AVAILABILITY OF CONTINUOUSLY-OPERATED, COHERENT, MULTIFUNCTIONAL SYSTEMS

by

Alberto Sols

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Approved :

J. A. Nachlas (Chairman)

B. S. Blanchard

W. J. Fabrycky

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Alberto Sols

Committee Chairman : Dr. Joel A. Nachlas
Industrial and Systems Engineering

(Abstract)

Modern systems are characterized by a multifunctional capability. They are designed to accomplish not one but a series of missions, each one requiring the performance of certain functions. Each of those functions requires the support of some system elements. The fact that an element is not available at certain point in time by no means implies that the entire system is "down" at that moment as the traditional availability definition and formulation requires. Depending on what the mission-function and function-element support requirements are, the "down" condition of a certain element may prevent the accomplishment of some system missions, but some others will still be available. Moreover, the traditional approach assumes that the time to failure and the time to repair associated to each element follow both a negative exponential distribution. Therefore, a more comprehensive treatment of the concept of system availability is required.

All the necessary assumptions to enable the definition and quantification of availability figures of merit are listed. Then, definitions are established for availability and degraded availability at different levels in the system structure, from element to system. In addition, some related concepts such as mission reliability and dependability are defined. The developed model enables the prediction of the defined availability figures of merit. The foundation of the model is the renewal process associated with each system element and the links that specify the mission-function and function-element support requirements. The formulation for some related concepts is also presented.

Some well-known pairs of distributions are considered and the general expressions are particularized for them. Finally, an example is conducted in order to show the applicability of the derived expressions and to compare the obtained results with those obtained using the traditional approach.

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CHAPTER 1 - INTRODUCTION

1.1 Multifunctional Systems.

Modern systems are usually characterized by a multifunctional capability. That is, they are designed and developed to perform not just one but a set of missions. As a typical example of a multifunctional system we may think of a frigate which can be assigned a variety of missions: escort merchant convoys, escort larger battleships, patrol defined areas, and so on. Each type of mission requires the performance of some of the functions that the frigate is capable of accomplishing. We may also consider a high-fidelity entertainment system that can be assigned several missions based on the different functions it can accomplish : play music from compact discs, play music from cassettes, play music from records, and play music from the radio.

1.2 Statement of the Problem.

The accomplishment of each of a system's functions will require specific hardware, software, human, and other resources. Some of those resources may be shared by some of the functions and some others may be function-specific. Therefore, it may happen that some of a system's functions cannot be accomplished at a time that some others can. In the frigate example, a failure in the sonar systems may imply the temporary loss of its antisubmarine warfare capability but that will have no effect on the anti-surface and anti-air warfare ones. Similarly, in the high-fidelity system example, a problem in the compact disc player may prevent its utilization temporarily but the other music sources could still be used.

Multifunctional capability means that system availability can be no longer defined on a binary basis. It is not appropriate to state that either the system is available or it is not. For multifunctional systems, a function or capability may be available at a specific time in which some other is not. In other words, system availability is, among other things, mission-dependent.

A further complication is the fact that most systems can operate in an acceptable way under certain degraded conditions. The two basic kinds of degraded modes of operation are the following :

(a) system-inherent degraded modes. For some systems, such as a frigate, the accomplishment of a specific function does not rely on a single piece of equipment. In a frigate, several types of detectors and weapons are required in addition to the data-processing and decision-making means to provide an antisubmarine functional capability. A failure in one of the elements of the system will, in some cases, result in a partial loss of the function fulfillment capability and in some others it will mean a total loss. All elements of the system can be defined on a working-failed basis.

(b) element-inherent degraded modes. Some elements may admit themselves a degraded mode of operation resulting in a degraded mode of operation at system level. Those elements cannot be defined on a binary (working-failed) basis.

In summary, a failure in an element of a multifunctional system may have the three following effects at the system level :

(a) the system loses the capability of performing all its functions;

(b) the system loses the capability of performing one of its functions; or

(c) the system suffers a degradation in the capability of performing one or more functions.

The traditional availability formulation could be applied to a multifunctional system. Nevertheless, the figure that would result would not really be indicative of the extent to which the user can expect his needs to be fulfilled throughout the useful life of the system. The fact that multifunctional systems can temporarily lose a specific capability while retaining the others intact and that some functions can sometimes be accomplished in degraded modes poses the necessity for a more comprehensive treatment. A new model is required for the availability of multifunctional systems.

1.3 Literature Review.

A significant number of relevant books written in the fields of systems engineering, logistics, reliability, maintainability, and renewal theory have been reviewed (most of them are mentioned in the Literature Cited and Additional References). In addition, the papers published during the last two decades in those fields in relevant journals were reviewed. It was found that only minor references to the problem were made. No author provided either a proper definition of the problem or a mathematical approach for solving it.

As a result of the performed literature review it is concluded that there has been no proper treatment of availability of multifunctional systems. It is a challenging problem that has not been addressed.

1.4 Objectives of the Research.

Four clearly defined objectives are pursued in this research :

(1) to identify and describe in detail the limitations of the traditional availability formulation, especially when applied to continuously-operated, coherent, multifunctional systems that admit system-inherent degraded modes of operation;

(2) to thoroughly review the approaches proposed in the last few years for dealing with multifunctional systems;

(3) to propose the adequate terminology and definitions for the concepts involved, setting the appropriate boundaries among availability and other similar concepts (such as dependability and operational readiness);

(4) to establish a model for the estimation of system availability during the design phase for continuously-operated, coherent multifunctional systems.

1.4.1 Approach.

The first step is to give an overview of the traditional availability approach, reviewing the different formulations. Then the major drawbacks of the traditional availability formulation are highlighted. Next, the assumptions and definitions required to develop the proposed approach will be stated. Finally a model is proposed in order to predict availability figures of merit for continuously-operated, coherent, multifunctional systems. The model is applied to a selected case and the results obtained are compared to those obtained using the traditional approach.

1.4.2 Potential Applications of the Results.

There is tremendous potential for the application of a formulation for availability of multifunctional systems since most of the systems designed and developed today fall under that category. That formulation would enable the proper specification of availability requirements for new systems, the design of such requirements into the systems, and the prediction and measurement of such figures of merit.

1.5 Summary of Results.

The developed model enables the prediction of availability figures of merit for multifunctional systems. Availability is defined and calculated at the system, function, mission and system level. Moreover, degraded availability is defined and calculated at the function, mission and system level. Also, two availability-related concepts (mission reliability and mission dependability) are also defined and formulated. The expressions obtained are valid for a system with a series configuration, whose elements may exhibit any distribution of times to failure and times to repair. Those expressions are then particularized for some pairs of distributions. Finally, two examples are constructed and analyzed to illustrate the applicability of the developed model.

CHAPTER 2 - IDENTIFICATION OF THE LIMITATIONS OF THE TRADITIONAL APPROACH TO AVAILABILITY DEFINITION AND FORMULATION

2.1 Introduction.

The purpose of this chapter is twofold : to give an overview of the traditional approach to availability definition and formulation and to highlight its main limitations, especially when applied to multifunctional systems.

2.2 Overview of the Traditional Approach.

Availability of a system is traditionally defined as the probability that it is operating satisfactorily at time t [1,2,3]. That is, availability (or point availability) is then expressed as :

$$A(t)=P[x(t)=1] \tag{2.1}$$

where x(t) is a binary status variable with value 1 if the system is functioning and 0 if it is failed. From that initial definition, the same referenced authors derive three other availability figures of merit :

a) limiting or steady-state availability,

$$A_l = \lim_{t \to \infty} A(t) \tag{2.2}$$

b) average availability in [0,T],

$$A_v = \frac{\int_0^T A(t)dt}{T} \tag{2.3}$$

c) limiting average availability in [0,T],

$$A_{lv} = \lim_{T \to \infty} \frac{\int_0^T A(t)dt}{T} \tag{2.4}$$

When element repair is permitted and the distribution of the random variables time to failure and time to repair are both negative exponential, the probability that an element is working at time t is given by [4,5,6,7] :

$$A = P[x(t) = 1] = \frac{\mu}{\mu + \lambda} + \frac{\lambda}{\mu + \lambda} e^{-(\lambda+\mu)t} \tag{2.5}$$

where λ is the failure rate and μ is the repair rate. When time tends to infinity, the limiting or steady-state availability becomes

$$A = \frac{\mu}{\mu + \lambda} \tag{2.6}$$

which can be also expressed as a function of the mean time between failure MTBF (reciprocal of the failure rate) and the mean time to repair MTTR (reciprocal of the repair rate),

$$A = \frac{\mathrm{MTBF}}{\mathrm{MTBF} + \mathrm{MTTR}} \tag{2.7}$$

This result, obtained at the element level, is generalized at the system level assuming that all its elements have times to failure and times to repair following negative exponential distributions. Other distributions, such as the log-normal for times to repair, have been considered in the past; nevertheless, the only traditional formulation available is for the case of negative exponential distributions of the time to failure and the time to repair.

Some authors [8,9,10] make a distinction among inherent availability A_i (that considers corrective maintenance actions only), achieved availability A_a (that considers both preventive and corrective maintenance actions), and operational availability A_o (that considers preventive and corrective maintenance actions and also logistics and administrative delay times). The formulation used for the three measures of availability is the following :

$$A_i = \frac{MTBF}{MTBF + MTTR} \tag{2.8}$$

$$A_a = \frac{MTBM}{MTBM + \overline{M}} \tag{2.9}$$

$$A_o = \frac{MTBM}{MTBM + MDT} \tag{2.10}$$

where MTBM is the mean time between maintenance, $\overline{M}$ is the mean active maintenance time, and MDT is the mean maintenance down time. Note that inherent availability's expression (2.8) is the same as equation (2.7) due to the fact that it accounts only for corrective maintenance actions performed on the considered element or system.

2.3 Limitations of the Traditional Approach.

The following are the main limitations identified with the traditional definition and formulation of availability.

2.3.1 Strict Application of the Traditional Approach to Multifunctional Systems.

The estimated availability would be very low (much lower probably than the actual availability) if every time a component has failed the entire system is considered to be unavailable.

2.3.2 Assumption that the Useful Life Length is an Integer Multiple of the Sum of the Time to Failure and the Time to Repair.

If the length of the useful life of the system is not an integer multiple of the sum of the time to failure and the time to repair, then the approximation given by the traditional formulation will be smaller than the actual value. Several examples of operational availability computed both directly and using the traditional formula (equation 2.10) are shown in Tables 2.1 and 2.2. The differences are not very significant, but certainly the length of the useful life does not have to be an integer multiple of the sum of the time to failure and the time to repair.

2.3.3 Availability not a Function of the Useful Life Length.

Experience shows that the longer the useful life of a given system, the lower its average availability. The traditional approach does not consider the expected useful life of the system in the estimation of the system availability.

2.3.4 Assumption of Negative Exponential Distributions for both Time-to-Failure and Time-to-Repair.

It is assumed in the traditional formulation that all elements in the system exhibit a negative exponential distribution for both the time to failure and the time to repair. That is rather restrictive because many systems have components whose time to failure and time to repair follow other distributions.

MTBF = 38 weeks

MDT = 2 weeks

$$A_o\ (I) = \frac{MTBF}{MTBF + MDT}$$

$$A_o\ (II) = \frac{\text{up time}}{\text{total time}}$$

Table 2.1 - Operational Availability Calculations.

Life(years)	Ao (I)	Ao (II)	Ao(II)/Ao(I)
2	0.950000	0.961538	1.012146
5	0.950000	0.953846	1.004049
6	0.950000	0.955128	1.005398
9	0.950000	0.952991	1.003149
12	0.950000	0.950769	1.000810
16	0.950000	0.951923	1.002024

MTBF = 52 weeks

MDT = 2 weeks

$$A_o\ (I) = \frac{MTBF}{MTBF + MDT}$$

$$A_o\ (II) = \frac{\text{up time}}{\text{total time}}$$

Table 2.2 - Operational Availability Calculations.

Life(years)	Ao (I)	Ao (II)	Ao(II)/Ao(I)
3	0.962963	0.974359	1.011834
4	0.962963	0.971154	1.008506
5	0.962963	0.969231	1.006509
8	0.962963	0.966346	1.003513
9	0.962963	0.965812	1.002959
10	0.962963	0.965385	1.002515

2.3.5 Assumption that the System Availability will Reach a Steady State.

Following the preceding limitation, if other distributions are considered it may be no longer true that the system limiting availability will exist (that is, that there will be a steady state or limiting availability).

2.4 Summary.

The traditional availability definition and formulation is based on restrictive assumptions that reduce its applicability to many modern systems. A more comprehensive model that considers the length of the useful life, the characteristics of multifunctional systems, and any potential distribution for the time to failure and the time to repair of the system elements is required.

CHAPTER 3 - PROPOSED MODEL

3.1 Introduction.

Modern systems are much more than just the prime piece of hardware or equipment. Modern systems comprise both the prime equipment and its logistic support elements. Namely, these are the facilities required to support the system operation and/or maintenance, the required spares and repair parts, the necessary documentation, the necessary test and support equipment, the required computer resources, and the personnel required to operate and maintain the prime equipment and the other appropriate logistic support elements.

