

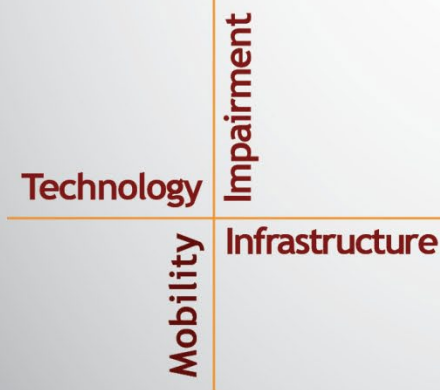
NSTSCCE

National Surface Transportation Safety Center for Excellence

Development of a Nighttime Visual Performance Model by Examining Distributions of Detection Distances

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EXECUTIVE SUMMARY

Modeling the visual performance of drivers at night is complex. In addition to factors like luminance, contrast, observer age, and object size, research has shown that the motion of the object and the expectancy (anticipation of the presence of an object) of the observer play an important role in the observer's ability to detect an object on the roadway at night. Thus, it is important for a visual performance model to account for these factors. However, accounting for these factors could result in highly complex models, as accurately measuring driver expectancy and attention is difficult. A probabilistic approach to modeling nighttime driver visual performance could offer promise.

In a probabilistic modeling approach, the variable of interest is treated as a random variable and the probability distribution of this variable is studied as a response to different conditions. In the case of night driving, we propose to use the detection distance of an object (such as a pedestrian) as the variable of interest. Detection distance is a measure of the reaction time of the driver. By studying the distribution of detection distances of objects under different lighting conditions, we can accurately understand the change in the detection probability of an object as a driver approaches an object.

Researchers in the field of cognitive psychology have started studying reaction time (RT) distributions to better understand human response and perception. Like RT, detection distance is bounded by zero at one end and extends to almost infinity (theoretically) at the other end. This distribution behavior is accurately described by the Weibull function. RTs in different kinds of cognitive tasks have been successfully modeled using the Weibull distribution.

The current report has two goals. The first goal is to test if the detection distance distributions are accurately defined by the Weibull distribution. The second goal is to understand how different light levels affect the detection distance distributions of a child-sized mannequin. This will be accomplished by performing a distribution analysis involving fitting a Weibull distribution to the detection distance data. The distribution fit will indicate how parameters like shape and scale vary across different conditions and their practical impacts on driver visual performance.

The results of the study showed that the Weibull distribution could be used to fit the detection distance data, and that changing the light level definitely influenced the parameters of the distribution. An increase in light level increased the scale parameter and caused the detection distance distribution to stretch out from the pedestrian's location. The results of the study also showed that both the scale and shape parameters could be used to compare the effectiveness of different lighting systems or interventions. The survivor functions of the detection distance data from the fitted Weibull distribution could be used to compare the effectiveness of a lighting system or a countermeasure by calculating the percentage of the population that detected the pedestrian from a distance greater than the stopping sight distance.

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LIST OF ABBREVIATIONS AND SYMBOLS

CDF	cumulative distribution function
DAS	data acquisition system
DGPS	Differential Global Positioning System
PDF	probability density function
RT	reaction time
SF	survivor function

CHAPTER 1. INTRODUCTION

Modeling the visual performance of drivers at night is complex. In addition to factors like luminance, contrast, observer age, and object size, research has shown that the motion of the object and expectancy (i.e., the observer's anticipation of the presence of an object) play an important role in the observer's ability to detect an object on the roadway at night. Thus, it is important for a visual performance model to account for these factors. However, doing so could result in highly complex models, as accurately measuring driver expectancy and attention is difficult. Detection distance is frequently used as a measure of visual performance in research on nighttime roadway visibility (Bhagavathula & Gibbons, 2013; Gibbons et al., 2012; Gibbons et al., 2008; Hills, 1975; Shinar, 1985). In a standard plot of detection distance versus lighting condition, each column or data point represents the mean detection distance of several participants across different lighting conditions (Figure 1).

