APPENDIX A

This mathematical appendix derives the political equilibrium production and trade policies under *cooperation* with a *global production* externality.

Home and foreign country exhibit similar economic and political structures. Variables and parameters for the foreign country are denoted with an asterisk (*). Supply for industry i in the home country is given by Hotelling's lemma

$$(A1) \quad X_i(p_i^s) = \Pi_i'(p_i^s)$$

Each individual j maximizes direct utility

$$(A2) \quad u_j = c_{0j} + \sum_{i=1}^{n} u_{ij}(c_{ij}) + u_{Ej}(E),$$

where c_{0j} is the consumption of the numeraire good, c_{ij} is the consumption of good i by individual j, and $u_{Ej}(E)$ is the utility that individual j derives from the state of the environment as determined by the externality E. The externality is global and is generated by production of one or more nonnumeraire goods e in one or both such that $\frac{\partial E(X_e, X_e^*)}{\partial X_e} = E_X > 0$ and $\frac{\partial E(X_e, X_e^*)}{\partial X_e^*} = E_{X^*} > 0$. All $u_{ij}(.)$ are assumed to be increasing and concave function. Further assume that the externality is negative, that is $\frac{\partial u_{Ej}}{\partial E} = u_{Ej}' < 0$. All consumers are assumed to be identical and the indirect utility of a representative consumer is given by

$$(A3) \quad v(p^d, y, E) = y + s(p^d) + u_E(E),$$

where $p^d = (p_1^d, p_2^d, ..., p_n^d)$ is the vector of consumer prices for nonnumeraire goods, and y represents her income. Consumer surplus from all nonnumeraire goods is

$$(A4) \quad s(p^d) = \sum_{i=1}^{n} u_i[d_i(p_i^d)] - \sum_{i=1}^{n} p_i^d d_i(p_i^d).$$

Using Roy's Identity yields individual demand for good i

$$\text{(A5)} \quad d_i(p_i^d) = -\frac{\partial s(p^d)}{\partial p_i^d}.$$

Total demand for good i in the home country is $D_i = Nd_i$ and $\frac{\partial D_i}{\partial p_i^d} = D_i' < 0$. The price equilibrium conditions for supply and demand in the home and foreign country are

$$(A6a) \quad p_i^s = \frac{\theta_i}{\tau_i} p_i^w$$

$$(A6b) \quad p_i^{s*} = \frac{\theta_i^*}{\tau_i^*} p_i^w$$

$$(A6c) \quad p_i^d = \theta_i p_i^w$$

$$(A6d) \quad p_i^{d*} = \theta_i^* p_i^w \,.$$

The net revenue of the government in the home country is generated by domestic and trade policies. Net per-capita transfer is then

$$(A7) \quad r(\tau,\theta,\tau^*,\theta^*) = \tfrac{1}{N}\sum_i p_i^s(\tau_i - 1)X_i(p_i^s) + \tfrac{1}{N}\sum_i p_i^w(\theta_i - 1)[D_i(p_i^d) - X_i(p_i^s)].$$

The governments cooperate when they set production and trade policies and efficiency requires that they maximize the weighted sum

$$(A8) \quad A^*G + AG^* = A^*[\textstyle\sum_{i\in L} C_i(\tau,\theta,\tau^*,\theta^*) + aW(\tau,\theta,\tau^*,\theta^*)]$$
$$+A[\textstyle\sum_{i\in L^*} C_i^*(\tau,\theta,\tau^*,\theta^*) + a^*W^*(\tau,\theta,\tau^*,\theta^*)]$$

where $A = (\alpha_L + a)$ and $A^* = (\alpha_L^* + a^*)$. The first order condition for the cooperative equilibrium is then

$$(A9) \quad \begin{array}{l} A^*\left(\sum_{i\in L} \nabla_\beta W_i(\tau,\theta,\tau^*,\theta^*) + a\nabla_\beta W(\tau,\theta,\tau^*,\theta^*)\right) + \\ A\left(\sum_{i\in L^*} \nabla_\beta W_i^*(\tau^*,\theta^*,\tau,\theta) + a^*\nabla_\beta W^*(\tau^*,\theta^*,\tau,\theta)\right) = 0, \end{array}$$

where $\beta = \tau, \tau^*, \theta, \theta^*$.

Substituting in for the partial derivatives in (A9) yields

$$
(A10)\quad
\begin{aligned}
&A^*(\textstyle\sum_{i\in L}\nabla_\beta\{l_i+\Pi_i(p_i^s)+N_i[r(\tau,\theta,\tau^*,\theta^*)+\frac{1}{N}R+s(p^d)+u_E(X_e,X_e^*)]\}\\
&+a\nabla_\beta\{l+\sum_{i=1}^{n}\Pi_i(p_i^s)+Nr(\tau,\theta,\tau^*,\theta^*)+R+Ns(p^d)+Nu_E(X_e,X_e^*)\})\\
&+A(\textstyle\sum_{i\in L^*}\nabla_\beta\{l_i^*+\Pi_i^*(p_i^{s*})+N_i^*[r^*(\tau^*,\theta^*,\tau,\theta)-\frac{1}{N^*}R+s(p_i^{d*})+u_E^*(X_e,X_e^*)]\}\\
&+a^*\nabla_\beta\{l^*+\sum_{i=1}^{n}\Pi_i^*(p_i^{s*})+N_i^*r^*(\tau^*,\theta^*,\tau,\theta)-R+N_i^*s^*(p_i^{d*})+N_i^*u_E^*(X_e,X_e^*)\})=0.
\end{aligned}
$$

Adding and subtracting $A^*\alpha_L\Pi_i$ and $A\alpha_L^*\Pi_i^*$, and noting that labor supply is unchanged, yields

$$
(A11)\quad
\begin{aligned}
&\nabla_\beta\{A^*(I_{iL}-\alpha_L)\Pi_i(p_i^s)+A^*A[\Pi_i(p_i^s)+Nr(\tau,\theta,\tau^*,\theta^*)+R+Ns(p^d)+Nu_E(X_{e,}X_e^*)]\}+\\
&\nabla_\beta\{A(I_{iL*}^*-\alpha_L^*)\Pi_i^*(p_i^{s*})+AA^*[\Pi_i^*(p_i^{s*})+N^*r^*(\tau^*,\theta^*,\tau,\theta)-R+N^*s(p^{d*})+N^*u_e^*(X_{e,}X_e^*)]\}=0.
\end{aligned}
$$

Note that an international transfer R drops out of the maximization problem because it does not affect efficiency. First consider the first-order condition for the home country's production policy for good i, that is for $\beta=\tau_i$.Denoting $M_i=D_i-X_i$, substituting (A7) into (A10), and then using (A1) and (A5) gives

