Chapter 1 -- Introduction

1.1 Aft pressure bulkhead

On a large transport aircraft, the pressurized cabin is closed at the aft end by a dome-shaped pressure bulkhead as is shown in Fig. 1.1. In general, there is an abrupt change in curvature at the joint where the fuselage and dome meet which, under the internal pressure load, gives rise to large local stresses. The geometric discontinuity in curvature at the joint causes a bending boundary layer in each shell; this response is typical whenever domes are used as pressure vessel end closures.

Fig. 1.1 Aft Pressure Bulkhead on a Large Transport Airplane

In the special case of an unstiffened circular cylindrical shell closed by an unstiffened hemispherical dome, both of which are made of the same metallic material, it is possible to eliminate the bending boundary layer at the joint (Williams, 1960). The bending boundary layer is eliminated by matching the radial displacements at the joint from shell membrane theory, which leads to the relationship between the thickness of the spherical shell, t_s, and the thickness of the cylindrical shell, t_c, given by

$$\frac{t_s}{t_c} = \frac{1-\nu}{2-\nu}$$

where ν is Poisson's ratio of the material. However, transport aircraft are semi-monocoque structures, with internal stiffening of the pressure cabin necessary to carry service loads other than internal pressure. The presence of internal stiffeners leads to the phenomenon of skin "pillowing" between the stiffeners under the pressure load, which causes additional bending boundary layers in the shell wall adjacent to the stiffeners. The two basic design considerations for joining the dome to the rear fuselage are (Niu, 1989)

- Owing to the comparatively heavy membrane force involved, it is desirable to avoid radial offset between the shell and dome skins.

- There must not, in the neighborhood of the joint, be any reduction in the longitudinal bending stiffness of the fuselage wall, upon the maintenance of which the elastic stability of the wall depends.

This second consideration implies that the joint between the aft section fuselage and the dome is part of the dome design.

1.2 Some analysis issues and the objective of the work

In the interest of efficient design, the analysis of the dome should include the effects of geometric nonlinearity. The presence of geometric discontinuities and meridians with variable curvature (e.g., ellipsoidal and torispherical domes) in internally pressurized shells can result in a state of meridional tension combined with localized hoop compression. Design of domes must therefore consider the possibility of buckling. It is known that shell behavior is sensitive to prebuckling geometric nonlinearity, the effect of which is to increase the buckling load for the material response in the linear elastic range. For example, Flores and Godoy (1990) have demonstrated for a cylinder capped by an ellipsoidal dome that a linear analysis predicts buckling at a much lower pressure than is found from a geometrically nonlinear analysis. Adachi and Benicek (1964) showed experimentally that estimates of buckling pressures of internally pressurized domes obtained by nonlinear analysis are superior to those obtained by linear analysis. Furthermore, Flores and Godoy (1990) also show that the linear analysis predicts maximum limiting values for all stresses, in some cases overestimating stresses by fifty percent. The conservative nature of the linear analysis thus leads to a final design which is heavier than it needs to be. A geometrically nonlinear analysis is therefore recommended for weight-critical applications, such as aircraft structures.

In addition to the geometrically nonlinear analysis, a complete analysis of composite shells should allow for accurate calculation of gradients of the stress resultants within the shell. This requirement arises due to the influence of these gradients on interlaminar stresses, as pointed out by Foster and Johnson (1991).

In the analysis of circumferentially closed shells, it is common to assume a Fourier trigonometric series expansion in the circumferential coordinate for all dependent variables. This assumption makes sense because the response must clearly be periodic, with period of at most 2π radians. The simplest such assumption is for the case of axially symmetric response, which may occur if the geometry, material properties and loading are axially symmetric. It may be seen by looking at Fig. 1.1 that the dome does not usually possess axial symmetry of geometry, owing to the presence of meridional stiffeners. We note however that the figure is of an aluminum dome, which requires meridional stiffening as a result of construction techniques: the dome is formed from a set of triangular gores, that are bonded together at the edges. No such joining is necessary when the dome is filament wound from advanced composite material or constructed using tow placement techniques -- in those cases the meridional stiffeners might not be required. For this work, we will consider domes with ring stiffeners, only. This simplified configuration does allow for axisymmetry of response. To this axially symmetric response, higher harmonics may be added to get an overall response; the solution for the axisymmetric case is thus a necessary prerequisite for a more general analysis.

The **objective** of the current work is to develop an analysis of the axially symmetric response of laminated composite material pressure domes which captures the gradients of the stress resultants near geometric and material discontinuities. Both linear and geometrically nonlinear axially symmetric equilibrium states are considered. Additionally, the analysis is to include anisotropic wall construction, a general, single-valued meridional curve, ring stiffeners, and elastic boundary conditions at the joints between the dome and cylindrical shell and the dome and the cover plate that closes the apex end of the dome. The mathematical model is depicted in Fig. 1.2.