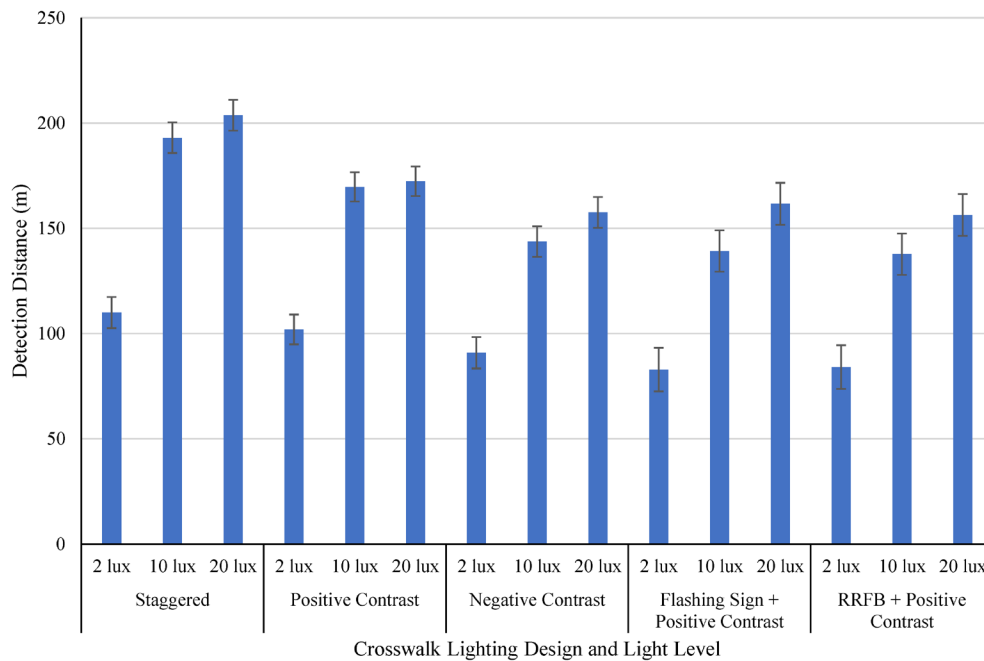


Figure 1. Chart. Example plot of detection distance and lighting level (Bhagavathula & Gibbons, 2023).

The mean detection distance is merely a measure of the central tendency of the underlying detection distance distribution. In research on nighttime roadway visibility, the distributions of detection distance have rarely been studied. In contrast, reaction/response times and their distributions have been extensively studied in psychology (Heathcote et al., 1991; Hockley, 1984; Logan, 1992; Palmer et al., 2011; Spieler et al., 2000). Detection distances are dependent on reaction times; therefore, they should follow similar distributions and trends.

There are two major problems with using means for analysis. First, when using means we are assuming that the underlying distribution is normal. Research has shown that reaction times (RTs) are not normally distributed but rather are positively skewed (Cousineau et al., 2002; Heathcote et al., 1991; Hockley, 1984; Palmer et al., 2011; Van Zandt, 2000). Second, different

conditions in an experiment might affect different sections of the same distribution. A change in the mean RT could be because of a change in the shape/shift/scale of the distribution (Spieler et al., 2000). Conversely, distributions with different shape/scale could have the same mean RTs. Because of these two major issues, researchers in the field of cognitive psychology have started studying RT distributions to better understand human response and perception (Cousineau et al., 2002; Heathcote et al., 1991; Hockley, 1984; Johnson et al., 1994; Logan, 1992; Ratcliff, 1978).

A probabilistic approach to model nighttime driver visual performance could offer a better approach. In such an approach, the variable of interest is treated as a random variable, and the probability distribution of this variable is studied as a response to different conditions. In the case of night driving, we propose to use the detection distance of an object (such as a pedestrian, mannequin, or standard visibility target located either in real road or a full-scale test track) as the variable of interest. By studying the distribution of detection distances of objects under different lighting conditions, we can accurately understand the change in the detection probability of an object as a driver approaches it. For instance, in a night driving situation, for a driver approaching an object on the roadway, the probability of detection increases as the vehicle approaches it. By measuring the object detection distance, a distribution of the detection distances for that object can be developed, which will help us to understand the probability of detecting the object at difference distances on a vehicle's approach.

Like RT, detection distance is bounded by zero at one end and extends to a very long distance, depending on multiple factors like human visual capabilities, earth's curvature, air quality, weather, etc. This distribution behavior is accurately described by the Weibull function (Logan, 1992). RTs in different kinds of cognitive tasks have been successfully modeled using the Weibull distribution (Cousineau et al., 2002; Heathcote et al., 1991; Hockley, 1984; Logan, 1992; Palmer et al., 2011; Ratcliff, 1978).

The goals of this report are to examine the detection distance distributions for a pedestrian at a crosswalk on the roadway at night and to assess how changing the light level affects the shapes of these distributions. This will be accomplished by performing a distribution analysis involving fitting a Weibull distribution to the detection distance data for a child-sized mannequin at different light levels. The distribution fit will indicate how parameters like shape and scale vary across different conditions and their practical impacts on visual performance and driver behavior. Furthermore, using the entire distribution to evaluate the performance of a new intervention (for example, a new lighting system, lighting level, etc.) can help determine the tails of the distribution in addition to the means, especially the lower tail or the fifth/tenth percentile as it shows the worst or highest risk performance. This will also help not only in determining the effectiveness of interventions but also in designing safer interventions that will account for the worst performers.