Let us momentarily define system availability as the percentage of the system useful life in which the prime equipment is capable of performing that for which it was designed. The availability of a system depends basically on three components : the system design, the system production, and the system's logistic support capability. The first accounts basically for the reliability and maintainability designed into the system. The second accounts for the potential impacts that the production process may have on the designed features, and the third depends on the probability of having the required human and material resources available where and when required. Therefore, the probability that the prime equipment will be capable of performing as desired will depend on both its state and that of its logistic support elements. Consider a battletank. Even if all the tank assemblies are working and if the tank crew is ready, if the tank's radar is momentarily inoperative, the tank cannot perform the missions for which it was designed. System availability is affected by both characteristics inherent to the prime equipment (such as reliability and maintainability) and characteristics inherent to the logistic support elements.

In the traditional availability definition and formulation, the useful life of a system is divided into two types of periods : those in which the system is available (A), and those in which it is not ($\overline{A}$), as depicted in Figure 3.1 (a).

Let us consider a multifunctional system designed to fulfill a series of specified missions. It is assumed that each of those missions requires that a certain set of the system's functions be performed. Moreover, the performance of each function requires that a certain subset of the elements of the system be in proper operating condition. Since only one mission will be accomplished at a time, only a subset of functions (and therefore of elements) are required at a time. This means that the system availability computed with this perspective in mind will be higher, as depicted in Figure 3.1 (b). Furthermore, some functions may be performed in a degraded mode of operation which means that there is an intermediate state between the system being available and not being available; the system may be capable of performing the assigned mission in a degraded mode and consequently there will be a degraded availability, as shown in Figure 3.1 (c).

In order to establish the framework for the development of a model for the availability of continuously-operated coherent multifunctional systems, the first step is to state the necessary assumptions and to establish the necessary definitions. The notation used is

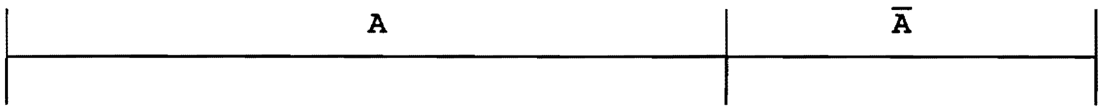

Figure 3.1 (a) Traditional Approach.

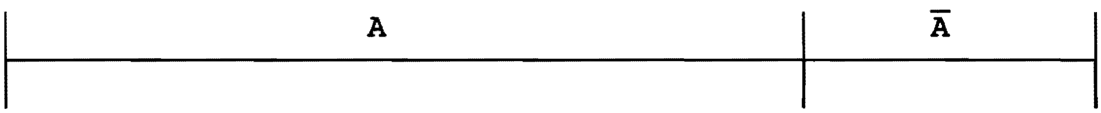

Figure 3.1 (b) Consideration of Multifunctional Capabilities.

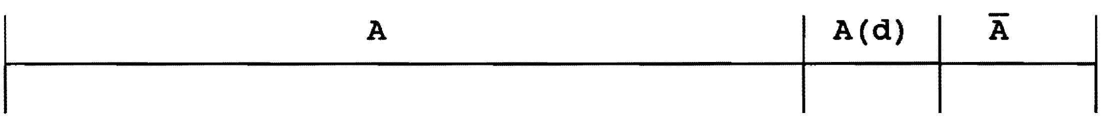

Figure 3.1 (c) Consideration of Degraded Availability.

introduced, and the basic algebraic structure for the estimation of system availability is outlined; that algebraic structure consists of the element, function, mission, and system status functions and reliability functions. The availability of a system can be expected to depend on that of each of its elements. Element availability is a consequence of two processes : the distribution of the times to failure and the distribution of the times to repair. A general expression based on the convolution of those two distributions (that could be solved at least by numerical methods) is derived. Finally, some well-known pairs of distributions are analyzed.

Availability is calculated, as defined, at the element, function, mission, and system level. For the last three levels, a degraded availability is also computed. This provides the systems engineer with visibility of both the true and degraded availability through the structure of the considered system, as shown in Figure 3.2.

3.2 Assumptions.

Assumption 1. When a function is not being performed because the mission the system is engaged in does not require it, then the elements that support that particular function and that do not support any other function being performed do not age.

Assumption 2. The system cannot perform more than one mission at a time.

Assumption 3. The elements required to support a function constitute a series arrangement and operate during the entire function time.

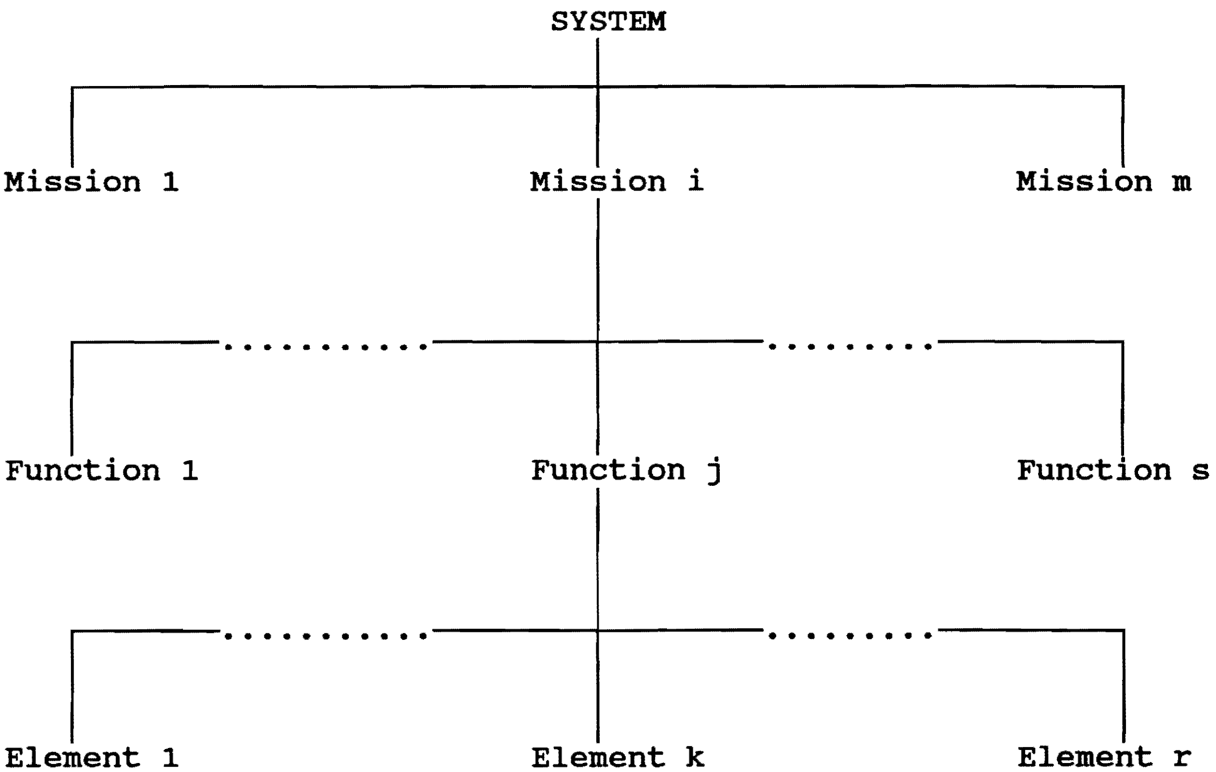

Figure 3.2 System Mission, Function and Element Structure.

Assumption 4. The functions that support a mission constitute a series arrangement and are provided throughout the entire mission duration.

Assumption 5. The state of any element of the system is binary (that is, any system element at any point in time is either working (or capable of) or failed).

Assumption 6. The system elements are considered to be statistically independent.

Assumption 7. At any point in time, the state of the system is completely determined by the state of its elements.

Assumption 8. The distribution of the time to failure of the system elements is the same for all the missions for which the system is designed.

Assumption 9. Preventive maintenance actions can be performed simultaneous to normal system operation and consequently the functions requiring those preventive maintenance actions are available during the time in which the latter are being executed.

Assumption 10. The distribution of the time to repair of the system elements is the same for all the missions for which the system is designed.

Assumption 11. The number of times each type of mission is going to be requested and the average duration of each mission type are known.

3.3 Definitions.

Definition 1. A system is an assemblage of elements forming a unitary whole, designed, developed and deployed to fulfill an identified and defined need.

Definition 2. An element is the lowest item considered (for these analyses) in each branch of the hardware breakdown structure of the system.

Definition 3. A mission is any definite set of tasks the system has been designed to accomplish.

Definition 4. A system function is a specific action provided by a subset of system elements and required to achieve a given objective.

Definition 5. A system is continuously operated if at any point in time during its useful life there is at least one of the system missions being solicited.

Definition 6. A system is intermittently operated if at a certain point in time during its useful life none of the system missions are being solicited.

Definition 7. A system is coherent if every element is required to support at least one function.

Definition 8. A support environment is ideal if the probability of having the required human and material resources for the performance of the required maintenance actions, at the required time, and at the required place is always one.

Definition 9. A support environment is real if the probability of having the required human and material resources for the performance of the required maintenance actions, at the required time, and at the required place is always less than one.

Definition 10. Availability (A) of a multifunctional system is the average probability that the system's prime equipment will be ready for the mission it is called for at any point in time in an ideal support environment.

From now on, whenever system availability is mentioned, it will in fact mean system's prime equipment availability in an ideal support environment.

Definition 11. Mission M_i availability (Am_i) is the average probability that the system will be ready to undertake that particular mission at any point in time in an ideal support environment.

Definition 12. Function F_i availability (Af_i) is the average probability that that particular function can be undertaken at any point in time in an ideal support environment.

Definition 13. Element E_k availability (Ae_k) is the average probability that that particular element is working (or capable of) at any point in time in an ideal support environment.

Definition 14. Degraded availability ($A^{(d)}$) of a multifunctional system is the average probability that at any point in time the system, operating in an ideal support environment, will be capable of engaging in the required mission with at least one of the required supporting functions being in one of its predefined acceptable degraded modes of operation.

Definition 15. Degraded mission M_i availability ($Am_i^{(d)}$) is the average probability that the system will be ready to undertake that particular mission at any point in time, in an ideal support environment, with at least one of its supporting functions being in one of its predefined acceptable, degraded modes of operation.

Definition 16. Degraded function F_i availability ($Af_i^{(d)}$) is the average probability that the system will be ready to undertake that particular function at any point in time, in an ideal support environment, in one of its predefined acceptable, degraded modes of operation.

Definition 17. A function is in an acceptable, degraded mode of operation when the extent to which it is accomplished, although not complete, still partially satisfies the user.

Definition 18. Mission Reliability (MRm_i) is the probability that a system will complete an assigned mission M_i without failures, given that it was operating satisfactorily at the beginning of that mission.

Definition 19. Allowable Repair Time (RT_i) is the maximum time that a repair action should take upon a failure of a system element E_i to restore it to proper operating condition.

Definition 20. Mission Dependability (Dm_i) is the probability that a system will complete an assigned mission M_i, given that it was operating satisfactorily at the beginning of that mission and that in the case that failures would happen, the system could be restored to proper operating condition in less that a given allowable repair time.

Definition 21. Operational Maintainability (Mo) is the probability that when a failure occurs the system will be repaired to proper operating conditions in less than a given allowable repair time.

Definition 22. Operational Readiness (OR) is the availability of a system in a real support environment.

Definition 23. Degraded Operational Readiness (ORd) is the degraded availability of a system in a real support environment.

3.4 Notation.

L	system useful life
M_i	i-th type of system mission (i=1,...,m)
NM_i	number of times the system is assigned mission type Mi
MT_i	average duration of mission type M_i
F_j	j-th system function (j=1,...,s)
x_j	number of degraded modes of function F_j

y_i number of degraded modes of mission M_i

E_k i-th system element (k=1,...,r)

α_i susbset of system functions required to support mission M_i

β_i^0 susbset of system elements required to support function F_i

β_i^k susbset of system elements required to support function F_i in its k-th degraded mode

χ_i^0 susbset of system elements required to support mission M_i

χ_i^k susbset of system elements required to support mission M_i in its k-th degraded mode

ϕ_j function F_j status function

ϕ_j^k k-th degraded mode of function F_j status function

$\phi_j^{(d)}$ degraded function F_j status function

φ_i mission M_i status function

φ_i^k k-th degraded mode mission M_i status function

$\varphi_i^{(d)}$ degraded mission M_i status function

ψ system status function

$p_i(t)$ probability that element E_i is working at time t (no repair is permitted)

$Pr_i(t)$ probability that element E_i is working at time t (repair is permitted)

$R_{F_i}(t)$ function F_i reliability function

$R_{M_i}(t)$ mission M_i reliability function

$\lambda_k(t)$ failure rate of element E_k

$\mu_k(t)$ repair rate of element E_k

EF_k cumulative distribution function of time to failure of element E_k

ER_k cumulative distribution function of time to repair of element E_k

RT_k	deterministic repair time of element E_k
H_k	distribution of the sum of the failure time and the repair time of element E_k
M_{H_k}	renewal function of element E_k
A	system availability
$A^{(d)}$	system degraded availability
Am_i	availability of mission M_i
$Am_i^{(d)}$	degraded availability of mission M_i
Af_j	availability of function F_j
$Af_j^{(d)}$	degraded availability of function F_j
Ae_k	availability of element E_k
Dm_i	mission M_i dependability
MRm_i	mission M_i reliability

3.5 Basic Algebraic Structure.

3.5.1 Element, Function, Mission, and System Status Functions.

The development of the availability formulation requires of the establishment of the adequate algebraic structure. First the element, function, mission, and system status functions are defined. Then the element, function, mission and system reliability are defined.

