Essays on Pricing and Promotional Strategies

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(ABSTRACT)

This dissertation contains three essays on theoretical analysis of pricing and promotional strategies. Chapter 1 serves as a brief introduction that provides a motivation and an overview of the topics covered in the subsequent chapters.

In Chapter 2, we study optimal couponing strategies in a differentiated duopoly with repeat purchase. Both firms can distribute defensive coupons alone, defensive and offensive coupons together, or mass media coupons. They can also determine how many coupons to offer. Allowing consumers to change their tastes for the firms' products over time, we find that the optimal couponing strategy for the firms is to only distribute coupons to all of the customers who buy from them. The effects of intertemporally constant preferences and consumer myopia on the profitability of the optimal couponing are investigated as well.

Chapter 3 examines the profitability of behavior-based price discrimination (BBPD) by duopolists producing horizontally differentiated experience goods. We consider a three-stage game in which the firms first make price discrimination decisions followed by two-stage pricing decisions. The main findings are: (i) there are two subgame perfect Nash equilibria where both firms do not collect information about consumers' purchase histories so that neither firm price discriminates and where both firms collect consumer information to practice BBPD; and (ii) BBPD is more profitable than uniform pricing if sufficiently many consumers have a poor experience with the firms' products. The asymmetric case where one firm produces experience goods and the other search goods is also investigated.

Chapter 4 provides a possible explanation of the fact that one ticket price is charged for all movies (regardless of their quality) in the motion-picture industry. Considering a model à la Hotelling in which moviegoers form their beliefs about movie quality through pricing schemes to which an exhibitor commits, we characterize the conditions under which committing to uniform pricing is more profitable than committing to variable pricing. The welfare consequences of a uniform pricing commitment and some extensions of the model are discussed as well.

To My Parents

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Chapter 1

Introduction

In practice, firms are using various pricing and promotional strategies to increase profits. The strategies generally take forms of price discrimination, including: charging different prices based on buyers' observable characteristics such as age, gender, and occupation (third-degree price discrimination, group pricing); offering various combinations of price and quantity (or price and quality) and having consumers self-select (second-degree price discrimination, menu pricing); selling two or more products in a single package (bundling); giving a discount when bringing a coupon (couponing).

Due to the development of more sophisticated methods for acquiring, storing, and analyzing consumer information, firms can now charge different prices depending on whether customers have previously purchased products from them and offer (future) discounts to selected customers. In the software industry, for example, *Microsoft*, *Symantec*, and *Adobe* sometimes offer lower upgrade prices to their existing users. *McDonald's* and *Starbucks* often give buy one get one free coupons. On the other hand, long-distance carriers such as *AT&T* and *MCI* often charge lower prices to competitors' customers. As recent advances in information technology (IT) take price discrimination to a new level, we need to delve into more of such pricing tactics.

The main purpose of this dissertation is to examine which pricing or promotional strategy firms should adopt when consumer information is available. In order to do that, we first try to answer the following two questions:

- Who should firms distribute coupons to in order to maximize profits?

- When is it profitable for firms to use consumers' past purchase information for discriminatory pricing?

The first question will be tackled in Chapter 2 of this dissertation. Depending on the method of distribution, coupons can be divided into mass media coupons and targeted coupons. Targeted coupons then take two forms: defensive coupons and offensive coupons (Shaffer and Zhang, 1995). Mass media coupons are randomly distributed to consumers. Defensive coupons are offered to retain firms' own customers, while offensive coupons are offered to poach rival firms' customers. In the previous couponing literature, mass media coupons are mainly studied (Narasimhan, 1984).

Recent advances in IT enable firms to send targeted coupons to selected customers. However, as reviewed in Chapter 2, much of the existing literature on targeted coupons has focused on offensive coupons. There are two reasons for this: (i) consumers have to pay costs to switch between firms (Bester and Petrakis, 1996; Chen, 1997), and (ii) they prefer a specific firm's product (Fudenberg and Tirole, 2000). In either case, luring rival firms' customers through coupons (discounts) arises as the equilibrium outcome. In addition, offensive couponing with exogenous switching costs and/or constant preferences leads to lower firm profits.

Despite the extensive economic literature on couponing, the use of both defensive and offensive coupons has received little attention. In Shaffer and Zhang (1995) and Kosmopoulou, Liu, and Shuai (2012), firms can send defensive and offensive coupons together. However, these two studies are based on models with single purchase so that dynamic properties of consumer preferences are ignored. In Chapter 2, we will try to fill the gap existing in the couponing literature by enlarging firms' couponing strategy space and allowing for changing preferences across purchase occasions.

Employing a differentiated product duopoly model with repeat purchase, where all consumers change their tastes over time, we find that the firms' optimal couponing strategy is to only distribute coupons to all of the customers who buy from them.

A well-established result in the price discrimination literature is that price discrimination in oligopoly intensifies competition and leads to lower profits (Thisse and Vives, 1988; Shaffer and Zhang, 1995; Bester and Petrakis, 1996; Liu and Serfes, 2004). One important ingredient for price discrimination to have profit-reducing effects is best-response asymmetry, i.e., one firm's strong market is the other firm's weak market (Corts, 1998; Armstrong, 2006). Although there are extensive studies on oligopolistic price discrimination, only a few show that price discrimination can boost profits (Esteves, 2009; Shin and Sudhir, 2010; Liu and Shuai, 2013).

As IT advances, firms can segment customers on the basis of their purchase histories and price discriminate accordingly. This form of price discrimination has been named in the literature as behavior-based price discrimination (BBPD). In Chapter 3, we will answer the second question by characterizing the conditions under which BBPD in markets exhibiting best-response asymmetry is more profitable than uniform pricing.

Considering duopolists producing horizontally differentiated experience goods and a three-stage game where they first make price discrimination decisions followed by two-period pricing decisions, we show that BBPD enables the firms to make more profits than uniform pricing when sufficiently many consumers experience a bad fit with their products. The intuition behind this result is that as many consumers' valuations are expected to fall over time, the firms can each raise their first-period price via BBPD, which results in higher profits in the first period.

Finally, in Chapter 4 of this dissertation, we turn our attention to a pricing tactic used in the movie business. Two puzzling phenomena have been observed in the motion-picture industry: (i) movie theaters in the United States practice several price discrimination schemes such as discounts for seniors and students, whereas they charge the same ticket price for all movies (the movie puzzle); (ii) most moviegoers prefer a Saturday night movie to a Monday night movie, while they usually pay one price for the movie tickets, regardless of the day of the week (the show-time puzzle). Here, we will try to solve the first puzzle only by answering the next question:

- Why do all movie tickets cost the same?

Such price uniformity across movies is puzzling in light of the potential profitability of prices that vary with demand characteristics. One would expect that exhibitors can be better off by charging more for blockbusters. Surprisingly, only a few studies attempt to explain why movie theaters employ uniform pricing (Orbach and Einav, 2007; Chen, 2009; Courty, 2011).

Motion pictures are uncertain products in the sense that it is difficult for movie theaters to estimate which movie will be a hit or flop before screening it (De Vany and Walls, 1999). Likewise, moviegoers are uninformed of the quality of movies before viewing it and thus their decisions about which movie to see rely on factors other than movie quality. Here, we consider the situation where moviegoers form their beliefs about movie quality based on pricing schemes to which an exhibitor commits.

The main finding of Chapter 4 is that there exists a range of moviegoer's beliefs in which committing to uniform pricing is more profitable than committing to variable pricing. This range can be characterized by a low belief that a movie for which the exhibitor commits to charge a high price is of high quality (equivalently, a relatively high demand for a movie believed to be of lower quality). Therefore, (committing to) uniform pricing observed in the motion-picture industry reflects that audiences still remain uncertain about a movie's quality even though movie theaters (commit to) charge a high ticket price for the movie.

Chapter 2

On Optimal Couponing Strategies

2.1 Introduction

Couponing is a widely used promotional strategy in firms' competition. The most recent NCH (2011) report says that 332 billion coupons were distributed in 2010, the largest single-year distribution quantity ever recorded in the U.S.

Depending on the method of distribution, coupons can be divided into *mass media coupons* and *targeted coupons*. Targeted coupons then take two forms: *defensive coupons* and *offensive coupons* (Shaffer and Zhang, 1995). Mass media coupons are randomly distributed to consumers. Defensive coupons are distributed to retain firms' own customers, while offensive coupons are offered to poach rival firms' customers.

Previous studies on couponing consider mass media coupons, which leads to market segmentation through consumers' self-selection. Narasimhan (1984), among others, shows that coupons serve as a price discrimination vehicle, charging a lower price to coupon users (more price-elastic consumers).

As information about customers becomes more available and more accurate due to better technology, firms can now send targeted coupons to selected customers. Using a model of product differentiation à la Hotelling, Shaffer and Zhang (1995) examine the effect of targeted coupons on firm profits, prices, and coupon face values. Their main finding is that targeted coupons deteriorate firm profits due to increased competition for potential brand switchers. Bester and Petrakis (1996) study sales promotion via coupons in a duopoly. In their model, the role of (offensive) coupons is to reduce consumer switching costs.[1] They show that, in equilibrium, couponing intensifies competition between firms, and hence lowers their profits. Regarding the firms' ability to segment customers (information quality), Liu and Serfes (2004) show that for high levels of information quality, acquiring information to price discriminate is each firm's dominant strategy. In this case, a prisoner's dilemma emerges since equilibrium profits are lower under targeted pricing than under uniform pricing.

In the context of price discrimination based on purchase history, Chen (1997) considers a two-period homogeneous product duopoly model. Here consumers incur costs when switching one firm to another, which enables firms to segment and price discriminate consumers. He shows that in equilibrium, each firm charges a lower price to the competitor's customers than to its own customers in the second period (paying customers to switch) and that the discriminatory pricing lowers profits. Fudenberg and Tirole (2000) analyze a two-period duopoly model in which consumers have different preferences for firms' products and each firm can set different prices in period 2, depending on whether or not consumers have bought its product in period 1. They find that each firm can poach the rival firm's customers by charging them a lower price (customer poaching) and that the difference in the prices charged to loyal and switching customers reduces firm profits.[2]

As reviewed above, the existing literature on targeted coupons has mainly studied offensive couponing. There are two reasons for this: (i) consumers have to pay costs to switch between firms (Bester and Petrakis, 1996; Chen, 1997), and (ii) they prefer a specific firm's product (Fudenberg and Tirole, 2000). In either case, enticing brand switching through coupons (discounts) arises as the equilibrium outcome. The intuition for this is that consumers who bought from a competitor are revealed to have a lower relative preference for the firm's product (or a relatively high cost to switch to the firm), and so profit maximization requires the firm to charge a lower price to them. In addition, offensive couponing with exogenous switching costs and/or constant preferences leads to lower firm profits.[3] Caminal

[1]Based on a two-period differentiated product duopoly model, Klemperer (1987) examines the effects of consumer switching costs on the competitiveness of markets. He shows that an increase in the proportion of consumers whose tastes change between the periods makes a market more competitive, but that prices and profits in both periods are higher than in a market without switching costs if all consumers' tastes remain constant.

[2]Taylor (2003) extends Chen (1997) to multiple periods and multiple firms, and Villas-Boas (1999) provides an analysis similar to Fudenberg and Tirole (2000) but in a duopoly with infinitely lived firms and overlapping generations of consumers.

[3]A well-established result in the price discrimination literature is that oligopolistic price discrimination intensifies competition and leads to lower profits (see, among others, Thisse and Vives, 1988; Shaffer and

and Matutes (1990), on the other hand, consider the situation in which couponing endogenously creates switching costs for consumers by rewarding loyalty. Employing a two-period differentiated product duopoly model with independent preferences over time, they show that defensive couponing is more profitable than if coupons were not allowed.

Despite the extensive economic literature on couponing, the use of both defensive and offensive coupons has received little attention. In Shaffer and Zhang (1995) and Kosmopoulou, Liu, and Shuai (2012), firms can send defensive and offensive coupons together. However, these two studies are based on models with single purchase so that dynamic properties of consumer preferences are ignored, which may affect firms' couponing strategies.[4] Here we will try to fill the gap existing in the couponing literature by enlarging firms' couponing strategy space and allowing for changing preferences across purchase occasions.

The objective of the present study is to find the optimal couponing strategy for firms in a differentiated product duopoly with repeat purchase. Specifically, firms can send defensive coupons alone, defensive and offensive coupons together, or mass media coupons. They can also determine how many coupons to offer. This modeling approach differs from Caminal and Matutes (1990) in that they consider defensive coupons only where no decisions on the number of coupon offers are made by firms. However, sending out offensive coupons alone will not be considered as firms' couponing strategy because, rather than assuming exogenous switching costs and fixed preferences, we assume that consumers incur no costs when switching between firms' products and that their preferences are independent across periods. With regard to independent preferences, Chen and Pearcy (2010) show that when commitment to future prices is possible, firms reward consumer loyalty if intertemporal preference dependence is low, but pay consumers to switch if preference dependence is high.[5] We then incorporate fixed preferences and consumer myopia into the model to see how they

Zhang, 1995; Bester and Petrakis, 1996; Liu and Serfes, 2004). One important environment for price discrimination to intensify competition and to reduce firm profits is best-response asymmetry, i.e., one firm's strong market is the other firm's weak market (see Corts, 1998).

[4]Drawing on a data set with information on shelf prices and available coupons for 25 ready-to-eat breakfast cereals, Nevo and Wolfram (2002) find that lagged coupons are positively correlated with current sales, suggesting that coupons are used to induce repurchase.

[5]One issue in the literature on price discrimination is when firms should offer a lower price to loyal consumers or to new consumers. Shaffer and Zhang (2000) show that when demand is asymmetric, it is optimal that one firm charges a lower price to its rival's customers and the other charges a lower price to its own customers. In Shin and Sudhir (2010), paying customers to stay is optimal when both heterogeneity in purchase quantities and preference stochasticity are sufficiently high. De Nijs and Rhodes (2013) find that if over half of consumers believe their existing product is inferior (resp. superior) to the other one, firms offer a lower (resp. higher) price to their loyal consumers.

affect the profitability of the optimal couponing.

Focusing on a symmetric equilibrium, we obtain the following results. (i) Each firm's optimal couponing strategy is to send (defensive) coupons to all of its own customers. (ii) Sending out (offensive) coupons to a rival firm's customers is detrimental to firm profits. (iii) Offering mass media coupons is not profitable compared with the case where there are no coupons. (iv) The existence of consumers with constant tastes makes firms using the optimal couponing strategy better off. (v) In case the number of myopic consumers is large enough, the optimal couponing leads to lower profits than in the absence of coupons.

The rest of the paper is structured as follows. Section 2.2 sets up the model. In Section 2.3, firms' couponing strategies are analyzed to find the optimal one. Section 2.4 extends the model by allowing some consumers to have fixed preferences over time and considering consumer myopia. Section 2.5 concludes.

2.2 The model

Consider the following two-period differentiated duopoly model à la Hotelling.[6] Two firms (A and B) produce competing goods at a constant marginal cost, which we normalize to zero for simplicity. Firm A is located at point 0 and firm B at point 1 of the unit interval. In each period, there is a continuum of consumers uniformly distributed on the interval $[0, 1]$ with a unit mass. Each consumer is identified by her location on the interval, which corresponds to her ideal product. Consumers buy at most one unit of the good in each period and are willing to pay at most v. We assume that v is sufficiently high for non-purchase to be dismissed. A consumer located at $x \in [0, 1]$ incurs a disutility of tx when purchasing from firm A, and of $t(1 - x)$ when purchasing from firm B, where $t > 0$ measures the per-unit distaste's cost of buying away from her ideal product.

Each consumer's location in period 2 is allowed to vary over time randomly and independently of their first-period location (we will relax this assumption in Section 2.4).[7] In other words,

[6] The current two-period model captures the essential features of an infinite-horizon model since consumers' tastes change through time. Nevertheless, it would be interesting to extend our model to an infinite-horizon overlapping generations model with couponing.

[7] A consumer's first-period choice contains no information about her second-period preference. Hence in equilibrium, there is no price discrimination by purchase history. Instead, firms can use coupons or long-term contracts (commitment to future prices).

consumers all change preferences from period 1 to period 2 and do not know their second-period preferences in period 1. For example, a consumer's preferences for different airlines may vary from one period to the next as travel plans change. A customer may change her preferred shopping venue depending on whether shopping trip starts from home or work. All consumers are also forward-looking in the sense that in period 1, they buy from the firm that maximizes the sum of their first-period surpluses and their expected surpluses in period 2, anticipating the second-period prices, given the firms' couponing strategies (consumer myopia will be considered in Section 2.4). Throughout the paper, for $i \in \{A, B\}$, consumers who buy from firm i in period 1 will be called firm i's consumers and those who repurchase from firm i in period 2 will be called firm i's loyal consumers.

Prior to price competition, both firms can choose whether to send defensive coupons alone, defensive and offensive coupons together (hereafter *mixed coupons*), or mass media coupons. Here coupons redeemed in period 2 are discounts in absolute value for consumers who receive them in period 1. Defensive coupons are sent to a firm's own customers, while mixed coupons are distributed to both a firm's and its rival's customers. Mass media coupons are randomly distributed to consumers. We then have the following definition for firm $i \in \{A, B\}$:[8]

Definition 2.1. (1) Firm i is said to use a *defensive couponing strategy* $(\eta^i, 0)$ when it distributes coupons to a fraction $\eta^i \in (0, 1]$ of its own consumers and gives no coupons to its rival's consumers. In such a case η^i will be referred to as firm i's *defensive couponing intensity*. (2) Firm i is said to use a *mixed couponing strategy* $(1, \theta^i)$ when it distributes coupons to all of its consumers and a fraction $\theta^i \in [0, 1)$ of the rival firm's consumers. θ^i will be called firm i's *offensive couponing intensity*. (3) Firm i is said to use a *mass media couponing strategy* (μ^i) when it sends coupons randomly to a fraction $\mu^i \in (0, 1)$ of consumers distributed on the interval $[0, 1]$. We will call μ^i firm i's *mass media couponing intensity*.

We assume that each consumer is equally likely to receive a coupon and that trading coupons is not possible.[9] As can be seen from Definition 2.1, each couponing strategy is characterized by its intensity. Note that we generalize the model of Caminal and Matutes (1990) where only the case $(\eta^i, 0) = (1, \theta^i) = (1, 0)$ is studied. Let α_A denote firm A's market share in

[8] A similar definition can be found in Kosmopoulou, Liu, and Shuai (2012).

[9] Kosmopoulou, Liu, and Shuai (2012) examine the impact of coupon trading on equilibrium prices, promotion intensities (coupon face value and probability of receiving coupons), and profits. They find that when the fraction of coupon traders increases, firms respond by sending fewer coupons to consumers and reducing the face value of coupons, which leads to higher equilibrium prices and profits.

period 1 so that all consumers with $x \in [0, \alpha_A]$ have bought from firm A in the first period, whereas those with $x \in (\alpha_A, 1]$ have bought from firm B.

Given the firms' couponing strategies, the timing of the game is then as follows:

- Period 1: Firm $i \in \{A, B\}$ sets a first-period price (p_1^i), a coupon (face) value ($r^i \geq 0$), and its couponing intensity (η^i, θ^i, or μ^i) to maximize the sum of profits in periods 1 and 2, resulting in a portion α_A of consumers purchasing from firm A and the remaining portion $1 - \alpha_A$ purchasing from firm B.

- Period 2: Firm i chooses a second-period price (p_2^i). Each consumer decides whether or not to be loyal depending on her new location (preference) and the effective prices ($p_2^i - r^i$).

The following example is useful to clarify the firms' couponing strategies:

Example 2.1. Suppose that the two firms employ $(1, \theta^A) = (1, \theta^B) = (1, \frac{1}{3})$. Consider a consumer who is located at x_1 and bought from firm A in period 1, i.e. $x_1 \in [0, \alpha_A]$. In period 2, if she buys from firm A again, she will pay $p_2^A - r^A$. On the other hand, if she buys from firm B, she will pay $p_2^B - r^B$ (resp. p_2^B) with probability $\frac{1}{3}$ (resp. $\frac{2}{3}$). Thus, when this consumer buys from firm A (resp. B) in period 2, she enjoys utilities $v - tx_1 - p_1^A$ and $v - tx_2 - (p_2^A - r^A)$ (resp. $v - t(1 - x_2) - (p_2^B - \frac{1}{3}r^B)$) in periods 1 and 2, respectively, where x_2 is her new location (preference) in period 2.

For simplicity, the discount factor for both firms and consumers is assumed to be 1.[10] In addition, we assume that there is no cost of distributing defensive and mass media coupons, but that for firm i it costs $F(\theta^i)$ to send mixed coupons, where $F(\theta^i)$ takes the form of $\frac{c}{2}(\theta^i)^2$, $c > 0$. The cost of distributing mixed coupons, increasing and convex in θ^i, captures the idea that it is more costly to send coupons to consumers far away than to those near firms. Since the firms are *ex ante* symmetric, we focus on (pure-strategy) symmetric equilibria in which both firms offer defensive coupons, mixed coupons, or mass media coupons.

[10]The assumption does not affect the main results we will present.

2.3 Analysis

There are three subgames to consider, corresponding to the following scenarios: (i) both firms send defensive coupons; (ii) both firms send mixed coupons; and (iii) both firms distribute mass media coupons. For each subgame, we will use the subgame perfect Nash equilibrium as the solution concept and proceed by backward induction. We first investigate the firms' price competition, taking as given their choices of couponing intensities. Then the firms' decisions on how many coupons to offer are examined.

2.3.1 Defensive couponing

Consider first the case where both firms use defensive couponing strategies. It is assumed for a while that the firms' defensive couponing intensities are given as $\eta^A = \eta^B = \eta$. We start by constructing the demand for firm A in the second period. In period 2, α_A, r^i, and η are given. Recall that all consumers redraw their taste parameter at the beginning of period 2.

Consumers in period 2 can be divided into the following four groups, depending on which firm they bought from and whether they received coupons in period 1:

- (D1) Consumers in $[0, \alpha_A]$ with firm A's coupons; x_{10}; η
- (D2) Consumers in $[0, \alpha_A]$ without coupons; x_{00}; $1 - \eta$
- (D3) Consumers in $(\alpha_A, 1]$ with firm B's coupons; x_{01}; η
- (D4) Consumers in $(\alpha_A, 1]$ without coupons; x_{00}; $1 - \eta$.

Let us denote by x_{ab}, with $a, b \in \{0, 1\}$, a consumer who is indifferent between buying from A and buying from B in period 2. Specifically, x_{10} (resp. x_{01}) is the indifferent consumer of the group in which consumers receive coupons from firm A (resp. B). x_{00} denotes the indifferent consumer of the group without coupons. These indifferent consumers are then defined as follows:[11]

[11]Switching occurs in period 2 if $0 < x_{01} \leq x_{00} \leq x_{10} < 1$ or $r^B - t < p_2^B - p_2^A < t - r^A$, which is satisfied in the symmetric equilibrium.

$$
\begin{aligned}
v - tx_{10} - (p_2^A - r^A) &= v - t(1 - x_{10}) - p_2^B \\
v - tx_{00} - p_2^A &= v - t(1 - x_{00}) - p_2^B \\
v - tx_{01} - p_2^A &= v - t(1 - x_{01}) - (p_2^B - r^B).
\end{aligned}
$$

Example 2.2. Consider consumers belonging to group D3. They bought from firm B and received coupons in period 1. In period 2, a consumer in this group prefers purchasing from firm A to purchasing from firm B (i.e., she switches to firm A) if her new taste parameter is smaller than x_{01}: $x \leq x_{01} = \frac{p_2^B - p_2^A - r^B + t}{2t}$.