CHAPTER 2. METHODS

A human factors evaluation was conducted on the Virginia Smart Roads Highway at night in relatively clear weather conditions (no rain/snow/fog). The Smart Roads Highway is a 2.2-mile-long, controlled-access research facility built to U.S. highway specifications. A realistic midblock crosswalk was simulated on the road. A series of crosswalk light levels that are representative of currently used design were installed on the Virginia Smart Roads Highway (see Figure 2 and Figure 3).

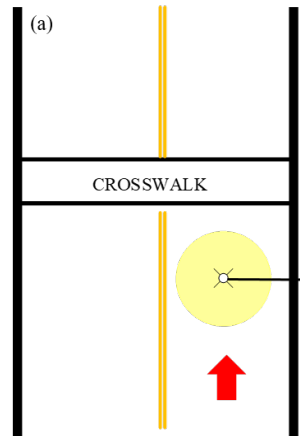


Figure 2. Illustration. Crosswalk lighting design that was evaluated in the study. Red arrow represents direction of vehicle approach.



Figure 3. Photo. Crosswalk lighting design that was evaluated in the study.

PARTICIPANTS

Twenty-four participants were recruited to participate in the study. Two participant age groups (18–35 years and 65+ years) were used to account for the changes in visual capabilities of the participants as they age. Each of these age groups was also gender balanced. All the participants had a valid U.S. driver’s license and a visual acuity of at least 20/40 (measured with Early Treatment Diabetic Retinopathy Study chart with an illuminator cabinet). All experimental activities were approved by the Virginia Tech Institutional Research Board (IRB# 20-332). Participants were paid \$30 per hour for their participation in the study.

EXPERIMENTAL DESIGN

The experimental design is a within-subjects experiment as shown in Table 1. The experiment was planned for three sessions spread across three nights. The order or the presentation of the experimental conditions was counterbalanced across participants with a set number of experimental conditions planned for each session.

Table 1. Experimental design – Independent variables for midblock crosswalk.

Independent Variable	Levels
Light level (average vertical illuminance)	No Light – 0.4 lux 2 lux 10 lux 20 lux

Independent Variable

Light Level: All of the overhead crosswalk lighting designs were illuminated to four light levels based on the average vertical illuminance (see Table 1). The low light level was established at 2 lux. The medium light level was established at 10 lux and is based on the research from Bullough and Skinner (2015). The high light level was established at 20 lux based on research from Edwards and Gibbons (2008). Light level was a within-subjects variable.

Dependent Variable

Crosswalk light levels were assessed by measuring detection distance, the distance at which pedestrians are visible and identifiable to a driver, in a detection task.

PROCEDURE

Three experimental sessions were experienced by each participant spread across three nights of data collection. In the first session, which included both orientation to the experiment and data collection, participants signed an informed consent document and their visual acuities were checked to see if they met the requirements for the study. After a participant provided consent, the in-vehicle experimenter escorted the participant to the experimental vehicle parked outside. The experimenter had the participant sit in the driver’s seat of the vehicle and demonstrated the seat and steering wheel adjustments. The experimenter then asked the participant to make adjustments as needed and to buckle their safety belt.

The in-vehicle experimenter entered the back seat of the vehicle and prepared the data collection equipment. The data acquisition system (DAS) recorded vehicle speed, GPS, and other network data from the vehicle, as well as video and audio inside and outside the vehicle. Once the DAS was ready, the experimenter instructed the participant to drive to the Smart Roads. A speed limit of 35 mi/h (56 km/h) was established for the study, and the in-vehicle experimenter kept track of the participant's speed during the experimental session.

The first lap of the first session was a practice lap. The practice lap was used to familiarize participants with where they would turn around, where the crosswalk was located, and to give them an opportunity to see the mannequins so they knew what to look for.

In each experimental session, the experimental trials began once the participants indicated they were ready. Each time the participant drove through the test area, a different light level would be presented at the midblock crosswalk. Additionally, the mannequins appeared at different locations relative to the crosswalks. As the participant drove, they were instructed to say the word "pedestrian," "kid," or "child" (whichever was easiest for them to remember) whenever they saw one of the child-sized mannequins. The in-vehicle experimenter pressed a handheld button each time a mannequin was identified. This button presses were used as indices in the data stream recorded by the DAS. These indices were used to determine the exact point of detection by the participant by watching and listening to the audio-video stream recorded by the DAS. The DGPS coordinates of the mannequin positions were pre-recorded. The DGPS coordinates at the point of detection between the car and the mannequin were used to calculate the detection distance, which was used as the dependent measure. The accuracy of the detection distances calculated using the DGPS system was about 0.2 m (0.66 ft.). During pilot testing, full-versus child-sized mannequins were evaluated for their suitability. Child-sized mannequins were chosen as they are smaller and more difficult to detect. Each mannequin was 1.2 meters (46 inches) in height. Child-sized mannequins were outfitted in gray-colored scrubs, as shown in Figure 4. Grey was chosen because it is a neutral color and it is rendered similarly under different illuminance levels. Mannequins were located at the entry, middle, and exits of the midblock crosswalk (see Figure 5).

