as significant predictors for achievement in geometry for boys and girls. For boys the Surface Development Test emerged as a

significant predictor for the final grade. The Surface Development Test is a measure of visualization, skill in organizing component parts into a whole configuration. For girls, the Cube Comparisons Test was a significant predictor of the final grade. Cube Comparisons is a measure of orientation, skill in imagining how a figure would look from different perspectives.

Both visualization and orientation are skills which, intuitively, would be important for <sup>Coordinate</sup> geometry problem solving. As a branch of geometry measuring spatial relationships between lines, points, angles, and figures, <sup>Shapes</sup> skill in visualizing transformations and orientations of these spatial relationships would facilitate problem solving efforts. Why the tests of Surface Development and Cube Comparisons predict geometry differently for boys and girls is not explainable at the present time, but suggests that differential skills or strategies contribute to performance in geometry problem solving for boys and girls. Whether this is unique to this sample, or generalizable to other school populations is worthy of further investigation. It should be noted that the small N complicates the interpretation of the separate regression analyses for boys and girls.

The lack of gender differences on the individual cognitive measures supports a recent meta-analysis which indicated declining gender differences in cognitive abilities (Hyde et al., 1990; Linn & Hyde, 1989). The size of this study, however, was too small to permit conclusions about a decline in differences.

The strength of the logical reasoning test (Diagramming Relations Test) and the Necessary Arithmetic Operations Test to predict geometry achievement in both boys and girls demonstrated in this study warrants further investigation. The Diagramming Relations Test emphasizes in its instructions the use of a visual strategy, Venn diagrams. In other words it instructs subjects to draw relationships between three items with the use of overlapping circles. A verbal reasoning test, such as the Nonsense Syllogisms Test, assesses analytical skills needed in geometry problem solving without the confounding influence of a visuospatial strategy. Use of this additional measure would provide more useful information about the contribution of both verbal and visual logical reasoning skills to geometry problem solving separate from visuospatial processes. Reasonable hypotheses would be that both are important, or that visuospatial logical thinking is more important than verbal logical

thinking since geometry is a branch of mathematics that combines both visuospatial and logical thinking processes.

The emergence of Necessary Arithmetic Operations Test as a significant predictor for both boys and girls final grade indicate the importance of knowledge of arithmetic operations and general intelligence in solving geometry problems. As a branch of mathematics where arithmetic operations are applied to solving spatial problems it seems quite logical that this knowledge would be significantly useful in geometry problem solving. Since the Necessary Arithmetic Operations Test loads near tests of general intelligence, general ability distinguishes better geometry problem solvers in this sample. Further review of the literature, previously described, revealed additional cognitive skills that could be predictive of achievement in geometry. These cognitive skills include disembedding, orientation, flexibility of closure, and short-term memory capacity.

The pilot study did not include a standardized measure of geometry achievement. Such a measurement instrument, in addition to classroom measures of achievement, is needed due to the many unknown and unassessed variables (eg., motivation, time management skills, parental assistance)

possibly influencing classroom measures. Another problem with using only classroom measures is that different teachers have different homework assignments and tests emphasizing different facets of geometry problem solving possibly not focussed on as strongly by other teachers. Additionally, different teachers also have different reward structures which may reward behaviors other than geometry problem solving performance, such as punctuality (Slavin, 1977). Interpretation of results employing classroom measure is therefore problematic.

## THESIS RESEARCH

The thesis research is a further investigation of the cognitive skills needed in geometry. This study included the visuospatial measures of disembedding and orientation revealed by the literature review which were not included in the pilot study. In addition, this research extended the pilot study with the following measures: a Gestalt closure test; a verbal logical test; a measure of short-term memory span; and a standardized geometry achievement test.

A problem with previous studies investigating relations between visuospatial skills and geometry problem solving (and mathematics in general) is that researchers have tended to use only one visuospatial measure and therefore have only assessed one facet of visuospatial processing. Incorrectly, they have generalized to the overall visuospatial ability rather than limiting their discussion to the subfactor under investigation. Such over-generalization has led to misinterpretations about the underlying influence of visuospatial skills on geometry problem solving. Unless a researcher uses instruments which measure the different subfactors of visuospatial skills, global statements about relations between the visuospatial skills and geometry problem solving are unwarranted.

The present study takes a multidimensional approach by including measures of each of the four major visuospatial subfactors. Such an approach will help reveal the particular visuospatial skill, or skills, which are most important to geometry problem solving. A review of the literature strongly suggested that visualization would best predict geometry problem solving performance. Yet, the pilot study suggested potentially different visuospatial skills moderate performance in boys and girls, as did also Battista (1990).

The logical reasoning test employed in the pilot study emphasized a visuospatial strategy. It is difficult to interpret whether the visuospatial processes or the deductive processes were significantly related to geometry problem solving. Since the premises in geometry problem solving are often given in verbal form, it is proposed here skill in verbal logical reasoning will facilitate geometry problem solving performance. The inclusion of both a verbal and a visuospatial logical reasoning test will permit evaluations of the importance of both or only one reasoning process for geometry problem solving.

