

A New Multi-Scale State Estimation Framework for the Next Generation of Power Grid EMS

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Abstract—Accurate system state information under various operation conditions is a prerequisite for power grid monitoring and efficient control. To achieve that goal, a new multi-scale state estimation framework is proposed, paving the way for the development of next generation of energy management system (EMS). The developed framework consists of three key components, namely the static state estimation (SSE) module, the dynamic state estimation (DSE) module, the interfaces and switching logics between the two modules. Specifically, the singular spectrum analysis (SSA)-based change point detection approach is developed to monitor the system continuously. If no event is detected by the SSA, the robust SSE using both SCADA and PMU measurements is executed. Otherwise, the event is declared and the results from SSE are used to derive the initial condition for DSE. During the transient process, only PMU-based DSE is executed for system monitoring and it will be terminated when SSA does not detect any change point of the system. After that, the DSE results are forwarded for SSE initialization and bus voltage magnitude and angle estimations. Simulation results carried out on the IEEE 39-bus system demonstrate the effectiveness and benefits of the proposed framework.

Index Terms—Power system state estimation, energy management system, dynamic state estimation, singular spectrum analysis, event detection.

I. INTRODUCTION

DUE to the increased penetration of variable generation, demand response, new power market structure and unexpected events, the behavior of modern power systems is becoming more stochastic and dynamic [1]. It is critically important to understand the current and predict near-future operational states to enhance system resilience and reliability so that grid operators can take preventive actions to mitigate the impact. The state-of-the-art energy management system (EMS) system being adopted by the power industry relies heavily on the static state estimation (SSE) to gain situational awareness of power grid [2]. With the assumption that the power system is under quasi-steady states, SSE leverages spatial redundancy of measurements to estimates a single snapshot of current system operation conditions, such as bus voltage magnitudes and angles. SSE also helps identify and correct measurement and topology errors. The complete, coherent and reliable database obtained from SSE enables many EMS applications, such as contingency analysis, voltage stability assessment, optimal

power flow, to name a few. Typical SSE methods include the weighted least squares (WLS) [3], the weighted least absolute value (WLAV) method [4], and the generalized maximum-likelihood (GM)-estimator [5], [6], etc. However, the quasi-steady states assumptions may be violated in the presence of quick and severe system changes due to faults, sudden changes of renewable energy injections, etc. As a result, SSE used in today's EMS should be reassessed and reinforced with new monitoring tools, such as dynamic state estimation (DSE).

Focusing on electromechanical dynamics, DSE is able to capture rapid dynamic changes in system states, allowing the system operators to perform online control and protection. DSE is typically developed based on the Kalman filter framework. It consists of two main steps, namely the state prediction based on the nonlinear differential equations that model the generator and its associated control devices, and the state filtering that processes the state prediction and the incoming measurements simultaneously. To date, several different DSEs have been proposed in the literature, such as the extended Kalman filter (EKF) [7], the unscented Kalman filter (UKF) [8], the ensemble Kalman filter (EnKF) and the particle filter (PF) [9], to name a few. The enhancement of their robustness to bad data, model parameter errors, non-Gaussian noise and cyber attacks has also been studied in [10]–[12]. Although DSE is promising, there exists a grand challenge about how to integrate it with the existing SSE in the today's EMS for power system monitoring over wide temporal and spatial ranges.

This paper proposes a new multi-scale state estimation framework that effectively integrates the SSE and DSE together, enabling the development of next generation of EMS. Specifically, the singular spectrum analysis (SSA)-based change point detection approach [13], [14] is extended to detect the changes of system operational conditions and to identify the dominant dynamics so that proper estimators can be selected and combined to obtain the optimal performance. The SSA is a non-parametric and powerful method for time-series analysis. Its main idea is to perform singular value decomposition (SVD) of the trajectory matrix that is obtained from the original series with a subsequent reconstruction of the time series. It is motivated by the fact that at a certain time instant, if the mechanism of generating the time series has changed, the distance between the subspace spanned by certain eigen-vectors of the lag-covariance matrix and the lagged vectors will increase. Thus, a statistical test based on the SSA can be applied for deciding whether to use the SSE or the DSE. Specifically, if no event is detected by the SSA, the system is operated under normal conditions and the robust SSE using

