DOES THE RELITIVE PRICE OF NON-TRADED GOODS CONTRIBUTE TO THE SHORT-TERM VOLATILITY IN THE U.S./CANADA REAL EXCHANGE RATE? A STOCHASTIC COEFFICIENT ESTIMATION APPROACH

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(ABSTRACT)

This study uses a random coefficient estimation procedure to test the hypothesis that much of the volatility in the U.S./Canada real exchange rate over the time period 1971 through 1999 is due to the relative price of non-traded goods to traded goods. The model specification used in this study provides estimates of the sensitivity of movements in the U.S./Canada real exchange rate to movements in both the relative price of traded goods and the relative price of non-traded goods to traded goods in each of the two countries. I test for purchasing power parity in each of the two components of the model and address the question of volatility through the examination of the time profile of the respective coefficient estimates. The empirical results support the conclusion that the average value of the coefficient on the relative price of non-traded goods to traded goods component is smaller than that on the relative price of traded goods component. However, purchasing power parity in both components can not be rejected when the period of study is limited to 1971 through 1994. Furthermore, examination of the time profile of the random coefficients on the relative price of non-traded goods to traded goods component suggests that it is much more volatile and, therefore, quite significant in capturing the volatility in U.S./Canada real exchange rate movements.

With regard to purchasing power parity in both the traded goods component and the non-traded goods to traded goods component, these results are consistent with the implications of the theory of purchasing power parity. However, they are not entirely consistent with the evidence presented in recent literature. Specifically, evidence presented in recent studies can not support perfect purchasing power parity in either traded goods or non-traded goods and leads to the conclusion that non-traded goods are much less significant, if at all, in the determination of the U.S./Canada real exchange rate. This inconsistency with recent literature is most likely a result of the fact that the random coefficient modeling technique used in this study allows the coefficients to vary over time and, thereby, enables the volatility of both components to be captured in the model. Therefore, given the apparent significance of the relative price of non-traded goods to traded goods, the volatility of this component can logically be expected to significantly contribute to the volatility in the U.S./Canada real exchange rate.

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This thesis is dedicated to my wife, Kelly Thorne, and my son, Matthew Thorne. Thank you so much for your love, support, and patience.

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INTRODUCTION:

An interesting occurrence in the area of international economics is the deviation of U.S. short-term real exchange rates from the equilibrium values predicted by the theory of purchasing power parity and the underlying law of one price. The degree of this variation has been shown to be quite substantial since the demise of the Bretton-Woods System of currency convertibility in 1973 and, as a result, empirical models based on this theory have produced results inconsistent with observed short-term real exchange rates.[1] This inconsistency has prompted the questioning of both the overall ability of the theory of purchasing power parity to explain U.S. short-term real exchange rate movements and the relevance of non-traded goods in this theory. However, in approaching these two questions, it is important to consider the possibility that international trade patterns and real exchange rates may not be completely flexible in the short run and the degree of this flexibility may vary considerably and follow a random pattern over time.

Regarding the question of credibility of the theory purchasing power parity, DeGrawe (1996) and Krugman and Obstfeld (2000) present evidence supporting the conclusion that short-term variation in the U.S. real exchange rate is a common observance and, therefore, the theory of purchasing power parity describes a necessary long-term equilibrium condition. Evidence presented by DeGrawe (1996) on this subject suggests that the nominal exchange rates and real exchange rates of industrial countries have been highly correlated during the floating exchange rate period (after 1970). Canzoneri, et al (1999), also test for evidence of long-run purchasing power parity in traded goods and find little evidence against a unit root in the series for nominal exchange rates and the series for purchasing power parity rates. In their study, Canzoneri, et al conclude that nominal exchange rates and purchasing power parity rates are generally co-integrated, however, the data provides mixed results for a unit root in the difference between these series. In fact, when the U.S. dollar is the reference currency, Canzoneri, et al present evidence that the slopes of the co-integrating relationships vary considerably and are substantially different from one. Since the test for a unit root in the difference between nominal rates and purchasing power parity rates (a co-integrating slope of one) is rejected, the relationship can not be said to be stationary and strong evidence favoring the assumption of absolute purchasing power parity can not be confirmed.

In regard to the question of the relevance of non-traded goods in the theory of purchasing power parity, evidence presented by Engel (1999) leads to the conclusion that U.S. real exchange rate movements are essentially the result of movements in the relative price of traded goods.[2] When using the consumer price index as a measure of price, Engel finds that the drift, measured as the mean square error, in the real exchange rate is almost entirely due to the drift in the traded goods component in all cases except the U.S./Canada real exchange rate. The portion of the drift in the real exchange rate attributable to the traded goods component is .485 for the U.S./Canada rate compared to .999, .996, .993 and .857 for U.S./Japan, U.S./Germany, U.S./France and U.S./Italy, respectively. Therefore, the relative price of non-traded goods is thought to have little influence on U.S. real exchange rate movements. With regard to Canada, the traded goods component accounts for over 95% of the short-term movement in the real exchange rate, however, the importance of the traded goods component falls to 45% over a 15 year period and

[1] See Paul De Grauwe, "International Money" , (1996).
[2] See Engel, Charles, "Accounting for U.S. Real Exchange Rate Changes" (1999)

begins to rise again for years beyond the 15 year period. This result for the U.S./Canada rate is quite interesting and is the focus of study.

As evidenced by Engel (1999), the significance of movements in both the traded goods component and the non-traded goods component to movements in the U.S./Canada real exchange rate is not constant over time. This fact leads to the hypothesis to be explored in this study: that the significance and volatility of these two components is best captured through the use a random coefficient modeling procedure. This will allow for a more accurate forecast and provide insight into possible causes for the volatility of movements in the U.S./Canada real exchange rate. In this context, the examination of the time profile of the coefficients on each component will provide insight into the nature of the variation of the short term U.S./Canada real exchange rate from the equilibrium value consistent with the theory of purchasing power parity. This is accomplished by considering data over the time period of 1971 to 1999 and applying the random coefficient modeling procedure described by Swamy and Tavalas to a model of the U.S./Canada real exchange rate derived from the theory of purchasing power parity.[3]

Despite the apparent lack of short-term predictive power of models based on purchasing power parity and the relative insignificance of non-traded goods, as presented above, it is generally accepted that purchasing power parity is essential in explaining the adjustment path of short-term real exchange rate to the equilibrium long-term rate.[4] It is, however, probable that the lack of short-term predictive power is due, at least in part, to measurement error in price indices, the effect of omitted variables and the variation in the value of the coefficients on the model components over time. Considering this, the model specification presented in this study introduces random coefficients and avoids the not entirely plausible classical linear regression assumptions of constant coefficients, the capturing of the effect of all omitted variables in an independent error term, and the use of data that does not contain measurement errors. As will be explained in section VI of this paper, this specification also utilizes measures of relative productivity and relative output as concomitant variables to assist in describing the relationship between the explanatory variables and their respective coefficients.[5] This particular method of providing structure to the random process generating the coefficients is distinctly different than that of simple random walk or autoregressive processes and adds the additional benefit of incorporating into the model the information contained in the influence of these concomitant variables. Through the use of these concomitant variables and this modeling technique, it is shown that the relative price of traded goods and the relative price of non-traded goods to traded goods have distinct implications for movements in the U.S./Canada real exchange rate.

In particular, the time profile of the random coefficients on the relative price of traded goods is less volatile over the time period studied and the average value is greater than that of the

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[3] See Swamy, P.A.V.B and Tavalas George, "Random Coefficient Models" (2000) and Swamy P.A.V.B and Tavalas George, "Random Coefficient Models: Theory and Applications" (1995).
[4] The general acceptance of this theory is discussed in DeGrauwe, P, "International Money", (1996) and Krugman, Paul and Obstfeld, Maurice, "International Economics", (2000).
[5] A concomitant variable is a variable that is not included in the equation determining the relationship between the independent and dependent variables, but is used in the estimation of the coefficients on the independent variables by assisting in the explanation of correlation between these coefficients and the independent variables. For a complete discussion, see Swamy, P.A.V.B. and Travlas, George, "Random Coefficient Models" , (1999).

random coefficients on the relative price of non-traded goods to traded goods in each country. Over the period of 1971 to 1999, the time profile of the coefficients on the relative price of traded goods component is relatively stable and the periodic value of the coefficient ranges from 0.841 to 1.192 with an average value of 1.026. The time profile of the estimate of the direct effect of the relative price of traded goods on the U.S./Canada real exchange rate over the same time period includes periodic values ranging from 0.9882 to 1.033 with an average value of 1.014.[6] In contrast, the time profile of the random coefficients on the relative price of non-traded goods to traded goods component is steadily declining over this time period and the periodic value of the coefficient ranges from -1.060 to 1.480 with an average value of 0.661. The time profile of the estimate of the direct effect of this component on the U.S./Canada real exchange rate includes periodic values ranging from -1.933 to 1.886 with an average value of .520. Therefore, over the time period of 1971 through 1999, the variance of the coefficients on the relative price of traded good component from the mean value is considerably less than the variance of the coefficients on the relative price of non-traded to traded goods component from the mean value.

Another important result is that, even though the average value of the coefficients on the relative price of non-traded goods to traded goods is lower than that on the relative price of traded goods component, the values are greater than one in absolute value in many periods. In fact, purchasing power parity in both components can not be rejected when the period of study is limited to 1971-1994. As can be seen above, the coefficients on the relative price of non-traded goods to traded goods component range from having a substantially positive affect to having a substantial negative affect on the real exchange rate. In fact, of the 29 periods observed in this study, the value of the random coefficients are greater than the maximum value of the random coefficients on the relative price of traded goods in 7 of the 29 periods or 24% of the time in the study. The value of these random coefficient are, however, lower than the minimum value of the random coefficients on the relative price of traded goods in 15 of the 29 periods, or 52% of the time. It is this variance in the random coefficients on the relative price of non-traded goods to traded goods component in each country that captures much of the volatility of movements in the U.S./Canada real exchange rate as predicted by a model based on the theory of purchasing power parity. As expected, the random coefficients on the relative price of traded goods show much less variation from the average value. The empirical results are presented in section VI.

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[6] The calculation and significance of the direct effect vector is explained in section VI. Also see Swamy, P.A.V.B and George Tavalas, "Random Coefficient Models", 1995 and 2000.

I. BACKGROUND:

The foundation of this particular approach is the assumption that there is some degree of randomness inherent in the short-term movements of real exchange rates. The particular specification of the model used in this study captures a portion of the randomness inherent in the movements of the U.S./Canada real exchange rate by allowing the coefficients on inflation differentials in both the traded goods sector and non-traded goods sector to vary over time. The model used follows the common assumption that a change in the relative price of goods is a major factor determining the relative demand for domestic goods and foreign goods. The level of demand for these goods affects the demand for the respective currency and, therefore, the nominal exchange rate. This relationship is important to the value of the real exchange rate because the nominal exchange rate changes by an amount necessary to keep the real exchange rate at the equilibrium level. The logic of this statement is consistent with the assumption that, according to the theory of purchasing power parity, the fundamental determinants of real exchange rates are inflation and productivity. However, here the additional assumption is taken that responses to real price shocks include a random element. Therefore, through the use of measures of both productivity and output differentials, logic can be extended to the process describing the randomness of the coefficients in the model describing movements in the U.S./Canada real exchange rate. To more completely explain the basis for these assumptions, reference should be made to the theory of purchasing power parity and the law of one price, which are among the dominant constructs in international trade and finance theory[7].

The theory of purchasing power parity is an equilibrium relationship where a change in the nominal exchange rate is a result of a change in the relative rates of inflation between two countries. The degree of this change will be exactly what is required to maintain the relative purchasing power of a currency. In the long run, the theory of purchasing power parity and the law of one price imply that nominal exchange rates should adjust so that an identical product costs the same, in terms of a currency, in all countries. The only difference should be attributed to transportation costs, etc. This implication must be true if the purchasing power of a currency is to be maintained and inflation in not transferred from one country to another. In order for this to be the case, the nominal exchange rate must adjust to reflect differences in the price level between any two countries. In the absence of such an adjustment process, arbitrage opportunities would exist where a good could be purchased from the country where the price is relatively lower and sold in the country where the price is relatively higher. The reason is that inflation causes the real value of one unit of the domestic currency, in terms of purchasing power, to change relative to one unit of the foreign currency.

This simple theory is used to make long run predictions about the effects of inflation on nominal exchange rates and, therefore, predict if, and to what extent, a currency will appreciate or depreciate relative to another. That is, the currency of a particular country must be depreciating relative to the currency of another country if the former country is experiencing a greater rate of inflation than the latter. With purchasing power maintained through this adjustment process, the theory of purchasing power parity can also be thought of as simply an extension of the quantity theory of money. Specifically, due to the long run proportionality

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[7] See Krugman, Paul and Obstfeld, Maurice, "International Economics", (2000) and DeGrauwe, Paul, "International Money", (1996).

between money and prices, changes in the money stock have proportional affects on the overall price level. This change in the price level will affect both the domestic and foreign purchasing power of a currency and will require a proportional change in the nominal exchange rate for relative prices and the real exchange rate to remain at equilibrium levels.[8]

According to the theory of purchasing power parity, the equilibrium level of the nominal exchange rate can be written as:

$s_t = q_t\,(P/P^*)$ ***(Equation A)***

where:

- s_t is the nominal exchange rate, q_t is the real exchange rate, P is the domestic price level and P^* is the foreign price level.