This process of detections continued until the participant encountered all planned light levels for each session. The presentation of crosswalk light levels was counterbalanced. The presentation of mannequins was randomized with "blanks" (i.e., no pedestrian presentation) to keep the participants from guessing.



Figure 4. Photo. Child-sized mannequin wearing gray scrubs.

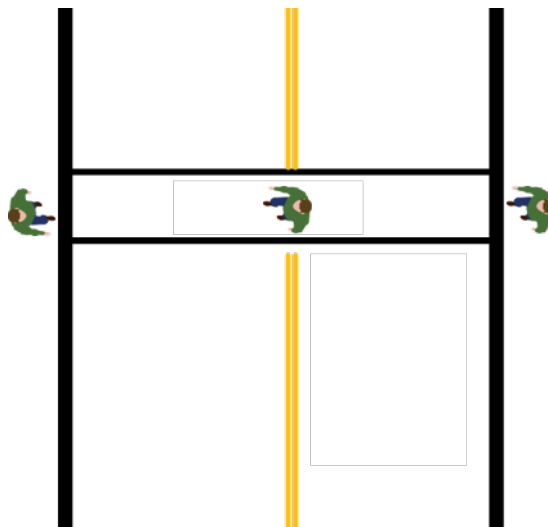


Figure 5. Diagram. Mannequin locations used in the midblock detection task.

Once the experimental session was complete, the participants were asked to drive back to the VTTI building and were then dismissed. Participants drove 20 laps during each session. Each session lasted 1.5 to 2 hours. Participants each took part in three experimental sessions, for a total participation time of approximately 6 hours.

ANALYSIS – DISTRIBUTION FITTING

A Weibull function with two parameters was used to fit the distribution. The probability density function (PDF), the cumulative distribution function (CDF), and the survivor function (SF) of a Weibull distribution with two parameters are given by:

$$PDF: y = f(x/a, b) = \frac{b}{a} \cdot x^{(b-1)} \cdot e^{-\left(\frac{x}{a}\right)^b} \quad (1)$$

$$CDF: y = f(x/a, b) = 1 - e^{-\left(\frac{x}{b}\right)^a} \quad (2)$$

$$SF: y = 1 - PDF = e^{-\left(\frac{x}{b}\right)^a} \quad (3)$$

where, a = scale parameter, b = shape parameter, and x is the random variable.

The PDF of the detection distance is a function that describes the probability that the given detection distance will take on a certain value. The CDF describes the probability that the detection distance will take on a specific value or less. The SF describes the probability the detection distance will take a value greater than a specific value; it is a complementary function of the CDF. The scale parameter is a . It has the same units as x . The larger the scale, the more spread out the distribution, as seen in Figure 6. The shape parameter is b , and is also called the Weibull slope. The shape parameter affects the behavior of the distribution. Figure 6 shows the change in the shape of the Weibull distribution with either a changing scale and/or a changing shape parameter, where the other parameter is kept constant.