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both SCADA and PMU measurements is executed. Otherwise, the event is declared and the results obtained from recent SSE are used to derive the initial condition for DSE. During the transient process, only PMU-based DSE is executed for power grid monitoring. The DSE will be terminated when no change point of the system is detected. After that, the DSE results are forwarded for SSE initialization and bus voltage magnitude and angle estimations. The developed framework is able to track the changes of system states over wide temporal and spatial ranges and significantly enhances the operator's situational awareness, yielding better system monitoring, protection and control.

The remainder of this paper is organized as follows: Section II presents the proposed framework, where the problem formulation is firstly introduced, followed by the derivation of the connections and logics between SSE and DSE. Section III shows some illustrative results. Section IV concludes the paper.

II. PROPOSED MULTI-SCALE ESTIMATION FRAMEWORK

Unlike the literature that deals with SSE and DSE separately, this paper develops a new framework to analyze the connections and logics between SSE and DSE. These will be used together with the SSA-based change point detection approach for power grid multi-scale state monitoring. To this end, a problem reformulation will be firstly presented. Then, the use of SSA for system event detection is discussed, followed by the introduction of SSE and DSE modules.

A. Problem Reformulation

A power system can be described by a set of continuous-time nonlinear differential and algebraic (DAE) equations shown below [15]:

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{y}(t), \mathbf{u}(t), \mathbf{p}) \quad (1)$$

$$\mathbf{0} = \mathbf{g}(\mathbf{x}(t), \mathbf{y}(t), \mathbf{u}(t), \mathbf{p}) \quad (2)$$

where $\mathbf{x} \in \mathbb{R}^n$ represents the system dynamic state vector, such as the internal states of machines; $\mathbf{y} \in \mathbb{R}^m$ denotes the algebraic state vector, such as bus voltage magnitude and angle; \mathbf{u} is the input vector that drives the state transition; \mathbf{p} are model parameters; \mathbf{f} and \mathbf{g} are nonlinear functions.

After time discretization, the aforementioned continuous-time DAE model can be organized as

$$\mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{y}_{k-1}, \mathbf{u}_k, \mathbf{p}) + \mathbf{w}_k, \quad (3)$$

$$\mathbf{0} = \mathbf{g}(\mathbf{x}_k, \mathbf{y}_k, \mathbf{u}_k, \mathbf{p}), \quad (4)$$

where \mathbf{w}_k represents model approximation errors. The algebraic variables \mathbf{y}_k are either directly measured by PMUs or can be deduced from other measured quantities, such as real and reactive power injections. It is worth noting that \mathbf{g} is nonlinear and its inverse function $\mathbf{y}_k = \mathbf{g}^{-1}(\mathbf{x}_k, \mathbf{u}_k, \mathbf{p})$ may not be always available. One way to deal with that is to treat the equality constraint (4) as pseudo-measurements and process them together with the regular PMU measurements. As a result, the general state-space model is expressed as follows:

$$\mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{y}_{k-1}, \mathbf{u}_k, \mathbf{p}) + \mathbf{w}_k, \quad (5)$$

$$\mathbf{z}_k = \mathbf{h}(\mathbf{x}_k, \mathbf{y}_k, \mathbf{u}_k, \mathbf{p}) + \mathbf{v}_k, \quad (6)$$

where \mathbf{z}_k is the measurement vector, including the pseudo-measurements, the measured algebraic variables, real and reactive power injections, etc.; \mathbf{h} is the nonlinear measurement function; \mathbf{w}_k and \mathbf{v}_k are called the system process and measurement noise, and assumed to have zero mean and covariance matrices, \mathbf{Q}_k and \mathbf{R}_k , respectively.