It is assumed that the status of any element of the system is binary. To indicate the state of the i-th system element E_i, we assign it a binary indicator variable x_i, so that for $i=1,...,r$:

$$x_i(t)=\begin{cases}1 & \text{if element } E_i \text{ is working (or capable of) at time t}\\ 0 & \text{otherwise}\end{cases}$$

It is necessary for the end user to define a mission-function matrix {MF} in order to represent the functions required to support a particular mission :

$$\mathrm{mf}_{ij}=\begin{cases}1 & \text{if function } F_j \text{ required to support mission } M_i\\ 0 & \text{otherwise}\end{cases}$$

Moreover, a function-element {FE} matrix can be defined to represent the elements required to provide a certain function :

$$\mathrm{fe}_{ijk}\begin{cases}(k=0)=\begin{cases}1 & \text{element } E_j \text{ required to support } F_i\\ 0 & \text{otherwise}\end{cases}\\ (k=1,\ldots,x_i)=\begin{cases}1 & \text{element } E_j \text{ required to support k - th degraded mode of } F_i\\ 0 & \text{otherwise}\end{cases}\end{cases}$$

Based on the above matrices we can construct a mission-element {ME} matrix to represent the elements required to truly support a given mission. Since the different functions required to support the given mission may be accomplished in different degraded modes and since not all combinations of degraded modes of functions accomplishment may be considered acceptable for mission performance, it is then necessary that the end user specifies the specific combinations of acceptable degraded modes of operation for the functions supporting a mission. Then :

$$me_{ijk}\begin{cases}(k=0)=\begin{cases}1 \text{ element } E_j \text{ required to support mission } M_i\\ 0 \text{ otherwise}\end{cases}\\ (k=1,\dots,y_i)=\begin{cases}1 \text{ element } E_j \text{ required to support k - th}\\ \quad \text{degraded mode of accomplishment of } M_i\\ 0 \text{ otherwise}\end{cases}\end{cases}$$

It is clear that :

$$me_{ij0}=\coprod_{k=1}^{s} mf_{ik}fe_{kj0} \tag{3.1}$$

Define α_i as the subset of system functions required to support mission M_i :

$$\alpha_i=\{\, j \,/\, mf_{ij}=1 \,\} \qquad (i=1,\dots,n;\ j=1,\dots,s) \tag{3.2}$$

Define β_i^0 as the subset of elements required to support function F_i :

$$\beta_i^0=\{\, j \,/\, fe_{ij0}=1 \,\} \qquad (i=1,\dots,n;\ j=1,\dots,s) \tag{3.3}$$

Define β_i^k (k=1,...,p_i) as the subsets of elements required to provide function F_i in its k-th degraded mode :

$$\beta_i^k = \{ j \,/\, fe_{ijk}=1 \} \qquad (i=1,...,n;\ j=1,...,s;\ k=1,...,p_i) \tag{3.4}$$

Define χ_i^0 as the subset of elements required to support mission M_i :

$$\chi_i^0 = \{ j \,/\, me_{ij0}=1 \} \qquad (i=1,...,n;\ j=1,...,r) \tag{3.5}$$

Define χ_i^k as the subsets of elements required to provide mission M_i in its k-th degraded mode :

$$\chi_i^k = \{ j \,/\, fe_{ijk}=1 \} \qquad (i=1,...,n;\ j=1,...,r;\ k=1,...,q_i) \tag{3.6}$$

It is possible to derive for each system function F_j (j=1,...,s) a function status function ϕ_j, defined as follows :

$$\phi_j(t) = \prod_{x_i \in \beta_i^0} x_i(t) \tag{3.7}$$

The function status functions ϕ_j are also binary :

$$\phi_j(t) = \begin{cases} 1 & \text{function } F_j \text{ available at time t} \\ 0 & \text{otherwise} \end{cases}$$

Moreover, it is possible to construct for each system function F_j (j=1,...,s) in its k-th degraded mode of operation the corresponding k-th degraded mode function status function :

$$\phi_j^k(t) = [1-\phi_j(t)]\prod_{i\in\beta_j^k} x_i(t) \tag{3.8}$$

which is binary :

$$\phi_j^k(t) = \begin{cases} 1 & \text{function } F_j \text{ is in its k - th degraded mode at time t} \\ 0 & \text{otherwise} \end{cases}$$

The term $[1-\phi_j(t)]$ in equation (3.8) ensures that function F_j is not available (in the fully operational or non-degraded mode).

It is also possible to construct the degraded function status function for each system function F_j (j=1,...,s) :

$$\phi_j^{(d)}(t) = [1-\phi_j(t)]\coprod_{k=1}^{p_j}\phi_j^k(t) = [1-\phi_j(t)]\coprod_{k=1}^{p_j}\prod_{i\in\beta_j^k} x_i(t) \tag{3.9}$$

which are also binary :

$$\phi_j^{(d)}(t) = \begin{cases} 1 & \text{function } F_j \text{ is in a degraded mode at time t} \\ 0 & \text{otherwise} \end{cases}$$

The term $[1 - \phi_j(t)]$ in equation (3.9) ensures that function F_j is not available (in the fully operational or non-degraded mode). Equation (3.9) states that for function F_j to be in a degraded mode it has to be that at least the subset of elements that define one of its acceptable modes of degraded operation are all working at time t.

For each system mission M_j (j=1,...,n), the mission status function φ_j can be built upon the element status functions :

$$\varphi_j(t) = \prod_{i \in \chi_j^0} x_i(t) \qquad (3.10)$$

The mission status functions φ_j are also binary :

$$\varphi_j(t) = \begin{cases} 1 & \text{mission } M_j \text{ available at time t} \\ 0 & \text{otherwise} \end{cases}$$

It is also possible to construct for each system mission M_j (j=1,...,n) in its k-th degraded mode of operation the corresponding k-th degraded mode mission status function :

$$\varphi_j^k(t) = [1 - \varphi_j(t)] \prod_{i \in \chi_j^k} x_i(t) \qquad (3.11)$$

which is also binary :

$$\varphi_j^k(t) = \begin{cases} 1 & \text{mission } M_j \text{ is in its k-th degraded mode at time t} \\ 0 & \text{otherwise} \end{cases}$$

The term $[1-\varphi_j(t)]$ in equation (3.11) ensures that mission M_j is not available (in the fully operational or non-degraded mode).

Moreover, a degraded mission status function can be constructed for each system mission M_j (i=1,...,n) :

$$\varphi_j^{(d)}(t) = [1-\varphi_j(t)]\coprod_{k=1}^{q_i} \varphi_i^k(t) = [1-\varphi_j(t)]\coprod_{k=1}^{q_i}\prod_{i\in\chi_j^k} x_i \tag{3.12}$$

which is also binary :

$$\varphi_j^{(d)}(t) = \begin{cases} 1 & \text{mission } M_j \text{ has at least one supporting function in a degraded mode at time t} \\ 0 & \text{otherwise} \end{cases}$$

The term $[1-\varphi_j(t)]$ in equation (3.12) ensures that mission M_j is not available (in the fully operational or non-degraded mode). Equation (3.12) states that for mission M_j to be in a degraded mode it has to be that at least the subset of elements that define one of its acceptable modes of degraded operation are all working at time t.

A system status function ψ can also be constructed :

$$\psi(t) = \prod_{i=1}^{r} x_i(t) \tag{3.13}$$

The system status function is also binary :

$$\psi(t) = \begin{cases} 1 & \text{system completely available at time t} \\ 0 & \text{otherwise} \end{cases}$$

3.5.2 Element, Function, Mission, and System Reliability Functions.

Denote the reliability of element E_i (the probability that it works, or can work, at time t) by p_i:

$$p_i\,(t) = P[\,x_i\,(t) = 1] = 1 - EF_i(t) \tag{3.14}$$

Therefore, for any system function F_j it is possible to define the function reliability function as :

$$R_{F_j} = \prod_{i \in \beta_j^0} P[x_i(t) = 1] = \prod_{i \in \beta_j^0} p_i(t) = \prod_{i \in \beta_j^0} [1 - EF_i(t)] \tag{3.15}$$

For any system mission M_j, the mission reliability function is given by :

$$R_{M_j} = \prod_{i \in \chi_j^0} P[x_i = 1] = \prod_{i \in \chi_j^0} p_i(t) = \prod_{i \in \chi_j^0} [1 - EF_i(t)] \tag{3.16}$$

Due to the system series configuration, the overall system reliability function is :

$$R = \prod_{i=1}^{r} p_i(t) = \prod_{i=1}^{r} [1 - EF_i(t)] \tag{3.17}$$

3.6 Convolution of the Time-to-Failure Distribution and the Time-to-Repair Distribution. The Renewal Process.

Every system element E_i is subject to two distributions : the time-to-failure distribution EF_i (t) and the time-to-repair distribution ER_i (t). Both the time to failure and the time to repair are random variables. The sequence of periods of operation terminated by failures and periods during which the element is repaired constitute a renewal process. A renewal process is "a sequence of independent, identically distributed, non-negative random variables which, with probability 1, are not all zero" [11]. Renewal theory is the base, together with the already derived status and reliability functions, upon which the availability formulation is constructed.

The distribution H_i (t) of the sum of the two random variables time to failure and time to repair is the convolution of their corresponding distributions [12,13] :

$$H_i(t) = \int_{0}^{t} EF_i(t - x) dER_i(x) \tag{3.18}$$

or equivalently :

$$H_i(t)=\int_0^t EF_i(x)dER_i(t-x)=\int_0^t ER_i(t-x)dEF_i(x)=\int_0^t ER_i(x)dEF_i(t-x) \quad (3.19)$$

If we denote by $N_i(t)$ the number of renewals in the time interval [0,t], the expected value of $N_i(t)$ is called the renewal function. Denoting by $M_{H_i}(t)$ the renewal function corresponding to the distribution $H_i(t)$, then [14,15,16,17] :

$$M_{H_i}(t)=E[N_i(t)] \quad (3.20)$$

The expected value of $N_i(t)$ is :

$$E[N_i(t)]=\sum_{k=1}^{\infty}P[N_i(t)\geq k] \quad (3.21)$$

The probability of at least n renewals is equal to the n-fold convolution of the underlying distribution $H_i(t)$ of the renewal process [18,19] :

$$P[N_i(t)\geq n]=H_i^{(n)}(t) \quad (3.22)$$

Therefore, the renewal function is given by :

$$M_{H_i}(t)=\sum_{k=1}^{\infty}H_i^{(n)}(t) \quad (3.23)$$

It can be extended to :

$$M_{H_i}(t) = H_i(t) + \int_0^t M_{H_i}(t-x)dH_i(x) \tag{3.24}$$

This expression is known as the fundamental renewal equation [20,21,22]. This equation permits the analysis of renewal processes. Frequently, the solution requires the utilization of the Laplace or Laplace-Stieltjes transforms.

For a given function G(t), the Laplace transform is :

$$G^*(s) = \int_0^\infty e^{-st} G(t)dt \tag{3.25}$$

and the Laplace-Stieltjes transform is :

$$G_s^*(s) = \int_0^\infty e^{-st} dG(t) \tag{3.26}$$

It is necessary to differentiate among the elapsed time and the time an element has been actually requested to work :

- t total elapsed time
- τ_k cumulative time that element E_k has been requested to work

The cumulative time τ_k that element E_k has been requested to operate can be expressed as a function of the total elapsed time t :

$$\tau_k = \left[\sum_{i=1}^{m} \frac{NM_i MT_i}{L} me_{ik} \right] t \qquad (3.27)$$

Then, the probability that an element E_i is working (or capable of) at time t is given by :

$$\Pr[x_i(t) = 1] = \overline{EF_i}(\tau_i) + \int_0^{\tau_i} M_{H_i}(\tau_i - x) d\overline{EF_i}(x) \qquad (3.28)$$

Note the distinction used in notation to represent the probability that an element is working at time t when repair is permitted ($\Pr[x_i(t)=1]$) and the notation used in equation (3.14) to represent the probability that an element is working at time t when repair has not been considered ($P[x_i(t)=1]$).

The probability that a function F_i can be undertaken at time t depends on the probability that the elements required to support it are capable of working at that time :

$$\Pr[\phi_i(t) = 1] = \prod_{j \in \beta_i^0} \Pr[x_j(t) = 1] = \prod_{j \in \beta_i^0} \left\{ \overline{EF_j}(\tau_j) + \int_0^{\tau_j} M_{H_j}(\tau_j - x) d\overline{EF_j}(x) \right\} \qquad (3.29)$$

The probability that a mission M_i can be undertaken at time t depends on the probability that the elements required to support it can work at that time :

$$\Pr[\varphi_i(t)=1] = \prod_{j\in\chi_i^0} \Pr[x_j(t)=1] = \prod_{j\in\chi_i^0} \left\{ \overline{EF}_j(\tau_j) + \int_0^{\tau_j} M_{H_j}(\tau_j - x)d\overline{EF}_j(x) \right\}$$

(3.30)

3.7 Availability Formulation.

According to the given definitions and based on the results of the previous section, the availability of a multifunctional system is expressed as follows :

$$A = \frac{\int_0^L P(t)dt}{L} \tag{3.31}$$

where P(t) is the probability that the system will be ready to undertake the required mission at time t. This can be expressed as :

$$P(t)=\sum_{i=1}^{n} \{ \ P(M_i \text{ required})\ P(M_i \text{ can be undertaken}) \} \tag{3.32}$$

The probability that mission M_i is required can be expressed as the total time the system is expected to be engaged in mission M_i over the length of the useful life of the system :

$$P(M_i \text{ is required}) = \frac{NM_i\, MT_i}{L} \tag{3.33}$$

The probability that mission M_i can be undertaken is the product of the probabilities that the elements required to support it are available. Consequently P(t) can be expressed as :

$$P(t)=\sum_{i=1}^{n} \left\{ \frac{NM_i\, MT_i}{L} \prod_{k\in\chi_i^0} \Pr[x_k(t)=1] \right\} \tag{3.34}$$

and the system availability can be expressed as :

$$A = \frac{\int_0^L \left[\sum_{i=1}^{n} \left\{ \frac{NM_i\, MT_i}{L} \prod_{k\in\chi_i^0} \Pr[x_k(t)=1] \right\} \right] dt}{L} \tag{3.35}$$

The above expression quantifies the availability of continuously-operated, coherent, multifunctional systems.