$$
(A12)\quad
\begin{aligned}
&A^*(I_{iL}-\alpha_L)X_i\frac{\partial p_i^s}{\partial\tau_i}\\
&+A^*A[X_i\frac{\partial p_i^s}{\partial\tau_i}+(\tau_i-1)X_i\frac{\partial p_i^s}{\partial\tau_i}+p_i^sX_i+p_i^s(\tau_i-1)X_i'\frac{\partial p_i^s}{\partial\tau_i}-D_i\frac{\partial p_i^d}{\partial\tau_i}]\\
&+A^*A[\frac{\partial p_i^w}{\partial\tau_i}(\theta_i-1)M_i+p_i^w(\theta_i-1)(D_i'\frac{\partial p_i^d}{\partial\tau_i}-X_i'\frac{\partial p_i^s}{\partial\tau_i})]\\
&+A^*A[N'u_E'E_XX_i'\frac{\partial p_i^s}{\partial\tau_i})+N'u_E'E_{X*}X_i'^*\frac{\partial p_i^{s*}}{\partial\tau_i}]\\
&+A(I_{iL*}^*-\alpha_L^*)X_i^*\frac{\partial p_i^{s*}}{\partial\tau_i}\\
&+AA^*[X_i^*\frac{\partial p_i^{s*}}{\partial\tau_i}+(\tau_i^*-1)X_i^*\frac{\partial p_i^{s*}}{\partial\tau_i}+p_i^{s*}(\tau_i^*-1)X_i'^*\frac{\partial p_i^{s*}}{\partial\tau_i}-D_i^*\frac{\partial p_i^{d*}}{\partial\tau_i}]\\
&+AA^*[\frac{\partial p_i^w}{\partial\tau_i}(\theta_i^*-1)M_i^*+p_i^w(\theta_i^*-1)(D_i'^*\frac{\partial p_i^{*d}}{\partial\tau_i}-X_i'^*\frac{\partial p_i^{s*}}{\partial\tau_i})]\\
&+AA^*[N'^*u_E'^*E_X^*X_i'^*\frac{\partial p_i^{s*}}{\partial\tau_i})+N'^*u_E'^*E_XX_i'\frac{\partial p_i^s}{\partial\tau_i}]=\\
&A^*(I_{iL}-\alpha_L)X_i\frac{\partial p_i^s}{\partial\tau_i}\\
&+A^*A[(\tau_i-1)X_i'\frac{\partial p_i^s}{\partial\tau_i}\frac{\theta_i}{\tau_i}p_i^w]+A^*A[\frac{\theta_i}{\tau_i}p_i^wX_i+\tau_iX_i\frac{\partial p_i^s}{\partial\tau_i}-\theta_iX_i\frac{\partial p_i^w}{\partial\tau_i}]\\
&+A^*A[p_i^w(\theta_i-1)(D_i'\frac{\partial p_i^d}{\partial\tau_i}-X_i'\frac{\partial p_i^s}{\partial\tau_i})]\\
&+A^*A[N'u_E'E_XX_i'\frac{\partial p_i^s}{\partial\tau_i})+N'u_E'E_{X*}X_i'^*\frac{\partial p_i^{s*}}{\partial\tau_i}]\\
&+AA^*(I_{iL*}^*-\alpha_L^*)X_i^*\frac{\partial p_i^{s*}}{\partial\tau_i}\\
&+AA^*[\tau_i^*X_i^*\frac{\partial p_i^{s*}}{\partial\tau_i}-\theta_i^*X_i^*\frac{\partial p_i^w}{\partial\tau_i}+(\tau_i^*-1)X_i'^*\frac{\partial p_i^{s*}}{\partial\tau_i}\frac{\theta_i^*}{\tau_i^*}p_i^w]\\
&+AA^*[p_i^w(\theta_i^*-1)(D_i'^*\frac{\partial p_i^{*d}}{\partial\tau_i}-X_i'^*\frac{\partial p_i^s}{\partial\tau_i})]\\
&+AA^*[N'^*u_E'^*E_X^*X_i'^*\frac{\partial p_i^{s*}}{\partial\tau_i})+N'^*u_E'^*E_XX_i'\frac{\partial p_i^s}{\partial\tau_i}]=0.
\end{aligned}
$$

Differentiating the price equilibrium conditions (A6) with respect to τ_i yields

$$(A13a)\quad \frac{\partial p_i^s}{\partial \tau_i} = \frac{\theta_i}{\tau_i}\frac{\partial p_i^w}{\partial \tau_i} - \frac{p_i^w \cdot \theta_i}{\tau_i^2}$$

$$(A13b)\quad \frac{\partial p_i^{s*}}{\partial \tau_i} = \frac{\theta_i^*}{\tau_i^*}\frac{\partial p_i^w}{\partial \tau_i}$$

$$(A13c)\quad \frac{\partial p_i^d}{\partial \tau_i} = \theta_i \frac{\partial p_i^w}{\partial \tau_i}$$

$$(A13d)\quad \frac{\partial p_i^{d*}}{\partial \tau_i} = \theta_{ii}^* \frac{\partial p_i^w}{\partial \tau_i}.$$

Using equations (A13), and collecting terms, the first order condition for the production policy (A12) can be written as

$$
\begin{aligned}
& A^*(I_{iL} - \alpha_L) X_i (\frac{\theta_i}{\tau_i}\frac{\partial p_i^w}{\partial \tau_i} - \frac{p_i^w \theta_i}{\tau_i^2}) + \\
& A^* A[(\tau_i - 1) X_i' \frac{\theta_i}{\tau_i} p_i^w (\frac{\theta_i}{\tau_i}\frac{\partial p_i^w}{\partial \tau_i} - \frac{p_i^w \theta_i}{\tau_i^2})] + \\
& A^* A[p_i^w (\theta_i - 1)(D_i' \frac{\partial p_i^w}{\partial \tau_i}\theta_i - X_i'(\frac{\theta_i}{\tau_i}\frac{\partial p_i^w}{\partial \tau_i} - \frac{p_i^w \theta_i}{\tau_i^2}))] \\
& + A^* A[N u_E' E_X X_i'(\frac{\theta_i}{\tau_i}\frac{\partial p_i^w}{\partial \tau_i} - \frac{p_i^w \theta_i}{\tau_i^2})) + N u_E' E_{X*} X_i'^* \frac{\theta_i^*}{\tau_i^*}\frac{\partial p_i^w}{\partial \tau_i}] + \\
(A14)\quad & A(I_{iL*}^* - \alpha_L^*) X_i^* \frac{\theta_i^*}{\tau_i^*}\frac{\partial p_i^w}{\partial \tau_i} \\
& + AA^*[(\tau_i^* - 1) X_i'^* \frac{\theta_i^*}{\tau_i^*}\frac{\partial p_i^w}{\partial \tau_i}\frac{\theta_i^*}{\tau_i^*} p_i^w] \\
& + AA^*[p_i^w (\theta_i^* - 1)\theta_i^* M_i'^* \frac{\partial p_i^w}{\partial \tau_i}] \\
& + AA^*[N^* u_E'^* E_{X*} X_i'^* \frac{\theta_i^*}{\tau_i^*}\frac{\partial p_i^w}{\partial \tau_i} + N^* u_E'^* E_X X_i'(\frac{\theta_i}{\tau_i}\frac{\partial p_i^w}{\partial \tau_i} - \frac{p_i^w \theta_i}{\tau_i^2})] \\
& = 0.
\end{aligned}
$$

To get $\frac{\partial p_i^w}{\partial \tau_i}$ totally differentiate the world market equilibrium condition

$D_i(\theta_i p_i^w) - X_i(\frac{\theta_i p_i^w}{\tau_i}) = -[D_i^*(\theta_i^* p_i^w) - X_i^*(\frac{\theta_i^* p_i^w}{\tau_i^*})]$, where the asterisk stands for the variables in the foreign country. This yields

$$(A15)\quad \frac{\partial p_i^w}{\partial \tau_i} = -\frac{(X_i' \;{}^{\theta_i}\!/_{\tau_i^2})\, p_i^w}{\theta_i\left(D_i' - {}^{X_i'}\!/_{\tau_i}\right) + \theta_i^*\left(D_i'^* - {}^{X_i'^*}\!/_{\tau_i^*}\right)} = -\frac{(X_i' \;{}^{\theta_i}\!/_{\tau_i^2})\, p_i^w}{\theta_i M_i' + \theta_i^* M_i'^*},$$

where $M_i' = D_i' - {}^{X_i'}\!/_{\tau_i}$ and $M_i'^* = D_i'^* - {}^{X_i'^*}\!/_{\tau_i^*}$. Differentiating the price equilibrium conditions, $p_i^s = \frac{\theta_i}{\tau_i} p_i^w$ and $p_i^d = \theta_i p_i^w$, and using equation (15) yields

$$(16a)\quad \frac{\partial p_i^s}{\partial \tau_i} = -\frac{p_i^w \theta_i (\theta D_i' + \theta_i^* M_i'^*)}{\tau_i^2(\theta_i M_i' + \theta_i^* M_i'^*)}$$

$$(16b)\quad \frac{\partial p_i^d}{\partial \tau_i} = -\frac{\theta_i (X_i' \;{}^{\theta_i}\!/_{\tau_i^2})\, p_i^w}{\theta_i M_i' + \theta_i^* M_i'^*}$$

$$(16c)\quad \frac{\partial p_i^s}{\partial \theta_i} = \frac{\theta_i^* M_i'^* p_i^w}{\tau_i(\theta_i M_i' + \theta_i^* M_i'^*)}$$

$$(16d)\quad \frac{\partial p_i^d}{\partial \theta_i} = \frac{\theta_i^* M_i'^* p_i^w}{\theta_i M_i' + \theta_i^* M_i'^*}.$$

