SOME STUDIES IN SIMULTANEOUS FAILURE IN EQUIPMENT ITEMS

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(ABSTRACT)

This study can be classified under the subject of equipment item replacement analysis. Simultaneous failure, SF, of the components of an equipment item, EI, is the topic of this thesis. Examination of the possibility of designing components of an EI for SF is one objective of the study. The motive for this objective is the belief that SF designs of EI minimize the total cost of acquiring and operating the EI.

Examination of the strength of materials reveals that design life of components is not sufficiently flexible to realize SF, and design requirements can predetermine design life. This is true for mechanical components such as links, gears, and bearings. Hence it was concluded that SF of the mechanical components of an EI, cannot be easily achieved.

The second objective of the study was to formulate a model for optimizing the design of the components of an EI with life considerations. A mechanical reliability model was first modified by the inclusion of fatigue stress-strength relationships, and the theory of Cumulative Damage. By mathematical manipulation to suit the principles of Lagrange's Method of Undetermined Multipliers, an optimization model has been developed. This model enables system and component design constraints and requirements to be included in the optimization process.

ACKNOWLEDGEMENTS

I would like to express my sincere thanks to Dr. W. J. Fabrycky, major advisor and chairman, for his unlimited patience, understanding and expert advise throughout this effort. I also wish to express my appreciation to Prof. Benjamin S. Blanchard, and Dr. Kostas Triantis for their time and advice. I also wish to thank Dr. Timothy J. Greene for substituting for Dr. Kostas Triantis in his absence.

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1.0 INTRODUCTION

"Have you heard of the wonderful One Hoss Shay,
That was built in such a logical way,
It ran a hundred years to a day,
And then, of a sudden, it -

..
There is always somewhere a weakest spot.

..
A chaise breaks down, but doesn't wear out
'n' the way t' fix, uz I maintain,
Is only jest
'T' make that place uz strong as the rest."
..
..
She was a wonder, and nothing less!

"The Deacon's Masterpiece"
Oliver Wendell Holmes

The One Hoss Shay or One Horse Cart, represents a perfectly designed equipment item. The One Horse Cart was used by the Deacon for a hundred years, at which point in time, all its parts failed. Before the Deacon perfected his One Hoss Shay failures occurred at random in the chassis, the springs, the crossbar, the hub, and other parts of the Horse Cart. In order to overcome these randomly occuring failures, the Deacon 'designed' each of the parts to be of equal strength. This resulted in a Hoss Shay with parts which failed simultaneously at exactly 100 years. Imagine modern equipment items, such as computers or machine tools, designed like the wonderful One Hoss Shay. Such designs can be called perfect designs.

The subject of this study is equipment item replacement analysis. Replacement of an equipment item is necessitated for various reasons - frequent failures in component parts, extensive maintenance requirements, poor performance relative to newer but similar equipment, technological obsolescence, or economic necessity.

The classic repair/replace dilemma is present in any replacement analysis. Such situations are influenced by a need for cost minimization or profit maximization. Cost factors in such analysis are acquisition cost, operations and maintenance costs, repair cost, and cost of technological obsolescence. The repair/replace dilemma is decided by comparing the cost of acquiring and operating a new equipment item, or challenger, with the cost of operating the current item, or defender. A replacement analysis may be made with or without failure in the equipment item.

The problem to be addressed in this thesis is the planned simultaneous failure, SF, of the components of an equipment item, EI. It is believed that planned SF of an EI minimizes the total cost of its acquisition and operation. This belief is the driving motive for this study. The examination of the achievability of SF in EI designs is one objective of this study.

1.1 DEFINITIONS

Before proceeding, some frequently used terms are defined.

COMPONENT: This term describes a part of an equipment item such as a spring, gear , or bearing. Assemblies are comprised of one or more such components.

FAILURE of an equipment item occurs when it is incapable of satisfactory performance, due to one or more component parts reaching their limiting conditions of wear and tear.

SIMULTANEOUS FAILURE refers to the phenomenon of all the components of an equipment item failing at the same point in time.

REPAIR describes the activity of fixing a failure that restores the equipment item to operating status. This activity consists of replacing a failed component with an identical but new part. Only equipment items can be repaired whereas components can only be replaced.

LIFE is the design life "built", or designed, into component parts. Life will also be referred to as "Design Life". A component part has completed its design life when it reaches its limiting conditions of wear.

CONSUMABLES are components or equipment items that are "consumed" with use, and may only be replaced upon completing their design lives. Bearings are examples of consumables, because they are replaced upon reaching their limiting conditions of wear i.e. they are consumed with use. Equipment items with simultaneous failure are also consumables.

A REPAIRABLE EQUIPMENT ITEM, (REI), refers to an equipment item that is only repaired upon failure, because its parts experience non-simultaneous failure. Some examples of REI are automobiles, aircraft, and computers.

A CONSUMABLE EQUIPMENT ITEM, (CEI), refers to an equipment item that is not repaired but replaced upon failure, because the equipment item experiences simultaneous failure of its component parts. Consumable equipment items may also be referred to as consumables.

ANNUAL EQUIVALENT COST is a series of equal annual costs equivalent to a set of non-uniform costs, such as present and future costs.

FIRST COST of an equipment item is its acquisition cost.

1.2 THE EQUIPMENT ITEM

All equipment items mentioned in this study are comprised of assemblies, sub-assemblies, and components as illustrated in Figure 1. Observe that the basic parts of an EI are its components. For an EI to be functional, its components should be capable of satisfactory performance. Components form sub-assemblies, which in turn, form assemblies. All mechanical component parts are assumed to be consumables. In other words they are disposed off or replaced upon completing their design lives instead of being rebuilt.

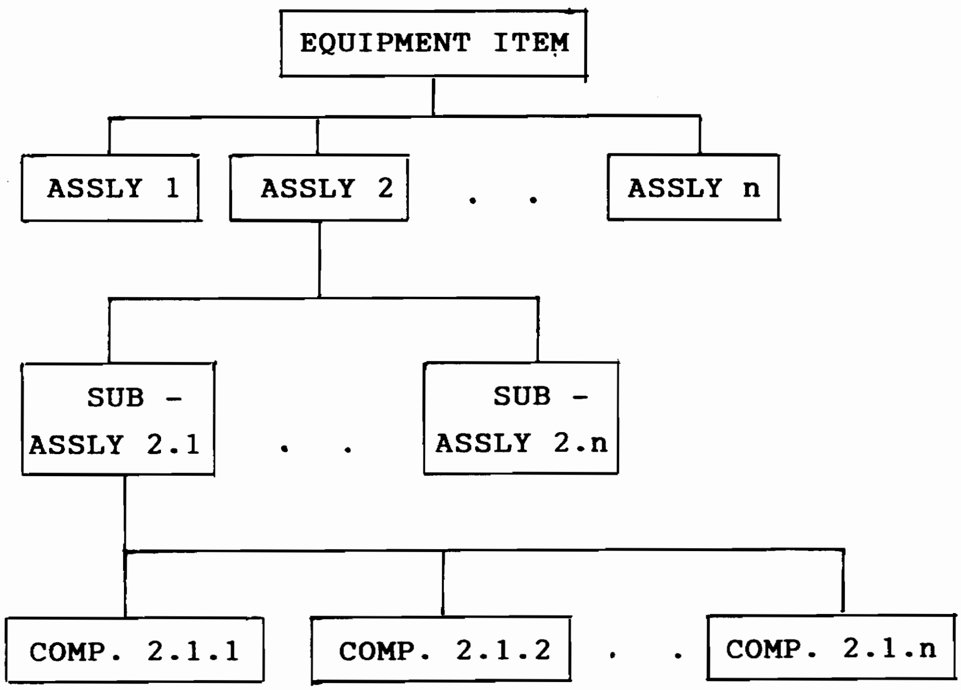


Figure 1. Structure of an equipment item (EI)

Component parts are assumed to fail at the end of their design lives. However in reality, failure can occur at any time for various reasons. For simplicity, steady state conditions are assumed for this study i.e. operating conditions and hence stresses can be accurately predicted.

All EI will be classified into two types - Repairables and Consumables. Repairable equipment items, REIs, are those items that experience component failures at different times. Such failures occur because the component parts have unequal lives. Consumable equipment items, CEIs, are not repaired upon failure, but are only replaced because their component parts experience simultaneous failure. The concept of CEI is largely theoretical as no such EI is known, which experiences simultaneous failure of all its components. It will be assumed that replacing such CEIs is better than repairing all its failed components.

1.3 THESIS OBJECTIVES

The primary objective of this thesis is to examine if simultaneous failure in equipment item designs is achievable. The motive for this objective is the belief that the acquisition and operation of an EI with SF of component parts is economically more favorable than a similar EI which experiences Non-Simultaneous Failure, NSF.

The belief of economically favorable SF designs came from the concept of the perfectly designed One Horse Cart, in the poem, The One Hoss Shay. By avoiding all repair and replacement during the operating life cycle, SF designs appear to be economically favorable.

The second objective of this study is to develop a rationale for a multi-component equipment item design optimization model which allows for (design) life considerations during design.

1.4 LITERATURE REVIEW

Literature on the subject of replacement is abundant. The focus of the literature is economic analysis of repair and replacement, and failure prediction and simulation. Literature on the economics of repair and replacement can be found from as early as 1940.

Just as literature is abundant on the issues of economics of repair and replacement, the topic of failure prediction and simulation has been well researched. Failure prediction of components and equipment items using the mathematics of probability distributions is an area of research that has received much attention. Similarly, the area of design life prediction by considering wear and strength as distributions with the passage of time and use, has been well covered.

However, the area of design life estimation and prediction by a consideration of actual operating stresses and strengths has not received much attention. Models to design components of equipment item (or items) for SF are scarce. One possible reason for the lack of treatment of this subject is the difficulty of gathering data under operating conditions for components and equipment items.

The focus of this study, as stated in the objectives, is to examine for design life flexibility to achieve SF. Another objective is to develop the rationale for EI design with component design life consideration. One source for design life information with stressand strength consideration is design engineering texts and handbook (Shigley and Mischke, and Collins). This source has been used in the following chapters to study the flexibility of component design lives and to develop the rationale for the optimization of equipment item design with life considerations.

The economics of repair and replacement, and failure prediction models, it was felt by the author, does not relate to the focus of the study. Hence this literature has not been reviewed here.

1.5 THESIS OUTLINE

In Chapter 2, the issues in economic replacement analyses are illustrated by four numerical examples. Two EIs, a chain and a multi-component EI, are analyzed to select the most favorable alternative from a set of SF and NSF alternatives, for each EI. The conditions for a favorable alternative are then generalized and expressed in algebraic form.

Component life is determined by the moment of failure. Failure is the result of operating stresses. Stress resistance is designed into the component as its strength. Thus, by examining the strength to resist failure causing stresses, the life capacity of a component may be examined.

Chapter 3 examines the strength of materials from the perspective of (design) life. The examination of strength of materials revealed that a designer's control over (design) life is limited, because of the nature of material response to stresses.

Having examined the strength of materials, the stress-strength relationship, and the theory of cumulative damage were introduced to the mechanical reliability model in Chapter 4. The mechanical reliability model in Chapter 4 is the basis for the development of the multicomponent equipment item optimization model.

In Chapter 5 the principles of cumulative damage, strength, and reliability are formulated in accordance with Lagrange's Method of Undetermined Multipliers to form a mathematical framework for examining EI design. The framework enables all components of an EI to be examined simultaneously, with all design constraints that may exist. Chapter 6 presents the summary, conclusions, and recommendations for further research.

The thesis follows the logic illustrated in Figure 2.

TOTAL COST OF ACQUISITION AND OPERATION OF EIs MAY BE MINIMIZED WITH A SF DESIGN

IS SIMULTANEOUS FAILURE ACHIEVABLE?

LIFE IS DETERMINED BY THE MOMENT OF FAILURE.
FAILURE IS DETERMINED BY THE STRESS MAGNITUDE.
STRESS RESISTANCE DEPENDS UPON THE STRENGTH OF MATERIALS.
THUS, STRENGTH DETERMINES LIFE.