Since a fraction η of each firm's consumers receive coupons, the second-period demand of firm A is given by

$$
q_2^A(p_2^A, p_2^B) = \underbrace{\alpha_A \eta x_{10}}_{\text{in D1}} + \underbrace{\alpha_A(1-\eta)x_{00}}_{\text{in D2}} + \underbrace{(1-\alpha_A)\eta x_{01}}_{\text{in D3}} + \underbrace{(1-\alpha_A)(1-\eta)x_{00}}_{\text{in D4}}.
$$

Then, in period 2, a fraction $\alpha_A \eta x_{10}$ of consumers buy from firm A at the discounted price $p_2^A - r^A$, while another fraction $\alpha_A(1-\eta)x_{00} + (1-\alpha_A)\eta x_{01} + (1-\alpha_A)(1-\eta)x_{00}$ buy from firm A at the full price p_2^A. Thus, firm A's second-period maximization problem can be written as

$$
\max_{p_2^A} \pi_2^A = (\alpha_A \eta x_{10})(p_2^A - r^A) + [\alpha_A(1-\eta)x_{00} + (1-\alpha_A)\eta x_{01} + (1-\alpha_A)(1-\eta)x_{00}]\, p_2^A. \tag{2.1}
$$

The first-order condition for the problem (2.1) gives firm A's best-response function. We can proceed in a similar way for firm B and then solve the system of the two best-response functions to find the second-period equilibrium prices. Substituting the second-period equilibrium prices into π_2^A in (2.1) yields the equilibrium profit for firm A in period 2, denoted by $\hat{\pi}_2^A(\alpha_A, r^A, r^B; \eta)$. Assuming that all consumers are forward-looking, we can now express firm A's profit maximization problem in period 1 as

$$
\max_{p_1^A, r^A} \pi^A = \pi_1^A + \hat{\pi}_2^A, \tag{2.2}
$$

where $\pi_1^A = p_1^A \alpha_A(p_1^A, p_1^B, r^A, r^B; \eta)$.

Taking the first-order conditions for the problem (2.2) and imposing symmetry, we can characterize the equilibrium values of the game, as presented in the following lemma:

Lemma 2.1. *Suppose that firms A and B employ a defensive couponing strategy* $(\eta, 0)$. *In the equilibrium,*

(i) The first-period prices are

$$p_1^A = p_1^B = p_1^d = t + \frac{2t\eta(1+\eta)}{(2+\eta)^2}. \tag{2.3}$$

(ii) The values of the coupons are

$$r^A = r^B = r^d = \frac{2t}{2+\eta}. \tag{2.4}$$

(iii) The second-period prices are

$$p_2^A = p_2^B = p_2^d = t + \frac{t\eta}{2+\eta}. \tag{2.5}$$

(iv) The firms' profits are

$$\pi^A = \pi^B = \pi^d = t + t\left(\frac{\eta}{2+\eta}\right)^2. \tag{2.6}$$

Proof. See Appendix. □

From Lemma 2.1, we can see that in the equilibrium, as the firms distribute more coupons to their own customers, both the first- and second-period prices rise: $\frac{dp_1^d}{d\eta} > 0$ and $\frac{dp_2^d}{d\eta} > 0$. The intuition for the increase in the first-period price goes as follows. As more coupons are sent out, more consumers with the coupons accept to pay a higher price in period 1 since they anticipate their loyalty will be rewarded in period 2. The equilibrium profit also increases with the defensive couponing intensity: $\frac{d\pi^d}{d\eta} > 0$. On the other hand, the equilibrium value of a coupon decreases as the defensive couponing intensity increases: $\frac{dr^d}{d\eta} < 0$. It is noteworthy that if coupons are not used, the game is similar, in each period, to the standard Hotelling model so that for $j \in \{1, 2\}$, $p_j^A = p_j^B = p^n = t$, $\pi_j^A = \pi_j^B = \pi_j^n = \frac{t}{2}$, and $\pi^A = \pi^B = \pi^n = t$.

Here we use the superscript n to denote the equilibrium values in the case of no coupons. Hence, the prices in both periods are higher than in the absence of coupons, although loyal consumers with coupons pay a lower price in period 2: $p_2^d - r^d = t + \frac{t(\eta-2)}{2+\eta} < p^n = t$, for all $\eta \in (0,1]$. This result contrasts with the one of Villas-Boas (1999) who shows that dynamic price discrimination lowers all prices. In addition, defensive couponing allows the firms to increase their profits compared with the case where no coupons are used: $\pi^d = t + t\left(\frac{\eta}{2+\eta}\right)^2 > \pi^n = t$, for all $\eta \in (0,1]$. Unlike the general results in the oligopolistic price discrimination literature, dynamic price discrimination by coupons considered here boosts firm profits.[12]

It can also be checked that as the firms offer more coupons to their customers, more customers are less tempted to switch to a rival firm. Formally, $\eta x_{10} + (1-\eta)x_{00}$ (resp. $\eta x_{01} + (1-\eta)x_{00}$) is the fraction of firm A's (resp. B's) consumers who decide to buy from firm A in period 2. Then we have $\frac{d[\eta x_{10}+(1-\eta)x_{00}]}{d\eta} > 0$ (resp. $\frac{d[\eta x_{01}+(1-\eta)x_{00}]}{d\eta} < 0$), which leads more consumers to benefit from coupons in period 2. In this sense, the second period becomes more competitive as more coupons are sent out. Figure 2.1, drawn for $t = 1$, shows that the equilibrium profit in period 2 (π_2^d) is decreasing with η and lower than without couponing. Note that lower second-period profits are more than compensated by higher first-period profits.

Considering the firms' choices of defensive couponing intensities, we can now formulate the following result:

Proposition 2.1. *In the unique symmetric subgame perfect Nash equilibrium of defensive couponing, both firms distribute coupons to all of their own consumers.*

Proof. See Appendix. □

Proposition 2.1 says that it is optimal for each firm to choose the defensive couponing intensity of 1 in the first period.[13] This result with (2.3), (2.4), and (2.5) constitutes the subgame perfect outcome of defensive couponing.

In terms of competitiveness of coupons, defensive couponing softens competition regardless of the number of coupons offered. However, Caminal and Claici (2007) argue that loyalty-rewarding programs intensify competition unless the number of firms is sufficiently small and firms are restricted to use lump-sum coupons. Our result complements that of Caminal and

[12] See footnote 3.
[13] Recall that the firms incur no cost when distributing defensive coupons.

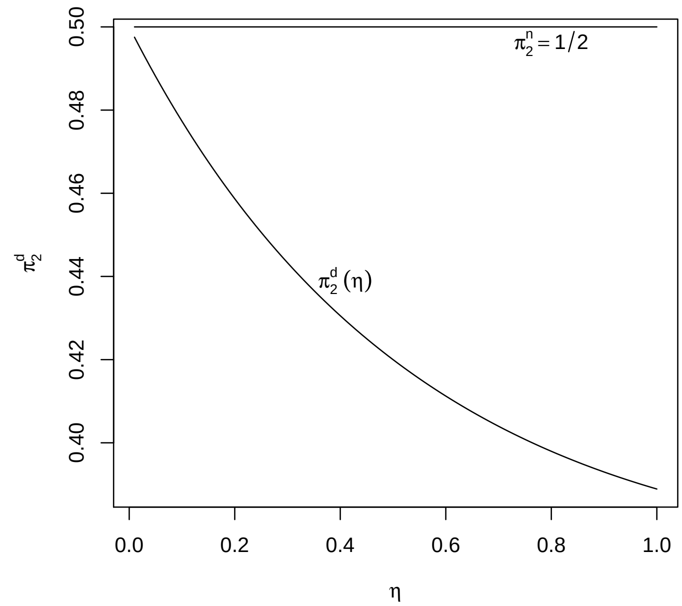

Figure 2.1: Profit when η varies ($t = 1$)

Claici (2007) in the sense that, in a duopoly, couponing for rewarding loyalty becomes more anti-competitive as firms send out more coupons.[14]

2.3.2 Mixed couponing

We next turn to the case in which both firms employ a mixed couponing strategy with $\theta^A = \theta^B = \theta$. As before, we analyze the game by first deriving the demand for firm A in the second period.

Consumers in period 2 can be segmented into the following four groups, depending on their first-period choices and the firms' offensive couponing intensity:

- (M1) Consumers in $[0, \alpha_A]$ with both firms' coupons; x_{11}; θ
- (M2) Consumers in $[0, \alpha_A]$ with only firm A's coupons; x_{10}; $1 - \theta$

[14] In her empirical study on the airline industry, Lederman (2007) finds that frequent flyer programs (FFPs) increase demand for airlines, and interprets this finding as evidence that FFP reinforces firms' market power. Fong and Liu (2011) show that rewarding loyalty makes tacit collusion easier to sustain.

- (M3) Consumers in $(\alpha_A, 1]$ with both firms' coupons; x_{11}; θ
- (M4) Consumers in $(\alpha_A, 1]$ with only firm B's coupons; x_{01}; $1-\theta$.

The indifferent consumers x_{10} and x_{01} are the same as in defensive couponing, while x_{11} is the indifferent consumer of the group in which consumers receive coupons from both firms and defined as[15]

$$v - tx_{11} - (p_2^A - r^A) = v - t(1-x_{11}) - (p_2^B - r^B).$$

Thus, the second-period demand of firm A is

$$q_2^A(p_2^A, p_2^B) = \underbrace{\alpha_A\theta x_{11}}_{\text{in M1}} + \underbrace{\alpha_A(1-\theta)x_{10}}_{\text{in M2}} + \underbrace{(1-\alpha_A)\theta x_{11}}_{\text{in M3}} + \underbrace{(1-\alpha_A)(1-\theta)x_{01}}_{\text{in M4}}.$$

In period 2, a fraction $\alpha_A\theta x_{11} + \alpha_A(1-\theta)x_{10} + (1-\alpha_A)\theta x_{11}$ of consumers buy from firm A at the discounted price $p_2^A - r^A$, while another fraction $(1-\alpha_A)(1-\theta)x_{01}$ buy from firm A at the full price p_2^A. Hence, we can write the second-period maximization problem of firm A as

$$\max_{p_2^A} \pi_2^A = \left[\alpha_A\theta x_{11} + \alpha_A(1-\theta)x_{10} + (1-\alpha_A)\theta x_{11}\right](p_2^A - r^A) + \left[(1-\alpha_A)(1-\theta)x_{01}\right]p_2^A. \tag{2.7}$$

Proceeding similarly for firm B and solving for the Nash equilibrium, we obtain the equilibrium profit for firm A in period 2, denoted by $\tilde{\pi}_2^A(\alpha_A, r^A, r^B; \theta)$. With forward-looking consumers, firm A's profit maximization problem in period 1 is

$$\max_{p_1^A, r^A} \pi^A = \pi_1^A + \tilde{\pi}_2^A, \tag{2.8}$$

where $\pi_1^A = p_1^A \alpha_A(p_1^A, p_1^B, r^A, r^B; \theta)$.

[15]Switching occurs in period 2 if $0 < x_{01} \leq x_{11} \leq x_{10} < 1$ or $r^B - t < p_2^B - p_2^A < t - r^A$, which is satisfied in the symmetric equilibrium.

Taking the first-order conditions for the problem (2.8) and imposing symmetry yields the next lemma, which characterizes the equilibrium values of the game:

Lemma 2.2. *Suppose that firms A and B use a mixed couponing strategy* $(1, \theta)$. *In the equilibrium,*

(i) The first-period prices are

$$p_1^A = p_1^B = p_1^m = t + \frac{2t(1-\theta)(2-3\theta)}{9(1+\theta)^2}. \tag{2.9}$$

(ii) The values of the coupons are

$$r^A = r^B = r^m = \frac{2t}{3(1+\theta)}. \tag{2.10}$$

(iii) The second-period prices are

$$p_2^A = p_2^B = p_2^m = \frac{4t}{3}. \tag{2.11}$$

(iv) The firms' profits are

$$\pi^A = \pi^B = \pi^m = t + \frac{t(1-\theta)(1-3\theta)}{9(1+\theta)^2} - F(\theta). \tag{2.12}$$

Proof. See Appendix. □

Lemma 2.2 gives results that are different from those of Lemma 2.1. In the equilibrium, the first-period price initially decreases and then increases in the offensive couponing intensity, approaching p^n (see Figure 2.2). The reason is that coupons sent out offensively in mixed couponing intensify price competition in period 1 since each firm should lower a first-period price to prevent its own customers from being poached by the rival firm's (offensive) coupons. The second-period price remains the same regardless of θ. The equilibrium value of a coupon, however, decreases with the offensive couponing intensity as in defensive couponing. Finally, as can be seen in Figure 2.3, the equilibrium profit decreases as the firms distribute more coupons to their rival firm's customers. Moreover, if both firms send coupons to more than

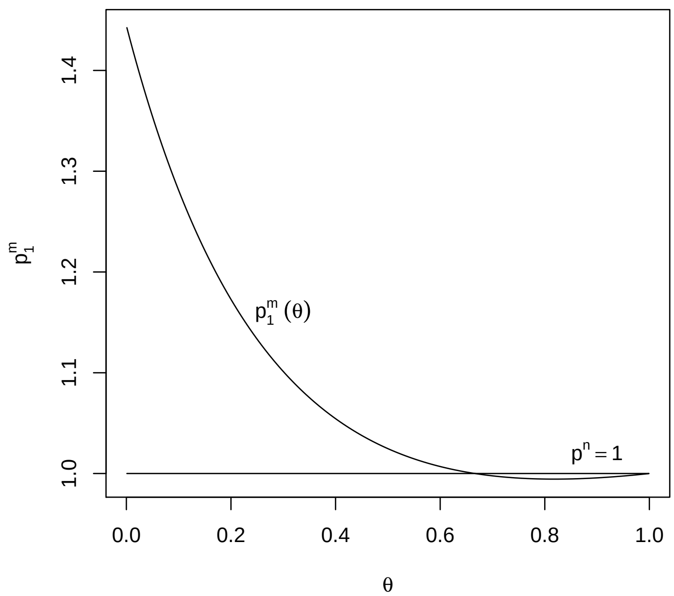

Figure 2.2: Price when θ varies ($t = 1$)

a fraction θ^* of the rival firm's customers, then mixed couponing is less profitable than in the case of no coupons, where θ^* is the solution to the equation $\frac{(1-\theta)(1-3\theta)}{[3\theta(1+\theta)]^2} = \frac{c}{2t}$.[16]

Considering the firms' choices of offensive couponing intensities, we can draw the next result:

Proposition 2.2. *In the unique symmetric subgame perfect Nash equilibrium of mixed couponing, the firms do not distribute any coupons to their rival's consumers.*

Proof. See Appendix. □

Proposition 2.2 states that it is optimal for each firm to choose the offensive couponing intensity of 0 in period 1. This result with (2.9), (2.10), and (2.11) also constitutes the subgame perfect outcome of mixed couponing. Under mixed couponing, the coupons sent to each firm's customers defensively increase the cost of attracting the rival firm's customers, because the discount required to entice an additional customer should cover this defensive

[16]Recall that when firm i sends mixed coupons so that a fraction $\theta^i \in [0, 1)$ of the competitor's customers receive firm i's offensive coupons, it has to pay the cost $F(\theta^i) = \frac{c}{2}(\theta^i)^2$, $c > 0$.

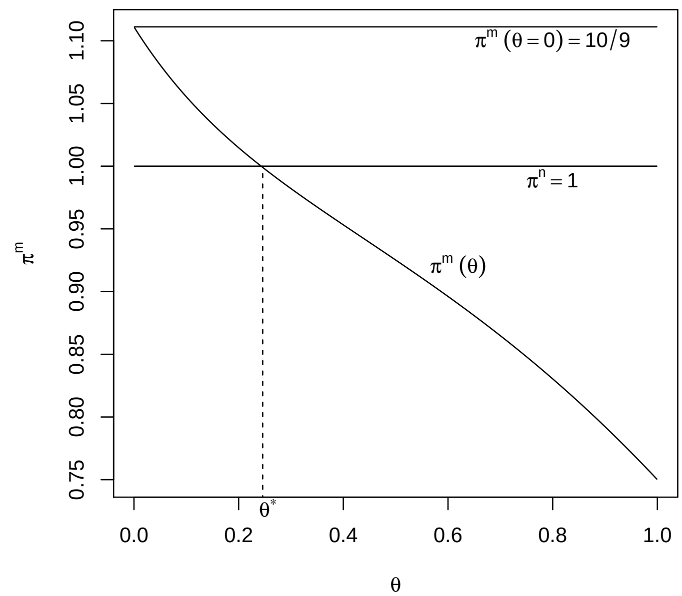

Figure 2.3: Profit when θ varies ($t = 1$, $c = \frac{1}{2}$)

coupon. Hence, the defensive coupons in mixed couponing reduce the firms' incentive to offer coupons offensively.

2.3.3 Mass media couponing

Finally, the case where both firms use a mass media couponing strategy with the same intensity is analyzed. Relegating all the expressions appearing in mass media couponing to the appendix, the equilibrium values of the game can be found as follows:

Lemma 2.3. *Suppose that firms A and B employ a mass media couponing strategy (μ). In the equilibrium,*

(i) The first-period prices are

$$p_1^A = p_1^B = p_1^s = t. \tag{2.13}$$

(ii) The values of the coupons are

$$r^A = r^B = r^s = 0. \tag{2.14}$$

(iii) The second-period prices are

$$p_2^A = p_2^B = p_2^s = t. \tag{2.15}$$

(iv) The firms' profits are

$$\pi^A = \pi^B = \pi^s = t. \tag{2.16}$$

Proof. See Appendix. □

Under mass media couponing, as can be seen from Definition 2.1, the probability that consumers receive coupons from each firm is the same, regardless of which firm they buy from. That is, all consumers have the same expected second-period surplus from buying from either firm in period 1. Hence, consumers rely only on current prices when making their first-period purchasing decisions. Recall that each firm's market share in period 1 (α_A and $1 - \alpha_A$ for firms A and B, respectively) is a function of the first-period prices and coupon values in the case of both defensive and mixed couponing. As a result, mass media couponing does not mitigate price competition in period 1 so that the firms have no incentive to issue coupons: $r^s = 0$. Here the outcome of the game is the same as in the absence of coupons.

From Lemma 2.3, and considering the firms' choices of mass media couponing intensities, the following proposition is obtained:

Proposition 2.3. *Mass media couponing is not profitable compared with the case of no coupons, regardless of its intensity. Furthermore, there are infinitely many symmetric subgame perfect Nash equilibria in the case of mass media couponing.*

Proof. See Appendix. □

Combining Propositions 2.1, 2.2, and 2.3, we can derive the result on the optimality of couponing strategies as follows:

Proposition 2.4. *Consider a differentiated product duopoly with repeat purchase, where all consumers change their preferences over time. Then the optimal couponing strategy is such that firms only offer coupons to all of the consumers who buy from them.*

This result is contrary to that of Kosmopoulou, Liu, and Shuai (2012) who find that in equilibrium firms have no incentive to distribute defensive coupons. The difference of the results between their study and the current one comes from the fact that, in our model, we have repeat purchase and consumer preferences are assumed to be independent over time, while Kosmopoulou, Liu, and Shuai (2012) use a model with single purchase.[17]

As discussed in Chen and Pearcy (2010), depending on intertemporal preference dependence, firms would implement the different forms of dynamic pricing. In the airline industry, for example, consumer loyalty will be rewarded as consumers are likely to change their preferences for different airlines. On the other hand, long-distance telephone companies will involve paying customers to switch as their preferences are unlikely to vary over time. In this vein, Proposition 2.4 has implications for marketing devices such as frequent-flyer programs (by airline companies) and frequent-stay programs (by hotels).

2.4 Extensions

In this section we extend the model by relaxing the assumptions: (i) consumer preferences are independent across periods and (ii) all consumers are forward-looking. In other words, we now explore how the existence of consumers with fixed preferences and that of myopic consumers affects the profitability of the optimal couponing. All relevant expressions will be provided in the appendix.

2.4.1 Constant preferences

Consider the case in which a fraction $\delta \in (0, 1)$ of consumers have intertemporally constant preferences for the firms' products, whereas the remaining fraction $1 - \delta$ change their tastes over time. Then, the symmetric subgame perfect outcome of the game is given in the following lemma:

[17] In Shaffer and Zhang (1995), each firm uses offensive couponing when the cost of coupon targeting is relatively high, and adjusts its strategy by sending more defensive coupons as the cost declines.

Lemma 2.4. *Suppose that firm $i \in \{A, B\}$ uses the optimal couponing strategy, $(\eta^i, 0) = (1, \theta^i) = (1, 0)$, and that a fraction $\delta \in (0, 1)$ of consumers have fixed preferences across periods. In the symmetric subgame perfect equilibrium,*

(i) The first-period prices are

$$p_1^A = p_1^B = p_1 = t - \delta \left[\frac{2(t - \delta)}{3(1 - \delta)} - t \right] + \left(\frac{2\delta}{3t} + \delta \right) r + \left(\frac{1 - \delta}{t} \right) r^2. \tag{2.17}$$

(ii) The value of each firm's coupon is the solution to the equation[18]

$$\frac{3(1-\delta)}{2t^2} r^3 + \left[\frac{(2-\delta)(3\delta - 1)}{2t} + \frac{\delta}{t^2} \right] r^2 + \left[\frac{\delta(2-\delta)(3\delta-1)}{3t(1-\delta)} + \frac{(1-\delta)(3+\delta)}{2} \right] r \tag{2.18}$$
$$- t(1 - \delta^2) - \frac{2\delta^2}{3} = 0.$$

(iii) The second-period prices are

$$p_2^A = p_2^B = p_2 = \frac{t}{1 - \delta} + \frac{r}{2}. \tag{2.19}$$

(iv) The firms' profits are then determined by

$$\pi^A = \pi^B = \pi = \frac{p_1}{2} + \frac{t}{2(1 - \delta)} - \frac{\delta}{4} r - \left(\frac{1 - \delta}{4t} \right) r^2. \tag{2.20}$$

Proof. See Appendix. □

From Lemma 2.4, we can check that, in the equilibrium, the value of a coupon decreases in δ, approaching 0 as δ goes to 1 (see Figure 2.4). The reason is that the presence of consumers with constant tastes makes a market less competitive so that each firm has less incentive to offer coupons to retain its customers. Note also that, in the equilibrium, the first-period price initially increases and then decreases, while the second-period price monotonically rises as δ increases. Finally, as can be seen in Figure 2.5, the equilibrium profit increases as more consumers have constant preferences, which immediately gives the following result:

[18] The cubic equation (2.18) has one real root for all $t > 0$ and all $\delta \in (0, 1)$, whose proof is available upon request.

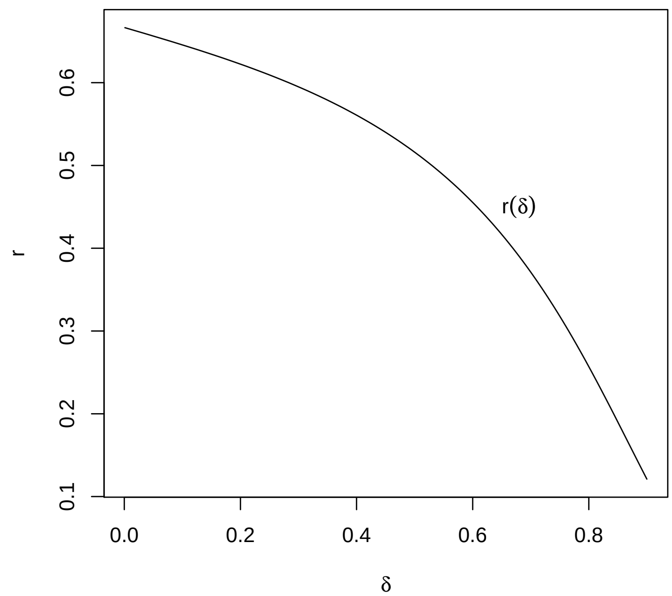

Figure 2.4: Coupon when δ varies ($t = 1$)

Proposition 2.5. *Under the assumptions of Lemma 2.4, an increase in the proportion of consumers whose intertemporal preferences are constant raises the firms' profits.*

2.4.2 Consumer myopia

We now examine the situation where some consumers are myopic in the sense that they only care about first-period prices when making their purchasing decisions in period 1. Specifically, a fraction $\lambda \in (0, 1)$ of consumers are myopic, while the complementary fraction $1 - \lambda$ are forward-looking. The following lemma presents the symmetric subgame perfect outcome of the game:

Lemma 2.5. *Suppose that firm $i \in \{A, B\}$ uses the optimal couponing strategy, $(\eta^i, 0) = (1, \theta^i) = (1, 0)$, and that a fraction $\lambda \in (0, 1)$ of consumers are myopic. In the symmetric subgame perfect equilibrium,*

(i) The first-period prices are

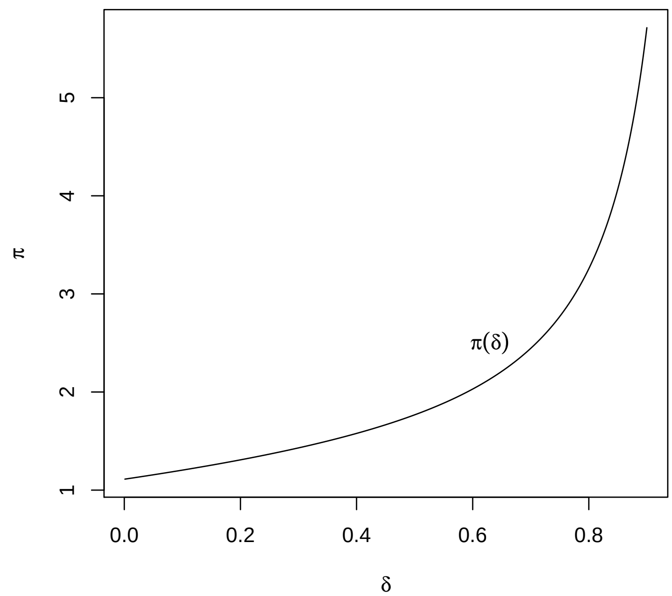

Figure 2.5: Profit when δ varies ($t = 1$)

$$p_1^A = p_1^B = p_1 = \frac{1}{\frac{\lambda}{t} + \frac{t(1-\lambda)}{t^2+r^2}}. \tag{2.21}$$

(ii) The value of each firm's coupon is the solution to the equation[19]

$$r^3 + \left(\frac{t^2}{\lambda}\right) r - \frac{2t^3(1-\lambda)}{3\lambda} = 0. \tag{2.22}$$

(iii) The second-period prices are

$$p_2^A = p_2^B = p_2 = t + \frac{r}{2}. \tag{2.23}$$

(iv) The firms' profits are then determined by

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[19] $r^3 + \left(\frac{t^2}{\lambda}\right) r - \frac{2t^3(1-\lambda)}{3\lambda}$ is monotone increasing in r since $\frac{t^2}{\lambda} > 0$. Hence, (2.22) has one real root for all $t > 0$ and all $\lambda \in (0, 1)$.

$$\pi^A = \pi^B = \pi = \frac{p_1}{2} + \frac{t}{2} - \frac{r^2}{4t}. \tag{2.24}$$

Proof. See Appendix. □

Note first that in the equilibrium, the value of a coupon decreases in λ, approaching 0 as λ goes to 1 (see Figure 2.6). Both the first- and second-period prices also decrease to $p^n = t$ as λ goes to 1. The intuition behind the results is that each firm is able to charge a higher first-period price to its consumers because forward-looking consumers can agree to pay the price as they expect their loyalty to be rewarded (by defensive coupons) in period 2. However, when it comes to myopic consumers, coupons cannot entice them to accept a higher price since they only care about current prices in making purchasing decisions in period 1. Therefore, as more consumers are myopic, price competition in period 1 becomes more aggressive and offering coupons is less attractive for the firms.

