

# **MODELING PERMISSIVE LEFT-TURN GAP ACCEPTANCE BEHAVIOR AT SIGNALIZED INTERSECTIONS**

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# **MODELING PERMISSIVE LEFT-TURN GAP ACCEPTANCE BEHAVIOR AT SIGNALIZED INTERSECTIONS**

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## **ABSTRACT**

The research presented in this thesis, studies driver gap acceptance behavior for permissive left turn movements at signalized intersections. The study characterizes the impact of different independent variables affecting a driver's gap acceptance decision. The research presented is based on field data gathered from the Depot and Franklin Street signalized intersection in Christiansburg, Virginia. The thesis attempts to model the gap acceptance behavior using three different approaches, a deterministic statistical approach, a stochastic approach, and a psycho-physical approach.

First, the deterministic statistical modeling approach is conducted using logistic regression to characterize the impact of a number of variables on driver gap acceptance behavior. The variables studied are the gap duration, the driver's wait time in search of an acceptable gap, the time required to travel to clear the conflict point, and the rain intensity. The model demonstrates that drivers become more aggressive as they wait longer in search of an acceptable gap. Alternatively, the results show that drivers become less aggressive as the rain intensity increases.

Considering stochastic gap acceptance, two stochastic approaches are compared, namely: a Bayesian and a Bootstrap approach. The study demonstrates that the two methods estimate consistent model parameters but the Bayesian estimated parameters are less skewed. The study develops a procedure to model stochastic gap acceptance behavior while capturing model parameter correlations without the need to store all parameter combinations. The model is then implemented to estimate stochastic opposed saturation flow rates.

Finally, the third approach uses a psycho-physical modeling approach. The physical component captures the vehicle constraints on gap acceptance behavior using vehicle dynamics models while the psychological component models the driver deliberation and decision process. The results of this model are compared to the statistical logistic modeling approach. The results demonstrate that the psycho-physical model is superior as it produces consistent performance for accepted and rejected gap predictions although it is only constructed using accepted gaps.

In general, the three proposed models capture gap acceptance behavior for different vehicle types, roadway surface conditions, weather effects and types of control which could affect the driver gap acceptance behavior. These findings can be used to develop weather responsive traffic signal timings and can also be integrated into emerging IntelliDrive systems.

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## **ATTRIBUTION**

Prof. Hesham Rakha (Professor, Department of Civil Engineering, Virginia Tech and Director, Center for Sustainable Mobility VTTI) is the primary Advisor and Committee Chair. Prof. Rakha provided the research direction for all work during the author's graduate study. Furthermore, Prof. Rakha also directed the composition of the chapters, extensively edited each chapter in this thesis and is the corresponding author of all papers published which will be discussed in Chapters 4, 5 and 6.

Dr. Shereef Sadek (Virginia Tech Transportation Institute, Virginia Tech) worked in collaboration with the author on a project supported from Federal Highway Administration (FHWA). Dr Shereef helped in data reduction procedures and cooperated in writing the papers presented in Chapters 4 and 5.

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## CHAPTER 1 : INTRODUCTION

Congestion mitigation in urban areas is an important issue that needs addressing in our modern society. Numerous problems are associated with interrupted traffic flow; include vehicle and personal delay that lead to significant loss in resources.

One of the factors that affect the capacity and saturation flow rate at signalized and non-signalized intersections is gap acceptance behavior. Gap acceptance is defined as the process that occurs when a traffic stream (known as the opposed flow) has to cross another traffic stream (known as the opposing flow) or merge with the opposing flow. Gap acceptance behavior appears when vehicles on a minor approach need to cross a major street at a two-way stop controlled intersection, or when vehicles have to make a left turn through an opposing through movement at a signalized intersection. Alternatively, gap acceptance is applied when vehicles approaching a roundabout have to merge with vehicles traveling in the roundabout, or when vehicles merging onto a freeway have to find gaps in the freeway flow. Generally gap acceptance analysis can be divided into two types of analyses: (1) crossing gap acceptance at intersections or (2) gap acceptance during merging or lane-change maneuvers.

Within the context of crossing gap acceptance, a gap is defined as the elapsed-time interval between arrivals of successive vehicles in the opposing flow at a specified reference point in the intersection area. The minimum gap that a driver is willing to accept is generally called the critical gap. The Highway Capacity Manual (HCM 2000) [1] defines the critical gap as the *“minimum time interval between the front bumpers of two successive vehicles in the major traffic stream that will allow the entry of one minor- street vehicle.”* When more than one minor street vehicle use the same gap, the time headway between the two minor street vehicles is called the follow-up time. In general, the follow-up time is shorter than the critical gap and equals the inverse of the saturation flow rate.

### 1.1 Problem Statement

Driver gap acceptance behavior traffic situation is considered a decision making traffic process. The primary concern of the analyst is to understand how drivers make their decisions with regards to accepting or rejecting a given gap and which factors affect these decisions.

There are many factors affecting the gap acceptance decision and several studies in the literature have investigated the impact of different factors. These factors include: day versus nighttime effects [2], the speed of the opposing vehicle [3, 4], the type of intersection control (yield versus stop sign) [5], the driver sight distance [6], the geometry of the intersection, the trip purpose, and the expected waiting time [7].

Mathematical representations of the gap acceptance process are an important component of traffic simulation software. In an attempt to provide a more realistic representation of this behavior, these mathematical descriptions have become more complex by including driver behavior parameters, such as 'impatience' or 'aggressiveness' associated with various parameters. Several methods have been developed to model this behavior including empirical analysis and theoretical logit and probit models.

Hence, the process is difficult because drivers do not know the number and size of gaps that will be offered to them a priori. The variables affecting the gap acceptance/rejection decision may be difficult to identify and the driver choice could be risky. However, most of the research has centered on the prediction of delay rather than the behavior of drivers [8].

The basic differences among the different studies were the underlying assumptions about driver behavior (consistent or inconsistent), the mathematical distributions fitted to accepted gaps and the type of the developed gap acceptance model (deterministic versus probabilistic). Three basic driver behavioral models have formed the basis for previous studies [9], namely:

- a. "Fixed critical" or "Deterministic" gap concept which implies that there is a minimum critical lag or gap,  $(t_c)$ , which is identical for all drivers in a population. A driver will accept a lag or gap in the major stream only if it is equal to or greater than the critical gap,  $(t_c)$ . In this case, the gap acceptance function is a step function;
- b. "Consistent driver" concept which implies that each individual driver has his own constant critical lag or gap size, which is fixed for the driver but distributed over the population (inter-driver variability). This pattern of behavior implies absolute consistency for each individual driver although individuals may vary from each other ;

- c. “Inconsistent driver” concept which implies that variability in gap acceptance behavior exists within and between individuals as well (intra-driver variability).

It can be stated that previous research has made simplifying assumptions and failed to capture the impact of various factors on gap acceptance behavior. This research effort is a modest attempt to address some of these issues. The factors that are studied include:

- a. Driver aggressiveness and impatience as a function of the time he/she spend waiting in search of an acceptable gap;
- b. The buffer of safety that driver is willing to accept in order to avoid a conflict with a conflicting opposing vehicle ;
- c. The vehicle characteristics (e.g. vehicle power, mass and engine capacity);
- d. Weather impact (rain intensity) and roadway surface condition (dry or wet) effect on gap acceptance behavior;
- e. Distance of travel to clear the conflict point (first or second opposing lane); and
- f. Intra- and inter-differences in driver behavior while capturing the underlying correlations between various factors.

## **1.2 Research Objectives**

This research focuses on modeling driver gap acceptance behavior for permissive left-turn movements at signalized intersections. Therefore, the main objectives of this research could be summarized as follow:

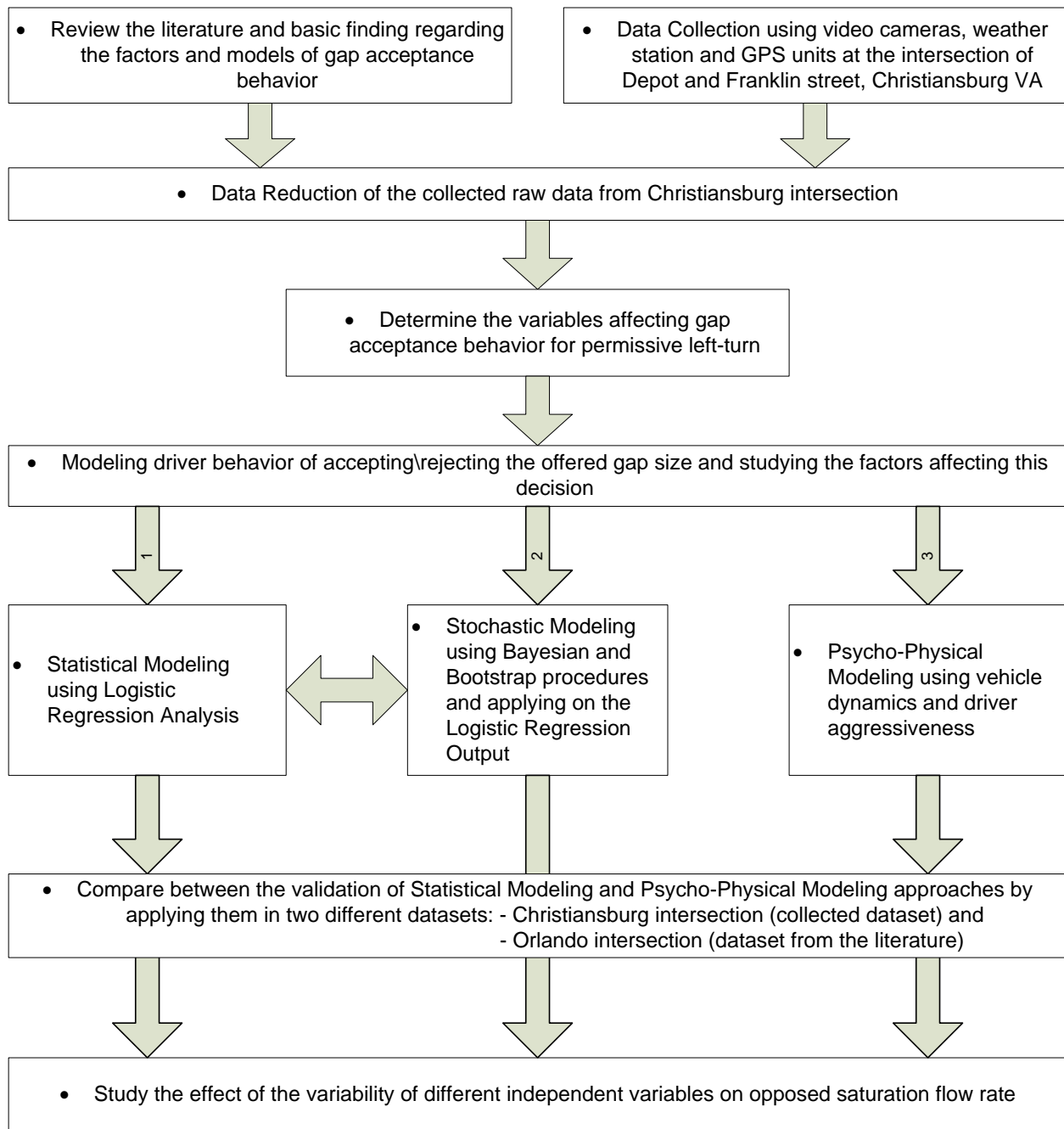
- a. Investigate the influence of driver waiting time, distance travelled to clear the conflict point, and rain intensity on left-turn gap-acceptance behavior;
- b. Develop a statistical modeling approach using logistic regression to fit gap acceptance behavior data to model driver gap acceptance behavior and compute the driver-specific critical gap;
- c. Compute opposed saturation flow rate weather adjustment factors;

- d. Develop two stochastic approaches (Boot strapping and Bayesian) for the modeling of stochastic driver gap acceptance behavior that capture intra- and inter-differences in driver behavior;
- e. Develop a framework for modeling inter- and intra- differences in driver behavior while capturing the correlation between various factor;
- f. Study the impact of a stochastic gap acceptance behavior on opposed saturation flow rates;
- g. Construct a Psycho-Physical model of gap acceptance behavior. The model is physical because it captures the physical constraints presented in the vehicle dynamics. Alternatively, the model is psychological because it captures the human's psychological deliberation and desired safety buffer to avoid a collision with an on-coming vehicle.

### **1.3 Research Methodology**

The research that is presented in this thesis involves several tasks, as follow: review and synthesize the literature, gather field data, reduce and analyze the field data, model driver gap acceptance behavior and the study the effect of different variables on driver gap acceptance behavior. The research methodology is summarized in Figure 1.1.

The research begins with the literature review in order to identify the current state-of-art in modeling driver gap acceptance behavior and identifying the needs for further research. Subsequently, field data are gathered at the Depot and Franklin Street intersection located in Christiansburg Virginia. The data reduction and analysis of the raw data step was followed in order to identify different control variables affecting the gap acceptance decision. Three modeling frameworks are considered in this thesis: a statistical, a stochastic and finally a psycho-physical approach. The models validation was conducted on two datasets. The first dataset is the same dataset that was used in the development of the models while the second dataset was provided by the University of Central Florida for a signalized intersection in Orlando, FL. Finally, the impact of gap acceptance behavior on opposed saturation flow rates was derived.



**Figure 1.1: The research methodology**

## **1.4 Research Contributions**

By accomplishing the research objectives, this research will benefit in different ways. By investigating the factors that impact gap acceptance behavior, it can assist in developing more accurate capacity models for signalized intersection with permissive left turn movement. In addition, the research is beneficial in the design of highways, traffic operation and analysis, and travel safety modeling. The constructed models can also be generalized and applied in the modeling of driver gap acceptance behavior at yield, stops signs and roundabouts.

It is anticipated that this research will contribute in the future of intelligent transportation system (ITS), IntelliDrive systems, and vehicle to vehicle communications. Furthermore, the proposed approaches for modeling gap acceptance behavior could be used in the Intersection Decision Support (IDS) and improve the feasibility of stop sign gap assistance systems.

## **1.5 Thesis Layout**

This thesis is organized into seven chapters:

The first chapter includes a general description of gap acceptance phenomenon, statement of the problem, research objectives, research methodology and research contributions.

The second chapter presents the necessary definitions and summarizes the basic findings of the current state-of-art procedures in modeling driver gap acceptance behavior along with the areas that require additional research.

The third chapter presents the studied intersection (the intersection of Depot and Franklin Street, Christiansburg, VA). Moreover, it shows the data collection procedures and data reduction preliminary results.

The fourth chapter shows the different proposed statistical models along with the model calibration results. Subsequently, the predicted critical gap from each model is presented and the influence of waiting time and rain intensity is studied. Finally, the saturation flow adjustment factor due to rain intensity and opposing flow impact is described.

The fifth chapter presents the boot strapping and the Bayesian statistics approaches for gap acceptance modeling. A comparison between the two approaches is then presented. At the end,

the modeling of stochastic opposed saturation flow rate is discussed using the proposed gap acceptance modeling framework.

The sixth chapter develops a psycho-physical framework for modeling driver left turn gap acceptance behavior. The physical component of the model explicitly captures the vehicle constraints on driving behavior using a vehicle dynamics model, while the psychological component explicitly models the driver's deliberation in accepting/rejecting a gap.

The final chapter provides overall conclusions as well as recommendations for further research.

Noteworthy is the fact that chapters four, five and six are papers that have either been submitted or will be submitted for publication. These papers are as follows:

- a. Zohdy I., S. Sadek and H. Rakha, "Empirical Analysis of Wait time and Rain Intensity Effects on Driver Left-Turn Gap Acceptance Behavior," accepted for presentation at the TRB 89<sup>th</sup> Annual Meeting and being considered for publication in the Transportation Research Record
- b. Rakha H., S. Sadek and I. Zohdy, "Modeling Stochastic Left-Turn Gap Acceptance Behavior," accepted for presentation at the TRB 89<sup>th</sup> Annual Meeting.
- c. Zohdy I. and H. Rakha, "Modeling Left-Turn Gap Acceptance Behavior: A Psycho-Physical Approach," to be submitted.

## CHAPTER 2 : LITERATURE REVIEW

This chapter provides the needed definitions, a review of related research, and identifies areas in which the literature should be expanded. The chapter is divided into four main subsections, which are: basic definitions, critical gap calculation methods, modeling gap acceptance behavior and summary & conclusions.

### 2.1 Basic Definitions

This section clarifies the basic relevant definitions presented in the different literature as follow:

#### 2.1.1 Gap

The gap is defined generally in most of the studies as “*the elapsed time interval (time headway) between arrivals of two successive vehicles in the major stream at the same reference point*” [1-5, 7-18].

Some of the literature sources define the gap at intersections as “*the time elapsed between the rear bumper of one vehicle and the front bumper of the following vehicle passing a given point*” [19, 20] but usually this definition is applied for the gap acceptance in changing lane scenario [21-23].

#### 2.1.2 Lag

The lag is defined as “*the interval from the time a side-street car reaches the intersection (or the head of the line, if there is a line of waiting cars) until the passage of the next main-street car*” [5, 10, 11, 20, 24]. In other words lag is the portion of the last gap in the major stream remaining when a vehicle at minor road reaches the intersection point from which it is ready to execute the desired maneuver.

#### 2.1.3 Critical Gap (or Lag)

There is no single definition of the term "critical gap" [9, 13] but it is usually considered as the minimum acceptable gap for a driver. The critical gap is computed using different methods of calculation, as will be described in more detail in the following section of this chapter. Generally, the critical gap includes a deterministic and probabilistic component.

Some of the literature consider the critical gap as a deterministic parameter (e.g. Raff [10]) the critical lag (modified to critical gap in some other literature) is defined as “*the value of time (t) for which the total number of accepted lags shorter than (t) is equal to the total number of rejected lags longer than (t).*” Mason et al. [15] estimated the value of the critical gap using the classical Greenshields method which is identified as “*the gap range that has an equal number of acceptances and rejection*”. Moreover, the Highway Capacity Manual (HCM 2000) [1] considers the critical gap as the “*minimum time interval between the front bumpers of two successive vehicles in the major traffic stream that will allow the entry of one minor- street vehicle.*”

Alternatively, some of the literature consider the critical gap as a stochastic variable and define it as “*the gap size corresponding to probability of acceptance 50 percent (i.e. probability of acceptance is equal to the probability of rejection)*” [4, 6, 7, 25].

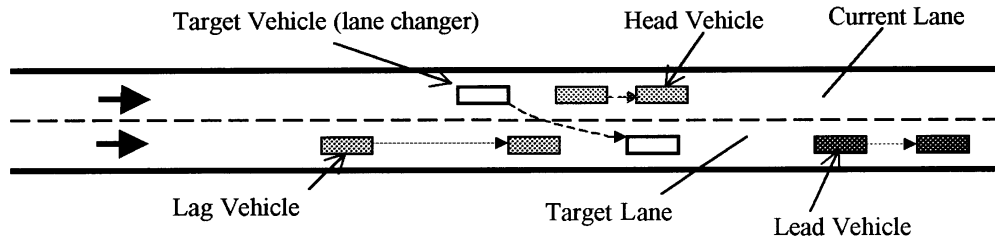
#### **2.1.4 Follow up time**

The follow-up time is considered in HCM 2000 [1] as “*the time headway between the two minor street vehicles accepting the same gap size*”. In other words, it is the discharge time headway for the unopposed saturation flow rate (i.e. when the opposing flow is zero). In general, the follow-up time is shorter than the critical gap and equals the inverse of the saturation flow rate.

## **2.2 Critical Gap Calculation Methods**

Since the critical gap of a driver as mentioned before cannot be measured directly, censored observations (i.e., accepted and rejected gaps) are used to compute critical gaps, as will be described later. The minimum gap that a driver is willing to accept is generally called the critical gap. For more than three decades research efforts have attempted to model driver gap acceptance behavior, either through deterministic or by probabilistic means. The deterministic critical values are treated as a single threshold for accepting or rejecting gaps. Alternatively, probabilistic models solve some of the inconsistency elements in gap acceptance behavior by using a statistical treatment of opposed driver gap acceptance behavior, i.e. the driver’s perception of a minimum acceptable gap is treated as a random variable. For the case of gap acceptance in changing lanes, the critical gap value is divided to a leading critical gap and trailing critical gap. The leading critical gap is the minimum gap between the rear bumper of an assumed leader in the target lane and the front bumper of a subject vehicle below which the subject vehicle will not make a lane change. Similarly, a trailing critical gap is the minimum gap between the rear bumper of a subject

vehicle and the front bumper of an assumed follower in the target lane below which the subject vehicle will not perform a lane change. Figure 2.1 is shown the different vehicles affecting the decision of the lane changing behavior. For the case of crossing a traffic stream, the critical gap as mentioned before could be considered as the minimum time headway accepted by the driver to cross a major road.



**Figure 2.1: Definitions of vehicles affecting lane-changing behavior (Source [26])**

Furthermore, for permissive left-turn traffic gap acceptance, the HCM 2000[1] estimates the opposed saturation flow rate based on the critical gap and follow-up time. HCM 2000 considers the critical gap accepted by left-turn drivers as a deterministic value equal to 4.5 s at signalized intersections with a permitted left-turn phase. This value is independent of the number of opposing-through lanes to be crossed by the opposed vehicles. In addition, the American Association of State Highway and Transportation Officials (AASHTO, 2001) [27], classifies the left turning movements from the major road across opposing traffic as Case *F*. The AASHTO (2001) recommends that for case *F* opposed movements that the critical gap for left-turning passenger cars be set equal to 5.5 s (for passenger cars) and for left-turning vehicles that cross more than one opposing lane to add an additional 0.5 s for each additional lane of travel.

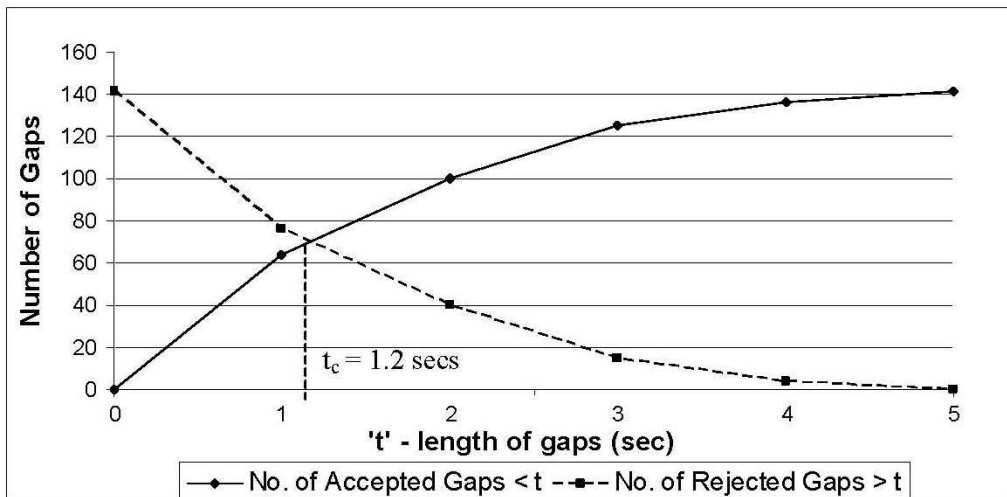
In the different literature, it is used both deterministic and stochastic methods to determine critical gap value from the field data which will be summarized in the following sections.

### 2.2.1 Deterministic Methods

Three main deterministic methods are used to estimate critical gap for the mandatory lane changing and for crossing an intersection case, namely: Raff's method, Greenshield's method and acceptance curve method as described in the following sections.

#### 2.2.1.1 Raff's method

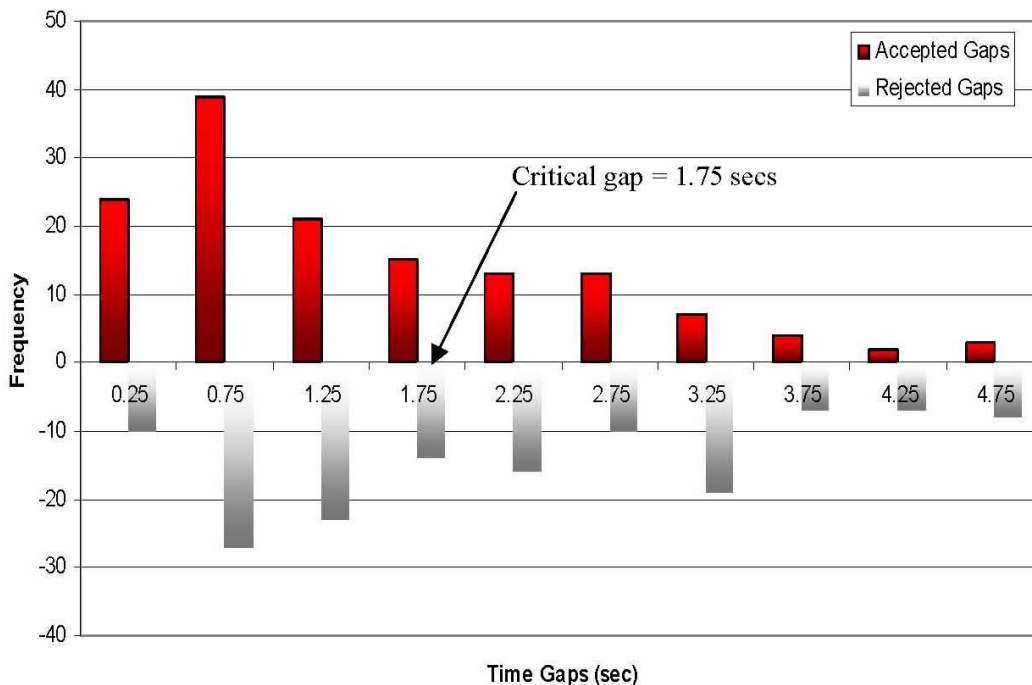
Raff method defines the critical gap to be the size of gap that results in an equal number of accepted shorter gaps and longer rejected gaps. The critical gap is defined graphically by plotting two curves; one indicating the cumulative frequency of accepted gaps, and the other indicating the cumulative frequency of rejected gaps. The critical gap is the gap duration, on the x-axis, at which the two curves intersect. The original Raff definition only uses lag acceptance and rejection data as illustrated in Figure 2.2. This approach is considered statistically wasteful by some researchers, because useful gap acceptance and rejection data are omitted. There were two approaches to remedy this shortcoming. First, combining the gap and lag data based on the notion that there is no statistical significance between lag and gap data. Second, as an alternative approach is to separate the lag and gap data into "lag-only" and "gap-only" curves [12-14, 28, 29].



**Figure 2.2: Example Application of Raff's Method for Calculating the Critical Gap (Source [28])**

### 2.2.1.2 Greenshield's method

The Greenshield's method estimates the value of the critical gap using the classical Greenshields method (as illustrated in Figure 2.3), they employed a histogram to represent the total number of acceptances and rejections for each gap range (usually for each 0.5 s). The vertical axis of the histogram represents the number of gaps accepted (positive value) or rejected (negative value) of a certain gap range and the horizontal axis represents the gap-size range. The gap range having an equal number of accepted and rejected gaps is the range of the critical gap size, and the mean of this range is the size of the critical gap [14, 15, 28]. If none of the gap ranges have an equal number of acceptances and rejections, then the gap range that has the closest number of acceptances and rejections is used as the critical gap range, and the mean of this range is the critical gap. Small sample sizes may affect and distort this analysis



**Figure 2.3 : Example Application of the Greenshield's Method to Compute the Critical Gap (Source [28])**

### 2.2.1.3 Acceptance Curve method

It is used the Acceptance Curve method to estimate the value of the critical gap where the dependent variables of an “S-shaped” response variable curve, with  $y = 0$  and  $y = 1$  being the asymptotes, are the cumulative probability of accepting a gap of a specific length. The x-value corresponding to the 50% probability is used as the critical gap. The acceptance curve method identifies the gap size with a 0.5 probability (50 percent chance) of acceptance by the drivers. The probability of acceptance is calculated as  $P_i = n_i/N$ , where  $P_i$  is the probability of accepting a gap (seconds) of length  $i$ ,  $n_i$  is the number of gaps accepted, length  $i$ , and  $N$  is the total number of events of gap acceptance [14, 18, 28]. Figure 2.4 graphically shows the Accepted Curve method.

## 2.2.2 Stochastic Methods (Probabilistic model)

Three stochastic methods used to estimate are used to estimate critical gap for the mandatory lane changing and for crossing an intersection case, namely: maximum likelihood estimation, Logit method and Probit method, which will be presented in the following sections.

### 2.2.2.1 Maximum likelihood estimation

The gap acceptance process is treated as a binary function with accepted gaps represented as one and rejected gap represented as zero. The likelihood function is defined as the joint probability (density) function of observable random variables. Thus, maximum likelihood is estimated by getting the values of the parameters for a certain distribution function that maximize the likelihood function [14, 28]. The log of the maximum likelihood function for the critical gap is a function of accepted and rejected gaps as Equation(1):

$$L = \sum \ln [F(b_i) - F(a_i)] \quad (1)$$

where

$L$  = likelihood for critical gap,

$F(b_i)$  = cumulative distribution function (cdf) of accepted gaps,

$F(a_i)$  = cdf of largest rejected gaps,

$b_i$  and  $a_i$  = gap accepted and largest gap rejected by a driver, respectively.