LIFE CAPACITY IS EXAMINED THROUGH THE STRENGTH OF MATERIALS

EXAMINATION REVEALS THAT THE CONTROL IS EITHER INFLUENCED BY DESIGN CONSTRAINTS, OR IS A DESIGN INDEPENDENT PARAMETER

CONCLUSION: SIMULTANEOUS FAILURE WILL BE DIFFICULT TO ACHIEVE

TO DEVELOP A MULTI-COMPONENT EI DESIGN MODEL, WITH LIFE CONSIDERATIONS, RELIABILITY ESTIMATION IS NECESSARY

USING FATIGUE STRESS AND STRENGTH RELATIONSHIPS AND CUMULATIVE DAMAGE ESTIMATION METHODS, THE BASIS FOR DESIGN LIFE CONSIDERATION IN DESIGN IS ESTABLISHED

ADDING LAGRANGE'S METHOD OF UNDETERMINED MULTIPLIERS TO THE ABOVE, A MULTI-COMPONENT EI DESIGN MODEL IS DEVELOPED

Figure 2. Logic flowchart of thesis.

2.0 SIMULTANEOUS FAILURE IN EQUIPMENT ITEMS

In this chapter some numerical examples of replacement situations with SF and NSF are illustrated. These replacement situations are also generalized algebraically for determining the necessary conditions for the favorability of a SF situation.

The numerical analyses consider only the costs of acquisition and operation of the EIs. At this stage, a designers objective is to examine the various alternatives to identify the alternative with the lowest cost of acquisition and operation.

The role of factors such as repair cost, time value of money, time horizon, and acquisition cost in such analyses are also illustrated through the examples. SF is favored in only two of the four different situations analyzed. This is an indication of the unpredictable nature of such economic analyses. Favorability of a particular alternative must not be assumed in such replacement situations.

Although the results of the numerical analyses do not prove the favorability of a SF alternative, the belief that a SF alternative can minimize the total cost of acquisition and operation of an EI cannot be rejected. However, only after a careful analysis should an alternative be chosen as the most favorable alternative.

In the algebraic generalization of replacement situations, the conditions necessary for the favorability of a particular alternative are identified. These conditions are applicable to both SF and NSF alternatives in any repair/replace situation.

2.1 ECONOMIC ANALYSIS OF SIMULTANEOUS FAILURE

In this section economic analyses of two simple EIs are performed to highlight some issues in replacement studies and will cover EIs from simple chains to multi-assembly items. The purpose of these analyses is to numerically demonstrate the notion that SF designs can minimize total costs of acquisition and operation. It has also been numerically demonstrated that a SF alternative is not the best alternative in every repair/replace situation.

A chain is probably the simplest EI since all its component parts or links are identical in shape, size, and material properties, and, there are no assemblies and subassemblies.

This is the most significant difference between a chain and most other EIs, and is illustrated in Figures 3 and 4.

Despite these differences, for the purposes of a replacement analysis, a chain and other EIs are similar. The component skeletons of a chain and other EIs are identical. This is observed in Figures 3 and 4. Hence, failure in a link is treated like a failure in a component part of another EI. In other words, the failed part is either repaired or replaced and the cost of repair or replacement is accounted. To demonstrate this similarity, replacement analyses of a chain and an EI are performed here. The analyses includes SF and NSF alternatives for the chain and the EI.

The following assumptions are made for the analyses:

1. Component acquisition cost increases, either linearly or non-linearly, with an increase in design life.
2. The EI acquisition cost is the sum of the cost of its components and the cost to assemble them.
3. The cost of operating the EI is constant for all design alternatives of EI.
4. Upon failure of component parts, either the component(s) or the EI may be replaced.

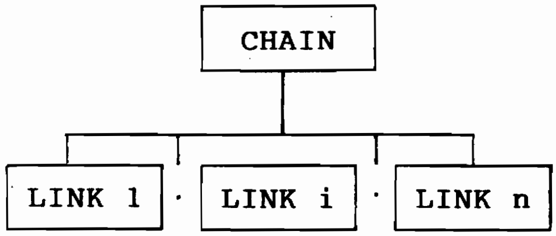


Figure 3. Structure of a chain.

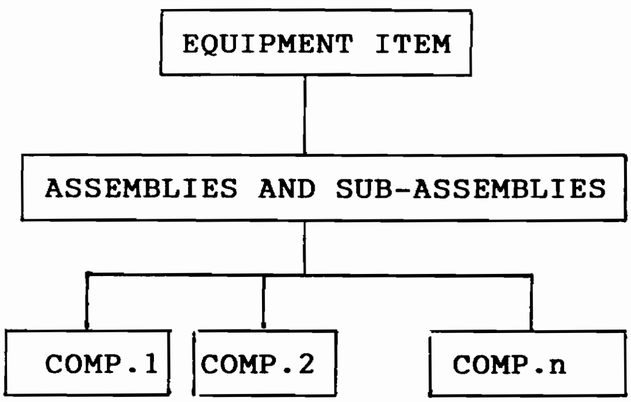


Figure 4. Structure of an equipment item to show its similarity to a chain.

5. Replacing a component is represented by a repair cost.

6. The annual equivalent cost measure is used to evaluate the alternatives (Theusen, G. J. & Fabrycky, W. J, pp 40 & 41).

7. With simultaneous failure and equipment replacement, no facility is required to replace the equipment.

8. No labor cost is generated when replacing EIs in assumption 7.

Validation of assumptions:

Assumption 1 : Increasing cost with increase in design life.

The life of a component may be increased by use of a stronger material, or by an increase in size. Often, stronger materials are more "expensive", which justifies an increase in acquisition cost for an increase in design life.

Assumption 2 : Acquisition cost of REI = Component Cost + Assembly Cost.

The manufacturing cost is only a fraction of the total cost of any finished product. Some of the other costs include : assembly costs and value added costs. Assembly costs included in the analysis are representative of such costs.

Assumption 3 : Cost of operating an equipment item is constant for all design alternatives.

The component parts of EIs of all the alternatives are assumed to have the same performance characteristics except for their design lives. Hence the assumption that all alternatives have constant operating costs.

Assumption 4 : Either components or the EI itself may be replaced upon failure.

This outlines the options available upon failure.

Assumption 5 : Replacing a component causes a Repair Cost.

This may be expected of most EIs.

Assumption 6 : Use of Annual Equivalent Cost for evaluating alternatives.

Other measures such as Present Cost, and Future Cost may also be used to evaluate alternatives. However this was chosen over the others, for ease of comparison.

Assumption 7 : Replacement of equipment items without the use of a facility.

This is a simplifying assumption.

Assumption 8 : No labor cost is associated with replacing an equipment item.

Since it is assumed that failed equipment items are replaced by procuring another, no labor (cost) is assumed to be required to replace it.

The cost breakdown structure is as follows:

AEC = Annual Equivalent Cost
= first cost + cost of operation and repair

First Cost = Component acquisition cost + Assembly cost

Operations Cost = Cost of all hardware and personnel incurred in order to keep components and, hence, the equipment item(s) functioning

Repair Cost = Cost of all hardware and personnel to repair equipment item(s) upon failure

Note that it is assumed that spares and inventory costs, salvage value and disposal costs are negligible and are not included in the analysis. Since the economic analyses have been performed only to demonstrate the unpredictability of the outcome of repair/replacement situations, and highlight some of the issues in these studies, the cost definitions are not very rigid.

2.1.1 CHAIN ANALYSIS

A simple economic analysis is performed here to examine the favorability of SF and NSF alternatives of a chain. A chain is assumed to have 'n' links. Two options exist - replace the chain upon failure of a link (or links), or, repair the failed link (or links). To repair failed links, it is assumed that a repair facility is required. This generates a repair facility cost.

The cost factors assumed are:

Cost of a repair facility = \$150

Cost of a chain = \$2,000

Cost to repair a failed link = Cost of new link + Cost of labor to replace the link = \$75

Two scenarios are considered for the use of the chain : An infinite horizon, and a finite horizon. The situations considered for each scenario are - SF of the links, and NSF of the links. SF of links occurs after 4 years. In the NSF situation, one link is assumed to fail every year.

Scenario 1 - Infinite Horizon

For this scenario, it is assumed that the chain is required for an infinite time period.

Alternative 1(A) - Simultaneous Failure

AEC to replace the chain every 4 years, upon SF is:

$$= \$2,000(A/P,10,\infty) + \$2,000(P/F,10,4)(A/F,10,\infty) + \$2,000(P/F,10,8)(A/P,10,\infty) + \ldots + \$2,000(P/F,10,\infty)(A/P,10,\infty)$$

$$= \$2000(A/P,10,\infty)[\ 1 + (P/F,10,4) + \ldots + (P/F,10,\infty)]$$

$$= \$600$$

Alternative 1(B) - Non-Simultaneous Failure

AEC to repair a failed link every year:

$$= \$2000(A/P,10,\infty) + \$150 + \$75$$

$$= \$425$$

From the above analysis, Alternative B is more favorable than Alternative A because it has a lower AEC to acquire and operate the EI. If more than one link fails every year, the AEC of the NSF alternative will increase and could cause a decision reversal.

Scenario 2 - Finite Horizon

In this scenario the chain is required for only 10 years.

Alternative 2(A) - Simultaneous Failure

The AEC to replace the chain every 4 years is:

AEC = \$2,000(A/P,10,10) + \$2,000(A/P,10,10)(P/F,10,4) +
\$2,000(A/P,10,10)(P/F,10,8)

= \$2,000(A/P,10,10)[1 + (P/F,10,4) + (P/F,10,8)]

= \$430

Alternative 2(B) - Non-Simultaneous Failure

AEC = \$2,000(A/P,10,10) + \$100 + \$75

= \$550

In this scenario, Alternative A is more favorable than Alternative B because of the lower AEC of Alternative A. From the two analyses, it is seen that the favorability of an alternative can depend upon the time value of money, the first cost, the planning horizon, the SF life, and the repair facility cost.

2.1.2 MULTI-COMPONENT EQUIPMENT ITEM ANALYSIS

In the following paragraphs, an economic analysis of an EI is performed to compare simultaneous failure alternatives with non-simultaneous failure alternatives. The EI is assumed to have only two components. The acquisition cost and design life of each of these components is listed in Table 1.

Table 1. Component design life and acquisition cost

LIFE (YEARS)	ACQUISITION COST OF COMPONENT A ($)	ACQUISITION COST OF COMPONENT B ($)
2	100	100
3	200	175
4	300	225
5	400	250

Other numerical assumptions are :

Annual Equivalent Repair Facility Cost = $100

Annual Equivalent Operations Cost = $ 75

Labor cost / hour = $ 50

Consider the REI with component lives of 2 and 5 years respectively. It is assumed that the component with the 2 year life is replaced upon failure. The component with the 5 year life functions till the end of year 5. Upon failure of the 2 year life component, the EI is repaired, This is done at the end of year 2 and year 4. Because of the need for repair, a repair facility is assumed to be required, along with a labor cost to replace the component. In order to evaluate the cost of acquiring and operating this REI all costs are converted to the Annual Equivalent Cost measure. It is assumed that the time value of money is 10%. This is illustrated in Figure 5(a).

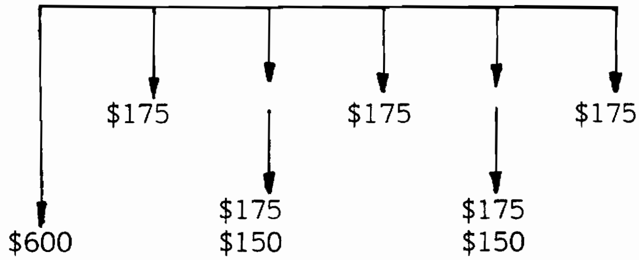
AEC = First cost + Repair Facility Cost + Operations Cost

+ Cost of replacing failed component in years 2 and 4.