In addition, as can be seen in Figure 2.7, the equilibrium profit first falls and then rises in λ, approaching π^n. Using this fact and solving for λ from $\pi = \pi^n$, the following result is obtained:

Proposition 2.6. *Under the assumptions of Lemma 2.5, there exists a critical fraction of myopic consumers, $\lambda^* = \frac{9}{19}$, such that if $\lambda > \lambda^*$ then the optimal couponing leads to lower profits than in the absence of coupons.*

This shows that if there are enough myopic consumers in a market, the optimal couponing is rather detrimental to firm profits. In this case it is better for the firms not to distribute any coupons.

According to NCH (2011), the number of digital coupon offers increased by 37% in 2010, the largest increase across all types of coupon media. The above result provides a possible explanation as to why more and more firms use Internet websites to distribute coupons. Consumers searching for coupons on Internet websites are more likely to be forward-looking as they are seeking future discounts via coupons, and thus firms might be able to screen out myopic consumers by distributing coupons through Internet websites.

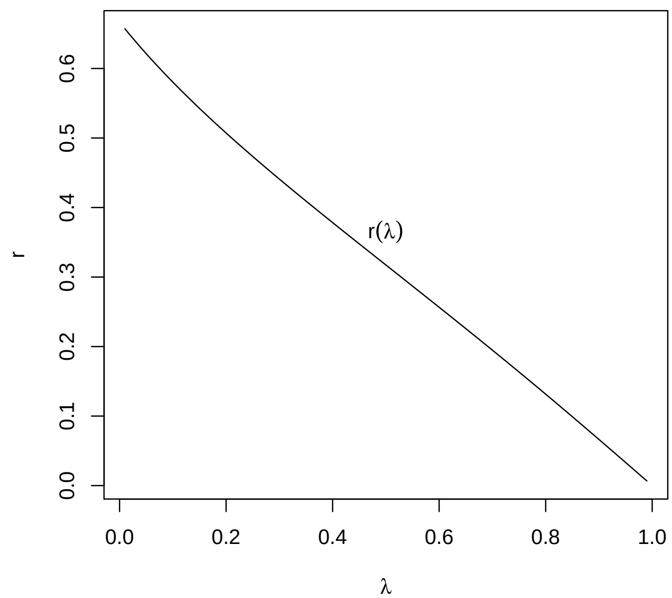

Figure 2.6: Coupon when λ varies ($t = 1$)

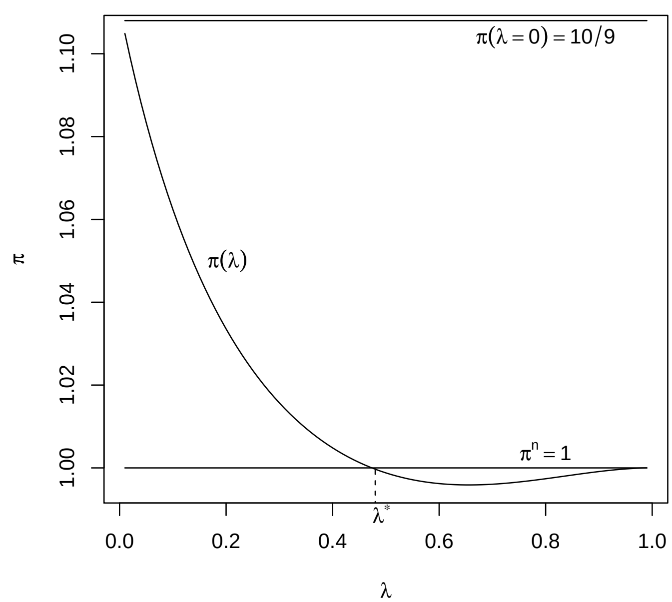

Figure 2.7: Profit when λ varies ($t = 1$)

2.5 Conclusion

Based on a differentiated product duopoly model with repeat purchase, targeted (defensive and offensive) coupons and mass media coupons are explored to find the optimal couponing strategy. By allowing for independent preferences between purchases, we arrive at the following results.

First, defensive couponing allows the firms to increase their profits compared with the case of no coupons, regardless of how many coupons are distributed. Each firm's profit is then maximized when it offers coupons to all of its own customers. The intuition behind this result is that consumers with coupons accept to pay a higher price in period 1 as they anticipate their loyalty will be rewarded in period 2. In addition, as more coupons are offered defensively, the second period becomes more competitive although the prices in both periods rise. This is because in period 2, more consumers decide to remain loyal as the number of coupons distributed increases, and the effective price paid by them is lower than without coupons.

Second, under mixed couponing, sending out coupons to poach a rival firm's customers reduces firm profits. Moreover, it leads to lower profits than without couponing when too many coupons are distributed to a competitor's customers. The reason is that each firm should lower their first-period price to prevent the rival's offensive coupons from luring away its customers, which results in intensified competition in period 1.

Third, randomly distributed coupons are not conducive to the firms' profits. Under this mass media couponing, the probability of receiving coupons from each firm is the same, independent of consumers' choices in period 1. Thus, consumers only care about first-period prices when making their purchasing decisions in period 1, which leads to fierce price competition in the first period. From the above results, therefore, it turns out that, if no consumers are myopic, the optimal couponing strategy for the firms is to only distribute coupons to all of the customers who buy from them.

It is also shown that, when the firms use the optimal couponing strategy, an increase in the proportion of consumers whose preferences are time-invariant boosts their profits. The reason is that the presence of consumers with time-invariant tastes softens competition between the firms.

Finally, the optimal couponing can be less profitable than in the absence of coupons if there are sufficiently many myopic consumers in a market. This is due to the fact that rewarding loyalty through defensive coupons cannot induce myopic consumers to agree to a higher first-period price since only current prices matter for their purchasing decisions in period 1. Hence, the existence of myopic consumers makes price competition more intense. This suggests that when price discriminating by defensive coupons, firms can achieve higher profits by screening out myopic consumers.

2.6 Appendix

Proof of Lemma 2.1

From the first-order condition for the problem (2.1), we find firm A's best-response function:

$$p_2^A(p_2^B) = \frac{1}{2}\left[t + p_2^B + 2\alpha_A \eta r^A - (1-\alpha_A)\eta r^B\right]. \tag{2.25}$$

Similarly, firm B's best-response function can be found as

$$p_2^B(p_2^A) = \frac{1}{2}\left[t + p_2^A + 2(1-\alpha_A)\eta r^B - \alpha_A \eta r^A\right]. \tag{2.26}$$

Solving (2.25) and (2.26) simultaneously gives the second-period equilibrium prices:

$$p_2^A = t + \alpha_A \eta r^A \;\text{ and }\; p_2^B = t + (1-\alpha_A)\eta r^B. \tag{2.27}$$

We can see that the second-period prices increase both in the firm's first-period market share and in the coupon value. Obviously, they are also increasing in the defensive couponing intensity. The equilibrium profit for firm A in period 2 is then calculated as

$$\hat{\pi}_2^A = \frac{t}{2} - \frac{1}{2t}\left[\alpha_A(1-\alpha_A)\eta^2 r^A r^B + \alpha_A(1-\alpha_A\eta)\eta(r^A)^2\right]. \tag{2.28}$$

Now we need to compute firm A's first-period market share α_A, which depends on p_1^i and r^i. The indifferent consumer, $\hat{x}$, is such that the sum of the difference in her first-period surpluses from buying from firms A and B (denoted by $\Delta S_1 = S_1^A - S_1^B$) and the difference in her expected second-period surpluses (denoted by $\Delta S_2 = S_2^A - S_2^B$) is equal to zero. The first-period surplus difference is simply given by

$$\Delta S_1 = S_1^A - S_1^B = (v - t\hat{x} - p_1^A) - (v - t(1-\hat{x}) - p_1^B) = t - 2t\hat{x} + p_1^B - p_1^A.$$

Consumers do not know which will be their taste parameter in period 2. Note also that a fraction η of each firm's consumers receive coupons. Thus, the expected second-period surplus from buying from firm A in period 1 can be written as

$$\begin{aligned} S_2^A = \eta & \left[\int_0^{x_{10}} \left(v - tx - (p_2^A - r^A)\right) dx + \int_{x_{10}}^1 \left(v - t(1-x) - p_2^B\right) dx\right] \\ & + (1-\eta)\left[\int_0^{x_{00}} \left(v - tx - p_2^A\right) dx + \int_{x_{00}}^1 \left(v - t(1-x) - p_2^B\right) dx\right]. \end{aligned}$$

For consumers who receive (resp. do not receive) coupons from firm A, if the new preference is below x_{10} (resp. x_{00}), the consumer buys again from firm A; otherwise, she buys from firm B. In the same vein, the expected second-period surplus from buying from firm B in period 1 is

$$\begin{aligned} S_2^B = \eta & \left[\int_0^{x_{01}} \left(v - tx - p_2^A\right) dx + \int_{x_{01}}^1 \left(v - t(1-x) - (p_2^B - r^B)\right) dx\right] \\ & + (1-\eta)\left[\int_0^{x_{00}} \left(v - tx - p_2^A\right) dx + \int_{x_{00}}^1 \left(v - t(1-x) - p_2^B\right) dx\right]. \end{aligned}$$

After some algebra, we have

$$\begin{aligned} S_2^A &= \eta\left(tx_{10}^2 + v - p_2^B - t + \frac{t^2}{2}\right) + (1-\eta)\left(tx_{00}^2 + v - p_2^B - t + \frac{t^2}{2}\right) \\ S_2^B &= \eta\left(tx_{01}^2 + v - p_2^B + r^B - t + \frac{t^2}{2}\right) + (1-\eta)\left(tx_{00}^2 + v - p_2^B - t + \frac{t^2}{2}\right), \end{aligned}$$

which gives

$$\Delta S_2 = S_2^A - S_2^B = t\eta(x_{10}^2 - x_{01}^2) - \eta r^B.$$

Since $\hat{x}$ is defined by $\Delta S_1 + \Delta S_2 = 0$ and $\hat{x} = \alpha_A$, the first-period market share of firm A, α_A, is given implicitly by

$$t - 2t\alpha_A + p_1^B - p_1^A = \eta r^B + t\eta(x_{01}^2 - x_{10}^2), \tag{2.29}$$

where $x_{10} = \frac{dp_2 + r^A + t}{2t}$, $x_{01} = \frac{dp_2 - r^B + t}{2t}$, and $dp_2 = p_2^B - p_2^A = (1-\alpha_A)\eta r^B - \alpha_A \eta r^A$.

The first-order conditions for the problem (2.2) are

$$\begin{cases} \frac{\partial \pi^A}{\partial p_1^A} = \alpha_A + p_1^A \frac{\partial \alpha_A}{\partial p_1^A} - \frac{1}{2t}\left[(1-2\alpha_A)\eta^2 r^A r^B + (1-2\alpha_A\eta)\eta(r^A)^2\right]\frac{\partial \alpha_A}{\partial p_1^A} = 0 \\ \frac{\partial \pi^A}{\partial r^A} = p_1^A \frac{\partial \alpha_A}{\partial r^A} - \frac{1}{2t}\left[\alpha_A(1-\alpha_A)\eta^2 r^B + 2\alpha_A(1-\alpha_A\eta)\eta r^A\right] \\ \qquad -\frac{1}{2t}\left[(1-2\alpha_A)\eta^2 r^A r^B + (1-2\alpha_A\eta)\eta(r^A)^2\right]\frac{\partial \alpha_A}{\partial r^A} = 0. \end{cases} \tag{2.30}$$

Here the values of $\frac{\partial \alpha_A}{\partial p_1^A}$ and $\frac{\partial \alpha_A}{\partial r^A}$ are obtained by using the implicit function theorem for (2.29). We focus on a symmetric equilibrium so that $p_1^A = p_1^B = p_1$, $r^A = r^B = r$, and $\alpha_A = \frac{1}{2}$. Then we have

$$\frac{\partial \alpha_A}{\partial p_1^A} = \frac{-t}{2(t^2+\eta^2 r^2)} \quad \text{and} \quad \frac{\partial \alpha_A}{\partial r^A} = \frac{\eta[t+(1-\eta)r]}{4(t^2+\eta^2 r^2)}. \tag{2.31}$$

Replacing $\frac{\partial \alpha_A}{\partial p_1^A}$ and $\frac{\partial \alpha_A}{\partial r^A}$ in (2.30) by (2.31) and solving the system gives (2.3) and (2.4). (2.5) is obtained by plugging (2.4) into (2.27). With (2.3), (2.4), and (2.28) we get (2.6). $\square$

Proof of Proposition 2.1

To prove the proposition, it suffices to show that $\forall \eta^i \in (0,1)$, $\frac{\partial \pi^i}{\partial \eta^i} > 0$ at a symmetric equilibrium, $i \in \{A, B\}$. Here, we will see whether firm A has incentive to deviate from $\eta^A = \eta^B = \eta \in (0,1)$. Considering η^A and η^B, the problem (2.1) becomes

$$\max_{p_2^A} \pi_2^A = (\alpha_A \eta^A x_{10})(p_2^A - r^A) + \left[\alpha_A(1-\eta^A)x_{00} + (1-\alpha_A)\eta^B x_{01} + (1-\alpha_A)(1-\eta^B)x_{00}\right]p_2^A.$$

The first-order condition for the above problem gives firm A's best-response function. Proceeding similarly for firm B and solving the two best-response functions, we obtain

$$p_2^A = t + \alpha_A \eta^A r^A \quad \text{and} \quad p_2^B = t + (1-\alpha_A)\eta^B r^B.$$

The equilibrium profit for firm A in period 2 is then calculated as

$$\hat{\pi}_2^A = \frac{t}{2} - \frac{1}{2t}\left[\alpha_A(1-\alpha_A)\eta^A\eta^B r^A r^B + \alpha_A(1-\alpha_A\eta^A)\eta^A(r^A)^2\right].$$

Using the same procedure as in Lemma 2.1 yields the first-period market share of firm A given implicitly by

$$t - 2t\alpha_A + p_1^B - p_1^A = \eta^B r^B + t\eta^B x_{01}^2 - t\eta^A x_{10}^2 + t(\eta^A - \eta^B)x_{00}^2.$$

Applying the implicit function theorem and evaluating at a symmetric equilibrium, we have

$$\frac{\partial \alpha_A}{\partial \eta^A} = \frac{2tr + (1-2\eta)r^2}{8(t^2 + \eta^2 r^2)}.$$

Note that $\pi^A = p_1^A \alpha_A + \hat{\pi}_2^A$. A few lines of computations can show that

$$\frac{\partial \pi^A}{\partial \eta^A} = p_1^A \frac{\partial \alpha_A}{\partial \eta^A} + \frac{\partial \hat{\pi}_2^A}{\partial \eta^A} = \frac{t}{2(2+\eta)^2} > 0,$$

at $p_1^A = p_1^B = p_1^d$, $r^A = r^B = r^d$, and $\eta^A = \eta^B = \eta \in (0,1)$. $\square$

Proof of Lemma 2.2

From the first-order condition for the problem (2.7), we find firm A's best-response function:

$$p_2^A(p_2^B) = \frac{1}{2}\left(t + p_2^B + 2r^A\Phi - r^B\Psi\right), \tag{2.32}$$

where $\Phi = \alpha_A + (1-\alpha_A)\theta$ and $\Psi = \alpha_A\theta + (1-\alpha_A)$.

Similarly, firm B's best-response function can be found as

$$p_2^B(p_2^A) = \frac{1}{2}\left(t + p_2^A + 2r^B\Psi - r^A\Phi\right). \tag{2.33}$$

Solving (2.32) and (2.33), we obtain the second-period equilibrium prices:

$$p_2^A = t + r^A\Phi \quad \text{and} \quad p_2^B = t + r^B\Psi. \tag{2.34}$$

The equilibrium profit for firm A in period 2 is then

$$\tilde{\pi}_2^A = \frac{t}{2} - \frac{1}{2t}\left[r^A(1-\Phi)(r^A\Phi + r^B(1-\Psi))\right]. \tag{2.35}$$

As in Lemma 2.1, we need to compute firm A's first-period market share α_A. Let $\tilde{x}$ be the indifferent consumer. Assuming forward-looking consumers, $\tilde{x}$ is such that $\Delta S_1 + \Delta S_2 = 0$. The first-period surplus difference is given by

$$\Delta S_1 = S_1^A - S_1^B = (v - t\tilde{x} - p_1^A) - (v - t(1-\tilde{x}) - p_1^B) = t - 2t\tilde{x} + p_1^B - p_1^A.$$

The expected second-period surpluses from buying from firms A and B in period 1 can be, respectively, expressed as

$$\begin{aligned}
S_2^A = {}& \theta\left[\int_0^{x_{11}}\left(v - tx - (p_2^A - r^A)\right)dx + \int_{x_{11}}^1\left(v - t(1-x) - (p_2^B - r^B)\right)dx\right] \\
& + (1-\theta)\left[\int_0^{x_{10}}\left(v - tx - (p_2^A - r^A)\right)dx + \int_{x_{10}}^1\left(v - t(1-x) - p_2^B\right)dx\right] \\
S_2^B = {}& \theta\left[\int_0^{x_{11}}\left(v - tx - (p_2^A - r^A)\right)dx + \int_{x_{11}}^1\left(v - t(1-x) - (p_2^B - r^B)\right)dx\right] \\
& + (1-\theta)\left[\int_0^{x_{01}}\left(v - tx - p_2^A\right)dx + \int_{x_{01}}^1\left(v - t(1-x) - (p_2^B - r^B)\right)dx\right].
\end{aligned}$$

A few lines of computations establish that

$$S_2^A = \theta\left(tx_{11}^2 + v - p_2^B + r^B - \frac{t}{2}\right) + (1-\theta)\left(tx_{10}^2 + v - p_2^B - \frac{t}{2}\right)$$
$$S_2^B = \theta\left(tx_{11}^2 + v - p_2^B + r^B - \frac{t}{2}\right) + (1-\theta)\left(tx_{01}^2 + v - p_2^B + r^B - \frac{t}{2}\right),$$

which gives

$$\Delta S_2 = S_2^A - S_2^B = t(1-\theta)(x_{10}^2 - x_{01}^2) - (1-\theta)r^B.$$

Since $\tilde{x}$ is defined by $\Delta S_1 + \Delta S_2 = 0$ and $\tilde{x} = \alpha_A$, the first-period market share of firm A is given implicitly by

$$t - 2t\alpha_A + p_1^B - p_1^A = (1-\theta)r^B + t(1-\theta)(x_{01}^2 - x_{10}^2), \tag{2.36}$$

where $x_{10} = \frac{dp_2 + r^A + t}{2t}$, $x_{01} = \frac{dp_2 - r^B + t}{2t}$, and $dp_2 = p_2^B - p_2^A = r^B\Psi - r^A\Phi$.

The first-order conditions for the problem (2.8) are then

$$\begin{cases} \frac{\partial \pi^A}{\partial p_1^A} = \alpha_A + p_1^A \frac{\partial \alpha_A}{\partial p_1^A} - \frac{1}{2t}\big[(\theta-1)r^A(r^A\Phi + r^B(1-\Psi)) \\ \qquad +(1-\theta)r^A(r^A+r^B)(1-\Phi)\big]\frac{\partial \alpha_A}{\partial p_1^A} = 0 \\ \frac{\partial \pi^A}{\partial r^A} = p_1^A \frac{\partial \alpha_A}{\partial r^A} - \frac{1}{2t}\left[2r^A\Phi(1-\Phi) + r^B(1-\Phi)(1-\Psi)\right] \\ \qquad -\frac{1}{2t}\left[(\theta-1)r^A(r^A\Phi + r^B(1-\Psi)) + (1-\theta)r^A(r^A+r^B)(1-\Phi)\right]\frac{\partial \alpha_A}{\partial r^A} = 0. \end{cases} \tag{2.37}$$

Using the implicit function theorem for (2.36) and imposing symmetry (i.e., $p_1^A = p_1^B = p_1$, $r^A = r^B = r$, and so $\alpha_A = \frac{1}{2}$), we obtain

$$\frac{\partial \alpha_A}{\partial p_1^A} = \frac{-t}{2[t^2 + (1-\theta)^2 r^2]} \quad \text{and} \quad \frac{\partial \alpha_A}{\partial r^A} = \frac{(1-\theta)(t-\theta r)}{4[t^2 + (1-\theta)^2 r^2]}. \tag{2.38}$$

Replacing $\frac{\partial \alpha_A}{\partial p_1^A}$ and $\frac{\partial \alpha_A}{\partial r^A}$ in (2.37) by (2.38) and solving the system, we get (2.9) and (2.10). Plugging (2.10) into (2.34) gives (2.11). Using (2.9), (2.10), (2.35), and $F(\theta)$, we obtain (2.12). $\square$

Proof of Proposition 2.2

To prove the proposition, it is enough to show that $\forall \theta^i \in (0,1)$, $\frac{\partial \pi^i}{\partial \theta^i} < 0$ at a symmetric equilibrium, $i \in \{A, B\}$. In what follows, we will check whether firm A has incentive to deviate from $\theta^A = \theta^B = \theta \in (0,1)$. Considering θ^A and θ^B, the problem (2.7) becomes

$$\max_{p_2^A} \pi_2^A = \left[\alpha_A \theta^B x_{11} + \alpha_A(1-\theta^B)x_{10} + (1-\alpha_A)\theta^A x_{11}\right](p_2^A - r^A) + \left[(1-\alpha_A)(1-\theta^A)x_{01}\right] p_2^A.$$

The first-order condition for the above problem gives firm A's best-response function. Proceeding similarly for firm B and solving the two best-response functions, we obtain

$$p_2^A = t + r^A \Phi' \ \text{ and } \ p_2^B = t + r^B \Psi',$$

where $\Phi' = \alpha_A + (1-\alpha_A)\theta^A$ and $\Psi' = \alpha_A \theta^B + (1-\alpha_A)$.