As discussed earlier, the importance of short-term memory span has been demonstrated in reading, the Raven

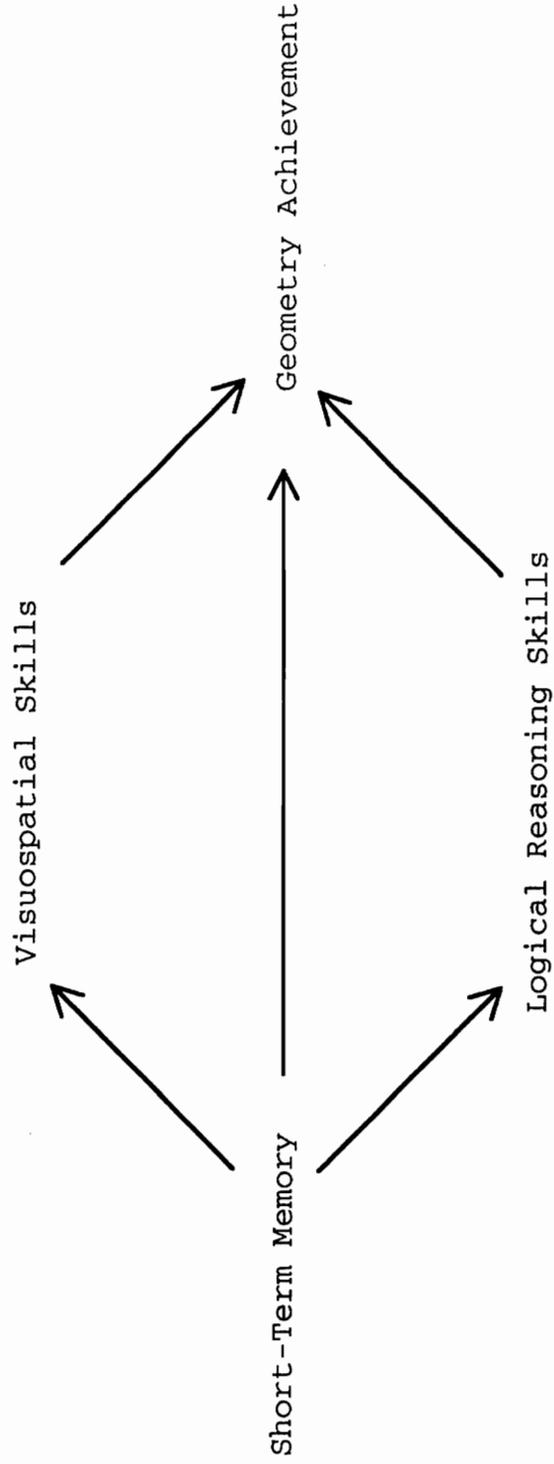
Progressive Matrices Test, as well as other cognitive tasks (Carpenter et al., 1990; Oakhill et al., 1988). Given these results it was anticipated that short-term memory span for a list of words would be positively related to geometry achievement.

It is proposed here the givens and goals of a problem first enter the phonological store. An initial internal representation of the problem is formed in working memory. Problem solving knowledge in how to solve the problem enters working memory from long-term memory and interacts with the initial representation in working memory. It is hypothesized that geometry problem solvers who have greater short-term memory spans will therefore be able to hold more information in working memory. This capacity will aid in performance on geometry problem solving tasks.

This research proposes a model of achievement in geometry. As can be seen in Figure 1, this model predicts span of short-term memory, visuospatial skills, and logical reasoning skills have a positive, direct effect on achievement in geometry problem solving. In addition, the model predicts span of short-term memory has an indirect,

Figure 1

Hypothesized Model of Achievement in Geometry.



positive effect on achievement in geometry by direct, positive effects on both visuospatial skills and logical thinking skills.

Hypotheses for the thesis research are the following:

(1) Subjects who are both high in visuospatial and logical reasoning skills will demonstrate higher achievement in <sup>achievement</sup> geometry relative to those who are low in both visuospatial and logical reasoning skills (Battista, 1990).

(2) Subjects who are both high in visuospatial and logical reasoning skills will demonstrate higher in geometry compared to subjects who are low in just one of the skills of visuospatial or logical reasoning (Battista, 1990).

(3) Subjects who have greater short-term memory spans will demonstrate higher scores on a measure of geometry achievement than subjects who have lower spans of short-term memory (Daneman & Carpenter, 1980; Carpenter et al, 1990).

(4) A path analysis will demonstrate that there are significant direct, positive effects of visuospatial skills, logical reasoning skills, and span of short-term memory on achievement in a standardized test of <sup>math</sup> geometry (Baddeley, 1990).

(5) A path analysis will show indirect, positive effects of span of short-term memory on geometry problem solving by its positive, direct effects on visuospatial skills and logical reasoning skills.

(6) Gender differences favoring boys will be found on the measure of speeded rotations, but not on measures of orientation, disembedding, flexibility of closure, logical reasoning, or short-term memory span (Feingold, 1988; Linn & Petersen, 1985; Wasserstein, Weiss, Rosen, Gerstman, & Costa, 1980).