= $600(A/P,10,5) + $100 + $75 + $150(A/P,10,5)

(A/P,10,2) + $150(A/P,10,5)(P/F,10,4)

= $498

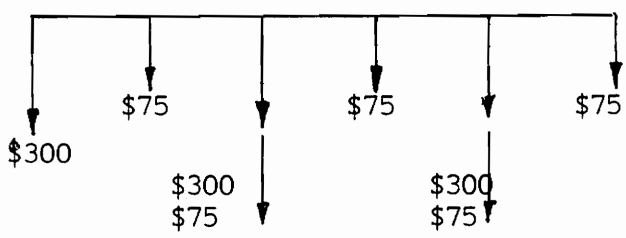


$600 = First Cost of Equipment Item
= Assembly Cost + First Cost of Components
= $100 + ($100 + $400)

$175 = Repair Facility Cost + Operating Cost
= $100 + $75

$150 = First Cost of a Component + Repair Cost
= $100 + $50

Figure 5(a). Cash flows for an Equipment Item with component lives of 2 years and 5 years respectively.



$300 = First Cost of Equipment Item
= Assembly Cost + First Cost of Components
= $100 + ($100 + $100)

$75 = Operating Cost

Figure 5(b). Cash Flows for an Equipment Item with simultaneous failure at 2 years.

Consider a CEI with 2 components, each with lives of 2 years. For this case, no repair facility is assumed to be required because of equipment replacement upon (simultaneous) failure. The cash flows for this alternative are illustrated in Figure 5(b). The cost of the assembly is $100.

AEC = First cost + Cost of new equipment in years 2 and 4

AEC = $300(A/P 10, 5) + $300(A/P 10,5)(P/F 10,2) + $300(A/P 10,5)(P/F 10, 4) + $75

= $273

In order to examine the advantages of an EI with simultaneous failure over one with non-simultaneous failure, various alternatives, each with a different combination of component lives is considered for acquisition and operation. The various alternatives are evaluated using the annual equivalent cost measure. The alternatives with non-simultaneous failure of components are assumed to require a repair facility. For simultaneous failure, the components are assumed to require no repair facility if the equipment is replaced upon failure of both components. If only the components are replaced and not the equipment upon simultaneous failure of the components, a repair facility is required. In Table 2, various alternatives with non-simultaneous failure are compared, using the annual equivalent cost measure.

Table 2. Cost comparison for non-simultaneous failure

COMPONENT LIFE		ANNUAL EQUIVALENT COST	
		COST OF ASSEMBLY	
A	B	$500	$100
(YEARS)		($)	($)
2	5	498	392
3	5	503	398
4	5	532	427
5	2	472	366
5	3	488	382
5	4	528	422

Table 3. Cost comparison for simultaneous failure

COMPONENT LIFE		ANNUAL EQUIVALENT COST	
		COST OF ASSEMBLY	
A	B	$500	$100
(YEARS)			
2	2	538	273
3	3	560	294
4	4	541	363
5	5	391	286

In Table 3, the alternatives with simultaneous failure and equipment replacement upon failure are compared. (Assumption 7 is used for these calculations).

In the above example, it is assumed that the desired life is 5 years for all alternatives. It must be mentioned that many other alternatives exist, if all combinations of component lives are considered. However, if only the alternatives considered above are evaluated, the results in Tables 2 and 3 are indicative of the favorability of simultaneous failure over non-simultaneous failure.

Consider the results in Table 2. Table 2 provides the results of nonsimultaneous failure of components of the REI.

For these calculations it was assumed that components are replaced upon failure, and the REI operated for 5 years. In Table 3, the results of the equipment replacement with SF designs are tabulated. From the results of this evaluation, it is seen that simultaneous failure is favorable for component lives of 5 years when assembly costs are $500. When assembly costs are $100, simultaneous failure is best with component lives of 2 years.

The favorability of simultaneous failure is observed for EI with high ($500) and low ($100) cost of assembly. In both situations, the most favorable situation occurs when there are no repair facility costs. This occurrence is an outcome of all the factors assumed for the analysis. Because of the nature of such economic analysis, it is also difficult to conclude that the absence of a repair facility cost caused these alternatives to be more favorable. Other factors such as the time value of money, the acquisition cost to design life relationship, and the cost of repair labor may have influenced this outcome.

Let us assume that no repair facility is maintained and all repair is performed at an outside facility. Assume that this cost is $200. The AECs for this situation are tabulated in Table 4 below. A comparison of AECs in Table 3 and Table 4 indicates that NSF alternatives are now favored over SF alternatives, for both the $500 assemblies and the $100 assemblies.

Table 4. Cost comparison for non-simultaneous failure

COMPONENT LIFE		ANNUAL EQUIVALENT COST	
A	B	COST OF ASSEMBLY	
(YEARS)		$500	$100
2	5	343	238
2	4	329	224
2	3	367	261
5	2	383	277
5	3	357	252
5	4	373	267

In the chain example, decision reversal occured when the planning horizon changed from infinity to 10 years. For the EI example, a change in repair cost caused a decision reversal. These four examples bring out the need for an examination of each replacement situation for favorability of a SF alternative. It should not be assumed that SF alternatives will be most favorable.

2.2 ALGEBRAIC ANALYSIS OF SIMULTANEOUS FAILURE

In this section, various failure situations are algebraically analysed to determine the conditions necessary for the favorability of simultaneous failure. Some of the simplifying assumptions made in the previous section are carried over to this section.

As in Section 2.1, in this section, it is assumed that no facility is required to replace an equipment item.

In the numerical analyses, it was possible to differentiate equipment items with various component combinations by their first costs. In this section , equipment items with different component lives have different first costs. As before, operating costs are assumed to be constant but are not included in the analysis.

In the numerical analyses, failure in a link and a component part (of the two component equipment item) are treated alike. Either a repair facility is used to replace the failed component, or the equipment item is replaced and no repair facility is required. EI with a few or more components, including chains, may be analyzed using the same approach.

Consider a multi-component EI. Through a step-by-step analysis of different failure situations, the conditions for a favorable alternative are identified.

CASE I: Failure At 'H'.

Assume that it is possible to design an equipment item that lasts, for a very long period of time, say 'H' years, similar to the Deacon's Hoss Shay that lasted a hundred years. Let

Cost of the EI, with failure at 'H' = \$ P

AEC of this EI = $\$(A)_H$

If SF occurs at H, then the EI is a CEI. The cash flows for this situtation are illustrated in Figure 6(a).

CASE II: Non-Simultaneous Failure (NSF) At H/2.

Assume that one or more, but not all, components fail before time 'H', say at time at (H/2). Two options exist: either replace the failed component(s), or replace the equipment item. Let

First Cost of REI = $\$ P_{ttff}$

Cost to repair the REI, including downtime, = \$ R

where ttff = time to first failure

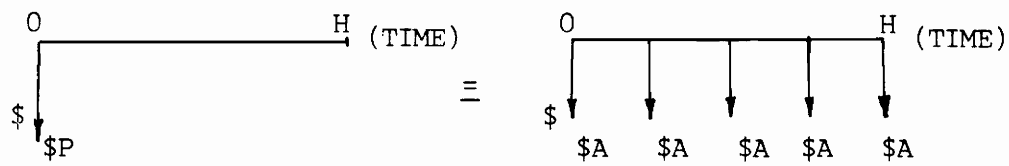


Figure 6(a). Cash flows for Simultaneous Failure at H

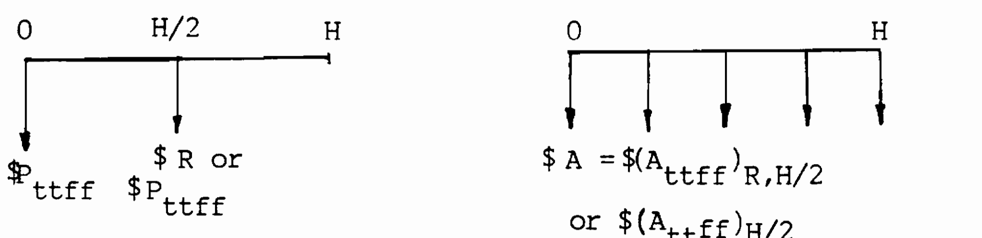


Figure 6(b). Cash flows for Non-simultaneous Failure at H/2

Figure 6(c). Cash flows for Simultaneous Failure at H/2

(A) If the REI is replaced at (H/2):

AEC to use the REI for H years

$$= (P_{ttff} + \$P_{ttff}(P/F,i,H/2))(A/P,i,H)$$

$$= \$(A_{ttff})_{H/2}$$

The cash flows in this situation are illustrated in Figure 6(b). Since failure occurs at (H/2), replacement is made once by an identical REI.

(B) If the REI is repaired at (H/2):

AEC to use the REI for H years

$$= (\$P_{ttff} + \$R(P/F,i,H/2))(A/P,i,H)$$

$$= (A_{ttff})_{R,H/2}$$

The cash flows for this situation are illustrated in Figure 6(b). It is assumed that the REI fails only once at (H/2).

It is desirable to replace the EI only if:

$$\$(A_{ttff})_{H/2} < \$(A_{ttff})_{R,H/2} \qquad (2.1)$$

CASE III: Simultaneous Failure at H/2.

If simultaneous failure occured at time H/2:

Cost of EI with SF at H/2 $= \$P_{ttsf}$

Where ttsf = time to simultaneous failure

AEC to use the CEI for H years

$$= \$P_{ttsf} + \$P_{ttsf}(P/F,i,H/2)(A/P,i,H)$$

$$= \$(A_{ttsf})_{H/2}$$

This situation is illustrated in Figure 6(c).

In cases II and III, the CEI is favored over the REI if:

$$\$(A_{ttsf})_{H/2} < \$(A_{ttff})_{H/2}$$

$$\$(A_{ttsf})_{H/2} < \$(A_{ttff})_{R,H/2} \quad (2.2)$$

In cases I, II, and III, the CEI, with failure at 'H' years, is best if:

$$\$(A)_H < \$(A_{ttsf})_{H/2}$$

$$\$(A)_H < \$(A_{ttff})_{H/2}$$

$$\$(A)_H < \$(A_{ttff})_{R,H/2} \quad (2.3)$$

Continuing in this manner, assume that EI fails at time H/4. The CEI with failure at 'H' is best if:

$$\$(A)_H < \$(A_{ttsf})_{H/4}$$

$$\$(A)_H < \$(A_{ttff})_{H/4}$$

$$\$(A)_H < \$(A_{ttff})_{R,H/4} \quad (2.4)$$

where R = Cost of all repairs, including downtime

Replacement is made by identical units at each failure.

CASE IV: Failure at 't'.

Assuming that failure occurs at a random time 't' < H (Failure can be either simultaneous or non-simultaneous. Although failure is random, it is assumed that replacement is made with identical units).

(A) If Failure Is Simultaneous:

Cost of EI = $ P_{ttsf}

AEC of operating the EI until time H

= $ $P_{ttsf}(1 + \ldots + (P/F, i, t_{H/t}))$

= $ $(A_{ttsf})_{H/t}$

(B) If Failure is Non-Simultaneous:

(1)Repair Case: When the REI is repaired upon failure:

Cost of REI = $ $(P_{ttff})_{H/t}$

Cost of a repair = $ R_i

AEC of using the REI until time t

= [$($P_{ttff}$) + Σ($$R_j$ (P/F,i,t_j))][A/P,i,H]

$$= \$(A_{ttff})_{R,H/t}$$

(2) If the EI is replaced at each failure:

Cost of REI $= \$(P_{ttff})_{H/t}$

AEC of using the REI until time H

$$= \$(P_{ttff})(1 + (P/F,i,t_1) + .. + (P/F,i,t_t)$$

$$= \$(A_{ttff})_{H/t}$$

Consider the situation when

$$\$\ (P)_H \quad < \quad \$\ (P_{ttsf})_{H/t}$$

$$\$\ (P)_H \quad < \quad \$\ (P_{ttff})_{H/t}$$

Then

$$\$\ (A)_H \quad < \quad \$\ (A_{ttsf})_{H/t}$$

$$\$\ (A)_H \quad < \quad \$\ (A_{ttff})_{H/t}$$

$$\$\ (A)_H \quad < \quad \$\ (A_{ttff})_{R,H/t} \qquad (2.5)$$

In this situation, SF for the EI at 'H' is the best alternative. This is also the best case situation. If

$$\$(P)_H \quad > \quad \$(P_{ttsf})_{H/t}$$

$$\$(P)_H \quad > \quad \$(P_{ttff})_{H/t}$$

Then

$$\$(A)_H \quad <>= \quad \$(A_{ttsf})_{H/2}$$

$$\$(A)_H \quad <>= \quad \$(A_{ttff})_{H/t}$$

$$\$(A)_H \quad <>= \quad \$(A_{ttff})_{R,H/t} \qquad (2.6)$$

In other words, the AEC of an EI with SF at H may be less than, greater than, or equal to, the AEC of an EI with NSF.