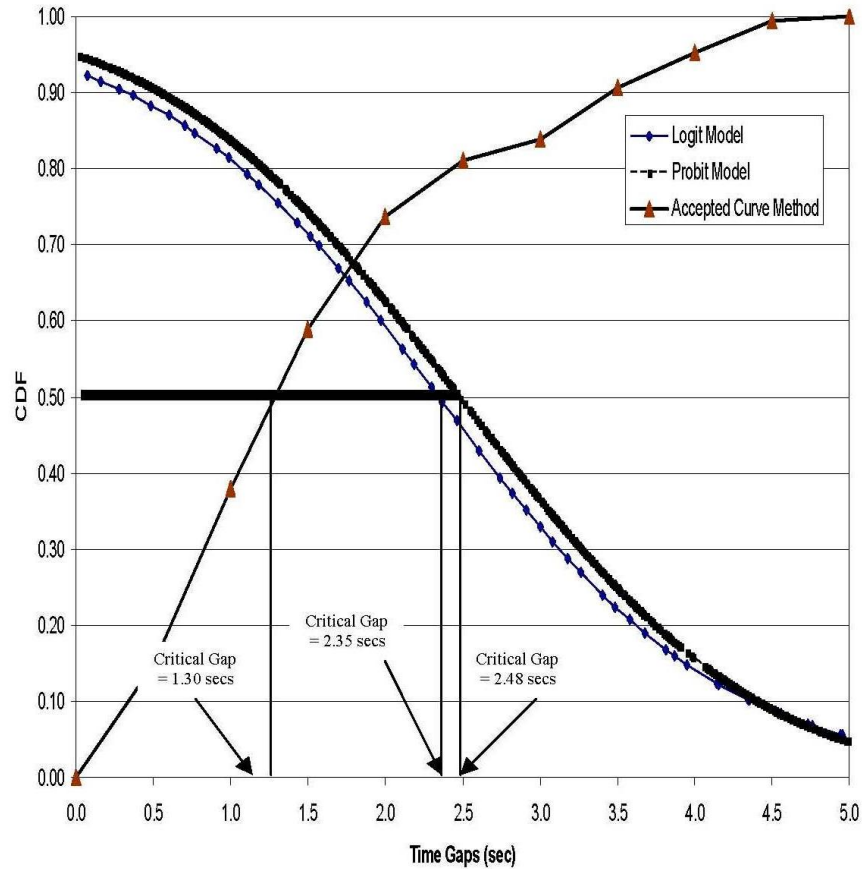
### *2.2.2.2 The Logit method*

It is used for prediction of the probability of occurrence of an event by fitting data to a logistic curve. It is a generalized linear model used for binomial regression and it is called Logistic regression. Like many forms of regression analysis, it makes use of several predictor variables that may be either numerical or categorical. The response curve, which is the percentage of gap acceptance, is determined and plotted against the gap size [6, 14, 25, 28, 30]. A fitted linear line can then be plotted on the chart to identify the value of gap that gives 0.5 probability of acceptance of a gap size as shown in Figure 2.4.

### *2.2.2.3 The Probit method*

It is a statistical technique used to treat the percentages of a population making all-or-nothing (binomial) responses to increasingly severe values of a stimulus. In the context of gap acceptance studies, the value of a stimulus is the size of the gap. In some statistical literatures, the logit model and Probit model are considered the same, although the binary logit model is more popular than the binary probit model, The main difference between the two models pertains to the cumulative distribution function that is used to define the choice probabilities. Unlike the binary probit model, the cumulative distribution function (CDF) of the logit model is not in integral form different from the probit model CDF which takes a cumulative normal distribution function [7, 14, 28, 31]. The parameters of the probit models are estimated using maximum-likelihood procedures.

A fitted linear line can then be plotted on the chart to identify the value of the gap that produces Probit (P) equal to 0.5. This value is considered as the median value of the stimulus of the critical gap shown in Figure 2.4.



**Figure 2.4 : Critical Time Gaps using Acceptance Curve method, Logit method and Probit method (Source [28])**

In general, the critical gap is considered as a function of accepted and rejected gaps and depends on many parameters affecting the driver behavior as aggressiveness, impatience, etc. The drivers behave differently under diverse traffic, geometric, and environmental conditions. Similarly, the same driver can behave differently under varying traffic and control conditions. Due to this stochastic nature of driver behavior, it is imperative to model critical gaps, mathematically, for use in microscopic traffic simulation models to get realistic results.

Current simulation models like INTEGRATION [32-36], MITSIM [21], CORSIM [37], VISSIM [38], and SITRAS [39] use critical gaps in different ways in lane changing models. INTEGRATION and MITSIM [21] assume the distribution of critical gaps to have a lognormal distribution. Both distance leading and trailing critical gaps are considered separately. The gap acceptance decision is affected by various variables including the relative speeds of the subject

vehicle with respect to the assumed leader and the assumed follower. Critical gaps may be affected by other variables such as the path plan. CORSIM [37] defines critical gaps by using risk factors. The risk factor is defined by the rate of deceleration a driver will apply to avoid collision if his leader brakes to a complete stop. Risk factors are evaluated for every lane change between the subject vehicle and the assumed leader and between the subject vehicle and the assumed follower. The estimated risk factor is then compared to an acceptable risk factor that depends on the type of lane change, type of drive, and urgency of the lane change. In SITRAS [39], a critical gap is defined as the minimum acceptable gap and is equal to the sum of minimum safe constant gap and product of a constant with relative speed. In VISSIM [38], the psychophysical model presents critical gaps as thresholds depending on the relative speeds of the subject vehicle and the assumed leader and the subject vehicle and the assumed follower.

## **2.3 Gap acceptance Modeling Behavior**

Gap acceptance behavior occurs when vehicles on a minor approach need to cross a major street at a two-way stop controlled intersection, or when vehicles have to make a left turn through an opposing through movement at a signalized intersection. Alternatively, gap acceptance is applied when vehicles approaching a roundabout have to merge with vehicles traveling in the roundabout, or when vehicles merging onto a freeway have to find gaps in the freeway flow. Generally gap acceptance analysis can be divided into two types of analyses: (1) crossing gap acceptance at intersections (which is the research of this thesis) or (2) gap acceptance during merging or lane-change maneuvers. This section describes both approaches to modeling gap acceptance with focusing on the crossing gap acceptance modeling.

### **2.3.1 Crossing Gap Acceptance Modeling**

Within the context of crossing gap acceptance, a gap is defined as the elapsed-time interval between arrivals of successive vehicles in the opposing flow at a specified reference point in the intersection area.

The simplest form of crossing gap acceptance modeling was originally formulated as a deterministic formulation [10, 11]. This function assumes that each driver accepts an identical gap as

$$f(g) = H(g - t)$$

$$H(g - t) = \begin{cases} 0 & g < t \\ 1 & g \geq t \end{cases} \quad (2)$$

where  $g$  is the gap size (s) and  $t$  is the critical gap size (s). The original Raff definition only uses lag acceptance and rejection data. This approach is considered statistically wasteful by some researchers, because useful gap acceptance and rejection data are omitted. There are two approaches to remedy this shortcoming. Fitzpatrick [29] decided to combine the gap and lag data based on the notion that there is no statistical significance between lag and gap data. An alternative approach is to separate the lag and gap data into “lag-only” and “gap-only” curves.

Mason et al. [15] estimated the value of the critical gap using the classical Greenshields method (Figure 10), they employed a histogram to represent the total number of acceptances and rejections for each gap range. The critical gap as mentioned before is identified as the gap range that has an equal number of acceptances and rejections. Small sample sizes may affect and distort this analysis

Maze [18] used the Acceptance Curve method to estimate the value of the critical gap where the dependent variables of an “S-shaped” response variable curve, with  $y = 0$  and  $y = 1$  being the asymptotes, are the cumulative probability of accepting a gap of a specific length. The acceptance curve method identifies the gap size with a 0.5 probability (50 percent chance) of acceptance by the drivers. The probability of acceptance is calculated as  $P_i = n_i/N$ , where  $P_i$  is the probability of accepting a gap (seconds) of length  $i$ ,  $n_i$  is the number of gaps accepted, length  $i$ , and  $N$  is the total number of events of gap acceptance.

Solberg and Oppenlander [40] extended the modeling of gap acceptance behavior by considering a probit model. The acceptance or rejection of a time gap is represented by an all-or-nothing or binomial response, which is dependent on the size of the gap. The probit of the expected proportion accepting a time gap is described by the following linear equation

$$Y = 5 + \frac{X - \mu}{\sigma} \quad (3)$$

Where  $Y$  is the probit of proportion of drivers accepting a time gap;  $X$  is the natural logarithm of the time gap;  $\mu$  is the mean tolerance distribution; and  $\sigma$  is the standard deviation of tolerance distribution.

Pant and Balakrishnan [16] described the development of a neural network and a binary-logit model for predicting accepted or rejected gaps at rural, low-volume two-way stop-controlled intersections. The study found that the type of control (stop-versus stop plus intersection-control beacon) had an effect on the driver's decision to accept or reject a gap. Other significant variables included the vehicular speed, turning movements (left, through, and right) in both the major and minor directions, size of gap, service time, stop type (rolling or complete), queue in the minor direction, and the existence of a vehicle in the opposite approach. A logit model computes the probability of choice  $i$  (in this case accepting a gap) as

$$P_k \ i = \frac{e^{U_i}}{1 + e^{U_i}} \quad (4)$$

Where  $U$  is the utility of an accepted gap. Since the driver has only two choices (accept or reject a gap), the probability of rejecting a gap is computed as  $1 - P_k(i)$ . The logit model is derived from the assumption that the error terms of the utility functions are independent and identically Gumbel distributed. These models were first introduced in the context of binary choice models, where the logistic distribution is used to derive the probability. Their generalization to more than two alternatives is referred to as multinomial logit models. Alternatively, a neural network can be characterized as a computational model which is composed of a highly interconnected mesh of nonlinear elements and whose structure is inspired by the biological nervous system. Pant and Balakrishnan [16] concluded that the neural network performed better than the logit model in predicting accepted and rejected gaps. The utility function for the logit model was estimated as

$$\begin{aligned} U_i = & -0.47567 \times CTYP + 0.44116 \times UGAP + 0.02333 \times SPD \\ & + 0.95405 \times MJTR1 + 1.87415 \times MJTR2 \\ & + 0.08779 \times SRVT - 0.09484 \times MNTR1 \\ & + 1.73174 \times MNTR2 - 6.13626 \times STYP \\ & + 0.40523 \times QUE + 0.33206 \times EXIS \end{aligned} \quad (5)$$

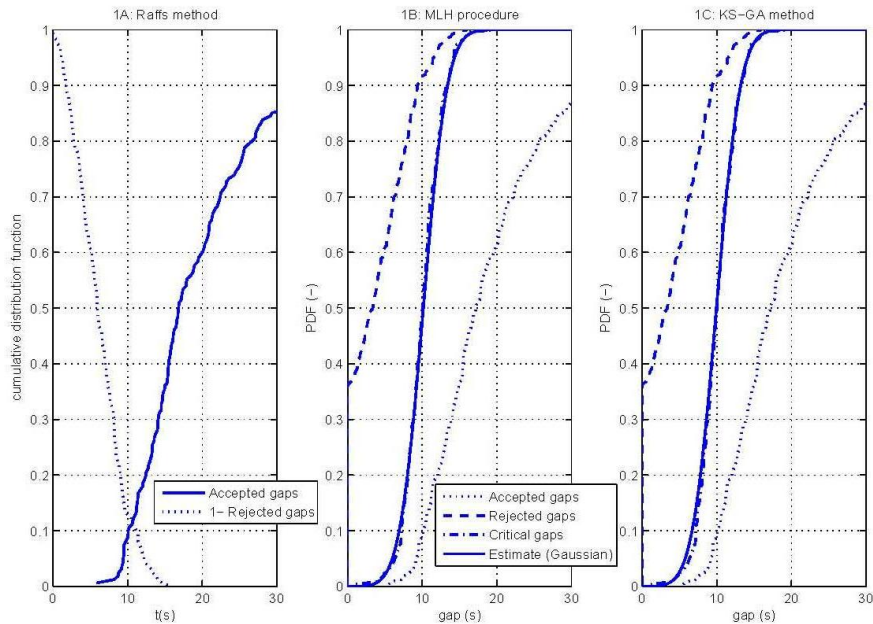
Where  $CTYP$  is the control type (1 =Stop, 0 = Stop plus Beacon);  $UGAP$  is the utilized gap (s);  $SPD$  is the speed in the major direction (mph);  $MJTR1$ ,  $MJTR2$  is the turning movement counts in

the major directions [(0,0) = through movements; (1,0) = left movement; and (0,1)= right movement]; *SRVT* is the service time (s); *MNTR1*, *MNTR2* is the turning movement in minor directions [(0,0) = through movement; (1,0) = left movement; and (0,1) = right movement]; *QUE* is flag indicating the existence of a queue on the minor street approach (1 = Yes, 0 = No); *STYP* is the stop type (1 = Complete, 0 = Rolling); and *EXIS* is a flag indicating the existence of vehicle on opposite approach (1 = Yes, 0 = No). The coefficients of all variables in the model were tested at a 0.05 level of significance. The results showed that all variables were significant.

Hamed and Easa [7] developed a system of disaggregate models that account for the effect of intersection, driver, and traffic characteristics on gap acceptance for left turn maneuvers at urban T-intersections controlled by stop signs on the minor roads. In most studies, the critical gap at a particular intersection is assumed to be the same for all drivers. However, Hamed and Easa [7] assumed the critical gap to be a function of driver socioeconomic characteristics, level of service, intersection geometry, trip purpose and expected waiting time at the head of the queue. Driver gap acceptance was treated as a discrete choice problem. Each driver had two choices: either to accept a gap and move into the intersection or reject the available gap and wait for the next one.

Hegeman and Hoogendoorn [41] presented a new simulation-based approach to estimate the distribution of critical gaps. The proposed, so called, KS-GA method enables estimation of other behavior parameters simultaneously with the critical gap distribution parameters. The approach is based on minimization of the Kolmogorov-Smirnov distance between the simulated and observed accepted gaps, and the simulated and observed rejected gaps. The optimization is performed using a Genetic Algorithm. Hegeman and Hoogendoorn [41] also compared the new method with two other gap distribution estimation methods. The first method is the method of Raff and the second is the maximum likelihood procedure (ML procedure), which is selected for the evaluation of critical gaps in the HCM [1]. The verification and cross comparison of the KS-GA method with Raff's method and the ML procedure was performed for different sets of parameter values, using parametric bootstrap. For all three methods, the same sets of parameter values were applied and results were compared (Figure 2.5). The strength of this new method compared to existing methods, is its flexibility towards inter and intra-driver differences in gap acceptance, for instance due to increasing impatience of drivers. The KSGA method enables estimation of

parameters related to these differences such as an impatience factor, simultaneously with the critical gap distribution parameters.



**Figure 2.5: Example of Estimation of Raff's Method and the ML Procedure (Source [41])**

Miller [12] evaluated nine different methods to estimate gap acceptance parameters. The author claimed that the disadvantage of the method of maximum likelihood is that a specific distribution must be assumed for the critical gaps and quite often though, estimators obtained by using the method of maximum likelihood are still good estimators even though the distribution which was assumed in order to obtain them was unrealistic. Miller also showed that methods of analysis of gap acceptance data which use only lag information are inferior to those which make use of all gaps, for two reasons: they use less information and are consequently less precise and the measurement of lags is usually less precise than that of gaps.

In addition, a comparative study by Hewitt [24] and [42] showed that the maximum-likelihood method was complicated, but that it gave an accurate estimation of the critical gap mean and standard deviation. The study concluded that the size of critical gap is determined by the driver's characteristics and style of driving, but will also vary with the design of the intersection, the type and speed of the trailing vehicle forming the gap and other factors such as weather. It is assumed that each driver has a fixed critical gap, and only variation is that between drivers. The difference

between the critical gap and the lag distributions increases when the main road flow decreases, with heavy flow on the main road fewer lags are accepted and the critical gap distribution is closer to the critical time distribution. Also, Hewitt [24] noted that drivers who will accept a short gap also are likely to accept a short lag. Risk-taking drivers are more likely to accept the initial lag and thus not be represented in the subset of drivers whose gap acceptance behavior is observed.

The impact of the speed of the opposing vehicle has been a controversial factor in the study of gap acceptance and critical gap. Simulation studies by Sinha and Tomiak [3] and Brilon [4] indicated that the critical gap increases as the speed increases. On the contrary, experimental studies on gap acceptance by Ashworth and Bottom [8] revealed a negative effect of speed on gap acceptance and found that a driver waiting for a longer time accepted a shorter when compared to others.

Tsongos [2] showed that there was no difference in gaps between day and night. Studies by Ashton [17] and Maze [18] found that the length of queuing time did not affect gap acceptance. A study by Polus [5] found that mean gaps and lags may be influenced by the type of sign control (yield versus stop).

Abu-Sheikh [9] investigated the effects of the gap, vehicle, traffic, trip and driver inter-influence factors and characteristics on driver gap acceptance behavior at priority intersections. The author studied the effects of these variables under real life driving situation and data was collected based on the simultaneous use of both of video recording and field administrated interviews. Logistic regression is built using the variables: driver sex, age, driving experience, total delay, volume, trip duration and speed.

Davis and Swenson [25] presented different logit gap acceptance models by combining different independent variables (distance to the intersection, time to reach the intersection and speed for the opposing vehicles) and compare between them. They claimed that, for a given gap duration, the probability that a driver accepts that gap increases as the speed of the opposing vehicle increases and could be expected if distance were the primary determinant of gap acceptance.

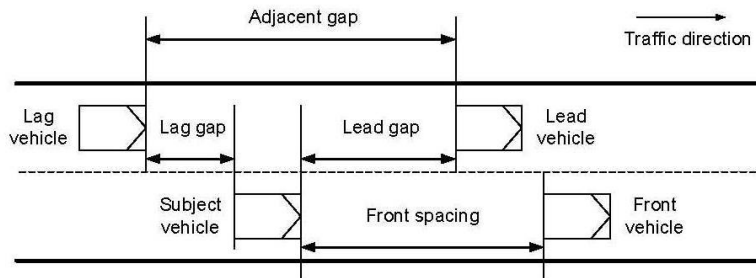
Yan and Radwan [6] studied the effect of restricted sight distances on the gap acceptance behavior on permitted left turn operation and the left turn capacity. Also, they showed that the

sight obstruction due the opposing turning vehicles may contribute to significant increments of the critical gap and follow-up time because drivers need more response time to decide to accept the available gap compared to the drivers with sufficient sight distances. This could result in an extra traffic delay and a capacity reduction in the left-turn lane.

### 2.3.2 Gap Acceptance during Merging and Lane-changing

Lane-changing behavior is usually modeled in two steps: (a) the decision to consider a lane change, and (b) the decision to execute the lane change. In most models, lane changes are classified as either mandatory (MLC) or discretionary (DLC). MLCs are performed when the driver must leave the current lane while DLCs are performed to improve driving conditions (e.g. increase speed). Gap acceptance models are used to model the execution of lane changes.

Toledo et al. [21] presented an integrated lane-changing model, which allows joint evaluation of mandatory and discretionary considerations. The model parameters were estimated using detailed vehicle trajectory data. To execute a lane change, the drivers assesses the positions and speeds of the lead and lag vehicles in the target lane (Figure 2.6) and decides whether the gap between them is sufficient to execute the lane change.



**Figure 2.6: Definitions of Front, Lead and Lag Vehicles and their Relations (gaps) with the Subject Vehicle (Source [21])**

As was described earlier, gap acceptance models are formulated as binary choice problems, in which drivers decide whether to accept or reject an available gap by comparing to a critical gap (minimum acceptable gap). Critical gaps are modeled as random variables to capture variations between and within drivers.

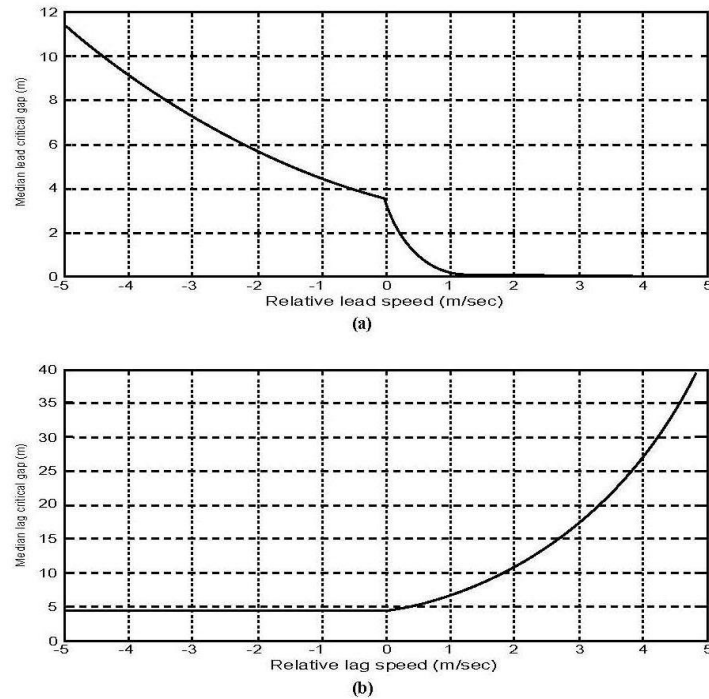
Toledo et al. [21] proposed a model that integrates mandatory and discretionary considerations into a single utility model to overcome the limitations of existing models that separate between MLC and DLC and therefore suffer from two important weaknesses:

- a. They do not capture trade-offs between mandatory and discretionary considerations
- b. These models assume that the existence (or nonexistence) of an MLC situation is known (i.e., drivers start responding to the MLC situation at a certain point, often defined by the distance from the point where they have to be in a specific lane).

The author also analyzed the relation between the lead and lag critical gaps and the corresponding relative lead and lag speeds, respectively. They deduced that the lead critical gap decreases with a decrease in the relative lead speed—that is, the lead critical gap is larger when the subject vehicle is faster relative to the lead vehicle and the effect of the relative speed is strongest when the lead vehicle is faster than the subject vehicle, as illustrated in Figure 2.7. In this case, the lead critical gap quickly diminishes as a function of the speed difference. This result suggests that drivers perceive very little risk from the lead vehicle when the gap is increasing. Inversely, the lag critical gap increases with the relative lag speed: the faster the lag vehicle is relative to the subject vehicle, the larger the lag critical gap is. In contrast to the lead critical gap, the lag gap does not diminish when the subject is faster, as illustrated in Figure 2.7. A possible explanation is that drivers may maintain a minimum critical lag gap as a safety buffer because their perception of the lag gap is not as reliable as it is for the lead gap.

Kim et al. [23, 43] proposed a lane changing gap acceptance model for merging vehicles. The model was developed using field data collected from two different on-ramp freeway sections with different acceleration lane lengths. The trailing gap and the relative speed to the trailing vehicle were examined in this research. It was found that the distributions of the critical trailing gaps follow a normal distribution and that the means and the variances of the distributions changed depending on the location of the acceleration lane. The model validation tests were performed with the field data with respect to: (a) the relationship between the merging location and the traffic stream density, (b) the relative speed between the merging and trailing vehicles, and (c) the distribution of trailing gaps accepted. The primary objective of the study was to propose a lane-changing gap acceptance model for simulation within the structure of current traffic simulation software. Kim et al. [23] found from the field data that the 85 % of the drivers merge to the main

lane from the acceleration lane when: (a) the leading gap is longer than or equal to 1.90 sec, (b) trailing gap is longer than or equal to 3.3 sec, and (c) the relative speed to the trailing vehicle is within  $\pm 10.5$  km/h. The field data also showed that the locations of merging changed depending on the traffic stream density of the freeway lane adjacent to the acceleration lane.



**Figure 2.7: Median (a) Lead and (b) Lag Critical Gaps as a Function of Relative Speeds (Source [21])**

Kovvali and Zhang [44] analyzed the gap acceptance behavior on freeway segments by conducting a detailed analysis of the vehicle trajectory data obtained through Next Generation SIMulation (NGSIM) [45] program. The paper classified accepted gaps from the vehicle trajectory data into mandatory lane change (MLC) and discretionary lane change (DLC) gaps, and conducted a detailed correlation and regression analysis. The analysis showed that MLC maneuvers require smaller gaps than DLC maneuvers. Also, if multiple lanes need to be traversed for an MLC, drivers are more inclined to accept smaller gaps. Presence of heavy vehicles as subject or lag vehicles also tend to increase accepted gaps for both MLC and DLC maneuvers,

while front and lead vehicle types do not have a significant effect. The model looked at explanatory variables such as speeds, accelerations, vehicle types, decision points, etc., for subject, lead and lag vehicles to develop a gap acceptance logic that can be incorporated into any lane changing model. Kovvali and Zhang [44] developed a model of the format

$$g = e^{\alpha + \beta^T x} \quad (6)$$

Where  $g_{MLC}$  is the accepted total gap when making a mandatory lane change (ft);  $\alpha$  is a model constant; “ $\beta$ ” is the corresponding vector of coefficients; and  $x$  is the vector of explanatory variables.

In the case of mandatory lane changes the model was

$$g_{MLC} = e^{1.939 - 0.095N + 0.631TP_{sub} + 0.285TP_{lag} + 0.038V_{sub}} \quad (R^2 = 0.495). \quad (7)$$

Where  $N$  is the number of lane changes required to exit the freeway from the off-ramp;  $TP_{sub}$  is the subject vehicle type (1=motorcycle, 2=automobile, 3=heavy vehicle);  $TP_{lag}$  is the lag vehicle type (1=motorcycle, 2=automobile, 3=heavy vehicle); and  $V_{sub}$  is the subject vehicle speed (in mph).

The regression model for accepted gaps when making discretionary lane changes is (all explanatory variables in the equation are significant at the 5% level of significance) cast as

$$g_{DLC} = e^{2.015 + 0.603TP_{sub} + 0.351TP_{lag} + 0.028V_{sub} - 0.005RAcc_{leadlag}} \quad (R^2 = 0.389). \quad (8)$$

Where  $RAcc_{leadlag}$  is the relative acceleration between the lead and lag vehicle (in ft/sec<sup>2</sup>).

Regression analysis revealed that the variance of observed variables, including MLC-specific variables, vehicle types, speed, relative speed, acceleration, and relative acceleration, can explain the 49.5% and 38.9% for MLC and DLC gap acceptance variance, respectively. These values suggest that some other unobserved factors, such as gender, age, trip purpose, weather, etc., also play a role in gap acceptance when conducting lane changes on freeways.

Goswami and Bham [28] presented preliminary analysis of driver lane change behavior using NGSIM data and proposed statistical distributions for accepted time gaps in mandatory lane change maneuvers. Critical gaps for lane change maneuvers are estimated based on accepted and

rejected gaps. The distributions of accepted and critical time gaps can be utilized in modeling the lane change behavior of drivers in microscopic traffic simulation models. Goswami and Bham [28] used both deterministic and stochastic methods to determine leading and trailing critical gaps from field data. The deterministic methods that are used to estimate leading and trailing critical gaps for mandatory lane changes for the data sets are the Raff, Greenshield's, and Acceptance curve methods and regarding the stochastic methods are ML, Logit and Probit methods. The results from these methods were compared and critical gaps, as a function of lane change location, were presented earlier. The methods are useful for freeway lane change situations, as they make use of accepted and rejected gaps.

Toledo et al. [46] presented a new lane-changing model that incorporates an explicit choice of a target lane. This approach differs from those used in existing models that assume drivers evaluate the current and adjacent lanes and chooses a direction of change (or no change) on the basis of the utilities of these lanes only; these myopic models can explain only one lane change at a time. While the proposed model is applicable to any freeway situation, it is most useful in cases in which there are large differences in the level of service among the lanes, such as in presence of exclusive lanes. The model structure can also capture drivers' preferences for specific lanes, such as in the case in which travel lanes and passing lanes are defined. The lane choice set is therefore dictated by the current position of the vehicle and in multilane facilities would be restricted to a subset of the available lanes. The model hypothesizes two levels of decision making: (1) the target lane choice model and (2) the gap acceptance model.

The target lane choice set constitutes all available lanes to which the driver is eligible to move. In the presence of exclusive lanes, the choice set would depend on the eligibility of the vehicle to enter the exclusive lanes, and thus the choice set is not the same for all drivers. The target lane utilities are affected by the lane attributes, such as the density and the speed of traffic in the lane and the presence of heavy vehicles, and variables that relate to the path plan, such as the distance to a point where the driver needs to be in a specific lane and the number of lane changes required to go from the target lane to the correct lane. The utilities of the various lanes are given by

$$U_{i,n,t}^{TL} = \beta_i^{TL} X_{i,n,t}^{TL} + \alpha_i^{TL} v_n + \varepsilon_{i,n,t}^{TL} \quad \forall i . \quad (9)$$

Where  $U_{i,n,t}^{TL}$  is the utility of lane  $i$  for a target lane ( $TL$ ) for driver  $n$  at time  $t$ ;  $X_{i,n,t}^{TL}$  vector of explanatory variables that affect the utility of lane  $i$ ;  $\beta_i^{TL}$  is a vector of parameters;  $v_n$  is the individual-specific latent variable assumed to follow some distribution in the population;  $\alpha_i^{TL}$  are model parameters; and  $\varepsilon_{i,n,t}^{TL}$  is a random term associated with the target lane utilities.