(7) There will be gender differences favoring boys on measures of achievement in geometry (Hyde et al, 1990).

#### Method

##### Subjects

Subjects were ~~129~~<sup>100</sup> students in high school <sup>75</sup> geometry <sup>Wadeley</sup> classes. Subjects ranged in age from 15 to 19 (mean = 15.92, S.D. = 1.05). Subjects were students in grades <sup>5-8</sup> through 12. Subjects were students in two schools. School 1 was located in a rural area of southwest Virginia and included 59 subjects from three classes. All three classes had the same teacher. School 2 was located in a suburban area of southwest Virginia. Students from two teachers' classes participated at this school. Both teachers had two

classes each. One teacher had a total of 42 subjects, and the other teacher had a total of 28 subjects.

Since testing was administered over a period of three days a few subjects were absent on one, or two of the testing days. These students' scores were dropped from the analysis. There were 110 subjects remaining, with 51 girls and 59 boys. Permission to participate was obtained from the parents and students. Please see Appendix 1 for a copy of the consent form. In addition, this research was approved by the Human Subjects Committee of the Psychology Department and the Institutional Review Board of Virginia Tech, and the participating school and county administrators.

### Instruments

Subjects were administered the following tests from the Revised Kit of Factor-Referenced Cognitive Tests (Ekstrom et al., 1976): Two logical reasoning tests (Diagramming Relations Test, and Nonsense Syllogisms Test), the Necessary Arithmetic Operations Test, four visuospatial tests (Paper Folding Test, Card Rotations Test, Surface Development Test, Hidden Patterns Test), and a measure of Gestalt closure (Gestalt Completion Test). In addition, the Mental Rotations Test (Vandenberg & Kuse, 1978) was administered.

The reliabilities for the Diagramming Relations Test, the Necessary Arithmetic Operations Test, the Paper Folding Test, the Surface Development Test, and the Mental Rotations Test were reported earlier in the pilot study. The cognitive tests added to this research have also demonstrated moderate to strong reliability: Nonsense Syllogisms Test,  $r = .64$ ; Gestalt Completion Test,  $r = .85$ ; Card Rotations Test,  $r = .87$ ; Hidden Patterns Test,  $r = .90$  (Ekstrom et al., 1976).

The Rey Auditory Memory Test (Rey, 1964) was included to test subjects' short-term memory span. A list of 15 words was read via an audiotape at a constant rate of one word a second. The subjects' task was to recall the words in the correct sequence on an answer sheet for each of five trials. Subjects were given 30 seconds to write down their responses. The number of words correctly recalled was recorded as their score. The Rey Auditory Memory Test has demonstrated respectable test-retest reliability, with correlations ranging from .77 to .90 (Shapiro & Harrison, 1990).

Year-end geometry achievement was assessed by the New York Regents Geometry Exam (Kaplan, 1981) and z-score transformations of the final grade assigned by the teacher.

### Procedure

Tests were administered two weeks before the end of the school year by the researchers over a period of three consecutive days for two of the teachers, one in School 1 and the other in School 2. Test administration took three class periods. Class periods were approximately 50 minutes. In order to meet the scheduling requirements of one of the teachers at School 2, this teacher's students were administered the tests in one day in three consecutive periods.

### Results

#### Descriptive Data

Table 5 presents the means and standard deviations of the measures of the cognitive skills and geometry for the total sample, as well as each separate school.

In general, few differences were found between schools on the ten measures of cognitive skills. The only exceptions were the New York Regents Geometry Exam, the Rey Auditory Memory Test, and the Diagramming Relations Test. An analysis of variance revealed that there was a significant difference between schools on the New York Regents Geometry Exam  $F(2,119) = 109, p = .007$ , the Rey Auditory Memory Exam  $F(2,108) = 5.57, p = .005$ , and

Table 5

Means and Standard Deviations of the Cognitive and Geometry Measures for the Total Sample, School 1, and School 2

Measure	Total Sample		School 1		School 2	
	N = 110		N = 55		N = 56	
	Mean	SD	Mean	SD	Mean	SD
Rey Auditory Memory	9.93	1.51	9.51	1.53	10.35	1.36
Nonsense Syllogisms	1.36	4.16	1.64	4.07	1.17	4.16
Diagramming Relations	12.32	6.24	10.83	6.13	13.86	5.88
Necessary Arithmetic	6.09	2.56	5.64	2.51	6.57	2.57
Surface Development	13.06	7.80	10.74	7.74	15.17	7.30
Paper Folding	9.93	4.43	8.67	5.04	11.16	3.36
Gestalt Completion	13.90	2.86	13.53	2.85	14.21	2.83
Mental Rotations	9.72	5.43	9.37	5.12	9.99	5.75
Hidden Pattern	80.62	28.38	76.35	28.93	84.88	17.06
Card Rotations	51.75	18.79	47.42	19.58	55.62	17.04
New York Regents	11.70	5.86	7.74	3.62	14.05	5.35
Final Grade	79.87	12.03	78.41	13.77	81.33	9.86

Diagramming Relations  $F(2,89) = 4.79, p = .01$ . Subsequent mean comparisons revealed the students in the rural high school scored significantly lower on these three measures than those students in the suburban high school.