When there exist large disturbances in the system, such as three-phase fault, sudden changes, topology switching, etc., DSE-based on (5)-(6) should be used to describe the dynamics. In this context, PMU measurements may be the only source of information to carry out DSE. At the local level (e.g. generation station, substation or FACTS device site), digital recorders or protection devices [16], also referred to as intelligent electronic devices, can provide the required synchronized information to execute DSE. It should be noted that the recent SSE results before the occurrence of disturbance are advocated to initialize the DSE for achieving better convergence speed.

When the system operating point changes exclusively due to slow and smooth load/renewable generation changes, it is said to be under normal operation condition. In this scenario, the generators and other controllers are able to almost "instantly" absorb these slow changes, yielding negligible changes of the dynamic states. However, the algebraic state variable \mathbf{y}_k that contains bus voltage magnitude and angle are involving following the changes of loads and renewable generations. As the dynamics of the system are negligible, by substituting $\mathbf{x}_k = \mathbf{x}_{k-1}$ into (6) and removing (5), we obtain the following well-known SSE model:

$$\mathbf{z}_k = \mathbf{h}(\mathbf{y}_k, \mathbf{u}_k, \mathbf{p}) + \mathbf{v}_k. \quad (7)$$

It is worth pointing out that \mathbf{x}_{k-1} obtained from the DSE can be substitute into (4) to infer a priori information of \mathbf{y}_k via

$$\begin{aligned} \mathbf{0} &= \mathbf{g}(\mathbf{x}_{k-1}, \mathbf{y}_k, \mathbf{u}_k, \mathbf{p}) + \mathbf{G}_k(\mathbf{x}_k - \mathbf{x}_{k-1}) \\ &= \mathbf{g}(\mathbf{x}_{k-1}, \mathbf{y}_k, \mathbf{u}_k, \mathbf{p}) \end{aligned} \quad (8)$$

where (8) is just the power flow equation that can be solved by the Newton method. The priori information not only enhances the convergence speed of the SSE but also provides redundant knowledge for better bad data and topology error detection. We will investigate this in our future work.

B. Change-Point Detection by SSA

From the above derivations, it becomes clear that the switching logic between the SSE and the DSE is determined by the system event/disturbance. To this end, we extend the SSA [13], [14] for power system event detection using time series PMU measurements. Its main idea is that if the mechanism of generating the PMU time series has changed at a certain point, the distance between the subspace spanned by certain eigenvectors of the lag covariance matrix and the lagged vectors will increase. Thus, a statistical test can be performed for change detection. In power systems, if event happens, the temporal and spatial correlations of the PMU measurement time series will change, which will be characterized by the SSA as explained next.

Assume that the PMU measurements are received as a time series $x_n, x_{(n+1)}, \dots, x_N$, where n is the iteration number and $N - n + 1$ is the length of the time series; M is the lag of the

time series. The procedures of using the SSA for change-point detection can be summarized by the following three steps:

Step 1: Construct an l -dimensional space

- Construct the trajectory matrix

$$\mathbf{X}^n = \begin{bmatrix} x_{n+1} & x_{n+2} & x_{n+3} & \dots & x_{n+K} \\ x_{n+2} & x_{n+3} & x_{n+4} & \dots & x_{n+K+1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{n+M} & x_{n+M+1} & x_{n+M+2} & \dots & x_{n+N} \end{bmatrix} \quad (9)$$

where $K = N - M + 1$.

- Perform the SVD on the lag-covariance matrix $\mathbf{R}_n = \mathbf{X}^n(\mathbf{X}^n)^T$ to find its eigenvalue λ and eigenvector \mathbf{U} ;
- Select a group of l largest eigenvalues and their associated eigenvectors, which is denoted as \mathbf{I} and its corresponding eigenvectors are \mathbf{U}_I ;

Step 2: Construct the test matrix $\mathbf{X}_T^n \in \mathbb{R}^{M \times Q}$, whose j th column vector is \mathbf{X}_j , $j = p + 1, \dots, p + Q$, where p and Q are appropriately chosen constants. Formally, we have

$$\mathbf{X}_T^n = \begin{bmatrix} x_{n+p+1} & x_{n+p+2} & x_{n+p+3} & \dots & x_{n+q} \\ x_{n+p+2} & x_{n+p+3} & x_{n+p+4} & \dots & x_{n+q+1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{n+p+M} & x_{n+p+M+1} & x_{n+p+M+2} & \dots & x_{n+q+M-1} \end{bmatrix} \quad (10)$$

where $q = p + Q$.