The gap acceptance model captures a driver's choice whether the available gap in the adjacent lane in the change direction can be used to complete the lane change or not. The driver evaluates the available lead and lag gaps, which are defined by the clear spacing between the rear of the lead vehicle and the front of the subject vehicle and between the rear of the subject vehicle and the front of the lag vehicle, respectively, and compare them with the corresponding critical gaps. Critical gaps are assumed to follow lognormal distributions to ensure that they are always non-negative as

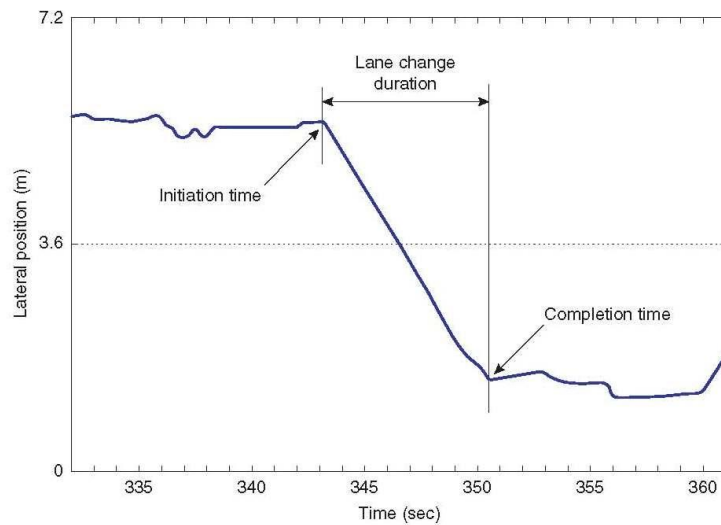
$$\ln(G_{nt}^{gd}) = \beta^{gt} X_{nt}^{gd} + \alpha^g v_n + \varepsilon_{nt}^{gd} \quad \text{for } g \in \{\text{lead, lag}\} \text{ and } d \in \{\text{right, left}\} \quad (10)$$

Where  $G_{nt}^{gd}$  is the critical gap  $g$  in the direction of change  $d$  (m);  $X_{nt}^{gd}$  is the vector of explanatory variables;  $\beta^{gt}$  is corresponding vector of parameters;  $\varepsilon_{nt}^{gd}$  is a random term;  $\alpha^g$  is a parameter of the driver-specific random term of  $v_n$ . The gap acceptance model assumes that the driver must accept both the lead gap and the lag gap to change lanes.

Tijerina et al. [47] used observers who accompanied the driver in the vehicle. The observers gave driving instructions and recorded drivers' actions. The study included 39 drivers who drove both on highways and on urban streets. For the urban streets, lane-change durations were between 3.5 and 6.5 s, with a mean of 5.0 s. For highways the duration ranged from 3.5 s to 8.5 s with a mean of 5.8 s.

Toledo and Zohar [22] used trajectory data at a high time resolution that were collected from high mounted cameras to estimate lane-change duration models. They mentioned that there are significant limitations and potential biases in most studies. In most cases, human observers or obtrusive equipment, such as eye markers were used to collect data. Their presence may have affected driver's behavior and biased the results. In one case, data were collected by using a driving simulator and not in naturalistic driving, which may negatively affect the realism of the

driving experience and the fidelity of the data collected. Another difficulty is that the definitions of the initiation and completion of lane changes differ in the various studies. For example, in some cases it is assumed that lane changes are initiated when the driver decides to change lanes. This ambiguous definition may be interpreted differently by different drivers and lead to larger variability of the lane-changing durations. Moreover, this definition is not appropriate for use in microscopic traffic simulation, in which driving behavior models are mostly based on the observable positions and speeds of vehicles and not on their intentions. Human factors used in some of the studies to explain lane-change durations, such as eye and head movements, are also not applicable in the context of traffic simulation. Lane change initiation and completion points can be defined as the time instances when the lateral movement of the subject vehicle begins and ends, respectively. Figure 2.8 demonstrates these points on the trajectory of one of the vehicles in a dataset. The lane-change duration is the time lapse between its initiation and completion.



**Figure 2.8 : Definition of lane change initiation and completion time points (Source [22])**

Toledo and Zohar [22] indicated that the results of lane changes are not instantaneous events as most microscopic traffic simulations model them but may have durations in the range of 1.0 to 13.3 s, with a mean of 4.6 s. Lane change durations are affected by traffic conditions captured by the traffic density, by the direction of the change, and by other vehicles around the subject vehicle.

Goswami et al. [48] suggested that it is important to compare simulation results to field data. On a multilane freeway it is important to compare the number of lane changes in each lane. It is important to study the frequency of lane changes on multilane freeways to understand and develop capacity models for freeways and improve the geometrics of the multilane freeways to increase safety. Therefore, they studied a half mile section of the I-80 five-lane freeway in California. An on-ramp and a shoulder lane drop is studied and they determined the number of lane-changes in terms of origin and destination lanes and their percentages with respect to the entrance volume were presented for each freeway lane to use these analysis data as guidelines for traffic simulation results. Comparisons of lane change frequency, towards the shoulder lane and towards the median lane were carried out is presented in Table 2.1 (lane 1 is the median lane and lane 6 is the shoulder lane).

**Table 2.1: Number of Lane-change from Origin to Destination as Percentage of Entering Volume (Source [48] )**

To From	No. of Vehicles Entering	1	2	3	4	5	6	Off- Ramp (8)
1	<b>870</b>	<b>736</b> (84.6%)	108 (12.4%)	21 (2.5%)	1 (0.1%)	1 (0.1%)	0	2 (0.2%)
2	<b>773</b>	105 (13.7%)	<b>506</b> (65.5%)	117 (15%)	30 (3.9%)	9 (1.1%)	0	6 (0.7%)
3	<b>591</b>	28 (4.9%)	164 (27.7%)	<b>268</b> (45.3%)	100 (16.9%)	14 (2.4%)	0	16 (2.7%)
4	<b>715</b>	20 (2.8%)	96 (13.4%)	228 (31.8%)	<b>253</b> (35.4%)	67 (9.2%)	0	52 (7.3%)
5	<b>719</b>	20 (2.8%)	42 (5.6%)	110 (15.4%)	255 (35.5%)	<b>219</b> (30.4%)	0	74 (10.3%)
6	<b>751</b>	2 (0.2%)	5 (0.6%)	28 (3.6%)	99 (13.1%)	394 (52.4%)	0	224 (29.8%)
On- Ramp(7)	<b>307</b>	6 (1.9%)	10 (3.2%)	37 (12%)	86 (28%)	155 (50.5%)	0	12 (3.9%)
<b>Total</b>	<b>4726</b>	<b>917</b>	<b>931</b>	<b>809</b>	<b>824</b>	<b>859</b>	<b>0</b>	<b>386</b>

Note: Number of through vehicles in each lane is in bold.

## 2.4 Summary and Conclusions

The literature review presented in this chapter provides the current state-of-art for the gap acceptance behavior models in case of lane changing and crossing scenarios. Most of the previous

gap acceptance studies were mainly concerned about establishing proper statistical distributions that fit the accepted gaps.

There are many factors affecting the gap acceptance decision and several studies in the literature have investigated the impact of different factors. These factors include: day versus nighttime effects [2], the speed of the opposing vehicle [3, 4], the type of intersection control (yield versus stop sign) [5], the driver sight distance [6], the geometry of the intersection, the trip purpose, and the expected waiting time [7].

Hence, the process is difficult because drivers do not know the number and size of gaps that will be offered to them a priori. The variables affecting the gap acceptance/rejection decision may be difficult to identify and the driver choice could be risky. However, most of the research has centered on the prediction of delay rather than the behavior of drivers [8].

The basic differences among the different studies were the underlying assumptions about driver behavior, the mathematical distributions fitted to accepted gaps and the type of the developed gap acceptance model. Three basic driver behavioral models have formed the basis for previous studies [9], namely: Deterministic, consistent driver and inconsistent driver.

Functions developed to explain the variation in driver gap acceptance behavior were limited to the use of few quantitative variables. Consequently, after reviewing the literature, it can be concluded that the effects of many other important factors affecting gap acceptance behavior are either completely neglected or only qualitatively described. This research is limited to modeling gap acceptance behavior in case of permissive left-turns at signalized intersections. The proposed research expands the domain of knowledge by introducing new variables into the decision process. The variables include the offered gap size, the waiting time, travel time to the conflict point, the vehicle characteristics, the driver's aggressiveness and the rain intensity impact. In addition, three proposed methods are presented in order to capture the effect of these variables which will be discussed in detail in the following chapters. A quantitative analysis of this subject directly benefits the users of traffic simulation packages by increasing the accuracy of the modeling process.

## CHAPTER 3 : CASE STUDY AND DATA REDUCTION

This chapter describes the study site that was utilized to conduct the study. In terms of the chapter layout, initially the study site is described followed by a description of the data collection effort. Subsequently, the data analysis procedures are presented followed by the data reduction and preliminary results.

### 3.1 Study Site Description

The study site that is considered for modeling permissive left-turn gap acceptance behavior is the signalized intersection of Depot Street and North Franklin Street (Business Route 460) in Christiansburg, Virginia. An aerial photo of the intersection is shown in Figure 3.1. It consists of four approaches at approximately 90° angles. The posted speed limit for the eastbound and northbound approaches was 35 mph and for the westbound and southbound approaches was 25 mph at the time of the study.

The signal phasing of the intersection at the time of the study included three phases, two phases for the Depot street North and South (one phase for each approach) and one phase for the North Franklin Street (two approaches discharging during the same phase) with a permissive left turn phase. The entering average annual daily traffic (AADT) for this intersection was 26,671 vehicles per day and there was an average of 11 accidents per year reported at this intersection.



Figure 3.1: Aerial view of Depot & Franklin intersection (Source [49])

## 3.2 Data Collection

The data used in this study were collected as part of the CICAS-V (Cooperative Intersection Collision Avoidance Systems) data collection effort [49]. The DAS (Data Acquisition System) employed a suite of hardware and software to record information about vehicles that approached the site. Data were acquired for two months (from June to August 2007). In the CICAS-V data collection effort, data were collected and analyzed with the goal of understanding how drivers approach intersections under various speeds and environmental conditions. The data acquisition system (DAS) employed a suite of hardware and software to record information about vehicles that approached the test sites. The sensing network was a distributed subsystem of components that provided raw inputs to the processing stack at a rate of 20 Hz (each 1/20<sup>th</sup> of second is recorded). The data acquisition hardware consisted of multiple components as follows:

- a. Video cameras to collect the visual scene. The video cameras were mounted on each of the four traffic signal mast arms to provide an image of the entire intersection environment at a frequency of 20 Hertz giving uncertainty of  $\pm 0.05$  sec in measured time. Figure 3.2 is showing a screen shot for the recording video of the intersection which is consisted of four quadrant (a quadrant for each approach);
- b. Weather stations. The weather station provided weather information once each minute as shown in Figure 3.3. The collected weather data included long-term rain fall, daily rain fall, wind direction, wind speed, average wind speed, temperature, barometric pressure, and humidity level. The rain fall intensity is the only concerned output from the weather station data, in this research;
- c. GPS units provided synchronized global time. A basic GPS system was employed at each intersection primarily for acquiring an accurate global time. This time information was used to stamp the data and to relate to the weather station outputs.

The processing stack processed the sensor data and assembled the dataset in real-time while archiving to binary data and compressed video files. Data subsets were extracted from the archived data using in-house written codes for analysis purposes.



**Figure 3.2 : a screen shot from the recording video of the intersection of Depot and Franklin st.**



**Figure 3.3: The weather station posted in the intersection of Depot and Franklin st.**

### 3.3 Data Analysis Procedures

In the CICAS-V dataset, the data were stored in different files; each file included a single hour of data with a unique File ID. The data were collected at a frequency of 20 Hz (once every 1/20<sup>th</sup> of a second). Filtering techniques and triggers are used to extract classification parameters from the selected datasets (.e.g., deceleration levels, distance from an intersection, weather condition, etc.). Data storage space, data storage configuration, data reduction capabilities, and computing requirements are integrated issues that vary widely for each analysis. The data storage required for each database depends upon the type and size of data that are available and needed for each analysis. The computing requirements necessary for data manipulation and analysis are a result of the data size, data storage configuration, reduction needs, and analysis requirements. Obviously, this can become more complicated when working across databases (e.g. traffic and weather data). The usage of the CICAS-V data had some limitations [50] as follow:

- a. They only include data for vehicles within 100m of the intersection. This may deem the data unusable or of limited use for the characterization of lane-changing behavior;
- b. The behavior of the vehicle inside the intersection is not captured. This becomes an issue for the characterization of driver acceleration behavior;
- c. The data only include two approaches with permissive left turns for the use in the characterization of gap acceptance behavior.

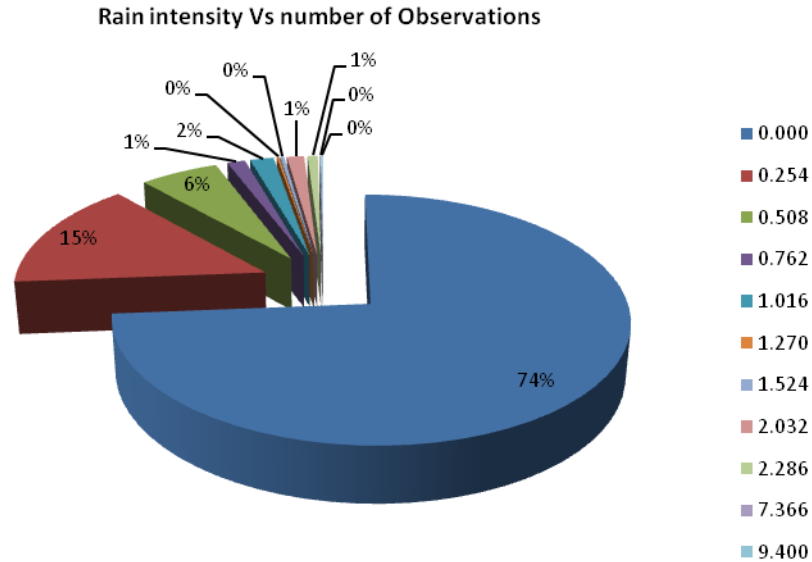
The video data were reduced manually by recording the time instant at which a subject vehicle initiated its search to make a left turn maneuver, the time step at which the vehicle made its first move to execute its left turn maneuver, and the time the left turning vehicle reached each of the conflict points. In addition, the time stamps at which each of the opposing vehicles passed the conflict points were identified. The final dataset that was analyzed consisted of a total of 2,730 gaps of which 301 were accepted and 2,429 were rejected. It was excluded more than 400 observations of gaps ending by red light signal, i.e. the gap is offered between a vehicle and no second opposing vehicle due to ending of the green phase. These 2,730 observations included 2,017 observations for dry conditions and 713 observations for different rain intensity levels (from 0.254 cm/h up to 9.4 cm/h). Table 4.1 summarizes the field data breakdown.

**Table 3.1: Total Observations Breakdown**

Total Observations (percentage %)					
Rain Intensity (cm/hr)	Total Accepted Gap		Total Rejected Gaps		Sum
	First lane	Second lane	First lane	Second lane	
0.000	70 (2.50%)	159 (5.82%)	509 (18.60%)	1279 (46.85%)	2017 (73.88%)
0.254	11 (0.40%)	30 (1.09%)	93 (3.40%)	268 (9.82%)	402 (14.74%)
0.508	5 (0.18%)	8 (0.29%)	40 (1.46%)	108 (3.95%)	161 (5.90%)
0.762	2 (0.07%)	6 (0.22%)	6 (0.29%)	24 (0.88%)	38 (1.39%)
1.016	2 (0.07%)	0 (0.00%)	5 (0.18%)	27 (0.99%)	34 (1.24%)
1.270	1 (0.04%)	1 (0.04%)	1 (0.04%)	3 (0.11%)	6 (0.22%)
1.524	1 (0.04%)	0 (0.00%)	1 (0.04%)	6 (0.22%)	8 (0.29%)
2.032	1 (0.04%)	2 (0.07%)	13 (0.48%)	19 (0.69%)	35 (1.28%)
2.286	0 (0.00%)	2 (0.07%)	5 (0.18%)	17 (0.63%)	24 (0.88%)
7.366	0 (0.00%)	0 (0.00%)	1 (0.04%)	1 (0.04%)	2 (0.07%)
9.400	0 (0.00%)	0 (0.00%)	1 (0.04%)	2 (0.07%)	3 (0.11%)
<b>Sum</b>	93 (3.41%)	208 (7.62%)	675 (24.72%)	1754 (64.25%)	2730 (100%)

The table shows the total number of rejected gaps (89%) is much greater than the total number of accepted gaps (11%). This is rational given that a driver typically rejects many gaps before accepting a gap. In addition, the number of offered gaps in the second lane (72%) is greater than the first lane (28%) since the first lane is a shared lane and thus does not have the level of utilization as the second lane.

Moreover, it is noticeable that 74% of the observations were observed for dry conditions (0 cm/hr), 21% of the observations were observed for rain intensities ranging between 0.254 cm/hr and 0.508 cm/hr and the remaining 5% were observed during rain intensities varying between 0.762 cm/hr and 9.4 cm/hr. Figure 3.4 is illustrating the percentage distribution (rounded to the nearest integer) of the total observations for different rain intensities.

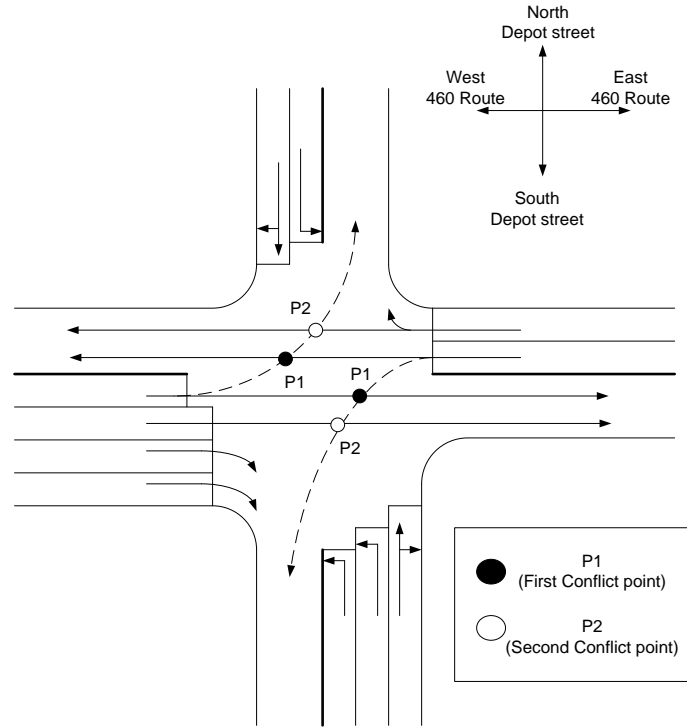


**Figure 3.4: The percentage of observations distribution over different rain intensity (cm/hr)**

### 3.4 Preliminary Results

The variables that were used in this study included the gap size, driver decision (accept or reject), the opposing lane number for the accepted/rejected gap, driver wait time in search of an acceptable gap, travel time to reach the conflict points, and rain intensity. The gap size “t” is a continuous variable measured in seconds and defined as the time headway difference between the passage of the front bumper of a lead vehicle and the following vehicle at a reference point which is taken as the conflict point (P1 or P2) in the opposing direction.

The analysis assumed that left turning vehicles heading for South Depot Street were similar in gap acceptance behavior to left turning vehicles heading for North Depot Street, as illustrated in Figure 3.5.



**Figure 3.5: Intersection of Depot Street and North Franklin Street (Business Route 460).**

Gaps recorded were the gaps offered to the first vehicle in the queue once their right-of-way permitted them to search for an acceptable gap (i.e. once the signal indication was green or they became the first vehicle in the queue). The wait time was computed from the first instant they were eligible to search for an acceptable gap until a gap was accepted. The Gap decision was recorded as a binary variable (0=rejection and 1=acceptance). A frequency plot for accepted and rejected gaps for dry and rain conditions are shown in Figure 4.2.a-b, respectively. The opposing lane number “L” is an indicator variable (0 or 1) to represent the lane in which the gap was present (L=0 for first lane and L=1 for second lane). The waiting time is a continuous variable “w” measured in seconds and it is defined as the summation of all rejected gap sizes offered to driver before accepting a gap, frequency plot is shown in Figure 4.2.c. The minimum waiting time measured was 0 sec and the maximum value was 60 sec. The travel time “ $\tau_i$ ” is a continuous variable measured in seconds and is defined as the time taken by left turning vehicles to reach conflict point  $P_i$  Figure 4.2.d. The 50<sup>th</sup> percentile (median) value for the first lane travel time was 2.3 s and for the second lane was 3.5 s. Rain intensity is also a continuous variable “r” measured in cm/h. Figure 4.2.e-f shows the rain intensity frequency plot for the first and second lanes,

respectively. The minimum rain intensity recorded was 0 cm/h (dry condition) and the maximum rain intensity was 9.4 cm/h with a weighted mean 0.1654 cm/h.

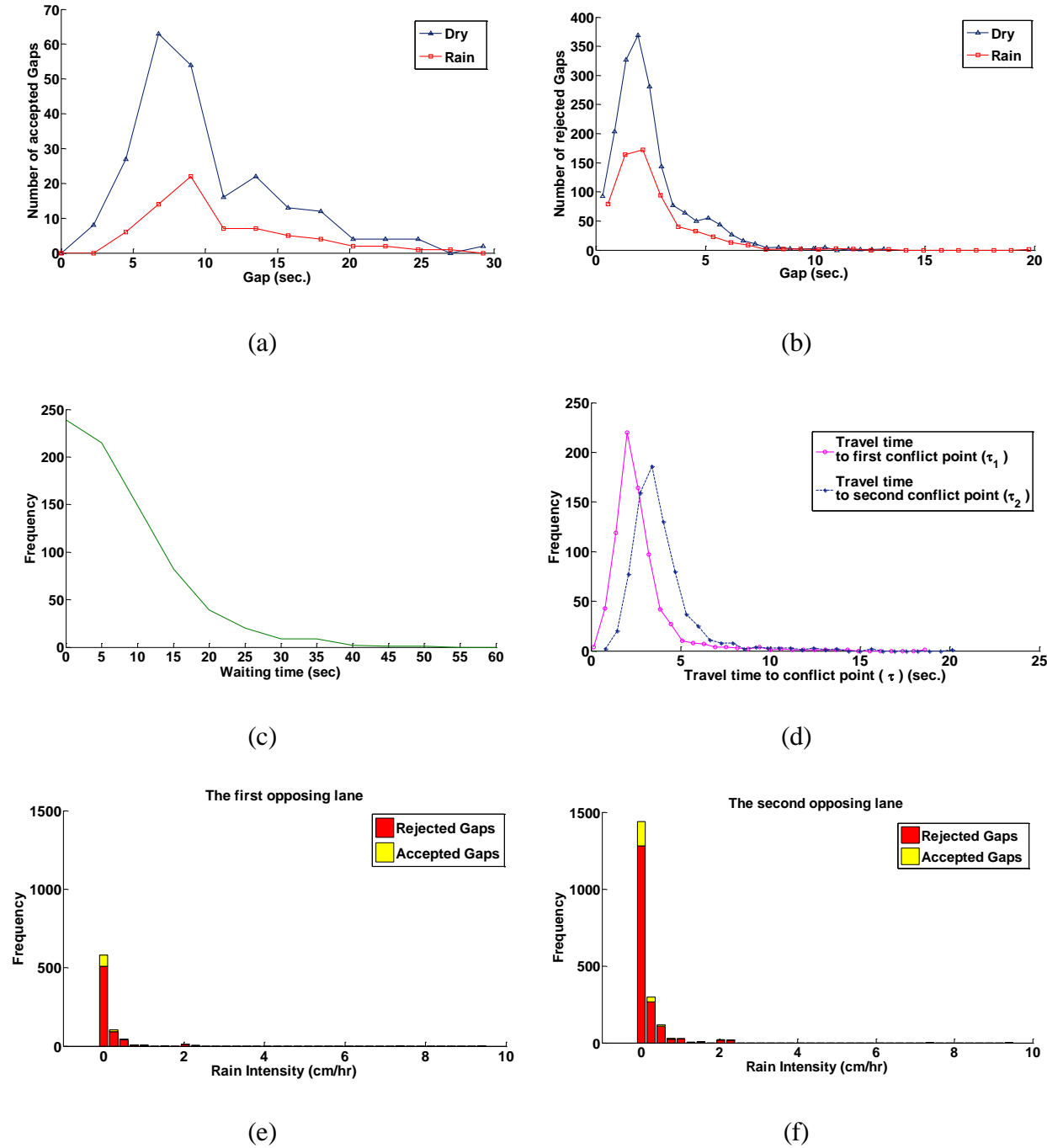
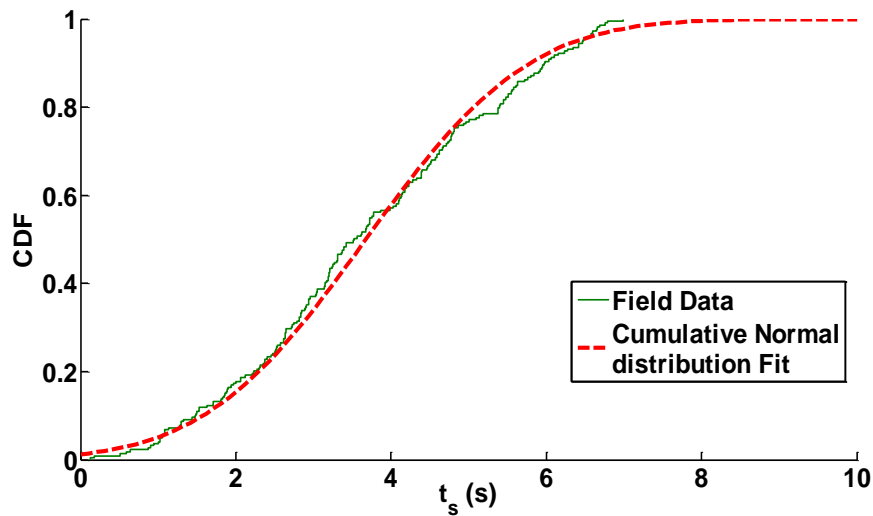
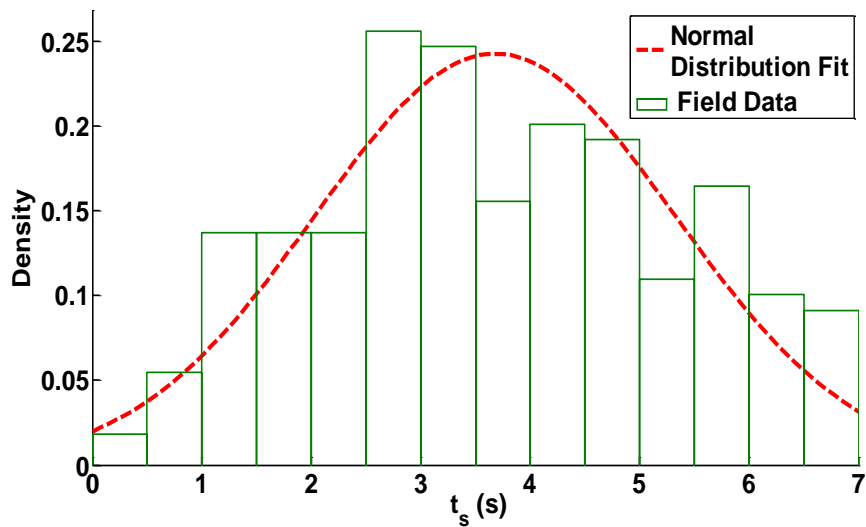


Figure 3.6: Data Frequencies for Different Rain Intensity Levels and Opposing Lanes

The additional gap duration required by a driver to avoid a conflict with an opposing vehicle is termed in this research, the buffer of safety ( $t_s$ ). The density distribution of the  $t_s$  from the collected data is illustrated in Figure 3.7(a) and the cumulative distribution function in Figure 3.7 (b). The distribution of the  $t_s$  can be approximated for a normal distribution with mean ( $\mu$ ) equal to 3.679 s and standard deviation ( $\sigma$ ) equal to 1.645 s.



(a)



(b)

**Figure 3.7: Distribution of buffer of safety ( $t_s$ ) from the field data**

The data reduction and analysis of the raw data was followed in order to identify different control variables affecting a driver's gap acceptance decision. Subsequently, three proposed modeling frameworks were considered in this thesis: a deterministic, a stochastic and finally a psycho-physical approach, will be presented in the following chapters.

## **CHAPTER 4 : STATISTICAL MODELING APPROACH**

Zohdy, I., S. Sadek and H. Rakha, “Empirical Analysis of Wait time and Rain Intensity Effects on Driver Left-Turn Gap Acceptance Behavior,” accepted for presentation at the TRB 89<sup>th</sup> Annual Meeting and being considered for publication in the Transportation Research Record

### **4.1 Abstract**

In this paper an empirical study is conducted to quantify the impact of a number of variables on driver left turn gap acceptance behavior. The variables that are considered include the gap duration, the driver’s wait time in search of an appropriate acceptable gap, the time traveled by a driver to clear a conflict point, and the rain intensity. Gap acceptance data for a permissive left turn maneuver at a signalized intersection were collected over a two-month period. Using the data logistic regression models were calibrated to the data. The models reveal that the acceptable gap duration decreases as a function of the driver’s wait time and increases as the rain intensity increases. The study then demonstrates how the critical gap is influenced by a number of parameters including the wait time and rain intensity. The study demonstrates how rain intensity affects the permissive left-turn saturation flow rate. It is anticipated that these findings can be used to develop weather-specific traffic signal timings that account for changes in traffic stream saturation flow rates due to changes in critical gap values as a function of weather conditions.

### **4.2 Introduction**

Gap acceptance is defined as the process that occurs when a traffic stream (known as the opposed flow) has to either, cross or merge with another traffic stream (known as the opposing flow). Examples of gap acceptance behavior occurs when vehicles on a minor approach need to cross a major street at a two-way stop controlled intersection, when vehicles have to make a left turn through an opposing through movement at a signalized intersection, or when vehicles merging onto a freeway have to find gaps in the mainline freeway flow.

Generally gap acceptance analysis can be divided into two types: 1) crossing gap acceptance at intersections; or 2) gap acceptance during merging or lane-change maneuvers. This paper focuses on crossing gap acceptance behavior for permissive left turns.

A gap is defined as the elapsed-time interval between arrivals of successive vehicles in the opposing flow at a specified reference point in the intersection area. The minimum gap that a driver is willing to accept is generally called the critical gap. The Highway Capacity Manual (HCM) (2000) [1] defines the critical gap as the “*minimum time interval between the front bumpers of two successive vehicles in the major traffic stream that will allow the entry of one minor-street vehicle.*” When more than one opposed vehicle uses a gap, the time headway between the two opposed vehicles is called the follow-up time. In general, the follow-up time is shorter than the critical gap and equals the inverse of the unopposed saturation flow rate.