The equilibrium profit for firm A in period 2 is then calculated as

$$\tilde{\pi}_2^A = \frac{t}{2} - \frac{1}{2t}\left[r^A(1-\Phi')(r^A\Phi' + r^B(1-\Psi'))\right].$$

Using the same procedure as in Lemma 2.2 yields the first-period market share of firm A given implicitly by

$$t - 2t\alpha_A + p_1^B - p_1^A = (1-\theta^B)r^B + t(1-\theta^A)x_{01}^2 - t(1-\theta^B)x_{10}^2 + t(\theta^A - \theta^B)x_{11}^2.$$

Applying the implicit function theorem and evaluating at a symmetric equilibrium, we have

$$\frac{\partial \alpha_A}{\partial \theta^A} = \frac{-2tr - (1-2\theta)r^2}{8[t^2 + (1-\theta)^2 r^2]}.$$

Note that $\pi^A = p_1^A \alpha_A + \tilde{\pi}_2^A$. After some computations, we can check that

$$\frac{\partial \pi^A}{\partial \theta^A} = p_1^A \frac{\partial \alpha_A}{\partial \theta^A} + \frac{\partial \tilde{\pi}_2^A}{\partial \theta^A} = -\frac{t}{6(1+\theta)^2} < 0,$$

at $p_1^A = p_1^B = p_1^m$, $r^A = r^B = r^m$, and $\theta^A = \theta^B = \theta \in (0,1)$. $\square$

Proof of Lemma 2.3

Let us begin by deriving the demand for firm A in the second period. In period 1, all consumers have the same probability of receiving coupons from each firm, regardless of which firm they buy from. Thus consumers in period 2 can be segmented into the following four groups:

- Consumers with both firms' coupons; x_{11}; μ^2
- Consumers with only firm A's coupons; x_{10}; $\mu(1-\mu)$
- Consumers with only firm B's coupons; x_{01}; $(1-\mu)\mu$
- Consumers without coupons; x_{00}; $(1-\mu)^2$.

The second-period demand of firm A is then given by

$$q_2^A(p_2^A, p_2^B) = \mu^2 x_{11} + \mu(1-\mu)x_{10} + (1-\mu)\mu x_{01} + (1-\mu)^2 x_{00}.$$

Here, a fraction $\mu^2 x_{11} + \mu(1-\mu)x_{10}$ of consumers buy from firm A at $p_2^A - r^A$, while another fraction $(1-\mu)\mu x_{01} + (1-\mu)^2 x_{00}$ buy from firm A at p_2^A. Hence, firm A's second-period maximization problem is

$$\max_{p_2^A} \pi_2^A = \left[\mu^2 x_{11} + \mu(1-\mu)x_{10}\right](p_2^A - r^A) + \left[(1-\mu)\mu x_{01} + (1-\mu)^2 x_{00}\right] p_2^A. \tag{2.39}$$

From the first-order condition for the problem (2.39), we find firm A's best-response function:

$$p_2^A(p_2^B) = \frac{1}{2}\left(t + p_2^B + 2\mu r^A - \mu r^B\right). \tag{2.40}$$

Similarly, firm B's best-response function can be found as

$$p_2^B(p_2^A) = \frac{1}{2}\left(t + p_2^A + 2\mu r^B - \mu r^A\right). \tag{2.41}$$

Solving (2.40) and (2.41) simultaneously gives the second-period equilibrium prices:

$$p_2^A = t + \mu r^A \quad \text{and} \quad p_2^B = t + \mu r^B. \tag{2.42}$$

Note that the second-period prices are independent of each firm's first-period market share. The equilibrium profit for firm A in period 2 is then

$$\bar{\pi}_2^A = \frac{t}{2} - \frac{\mu(1-\mu)(r^A)^2}{2t}. \tag{2.43}$$

Finally, we can set up firm A's profit maximization problem in period 1:

$$\max_{p_1^A, r^A} \pi^A = p_1^A \alpha_A + \bar{\pi}_2^A. \tag{2.44}$$

The first-order conditions for the problem (2.44) are

$$\begin{cases} \frac{\partial \pi^A}{\partial p_1^A} = \alpha_A + p_1^A \frac{\partial \alpha_A}{\partial p_1^A} = 0 \\ \frac{\partial \pi^A}{\partial r^A} = p_1^A \frac{\partial \alpha_A}{\partial r^A} - \frac{\mu(1-\mu) r^A}{t} = 0. \end{cases} \tag{2.45}$$

Since the probability that consumers receive coupons from each firm is the same, independent of their first-period choices, all consumers have the same expected second-period surplus from buying from each firm in period 1, i.e. $\Delta S_2 = 0$. Thus the first-period market share of firm A, α_A, is defined by

$$\Delta S_1 = t - 2t\alpha_A + p_1^B - p_1^A = 0,$$

which leads to

$$\frac{\partial \alpha_A}{\partial p_1^A} = -\frac{1}{2t} \quad \text{and} \quad \frac{\partial \alpha_A}{\partial r^A} = 0. \tag{2.46}$$

Plugging (2.46) into (2.45) gives (2.13) and (2.14). Recall here that $0 < \mu < 1$. (2.15) is obtained from (2.42) and (2.16) is derived from (2.13), (2.14), and (2.43). □

Proof of Proposition 2.3

To prove the second part of the proposition, it suffices to show that $\forall \mu^i \in (0,1)$, $\frac{\partial \pi^i}{\partial \mu^i} = 0$ at a symmetric equilibrium, $i \in \{A, B\}$. We will see whether firm A has incentive to deviate from $\mu^A = \mu^B = \mu \in (0,1)$. Considering μ^A and μ^B, the equilibrium profit for firm A in period 2 becomes

$$\bar{\pi}_2^A = \frac{t}{2} - \frac{\mu^A(1-\mu^A)(r^A)^2}{2t}.$$

From Lemma 2.3, the first-period market share of firm A is

$$\alpha_A = \frac{p_1^B - p_1^A + t}{2t}.$$

The reason is that even if $\mu^A \neq \mu^B$, the expected second-period surplus from buying from each firm in period 1 is the same because coupons are distributed randomly.

Then we can check that

$$\frac{\partial \pi^A}{\partial \mu^A} = p_1^A \frac{\partial \alpha_A}{\partial \mu^A} + \frac{\partial \bar{\pi}_2^A}{\partial \mu^A} = 0,$$

at $p_1^A = p_1^B = p_1^s = t$, $r^A = r^B = r^s = 0$, and $\mu^A = \mu^B = \mu \in (0,1)$. □

Proof of Lemma 2.4

Since we are considering the optimal couponing and a fraction δ of consumers have constant tastes, the second-period demand of firm A is

$$q_2^A(p_2^A, p_2^B) = \delta\alpha_A + (1-\delta)\left[\alpha_A x_{10} + (1-\alpha_A)x_{01}\right].$$

Thus, firm A's second-period maximization problem is

$$\max_{p_2^A} \pi_2^A = \left[\delta\alpha_A + (1-\delta)\alpha_A x_{10}\right](p_2^A - r^A) + (1-\delta)(1-\alpha_A)x_{01}p_2^A.$$

The first-order condition for the above problem gives firm A's best-response function. Proceeding similarly for firm B and solving the two best-response functions, we obtain

$$p_2^A = t + \alpha_A r^A + \Gamma \quad \text{and} \quad p_2^B = t + (1-\alpha_A)r^B + \Gamma',$$

where $\Gamma = \frac{2t\delta}{3(1-\delta)}(1+\alpha_A)$ and $\Gamma' = \frac{2t\delta}{3(1-\delta)}(2-\alpha_A)$.

The second-period equilibrium profit for firm A is then

$$\pi_2^A = (1-\delta)\left[\hat{\pi}_2^A(\eta = 1) + \frac{t\Gamma' + \Gamma\Gamma' - \Gamma^2}{2t}\right] + \delta\alpha_A\left[t - (1-\alpha_A)r^A + \Gamma\right].$$

Finally, firm A's profit maximization problem in period 1 is

$$\max_{p_1^A, r^A} \pi^A = p_1^A\alpha_A + \pi_2^A.$$

The first-period surplus difference is given as before, while the expected second-period surpluses from buying from firms A and B in period 1 are now, respectively, given by

$$S_2^A = \delta\left[v - tx - (p_2^A - r^A)\right] + (1-\delta)\left[\int_0^{x_{10}}(v - tx - (p_2^A - r^A))dx + \int_{x_{10}}^1 (v - t(1-x) - p_2^B)dx\right]$$
$$S_2^B = \delta\left[v - t(1-x) - (p_2^B - r^B)\right] + (1-\delta)\left[\int_0^{x_{01}}(v - tx - p_2^A)dx + \int_{x_{01}}^1 (v - t(1-x) - (p_2^B - r^B))dx\right].$$

Following the same reasoning as in Lemma 2.1, we can compute the first-period market share of firm A which is implicitly given by

$$t - 2t\alpha_A + p_1^B - p_1^A = (1-\delta)(r^B + tx_{01}^2 - tx_{10}^2) - \delta(p_2^B - p_2^A + r^A - r^B + t - 2t\alpha_A).$$

Using the implicit function theorem and evaluating at a symmetric equilibrium yields

$$\frac{\partial \alpha_A}{\partial p_1^A} = \frac{-t}{2\left[t^2(1+\delta) + \left(r + \frac{2\delta}{3(1-\delta)}\right)(r + \delta(t-r))\right]}$$
$$\frac{\partial \alpha_A}{\partial r^A} = \frac{t}{4\left[t^2(1+\delta) + \left(r + \frac{2\delta}{3(1-\delta)}\right)(r + \delta t)\right]}.$$

Plugging these into the first-order conditions for firm A's profit maximization problem in period 1 and solving the system, we get (2.17) and (2.18). (2.19) and (2.20) are obtained as in Lemma 2.1. □

Proof of Lemma 2.5

Let $\tilde{\alpha}_A$ and $\hat{\alpha}_A$ be firm A's first-period market shares with myopic and forward-looking consumers, respectively. Since myopic consumers only care about current prices when making their purchasing decisions in period 1, $\tilde{\alpha}_A$ is simply given by

$$\tilde{\alpha}_A = \frac{p_1^B - p_1^A + t}{2t}.$$

Here we are considering the optimal couponing strategy so that $\hat{\alpha}_A$ comes from (2.29) with $\eta = 1$. That is,

$$\hat{\alpha}_A = \frac{4t(t + p_1^B - p_1^A) + (r^A + r^B)^2 + 2t(r^A - r^B)}{2[4t^2 + (r^A + r^B)^2]}.$$

With a fraction λ of consumers being myopic, the first-period market share of firm A, α_A, is determined by

$$\alpha_A = \lambda\tilde{\alpha}_A + (1-\lambda)\hat{\alpha}_A.$$

Imposing symmetry, we have

$$\frac{\partial \alpha_A}{\partial p_1^A} = \lambda\left(\frac{-1}{2t}\right) + (1-\lambda)\left[\frac{-t}{2(t^2+r^2)}\right] \quad \text{and} \quad \frac{\partial \alpha_A}{\partial r^A} = (1-\lambda)\left[\frac{t}{4(t^2+r^2)}\right].$$

Substituting these and $\eta = 1$ into (2.30) yields (2.21) and (2.22). (2.23) and (2.24) are obtained as in Lemma 2.1. □

Chapter 3

Behavior-Based Price Discrimination with Experience Goods

3.1 Introduction

In the price discrimination literature, it is well established that oligopolistic price discrimination intensifies competition and leads to lower profits (see, among others, Thisse and Vives, 1988; Shaffer and Zhang, 1995; Bester and Petrakis, 1996; Liu and Serfes, 2004). One important environment for price discrimination to intensify competition and to reduce firm profits is best-response asymmetry, i.e., one firm's strong market is the other firm's weak market (Corts, 1998; Armstrong, 2006). This best-response asymmetry is often introduced by employing a Hotelling model of product differentiation, where consumers are heterogeneous on a single dimension.[1]

Despite the extensive economic literature on oligopolistic price discrimination, there exist a few works that produce results opposite to the above studies. Using a two-dimensional Hotelling model, Esteves (2009) shows that price discrimination can increase profits if firms have information about consumer preferences only in the less differentiated dimension. In Shin and Sudhir (2010) where a two-period Hotelling model is employed, pricing based on customers' past purchase behavior can be profitable when either heterogeneity in purchase quantities or preference stochasticity is sufficiently high. Liu and Shuai (2013) develop a

[1] Firms are better off when they price discriminate according to "choosiness" (transportation cost). This case is an example of price discrimination under best-response symmetry (see Armstrong, 2006).

multi-dimensional Hotelling model to examine the profitability of discriminatory pricing. The authors find that when firms price discriminate on one and the same dimension, they make more profits than under uniform pricing.

Due to the development of more sophisticated methods for acquiring, storing, and analyzing consumer information, firms can now segment customers on the basis of their purchase histories and price discriminate accordingly.[2] In the present study, we investigate the profit effects of behavior-based price discrimination (BBPD) when firms produce experience goods. This paper is thus related to two strands of the literature. One is the literature on BBPD under best-response asymmetry. The other is the literature on experience good pricing.

Regarding the literature on experience good pricing, Villas-Boas (2004) uses a two-period model where consumers learn in the first period about the product they purchase and then choose between the competing products in the second period. He shows that a firm is better (worse) off in the future from having a greater market share today if there is a greater mass of valuations above (below) the mean. Considering duopoly competition with an infinite horizon model in an experience goods market, Villas-Boas (2006) also finds that steady-state prices and profits are higher the greater the probability of perfect product fit. However, the issue of price discrimination is not considered in his studies. Bang, Kim, and Yoon (2011) study price discrimination with a good neither perfectly a search good nor perfectly an experience good. They consider the situation in which buyers' prior valuations are initially observable to seller(s) but buyers further draw a private signal which may give them additional information about a product. In this setting, they explain the possibility of reverse price discrimination where a higher price is charged to low valuation buyers.

In the literature on BBPD, Chen (1997) considers a two-period homogeneous product duopoly model. Here consumers incur costs when switching from one firm to another, which enables firms to segment and price discriminate consumers. He shows that in equilibrium, each firm charges a lower price to the competitor's customers than to its own customers in the second period (paying customers to switch) and that price discrimination lowers the firms' profits. Fudenberg and Tirole (2000) analyze a two-period duopoly model in which consumers have different preferences for the firms' products and each firm can set different prices in period 2, depending on whether or not consumers have bought its product in period 1. They find that each firm can poach the rival firm's customers by charging them a lower price and that the

[2]For surveys on behavior-based price discrimination, see Chen (2005) and Fudenberg and Villas-Boas (2007).

difference in the prices charged to loyal and switching customers reduces the firms' profits.[3] The key difference between Fudenberg and Tirole (2000) and the present study is that while they propose a model with search goods, our model comes with experience goods.[4]

The paper that is closest to ours is De Nijs and Rhodes (2013). They study BBPD with experience goods, but focus on the issue of when firms should offer a lower price to loyal customers or to new customers.[5] In their model, firms are still worse off when price discriminating.

The purpose of this study is to characterize the conditions under which BBPD in markets exhibiting best-response asymmetry is more profitable than uniform pricing. Considering duopolists producing horizontally differentiated experience goods and a three-stage game where they first make price discrimination decisions followed by two-period pricing decisions, we show that BBPD under best-response asymmetry can boost the firms' profits when sufficiently many consumers have a poor experience with the firms' products. The asymmetric case in which one firm produces experience goods and the other search goods is investigated as well.

The rest of the paper unfolds as follows. Section 3.2 sets up the model. In Section 3.3, the case in which two firms produce experience goods is analyzed. Section 3.4 examines the asymmetric case. Section 3.5 concludes.

3.2 The model

Two firms (A and B) produce at zero marginal cost horizontally differentiated experience goods (the case where one firm produces experience goods and the other search goods will

[3]Taylor (2003) extends Chen (1997) to multiple periods and multiple firms, and Villas-Boas (1999) provides an analysis similar to Fudenberg and Tirole (2000) but in a duopoly with infinitely lived firms and overlapping generations of consumers.

[4]A search good is one whose quality can be easily evaluated by consumers prior to purchase (see Nelson, 1970).

[5]One issue in the literature on price discrimination is when firms should offer a lower price to loyal consumers or to new consumers. Shaffer and Zhang (2000) show that when demand is asymmetric, it is optimal that one firm charges a lower price to its rival's customers and the other charges a lower price to its own customers. Chen and Pearcy (2010) find that when commitment to future prices is possible, firms reward consumer loyalty if intertemporal preference dependence is low, but pay consumers to switch if preference dependence is high. In Shin and Sudhir (2010), paying customers to stay is optimal when both heterogeneity in purchase quantities and preference stochasticity are sufficiently high. Finally, De Nijs and Rhodes (2013) find that if over half of consumers believe their existing product is inferior (resp. superior) to the other one, firms offer a lower (resp. higher) price to their loyal consumers.

be considered in Section 3.4). Firm A is located at point 0 and firm B at point 1 of the unit interval. There are three periods, 0, 1 and 2: each consumer lives for two periods (1 and 2) and demands at most one unit of the product each period. In periods 1 and 2, there is a continuum of consumers uniformly distributed on the interval $[0, 1]$ with a unit mass. A consumer's location or preference, $x \in [0, 1]$, is known by each consumer before purchasing a product and constant over the two periods. The consumer incurs a disutility of tx when purchasing from firm A, and of $t(1-x)$ when purchasing from firm B, where $t > 0$ measures the disutility (per unit of distance) of buying away from her ideal product.

Consumers are initially uncertain about their valuations for a product, and only learn about them after experiencing (buying) the product. Let θ_L (resp. θ_H) be the value each consumer attaches to a previously purchased product when she is dissatisfied (resp. satisfied) with it, where $\theta_L < \theta_H$. Without loss of generality, θ_L is normalized to zero. Suppose that for a product that has never been experienced, consumers all expect it to be unsatisfactory with probability $\omega \in (0, 1)$. We denote by $\theta := \omega\theta_L + (1-\omega)\theta_H \overset{\theta_L=0}{=} (1-\omega)\theta_H$ the (expected) value placed on an untried product. Then a type (s, x) consumer, $s \in \{\theta_L, \theta, \theta_H\}$, has a value for firm A's product of $v + s - tx$ and she has a value for firm B's product of $v + s - t(1-x)$, where v is sufficiently large so that in equilibrium all consumers purchase one of the products in each period. Here v represents the product value without uncertainty, while s captures the experience-related value. The following example is helpful to better understand:

Example 3.1. Consider a consumer of type (s, x) who buys from firm A in period 1 at, say, price p. Since this consumer has never experienced the product before, the expected net benefit of purchasing firm A's product is $v + \theta - tx - p$. If the consumer is satisfied (resp. dissatisfied) with firm A's product in period 1 and continues to purchase it in period 2 at, say, price q, then she enjoys the net benefit $v + \theta_H - tx - q$ (resp. $v - tx - q$). In period 2, however, if the consumer switches to firm B's product priced at, say, r, she expects to obtain the net benefit $v + \theta - t(1-x) - r$ since the product has never been tried before.

To see the effects of consumer expectation and experience on firms' behavior, we assume that in period 2 (when each consumer has experienced one of the products), a proportion $\lambda \in (0, 1)$ of consumers are dissatisfied with a previously purchased product and that the remaining proportion $1 - \lambda$ are satisfied with it. Note that ω is associated with s before experiencing a product, while λ is associated with s after.[6]

[6]Shapiro (1983) considers a similar setup in which consumers can initially either over- or underestimate the quality of experience goods.

The timing of the game is then as follows:

- Period 0: Firms, simultaneously and independently, make decisions on whether to collect consumer information to price discriminate. We assume that there is no information acquisition cost.
- Period 1: After observing each other's information acquisition decisions, firms A and B simultaneously choose first-period prices p_{A1} and p_{B1}, respectively. Consumers then decide which firm to buy from.
- Period 2: Firms simultaneously set second-period prices. If firm $i \in \{A, B\}$ collected consumer information in period 0 so that it can identify previous customers, firm i offers prices p_{i2} to its own past customers and r_{i2} to customers who purchased from the rival in period 1. Otherwise, $p_{i2} = r_{i2} = q_{i2}$ (when distinction is needed, p_{i2} will be referred to as a second-period price, while r_{i2} will be referred to as the poaching price). Consumers decide which firm to buy from given the prices.

The firms cannot observe consumers' preferences, but they can use consumers' first-period purchase histories (consumer information) to price accordingly. Here it is assumed that once firms collect consumer information, they always practice price discrimination. The discount factor for both firms and consumers is assumed to be 1 for simplicity. Finally, the following assumption on the parameters of the model will be maintained throughout the paper:

Assumption 3.1. $\theta_H < \frac{t}{4} + \frac{31\theta}{92}$.

The assumption implies that the benefit of experiencing a satisfactory product is not too large and guarantees that in equilibrium, there exist consumers who switch to an untried product in period 2 even if they are satisfied with a previously purchased product.

3.3 Analysis

In this section the case where both firms produce experience goods is analyzed. We first investigate the firms' price competition, taking as given their choices of collecting consumer information in period 0. Then the firms' decisions on whether to collect consumer information to price discriminate are examined. There are four subgames to consider, corresponding to the following scenarios: no firm collects information about consumers (NN); only firm A collects information and price discriminates (CN); only firm B collects information and price discriminates (NC); and both firms collect information and price discriminate (CC). Here C and N stand for "collect" and "do not collect consumer information," respectively. We will use subgame perfect Nash equilibrium (SPNE) as the solution concept and proceed by backward induction; this means that we have to solve each of the above subgames.

3.3.1 Subgame NN

When neither firm acquires consumer information, the firms cannot identify previous customers and price discriminate based on their purchase histories. Thus, the first-period pricing game is independent of the second-period one. The equilibrium for subgame NN is then simply two repetitions of the static equilibrium of the standard Hotelling model, which yields the following equilibrium prices and profits:

$$p_{A1}^{NN} = p_{B1}^{NN} = t,\ q_{A2}^{NN} = q_{B2}^{NN} = t,\ \text{and } \pi_A^{NN} = \pi_B^{NN} = t.$$

3.3.2 Subgame CN

In subgame CN, only firm A collects consumer information to price discriminate. Thus, in period 2, firm A charges p_{A2} and r_{A2} to its own customers and to firm B's previous customers, respectively, while firm B charges q_{B2} to all customers. Suppose that in period 1, the market splits at $\hat{x}_1$ so that all consumers to the left (resp. right) of $\hat{x}_1$ buy from firm A (resp. firm B). We start our analysis by investigating the firms' competition in period 2 taking $\hat{x}_1$ as given.