#### Gender Differences

As can be seen in Table 6, there were few gender differences on the cognitive skills tests. As expected, differences were found on the measure of speeded rotations, the Mental Rotations Test. Unexpectedly, differences were found on measures of short-term memory, orientation, and Gestalt closure. Boys had significantly higher scores than girls on the Mental Rotations Test, the Card Rotations Test, the Gestalt Completion Test. Girls had significantly higher scores on the Rey Auditory Memory Test. Interestingly, girls scored significantly higher on the in-class final geometry test than boys, but there were no differences between boys and girls on the standardized New York Regents Geometry Exam.

#### Correlations between Measures

Table 7 displays the correlations between measures of visuospatial skills, analytical skills, and geometry achievement. As can be seen, there were a number of

Table 6

Means and Standard Deviations of the Cognitive and Geometry Measures for the Boys and Girls

Measure	Boys N = 52		Girls N = 59		F	p=
	Mean	SD	Mean	SD		
Rey Auditory Memory	9.49	1.43	10.34	1.47	10.04	.002
Nonsense Syllogisms	1.66	4.88	1.09	4.41	.07	N.S.
Diagramming Relations	12.18	6.57	12.47	6.01	.63	N.S.
Necessary Arithmetic	6.14	2.78	6.05	2.38	.01	N.S.
Surface Development	13.46	7.80	12.71	7.83	.98	N.S.
Paper Folding	9.49	4.43	10.31	3.89	.02	N.S.
Gestalt Completion	14.24	2.67	13.59	3.02	4.69	.033
Mental Rotations	12.39	5.08	7.37	4.62	30.74	.001
Hidden Pattern	79.06	31.89	82.00	25.12	.17	N.S.
Card Rotations	55.06	18.59	48.67	18.61	7.66	.007
New York Regents	12.12	5.37	11.31	6.31	.13	N.S.
Final Grade	77.64	13.78	82.15	12.88	4.91	.029

Table 7  
Correlations between Measures of Cognitive Skills and Geometry Achievement

	PAPR	SURF	HIDN	CARD	GSTLT	NSYL	DREL	REY	ARITH	NYRG	GRADE
MRT	.43**	.47**	.30**	.40**	.34**	-.02	.16	-.07	.10	.16	-.07
PAPR		.57**	.56**	.42**	.36**	-.01	.60**	.03	.26**	.35**	.30**
SURF			.34**	.41	.33	.02	.51**	.03	.27*	.52**	.25**
HIDN				.36**	.31	.16	.25*	.13	.16	.32**	.07
CARD					.28**	.09	.34**	-.03	.09	.29**	-.05
GSTLT						.09	.34**	-.05	.23*	.17	-.15
NSYL							.25*	.06	-.01	-.03	.07
DREL								.10	.23*	.46**	.26**
REY									.23*	.34**	.18
ARITH										.33**	.20*
NYRG											.31**

\*p < .05, \*\*p < .01

MRT	Mental Rotations Test	REY	Auditory Memory Test
PAPR	Paper Folding Test	DREL	Diagramming Relations Test
SURF	Surface Development Test	ARITH	Necessary Arithmetic Operations Test
HIDN	Hidden Patterns Test	NYRG	Necessary Arithmetic Operations Test
CARD	Card Rotations Test	GRADE	Final Grade
GSTLT	Gestalt Closure Test		
NSYL	Nonsense Syllogisms Test		

significant correlations between measures. These intercorrelations were employed in the multiple linear regression analyses and the factor analyses described in the following sections.

#### Multiple Linear Regression Analysis

Separate multiple linear regression analyses were performed using the New York Regents Geometry Exam and z-scores for the final grade as the criterion. The predictors used in the regression model were the visuospatial measures (Surface Development Test, Paper Folding Test, Gestalt Completion Test, Mental Rotations Test, Hidden Patterns Test, and Card Rotations Test), logical reasoning measures (Diagramming Relations Test and Nonsense Syllogisms Test), the Rey Auditory Memory Test, and the Necessary Arithmetic Operations Test, and gender.

New York Regents Exam. As can be seen from Table 8, there was a significant relationship between the New York Regents Exam and Surface Development, Diagramming Relations, and Rey Auditory Memory,  $F(10,72) = 6.01, p = .0001$ . The R Squared of .45 indicates a moderate relationship.

Separate analyses for boys and girls revealed highly different patterns of predictor variables (the cognitive skill measures) for this criterion (New York Regents

Geometry Exam) for the genders. For boys, the Rey Auditory Memory Test, Nonsense Syllogisms Test, and Diagramming Relations Test were significant predictors for the New York Regents Geometry Exam,  $F(10,28) = 4.64$ ,  $p = .0006$ . The R squared was .62. The analyses revealed a negative beta for the Nonsense Syllogisms Test indicating that boys who score high on this test attained lower scores on the New York Regents Geometry Exam than did boys who scored low on the Nonsense Syllogisms Test. For girls, the New York Regents Geometry Exam was only predicted by the Surface Development Test,  $F(10,32) = 3.66$ ,  $p = .003$ . The R squared was .53.