Using equations (A15) and (A16) and solving implicitly for the home production policies yields

$$(A17)\quad (\tau_i - 1) = -\frac{(I_{iL} - \alpha_L)}{(a + \alpha_L)} \frac{X_i}{p_i^s X_i'}$$

$$-(\theta_i - 1)\left[\frac{\tau_i^2 p_i^w \theta_i M_i'(-p_i^w X_i' \theta_i/\tau_i^2) + X_i' p_i^w \theta_i p_i^w (\theta_i M_i' + \theta_i^* M_i'^*)}{-X_i' p_i^w \theta_i/\tau_i \, \theta_i p_i^w (\theta_i D_i' + \theta_i^* M_i'^*)}\right]$$

$$-\frac{(I_{iL*}^* - \alpha_L^*)}{(a^* + \alpha_L^*)} \frac{X_i^* \theta_i^*/\tau_i^*}{p_i^s (\theta_i D_i' + \theta_i^* M_i'^*)} - (\tau_i^* - 1) \frac{X_i'^* \theta_i^{*2}/\tau_i^{*2}}{\theta_i/\tau_i (\theta_i D_i' + \theta_i^* M_i'^*)}$$

$$-(\theta_i^* - 1)\left[\frac{\theta_i^* M_i'^*}{\theta_i/\tau_i (\theta_i D_i' + \theta_i^* M_i'^*)}\right]$$

$$-\frac{(Nu_E' + N^* u_E'^*) E_X}{p_i^s} - \frac{(Nu_E' E_{X*} + N^* u_E'^*) E_{X*} X_i'^* \theta_i^*/\tau_i^*}{p_i^s (\theta_i D_i' + \theta_i^* M_i'^*)}$$

$$(A18)\quad (\tau_i - 1) = -\frac{(I_{iL} - \alpha_L)}{(a + \alpha_L)} \frac{X_i}{p_i^s X_i'} - \frac{(I_{iL*}^* - \alpha_L^*)}{(a^* + \alpha_L^*)} \frac{X_i^* \theta_i^*/\tau_i^*}{p_i^s (\theta_i D_i' + \theta_i^* M_i'^*)}$$

$$-(\tau_i^* - 1) \frac{p_i^w X_i'^* \theta_i^{*2}/\tau_i^{*2}}{p_i^s (\theta_i D_i' + \theta_i^* M_i'^*)}$$

$$+[(\theta_i - 1) - (\theta_i^* - 1)]\left[\frac{p_i^w \theta_i^* M_i'^*}{p_i^s (\theta_i D_i' + \theta_i^* M_i'^*)}\right]$$

$$-\frac{(Nu_E' + N^* u_E'^*) E_X}{p_i^s} - \frac{(Nu_E' + N^* u_E'^*) E_{X*} X_i'^* \theta_i^*/\tau_i^*}{p_i^s (\theta_i D_i' + \theta_i^* M_i'^*)}.$$

The foreign production policy can be derived in a similar manner as the home production policy

$$(A19)\quad (\tau_i^* - 1) = -\frac{(I_{iL*}^* - \alpha_L^*)}{(a^* + \alpha_L^*)} \frac{X_i^*}{p_i^{s*} X_i'^*} - \frac{(I_{iL} - \alpha_L)}{(a + \alpha_L)} \frac{X_i' \theta_i/\tau_i}{p_i^{s*} (\theta_i^* D_i'^* + \theta_i M_i')}$$

$$-(\tau_i - 1) \frac{p_i^w X_i' \theta_i^2/\tau_i^2}{p_i^{s*} (\theta_i^* D_i'^* + \theta_i M_i')}$$

$$+[(\theta_i^* - 1) - (\theta_i - 1)]\left[\frac{p_i^w \theta_i M_i'}{p_i^{s*} (\theta_i^* D_i'^* + \theta_i M_i')}\right]$$

$$-\frac{(Nu_E' + N^* u_E'^*) E_{X*}}{p_i^{s*}} - \frac{(Nu_E' + N^* u_E'^*) E_X X_i' \theta_i/\tau_i}{p_i^{s*} (\theta_i^* D_i'^* + \theta_i M_i')}$$

Equations (A18) and (A19) are the same as equations (20a) and (20b) in chapter 4. Now consider the first-order condition for the trade policy for industry i, that is for $\beta = \theta_i$ in (A10). Substituting (A7) into (A10), and using (A1) and (A5) gives

$$
\begin{aligned}
(A20) \quad & A^*(I_{iL} - \alpha_L) X_i \frac{\partial p_i^s}{\partial \theta_i} \\
& + A^* A[X_i \frac{\partial p_i^s}{\partial \theta_i} + (\tau_i - 1) X_i \frac{\partial p_i^s}{\partial \theta_i} + p_i^s (\tau_i - 1) X_i' \frac{\partial p_i^s}{\partial \theta_i} - D_i \frac{\partial p_i^d}{\partial \theta_i}] \\
& + A^* A[\frac{\partial p_i^w}{\partial \theta_i}(\theta_i - 1) M_i + p_i^w M_i + p_i^w (\theta_i - 1)(D_i' \frac{\partial p_i^d}{\partial \theta_i} - X_i' \frac{\partial p_i^s}{\partial \theta_i})] \\
& + A^* A[N' u_E' E_X X_i' \frac{\partial p_i^s}{\partial \theta_i}) + N' u_E' E_{X*} X_i'^* \frac{\partial p_i^{s*}}{\partial \theta_i}] \\
& + A(I_{iL*}^* - \alpha_L^*) X_i^* \frac{\partial p_i^{s*}}{\partial \theta_i} \\
& + AA^*[X_i^* \frac{\partial p_i^s}{\partial \theta_i} + (\tau_i^* - 1) X_i^* \frac{\partial p_i^{s*}}{\partial \theta_i} + p_i^{s*} (\tau_i^* - 1) X_i'^* \frac{\partial p_i^{*s}}{\partial \theta_i} - D_i^* \frac{\partial p_i^{d*}}{\partial \theta_i}] \\
& + AA^*[\frac{\partial p_i^w}{\partial \theta_i}(\theta_i^* - 1) M_i^* + p_i^w (\theta_i^* - 1)(D_i'^* \frac{\partial p_i^{d*}}{\partial \theta_i} - X_i'^* \frac{\partial p_i^{s*}}{\partial \theta_i})] \\
& + AA^*[N'^* u_E'^* E_X^* X_i'^* \frac{\partial p_i^{s*}}{\partial \theta_i}) + N'^* u_E'^* E_X X_i' \frac{\partial p_i^s}{\partial \theta_i}] = 0.
\end{aligned}
$$

Differentiating the price equilibrium conditions (A6) with respect to θ_i yields

$$(A21a) \quad \frac{\partial p_i^s}{\partial \theta_i} = \frac{1}{\tau_i}(\theta_i \frac{\partial p_i^w}{\partial \theta_i} + p_i^w)$$

$$(A21b) \quad \frac{\partial p_i^{s*}}{\partial \theta_i} = \frac{\theta_i^*}{\tau_i^*} \frac{\partial p_i^w}{\partial \theta_i}$$

$$(A21c) \quad \frac{\partial p_i^d}{\partial \theta_i} = \theta_i \frac{\partial p_i^w}{\partial \theta_i} + p_i^w$$

$$(A21d) \quad \frac{\partial p_i^{d*}}{\partial \theta_i} = \theta_i^* \frac{\partial p_i^w}{\partial \theta_i}$$