Step 3: Calculate the statistics for detecting the changes

- Calculate the Euclidean distance $D(n, I, p, Q)$ between the column vectors of \mathbf{X}_T^n and the subspace spanned by \mathbf{U}_I via

$$D_{n,I,p,Q} = \sum_{j=p+1}^q (\mathbf{X}_j^T \mathbf{X}_j - \mathbf{X}_j^T \mathbf{U}_I \mathbf{U}_I^T \mathbf{X}_j) \quad (11)$$

- Calculate the normalized distances S_n as follows:

$$\hat{D}_{n,I,p,Q} = \frac{1}{MQ} D_{n,I,p,Q}, \quad (12)$$

$$S_n = \hat{D}_{n,I,p,Q} / \mu_{n,I}, \quad (13)$$

where $\mu_{n,I}$ is an estimator of the normalized sum of squared distances $\hat{D}_{j,l,p,q}$ at the time intervals $[j + 1, j + m]$ when the hypothesis of no change is accepted and $\mu_{n,I} = \hat{D}_{m,I,0,K}$.

- Calculate the cumulative sum control chart statistics

$$W_{n+1} = \left(W_n + S_{n+1} - S_n - \frac{\kappa}{\sqrt{MQ}} \right)^+ \quad (14)$$

where $W_1 = S_1$; $(a)^+ = \max(0, a)$ for any $a \in \mathbb{R}$ and κ is usually chosen to be $1/\sqrt{9MQ}$ [13].

To demonstrate the effectiveness of the SSA method for system event detection, the recorded power flow data from Oregon to California right before the power outage on Aug 10th, 1996 are used for test. There are two events occurred before the blackout at around 400s and 700s, respectively. The SSA-based event detection results are shown in Fig. 1. It can be observed that the SSA method is able to flag correctly the events as well as record the starting and ending times of the corresponding events. The latter enable the reliable switching between the SSE and the DSE.

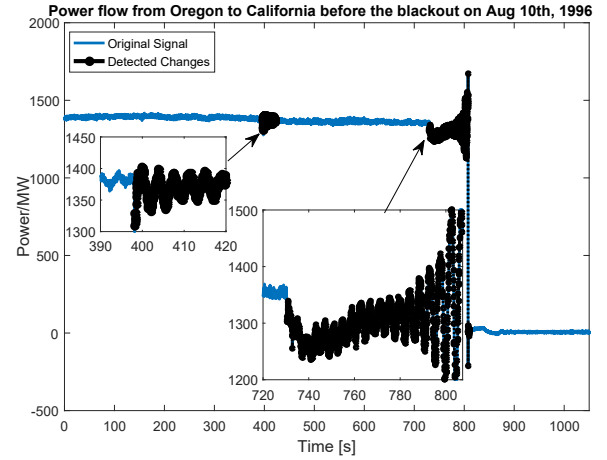


Fig. 1. SSA-based approach for event detection of the 1996 western coast blackout.

C. SSE and DSE Modules

If the SSA-based change point detection method does not flag any event, SSE module is executed based on (8) and the a priori information of \mathbf{y}_k . When the DSE results are not available, a flat-start voltage profile for each bus is adopted to initialize the SSE. To deal with various types of bad data, we advocate the use of the generalized Maximum-likelihood (GM)-estimator [5], [6]. It aims to minimize the following objective function

$$J(\mathbf{y}) = \sum_{i=1}^m \omega_i^2 \rho(r_{S_i}), \quad (15)$$

where ω_i is the weight to bound the influence of bad data, including vertical outliers and bad leverage points; $\rho(\cdot)$ denotes the cost function; $r_{S_i} = r_i / \sigma_i \omega_i$ is the standardized residual; $r_i = z_i - h_i(\hat{\mathbf{y}})$; σ_i is the standard deviation of i th measurement; $\rho(\cdot)$ is the Huber convex cost function. The detailed solution of (15) can be found in [5], [6].