Classroom Geometry Performance. For the total sample, Paper Folding and Gestalt Completion were significant predictors of the final grade,  $F(10,83) = 3.31$ ,  $p = .001$ . The R Squared was .28. The Gestalt Completion Test has a negative beta indicating that those students who are high scorers on this measure achieved lower final grades.

Separate analyses were performed for boys and girls. The regression equation for the boys with all the cognitive measure used as predictive variables for the final grade was not significant. The Card Rotations Test significantly predicted the final grade for the girls,  $F(10,39) = 2.61$ ,  $p = .02$ . The R Squared was .40.

Table 8

Results of Multiple Linear Regression Analyses with the New York Regents Exam as Criterion and Cognitive Skills as Predictors for the Total Sample, Boys, and Girls

Variable	B	SE B	Beta	t	p=
Total Sample (N = 110)					
R Squared = .45					
Rey Auditory Memory	1.03	.354	.266	8.49	.004
Surface Development	.302	.090	.403	11.42	.001
Diagramming Relations	.240	.119	.255	4.04	.048
Boys (N = 59)					
R Squared = .62					
Rey Auditory Memory	2.27	.547	.605	4.15	.001
Nonsense Syllogisms	-0.50	-.229	-.361	-2.18	.038
Diagramming Relations	.388	.163	.475	2.39	.024
Girls (N = 51)					
R Squared = .53					
Surface Development	.443	.131	.550	3.39	.002

### Path Analysis

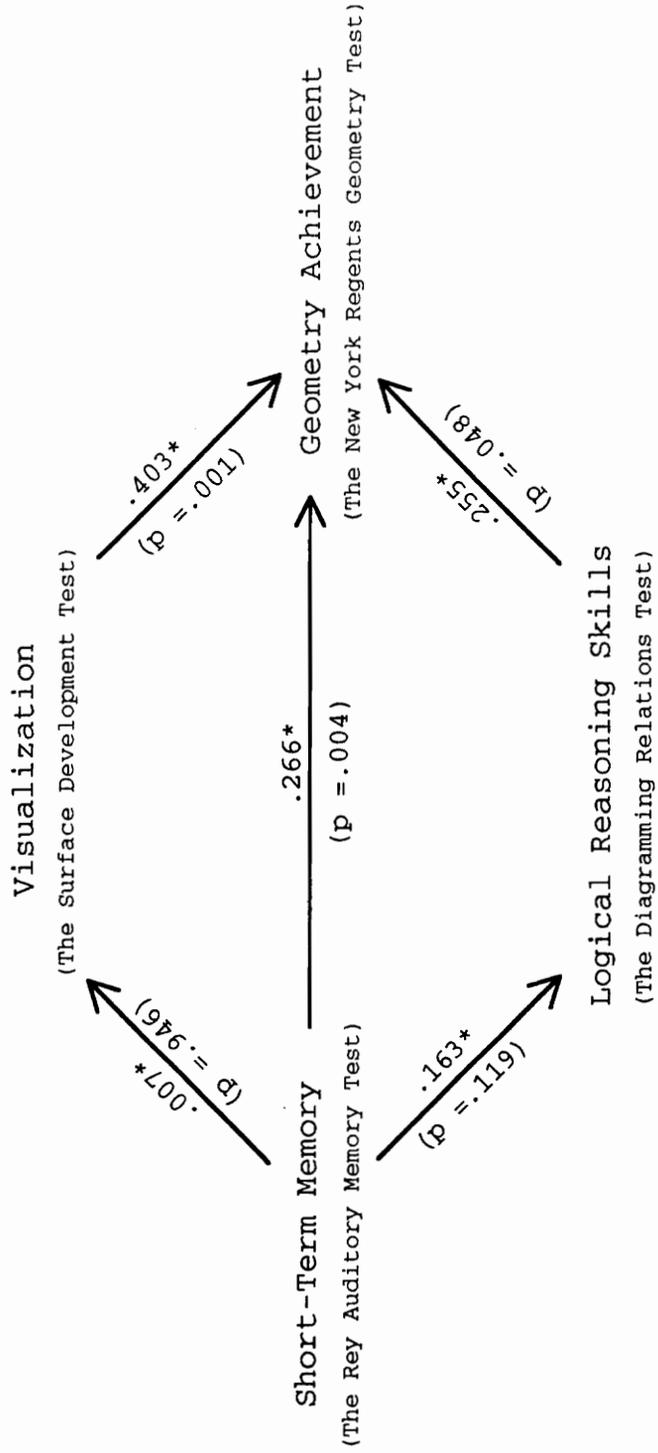
A path analysis revealed, as predicted and presented in Figure 2, significant direct positive effects for Surface Development Test ( $B = .403$ ,  $p = .001$ ), Diagramming Relations Test ( $B = .255$ ,  $p = .048$ ), and the Auditory Memory Test ( $B = .266$ ,  $p = .004$ ). The indirect effects of span of short-term memory as measured by the Auditory Memory Test on the Surface Development Test ( $B = .007$ ,  $p = .946$ ) and the Diagramming Relations Test ( $B = .163$ ,  $p = .119$ ) were not significant.