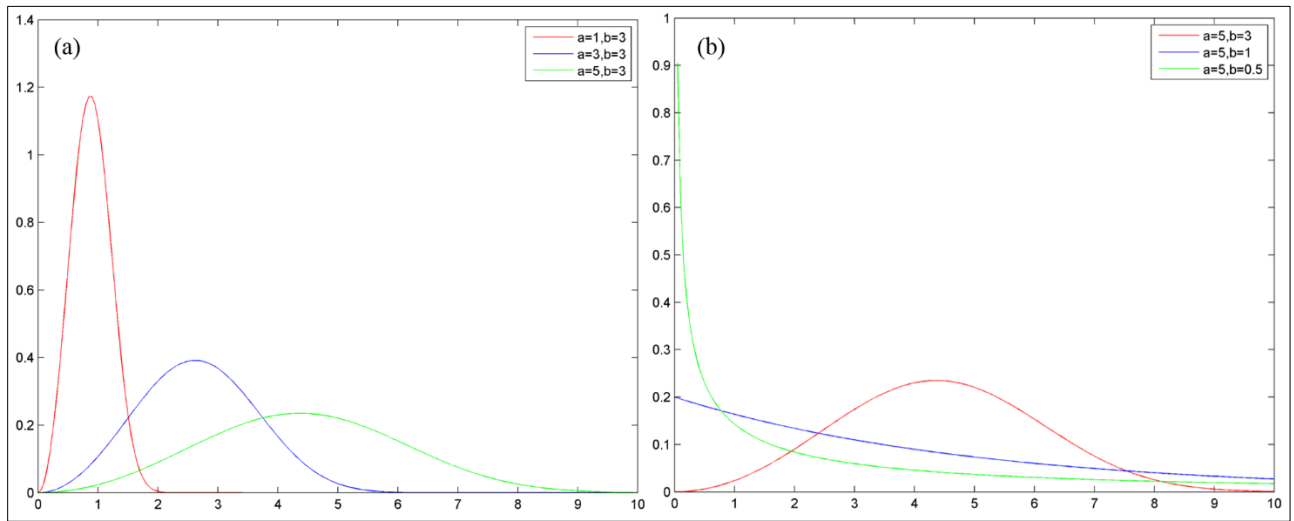


Figure 6. Graphs. Change in the shape of the probability distribution functions of Weibull distribution by changing the scale parameter (a) and the shape parameter (b).

Anderson-Darling goodness of fit was used to assess whether the detection distance followed a Weibull distribution (“Anderson–Darling Test,” 2008; Glen, 2023; National Institute of Standards and Technology, 2021). Statistical significance was established at $p < 0.05$. If the detection distance data fit the Weibull distribution, the goodness-of-fit test would not be significant (p -value should be greater than 0.05); this is similar to other goodness-of-fit tests for normality like Shapiro-Wilk test (Ghasemi & Zahediasl, 2012). If the detection distance data fit the Weibull distribution, the shape and scale parameters of the fitted Weibull distribution were calculated to assess the effects of changing lighting configurations and illuminances. Detection

distance data were fit using JMP. Detection distance data from the four light levels were used for the distribution fitting. Overall, four fits were generated.

CHAPTER 3. RESULTS

Table 2 shows the results from the Anderson-Darling goodness-of-fit test, which indicated that the Weibull distribution fit the detection distance data as evidenced by the lack of significance (Glen, 2023; National Institute of Standards and Technology, 2021).

Table 2. Anderson-Darling goodness of fit results.

Light Level	A2	P-Value
No Lighting	0.39	0.39
2 lux	0.59	0.13
10 lux	0.38	0.42
20 lux	0.57	0.14

Differences in the scale and shape parameters of the fitted distributions were dependent on the lighting configuration and illuminance. The SFs of the child-size mannequin for each light level with the relationship to illuminance are shown in Figure 7. Here it is noteworthy that the light level relationships for 10 and 20 lux overlap, indicating that the scale parameter would be the same and that the influence of lighting on detection has plateaued and adding more lighting beyond 10 lux does not impact the detection of the object.

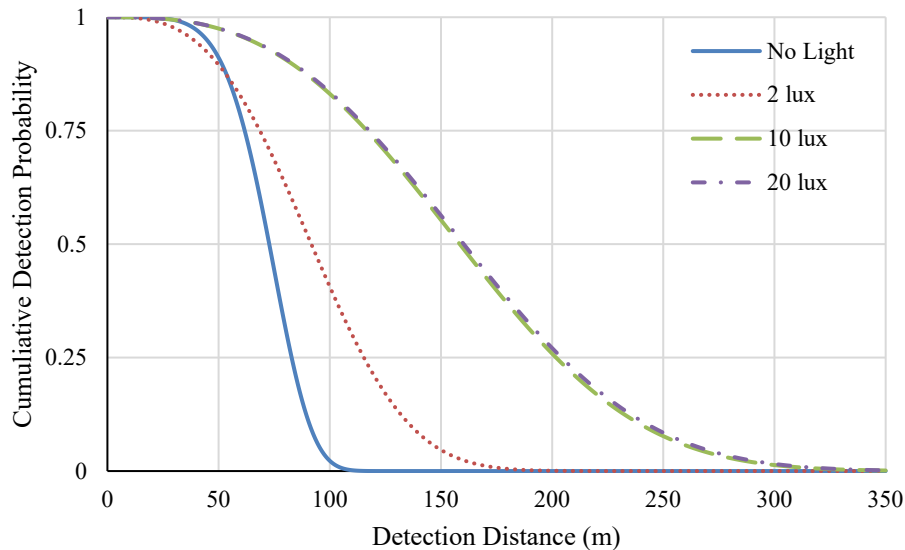


Figure 7. Graph. SF of detection distance of child-sized mannequin at different light levels.