In this context, the real exchange rate can be thought of as a factor of proportionality allowing for equality between the nominal exchange rate and the ratio of price levels. The implication here is that, at equilibrium, the real exchange rate should remain constant if the additional assumption is taken that that the real demand for foreign goods remains constant. This assumption is necessary because it has been shown that the proportionality of this long-run equilibrium relationship will only hold to the extent that the exogenous disturbances in the price levels are created by monetary factors[9]. Therefore, the fact that the proportionality does not hold if the disturbance is real is extremely relevant given empirical data shows that real disturbances are common and the real demand for foreign goods is not constant.[10] An implication of this observation is that, when non-monetary factors are considered, real trade preferences may be affected by changes in relative prices and, therefore, the forces of supply and demand must force the real exchange rate to an equilibrium level. This is accomplished through changes in the nominal exchange rate of a magnitude sufficient to reflect the change in purchasing power that the currency must command over both domestic and foreign good. However, the process is likely to be long in duration and consist of substantial short-term variations from this equilibrium level.[11]

With this taken into consideration, the theory of purchasing power parity implies that movements in the real exchange rate, which is equivalent to inflation adjusted nominal rates, should reflect changes in the real demand for domestic and foreign goods. To show this, the previously stated equilibrium relationship in *Equation A* can be transformed in terms of the level of the real exchange rate and written as follows:

$q_t = s_t\,(P^*/P)$ ***(Equation B)***

It can then be seen that shocks creating changes in the ratio of prices affect the real exchange rate and the nominal exchange rate can be thought of as a factor of proportionality or, equivalently,

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[8] Gustav Cassel shows that the theory of purchasing power parity holds only if the sources of price disturbances are monetary. See Cassel, Gustav, "Money and Foreign Exchange after 1916" (1922).
[9] See Cassel, Gustav, "Money and Foreign Exchange after 1916" (1922).
[10] See De Grauwe, Paul , "International Money", (1996) for detailed discussion.
[11] See DeGrauwe, Paul, "International Money", (1996)

that which is required to bring the real exchange rate to its equilibrium level. From this representation of the purchasing power parity condition, a real shock, such as changes in relative productivity or relative output, can be incorporated into the relationship to determine the extent that the shock affects the real exchange rate, given a particular nominal exchange rate.

However, a major consideration in the use of this theory is that some goods and services can not be traded across national frontiers (land, buildings) and many can be traded only to a limited extent. In fact, Engel (1999) concluded that the inflation rate for goods that are not tradable have little bearing on real exchange rates.[12] The implication of the study by Engel is that real exchange rates are determined primarily by the exchange of traded goods and, therefore, the theory of purchasing power parity and the law of one price apply mainly to tradable goods. This distinction between traded goods and non-traded goods expands upon the theory of purchasing power parity by addressing the fact that not all goods are traded in international markets, however, they are included in the overall price level of a particular country. This notion is best represented through the use of the Balassa-Samuelson hypothesis of exchange rate determination.

This prevalent hypothesis is based on the theory of purchasing power parity and explains real exchange rate movements in terms of sectoral productivities[13]. The first component of the hypothesis is the assumption that the relative price of non-traded goods reflects the relative labor productivities in the traded and non-traded sectors of an economy. The second component is the assumption of purchasing power parity for traded goods. With these components considered, this hypothesis suggests that changes in relative inflation rates and, therefore, changes in the real exchange rate depend not only on the price of traded goods, but also the relative price of non-traded goods. It is further suggested that changes in the relative price of goods in these two sectors are proportional to relative productivity, which is defined as the ratio of marginal costs or, equivalently, the ratio of marginal labor products.[14] By considering the importance of relative productivity, the logic of this hypothesis has important implications to the adjustment process of real exchange rates to their equilibrium values and, therefore, the proper selection of concomitant variables in the model presented in section IV.

First, the Balassa-Samuelson hypothesis implies that the relative price of non-traded goods, both within the domestic country and relative to the foreign country, must be increasing if the ratio of traded goods productivity to non-traded goods productivity is increasing more rapidly in the domestic country than in the foreign country. This is so because the relative price of non-traded goods in the domestic country increases due to the increase in productivity in the traded goods sector and the resulting reduction in the unit cost of traded goods. The lower unit price that results from the increase in productivity makes the traded goods of the domestic country

[12] See Engle, Charles, "Accounting for the U.S. Real Exchange Rate", (1999)
[13] See Balassa, B, "The Purchasing Power Parity Doctrine", (1964); Samuelson, "Theoretical Notes on Trade Problems", (1964); and Asea. PK, et al, "The Balassa-Samuelson Model: A General Equilibrium Appraisal", (1994).
[14] It has been suggested by Canzoneri, et al, that the ratio of marginal costs is proportional to the ratio of average labor products in the traded and non-traded sectors of the economy. See Canzoneri, Matthew, Cumby, Robert E., Diba, Behzad, " Relative Labor Productuvuty and the Real Exchange Rate: Evidence for a Panel of OEDC Countries", (1999)

more attractive to foreign countries.[15] This creates an increase in foreign demand for domestic traded goods, and domestic currency, that forces the price of the domestic currency up to the equilibrium level. Therefore, the stronger domestic currency is due to the adjustment in the nominal exchange rate required to equalize the price of traded goods. Without a corresponding productivity increase of the same magnitude in the non-traded goods sector of the domestic economy, the price of non-traded goods will become more expensive in terms of the foreign currency due to the higher price of the domestic currency. Assuming that the ratio of traded goods prices to non-traded goods prices in the foreign country has remained unchanged, the stronger domestic currency will also cause both the traded and non-traded goods of the foreign country to be relatively less expensive.

Second, due to the lower unit cost that results from the increased productivity in the traded goods sector, wages can grow at a faster rate in the traded goods sector and the economy can still remain competitive internationally. This is so because of the assumption that the price of traded goods must be the same in each country. However, this increase in production efficiency and wages is not transferred to the non-traded goods sector. It is this concept of traded goods price equality, in terms of the purchasing power of a currency, that is thought of as being vital in explaining the adjustment process of the real exchange rate. It is shown in this study that the inclusion of non-trade goods, for which no such equality relationship is defined, is also of significant importance.

[15] The increase in efficiency may result in a reduction in the unit cost of a product, a product of superior quality and comparable cost or both. In any case it is likely that the demand for the product and, therefore, demand for the currency will increase.

II. BASIS:

As previously stated, recent literature has questioned both the importance of the relative price of non-traded goods as a determinant of the real exchange rate and the overall validity of the purchasing power parity theory in explaining real exchange rate movements, especially when the U.S. dollar is the reference currency. This questioning has been supported in recent studies and it has been shown that many of the econometric models based on the theory of purchasing power parity and the Balassa-Samuelson hypothesis have not faired well with empirical data.[16] Considering these theories and the information in recent studies, the question to be addressed in this study also concerns the significance of the effects of movements in the relative price of traded goods and movements in the relative price of non-traded goods to traded good on movements in the U.S./Canada real exchange rate. This study, however, attempts to address the fact that the sensitivity of real exchange rate movements to movements in these two components is likely to include a random element. Therefore, applying the random coefficient estimation procedure described by Swamy and Tavalis to a model of real U.S./Canada exchange rate determination based on the theory of purchasing power parity will allow for the assessment of the time-varying behavior of both components of the model. The basis for the assumption of randomness in the coefficients of these components of real exchange rate determination will be discussed for the remainder of this section and again in section IV after the assumptions of the model have been established.

To begin, the observed volatility in market determined short-term exchange rates provide conceivably the most compelling evidence supporting the assumption of randomness inherent in short-term exchange rate values. The assumption of random coefficients seems prudent given the fact that the real exchange rates have deviated substantially from the values predicted by purchasing power parity and have been extremely volatile since the currencies of the world's industrial countries have been allowed to float. In fact, De Grauwe (1996) presents evidence that the observance of substantial volatility in short-term exchange rates around the long-term trend is largely the result of the fact that exchange rates have been market determined rather than fixed by government or central bank policy. This is especially true since the end of the Bretton-Woods System of currency convertibility in 1973. Since the demise of this system, exchange rates of major international currencies have been determined, to various extents, by the market forces of supply and demand and have been allowed to follow a controlled float.[17] As a result, exchange rates have become much more volatile and short-term movements have deviated substantially from the equilibrium value consistent with purchasing power parity[18].

To consider the nature of the short-term variability of exchange rate movements, Paul De Grauwe (1996) conducted a study that compared movements in exchange rates with two benchmarks: expected exchange rate movements and changes in the price level.[19] In the context of this study, the relevance of the expected exchange rate follows from the assumption of rational

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[16] See De Grauwe, Paul, "International Money" , (1996)

[17] The term "controlled" or "dirty" float is used to address the fact that foreign exchange intervention by national central banks or governments is still prominent to prevent major changes in the value of the currency of their respective country.

[18] See De Grauwe, Paul, "International Money", (1996)

[19] See De Grauwe, Paul, "International Money", (1996)

expectations and the efficient market hypothesis. This basically implies that people are rational and that investment and import/export decisions of market participants are based on the predictability of short-term exchange rate movements. Following from this implication, De Grauwe presents the position that the best measure of the short-term exchange rate expected by the market is represented by the forward premium in the currency options market. This position is supported by another dominant theory in international economics: the interest parity theory.

The interest parity theory basically states that the short-term exchange rate must be such that deposits of all currencies offer the same expected rate of return. However, De Grauwe (1996) presents evidence of a substantial difference in observed changes of short-term exchange rates and forward premiums since exchanges rates have been allowed to float. The implication of this data is that the forward rate is a biased predictor of short-term exchange rate movements. The bias is proven by the fact that movements in the short-term exchange rate deviated substantially from movements in the forward premium in both magnitude and direction and, therefore, the forward market rate could not be an accurate predictor of the exchange rate movements. This supports the conclusion that the changes in the short-term exchange rate are largely unexpected or included a substantial random component. A similar result occurred when exchange rate movements are compared to movements in inflation differentials. De Grauwe also presents evidence leading to the conclusion that movements in the short-term exchange rate are much larger than movements in inflation differentials. Additionally, De Grauwe presented evidence that deviations of actual short-term exchange rates from the equilibrium value predicted by purchasing power parity followed trends of several years in duration and often the actual short-term exchange rate moved in the opposite direction of that predicted by purchasing power parity.

The nature of short-term real exchange rate variability is also shown by De Grauwe to have very long cycles of divergence from the values predicted by purchasing power parity and, therefore, the adjustment to long-run purchasing parity equilibrium is thought to be quite long in duration. This particular conclusion supports the idea that the theory of purchasing power parity primarily describes a long-term equilibrium relationship. However, this study provides evidence supporting the position that the productivity in the traded goods and non-traded goods sectors of the economy are of significant importance in determining this adjustment process. Specifically, the proportionality between the changes in inflation differentials and changes in real exchange rate movements are affected by real economic shocks that result from changes in productivity and will, thereby, affect the equilibrium exchange rate.

Paul De Grauwe considers this fact by constructing productivity-corrected purchasing power parity values against which observed values are compared. The results of De Grauwe's study are that the theory of purchasing power parity and the use of production differentials combined to give an accurate explanation of long-run trends in exchange rate movements, however, are not able to explain short-term movements. In addition, significant deviations are present between the real exchange rate and the productivity corrected rates. In contrast to De Grauwe's approach, this study will integrate the productivity differentials into the model to assist in explaining the relationship between the relative price of traded goods and the relative price of non-traded goods to traded goods and their respective coefficients. The implication of using this model specification is that the results will reflect the influence of productivity trends on the coefficient estimates over time and assist in explaining the direct effect of each component on the real exchange rate.

With regard to the significance of the effect of movements in the relative prices of non-traded goods, Engel (1999) conducted a study that examined the relevance of non-traded goods in the determination of real exchange rates. The study measured the proportion of the U.S. real exchange rate movements that could be attributed to movements in the relative prices of non-traded goods.[20] This is done by separating the changes in the overall price level into specific traded goods and non-traded goods components and, thereby, expressing the theory of purchasing power parity in such a manner as to separate the adjustment for the inflation differential in terms of these two components. In this study, Engel develops various price indices in order to account for the relative prices of traded and non-traded goods. A distinct price index is created to measure non-traded good prices by using each of the following: consumer price indices, output prices, price deflators for personal consumption expenditures, aggregate producer price indices, and consumer prices adjusted for marketing services. The difficulty in Engel's study appears to be in the determination of a precise price index for non-traded and traded goods. However, Engel concludes that, even with various measures of the relative price of non-traded goods, the non-traded goods component is of little importance in determining the real exchange rate, or, equivalently, that the real exchange rate is determined almost entirely by the relative price of traded goods. In this study, I use same rationale as Engel in distinguishing between traded and non-traded goods. The assumption is that traded goods consist of all manufactured goods and food that can reasonably be traded outside the geographical boundary of a country without the need for exorbitant transportation costs. The implication here is that trade of certain goods would necessitate transportation cost of a magnitude that would render the transaction economically unfeasible. It is also assumed that non-traded goods consist of housing and services. This distinction is difficult due to international consumption of services and the consumption of services by foreign tourist. However, for the purposes of this study, this distinction between traded and non-traded goods will serve as a good proxy.[21] The composition of the traded good index and the non-traded goods index for the U.S. and Canada is presented in Section V and in Appendix A.