### Factor Analysis

Separate factor analyses with principal components using a varimax rotation were performed, one using the New York Regents Exam and one using the final classroom grade (Table 9 and 10). Factor analysis employing an oblique rotation confirmed the varimax rotation analysis. Three factors with eigenvalues over 1.0 were revealed in each. Variables with factor loadings showing a value of .40 or higher were included in a factor.

The New York Regents Geometry Exam. As can be seen in Table 9, factor analysis including the cognitive measures with the New York Regents Geometry Exam produced three factors. The first factor was defined by the Diagramming

Figure 2  
Path Analysis of Achievement in Geometry.



\* Standardized regression coefficient

Relations Test, all visuospatial measures, and the New York Regents Geometry Exam. The Diagramming Relations Test, the Necessary Arithmetic Operations Test, the Rey Auditory Memory Test, and the New York Regents Exam loaded on Factor 2. The last factor had the Nonsense Syllogisms Test and the Hidden Patterns Test loading on it.

Classroom Geometry Performance. A factor analysis including the cognitive measures with the final grade produced three factors. These results are displayed in Table 10.

The first factor loaded all the visuospatial measures with the Diagramming Relations Test. Factor 2 included the Diagramming Relations Test, Necessary Arithmetic Operations Test, the Rey Auditory Memory Test, and the final grade. The third factor loaded the Hidden Patterns Test, the Gestalt Completion Test, and the Nonsense Syllogisms Test.

Table 9

Factor Analysis of the Cognitive Tests and the New York Regents  
Geometry Exam

Measure	Factor 1	Factor 2	Factor 3	Communalities
Rey Auditory Memory	-.25	.66	.20	.54
Nonsense Syllogisms	-.07	.10	.84	.73
Diagramming Relations	.50	.46	.38	.61
Necessary Arithmetic	.02	.68	.08	.46
Surface Development	.77	.31	-.08	.70
Paper Folding	.71	.17	.24	.59
Mental Rotations	.69	-.20	.03	.49
Hidden Pattern	.50	.16	.50	.52
Card Rotations	.66	-.01	-.13	.46
Gestalt Completion	.58	-.13	.35	.48
New York Regents	.42	.77	.12	.78
Eigenvalue	3.59	1.64	1.13	
Percent Variance Accounted for	33%	15%	10%	

Table 10

Factor Analysis of the Cognitive Tests and the Final Grade

Measure	Factor 1	Factor 2	Factor 3	Communality
Rey Auditory Mem.	-.22	.55	.36	.48
Nonsense Syllogisms	-.04	.10	.76	.59
Diagramming Relations	.50	.41	.37	.56
Necessary Arithmetic	.05	.61	.08	.39
Surface Development	.80	.28	-.03	.72
Paper Folding	.74	.25	.19	.65
Gestalt Completion	.51	-.18	.42	.48
Mental Rotations	.74	-.10	-.04	.55
Hidden Pattern	.48	-.01	.59	.57
Card Rotations	.64	-.20	.05	.45
Final Grade	.06	.78	-.17	.64
Eigengalue	3.29	1.66	1.12	
Percent Variance Accounted for	30%	15%	10%	

## Discussion

Model of Achievement in Geometry

The results clearly show, as anticipated, high school geometry performance is predicted by certain visuospatial factors and logical reasoning skills, and by the capacity to hold information in short-term memory. Specifically, The New York Regents Geometry Exam was significantly predicted by the Surface Development Test, The Diagramming Relations Test, and the Rey Auditory Memory Test as demonstrated by the multiple linear regression analysis and the path analysis.

The factor analyses provide support for the conclusions drawn from the regression and path analyses. The loading of the New York Regents Geometry Exam with the visuospatial measures and one of the analytical reasoning measures, and the Necessary Arithmetic Operations Test and the Rey Auditory Memory Test indicate, in addition to a knowledge of arithmetic operations, that visuospatial and analytical reasoning skills and short-term memory capacity are important for performance on this geometry test.

Clearly, there is convergent evidence from the multiple regression, path, and factor analyses that the skills of visualization, deductive reasoning, and the capacity to hold

larger amounts in short-term memory are important for geometry problem solving.

Visuospatial factors. Visualization as measured by the Surface Development Test emerged as the visuospatial subfactor which predicts performance on the New York Regents Exam. The Surface Development Test measures skill in visually organizing two-dimensional figures in component parts into a whole three-dimensional figure. This skill is more important for processing information that is concerned with the "properties, measurement, and relationships of points, lines, angles, and figures in space" (Urdang, 1968) than the skills of speeded rotations, or disembedding. Salthouse, Babcock, Mitchell, Palmon, and Skovronek (1990) found subjects who could preserve information while that information or some other information is being processed performed better on the Surface Development Test. The results in this study suggest that the ability to preserve a mental representation of a figure while manipulating that figure, or performing mathematical operations, distinguishes better geometry problem solvers from the less efficient.