Differences in the scale and shape parameters of the fitted distributions were dependent on the light level. Figure 8 and Figure 10 illustrate the mannequins' scale and shape parameters in the different light levels at the crosswalk. The scale parameter (a) of the mannequins increases with increase in the light level. The scale parameter closely followed the mean detection distance of the object (see Figure 9). The shape parameter decreased with increase in the light level.

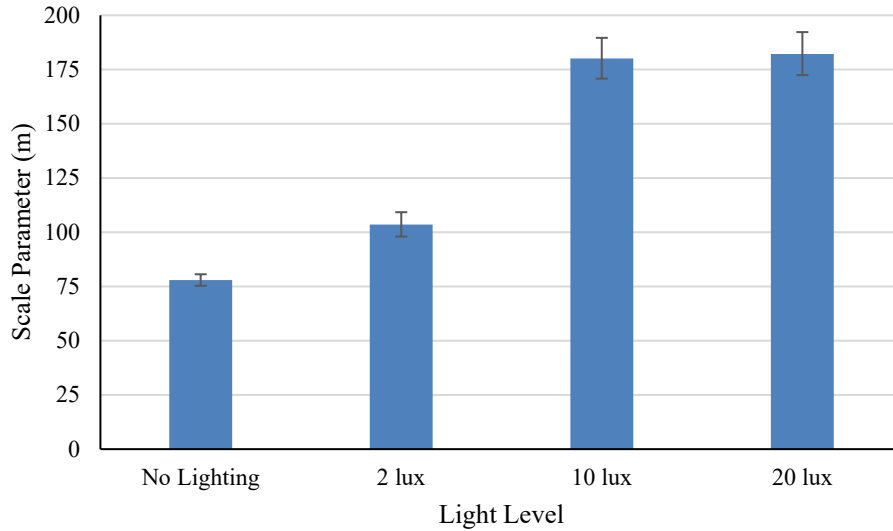


Figure 8. Chart. Scale parameter for the child-sized mannequin at different light levels. Error bars denote standard errors.

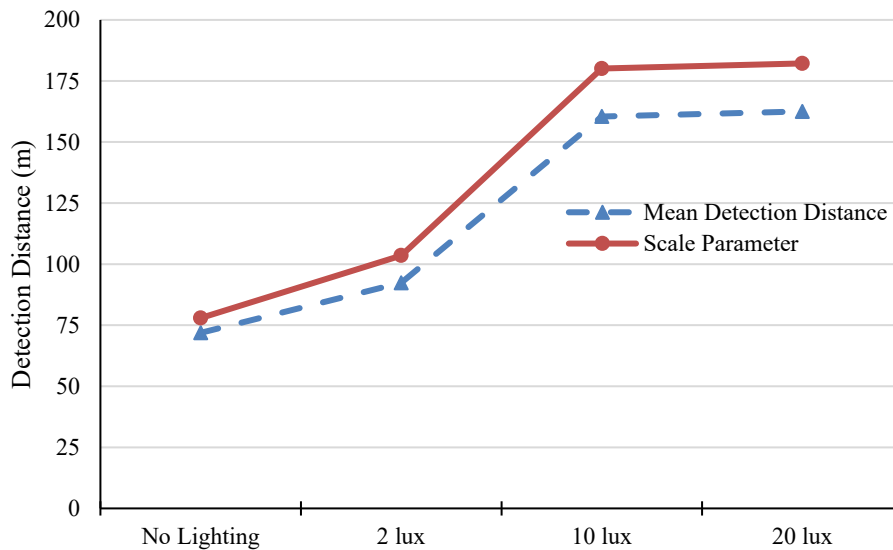


Figure 9. Graph. Scale parameter for every light level follows the mean detection distance.

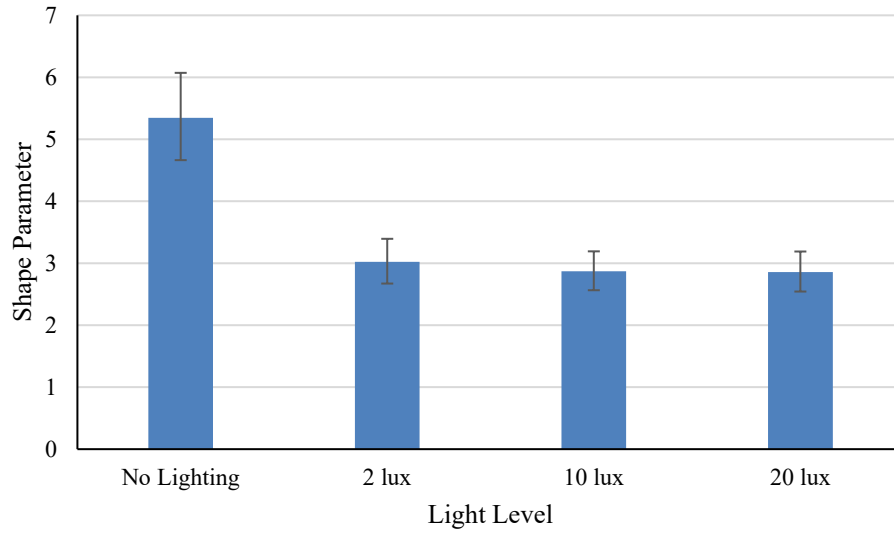


Figure 10. Chart. Shape parameter for the child-sized mannequin at different light levels. Error bars denote standard errors.