The degraded system availability is expressed as :

$$A^{(d)} = \frac{\int_0^L \left[\sum_{i=1}^{n} \left\{ \frac{NM_i\, MT_i}{L} [(1-\varphi_i)\varphi_i^{(d)}] \left[\coprod_{k=1}^{q_i} \prod_{h\in\chi_i^k} \Pr[x_h(t)=1] \right] \right\} \right] dt}{L} \tag{3.36}$$

where the factors $[(1-\varphi_i)\varphi_i^{(d)}]$ ensure that the required mission M_i is being performed with at least one of its supporting function being accomplished in a degraded mode.

The availability of mission M_i can be then expressed as :

$$Am_i = \frac{\int_0^L \Pr[\varphi_i(t)=1]dt}{L} = \frac{\int_0^L \left[\prod_{k\in\chi_i^0} \Pr[x_k(t)=1]\right]dt}{L} \tag{3.37}$$

The degraded mission M_i availability is expressed as :

$$Am_i^{(d)} = \frac{\int_0^L \left[\Pr[\varphi_i^{(d)}(t)=1\right]dt}{L}[(1-\varphi_i)\varphi_i^{(d)}] \tag{3.38}$$

That is :

$$Am_i^{(d)} = \frac{\int_0^L \left[\coprod_{k=1}^{q_i} \prod_{h\in\chi_i^k} \Pr[x_h(t)=1]\right]dt}{L}[(1-\varphi_i)\varphi_i^{(d)}] \tag{3.39}$$

where the factor $[(1-\varphi_i)\varphi_i^{(d)}]$ ensures that mission M_i is being performed with at least one of its supporting functions being accomplished in a degraded mode.

The availability of function F_i can be then expressed as :

$$Af_i = \frac{\int_0^L \Pr[\phi_i(t) = 1]dt}{L} = \frac{\int_0^L \left[\prod_{j \in \beta_i^0} \Pr[x_j(t) = 1]\right]dt}{L} \quad (3.40)$$

The degraded availability of function F_i is :

$$Af_i^{(d)} = \frac{\int_0^L \left[\coprod_{k=1}^{p_i} \prod_{j \in \beta_i^k} \Pr[x_j(t) = 1]\right]dt}{L}[(1-\phi_i)\phi_i^{(d)}] \quad (3.41)$$

where the factor $[(1-\phi_i)\phi_i^{(d)}]$ ensures that function F_i is being performed in a degraded mode.

The availability of element E_i can be then expressed as :

$$Ae_i = \frac{\int_0^L \Pr[x_i(t) = 1]dt}{L} \quad (3.42)$$

The above expressions constitute the formulation, based on the adopted definitions, of availability and degraded availability at the system, the mission, and the function level and also the availability at the element level. They are general expressions that depend on the particular failure and repair distributions of the system elements. The next sections shows particularizations of those expressions for some well-known pairs of distributions.

3.7.1 Negative Exponential Time-to-Failure Distribution and Deterministic Repair Time.

If the distribution of the times to failure is negative exponential, then the failure distribution for element E_i is :

$$EF_i(t) = 1 - e^{-\lambda_i t} \qquad (3.43)$$

where λ_i is the failure rate of the i-th element. If the repair times are deterministic, for element E_i the repair time will be RT_i. In this case the process of repairing the item is a Poisson process and the probability that it is working at time t is given by [23,24,25,26] :

$$\Pr[x_k(t) = 1] = \sum_{x=0}^{\infty} e^{-\lambda_k \tau_k} \frac{(\lambda_k \tau_k)^x}{x!} \qquad (3.44)$$

If Mo_k is the operational maintainability of E_k or probability that the repair of the element will be accomplished in less than the allowable repair time RT_k, then equation (3.44) can be rewritten as follows :

$$\Pr[x_k(t) = 1] = \sum_{x=0}^{\infty} e^{-\lambda_k \tau_k} \frac{(\lambda_k \tau_k)^x}{x!} Mo_k^x \qquad (3.45)$$

The availability of E_k is given by :

$$Ae_k = \frac{\int_0^L \left\{ \sum_{x=0}^{\infty} e^{-(\lambda_k \tau_k)} \frac{(\lambda_k \tau_k)^x}{x!} Mo_k^x \right\} dt}{L} \tag{3.46}$$

The availability of function F_k is given by equation (3.40). Since

$$\Pr[\phi_k(t) = 1] = \prod_{i \in \beta_k^0} \Pr[x_i(t) = 1] = \prod_{i \in \beta_k^0} \sum_{x=0}^{\infty} e^{-\lambda_i \tau_i} \frac{(\lambda_i \tau_i)^x}{x!} Mo_i^x \tag{3.47}$$

then

$$Af_k = \frac{\int_o^L \{ \prod_{j \in \beta_k^0} \sum_{x=0}^{\infty} e^{-(\lambda_j \tau_j)} \frac{(\lambda_j \tau_j)^x}{x!} Mo_j^x \} dt}{L} \tag{3.48}$$

Similarly, the degraded availability of function F_k is :

$$Af_k^{(d)} = \frac{\int_o^L \left[\coprod_{i=1}^{p_k} \prod_{j \in \beta_k^i} \sum_{x=0}^{\infty} e^{-\lambda_j \tau_j} \frac{(\lambda_j \tau_j)^x}{x!} Mo_j^x \right] dt}{L} [(1-\phi_k)\phi_k^{(d)}] \tag{3.49}$$

The availability of mission M_k is given by equation (3.37). Since

$$\Pr[\varphi_k(t) = 1] = \prod_{j \in \chi_k^0} \Pr[x_j(t) = 1] = \prod_{j \in \chi_k^0} \sum_{x=0}^{\infty} e^{-(\lambda_j \tau_j)} \frac{(\lambda_j \tau_j)^x}{x!} Mo_j^x \tag{3.50}$$

then :

$$\mathrm{Am}_k = \frac{\int_0^L \{\prod_{j \in \chi_k^0} \sum_{x=0}^{\infty} e^{-(\lambda_j \tau_j)} \frac{(\lambda_j \tau_j)^x}{x!} \mathrm{Mo}_j^x\} dt}{L} \tag{3.51}$$

Similarly, the degraded availability of mission M_k is :

$$\mathrm{Am}_k^{(d)} = \frac{\int_0^L \left\{ \coprod_{l=1}^{q_k} \prod_{h \in \chi_k^l} \sum_{x=0}^{\infty} e^{-\lambda_h \tau_h} \frac{(\lambda_h \tau_h)^x}{x!} \mathrm{Mo}_h^x \right\} dt}{L} [(1-\varphi_k)\varphi_k^{(d)}] \tag{3.52}$$

The availability of the system is given by equation (3.35). Since

$$P(t) = \sum_{i=1}^{n} \{ \frac{\mathrm{NM_i\, MT_i}}{L} \prod_{k \in \chi_i^0} \Pr[x_k(t) = 1] \}$$

and

$$\Pr[x_k(t) = 1] = \sum_{x=0}^{\infty} e^{-\lambda_k \tau_k} \frac{(\lambda_k \tau_k)^x}{x!} \mathrm{Mo}_k^x$$

it follows that :

$$A = \frac{\int_0^L \left[\sum_{i=1}^{n} \left\{ \frac{NM_i MT_i}{L} \prod_{k \in \chi_i^0} \sum_{x=0}^{\infty} e^{-(\lambda_k \tau_k)} \frac{(\lambda_k \tau_k)^x}{x!} Mo_k^x \right\} \right] dt}{L} \tag{3.53}$$

Similarly, the degraded availability of the system is :

$$A^{(d)} = \frac{\int_0^L \left[\sum_{i=1}^{n} \frac{NM_i MT_i}{L} [(1-\varphi_i)\varphi_i^{(d)}] \prod_{k=1}^{q_i} \prod_{h \in \chi_i^k} \sum_{x=0}^{\infty} e^{-(\lambda_h \tau_h)} \frac{(\lambda_h \tau_h)^x}{x!} Mo_h^x \right] dt}{L} \tag{3.54}$$

3.7.2. Negative Exponential Time-to-Failure Distribution and Negative Exponential Time-to-Repair Distribution.

If we assume that both the failure time and the repair time for element E_k follow a negative exponential distribution, then the probability that element E_k is working at time t is a well known result of renewal theory (see equation (2.5)),

$$\Pr[x_k(t) = 1] = \frac{\mu_k}{\mu_k + \lambda_k} + \frac{\lambda_k}{\mu_k + \lambda_k} e^{-(\lambda_k + \mu_k)\tau_k}$$

and therefore the availability of element E_k is given by

$$Ae_k = \frac{\int_0^L \left\{ \frac{\mu_k}{\mu_k + \lambda_k} + \frac{\lambda_k}{\mu_k + \lambda_k} e^{-(\lambda_k + \mu_k)\tau_k} \right\} dt}{L} \tag{3.55}$$

The availability of function F_k is given by equation (3.40). Since

$$\Pr[\phi_k(t) = 1] = \prod_{i \in \beta_k^0} \Pr[x_i(t) = 1] = \prod_{i \in \beta_k^0} \left\{ \frac{\mu_i}{\mu_i +_i} + \frac{\lambda_i}{\mu_i + \lambda_i} e^{-(\lambda_i + \mu_i)\tau_i} \right\}$$

then

$$Af_k = \frac{\int_0^L \left[\prod_{i \in \beta_k^0} \left\{ \frac{\mu_i}{\mu_i + \lambda_i} + \frac{\lambda_i}{\mu_i + \lambda_i} e^{-(\lambda_i + \mu_i)\tau_i} \right\} \right] dt}{L} \tag{3.56}$$

Similarly, the degraded availability of function F_k is :

$$Af_k^{(d)} = \frac{\int_0^L \left[\coprod_{i=1}^{p_k} \prod_{j \in \beta_k^i} \left\{ \frac{\mu_j}{\mu_j + \lambda_j} + \frac{\lambda_j}{\mu_j + \lambda_j} e^{-(\lambda_j + \mu_j)\tau_j} \right\} \right] dt}{L} [(1 - \varphi_k)\varphi_k^{(d)}] \tag{3.57}$$

The availability of mission M_k is given by equation (3.37). Since

$$\Pr[\varphi_k(t) = 1] = \prod_{i \in \chi_j^0} \Pr[x_i(t) = 1] = \prod_{i \in \chi_j^0} \left\{ \frac{\mu_i}{\mu_i + \lambda_i} + \frac{\lambda_i}{\mu_i + \lambda_i} e^{-(\lambda_i + \mu_i)\tau_i} \right\}$$

then,

$$\mathrm{Am}_k = \frac{\int_0^L \left[\prod_{j \in \chi_k^0} \left[\frac{\mu_j}{\mu_j + \lambda_j} + \frac{\lambda_j}{\mu_j + \lambda_j} e^{-(\lambda_j + \mu_j)\tau_j} \right] \right] \mathrm{dt}}{\mathrm{L}} \qquad (3.58)$$

Similarly, the degraded availability of mission M_k is :

$$\mathrm{Am}_k^{(d)} = \frac{\int_0^L \left\{ \prod_{i=1}^{q_k} \prod_{j \in \chi_k^i} \left\{ \frac{\mu_j}{\mu_j + \lambda_j} + \frac{\lambda_j}{\mu_j + \lambda_j} e^{-(\lambda_j + \mu_j)_j} \right\} \right\} \mathrm{dt}}{\mathrm{L}} [(1-\varphi_k)\varphi_k^{(d)}] \qquad (4.59)$$

The availability of the system is given by equation (3.35). Since

$$\mathrm{P(t)} = \sum_{i=1}^{n} \left\{ \frac{\mathrm{NM_i\ MT_i}}{\mathrm{L}} \prod_{k \in \chi_i^0} \Pr[\mathrm{x_k(t)} = 1] \right\}$$

and

$$\Pr[\mathrm{x_k(t)} = 1] = \frac{\mu_k}{\mu_k + \lambda_k} + \frac{\lambda_k}{\mu_k + \lambda_k} e^{-(\lambda_k + \mu_k)\tau_k}$$

it follows that :

$$A = \frac{\int_0^L \left[\sum_{i=1}^{n} \left\{ \frac{NM_i MT_i}{L} \prod_{k \in \chi_i^0} \left\{ \frac{\mu_k}{\mu_k + \lambda_k} + \frac{\lambda_k}{\mu_k + \lambda_k} e^{-(\lambda_k + \mu_k)\tau_k} \right\} \right\} \right] dt}{L} \qquad (3.60)$$

Similarly, the degraded availability of the system is :

$$A^{(d)} = \frac{\int_0^L \left[\sum_{i=1}^{n} \left\{ \frac{NM_i MT_i}{L} [(1-\varphi_i)\varphi_i^{(d)}] \coprod_{k=1}^{q_i} \prod_{h \in \chi_i^k} [\frac{\mu_h}{\mu_h + \lambda_h} + \frac{\lambda_h}{\mu_h + \lambda_h} e^{-(\lambda_h + \mu_h)\tau_h}] \right\} \right] dt}{L} \qquad (3.61)$$

3.7.3 Weibull Time-to-Failure Distribution and Negative Exponential Time-to-Repair Distribution.

The convolution of a Weibull distribution and a negative exponential distribution is at least approximately a Weibull distribution [27]. Moreover, the renewal function for the underlying Weibull distribution can be represented by the Lomnicki approximation. Therefore, the case of a Weibull time-to-failure distribution and a negative exponential time-to-repair distribution can be solved numerically, using the general expressions derived in Section 3.7.