Since the critical gap of a driver cannot be measured directly, censored observations (i.e., accepted and rejected gaps) are used to compute critical gaps, as will be described later. For more than three decades research efforts have attempted to model driver gap acceptance behavior, either through deterministic or by probabilistic means. The deterministic critical values are treated as a single threshold for accepting or rejecting gaps. Examples of deterministic methods include Raff’s [10, 14, 29] and Greenshield’s [15, 51] methods. The stochastic or probabilistic approach to modeling gap acceptance behavior involves constructing either logit [6] or probit models [7, 40] using some maximum likelihood calibration technique. The fundamental assumption is that drivers will accept all gaps that are larger than the critical gap and reject all smaller gaps. The population critical gap is then computed as the median of all driver critical gaps, as discussed later in the paper. Although various researchers have used different definitions for the critical gap, the deterministic model has been the conventional approach of gap acceptance studies. As an alternative, probabilistic models solve some of the inconsistency elements in gap acceptance behavior by using a statistical treatment of opposed driver gap acceptance behavior. In other words a driver’s perception of a minimum acceptable gap is treated as a random variable [13].

For permissive left-turn traffic, The HCM 2000 estimates the opposed saturation flow rate based on the critical gap and follow-up time. The HCM considers the critical gap accepted by left-turn drivers as a deterministic value equal to 4.5 s at signalized intersections with a permitted left-turn phase and this value is independent of the number of opposing-through lanes to be crossed by the opposed vehicles.

The American Association of State Highway and Transportation Officials (AASHTO, 2001) [27], classifies the left turning movements from the major road across opposing traffic as Case *F*. The AASHTO (2001) recommends that for case *F* opposed movements that the critical gap for left-turning passenger cars be set equal to 5.5 s (for passenger cars) and for left-turning vehicles that cross more than one opposing lane to add an additional 0.5 s for each additional lane of travel.

Several studies in the literature have investigated the impact of different factors on driver gap acceptance behavior. These factors include day and nighttime effects [2], the speed of the opposing vehicle [3, 4], the type of intersection control (yield versus stop sign) [5], the driver sight distance [6], the geometry of the intersection, the trip purpose, and the expected waiting time [7]. In addition, some of the literatures have examined the factors affecting the left turn and gap acceptance crash patterns at intersections. Individual characteristics, vehicle capability and intersection design factors, are considered the main factors contributed to the probability that a left-turn or gap acceptance crash will occur [52].

### **4.3 Study Objectives and Paper Layout**

The main objectives of the study are to investigate the influence of driver waiting time, distance travelled to clear the conflict point, and rain intensity on left-turn gap-acceptance behavior. In addition, logit models are fit to the data to model driver gap acceptance behavior and compute the driver-specific critical gap. It is hypothesized that (a) driver's become more aggressive as they wait longer in search of an acceptable gap; (b) driver' require longer gaps under rainy conditions; and (c) the size of a driver's acceptable gap increases as the distance to the conflict point increases.

In terms of the paper layout, initially the study site and data acquisition procedures are described followed by a description of the different proposed models along with model calibration results. Subsequently, the predicted critical gap from each model is presented and the influence of waiting time and rain intensity is studied. Finally, conclusions and recommendations are presented.

#### **4.4 Study Site Description and Data Acquisition**

The study site that was considered in this study was the signalized intersection of Depot Street and North Franklin Street (Business Route 460) in Christiansburg, Virginia. A schematic of the intersection is shown in Figure 4.1. It consists of four approaches at approximately 90° angles. The posted speed limit for the eastbound and northbound approaches was 35 mph and for the westbound and southbound approaches was 25 mph at the time of the study.

The signal phasing of the intersection included three phases, two phases for the Depot street North and South (one phase for each approach) and one phase for the North Franklin Street (two approaches discharging during the same phase) with a permissive left turn phase. The entering average annual daily traffic (AADT) for this intersection was 26,671 vehicles per day and there was an average of 11 accidents per year reported at this intersection.

The data used in this study were collected as part of the CICAS-V data collection effort [49]. The Data Acquisition System (DAS) employed a suite of hardware and software to record information about vehicles that approached the site. Data were acquired for two months (from June to August 2007).

The data acquisition hardware consisted of multiple components as follows:

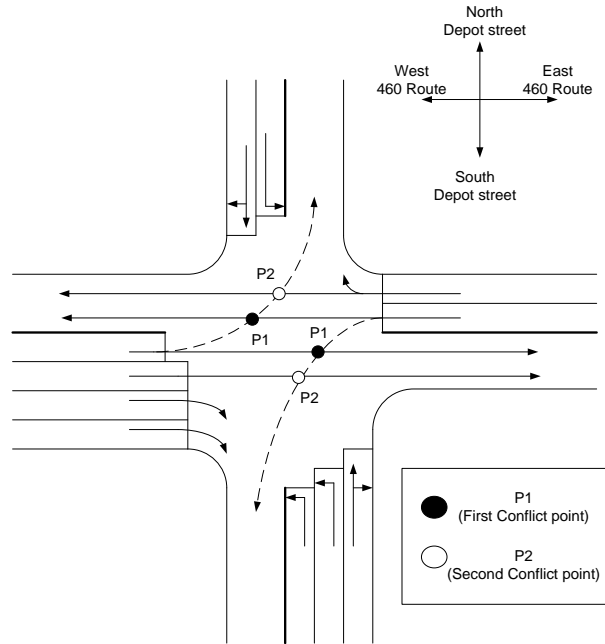
- a. Video cameras to collect the visual scene. There were four cameras installed at the intersection (one camera for each approach) to provide video feed of the entire intersection environment at 20 frames per second.
- b. Weather stations. The weather station provided weather information once each minute. The collected weather data included long-term rain fall, daily rain fall, wind direction, wind speed, average wind speed, temperature, barometric pressure, and humidity level.
- c. GPS units provided synchronized global time. A basic GPS system was employed at each intersection primarily for acquiring an accurate global time. This time information was used to stamp the data.

The processing stack processed the sensor data and assembled the dataset in real-time while archiving to binary data and compressed video files. Data subsets were extracted from the archived data using in-house written codes for analysis purposes.

## 4.5 Data Reduction Results

The variables that were used in this study included the gap size, driver decision (accept or reject), the opposing lane number for the accepted/rejected gap, driver wait time in search of an acceptable gap, travel time to reach the conflict points, and rain intensity. The gap size “ $t$ ” is a continuous variable measured in seconds and defined as the time headway difference between the passage of the front bumper of a lead vehicle and the following vehicle at a reference point which is taken as the conflict point (P1 or P2) in the opposing direction.

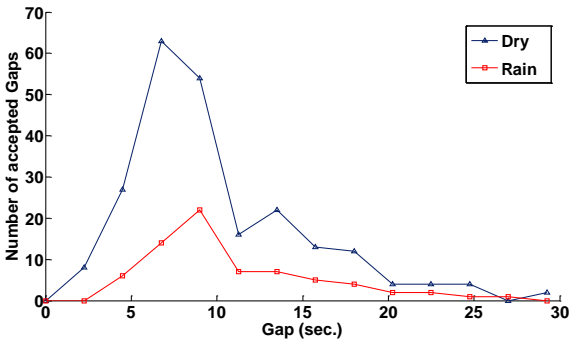
The analysis assumed that left turning vehicles heading for South Depot Street were similar in gap acceptance behavior to left turning vehicles heading for North Depot Street, as illustrated in Figure 4.1. Gaps recorded were the gaps offered to the first vehicle in the queue once their right-of-way permitted them to search for an acceptable gap (i.e. once the signal indication was green or they became the first vehicle in the queue). The wait time was computed from the first instant they were eligible to search for an acceptable gap until a gap was accepted. The Gap decision was recorded as a binary variable (0=rejection and 1=acceptance). A frequency plot for accepted and rejected gaps for dry and rain conditions are shown in Figure 4.2.a-b, respectively. The opposing lane number “ $L$ ” is an indicator variable (0 or 1) to represent the lane in which the gap was present ( $L=0$  for first lane and  $L=1$  for second lane). The waiting time is a continuous variable “ $w$ ” measured in seconds and it is defined as the summation of all rejected gap sizes offered to driver before accepting a gap, frequency plot is shown in Figure 4.2.c. The minimum waiting time measured is 0 sec and the maximum value was 60 sec. The travel time “ $\tau_i$ ” is a continuous variable measured in seconds and is defined as the time taken by left turning vehicles to reach conflict point  $P_i$  Figure 4.2.d. The 50<sup>th</sup> percentile (median) value for the first lane travel time was 2.3 s and for the second lane was 3.5 s. Rain intensity is also a continuous variable “ $r$ ” measured in cm/h. Figure 4.2.e-f shows the rain intensity frequency plot for the first and second lanes, respectively. The minimum rain intensity recorded was 0 cm/h (dry condition) and the maximum rain intensity was 9.4 cm/h with a weighted mean 0.1654 cm/h.



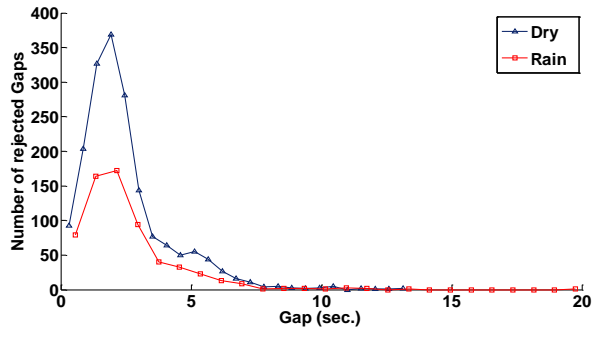
**Figure 4.1: Intersection of Depot Street and North Franklin Street (Business Route 460).**

In the dataset, 2,730 gap decisions from a total of 301 left-turning movements were recorded, after excluding accepted gaps terminated by the red signal indication (no following vehicle). The 2,730 observations included 2,017 observations for dry conditions and 713 observations for rain conditions. Table 4.1 summarizes the field data breakdown.

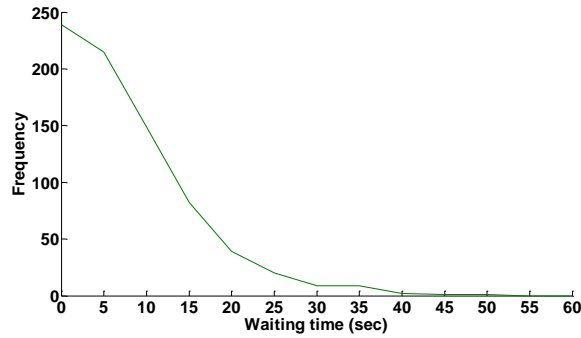
Although the left turning vehicles on the westbound approach had to cross four lanes while those on the eastbound approach only crossed two lanes, an analysis of the gap acceptance behavior for points P1 and P2 revealed no statistically significant differences between the two approaches. Consequently both approaches were pooled together in conducting the analysis. Furthermore, because only few observations were observed for the westbound third and fourth opposing lanes these observations were excluded from the analysis.



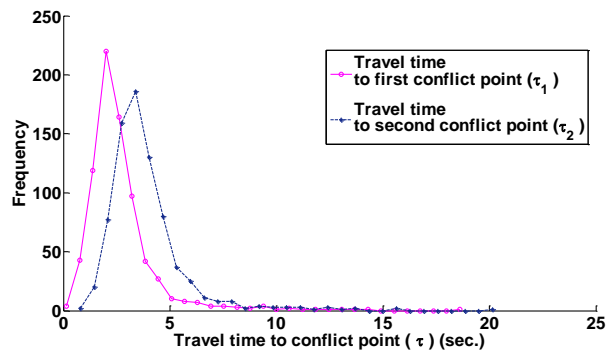
(a)



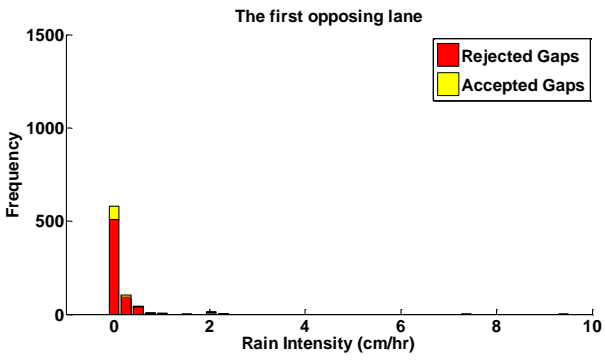
(b)



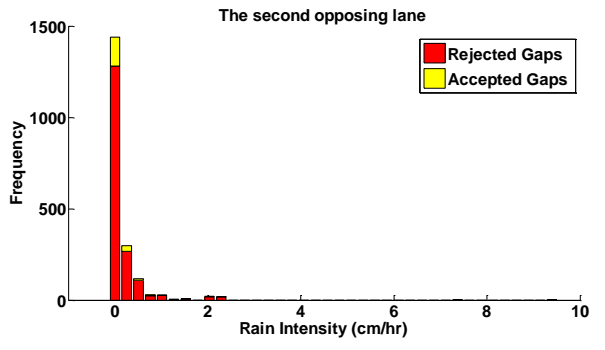
(c)



(d)



(e)



(f)

**Figure 4.2: Data Frequencies for Different Rain Intensity Levels and Opposing Lanes**

**Table 4.1: Total Observations Breakdown**

Total Observations (percentage %)					
Rain Intensity (cm/hr)	Total Accepted Gap		Total Rejected Gaps		Sum
	First lane	Second lane	First lane	Second lane	
0.000	70 (2.50%)	159 (5.82%)	509 (18.60%)	1279 (46.85%)	2017 (73.88%)
0.254	11 (0.40%)	30 (1.09%)	93 (3.40%)	268 (9.82%)	402 (14.74%)
0.508	5 (0.18%)	8 (0.29%)	40 (1.46%)	108 (3.95%)	161 (5.90%)
0.762	2 (0.07%)	6 (0.22%)	6 (0.29%)	24 (0.88%)	38 (1.39%)
1.016	2 (0.07%)	0 (0.00%)	5 (0.18%)	27 (0.99%)	34 (1.24%)
1.270	1 (0.04%)	1 (0.04%)	1 (0.04%)	3 (0.11%)	6 (0.22%)
1.524	1 (0.04%)	0 (0.00%)	1 (0.04%)	6 (0.22%)	8 (0.29%)
2.032	1 (0.04%)	2 (0.07%)	13 (0.48%)	19 (0.69%)	35 (1.28%)
2.286	0 (0.00%)	2 (0.07%)	5 (0.18%)	17 (0.63%)	24 (0.88%)
7.366	0 (0.00%)	0 (0.00%)	1 (0.04%)	1 (0.04%)	2 (0.07%)
9.400	0 (0.00%)	0 (0.00%)	1 (0.04%)	2 (0.07%)	3 (0.11%)
<b>Sum</b>	<b>93 (3.41%)</b>	<b>208 (7.62%)</b>	<b>675 (24.72%)</b>	<b>1754 (64.25%)</b>	<b>2730 (100%)</b>

#### 4.6 Logistic Regression models

As indicated before, the primary objective of this study was to quantify the dependence of gap acceptance decisions on wait time, rain intensity, and travel time. Given that the response variable is discrete (0 or 1) while the explanatory variables are continuous, a logistic model was fit to the data to estimate the probability of accepting a gap ( $p$ ) as shown in Equation (11):

$$p = \frac{e^{U(x)}}{1 + e^{U(x)}} \quad (11)$$

$$U(x) = \text{logit}(p) = \ln \left[ \frac{p}{1-p} \right] = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_n x_n \quad (12)$$

Where;  $p$  is the probability of accepting a gap;  $x_1, x_2, \dots, x_n$  are the explanatory variables; and  $\beta_0, \beta_1, \beta_2, \dots, \beta_n$  are the estimated regression coefficients.

In this study, different models were considered to capture gap acceptance behavior in terms of different variables (gap size, waiting time, accepted gap lane number, and rain intensity). By applying different statistical approaches to variable elimination, and treating the accepted and

rejected decision as a binary choice (0 or 1), and assuming a logit link function for the generalized linear model (GLM), the three following models were developed:

#### 4.6.1 Model 1, (M1)

$$\text{logit}(p) = \beta_o + \beta_1 g + \beta_2 w + \beta_3 L + \beta_4 r + \beta_5 Lg \quad (13)$$

#### 4.6.2 Model 2, (M2)

$$\text{logit}(p) = \beta_o + \beta_1 (g - \bar{\tau}_t) + \beta_2 (w) + \beta_3 (r) \quad (14)$$

#### 4.6.3 Model 3, (M3)

$$\text{logit}(p) = \beta_o + \beta_1 \frac{g}{\bar{\tau}_t} + \beta_2 \frac{w}{\bar{\tau}_t} + \beta_3 \frac{r}{\bar{r}} \quad (15)$$

Where;  $\text{logit}(p) = \ln(p/(1-p))$ ;  $p$  is probability of accepting a gap;  $g$  is the gap size offered to the opposed vehicle (s);  $w$  is the waiting time to accept a gap (s);  $r$  is the rain intensity (cm/h);  $\bar{r}$  is the average rain intensity at the site (in this case 0.1654 cm/hr);  $L$  is the lane indicator variable for the lane number of the accepted gap (0=First lane, 1=Second lane);  $\tau_t$  the median travel time to a conflict point (for the first lane “P<sub>1</sub>”=2.3 s and for the second lane “P<sub>2</sub>” = 3.5 s). The estimated parameters for the three proposed models are shown in Table 4.2. For model 1 (M1), the independent variables presented are the gap size, waiting time to accept a gap, lane number as an indicator variable, rain intensity and the interaction term between gap size and the lane number.

In the case of the second model (M2), the independent variables include the difference between the gap size and the travel time to the conflict point (which is the time remaining for a left turn driver to clear the conflict point), the waiting time in search of an acceptable gap, and the rain intensity.

In the case of the third model (M3), the independent variables are dimensionless in order to generalize and normalize the random error. Initially, the gap size was normalized by dividing by the travel time plus some buffer of safety that is computed as a fraction of the gap size. The waiting time was normalized by dividing by the mean green time. However, from a practical standpoint this model could not be used at a stop sign given that the green time would be infinity.

Consequently the M3 was proposed where the gap size and waiting time are normalized by dividing by the travel time to the respective conflict point. The rain intensity is normalized by the mean rain intensity at the location. It is anticipated that this model could give a good estimation for the effect of these independent variables on gap acceptance at other intersections using the respective intersection characteristics; e.g. the effect of the rain intensity of 0.5 cm/h to mean rain intensity equal 1 cm/h is the same as the effect of a rain intensity of 1 cm/h at a location where the mean rain intensity is 2 cm/h.

**Table 4.2: The estimated parameters for the three proposed models and the statistics tests**

	<i>Term</i>	$\beta_i$	<i>Estimated mean values</i>	<i>Std Error</i>	<i>L-R ChiSquare</i>	<i>Prob&gt;ChiSq (p values)</i>	<i>Lower CL</i>	<i>Upper CL</i>
<b>Model 1 (M1)</b>	Intercept	$\beta_o$	-7.237	0.626	721.177	<.0001	-8.600	-6.125
	$g$	$\beta_1$	1.009	0.101	374.643	<.0001	0.827	1.228
	$w$	$\beta_2$	0.034	0.009	10.752	0.0010	0.014	0.053
	$L$	$\beta_3$	1.332	0.668	4.649	0.0311	0.113	2.760
	$r$	$\beta_4$	-0.666	0.231	10.850	0.0010	-1.201	-0.255
	$L^*g$	$\beta_5$	-0.281	0.109	7.938	0.0048	-0.513	-0.080
<b>The whole model tests:</b> LogLikelihood= -389.614, ChiSquare= 1111.491, Prob>ChiSq (p-value) <0.0001								
<b>Model 2 (M2)</b>	Intercept	$\beta_o$	-3.677	0.164	1113.235	<.0001	-4.011	-3.367
	$g - \bar{\tau}_i$	$\beta_1$	0.771	0.038	1078.144	<.0001	0.698	0.850
	$w$	$\beta_2$	0.033	0.009	10.498	0.0012	0.014	0.053
	$r$	$\beta_3$	-0.623	0.235	9.907	0.0016	-1.167	-0.217
<b>The whole model tests:</b> LogLikelihood= -399.031, ChiSquare= 1092.656, Prob>ChiSq (p-value) <0.0001								
<b>Model 3 (M3)</b>	Intercept	$\beta_o$	-5.650	0.233	1912.931	0.0000	-6.124	-5.211
	$g / \bar{\tau}_i$	$\beta_1$	2.160	0.106	1024.739	<.0001	1.959	2.376
	$w / \bar{\tau}_i$	$\beta_2$	0.065	0.029	4.727	0.0297	0.007	0.121
	$r / \bar{r}$	$\beta_3$	-0.109	0.034	10.826	0.0010	-0.186	-0.043
<b>The whole model tests:</b> LogLikelihood= -424.961, ChiSquare= 1040.796, Prob>ChiSq (p-value) <0.0001								

## 4.7 Comparison of Models

In comparing the different models (M1, M2 & M3), two criteria were considered: (a) Success Rate factor (SR) and (b) Bayesian information criterion (BIC). These criteria are briefly described.

The SR is defined as the percentage of observations with acceptance/rejection outcomes that are identical to field responses. The model with the largest SR is the best model. In computing the SR for each model, the probability of accepting a gap was computed for each of the 2730 observations using the explanatory variables and rounding the response to 0 or 1 (0 if the resulted probability is below 50% and 1 if it is equal or above) and compared to the field observed response binary choice.

The negative log likelihood (or, equivalently, the deviance) can be used as a measure of how well a model fits a data set, with smaller values being indicative of a better fit. However, due to the difference in number of parameters from one model to the other, this criterion will be biased in favor of less parsimonious models; therefore, the Bayesian Information Criterion (BIC) is used as was suggested in the literature [25]. The BIC is a value which adds a penalty as the complexity of the model increases where the complexity refers to the number of parameters, as

$$BIC = -2 \times LL + p \ln(N) \quad (16)$$

Where;  $LL$  is the posterior expected log likelihood,  $p$  is the number of parameters used by the model, and  $N$  is the number of datum points (number of observations). Table 4.3 presents the SR and the BIC results for the proposed models.

**Table 4.3: Model Evaluation Criterion Summary**

Measurements	Models		
	M 1	M 2	M 3
SR1 (% success for estimating observed accepted gaps)	64	64	58
SR2 (% success for estimating observed rejected gaps)	98	98	98
SR3 (% success for estimating all observed gaps)	95	95	93
BIC	799.845	811.807	863.668

As demonstrated in Table 4.3, the M1 and M2 models have the same success rates for observed accepted gaps (64%), observed rejected gaps (98%) and the total observed gaps (weighted mean of accepted and rejected gaps) value (95%). In the case of the M3 model, the SR values are less than the other models. It is worth noting that for M3, if the estimated required gap and the average green time are used to normalize the offered gap and wait time respectively, a higher SR is obtained. However, as was mentioned earlier this model was deemed impractical.

Referring to the BIC values, the M1 model is superior to the M2 and M3 models. In summary, each of the three models provides similar inferences concerning the relation between gap acceptance, waiting time, rain intensity, and lane number (or travel time). Each of these models predicts that, for a given gap size and waiting time, the probability that a driver accepts a gap decreases as the rain intensity increases and at a given gap size and rain intensity level the probability of accepting a gap increases as the waiting time increases for a specific opposing lane. To illustrate this effect, Figure 4.3 displays the probability of gap acceptance versus gap size for each of the three models, in which the waiting time and rain intensity are set to 0. Gap acceptance probabilities are computed using the model parameters given in Table 4.2.

#### 4.8 Critical Gap estimation Based on Logistic Regression

Generally, the larger the available gaps, the higher the probability that drivers will accept the gaps. The critical gap is defined as the gap size that has equally likely to be accepted and rejected; and thus corresponds to the median of the probability of accepted gaps, as shown in Figure 4.3. The different critical gap values for the three proposed models are summarized in Table 4.4 (for a waiting time and rain intensity of zero).

**Table 4.4: The different critical gap values for each model**

	(t <sub>c</sub> ) Critical Gap (s)	
	First Lane	Second Lane
<b>Model 1</b>	7.2	8.1
<b>Model 2</b>	7.1	8.3
<b>Model 3</b>	6.0	9.2

The critical gap values that are presented in Table 4.4 are significantly larger than the HCM [1] recommended value of 4.5 s. One possible explanation is that the geometry of the intersection was such that the approaches involved some curvature and thus drivers might have had a hard time establishing which lane vehicles the opposing vehicles were in and which movement they were executing. An alternative explanation could be that the population density in the Town of Christiansburg is very small compared to large cities and thus drivers may be less aggressive. In addition, an earlier study [6] showed that the opposing vehicles turning left can block a driver's view of oncoming traffic, which results in larger accepted gap sizes. Specifically, the study indicates that in case of no opposing left turn vehicle (no sight blockage), the critical gap is 5.6 s and it increases by 2.1 s in the case of sight blockage. This might be a factor in the current study.

Regarding the difference between the critical gap value between the first and second lane, it is noticeable that for the three models the difference is greater than the 0.5 s value recommended in the AASHTO design procedures [27]. However, it should be noted that in the case of the M1 and M2 models this difference is considered reasonable (0.9 s and 1.2 s respectively). This value is consistent with the difference in median travel times of 1.2 s. The M3 model appears to over-estimate this difference (3.2 s).

#### 4.9 Effect of Waiting Time and Rain Intensity On Critical Gap Values

The critical gap can be computed by setting the probability of accepting a gap equal to 0.5 which entails setting the logit to zero. Consequently the critical gap ( $t_c$ ) for models M1, M2, and M3 can be computed using Equations (17) through(19), respectively.

$$t_c = -\frac{\beta_0}{(\beta_1 + \beta_5 L)} - \frac{\beta_2}{(\beta_1 + \beta_5 L)} w - \frac{\beta_3}{(\beta_1 + \beta_5 L)} L - \frac{\beta_4}{(\beta_1 + \beta_5 L)} r \quad (17)$$

$$t_c = \bar{\tau}_t - \frac{\beta_0}{\beta_1} - \frac{\beta_2}{\beta_1} w - \frac{\beta_3}{\beta_1} r \quad (18)$$

$$t_c = -\frac{\beta_0}{\beta_1} \bar{\tau}_t - \frac{\beta_2}{\beta_1} w - \frac{\beta_3 \bar{\tau}_t}{\beta_1 \bar{r}} r \quad (19)$$

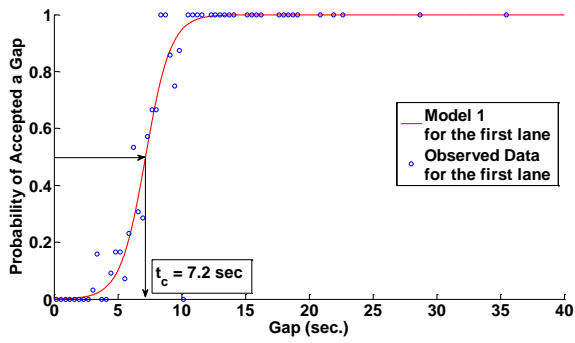
In exploring the relationship between the critical gap and wait time or rain intensity, the critical gap is plotted versus the wait time and rain intensity at constant rain intensity levels and wait

times, respectively. In general, the relationship between the waiting time and critical gap size “ $t_c$ ” is a linear decay function, i.e. when the driver waits longer, he/she becomes more aggressive and thus the critical gap decreases. Increasing the rain intensity results in less aggressive driver behavior and an increase in the critical gap. The effect of the rain intensity and waiting time on the critical gap value for each lane (first lane and second lane) is shown in Figure 4.4.a-f. It is assumed in plotting the relation between waiting time and the critical gap, or the rain intensity and the critical gap that the other independent variable is held constant, to show the effect of each of these independent variables on the critical gap value for the first and the second lane.

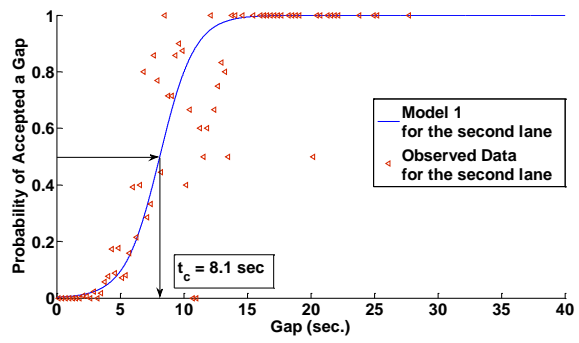
In the case of the M1 model (Figure 4.a) the slope of the line for the second lane is much steeper than that for the first lane. This difference in slopes results in the intersection of these two lines at a waiting time of approximately 70 s, implying that a driver is willing to accept a shorter gap in the second lane, which appears to be unrealistic. Figure 4.b shows that the difference between the critical gap size for the first and the second lane is 0.9 s at rain intensity 0 cm/h and the difference increases with rain intensity. The difference becomes 3.5 s at rain intensity of 10 cm/h.

In the case of the M2 model (shown in Figure 4.c-d) by increasing the waiting time or the rain intensity, the difference between the critical gap value for the first and second lane remains constant and equal to 1.2 s which is equal to the difference in travel time between the first and second conflict points ( $P_1$  &  $P_2$ ).

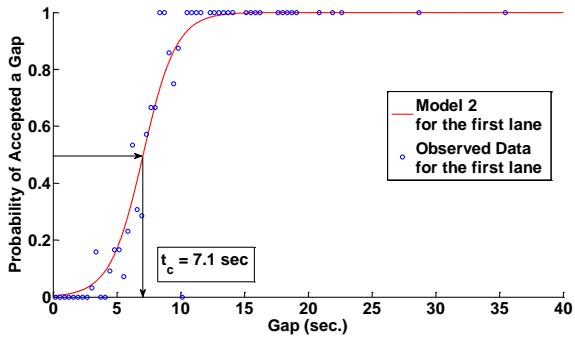
In the case of the M3 model (shown in Figure 4.e-f) the difference in critical gap for the first and second lane is equal to 3.2 s at a waiting time and rain intensity of zero. Increasing the waiting time, the difference remains constant, while increasing the rain intensity increases the difference reaching 6.8 s at rain intensity 10 cm/h. This difference seems to be very high.