CHAPTER 4. DISCUSSION

The goals of this report were to examine the detection distance distributions of a pedestrian on the roadway at night and to understand the effects of light levels on the parameters of the distribution. Two major findings are evident. First, it was determined that Weibull distribution could be used to fit the detection distance data. Second, the results showed that changing the light level definitely influences the parameters of the distribution.

Increase in light level increases the scale parameter on the Weibull distribution fit. Higher light levels also make the detection distance distribution stretch out from the location of the pedestrian. This could be attributed to the increased availability of visual information in the presence of roadway lighting, which makes pedestrians visible from further away and therefore increases the range of distances from which they can be seen. There is also a strong correlation between the scale parameter and the mean detection distance of the object. As illustrated in Figure 9, the scale parameter seems to be directly affected by the mean detection distance. This may indicate that an increase in a pedestrian's visibility is associated with a higher scale parameter value.

The effect of lighting on the shape parameter tends to affect the spread of the distribution. A lower scale parameter value resulted in a more positively skewed distribution, as observed in the no lighting condition. Because of this, the shape parameter mainly controls the behavior of the distribution's tails. Increase in the light level resulted in lowering the shape parameter with a massive decrease from no lighting to the 2-lux light level.

Both the scale and shape parameters could be used to compare the effectiveness of different kinds of interventions, such as new versus existing lighting systems. Distribution analysis helps understand the performance of drivers at the tails of the distribution. It is important to know the performance of the lower tails of the population because crashes often happen at tails of the distribution, where the drivers detect the pedestrian late and do not have enough time and distance to come to a stop to avoid a collision. Furthermore, for two distributions with the same scale parameter but different shape parameters, the distribution with the higher shape parameter could be deemed more effective, as its lower tail population would have significantly longer detection distances than the lower tail population of the distribution with the lower shape parameter. For example, the pedestrian detection distances in different light levels could be compared using the SF to compare the detection distances of the lower tail (here 10th percentile) of the population (Figure 11). The Weibull distribution of the detection distances in the lower light levels has a lower scale parameter than the distribution in the higher light levels. In this case, 90% of the population detected the mannequin in the higher light level (20 lux) at a distance of 82 meters. In the no lighting conditions, 90% of the population detected the mannequin at 51 meters, a distance 60.8% shorter than in the highest light level.

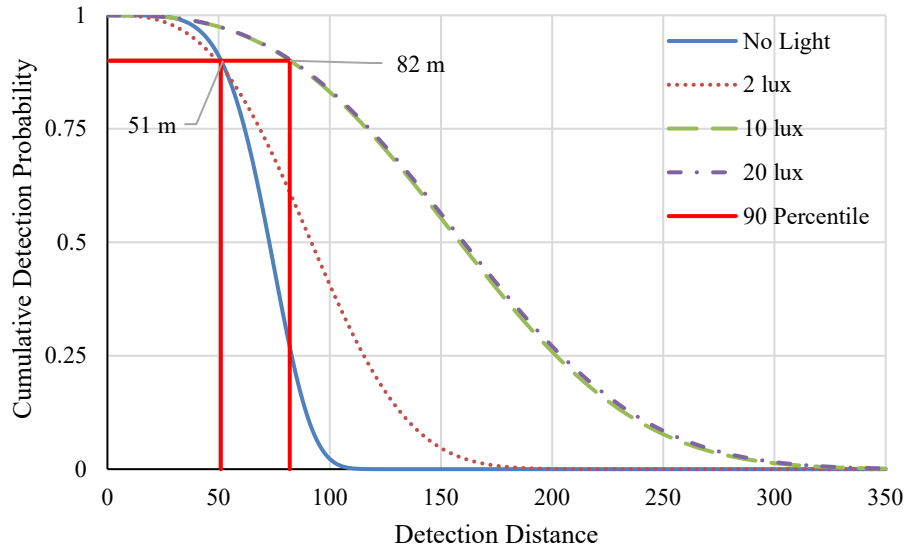


Figure 11. Graph. SFs of the child-sized mannequin at different light levels indicating the distance at which the 90th-percentile of drivers detected the mannequin.