Substituting (A21) into (A20) and collecting terms yields

$$
\begin{aligned}
& A^*(I_{iL}-\alpha_L)X_i\frac{1}{\tau_i}(\theta_i\frac{\partial p_i^w}{\partial\theta_i}+p_i^w) \\
& +A^*A[\tau_i X_i\frac{1}{\tau_i}(\theta_i\frac{\partial p_i^w}{\partial\theta_i}+p_i^w)-p_i^w X_i-\theta_i X_i\frac{\partial p_i^w}{\partial\theta_i}] \\
& +A^*A[-D_i(\theta_i\frac{\partial p_i^w}{\partial\theta_i}+p_i^w)+p_i^w D_i+\theta_i D_i\frac{\partial p_i^w}{\partial\theta_i}] \\
& +A^*A[(\tau_i-1)X_i'\frac{\theta_i}{\tau_i}p_i^w\frac{1}{\tau_i}(\theta_i\frac{\partial p_i^w}{\partial\theta_i}+p_i^w)] \\
& +A^*A[p_i^w(\theta_i-1)(D_i'-X_i'/\tau_i)(\theta_i\frac{\partial p_i^w}{\partial\theta_i}+p_i^w)] \\
& +A^*A[N'u_E'E_X X_i'\frac{1}{\tau_i}(\theta_i\frac{\partial p_i^w}{\partial\theta_i}+p_i^w))+N'u_E'E_{X*}X_i'^*\frac{\theta_i^*}{\tau_i^*}\frac{\partial p_i^w}{\partial\theta_i}] \\
(A22)\quad & +A(I_{iL}^*-\alpha_L^*)X_i^*\frac{\theta_i^*}{\tau_i^*}\frac{\partial p_i^w}{\partial\theta_i} \\
& +AA^*[\tau_i^*X_i^*\frac{\theta_i^*}{\tau_i^*}\frac{\partial p_i^w}{\partial\theta_i}-\theta_i^*X_i^*\frac{\partial p_i^w}{\partial\theta_i}] \\
& +AA^*[-D_i^*\theta_i^*\frac{\partial p_i^w}{\partial\theta_i}+D_i^*\theta_i^*\frac{\partial p_i^w}{\partial\theta_i}] \\
& +AA^*[(\tau_i^*-1)p_i^w X_i'^*(\frac{\theta_i^*}{\tau_i^*})^2\frac{\partial p_i^w}{\partial\theta_i}] \\
& +AA^*[p_i^w(\theta_i^*-1)\theta_i^*(D_i'^*-X_i'^*/\tau_i^*)\frac{\partial p_i^w}{\partial\theta_i}] \\
& +AA^*[N'^*u_E'^*E_X^*X_i'^*\frac{\theta_i^*}{\tau_i^*}\frac{\partial p_i^w}{\partial\theta_i})+N'^*u_E'^*E_X X_i'\frac{1}{\tau_i}(\theta_i\frac{\partial p_i^w}{\partial\theta_i}+p_i^w)] \\
& =0.
\end{aligned}
$$

Total differentiation of the market equilibrium condition yields

$$
(A23)\quad \frac{\partial p_i^w}{\partial\theta_i}=-\frac{(D_i'-X_i'/\tau_i)\,p_i^w}{\theta_i\left(D_i'-X_i'/\tau_i\right)+\theta_i^*\left(D_i'^*-X_i'^*/\tau_i^*\right)}=-\frac{M_i'p_i^w}{\theta_i M_i'+\theta_i^*M_i'^*}
$$

Substituting equation (A23) into equation (A22) and solving for the difference in trade policies yields

$$
(A24)\quad
\begin{aligned}
(\theta_i - 1) - (\theta_i^* - 1) = &-\frac{(I_{iL} - \alpha_L)}{(a + \alpha_L)} \frac{X_i}{p_i^w \tau_i M_i'} + \frac{(I_{iL}^* - \alpha_L^*)}{(a^* + \alpha_L^*)} \frac{X_i^*}{p_i^w \tau_i^* M_i'^*} \\
&-(\tau_i - 1) \frac{p_i^s X_i'}{p_i^w \tau_i M_i'} + (\tau_i^* - 1) \frac{p_i^{s*} X_i'^*}{p_i^w \tau_i^* M_i'^*} \\
&- \frac{(Nu_E' + N^* u_E'^*) E_X X_i'}{p_i^w \tau_i M_i'} + \frac{(Nu_E' + N^* u_E'^*) E_{X*} X_i'^*}{p_i^w \tau_i^* M_i'^*}.
\end{aligned}
$$

Equation (A24) is the same as equation (21) in chapter 4. Equations (A18), (A19), and (A24) determine the home and foreign countries' production policies and the difference between their trade policies. Substituting equation (A24) into the expressions for the production policies (A18) and (A19) yields for the home country's production policy

$$
(A25)\quad
\begin{aligned}
(\tau_i - 1) = &-\frac{(I_{iL} - \alpha_L)}{(a + \alpha_L)} \frac{X_i}{p_i^s X_i'} - \frac{(I_{iL}^* - \alpha_L^*)}{(a^* + \alpha_L^*)} \frac{X_i^* \, {}^{\theta_i^*}\!/\!_{\tau_i^*}}{p_i^s (\theta_i D_i' + \theta_i^* M_i'^*)} \\
&-(\tau_i^* - 1) \frac{p_i^w X_i'^* \, {}^{\theta_i^{*2}}\!/\!_{\tau_i^{*2}}}{p_i^s (\theta_i D_i' + \theta_i^* M_i'^*)} \\
&+\left[\frac{p_i^w \theta_i^* M_i'^*}{p_i^s (\theta_i D_i' + \theta_i^* M_i'^*)}\right]\left\{-\frac{(I_{iL} - \alpha_L)}{(a + \alpha_L)} \frac{X_i}{p_i^w \tau_i M_i'} + \frac{(I_{iL}^* - \alpha_L^*)}{(a^* + \alpha_L^*)} \frac{X_i^*}{p_i^w \tau_i^* M_i'^*}\right. \\
&-(\tau_i - 1) \frac{p_i^s X_i'}{p_i^w \tau_i M_i'} + (\tau_i^* - 1) \frac{p_i^{s*} X_i'^*}{p_i^w \tau_i^* M_i'^*} \\
&\left.- \frac{(Nu_E' + N^* u_E'^*) E_X X_i'}{p_i^w \tau_i M_i'} + \frac{(Nu_E' + N^* u_E'^*) E_{X*} X_i'^*}{p_i^w \tau_i^* M_i'^*}\right\} \\
&- \frac{(Nu_E' + N^* u_E'^*) E_X}{p_i^s} - \frac{(Nu_E' + N^* u_E'^*) E_{X*} X_i'^* \, {}^{\theta_i^*}\!/\!_{\tau_i^*}}{p_i^s (\theta_i D_i' + \theta_i^* M_i'^*)}.
\end{aligned}
$$

Collecting terms, an implicit expression for the political equilibrium production policy for the home country is

$$
(A26)\quad (\tau_i - 1) = -\frac{(I_{iL} - \alpha_L)}{(a + \alpha_L)} \frac{X_i}{p_i^s X_i'} - \frac{(Nu_E' + N^* u_E'^*) E_X}{p_i^s}.
$$

The political equilibrium production policy for the foreign country is implicitly given by

$$
(A27)\quad (\tau_i^* - 1) = -\frac{(I_{iL*}^* - \alpha_L^*)}{(a^* + \alpha_L^*)} \frac{X_i^*}{p_i^{s*} X_i'^*} - \frac{(Nu_E' + N^* u_E'^*) E_{X*}}{p_i^{s*}}.
$$

Substituting equations (A26) and (A27) back into equation (A25) yields the difference of the political equilibrium trade policies

$$(A28) \quad (\theta_i - 1) - (\theta_i^* - 1) = 0.$$

Equations (A26) and (A27) are the same as equations (22a) and (22b) in chapter 4, while (A28) is the same as equation (23).

APPENDIX B

This mathematical appendix derives the political equilibrium consumption and trade policies under *cooperation* with a *global consumption* externality.

Each individual j maximizes $u_j = c_{0j} + \sum_{i=1}^{n} u_{ij}(c_{ij}) + u_{Ej}(E)$,

c_{0j} consumption of the numeraire good by individual j

c_{ij} consumption of good i by individual j

$u_{Ej}(E)$ utility that individual j derives from the state of the environment

E externality

$u_{ij}(.)$ increasing and concave functions

$\frac{\partial u_{Ej}}{\partial E} = u'_{Ej} < 0$ negative externality

$$E = \sum_{\substack{k=1 \\ k \neq j}}^{N} c_{ek} + \sum_{k=1}^{N^*} c^*_{ek} = \sum_{\substack{k=1 \\ k \neq j}}^{N} c_{ek} = (N-1)\, c_e + N^* c^*_e$$

$$\frac{\partial E}{\partial c_e} = E_c = (N-1)$$

$$\frac{\partial E}{\partial c^*_e} = E_{c^*} = N^*$$

$p^d = (p^d_1, p^d_2, ..., p^d_n)$ consumer price vector for the home country

$v(p^d, y, E) = y + s(p^d) + u_E(E)$ indirect utility of representative consumer

$s(p^d) = \sum_{i=1}^{n} u_i[d_i(p^d_i)] - \sum_{i=1}^{n} p^d_i d_i(p^d_i)$ consumer surplus from nonnumeraire goods

$d_i(p_i^d) = -\frac{\partial s}{\partial p_i^d}$ individual demand for good i in the home country

$D_i(p_i^d) = Nd_i(p_i^d)$ total demand for good i in the home country

$\frac{\partial D_i}{\partial p_i^d} = D_i' < 0$.