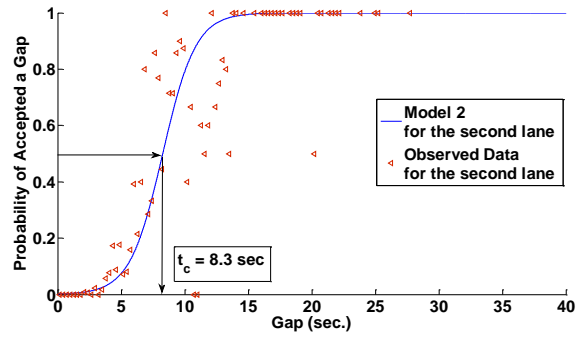
(a)



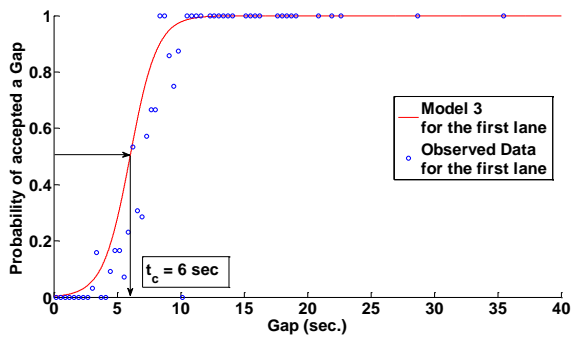
(b)



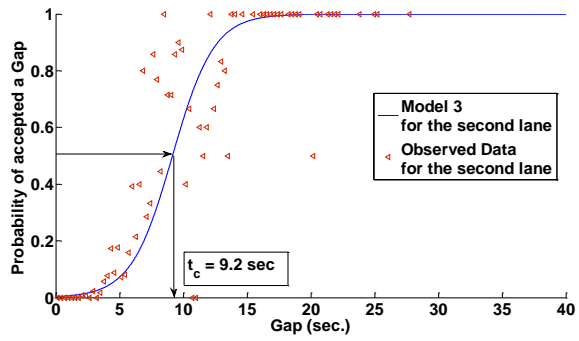
(c)



(d)

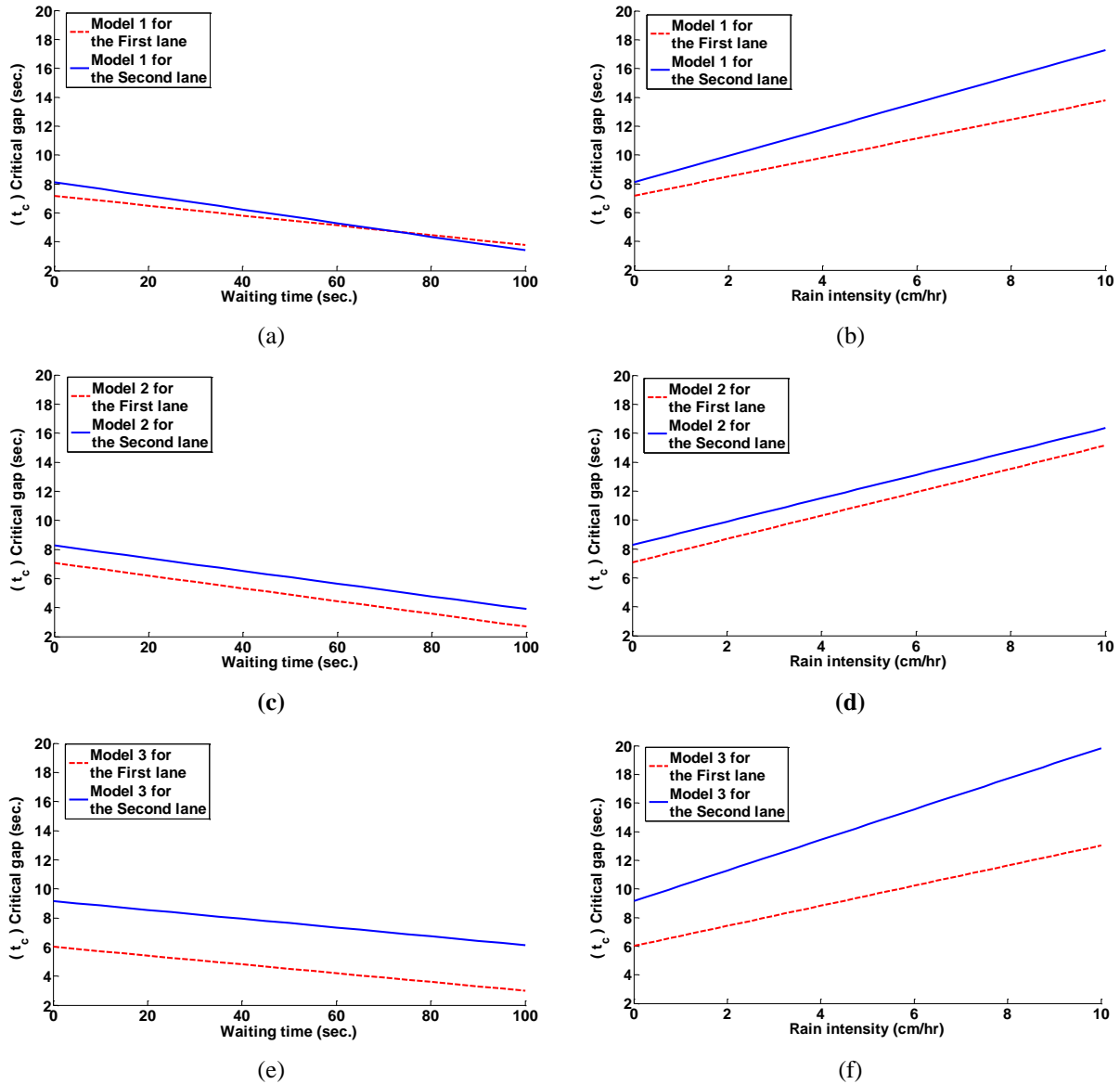


(e)



(f)

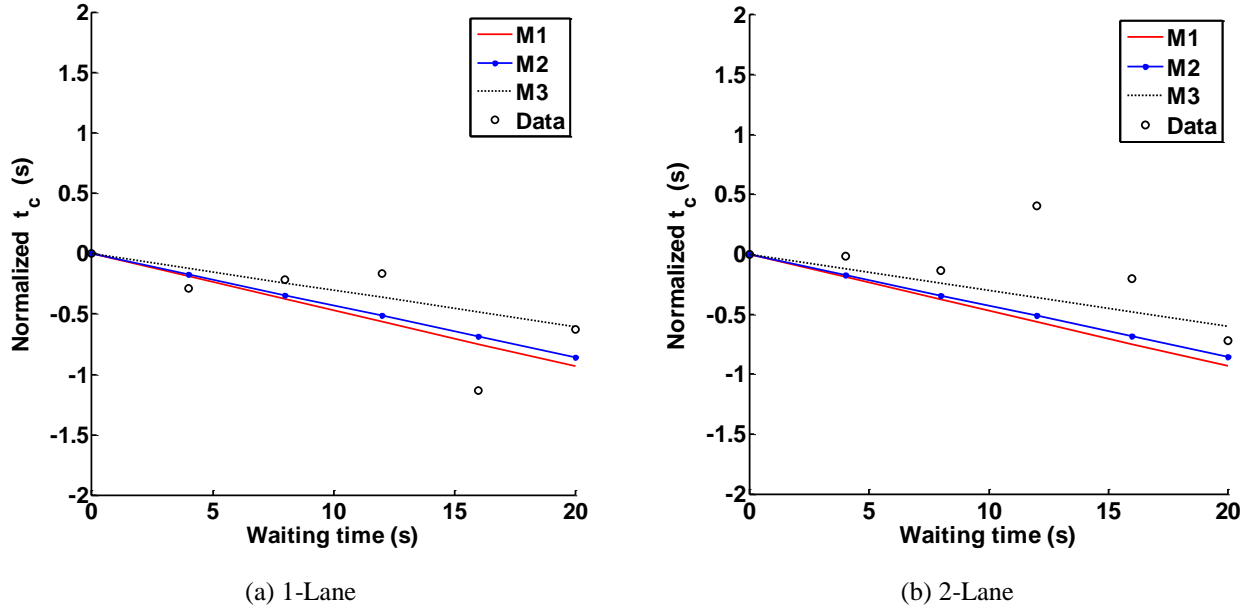
**Figure 4.3: The Accepted Probability and Critical Gap Values for Different Models**



**Figure 4.4: Effect of waiting time and rain intensity on critical gap value for different models**

Figure 4.5 shows a comparison between the model estimates of the critical gap and the calculated values from the field data. The critical gap was estimated from the field data by sorting the data based on the waiting time. The data were then binned into equally sized bins (i.e. equal number of observations in each bin). The average waiting time and the median accepted gap (critical gap) were computed. This process was repeated multiple times to converge to the desired statistics. As demonstrated in Figure 4.5, the waiting time is plotted against the normalized critical gap (the

critical gap value for each model minus the critical gap value at zero waiting time), it is noticeable that the models capture the correct trend, however the absolute do not necessarily match.



**Figure 4.5: Comparison of Field Observed and Model Estimated Critical Gaps**

#### 4.10 Impact on Opposed Saturation Flow Rates

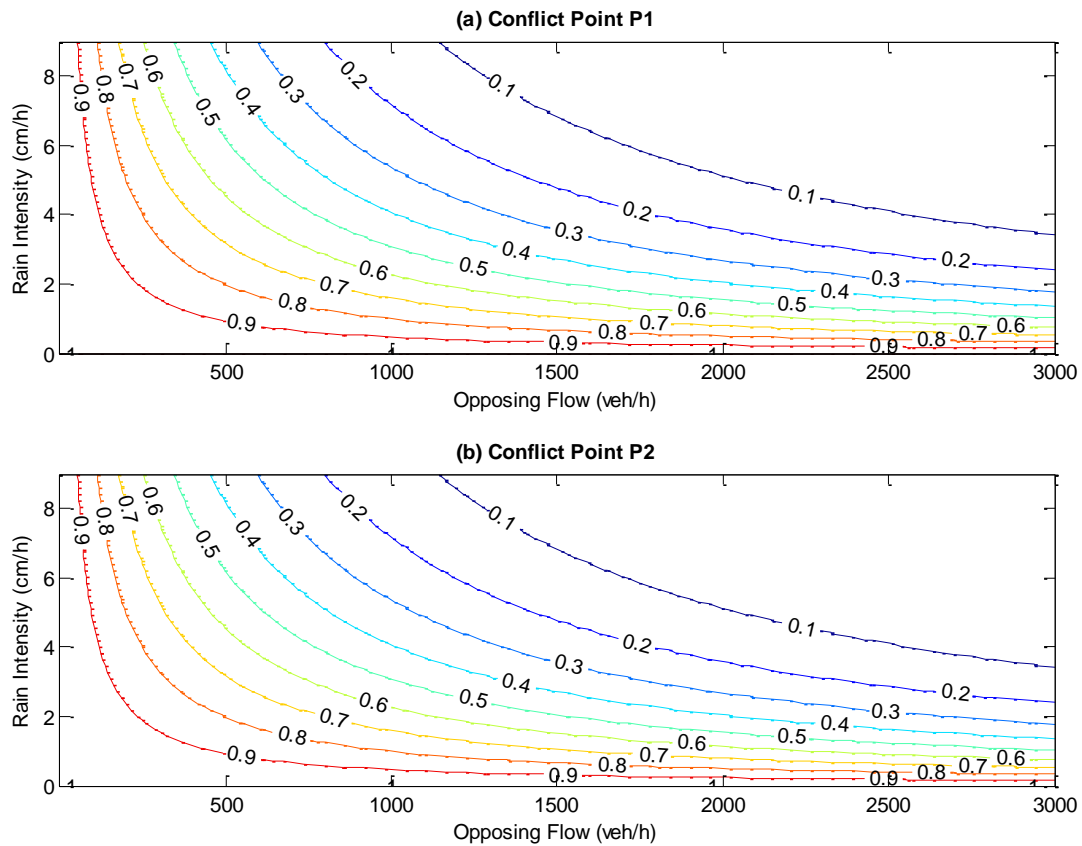
Once the critical gap is computed, the opposed saturation flow rate ( $s$ ) can be computed using Equation (20). Here  $v_0$  is the opposing flow (veh/h),  $t_c$  is the critical gap size (s) which can be computed using one of the three models that were presented earlier (M1, M2, and M3), and  $t_f$  is the follow-up time (s). The follow-up time is the discharge time headway for the unopposed saturation flow rate (i.e. when the opposing flow is zero).

$$s = v_0 \frac{e^{\frac{-v_0 t_c}{3600}}}{1 - e^{\frac{-v_0 t_f}{3600}}} \quad (20)$$

The critical gap size and corresponding opposed saturation flow rate was computed for various rain intensity and opposing flow levels using the M2 model. Opposed saturation flow rate adjustment factors ( $F$ ) (saturation flow rate relative to the zero rain intensity saturation flow rate) were then computed, as illustrated in Figure 4.6 and Equation (21) where  $s_w$  is the increment

weather opposed saturation flow rate. The figure clearly demonstrates a decrease in the opposed saturation flow rate as the rain intensity and opposing flow increases. The figure also demonstrates that the relative effect of rain intensity on the saturation flow rate increases as the opposing flow rate increases. Noteworthy is the fact that the nomographs are identical for both conflict points given that the average travel time only alters the absolute values of the critical gaps; however the relative changes in saturation flow rates are identical.

$$s_w = Fs \tag{21}$$



**Figure 4.6: Variation in Opposed Saturation Flow Rate Adjustment Factor as a Function of Rain Intensity and Opposing Flow**

## **4.11 Conclusions and Recommendations for Further Research**

The study analyzed field data at a signalized intersection to study left-turn gap acceptance behavior using three logit models. The proposed models tested the dependence of gap acceptance behavior on the gap size, the time the driver waits in search of a gap, and the rain intensity. The main findings are, although the first model is best from a statistical standpoint, it produces unrealistic behavior with regards to wait times. The second model is comparable to the first model in terms of statistical measure performance with the advantage that it produces realistic behavior with regards wait times and rain intensity levels. The third model produces differences in critical gap values across different lanes that are much larger than what is reported in the literature and is inconsistent with the field observations. In addition, the statistical tests show that it is inferior to the other models.

The models demonstrate that drivers become more aggressive as they wait longer in search of a gap. Specifically, the critical gap is found to decay in a linear fashion as the wait time increases. Furthermore, the study results demonstrate that rain intensity results in a linear increase in the critical gap as the rain intensity increases. It should be noted that the results and models were developed using data from a single intersection. Consequently, further data collection and validation is required. Finally, these findings have implications on the design of traffic signal timings that include permissive left turn movements. Using the study findings inclement weather signal timings can be implemented within traffic signal controllers that account for the reduction in opposed saturation flow rates. Furthermore, these results can also be integrated within emerging IntelliDrive systems.

## **4.12 Acknowledgements**

The authors acknowledge the valuable input from Dr. Roemer Alfelor and Dr. David Yang of the Federal Highway Administration, Daniel Krechmer of Cambridge Systematics, Inc., and Dr. Feng Guo in the Statistics Dept. at Virginia Tech. Finally, the authors also acknowledge the financial support provided by the FHWA in conducting this research effort.

## CHAPTER 5 : STOCHASTIC MODELING APPROACH

Rakha H., S. Sadek and I. Zohdy, "Modeling Stochastic Left-Turn Gap Acceptance Behavior," accepted for presentation at the TRB 89<sup>th</sup> Annual Meeting.

### 5.1 Abstract

A Bayesian and Bootstrap logistic left-turn gap acceptance model is developed using 2,730 field observations (301 accepted and 2,429 rejected gaps). The variables that are considered in the model include the gap duration; the driver's wait time in search of an appropriate acceptable gap; the time traveled by a driver to clear the conflict point; and the rain intensity. The model demonstrates that the acceptable time gap decreases as a function of the driver's wait time and increases as the rain intensity increases. The Bayesian and Bootstrap approaches are demonstrated to estimate consistent model parameters. A procedure for modeling the Bayesian realizations that captures parameter correlations without the need to store all parameter combinations is developed. The proposed procedure is demonstrated to produce results that are consistent with the use of the Bayesian realizations. The study then demonstrates how the model produces stochastic realizations of opposed saturation flow rates using a Monte Carlo simulation example application.

### 5.2 Introduction

Gap acceptance is defined as the process that occurs when a traffic stream (known as the opposed flow) has to either cross or merge with another traffic stream (known as the opposing flow). Examples of gap acceptance behavior occur when vehicles on a minor approach cross a major street at a two-way stop controlled intersection, when vehicles make a left turn through an opposing through movement at a signalized intersection, or when vehicles merge onto a freeway. This paper focuses on crossing gap acceptance behavior for permissive left turns.

A gap is defined as the elapsed-time interval between arrivals of successive vehicles in the opposing flow at a specified reference point in the intersection area. The minimum gap that a driver is willing to accept is generally called the critical gap. The Highway Capacity Manual (HCM) (2000) [1] defines the critical gap as the "*minimum time interval between the front bumpers of two successive vehicles in the major traffic stream that will allow the entry of one*

*minor-street vehicle.*” When more than a single opposed vehicle uses a gap, the time headway between successive opposed vehicles is defined as the follow-up time. In general, the follow-up time is shorter than the critical gap and equals the inverse of the unopposed saturation flow rate. Since the critical gap of a driver cannot be measured directly, censored observations (i.e., accepted and rejected gaps) are used to compute critical gaps, as will be described later. For more than three decades research efforts have attempted to model driver gap acceptance behavior, using either deterministic or probabilistic methods. The deterministic critical values are treated as a single threshold for accepting or rejecting gaps. Examples of deterministic methods include the Raff’s [10, 14, 29] and Greenshield’s [15, 51] methods. The stochastic or probabilistic approach to modeling gap acceptance behavior involves constructing either a logit [6] or probit model [7, 40] using some maximum likelihood calibration technique. The fundamental assumption is that drivers will accept all gaps that are larger than the critical gap and reject all smaller gaps. Although various researchers have used different definitions for the critical gap, the deterministic model has been the conventional approach of gap acceptance studies. As an alternative, probabilistic models solve some of the inconsistency elements in gap acceptance behavior by using a statistical treatment of opposed driver gap acceptance behavior. In other words a driver’s perception of a minimum acceptable gap is treated as a random variable [13].

In the case of permissive left-turn maneuvers, the HCM estimates the opposed saturation flow rate based on the critical and follow-up gap durations. The critical gap accepted by left-turn drivers is 4.5 s, according to the HCM, and the average follow-up time between continuous left-turn vehicles is 2.5 s at signalized intersections with a permitted left-turn phase. These values are independent of the number of opposing-through lanes to be crossed by the opposed vehicles. The American Association of State Highway and Transportation Officials (AASHTO, 2001) [27], classifies the left turning movements from the major road across opposing traffic as Case *F*. The AASHTO (2001) recommends that critical gap for left-turning passenger cars be set equal to 5.5 s and for left-turning vehicles that cross more than one opposing lane to add an additional 0.5 s for each additional lane of travel.

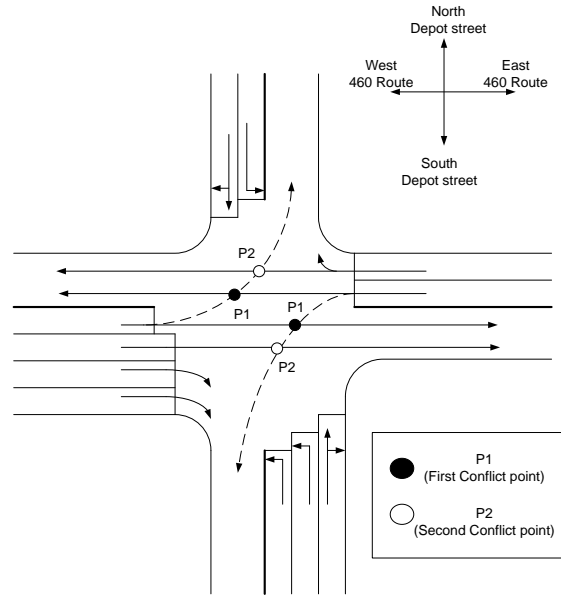
### **5.3 Study Objectives and Paper Layout**

The objectives of this study are: (a) to develop two stochastic approaches (boot strapping and Bayesian) for the modeling of driver gap acceptance behavior; (b) to compare the stochastic modeling behavior; and (c) to study the impact of stochastic gap acceptance behavior on the opposed saturation flow rate.

In terms of the paper layout initially the data gathering procedures and a description of the data is provided followed by a discussion of the boot strapping approach to modeling gap acceptance behavior. Subsequently, a Bayesian approach for modeling driver gap acceptance behavior is presented followed by a comparison of the two approaches. The modeling of stochastic opposed saturation flow rate is then discussed using the proposed stochastic gap acceptance modeling framework. Finally, the conclusions of the paper are presented together with recommendations for further research.

### **5.4 Data Gathering and Extraction**

The study site that was considered in this study was the signalized intersection of Depot Street and North Franklin Street (Business Route 460) in Christiansburg, Virginia, as illustrated in Figure 4.1. The intersection consisted of four approaches at approximately 90° angles. The posted speed limit for the eastbound and northbound approaches was 35 mi/h and for the westbound and southbound approaches was 25 mi/h. The signal phasing of the intersection included three phases, two phases for the Depot street North and South (one phase for each approach) and one phase for the North Franklin Street (two approaches discharging during the same phase) with permissive left turn movements. The entering average annual daily traffic (AADT) for this intersection was 26,671 veh/day and there was an average of 11 accidents per year reported at this intersection.



**Figure 5.1: Study Intersection**

The intersection was equipped with an infrastructure-based radar and digital video data collection system that measured a variety of state and kinematic information (such as vehicle acceleration and velocity). The video cameras were mounted on each of the four traffic signal mast arms to provide an image of the entire intersection environment at a frequency of 20 Hertz. In addition, a weather station was installed on one of the signal masts that provided weather information each minute. The collected weather data included long-term rain fall, daily rain fall, wind direction, wind speed, average wind speed, temperature, barometric pressure, and humidity level.

The video data were reduced manually by recording the time instant at which a subject vehicle initiated its search to make a left turn maneuver, the time step at which the vehicle made its first move to execute its left turn maneuver, and the time the left turning vehicle reached each of the conflict points. In addition, the time stamps at which each of the opposing vehicles passed the conflict points were identified. The final dataset that was analyzed consisted of a total of 2,730 gaps of which 301 were accepted and 2,429 were rejected. These 2,730 observations included 2,017 observations for dry conditions and 713 observations for different rain intensity levels, as summarized in Table 4.1.

**Table 5.1 : Total Observations Breakdown**

Rain Intensity (cm/hr)	Total Accepted Gaps		Total Rejected Gaps		Sum
	First lane	Second lane	First lane	Second lane	
0.000	70	159	509	1279	2017
0.254	11	30	93	268	402
0.508	5	8	40	108	161
0.762	2	6	6	24	38
1.016	2	0	5	27	34
1.270	1	1	1	3	6
1.524	1	0	1	6	8
2.032	1	2	13	19	35
2.286	0	2	5	17	24
7.366	0	0	1	1	2
9.400	0	0	1	2	3
<b>Sum</b>	93	208	675	1754	2730

### 5.5 Model Parameter Estimation using Bootstrapping(BS)

Given that the driver response is a discrete variable (reject or accept) while the independent variables are continuous, a logistic model was fit to the data. Three multivariate models were evaluated and compared. The final model that was selected was of the form

$$\text{logit}(p) = \beta_0 + \beta_1(g - \tau_t) + \beta_2(w) + \beta_3(r). \quad (22)$$

Where  $\text{logit}(p)$  equals  $\ln(p/(1-p))$ ;  $p$  is probability of accepting a gap;  $g$  is the gap size offered to the opposed vehicle (s);  $w$  is the duration of time that the driver waits in search of an acceptable gap (s);  $r$  is the rain intensity (cm/h); and  $\tau_t$  is the median travel time to the conflict point (2.3 s in the case of the first conflict point and 3.5 s for the second). In calibrating the model to the field data using a generalized linear model (GLM) the model coefficients ( $\beta_0$ ,  $\beta_1$ ,  $\beta_2$ , and  $\beta_3$ ) were estimated at -3.677, 0.771, 0.033, -0.623, respectively. The 95 percent confidence limits were estimated at (-4.011, -3.367), (0.698, 0.850), (0.014, 0.053), and (-1.167, -0.217), respectively.

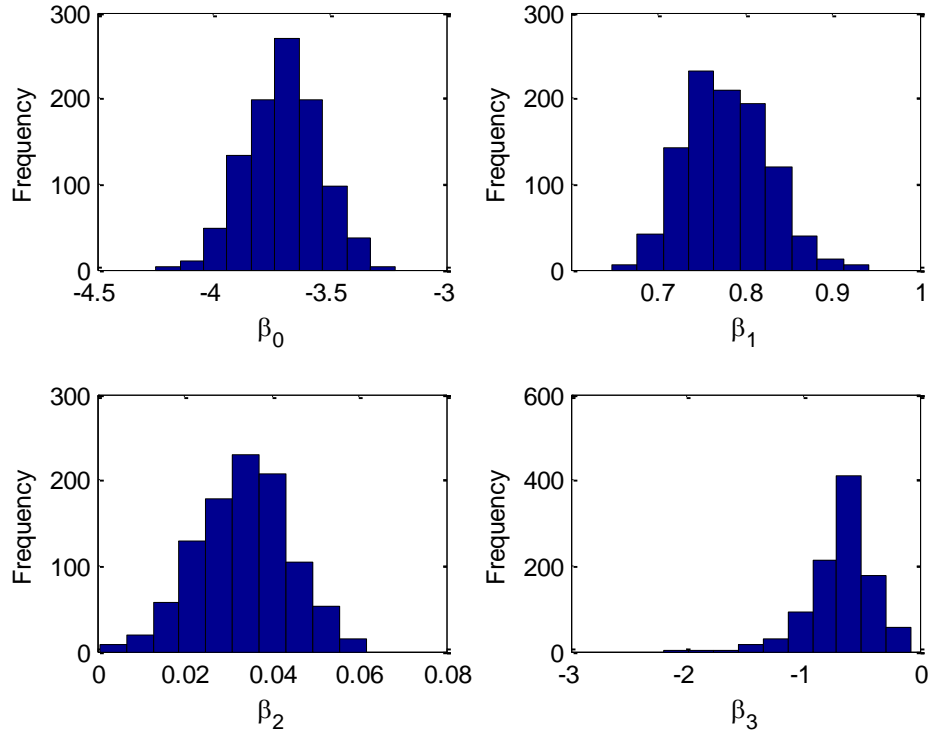
Based on Equation (22), the critical gap can then be computed by setting the probability of accepting a gap to 0.5 which results in a logit function that equals zero. Consequently the critical gap ( $t_c$ ) can be computed as

$$t_c = \tau_t - \frac{\beta_0}{\beta_1} - \frac{\beta_2}{\beta_1} w - \frac{\beta_3}{\beta_1} r = \alpha_0 + \alpha_1 w + \alpha_2 r. \quad (23)$$

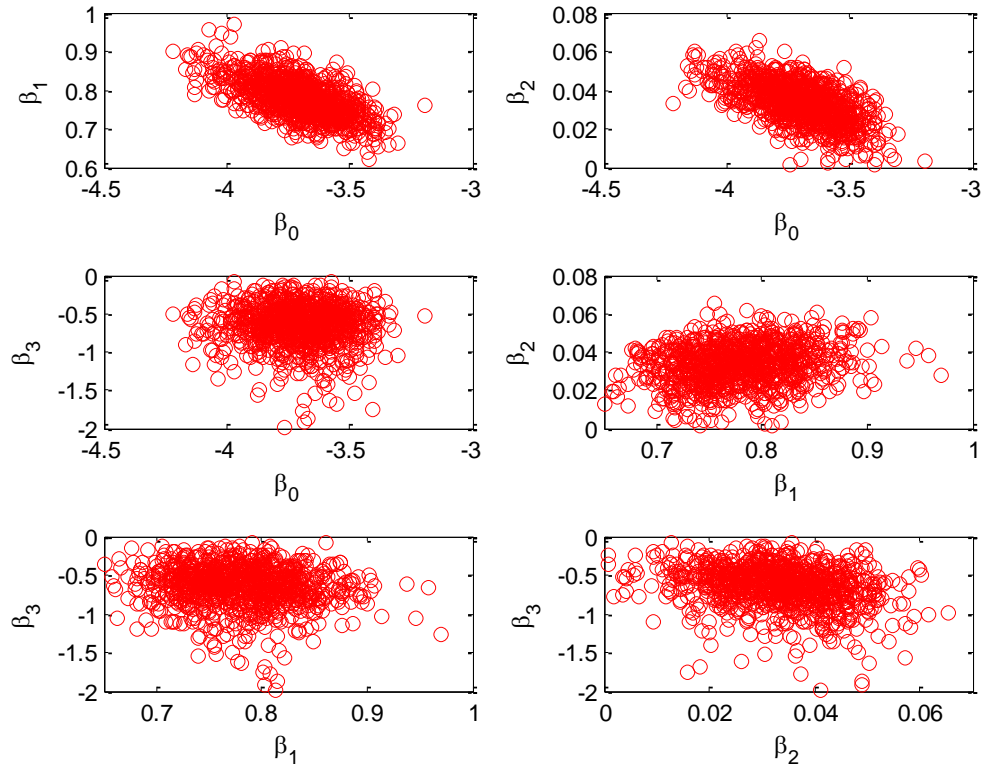
By applying the calibrated logit model coefficients ( $\beta_0$ ,  $\beta_1$ ,  $\beta_2$ , and  $\beta_3$ ) to Equation (23) the critical gap coefficients ( $\alpha_0$ ,  $\alpha_1$ , and  $\alpha_2$ ) are computed to be 7.07, -0.04, and 0.81 for the first conflict point and 8.27, -0.04, 0.81 for the second conflict point, respectively. This implies that the critical gap decreases as the driver waits longer in search of an acceptable gap (i.e. drivers become more aggressive as they wait longer). Alternatively, the critical gap increases as the rain intensity increases (i.e. drivers become less aggressive as the rain intensity increases).