This is decided by:

(1) $[\$(P) - \$(P_{ttsf})_{H/t}]$

(2) $[\$(P) - \$(P_{ttff})_{H/t}]$

(3) $R

(4) Time value of money

(5) H

(1) and (2) are the dollar difference's in first costs of EI with SF and NSF at H and (H/t). The greater the difference the greater the amount allowed for repair, or replacement.

Assuming that SF at the service life (H) is not achievable, SF at a life less than H could be the best alternative if:

$$\$\,(A_{ttsf})_{H/t} \quad < \quad \$\,(A_{ttff})_{H/t}$$

$$\$\,(A_{ttsf})_{H/t} \quad < \quad \$\,(A_{ttff})_{R,H/t} \qquad (2.7)$$

If an EI with SF is the best alternative, it may be economically benificial to further the analysis for the best SF life. In the numerical example in Section 2.1, for an assembly cost of $100, SF at 2 years was better than SF at 5 years. For this situation,

First Cost (2 years SF life) = $300

First Cost (5 years SF life) = $750

Although the EI with SF life of 2 years was replaced at the end of years 2 and 4 it was still a better alternative economically.

2.3 SUMMARY

In Section 2.1, the desirability, in some situations, of simultaneous failure designs was demonstrated. It was also demonstrated how a change in the planning horizon, or repair cost caused a decision reversal. The difficulties faced in most economic analysis were also illustrated in Section 2.1. The nature of factors such as - cost of labor, cost of material, and, cost of repair, and the nature of relationships between design life and first cost, make it difficult to generalize any results from economic analyses. Most of these factors are market determined, and are independent of the control of the designer.

In spite of the presence of such factors, the notion that SF alternatives minimize the costs of acqusition and operation of EI persists. It is always difficult to come to an overarching conclusion from an economic analysis. Hence every EI replacement situation must be examined for the most favorable alternative.

In Section 2.2, the costs and failure situations are generalized using algebraic expressions. The assumptions made in Section 2.1 are carried over to this section. The conditions generalized in Section 2.2 for the favorability of an alternative will hold for any replacement analysis situation. Although the analysis is performed here using AEC as the basis of comparison, the conditions will not change if either the Present Equivalent or Future Equivalent bases of comparison are used. The influence of factors such as the time value of money should never be underestimated.

Because of these difficulties, it was decided to examine the feasibility of achieving (economic benefits from) simultaneous failure from a designer's perspective. By examining the strength of materials, as done in Chapter 3, it is possible to study the flexibility of design life for some materials and mechanical components. The extent of flexibility observed is used to determine if SF can be realized.

3.0 STRENGTH OF MATERIALS FROM A LIFE PERSPECTIVE

In Chapter 2, the possibility of cost minimization with SF designs was investigated. The importance of economic factors in such analyses was demonstrated by numerical examples. It also showed why it is difficult to prove conclusively the favorability of SF designs. An algebraic interpretation of the factors and conditions for the favorability of an alternative was also performed.

The purpose of this chapter is to examine the flexibility of design life of some commonly used mechanical components. This is done by examining the fatigue strength properties of the materials used in producing (manufacturing) such components. From the flexibility of the design life capacities of these materials, the achievability of SF designs will be determined.

The flexibility of the design life of three mechanical components, a link, a spur gear, and a compression spring, and some other materials are also examined. Such a study will help determine if simultaneous failure designs should be pursued for realizing any economic benefits. This chapter represents a design engineer's examination for the realization of simultaneous failure before a detail design stage.

Section 3.1 briefly reviews mechanical failure and modes of such failure. Some methods to measure fatigue are described in Section 3.2. Section 3.3 examines the strengths of commonly used materials.

3.1 MECHANICAL FAILURE MODES

In this section, mechanical failure modes and their causes are briefly examined. The importance of fatigue in life determination is also demonstrated.

Mechanical failure may be defined as any change in size, shape, or material property that renders it incapable of satisfactory performance. The following are some of the most commonly observed modes of mechanical failure :

Deformation	Creep
Yielding	Impact
Fracture	Fatigue
Fretting	Wear

Some common manifestations of failure are:

Elastic deformation
Plastic deformation
Material Change - Metallurgical
- Chemical
- Nuclear

Failure results from static or dynamic operating loads and environments over the service life. A designer has to consider all possible modes of failure to ensure reliable operation. Ideally this would be possible if the results of strength tests, for all these failure situations were available. Practically, a designer has to work by extrapolating the results of simple strength tests, based on empiricism.

The determination of design life, involves the consideration of strength under static and dynamic circumstances or loads. Under static considerations, failure often occurs by shearing. However, most static strength determinations do not determine life. But static stresses cannot be ignored in the determination of strength and life.

A majority of engineering designs involve parts subjected to fluctuating or cyclic stresses, or fatigue stresses. Such stresses cause irreversible changes in material properties, often hard to detect, until failure occurs (Collins, pp. 164). Machine members are often found to have failed under the action of repeated or fluctuating stresses. Careful analysis reveals that the actual maximum fatigue stresses were below the ultimate strength of the material and frequently even below the yield strength. The most distinguishing characteristic of these failures has been that the stresses have been repeated a large number of times (Shigley, Chapter 4).

Thus it will be assumed that fatigue stresses determine design life. Some fatigue properties of materials will be examined in the following sections.

3.2 FATIGUE STRENGTH MEASUREMENT

Some methods to measure fatigue strength are outlined in this section. This will enable the measurement of component design life, towards the realization of simultaneous failure. The observations of this analysis are used in developing the rationale for component and equipment design optimization in the following chapters.

Design Life is assumed to be determined by the fatigue resisting strength of the design, if fatigue stresses are the only operating stresses. In most situations, besides fatigue stresses, several static and environmental stresses also exist, such as effects of corrosion and shear, which interact with fatigue stresses, making the prediction of failure difficult. Fluctuating or cyclic loading, that causes fatigue failure is illustrated in Figures 7(a) and 7(b). A common method of

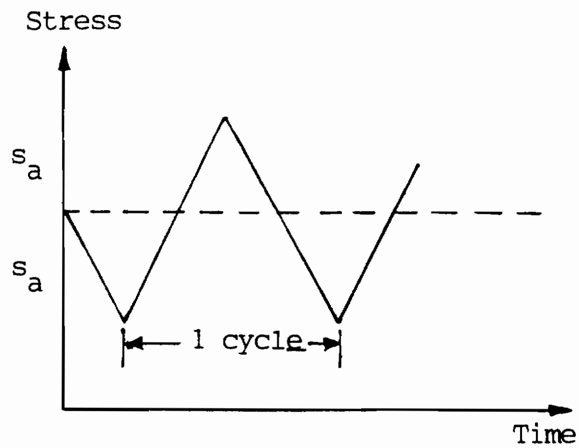


Figure 7(a). Constant amplitude Stress-Time pattern

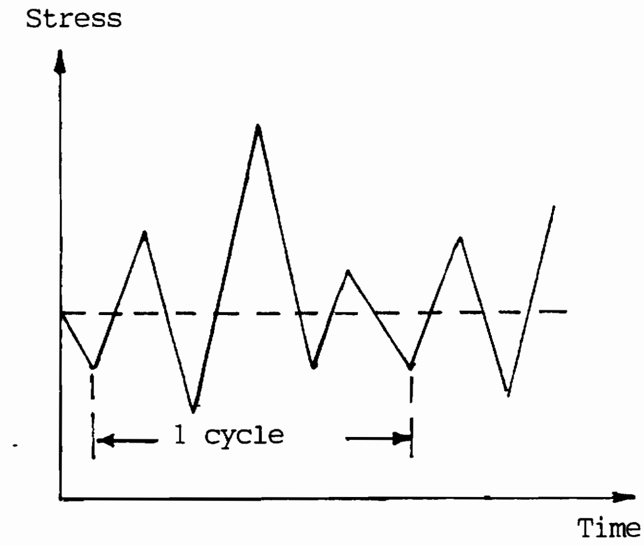


Figure 7(b). Repeated, fluctuating amplitude Stress-Time pattern fatigue strength measurement is the S-N curve, shown in Figure 8. These are log-log plots of cyclic stress amplitudes vs. life in cycles.

Life is often measured as the number of cycles of load for most mechanical design considerations. A cyclic load is a completely reversed load, that can be compared to a sinusoidal curve that has a maximum and a minimum of similar magnitude and opposite sign. This is shown in Figure 7(a).

Because of the scatter of fatigue life and strength, a family of S-N curves exists for most materials, with probability of failure as the parameter. The S-N-P curves, named after Stress-Life-Probability curves, show points of equal probability of failure, as in Figure 9. The S-N-P curves constitute design information of fundamental importance for mechanical parts subjected to repeated loading.

3.3 FATIGUE STRENGTH AND DESIGN LIFE

Using the methods described in Section 3.2, the design life flexibility of a link, a gear, a spring, and some commonly used materials in mechanical components are examined in this section.

Experimental data on the Stress-Life relationship is the first evidence of the rigid nature of Design Life. From the data in Table 5, the S-N data for 99% probability of failure

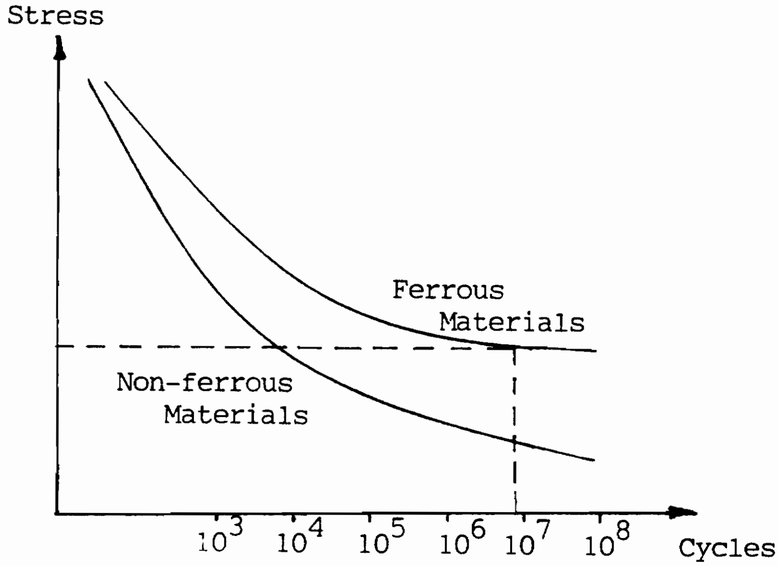


Figure 8 . Plot of Stess-Life (S-N) data for two types of material response to cyclic loading.

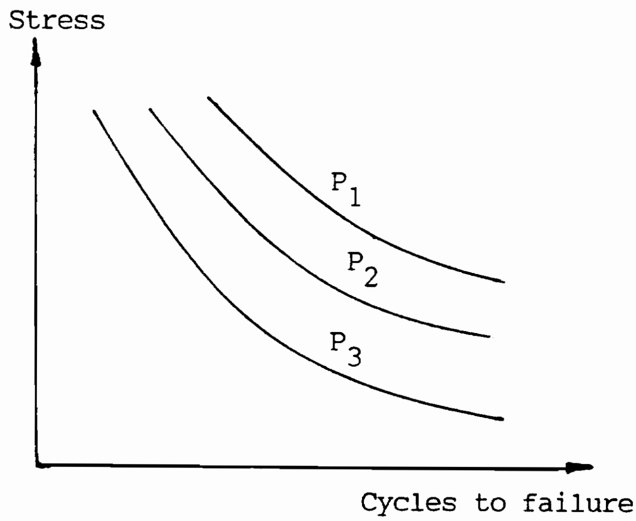


Figure 9 . Family of S-N-P curves (Adapted from Collins, pp 183).

for 4340 R/c 35 steel, there exists a definite life in cycles for a particular level of stress amplitude i.e. irrespective of the size of the member, there is a one to one mapping of operating load and design life.