Both, home and foreign country exhibit a similar structure. Variables and parameters for the foreign country are denoted with an asterisk (*). The price equilibrium conditions for supply and demand are

$$(B1a) \quad p_i^s = \theta_i p_i^w$$

$$(B1b) \quad p_i^s * = \theta_i^* p_i^w$$

$$(B1c) \quad p_i^d = \tau_i \theta_i p_i^w$$

$$(B1d) \quad p_i^d * = \tau_i^* \theta_i^* p_i^w$$

The net revenue of the home government is

$$(B2) \quad r(\tau,\theta) = \tfrac{1}{N}\sum_i p_i^s(\tau_i - 1) D_i(p_i^s) + \tfrac{1}{N}\sum_i p_i^w(\theta_i - 1)[D_i(p_i^d) - X_i(p_i^s)].$$

The governments cooperate when they set production and trade policies and efficiency requires that they maximize the weighted sum

$$(B3) \quad W = A^*G + AG^* = A^*[\textstyle\sum_{i\in L} C_i(\tau,\theta,\tau^*,\theta^*) + aW(\tau,\theta,\tau^*,\theta^*)]$$
$$+A[\textstyle\sum_{i\in L^*} C_i^*(\tau,\theta,\tau^*,\theta^*) + a^*W^*(\tau,\theta,\tau^*,\theta^*)]$$

where $A = (\alpha_L + a)$ and $A^* = (\alpha_L^* + a^*)$. The first order condition for the cooperative equilibrium is then

$$(B4) \quad \begin{array}{l} A^*\left(\sum_{i\in L}\nabla_\beta W_i(\tau,\theta,\tau^*,\theta^*) + a\nabla_\beta W(\tau,\theta,\tau^*,\theta^*)\right) + \\ A\left(\sum_{i\in L^*}\nabla_\beta W_i^*(\tau^*,\theta^*,\tau,\theta) + a^*\nabla_\beta W^*(\tau^*,\theta^*,\tau,\theta)\right) = 0, \end{array}$$

where $\beta = \tau, \tau^*, \theta, \theta^*$.

Substituting in for the partial derivatives in (B4) yields

$$(B5)\quad \begin{aligned} & A^*(\textstyle\sum_{i\in L}\nabla_\beta\{l_i + \Pi_i(p_i^s) + N_i[r(\tau,\theta,\tau^*,\theta^*) + \frac{1}{N}R + s(p^d) + u_E(X_e, X_e^*)]\} \\ & + a\nabla_\beta\{l + \sum_{i=1}^n \Pi_i(p_i^s) + Nr(\tau,\theta,\tau^*,\theta^*) + R + Ns(p^d) + Nu_E(X_e, X_e^*)\}) \\ & + A(\textstyle\sum_{i\in L^*}\nabla_\beta\{l_i^* + \Pi_i^*(p_i^{s*}) + N_i^*[r^*(\tau^*,\theta^*,\tau,\theta) - \frac{1}{N^*}R + s(p_i^{d*}) + u_E^*(X_e, X_e^*)]\} \\ & + a^*\nabla_\beta\{l^* + \sum_{i=1}^n \Pi_i^*(p_i^{s*}) + N_i^* r^*(\tau^*,\theta^*,\tau,\theta) - R + N_i^* s^*(p_i^{d*}) + N_i^* u_E^*(X_e, X_e^*)\}) = 0. \end{aligned}$$

Adding and subtracting $A^*\alpha_L\Pi_i$ and $A\alpha_L^*\Pi_i^*$, and noting that labor supply is unchanged, yields

$$(B6a)\quad \begin{aligned} & \nabla_\beta\{A^*(I_{iL} - \alpha_L)\Pi_i(p_i^s) + A^*A[\Pi_i(p_i^s) + Nr(\tau,\theta,\tau^*,\theta^*) + R + Ns(p^d) + Nu_E(E)]\} \\ & \nabla_\beta\{A(I_{iL^*}^* - \alpha_L^*)\Pi_i^*(p_i^{s*}) + AA^*[\Pi_i^*(p_i^{s*}) + N^*r^*(\tau^*,\theta^*,\tau,\theta) - R + N^*s(p^{d*}) + N^*u_E^*(E^*)]\} = 0. \end{aligned}$$

Since the transfer R just cancels out, equation (A11a) becomes

$$(B6b)\quad \begin{aligned} & \nabla_\beta\{A^*(I_{iL} - \alpha_L)\Pi_i(p_i^s) + A^*A[\Pi_i(p_i^s) + Nr(\tau,\theta,\tau^*,\theta^*) + Ns(p^d) + Nu_E(E)]\} \\ & \nabla_\beta\{A(I_{iL^*}^* - \alpha_L^*)\Pi_i^*(p_i^{s*}) + AA^*[\Pi_i^*(p_i^{s*}) + N^*r^*(\tau^*,\theta^*,\tau,\theta) + N^*s(p^{d*}) + N^*u_E^*(E^*)]\} = 0. \end{aligned}$$

where $\beta = \tau, \tau^*, \theta, \theta^*$. First consider the first-order condition for the home country's consumption policy, that is for $\beta = \tau_i$. Denoting $M_i = D_i - X_i$ gives

$$
(B7)\quad
\begin{aligned}
&A^{*}(I_{iL}-\alpha_{L})X_i\frac{\partial p_i^s}{\partial \tau_i}\\
&+A^{*}A[X_i\frac{\partial p_i^s}{\partial \tau_i}+(\tau_i-1)D_i\frac{\partial p_i^s}{\partial \tau_i}+p_i^s D_i+p_i^s(\tau_i-1)D_i'\frac{\partial p_i^s}{\partial \tau_i}-D_i\frac{\partial p_i^d}{\partial \tau_i}]\\
&+A^{*}A[\frac{\partial p_i^w}{\partial \tau_i}(\theta_i-1)M_i+p_i^w(\theta_i-1)(D_i'\frac{\partial p_i^d}{\partial \tau_i}-X_i'\frac{\partial p_i^s}{\partial \tau_i})]\\
&+A^{*}A[Nu_E'(N-1)d_i'\frac{\partial p_i^d}{\partial \tau_i}+Nu_E'N^{*}d_i'^{*}\frac{\partial p_i^{d*}}{\partial \tau_i}]\\
&+A(I_{iL*}^{*}-\alpha_{L}^{*})X_i^{*}\frac{\partial p_i^{s*}}{\partial \tau_i}\\
&+A^{*}A[X_i^{*}\frac{\partial p_i^{s*}}{\partial \tau_i}+(\tau_i^{*}-1)D_i^{*}\frac{\partial p_i^{s*}}{\partial \tau_i}+p_i^{s*}(\tau_i^{*}-1)D_i'^{*}\frac{\partial p_i^{s*}}{\partial \tau_i}-D_i^{*}\frac{\partial p_i^{d*}}{\partial \tau_i}]\\
&+A^{*}A[\frac{\partial p_i^w}{\partial \tau_i}(\theta_i^{*}-1)M_i^{*}+p_i^w(\theta_i^{*}-1)(D_i'^{*}\frac{\partial p_i^{*d}}{\partial \tau_i}-X_i'^{*}\frac{\partial p_i^s}{\partial \tau_i})]\\
&+A^{*}A[N^{*}u_E'^{*}(N^{*}-1)d_i'^{*}\frac{\partial p_i^{d*}}{\partial \tau_i}+N^{*}u_E'^{*}Nd_i'\frac{\partial p_i^d}{\partial \tau_i}]=0.
\end{aligned}
$$