When there are events detected by the SSA-based method, the SSE cannot be used to monitor the power system dynamics as the SCADA measurements are slow and non-synchronized. To this end, the SSE has to be switched to the DSE. On the other hand, initialization of the states is usually required for the DSE. If the flat start is leveraged, the DSE may take too long to track the true states of the system and during that period, key grid dynamics are missed. In addition, there might be convergence issues for the DSE with poor state initialization in presence of strong system nonlinearity. To deal with that, we propose to leverage the recent SSE results for state initialization of the DSE. This is achieved by substituting the estimated $\hat{\mathbf{y}}_k$ into (4) and solving the following power flow equations to obtain \mathbf{x}_k

$$\mathbf{0} = \mathbf{g}(\mathbf{x}_k, \hat{\mathbf{y}}_k, \mathbf{u}_k, \mathbf{p}). \quad (16)$$

When the PMU measurement \mathbf{z}_{k+1} is available, we will rely on the following dynamic model for the DSE:

$$\mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k, \mathbf{y}_k, \mathbf{u}_{k+1}, \mathbf{p}) + \mathbf{w}_{k+1}, \quad (17)$$

$$\mathbf{z}_{k+1} = \mathbf{h}(\mathbf{x}_{k+1}, \mathbf{y}_{k+1}, \mathbf{u}_{k+1}, \mathbf{p}) + \mathbf{v}_{k+1}, \quad (18)$$

Specifically, the robust GM-UKF proposed in our previous work [11] is used to obtain the dynamic states of the system.

III. NUMERICAL RESULTS

Extensive simulations are carried out on the IEEE 10-machine 39-bus test system to validate the effectiveness of the proposed framework. Specifically, the transmission Line 15-16 is tripped to create a large disturbance; the time-domain simulation results are taken as the true values of the state variables and used to simulate PMU measurements; the latter include the voltage and current phasors, and the real and reactive power injections at each generator's terminal bus; then noise is added to them to imitate realistic metered values; the measurement noise is assumed to be distributed with zero mean and covariance matrix $10^{-5} \mathbf{I}_{m \times m}$; a Gaussian random variable with zero mean and covariance matrix $4 \times 10^{-5} \mathbf{I}_{n \times n}$ is assumed for system process noise; all the generators are assumed to be the detailed two-axis generator model with IEEE-DC1A exciter and TGOV1 turbine-governor, whose parameter values are taken from [17]; the break point of the Huber cost function is set to 2. As for the SCADA measurements, the power flow results are treated as the true values and Gaussian noises with zero mean and standard deviation 10^{-2} are added to the obtained power injections and flows to simulate realistic measurements. The SCADA and PMU measurements are updated every 2s and 1/60s, respectively. Due to space limitation, only the estimated rotor angle δ_{5-1} , rotor speed ω_5 , exciter field voltage E_{fd5} and governor mechanical power TM_5 of Generator 5 are shown for illustration purpose.

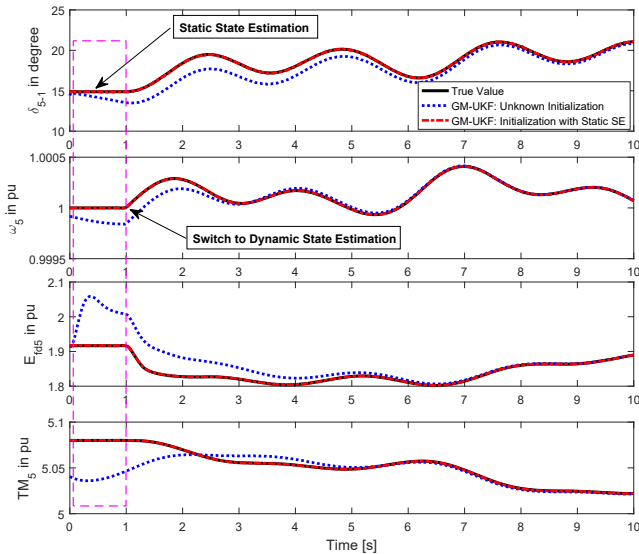


Fig. 2. Benefits of using SSE results to speed up the convergence speed of DSE in presence of large disturbance.