The model coefficients were also calibrated using a Bootstrap approach in order to generate stochastic estimates of the model coefficients. This was achieved by re-sampling the observations with repetitions and estimating the model parameters for each sample. Specifically, the observed gaps were randomly sampled by generating  $N$  uniformly distributed random numbers between 0 and  $N$ , where  $N$  is the total number of observations. These random numbers were then rounded to the nearest integer to identify the observations to be included in the sample. Each sample was used to calibrate four model parameters using Equation (22). The calibration, which was conducted in Matlab using the GLM procedure, was repeated 50,000 times. The calibrated model parameter distributions were then generated. Figure 5.2 illustrates the parameter distributions considering a sample size of 1,000 realizations in order to be consistent with the results of the following section. Noteworthy is the fact that in some rare instances the wait time coefficient ( $\beta_2$ ) was less than zero, indicating that for these realizations the model critical gap increases as the driver wait time increases. These instances represent unrealistic realizations of the model parameters.

Figure 5.3 also shows the inter-dependence of the model parameters. In each subplot the relationship between two parameters can be established. It is clear that a relationship between  $\beta_1$  and  $\beta_0$  and between  $\beta_2$  and  $\beta_0$  can be deduced. However, there does not appear to be a relationship between  $\beta_3$  and  $\beta_0$ .



**Figure 5.2: Model Coefficient Distributions using Bootstrapping**



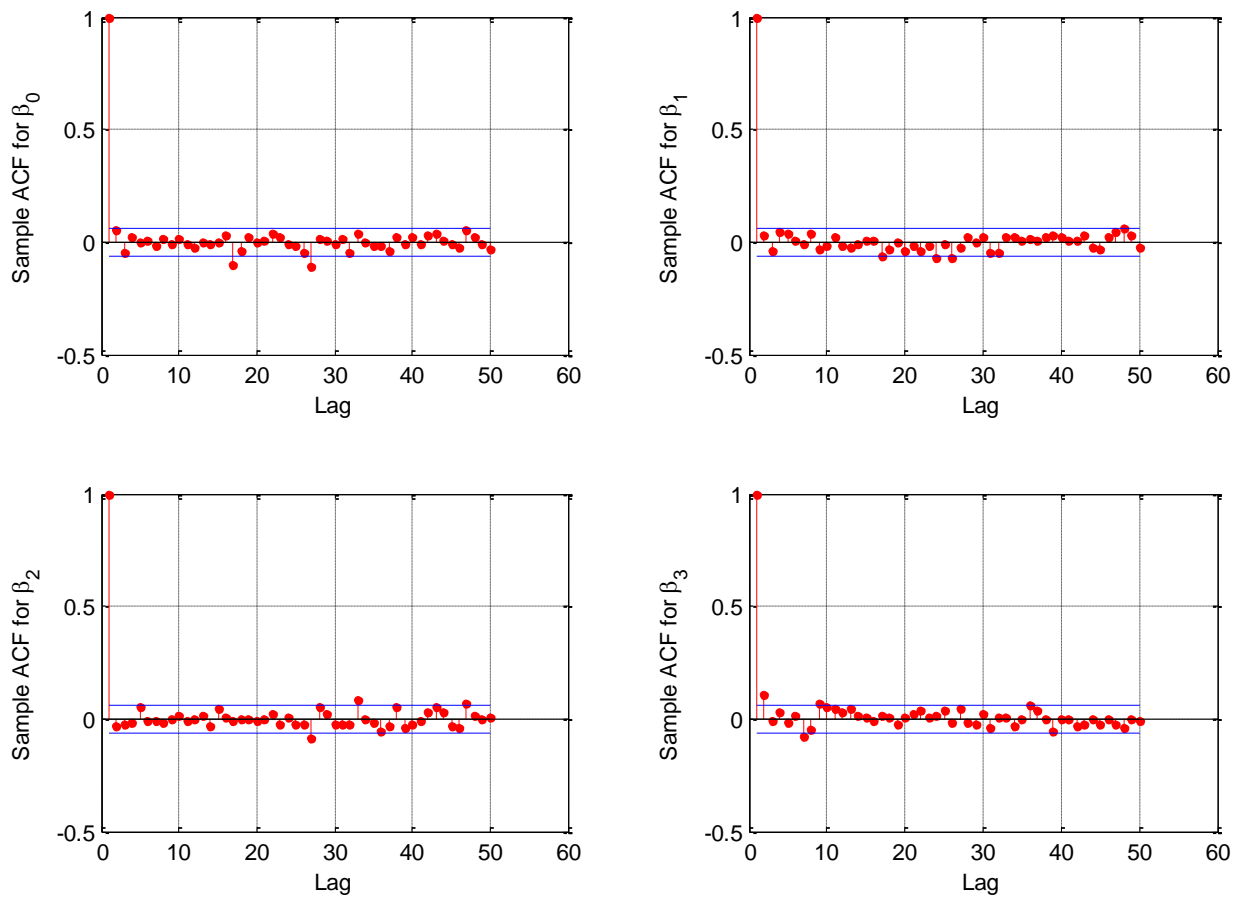
**Figure 5.3 Inter-dependence of Model Parameters using Bootstrapping**

## 5.6 Estimation of Model Parameters using Bayesian Statistics

Statistical inferences are usually based on maximum likelihood estimation (MLE). MLE selects the parameters that maximize some likelihood function. In MLE, parameters are assumed to be unknown and fixed. In Bayesian statistics, the uncertainty about the unknown parameters is quantified so that the unknown parameters are regarded as random variables. Bayesian estimates of the model parameters, along with their posterior distributions, were estimated using the Markov Chain Monte Carlo (MCMC) slice algorithm implemented within the Matlab software. Monte Carlo methods are often used in Bayesian data analysis to generate the posterior distributions when these distributions cannot be generated analytically. The approach generates random samples from these distributions to estimate the posterior distribution or derived statistics (e.g. mean, median, and standard deviation). The slice sampling algorithm that is implemented within Matlab is an algorithm designed to sample from a distribution with an arbitrary density function, known only up to a constant of proportionality – exactly what is needed for sampling from a complicated posterior distribution whose normalization constant is unknown. The algorithm does not generate independent samples, but rather a Markovian sequence whose stationary distribution is the target distribution. This algorithm differs from other well-known MCMC algorithms because only the scaled posterior need to be specified – no proposal or marginal distributions are needed.

The MCMC slice algorithm requires an initial solution for the model parameters, the prior distribution, the number of samples to be generated, a burn-in rate, and a thinning ratio. The initial solution was assumed to be the solution generated using the GLM logistic model. The prior distributions for the model parameters were assumed to be “non-informative” normal distributions with means equal to 0 and variances equal to  $10^6$ . The number of samples was set at 1000, the burn-in rate was set at 100, and the thinning ratio was set at 40. These parameters translate into a total sample size of 44,000 simulations, including a burn-in sample size of 4,000 observations. Of the remaining 40,000 sample simulations 1,000 are selected using a thinning rate of 40 (i.e. selecting 1 in every 40 simulations). Inspection of the autocorrelation functions for each of the four coefficients demonstrates that the sample size is sufficient for convergence, as illustrated in Figure 5.4. The solid horizontal lines represent the 95% confidence limits while the

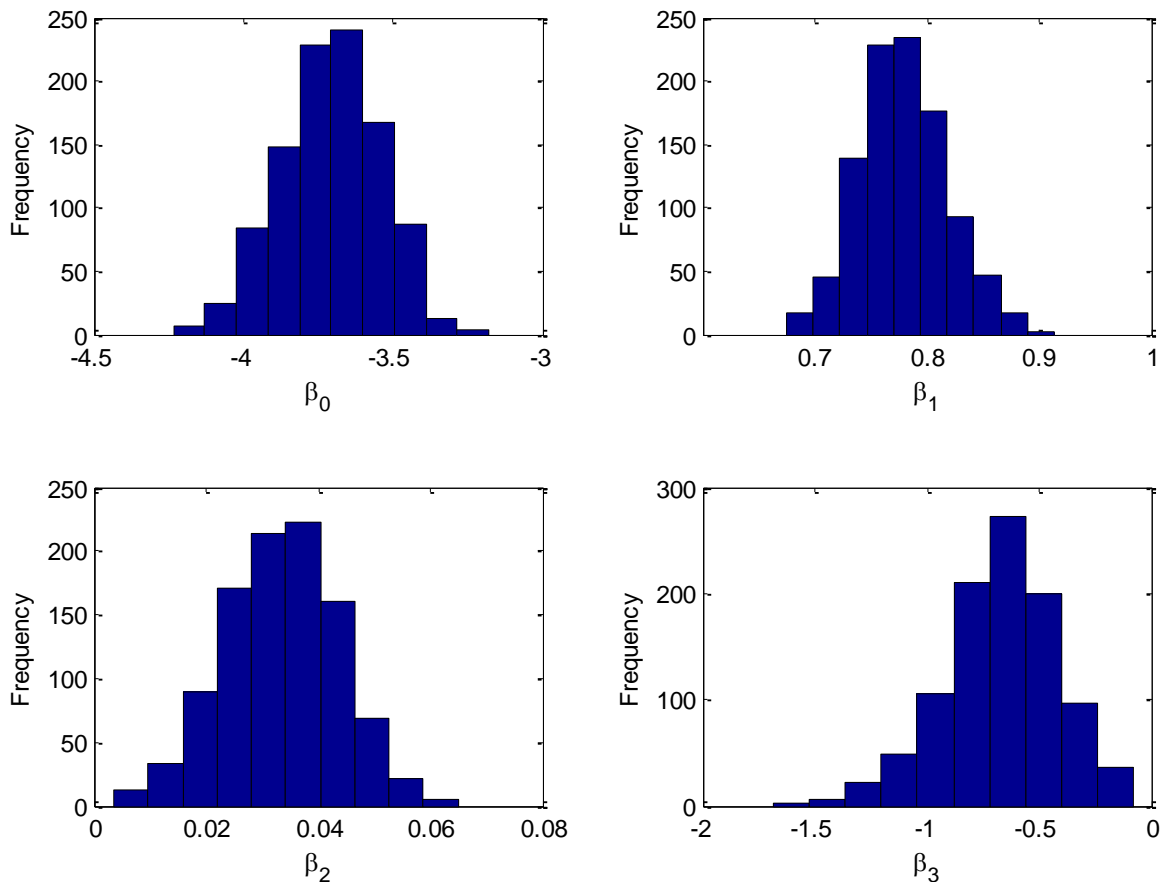
stem lines represent the autocorrelation function. As would be expected an autocorrelation of 1.0 is observed for a lag of 0 while the values are within the confidence limits for larger lag times.



**Figure 5.4: Model Parameter Autocorrelation Function Variation**

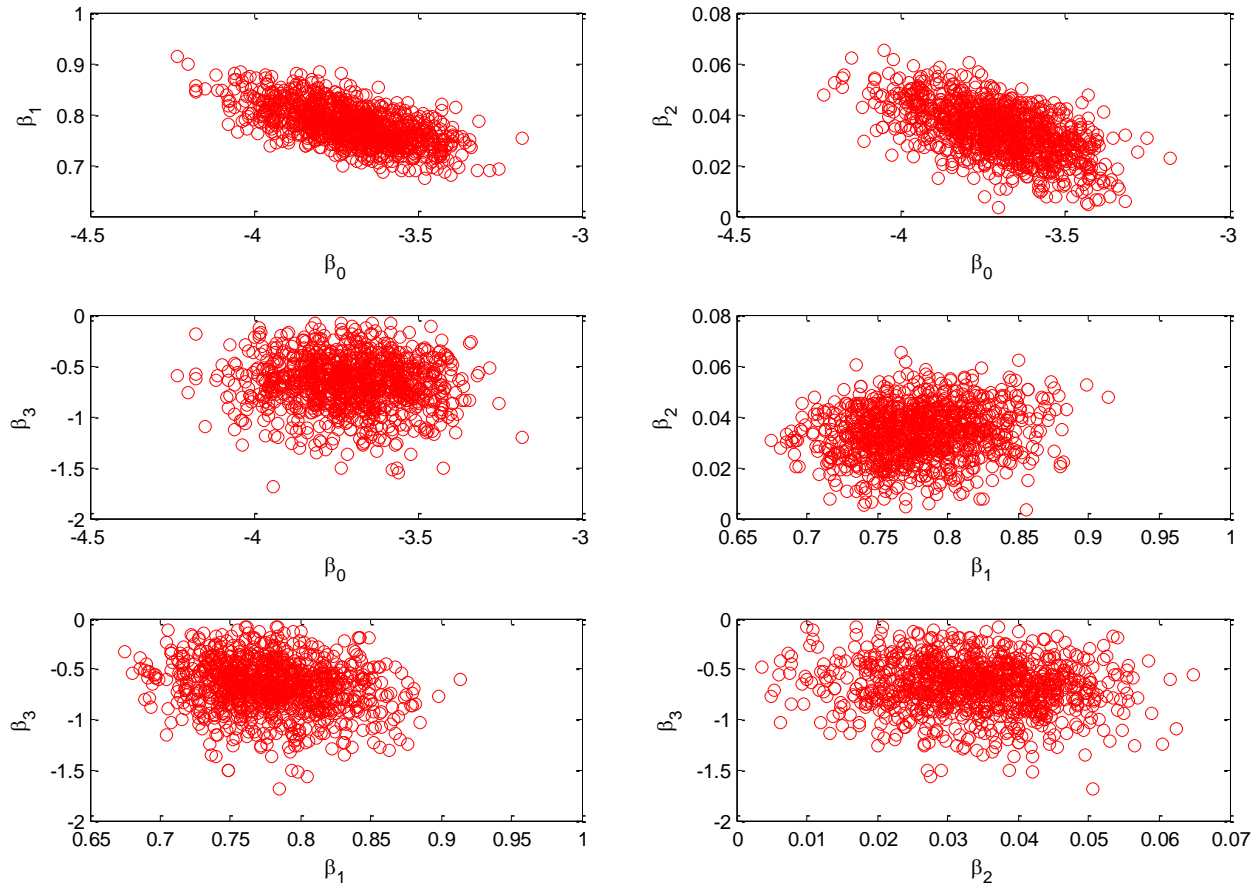
A Kolmogorov-Smirnov goodness of fit test concluded that there was insufficient evidence to reject the null hypothesis that the model parameter distributions were different from the normal distribution at a level of significance of 0.05, as illustrated in Figure 5.5. A close analysis of the figure demonstrates that the distribution of the rain intensity parameter,  $\beta_3$ , is slightly left skewed while the gap parameter,  $\beta_1$ , appears to be slightly right skewed. The model parameters are consistently negative in the case of the  $\beta_0$  and  $\beta_3$  parameters while they are consistently positive in the case of the  $\beta_1$  and  $\beta_4$  parameters. Consequently, the parameter distributions are logical in terms of their signs (i.e. consistently negative for  $\beta_0$  and  $\beta_3$  parameters and consistently positive

for  $\beta_1$  and  $\beta_2$  parameters). These parameter distributions provide better estimates of the model coefficients when compared to the Bootstrap approach. Specifically, the distribution of the  $\beta_1$  and  $\beta_3$  parameters are less skewed compared to the Bootstrap coefficient estimates.



**Figure 5.5: Model Parameter Distributions**

As was done with the Bootstrap analysis, Figure 5.6 illustrates the interaction between the four model coefficients ( $\beta_0$ ,  $\beta_1$ ,  $\beta_2$ , and  $\beta_3$ ) as generated from the Bayesian approach. As was the case with the Bootstrap analysis there appears to be a linear relationship between  $\beta_1$  and  $\beta_0$  and between  $\beta_2$  and  $\beta_0$ . There also appears to be some evidence for a linear relationship between  $\beta_3$  and  $\beta_2$ . In comparing Figure 5.6 to Figure 5.3 it appears that the level of noise generated using the Bayesian approach is less than that using the Bootstrap approach. This could be attributed to the fact that the Bayesian approach used a thinning process that ensured that there was no autocorrelation between model parameters.



**Figure 5.6: Inter-dependence of Bayesian Model Parameters**

## 5.7 Comparison of Bootstrapping and Bayesian Approaches

This section presents a limited comparison of parameter statistics generated using the Bootstrap and Bayesian approaches. The comparison is made in terms of parameter means ( $\mu$ ), parameter standard deviation ( $\sigma$ ), and Kurtosis which is a measure of the distribution skewness ( $K$ ) as well as their respective probability and cumulative distributions.

Table 5.2 summarizes the mean, standard deviation, and Kurtosis parameter estimates for all model parameters. The first four rows in the table provide the statistics for different Bayesian runs, while the last row shows the statistics obtained from Bootstrapping. The scenarios include: (a) Uniform, where the prior is a uniform distribution ranging from -100 to 100 (i.e. non-informative); (b) Normal 1, where the prior is  $N(0, 1000)$ ; (c) Normal 2, where the prior is

$N(\mu_{BS}, \sigma_{BS})$ ; (d) Normal 3, where the prior is  $N(\mu_{BS}, \sigma_{BS})$ ,  $n_{\text{samples}}= 10,000$  and  $n_{\text{thin}}=1$ ; and (e) Bootstrap, is a Bootstrap approach.

**Table 5.2: Parameter Statistics Comparison**

	$\mu$				$\sigma$				K			
	$\beta_0$	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_0$	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_0$	$\beta_1$	$\beta_2$	$\beta_3$
<b>Uniform</b>	-3.709	0.7799	0.0337	-0.682	0.1689	0.0390	0.0100	0.2530	-0.087	0.205	-0.021	-0.463
<b>Normal 1</b>	-3.706	0.7792	0.0335	-0.671	0.1649	0.0390	0.0102	0.2526	-0.129	0.241	-0.107	-0.419
<b>Normal 2</b>	-3.704	0.7791	0.0335	-0.657	0.1021	0.0275	0.0069	0.1798	-0.086	-0.077	-0.045	-0.045
<b>Normal 3</b>	-3.706	0.7794	0.0337	-0.669	0.1015	0.0275	0.0068	0.1765	-0.115	0.0679	-0.039	-0.191
<b>Bootstrap</b>	-3.710	0.7796	0.0336	-0.665	0.1554	0.0483	0.0106	0.2733	-0.202	0.225	-0.237	-1.127

In general both techniques result in comparable results for the coefficient  $\beta_0$ ,  $\beta_1$ , and  $\beta_2$ ; however, the  $\beta_3$  estimates are marginally different. An important observation is that the Bayesian analysis converges approximately to the same values no matter what the prior distribution is except for the  $\beta_3$  parameter. The Bootstrapping distributions are more skewed especially for the  $\beta_3$  parameter.

A comparison of the parameter density functions showed similar distributions were obtained from both techniques for parameters  $\beta_0$  and  $\beta_1$ . However, differences were observed for the  $\beta_2$  and  $\beta_3$  parameters. A Kolmogorov-Smirnov test was applied to compare the parameter probability density functions for the various parameters. The test results showed that for the first three coefficients there was insufficient evidence to reject the hypothesis that the parameters were from the same distribution at a level of significance of 5 percent except the last two (Normal 3 and Bootstrap approaches). The distribution obtained from Bootstrapping for  $\beta_3$  is different from all others.

It should also be noted that in the case of the  $\beta_2$  parameter the Bayesian approach produces a normally distributed function with a mean of 0.0337 and a standard deviation of 0.0102. Consequently all  $\beta_2$  parameter realizations are positive. Similarly, the Bootstrap approach has a 0.0% probability of producing negative  $\beta_2$  parameter estimates given that the  $\beta_2$  parameter mean is 0.0336 with a standard deviation of 0.0106. All model parameter estimates are close to the GLM model estimates of -3.677, 0.771, 0.033, and -0.623, respectively regardless of what the prior distribution is. In summary, the results suggest that a Bayesian approach for estimating model parameters is consistent a Bootstrap approach, however it does produce less variability in the model parameter estimates. Consequently, the remainder of the paper considers the sole use of a Bayesian approach.

## 5.8 Model Application

In modeling driver gap acceptance behavior realizations of the  $\beta_i$  parameters cannot be generated independently; instead the correlations between these parameters must be captured. Figure 5.6 demonstrates that there are definite correlations between the  $\beta_1$  and  $\beta_0$  and  $\beta_2$  and  $\beta_0$  parameters. The modeling of parameter interactions can be achieved by either storing the set of parameter realizations from the Bayesian model, which is computationally intensive. Alternatively, the interaction can be captured through the use of joint probability functions; however, this is typically difficult to implement given that the joint probability is a five dimensional function when one considers the four parameters. Consequently, a novice approach was adopted that involves generating a normally distributed random number for the  $\beta_0$  parameter as

$$\beta_0 \sim N(\mu_{\beta_0}, \sigma_{\beta_0}). \quad (24)$$

A regression model is then fit between the  $\beta_1$  and  $\beta_0$  parameters with a normally distributed error term ( $e_1$ ) as

$$\beta_1 = a_{\beta_1} + b_{\beta_1} \beta_0 + e_1 \quad \text{where } e_1 \sim N(0, \sigma_{e_1}). \quad (25)$$

The dependency of  $\beta_2$  on  $\beta_0$  and  $\beta_1$  can be captured by fitting a regression model of the form

$$\beta_2 = a_{\beta_2} + b_{\beta_2} \beta_0 + c_{\beta_2} \beta_1 + e_2 = (a_{\beta_2} + a_{\beta_1} c_{\beta_2}) + (b_{\beta_2} + b_{\beta_1} c_{\beta_2}) \beta_0 + (c_{\beta_2} e_1 + e_2). \quad (26)$$

This relationship can be reduced to a relationship between  $\beta_2$  on  $\beta_0$  using the relationship between  $\beta_1$  and  $\beta_0$  that was established in Equation (25). The error term remains normally distributed given that it is the summation of two normally distributed variables with zero mean. The final model is cast as

$$\beta_2 = a'_{\beta_2} + b'_{\beta_2} \beta_0 + e'_2 \quad \text{where } e'_2 \sim N(0, \sigma_{e'_2}). \quad (27)$$

Similarly the relationship between  $\beta_3$  and the other variables can be reduced to a relationship between  $\beta_3$  and  $\beta_0$  as

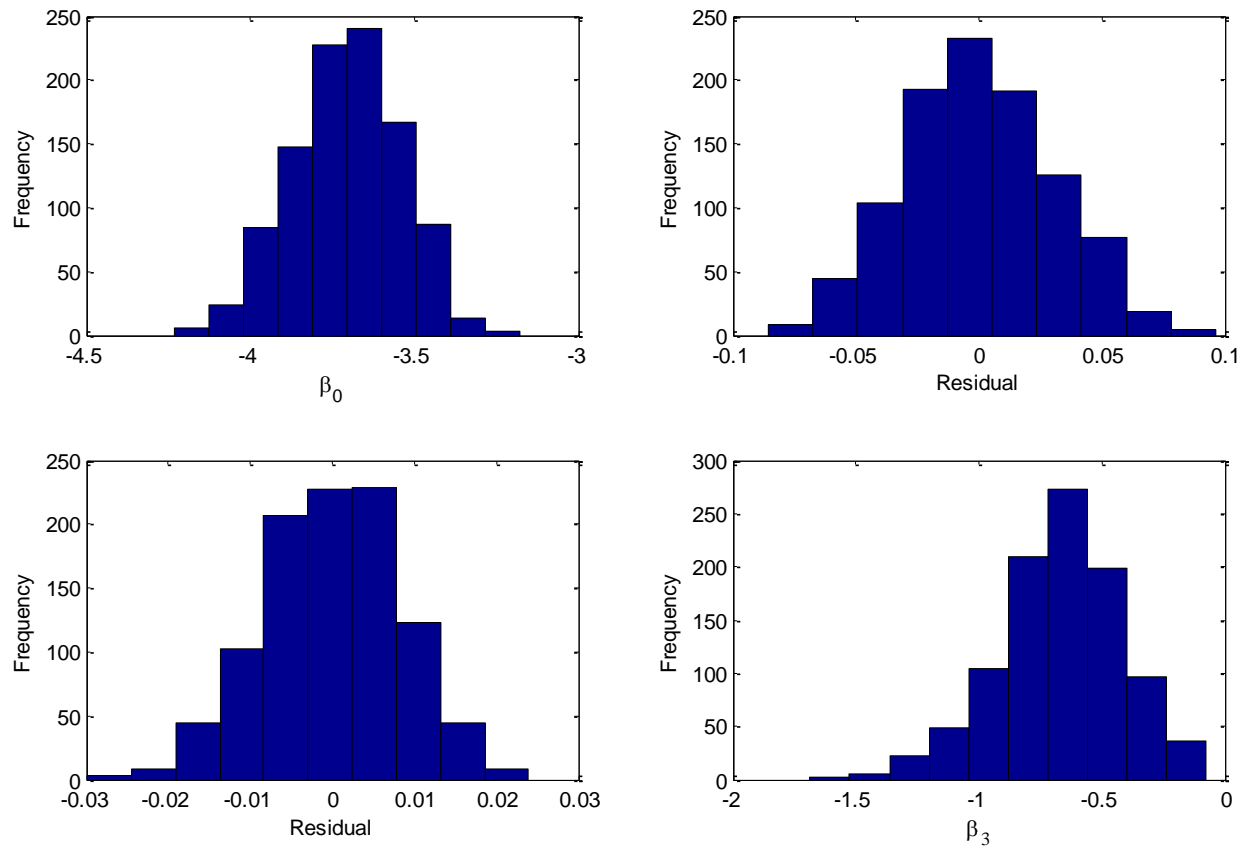
$$\beta_3 = a_{\beta_3} + b_{\beta_3} \beta_0 + c_{\beta_3} \beta_1 + d_{\beta_3} \beta_2 + e_3 = a'_{\beta_3} + b'_{\beta_3} \beta_0 + e'_3 \quad \text{where } e'_3 \sim N(0, \sigma_{e'_3}). \quad (28)$$

Where  $a_x$ ,  $b_x$ ,  $c_x$ , and  $d_x$  are regression model parameters for variable  $x$ . The application of this approach to the model parameters concluded that there was no linear relationship between  $\beta_3$  on  $\beta_0$ , as demonstrated in Table 5.3. Consequently, the final model requires the calibration of a total of 12 parameters.

**Table 5.3: Regression Model Coefficients**

	$\alpha_0$	$\alpha_1$	$\mu$	$\sigma$
$\beta_0$			-3.706	0.1649
$\beta_1$	0.2384	-0.1459	0.000	0.0307
$\beta_2$	-0.0986	-0.0356	0.000	0.0084
$\beta_3$			-0.671	0.2526

The four normal distributions for the  $\beta_0$ ,  $e_1$ ,  $\beta_3$ , and  $e_3$  parameters are illustrated in Figure 5.7. A Kolmogrov-Smirnov goodness of fit test concluded that there was insufficient evidence to reject the null hypothesis that the distributions were from the normal distribution at a level of significance of 0.05.



**Figure 5.7: Model Parameter and Residual Error Distributions**

The model parameter credible intervals were computed, as demonstrated in Table 5.4. In addition the credible interval for the critical gap for the 1-lane and 2-lane movements are also computed. If we consider the Bayesian credible intervals are treated as frequentist confidence intervals then it is evident that the critical gaps are statistically different. This difference is approximately 1.1 s, which is equal to the difference in travel times to traverse the two conflict points.