As consumer learning occurs in period 2, there are two groups of consumers in each firm's former customers. One is a group of consumers who are dissatisfied with a product they bought in period 1, and the other is a group of consumers who are satisfied with it. Let us denote by $\hat{x}_i^j$, with $j \in \{d, s\}$, a consumer who bought from firm i in period 1 and is indifferent between buying from firm A and buying from firm B in period 2. Specifically, $\hat{x}_A^d$ (resp. $\hat{x}_A^s$) is the second-period indifferent consumer of the group in which consumers bought firm A's product and are dissatisfied (resp. satisfied) with it. Similarly, $\hat{x}_B^d$ (resp. $\hat{x}_B^s$) is the second-period indifferent consumer of the group in which consumers bought firm

B's product and are dissatisfied (resp. satisfied) with it. These indifferent consumers are then defined as follows:

$$\begin{aligned}
&v - t\hat{x}_A^d - p_{A2} = v + \theta - t(1 - \hat{x}_A^d) - q_{B2} \\
&v + \theta_H - t\hat{x}_A^s - p_{A2} = v + \theta - t(1 - \hat{x}_A^s) - q_{B2} \\
&v + \theta - t\hat{x}_B^d - r_{A2} = v - t(1 - \hat{x}_B^d) - q_{B2} \\
&v + \theta - t\hat{x}_B^s - r_{A2} = v + \theta_H - t(1 - \hat{x}_B^s) - q_{B2}.
\end{aligned}$$

The second-period demand functions of firms A and B respectively are

$$D_{A2}(\cdot) = \underbrace{\lambda\hat{x}_A^d + (1-\lambda)\hat{x}_A^s}_{\text{in } A'\text{s turf}} + \underbrace{\lambda(\hat{x}_B^d - \hat{x}_1) + (1-\lambda)(\hat{x}_B^s - \hat{x}_1)}_{\text{in } B'\text{s turf}} \tag{3.1}$$

$$D_{B2}(\cdot) = \underbrace{\lambda(1 - \hat{x}_B^d) + (1-\lambda)(1 - \hat{x}_B^s)}_{\text{in } B'\text{s turf}} + \underbrace{\lambda(\hat{x}_1 - \hat{x}_A^d) + (1-\lambda)(\hat{x}_1 - \hat{x}_A^s)}_{\text{in } A'\text{s turf}}. \tag{3.2}$$

Hence, A's and B's second-period profit maximization problems can be respectively written as

$$\max_{p_{A2}, r_{A2}} \pi_{A2} = p_{A2}\left[\lambda\hat{x}_A^d + (1-\lambda)\hat{x}_A^s\right] + r_{A2}\left[\lambda(\hat{x}_B^d - \hat{x}_1) + (1-\lambda)(\hat{x}_B^s - \hat{x}_1)\right] \tag{3.3}$$

$$\max_{q_{B2}} \pi_{B2} = q_{B2}\left[\lambda(1 - \hat{x}_B^d) + (1-\lambda)(1 - \hat{x}_B^s) + \lambda(\hat{x}_1 - \hat{x}_A^d) + (1-\lambda)(\hat{x}_1 - \hat{x}_A^s)\right]. \tag{3.4}$$

The first-order conditions for the problems (3.3) and (3.4) give both firms' best-response functions. We then solve the system of the two best-response functions to find the second-period equilibrium prices. Substituting the second-period equilibrium prices into the profit functions yields the equilibrium profits for firms A and B in period 2, denoted by π_{A2}^{CN} and π_{B2}^{CN} respectively. Finally, we can express both firms' profit maximization problems in period 1 as follows:

$$\max_{p_{A1}} \pi_A = p_{A1}\hat{x}_1 + \pi_{A2}^{CN} \tag{3.5}$$

$$\max_{p_{B1}} \pi_B = p_{B1}(1 - \hat{x}_1) + \pi_{B2}^{CN}. \tag{3.6}$$

Taking the first-order conditions for the problems (3.5) and (3.6), we can characterize the equilibrium values of the game, as presented in the following lemma:

Lemma 3.1. *Suppose only firm A collects information about consumers and price discriminates. The equilibrium in the two-stage pricing game is characterized by the following:*

(i) The firms' first-period prices are

$$p_{A1}^{CN} = \frac{2t}{3} - \frac{\Omega}{2} \quad \text{and} \quad p_{B1}^{CN} = \frac{11t}{12}. \tag{3.7}$$

(ii) The firms' second-period prices are

$$\begin{cases} p_{A2}^{CN} = \frac{3t}{4} + \frac{\Omega}{2} \\ r_{A2}^{CN} = \frac{t}{4} - \frac{\Omega}{2} \end{cases} \quad \text{and} \quad q_{B2}^{CN} = \frac{t}{2}. \tag{3.8}$$

(iii) The firms' profits are

$$\pi_A^{CN} = \frac{31t}{48} + \frac{\Omega^2}{4t} \quad \text{and} \quad \pi_B^{CN} = \frac{17t}{24}, \tag{3.9}$$

where $\Omega := \lambda(\theta_L - \theta) + (1-\lambda)(\theta_H - \theta) \overset{\theta_L=0}{=} (1-\lambda)\theta_H - \theta$.

Proof. See Appendix. □

Note first that the equilibrium values for firm A depend on the value of Ω, whereas firm B's equilibrium values do not. For any $\Omega \in (-\theta, \theta_H - \theta)$, we have that $p_{A2}^{CN} > q_{B2}^{CN} > r_{A2}^{CN}$. Thus each firm is poaching its competitor's previous customers. In period 1, we substitute for the equilibrium values and obtain $\hat{x}_1 = \frac{1}{2}$. The market allocation in period 2 is then described by $\hat{x}_A^d = \frac{3}{8} - \frac{\Omega}{4t} - \frac{\theta}{2t}$, $\hat{x}_A^s = \frac{3}{8} - \frac{\Omega}{4t} + \frac{\theta_H}{2t} - \frac{\theta}{2t}$, $\hat{x}_B^d = \frac{5}{8} + \frac{\Omega}{4t} + \frac{\theta}{2t}$, and $\hat{x}_B^s = \frac{5}{8} + \frac{\Omega}{4t} + \frac{\theta}{2t} - \frac{\theta_H}{2t}$. That is, among consumers who are dissatisfied with firm A's product, those with $x \in [0, \hat{x}_A^d)$ (resp. $x \in [\hat{x}_A^d, \hat{x}_1)$ continue to buy from firm A (resp. switch to firm B). For consumers who are satisfied with firm A's product, it can be expressed by replacing $\hat{x}_A^d$ with $\hat{x}_A^s$. Similarly for firm B.[7]

[7] Assumption 3.1 guarantees that in the equilibrium, switching occurs in period 2 (i.e., $0 < \hat{x}_A^d < \hat{x}_A^s < \hat{x}_1 < \hat{x}_B^s < \hat{x}_B^d < 1$).

It can be easily verified that the firms set lower prices in both periods compared with the situation where they do not collect consumer information for BBPD ($p_{i1}^{NN} > p_{B1}^{CN} > p_{A1}^{CN}$ and $q_{i2}^{NN} > p_{A2}^{CN} > q_{B2}^{CN} > r_{A2}^{CN}$). This result is in line with that of Villas-Boas (1999) who shows that dynamic price discrimination lowers all prices. Thus, both firms achieve lower profits than under uniform pricing.

3.3.3 Subgame NC

The analysis of subgame NC is symmetric to that of subgame CN, and thus we omit it.

3.3.4 Subgame CC

When both firms collect consumer information for price discrimination, firm $i \in \{A, B\}$ charges p_{i2} and r_{i2} to its own customers and to the competitor's previous customers in period 2, respectively. In this case, the indifferent consumers in period 2 are defined as follows:

$$
\begin{aligned}
v - t\hat{x}_A^d - p_{A2} &= v + \theta - t(1 - \hat{x}_A^d) - r_{B2} \\
v + \theta_H - t\hat{x}_A^s - p_{A2} &= v + \theta - t(1 - \hat{x}_A^s) - r_{B2} \\
v + \theta - t\hat{x}_B^d - r_{A2} &= v - t(1 - \hat{x}_B^d) - p_{B2} \\
v + \theta - t\hat{x}_B^s - r_{A2} &= v + \theta_H - t(1 - \hat{x}_B^s) - p_{B2}.
\end{aligned}
$$

The firms' demand functions are given by (3.1) and (3.2). A's and B's second-period profit maximization problems respectively are then

$$\max_{p_{A2}, r_{A2}} \pi_{A2} = p_{A2}\left[\lambda\hat{x}_A^d + (1-\lambda)\hat{x}_A^s\right] + r_{A2}\left[\lambda(\hat{x}_B^d - \hat{x}_1) + (1-\lambda)(\hat{x}_B^s - \hat{x}_1)\right] \tag{3.10}$$

$$\max_{p_{B2}, r_{B2}} \pi_{B2} = p_{B2}\left[\lambda(1-\hat{x}_B^d) + (1-\lambda)(1-\hat{x}_B^s)\right] + r_{B2}\left[\lambda(\hat{x}_1 - \hat{x}_A^d) + (1-\lambda)(\hat{x}_1 - \hat{x}_A^s)\right]. \tag{3.11}$$

Proceeding similarly as in subgame CN and denoting the second-period equilibrium profit for firm i by π_{i2}^{CC}, we can set up both firms' profit maximization problems in period 1 as follows:

$$\max_{p_{A1}} \pi_A = p_{A1}\hat{x}_1 + \pi_{A2}^{CC} \tag{3.12}$$

$$\max_{p_{B1}} \pi_B = p_{B1}(1-\hat{x}_1) + \pi_{B2}^{CC}. \tag{3.13}$$

The first-order conditions for the problems (3.12) and (3.13) easily lead to the following results:

Lemma 3.2. *Suppose both firms collect information about consumers and price discriminate. The equilibrium in the two-stage pricing game is characterized as follows:*

(i) The firms' first-period prices are

$$p_{A1}^{CC} = p_{B1}^{CC} = \frac{4t}{3} - \frac{2\Omega}{3}. \tag{3.14}$$

(ii) The firms' second-period prices are

$$\begin{cases} p_{A2}^{CC} = p_{B2}^{CC} = \frac{2t}{3} + \frac{\Omega}{3} \\ r_{A2}^{CC} = r_{B2}^{CC} = \frac{t}{3} - \frac{\Omega}{3}. \end{cases} \tag{3.15}$$

(iii) The firms' profits are

$$\pi_A^{CC} = \pi_B^{CC} = \frac{17t}{18} - \frac{2\Omega}{9} + \frac{\Omega^2}{9t}. \tag{3.16}$$

Proof. See Appendix. □

From Lemma 3.2, we can see that in the equilibrium, each firm is poaching its competitor's former customers since for any $\Omega \in (-\theta, \theta_H - \theta)$, $p_{i2}^{CC} > r_{i2}^{CC}$. In period 1, we substitute for the equilibrium values and obtain $\hat{x}_1 = \frac{1}{2}$. The market allocation in period 2 is described by $\hat{x}_A^d = \frac{1}{3} - \frac{\Omega}{3t} - \frac{\theta}{2t}$, $\hat{x}_A^s = \frac{1}{3} - \frac{\Omega}{3t} + \frac{\theta_H}{2t} - \frac{\theta}{2t}$, $\hat{x}_B^d = \frac{2}{3} + \frac{\Omega}{3t} + \frac{\theta}{2t}$, and $\hat{x}_B^s = \frac{2}{3} + \frac{\Omega}{3t} + \frac{\theta}{2t} - \frac{\theta_H}{2t}$. Thus, among consumers who are dissatisfied with firm B's product, those with $x \in (\hat{x}_B^d, 1]$ (resp. $x \in (\hat{x}_1, \hat{x}_B^d])$ continue to buy from firm B (resp. switch to firm A). For consumers who are

		Firm B	
		C	N
Firm A	C	π_A^{CC}, π_B^{CC}	π_A^{CN}, π_B^{CN}
	N	π_A^{NC}, π_B^{NC}	π_A^{NN}, π_B^{NN}

Table 3.1: Price discrimination decisions

satisfied with firm B's product, it can be expressed by replacing $\hat{x}_B^d$ with $\hat{x}_B^s$. Similarly for firm A.[8]

We also find that the firms set a higher price in the first period ($p_{i1}^{CC} > p_{i1}^{NN}$) and lower prices in the second period ($r_{i2}^{CC} < p_{i2}^{CC} < q_{i2}^{NN}$) than the case where they do not collect consumer information for BBPD. This result is the same as that of Fudenberg and Tirole (2000) who show that BBPD raises the first-period price but reduces the second-period prices. It is noteworthy that if the firms produce search goods (i.e., $\theta_L = \theta_H$), then the equilibrium prices are $p_{i1}^* = \frac{4t}{3}$, $p_{i2}^* = \frac{2t}{3}$, and $r_{i2}^* = \frac{t}{3}$. Hence, compared to the case of search goods, the first-period equilibrium price and the equilibrium poaching price rise ($p_{i1}^{CC} > p_{i1}^*$ and $r_{i2}^{CC} > r_{i2}^*$) while the second-period equilibrium price falls ($p_{i2}^{CC} < p_{i2}^*$) when there exist sufficiently many dissatisfied consumers (formally $\lambda > \frac{\theta_H - \theta}{\theta_H}$). The reasons for these results are as follows. Each firm has room to raise its poaching price as more consumers have a poor experience with the competitor's product. However, the presence of dissatisfied consumers with its own product intensifies price competition in period 2 since each firm should lower a second-period price to prevent the consumers from switching to the rival firm. The effects of BBPD on the profits will be discussed in the next subsection.

3.3.5 Price discrimination decisions

Having derived the equilibrium profits for all subgames, we can now examine the firms' price discrimination decisions. In period 0, the firms simultaneously decide whether to acquire consumer information and price discriminate. In practice, firms are likely to make pricing decisions more often than discrimination decisions, supporting a sequential discrimination-then-pricing game. The game is represented by the following payoff matrix:

[8] Assumption 3.1 guarantees that in the equilibrium, switching occurs in period 2 (i.e., $0 < \hat{x}_A^d < \hat{x}_A^s < \hat{x}_1 < \hat{x}_B^s < \hat{x}_B^d < 1$).

The results are summarized below:

Proposition 3.1. *Consider two firms A and B producing experience goods. In the three-stage game in which the firms first make price discrimination decisions followed by two-period pricing decisions, there are two (pure strategy) SPNE. In the first SPNE, both firms do not collect consumer information so that neither firm price discriminates (N, N). In the other SPNE, both firms collect consumer information and price discriminate (C, C).*

Proof. See Appendix. □

Regarding the relation between the firms' ability to segment consumers (information quality) and price discrimination decisions, Liu and Serfes (2004) show that for low levels of information quality, the unique equilibrium is for neither firm to acquire consumer information, whereas when the information becomes more refined, acquiring information to price discriminate is each firm's dominant strategy. As in our case, there is no asymmetric equilibrium, where one firm acquires information and the other does not. Interestingly, equilibrium profits are lower under discriminatory pricing than under uniform pricing so that a prisoner's dilemma emerges when the information quality is high. In our setting, however, the prisoner's dilemma aspect of price discrimination falls away.

Then the natural question is: which pricing scheme is more profitable? The following proposition answers this question:

Proposition 3.2. *Consider the game described in Proposition 3.1. (1) Suppose $\theta > k_1 t$, where $k_1 = \sqrt{\frac{3}{2}} - 1$. Then there exists a critical fraction of dissatisfied consumers, $\lambda^* = \frac{\theta_H - \theta + k_1 t}{\theta_H}$, such that for $i \in \{A, B\}$, if $\lambda > \lambda^*$, $\pi_i^{CC} > \pi_i^{NN}$. (2) Suppose $\theta \leq k_1 t$. Then $\pi_i^{CC} < \pi_i^{NN}$, $i \in \{A, B\}$.*

Proof. See Appendix. □

The first part of the results in Proposition 3.2 shows the possibility that BBPD enables the firms to make more profits than uniform pricing when sufficiently many consumers experience a bad fit with their products. The intuition behind this goes as follows. The firms try to extract higher profits in the first period if it is expected that consumers' valuations for their products will decrease over time. BBPD then allows the firms to increase a first-period price for this. This result contrasts with De Nijs and Rhodes (2013) where BBPD by experience good-producing firms reduces the profits. The second result of the proposition tells us that when the (expected) value of an untried product is small enough, the firms are better off by pricing uniformly.

3.4 The asymmetric case

In this section we consider the situation in which consumers are initially uncertain about their valuations for firm A's product, while they are fully informed of the value of firm B's product. In other words, firms A and B produce experience and search goods, respectively. This can occur in a market where firm A is an entrant and firm B is an incumbent as the quality of an entrant's product is unlikely to be revealed at the outset.[9]

In order for the model to capture such a situation, suppose that the gross benefit consumers obtain from buying firm B's product is $v+\bar{\theta}$ in periods 1 and 2, and that consumers initially expect firm A's product to have the same value as firm B's product.[10] Note here that $\bar{\theta}$ is the value of firm B's product that has been learned through experience prior to A's entry on the market, and that it is the value consumers place on the untried product of firm A in period 1.[11] Recall that $\lambda \in (0,1)$ is the fraction of consumers who are dissatisfied with firm A's product in period 2. We denote the corresponding subgames analyzed in this section by NN', CN', NC', and CC'.

3.4.1 Subgame NN'

In subgame NN', no information about consumers' first-period purchases is acquired by the firms. Thus, the firms cannot identify previous customers so that each charges the same price to all customers in period 2. Since the respective pricing games of periods 1 and 2 are

[9] In the context of the effects of BBPD on entry and welfare, Gehrig, Shy, and Stenbacka (2011) explore BBPD by an incumbent in an asymmetric market where the incumbent has a strategic advantage based on switching costs, while an entrant cannot price discriminate because of having no access to information about consumers' purchase histories. They show that: (i) consumer surplus is higher with uniform pricing than with BBPD; (ii) the entry decision is invariant to whether the incumbent implements BBPD or uniform pricing; and (iii) BBPD by the incumbent improves social welfare, which comes from the increase in the incumbent's profit at the expense of consumer surplus.

[10] Even if we assume that consumers initially expect firm A's product to be unsatisfactory with probability $\omega \in (0,1)$ as in the symmetric case, the main results of this section do not change.

[11] In this section, we assume that $\theta_H < \frac{t}{4} + \frac{31\bar{\theta}}{92}$, which corresponds to Assumption 3.1.

independent, $\hat{x}_1 = \frac{1}{2}$ and the indifferent consumer in period 2, $\hat{x}_2$, is defined as

$$v + \frac{1}{2}(1-\lambda)\theta_H + \frac{1}{2}\bar{\theta} - t\hat{x}_2 - q_{A2} = v + \bar{\theta} - t(1-\hat{x}_2) - q_{B2}.$$

It is then easy to show that the equilibrium prices and profits are

$$\begin{aligned}
&p_{A1}^{NN'} = p_{B1}^{NN'} = t \\
&q_{A2}^{NN'} = t + \frac{\bar{\Omega}}{6} \text{ and } q_{B2}^{NN'} = t - \frac{\bar{\Omega}}{6} \\
&\pi_A^{NN'} = t + \frac{\bar{\Omega}}{6} + \frac{\bar{\Omega}^2}{72t} \text{ and } \pi_B^{NN'} = t - \frac{\bar{\Omega}}{6} + \frac{\bar{\Omega}^2}{72t},
\end{aligned}$$

where $\bar{\Omega} := \lambda(\theta_L - \bar{\theta}) + (1-\lambda)(\theta_H - \bar{\theta}) \overset{\theta_L=0}{=} (1-\lambda)\theta_H - \bar{\theta}$.

We can see that when both firms price uniformly, the experience good-producing firm (firm A) makes more profits than the search good-producing firm (firm B) insofar as $\lambda < \frac{\theta_H - \bar{\theta}}{\theta_H}$.

3.4.2 Subgame CN'

In subgame CN', only firm A collects consumer information for BBPD. Hence, in period 2, firm A offers p_{A2} and r_{A2} to its own customers and to firm B's previous customers, respectively, while firm B offers q_{B2} to all customers. Since all consumers are certain about the quality of firm B's product, the second-period indifferent consumers are defined as follows:

$$\begin{aligned}
&v - t\hat{x}_A^d - p_{A2} = v + \bar{\theta} - t(1-\hat{x}_A^d) - q_{B2} \\
&v + \theta_H - t\hat{x}_A^s - p_{A2} = v + \bar{\theta} - t(1-\hat{x}_A^s) - q_{B2} \\
&v + \bar{\theta} - t\hat{x}_B - r_{A2} = v + \bar{\theta} - t(1-\hat{x}_B) - q_{B2}.
\end{aligned}$$

The second-period demand functions of firms A and B respectively are

$$D_{A2}(\cdot) = \underbrace{\lambda\hat{x}_A^d + (1-\lambda)\hat{x}_A^s}_{\text{in } A'\text{s turf}} + \underbrace{(\hat{x}_B - \hat{x}_1)}_{\text{in } B'\text{s turf}} \tag{3.17}$$

$$D_{B2}(\cdot) = \underbrace{(1-\hat{x}_B)}_{\text{in } B'\text{s turf}} + \underbrace{\lambda(\hat{x}_1 - \hat{x}_A^d) + (1-\lambda)(\hat{x}_1 - \hat{x}_A^s)}_{\text{in } A'\text{s turf}}. \tag{3.18}$$

Thus, the second-period profit maximization problems of firms A and B can be respectively written as

$$\max_{p_{A2}, r_{A2}} \pi_{A2} = p_{A2}\left[\lambda\hat{x}_A^d + (1-\lambda)\hat{x}_A^s\right] + r_{A2}(\hat{x}_B - \hat{x}_1) \tag{3.19}$$

$$\max_{q_{B2}} \pi_{B2} = q_{B2}\left[(1-\hat{x}_B) + \lambda(\hat{x}_1 - \hat{x}_A^d) + (1-\lambda)(\hat{x}_1 - \hat{x}_A^s)\right]. \tag{3.20}$$

Proceeding similarly as in Section 3.3 and denoting the second-period equilibrium profit for firm i by $\pi_{i2}^{CN'}$, we can set up both firms' profit maximization problems in period 1 as follows:

$$\max_{p_{A1}} \pi_A = p_{A1}\hat{x}_1 + \pi_{A2}^{CN'} \tag{3.21}$$

$$\max_{p_{B1}} \pi_B = p_{B1}(1-\hat{x}_1) + \pi_{B2}^{CN'}. \tag{3.22}$$

The next lemma presents the equilibrium prices and profits for subgame CN':

Lemma 3.3. *Suppose only firm A (the experience good-producing firm) collects information about consumers and price discriminates. The equilibrium in the two-stage pricing game is characterized by the following:*

(i) The firms' first-period prices are

$$p_{A1}^{CN'} = \frac{2t}{3} - \frac{11\bar{\Omega}}{92} \quad \text{and} \quad p_{B1}^{CN'} = \frac{11t}{12} - \frac{7\bar{\Omega}}{46}. \tag{3.23}$$

(ii) The firms' second-period prices are

$$\begin{cases} p_{A2}^{CN'} = \frac{3t}{4} + \frac{39\bar{\Omega}}{92} \\ r_{A2}^{CN'} = \frac{t}{4} - \frac{11\bar{\Omega}}{92} \end{cases} \quad \text{and} \quad q_{B2}^{CN'} = \frac{t}{2} - \frac{7\bar{\Omega}}{46}. \tag{3.24}$$

(iii) The firms' profits are

$$\pi_A^{CN'} = \frac{31t}{48} + \frac{71\bar{\Omega}}{276} + \frac{777\bar{\Omega}^2}{8464t} \quad \text{and} \quad \pi_B^{CN'} = \frac{17t}{24} - \frac{37\bar{\Omega}}{138} + \frac{63\bar{\Omega}^2}{2116t}. \tag{3.25}$$

Proof. See Appendix. □

Since $p_{A2}^{CN'} > q_{B2}^{CN'} > r_{A2}^{CN'}$ for any $\bar{\Omega} \in (-\bar{\theta}, \theta_H - \bar{\theta})$, each firm is poaching its competitor's previous customers in period 2 (see footnotes 7 and 11). It can be also checked that the firms set lower prices in both periods compared with the situation where they use uniform pricing ($p_{i1}^{NN'} > p_{B1}^{CN'} > p_{A1}^{CN'}$, $q_{A2}^{NN'} > p_{A2}^{CN'} > r_{A2}^{CN'}$, and $q_{B2}^{NN'} > q_{B2}^{CN'}$).

3.4.3 Subgame NC'

When only firm B collects consumer information, firm B charges p_{B2} and r_{B2} to its own customers and to firm A's former customers, respectively, while firm A charges q_{A2} to all customers in period 2. Hence the second-period indifferent consumers are defined as follows:

$$\begin{aligned}
v - t\hat{x}_A^d - q_{A2} &= v + \bar{\theta} - t(1 - \hat{x}_A^d) - r_{B2} \\
v + \theta_H - t\hat{x}_A^s - q_{A2} &= v + \bar{\theta} - t(1 - \hat{x}_A^s) - r_{B2} \\
v + \bar{\theta} - t\hat{x}_B - q_{A2} &= v + \bar{\theta} - t(1 - \hat{x}_B) - p_{B2}.
\end{aligned}$$

The firms' demand functions are given by (3.17) and (3.18). Then we can respectively construct the second- and first-period profit maximization problems of the firms as follows:

$$\max_{q_{A2}} \pi_{A2} = q_{A2}\left[\lambda\hat{x}_A^d + (1-\lambda)\hat{x}_A^s + (\hat{x}_B - \hat{x}_1)\right] \tag{3.26}$$

$$\max_{p_{B2}, r_{B2}} \pi_{B2} = p_{B2}(1 - \hat{x}_B) + r_{B2}\left[\lambda(\hat{x}_1 - \hat{x}_A^d) + (1-\lambda)(\hat{x}_1 - \hat{x}_A^s)\right], \tag{3.27}$$

and

$$\max_{p_{A1}} \pi_A = p_{A1}\hat{x}_1 + \pi_{A2}^{NC'} \tag{3.28}$$

$$\max_{p_{B1}} \pi_B = p_{B1}(1 - \hat{x}_1) + \pi_{B2}^{NC'}, \tag{3.29}$$

where $\pi_{i2}^{NC'}$ is the second-period equilibrium profit for firm i.