Differentiating the price equilibrium conditions (1) with respect to τ_i yields

$$(B8a)\quad \frac{\partial p_i^s}{\partial \tau_i}=\theta_i\frac{\partial p_i^w}{\partial \tau_i}$$

$$(B8b)\quad \frac{\partial p_i^{s*}}{\partial \tau_i}=\theta_i^{*}\frac{\partial p_i^w}{\partial \tau_i}$$

$$(B8c)\quad \frac{\partial p_i^d}{\partial \tau_i}=\tau_i\theta_i\frac{\partial p_i^w}{\partial \tau_i}+\theta_i p_i^w$$

$$(B8d)\quad \frac{\partial p_i^{d*}}{\partial \tau_i}=\tau_i^{*}\theta_i^{*}\frac{\partial p_i^w}{\partial \tau_i}$$

Using equations (B8), and collecting terms, the first order condition for the consumption policy (B7) can be written as

$$
\begin{aligned}
(B9)\quad & A^*(I_{iL}-\alpha_L)X_i(\theta_i\frac{\partial p_i^w}{\partial\tau_i}) \\
& +A^*A[X_i(\theta_i\frac{\partial p_i^w}{\partial\tau_i})-D_i(\theta_i p_i^w+\tau_i\theta_i\frac{\partial p_i^w}{\partial\tau_i})+(\tau_i-1)\theta_i p_i^w D_i'(\theta_i p_i^w+\tau_i\theta_i\frac{\partial p_i^w}{\partial\tau_i})] \\
& +A^*A[\theta_i p_i^w D_i+\tau_i\theta_i D_i\frac{\partial p_i^w}{\partial\tau_i}-\theta_i D_i\frac{\partial p_i^w}{\partial\tau_i}+\theta_i D_i\frac{\partial p_i^w}{\partial\tau_i}-D_i\frac{\partial p_i^w}{\partial\tau_i}-X_i\theta_i\frac{\partial p_i^w}{\partial\tau_i}+X_i\frac{\partial p_i^w}{\partial\tau_i}] \\
& +A^*A[p_i^w(\theta_i-1)(D_i'(\theta_i p_i^w+\tau_i\theta_i\frac{\partial p_i^w}{\partial\tau_i})_i-X_i'\theta_i\frac{\partial p_i^w}{\partial\tau_i})] \\
& +A^*A[Nu_E'(N-1)d_i'(\theta_i p_i^w+\tau_i\theta_i\frac{\partial p_i^w}{\partial\tau_i})+Nu_E'N^*d_i'^*\tau_i^*\theta_i^*\frac{\partial p_i^w}{\partial\tau_i}] \\
& +A(I_{iL*}^*-\alpha_L^*)X_i^*\theta_i^*\frac{\partial p_i^w}{\partial\tau_i} \\
& +AA^*[X_i^*\theta_i^*\frac{\partial p_i^w}{\partial\tau_i}-D_i^*\tau_i^*\theta_i^*\frac{\partial p_i^w}{\partial\tau_i}+D_i^*\tau_i^*\theta_i^*\frac{\partial p_i^w}{\partial\tau_i}-D_i^*\theta_i^*\frac{\partial p_i^w}{\partial\tau_i}] \\
& +AA^*[(\tau_i^*-1)p_i^{s*}D_i'^*\tau_i^*\theta_i^*\frac{\partial p_i^w}{\partial\tau_i}+(\theta_i^*-1)p_i^w(D_i'^*\tau_i^*\theta_i^*\frac{\partial p_i^w}{\partial\tau_i}-X_i'^*\theta_i^*\frac{\partial p_i^w}{\partial\tau_i})] \\
& +AA^*[D_i^*\theta_i^*\frac{\partial p_i^w}{\partial\tau_i}-D_i^*\frac{\partial p_i^w}{\partial\tau_i}-X_i^*\theta_i^*\frac{\partial p_i^w}{\partial\tau_i}+X_i^*\frac{\partial p_i^w}{\partial\tau_i}] \\
& +A^*A[N^*u_E'^*d_i'^*(N^*-1)\tau_i^*\theta_i^*\frac{\partial p_i^w}{\partial\tau_i}+N^*u_E'^*Nd_i'(\theta_i p_i^w+\tau_i\theta_i\frac{\partial p_i^w}{\partial\tau_i})=0.
\end{aligned}
$$

And thus

$$
\begin{aligned}
(B10)\quad & A^*(I_{iL}-\alpha_L)X_i(\theta_i \frac{\partial p_i^w}{\partial \tau_i}) \\
& + A^* A[(\tau_i - 1)\theta_i p_i^w D_i'(\theta_i p_i^w + \tau_i \theta_i \frac{\partial p_i^w}{\partial \tau_i})] \\
& + A^* A[p_i^w(\theta_i - 1)(D_i'(\theta_i p_i^w + \tau_i \theta_i \frac{\partial p_i^w}{\partial \tau_i})_i - X_i'\theta_i \frac{\partial p_i^w}{\partial \tau_i})] \\
& + A^* A[Nu_E'(N-1)d_i'(\theta_i p_i^w + \tau_i \theta_i \frac{\partial p_i^w}{\partial \tau_i}) + Nu_E' N^* d_i'^* \tau_i^* \theta_i^* \frac{\partial p_i^w}{\partial \tau_i}] \\
& + A(I_{iL*}^* - \alpha_L^*)X_i^* \theta_i^* \frac{\partial p_i^w}{\partial \tau_i} \\
& + AA^*[(\tau_i^* - 1)\theta_i^* p_i^w D_i'^* \tau_i^* \theta_i^* \frac{\partial p_i^w}{\partial \tau_i}] \\
& + AA^*[(\theta_i^* - 1)p_i^w(D_i'^* \tau_i^* \theta_i^* \frac{\partial p_i^w}{\partial \tau_i} - X_i'^* \theta_i^* \frac{\partial p_i^w}{\partial \tau_i})] \\
& + AA^*[N^* u_E'^*(N^*-1)d_i'^* \tau_i^* \theta_i^* \frac{\partial p_i^w}{\partial \tau_i} + N^* u_E'^* N d_i'(\theta_i p_i^w + \tau_i \theta_i \frac{\partial p_i^w}{\partial \tau_i})] = 0.
\end{aligned}
$$

To get $\frac{\partial p_i^w}{\partial \tau_i}$ totally differentiate the world market equilibrium condition $D_i(\tau_i \theta_i p_i^w) - X_i(\theta_i p_i^w) = -[D_i^*(\tau_i^* \theta_i^* p_i^w) - X_i^*(\theta_i^* p_i^w)]$. This yields

$$
(B11) \quad \frac{\partial p_i^w}{\partial \tau_i} = -\frac{M_i' p_i^w}{\theta_i M_i' + \theta_i^* M_i'^*},
$$

where $M_i' = \tau_i D_i' - X_i'$ and $M_i'^* = \tau_i^* D_i'^* - X_i'^*$.

Substituting equation (B11) for $\frac{\partial p_i^w}{\partial \tau_i}$ collecting terms, and then solving for the consumption policy in the home country yields

$$(B12)\quad (\tau_i - 1) = -\frac{(I_{iL} - \alpha_L)}{(a+\alpha_L)}\frac{X_i}{p_i^w(\theta_i X_i' - \theta_i^* M_i'^*)} - \frac{(I_{iL*}^* - \alpha_L^*)}{(a^* + \alpha_L^*)}\frac{\theta_i^* X_i^*}{p_i^w \theta_i(\theta_i X_i' - \theta_i^* M_i'^*)}$$
$$-(\tau_i^* - 1)\frac{\tau_i^* \theta_i^{*2} D_i'^*}{\theta_i(\theta_i X_i' - \theta_i^* M_i'^*)} + [(\theta_i - 1) - (\theta_i^* - 1)][\frac{\theta_i^* M_i'^*}{\theta_i(\theta_i X_i' - \theta_i^* M_i'^*)}]$$
$$-\frac{u_E'(N-1)}{p_i^w \theta_i} - \frac{u_E'^* N^*}{p_i^w \theta_i} - \frac{N u_E' D_i'^* \tau_i^* \theta_i^*}{p_i^w \theta_i(\theta_i X_i' - \theta_i^* M_i'^*)} - \frac{u_E'^*(N^*-1) D_i'^* \tau_i^* \theta_i^*}{p_i^w \theta_i(\theta_i X_i' - \theta_i^* M_i'^*)}$$