In this case, for all system elements the time-to-failure distribution and the time-to-repair distribution are given by :

$$EF_i = 1 - e^{-(\alpha_i t^{\beta_i})}$$

$$ER_i = 1 - e^{-(\mu_i t)}$$

Let us consider a system such that the time to failure of a certain element E_i exhibits a Weibull distribution with the following parameters :

$$\begin{cases} \alpha_i = 1.5 \\ \beta_i = 1.25 \end{cases} \quad (3.62)$$

The mean of that distribution is given by [28] :

$$m_i = \alpha_i^{-\left(\frac{1}{\beta_i}\right)} \Gamma(1 + \frac{1}{\beta_i}) \quad (3.63)$$

Therefore,

m_i=0.6734

The characteristic life of the distribution is the time t that satisfies :

$$\mathrm{Ln}(\alpha_i) + \beta_i \,\mathrm{Ln}(t) = 0 \quad (3.64)$$

Therefore,

$Ln(1.50) + 1.25\ Ln(t) = 0$

and

$t = 0.723.$

For the purpose of constructing a plausible example let us select the parameter of the negative exponential distribution representing the behavior of the time to repair such that it is approximately twenty times larger than the reciprocal of the mean of the Weibull distribution. Then :

$\mu_i = 30$

The next step is to compute equation (3.38), which in this particular case can be rewritten as follows :

$$H_i(t) = \int_0^t [(1 - e^{-\alpha_i (t-x)^{\beta_i}})\mu_i e^{-\mu_i x} dx \qquad (3.65)$$

Equation (3.65) is solved numerically for several values of t. The program used to obtain the values of $H_i(t)$ is listed in Appendix I and the results are shown in Table 3.1.

Table 3.1 - Calculated Values of $H_i(t)$.

t	$H_i(t)$
0.20	0.14809768
0.25	0.19897660
0.30	0.24971871
0.35	0.29952155
0.40	0.34786355
0.45	0.39439691
0.50	0.43889224
0.55	0.48120551
0.60	0.52125634
0.65	0.55901260
0.70	0.59447900

The values of the parameters alpha and beta that approximate that distribution are obtained by the Weibull distribution linear regression estimation method in which :

$$\mathrm{Ln}\left(\mathrm{Ln}\left(\frac{1}{1-\mathrm{H(t)}}\right)\right)=\mathrm{Ln}(\alpha)+\beta\,\mathrm{Ln(t)} \tag{3.66}$$

The linear regression between variables x and y is defined by :

$$y = a + bx \tag{3.67}$$

where a and b, and the correlation coefficient are obtained by the method of least squares as follows [29]:

$$b=\frac{n\sum_{i=1}^{n}x_i y_i-\left(\sum_{i=1}^{n}x_i\right)\left(\sum_{i=1}^{n}y_i\right)}{n\sum_{i=1}^{n}x_i^2-\left(\sum_{i=1}^{n}x_i\right)^2} \tag{3.68}$$

and

$$a=\frac{\sum_{i=1}^{n}y_i-b\sum_{i=1}^{n}x_i}{n} \tag{3.69}$$

The correlation coefficient is given by :

$$r = \frac{n\sum_{i=1}^{n} x_i y_i - \left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)}{\sqrt{\left\{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2\right\}\left\{n\sum_{i=1}^{n} y_i^2 - \left(\sum_{i=1}^{n} y_i\right)^2\right\}}} \tag{3.70}$$

Using the program listed in Appendix II and the calculated values of $H_i(t)$, the values obtained for the parameters of the resulting Weibull distribution are :

$$\begin{cases} \alpha_{H_i} = 1.491 \\ \beta_{H_i} = 1.374 \end{cases}$$

The corresponding correlation coefficient is :

r=0.999

and consequently $H_i(t)$ can be expressed as :

$$H_i(t) = 1 - e^{-\alpha_{H_i}(t)^{\beta_{H_i}}} = 1 - e^{-1.491(t)^{1.374}} \tag{3.71}$$

The renewal function of a weibull distribution can be expressed through the Lomnicki approximation [30].

Therefore, the renewal function can be written as :

$$M_H(t) = e^{-\alpha_H (t)^{\beta_H}} \sum_{k=1}^{\infty} \left[\frac{(\alpha_H t^{\beta_H})^k}{k!} \sum_{s=1}^{k} c(s) \right] \tag{3.72}$$

where c(s) are the coefficients of the Lomnicki approximation. The Lomnicki approximation is summarized in Appendix III and a program written to compute the coefficients is shown in Appendix IV. For the obtained values of α_H and β_H the first fifteen coefficients of the Lomnicki approximation are :

c(1) = 1.00000000	c(9) =0.44249046
c(2) = 0.67612016	c(10)=0.42943418
c(3) = 0.61144459	c(11)=0.41747487
c(4) = 0.55988199	c(12)=0.40691939
c(5) = 0.52397430	c(13)=0.39623702
c(6) = 0.49690351	c(14)=0.38345894
c(7) = 0.47534198	c(15)=0.36494642
c(8) = 0.45754746	

Equation (3.72) involves a summation of infinite terms. Nevertheless, since terms become smaller it is possible to truncate it at the desired level of accuracy. Fifteen terms is usually enough, because :

$$\frac{1}{15\,!} = 7.6 \times 10^{-13}$$

which is small enough for most practical purposes.With the renewal function obtained by substititing the corresponding values in equation (3.72), it is possible to obtain by numerical integration the value of the probability of a certain element working by a certain time as given by equation (3.29). Once the probability of any element working as a function of time is known, all the developed availability figures of merit can be obtained by numerical integration, using equations (3.35) through (3.41).

3.7.4 Rayleigh Time-to-Failure Failure Distribution and Negative Exponential Time-to-Repair Distribution.

The Rayleigh distribution is a Weibull distribution in which the shape parameter is [31]:

$$\beta = 2$$

The case of a Rayleigh time-to-failure distribution and negative exponential time- to-repair distribution can be then treated as the case of a Weibull time-to-failure distribution with $\beta = 2$ and negative exponential time-to-repair distribution, shown in the previous section.

3.8 Single Purpose Systems.

The general results obtained in Section 3.7 can be particularized for the case of a system performing only one type of mission. In the case that only one type of mission can be assigned to the system, then :

$$\alpha = \{i / i = 1,\dots,s\} \qquad (3.73)$$

and also

$$\chi = \{i / i = 1,\dots,r\} \qquad (3.74)$$

with β_i^0 and β_i^k still defined by equations (3.3) and (3.4), respectively.

Then the general expressions for availability and degraded availability derived in Section 3.7 are expressed as follows.

Since there is only one mission, the availability of that unique mission will be the availability of the entire system. Then, the availability at the system (equal to that of the unique mission) is :

$$A = \frac{\int_0^L \left\{ \prod_{i=1}^{r} \Pr[x_i(t) = 1] \right\} dt}{L} \qquad (3.75)$$

The degraded availability at the system (unique mission) level is :

$$A^{(d)} = \frac{\int_0^L \left\{ \coprod_{k=1}^{q} \prod_{h \in \chi^k} \Pr[x_h(t) = 1] \right\} dt}{L} \qquad (3.76)$$

The availability of function F_i (Af_i), the degraded availability of function F_i ($Af_i^{(d)}$), and the availability of element E_i (Ae_i) are still defined by equations (3.40), (3.41) and (3.42) respectively.

If availability had been defined as point availability and not average availability, equation (3.75) would be :

$$A(t) = \left\{ \prod_{i=1}^{r} \Pr[x_i(t) = 1] \right\} \tag{3.77}$$

Under the traditional availability formulation, the availability at time t of a series system is the product of the availabilities at that time of its elements. That is, it is the product of the probabilities that the system elements are working at time t. That is exactly what equation (3.77) represents.

In the particular case that both the time to failure and the time to repair for all elements follow negative exponential distributions, then equation (3.77) can be rewritten as :

$$A(t) = \prod_{i=1}^{r} \left\{ \frac{\mu_i}{\mu_i + \lambda_i} + \frac{\lambda_i}{\mu_i + \lambda_i} e^{-(\lambda_i + \mu_i)t} \right\} \tag{3.78}$$

Equation (3.78) is the same expression obtained in the traditional availability formulation for the point availability of series systems whose elements exhibit a time to failure and a time to repair following negative exponential distributions (which is widely referenced in Section 2.2).

It can be therefore concluded that the traditional availability formulation is a special case of the proposed model when the considered system has been designed to accomplish only one type of mission.

3.9 Formulation of Related Concepts.

Two system effectiveness figures of merit closely related to availability are mission reliability and mission dependability. They were defined in Section 3.3, and the developed model enables their formulation.

3.9.1 Mission Dependability.

According to the given definition, the dependability of mission M_i is given by :

$$Dm_i^{[T_1,T_2]} = \left\{ \prod_{k \in \chi_i^0} \Pr[x_k(T_1) = 1] \right\} \frac{\int_{T_1}^{T_2} \prod_{k \in \chi_i^0} \Pr[x_k(t) = 1] \, dt}{T_2 - T_1} \tag{3.79}$$

The first term in the right-hand side of the above expression reflects the requirement that the system be working at the time the mission is required; the second term reflects the average probability that the mission will be completed, allowing for repair of failed elements.

Since the concept of mission dependability requires that system be restored after a failure to proper operating condition in less than a given allowable repair time (which will happen with a probability called operational maintainability), the probability that an element is working at time t is still given by equation (3.28), but the renewal function given in equation (3.23) has to be rewritten as follows :

$$M_{H_i}(t) = \sum_{k=1}^{\infty} \{H_i^{(n)}(t)\, Mo_i^n\} \tag{3.80}$$

3.9.2 Mission Reliability.

According to the given definition, the mission reliability of mission M_i is given by :

$$MRm_i^{[T_1,T_2]} = \left\{ \prod_{k \in \chi_i^0} Pr[x_k(T_1) = 1] \right\} \frac{\int_{T_1}^{T_2} \prod_{k \in \chi_i^0} P[x_k(t) = 1]\, dt}{T_2 - T_1} \tag{3.81}$$

The first term in the right-hand side of the above expression reflects the requirement that the system be working at the time the mission is required; the second term reflects the average probability that the mission will be completed, when repair of failed elements is not permitted.

CHAPTER 4 - EXAMPLES

4.1 Introduction.

Two examples are provided to show the applicability of the developed model. In order to be able to apply also the traditional approach and thus compare the results, it is assumed that all the elements of the selected system exhibit a negative exponential distribution for both the time to failure and the tine to repair.

4.2 Frigate Example.

Let us select a frigate as the multifunctional system to which to apply the developed availability formulation. The calculations are performed with the program written for that purpose. The program is listed in Appendix V. The frigate may be assigned different missions and each of those missions requires the performance of different functions. Each function is supported by certain system elements. Specifically, the frigate may be assigned the following seven types of missions :

1) Seagoing training.
2) Surface combat training.
3) Submarine combat training.
4) Air combat training.

5) Combat training.

6) Patrol.

7) Escort aircraft/carrier.

The following five system functions are assumed to be required to support those missions :

1) Anti-surface warfare capability.

2) Anti-submarine warfare capability.

3) Anti-air warfare capability.

4) Command and control.

5) Propulsion.

Thirteen elements are required to support the above-mentioned functions :

1) Surface radar.

2) Combat information center.

3) Gun.

4) Sonar.

5) Depth charge.

6) Surveillance radar.

7) Machine gun.

8) Navigation radar.

9) Rudder.

10) Main Control Paanel.

11) Engines.

12) Propeller.

13) Reduction gear.

The specific mission-function and function-element support requirements are shown in Tables 4.1 and 4.2, respectively.

Table 4.1 - Frigate Mission-Function Support Requirements.

Functions / Missions	Antisurface warfare capability	Antisubmarine warfare capability	Antiair warfare capability	Command and control	Propulsion
Seagoing training	0	0	0	1	1
Surface combat training	1	0	0	1	1
Submarine combat training	0	1	0	1	1
Air combat training	0	0	1	1	1
Combat training	1	1	1	1	1
Patrol	1	1	1	1	1
Escort aircraft/ carrier	1	1	0	1	1

Table 4.2 - Frigate Function-Element Support Requirements.