**Table 5.4: Model Parameter Credible Intervals**

	Q <sub>0.025</sub>	Mean	Q <sub>0.975</sub>
$\beta_0$	-4.034	-3.706	-3.406
$\beta_1$	0.708	0.779	0.860
$\beta_2$	0.012	0.034	0.053
$\beta_3$	-1.234	-0.671	-0.197
$t_{c,P1}$	6.733	7.111	7.508
$t_{c,P2}$	7.833	8.211	8.608

The proposed procedure for modeling stochastic driver gap acceptance behavior without the need to store the set of  $\beta_i$  realizations can be summarized as follows:

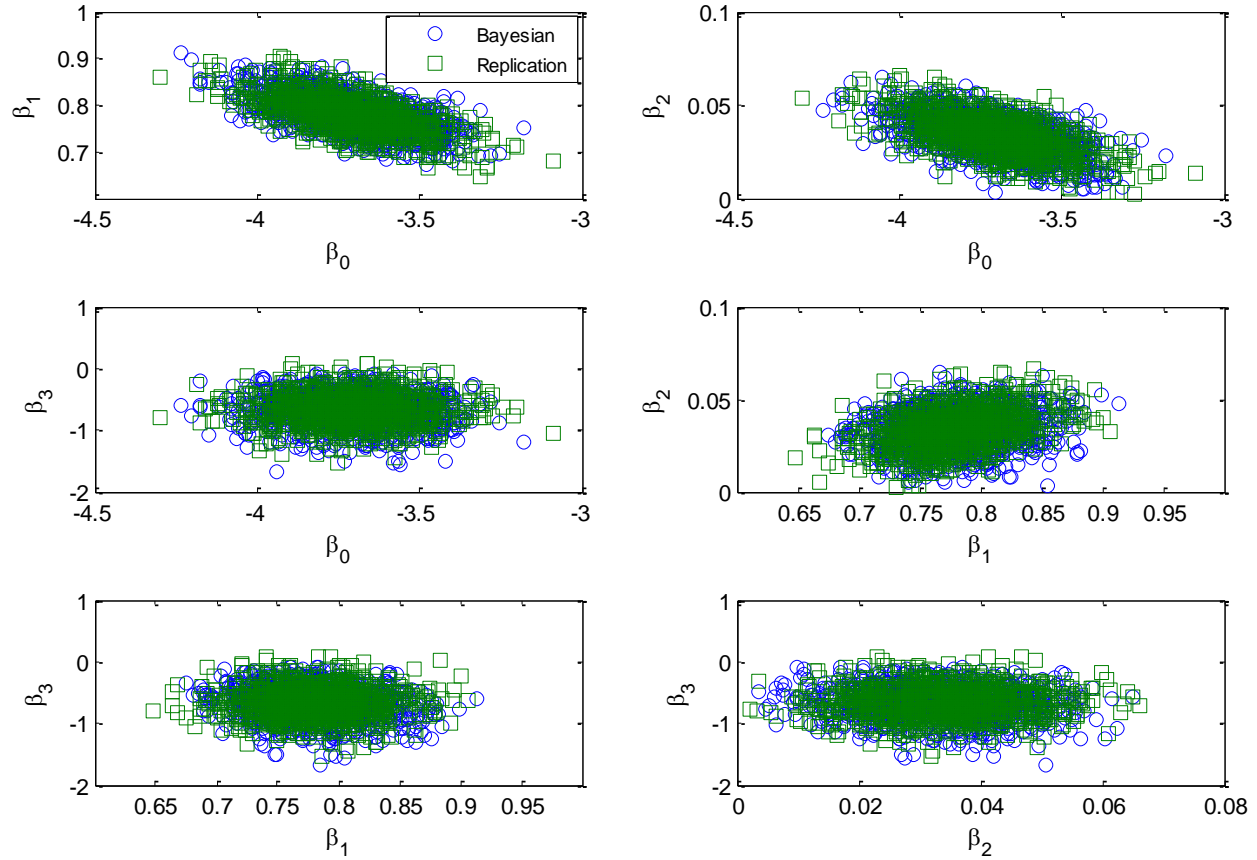
Generate the  $\beta_0$  coefficient from a normal distribution  $N(\mu_0, \sigma_0)$ .

Compute the  $\beta_1$  coefficient using Equation (25) with the coefficients of Table 5.3.

Compute the  $\beta_2$  coefficient using Equation (27) with the coefficients of Table 5.3.

Compute the  $\beta_3$  coefficient using Equation (28) with the coefficients of Table 5.3.

In order to validate the proposed procedure, the procedure was repeated 1000 times and compared to the coefficient combination realizations that were derived from the Bayesian approach, as illustrated in Figure 5.8. It is clear that the replication replicates the parameter interactions in a similar fashion to the initial data.



**Figure 5.8: Comparison between Bayesian and Generated Parameter Inter-relations.**

Table 5.5 also shows a comparison between the predicted statistics from the Bayesian distributions and the generated ones for all four parameters. The parameter estimates clearly demonstrate the validity of the proposed procedure. This procedure is simple to estimate and only requires the storage of 12 parameters.

**Table 5.5: Comparison of Bayesian and Generated Parameter Distribution Statistics**

	$\mu$				$\sigma$			
	$\beta_0$	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_0$	$\beta_1$	$\beta_2$	$\beta_3$
Bayesian	-3.710	0.779	0.034	-0.671	0.165	0.039	0.0102	0.253
Generated	-3.710	0.780	0.034	-0.666	0.167	0.039	0.0105	0.247

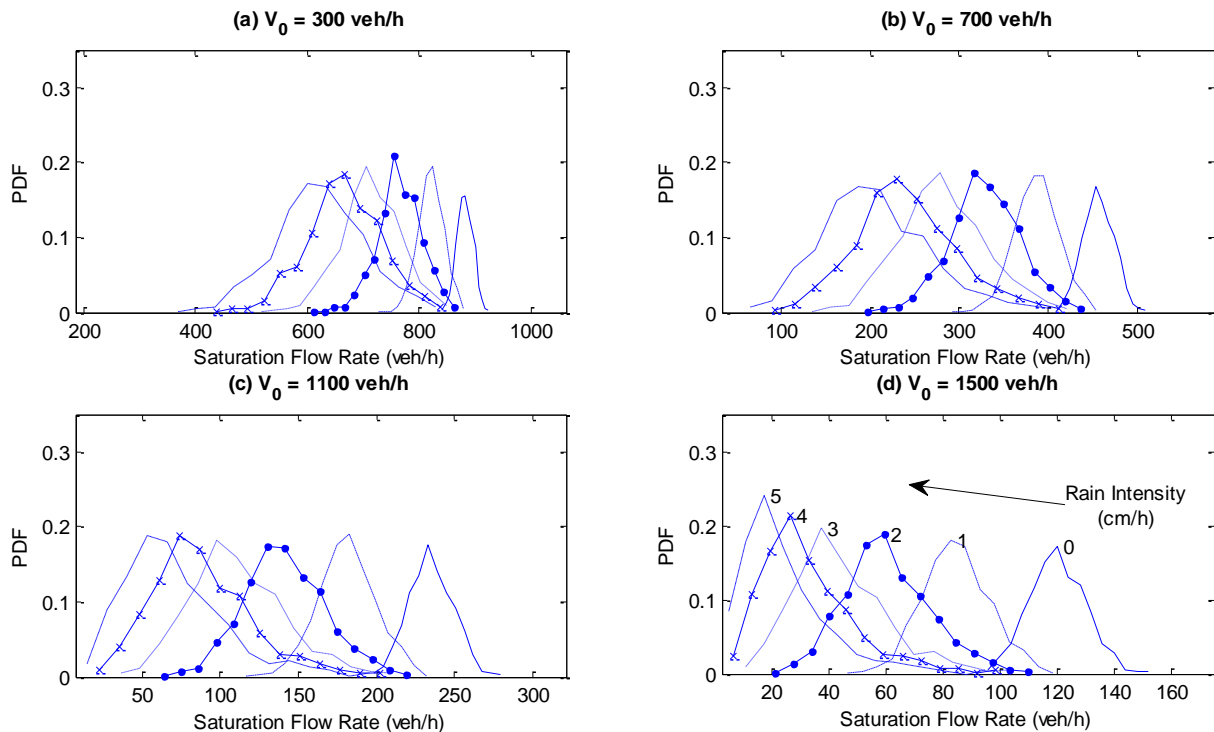
## 5.9 Estimation of Stochastic Saturation Flow Rates

Once the critical gap is computed, the opposed saturation flow rate ( $s$ ) can be computed using Equation (29). Here  $v_0$  is the opposing flow (veh/h),  $t_c$  is the critical gap size (s) which is

computed using Equation (23), and  $t_f$  is the follow-up time (s). The follow-up time is the discharge time headway for the unopposed saturation flow rate (i.e. when the opposing flow is zero).

$$s = v_0 \frac{e^{\frac{-v_0 t_c}{3600}}}{1 - e^{\frac{-v_0 t_f}{3600}}} \quad (29)$$

The critical gap size and corresponding opposed saturation flow rate was computed for various opposing flows by applying a Monte Carlo simulation using 1000 samples of the Bayesian model coefficients, as illustrated in Figure 5.9. The figure demonstrates that the estimated opposed saturation flow rate is stochastic and that the mean saturation flow rate decreases as the opposing flow increases while the saturation flow rate dispersion increases as the opposing flow increases. Similarly, the opposed saturation flow rate decreases as the rain intensity increases and the dispersion in the saturation flow rates increases as the rain intensity increases.



**Figure 5.9: Variation in Saturation Flow Rate as a Function of Opposing Flow and Rain Intensity**

## **5.10 Conclusions**

A Bayesian and Bootstrap logistic left-turn gap acceptance model was developed using 2,730 field observations (301 accepted and 2,429 rejected gaps). The variables that were considered in the model include the gap duration; the driver's wait time in search of an appropriate acceptable gap; the time traveled by a driver to clear the conflict point; and the rain intensity. The Bayesian approach was found to be consistent with Bootstrap approach in modeling stochastic gap acceptance behavior producing logical coefficient values. A procedure for modeling the Bayesian realizations that captures parameter correlations without the need to store all parameter combinations was developed. The proposed procedure was demonstrated to produce parameter realizations consistent with the use of those produced from the Bayesian modeling. A Monte Carlo simulation was executed to study the effect of the proposed model on the opposed saturation flow rate. The results demonstrate that the model produces stochastic saturation flow rates and that the variance of the saturation flow rate increases as the opposed flow and rain intensity increases.

## **5.11 Acknowledgements**

The authors acknowledge the valuable input from Dr. Roemer Alfelor and Dr. David Yang of the Federal Highway Administration, Daniel Krechmer of Cambridge Systematics, Inc., and Dr. Feng Guo in the Statistics Dept. at Virginia Tech. Finally, the authors also acknowledge the financial support provided by the FHWA in conducting this research effort.

## **CHAPTER 6 : PSYCHO-PHYSICAL MODELING APPROACH**

Zohdy I. and H. Rakha, “Modeling Left-Turn Gap Acceptance Behavior: A Psycho-Physical Approach,” to be submitted.

### **6.1 Abstract**

The paper develops a psycho-physical framework for modeling driver left turn gap acceptance behavior. The physical component of the model explicitly captures the vehicle constraints on driving behavior using a vehicle dynamics model, while the psychological component explicitly models the driver’s deliberation in accepting/rejecting a gap. The model is developed using 301 accepted gaps and subsequently validated on 2,429 rejected gaps at the same test site in addition to 1,485 gap decisions (323 accepted and 1,162 rejected) at another site. The model predictions are then compared to a more traditional statistical logit modeling model. The variables that are considered in the logit modeling framework include the gap duration; the driver’s wait time in search of an appropriate acceptable gap; the time traveled by a driver to clear the conflict point; and the rain intensity. The results demonstrate that the psycho-physical model is superior to standard statistical logit model approaches because it produces consistent performance for accepted and rejected gaps (correct predictions of approximately 90%). The model produces consistent results at different sites, and most importantly the model can be generalized to capture different vehicle, roadway, movement, intersection characteristics, and weather effects on driver gap acceptance behavior.

### **6.2 Introduction**

Gap acceptance is defined as the process that occurs when a traffic stream (known as the opposed flow) has to either cross or merge with another traffic stream (known as the opposing flow). Examples of gap acceptance behavior occur when vehicles on a minor approach cross a major street at a two-way stop controlled intersection, when vehicles make a left turn through an opposing through movement at a signalized intersection, or when vehicles merge onto a freeway. This paper focuses on crossing gap acceptance behavior for permissive left turns.

A gap is defined as the elapsed-time interval between arrivals of successive vehicles in the opposing flow at a specified reference point in the intersection area. The minimum gap that a

driver is willing to accept is generally called the critical gap. The Highway Capacity Manual (HCM) (2000) [1] defines the critical gap as the “*minimum time interval between the front bumpers of two successive vehicles in the major traffic stream that will allow the entry of one minor-street vehicle.*” Since the critical gap of a driver cannot be measured directly, censored observations (i.e., accepted and rejected gaps) are used to compute critical gaps, as will be described later. For more than three decades research efforts have attempted to model driver gap acceptance behavior, using either deterministic or probabilistic methods. The deterministic critical values are treated as a single threshold for accepting or rejecting gaps. Examples of deterministic methods include the Raff’s [2-4] and Greenshield’s [15, 51] methods. The stochastic or probabilistic approach to modeling gap acceptance behavior involves constructing either a logit [6] or probit model [7, 40] using some maximum likelihood calibration technique. The fundamental assumption is that drivers will accept all gaps that are larger than the critical gap and reject all smaller gaps. Although various researchers have used different definitions for the critical gap, the deterministic model has been the conventional approach of gap acceptance studies. As an alternative, probabilistic models solve some of the inconsistency elements in gap acceptance behavior by using a statistical treatment of opposed driver gap acceptance behavior. In other words a driver’s perception of a minimum acceptable gap is treated as a random variable [13].

In the case of permissive left-turn maneuvers, the HCM 2000 estimates the opposed saturation flow rate based on the critical and follow-up gap durations. The HCM 2000 considers the critical gap accepted by left-turn drivers as a deterministic value equal to 4.5 s at signalized intersections with a permitted left-turn phase. This value is independent of the number of opposing-through lanes to be crossed by the opposed vehicles. The American Association of State Highway and Transportation Officials (AASHTO, 2001) [27], classifies the left turning movements from the major road across opposing traffic as Case *F*. The AASHTO (2001) recommends that critical gap for left-turning passenger cars be set equal to 5.5 s and for left-turning vehicles that cross more than one opposing lane to add an additional 0.5 s for each additional lane of travel.

The objective of this study is to develop a psycho-physical model of driver gap acceptance behavior. The model is physical because it captures the vehicle constraints on driver gap

acceptance behavior. Alternatively, the model is psychological because it captures the human's psychological deliberation in gap acceptance behavior.

In terms of the paper layout initially the data gathering procedures and a description of the data is provided followed by a discussion of the traditional statistical approach to modeling gap acceptance behavior. Subsequently the constraints of vehicle dynamics on driver gap acceptance behavior are presented followed by a comparison of the two approaches. The application of the psycho-physical model on a different data set is then discussed and the modeling of differences in driver behavior. Finally, the summary conclusions of the paper are presented.

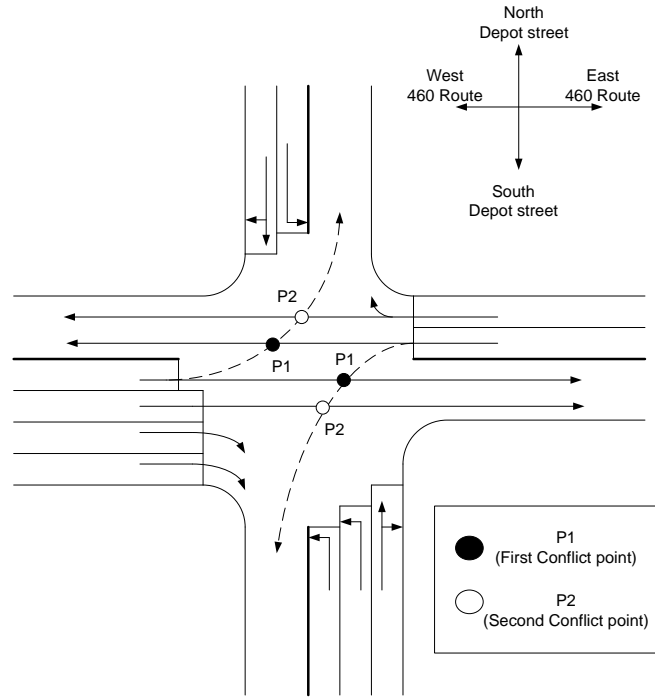
### **6.3 Study Site and Data Extraction**

The site that was considered in this study was the signalized intersection of Depot Street and North Franklin Street (Business Route 460) in Christiansburg, Virginia, as illustrated in Figure 6.1. The intersection consisted of four approaches at approximately 90° angles. The posted speed limit for the eastbound and northbound approaches was 35 mi/h and for the westbound and southbound approaches was 25 mi/h. The signal phasing of the intersection included three phases, two phases for the Depot Street North and South (one phase for each approach) and one phase for the North Franklin Street (two approaches discharging during the same phase) with permissive left turn movements. The entering average annual daily traffic (AADT) for this intersection was 26,671 veh/day and there was an average of 11 accidents per year reported at this intersection.

The intersection was equipped with an infrastructure-based radar and digital video data collection system. The video cameras were mounted on each of the four traffic signal mast arms to provide an image of the entire intersection environment at a frequency of 20 Hertz. In addition, a weather station was installed on one of the signal masts that provided weather information each minute. The collected weather data included long-term rain fall, daily rain fall, wind direction, wind speed, average wind speed, temperature, barometric pressure, and humidity level.

The video data were reduced manually by recording the time instant at which a subject vehicle initiated its search to make a left turn maneuver, the time step at which the vehicle made its first move to execute its left turn maneuver, and the time the left turning vehicle reached each of the conflict points. In addition, the time stamps at which each of the opposing vehicles passed the conflict points were identified. The final dataset that was analyzed consisted of a total of 2,730

gaps of which 301 were accepted and 2,429 were rejected. These 2,730 observations included 2,017 observations for dry conditions and 713 observations for different rain intensity levels (from 0.254 cm/h up to 9.4 cm/h).



**Figure 6.1: Study Intersection**

## 6.4 Traditional Statistical Models

It is common to use statistical models to capture driver gap acceptance behavior. Given that the driver response is a discrete variable (reject or accept) while the independent variables are continuous, a logistic model is typically fit to the data. Many multivariate models were evaluated and compared. The final model that was selected was of the form

$$\text{logit}(p) = \beta_o + \beta_1(g - \tau_t) + \beta_2(w) + \beta_3(r). \quad (30)$$

Where  $\text{logit}(p)$  equals  $\ln(p/(1-p))$ ;  $p$  is probability of accepting a gap;  $g$  is the gap size offered to the opposed vehicle (s);  $w$  is the duration of time that the driver waits in search of an acceptable gap (s);  $r$  is the rain intensity (cm/h); and  $\tau_t$  is the median travel time to the conflict point (2.3 s in the case of the first conflict point P1 and 3.5 s for the second conflict point P2). In calibrating the

model to the field data using a generalized linear model (GLM) the model coefficients ( $\beta_0, \beta_1, \beta_2,$  and  $\beta_3$ ) were estimated at -3.677, 0.771, 0.033 and -0.623, respectively. The 95 percent confidence limits were estimated at (-4.011, -3.367), (0.698, 0.850), (0.014, 0.053), and (-1.167, -0.217), respectively.

Based on Equation (22), the critical gap can then be computed by setting the probability of accepting a gap to 0.5 which results in a logit function of zero value. Consequently the critical gap ( $t_c$ ) is computed as

$$t_c = \tau_t - \frac{\beta_0}{\beta_1} - \frac{\beta_2}{\beta_1} w - \frac{\beta_3}{\beta_1} r = \alpha_0 + \alpha_1 w + \alpha_2 r. \quad (31)$$

By applying the calibrated logit model coefficients ( $\beta_0, \beta_1, \beta_2,$  and  $\beta_3$ ) to Equation (23) the critical gap coefficients ( $\alpha_0, \alpha_1,$  and  $\alpha_2$ ) are computed to be 7.07, -0.04, and 0.81 for the first conflict point and 8.27, -0.04, 0.81 for the second conflict point, respectively. Therefore, at zero waiting time ( $w=0$ ) and dry condition ( $r=0$ ), the critical gap values are 7.1 and 8.3 s for P1 and P2 respectively. Equation (23) implies that the critical gap decreases as the driver waits longer in search of an acceptable gap (i.e. drivers become more aggressive as they wait longer). Alternatively, the critical gap increases as the rain intensity increases (i.e. drivers become less aggressive as the rain intensity increases).

## 6.5 Proposed Psycho-Physical Modeling Framework

Driver gap acceptance behavior entails estimating the duration of time it would take the subject vehicle to clear a conflict point and avoid a collision with an oncoming opposing vehicle. Typically, the driver requires some additional buffer of safety to ensure that no collision occurs. Consequently, the modeling of driver gap acceptance behavior requires the modeling of driver acceleration behavior and the additional buffer of safety the driver requires to accept a gap as will be described in the following sections.

## 6.6 Modeling Vehicle dynamics

Vehicle acceleration is governed by vehicle dynamics. Vehicle dynamics models compute the maximum vehicle acceleration levels from the resultant force acting on a vehicle, as

$$a = \frac{F - R}{m} \quad (32)$$

Where  $a$  is the vehicle acceleration ( $\text{m/s}^2$ ),  $F$  is the resultant force (N),  $R$  is the total resistance force (N), and  $m$  is the vehicle mass (kg).

Given that acceleration is the second derivative of distance with respect to time, Equation (32) resolves to a second-order Ordinary Differential Equation (ODE) of the form

$$\frac{d^2x}{dt^2} = f\left(\frac{dx}{dt}, x\right). \quad (33)$$

The vehicle tractive force is computed as

$$F_T = 3600 \beta \eta \frac{P}{u} f_p. \quad (34)$$

Here  $F_T$  is the engine tractive force (N),  $\beta$  is a gear reduction factor that will be described later (unitless),  $\eta$  is the driveline efficiency (unitless),  $P$  is the vehicle power (kW), and  $u$  is the vehicle speed (km/h), and  $f_p$  is the throttle level that the driver is willing to employ. Given that the tractive effort tends to infinity as the vehicle speed tends to zero, the tractive force cannot exceed the maximum force that can be sustained between the vehicle's tractive axle tires and the roadway surface, which is computed as

$$F_{max} = m_{ta} g \mu. \quad (35)$$

Here  $m_{ta}$  is the mass of the vehicle on the tractive axle (kg),  $g$  is the gravitational acceleration ( $9.8066 \text{ m/s}^2$ ), and  $\mu$  is the coefficient of roadway adhesion and also known as the coefficient of friction (unitless).

Typical axle mass distributions for different truck types were presented in an earlier publication and thus are not discussed further [53]. The resultant force is then computed as the minimum of the two forces as

$$F = \min(F_T, F_{max}). \quad (36)$$

Rakha and Lucic [54] introduced the  $\beta$  factor into Equation (34), in order to account for the gear shift impacts at low traveling speeds when trucks are accelerating. While the variable power

factor does not incorporate gear shifts explicitly, it does account for the major behavioral characteristics that result from gear shifts, namely the reductions of power. Specifically, the factor is a linear function of vehicle speed with an intercept of  $1/u_0$  and a maximum value of 1.0 at  $u_0$  (optimum speed or the speed at which the vehicle attains its full power) as

$$\beta = \frac{1}{u_0} \left[ 1 + \min(u, u_0) \left( 1 - \frac{1}{u_0} \right) \right]. \quad (37)$$

The intercept guarantees that the vehicle has enough power to accelerate from a complete stop. The calibration of the variable power factor was conducted by experimenting with different truck and weight combinations to estimate the speed at which the vehicle power reaches its maximum (termed the optimum speed). The optimum speed was found to vary as a function of the weight-to-power ratio (for weight-to-power ratios ( $w$ ) ranging from 30 to 170 kg/kW) as

$$u_0 = 1164w^{-0.75}. \quad (38)$$

Here  $w$  is the weight-to-power ration in kg/kW. Rakha and Snare [55] demonstrated that the gear shift parameter  $\beta$  is not required for the modeling of light-duty vehicle acceleration behavior (weight-to-power is less than 30 kg/kW).

Three resistance forces are considered in the model, namely the aerodynamic, rolling, and grade resistance forces [53, 56]. The first resistance force is the aerodynamic resistance that varies as a function of the square of the air speed. Although a precise description of the various forces would involve the use of vectors, for most transportation applications scalar equations suffice if the forces are considered to only apply in the roadway longitudinal direction. For the motion of a vehicle in still air, the air speed equals the vehicles speed as

$$R_a = \frac{\rho}{2 \times 3.6^2} C_d C_h A u^2 = c_1 C_d C_h A u^2, \quad (39)$$

Where  $\rho$  is the density of air at sea level and a temperature of 15°C (59°F) (equal to 1.2256 kg/m<sup>3</sup>),  $C_d$  is the drag coefficient (unitless),  $C_h$  is a correction factor for altitude (unitless), and  $A$  is the vehicle frontal area (m<sup>2</sup>). Given that the air density varies as a function of altitude, the  $C_h$  factor can be computed as

$$C_h = 1 - 8.5 \times 10^{-5} H . \quad (40)$$

Typical values of vehicle frontal areas for different vehicle types and typical drag coefficients are provided in the literature [53].

The second resistance force is the rolling resistance, which is a linear function of the vehicle speed and mass, as

$$R_r = C_r (c_2 u + c_3) \frac{mg}{1000} . \quad (41)$$

Typical values for the rolling coefficients ( $C_r$ ,  $c_2$ , and  $c_3$ ), as a function of the road surface type, condition, and vehicle tires, are provided in the literature [53]. Generally, radial tires provide a resistance that is 25 percent less than that for bias ply tires.

The third and final resistance force is the grade resistance, which accounts for the proportion of the vehicle weight that resists the movement as a function of the roadway grade ( $i$ ) as

$$R_g = mgi . \quad (42)$$

Having computed the various resistance forces, the total resistance force is computed as

$$R = R_a + R_r + R_g . \quad (43)$$

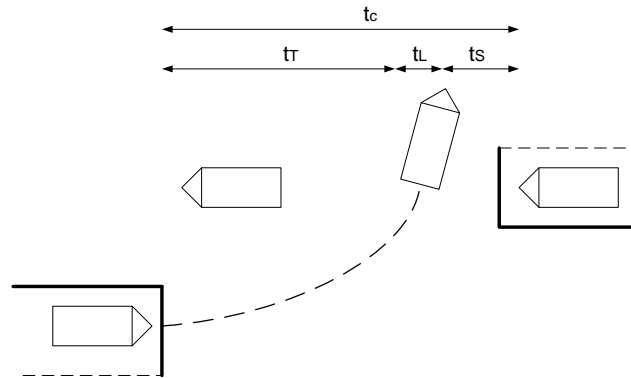
Using vehicle dynamics models, the maximum possible acceleration a vehicle is willing to exert could be calculated based on Equation(32).

## 6.7 Psychological Modeling Component

The proposed modeling approach can be categorized as a psycho-physical model given that the model captures the psychological deliberation of the driver in addition to the physical constraints imposed by the vehicle. In addition, the model captures the interface between the vehicle tires and the roadway surface. The proposed model considers the driver specific critical gap (the minimum gap a driver is willing to accept) for each driver, is the summation of the travel time to reach the conflict point, the time needed to clear the length of the vehicle and an additional buffer of safety time as

$$t_c = t_T + t_L + t_S \quad (44)$$

Where;  $t_c$  is the critical gap duration for a specific driver,  $t_T$  is the travel time to conflict point,  $t_L$  is the conflict zone clearance time and  $t_S$  is the duration of the buffer of safety to avoid a conflict with an oncoming vehicle, as illustrated in Figure 6.2. Each term of this equation will be described in detail in the following sections.



**Figure 6.2: The proposed critical gap value for the psycho-physical model**

### 6.7.1 Travel Time to Conflict Point ( $t_T$ )

Knowing the type of vehicle entering the intersection and the roadway surface condition (wet or dry), the travel time to reach the conflict can be computed by numerically solving a second-order differential equation, as summarized in Figure 6.. This travel time is a function of the distance traveled to the conflict point and the level of acceleration the driver/vehicle is willing to exert. As presented earlier in Equation (32), the acceleration of a vehicle is computed based on the resultant force (difference between the tractive forces and the resistance forces), the road surface condition (coefficient of road adhesion), and the specifications of the vehicle (dimensions, power of engine, mass, mass on tractive axle, etc.).

### 6.7.2 Conflict Zone Clearance Time ( $t_L$ )

After determining the time and distance to reach the conflict point; the speed of the vehicle is used to estimate the additional time required to travel the vehicle length in order to clear the conflict point. This time is termed the conflict point clearance time ( $t_L$ ).

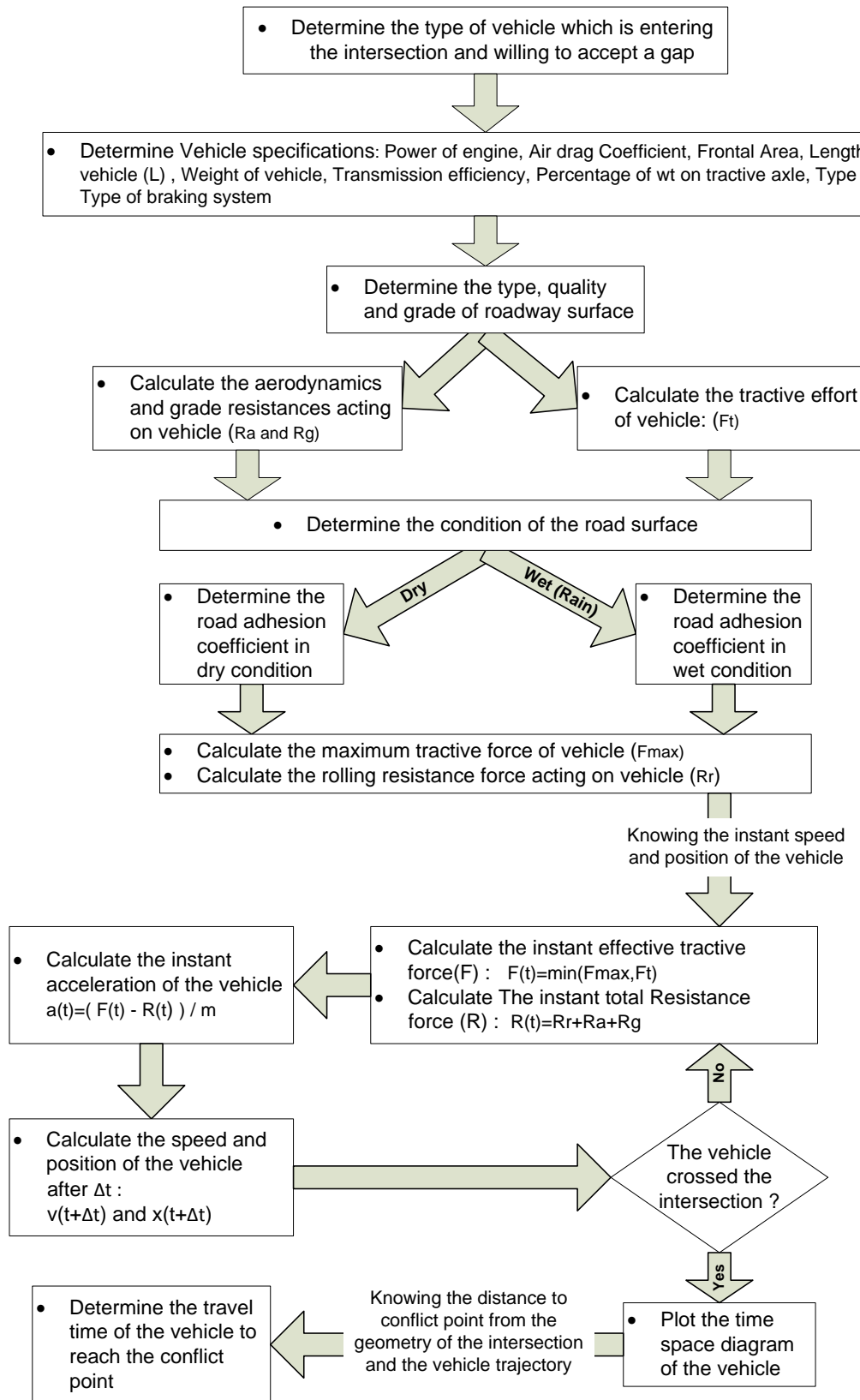
### 6.7.3 Safety Buffer Time ( $t_s$ )

The buffer of safety is defined as the additional gap duration required by a driver to ensure that no conflict results from a gap acceptance maneuver. This additional time serves as a minimum safety buffer to avoid a collision. The density distribution of the  $t_s$  from the collected data is illustrated in Figure 6.(a) and the cumulative distribution function in Figure 6.(b). The distribution of the  $t_s$  can be approximated for a normal distribution with mean ( $\mu$ ) equal to 3.679 s and standard deviation ( $\sigma$ ) equal to 1.645 s. Given that we are attempting to identify the minimum additional gap the 5<sup>th</sup> percentile buffer of safety was considered in the analysis. The buffer of safety was found to vary as a function of the travel time to the conflict point ( $t_T$ ) and roadway surface condition. Consequently, a relationship between the buffer of safety (5<sup>th</sup> percentile additional gap time) and the  $t_T$  and the roadway surface condition was established, as illustrated in Figure 6.(c). The 5<sup>th</sup> percentile additional gap time was used to ensure that the minimum additional time is only considered. Longer times would only represent the remaining gap duration not the minimum additional time that a driver is willing to accept.