The foreign country's consumption policy can implicitly be derived in a similar manner as the home country's consumption policy. The result is

$$(B13)\quad (\tau_i^* - 1) = -\frac{(I_{iL*}^* - \alpha_L^*)}{(a^* + \alpha_L^*)}\frac{X_i^*}{p_i^w(\theta_i^* X_i'^* - \theta_i M_i')} - \frac{(I_{iL} - \alpha_L)}{(a+\alpha_L)}\frac{\theta_i X_i}{p_i^w \theta_i^*(\theta_i^* X_i'^* - \theta_i M_i')}$$
$$-(\tau_i - 1)\frac{\tau_i \theta_i^2 D_i'}{\theta_i^*(\theta_i^* X_i'^* - \theta_i M_i')} + [(\theta_i^* - 1) - (\theta_i - 1)][\frac{\theta_i M_i'}{\theta_i^*(\theta_i^* X_i'^* - \theta_i M_i')}]$$
$$-\frac{u_E'^*(N^*-1)}{p_i^w \theta_i^*} - \frac{u_E' N}{p_i^w \theta_i^*} - \frac{N^* u_E'^* D_i' \tau_i \theta_i}{p_i^w \theta_i^*(\theta_i^* X_i'^* - \theta_i M_i')} - \frac{u_E'(N-1) D_i' \tau_i \theta_i}{p_i^w \theta_i^*(\theta_i^* X_i'^* - \theta_i M_i')}.$$

Equations (B12) and (B13) are the same as equations (26a) and (26b) in chapter 4.
Now consider the first-order condition for the home country's trade policy θ_i

$$
(B14) \quad
\begin{aligned}
& A^*(I_{iL}-\alpha_L)X_i \frac{\partial p_i^s}{\partial \theta_i} \\
& + A^* A[X_i \frac{\partial p_i^s}{\partial \theta_i} + (\tau_i - 1)D_i \frac{\partial p_i^s}{\partial \theta_i} + p_i^s(\tau_i - 1)D_i' \frac{\partial p_i^d}{\partial \theta_i} - D_i \frac{\partial p_i^d}{\partial \theta_i}] \\
& + A^* A[\frac{\partial p_i^w}{\partial \theta_i}(\theta_i - 1)M_i + p_i^w M_i + p_i^w(\theta_i - 1)(D_i' \frac{\partial p_i^d}{\partial \theta_i} - X_i' \frac{\partial p_i^s}{\partial \theta_i})] \\
& + A^* A[Nu_E'(N-1)d_i' \frac{\partial p_i^d}{\partial \theta_i} + Nu_E' N^* d_i'^* \frac{\partial p_i^{d*}}{\partial \theta_i}] \\
& + A(I_{iL*}^* - \alpha_L^*)X_i^* \frac{\partial p_i^{s*}}{\partial \theta_i} \\
& + AA^*[X_i^* \frac{\partial p_i^s}{\partial \theta_i} + (\tau_i^* - 1)D_i^* \frac{\partial p_i^{s*}}{\partial \theta_i} + p_i^{s*}(\tau_i^* - 1)D_i'^* \frac{\partial p_i^{*d}}{\partial \theta_i} - D_i^* \frac{\partial p_i^{d*}}{\partial \theta_i}] \\
& + AA^*[\frac{\partial p_i^w}{\partial \theta_i}(\theta_i^* - 1)M_i^* + p_i^w(\theta_i^* - 1)(D_i'^* \frac{\partial p_i^{d*}}{\partial \theta_i} - X_i'^* \frac{\partial p_i^{s*}}{\partial \theta_i})] \\
& + AA^*[N^* u_E'^*(N^* - 1)d_i'^* \frac{\partial p_i^{d*}}{\partial \theta_i} + N^* u_E'^* N d_i' \frac{\partial p_i^d}{\partial \theta_i}] = 0.
\end{aligned}
$$

Differentiating the price equilibrium conditions (B1) with respect to θ_i yields

$$(B15a) \quad \frac{\partial p_i^s}{\partial \theta_i} = \frac{1}{\tau_i}(\theta_i \frac{\partial p_i^w}{\partial \theta_i} + p_i^w)$$

$$(B15b) \quad \frac{\partial p_i^{s*}}{\partial \theta_i} = \frac{\theta_i^*}{\tau_i^*} \frac{\partial p_i^w}{\partial \theta_i}$$

$$(B15c) \quad \frac{\partial p_i^d}{\partial \theta_i} = \theta_i \frac{\partial p_i^w}{\partial \theta_i} + p_i^w$$

$$(B15d) \quad \frac{\partial p_i^{d*}}{\partial \theta_i} = \theta_i^* \frac{\partial p_i^w}{\partial \theta_i}$$

Substituting equations (B15) into (B14), using the market equilibrium condition and collecting terms yields

$$
\begin{aligned}
(B16)\quad & A^*(I_{iL}-\alpha_L)X_i(\theta_i \frac{\partial p_i^w}{\partial \theta_i}+p_i^w) \\
& +A^*A[X_i(\theta_i \frac{\partial p_i^w}{\partial \theta_i}+p_i^w)-D_i(\tau_i\theta_i \frac{\partial p_i^w}{\partial \theta_i}+\tau_i p_i^w)+\tau_i D_i(\theta_i \frac{\partial p_i^w}{\partial \theta_i}+p_i^w)-D_i(\theta_i \frac{\partial p_i^w}{\partial \theta_i}+p_i^w)] \\
& +A^*A[(\tau_i-1)\theta_i p_i^w D_i'(\tau_i\theta_i \frac{\partial p_i^w}{\partial \theta_i}+\tau_i p_i^w)+p_i^w D_i-p_i^w X_i+D_i\theta_i \frac{\partial p_i^w}{\partial \theta_i}-D_i \frac{\partial p_i^w}{\partial \theta_i}-X_i\theta_i \frac{\partial p_i^w}{\partial \theta_i}+X_i \frac{\partial p_i^w}{\partial \theta_i}] \\
& +A^*A[p_i^w(\theta_i-1)(\tau_i D_i'-X_i')(\theta_i \frac{\partial p_i^w}{\partial \theta_i}+p_i^w)] \\
& +A^*A[Nu_E'(N-1)d_i'(\tau_i\theta_i \frac{\partial p_i^w}{\partial \theta_i}+\tau_i p_i^w)+Nu_E'N^*d_i'^*\tau_i^*\theta_i^* \frac{\partial p_i^w}{\partial \theta_i}] \\
& +A(I_{iL}^*-\alpha_L^*)X_i^*\theta_i^* \frac{\partial p_i^w}{\partial \theta_i} \\
& +AA^*[X_i^*\theta_i^* \frac{\partial p_i^w}{\partial \theta_i}-D_i^*\tau_i^*\theta_i^* \frac{\partial p_i^w}{\partial \theta_i}+D_i^*\tau_i^*\theta_i^* \frac{\partial p_i^w}{\partial \theta_i}-D_i^*\theta_i^* \frac{\partial p_i^w}{\partial \theta_i}] \\
& +AA^*[(\tau_i^*-1)p_i^w\tau_i^*D_i'^*(\theta_i^*)^2 \frac{\partial p_i^w}{\partial \theta_i}] \\
& +AA^*[D_i^*\theta_i^* \frac{\partial p_i^w}{\partial \theta_i}-D_i^* \frac{\partial p_i^w}{\partial \theta_i}-X_i^*\theta_i^* \frac{\partial p_i^w}{\partial \theta_i}+X_i^* \frac{\partial p_i^w}{\partial \theta_i}+(\theta_i^*-1)p_i^w(D_i'^*\tau_i^*\theta_i^* \frac{\partial p_i^w}{\partial \theta_i}-X_i^*\theta_i^* \frac{\partial p_i^w}{\partial \theta_i})] \\
& +AA^*[N^*u_E'^*(N^*-1)d_i'^*\tau_i^*\theta_i^* \frac{\partial p_i^w}{\partial \theta_i}+N^*u_E'^*Nd_i'(\tau_i\theta_i \frac{\partial p_i^w}{\partial \theta_i}+\tau_i p_i^w)]=0.
\end{aligned}
$$