Table 5. S-N data for 99% probability of failure

Stress Amplitude kpsi	Design Life cycles
168	100
160	1,360
150	3,600
140	7,100
130	14,200
120	28,000
110	66,000
100	110,000
90	216,000
80	440,000
70	1,980,000
68	∞

(Adapted from Collins, Chapter 8, Table 8.2)

The property of 4340 R/c 35 steel is illustrated in the design of a simple mechanical link. A link, made from 4340 R/c 35 steel is subjected to a fatigue causing load of 22,000 lbs. The stress-life-size distribution is tabulated in Table 5. By interpolating between the data in Table 5, the results in Table 6 are obtained. To determine size, the simple relationship

$$\text{Stress} = (\text{Load}/\text{Area})$$

is used. The results reflect a predetermined life to size relationship that exists for this material.

Table 6. Stress-life distribution for a link

Design Life (cycles)	Stress (kpsi)	Size (sq.in.)
4,500	142	0.155
9,000	132	0.166
13,500	127	0.173
18,500	122	0.180
22,500	118	0.186
33,750	113	0.194
45,000	108	0.202
∞	68	0.323

In Table 5, it is observed that when area is slightly more than doubled, life increases from 4,500 cycles to infinite cycles. Thus, if a design situation requires that a link be made with a size of 0.323 square inches or more, life cannot be controlled.

Consider another example, that of a spur gear. A 20* full depth spur (pinion) gear for a 100 hp, 1120 rpm motor, is required for a 4:1 reduction gear train. The pinion is to be made from UNS G 10400 steel, heat-treated and drawn to 1000 F. Estimate the size of a tooth of the gear.

(Shigley, pp. 28). The fatigue strength-life relationship for UNS G 10400 steel is given in Table 7.

Table 7. S-N data for UNS G 10400 steel

Design Life (cycles)	Stress Amplitude (kpsi)
15,100	45.0
41,300	39.1
408,300	30.9
897,400	28.9
1,696,000	28.05
2,692,000	27.4
10,012,000	27.25
10,089,000	27.0
∞	26.0

In Table 8 below, using the S-N data from Table 6, the sensitivity of a spur gear life to size is examined.

Table 8. Stress life distribution for a gear

Design Life (cycles)	Stress (kpsi)	Size (inches)
15,000	51.86	2.0
24,000	41.49	2.5
247,300	34.5	3.0
408,000	30.8	3.5
∞	26.1	4.0

(See Appendix A for a detailed solution)

From Table 8 it is seen that an increase in gear tooth size from 2.0" to 4.0" increases life from less than 15,000 cycles to ∞. For stresses greater than 30 kpsi, life is extremely sensitive, increasing rapidly to ∞.

In the above examples, each size of the component has a fixed life. Very large increases in life (> 100%) result from increases of (8-20)% in size. Thus it appears that design life will be difficult to control, when design constraints such as size, weight, or volume requirements have to be satisfied.

Another indication of the infinite life capacity of materials is found in spring steels. For spring steels used in the manufacture of all kinds of wire springs, tensile strength, size, and material type do not affect life capacity, when wire size is less than 3/8" (Shigley, pp. 306).

For such spring steels, design life is a design-independent parameter. To illustrate the inflexibility of Design Life in the design of a compression spring consider the following example. A compression spring is to be made using a No. 13 W & M gauge (0.091 in.) music-wire with an outer-diameter of 9/16 in., free-length of 3/8 in., 21 active coils, and squared and ground ends. The spring is to be assembled with a preload of 10 lbs, and will operate at a maximum load of 60 lbs. (Example from Shigley, pp. 307).

The torsional shear stress depends upon the design dependent parameters - outer diameter, the wire diameter, number of coils and operating loads. The torsional shear stress of a compression spring is the fatigue stress, and is used as the design stress in examining the sensitivity of design life.

Table 9. Stress-life distribution for a spring

Wire dia (in.)	Area $X10^{-3}$ (sq.in)	Finite Life Strength kpsi	Shear Stress kpsi	Design Life N
0.081	5.2	61.08	49.2	52960
0.086	5.85	61.6	41.4	50329
0.091	6.5	61.4	34.9	50000
0.094	7.05	61.06	31.8	49800
0.0996	7.85	61.14	26.66	50000

(See Appendix B for a detailed solution)

From the results in Table 9, it is observed that despite a 20% change in size, design life is almost unchanged (≡ 50000 cycles) because the fatigue strength of the spring steel is almost unchanged (≡ 61 kpsi). This is a good indication of the inflexibility of design life.

Yet another indicator of the inflexibility of design life is evident in wrought and cast steels. The fatigue endurance limit is the limit stress at or below which life of the member under fatigue is infinite. From data on the fatigue endurance limits and tensile strengths of wrought and cast steels, it can be seen that the endurance limit of the steels is 50% of the tensile strengths, when tensile strengths are less than 200 kpsi. When tensile strengths are greater than 200 kpsi, the endurance limits are, for practical purposes, constant at 100 kpsi. When tensile strengths are greater than 200 kpsi, and fatigue stresses are less than 100 kpsi, life is infinite for such steels

3.4 SUMMARY

From the examples in Section 3.3 and evidence of the resistance to fatigue stresses of materials, it may be concluded that SF is difficult to achieve for these fatigue situations. However, there is insufficient evidence to state that it is impossible to achieve SF for other design situations. Before coming to such a conclusion, it would require every possible material, intended for use in the design of the components of an equipment item, be examined for design life flexibility.

The link, gear, and spring designs showed why SF can be difficult to achieve. For example, consider the link, the gear, and the spring in the previous examples to be designed for SF. If the design conditions of the spring allow for a ± 10% variation in cross-section, and a ± 10% change in outer diameter, the worst case life is only 50,000 cycles. In such a situation, the link and gear would have to be designed for at least 50,000 cycles. While the link may be designed for this life, a gear would have a life of ∞, with the given design constraints.

In an EI with several component parts, design constraints that restrict the life of a component, may preclude the realization of SF. Although larger sizes may increase acquisition costs, savings can result by avoiding repair and repair facility costs. For example, a gear with a tooth width of 4.0" has infinite life, while a gear with 3.0" has a life on the order of 100×10^3 cycles of life. The larger gear may be replaced upon completion of the mission, although it has not failed, and avoid failure during the mission.

Observe from Section 3.3 that design life is not proportional to size or stress. Increases in size produce disproportionate increases in life. Similarly, for increases in fatigue stress, disproportionate decreases in design life occur. These observations are captured in the stress-life relationship in Chapter 4. Since design life is also disproportionately related to stress, damage will be greater at higher stress, than at lower stress. This concept of non- linear damage accumulation is demonstrated in Chapter 4, by the Cumulative Damage theory.

Assume that a designer is advised to design an equipment item for simultaneous failure after an economic analysis. Towards this design objective, the first step is to gather the fatigue properties of the materials required or desired for use in the EI design. An analysis of the fatigue properties at this stage would prevent the commitment of resources to detail design, if SF is not achievable. If SF at the required life cannot be achieved, the economic favorability of SF at other lives should be examined. If a SF life is decided upon, and is not perfectly achievable, it should be attempted in the limit i.e. design each component for a life as close to SF life as possible. This iterative process, of selecting an alternative from an economic analysis, and verifying its achievability could in itself prevent economic waste.

Having examined the achievability of SF in EI design, Chapters 4 and 5 develop a framework for the optimization of multi-component EI design. The first step towards this objective is to establish a model for the estimation of reliability or probability of fatigue failure. This estimate of failure probability is made from a consideration of the operating loads acting on the components of the equipment item.

4.0 RELIABILITY, CUMULATIVE DAMAGE AND LIFE STRENGTH

Having examined the flexibility of design life of some commonly used materials in component designs, the following chapters will attempt to develop a framework for a multi-component equipment item design optimization model with life consideration.

The design stage of analysis requires a systematic and rational method of analysis. This stage must achieve the goal of ensuring operational requirements with some level of reliability over the life of the equipment. Towards this goal, a preliminary form of system design has to be conceived. This design is subject to change as the design progresses to the detailed design stage. Such an analysis will allow for the allocation of resources towards a detail design only when system requirements are satisfied.

To develop an initial estimate of the size and configuration of the equipment item design, loading, reliability and life considerations should be included in the analysis. This is necessary to ensure the required probability of success or reliability during the operating life.

The purpose of this chapter is to take the first step towards developing a model for a preliminary design analysis. Using a mechanical reliability model as its basis, it will introduce fatigue stress and strength considerations, and the theory of cumulative damage in the analysis.

The scope and limitations of this model are detailed in Section 4.1. In Sections 4.2 and 4.3, the reliability model is modified with the fatigue stress-strength relationship and the theory of Cumulative Damage. This modified reliability model will help optimize the design of a single component, given the operating loads, material properties, and design constraints

4.1 SCOPE AND ASSUMPTIONS

In an operating environment, the loading that an equipment is subjected to, may be described more accurately by a probability distribution than by deterministic values. This also enables a probability of failure or reliability to be associated with the design, which more realistically describes the design.

The non-probabilistic and probabilistic viewpoints of an applied load are described by Figure 10. If a load L is comprised of a number of parts, L1, L2, ... ,Ln, the relation of L to each of these is described by the function F(L1, L2, Ln), so that the solution for L yields a single value. If however the parts L1, ... , Ln, are described by probability density functions, the solution for L is a

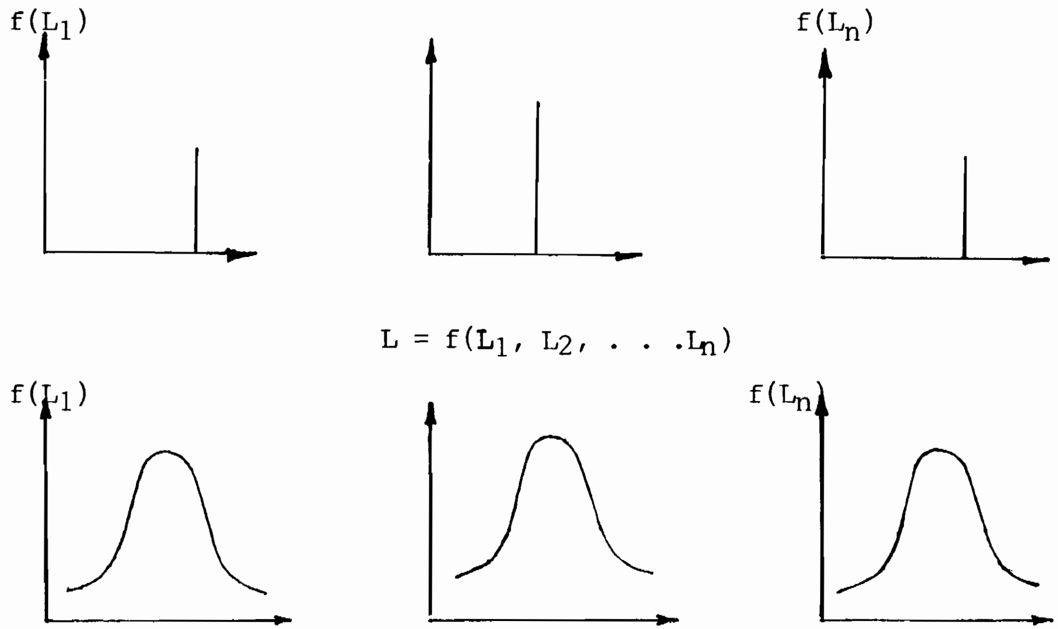


Figure 10. Probabilistic and Non-probabilistic loads.

single joint probability density function, that more realistically describes an operating environment.

The Central Limit Theorem is very important in statistics where estimations are made. It accounts for the role of the Normal Distribution in much of statistics. This theorem states:

For a sequence of independent, identically distributed random variables, with a finite mean and finite variance,

$$\sqrt{\{n(X' + \mu)/ \sigma\}} \;\tilde{}\; N(0,1)$$

where

X' = sample mean

n = sample size

μ = population mean

σ = sample standard deviation

$N(0,1)$ = Normal distribution with mean 0 and standard deviation of 1

The use of the normal distribution enables the following assumptions to be made about random variables:

1. Random variables are often normally distributed with a good degree of approximation (Svenson).

2. Sums of random variables, regardless of the underlying distributions, tend toward the normal distribution, by the Central Limit Theorem (Bowker).

