Passive range estimation using subarray parallax

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Consider a large aperture linear array which is detecting coherent radiation from a source. Suppose that the array is processed as a string of subarrays consisting of adjacent sensors. Assume that the signal is a plane wave whose coherence distance is of the order of the subarray lengths. There is a systematic difference among the subarray bearing estimates due to parallax. A statistical method for using the parallax effect to estimate range is presented. Approximate expressions for the variance and bias of the estimator are derived under the assumption that the gain of each subarray is large. These theoretical results are compared with values obtained using artificial data for several parameter values.

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INTRODUCTION

Consider a horizontal linear array of sensors which is detecting coherent radiation from a source. If the medium is horizontally homogeneous, then the received signal is a cylindrical wave plus noise. The maximum-likelihood estimator of range assuming a cylindrical wave plus Gaussian noise has been studied by Hahn[1] and Carter.[2] The variance of the estimator due to additive noise is proportional to R^4L^{-4}, where R is the range and L is the array length.[3] Inhomogeneities in the medium near the array distort the wave curvature, increasing the variance. Random variation of sound velocity results in a loss of signal coherence which also increases variance.[4] When R is large, moreover, the curvature is almost impossible to measure.

There is another method for estimating the range to a distant source using a large aperture array. This approach makes use of the parallax effect. Suppose that the array is processed as a string of subarrays consisting of adjacent sensors. If each subarray gain is sufficiently large to resolve the source, then there will be a systematic measurable difference among the bearing estimates due to parallax. For the parallax method we need only assume that the received signal is a plane wave whose coherence distance is of the order of the subarray lengths. In other words, the signal must appear as a coherent plane wave to the subarrays, but not to the whole array. This assumption about the signal is weaker than those made in the previously cited papers on focused arrays.

The range estimator using subarrays is presented in the next section. The method and its statistical properties also apply to a configuration of subarrays that cannot be connected together as a whole.

I. PARALLAX RANGE ESTIMATION

Suppose that the array consists of a line of J linear subarrays. An array of $M=JM_J$ sensors can be electronically processed as J subarrays of M_J adjacent sensors. Assume that the signal at each subarray is a plane wave plus Gaussian noise with a signal-to-noise ratio denoted ρ, and we know the sign of the source bearing. To simplify a comparison with the statistical results obtained from the cylindrical model, assume that the medium is horizontally homogeneous and the wave is narrow band with center frequency f and velocity c. Suppose that the jth subarray is sampled for an interval $T=N\delta$. Let x_j denote the coordinate of the center of the subarray on the array axis, and set the origin so that $\sum_{j=1}^{J} x_j=0$. Let $\hat{\psi}_j$ denote the maximum-likelihood estimator of bearing with respect to the axis *perpendicular* to the array at x_j (Fig. 1). If M_j, the number of sensors in subarray j is large, then the bearing that maximizes the energy of the output of a beamformer is maximum-likelihood.[5] If M_j *or* N is large,[6,7] $\hat{\psi}_j=\psi_j+(\cos\psi_j)^{-1}\epsilon_j$ where the distribution of the error ϵ_j is approximately Gaussian with mean *zero* and variance $\sigma_j^2=(\gamma^2/M_jL_j^2)\ \mathrm{rad}^2$, where L_j is the subarray length and

$$\gamma^2=3c^2/2Nf^2\rho\pi^2 . \qquad (1)$$

If the coherence distance of the noise is less than $x_{j+1}-x_j$, then ϵ_j and ϵ_{j+1} are uncorrelated.

Let x_0 denote the source's coordinate on the array

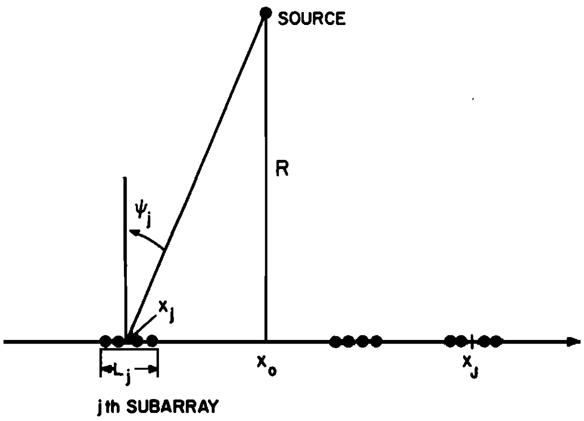

FIG. 1. Broadside source.

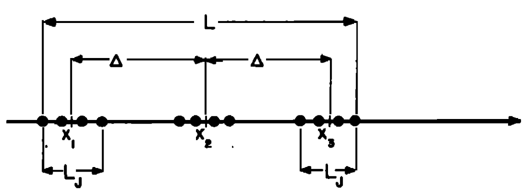

FIG. 2. Equally spaces subarrays.

TABLE I. Statistical properties of $\hat{R}$.