Lemma 3.4 presents the equilibrium prices and profits for subgame NC':

Lemma 3.4. *Suppose only firm B (the search good-producing firm) collects information about consumers and price discriminates. The equilibrium in the two-stage pricing game is characterized by the following:*

(i) The firms' first-period prices are

$$p_{A1}^{NC'} = \frac{11t}{12} + \frac{7\bar{\Omega}}{46} \quad \text{and} \quad p_{B1}^{NC'} = \frac{2t}{3} - \frac{35\bar{\Omega}}{92}. \tag{3.30}$$

(ii) The firms' second-period prices are

$$q_{A2}^{NC'} = \frac{t}{2} + \frac{7\bar{\Omega}}{46} \quad \text{and} \quad \begin{cases} p_{B2}^{NC'} = \frac{3t}{4} + \frac{7\bar{\Omega}}{92} \\ r_{B2}^{NC'} = \frac{t}{4} - \frac{35\bar{\Omega}}{92}. \end{cases} \tag{3.31}$$

(iii) The firms' profits are

$$\pi_{A}^{NC'} = \frac{17t}{24} + \frac{37\bar{\Omega}}{138} + \frac{63\bar{\Omega}^2}{2116t} \quad \text{and} \quad \pi_{B}^{NC'} = \frac{31t}{48} - \frac{71\bar{\Omega}}{276} + \frac{777\bar{\Omega}^2}{8464t}. \tag{3.32}$$

Proof. See Appendix. □

For any $\bar{\Omega} \in (-\bar{\theta}, \theta_H - \bar{\theta})$, $p_{B2}^{NC'} > q_{A2}^{NC'} > r_{B2}^{NC'}$. Thus each firm is poaching its competitor's past customers (see footnotes 7 and 11). As in subgame CN', the firms charge lower prices in both periods than the case where uniform pricing is used ($p_{i1}^{NN'} > p_{A1}^{NC'} > p_{B1}^{NC'}$, $q_{A2}^{NN'} > q_{A2}^{NC'}$, and $q_{B2}^{NN'} > p_{B2}^{NC'} > r_{B2}^{NC'}$).

3.4.4 Subgame CC'

Since both firms collect consumer information for BBPD in subgame CC', firm $i \in \{A, B\}$ respectively charges p_{i2} and r_{i2} to its own customers and to the competitor's previous customers in period 2. Hence the second-period indifferent consumers are defined as follows:

$$v - t\hat{x}_A^d - p_{A2} = v + \bar{\theta} - t(1 - \hat{x}_A^d) - r_{B2}$$
$$v + \theta_H - t\hat{x}_A^s - p_{A2} = v + \bar{\theta} - t(1 - \hat{x}_A^s) - r_{B2}$$
$$v + \bar{\theta} - t\hat{x}_B - r_{A2} = v + \bar{\theta} - t(1 - \hat{x}_B) - p_{B2}.$$

The firms' demand functions are given by (3.17) and (3.18). Then we can respectively construct the second- and first-period profit maximization problems of the firms as follows:

$$\max_{p_{A2}, r_{A2}} \pi_{A2} = p_{A2}\left[\lambda\hat{x}_A^d + (1-\lambda)\hat{x}_A^s\right] + r_{A2}(\hat{x}_B - \hat{x}_1) \tag{3.33}$$
$$\max_{p_{B2}, r_{B2}} \pi_{B2} = p_{B2}(1 - \hat{x}_B) + r_{B2}\left[\lambda(\hat{x}_1 - \hat{x}_A^d) + (1-\lambda)(\hat{x}_1 - \hat{x}_A^s)\right], \tag{3.34}$$

and

$$\max_{p_{A1}} \pi_A = p_{A1}\hat{x}_1 + \pi_{A2}^{CC'} \tag{3.35}$$
$$\max_{p_{B1}} \pi_B = p_{B1}(1 - \hat{x}_1) + \pi_{B2}^{CC'}, \tag{3.36}$$

where $\pi_{i2}^{CC'}$ is the second-period equilibrium profit for firm i.

The first-order conditions for the problems (3.35) and (3.36) lead immediately to the following results:

Lemma 3.5. *Suppose both firms collect information about consumers and price discriminate. The equilibrium in the two-stage pricing game is characterized as follows:*

(i) The firms' first-period prices are

$$p_{A1}^{CC'} = \frac{4t}{3} - \frac{5\bar{\Omega}}{24} \quad \text{and} \quad p_{B1}^{CC'} = \frac{4t}{3} - \frac{11\bar{\Omega}}{24}. \tag{3.37}$$

(ii) The firms' second-period prices are

$$\begin{cases} p_{A2}^{CC'} = \frac{2t}{3} + \frac{17\bar{\Omega}}{48} \\ r_{A2}^{CC'} = \frac{t}{3} - \frac{\bar{\Omega}}{24} \end{cases} \quad \text{and} \quad \begin{cases} p_{B2}^{CC'} = \frac{2t}{3} - \frac{\bar{\Omega}}{48} \\ r_{B2}^{CC'} = \frac{t}{3} - \frac{7\bar{\Omega}}{24}. \end{cases} \tag{3.38}$$

(iii) The firms' profits are

$$\pi_A^{CC'} = \frac{17t}{18} + \frac{23\bar{\Omega}}{144} + \frac{263\bar{\Omega}^2}{4608t} \quad \text{and} \quad \pi_B^{CC'} = \frac{17t}{18} - \frac{55\bar{\Omega}}{144} + \frac{263\bar{\Omega}^2}{4608t}. \tag{3.39}$$

Proof. See Appendix. $\square$

From Lemma 3.5, we can see that in the equilibrium, each firm is poaching its competitor's former customers since $p_{i2}^{CC'} > r_{i2}^{CC'}$ for any $\bar{\Omega} \in (-\bar{\theta}, \theta_H - \bar{\theta})$ (see footnotes 7 and 11). As in subgame CC, the firms set a higher price in the first period ($p_{i1}^{CC'} > p_{i1}^{NN'}$) and lower prices in the second period ($r_{i2}^{CC'} < p_{i2}^{CC'} < q_{i2}^{NN'}$) than under uniform pricing. Also, compared to the case of both firms producing search goods, the first-period and poaching prices rise ($p_{i1}^{CC'} > p_{i1}^*$ and $r_{i2}^{CC'} > r_{i2}^*$) and firm A's second-period price falls ($p_{A2}^{CC'} < p_{A2}^*$) when $\lambda > \frac{\theta_H - \bar{\theta}}{\theta_H}$. Unlike subgame CC, however, firm B (the search good-producing firm) charges a higher second-period price to its past customers ($p_{B2}^{CC'} > p_{B2}^*$). This is because under the condition, the first-period market share of firm B is greater than that of firm A so that firm B can raise a second-period price due to the increased demand.[12] In the following subsection, we will discuss the profitability of BBPD in the case of asymmetric firms.

3.4.5 Price discrimination decisions

Based on the derived equilibrium profits for all subgames, we can analyze the firms' price discrimination decisions. In period 0, the firms simultaneously decide whether to acquire consumer information and price discriminate. The results are summarized in the following proposition:

Proposition 3.3. *Consider two firms A and B where firm A produces experience goods and firm B produces search goods. In the three-stage game in which the firms first make price discrimination decisions followed by two-period pricing decisions, there are two (pure strategy) SPNE. In the first SPNE, both firms do not collect consumer information so that neither firm price discriminates (N, N). In the other SPNE, both firms collect consumer information and price discriminate (C, C).*

Proof. See Appendix. $\square$

[12] Note that in this case, $\hat{x}_1 = \frac{1}{2} + \frac{\bar{\Omega}}{32t}$. Thus if $\lambda > \frac{\theta_H - \bar{\theta}}{\theta_H} \Leftrightarrow \bar{\Omega} < 0$, $\hat{x}_1 < \frac{1}{2}$.

Even in this setting where the firms are not symmetric, only the symmetric equilibria can be supported as SPNE. The effects of BBPD on the firms' profits are then discussed below:

Proposition 3.4. *Consider the game described in Proposition 3.3. (1) Suppose $\bar{\theta} > k_2 t$, where $k_2 = \frac{16(2\sqrt{290}-31)}{199}$. Then there exists a critical fraction of dissatisfied consumers, $\lambda^{**} = \frac{\theta_H - \bar{\theta} + k_2 t}{\theta_H}$, such that if $\lambda > \lambda^{**}$, $\pi_A^{CC'} < \pi_A^{NN'}$ and $\pi_B^{CC'} > \pi_B^{NN'}$. If $\lambda < \lambda^{**}$, then $\pi_i^{CC'} < \pi_i^{NN'}$, $i \in \{A, B\}$. (2) Suppose $\bar{\theta} \leq k_2 t$. Then $\pi_i^{CC'} < \pi_i^{NN'}$, $i \in \{A, B\}$.*

Proof. See Appendix. $\square$

The first result of Proposition 3.4 says that in the equilibrium, the search good-producing firm can make more profits by employing BBPD than by employing uniform pricing when there are sufficiently many dissatisfied consumers with its competitor's experience goods. In this case, however, BBPD makes the experience good-producing firm worse off. The second result of the proposition states that if the experience-related value of firms' products is small enough, the firms are better off by pricing uniformly. In addition, from Proposition 3.4, we can draw the fact that for an experience good-producing firm competing with a search good-producing firm, BBPD always leads to lower profits than uniform pricing.

3.5 Conclusion

The present study tries to characterize the conditions under which behavior-based price discrimination in markets exhibiting best-response asymmetry is more profitable than uniform pricing. For this, two cases are considered. In the first case duopolists produce experience goods and in the second case one firm produces experience goods and the other produces search goods. Employing a three-period model in which firms first make price discrimination decisions followed by two-period pricing decisions, we arrive at the following results.

In the case of both firms producing experience goods, there are two subgame perfect Nash equilibria. In the first SPNE, both firms do not collect information about consumers' purchase histories so that neither firm price discriminates. In the other SPNE, both firms collect consumer information and price discriminate.

When both firms use BBPD, they set a higher price in the first period and lower prices in the second period than in the case of uniform pricing. In addition, compared to the case of both firms producing search goods, the first-period price and poaching price increase while the second-period price decreases when there are enough dissatisfied consumers in the market. The intuitions behind these results are as follows. Each firm can charge a higher poaching price as consumers experience a bad fit with the competitor's product. The existence of dissatisfied consumers with its own product makes price competition more intense because each firm should lower their second-period price to prevent the consumers from switching to the rival firm.

Finally, BBPD leads to higher profits than uniform pricing when sufficiently many consumers have a poor experience with the firms' products. The reason is that as many consumers' valuations are expected to fall over time, the firms can each raise their first-period price via BBPD, which results in higher profits in the first period.

When one firm produces experience goods and the other produces search goods, there also exist two subgame perfect Nash equilibria where neither firm acquires consumer information and where both firms collect consumer information for BBPD. As in the first case, under BBPD, the price set in the first period is higher and the prices set in the second period are lower than under uniform pricing. It also turns out that while detrimental to an experience good-producing firm, BBPD is conducive to profits for a search good-producing firm when there are enough dissatisfied consumers with its competitor's experience goods. This is because as more consumers have a bad fit with the experience goods, a firm producing search goods can have room to increase its prices in both periods. In practice, the value of an entrant's product is likely to be initially uncertain to customers. Thus, this model of the asymmetric firms can be applied to the situation where an incumbent and an entrant that are competing in price are considering what pricing scheme to adopt.

3.6 Appendix

Proof of Lemma 3.1

From the first-order conditions for the problems (3.3) and (3.4), we find both firms' best-response functions as follows:

$$
\begin{aligned}
p_{A2}(q_{B2}) &= \frac{1}{2}(q_{B2} + t + \Omega) \\
r_{A2}(q_{B2}) &= \frac{1}{2}(q_{B2} + t - \Omega - 2t\hat{x}_1) \\
q_{B2}(p_{A2}, r_{A2}) &= \frac{1}{4}(p_{A2} + r_{A2} + 2t\hat{x}_1).
\end{aligned}
$$

Solving the best-response functions simultaneously yields the second-period equilibrium prices:

$$
\begin{aligned}
p_{A2} &= \frac{2t}{3} + \frac{\Omega}{2} + \frac{t\hat{x}_1}{6} \\
r_{A2} &= \frac{2t}{3} - \frac{\Omega}{2} - \frac{5t\hat{x}_1}{6} \\
q_{B2} &= \frac{t}{3} + \frac{t\hat{x}_1}{3}.
\end{aligned}
\tag{3.40}
$$

Substituting the second-period equilibrium prices into the profit functions in (3.3) and (3.4), we can get the second-period equilibrium profits for firms A and B as follows:

$$
\pi_{A2}^{CN} = \left(\frac{2t}{3} + \frac{\Omega}{2} + \frac{t\hat{x}_1}{6}\right)\left(\frac{1}{3} + \frac{\Omega}{4t} + \frac{\hat{x}_1}{12}\right) + \left(\frac{2t}{3} - \frac{\Omega}{2} - \frac{5t\hat{x}_1}{6}\right)\left(\frac{1}{3} - \frac{\Omega}{4t} - \frac{5\hat{x}_1}{12}\right) \tag{3.41}
$$

$$
\pi_{B2}^{CN} = \left(\frac{t}{3} + \frac{t\hat{x}_1}{3}\right)\left(\frac{1}{3} + \frac{\hat{x}_1}{3}\right). \tag{3.42}
$$

Now we need to compute firm A's first-period market share $\hat{x}_1$. The consumer who is indifferent between the firms' products in period 1 foresees that if she buys from firm A in period 1, she will switch to firm B in period 2, whereas if she buys from firm B in period 1 she will switch to firm A in period 2. Thus, the indifferent consumer is defined by

$$
v + \theta - t\hat{x}_1 - p_{A1} + [v + \theta - t(1 - \hat{x}_1) - q_{B2}] = v + \theta - t(1 - \hat{x}_1) - p_{B1} + [v + \theta - t\hat{x}_1 - r_{A2}]. \tag{3.43}
$$

Plugging the second-period equilibrium prices into (3.43) gives

$$
\hat{x}_1 = \frac{12(p_{B1} - p_{A1}) + 4t - 6\Omega}{14t}. \tag{3.44}
$$

From (3.41), (3.42), and (3.44) we have

$$\begin{aligned}
\frac{\partial \pi_{A2}^{CN}}{\partial p_{A1}} &= 2\frac{\partial \hat{x}_1}{\partial p_{A1}}\left(-\frac{2t}{9}+\frac{\Omega}{4}+\frac{13t\hat{x}_1}{36}\right)\\
\frac{\partial \pi_{B2}^{CN}}{\partial p_{B1}} &= 2\frac{\partial \hat{x}_1}{\partial p_{B1}}\left(\frac{t}{9}+\frac{t\hat{x}_1}{9}\right)\\
\frac{\partial \hat{x}_1}{\partial p_{A1}} &= -\frac{6}{7t}\\
\frac{\partial \hat{x}_1}{\partial p_{B1}} &= \frac{6}{7t}.
\end{aligned} \tag{3.45}$$

The first-order conditions for the problems (3.5) and (3.6) are then

$$\begin{aligned}
\frac{\partial \pi_A}{\partial p_{A1}} &= \hat{x}_1 + p_{A1}\frac{\partial \hat{x}_1}{\partial p_{A1}} + \frac{\partial \pi_{A2}^{CN}}{\partial p_{A1}} = 0\\
\frac{\partial \pi_B}{\partial p_{B1}} &= 1-\hat{x}_1 - p_{B1}\frac{\partial \hat{x}_1}{\partial p_{B1}} + \frac{\partial \pi_{B2}^{CN}}{\partial p_{B1}} = 0.
\end{aligned}$$

Solving the system with (3.45) gives (3.7). It immediately leads to $\hat{x}_1 = \frac{1}{2}$. Replacing $\hat{x}_1$ in (3.40) with $\frac{1}{2}$, we get (3.8). Substituting the equilibrium values of p_{i1}, $\hat{x}_1$, and π_{i2} into the profit functions in (3.5) and (3.6), we obtain (3.9). □

Proof of Lemma 3.2

From the first-order conditions for the problems (3.10) and (3.11), we find both firms' best-response functions as follows:

$$\begin{aligned}
p_{A2}(r_{B2}) &= \frac{1}{2}(r_{B2}+t+\Omega)\\
r_{A2}(p_{B2}) &= \frac{1}{2}(p_{B2}+t-\Omega-2t\hat{x}_1)\\
p_{B2}(r_{A2}) &= \frac{1}{2}(r_{A2}+t+\Omega)\\
r_{B2}(p_{A2}) &= \frac{1}{2}(p_{A2}-t-\Omega+2t\hat{x}_1).
\end{aligned}$$

Solving the best-response functions simultaneously yields the second-period equilibrium prices:

$$
\begin{aligned}
p_{A2} &= \frac{t}{3} + \frac{\Omega}{3} + \frac{2t\hat{x}_1}{3} \\
r_{A2} &= t - \frac{\Omega}{3} - \frac{4t\hat{x}_1}{3} \\
p_{B2} &= t + \frac{\Omega}{3} - \frac{2t\hat{x}_1}{3} \\
r_{B2} &= -\frac{t}{3} - \frac{\Omega}{3} + \frac{4t\hat{x}_1}{3}.
\end{aligned}
\tag{3.46}
$$

Substituting the second-period equilibrium prices into the profit functions in (3.10) and (3.11), we can get the second-period equilibrium profits for firms A and B as follows:

$$
\pi_{A2}^{CC} = \left(\frac{t}{3} + \frac{\Omega}{3} + \frac{2t\hat{x}_1}{3}\right)\left(\frac{1}{6} + \frac{\Omega}{6t} + \frac{\hat{x}_1}{3}\right) + \left(t - \frac{\Omega}{3} - \frac{4t\hat{x}_1}{3}\right)\left(\frac{1}{2} - \frac{\Omega}{6t} - \frac{2\hat{x}_1}{3}\right) \tag{3.47}
$$

$$
\pi_{B2}^{CC} = \left(t + \frac{\Omega}{3} - \frac{2t\hat{x}_1}{3}\right)\left(\frac{1}{2} + \frac{\Omega}{6t} - \frac{\hat{x}_1}{3}\right) + \left(-\frac{t}{3} - \frac{\Omega}{3} + \frac{4t\hat{x}_1}{3}\right)\left(-\frac{1}{6} - \frac{\Omega}{6t} + \frac{2\hat{x}_1}{3}\right). \tag{3.48}
$$

We then need to compute firm A's first-period market share $\hat{x}_1$. As in Lemma 3.1, the indifferent consumer is defined by

$$
v + \theta - t\hat{x}_1 - p_{A1} + [v + \theta - t(1 - \hat{x}_1) - r_{B2}] = v + \theta - t(1 - \hat{x}_1) - p_{B1} + [v + \theta - t\hat{x}_1 - r_{A2}]. \tag{3.49}
$$

Plugging the second-period equilibrium prices into (3.49) gives

$$
\hat{x}_1 = \frac{3(p_{B1} - p_{A1}) + 4t}{8t}. \tag{3.50}
$$

From (3.47), (3.48), and (3.50) we have

$$
\begin{aligned}
\frac{\partial \pi_{A2}^{CC}}{\partial p_{A1}} &= 2\frac{\partial \hat{x}_1}{\partial p_{A1}}\left(-\frac{5t}{9}+\frac{\Omega}{3}+\frac{10t\hat{x}_1}{9}\right)\\
\frac{\partial \pi_{B2}^{CC}}{\partial p_{B1}} &= 2\frac{\partial \hat{x}_1}{\partial p_{B1}}\left(-\frac{5t}{9}-\frac{\Omega}{3}+\frac{10t\hat{x}_1}{9}\right)\\
\frac{\partial \hat{x}_1}{\partial p_{A1}} &= -\frac{3}{8t}\\
\frac{\partial \hat{x}_1}{\partial p_{B1}} &= \frac{3}{8t}.
\end{aligned}
\tag{3.51}
$$

The first-order conditions for the problems (3.12) and (3.13) are

$$
\begin{aligned}
\frac{\partial \pi_A}{\partial p_{A1}} &= \hat{x}_1 + p_{A1}\frac{\partial \hat{x}_1}{\partial p_{A1}} + \frac{\partial \pi_{A2}^{CC}}{\partial p_{A1}} = 0\\
\frac{\partial \pi_B}{\partial p_{B1}} &= 1-\hat{x}_1 - p_{B1}\frac{\partial \hat{x}_1}{\partial p_{B1}} + \frac{\partial \pi_{B2}^{CC}}{\partial p_{B1}} = 0.
\end{aligned}
$$

Solving the system with (3.51) gives (3.14). It immediately leads to $\hat{x}_1 = \frac{1}{2}$. Replacing $\hat{x}_1$ in (3.46) with $\frac{1}{2}$, we get (3.15). Substituting the equilibrium values of p_{i1}, $\hat{x}_1$, and π_{i2} into the profit functions in (3.12) and (3.13), we obtain (3.16). $\square$

Proof of Proposition 3.1

From Assumption 3.1, we know that $t > \Omega$. Then, it is easy to verify that $\pi_A^{CC} > \pi_A^{NC}$ and $\pi_A^{NN} > \pi_A^{CN}$ (simultaneously $\pi_B^{CC} > \pi_B^{CN}$ and $\pi_B^{NN} > \pi_B^{NC}$), which shows the claim. $\square$

Proof of Proposition 3.2

Recall first that $t > \Omega$. From $\pi_i^{CC} = \frac{17t}{18} - \frac{2\Omega}{9} + \frac{\Omega^2}{9t} > \pi_i^{NN} = t$, we have $\Omega = (1-\lambda)\theta_H - \theta < t\left(1-\sqrt{\frac{3}{2}}\right) = -tk_1$, which reduces to $\lambda^* = \frac{\theta_H - \theta + k_1 t}{\theta_H} < \lambda$. Here $0 < \lambda^* < 1$ when $\theta > k_1 t$. If $\theta \leq k_1 t$, then $\lambda^* \geq 1$ so that λ is always less than λ^*, which yields $\pi_i^{CC} < \pi_i^{NN}$. $\square$

Proof of Lemma 3.3

The first-order conditions of the problems (3.19) and (3.20) lead to the following best-response functions:

$$
\begin{aligned}
p_{A2}(q_{B2}) &= \frac{1}{2}(q_{B2} + t + \bar{\Omega}) \\
r_{A2}(q_{B2}) &= \frac{1}{2}(q_{B2} + t - 2t\hat{x}_1) \\
q_{B2}(p_{A2}, r_{A2}) &= \frac{1}{4}(p_{A2} + r_{A2} - \bar{\Omega} + 2t\hat{x}_1).
\end{aligned}
$$

Solving the best-response functions simultaneously yields the second-period equilibrium prices:

$$
\begin{aligned}
p_{A2} &= \frac{2t}{3} + \frac{5\bar{\Omega}}{12} + \frac{t\hat{x}_1}{6} \\
r_{A2} &= \frac{2t}{3} - \frac{\bar{\Omega}}{12} - \frac{5t\hat{x}_1}{6} \\
q_{B2} &= \frac{t}{3} - \frac{\bar{\Omega}}{6} + \frac{t\hat{x}_1}{3}.
\end{aligned}
\tag{3.52}
$$

Substituting the second-period equilibrium prices into the profit functions in (3.19) and (3.20), we can get the second-period equilibrium profits for firms A and B as follows:

$$
\pi_{A2}^{CN'} = \left(\frac{2t}{3} + \frac{5\bar{\Omega}}{12} + \frac{t\hat{x}_1}{6}\right)\left(\frac{1}{3} + \frac{5\bar{\Omega}}{24t} + \frac{\hat{x}_1}{12}\right) + \left(\frac{2t}{3} - \frac{\bar{\Omega}}{12} - \frac{5t\hat{x}_1}{6}\right)\left(\frac{1}{3} - \frac{\bar{\Omega}}{24t} - \frac{5\hat{x}_1}{12}\right) \tag{3.53}
$$

$$
\pi_{B2}^{CN'} = \left(\frac{t}{3} - \frac{\bar{\Omega}}{6} + \frac{t\hat{x}_1}{3}\right)\left(\frac{1}{3} - \frac{\bar{\Omega}}{6t} + \frac{\hat{x}_1}{3}\right). \tag{3.54}
$$

The first-period indifferent consumer is then defined by (3.43) with $\bar{\theta}$ instead of θ. Plugging the second-period equilibrium prices gives

$$
\hat{x}_1 = \frac{12(p_{B1} - p_{A1}) + 4t + \bar{\Omega}}{14t}. \tag{3.55}
$$