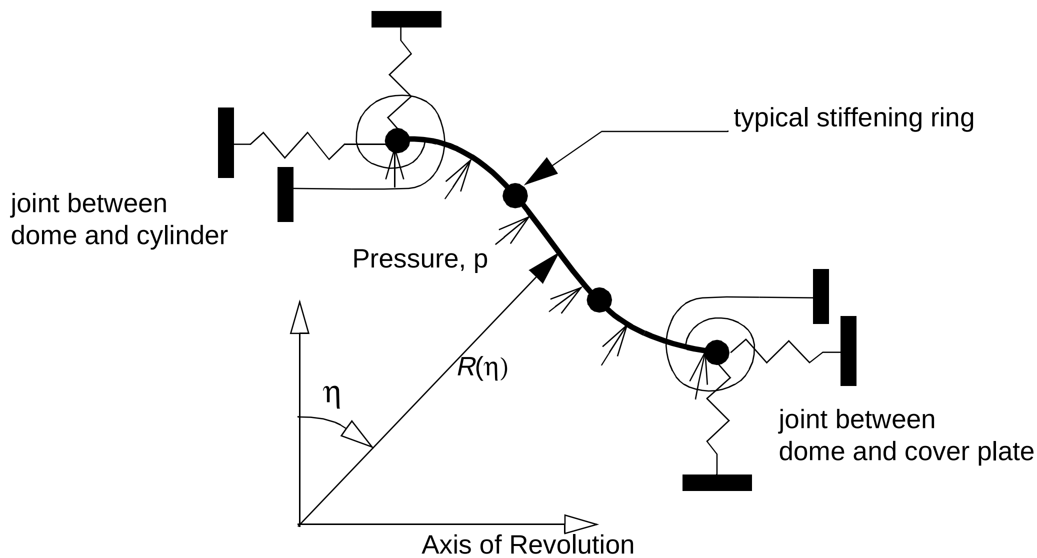

Fig. 1.2 Mathematical Idealization of a Dome

Of course, many shell problems have already been solved by use of the finite element method. As a practical matter, though, the gradients of the stress resultants are difficult to calculate accurately by the finite element method. The displacement-based finite element method uses a “weak” formulation, wherein interpolation functions for displacements are chosen to be of the lowest possible order allowing satisfaction of kinematic admissibility. These chosen interpolation functions are often insufficient for calculation of the gradients of the high-order resultants. Specifically, the gradient of the transverse shear stress resultant is dependent upon the fourth derivative of the normal displacement, while interpolation functions are normally chosen to have continuous first derivatives, at most. On the other hand, it is possible to use a finite element implementation based on the method of weighted residuals allowing for accurate calculation of the gradients of the stress resultants, but it is difficult to choose appropriate interpolation functions. It is thus an additional objective of this work to provide an alternative to the finite element method for analysis of these domes.

1.3 Method of approach

The steps in this analysis of the composite pressure dome are as follows:

- *Formulate the state vector equations for the linear axisymmetric response*. The state vector includes only generalized displacements and generalized forces that are contained in the boundary conditions on the meridional faces. The state vector equations are a set of first order, ordinary differential equations with variable coefficients, and along with the boundary conditions and transition conditions at the ring stiffeners, constitute a two-point boundary value problem. A set of linear algebraic equations are obtained as a by-product of the derivation of the linear state vector equation, which allow for calculation of the generalized forces on the circumferential faces. The equations are derived by use of the mixed variational principle of Reissner (1950).

- *Implement the stabilized marching technique* of the multiple shooting method to effect a numerical solution of the state vector equations.

- *Formulate the state vector equations for the geometrically nonlinear axisymmetric response*. This form of the state vector equations is obtained by direct manipulation of the field equations for the static response of shells of revolution.

- *Implement Newton's method within the stabilized marching technique* to solve the nonlinear equilibrium equations. Newton's method is directly applied to the nonlinear state vector equation, resulting in a set of equations for the "corrections" in the linear state vector form, for each iteration step.

Some of the technical literature to support the method of approach is presented in the following section.