The process of visualization is important for achievement in the classroom as demonstrated by the Paper Folding Test as a significant predictor of the final grade

in geometry. The Paper Folding Test is another test of visualization which measures skill in perceiving a whole pattern from viewing only a part of that pattern. The finding that flexibility of closure as measured by Gestalt Completion was negatively related to the final grade was unexpected. As a measure of skill in abstracting a whole from an incomplete figure it was hypothesized this measure would be predictive of achievement in geometry since problem solvers are given incomplete information in a problem and must solve for unknowns, thereby creating a whole.

Logical reasoning skills. The Diagramming Relations Test measures deductive reasoning skills using Venn diagrams. The underlying skill measured by this test should facilitate the ability to "deduct properties . . . from their defining conditions" (Urdang, 1968, p. 552), and the results truly demonstrate this intuitive logic. It is interesting, and significant, that the Diagramming Relations Test was more predictive than the Nonsense Syllogisms Test for the whole sample. This indicates that individuals using logical reasoning emphasizing a visuospatial strategy are more successful problem solvers in geometry.

Short-term memory span. The results of this study demonstrate that better problem solvers as measured by the

New York Regency Exam and Final Grade had higher working memory spans. With a greater capacity to hold more information, these students were able to retain more of the "givens" or defining conditions of the problem in working memory which facilitated their problem solving efforts. Solving a mathematical problem involves the progression of several steps, from an initial stage of representation of the givens, to a stage of representing the goals of the problem, and then to intermediate levels where operations are performed to resolution of the problem (Mayer, 1991). Problem solvers who can hold more information at each stage would be better problem solvers.

The path analysis revealed, surprisingly, that the Auditory Memory Test did not indirectly affect achievement in geometry via direct effects on the Surface Development Test or the Diagramming Relations Test. It may be, given the spatial nature of these measures, that a verbal short-term memory measure was not a good measure of these effects. A spatial short-term memory test may have revealed these indirect effects.

#### Gender Differences

The regression analysis which revealed that Surface Development is predictive of the New York Regents Exam for

girls but not for boys is supportive of Battista's (1990) results showing that visualization is more predictive of girls' success in geometry than for boys'. This indicates that visualization plays a central role in performance in geometry for girls but not for boys. Previous researchers (eg. Battista, 1990; Fennema & Tartre, 1985) have found boys rely more on mental visualization strategies to solve problems and girls who are low in visualization rely more on drawing strategies. Visualization training may prove to be beneficial for girls' achievement in geometry.

Spatial orientation, as measured by the Card Rotations Test, was found to be differentially predictive of the final grade. This measure predicted performance on the final grade for girls but not for boys. This test measures the ability to imagine figures presented from different perspectives. It does seem reasonable this skill would be important for success in geometry, however, it is not clear why these differences between samples emerged in this research.

In this sample girls achieved higher grades on the final grade than boys, though there were no differences between the genders on the New York Regents Exam. This is an encouraging result, although the sample is small, and

supportive of national trends demonstrating a decline in gender differences. Additionally, there are problems, as discussed in an earlier section, with drawing inferences about achievement as measured by classroom instruments. There are unknown and unmeasured variables which influence success in classroom assessments which make interpretation of this result problematic. With these cautions in mind, it is hoped this is further evidence of a decline in the gender differences frequently found in problem solving (Hyde et al., 1990; Lummis & Stevenson, 1990).

The Nonsense Syllogisms and the Diagramming Relations Tests were both predictive for the New York Regents Exam for boys, but not for girls. This is evidence supportive of Battista's (1990) finding that logical reasoning is more predictive of boys' performance than girls'. These results show that boys who are capable of both visuospatial logical reasoning and verbal logical reasoning achieved higher scores on the New York Regents Geometry Exam. This suggests that logical reasoning plays a more fundamental role in boys geometry problem solving than it does for girls. Training in logical reasoning may help dissipate the differences between boys who are low in their logical reasoning skills

and those who are high, and improve their performance in geometry problem solving.

Gender differences favoring boys were found on the measure of speeded rotations, the Mental Rotations Test. This was anticipated since previous research had shown that gender differences on these measures have persisted (Linn & Petersen, 1985; Vandenberg & Kuse, 1978).

Unexpected gender differences were found on the measure of spatial orientation, the Card Rotations Test, and the measure of Gestalt closure, the Gestalt Completion Test (both favoring boys), and the Rey Auditory Memory Test (favoring girls). Previous research (Linn & Petersen, 1985) has shown gender differences on the Card Rotations Test, however, as described earlier, this difference appears to be declining. It was therefore expected this decline would be revealed in this study. Previous research has not demonstrated gender differences on the measure of Gestalt Completion Test (Wasserstein et al., 1980) and they were therefore not expected to be found in this sample.