Functions / Elements	Antisurface warfare capability	Antisubmarine warfare capability	Antiair warfare capability	Command and control	Propulsion
Surface radar	1	0	0	0	0
Combat information center	1	1	1	0	0
Gun	1	0	0	0	0
Sonar	0	1	0	0	0
Depth charge	0	1	0	0	0
Surveillance radar	0	0	1	0	0
Machine gun	0	0	1	0	0
Navigation radar	0	0	0	1	0
Rudder	0	0	0	1	0
Main control panel	0	0	0	1	0
Engines	0	0	0	0	1
Propeller	0	0	0	0	1
Reduction gear	0	0	0	0	1

Assumed values of the failure rate and of the reciprocal of the time to repair of each element are shown in Table 4.3.

The program is run for five sets of number of times each type of mission is requested and of average mission type duration. The number of times each mission is requested and its average duration determine the length of the useful life of the system. The five mentioned sets are selected so that the corresponding useful lives are, approximately, 5, 10, 15, 20 and 25 years. The five sets are shown in Tables 4.4, 4.5, 4.6, 4.7 and 4.8, respectively.

The program computes three values : the availability of the system based on the developed model; the average availability based on the traditional approach; and the limiting availability based on the traditional approach. The results are shown in Table 4.9. As can be seen, the traditional limiting availability approach does not consider the length of the useful life. The traditional average availability is higher than the traditional limiting availability and depends on the length of the useful life. Moreover the availability predictions based on the developed model are always higher as anticipated by the fact that not all the elements are required to support the specific mission being undertaken at any point in time.

Table 4.3 - Frigate Elements Failure Rate and Reciprocal of Time to Repair.

Element	Lambda (1/hours)	Mu (1/hours)
Surface radar	0.000250	0.333
Combat information center	0.000694	0.166
Gun	0.000198	0.083
Sonar	0.000375	0.200
Depth charge	0.000210	0.100
Surveillance radar	0.000278	0.333
Machine gun	0.000263	0.140
Navigation radar	0.000225	0.333
Rudder	0.000057	0.013
Main control panel	0.000555	2.000
Engines	0.001025	0.450
Propeller	0.000008	0.009
Reduction gear	0.000091	0.250

Table 4.4 - Frigate Set # 1.

Mission	Number of Times	Average Duration (hours)
Seagoing training	35	240
Surface combat training	14	120
Submarine combat training	14	120
Air combat training	14	120
Combat training	23	170
Patrol	7	720
Escort aircraft/carrier	24	1080

Table 4.5 - Frigate Set # 2.

Mission	Number of Times	Average Duration (hours)
Seagoing training	85	240
Surface combat training	30	120
Submarine combat training	30	120
Air combat training	30	120
Combat training	55	170
Patrol	15	720
Escort aircraft/carrier	40	1080

Table 4.6 - Frigate Set # 3.

Mission	Number of Times	Average Duration (hours)
Seagoing training	125	240
Surface combat training	45	120
Submarine combat training	45	120
Air combat training	45	120
Combat training	80	170
Patrol	22	720
Escort aircraft/carrier	60	1080

Table 4.7 - Frigate Set # 4.

Mission	Number of Times	Average Duration (hours)
Seagoing training	170	240
Surface combat training	57	120
Submarine combat training	57	120
Air combat training	57	120
Combat training	105	170
Patrol	30	720
Escort aircraft/carrier	75	1080

Table 4.8 - Frigate Set # 5.

Mission	Number of Times	Average Duration (hours)
Seagoing training	190	240
Surface combat training	70	120
Submarine combat training	70	120
Air combat training	70	120
Combat training	115	170
Patrol	35	720
Escort aircraft/carrier	95	1080

Table 4.9 - Frigate Results.

Set	Useful life length (years)	Availability (developed model)	Average availability (traditional approach)	Limiting availability (traditional approach)
1	5	0.9821196059	0.9774298344	0.9774158558
2	10	0.9824424973	0.9774229981	0.9774158558
3	15	0.9823959862	0.9774206643	0.9774158558
4	20	0.9824706586	0.9774195710	0.9774158558
5	25	0.9823310359	0.97741899514	0.9774158558

4.3 Automobile Example.

Let us consider now an automobile designed so that people can drive to the office and back home twice per day : to the office early in the morning, back home at mid day, to the office again, and back home in the late afternoon. There are eight different missions the automobile driver can undertake :

1) Drive in spring with daylight conditions.
2) Drive in spring with night-time conditions.
3) Drive in summer with daylight conditions.
4) Drive in summer with night-time conditions.
5) Drive in fall with daylight conditions.
6) Drive in fall with night-time conditions.
7) Drive in winter with daylight conditions.
8) Drive in winter with night-time conditions.

The following five system functions are assumed to be required to support those missions :

1) Heating.
2) Air conditioning.
3) Head illumination.
4) Power supply.
5) Steering.

Nine elements are required to support those functions :

1) Heat-system heat exchanger.
2) Compressor.
3) Air conditioning-system heat exchanger.
4) Left headlight.
5) Right headlight.
6) Transmission.
7) Engine.
8) Steering wheel.
9) Wheels.
10) Shift.

The specific mission-function and function-element support requirements are shown in Tables 4.10 and 4.11, respectively.

Assumed values of the failure rate and of the reciprocal of the time to repair of each element are shown in Table 4.12.

The program is run for four sets of number of times each type of mission is requested and of average mission type duration. The four sets are selected so that the corresponding useful lives are, approximately, 4, 6, 9 and 15 years. The four sets are shown in Tables 4.13, 4.14, 4.15 and 4.16, respectively.

The results are shown in Table 4.17. Again, the availability values obtained with the developed model are higher than those obtained with the traditional approach. Also, the differences between the traditional average availability and the traditional limiting availability are shown again.

Table 4.10 - Automobile Mission-Function Support Requirements.

Functions / Missions	Heating	Air conditioning	Head illumination	Power	Steering
Drive spring daylight	0	0	0	1	1
Drive spring night time	0	0	1	1	1
Drive summer daylight	0	1	0	1	1
Drive summer night time	0	0	1	1	1
Drive fall daylight	0	0	0	1	1
Drive fall night time	0	0	1	1	1
Drive winter daylight	1	0	0	1	1
Drive winter night time	1	0	1	1	1

Table 4.11 - Automobile Function-Element Support Requirements.

Functions / Elements	Heating	Air conditioning	Headlights	Power	Steering
Heating-S. heat exchanger	1	0	0	0	0
Compressor	0	1	0	0	0
Air cond.-S. heat exchanger	0	1	0	0	0
Right headlight	0	0	1	0	0
Left headlight	0	0	1	0	0
Transmission	0	0	0	1	0
Engine	0	0	0	1	0
Steering wheel	0	0	0	0	1
Wheels	0	0	0	0	1
Shift	0	0	0	0	1

Table 4.12 - Automobile Elements Failure Rate and Reciprocal of Time to Repair.

Element	Lambda (1/hours)	Mu (1/hours)
Heating-system heat exchanger	0.000462	0.150
Compressor	0.000750	0.312
Air conditioning-system heat exchanger	0.000462	0.150
Right headlight	0.000833	1.500
Left headlight	0.000833	1.500
Transmission	0.000312	0.166
Engine	0.001531	0.357
Steering wheel	0.000157	1.900
Wheels	0.000612	3.000
Shift	0.001047	0.575

Table 4.13 - Automobile Set # 1.

Mission	Number of Times	Average Duration (hours)
Drive spring daylight	800	0.40
Drive spring night time	800	0.50
Drive summer daylight	800	0.40
Drive summer night time	800	0.50
Drive fall daylight	800	0.40
Drive fall night time	800	0.50
Drive winter daylight	800	0.55
Drive winter night time	800	0.65

Table 4.14 - Automobile Set # 2.

Mission	Number of Times	Average Duration (hours)
Drive spring daylight`	1150	0.40
Drive spring night time	1150	0.50
Drive summer daylight	1150	0.40
Drive summer night time	1150	0.50
Drive fall daylight	1150	0.40
Drive fall night time	1150	0.50
Drive winter daylight	1150	0.55
Drive winter night time	1150	0.65

Table 4.15 - Automobile Set # 3.

Mission	Number of Times	Average Duration (hours)
Drive spring daylight	1680	0.40
Drive spring night time	1680	0.50
Drive summer daylight	1680	0.40
Drive summer night time	1680	0.50
Drive fall daylight	1680	0.40
Drive fall night time	1680	0.50
Drive winter daylight	1680	0.55
Drive winter night time	1680	0.65

Table 4.16 - Automobile Set # 4.

Mission	Number of Times	Average Duration (hours)
Drive spring daylight	2800	0.40
Drive spring night time	2800	0.50
Drive summer daylight	2800	0.40
Drive summer night time	2800	0.50
Drive fall daylight	2800	0.40
Drive fall night time	2800	0.50
Drive winter daylight	2800	0.55
Drive winter night time	2800	0.65

Table 4.17 - Automobile Results.

Set	Useful life length (years)	Availability (developed model)	Average availability (traditional approach)	Limiting availability (traditional approach)
1	4	0.9900138411	0.9822462347	0.9822192162
2	6	0.9899094356	0.9822380117	0.9822192162
3	9	0.9901404403	0.9826889339	0.9822192162
4	15	0.9901338403	0.9822312064	0.9822192162

CHAPTER 5 - CONCLUSIONS AND EXTENSIONS

5.1 Conclusions.

The objective of developing a mathematical model for predicting the availability of multifunctional systems has been met. The developed model enables the estimation of availability figures of merit at the system, mission, function, and element level. Moreover, degraded availability is also estimated at the system, mission and function level. Those estimations are valid for any distributions of the time to failure and the time to repair of the system's elements.

The assumptions that are required for the development of the availability definition and formulation reduce slightly the applicability of the results obtained. Nevertheless, the consideration of the multifunctional feature capability and the development of the availability formulation for any cumulative distribution functions for the time to failure and the time to repair of each system element provide a more accurate and comprehensive treatment of the problem.

In addition to that, the developed model enables the definition and formulation of related concepts such as mission reliability and mission dependability.

The apparent difficulty in dealing with the derived mathematical expressions can be easily overcome with the computational power of personal computers and programming languages. This is shown in the examples developed in Chapter 4 to illustrate the differences of the results obtained using traditional approach and by the proposed model, and in the treatment of the Weibull distribution of times to failure and negative exponential distribution of times to repair shown in Section 3.7.3.

As the examples of Chapter 4 show, the new approach offers availability figures of merit for multifunctional systems higher than those yielded by the traditional approach. Those results support and reinforce the statements made relative to the inadequacy of the traditional approach for representing the behavior of multifunctional systems from an availability point of view.

5.2 Extensions.

There are many ways to further improve the results obtained and shown in this thesis. The following are ways that, dealt with on an individual basis or in combinations, would improve the existing formulation :

1) Study more pairs of distributions (distribution of time to failure and distribution of time to repair) to complement the current set of pairs of distributions for which the general expressions have been particularized.

2) Consider that the elements that do not work while a certain mission that does not require their support is being performed age and introduce the appropriate quantitative

factors to reflect the effects of that aging on the element, function, mission, and overall system availability.

3) Consider systems arranged in ways other than in series, specifically parallel structures and k-out-of-n structures.

4) Consider that the functions required to support a particular mission or the elements required to support a particular function do not have to operate throughout the entire duration of the mission or the function, respectively.

5) Consider that the system elements can be in a degraded state rather than having their states defined in a binary manner.

6) Consider the case of multifunctional systems subject to intermittent operation.

7) Study the figures of merit that would be meaningful in the case of one-shot systems.

8) Consider the probability of each of the elements of logistic support of the prime equipment being available, so that the support environment is no longer considered ideal.