1.4 Supporting literature

1.4.1 Elasticity solutions and shell theory solutions

Analysis of the response of structural shells may be performed either by application of the theory of elasticity (mathematically three-dimensional, or 3D) or by the use of a specialized shell theory (mathematically two-dimensional, or 2D). The elasticity approach has been utilized in, for example, the following three references. Roy and Tsai (1988) have analyzed the linear response of thick-walled composite cylinders under internal pressure, with emphasis on the degradation of the material properties under repeated loading. They considered the shell to be in a state of generalized plane strain and derived a set of equations which were analytically solved, for the case of axisymmetrically loaded, balanced, angle-ply laminates. Yuan (1992) has also formulated the problem of linear response of composite laminate cylinders under internal pressure, but without the restriction to plane strain. In the solution, he uses Lekhnitskii's stress functions, and examines the effects of radius-to-thickness ratio and winding angle for a family of $[\pm\alpha]_S$ laminates. Ren (1995) has formulated the same problem, effecting solution by assuming Fourier series expressions for the displacement quantities. His paper goes on to compare the accuracy of the 3D elasticity solution to the solution of the classical shell theory, finding good agreement for thin shells. The same conclusion has been reached by Bhaskar and Varadan (1993a, 1993b). In Bhaskar and Varadan (1993a), the subject is cylindrical shell panels under transverse load, and in Bhaskar and Varadan (1993b), an exact solution is given for circular cylindrical shells under axisymmetric load. In each case, the cylinders are simply supported angle-ply laminates. All of the references cited above are similar in these regards: they are linear analyses for right circular cylindrical shells and circular cylindrical panels, and are valid away from the ends of the shell or panel only. Unfortunately, solutions of the equations of the 3D elasticity theory for doubly-curved shells, for nonlinear response or non-axisymmetric loading may be quite computationally expensive.

As an alternative to the 3D theory of elasticity, it is common to solve shell problems using specialized shell theories, which incorporate *a priori* the fact that the thickness dimension is much smaller than the other two dimensions of the body. A number of excellent books have been written on the subject of shell theory, among them the texts by Flügge (1973), Librescu (1975), Novozhilov (1959), and Mushtari and Galimov (1961). In addition to these reference texts, the text by Vasiliev (1993), while not specifically dedicated to shells, provides very good coverage of composite structures, including shells. Some comparisons of the results of various shell theories to the conclusions of exact theories have been given by Chandrashekhara, et. al. Chandrashekhara and Pavan Kumar (1995) state that the classical shell theories of Flügge, Love and Sanders are equally good for long, simply supported, cross-ply, circular cylindrical shells under static pressure loading, that the theory of Donnell is generally inaccurate for these same shells, and that inclusion of first-order transverse shear deformation effects generally improves solution accuracy. Chadrashekhara and Nanjunda Rao (1996) analyzed simply supported, orthotropic, circular cylindrical shell panels under radial patch loading. They conclude that for this case, all shell theories are inferior to the 3D elasticity approach. Nevertheless, they state that the shell theory is adequate, so long as the shells are thin and not too shallow.

1.4.2 Shells of revolution

Logan and Hourani (1983), Tutuncu and Winckler (1992) and Gramoll (1993) have reported on the membrane analysis of composite shells of revolution, with Tutuncu and Winckler and Gramoll independently reporting the inadequacy of the membrane theory for most cases.

Kalnins (1964) has utilized the classical (Love-Kirchhoff) theory to solve for the linear bending response of isotropic shells of revolution. Paliwal, Gupta and Jain (1992) used a semi-membrane analysis for bending of an ellipsoidal, orthotropic shell on an elastic foundation. All of these latest references used the classical linear theory.

A nonlinear solution to the problem of axisymmetric bending of isotropic shells has been given by Thurston (1961) for shallow, spherical shells. Nonlinear bending analyses for composite laminate shells may be found in Librescu (1975), Vasiliev (1993), Alwar and Narasimhan (1993), Barbero, Reddy and Teply (1990), and Rothert and Di (1994). Alwar and Narasimhan (1993) incorporates the first-order transverse shear deformation theory and solves the problem using the Chebyshev-Galerkin spectral method for annular, spherical, cross-ply domes. Barbero, Reddy and Teply (1990) and Rothert and Di (1994) incorporate higher-order theories, and solve by the finite element method, for cylinders and for a general shell of revolution, respectively. The utility of the shell theory approach lies in the ability to handle more complicated geometries and to more easily allow for geometrically nonlinear analysis, compared to the 3D theory of elasticity.

1.4.3 Variational formulation and the state vector equations

The references of the previous two paragraphs share a common point: they are based upon the solution of approximate equations of equilibrium of the shell, found by variation of the total potential energy of the shell structure with respect to displacements, only. It was pointed out in Reissner (1950, 1953) that use of an approximate set of kinematically admissible displacements in the principle of minimum potential energy yields a variational equation equivalent to differential equations of equilibrium. Similarly, use of an assumed stress field which satisfies equilibrium (statically admissible) in the principle of minimum complementary energy yields a variational equation equivalent to the stress-displacement relations. In this way, equilibrium equations are approximated in the principle of minimum potential energy with the stresses obtained from the exact stress-displacement equations by differentiating the displacements, and the stress-displacement equations are approximated in the minimum complementary energy principle from stresses satisfying the exact equilibrium equations Application of a mixed variational principle yields differential equations of equilibrium and stress-displacement relations with no preferential treatment of one set of equations over the other.