However, in contrast to Engel's approach, this study addresses the significance of non-traded goods in the context of the time profile of a random coefficient model and, therefore, addresses the trend. The random process generating these coefficients is given structure through the use of concomitant variables that assist in explaining the relationship between the coefficients and the respective variable. The selection of the proper concomitant variables is, therefore, extremely important and the criteria for the proper selection of concomitant variables are described briefly in section IV. The starting point, however, is with economic theory. The particular theory used in this study to provide insight into the selection of concomitant variables is the Balassa-Samuelson hypothesis. As presented in section I, this hypothesis explains real exchange rate movements in terms of sectoral productivity differentials.

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[20] See Engel, Charles, "Accounting for U.S. Real Exchange Rate Changes", (1999).

[21] Although some services can be considered a traded good, such as some financial services and the consumption of domestic services by foreign tourists, this is considered to be small enough as not to significantly distort traded goods and non traded goods price indices.

The results of a study by Canzoneri, et al (1999) lead to the conclusion that the Balassa-Samuelson hypothesis fails to explain short-run movements in the real exchange rate and, also, has problems explaining some long run exchange rates, especially when the U.S. dollar is the reference currency.[22] However, Canzoneri, et al present evidence that the relative price of non-traded goods generally reflects the relative labor productivities in the traded and non-traded sectors of the economy. In this case, Canzoneri, et al could find little evidence against a unit root in the series for relative productivities and relative prices of non-traded goods. However, the critical result here is that the series for relative prices of non-traded goods and the relative productivities in the traded and non-traded goods sectors are co-integrated and the slope of the co-integrating relationship is found to be generally close to one, as is required by Balassa-Samuelson. In the context of this study, the fact that this relationship exists reinforces the choice of measures of relative productivities as concomitant variables.

[22] See Canzoneri, et al, "Relative Labor Productivity and Real Exchange Rate in the Long-Run: Evidence for a Panel of OEDC Countries" (1999)

III. LOGICAL FOUNDATION:

The model used by Engel (1999), as well as others, to relate the relative prices of traded goods and non-traded goods between countries to the real currency exchange rate serves as the basis of this paper. As previously stated, an important assumption in the model is that the overall price level in a particular country relative to that of another is assumed to be associated with the price level in both the traded and non-traded goods sectors. The model will now be discussed.

To begin, it is assumed that all goods produced and consumed within a country can be classified as either tradable or non-tradable. It then follows that the overall price index in a particular country can be given by a geometrically weighted average of prices of traded goods and prices of non-traded goods. The price index of the home country can be represented by the following equation:

$$\mathbf{p_t = (1-\alpha)p_t^T + \alpha p_t^N} \qquad \textit{(Equation 1.0)}$$

where:

- p_t is the natural logarithm of the price index, p_t^T is the natural logarithm of the traded goods price index, p_t^N is the natural logarithm of the non-traded goods price index and α is the share that non-traded goods take in the price index.

When the same logic is extended to the foreign country *Equation 1.0* is written with an asterisk in the following manner:

$$\mathbf{p_t^* = (1-\beta)p_t^{T*} + \beta p_t^{N*}} \qquad \textit{(Equation 1.1)}$$

where:

- p_t*, p_t^T*, and p_t^N*, have the same description as p_t, p_t^T, and p_t^N, respectively, except that they now represent price indices in the foreign country. The share that non-traded goods take in the foreign price index is represented by β.

The real exchange rate between two countries can then be given rewriting *Equation B* in logarithmic form and using the results of *Equations 1.0 and 1.1* as follows:

$$\mathbf{q_t = s_t + p_t^* - p_t} \qquad \textit{(Equation 1.2)}$$

where:

- s_t is the natural logarithm of the domestic currency price of the foreign currency
- p_t^* is the natural logarithm of the overall price level in the foreign country
- p_t is the natural logarithm of the overall price level in the domestic country

Here the real exchange rate, in terms of the domestic currency, is shown to be the nominal exchange rate of the domestic country's currency with that of the foreign country plus the difference between the foreign country's price level and the domestic country's price level. It can be seen from *Equation 1.2* that the nominal exchange must decrease if a change in foreign monetary policy causes a change in the foreign price level of a greater magnitude than the change in the price level in the domestic country ($p_t^* > p_t$). This adjustment in the nominal rate is necessary to maintain the purchasing power of the domestic currency relative the foreign currency and keep the real rate constant. In other words, inflation is not transferred to the domestic country. It can also be seen that real price shocks, which affect real trade preferences, result in changes to the real exchange rate (q_t) if the change in the nominal rate (s_t) is not exactly proportional to the change in the price level differential.

Equation 1.2 will now be written as the sum of the two distinct components x_t and y_t as follows:

$$\mathbf{q_t = x_t + y_t} \qquad \textit{\textbf{(Equation 1.3)}}$$

where:

- $\mathbf{x_t = s_t + p_t^{T*} - p_t^T}$
- $\mathbf{y_t = \beta(p_t^{N*} - p_t^{T*}) - \alpha(p_t^N - p_t^T)}$

This representation of the model shows that the natural log of the real exchange rate is comprised of the relative price of goods that are traded between two countries (x_t) and the weighted difference of the relative price of non-traded goods to traded goods in each of the two countries (y_t). The form of the model represented in *Equation 1.3* will be used in this study. The main assumption of this model is that, as previously stated, the price level in one country relative to that in another can be associated with the relative prices of traded goods and the relative difference between the prices of non-traded goods to traded goods.

In regard to the above discussion, the significance of the elements in y_t of *Equation 1.3* of the model should be noted. As presented by Engel (1999) the non-traded goods component is a "relative relative" price. That is, the relative price of non-tradable goods to tradable goods in a particular country is relative to the relative price of non-tradable goods to tradable goods in another country. This presentation has a distinct advantage over most previous studies in that the previous studies address only the relative price of traded goods to non-traded goods in a particular country. In addition, Engel cites that previous studies do not consider the importance of non-traded goods prices in the overall change in the exchange rate. This representation considers the effect of movements in the price of non-traded goods relative to changes in prices of traded goods when considering overall changes in the real exchange rate. Engel (1999) concludes that relative prices of non-traded goods account for little of the movement of U.S. real exchange rates.[23] However, the results do vary more when the movements in the U.S. exchange rate are analyzed with respect to the United Kingdom and Canada.

The difference between this study and the study conducted by Engel is in the randomness of the coefficients on the independent variables x_t and y_t in *Equation 1.3*. The random coefficient modeling technique described by Swamy and Tavlas (1999) will be used to estimate this relationship and the time profile of the coefficients will be examined for the period of 1973-

[23] See Engel, Charles, "Accounting for U.S. Real Exchange Rates Changes", (1999)

1999.[24] The theory given by Balassa-Samuelson will be used to obtain the appropriate concomitant variables. In particular, I will use measures of manufacturing output per hour and average annual hours in manufacturing in both the United States and Canada to serve as the concomitant variables used to give structure to the process creating the randomness in the coefficients. As will be explained in the next section, the use of these measures as concomitant variables is an attempt to explain the correlation between movements in the relative price of traded goods and movements in the relative price of non-traded goods to traded goods and their respective coefficients. The use of these particular measures captures important elements affecting the average labor product in the traded goods sector and, therefore, productivity in the traded goods sector relative to productivity in the non-traded goods sector. The price indices described by Engel will be used in constructing the data sets.

[24] See Swamy, P.A.V.B. and George Tavlas, "Random Coefficient Models: Theory and Applications" (1999) and Swamy, P.A.V.B. and George Tavlas, "Random Coefficient Models" (2000)

IV. THE MODEL:

This study uses a random coefficient specification of *Equation 1.3* presented in section III. The particular model specification presented used is of the general form developed by Swamy and Tavlas (1995 and 2000)[25]. This specification is used to provide forecasts of the real exchange rate and a time profile of the coefficients representing the true economic relationships between the explanatory variables and the explained variable. The application of the general model of Swamy and Tavalas to *Equation 1.3* will be presented after the assumptions have been briefly discussed. The following section will discuss the generalized random coefficient model as presented by Swamy and Tavlas.[26]

The basis of this approach is that the weakness of many econometric models is due to the model specification and restrictions imposed by the ordinary least squares and generalized least squares regression models. In particular, the use of classical linear regression models requires assumptions that can be argued not to be true when using economic time series data. The most fundamental of these assumptions are: that a constant vector of coefficients can be used to relate the dependent and independent variables; that the effect of all excluded variables is captured through the use of a normally distributed and independent additive error term with an expected value of zero; the observed values of variables are true and do not include measurement error; and that the true functional form of the model is known. However, due to the very nature of economic time series data, it is unlikely that these assumptions can be supported. As a response to this problem, traditional random coefficient models have relaxed the assumption of a constant vector of coefficients. Swamy and Tavalas describe these models as first-generation models and present them as follows:

$$y = x_{t1}\beta_{t1} + \Sigma^{K}_{j=2} x_{tj}\beta_{tj} = \mathbf{x}_t'\boldsymbol{\beta}_t \qquad \textbf{\textit{(Equation 2.0)}}$$

where:

- $\mathbf{x}_t'$ is a row vector of K explanatory variables; $\boldsymbol{\beta}_t$ is a column vector of K coefficients; $x_{t1} = 1$ for all t; x_{tj} for j = 2…K are all explanatory variables and t serves as an index for time.

In *Equation 2.0*, the column vector $\boldsymbol{\beta}_t$ is changing and, in order to have a well-defined model, structure must be introduced into the process determining the manner in which $\boldsymbol{\beta}_t$ is changing. This structure can be defined by the variation of the coefficients from their mean value and $\boldsymbol{\beta}_t$ can be defined by the equation

$$\boldsymbol{\beta}_t = \overline{\boldsymbol{\beta}} + \varepsilon_t \qquad \textbf{\textit{(Equation 2.1)}}$$

where:

- $\overline{\boldsymbol{\beta}}$ in *Equation 2.1* is a vector of means and ε_t is the vector of disturbance terms. It is these disturbance terms that are the random elements.

[25] See Swamy, P.A.V.B . and George Tavlas "Random Coefficient Models", (1999)
[26] See Swamy, P.A.V.B and George Tavlas 'Random Coefficient Models", (1999) for a complete presentation of the generalized random coefficient model and derivation of *Equations 2.4 through 2.8*

Equation 2.1 can then be defined as

$$y = \mathbf{x}_t'\overline{\boldsymbol{\beta}} + \Sigma^{K}_{j=1} x_{tj}\varepsilon_{tj} \qquad \textbf{\textit{(Equation 2.2)}}$$

Serial correlation can be introduced into *Equation 2.2* when t indexes time as follows

$$\varepsilon_t = \Phi\varepsilon_{t-1} + a_t \qquad \textbf{\textit{(Equation 2.3)}}$$

where:

- Φ in *Equation 2.3* is a matrix describing the autoregressive process generating the coefficients and a_t is a traditional spherical disturbance.

With the above specification, the stochastic structure determining the coefficients can be made to vary as a function of the set of observable variables. Again, the random element is the error or disturbance term and it is assumed that the random components are identically and independently distributed with an expected value of zero.

Swamy and Tavalas (1995 and 2000) also present a second-generation random coefficient model that modifies the first-generation model to relax all of the restrictions mentioned above and introduces the use of concomitant variables to explain the randomness in the coefficients. Concomitant variables are variables that are not included in the equation, however, are used to explain the correlations between the coefficients and the explanatory variables. This modification makes the assumptions that the values of observed variables contain measurement error and omitted variables can not be assumed to be independent of other included variables. It is also likely that the indirect effect of omitted variables and the effects of measurement errors are not constant and, therefore, do not support the idea of constant coefficients. In this framework, each coefficient consists of the sum of three parts: a direct effect of the true value of the explanatory variables on the true value of the explained variable; an indirect effect due to the bias of omitted variables; and the effect of mismeasurement of the explanatory variables.

In contrast to the classical model and the generation-one random coefficient model, this model is general and, as such, will have a different functional form for different paths of variation in the coefficients and will be unrestricted as long as the variation in the coefficients is unrestricted. It can then be assumed that the model will describe the true relationship between the explanatory variables and the explained variables with a certain, but unknown, variation in the coefficients. It can also be assumed that if the restrictions are violated, as they are in the classical regression model and the first generation random coefficient model, the estimates will be subject to specification error.

As in the case of any model, specification error will be present if values of the observed variables contain measurement error. Since most variables are observed and, therefore, measurement errors are likely, or at least possible, the true values are unlikely to be included in the model. Specification error will also be present if the functional form of the model is assumed to be true when it is actually false. The relevance of this statement in the present context is that the true functional form of the equation is not known and little is likely to be known about the omitted variables. Due to this fact, the set of omitted variables can not be assumed to be independent of the included variables and may indeed change over time. The specification of the model described by Swamy and Tavlas (1999) includes time varying coefficients and it is more likely to include the true values of the variables and be of the correct functional form. Therefore, with this specification, it is more reasonable to assume that the model will be true and the coefficients will describe the true elasticities of the explanatory variables.