3. Some other distributions can also be approximated with the normal distribution. For example, the lognormal distribution (Bowker).

4. An algebra of the normal distribution is well developed, so that closure under the binary conditions of taking sums and products, subject to boundary conditions (Lamarre).

By assuming that operating loads are represented by random variables, normal distributions may be used to represent them.

When considering the distributions of material strength properties, the scatter is often found to be quite small. For example, the standard deviations of endurance limits for steel is not likely to exceed 8% of the mean (Shigley, pp. 192) . Several handbooks and authors (Shigley; Mischke) recommend the use of the normal distribution as a good approximation to represent strength distributions, although other distributions such as the Weibull, the Lognormal, and the Gamma may be used. The normal distribution is continuous from $-\infty$ to $+\infty$, indicating a zero strength in the distribution, but zero strength would occur at 20 to 30 sigmas below the mean. However, since values of the normal distribution beyond 6 to 7 sigma are often insignificant, the normal distribution is accepted as a good approximation.

4.2 FINITE LIFE STRENGTH IN MECHANICAL RELIABILITY

The following discussion is based upon the work of Kececioglu and Lamarre (Kececioglu and Lamarre). Reliability of mechanical components may be defined as the probability that a stress 's' will exceed a strength 'S'. As shown in Figure 11, reliability and unreliability can be visualized from strength and stress distributions.

Reliability $R = P(S > s)$ (4.1)

where S = Failure governing strength

s = Failure causing stress

Thus, f(s) and f(S) are distributions of independent random variables respectively. The objective is to calculate the probability of a stress 's', exceeding a strength 'S'.

Rearranging Equation (4.1):

$$R = P(S-s > 0) \tag{4.2}$$

or

$$R = P(\epsilon > 0) \tag{4.3}$$

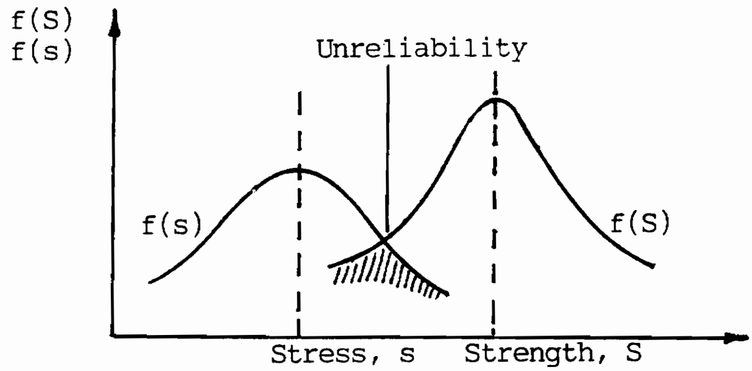


Figure 11. Failure governing Stress and Strength distributions

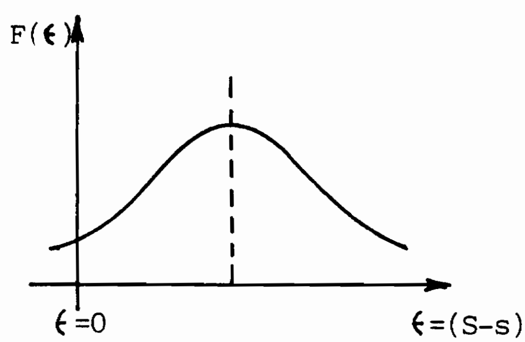


Figure 12. The Stress-Strength difference distribution

$$R = \int_0^{\infty} f(\epsilon)\, d\epsilon \qquad (4.4)$$

where $\epsilon = (S-s)$

S and s are assumed to be independent, identically distributed random variables here. Thus ϵ is a normally distributed random variable, as shown in Figure 13.

Since ϵ is normally distributed, the estimate of average reliability is given by :

$$R = \int_0^{\infty} [(1/\{\sigma\surd(2\pi)\}) \exp(-\tfrac{1}{2}(\epsilon - \epsilon'/ \sigma)^2)\, d\epsilon] \qquad (4.5)$$

where ϵ' = estimate of mean ϵ

$\sigma(\epsilon)$ = estimate of standard deviation of ϵ

From the properties of the normal distribution,

$$\epsilon' = (S' - s') \qquad (4.6)$$

$$\sigma(\epsilon)^2 = [\ \sigma(S')^2 + \sigma(s')^2\ ] \qquad (4.7)$$

where

S' = estimate of mean failure resisting strength

s' = estimate of mean failure causing stress

$\sigma(S')$ = estimate of standard deviation of S'

$\sigma(s')$ = estimate of standard deviation of s'

Most reliability models have two parameters, such as, σ and μ for the normal distribution, β and n for the Weibull distribution. These parameters are estimated by standard statistical techniques such as maximum likelihood method or the method of moments.

The coefficient of variation is a dimensionless parameter that considers the two parameters that define reliability factors. Mathematically, it may be expressed as

Coefficient of Variation = (Standard Deviation/ Mean)

The coefficient of variation determines the spread or shape of a distribution around the mean. The coefficient of variation can be conveniently summarized for a large class of materials and parts, similar to the damping ratio of a vibratory system. Therefore, design and replacement decisions can be made on the basis of limited information available about characteristic life, once a reasonable value of coefficient of variation is postulated or obtained from data in literature (The above discussion is from S.M.Pandit).

$\sigma(\epsilon)$ may now be expressed as :

$$\sigma(S,s)^2 = [\{\sigma(S')/S'\}^2(S)^2 + \{\sigma(s')/s'\}^2(s')^2] \quad (4.8)$$

where

$\{\sigma(S')/S'\}$, and $\{\sigma(s')/s'\}$ are coefficients of variation

To evaluate Equation (4.5), the standard normal tables may be used. To enable this, the standard normal deviate, Z, is determined as:

$$Z = (\epsilon - \epsilon')/\sigma(\epsilon') \qquad (4.9)$$

The limits of the integration, with Z are as follows:

for $\epsilon = \infty, \quad Z = (\infty - \epsilon')/(\sigma(\epsilon') = \infty \qquad (4.10)$

$\epsilon = 0, \quad Z = (0 - \epsilon')/(\sigma(\epsilon') = -\epsilon'/\sigma(\epsilon') \qquad (4.11)$

The reliability can be obtained from the standard normal tables using :

$$R = \int_{-Z}^{\infty} [\{ 1/(\sqrt{(2\pi)}) \} \exp(-t^2/2) \, dZ] \qquad (4.12)$$

where

$t = (\epsilon - \epsilon')/\sigma(\epsilon')$

The use of this approach is illustrated by the following example. Determine reliability given:

$S' = 170{,}000$ psi $\qquad \sigma(S') = 6{,}000$ psi

$s' = 112{,}600$ psi $\qquad \sigma(s') = 10{,}200$ psi

Solution:

$$\epsilon' = S' - s' = 57{,}400 \text{ psi}$$

$$\sigma(\epsilon') = 11{,}360 \text{ psi} \qquad (\text{ using Equation 4.7 })$$

Thus

$$[\, - (\epsilon')/\sigma(\epsilon')\,] = - [57{,}400/11{,}360]$$
$$= - 5.06$$

From Equation (4.12)

$$R = \int_{-5.06}^{\infty} [\{(1)/(\sqrt{2\pi})\} \exp(-t^2/2)]\, dZ$$

From the standard normal tables R = 0.9999997. That is, on an average, of 10^7 such components, 3 will fail.

The need to design for finite lives is almost common knowledge now. Finite life strength S_f or fatigue strength corresponding to a finite life, N, in cycles, can be calculated from the S-N diagram for the material in question. This equation of an S-N curve, in terms of the finite life strength, endurance limit strength, and design life, is given by (Shigley, pp 184):

$$\log S_f = - x\log N + y \tag{4.13}$$

where

S_f = Finite life strength

N = Design life in cycles

$$x = (1/3)\log(0.9S_{ut}/S_e) \tag{4.14}$$

$$y = \log(\{0.9S_{ut}\}^2/\{S_e\}) \tag{4.15}$$

S_{ut} = Ultimate tensile strength

S_e = Endurance limit

The ultimate tensile strength is the tensile stress the member can withstand before tensile failure starts to occur. Given S_{ut} and S_e, x and y may be calculated. Further, given either N or S_f, the other term may be determined. The reliability of a member in a state of stress is known from Equation (4.12).

Using the principles of the above derivation, the standard normal deviate, z, may be expressed as :

$$Z = [(s - S_f) / \sigma(s,S_f)] \tag{4.16}$$

where s is the operating stress.

Reliability may be determined by substituting Equation (4.16) for Z in Equation (4.12) as :

$$R = \int_{-(S_f-s)/\sigma(S_f,s)}^{\infty} [(1/(\sqrt{2\pi})) \exp(-t^2/2) dZ] \tag{4.17}$$

4.3 CUMULATIVE DAMAGE AND DESIGN LIFE

In this section, the principles of Cumulative Damage in fatigue life estimation are introduced and incorporated in the reliability model of Section 4.2. The theory of Cumulative Damage with fatigue-stress considerations will help estimate the design life capacity of a component.

Cyclic stresses causing fatigue failure are often non-uniform stresses spread over a range of amplitudes, as illustrated in Figure 7. These non-uniform stresses make the direct application of S-N-P diagrams, tensile strengths, and endurance limits data difficult. A basic postulate of fatigue investigators is that damage occurs at every operating stress cycle in the spectrum. The damage occurring is permanent, and gradually accumulating until it reaches a threshold or critical level, at which fatigue failure occurs. Despite this simple concept, difficulty is experienced in assessing the damage that occurs at any given stress amplitude in a cycle. Several theories have been postulated, and are known as Cumulative Damage, or CD, theories.

The earliest theory, the Palmgren Miner theory, considered damage accumulation to occur linearly. However it was soon discovered that fatigue damage accumulates nonlinearly, as shown in Figure 13 (Collins, pp 243). Despite the simplicity of the Palmgren-Miner hypothesis, that damage is directly proportional to the ratio of the actual cycles of

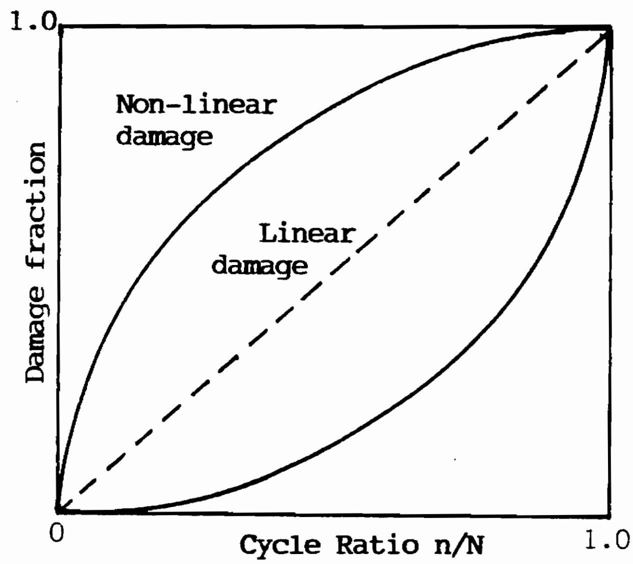


Figure 13. Non-linear damage accumulation.

stress to the maximum cycles of stress possible, a significant shortcoming exists. No influence of the order of application of the stresses is considered, which can give rise to considerable error in failure predictions

The non-linear damage theories, based on experimental evidence, express damage fraction as a function of stress amplitude levels, i.e. lesser damage for lower stress.