Steele and Kim (1992) have utilized the so-called Hellinger-Reissner mixed variational principle to derive the governing ordinary differential equations (ODE's) for linear response of axisymmetric shells in first-order state vector form. Their analysis uses the symmetric stress resultants of Sanders' classical shell theory (Sanders, 1959). They give results for isotropic, annular shells of revolution.

While Steele and Kim published a somewhat new method for deriving the governing ODE's for shells in state vector form, the actual expression of the ODE's in state vector form is not new. Kalnins (1964) obtained the equations in state vector form, as did Cohen (1974). Kalnins arrived at the state vector form of the equations by manipulation of the field equations for shells; Cohen does not show the derivation, but seems to simply take the equations of Kalnins. There are several advantages to be had by use of the mixed variational principle for the derivation of the state vector equation: the manipulations are somewhat simplified compared to beginning directly with the field equations, the method yields a state vector which contains only dependent variables which may be defined on meridional faces, and the method also yields a set of appropriate boundary conditions

1.4.4 Numerical solutions to the state vector equations

Kalnins (1964) solved the state vector equation by a method which he called the multisegment method of integration, now known as multiple shooting; Cohen (1964) solved the equations using a technique termed the field method. Each of these approaches has its relative merits. The field method is numerically stable, and seems to run faster than the shooting codes because the shooting technique requires segmentation of the solution domain in order to circumvent the problems of numerical instability. On the other hand, the shooting technique is relatively easy to program, can easily handle all boundary conditions, and is easily adapted to nonlinear problems, whereas the field method requires special handling for kinematic boundary conditions. For the purpose of this work, the multiple shooting technique has been selected.

A number of different implementations of the multiple shooting technique are available; Kalnins (1964), Keller (1968), Kant and Ramesh (1981), Ascher, Mattheij and Russell (1988), and Stoer and Bulirsch (1991) have all given different implementations for linear two-point BVP's. The texts of Keller, Ascher, et. al., and Stoer and Bulirsch go on to also describe the technique for nonlinear BVP's, each giving essentially the same technique. For the linear case, the stabilized marching technique of Ascher, et al. has the following advantages over the other possible implementations: it provides a rational means for selecting segment initial conditions in order to control the growth of unstable solution modes and improves the conditioning of the equations which must be solved for the superposition constants. Furthermore, the stabilized marching technique allows for reduced superposition in the case where the boundary conditions are separated. This last feature leads to a reduction in the number of integrations required, and thus improves the efficiency of the solution.

1.4.5 Buckling under internal pressure

It is known that doubly-curved shells of revolution can exhibit localized hoop compressive stresses when subjected to internal pressure loading, in the region of geometric or material discontinuities [see, e.g., Flügge (1973), pp. 325-351, Flores and Godoy (1990)]. Analysis for buckling of internally pressurized domes is thus an important topic. For isotropic shells, Bushnell (1985) has published an extensive study of buckling phenomena.

As reported by Bushnell (1985), The possibility of nonaxisymmetric buckling of internally pressurized domes was first noted by Galletly (1959). This predicted result was reinforced by Mescall (1962), who first successfully solved the equations for nonaxisymmetric buckling. Mescall’s results were for the linear elastic response case. Subsequent testing by Adachi and Benicek (1964) showed that the predictions from the analysis were greatly improved by the inclusion of geometric nonlinearities. Thurston and Holston (1966) were the first to account for moderately large axially symmetric prebuckling meridional rotations in the stability analysis. It is upon this last result that many successful buckling analysis codes have been developed, including BOSOR5, developed by Bushnell.

1.5 Organization of this document

The following chapters detail the procedure used for this work, and the results obtained. **Chapter two** contains a brief recitation of some aspects of basic shell theory which must be understood in order to follow the work. The chapter covers the theory of surfaces and provides a derivation of the shell theory to be used herein. **Chapter three** gives the derivation of the *state vector* form of the shell equations, which are to be solved in both linear and nonlinear forms, and the equations to be employed in Newton's method for solution of the nonlinear problem. **Chapter four** details the numerical solution technique to be utilized for numerical integration of the state vector equations (stabilized marching,) and some discussion of other numerical techniques used. **Chapter five** provides verification studies, showing the quality of the solutions obtained by the shooting technique employed for this work. **Chapter six** contains parametric studies of shells of revolution, specifically composite laminate ellipsoidal domes. The final chapter, **Chapter seven**, contains conclusions and recommendations for further work.