The second-generation model with two independent variables is as follows:

$$y_t = \alpha_{0t} + \alpha_{1t}x_{1t}^* + \alpha_{2t}x_{2t}^* + \Sigma^{nt}_{j=3}\, \alpha_{jt}x_{jt}^* \qquad \textbf{\textit{(Equation 2.4)}}$$

where:

- the α's represent the coefficients, x_{jt}^* represents all omitted variables, x_{1t}^* and x_{2t}^* are the included variables (the asterisks represent true values), n_t is the number of explanatory variables.

Swamy and Tavalas (1995 and 2000) interpret the coefficients as of the sum of three parts: (1) the direct effect of the true value of an explanatory variable on the true value of an explained variable, (2) the indirect effect (omitted-variable bias) due to the fact that the true value of the explanatory variable may affect the true values of the excluded variables, which affect the true value of the explained variable (The total effect that the excluded explanatory variable has on the explained variable includes the effect of the correlation of the excluded variable with the included explanatory variables and the effect of the excluded variable after the effects of the included variables have been removed) and (3) the effect of mismeasuring the explanatory variable. If each of the coefficients of the random coefficient model has this interpretation all conditions for the observability of stochastic laws as defined by Pratt and Schlaifer (1988) will be met and, according to Swamy and Tavalas (1999), will coincide with the underlying true economic relationship.[27] Since the observed variables are likely to contain measurement errors and little is known about the omitted variables, the question now is how to estimate this model.

The first step is to make the assumption that the omitted variables are correlated with the included explanatory variables. Swamy and Tavalas (1996 and 2000) present that x_{jt} in *Equation 2.4* can be stated as follows:

$$x_{jt}^* = \psi_{0jt} + \psi_{1jt}x_{1t}^* + \psi_{2jt}x_{2t}^* \qquad \textbf{\textit{(Equation 2.5)}}$$

where:

- ψ_{0jt} is the portion of the omitted variable remaining after the effects of the included explanatory variables have been removed and ψ_{1jt} and ψ_{2jt} represent the partial correlations between the omitted variables and the respective included explanatory variables.

[27] For a discussion of the conditions for the observability of stochastic laws see Pratt, John W. and Robert Schlaifer, "On the Nature and Discovery of Structure", (1984). Also see Swamy and Tavalas, "Random Coefficient Models" (1999) for application of these conditions to the random coefficient model.

When *Equation 2.5* is substituted into *Equation 2.4* the resulting equation is as follows:

$$y_t = (\alpha_{ot} + \Sigma^{nt}_{j=3}\, \alpha_{jt}\psi_{ojt}) + (\alpha_{1t} + \Sigma^{nt}_{j=3}\, \alpha_{jt}\psi_{1jt})x_{1t}^{*} + (\alpha_{2t} + \Sigma^{nt}_{j=3}\alpha_{jt}\psi_{2jt})x_{2t}^{*} \quad \textbf{\textit{(Equation 2.6)}}$$

where:

- the coefficients represent both the direct effect of x_{1t}^{*} and x_{2t}^{*} on y_t (α_{ot}, α_{1t}, α_{2t}) and the omitted variable bias ($\Sigma^{nt}_{j=k}\, \alpha_{jt}\psi_{0jt}$, $\Sigma^{nt}_{j=k}\, \alpha_{jt}\psi_{1jt}$, $\Sigma^{nt}_{j=k}\, \alpha_{jt}\psi_{k-1,jt}$).

The next step is to adjust the model for measurement error.

If *Equation 2.4* is written in the form

$$y_t = \gamma_{0t} + \gamma_{1t}x_{1t} + \gamma_{2t}x_{2t} \qquad \textbf{\textit{(Equation 2.7)}}$$

where x_{1t} and x_{2t} are written without an asterisk to represent observed values and *Equation 2.6* is adjusted to include these observable values, Swamy and Tavalas define the coefficients in *Equation 2.7* as follows:

$$\gamma_{ot} = (\alpha_{0t} + \Sigma^{nt}_{j=3}\, \alpha_{jt}\psi_{0jt} + v_{ot}),$$
$$\gamma_{1t} = (\alpha_{1t} + \Sigma^{nt}_{j=3}\, \alpha_{jt}\psi_{1jt})(1 - v_{1t}/x_{1t})$$
$$\gamma_{2t} = (\alpha_{2t} + \Sigma^{nt}_{j=3}\, \alpha_{jt}\psi_{2jt})(1 - v_{2t}/x_{2t})$$

where:

- v_{ot} represents the measurement error of y_t in *Equation 2.7* and v_{1t} and v_{2t} are the measurements errors associated with x_{1t} and x_{2t} in *Equation 2.7*, respectively.[28]

This term captures the effect of this measurement error as a random deviation from the true value and the coefficient is adjusted by this term.

With this specification, the coefficients will be random and will have interpretations that account for the direct effect (real economic relationship) of the explanatory variables on the explained variable, the effect of the correlation between omitted variables and included variables and the effect of mismeasurment errors in observed variables. It is important to note the following correlations that exist between the coefficients: γ_{1t} is a function of x_{1t}; γ_{2t} is a function of x_{2t} and all coefficients include α_{jt} as a source of variation. It is, however, the direct effect that is of interest because it is here that the true elasticities are provided. It now becomes necessary to separate the direct effects from the other effects contained in the interpretation of the coefficients. This is accomplished by obtaining estimates of α_{1t} and α_{2t}.

The estimation of the direct effects of the explanatory variables on the explained variable involves the use of concomitant variables to estimate the γ's. The assumptions related to this specification are that the concomitant variables explain the variation in the coefficients. It is also assumed that the error term has an expected value of zero for each variable and over all time periods and is some value of the error term of the previous period plus an additional error term.

[28] Swamy and Tavalis (1999 and 2000) provide a detailed explanation of coefficients in *Equation 2.7.*

According to Swamy and Tavalas, the coefficients of *Equation 2.7* satisfy the following equation:

$\gamma_{kt} = \pi_{ko} + \Sigma\, ^{p}{}_{j=1}\, \pi_{kj} z_{jt} + \varepsilon_{kt}$ ***(Equation 2.8A)***

where:

- z_{jt} are the concomitant variables and ε_{kt} the error (unexplained portion)

The error term is defined as:

$\varepsilon_{kt} = \phi_{kk}\varepsilon_{kt-1}$

where:

- ϕ_{kk} is the autocorrelation parameter and it is assumed that explanatory variables in *Equation 2.7* are independent of this error term.

Swamy and Tavalas rewrite *Equation 2.7* in terms of *Equation 2.8A* as follows:

$y_t = \pi_{00}+\Sigma\pi_{0j}z_{jt} + \pi_{10}x_{1t}+\Sigma\pi_{1j}z_{jt}x_{1t} + \pi_{k-1,t}x_{k-1,t}+\Sigma\pi_{k-1,j}z_{jt}x_{k-1,t} + e_{0t}+e_{1t}x_{1t}+e_{k-1,t}x_{k-1,t}$
(Equation 2.8B)

The set of concomitant variables used in the model must be chosen after examining the implications for the estimates of the direct effect portion of the coefficients. Among the validation criterion for choosing concomitant variables are that the set of concomitant variables chosen should have a high degree of explanatory power. Also, the signs and statistical significance of the direct effect estimates remain virtually unchanged as various sets of concomitants, other than the determinants of the direct effect, are put into the model.[29] This allows for confidence that the included explanatory variables have the assumed effects on the explained variable.

Equation 1.3 can now be written with the interpretations given in *Equation 2.7* as:

$q_t = \gamma_{0t} + \gamma_{1t}x_t + \gamma_{2t}y_t$ ***(Equation 2.9)***

where:

- x_t and y_t are as described in section III.

To be consistent with the theory of purchasing power parity and *Equation 1.3*, the value of $\gamma_{0t,}$ γ_{1t} and γ_{2t} in *Equation 2.9* should be equal to 0,1and 1, respectively. Therefore, appropriate test would be for $\gamma_{0t} = 0$, $\gamma_{1t} = 1$ and $\gamma_{2t} = 1$. However, as presented in section II, recent empirical studies have provided evidence that the value of γ_{2t} is not significantly different from zero when q_t in *Equation 2.9* represents the real exchange rate. In the case of the U.S./Canada real exchange rate, the non-traded goods component is shown in section II to be significantly more

[29] For a complete discussion of criteria for choosing concomitant variables see Swamy and Tavals, "Random Coefficient Models" (1999).

important in explaining the drift in the U.S./Canada real exchange rate than in explaining the drift in the U.S/Japan, U.S./Germany, U.S./France and U.S./Italy real exchange rates.

To derive the results presented in section VI, *Equation 2.9* is written as follows:

$q_t = \beta_t A'_t$ ***(Equation 2.10)***

where:

- β_t is a column vector of random coefficients with the same interpretations as the γ's in the above discussion and A'_t is a row vector of explanatory variables (x_t and y_t).

β_t in *Equation 2.10* is then defined as follows:

$\beta_t = \Pi z_t + L\varepsilon_t$ ***(Equation 2.11)***

where:

- Π is a K x P random coefficient matrix; zt is the P x 1 column vector of concomitants; ε_t is the error term; and L is an identity matrix .

The error term can be described as follows:

$\varepsilon_t = \Phi\varepsilon_{t-1} + a_t$ ***(Equation 2.12)***

with:

- Φ being the matrix representing the autocorrelation coefficients, which will be less than 1 in absolute value and a_t is the traditional spherical disturbance.

This specification of the model of the real exchange rate is appropriate when the complexity of modern domestic and international economies is considered. There are many interactions that occur within the international economy, some of which follow from market expectations, and the true functional form of the equation relating the explanatory variables and the real exchange rate is not likely to be known with any certainty. As presented in section II, the theory of rational expectations states that market participants will use all available information to derive a forecast of the future rate. This can be accomplished by forward-looking behavior in which current rates and expected future events are considered to forecast the expected future rate. Market participants may also use backward-looking behavior where current and past movements of the exchange rate are considered in making future prediction. The problem with these techniques is that market expectations can change frequently and reaction to various disruptions can be follow random patterns. The result is that exchange rates often deviate from the underlying fundamental variables and, therefore, become difficult to predict.

The fact that the construction of x_t and y_t depends on price indices for traded goods and non traded goods in each country and these indices are assigned a relative weight in the model supports the assumptions of the second-generation random coefficient model. First, measurement error is likely to be present in the value of the broad-based price indices. It is also quite probable that any two given traded goods or non-traded goods indices are not constructed in exactly the same manner with the same goods in the same percentages. Also, the measures used by various governments to report price levels are likely to be different in any given country and, due to cultural differences between countries, the weights of certain goods contained within the price index and the consumption patterns are likely to be different. Therefore, a relative change in the prices of a particular commodity can produce spurious conclusions.

There is also the view that price levels are relatively rigid in the short run and will not change abruptly, even in response to economic policy changes. It has been observed that it is often the case that the expectations of the future exchange rates overshoot the true value and the adjustment path toward the value predicted by purchasing power parity results in a random pattern.[30] The overshooting may be a result of short-term price rigidity as the short-term response to a disturbance may be greater that the long run equilibrium value. Since this reaction to market data and the ensuing adjustment path likely, or even possibly, includes a random element, the randomness of the coefficients should be assumed to be a necessary feature of the model. It is not realistic to assume that short-term price levels are completely flexible in the short run and spending patterns within countries and between countries, as well as relative wages, are likely to exhibit considerable short-term rigidity. Also, it is unlikely that a particular country will allow such shocks to affect its economy without implementing corrective policy aimed at maintaining domestic market equilibrium. It is important to note that prices of traded goods, and the ability to trade goods, are effected by transportation costs, monopolistic and oligopolistic market practices, tariffs and government actions restricting free trade. Changes in any of these events are often difficult to predict with any degree of accuracy.

The assumption of random coefficients is, therefore, appropriate considering the volatility in the short-term interest rates and the departure of these values from the values predicted by the theory of purchasing power parity.[31] Also, as presented in section II, theory dictates that the correlation between the relative prices of traded goods and non-traded goods both within a country and between countries is influenced by the rate of productivity change in the relative sectors. Since productivity affects the relative prices of goods, the appropriate measure of sectoral productivity can serve as a concomitant variable to help explain the correlation between the coefficients and the explanatory variables and provide structure to the process determining the value of these coefficients and the nature of the variability.

The model presented in *Equation 2.10* relaxes the restrictions listed above and specifies the model used in the study conducted by Engel (*Equation 1.3)* in a manner that accounts for the randomness inherent in the coefficients. This will increase the likelihood that the model captures the true functional form and coincides with the true underlying economic relationship. With this specification, the direct effect of the explanatory variables on the explained variable will be determined and, the indirect effect of omitted variables on included variables and mismeasurement of the explanatory variables can be ascertained.