A. Benefits of Initializing the DSE Using the SSE Results

The transmission Line 15-16 is tripped at $t = 0.5s$ and 10s system transients are recorded. To demonstrate the benefits of using recent SSE results before the disturbance for DSE, we have compared the scenarios, where they are used for state initialization or not. The results are shown in Fig. 2. It can be found that without proper state initialization of the DSE,

much longer time is required to converge to the true states as compared to the one that uses SSE results for initialization. In fact, the results do not come as a surprise. For nonlinear filters, the closer the initialization of the states to the true ones, the faster convergence they can achieve.

B. Benefits of Initializing the SSE Using the DSE Results

It is well-known in the literature that the iterative SSE using the Gauss-Newton method requires a good initialization to obtain rapidly converged estimation results [18]. This is precisely the case after the transient process, as the system is operating under unknown steady state conditions and the flat start may not be a good choice. To this end, additional tests are conducted. Specifically, following Fig. 2, another 6s time-domain simulations are added and the SSE is initiated at 10s. It is found that without using the results from the DSE, the SSE diverges while this is not the case for our proposed strategy. Another interesting test is that if the SSE is triggered at 16s, the estimation algorithm using a flat-start converges and takes 8 iterations. By contrast, starting the SSE from 10s using the DSE results and then leveraging the latest SSE run for initialization, our proposed method only takes 4 iterations to converge. It thus becomes clear that the DSE results are able to enhance the convergence of the SSE significantly.

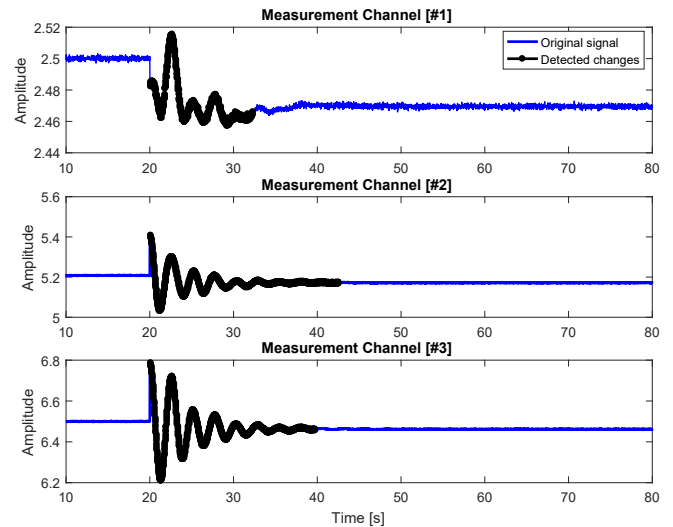


Fig. 3. Event detection of the SSA-based method.

C. Validation of the Proposed Framework

To validate the effectiveness of the proposed framework in transition between the SSE and the DSE interchangeably, we assume that at 20s, Line 16-21 is tripped and another 70s time-domain simulations are used for assessing the framework performance. The SSA-based event detection results are shown in Fig. 3. It is observed that the transmission line tripping induced transient responses have been effectively detected by the SSA method. After that, the SSE results obtained at 20s are used to calculate the dynamic states for initialization and the DSE is activated. The DSE results are displayed in Fig. 4. Thanks to the good initialization, the DSE is able to track the true dynamic states of the system at the very beginning of the