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APPENDIX I - PROGRAM TO CALCULATE THE VALUES OF THE CONVOLUTION OF A WEIBULL DISTRIBUTION AND A NEGATIVE EXPONENTIAL DISTRIBUTION

```
*
* INTEGRA   This program obtains H(t), the convolution of two
*           given distributions G1(t) and G2(t). G1(t) is a
*           weibull distribution, and G2(t) is a negative
*           exponential distribution.
*
*           Written in dBase IV by Alberto Sols in May 1992.

clear

* Settings

set talk off
set score off
set stat off
set deci to 8

use weibneg

* Declaration of public variables

public interval1,area1,limit,step,time1,time2
public abcisa1,ordinate1

* Declaration and initialization of variables

alfa=1.5
beta=1.25
mu=30
```

```
ordinate1=0
abcisa1=0
time1=0
time2=0
limit=0
step=0
interval1=0
area1=0

* Heading display and input of data

@ 2,2 say "INTEGRA Generates Series of H(t)"
@ 4,2 say "Time T1   : "
@ 4,13 get time1 picture "999999.999999"
@ 5,2 say "Time T2   : "
@ 5,13 get time2 picture "999999.999999"
@ 6,2 say "Step      : "
@ 6,13 get step picture "999999.999999"
@ 7,2 say "Interval1 : "
@ 7,13 get interval1 picture "999999.999999"
read

* Loop to calculate value of convolution function for a series
* of values of time t

limit=time1
do while .not. limit>time2
 abcisa1=0
 area1=0
 ordinate1=0
 @ 18,10 say "Limit       : "
 @ 18,22 say limit
 do while abcisa1<limit
   do gfuncion
   area1=area1+interval1*ordinate1
   @ 20,10 say "Abcisa1   : "
   @ 20,22 say abcisa1 picture "999999999999.99999999"
   @ 21,10 say "Ordinate1 : "
   @ 21,22 say ordinate1 picture "999999999999.99999999"
   @ 22,10 say "Area1     : "
   @ 22,22 say area1 picture "999999999999.99999999"
   abcisa1=abcisa1+interval1
 enddo
```

```
 append blank
 replace t with limit,ht with area1
 limit=limit+step
enddo
close all
quit

* Definition of function to be integrated

procedure gfuncion
ordinate1=(1-exp(-alfa*(limit-abcisa1)^beta))*mu*exp(-mu*abcisa1)
return
```

APPENDIX II - PROGRAM TO CALCULATE THE PARAMETERS OF THE WEIBULL DISTRIBUTION RESULTING OF THE CONVOLUTION OF A WEIBULL DISTRIBUTION AND A NEGATIVE EXPONENTIAL DISTRIBUTION

```
*
* WBNXLR  Program to calculate alfa and beta for the distribution
*         resulting of the convolution of a weibull distribution and
*         a negative exponential distribution.
*
*         Written in dBase IV by Alberto Sols in May 1992.

use weibneg
clear

* Settings

set stat off
set score off
set talk off
set deci to 8

* Declaration and initialization of variables

n=0
px=0
py=0
sigx=0
sigy=0
sigxy=0
sigx2=0
sigy2=0
```

```
r=0

n=reccount()

* Loop to calculate the double natural logarithm

go top
do while .not. eof()
  px=t
  py=ht
  qx=log(px)
  qy=log(log(1/(1-ht)))
  replace nept with qx,nepnep with qy
  skip
enddo

go top

* Loop to calculate the required parameters of the linear regression method (least squares)

do while .not. eof()
  py=nepnep
  px=nept
  @ 10,2 say "py : "
  @ 11,2 say "px : "
  @ 10,7 say px
  @ 11,7 say py
  sigx=sigx+px
  sigy=sigy+py
  sigxy=sigxy+px*py
  sigx2=sigx2+px*px
  sigy2=sigy2+py*py
  @ 12,2 say "sigx : "
  @ 12,9 say sigx
  @ 13,2 say "sigy : "
  @ 13,9 say sigy
  @ 14,2 say "sigxy: "
  @ 14,9 say sigxy
  @ 15,2 say "sigx2: "
  @ 15,9 say sigx2
  @ 16,2 say "sigy2: "
  @ 16,9 say sigy2
  skip
```

```
enddo

* Calculation of coefficients

b=(n*sigxy-sigx*sigy)/(n*sigx2-(sigx)^2)
a=(sigy-b*sigx)/n
r=(n*sigxy-sigx*sigy)/(((n*sigx2-sigx*sigx)*(n*sigy2-sigy*sigy))^0.5)
alfa=exp(a)

* Display of results

@ 4,4 say "Beta : "
@ 5,4 say "Alfa : "
@ 6,4 say "a    : "
@ 4,11 say b picture "999999.99999999"
@ 5,11 say alfa picture "999999.99999999"
@ 6,11 say a picture "999999.99999999"
@ 7,4 say "r    : "
@ 7,11 say r picture "999999.99999999"
quit
```

APPENDIX III - LOMNICKI APPROXIMATION TO WEIBULL RENEWAL FUNCTIONS

The coefficients of the Lomnicki approximation are calculated as follows :

$$s \geq k + 1 > 0$$

$$\gamma(s) = \frac{\Gamma(\beta s + 1)}{\Gamma(s + 1)}$$

$$b_0(s) = \gamma(s)$$

$$b_{k+1}(s) = \sum_{i=k}^{s-1} b_k(i)\gamma(s - i)$$

$$a_k(s) = \sum_{i=k}^{s} (-1)^{(k+i)} \binom{s}{i} \frac{b_k(i)}{\gamma(i)}$$

$$\phi_k(k) = a_k s(k)$$

$$\phi_k(s) = \sum_{i=k}^{s} a_i(s) - \sum_{i=k}^{s-1} a_i(s - 1)$$

$$c(s) = \sum_{k=1}^{s} \phi_k(s)$$

$$D_j(\alpha t^{\beta}) = \sum_{k=j}^{\infty} P_k(\alpha t^{\beta})$$

$$P_k(\alpha t^{\beta}) = \frac{(\alpha t^{\beta})^k}{k!} e^{-(\alpha t^{\beta})}$$

$$H(t) = \sum_{s=1}^{\infty} c(s) D_s(\alpha t^{\beta})$$

APPENDIX IV - PROGRAM TO CALCULATE THE COEFFICIENTS OF THE LOMNICKI APPROXIMATION

```
program Lomnicki;
{
 Program written to compute the coefficients of the Lomnicki
 approximation to Weibull renewal functions.

 Written by Alberto Sols in July 1992.
}
{$N+,E+}
const
  kmax = 10;
  smax = 10;
  alfa = 1.49101453;
  beta = 1.37417851;
  a1 = -0.5748646;
  a2 = +0.9512388;
  a3 = -0.6998578;
  a4 = +0.4245549;
  a5 = -0.1010678;
type
  firstmat = array [0..kmax,0..smax] of extended;
  secmat = array [1..kmax] of extended;
var
  i,j,k,s,flag : integer;
  val,val2 : real;
  Lomni : text;
  a,b,phi : firstmat;
  c : secmat;

procedure InitMat;
begin
  for s := 1 to smax do
```

```
    c[s] := 0;
  for s := 1 to smax do
    begin
      for k := 0 to kmax do
        begin
          a[s,k] := 0;
          b[s,k] := 0;
          phi[s,k] := 0
        end;
    end;
end; {InitMat}

function BigGamma (arg:real) : extended;
{
  Computes the gamma function for a given argument.
}
begin
  val := 1;
  val2 := 1;
  BigGamma := 1;
  flag := 1;
  while flag=1 do
    begin
      if arg <2 then
        begin
          if arg=1 then
             arg := 1
            else
             arg := arg-1;
          val2 := 1+a1*arg+a2*exp(2*ln(arg))+a3*exp(3*ln(arg));
          val2 := val2+a4*exp(4*ln(arg))+a5*exp(5*ln(arg));
          val := val*val2;
          BigGamma := val;
          flag := 0;
        end;
      if flag=1 then
        begin
          val := val*(arg-1);
          arg := arg-1;
          BigGamma := val;
        end;
    end;
end; {BigGamma}
```

```
function LowGamma (arg:real) : extended;
{
 Computes the lowercase gamma function defined by the Lomnicki approximation.
}
begin
 LowGamma := BigGamma(arg*beta+1)/BigGamma(arg+1);
end;

function Comb (arg1,arg2 : real ) : extended;
{
 Computes the combinatorial of the two given arguments -> combinations of arg1
 elements, taking arg2 at a time.
}
begin
 Comb := BigGamma(s+1)/(BigGamma(i+1)*BigGamma(s-i+1))
end;

procedure CompMatA;
{
 Computes the elements of the A matrix in the Lomnicki approximation.
}
begin
 for s := 1 to smax do
  begin
   for k := 0 to kmax do
    begin
     for i := k to s do
      if int((k+i)/2)-((k+i)/2) = 0 then
       flag := 1
      else
       flag := -1;
      a[k,s] := flag*b[i,k]*comb(s,i)/LowGamma(i)
    end;
  end;
end; {CompMatA}

procedure CompMatB;
{
 Computes the elements of the B matrix in  the Lomnicki approximation.
}
begin
 for s := 0 to smax do
```

```
    b[s,0] := LowGamma(s);
  for s := 1 to smax do
    begin
      for k := 0 to kmax do
        begin
          for i := k to s-1 do
            begin
              b[s,k] := b[s,k-1]*LowGamma(s-i)
            end;
        end;
    end;
end; {CompMatB}

procedure CompMatPhi;
{
  Computes the elements of the PHI matrix in  the Lomnicki approximation.
}
begin
  for s := 1 to smax do
    begin
      for k := 0 to kmax do
        begin
          for i:= k to s do
            begin
              phi[s,k] := phi[s,k]+a[s,i]
            end;
          for i := k to s-1 do
            begin
              phi[s,k] := phi[s,k]-a[s-1,i]
            end;
        end;
    end;
end; {CompMatPhi}

procedure CompVectC;
{
  Computes the elements of C vector in the Lomnicki approximation.
}
begin
  for s := 1 to smax do
    begin
      for k := 1 to s do
        begin
```

```
      c[s] := c[s]+phi[s,k]
     end;
   end;
end; {CompVectC}

procedure WriteVectC;
{
 Writes vector C to output file "lomnicki.txt".
}
begin
 ReWrite (Lomni);
 WriteLn (Lomni,'Coefficients of Lomnicki approximation');
 WriteLn (Lomni,'--------------------------------------');
 WriteLn (Lomni,' ');
 WriteLn (Lomni,'alfa = ',alfa:10:8);
 WriteLn (Lomni,'beta = ',beta:10:8);
 WriteLn (Lomni,' ');
 for s := 1 to smax do
  WriteLn (Lomni,'c[',s:2,'] = ',c[s]);
end; {WriteVectC}

begin {Lomnicki}
 Assign (Lomni,'c:\pascal\lomnicki.txt');
 InitMat;
 CompMatA;
 CompMatB;
 CompMatPhi;
 CompVectC;
 WriteVectC;
end. {Lomnicki}
```