As in *Equation 1.3* presented in section III, the real exchange rate will be described as the sum of two components: one describing the nominal rate plus the difference between the price indices in traded goods and the other representing the difference between the relative price of

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[30] See DeGrauwe, Paul, "International Money", (1996)
[31] See DeGrauwe, Paul, "International Money" (1996) for examples.

non traded goods to traded goods in each country. With this description, the model may be thought of as determining the real exchange rate by adjusting the nominal exchange rate by the two components described above and the relevance of each component can then be determined. With this specification of the model, the effect of the two components will vary over time and, therefore, the short-term random effects and the underlying long-term trend can be observed. The process generating the randomness in the coefficient estimates will be given structure by the inclusion of measures of the changes in the relative productivity in the traded goods sector as concomitant variables. The particular measures used will be the change in relative manufacturing output per hour and the change in relative annual hours in manufacturing. These particular measures are used in order to incorporate the evidence presented by Canzoneri, et al into this specification of the model. The fact that the relative price of non-tradable goods generally reflects the relative labor productivity in the traded and non-traded goods sectors and that these two series have been shown to be co-integrated reinforces the case for using these variables as concomitant variables.[32] With the inclusion of these concomitant variables in *Equation 2.10*, forecasts of the U.S./Canada real exchange rate are made and estimates of the time profile of the coefficient vectors and the direct effect are obtained.

[32] See Canzoneri, et al (1999) for details

V. DATA:

The data for the nominal exchange rate between the U.S. Dollar and the Canadian Dollar is obtained from Pacific Exchange Rate Service and the natural logarithm of this rate is taken to provide the appropriate measure.[33] The price indices used in this study are constructed from the appropriate consumer price indices in the U.S. and Canada. The data used to construct the price index for traded goods and non-traded goods in the U.S. is obtained from U.S. Bureau of Labor Statistics for the years 1971 through 1999.[34] The measure of the traded goods price index for the U.S. is constructed by taking the natural logarithm of the yearly average of the consumer price index of "All Goods less Food" and the consumer price index of "Food". The measure of the non-traded goods index for the U.S. is constructed by taking the natural logarithm of the yearly average of the consumer price index of "Other Goods and Services" and the consumer price index of "Housing". The data used to construct the price index for traded and non-traded goods in Canada is obtained from Statistics Canada (official government statistics for Canada) for the years 1971 through 1999.[35] The measure of the traded goods price index for Canada is constructed by taking the natural logarithm of the yearly average of the consumer price index of "All Goods excluding Food and Energy " and the consumer price index of "Food". The measure of the non-traded goods index for Canada is constructed by taking the natural logarithm of the yearly average of the consumer price index of "Services less Housing " and the consumer price index of "Housing".

I have chosen two concomitant variables to assist in explaining the correlation between the relative price of traded goods between the U.S. and Canada and relative price of non-traded goods to traded goods both within the U.S and relative to Canada and their respective coefficients. The concomitant variables are the difference between U.S and Canada manufacturing output per hour and the difference between the U.S. and Canada average annual hours in manufacturing.[36] The combination of these particular measures, a measure of manufacturing efficiency and a measure of labor input in manufacturing, will provide a good indication of changes in the overall manufacturing output in the U.S. and Canada. The data is obtained from the U.S. Bureau of labor Statistics and is constructed by taking the natural logarithm of the difference between the U.S. index and the Canada index. As stated by DeGrauwe (1996), the selection of the appropriate base year is very important when comparing indices. Since the real exchange rate is shown to be approximately in equilibrium between 1971

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[33] U.S./Canada nominal exchange rate data for the period 1971 through 1999 is obtained from Pacific Exchange Rate Service. See Pacific Exchange Rate Service (2000).

[34] The series “All Goods less Food”, “Food”, "Other Goods and Services" and “Housing” for the U.S. are exact names of CPI series as constructed by the U.S. Bureau of Labor Statistics. The series “All Goods less Food” does not include energy and the series “Other Goods and Services” does not include housing. The data for these series is obtained from the U.S. Bureau of Labor Statistics for the years 1971 through 1999. See Bureau of Labor Statistics 2001.

[35] The series “All Goods excluding Food and Energy”, "Food”, “Services less Housing” and “Housing” for Canada are exact names of CPI series as constructed by Statistics Canada. The data for these series is obtained from Statistics Canada for the years 1971 through 1999. See Statistics Canada, 2001.

[36] U.S. and Canada data for the series Manufacturing Output per Labor Hour and Average Hours in Manufacturing is obtained from the U.S. Bureau of Labor Statistics for the years 1971 through 1999.

and 1973, all data is stated in terms of 1973 dollars.[37] The full data set is presented in ***Appendix A*** to this paper.

Construction of price indices for traded and non-traded goods for the U.S. and Canada is consistent with that used by Charles Engel (1999). However, it should be noted that the use of this data contains the same problems described by Engel (1999). Particularly, as noted in *section II*, the data is imperfect because it is possible, and quite probable, that some items classified as traded goods may actually be non-traded (restaurant meals). Also, consumer prices contain not only the price of the item, but also the value of the service that brings the good to the market. It is also the case that the non-traded measurement may contain some elements that are traded, such as financial services. However, it is commonly assumed that, despite these imperfections in the data, the above measurements provide a good proxy for the specific components. This assumption is taken to be valid in this study.

Engel (1999) considered the bias in the measurement of traded goods and concludes that, even when the possibility that a non-traded component is included in the measure representing the relative price of traded goods, the relative price of the non-traded goods contributes little to changes in the real exchange rate.[38] Engel determines the price of traded goods as a weighted average of the price of non-traded goods and the price of goods that are "truly traded" by allowing non-traded marketing services to enter into the equation. This then affects the weight that non-traded goods have in the overall price index as the value of α and β in *Equations 1.0 and 1.1* are adjusted to reflect the weight that the non-traded marketing services take in the measure of traded goods prices. Engel allows the measure of the weight of non-traded marketing services to vary randomly and finds no significant differences in the results. For the current study, I shall take this assumption to be valid; however, it is my thought that the use of the random coefficient model will reduce these imperfections by taking into account the effect of mismeasurement in the data. In fact, the results presented in this study show that the relative price of non-traded goods to traded goods between the U.S. and Canada is significant in determining movements in the short-term real exchange rate. If this is the case, then further analysis of bias in the traded goods index may be warranted.

However, the focus of the current analysis is not only on the average sensitivity of movements in the U.S./Canada real exchange rate to movements in the relative price of traded goods and the relative price of non-traded goods to traded goods, but also the time profile of the random coefficients. For this study, the U.S. will serve as the home country and Canada will serve as the foreign country. Therefore, p_t^T, p_t^N, and α in *Equation 1.0* will refer to measures in the U.S and p_t^{T*}, p_t^{N*}, and β in *Equation 1.1* will refer to measurements in Canada. Therefore, in *Equation 1.2*, q_t is described by the natural logarithm of the U.S. Dollar price of the Canadian Dollar (s_t), the price index in Canada (p_t*) and the price index in the U.S (p_t). The elements used in the construction of x_t and y_t in *Equation 1.3* have the same description as in *Equation 1.2*. The construction of x_t and y_t in *Equation 1.3* is presented in ***Appendix B*** to this paper. The value of the weights in the price index used in the construction of y_t for the U.S. and Canada are taken

[37] Paul DeGrauwe discusses the fact that real exchange rate is assumed to be in approximate equilibrium during the time period of 1971 to 1973. See DeGrauwe, Paul, "International Money", (1996).
[38] See Engel, Charles, "Accounting for U.S. Real Exchange Rate Changes", (1999).

from a regression similar to that given by Engel in his study where the interpretations of p_t^T, p_t^N, and α in *Equation 1.0* and p_t^T*, p_t^N*, and β in *Equation 1.1* are as follows:

$$\Delta(ai) = \phi_1\Delta(aglf \& f) + \phi_2\Delta(slh \& h) + \varepsilon \qquad \textbf{\textit{(Equation 3.0)}}$$

where:

- ai is the price index of All Items
- aglf & f is the average of the price index for All Goods less Food and the price index for Food
- slh is the price index for All Services less Housing and h is the price index for Housing
- ε is the error term

and:

- the lower case letters denote the natural logarithms.

In *Equation 3.0*, as presented by Engel, (aglf & f) represents the traded goods price index and (shl & h) represents the non-traded goods index. The indices constructed in this study are consistent with this representation. As stated above, I constructed the traded goods price index using the series of "All Goods less Food" and "Food" for the U.S. and "All Goods excluding Food and Energy" and "Food" for the Canada. I constructed the non-traded price indices using the series of "Other Goods and Services" and "Housing" for the U.S. and "Services less Housing" and "Housing" for Canada. The value of α in *Equation 1.0* and *Equation 1.3* is given by ϕ_2 in the OLS regression of *Equation 3.0* for U.S. data. The value of β in *Equation 1.1* and *Equation 1.3* is given by ϕ_2 in the OLS regression of *Equation 3.0* for Canada data. With this data set, the value of α is .3834 and the value of β is .4288. However, it is unlikely that the coefficients on the independent variables in this equation are fixed over the time period studied. In this study, the assumption will be that the values for α and β reasonably represent the average values that would be obtained if estimated with a stochastic coefficient specification and, therefore, reasonably represent the intended weights. The use of the OLS procedure in this case is consistent with the procedure used by Engel. The construction of this these weighs in presented in ***Appendix C*** to the paper.

VI. EMPIRICAL RESULTS:

Results are obtained using a random coefficient modeling procedure with and without the inclusion of concomitant variables and an OLS procedure. The concomitant variables included are the difference in the U.S and Canada manufacturing output per hour and the difference in the U.S. and Canada annual manufacturing hours. These measures of productivity and factor input will assist in explaining the correlation between the relative price of traded goods and the relative price of non-traded goods to traded goods and their respective coefficients, which are used to explain movements in the real exchange rate between the U.S. and Canada. Using data from the time period 1971 through 1999, the forecasting accuracy over the time period 1995 through 1999 is tested using four different versions of the random coefficient model. The time profile of the beta vector in *Equation 2.10* and the direct effect vector in *Equation 2.11* (Πz_t) is then examined over the period of 1971 to 1999. In the first test, the Φ matrix in *Equation 2.12* is non-zero and restricted to be diagonal. In the second test the Φ matrix in *Equation 2.12* is non-zero and unconstrained. The distinction between these two versions of the model is that with a non-diagonal Φ matrix, the off diagonal elements will be non-zero and there will be a non-zero covariance between the elements. In the third version, the Φ matrix in *Equation 2.12* is zero and the model will not be specified to include autocorrelation. In the final test, concomitant variables are not used.

FORECASTS:

The natural logarithms of the actual and estimated values of the U.S./Canada real exchange rate over the time period 1995 through 1999 are presented in *Table 1.0*. Also presented is the root mean square error of the five forecasted values.

TABLE 1.0
Actual and Forecast Values

	1995	**1996**	**1997**	**1998**	**1999**	**RMSE**
Actual	***-023932***	***-0.24568***	***-0.26783***	***-0.34299***	***-0.34902***	
OLS	-.27421	-.28659	-.31434	-.39883	-.41093	.049001
RCM Φ Diag	-.23333	-.23863	-.26861	-.35167	-.31211	.017454
RCM Φ Non-Diag	-.22957	-.23503	-.26582	-.35208	-.30364	.021697
RCM Φ = 0	-.22664	-.23218	-.26504	-.35598	-.30991	.020240
RCM (No Com)	-.29945	-.30805	-.33869	-.43037	-.44555	.076032

As expected, the use of the RCM with the inclusion of the concomitant variables provides estimates closer to the actual value of the natural logarithm of the real exchange rate. From the results presented in *Table 1.0*, it can be seen that the relaxation of the restrictions of the OLS model and the inclusion of the concomitant variables described above provides a more accurate forecast of the U.S/Canada real exchange rate than the OLS model. It will also be presented that the observation of the time profile of the random coefficients does assist in explaining the effect of movements in x_t and y_t on short-term movements in the U.S./Canada real exchange rate. The most accurate forecast and lowest root mean square error of the forecast is obtained when the specification of the RCM is such that the Φ matrix is non-zero and restricted to be diagonal. Therefore, further analysis will concentrate on the results of the RCM with concomitant variables and a diagonal Φ matrix. Reference will be made to the OLS results for comparison purposes.

TABLE 1.1
Coefficient Estimates and T-Ratios

Coefficient Estimates & T-Ratios

Model	Intercept	x_t	y_t
OLS	**-.022546**	**1.1386**	**0.043286**
	(-3.2004)	(14.441)	(0.20282)
	[0.0070449]	[0.078848]	[0.21342]
RCM Φ non-zero & diagonal	See Table 1.2		

Coefficient estimates are presented in bold type, t-ratios are in parenthesis, and standard errors are in brackets.

According the theory of purchasing power parity the coefficients on x_t and y_t in *Equation 1.3* should not be significantly different from 1. However, this is not supported by the data presented in Table 1.1 for the OLS model. In the OLS model, the value of the coefficient on x_t is 1.1386 and the t-ratio is 14.441. In contrast, the value of the coefficient on y_t, 0.043286, is not significantly different from zero and the t-ratio of 0.02028 implies that y_t is not significant. These results are, however, consistent with current studies insomuch as movements in the price of non-traded goods contribute little to movements in the real exchange rate and strong evidence favoring perfect purchasing power parity has not been provided. The results of the RCM with Φ non-zero and diagonal are presented below.