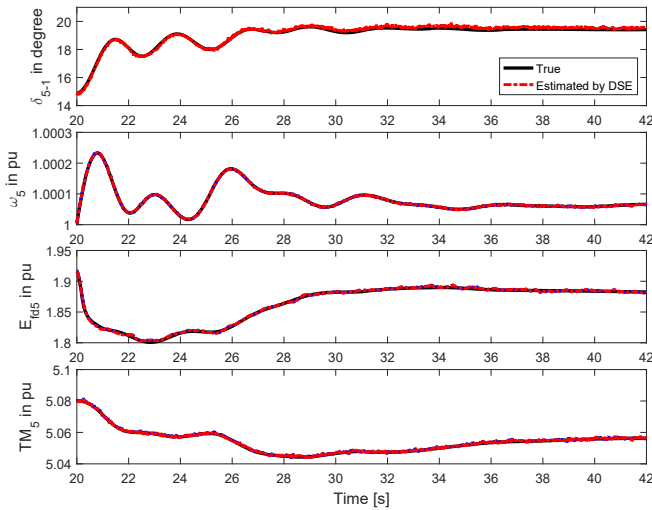


Fig. 4. DSE triggered by the SSA-based detection results at 20s.

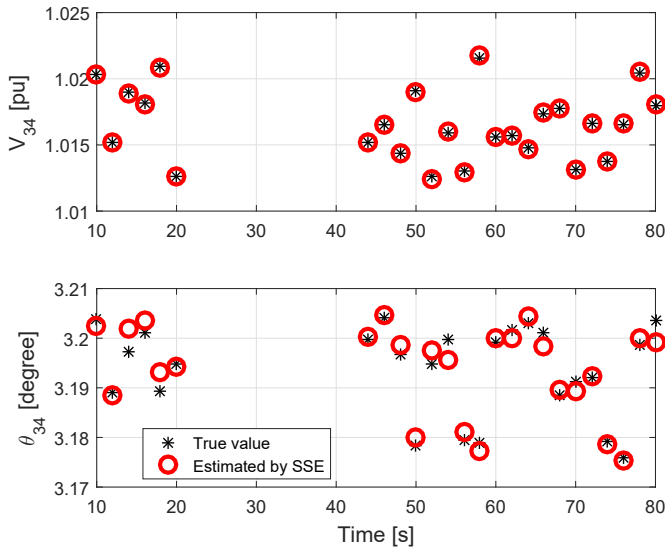


Fig. 5. Voltage magnitude and angle estimation of bus 34 by the SSE, where the results from 20s to 42s are blank as it is not executed due to the transient.

transient process. According to the SSA-based change point detection results, the system is operating under steady-state at 42s. Then, the DSE results are leveraged to calculate the bus voltage magnitude and angles of the system. These are used for the initialization of the SSE. Due to space limitation, only the estimated magnitude and angle at bus 34 are shown for illustration purpose. Its results are shown in Fig. 5. It is found that with the proper initialization via the DSE results, there is no convergence issue. Furthermore, the estimated results match well the true states. This further demonstrates the effectiveness of the SSA method for system change point detection. This is because if there are undetected system changes, the non-synchronized SCADA measurements would not reflect the actual system operations and therefore, the estimated results will be far away from the true states.

IV. CONCLUSIONS AND FUTURE WORK

In this paper, a new multi-scale state estimation framework is proposed that enables the effective integration of SSE and

DSE in EMS. We first derive the connections and logics between the SSE and the DSE from the dynamic system modeling prospective. Then, the SSA-based change point detection approach is developed to enable the reliable transitions between the SSE and the DSE. Specifically, if no event is detected, the system is operated under normal conditions and the robust SSE is executed. Otherwise, the event is declared and the DSE is triggered to monitor the system dynamic states. After the transient process, the DSE results are used for proper initialization of the SSE, yielding significantly improved convergence speed. Simulation results on the IEEE 39-bus system shows that the developed framework is able to track the changes of system states over wide temporal and spatial ranges. In the future, we will test the effectiveness of the proposed framework in realistic systems using field data.

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