Two non-linear damage theories are the Marco-Starkey theory (Marco-Starkey) and the Corten-Dolan theory (Corten-Dolan). The coefficients of damage, as expressed by each of them, for a spectrum of stresses is -

$$d = \sum_i (n_i/N_i)^m \geq 1 \qquad \text{(Marco-Starkey Theory)} \qquad (4.18)$$

n_i = number of cycles of stress at s_i

N_i = number of cycles to failure at s_i

m_i = stress dependent material constant

i = the ith stress in the spectrum, taking values from 1 to a finite number

d = coefficient of damage

Or

$$d = \sum_i (n_i/N_i)(s_i/s_1)^x \geq 1 \qquad \text{(Corten-Dolan Theory)} \qquad (4.19)$$

s_i = stress in operating environment

s_1 = highest operating stress amplitude

N_1 = number of cycles to failure at S_1

x = material constant

n_i = number of cycles at stress s_i

Other CD theories are the Henry theory, the Gatts theory and the Martin theory. Double linear CD theories have also been developed where two linear curves exist , one for crack initiation and one for crack propagation. The Manson Double Linear CD theory is one such theory (Manson, S.S.) (See Collins, Chapter 8, for more details).

The use of fatigue data and the concept of cumulative damage in Design Life analysis is illustrated in the following example (Collins, pp 269). A link made of 4340 steel, R/c 35 hardness, is subjected to three loads, 22,000 lbs, 12,000 lbs, and 6,500 lbs, for 12,000, 7,000, and 50,000 cycles respectively. This duty cycle is repeated three times during its life. Available fatigue data for 4340 steel at R/c 35 is presented below.

Table 10. S-N Data for 99% probability of failure

Stress Amplitude kpsi	Design Life cycles
168	100
160	1,360
150	3,600
140	7,100
130	14,200
120	28,000
110	66,000
100	110,000
90	216,000
80	440,000
70	1,980,000
68	∞

To begin the process, a cross-sectional area of the link is assumed so that the operating stresses may be evaluated. Note that the Corten Dolan CD theory is used here. Assuming an area of 0.15 sq. in., the stresses are determined as

$$s_1 = 22{,}000 / 0.15 = 146{,}000 \text{ psi}$$

$$s_2 = 12{,}000 / 0.15 = 80{,}000 \text{ psi}$$

$$s_3 = 6{,}600 / 0.15 = 43{,}333 \text{ psi}$$

Using $d = 5.67$,

$n_1 = 3{,}600$; $n_2 = 21{,}000$; $n_3 = 150{,}000$ cycles

Using Equation (4.19) from above :

$$d = \frac{(3,600)}{(4,950)} + \frac{(21,000)}{(4,950)}(80/146)^{5.67} + \frac{(150,000)}{(4,950)}(43.3/146)^{5.67}$$

$$= 0.898$$

(see Collins, pp 269, for a step-by-step solution)

Since d is less than 1, the cross-sectional area is larger than required. This implies a capacity for greater than required Design Life. By reducing the cross-sectional area, and by trial and error, the optimum size is obtained. (This method of analysis is applicable to members under axial loading only).

4.4 MECHANICAL RELIABILITY, CUMULATIVE DAMAGE, AND FINITE LIFE

The theory of cumulative damage can be extended to reliability and design life studies. When the strengths of materials can be approximated with normal distributions, the ensuing failure criteria can also be approximated to be normally distributed. Like the determination of ϵ, the damage coefficient can also be expressed as a difference of two terms.

From the principles of section 4.3, we have :

$$\epsilon = (D - d) \leq 0 \qquad (4.20)$$

where

D = Damage coefficient at stress s_1, for N_1 cycles

d = damage coefficient for a spectrum of stresses

Since S_f and s are normally distributed, it will be assumed that D and d are also normally distributed, and that D has a mean value of 1.

$$\epsilon = [1 - \sum_i (n_i/N_1)(s_i/S_1)^x] \qquad (4.21)$$

so that failure occurs if $\epsilon \leq 0$

Using the principles of the derivation from Section 4.2 and Section 4.3, we have:

$$\sigma(\epsilon') = (\sigma(D)^2 + \sigma(d)^2)^{\frac{1}{2}} \qquad (4.22)$$

where

D = estimate of damage coefficient with stress s_1 (The mean value of D may be assumed to be 1)

d = estimate of damage coefficient for the range of stresses

$\sigma(D)$ = estimate of the standard deviation of D

$\sigma(d)$ = estimate of the standard deviation of d

Using the coefficient of variation, discussed in Section 4.2:

$$\sigma(\epsilon') = [\{\sigma(D)/D\}^2(D)^2 + \{\sigma(d)/d\}^2(d)^2]^{\frac{1}{2}} \qquad (4.23)$$

Reliability may now be expressed as,

$$R = \int_{-Z}^{\infty} [1/(\sqrt{2\pi})\ \exp(-t^2/2)\ dZ\ ] \qquad (4.24)$$

where

$$t = [(D - d) - \epsilon']/[\sigma(D,d)]$$

and $Z = \epsilon'/\sigma(\epsilon')$

From the link, gear, and spring examples, it is observed that operating stresses, size, and life are inter-dependent.

Stress and strength may now be expressed as :

$$s = (L/a) \qquad (4.25)$$

$$S = Ka^b \qquad (4.26)$$

where

'K', 'b' are material constants

a is the cross-sectional area of the member under stress

Substituting Equations (4.25) and (4.26), in Equation (4.19)

$$d = \sum_i (n_i/N_1)(\{L_i/a\}/\{Ka^b\})^x] \qquad (4.27)$$

Thus, reliability is now a function of area, life (incycles), material strength, K, and, operating loads L.

Substituting Equation (4.27) in Equation (4.24)

$$R = \int_{-Z}^{\infty} [1/(\sqrt{2\pi})\ \exp(-t^2/2)\ dZ\ ] \qquad (4.28)$$

where

$$Z = [\ \{1 - \sum_i (n_i/N_1)(\{L_i/a\}/\{Ka^b\})^x]/\sigma(d,D)\}$$

4.5 SUMMARY

In this chapter, the mathematical model for mechanical reliability estimation was used as the basis for component design analysis with consideration of its operating conditions and the rate of damage accumulation in that environment.

The basis of the mathematical formulation in this chapter is the Normal Distribution assumption to represent stress and strength. The well developed algebra of the Central Limit Theorem was used in developing this rationale thus far.

Although other distributions can be used, as stated earlier, it will not affect the rationale of the analysis. Only the numerical results, and the form of the final solution will be different.

For a reasonably accurate preliminary design of a component the operating environment and system requirements should be considered. By mathematical manipulation, Equation (4.28) was derived. The use of fatigue stresses in damage estimation increases the accuracy of the preliminary (design) estimates.

Equation (4.28) represents the optimization model for a single component of a multi-component EI. With prior knowledge of the following - type of material, operating environment, and required reliability and design life, an estimate of the size may be obtained. This prevents the overdesign of components, because of the use of the theory of CD.

Equation (4.28) has its limitations. Besides the operating environment and requirements of a single component, the design constraints cannot be translated directly for use in such analysis. As stated earlier, it is important to consider system design requirements and constraints, during component design.

Lagrange's Method of Undetermined Multipliers, LMOUM, is an operations research technique that enables optimization to be performed with all given constraints. The principles of

LMOUM are used in Chapter 5 to develop Equation 4.28 further. The use of LMOUM in component design is illustrated in Chapter 5.

5.0 MULTI-COMPONENT EQUIPMENT ITEM DESIGN

In the previous chapter a method to optimize reliability, design life, and size, given a set of operating loads, for a single component an equipment item was outlined. However, there is a need to optimize these parameters relative to the other component members of the equipment item. In the following sections, a method to consider reliability, design life, size, strengths, and operating loads, for multi-components systems, simultaneously, will be outlined

Section 5.1 introduces the principles of Lagrange's Method of Undetermined Multipliers. LMOUM is useful in optimizing several simultaneous equations, relative to one variable. Adding LMOUM to the reliability, stess-strength and cumulative damage relationships, a design optimization rationale is established. This optimization is done with respect to a physical feature of the components of an equipment item. This is done in Section 5.2.

5.1 LAGRANGE'S METHOD OF UNDETERMINED MULTIPLIERS

An operations research technique, "Lagrange's Method of Undetermined Multipliers", or LMOUM, will be used for this analysis. LMOUM is defined as follows:

If it is required to find the optimum of a function:

$$w = F(x,y,z,u) \tag{5.1}$$

with the following constraining relationships :

$$\phi_1(x,y,z,u) = 0$$
$$\phi_2(x,y,z,u) = 0 \tag{5.2}$$

where z and u are functions of the independent variables x, and y, then the necessary conditions for an optimum or extremum are :

$$
\begin{aligned}
(\delta f/\delta x) + L_1(\delta\phi_1/\delta x) + L_2(\delta\phi_2/\delta x) &= 0 \\
(\delta f/\delta y) + L_1(\delta\phi_1/\delta y) + L_2(\delta\phi_2/\delta y) &= 0 \\
(\delta f/\delta z) + L_1(\delta\phi_1/\delta z) + L_2(\delta\phi_2/\delta z) &= 0 \\
(\delta f/\delta u) + L_1(\delta\phi_1/\delta u) + L_2(\delta\phi_2/\delta u) &= 0
\end{aligned} \tag{5.3}
$$

where

L_1, and L_2 are the undetermined Lagrange Multipliers.

The four equations in (5.3) along with the constraining relations of (5.2), may be used to determine the values of x, y, z, u, L_1, and L_2 (The above discussion is from Sokolnikoff and Pipes)

5.2 DESIGN OPTIMIZATION OF EQUIPMENT ITEM

Lagrange's Method of Undetermined Multipliers is well suited for this analysis, as it enables the consideration of independent and dependent design variables, in the process of design. It also enables the consideration of design constraints, which could be weight, or reliability, or size, and simultaneous consideration of several parameters of the design simultaneously.

To model the problem suitable to the Lagrange Formulation, the following relationships are used:

$$A = \theta(\text{size, material cost, labor cost,...}) \qquad (5.4)$$

where

A = Acquisition cost of the entire assembly

We know from Equation (4.11) that

$$R = f(\text{size, design life, loads, strength}) \qquad (5.5)$$

For an assembly of components, assuming a series reliability relationship between the various components:

$$R_s = \pi\ R_i \qquad (5.6)$$

This reliability relationship can be the constraining relationship for the Lagrange formulation. Using an undetermined multiplier, L, we have:

$$\delta A/\delta a + L(\delta R/\delta a) = 0 \qquad (5.7)$$

For an assembly of m components:

$$\delta A_i/\delta a_i + L(\delta R_i/\delta a_i) = 0 \qquad (5.8)$$

where 'i' is the 'ith' component of the m components

Equations (5.3) and (5.8) constitute a set of (m+1) equations in the unknowns L, a_1, a_2,...,a_m. This is the basic Lagrange formulation. To eliminate L, we use ratios,

$$\frac{\delta A_i/\delta a_i}{\delta A_j/\delta a_j} = \frac{\delta R_i/\delta a_i}{\delta R_j/\delta a_j} \qquad (5.9)$$

Now, $R = \Phi(Z)$ and $Z = f(a)$

Equation (4.20) may be expressed as

$$\frac{\delta R_i/\delta a_i}{\delta R_j/\delta a_j} = \frac{(\delta R_i/\delta Z_i)(\delta Z_i/\delta a_i)}{(\delta R_j/\delta Z_j)(\delta Z_j/\delta a_j)} \qquad (5.10)$$

Using Liebnitz rule to differentiate Equation (4.7)

$$(\delta R/\delta Z) = [-(1/\sqrt{2\pi})\exp(-Z^2/2)] \qquad (5.11)$$

Substituting Equations (5.11) and (5.10) in Equation (5.9)

$$\frac{\delta A_i/\delta a_i}{\delta A_j/\delta a_j} = \frac{[\exp(-Z_i^2/2)][\delta Z_i/\delta a_i)]}{[\exp(-Z_j^2/2)][\delta Z_j/\delta a_j)]} \tag{5.12}$$

$$\frac{dZ_i}{da_i} = \frac{d}{da_i}\left[\frac{(1 - d_i}{[\ C(D_i)^2 + C(d_i)^2 d_i^2)^{\frac{1}{2}}}\right] \tag{5.13}$$

$$= \frac{d(Z_i)}{d(d_i)}\ \frac{d(d_i)}{da_i} \tag{5.14}$$

$$\frac{dZ_i}{da_i} = \frac{[(C(D_i)^2 + C(d_i)^2 d_i^2)]d(d_i)}{[(C(D_i)^2 + C(d_i)^2 d_i^2)]\ da_i} \tag{5.15}$$

$$\frac{d(d_i)}{da_i} = \sum_l (n_{li}/N_l)(L_{li}/K_l)^{xi}\ (x_i(1-b_i)a_i^{xi(1-bi)-1}) \tag{5.16}$$