BETA VECTOR:

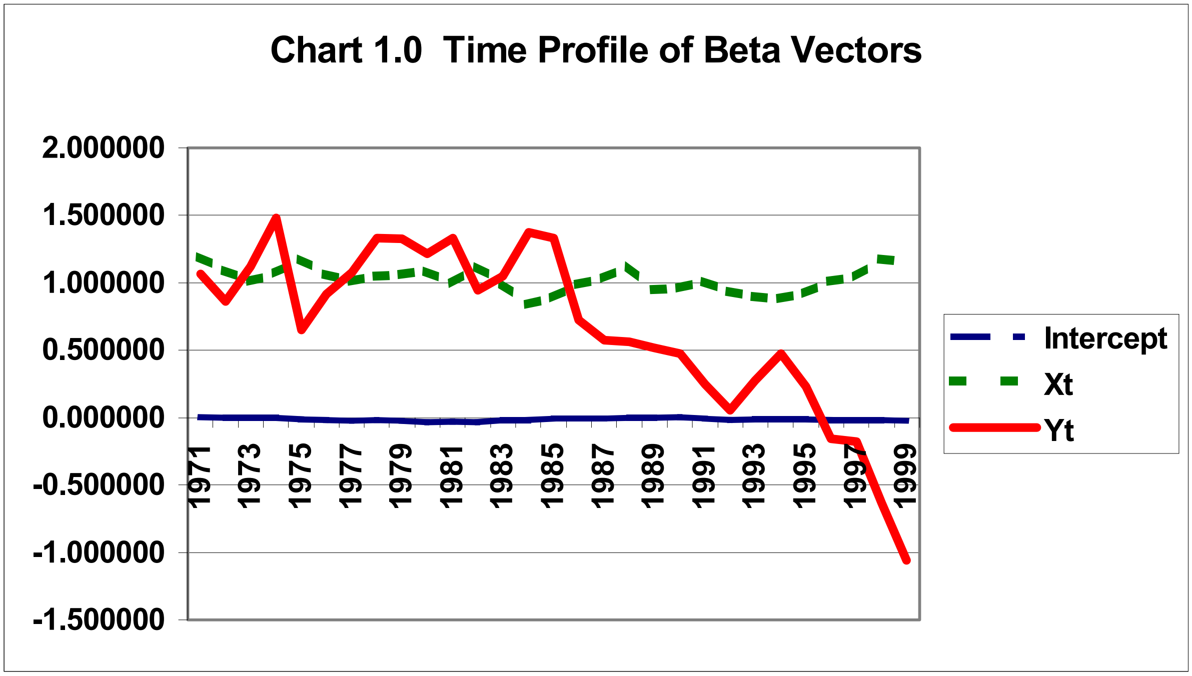
The Beta vector of the RCM is derived by adding an error term to the product of the Π matrix and the concomitant vector (z_t) in *Equation 2.11*. The random values in the beta matrix represent each coefficient as the sum of direct effect, indirect effect and the mismeasurement effect previously described. Therefore, the beta vector represents the trend in the elasticity of the real exchange rate with respect to the coefficients on x_t and y_t over the time period studied. The time profiles of the coefficient values on x_t and y_t are presented in *Table 1.2* and *Chart 1.0*.

TABLE 1.2
Beta Vectors
(Φ non-zero & diagonal)

	Intercept	X_t	Y_t
1971	0.006334	1.192300	1.065000
1972	-0.001860	1.090500	0.858820
1973	0.000001	1.009800	1.118200
1974	-0.004230	1.073300	1.480400
1975	-0.014856	1.175700	0.650860
1976	-0.020568	1.066200	0.914430
1977	-0.025653	1.009100	1.076500
1978	-0.019309	1.048100	1.329800
1979	-0.023360	1.057700	1.327900
1980	-0.032215	1.088200	1.215300
1981	-0.031884	0.994830	1.332300
1982	-0.035143	1.119500	0.946440
1983	-0.017555	0.997300	1.047000
1984	-0.016630	0.841060	1.375900
1985	-0.009634	0.885900	1.333000
1986	-0.006020	0.987230	0.721240
1987	-0.005809	1.033600	0.571180
1988	0.000541	1.127700	0.562710
1989	-0.001855	0.949670	0.520710
1990	0.002506	0.962640	0.474620
1991	-0.006384	1.010100	0.245800
1992	-0.017784	0.939980	0.051654
1993	-0.014159	0.902090	0.272480
1994	-0.012189	0.882080	0.475180
1995	-0.014219	0.922060	0.231970
1996	-0.017000	1.008200	-0.156170
1997	-0.016938	1.041200	-0.176260
1998	-0.019665	1.176200	-0.630880
1999	-0.024125	1.160500	-1.060400
Average	**-0.013781**	**1.025957**	**0.661230**
T-Ratio	-6.8636	58.9652	5.6421
Standard Error	.002008	.017399	.117196

The t-ratio and the standard error refer to the average value of the respective vectors for the period 1971 - 1999.

Due to the nature of the model, Swamy and Tavlas recommend consecutive iterations of the model to assure that the estimated periodic values of the stochastic coefficients converge.[39] The data in this model does converge to the results presented above after consecutive iterations. This assures the stability of the model and accuracy of the coefficient values presented. *Chart 1.0* gives a graphical presentation of the data in *Table 1.2.*



Observation of the time profile of the beta coefficients on x_t and y_t presented in *Table 1.2* and C*hart 1.0* show some interesting results. The coefficients on x_t show that, on average, the elasticity is higher than that on y_t and the trend is relatively stable over the time period. The value of the coefficient on x_t ranges in value from a maximum of 1.192 to a minimum of 0.841 with an average value of 1.026. In contrast, the trend of the coefficients on y_t shows much more variation and is declining over the time period. The value of the coefficient on y_t ranges from a maximum of 1.480 to a minimum of -1.060 with an average value of 0.661.

Table 1.3A lists descriptive statistics of the distribution of the elements in the beta vectors of x_t and y_t for the period 1971 through 1999. This distribution includes the forecast period of 1995 through 1999.

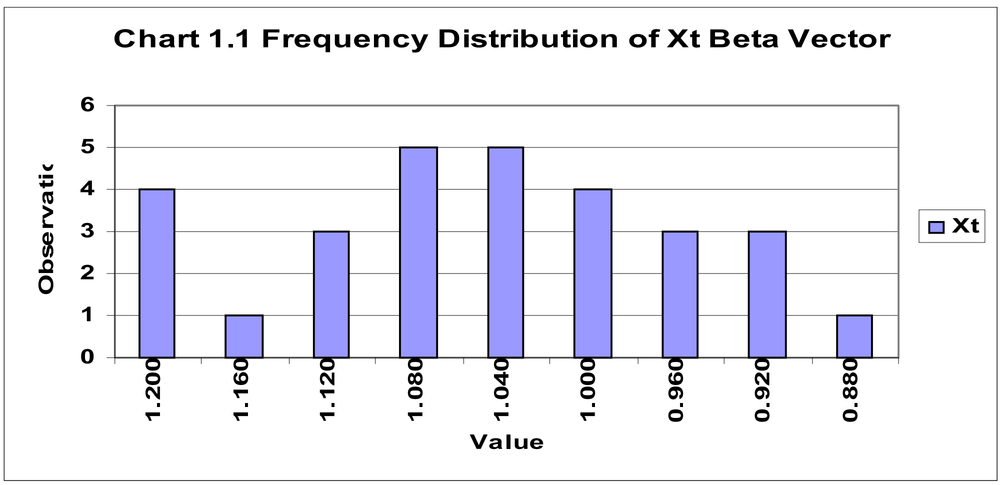
[39] See Swamy and Tavlas (1999 & 2000)

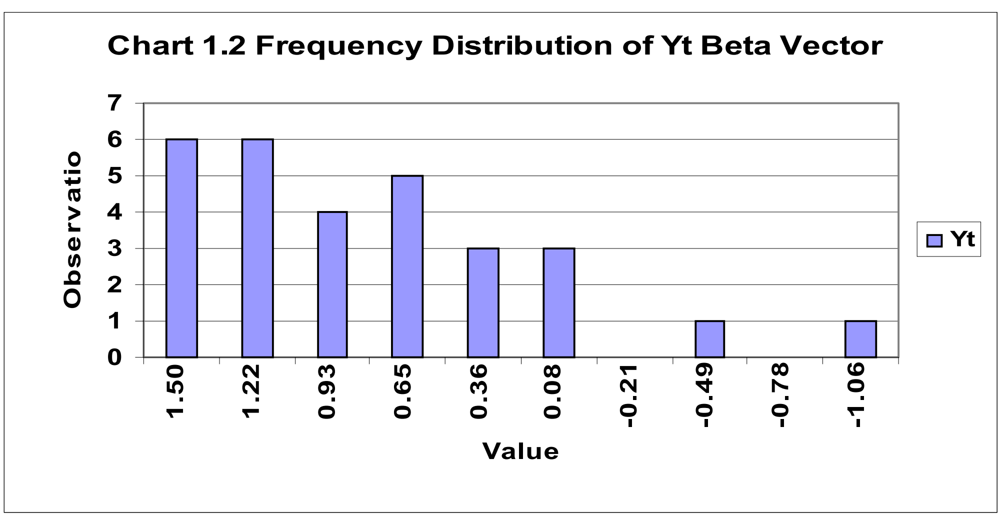
TABLE 1.3A
Descriptive Statistics of the Beta Vector of X_t and Y_t for the period 1971 through 1999

X_t Descriptive Statistics (includes forecast period 1995 - 1999)	
Mean	1.02596
Standard Error	0.01740
Median	1.01010
Standard Deviation	0.09370
Sample Variance	0.00878
Kurtosis	-0.60933
Skewness	-0.00676
Range	0.35124
Minimum	0.84106
Maximum	1.19230
Sum	29.75274
Count	29.00000

Y_t Descriptive Statistics (includes forecast period 1995 - 1999)	
Mean	0.66123
Standard Error	0.11720
Median	0.72124
Standard Deviation	0.63112
Sample Variance	0.39831
Kurtosis	0.68983
Skewness	-0.94978
Range	2.54080
Minimum	-1.06040
Maximum	1.48040
Sum	19.17568
Count	29.00000

Charts 1.1 through *1.4* show the frequency distribution and histogram of the beta vectors of x_t and y_t over the period 1971 through 1999.





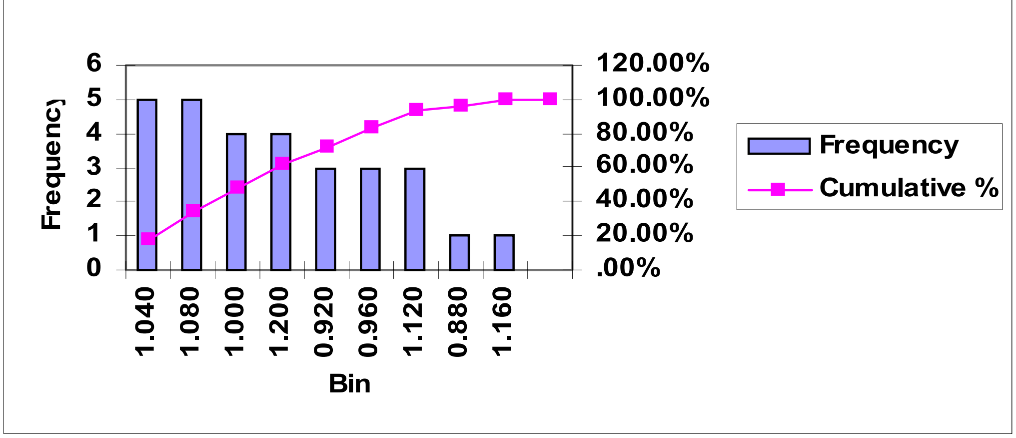


Chart 1.3 Histogram of Xt Beta Vector

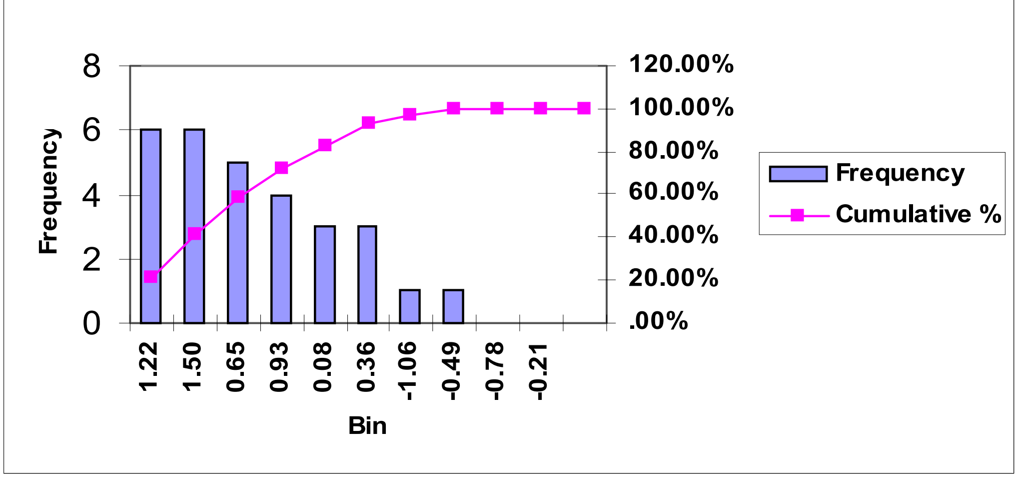


Chart 1.4 Histogram of Yt Beta Vector

Economic theory predicts that the average value of the coefficients on both x_t and y_t should be positive, which is true in each instance. Also, the t-ratios on the coefficients of both x_t and y_t indicate that the estimates are significant. These results are consistent with theory in that x_t in *Equation 1.3* represents the relative price of traded goods between two countries and, as such, changes in this measure can logically be expected to have a significant effect on the U.S./Canada real exchange rate. Specifically, perfect purchasing power parity implies that the coefficient on the relative price of traded goods should be equal to one and, at the .10 significance level, I can not reject the hypothesis that the coefficient is equal to one. I can reject this hypothesis at the .20 significance level.