From (3.53), (3.54), and (3.55) we have

$$\begin{aligned}
\frac{\partial \pi_{A2}^{CN'}}{\partial p_{A1}} &= 2\frac{\partial \hat{x}_1}{\partial p_{A1}}\left(-\frac{2t}{9}+\frac{5\bar{\Omega}}{72}+\frac{13t\hat{x}_1}{36}\right)\\
\frac{\partial \pi_{B2}^{CN'}}{\partial p_{B1}} &= 2\frac{\partial \hat{x}_1}{\partial p_{B1}}\left(\frac{t}{9}-\frac{\bar{\Omega}}{18}+\frac{t\hat{x}_1}{9}\right)\\
\frac{\partial \hat{x}_1}{\partial p_{A1}} &= -\frac{6}{7t}\\
\frac{\partial \hat{x}_1}{\partial p_{B1}} &= \frac{6}{7t}.
\end{aligned} \tag{3.56}$$

The first-order conditions for the problems (3.21) and (3.22) are

$$\begin{aligned}
\frac{\partial \pi_A}{\partial p_{A1}} &= \hat{x}_1 + p_{A1}\frac{\partial \hat{x}_1}{\partial p_{A1}} + \frac{\partial \pi_{A2}^{CN'}}{\partial p_{A1}} = 0\\
\frac{\partial \pi_B}{\partial p_{B1}} &= 1-\hat{x}_1 - p_{B1}\frac{\partial \hat{x}_1}{\partial p_{B1}} + \frac{\partial \pi_{B2}^{CN'}}{\partial p_{B1}} = 0.
\end{aligned}$$

Solving the system with (3.56) gives (3.23). It immediately leads to $\hat{x}_1 = \frac{1}{2}+\frac{\bar{\Omega}}{23t}$. Replacing $\hat{x}_1$ in (3.52) with $\frac{1}{2}+\frac{\bar{\Omega}}{23t}$, we get (3.24). Substituting the equilibrium values of p_{i1}, $\hat{x}_1$, and π_{i2} into the profit functions in (3.21) and (3.22), we obtain (3.25). $\square$

Proof of Lemma 3.4

The first-order conditions of the problems (3.26) and (3.27) lead to the following best-response functions:

$$\begin{aligned}
q_{A2}(p_{B2}, r_{B2}) &= \frac{1}{4}(p_{B2}+r_{B2}+2t+\bar{\Omega}-2t\hat{x}_1)\\
p_{B2}(q_{A2}) &= \frac{1}{2}(q_{A2}+t)\\
r_{B2}(q_{A2}) &= \frac{1}{2}(q_{A2}-t-\bar{\Omega}+2t\hat{x}_1).
\end{aligned}$$

Solving the best-response functions simultaneously yields the second-period equilibrium prices:

$$
\begin{aligned}
q_{A2} &= \frac{2t}{3} + \frac{\bar{\Omega}}{6} - \frac{t\hat{x}_1}{3} \\
p_{B2} &= \frac{5t}{6} + \frac{\bar{\Omega}}{12} - \frac{t\hat{x}_1}{6} \\
r_{B2} &= -\frac{t}{6} - \frac{5\bar{\Omega}}{12} + \frac{5t\hat{x}_1}{6}.
\end{aligned}
\tag{3.57}
$$

Substituting the second-period equilibrium prices into the profit functions in (3.26) and (3.27), we can get the second-period equilibrium profits for firms A and B as follows:

$$
\pi_{A2}^{NC'} = \left(\frac{2t}{3} + \frac{\bar{\Omega}}{6} - \frac{t\hat{x}_1}{3}\right)\left(\frac{2}{3} + \frac{\bar{\Omega}}{6t} - \frac{\hat{x}_1}{3}\right) \tag{3.58}
$$

$$
\pi_{B2}^{NC'} = \left(\frac{5t}{6} + \frac{\bar{\Omega}}{12} - \frac{t\hat{x}_1}{6}\right)\left(\frac{5}{12} + \frac{\bar{\Omega}}{24t} - \frac{\hat{x}_1}{12}\right) + \left(-\frac{t}{6} - \frac{5\bar{\Omega}}{12} + \frac{5t\hat{x}_1}{6}\right)\left(-\frac{1}{12} - \frac{5\bar{\Omega}}{24t} + \frac{5\hat{x}_1}{12}\right). \tag{3.59}
$$

The first-period indifferent consumer is defined by

$$
v + \bar{\theta} - t\hat{x}_1 - p_{A1} + [v + \bar{\theta} - t(1-\hat{x}_1) - r_{B2}] = v + \bar{\theta} - t(1-\hat{x}_1) - p_{B1} + [v + \bar{\theta} - t\hat{x}_1 - q_{A2}].
$$

Plugging the second-period equilibrium prices gives

$$
\hat{x}_1 = \frac{12(p_{B1} - p_{A1}) + 10t + 7\bar{\Omega}}{14t}. \tag{3.60}
$$

From (3.58), (3.59), and (3.60) we have

$$
\begin{aligned}
\frac{\partial \pi_{A2}^{NC'}}{\partial p_{A1}} &= 2\frac{\partial \hat{x}_1}{\partial p_{A1}}\left(-\frac{2t}{9} - \frac{\bar{\Omega}}{18} + \frac{t\hat{x}_1}{9}\right) \\
\frac{\partial \pi_{B2}^{NC'}}{\partial p_{B1}} &= 2\frac{\partial \hat{x}_1}{\partial p_{B1}}\left(-\frac{5t}{36} - \frac{13\bar{\Omega}}{72} + \frac{13t\hat{x}_1}{36}\right) \\
\frac{\partial \hat{x}_1}{\partial p_{A1}} &= -\frac{6}{7t} \\
\frac{\partial \hat{x}_1}{\partial p_{B1}} &= \frac{6}{7t}.
\end{aligned}
\tag{3.61}
$$

The first-order conditions for the problems (3.28) and (3.29) are then

$$\frac{\partial \pi_A}{\partial p_{A1}} = \hat{x}_1 + p_{A1}\frac{\partial \hat{x}_1}{\partial p_{A1}} + \frac{\partial \pi_{A2}^{NC'}}{\partial p_{A1}} = 0$$
$$\frac{\partial \pi_B}{\partial p_{B1}} = 1 - \hat{x}_1 - p_{B1}\frac{\partial \hat{x}_1}{\partial p_{B1}} + \frac{\partial \pi_{B2}^{NC'}}{\partial p_{B1}} = 0.$$

Solving the system with (3.61) gives (3.30). It immediately leads to $\hat{x}_1 = \frac{1}{2} + \frac{\bar{\Omega}}{23t}$. Replacing $\hat{x}_1$ in (3.57) with $\frac{1}{2} + \frac{\bar{\Omega}}{23t}$, we get (3.31). Substituting the equilibrium values of p_{i1}, $\hat{x}_1$, and π_{i2} into the profit functions in (3.28) and (3.29), we obtain (3.32). □

Proof of Lemma 3.5

From the first-order conditions for the problems (3.33) and (3.34), we find the firms' best-response functions as follows:

$$\begin{aligned}
p_{A2}(r_{B2}) &= \frac{1}{2}(r_{B2} + t + \bar{\Omega}) \\
r_{A2}(p_{B2}) &= \frac{1}{2}(p_{B2} + t - 2t\hat{x}_1) \\
p_{B2}(r_{A2}) &= \frac{1}{2}(r_{A2} + t) \\
r_{B2}(p_{A2}) &= \frac{1}{2}(p_{A2} - t - \bar{\Omega} + 2t\hat{x}_1).
\end{aligned}$$

Solving the best-response functions simultaneously yields the second-period equilibrium prices:

$$\begin{aligned}
p_{A2} &= \frac{t}{3} + \frac{\bar{\Omega}}{3} + \frac{2t\hat{x}_1}{3} \\
r_{A2} &= t - \frac{4t\hat{x}_1}{3} \\
p_{B2} &= t - \frac{2t\hat{x}_1}{3} \\
r_{B2} &= -\frac{t}{3} - \frac{\bar{\Omega}}{3} + \frac{4t\hat{x}_1}{3}.
\end{aligned} \tag{3.62}$$

Substituting the second-period equilibrium prices into the profit functions in (3.33) and

(3.34), we can get the second-period equilibrium profits for firms A and B as follows:

$$\pi_{A2}^{CC'} = \left(\frac{t}{3} + \frac{\bar{\Omega}}{3} + \frac{2t\hat{x}_1}{3}\right)\left(\frac{1}{6} + \frac{\bar{\Omega}}{6t} + \frac{\hat{x}_1}{3}\right) + \left(t - \frac{4t\hat{x}_1}{3}\right)\left(\frac{1}{2} - \frac{2\hat{x}_1}{3}\right) \tag{3.63}$$

$$\pi_{B2}^{CC'} = \left(t - \frac{2t\hat{x}_1}{3}\right)\left(\frac{1}{2} - \frac{\hat{x}_1}{3}\right) + \left(-\frac{t}{3} - \frac{\bar{\Omega}}{3} + \frac{4t\hat{x}_1}{3}\right)\left(-\frac{1}{6} - \frac{\bar{\Omega}}{6t} + \frac{2\hat{x}_1}{3}\right). \tag{3.64}$$

The first-period indifferent consumer is then defined by (3.49) with $\bar{\theta}$ instead of θ. Plugging the second-period equilibrium prices gives

$$\hat{x}_1 = \frac{3(p_{B1} - p_{A1}) + 4t + \bar{\Omega}}{8t}. \tag{3.65}$$

From (3.63), (3.64), and (3.65) we have

$$\begin{aligned}
\frac{\partial \pi_{A2}^{CC'}}{\partial p_{A1}} &= 2\frac{\partial \hat{x}_1}{\partial p_{A1}}\left(-\frac{5t}{9} + \frac{\bar{\Omega}}{9} + \frac{10t\hat{x}_1}{9}\right) \\
\frac{\partial \pi_{B2}^{CC'}}{\partial p_{B1}} &= 2\frac{\partial \hat{x}_1}{\partial p_{B1}}\left(-\frac{5t}{9} - \frac{2\bar{\Omega}}{9} + \frac{10t\hat{x}_1}{9}\right) \\
\frac{\partial \hat{x}_1}{\partial p_{A1}} &= -\frac{3}{8t} \\
\frac{\partial \hat{x}_1}{\partial p_{B1}} &= \frac{3}{8t}.
\end{aligned} \tag{3.66}$$

The first-order conditions for the problems (3.35) and (3.36) are

$$\begin{aligned}
\frac{\partial \pi_A}{\partial p_{A1}} &= \hat{x}_1 + p_{A1}\frac{\partial \hat{x}_1}{\partial p_{A1}} + \frac{\partial \pi_{A2}^{CC'}}{\partial p_{A1}} = 0 \\
\frac{\partial \pi_B}{\partial p_{B1}} &= 1 - \hat{x}_1 - p_{B1}\frac{\partial \hat{x}_1}{\partial p_{B1}} + \frac{\partial \pi_{B2}^{CC'}}{\partial p_{B1}} = 0.
\end{aligned}$$

Solving the system with (3.66) gives (3.37). It immediately leads to $\hat{x}_1 = \frac{1}{2} + \frac{\bar{\Omega}}{32t}$. Replacing $\hat{x}_1$ in (3.62) with $\frac{1}{2} + \frac{\bar{\Omega}}{32t}$ gives (3.38). Substituting the equilibrium values of p_{i1}, $\hat{x}_1$, and π_{i2} into the profit functions in (3.35) and (3.36), we obtain (3.39). □

Proof of Proposition 3.3

From $t > \bar{\Omega}$, it is easy to see that $\pi_A^{CC'} > \pi_A^{NC'}$, $\pi_A^{NN'} > \pi_A^{CN'}$, and $\pi_B^{CC'} > \pi_B^{CN'}$. Since $\theta_H < \frac{t}{4} + \frac{31\bar{\theta}}{92}$ (see footnote 11) and $\bar{\Omega} \in (-\bar{\theta}, \theta_H - \bar{\theta})$, we can show that $\pi_B^{NN'} > \pi_B^{NC'} \Leftrightarrow -\frac{23(\sqrt{325185}-150)t}{5935} \approx -1.62t < \bar{\Omega} < \frac{23(\sqrt{325185}+150)t}{5935} \approx 2.79t$, which proves the claim. $\square$

Proof of Proposition 3.4

From $\pi_B^{CC'} > \pi_B^{NN'} \Leftrightarrow \bar{\Omega} < -\frac{16(2\sqrt{290}-31)t}{199} = -k_2 t$ or $\bar{\Omega} > \frac{16(2\sqrt{290}+31)t}{199} \approx 5.24t$, we have $\lambda > \lambda^{**} = \frac{\theta_H - \bar{\theta} + k_2 t}{\theta_H}$ ($\because t > \bar{\Omega}$). Here $0 < \lambda^{**} < 1$ when $\bar{\theta} > k_2 t$. On the other hand, since $\theta_H < \frac{t}{4} + \frac{31\bar{\theta}}{92}$ (see footnote 11) and $\bar{\Omega} \in (-\bar{\theta}, \theta_H - \bar{\theta})$, $\pi_A^{CC'} < \pi_A^{NN'}$ $\forall \lambda \in (0, 1)$. If $\bar{\theta} \leq k_2 t$, then $\lambda^{**} \geq 1$ so that λ is always less than λ^{**}, which yields $\pi_i^{CC'} < \pi_i^{NN'}$. $\square$

Chapter 4

A Note on Uniform Pricing in the Motion-Picture Industry

4.1 Introduction

Movie theaters in the United States implement several price discrimination schemes such as discounts for seniors and students, while they charge the same ticket price for all movies.[1] Such price uniformity across movies is a puzzle because price variation over differentiated movies can be a profit-maximizing solution corresponding to different demand characteristics. One would expect that exhibitors can increase their profits by charging more for blockbusters. In the case of the digital music industry, using survey data on individuals' valuations of popular songs at iTunes where until recently most songs sold for $0.99, Shiller and Waldfogel (2011) find that alternatives to uniform pricing such as song-specific pricing, bundling, two-part tariffs and nonlinear pricing can raise both producer and consumer surplus.

Despite the extensive economic literature on pricing for differentiated products, there are surprisingly few studies on why movie theaters employ uniform pricing. Orbach and Einav (2007) conclude that exhibitors could increase profits by engaging in variable pricing and that the legal constraints on vertical arrangements between distributors and exhibitors make

[1]This phenomenon is referred to as the movie puzzle. Another puzzle in the motion-picture industry is the show-time puzzle, which refers to the lack of price variation between weekdays and weekends or across seasons (Orbach and Einav, 2007).

it difficult to engage in profitable price differentiation.[2] Chen (2009) considers the agency problem associated with concession sales between the exhibitors' profit maximization and the distributors' revenue maximization. He finds that the high profit mark-up from movie theaters' concession sales makes uniform pricing the profit-maximizing solution for exhibitors and that unless many successful event movies are expected, tiered pricing over regular and event movies will not benefit either exhibitors or distributors. Finally, Courty (2011) shows that a monopolist charges the same price for differentiated products when high quality products are likely to be assigned to low valuation consumers.

The movie business is risky as the market success of movies is not easily predicted. By showing that the probability distributions of movie box-office revenues and profits are characterized by heavy tails and infinite variance, De Vany and Walls (1999) conclude that there are no formulas for success in the motion-picture industry and that no amount of star power or marketing hype can make a movie a hit. Also, movies are experience goods in the sense that people do not know whether they will like a movie until they have seen it (Nelson, 1970). Thus, moviegoers decide which movie to see based on factors other than a movie's quality, which may be in the form of signals. The recent work of Moretti (2011) considers social learning in consumption of movies where movies' quality is *ex ante* uncertain and consumers hold a prior on quality, which they may update based on information from their peers. Using box-office data, he finds that social learning appears to have an important effect on profits in the movie industry.

In the context of price signaling, Wolinsky (1983) shows that considering a market in which the exact quality chosen by a firm is known only to the firm itself, prices serve as signals and each price-signal exceeds the marginal cost of producing the quality it signals. Milgrom and Roberts (1986) show that when consumers make repeat purchases, a high price combined with advertising enables a monopoly to signal its quality. High quality could also be signaled by a high price alone but this would reduce current demand, which is the basis of future demand. Bagwell and Riordan (1991) consider a situation where a monopoly signals quality to consumers when some consumers are informed about product quality. They find that in a one-period market, firms first signal high quality with prices higher than full information profit-maximization prices. As information about product quality is diffused, this price distortion decreases. Hence, high and declining prices signal a high quality product due to

[2] By the *Paramount* decrees, distributors are not allowed to vertically integrate theaters and any involvement of distributors in box-office pricing is prohibited.

an increasing number of informed consumers.

The present study considers the situation where moviegoers form their beliefs about movie quality through pricing schemes to which an exhibitor commits. Using Hotelling's model of product differentiation, we show that committing to uniform pricing is more profitable for an exhibitor than committing to variable pricing under certain conditions on moviegoer's beliefs. The welfare consequences of a uniform pricing commitment are investigated as well.

The remainder of the paper is organized as follows. Section 4.2 sets up the model. Section 4.3 presents the results of our work, and Section 4.4 extends the model by allowing an exhibitor to choose the location (genre) of movies and considering an arbitrary distribution of moviegoers. Section 4.5 concludes.

4.2 The model

Consider a multiplex in which an exhibitor is playing two differentiated movies located at the ends of the Hotelling unit interval, with movie 1 at point 0 and movie 2 at point 1 (we will relax this assumption in Section 4.4). We assume that the marginal cost of screening a movie for an additional audience is zero. There is a continuum of moviegoers uniformly distributed on the interval $[0, 1]$ with a unit mass (an arbitrary distribution will be considered in Section 4.4). Each moviegoer sees at most one movie. A moviegoer located at $x \in [0, 1]$ wants to see x kind (genre) of movie more than any other kind of movie. Thus, the moviegoer incurs a disutility of tx when seeing movie 1, and of $t(1 - x)$ when seeing movie 2, where $t > 0$ measures the per-unit transportation cost (or distaste's cost of seeing away from her ideal movie).

Motion pictures are uncertain products in the sense that it is difficult for movie theaters to estimate which movie will be a hit or flop before screening it (De Vany and Walls, 1999).[3] Suppose thus that each movie is of either high (H) or low (L) quality and that the exhibitor does not observe the exact quality of the movies prior to their release. However, since movie theaters can predict, to some extent, whether a movie will be a hit based on movie stars or marketing hype, we also suppose that the exhibitor expects movie 2 to be of high quality with higher probability than movie 1. This gives the exhibitor incentive to set a higher ticket

[3]Screenwriter William Goldman's famous saying about the movie industry is that "nobody knows anything" about the success of a movie.

price for movie 2. Nevertheless, the exhibitor can consider charging the same price for both movies as it is uncertain about how moviegoers will evaluate them. Indeed, one common explanation for price uniformity is that different ticket prices are likely to be perceived as quality signals and can deter moviegoers from seeing low-priced movies (Orbach and Einav, 2007). Letting p_i denote the ticket price of movie $i \in \{1, 2\}$, we then make the following definition:

Definition 4.1. An exhibitor is said to use uniform pricing (resp. variable pricing) when $p_1 = p_2 = p$ (resp. $p_1 < p_2$).

Due to the symmetry of the model, we will only consider that $p_1 < p_2$ in case of variable pricing. Of course, this comes from the assumption that the exhibitor expects movie 2 is more likely to be of high quality than movie 1. To incorporate the exhibitor's choice of the pricing schemes into the model, it is assumed that the exhibitor commits to whether it would employ uniform or variable pricing before setting movie ticket prices and that this commitment is binding.[4] In what follows, we denote by u (resp. d) uniform (resp. variable) pricing.

Likewise, moviegoers are uninformed of the quality of movies before viewing it and thus their decisions about which movie to see rely on factors other than movie quality. Here, we consider the situation where moviegoers form their expectations about movie quality based on the pricing schemes to which the exhibitor commits.

Let $\mu_i^j \in (0, 1)$ denote the belief (probability) moviegoers assign to the event that movie i is of high quality when the exhibitor commits to the pricing scheme $j \in \{u, d\}$. Also, let s_L (resp. s_H) be the basic value each moviegoer attaches to a low-quality (resp. high-quality) movie, where $s_H > s_L$. Then a moviegoer indexed by $x \in [0, 1]$ enjoys (expected) utility $\mu_i^j s_H + (1 - \mu_i^j)s_L - t(\mid x - i + 1 \mid) - p_i$ from seeing movie i under the pricing scheme j. If moviegoers do not see any movie, their utility is zero.

Our key assumption is that, conditional on the pricing schemes to which the exhibitor commits, moviegoers form their beliefs about the quality of movies in the following manner:

$$\begin{cases} \mu_1^u = \mu_2^u = b \\ \mu_1^d = b_1 < \mu_2^d = b_2, \end{cases} \tag{4.1}$$

[4] An example of such a commitment is to maintain or change customary pricing patterns.

where, without loss of generality, $b_1 < b < b_2$.

The belief formation (4.1) implies that if the exhibitor commits to uniform pricing, moviegoers believe the quality of the two movies is high with equal probability, whereas they believe movie 2 is more likely to be of high quality than movie 1 under a variable pricing commitment. Despite its simplicity, this belief formation captures the important stylized fact that a high price signals high quality (Wolinsky, 1983; Bagwell and Riordan, 1991).

For simplicity, s_L is normalized to zero. The utility of a moviegoer indexed by $x \in [0, 1]$ is then defined by

$$U_x \equiv \begin{cases} bs_H - tx - p & \text{if he sees movie 1; uniform} \\ bs_H - t(1-x) - p & \text{if he sees movie 2; uniform} \\ b_1 s_H - tx - p_1 & \text{if he sees movie 1; variable} \\ b_2 s_H - t(1-x) - p_2 & \text{if he sees movie 2; variable} \\ 0 & \text{if he does not see any movie.} \end{cases} \tag{4.2}$$

In sum, the interaction of the exhibitor and moviegoers is as follows:

- Stage 1: The exhibitor commits to whether it would use uniform or variable pricing.
- Stage 2: Conditional on the pricing scheme to which the exhibitor commits in stage 1, moviegoers form their beliefs about movie quality. The exhibitor chooses movie ticket prices according to the pricing commitment and then moviegoers decide which movie to see with their beliefs.

The following assumption on the parameters of the model will be maintained throughout the paper:

Assumption 4.1. $b_2 - b_1 < \frac{2t}{s_H}$.

The assumption guarantees that in equilibrium, there always exist moviegoers who prefer seeing movie 1 to 2, even if movie 1 is believed to be of lower quality because of a variable pricing commitment. In addition, the following definition will be useful in discussing our results:

Definition 4.2. A commitment to variable pricing is said to have a negative effect (resp. positive effect) on moviegoer's beliefs about the quality of movies if it leads to $b_1 + b_2 < 2b$ (resp. $b_1 + b_2 \geq 2b$).

By Definition 4.2, the negative effect of committing to variable pricing means that the sum of the movies' expected values under a variable pricing commitment is lower than under a uniform pricing commitment.

4.3 Analysis and results

To explore how the two pricing commitments affect the exhibitor's profit, moviegoer surplus, and social welfare, we begin this section with the analysis of the pricing schemes.