Let

$$E_{il} = (n_{li}/N_l)(L_{li}/K_l)^{xi} \tag{5.17}$$

$$B_i = x_i(1-b_i) \tag{5.18}$$

Substituting Equations (5.18), (5.17), (5.16), (5.15) in Equation (5.12)

$$\frac{\delta A_i/\delta a_i}{\delta A_j/\delta a_j} = \frac{[\exp(-Z_i^2/2)]}{[\exp(-Z_j^2/2)]} \times \frac{[(C(D_i)^2 + C(d_i)^2 d_i^2)]}{[(C(D_j)^2 + C(d_j)^2 d_j^2)]}$$

$$\times \frac{[(C(D_j)^2 + C(d_j)^2 d_j^2)]^{3/2}}{[(C(D_i)^2 + C(d_i)^2 d_i^2)]^{3/2}}$$

$$\times \frac{B_i a_i^{(Bi-1)}}{B_i a_j^{(Bj-1)}} \times \frac{\Sigma E_{il}}{\Sigma E_{jl}} \tag{5.19}$$

Substituting Equation (5.10) in Equation (5.19) above, to express LHS in terms of a, we have:

$$\frac{\delta A_i/\delta a_i}{\delta A_j/\delta a_j} = \frac{[\exp(-Z_i^2/2)]}{[\exp(-Z_j^2/2)]}$$

$$\times \quad \frac{[(C(D_i)^2 + C(d_i)^2\{(\Sigma A_{1i})a^{Bi}\}^2]}{[(C(D_j)^2 + C(d_j)^2\{(\Sigma A_{1j})a^{Bj}\}^2]}$$

$$\times \quad \frac{[(C(D_j)^2 + C(d_j)^2(\Sigma A_{1j}a^{Bj})^2]^{3/2}}{[(C(D_i)^2 + C(d_i)^2(\Sigma A_{1i}a^{Bi})^2)^{3/2}}$$

$$\times \quad \frac{B_i a_i^{(Bi-1)}}{B_j a_j^{(Bj-1)}} \quad \times \quad \frac{\Sigma E_{i1}}{\Sigma E_{j1}} \qquad (5.20)$$

Equations (5.20) and (5.6) now represent the set of m equations in the unknowns ($a_1, a_2, \ldots, a_m$) which may be solved for numerically using the various parametric values of n_i, N_i, S_i, s_i, $C(D_i)$, $C(d_i)$, and x_i. The controlling set of Equations (5.20) and (5.6) are functions of area, a_i.

5.3 SUMMARY

Equations (5.20) and (5.6) represent the model. The flexibility of LMOUM to pick the variable against which an equipment item may be designed provides the designer with a useful tool for design optimization. For a design situation when all relationships and constraints can be defined, LMOUM may be used to generate a good estimate of the design.

It is not possible to develop a general solution or relation expressing the area in terms of the other parameters. The above set of equations may be used however, with other parameters, to optimize one or more of the following acquisition cost, size, reliability, design life, operating loads, and material strength.

Design is an iterative decision making process that evolves through a set of design stages. Consider a repair/replace situation. An economic analysis may indicate that the alternative to replace the existing EI is most desirable. If the most desirable alternative is a SF alternative, the next step is to examine if this alternative can be realized in actual design. This preliminary analysis, to examine if the actual realization of the most desirable alternative is possible, requires a tool. The rationale of the model presented in this chapter is such a tool to perform this preliminary analysis.

If the preliminary analysis indicates that the most desirable alternative cannot be realized, the next step is to examine the design alternative that can be realized. The alternative, determined by the preliminary analysis tool, that can be realized should now be examined for its economic consequences.

6.0 SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

This chapter reviews the entire study. Section 6.1 summarizes each of the five previous chapters. In Section 6.2, the conclusions of the study are listed. Finally some recommendations for further study are suggested in Section 6.3.

6.1 SUMMARY

In this section, Chapters 1 to 5 are briefly reviewed.

As stated in Chapter 1, the subject of this study is replacement analysis. However, this study differs from most studies as it examines the concept of the perfect or ideal design. All EI were classified into either of two types, Repairable's or Consumable's. A Consumable Equipment Item, CEI, experiences SF of all its component parts i.e. all the components have equal design lives. Repairable Equipment Item, REI, have components that have unequal design lives, and hence non-simultaneous failure. The objective of the study was to examine if SF was realizable in a real world situation. As the Logic Flowchart showed, the flexibility of component design lives is a measure of the ability to achieve simultaneous failure. This is examined by the strengths of materials used in building mechanical components.

Chapter 2 demonstrates through four simplified numerical examples how simultaneous failure can minimize the total cost of EI acquisition and operation. It also identified the conditions which have to be satisfied for simultaneous failure to be the most favorable economic alternative. The economic factors, as, first costs, and time value of money, make it difficult for overarching conclusions to be made. Several assumptions were made to overcome the difficulties faced in such economic analysis. Although such assumptions simplify the economic analysis, they do not detract much from actual replacement analysis situations.

Chapter 3 briefly reviews the following : The role of fatigue stresses in determining design life; the measurement of such stresses; and the characteristic fatigue strength properties of some materials commonly used in mechanical component designs. Evidence from the design of a mechanical link, a gear, and a spring, can be used to conclude that design life of mechanical components are determined by design constraints such as operating loads, type of material, and reliability.

Chapter 4 linked principles of mechanical reliability, with the stress-life relationship and the theory of cumulative damage. The mathematical formulation developed can be used to generate a preliminary design of mechanical components. As was stated earlier in Chapter 4, reliability measurement is necessary to ensure that a designed equipment item or component will fulfill its required functions as specified. By using the fatigue stress-life relationship, and the theory of cumulative damage, in the reliability model, component life estimates may be made along with reliability and size estimates.

The model in Chapter 4 enables individual components to be optimized in regard to life, size, and reliability. To be able to simultaneously optimize the design of several components, along with any design constraints that may exist, Lagrange's Method of Undetermined Multipliers was added. As was shown in Chapter 5, the addition of LMOUM helped develop the proposed framework for multi-component design optimization.

6.2 CONCLUSIONS

It may be cautiously stated that EI cannot be designed with components that fail simultaneously. To be able to make this statement without qualification would require extensive research of EI components. For EI with mechanical components, however, such as gears, links and springs, it may be said that SF cannot be realized for finite lifes.

Every design is judged by its success in a competitive market environment. Simultaneous failure requires a complex, detailed analysis. Such an analysis may increase design costs significantly, which could offset any expected economic benefits from simultaneous failure. In other words, allocation of extensive resources may lead to the failure of EI designs (in the marketplace).

However, from the observations in Chapter 3 on the strength of materials, it is known that several components could possess an infinite life capacity. By merely avoiding failure, and hence replacement and repair, during the service life, designs with infinite component life may be economically beneficial. Since equipment item require components made from different materials, designing for a minimum design life may be a better design objective than that of designing for simultaneous failure.

Factors such as availability and acquisition cost can lead to the creation of a design with optimum performance characteristics, a greater than required Design Life, and yet be the best alternative economically. The logic of a penalty paid for unused design life in component parts of equipment item may have been true in the days of the Deacon and the One Hoss Shay. Such a logic does not appear to be justified with present design practice.

From the observations on the strengths of materials, it is inferred that components of the same size and shape may have very different Design Lives, because material properties can vary significantly.

The model to optimize size, weight, acquisition cost, reliability, strength, and stress (or loads), using Lagrange's Method of Undetermined Multipliers, requires the solution of some complex polynomial equations. However the rationale of the Lagrange Method is sound. By using constraining relationships, that often accompany design situations, a good estimate of design concepts may be obtained.

The representation of stress and strength by the normal distribution is a simplifying assumption at most. Engineering design authors, (Shigley; Carter) recommend this to be a sound assumption. It will be stated again that changing this assumption will only change the form of the solution, but the rationale will not change. Lagrange's Method of Undetermined Multipliers can still be used to develop the relationships for multi-component designs.

The assumption of an area to represent size of components may be argued. Many mechanical components, such as bearings and belts , are listed in handbooks by load ratings, diameters, (bore or outer), and speeds. The use of area may require a translation to another measure before a decision may be made.

6.3 RECOMMENDATIONS

With the increasing rate of technological change, and hence decreasing economic lives of equipment in many applications, the importance of optimizing resources and costs of designs cannot be ignored. The two areas which require further research are the economics of simultaneous failure, and, the realization of simultaneous failure. The following are recommendations or suggestions for further studies :

1. To examine the possibility of designing EI with different SF lives for different assemblies. Such designs may help to reduce economic penalties, although such designs may not be as desirable as a SF design.
2. To research a relationship between acquisition cost and size , or reliability. From the observations of this study it appears that acquisition cost is influenced more by size or reliability than by life. However, the author suggests that more evidence is required before conclusions may be made.
3. An examination of design life in electrical and electronic components, and assemblies, and the extent of control of design life by the designer, over such components. The examination of plastics is necessary, as the applications of plastics is increasing in engineering solutions.

4. Further examination of the conditions necessary for favorable simultaneous failure alternatives. This is necessary to determine the importance of the factors such as time value of money, and, planning horizon. Such a study may help reveal other important factors that influence the outcome of replacement situations.

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8.0 APPENDIX

A) SENSITIVITY OF GEAR TOOTH LIFE

The tooth fatigue stress in spur gears is expressed as:

$$\sigma = \frac{W_t P}{K_v F J} \qquad \text{(A1)}$$

where

W_t = Transmitted Load
P = Pitch
J = Form factor
K_v = Velocity factor
F = Tooth face width

In Equation A1,

W_t = 2500lbs
P = 4.5
J = 0.309
K_v = 0.312
F = 2" to 5"

Given all the parameters in Equation A1, σ may be determined for different values of F

For example:

For F = 2.0", σ = 51.86 kpsi

B) SENSITIVITY OF COMPRESSION SPRING LIFE

S_e = Endurance limit

$S_e = K_c K_e \times 45$
$= 0.814 \times 0.844 \times 45$
$= 30.8$ kpsi

where

K_c = stress concentration factor = 0.814

K_e = reliability factor = 0.844

are both obtained from handbooks, or design books.

Ultimate finite life strength , S_f, is given by

$$S_f = [10^b/N^m]$$

$$m = \frac{1}{3} \log \frac{0.9S_{su}}{S_e} = 0.146$$

$$b = \log \frac{(0.9S_{su})^2}{S_e} = 2.864$$

$$S_{ut} = A/d^m = 278 \text{ kpsi}$$

$$S_{su} = 0.6S_{ut} = 167 \text{ kpsi}$$

N = Design life desired / expected

S_{ut} = the ultimate tensile strength, determined from the limiting tensile strength

S_{su} = ultimate torsional shear strength determined from the limiting tensile strength

S_e = the endurance limit

A = 196

= constant for wire dia. from 0.004" - 0.026"

The torsional shear stress is given by:

$$s = (K_s 8FD)/\pi d^3$$

where

K_s = shear stress multiplication factor

= 1.097 from stress correction vs. spring index graph

F = load amplitude

= [F(max) - F(min)]/2 = 20 lbs

D = spring diameter = 9/16 in.

d = wire diameter = 0.091 in.

Using the above values, torsional shear stress is

= 34.9 kpsi

From Equation(4.17) we have

$R = 0.99$; $s = 34.9$; $-Z = -2.326$; $S_f = 61.4$;

Thus, $\sigma = 11.329$

Using this value of σ, we can determine S_f for various values of 's', the torsional-shear stress

Using Equation B1, we can now determine N, for different values of S_f, m, and b.

For example,

$$61.4 = 10^{.146}/(N)^{2.864}$$

$$N = 50{,}000 \text{ cycles}$$

For different values of S_f, we can now determine N. The results are tabulated in Table 9.

VITAE

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