

AN EXAMINATION OF THE VALIDITY OF STEADY  
STATE SHEAR STRENGTH DETERMINATION USING  
ISOTROPICALLY CONSOLIDATED UNDRAINED  
TRIAXIAL TESTS

Jonathan R. Porter

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Thomas L. Brandon, Chair  
J. Michael Duncan  
George M. Filz  
O. Hayden Griffin Jr.  
James R. Martin II

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# An Examination of the Validity of Steady State Shear Strength Determination Using Isotropically Consolidated Undrained Triaxial Tests

Jonathan R. Porter

## (ABSTRACT)

The assessment of the shear strength of soil deposits after the occurrence of large strains is an important issue for geotechnical engineers. One method for doing so, the steady state approach, is based on the assumption that the steady state undrained shear strength is a unique function of the *in situ* void ratio and effective stress. This method, which has been applied to liquefaction and flow failures, has been criticized because it may overestimate the *in situ* shear strength. The key to the steady state approach is accurate determination of the relationship between void ratio and effective stress at steady state. This is typically accomplished using conventional isotropically consolidated undrained (ICU) triaxial tests. The triaxial test was developed for measuring peak strengths, which typically occur at small strains, but steady state conditions typically occur at much larger strains. At large strain levels, the suitability of conventional triaxial testing procedures and error corrections is uncertain. The measured response at large strains may be inaccurate due to the influence of various testing errors. Furthermore, the true material response in the test specimen at large strains may not accurately represent *in situ* material behavior at large strains.

This research effort consisted of an experimental and analytical study to examine the validity of steady state undrained shear strength determination using conventional ICU triaxial tests. The analytical study addressed triaxial testing errors and conventional corrections that are applied to test data and their influence on the measured steady state parameters. Finite element analyses were conducted to investigate the influence of variations in restraint at the end platens on stress distributions in the sample and measured stress-strain response. The finite element analyses incorporated axisymmetric interface elements to model the friction characteristics between the end platens and the specimen ends. The experimental study focused on several sands that are susceptible to liquefaction. An interface direct shear test program was conducted in order to evaluate various schemes for reducing end platen friction. ICU triaxial tests were conducted on each material using both conventional and lubricated end platens.

Dedicated to my parents, Richard and Leona Porter,  
for their never-ending patience, understanding, love, and support.

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