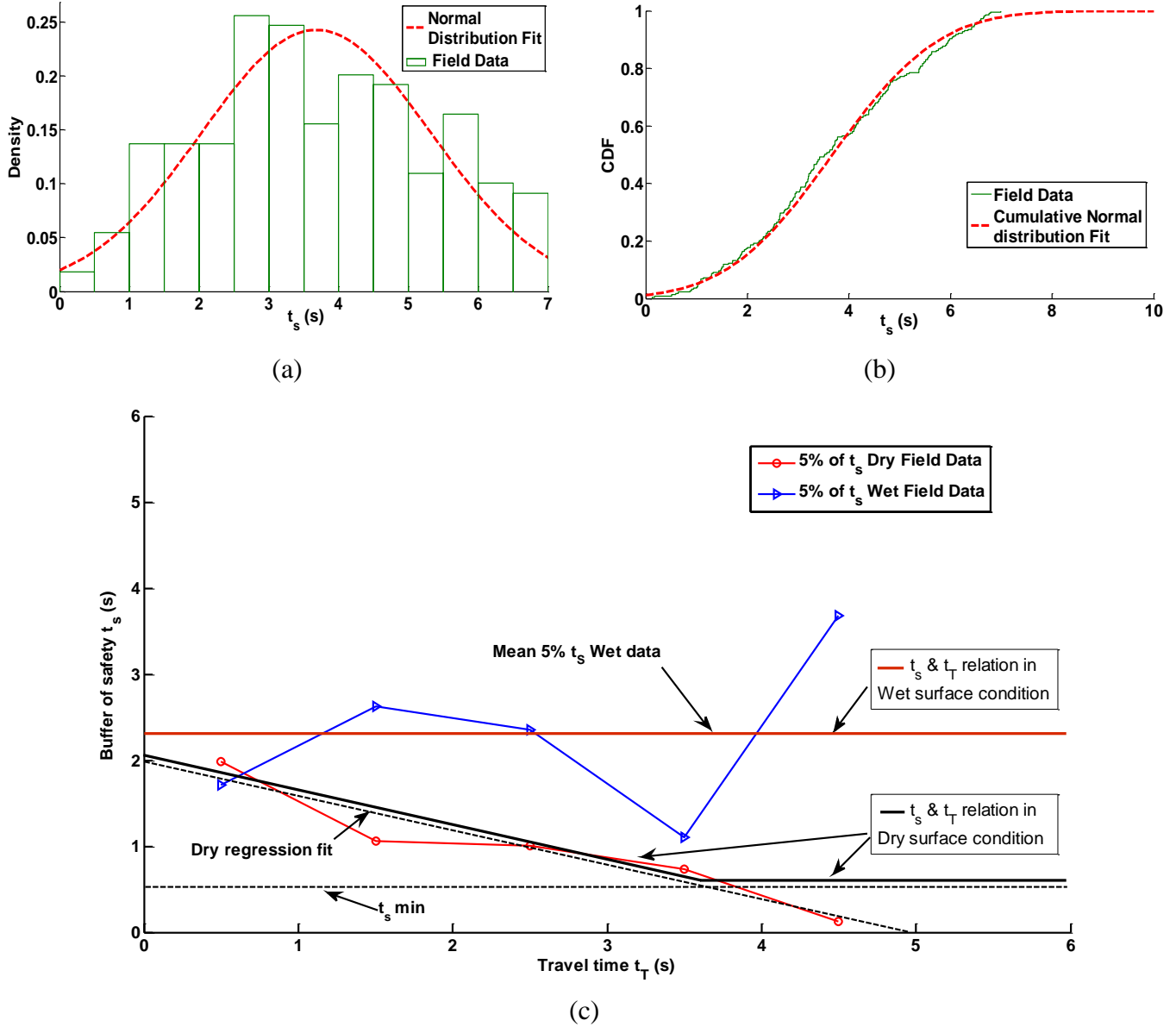
In the case of dry roadway conditions the  $t_s$  is computed as the minimum of a linear function of the travel time and a minimum required buffer of safety (0.5 s), where the regression line is of the function  $t_s = 1.99 - 0.404 t_T$  ( $R^2=90.4\%$  and  $\sigma = 0.2397$  s). Alternatively, in the case of a wet surface, because of a lack of relationship between  $t_s$  and the  $t_T$  ( $R^2 < 10\%$ ), the  $t_s$  is assumed to equal the 5<sup>th</sup> percentile  $t_s$  value for wet surface conditions, where the mean 5<sup>th</sup> percentile is 2.294 s with a standard deviation ( $\sigma$ ) of 0.868 s. The buffer of safety is thus computed as

$$t_s = \begin{cases} \max(1.99 - 0.404t_T, 0.5) + \varepsilon_1 & \text{Dry conditions} \\ 2.294 + \varepsilon_2 & \text{Wet conditions} \end{cases} \quad (45)$$

Here  $\varepsilon_1$  and  $\varepsilon_2$  are random error terms with a mean of zero and a standard deviation of 0.240 and 0.868, respectively. It should be noted that unlike the case of the logit model, the buffer of safety was not found to be impacted by the driver wait time (i.e. drivers were not observed to become more aggressive as they waited longer).



**Figure 6.3: The proposed steps for estimating the travel time of the vehicle to a conflict point**



**Figure 6.4: Distribution of buffer of safety ( $t_s$ ) from field data and its relation with the travel time ( $t_T$ ) for dry and wet conditions**

### 6.8 Typical Vehicle Gap Acceptance Scenario

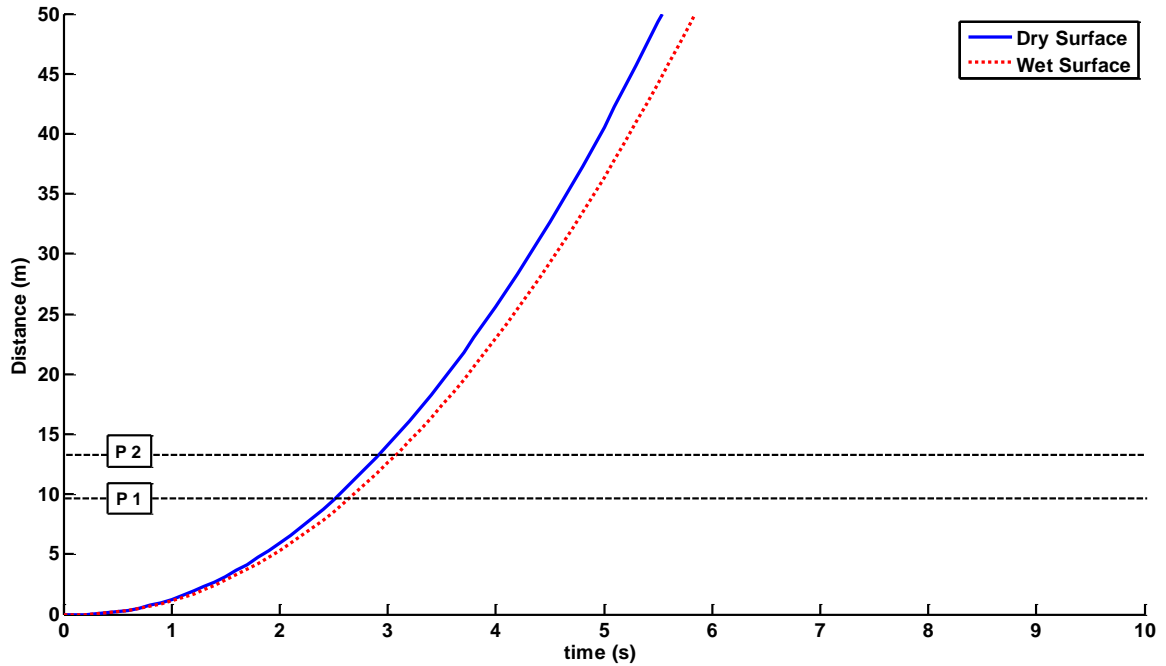
In order to compute the time to reach and clear the conflict point, vehicle parameters are required. A mid-sized vehicle was considered for the analysis (2006 Honda Civic-EX-Sedan) because it was felt that this vehicle would represent an average typical vehicle. The vehicle has an engine power of 140 Horse Power (Hp) and accelerates from a complete stop at the stop line on a level

asphalt surface (0% grade). Table 6.1 summarizes the vehicle specifications and parameters used in the model.

**Table 6.1: Vehicle Parameters of Study Vehicle**

	Parameter	Value	
<b>Tractive Force Parameters</b>	Power of engine (P)	140 Hp	
	Transmission Efficiency ( $\eta$ )	0.95	
	Total Weight (W)	1180 Kg	
	Mass on Tractive Axle ( $m_{ta}$ )	437 Kg	
	Roadway Adhesion ( $\mu$ )	Dry = 1.0 Wet= 0.8	
<b>Resistance Parameters</b>	Air Density ( $\rho$ )	1.2256 Kg/m <sup>3</sup>	
	Air Drag Coefficient ( $C_d$ )	0.3	
	Altitude Factor ( $C_h$ )	1.0	
	Frontal Area (A)	2.14 m <sup>2</sup>	
	Rolling Coefficient	Cr= 1.25	
		C <sub>2</sub> = 0.0328 C <sub>3</sub> = 4.575	

Incorporating the vehicle parameters of Table 6.1 in Equations (32) through (43) and setting the  $\beta$  and  $f_p$  parameters to 1.0, the ordinary differential equation was solved using the procedures summarized in Figure 6. to generate the time-space trajectory of the test vehicle, as illustrated in Figure 6.. Using the geometry of the intersection and assuming the trajectory of the left turn vehicle an ellipsoidal curve, the distance measured from the stop line of the left turn lane to the first conflict point (P1) and the second conflict point (P2) were 9 and 13 m, respectively. The travel time values ( $t_T$ ) to both conflict points for the dry and wet cases was computed in addition to the time needed to clear the conflict point ( $t_L$ ). Using the travel time values, the buffer of safety was computed using Equation (45). The critical gap value ( $t_c$ ) for both scenarios (dry and wet) was then computed using Equation (44).



**Figure 6.5: The time-space diagram of the typical case study vehicle**

Table 6.2 summarizes the different values of  $t_T$ ,  $t_L$ ,  $t_S$  and  $t_c$  for the Honda Civic EX vehicle. Depending on the surface condition (dry vs. wet), the driver either accepts or rejects the offered gap by comparing the size of the offered gap to the corresponding driver-specific critical gap ( $t_c$ ). The driver specific critical gap can be assumed to be a deterministic or stochastic variable as will be discussed later in the paper. It should be noted that the critical gaps produced from the psychophysical model are more consistent with what is reported in the literature (4.5 s) when compared to those derived from the statistical model. Furthermore, the results demonstrate a 0.2 s increase in the critical gap with the opposing flow is in the second lane. The introduction of rain results in a 1.6 s increase in the critical gap.

**Table 6.2: Mean Parameters Values for the Honda Civic EX Vehicle**

Time (s)	Conflict point	Dry	Wet
Travel time ( $t_T$ )	P1	2.397	2.667
	P2	2.851	3.176
Clear Vehicle time ( $t_C$ )	P1	0.611	0.687
	P2	0.511	0.574
Buffer of Safety time ( $t_S$ )	P1	1.022	2.294
	P2	0.838	2.294
Critical Gap time ( $t_C$ )	P1	4.030	5.648
	P2	4.200	6.044

It should be noted that because the vehicle is assumed to accelerate from a complete stop (zero speed) over a short distance, the governing propulsive force is the frictional force between the vehicle tires and pavement surface and not the engine propulsive force (refer to Equation (36)). Consequently the governing vehicle and roadway parameters are the mass of on the vehicle's propulsive axle (kg) and the coefficient of roadway adhesion (also known as the coefficient of roadway friction) and not the vehicle engine characteristics.

## 6.9 Model Validation and Comparison

The two models (statistical and psycho-physical) were evaluated and compared based on the Success Rate (SR) statistic. The SR is defined as the ratio of correct acceptance/rejection outcomes relative to the total number of observations analyzed. Consequently, a higher SR indicates a better model. In computing the SR for the statistical model, the probability of accepting a gap was computed for each observation using the observation specific explanatory variables. In order to generate a binary outcome (accept or reject), the probability of accepting a gap ( $p$  value) greater than 0.5 was assumed to result in an acceptance decision. Otherwise the gap was rejected. Alternatively, in the case of the psycho-physical model a gap was accepted if it was larger than the driver specific critical gap otherwise the gap was rejected. The SR was computed by comparing the acceptance/rejection decision to the field observed decision.

The models were applied to two datasets, namely: the Christiansburg intersection (shown in Figure 4.1) and a second dataset that was obtained from an earlier study [6] in Orlando, Florida. The objective of the Orlando study was to characterize the influence of the driver's sight distance on left turn gap acceptance decisions. Yan and Radwan [6] gathered data at the intersection of

Rouse Lk. Rd./Walmart and E. Colonial Drive located in Orange County in Orlando, Florida. This intersection had four level approaches at a 90° angle and a protected/permitted left turn signal phase for the major road, as shown in Figure 6.. The dataset consisted of a total of 1,485 gap decisions from a total of 323 left turning movements recorded for dry conditions. Given that the driver wait times were not recorded in the Orlando dataset, the average wait time from the Christiansburg dataset (7.6 s) was applied to the Orlando dataset. The results of the SR statistics for both datasets are summarized in Table 6.3.

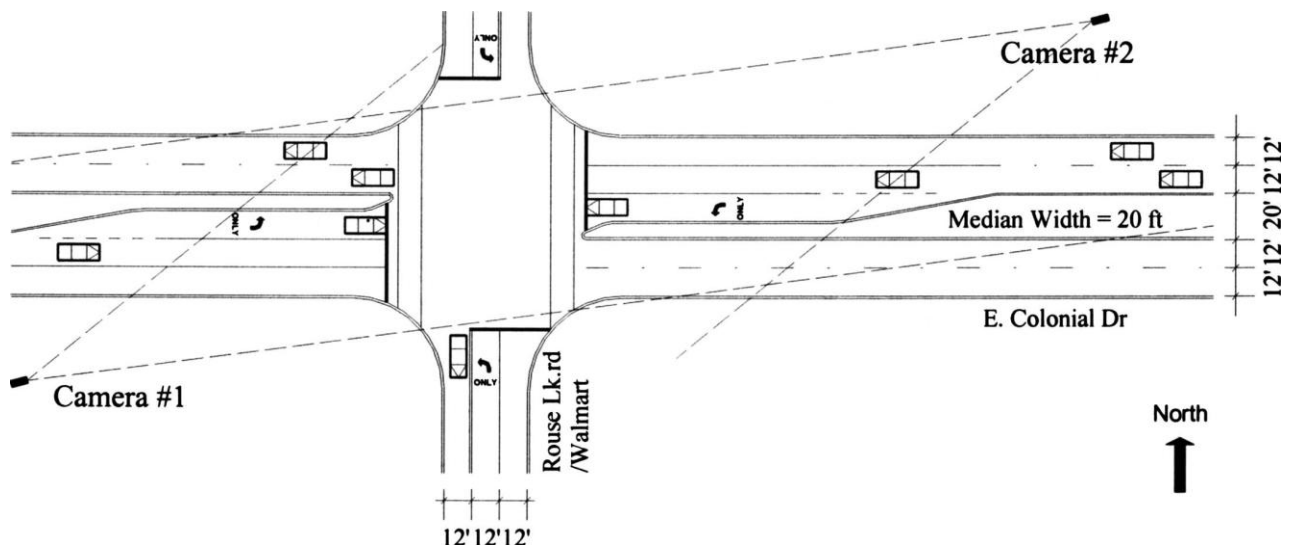


Figure 6.6: The intersection of Rouse Lk. Rd and E.Colonial Dr, Orlando, FL (source [6])

Table 6.3: Models Comparisons Summary

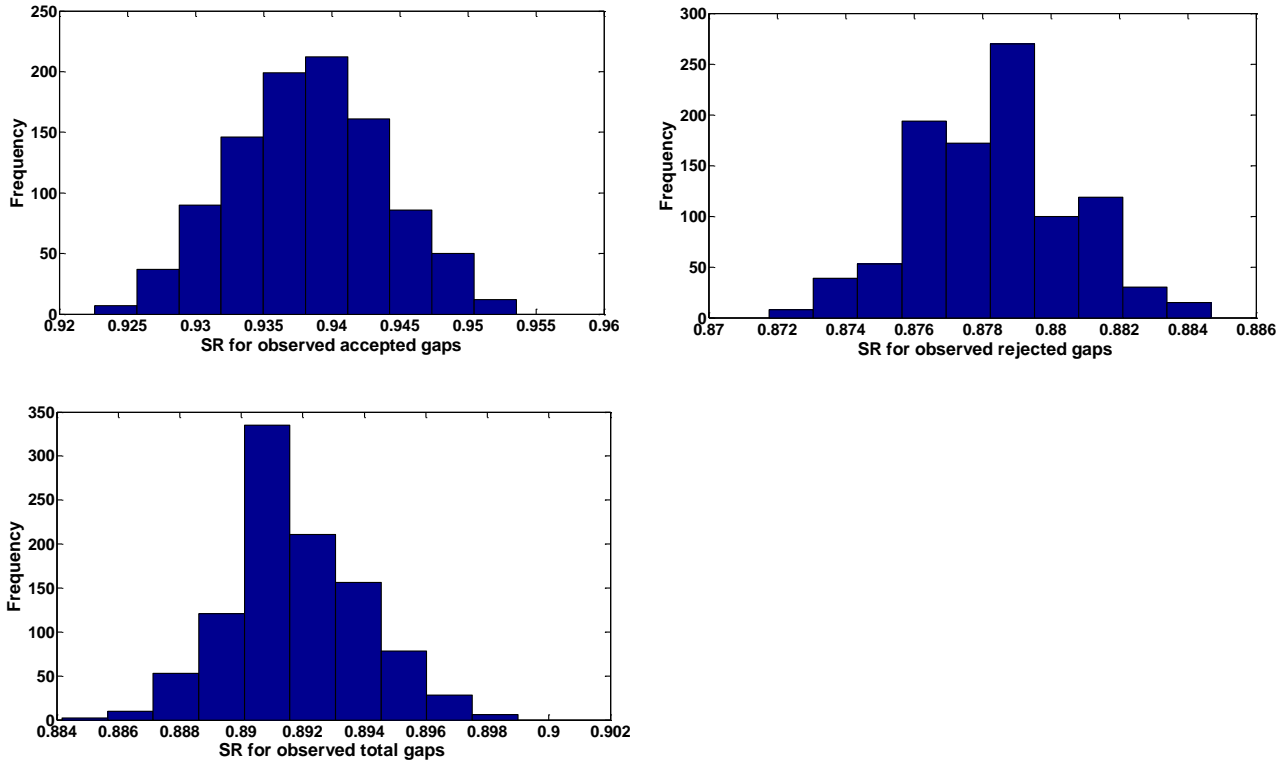
Measurements	Christiansburg Intersection (1 <sup>st</sup> dataset)		Orlando Intersection (2 <sup>nd</sup> dataset)	
	Statistical Model	Psycho-Physical Model	Statistical Model	Psycho-Physical Model
SR1 (SR for accepted gaps)	64%	94%	49%	94%
SR2 (SR for rejected gaps)	98%	88%	98%	88%
SR3 (SR for all gaps)	95%	89%	87%	90%

As summarized in Table 6.3, the SR for accepted gaps (SR1) was much higher in the case of the psycho-physical model compared to the statistical model for both datasets. Alternatively, the statistical model offered a higher success rate for the rejected gaps (SR2). The overall SR (SR3) was larger in the case of the statistical model for the first dataset but lower for the second dataset. The results seem to indicate that the psycho-physical model provides better predictions if one considers that the psycho-physical model was constructed solely using the accepted gaps from the first dataset. Furthermore, the psycho-physical model is recommended because it offers a prediction accuracy that is consistent for both accepted and rejected gaps (approximately 90%). Finally, the psycho-physical model also allows for the calibration of model parameters without the need to gather in-field gap acceptance decisions.

### **6.10 Modeling Stochastic Differences in Driver Behavior**

In capturing stochastic differences in driver behavior, the buffer of safety ( $t_s$ ) was modeled as a random variable. In case of a dry surface  $t_s$  follows a normal distribution a mean derived using Equation (45) and a standard deviation of 0.240 s. Alternatively, in the case of a wet surface the buffer of safety is a normally distributed variable with a mean of 2.294 and a standard deviation of 0.868 s.

A Monte Carlo simulation was run on the data considering a sample size of 1000 repetitions on the Orlando dataset. Given that each driver has a unique critical gap the SR varies depending on the value of the random number seed. Figure 6. shows the distribution of the SR values for all 1000 simulations. The mean SR is consistent with the values provided in Table 4.3 and vary between 92 and 95% in the case of accepted gaps and between 87 and 88% in the case of rejected gaps.



**Figure 6.7: Success Rate Distributions for Psycho-Physical Model on Orlando Intersection**

## 6.11 Summary and Conclusions

The research presented in this paper developed a psycho-physical framework for modeling driver left turn gap acceptance behavior. The physical component of the model explicitly captures the vehicle constraints on driving behavior using a vehicle dynamics model, while the psychological component explicitly models the driver’s deliberation in accepting/rejecting a gap. The model was developed using a 301 accepted gaps and subsequently validated on 2,429 rejected gaps at the same test site in addition to 1,485 gap decisions (323 accepted and 1,162 rejected) at another site. The model predictions were then compared to a more traditional statistical logit modeling model. The variables that were considered in the logit modeling framework included the gap duration; the driver’s wait time in search of an appropriate acceptable gap; the time traveled by a driver to clear the conflict point; and the rain intensity. The results demonstrate that the psycho-physical model is superior to standard statistical logit model approaches because it produces consistent performance for accepted and rejected gaps (correct predictions of approximately 90%), the model produces consistent results at different sites, and most importantly the model can

be generalized to capture different vehicle, roadway, movement, intersection characteristics, and weather effects on driver gap acceptance behavior.

## **6.12 Acknowledgements**

The authors acknowledge the valuable input from Dr. Roemer Alfelor and Dr. David Yang of the Federal Highway Administration, Daniel Krechmer of Cambridge Systematics, Inc., Dr. Shereef Sadek at the Virginia Tech Transportation Institute (VTTI), and Dr. Essam Radwan at the University of Central Florida for providing the second dataset that was used for validation purposes. Finally, the authors also acknowledge the financial support provided by the FHWA in conducting this research effort.

## **CHAPTER 7 : SUMMARY AND CONCLUSIONS**

The research presented in this thesis analyzed gap acceptance behavior for permissive left turn movements at signalized intersections. Three different modeling frameworks were developed including: deterministic statistical modeling, stochastic statistical modeling, and psycho-physical modeling. The conclusions of each approach will be presented in the following sections followed by recommendations for future research.

### **7.1 Statistical Modeling**

An empirical study was conducted to quantify the impact of a number of variables on driver left turn gap acceptance behavior. The variables that were considered include the gap duration, the driver's wait time in search of an appropriate acceptable gap, the time traveled by a driver to clear a conflict point, and the rain intensity. Three different logistic regression models were evaluated. The main findings were:

- a. The first model is better from a statistical standpoint although it produces unrealistic behavior with regards to wait times;
- b. The second model is comparable to the first model in terms of statistical measure performance. The second model is considered the best model as it produces realistic behavior with regards wait times and rain intensity levels;
- c. The third model produces differences in critical gap values across different lanes that are much larger than what is reported in the literature and is inconsistent with the field observations. In addition, the statistical tests show that it is inferior to the other models;
- d. All models reveal that the acceptable gap duration decreases as a function of the driver's wait and increases as the rain intensity increases. Furthermore, the size of a driver's acceptable gap increases as the distance to the conflict point increases.

The model demonstrates that drivers become more aggressive as they wait longer in search of an acceptable gap. Alternatively, the results show that drivers become less aggressive as the rain intensity increases. Finally, the study demonstrates how rain intensity affects the permissive left-turn saturation flow rate. It is anticipated that these findings can be used to develop weather-specific traffic signal timings that account for changes in traffic stream saturation flow rates due to changes in critical gap values as a function of weather conditions.

## **7.2 Stochastic Modeling**

Two stochastic modeling approaches were evaluated: a Bayesian and a Bootstrap approach. The study demonstrates that the two methods estimate consistent model parameters but the Bayesian estimated parameters are less skewed. The study develops a procedure to model stochastic gap acceptance behavior while capturing model parameter correlations without the need to store all parameter combinations. The variables that are considered in the model include the gap duration; the driver's wait time in search of an appropriate acceptable gap; the time traveled by a driver to clear the conflict point; and the rain intensity. The proposed approach involves generating four uniformly distributed random variables ( $\sim U(0,1)$ ) to generate four correlated model coefficient realizations. The main findings for using the Bayesian and Bootstrap procedures on the gathered data are:

- a. The Bayesian and Bootstrap procedures demonstrated consistent model parameters;
- b. The level of noise generated using the Bayesian approach is less than that using the Bootstrap approach. This could be attributed to the fact that the Bayesian approach used a thinning process that ensured that there was no autocorrelation between model parameters;
- c. In general both techniques result in comparable results for the coefficients of gap and waiting time; however, the coefficient of rain intensity estimates are marginally different;
- d. An important observation is that the Bayesian analysis converges approximately to the same values no matter what the prior distribution is except for the rain intensity parameter;
- e. The Bootstrapping distributions are more skewed than the Bayesian distributions especially for the rain intensity parameter.

In summary, the results suggest that a Bayesian approach for estimating model parameters is consistent with a Bootstrap approach; however it does produce less variability in the model parameter estimates. Consequently, the study proposes the use of a Bayesian procedure for the proposed approach. Moreover, a Monte Carlo simulation was executed to study the effect of the proposed model on the opposed saturation flow rate. The results demonstrate that the model produces stochastic saturation flow rates and that the variance of the saturation flow rate increases as the opposed flow and rain intensity increases.

### **7.3 Psycho-Physical Modeling**

Finally, a psycho-physical framework for modeling driver left turn gap acceptance behavior was developed. The physical component of the model explicitly captures the vehicle constraints on driving behavior using a vehicle dynamics model. The psychological component explicitly models the driver's deliberation in accepting/rejecting a gap. The model is developed using a dataset from an intersection in Christiansburg, VA, which is consisted of 301 accepted gaps and subsequently validated on 2,429 rejected gaps. The model was tested on a second dataset that was provided by the University of Central Florida for a signalized intersection in Orlando, FL. In this chapter, the critical gap value for each driver was considered the summation of three different time values: travel time to conflict point, clear time of conflict zone and the remaining time left by the driver as a buffer of safety. It was proposed a relationship between the travel time and the buffer of safety time value depending on the type of surface (dry or wet) and it is introduced a random error normally distributed to present the intra- and inter-driver variability. The model predictions are then compared to a more traditional statistical logit modeling model (presented before). The results demonstrate that the psycho-physical model is superior to standard statistical logit model approaches because it produces consistent performance for accepted and rejected gaps (correct predictions of approximately 90%). The model produces consistent results at different sites. In addition, the model can be generalized to capture different vehicle, roadway, movement, intersection characteristics, and weather effects on driver gap acceptance behavior.

## **7.4 Recommendations for Future Research**

The following areas of research should be pursued to expand the current research work on modeling gap acceptance behavior:

- 1- Gather more field data on gap acceptance behavior from different sites;
- 2- Apply the models to different traffic gap acceptance situations (e.g. stops sign control intersections and roundabouts);
- 3- Develop models capturing more weather conditions including snowy and icy conditions;
- 4- Evaluate the impact of different vehicle characteristics on driver gap acceptance behavior;
- 5- Quantify the impact of rain intensity on signal timing design;
- 6- Integrate the modeling of gap acceptance behavior with IntelliDrive systems, vehicle to vehicle communication and the intersection decision support (IDS).

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