	Mean bias $\langle \hat{R}/R - 1\rangle$	rms error $\langle(\hat{R}/R-1)^2\rangle^{1/2}$	Mean absolute error $\langle \lvert \hat{R}/R - 1 \rvert \rangle$	Mean standard deviation/R $\langle \sigma(\hat{R})\rangle/R$	Asymptotic standard deviation/R
			$\sigma_j = 0.5°$		
$R=280, J=14$	0.03	0.18	0.13	0.17	0.16
$R=560, J=28$	0.01	0.12	0.10	0.12	0.11
			$\sigma_j = 1.0°$		
$J=14$	0.10	0.50	0.30	0.49	0.32
$J=28$	0.08	0.32	0.22	0.31	0.23

axis and let R denote its distance from x_0, i.e., R is the *range perpendicular to the array axis*. Then $\tan\psi_j = (x_0 - x_j)/R$. Using the linear approximation to $\tan\hat{\psi}_j$ for small σ_j, $\tan\hat{\psi}_j = \tan\psi_j + (\cos\psi_j)^{-3}\epsilon_j$. Suppose that $R \gg L$ and $x_1 \leq x_0 \leq x_J$. Then $(\cos\psi_j)^{-3} = \{1 + [(x_0 - x_j)/R]^2\}^{3/2} \simeq 1$ for each j. Consequently,

$$\tan\hat{\psi}_j = a - bx_j + \epsilon_j\,, \qquad (2)$$

where $a = x_0R^{-1}$ and $b = R^{-1}$.

The weighted least-squares estimator of the slope b,

$$\hat{b} = \frac{-\sum_{j=1}^J M_j L_j^2 x_j(\tan\hat{\psi}_j - J^{-1}\sum_{j=1}^J \tan\hat{\psi}_j)}{\sum_{j=1}^J M_j L_j^2 x_j^2} \qquad (3)$$

is maximum-likelihood since the ϵ_j's are independent Gaussian errors. Moreover $\hat{b}$ is Gaussian with mean R^{-1} and variance $\gamma^2 \sum_{j=1}^J M_j L_j^2 x_j^2)^{-1}$. If γ/L_j is small, this approximation of the mean and variance is good even if the ϵ_j are not Gaussian.

Consider the range estimator $\hat{R} = \hat{b}^{-1}$. To simplify the asymptotics, assume that the array consists of equally spaced subarrays of equal length L_J with the same number of sensors, i.e., $x_{j+1} - x_j = \Delta$, $L_j = L_J$, and $M_j = M/J$ for each j (Fig. 2). The array length is then $L = (J-1)\Delta + L_J$. Since $\sum_{j=1}^J x_j = 0$, $x_j = [j - (J+1)/2]\Delta$. Thus, $\sum_{j=1}^J M_j L_j^2 x_j^2 = ML_J^2\Delta^2(J-1)(J+1)/12$. Using Taylor's formula,

$$\hat{R}/R = 1 - R(\hat{b} - b) + R^2(\hat{b} - b)^2 + O[R^3(\hat{b} - b)^3]\,. \qquad (4)$$

Assume that $\gamma R \ll M^{1/2} L_J J\Delta$. Then the higher order terms in (4) are small and

$$(E\hat{R})/R = 1 + R^2 E(\hat{b} - b)^2$$
$$\simeq 1 + [12\gamma^2 R^2/ML_J^2\Delta^2(J^2 - 1)]\,. \qquad (5)$$

From (4) the variance of $\hat{R}/R$ is

$$\sigma^2(\hat{R})/R^2 \simeq 12\gamma^2R^2/ML_J^2\Delta^2(J^2 - 1)\,, \qquad (6)$$

which is greater than the square of the bias $E\hat{R}/R - 1$ given the condition on γR.

If L is fixed, it is clear from (6) that the variance is minimized by using *two* abutting subarrays with $L_J = L/2$ and $\Delta = L/2$. In this case, $\sigma(\hat{R})/R \simeq 8\gamma M^{-1/2}L^{-2}R$. The optimal Hahn–Carter estimator[8] has a proportional standard deviation of $[8/(3)^{1/2}]\gamma M^{-1/2}L^{-2}R$. Thus my asymptotic standard deviation is $(3)^{1/2}$ times theirs, but they require the wave to be cylindrical for all values of R whereas I assume a plane wave. A cylindrical wave becomes a plane wave as $R \to \infty$. Now let $\Delta = L_J = L/J$ for $J \geq 3$. Then $\sigma(\hat{R})/R \simeq 2(3)^{1/2}L^{-2}R$. My estimator is then *less asymptotically efficient than the Hahn–Carter estimator for an equally spaced design.*

II. ARTIFICIAL DATA RESULTS

The proportional bias, rms error, mean absolute error, and standard deviation of $\hat{R}$ were estimated for $R/J\Delta = 20$ using 400 $\hat{R}$'s generated using Eqs. (2) and (3). An equally spaced subarray design was used with $\Delta = 1$, $x_0 = (J-1)/2$, $J = 14$ ($R = 280$), and $J = 28$ ($R = 560$). The ϵ_j were computed using the same pseudorandom Gaussian generator that is programmed in the TI58 calculator, with $\sigma_j = 0.5°$, and 1° for each j. The 95.5% confidence intervals for $\hat{\psi}_j$ are then $\psi_j \pm 1°$ and $\psi_j \pm 2°$, respectively. The results are given in Table I. The estimated $\sigma(\hat{R})/R$ are close to their asymptotic values, with the greatest discrepancy for $\sigma_j = 1°$. These results support the proposition that the asymptotic results can be used as a design tool for processing a long array as a sequence of subarrays to estimate the range of a distant source when the array gain is large.

ACKNOWLEDGMENT

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[8] From Eqs. (8), (10), and (11) in Ref. 2.