

# **Confidence Interval Estimation for Distribution Systems Power Consumption by using the Bootstrap Method**

by

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**Keywords:** Distribution Systems, Power Consumption Estimation, Confidence Intervals,  
Bootstrap Method.

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## **Abstract**

The objective of this thesis is to estimate, for a distribution network, confidence intervals containing the values of nodal hourly power consumption and nodal maximum power consumption per customer where they are not measured. The values of nodal hourly power consumption are needed in operational as well as in planning stages to carry out load flow studies. As for the values of nodal maximum power consumption per customer, they are used to solve planning problems such as transformer sizing. Confidence interval estimation was preferred to point estimation because it takes into consideration the large variability of the consumption values. A computationally intensive statistical technique, namely the bootstrap method, is utilized to estimate these intervals. It allows us to replace idealized model assumptions for the load distributions by model free analyses.

Two studies have been executed. The first one is based on the original nonparametric bootstrap method to calculate a 95% confidence interval for nodal hourly power consumption. This estimation is carried out for a given node and a given hour of the year. The second one makes use of the parametric bootstrap method in order to infer a 95% confidence interval for nodal maximum power consumption per customer. This estimation is realized for a given node and a given month. Simulation results carried out on a real data set are presented and discussed.

**Keywords:** Distribution Systems, Power Consumption Estimation, Confidence Intervals, Bootstrap Method.

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