The SFs of the detection distance data from the fitted Weibull distribution could also be used to compare the effectiveness of a lighting system or a countermeasure by calculating the percentage of the population that detected the mannequin from a distance greater than the stopping sight distance. Stopping sight distance is the distance required for the vehicle to come to a complete stop when travelling at a certain design speed. According to the American Association of State Highway and Transportation Officials (AASHTO), for a design speed of 35 mi/h, which is also the speed limit used in the study, the recommended stopping sight distance is 76.2 meters (AASHTO, 2001). The percentage of drivers who detected the pedestrian at distances greater than stopping sight distance could be used as a performance measure. This can be easily determined from the detection distance distributions by using the SFs (see Figure 12). Using the SFs of the pedestrian detection distances at different light levels, it is clearly evident that 91.5% of the drivers detected the mannequin at a distance greater than the stopping sight distance at higher light levels (20 lux and 10 lux), whereas only 36.7% of the drivers accomplished this in the absence of lighting. Finally, these results also show that the increase in the light level beyond 10 lux has no significant benefits in the increase of detection distances of mannequin. These results also confirm the results of another analysis (Bhagavathula & Gibbons, 2023) on the same data which explored statistical differences between detection distances at each light level and showed that there were no statistical differences between the 10-lux and the 20-lux light levels. However, it is important to note that no light levels were tested between 2 lux and 10 lux, and it is possible to see a plateau in detection distance (no increase in detection distance with increase in light level) at the light level.

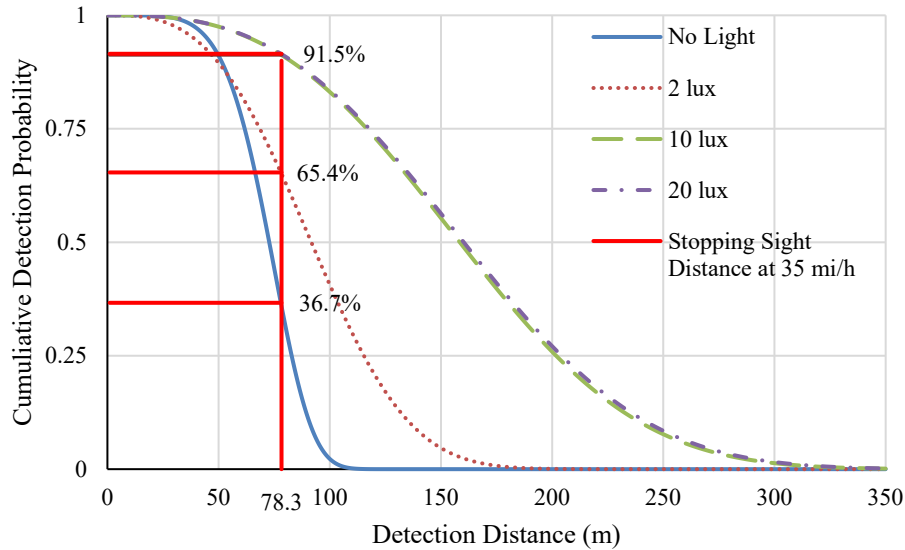


Figure 12. Graphs. SFs of the mannequin at different light levels used to calculate the percentage of drivers who detected the pedestrian at distances greater than the stopping sight distance.

It is noteworthy that there are some limitations to human vision. There is a detection threshold that is related to an object's contrast, the age of the observer, the size of the object, and observation time. The plateau in the data indicates that the threshold limit for the data has been reached. Using the shape parameter as an indicator of visual threshold can be a valuable tool in building an understanding of the needs in roadway lighting.

There is a limitation to using distribution analyses. Hundreds of trials would be required for each distribution to accurately estimate the true shape and scale parameters (Van Zandt, 2000). This could be very expensive and time-consuming. However, it is worth the effort to study and analyze the detection distance distributions if they can provide more detailed information about nighttime pedestrian visibility than analyzing means alone.

CHAPTER 5. CONCLUSION

An innovative approach to analyzing nighttime visibility data by examining detection distance distributions was considered. Weibull functions were used to capture the resulting empirical detection distance distributions. Results showed that:

- An increase in light level tends to increase the value of the scale parameters of the Weibull function and that an increase in the mannequin's visibility results in higher scale parameter values. This is also supported by the fact that the scale parameter is affected by the mean detection distance.
- The shape parameter tends to affect the tails of the distribution. The shape parameter might also be dependent on light level.
- The effectiveness of roadway lighting system can be assessed through new statistical approaches, particularly the visual threshold using the SF of the Weibull distributions.

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