However, the results are inconsistent with the theory of purchasing power parity with respect to the coefficient on y_t, which represents the weighted difference of the relative price of non-traded goods prices to traded goods prices in each country. Again, the theory of purchasing power parity implies that the coefficient on the relative price of non-traded goods to traded goods should be equal to one. In this study, I find that the coefficient on this component of the model is .661230, and with a standard error of .11796, I can reject the hypothesis that the coefficient on this component is equal to one at the .01 significance level. This is consistent with current literature in that movements in the relative price of non-traded goods to traded goods in the U.S.

and relative to Canada contributed considerably less to movements in the short-term real exchange rate than did movements in the relative price of traded goods. Specifically, the results of this study also lead to the conclusion that the average effect of the y_t component is less than that of the x_t component. However, the short-term effect, as evidenced by the values of the beta vector of y_t, is shown to be quite substantial, and in many periods the absolute value is greater than the absolute value of the coefficient on x_t. Also, the sign on the coefficients of y_t changes and indicates a substantially positive correlation in some periods and a substantially negative correlation in other periods over the time period studied. In fact, the values of this coefficient are greater than one in absolute value in many of the 29 periods observed in this study and are greater than the absolute value of the coefficient on the x_t component in 10 of the 29 periods, or 35% of the periods. Additionally, the values of the random coefficients on y_t are greater than the maximum value of the random coefficients on x_t in 7 of the 29 periods, or 24% of the periods in the study. The value of these random coefficient are, however, lower than the minimum value of the random coefficients on x_t in 15 of the 29 periods, or 52% of the time.

This variance in the random coefficients on y_t has important implications for the influence that movements in the relative prices of non-traded goods and traded goods have on movements in the real exchange rate. First, this component captures much of the volatility in the movements of the U.S./Canada real exchange rate as predicted by the model based on the theory of purchasing power parity. Second, this suggests that movements in the real exchange rate show increasingly negative correlation with movements in the relative price of non-traded to traded goods. This follows the logic of the model in that the expectation would be that if the term (P_t^N*-P_t^T*) is larger than the term (P_t^N-P_t^T), then the term y_t would be positive and the coefficient on y_t would be negative.

To again address the question of purchasing power parity, I analyze the profile of the coefficients over the period 1971 through 1994. This time period does not include the forecast period of 1995 through 1999. The descriptive statistics for x_t and y_t over this time period are presented in *Table 1.3B*.

TABLE 1.3B
Descriptive Statistics of the Beta Vector of X_t and Y_t for the period 1971 through 1994

X_t Descriptive Statistics for the estimation period 1971 - 1994	
Mean	**1.01852**
Standard Error	**0.01867**
Median	**1.00995**
Standard Deviation	**0.09148**
Sample Variance	**0.00837**
Kurtosis	**-0.42021**
Skewness	**-0.03388**
Range	**0.35124**
Minimum	**0.84106**
Maximum	**1.19230**
Sum	**24.44458**
Count	**24.00000**

Y_t Descriptive Statistics for the estimation period 1971 - 1994	
Mean	**0.87364**
Standard Error	**0.08387**
Median	**0.93044**
Standard Deviation	**0.41086**
Sample Variance	**0.16881**
Kurtosis	**-1.00909**
Skewness	**-0.31153**
Range	**1.42875**
Minimum	**0.05165**
Maximum	**1.48040**
Sum	**20.96742**
Count	**24.00000**

As is presented in *Table 1.3B,* average value of the beta vector for x_t over the estimation period 1971 through 1994 is 1.01852*,* which is closer to one than the average value over the time frame that included the forecast period. In this case, the hypothesis that the value of the coefficient on x_t over this period is equal to one can not be rejected at the .20 significance level. However, the most interesting result is with the value of the coefficient on y_t. When the forecast period is excluded from the analysis and only the estimation period of 1971 through 1994 is considered, the average value of y_t is .8736, which is also closer to one. In fact, during this period, I can not reject the hypothesis that the value of the coefficient on y_t is equal to one at the .10 significance level. The hypothesis can be rejected at the .20 significance level. This is a significant finding given that recent literature has concluded that non-traded goods contribute little to real exchange rate movements. It is also significant that the value of the coefficient on y_t over both time frames is significantly different than that suggested by the OLS modeling technique. The OLS procedure resulted in a value not significantly different from zero. This result is obviously quite different than the result provided with the random coefficient modeling specification used here. This difference is most likely due to the fact that the volatility in the periodic values of the coefficients on y_t is captured in the random coefficient model specification used in this study. The reason that this specification of the model is likely to provide a more accurate result is presented in section IV. Also, as presented in section IV, the beta vectors represent the total effect and, as such, contain indirect and mismeasurement effects and the error term $L\varepsilon_t$ in *Equation 2.11*. Therefore, the direct vector, the term Πz_t in *Equation 2.11*, should be analyzed to gain insight into the true economic relationship.

DIRECT EFFECT VECTOR:

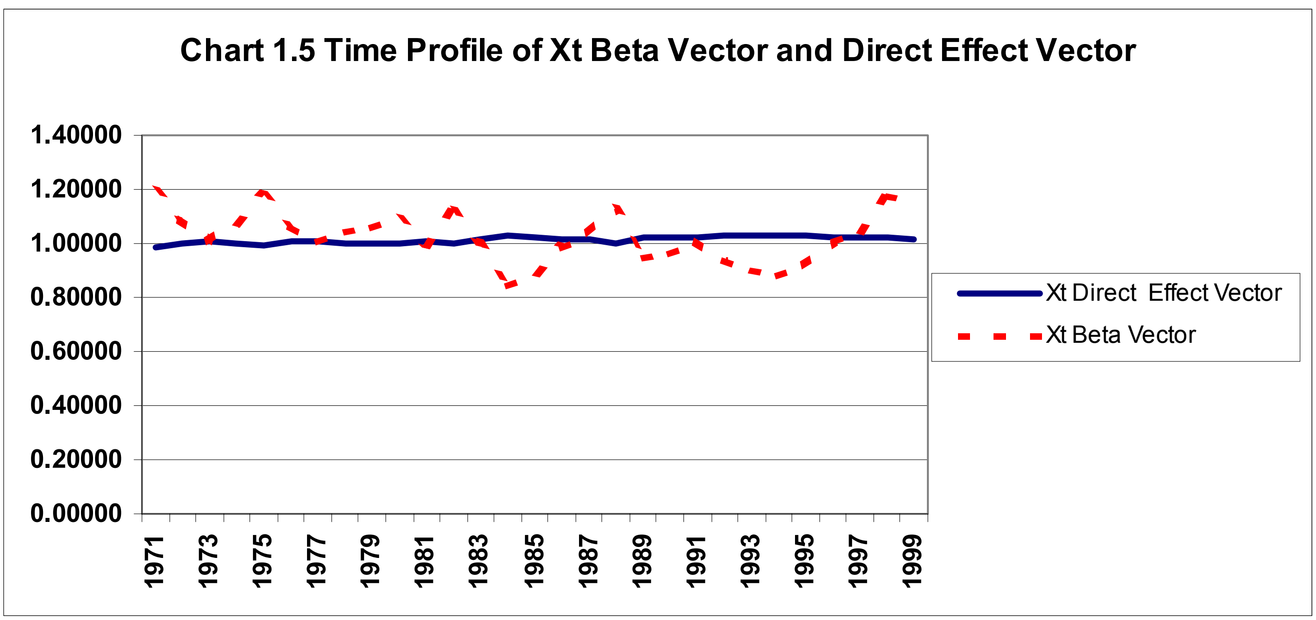
The concomitant variables used to obtain the beta vectors and direct effect vectors are the relative U.S. and Canada manufacturing output per hour (z_1) and the relative U.S. and Canada average annual manufacturing hours (z_2). The values of the Π matrix in *Equation 2.11* for the random coefficient model with Φ in *Equation 2.12* restricted to be diagonal are as follows in *Table 1.4*:

TABLE 1.4
Π Matrix

	Coefficient on Intercept, x_t and y_t	**Coefficients on Interaction with z_1**	**Coefficients on Interaction with z_2**
Intercept	**-0.0063972**	**-0.22794**	**-0.081847**
	(.-.50579)	(-.63991)	(-.82744)
X_t	**1.0098**	**-6.6101**	**0.98549**
	(18.434)	(-1.9346)	(1.1924)
Y_t	**1.1182**	**-14.696**	**-9.9169**
	(4.4014)	(-1.7181)	(-2.5157)

Coefficient estimates are presented in bold type and t-ratios are in parenthesis.

Given that the elements presented in *Table 1.4* represent the elements in the Π matrix in *Equation 2.11*, the coefficient on x_t in each period is expressed as the linear function 1.0098 + -6.6101(z_1) + .98549 (z_2) and the coefficient on y_t in each period is expressed as the linear function 1.1182 + -14.696 (z_1) + -9.9169 (z_2). The element π_{21}, 1.0098, represents the coefficient on x_t and the elements π_{22}, -6.6101, and π_{23}, 0.98549, represent the interactions of x_t with z_1 and z_2, respectively. Likewise, the element π_{31}, 1.1182, represents the coefficient on y_t and the elements π_{32}, -14.696, and π_{33}, -9.9169, represent the interactions of y_t with z_1 and z_2, respectively. In this case, the direct effect of x_t on q_t is best explained through the interaction with z_2 and the direct effect of y_t on q_t is best explained through the interaction with z_1.[40] The specific coefficients in the Π matrix relevant to the direct effect of x_t on q_t are π_{21}, 1.0098, and π_{23}, 0.98549, and the specific coefficients in the Π matrix relevant to the direct effect of y_t on q_t are π_{31}, 1.1182, and π_{32}, -14.696. The direct effect vector of x_t on q_t is obtained by adding the product of π_{23} and the z_2 value of the period to π_{21} in each of the periods (1.0098+0.98459(z_2)). The direct effect of y_t on q_t is obtained by adding the product of π_{32} and the z_1 of the period to π_{31} in each of the periods (1.1182+-14.696(z_1)). *Charts 1.5* and *1.6* show the beta vectors of x_t and y_t and the direct effect vectors resulting from the correlation of z_2 and z_1 with x_t and $y_{t,}$ respectively. The difference between the beta vector and the direct effect vector is equal to the indirect and mismeasurement effects and the error term $L\varepsilon_t$ in *Equation 2.11*.



[40] The direct effect of x_t on q_t as explained though the interaction with $z_1$and the direct effect of y_t on q_t as explained through the interaction with z_2 is presented in Appendix D

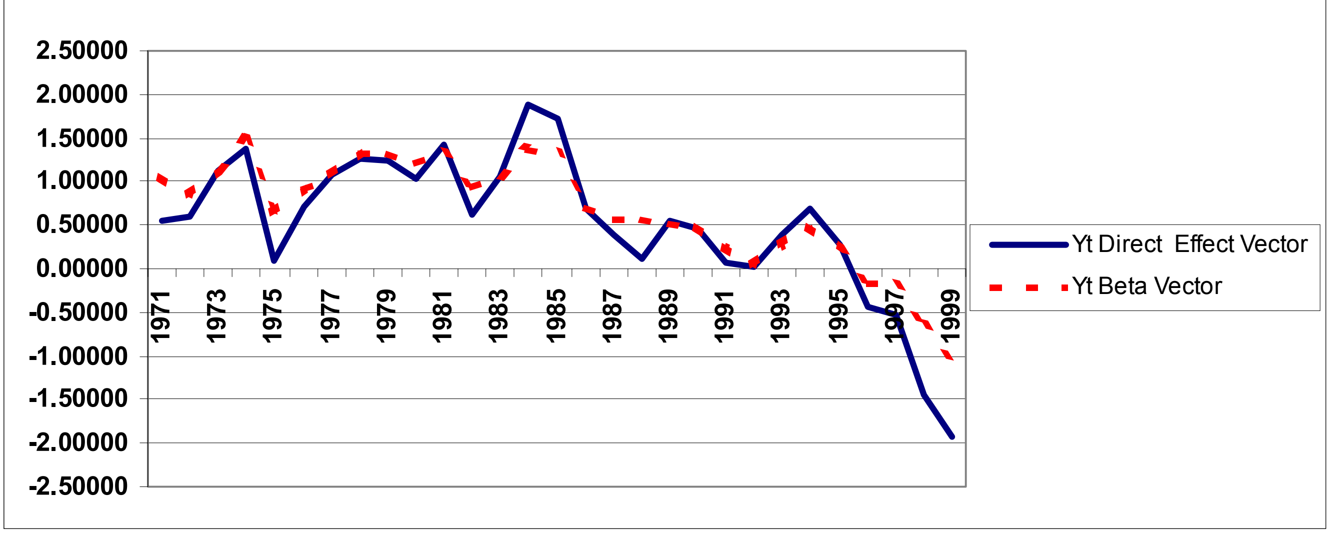


Chart 1.6 Time Profile of Yt Beta Vector and Direct Effect Vector

As presented in *Charts 1.5* and *1.6*, the periodic values and overall trend of the direct effect vectors of y_t and x_t on q_t are reasonably consistent with the beta vectors. Given the reason for the difference between the direct effect vector and the beta vector, as stated above, observation of *Charts 1.5 and 1.6* shows the accuracy of the coefficient values of the respective beta vectors in reflecting the direct effect of y_t and x_t on q_t. The indirect and mismeasurement effects contained in x_t in each period are captured by the function $(\pi_{22}z_1 + \varepsilon_t)$, or $-6.6101(z_1) + \varepsilon_t$ from *Table 1.4*. Similarly, the function for y_t is $(\pi_{33}z_2 + \varepsilon_t)$, or $-9.9169(z_2) + \varepsilon_t$ from *Table 1.4*.