APPENDIX V - PROGRAM TO CALCULATE THE AVAILABILITY OF A CONTINUOUSLY-OPERATED, COHERENT, MULTIFUNCTIONAL SYSTEM, ASSUMING THAT THE ELEMENTS' TIME TO FAILURE AND TIME TO REPAIR FOLLOW A NEGATIVE EXPONENTIAL DISTRIBUTION

```
program SysAv;
{
 This program computes availability of multifunctional systems based
 on the formulation developed by Alberto Sols in his Thesis "Availability
 of Continuously-Operated, Coherent, Multi-Functional Systems", presented
 at Virginia Polytechnic Institute and State University in partial
 fulfillment of the requirements for the degree of Master of Science
 in Systems Engineering.

 The distributions for both the time to failure and the time to repair
 of each system element are assumed to be negative exponential.

 It also computes system availability according to the traditional
 approach, so that both results can be compared.


            Written by Alberto Sols in June 1992.

}
{$N+,E+}
{
 The above statements are meant to emulate the numerical co-processor
 8087 and to allow calculations to be performed using the extended
```

```
 real type, for higher accuracy in calculations.
}
const
  {
   This section declares as constants several important parameters :
   m      -> number of system missions
   s      -> number of system functions
   r      -> number of elements of the system
   step   -> increment used in the algorithm that integrates
             a function
   lowlim -> lower limit of integration (0 by default)
   err1   -> value of argument x for which exp(-x) is negligible
             (exp(-1400)=9E-609)
   err2   -> value after which aux1 (explained below) can be
             taken as 0
  }
  m = 7;
  s = 5;
  r = 13;
  step = 10;
  lowlim = 0;
  err1 = 1400;
  err2 = 1E-500;
type
  lim0 = 0..1;
  lim1 = 1..m;
  lim2 = 1..s;
  lim3 = 1..r;
  lim4 = 1..2;
  firstmat = array [lim1,lim2] of real;
  secmat = array [lim2,lim3] of real;
  thirdmat = array [lim1,lim3] of real;
  fourmat = array [lim3] of extended;
  fivemat = array [lim1,lim4] of real;
  sixmat = array [lim1] of real;
var
  {
   The following are the variables used in the program :
   -> misp stores the percentage of useful life the system
      is in each type of mission
   -> mf stores the mission-function definition matrix
   -> fe stores the function-element definition matrix
   -> me stores the mission-element definition matrix
```

```
-> elambda stores the reciprocal of the MTBF of each element
-> emu stores the reciprocal of the MTTR of each element
-> tau stores the 'utilization' time for each element
-> mntd stores for each mission the number of times it is called for
   and the average duration time
-> L stores the useful life of the system, computed through
   number and duration of mission types
-> abcisa is the variable used to represent time when integrating
   functions
-> area is used to store the system availability computed through
   the new approach
-> tradav is used to store the system availability computed through
   the traditional approach (average availability)
-> limav is used to store the system availability computed through
   the traditional approach (limiting availability)
-> aux1 is used to store the value of element availability as a
   function of time in the algorithm that integrates system
   availability
-> aux2 is used to store the value of the product of availabilities
   for the elements that support a certain mission, in the algorithm
   that integrates system availability
-> aux3 is used to store the sigma of aux2, extended to all system
   missions
-> flag is used to detect when aux1 is too small (less than err2)
-> i,j,k and h are counters
-> OptSelect is used to take selected option in menus
-> FileMNT is a text file containing the number of times each
   mission type is called for, together with the average mission
   duration
-> FileMF is a text file containing the matrix that defines the
   mission-function links
-> FileFE is a text file containing the matrix that defines the
   function-element links
-> FileLM is a text file containing the matrix of the system
   elements' lambda and mu parameters
}
limav,tradav,area,aux1,aux2,aux3 : extended;
L,abcisa : real;
OptSelect,i,j,k,flag : integer;
h : longint;
mf : firstmat;
fe : secmat;
me : thirdmat;
```

```
  elambda,emu,tau : fourmat;
  mntd : fivemat;
  misp : sixmat;
  FileMNT,FileMF,FileFE,FileLM : text;

procedure BlankLines (k:integer);
{
 This procedure prints k blank lines on the screen.
}
begin {BlankLines}
  for i := 1 to k do
  WriteLn (' ')
end; {BlankLines}

procedure Heading;
{
  This procedure displays the initial heading of the program.
}
begin {Heading}
  BlankLines (4);
  WriteLn ('            AVAILABILITY of CONTINUOUSLY-OPERATED,');
  WriteLn ('             COHERENT, MULTIFUNCTIONAL SYSTEMS.');
  BlankLines(2);
  WriteLn ('                        by');
  BlankLines(2);
  WriteLn ('                    Alberto Sols');
  BlankLines(2);
  WriteLn ('                    (June 1992)')
end; {Heading}

procedure InitMatrix;
{
 This procedure initializes all the matrices used in the program.
}
begin {InitMatrix}
  for i := 1 to m do
    begin
      for j := 1 to s do
        mf[i,j] := 0
    end;
  for j := 1 to s do
    begin
      for k := 1 to r do
```

```
      fe[j,k] := 0
    end;
  for i := 1 to m do
    begin
      for k := 1 to r do
        me[i,k] := 0;
    end;
  for k := 1 to r do
    begin
      elambda[k] := 0;
      emu[k] := 0
    end;
  for i := 1 to m do
    begin
      mntd[i,1] := 0;
      mntd[i,2] := 0;
      misp[i] := 0
    end;
end; {InitMatrix}

procedure InputMenu1;
{
 Menu 1 - Takes selected input mode for number and duration of mission
        types.
}
begin {InputMenu1}
  WriteLn ('  Menu 1 - Select Number and Duration of Missions Input Mode.');
  BlankLines(1);
  WriteLn ('1. Input Number and Duration of Missions Through Screen.');
  WriteLn ('2. Read Number and Duration of Missions From File.');
  WriteLn (' ');
  WriteLn ('0. Quit SysAv.');
  BlankLines(2);
  ReadLn (OptSelect)
end; {InputMenu1}

procedure InputMenu2;
{
 Menu 2 - Takes selected input mode for the missions-functions and
        the functions-elements matrices.
}
begin {InputMenu2}
  WriteLn ('  Menu 2 - Select Input Mode for Mission-Function-Element Matrices.');
```

```
  BlankLines(1);
  WriteLn ('1. Input Matrices Through Screen.');
  WriteLn ('2. Read Matrices From Files.');
  BlankLines(2);
  ReadLn (OptSelect)
end; {InputMenu2}

procedure InputMenu3;
{
 Menu 3 - Takes selected input mode for the elements' lambda and mu.
}
begin {InputMenu3}
  WriteLn ('  Menu 3 - Select Elements Lambda and Mu Input Mode.');
  BlankLines(1);
  WriteLn ('1. Input Elements Lambda and Mu Through Screen.');
  WriteLn ('2. Read Elements Lambda and Mu From File.');
  BlankLines(2);
  ReadLn (OptSelect)
end; {InputMenu3}

procedure InputMNT;
{
 Takes as inputs the number of times each mission is called for,
 and the average duration of each mission type.
}
begin {InputMNT}
  WriteLn ('Enter Number of Times each Mission is Called for, ');
  WriteLn ('and Average Mission Duration.');
  BlankLines(1);
  WriteLn ('(Leave blank between numbers)');
  BlankLines(2);
  for i := 1 to m do
    begin
      Write ('# Times Mission M[',i:1,']  ','Average Duration M[',i:1,'] -> ');
      ReadLn (mntd[i,1],mntd[i,2])
    end;
end; {InputMNT}

procedure ReadMNT;
{
 Reads from a file the number of times each mission is called for and the
 average mission duration.
}
```

```
begin {ReadMNT}
 Reset (FileMNT);
 for i := 1 to m do
   begin
    Read (FileMNT,mntd[i,1]);
    ReadLn (FileMNT,mntd[i,2]);
   end;
end; {ReadMNT}

procedure InputMatrix;
begin {InputMatrix}
 {
  Takes as input the mission-function matrix :
  mf[i,j] = 1  -> Function Fj supports Mission Mi
  mf[i,j] = 0  -> otherwise
 }
 BlankLines(2);
 WriteLn ('Enter the Mission-Function Matrix.');
 BlankLines(2);
 for i := 1 to m do
   begin
    for j := 1 to s do
     begin
      Write ('Enter mf[',i:1,',',j:1,'] -> ');
      ReadLn (mf[i,j])
     end;
   end;
 {
  Takes as input the function-element matrix :
  fe[j,k] = 0  -> Element Ek supports Function Fj
  fe[j,k] = 0  -> otherwise
 }
 BlankLines(2);
 WriteLn ('Enter the Function-Element Matrix.');
 BlankLines(2);
 for j := 1 to s do
   begin
    for k := 1 to r do
     begin
      Write ('Enter fe[',j:1,',',k:1,'] -> ');
      ReadLn (fe[j,k])
     end;
   end;
```

```
end; {InputMatrix}

procedure ReadMatrix;
{
 Reads the mission-function and function-element matrices.
}
begin {ReadMatrix}
  {
  Reads mission-function matrix.
  }
  Reset (FileMF);
  for i := 1 to m do
    begin
      for j := 1 to s-1 do
        Read (FileMF,mf[i,j]);
      ReadLn (FileMF,mf[i,s]);
    end;
  {
  Reads function-element matrix.
  }
  Reset (FileFE);
  for j := 1 to s do
    begin
      for k := 1 to r-1 do
        Read (FileFE,fe[j,k]);
      ReadLn (FileFE,fe[j,r]);
    end;
end; {ReadMatrix}

procedure ComputeME;
{
 Computes the mission-element matrix.
}
begin {ComputeME}
  for i := 1 to m do
    begin
      for k := 1 to r do
        begin
          aux1 := 1;
          for j := 1 to s do
            aux1 := aux1*(1-mf[i,j]*fe[j,k]);
          me[i,k] := 1-aux1
        end;
```

```
  end;
end; {ComputeME}

procedure InputLM;
{
 Takes as inputs the values of lambda and mu for each element.
}
begin {InputLM}
 BlankLines(2);
 WriteLn ('Enter Lambda and Mu for Each Element.');
 BlankLines(1);
 WriteLn ('(Leave a Blank Between Them)');
 BlankLines(2);
 for k := 1 to r do
  begin
   Write ('Enter Lambda[',k:1,']  Mu[',k:1,']  ->  ');
   ReadLn (elambda[k],emu[k])
  end;
end; {InputLM}

procedure ReadLM;
{
 Reads elements' lambda and mu from file.
}
begin {ReadLM}
 Reset (FileLM);
 for k := 1 to r do
  begin
   Read (FileLM,elambda[k]);
   ReadLn (FileLM,emu[k])
  end;
end; {ReadLM}

procedure CompuTau;
{
 Computes the percentage of elapsed time each element has been
 requested to work.
}
begin {CompuTau}
 L := 0;
 for i := 1 to m do
  begin
   L := L + mntd[i,1]*mntd[i,2]
```

```
    end;
  for i := 1 to m do
    misp[i] := mntd[i,1]*mntd[i,2]/L;
  for k := 1 to r do
    begin
      for i := 1 to m do
        tau[k] := tau[k]+misp[i]*me[i,k]
    end;
end; {CompuTau}

function funcion (abcisa:real) : extended;
{
 Construction of the function to be integrated.
}
begin {funcion}
  funcion := 0;
  aux3 := 0;
  for i := 1 to m do
    begin
      aux2 := 1;
      for k := 1 to r do
        begin
          if me[i,k] = 1 then
            begin
              aux1 := elambda[k]/(elambda[k]+emu[k]);
              flag := 1;
              if ((elambda[k]+emu[k])*tau[k]*abcisa) > err1 then
                flag := 0;
              if aux1 < err2 then
                flag := 0;
              if flag = 0 then
                aux1 := 0
                else
                aux1 := aux1*exp(-(elambda[k]+emu[k])*tau[k]*abcisa);
              aux1 := aux1+emu[k]/(elambda[k]+emu[k]);
              aux2 := aux2*aux1;
            end;
        end;
      aux3 := aux3+aux2*misp[i];
    end;
  funcion := aux3
end; {funcion}
```

```
function funcion2(abcisa:real) : extended;
{
 This function is used to calculate the average availability
 based on the traditional approach, that assumes that all the
 elements are required all the time.
}
begin
  funcion2 := 0;
  aux2 := 1;
  for k := 1 to r do
    begin
      aux1 := elambda[k]/(elambda[k]+emu[k]);
      flag := 1;
      if (elambda[k]+emu[k])*abcisa > err1 then
         flag := 0;
      if aux1 < err2 then
         flag := 0;
      if flag = 0 then
         aux1 := 0
         else
         aux1 := aux1*exp(-(elambda[k]+emu[k])*abcisa);
      aux1 := aux1+(emu[k]/(elambda[k]+emu[k]));
      aux2 := aux2*aux1;
    end;
    funcion2 := aux2
end;

procedure Integra;
{
 It integrates a function between a lower and an upper limit,
 with a certain step, using a modified trapezoids rule.
 }
begin {Integra}
  aux1 := 0;
  area := 0;
  abcisa := 0;
  BlankLines(6);
  WriteLn ('Computing integral ...');
  BlankLines (22);
  WriteLn ('Lower Limit -> ',lowlim:16);
  WriteLn ('Upper Limit -> ',L:16:2);
  WriteLn ('Step --------> ',step:16);
  BlankLines(2);
```

```
 h := 0;
 while abcisa <= L do
   begin
     area := area+funcion(abcisa)*step;
     if int(h/100)*100-h = 0 then
       begin
         BlankLines(22);
         WriteLn ('Lower Limit ->  ',lowlim:14);
         WriteLn ('Upper Limit ->  ',L:17:2);
         WriteLn ('Step --------->  ',step:14);
         WriteLn ('Iteration --->  ',h:14);
         BlankLines(2)
       end;
     abcisa := abcisa+step;
     h := h+step
   end;
end; {Integra}

procedure TradApp;
{
 This procedure computes the traditional average availability.
}
begin
 tradav := 0;
 abcisa := 0;
 while abcisa < L do
   begin
     tradav := tradav+funcion2(abcisa)*step;
     abcisa := abcisa+step
   end;
 tradav := tradav/L;
 WriteLn ('Traditional Average Availability  ->  ',tradav:12:10)
end;

procedure AvRatio;
{
 This procedures computes and displays the ratio of the calculated
 availability figures of merit.
}
begin {AvRatio}
 BlankLines(2);
 WriteLn ('Availability/Trad. Average Av.    -> ',area/tradav:12:10);
 BlankLines(1);
```

```
  WriteLn ('Availability/Trad. Limiting Av.   -> ',area/limav:12:10);
  BlankLines(1);
  WriteLn ('Trad. Average Av./Trad. Lim. Av.  -> ',tradav/limav:12:10);
end; {AvRatio}

procedure LimAvail;
{
 This procedure computes the availability of the system, based
 on the traditional, limiting average availability formulation.
}
begin {LimAvail}
  limav := 1;
  for k := 1 to r do
    limav := limav*(emu[k]/(elambda[k]+emu[k]));
  BlankLines(2);
  WriteLn ('Traditional Limiting Availability -> ',limav:12:10);
end; {LimAvail}

begin {SysAv}
{
 Main body of the program.
}
  {
   Assignment of files used in the program.
  }
  Assign (FileMNT,'c:\pascal\fmnt.txt');
  Assign (FileMF,'c:\pascal\fmf.txt');
  Assign (FileFE,'c:\pascal\ffe.txt');
  Assign (FileLM,'c:\pascal\flm.txt');
  BlankLines(2);
  InitMatrix;
  Heading;
  BlankLines(4);
  InputMenu1;
  case OptSelect of
    1 : InputMNT;
    2 : ReadMNT;
  end;
  InputMenu2;
  case OptSelect of
    1 : InputMatrix;
    2 : ReadMatrix;
  end;
```

```
  ComputeME;
  InputMenu3;
  case OptSelect of
    1 : InputLM;
    2 : ReadLM;
  end;
  BlankLines (2);
  WriteLn ('Computing system element utilization times ....');
  CompuTau;
  BlankLines (2);
  Integra;
  BlankLines(2);
  area := area/L;
  WriteLn ('The system availability is        -> ',area:12:10);
  BlankLines(2);
  TradApp;
  LimAvail;
  AvRatio;
  BlankLines(2);
  WriteLn ('Press any key ...');
  ReadLn (aux3);
end. {SysAv}
```

VITA

Alberto Sols was born on October 21, 1960 in Madrid (Spain) to Alberto Sols and Maria Angeles Rodriguez-Candela. In 1985 he graduated with a Master of Science in Naval Architecture and Marine Engineering at Madrid Polytechnic University. From October 1983 to October 1989 he worked first for Construnaves and then for Aries Defensa. In November 1989 he joined ISDEFE, his current employer. In August 1991 he began his studies in the Systems Engineering Program at Virginia Polytechnic Institute and State University. In May 1992 he passed the exam to become a Certified Professional Logistician by the Society of Logistics Engineers.

He co-founded of the Spanish District of the Society of Logistics Engineers in 1988 serving afterwards two years as District Director and then one year as Madrid's Chapter Chairman.

Alberto married Mar Carlero in September 1988 and their son Alberto was born in December 1989.