Gender differences favoring girls were not expected on the Rey Auditory Memory Test since none had been reported in most of the literature reviewed for this research. However, the researchers reviewed (e.g., Baddeley, 1986; Cooney &

Swanson, 1990; Daneman & Carpenter, 1980; Yuill et al., 1989) did not report testing for gender differences in span of working memory. Lummis and Stevenson (1990) report the fifth grade girls in their cross-cultural sample of American, Taiwanese, and Japanese students had significantly higher scores on their auditory memory test (recall of a tapping pattern) than the boys. The girls in their study also had higher scores on a test of verbal memory. Such potential gender differences in span of working memory should be systematically evaluated in future research.

IMPLICATIONS FOR FUTURE RESEARCH  
AND EDUCATIONAL DEVELOPMENT

The findings in this research that visualization and logical reasoning predict performance in geometry provide support for Vernon's (1965) conceptualization that mathematics is related to logical and visuospatial processes. The finding that short-term memory capacity predicted performance on the New York Regents Exam is in accordance with Baddeley's (1986) model of working memory. These results suggest that individuals who are low in the skills of visualization and deductive reasoning may benefit from skills training in those areas to improve their performance in geometry.

The results from this research suggest that a componential analysis of geometry learning may provide more information about the cognitive skills that are important for achievement in high school geometry. Discussion with one of the teachers in the pilot study revealed that the first semester focussed on more conceptual learning such as definitions, logic, and solving proofs. The second semester she described as more "visuospatial and more inductive." She noted that there was less proof solving and use of logic. Computations had an emphasis on visuospatial

elements in geometry. In order to enhance the skills of students to complete these types of calculations she trained the students visuospatially. Her training included manipulation and visualization of "lego" type cubes. Possibly, this training dissipated the advantage the high visuospatial students had in performance on geometry tests and contributed to the lack of significant relationship between the visuospatial measures and geometry problem solving in the pilot study.

Research in the training of both visuospatial and logical reasoning skills may show that students would improve in geometry problem solving as a result of training. Results from Crawford's (1989) lab show that subjects who are trained in logical reasoning using Vein Diagrams, and other strategies, significantly improved their scores on both measures of visuospatial and logical reasoning skills. Indeed, the national standards for mathematics education emphasizes that all students should have experiences that facilitate the development of visuospatial and logical reasoning skills (National Council of Teachers of Mathematics, 1989).

Informal discussion with the teachers who participated in this research indicated that only one of these teachers

provided these learning experiences for the students. There also were no gender differences in this class. One of the teachers indicated she planned to incorporate more hands-on visuospatial training for her geometry students the following school year.

Baddeley's (1992) model of working memory also includes the subsidiary visuospatial scratch pad. A measure of visuospatial short-term memory was not employed in this research. Employment of such a measure may show individual differences on the visuospatial tests that were not revealed by the verbal test we employed, and may also be predictive of performance in geometry.

Greeno's (1980) research indicates that individuals who can successfully break geometry problems down into subgoals are better problem solvers. Perhaps breaking down a problem into subgoals allows these problem solvers to hold more in working memory. The implications of this are that research in training this strategy may prove beneficial for increasing the short-term memory capacity of individuals as well as improve their problem solving skills.

In summary, the results of this study strongly suggest that our educational system should develop training programs in visuospatial thinking and logical reasoning, both verbal

and visuospatial, at an early age. Such systematic training in these skills may well assist in overcoming lags in mathematical performance evident in the United States today.

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## Informed Consent Form

Visuospatial and Analytical Skills: Relationships to  
Achievement in High School Geometry

The purpose of this research is to investigate the relationship between various visuospatial and logic tests and geometry performance in order to gain a better understanding of the cognitive skills that are needed in geometry problem solving.

Participants will complete a series of visuospatial, logic, and geometry tests. Scores on these tests will be correlated with participants' geometry test grades and the final exam. It is expected that test sessions will require three class periods.

The information provided by this research may be used for research and educational purposes. The information relating to responses may be presented at scientific meetings and/or published in professional publications. Participants names will be removed from all tests and replaced with a code number. Therefore, a participant's name will not be identified with any test score or other data generated by this research. All information will be confidential. Your teachers will not be given your individual scores.

Participation in this research is voluntary and may be withdrawn at any time. If you are interested in the results they will be available from the researchers. However, since all data are anonymous and will be analyzed as such, information regarding individual responses will not be available.

This research project has been approved by the Human Subjects Committee, Psychology Department, Virginia Tech, and the Institutional Review Board, Virginia Tech. Any questions that you may have about the project should be directed to:

Marty Brown	Researcher	231-6581
Helen Crawford,	Research Director, and	231-6520
	Chair, Human Subjects Committee	
Ernest Stout	Institutional Review Board	231-5281

I hereby agree to voluntarily participate in the research project described above with the understanding that I may withdraw at any time.

Name (please print) \_\_\_\_\_

Signature \_\_\_\_\_ Date \_\_\_\_\_

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