4.3.1 Uniform pricing

Consider first the case where the exhibitor commits to uniform pricing. Let $\hat{x}^u$ denote a moviegoer who is indifferent between seeing movie 1 and movie 2. Given a price p and a belief b, $\hat{x}^u$ is determined by $bs_H - t\hat{x}^u - p = bs_H - t(1 - \hat{x}^u) - p$ in (4.2). Solving this condition gives $\hat{x}^u = \frac{1}{2}$, which means that all moviegoers indexed on $[0, \frac{1}{2}]$ will see movie 1, whereas all moviegoers indexed on $(\frac{1}{2}, 1]$ will see movie 2. The exhibitor can then maximize its profit by extracting all the surplus of this marginal moviegoer. The equilibrium values for price, profit, and $\hat{x}^u$ when committing to uniform pricing are thus

$$\begin{aligned} p^u &= bs_H - \frac{t}{2} \\ \pi^u &= bs_H - \frac{t}{2} \\ \hat{x}^u &= \frac{1}{2}. \end{aligned} \tag{4.3}$$

4.3.2 Variable pricing

Suppose now that the exhibitor commits to variable pricing. Recall that in the case of a variable pricing commitment, we have $b_1 < b_2$. Let $\hat{x}^d$ denote a moviegoer who is indifferent between seeing movie 1 and movie 2. Given prices (p_1, p_2) and beliefs (b_1, b_2), we have $\hat{x}^d = \frac{1}{2} + \frac{(b_1-b_2)s_H}{2t} + \frac{p_2-p_1}{2t}$ from $b_1 s_H - t\hat{x}^d - p_1 = b_2 s_H - t(1-\hat{x}^d) - p_2$ in (4.2). When the exhibitor chooses prices p_1 and p_2 to maximize $\pi = p_1\hat{x}^d + p_2(1-\hat{x}^d)$, it extracts all the surplus of the marginal moviegoer indexed by $\hat{x}^d$. The equilibrium values for prices, profit, and $\hat{x}^d$ under a variable pricing commitment are then[5]

$$\begin{aligned} p_1^d &= \frac{(3b_1+b_2)s_H}{4} - \frac{t}{2} \quad \text{and} \quad p_2^d = \frac{(b_1+3b_2)s_H}{4} - \frac{t}{2} \\ \pi^d &= \frac{(b_1+b_2)s_H}{2} + \frac{(b_2-b_1)^2 s_H^2}{8t} - \frac{t}{2} \\ \hat{x}^d &= \frac{1}{2} - \frac{(b_2-b_1)s_H}{4t}. \end{aligned} \tag{4.4}$$

4.3.3 Exhibitor's profit

To characterize the conditions under which the exhibitor has incentive to commit to uniform pricing, we calculate $\pi^u - \pi^d$. From (4.3) and (4.4) this calculation yields the following:

Proposition 4.1. *Suppose a commitment to variable pricing has a negative effect. Then committing to uniform pricing is more profitable than committing to variable pricing if moviegoer's beliefs about movie quality satisfy*

$$\frac{2b-(b_1+b_2)}{(b_2-b_1)^2} > \frac{s_H}{4t}. \tag{4.5}$$

Figure 4.1, drawn for $s_H = 2t$ and $b = \frac{1}{2}$, shows the ranges of moviegoer's beliefs for the model predictions.[6] Given that under a uniform pricing commitment, moviegoers expect the two movies to be of high quality with probability $\frac{1}{2}$, the exhibitor can be better off by committing to uniform pricing if moviegoers form their beliefs (b_1 and b_2) under a variable pricing commitment in region A+B (excluding the boundaries). For example, we can see that when movie 1 for which the exhibitor commits to charge a low price is believed to be of low quality with high enough probability, committing to uniform pricing is more likely to be profitable. However, if moviegoer's belief that the quality of the high-priced movie (movie 2) is high is large enough, then a variable pricing commitment would emerge as an optimal

[5] For details of the derivations, see Lemma 4.2 and its proof in Section 4.4.

[6] For the cases of $b = \frac{1}{3}$ and $b = \frac{2}{3}$, see Figures 4.2 and 4.3 in Appendix.

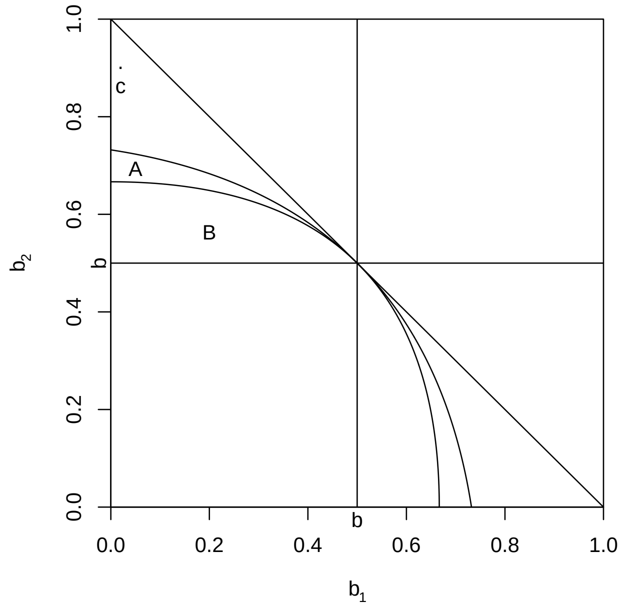

Figure 4.1: Belief ranges for the model predictions ($s_H = 2t$, $b = \frac{1}{2}$)

strategy (see, e.g., the point c in Figure 4.1). Therefore, (committing to) uniform pricing observed in the motion-picture industry reflects that audiences still remain uncertain about a movie's quality even though movie theaters (commit to) charge a high ticket price for the movie.

The following result can be directly obtained from Proposition 4.1:

Corollary 4.1. *Committing to variable pricing is more profitable than committing to uniform pricing whenever it has a positive effect.*

4.3.4 Moviegoer surplus

Consider now the effects of the two pricing commitments on moviegoer surplus. From (4.2) and (4.3), aggregate moviegoer surplus under a uniform pricing commitment is given by

$$CS^u = \int_0^{\hat{x}^u} [bs_H - tx - p^u]\, dx + \int_{\hat{x}^u}^1 [bs_H - t(1-x) - p^u]\, dx = \frac{t}{4}. \tag{4.6}$$

Using (4.2) and (4.4) gives aggregate moviegoer surplus under a variable pricing commitment as

$$\begin{aligned} CS^d &= \int_0^{\hat{x}^d} \left[b_1 s_H - tx - p_1^d\right] dx + \int_{\hat{x}^d}^1 \left[b_2 s_H - t(1-x) - p_2^d\right] dx \\ &= \frac{\left[t - \frac{1}{2}(b_2 - b_1)s_H\right]^2}{8t} + \frac{\left[t + \frac{1}{2}(b_2 - b_1)s_H\right]^2}{8t} \\ &= \frac{t}{4} + \frac{(b_2 - b_1)^2 s_H^2}{16t}. \end{aligned} \tag{4.7}$$

Subtracting (4.6) from (4.7) yields

$$CS^d - CS^u = \frac{(b_2 - b_1)^2 s_H^2}{16t} > 0.$$

Hence, we can formulate the following result:

Proposition 4.2. *Aggregate moviegoer surplus is higher under variable pricing than under uniform pricing, regardless of the effect of committing to variable pricing.*

The proposition says that a variable pricing commitment is desirable from the viewpoint of moviegoers. This is because committing to variable pricing partially allows moviegoers to acquire information about which movie will be better.

4.3.5 Social welfare

Except that a commitment to variable pricing has a positive effect, a uniform pricing commitment can generate a distributional conflict between the exhibitor and moviegoers. We therefore identify the conditions under which the exhibitor's profit associated with uniform pricing exceeds the loss in moviegoer surplus. To that end, define social welfare as the sum of aggregate moviegoer surplus and the exhibitor's profit.

Using (4.3) and (4.6), social welfare under a uniform pricing commitment is given by

$$W^u = bs_H - \frac{t}{4}. \tag{4.8}$$

(4.4) and (4.7) give social welfare under a variable pricing commitment as

$$W^d = \frac{(b_1 + b_2)s_H}{2} + \frac{3(b_2 - b_1)^2 s_H^2}{16t} - \frac{t}{4}. \tag{4.9}$$

Subtracting (4.9) from (4.8), we can draw the following result:

Proposition 4.3. *Suppose a commitment to variable pricing has a negative effect. Then committing to uniform pricing achieves higher social welfare if moviegoer's beliefs about movie quality satisfy*

$$\frac{2b - (b_1 + b_2)}{(b_2 - b_1)^2} > \frac{3s_H}{8t}.$$

Region B (excluding the boundaries) in Figure 4.1 presents the range of moviegoer's beliefs in which a uniform pricing commitment improves social welfare. Notice that the increase in social welfare by uniform pricing comes from the increase in the exhibitor's profit at the expense of moviegoer surplus.

From Proposition 4.3, the following result is also obtained:

Corollary 4.2. *Social welfare is higher under variable pricing compared with uniform pricing whenever a commitment to variable pricing has a positive effect.*

4.4 Extensions

In this section we extend the model by relaxing the assumptions: (i) movie 1 and movie 2 are located at the endpoints (0 and 1) of the unit interval, and (ii) moviegoers are uniformly distributed on the interval $[0, 1]$. In other words, we examine the effects of the choice of movie location (genre) and an arbitrary distribution of moviegoers over the interval on the profitability of a uniform pricing commitment.

4.4.1 Location choice

We first allow the exhibitor to choose the location of movies before setting movie ticket prices at stage 2.[7] The next lemma gives the exhibitor's optimal choice of movie location:

Lemma 4.1. *Let $\ell^j = (x_1^j, x_2^j)$ be the optimal location of movies 1 and 2 when committing to the pricing scheme $j \in \{u, d\}$, where x_i^j denotes a point at which movie i is located and $0 \leq x_1^j < x_2^j \leq 1$. Then, we have*

$$\ell^u = (x_1^u, x_2^u) = \left(\frac{1}{4}, \frac{3}{4}\right)$$
$$\ell^d = (x_1^d, x_2^d) = \left(\frac{1}{4} - \frac{(b_2 - b_1)s_H}{8t}, \frac{3}{4} - \frac{(b_2 - b_1)s_H}{8t}\right).$$

Proof. See Appendix. □

Lemma 4.1 implies that, when committing to the pricing scheme j, the original location of movies $(0, 1)$ is dominated by ℓ^j in the sense that $\pi_{\ell^j}^j > \pi^j$, where $\pi_{\ell^j}^j$ (resp. π^j) denotes the exhibitor's profit under a commitment to the pricing scheme j with the optimal movie location (resp. original movie location).

We can then examine the profitability and welfare consequences of each pricing commitment with its optimal movie location. The following proposition, in line with the previous results, summarizes the results on the exhibitor's pricing strategy when the movie location is endogenously determined:

Proposition 4.4. *Suppose that an exhibitor chooses the location (genre) of movies prior to setting movie ticket prices. Suppose also that a commitment to variable pricing has a negative effect. (i) Committing to uniform pricing then emerges as an optimal strategy if $\frac{2b-(b_1+b_2)}{(b_2-b_1)^2} > \frac{3s_H}{8t}$. (ii) It also achieves higher social welfare if $\frac{2b-(b_1+b_2)}{(b_2-b_1)^2} > \frac{7s_H}{16t}$. (iii) Aggregate moviegoer surplus is higher under variable pricing than under uniform pricing, regardless of the effect of committing to variable pricing.*

Proof. See Appendix. □

[7] This would apply, for example, to a movie industry that is vertically integrated and where producers (who choose the genres of movies to be produced) are also exhibitors.

4.4.2 Arbitrary distribution

Next, suppose that moviegoers are distributed on the interval $[0, 1]$ according to an arbitrary distribution function F with full support and density f. Here we restrict ourselves to distributions whose median is $\frac{1}{2}$ for simplicity. Assuming that the density function $f(x)$ is continuous and log-concave, we obtain the following results:

Lemma 4.2. *Suppose that an exhibitor commits to variable pricing. The indifferent moviegoer is characterized by the solution to the equation*

$$2\hat{x}^d - 1 = \frac{(b_1 - b_2)s_H}{t} + \frac{1 - 2F(\hat{x}^d)}{f(\hat{x}^d)}. \tag{4.10}$$

The unique equilibrium prices are then given by

$$p_1^d = b_1 s_H - t\hat{x}^d \ \text{ and } \ p_2^d = b_2 s_H - t(1 - \hat{x}^d). \tag{4.11}$$

The corresponding outcomes when committing to uniform pricing ($\hat{x}^u$ and p^u) can be also obtained by replacing b_1 and b_2 in (4.10) and (4.11) with b.

Proof. See Appendix. $\square$

Since $b_1 < b_2$, $\frac{1-2F(x)}{f(x)}$ is monotonically decreasing, and $\frac{1-2F(x)}{f(x)} = 0$ at $x = \frac{1}{2}$, we know that $\hat{x}^d < \frac{1}{2}$, and thus $F(\hat{x}^d) < F(\frac{1}{2}) = \frac{1}{2}$. Lemma 4.2 yields (4.3) and (4.4) when F is a uniform distribution on $[0, 1]$.

We can then state the following proposition:

Proposition 4.5. *Suppose a commitment to variable pricing has a negative effect. Then committing to uniform pricing is more profitable if*

$$F(\hat{x}^d) > \frac{1 - \sqrt{\mathcal{M}}}{2}, \tag{4.12}$$

where $\mathcal{M} = \frac{(2b - b_1 - b_2)s_H f(\hat{x}^d)}{t} > 0$ and $\hat{x}^d$ is the indifferent moviegoer under variable pricing.

Proof. See Appendix. □

Proposition 4.5 tells us that if, despite movie 1 being believed to be of lower quality than movie 2 due to a variable pricing commitment, the number of moviegoers seeing movie 1 ($F(\hat{x}^d)$) is greater than $\frac{1-\sqrt{\mathcal{M}}}{2}$ (and less than $\frac{1}{2}$), committing to uniform pricing makes the exhibitor better off. However, committing to variable pricing is more desirable for the exhibitor whenever it has a positive effect. Note that this result yields (4.5) when F is a uniform distribution on $[0, 1]$.

4.5 Conclusion

This work provides a possible explanation of the fact that one price is charged for all movies (regardless of their quality) in the motion-picture industry. Considering moviegoer's beliefs depending on the pricing schemes to which the exhibitor commits, it shows that there exists a range of moviegoer's beliefs in which committing to uniform pricing is profitable and improves social welfare. This range can be characterized by a low belief that a movie for which the exhibitor commits to charge a high price is of high quality (equivalently, a relatively high demand for a movie believed to be of lower quality), which reflects that moviegoers remain uncertain as to the quality of movies despite price differentials. A commitment to variable pricing is, however, more desirable in terms of the exhibitor's profit and social welfare insofar as it has a positive effect on moviegoer's beliefs. It is also more conducive to moviegoers. Finally, the profitability of a uniform pricing commitment holds even when introducing the exhibitor's choice of movie location (genre) and an arbitrary distribution of moviegoers.

4.6 Appendix

Proof of Lemma 4.1

Consider first the case of a variable pricing commitment. The optimal locations of the movies are determined such that

$$x_1^d = \frac{\hat{x}^d}{2} \quad \text{and} \quad x_2^d = \frac{1+\hat{x}^d}{2}, \tag{4.13}$$

which results in $(x_1^d, x_2^d) = \left(\frac{1}{4} - \frac{(b_2-b_1)s_H}{8t}, \frac{3}{4} - \frac{(b_2-b_1)s_H}{8t}\right)$. Then the optimal prices are

$$\tilde{p}_1^d = \frac{7b_1 s_H + b_2 s_H}{8} - \frac{t}{4} \quad \text{and} \quad \tilde{p}_2^d = \frac{b_1 s_H + 7b_2 s_H}{8} - \frac{t}{4}. \tag{4.14}$$

By setting movie 1's ticket price at $\tilde{p}_1^d$, the exhibitor can extract all the surplus of moviegoers located at 0 and $\hat{x}^d$. Similarly, the exhibitor can extract all the surplus of moviegoers located at $\hat{x}^d$ and 1 by charging $\tilde{p}_2^d$ for movie 2.

To show $x_1^d = \frac{\hat{x}^d}{2}$, suppose first that $x_1^d < \frac{\hat{x}^d}{2}$. Then it will not gain any new moviegoers on $[0, \frac{\hat{x}^d}{2}]$ but will lose some of those on $[\frac{\hat{x}^d}{2}, \hat{x}^d]$. In other words, if the exhibitor chooses movie 1 away from x_1^d, the only way it can continue to serve the entire market is by cutting movie 1's ticket price below $\tilde{p}_1^d$, which leads to lower profit. The same logic can be applied when $x_1^d > \frac{\hat{x}^d}{2}$. Thus $x_1^d = \frac{\hat{x}^d}{2}$. The proof of $x_2^d = \frac{1+\hat{x}^d}{2}$ is omitted since it is similar to that of $x_1^d = \frac{\hat{x}^d}{2}$.

In the case of a uniform pricing commitment, we can obtain

$$\begin{aligned}(x_1^u, x_2^u) &= \left(\frac{1}{4}, \frac{3}{4}\right) \\ \tilde{p}_1^u = \tilde{p}_2^u = \tilde{p}^u &= bs_H - \frac{t}{4},\end{aligned}$$

by replacing b_1 and b_2 in (4.13) and (4.14) with b. □

Proof of Proposition 4.4

By using the results of Lemma 4.1, we can easily calculate the exhibitor's profit and moviegoer surplus when the movie location is endogenously determined (see Tables 4.1 and 4.2 below). Formally,

$$\begin{aligned}\pi_{\ell^u}^u &= \tilde{p}^u \hat{x}^u + \tilde{p}^u(1 - \hat{x}^u) \\ \pi_{\ell^d}^d &= \tilde{p}_1^d \hat{x}^d + \tilde{p}_2^d(1 - \hat{x}^d),\end{aligned}$$

and

Location	Uniform (u)	Variable (d)
$(0,1)$	$\pi^u = bs_H - \frac{t}{2}$	$\pi^d = \frac{(b_1+b_2)s_H}{2} + \frac{(b_2-b_1)^2 s_H^2}{8t} - \frac{t}{2}$
(x_1^u, x_2^u)	$\pi_{\ell^u}^u = bs_H - \frac{t}{4}$	–
(x_1^d, x_2^d)	–	$\pi_{\ell^d}^d = \frac{(b_1+b_2)s_H}{2} + \frac{3(b_2-b_1)^2 s_H^2}{16t} - \frac{t}{4}$

Table 4.1: Profit with the optimal movie location

Location	Uniform (u)	Variable (d)
$(0,1)$	$CS^u = \frac{t}{4}$	$CS^d = \frac{t}{4} + \frac{(b_2-b_1)^2 s_H^2}{16t}$
(x_1^u, x_2^u)	$CS_{\ell^u}^u = \frac{t}{8}$	–
(x_1^d, x_2^d)	–	$CS_{\ell^d}^d = \frac{t}{8} + \frac{(b_2-b_1)^2 s_H^2}{32t}$

Table 4.2: Moviegoer surplus with the optimal movie location

$$
\begin{aligned}
CS_{\ell^u}^u &= 2 \times \left[\int_{x_1^u}^{\hat{x}^u} (bs_H - t \mid x - x_1^u \mid - \tilde{p}^u)\, dx + \int_{x_2^u}^{1} (bs_H - t \mid x_2^u - x \mid - \tilde{p}^u)\, dx \right] \\
CS_{\ell^d}^d &= 2 \times \left[\int_{x_1^d}^{\hat{x}^d} \left(b_1 s_H - t \mid x - x_1^d \mid - \tilde{p}_1^d\right) dx + \int_{x_2^d}^{1} \left(b_2 s_H - t \mid x_2^d - x \mid - \tilde{p}_2^d\right) dx \right].
\end{aligned}
$$

Then $\pi_{\ell^u}^u > \pi_{\ell^d}^d$ formulates (i). From $W_{\ell^u}^u = \pi_{\ell^u}^u + CS_{\ell^u}^u > W_{\ell^d}^d = \pi_{\ell^d}^d + CS_{\ell^d}^d$, we obtain (ii). Finally, (iii) can be derived from $CS_{\ell^d}^d > CS_{\ell^u}^u$. □

Proof of Lemma 4.2

At the optimal choice (p_1^d, p_2^d), the exhibitor extracts the entire surplus from the indifferent moviegoer $\hat{x}^d$:

$$b_1 s_H - t\hat{x}^d - p_1^d = b_2 s_H - t(1 - \hat{x}^d) - p_2^d = 0. \tag{4.15}$$

The problem faced by the exhibitor is given by

$$
\begin{aligned}
\max_{p_1, p_2} \pi = p_1 F &\left(\frac{1}{2} + \frac{(b_1 - b_2)s_H}{2t} + \frac{p_2 - p_1}{2t} \right) \\
&+ p_2 \left(1 - F\left(\frac{1}{2} + \frac{(b_1 - b_2)s_H}{2t} + \frac{p_2 - p_1}{2t} \right) \right).
\end{aligned}
$$

Using (4.15), this problem can be rewritten as

$$\max_{0\leq x\leq 1} \pi = (b_1 s_H - tx)F(x) + (b_2 s_H - t(1-x))(1-F(x)). \tag{4.16}$$

Differentiating the profit with respect to x yields

$$\frac{\partial \pi}{\partial x} = f(x)(b_1 - b_2)s_H + tf(x)\left(1 - 2x + \frac{1-2F(x)}{f(x)}\right).$$

Notice that $\frac{\partial \pi}{\partial x}\big|_{x=0} > 0$ and $\frac{\partial \pi}{\partial x}\big|_{x=1} < 0$. Moreover, $\frac{1-2F(x)}{f(x)}$ is monotonically decreasing since both F and $1-F$ are log-concave (Bagnoli and Bergstrom, 2005). Thus, there exists a unique point $\hat{x}^d$ for which $\frac{\partial \pi}{\partial x} = 0$, which is characterized by (4.10). $\square$

Proof of Proposition 4.5

Under the distribution F, Lemma 4.2 gives $\hat{x}^u = \frac{1}{2}$ and $F(\hat{x}^u) = \frac{1}{2}$. Hence the exhibitor's profit when committing to uniform pricing is given by

$$\pi^u = bs_H - \frac{t}{2}. \tag{4.17}$$

(4.16) yields the exhibitor's profit under a variable pricing commitment:

$$\begin{aligned}\pi^d &= (b_1 s_H - t\hat{x}^d)F(\hat{x}^d) + (b_2 s_H - t(1-\hat{x}^d))(1-F(\hat{x}^d)) \\ &= b_2 s_H + (b_1 - b_2)s_H F(\hat{x}^d) + t(F(\hat{x}^d)(1-2\hat{x}^d) + \hat{x}^d - 1). \end{aligned} \tag{4.18}$$

Subtracting (4.18) from (4.17), we have

$$\begin{aligned}\pi^u - \pi^d &= \frac{(2b - b_1 - b_2)s_H}{2} - t\left(\frac{1-2F(\hat{x}^d)}{f(\hat{x}^d)}\right)\left(\frac{1}{2} - F(\hat{x}^d)\right) \\ &= \frac{(2b - b_1 - b_2)s_H}{2} - \frac{t}{2}\left[\frac{(1-2F(\hat{x}^d))^2}{f(\hat{x}^d)}\right]. \end{aligned}$$

Here $1 - 2F(\hat{x}^d) > 0$ since $F(\hat{x}^d) < \frac{1}{2}$. Finally, from $\pi^u - \pi^d > 0$, we can arrive at (4.12). $\square$

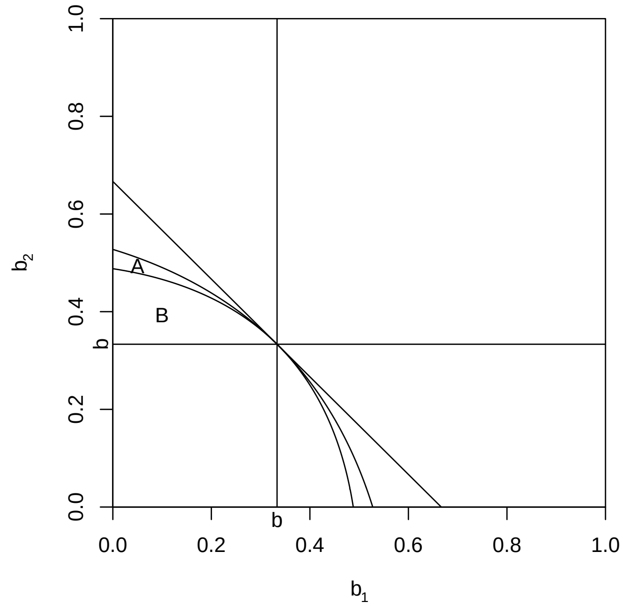

Figure 4.2: Belief ranges for the model predictions ($s_H = 2t$, $b = \frac{1}{3}$)

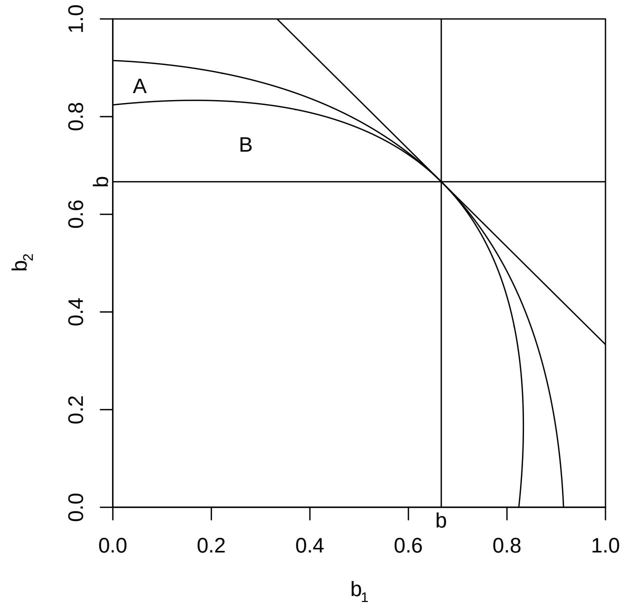

Figure 4.3: Belief ranges for the model predictions ($s_H = 2t$, $b = \frac{2}{3}$)

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