The average value of the direct effect of x_t on q_t over the period of 1971 to 1999 is 1.014 and the average value of the x_t beta vector over the same period is 1.026. The average value of the direct effect of y_t on q_t over the period of 1971 to 1999 is 0.520 and the average value of y_t beta vector over the same period is 0.661. As with the beta vectors, the variance of the direct effect vector of y_t on q_t is much greater than the variance of the direct effect vector of x_t on q_t. As can be seen in *Charts 1.5* and *1.6*, the trend of the direct effect vector is closely correlated with the trend in the beta vector for y_t, however, the deviation of the direct effect vector from the beta vectors appears to be greater for x_t. The maximum value of the elements of the direct effect vector of x_t on q_t is 1.033, compared to1.886 for the elements in the direct effect vector of y_t on q_t. The minimum value of the elements of the direct effect vector of x_t on q_t is 0.9882, compared to -1.933 for the elements in the direct effect vector of y_t on q_t.

As presented in section IV, the profiles of the direct effect vectors are extremely important is assessing the true economic relationship between the independent variables and the dependent variable. This is because these direct effect vectors do not contain the error term $L\varepsilon_t$ in *Equation 2.11* and do not include omitted variable bias and mismeasurement bias. In regard to x_t, further observation of *Chart 1.5* shows that the periodic values of the direct effect vector of x_t on q_t exhibit a very consistent pattern and do not significantly deviate from 1. This suggests near perfect purchasing power parity in the direct effect of x_t on q_t when indirect and mismeasurement effects are removed, or, stated differently, deviation from near perfect purchasing power parity in

x_t is the result of indirect and mismeasurement effects. The periodic values of the x_t beta vector consistently both overshoot and undershoot the values of the direct effect vector.

In regard to y_t, observation of *Chart 1.6* shows that the values of the direct effect vector of y_t on q_t tend to be less than the values of the beta vector. The exceptions being a large overshooting of the beta vector value over the time period of 1983 through 1986 and a much smaller overshooting over the period of 1993 to 1995. It is also important to notice that the direct effect vector is declining much more quickly during the period 1996 through 1999. The direct effect vectors of x_t and y_t on qt are presented in *Table 1.5*.

TABLE 1.5
Direct Effect Vectors

	Direct Effect Vector of x_t on q_t	Direct Effect Vector of y_t on q_t
	0.988157	0.561003
	1.003094	0.585598
	1.009800	1.118200
	0.997656	1.386482
	0.995208	0.103161
	1.005384	0.718594
	1.010393	1.069453
	1.002557	1.271594
	1.001401	1.243293
	0.999052	1.024544
	1.009029	1.418369
	0.998483	0.613527
	1.012189	1.065487
	1.027277	1.886219
	1.022321	1.713171
	1.017373	0.697236
	1.013528	0.389917
	1.002137	0.125631
	1.024393	0.555198
	1.023367	0.464243
	1.020345	0.058358
	1.031267	0.012023
	1.033215	0.382291
	1.033197	0.682315
	1.031267	0.279230
	1.025459	-0.424328
	1.021671	-0.537783
	1.018688	-1.453826
	1.017833	-1.932803
Average	1.013646	0.519876
Maximum	1.033215	1.886219
Minimum	0.988157	-1.932803

The descriptive statistics for the direct effect vectors of x_t and y_t are for the period 1971 through 1999 and for the period 1971 though 1994 are presented in Tables 1.6A and 1.6B, respectively.

TABLE 1.6A
Descriptive Statistics of the Direct Effect Vector of X_t and Y_t for the period 1971 through 1999

X_t Descriptive Statistics (Direct Effect Vector) (includes the forecast period 1994 - 1999)	
Mean	1.01365
Standard Error	0.00235
Median	1.01353
Standard Deviation	0.01266
Sample Variance	0.00016
Kurtosis	-1.02305
Skewness	-0.11723
Range	0.04506
Minimum	0.98816
Maximum	1.03321
Sum	29.39574
Count	29.00000

Y_t Descriptive Statistics (Direct Effect Vector) (includes the forecast period 1994 - 1999)	
Mean	0.51988
Standard Error	0.15726
Median	0.58560
Standard Deviation	0.84687
Sample Variance	0.71719
Kurtosis	1.92247
Skewness	-1.11978
Range	3.81902
Minimum	-1.93280
Maximum	1.88622
Sum	15.07640
Count	29.00000

TABLE 1.6B
Descriptive Statistics of the Direct Effect Vector of X_t and Y_t for the period 1971 through 1994

X_t Descriptive Statistics (Direct Effect Vector) for the estimation period 1971 - 1994	
Mean	1.01170
Standard Error	0.00264
Median	1.01010
Standard Deviation	0.01292
Sample Variance	0.00017
Kurtosis	-0.98578
Skewness	0.18788
Range	0.04506
Minimum	0.98816
Maximum	1.03321
Sum	24.28082
Count	24.00000

Y_t Descriptive Statistics (Direct Effect Vector) for the estimation period 1971 - 1994	
Mean	0.79775
Standard Error	0.10640
Median	0.68978
Standard Deviation	0.52125
Sample Variance	0.27170
Kurtosis	-0.61662
Skewness	0.34410
Range	1.87420
Minimum	0.01202
Maximum	1.88622
Sum	19.14591
Count	24.00000

As shown in *Table 1.6B*, the average value of the direct effect vector of x_t on q_t for the estimation period of 1971 though 1994, which does not include the forecast period of 1995 though 1999, is 1.012 compared to a value of 1.034 when the forecast period is included. The maximum and minimum value of the coefficient on x_t when the forecast period is not included in the analysis is 1.033 and 0.9882, respectively. The average value of the of the direct effect vector of y_t on q_t for the estimation period of 1971 though 1994 is 0.7978 compared to a value of 0.5200 when the forecast period is included. The maximum and minimum value of the coefficient on y_t when the forecast period is not included in the analysis is 1.886 and 0.012, respectively.

The critical finding relevant to the hypothesis being tested in this study is that, when considering the time profiles of both the beta vector and the direct effect vector of x_t and y_t, the y_t component is a significant component of the model. Specifically, the y_t component is significantly different from zero when the forecast period of 1995 through 1999 is included in the analysis and purchasing power parity can not be rejected when the forecast period is excluded from the analysis. However, there is significantly more volatility in both the beta vector and the direct effect vector of the y_t component of the model whether the forecast period is included or excluded from the analysis. *Table 1.7* is a summary chart of selected statistics of x_t and y_t.

TABLE 1.7
Summary of Selected Statistics of X_t and Y_t

	Including forecast period (1995 -1999)			
	Beta	standard error	Direct Effect	standard error
X_t	1.02596	0.0174	1.01365	0.00235
Y_t	0.66123	0.1172	0.51988	0.15726

	Excluding the forecast period (1995-1999)			
	Beta	standard error	Direct Effect	standard error
X_t	1.01852	0.01867	1.01170	0.00264
Y_t	0.87364	0.08387	0.79775	0.10640

CONCLUSION:

The dominant theory in current economic literature regarding the real exchange rate between two countries is the theory of purchasing power parity and the underlying law of one price. The scope of this study is to utilize the assumption of purchasing power parity and test the data on the U.S./Canada real exchange rate over the period 1971 through 1999 using a random coefficient model with the proper concomitant variables. The random coefficient model used in this study is of the general form presented by Swamy and Tavalas (1999), and, thereby, relaxes the restrictions of the OLS model. In particular, these restrictions are the assumption that the true functional form of the equation is known, the error term captures the effect of all excluded variables, the coefficients are fixed and the data does not include measurement errors. Swamy and Tavalas (1999) also present that this modeling approach provides results that more closely coincide with true economic interpretations of the data. The results presented in section IV indicate that the forecasted values of the real exchange are more accurate when these restrictions are removed and manufacturing output per hour and average annual hours in manufacturing are used as the concomitant variables explaining the correlation between the explanatory variables and their coefficients. It is also presented that this modeling approach provides random time profiles of the sensitivity of movements in the real exchange rate to movements in both the relative price of traded goods and movements in the relative price of non-traded goods to non-traded goods.

The results presented in section VI show that the forecast of the U.S./Canada real exchange rate for the time period 1995 through 1999 is most accurate when the specification of the model is such that the ϕ matrix is diagonal and non-zero in value. This matrix describes the serial correlation in the error term and, in this specification of the model, allows for autocorrelation in the process generating the coefficients, but does not allow for covariance between the matrix elements. The results of this model specification show that the average value of the random coefficients on the relative price of traded goods component and the relative price of non-traded to traded goods component over the period 1971 through 1999 is 1.026 and .661, respectively. Also, it is shown that the average value of the direct effect vectors of the relative price of traded goods component and the relative price of non-traded goods component over the same time period is 1.014 and .520, respectively. With regard to the relative price of non-traded goods component, the value is shown to be substantially different when the forecast period is excluded from the analysis. In this case the average value of the coefficient is .874 and the average value of the direct effect vector is .798.

These observations are consistent with previous studies in that the movements in the U.S./Canada real exchange rate are, as is true with the real exchange rate with other currencies, more sensitive to changes in the relative price of traded goods. It is, however, presented that the variation in the random coefficients on the relative price of non-traded goods to traded goods component is much larger than the variation in the random coefficients on the relative price of traded goods component over the same time period. The random coefficients on the relative price of non-traded goods to traded goods component decline steadily in value over the period and range from a maximum value of 1.480 to a minimum value of -1.060. The values of the direct effect vector over the same time period range in value from 1.886 to -1.933. In contrast, the random coefficients on the relative price of traded goods component are relatively stable in value over the period and range from a maximum vale of 1.192 to a minimum value of 0.841. The values of the direct effect vector range from 1.033 to .9882.

Perhaps the most important development here is that movements in the relative price of non-traded goods to traded goods both within the U.S. and relative to Canada is an important determinant of short-term movements in the real exchange rate. It is presented that the values of the coefficients exhibit a declining trend over the period 1984 though 1999 and have ranged from having a significantly positive effect to having a significantly negative effect. In both extreme cases, the absolute value of the coefficients is greater than one. The reasons for these results could lie in the geographic proximately and trade openness between the U.S. and Canada. Agreements such as the U.S.-Canada Free Trade Agreement of 1989 and the North American Free Trade Agreement of 1994 intended to promote increased and more efficient trade and investment through the reduction, or elimination, of tariffs and product quotas could have important implications to the analysis. In particular, the price and demand for both traded goods and non-traded goods in the U.S. and Canada is not as likely to be subject to the effects of restrictive trade policy regimes. Therefore, the time profile of the coefficients on both the relative price of traded goods and the relative price non-traded to traded goods should more accurately reflect true trade preferences.

The results presented in section VI are consistent with the theory of purchasing power parity and the law of one price in that the relative rates of inflation between countries are major factors in determining the real exchange rate. When the forecast period is excluded from the analysis, the evidence suggests that the value of the coefficients on both components is not significantly different than one. These results are not consistent with the conclusions of recent studies that changes in the real exchange rate are primarily determined by relative inflation in the traded goods sector. Therefore, an implication of this model specification is that both components are significant in explaining the movements in the real U.S./Canada real exchange rate. Also implied is that, due either to short term inflexibility of prices and consumer behavior, and/or measurement error, the short term adjustment of the real exchange rate to the long run equilibrium value predicted by the relative purchasing power parity theory is likely to follow a random pattern. This random short-term variation around the long run trend can be given structure by considering relative productivity and output of the traded goods sectors of the U.S. and Canada.

I conclude that, although movements in the U.S./Canada real exchange rate are, on average, more sensitive to movements in the relative price of traded goods, the theory of purchasing power parity applies to both traded goods and non-traded goods in determining the U.S./Canada real exchange rate. I further conclude that much of the volatility in the short-term movements in the U.S./Canada real exchange rate is attributable to movements in the relative price of non-traded goods to traded goods.

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