Total differentiation of the market equilibrium condition yields

$$
(B17)\quad \frac{\partial p_i^w}{\partial \theta_i} = -\frac{(D_i' - X_i'/\tau_i)\,p_i^w}{\theta_i\left(D_i' - X_i'/\tau_i\right)+\theta_i^*\left(D_i'^* - X_i'^*/\tau_i^*\right)} = -\frac{M_i' p_i^w}{\theta_i M_i' + \theta_i^* M_i'^*}.
$$

Substituting (B17) into (B16) and then solving implicitly for the difference in the home and foreign countries' trade policies gives

$$(B18)\quad \begin{aligned}(\theta_i - 1) - (\theta_i^* - 1) = &-\frac{(I_{iL} - \alpha_L)}{(a + \alpha_L)}\frac{X_i}{p_i^w M_i'} + \frac{(I_{iL}^* - \alpha_L^*)}{(a^* + \alpha_L^*)}\frac{X_i^*}{p_i^w M_i'^*} \\ &-(\tau_i - 1)\frac{p_i^d D_i'}{p_i^w M_i'} + (\tau_i^* - 1)\frac{p_i^{d*} D_i'^*}{p_i^w M_i'^*} \\ &-\frac{u_E'(N-1)D_i'\tau_i}{p_i^w M_i'} - \frac{u_E'^* N^* D_i'\tau_i}{p_i^w M_i'} \\ &+\frac{u_E' N D_i'^* \tau_i^* \theta_i^*}{p_i^w \theta_i^* M_i'^*} + \frac{u_E'^*(N^*-1)D_i'^*\tau_i^*\theta_i^*}{p_i^w\theta_i^* M_i'^*}.\end{aligned}$$

Equation (B18) and is the same as equation (27) in chapter 4. Equations (B12), (B13), and (B18) implicitly determine the home and foreign countries' consumption policies and the difference between their trade policies. Substituting equation (B18) into the expressions for the consumption policies (B12) and (B13) yields for the home country's consumption policy

$$(B19)\quad \begin{aligned}(\tau_i - 1) = &-\frac{(I_{iL} - \alpha_L)}{(a+\alpha_L)}\frac{X_i}{p_i^w(\theta_i X_i' - \theta_i^* M_i'^*)} - \frac{(I_{iL*}^* - \alpha_L^*)}{(a^* + \alpha_L^*)}\frac{\theta_i^* X_i^*}{p_i^w \theta_i(\theta_i X_i' - \theta_i^* M_i'^*)} \\ &-(\tau_i^* - 1)\frac{\tau_i^* \theta_i^{*2} D_i'^*}{\theta_i(\theta_i X_i' - \theta_i^* M_i'^*)} \\ &+[\frac{\theta_i^* M_i'^*}{\theta_i(\theta_i X_i' - \theta_i^* M_i'^*)}]\{-\frac{(I_{iL} - \alpha_L)}{(a+\alpha_L)}\frac{X_i}{p_i^w M_i'} + \frac{(I_{iL}^* - \alpha_L^*)}{(a^* + \alpha_L^*)}\frac{X_i^*}{p_i^w M_i'^*} \\ &-(\tau_i - 1)\frac{p_i^d D_i'}{p_i^w M_i'} + (\tau_i^* - 1)\frac{p_i^{d*} D_i'^*}{p_i^w M_i'^*} \\ &-\frac{D_i'\tau_i(u_E'(N-1) + u_E'^* N^*)}{p_i^w M_i'} + \frac{D_i'^*\tau_i^*\theta_i^*(Nu_E' + u_E'^*(N^*-1))}{p_i^w\theta_i^* M_i'^*}\} \\ &-\frac{(u_E'(N-1) + u_E'^* N^*)}{p_i^w\theta_i} - \frac{D_i'^*\tau_i^*\theta_i^*(Nu_E' + u_E'^*(N^*-1))}{p_i^w\theta_i(\theta_i X_i' - \theta_i^* M_i'^*)}.\end{aligned}$$

Collecting terms, yields an implicit expression for the home country's political equilibrium consumption policy

$$(B20)\quad (\tau_i - 1) = -\frac{(I_{iL} - \alpha_L)}{(a + \alpha_L)}\frac{X_i}{p_i^s X_i'} - \frac{(u_E'(N-1) + u_E'^* N^*)}{p_i^s}.$$

Similarly, an the political equilibrium consumption policy for the foreign country is given by

$$(B21) \quad (\tau_i^* - 1) = -\frac{(I_{iL^*}^* - \alpha_L^*)}{(a^* + \alpha_L^*)}\frac{X_i^*}{p_i^{s*} X_i'^*} - \frac{(u_E'^*(N^* - 1) + u_E' N)}{p_i^{s*}}.$$

Substituting equations (B20) and (B21) into equation (B18) and collecting terms yields the difference of the political equilibrium trade policies

$$(B22) \quad (\theta_i - 1) - (\theta_i^* - 1) = \frac{(I_{iL} - \alpha_L)}{(a + \alpha_L)}\frac{X_i}{p_i^w X_i'} - \frac{(I_{iL^*}^* - \alpha_L^*)}{(a^* + \alpha_L^*)}\frac{X_i^*}{p_i^{w*} X_i'^*}.$$

Equations (B20) and (B21) are the same as equations (28a) and (28b) in chapter 4, while equation (B22) is the same as equation (28) in chapter 4.

APPENDIX C

Proof to the proposition made in section 4.4. (Footnote 75)

Proposition.

Suppose governments have production and trade policies available, and the elasticity of output supply for the home and foreign industry is constant. Further assume that in the noncooperative equilibrium the importing country' import demand is less elastic than the exporting country's import demand. Then, importers (exporters) are worse (better) off under free trade than under noncooperation.

Proof.

Under noncooperation the equilibrium production and trade policies are implicitly given by (with similar expressions for the foreign industry)

$$(C1) \quad (\tau_i - 1) = -\frac{(I_{iL} - \alpha_L)}{(a + \alpha_L)} \frac{X_i}{p_i^s X_i'} = -\frac{(I_{iL} - \alpha_L)}{(a + \alpha_L)} \frac{1}{\varepsilon_{X_i, p_i^s}}$$

$$(C2) \quad (\theta_i - 1) = -\frac{M_i}{p_i^w \theta_i^* M_i'^*} = \frac{1}{\varepsilon_i^*}$$

Under cooperation the equilibrium production and trade policies are implicitly given by (with similar expressions for the foreign industry)

$$(C3) \quad (\tau_i - 1) = -\frac{(I_{iL} - \alpha_L)}{(a + \alpha_L)} \frac{X_i}{p_i^s X_i'}.$$

$$(C4) \quad (\theta_i - 1) - (\theta_i^* - 1) = 0.$$

In equilibrium the producer price for the home industry is

$$(C5) \quad p_i^s = \frac{\theta_i}{\tau_i} p_i^w(\tau_i, \tau_i^*, \theta_i, \theta_i^*)$$

And thus

$$(C6)\quad \begin{aligned} dp_i^s &= \frac{\theta_i}{\tau_i}\frac{\partial p_i^w}{\partial \tau_i}d\tau_i - \frac{\theta_i p_i^w}{\tau_i^2}d\tau_i + \frac{\theta_i}{\tau_i}\frac{\partial p_i^w}{\partial \tau_i^*}d\tau_i^* \\ &+ \frac{\theta_i}{\tau_i}\frac{\partial p_i^w}{\partial \theta_i}d\theta_i + \frac{p_i^w}{\tau_i}d\theta_i + \frac{\theta_i}{\tau_i}\frac{\partial p_i^w}{\partial \theta_i^*}d\theta_i^*. \end{aligned}$$

From equations (C1), (C3) and assuming constant supply elasticities, it follows that the production policies do not change from noncooperation to cooperation. Thus $d\tau_i = d\tau_i^* = 0$, and equation (C6) becomes

$$(C7)\quad dp_i^s = \frac{\theta_i}{\tau_i}\frac{\partial p_i^w}{\partial \theta_i}d\theta_i + \frac{p_i^w}{\tau_i}d\theta_i + \frac{\theta_i}{\tau_i}\frac{\partial p_i^w}{\partial \theta_i^*}d\theta_i^*.$$

The market clearing condition yields

$$(C8)\quad \frac{\partial p_i^w}{\partial \theta_i} = -\frac{(D_i' - X_i'/\tau_i)\,p_i^w}{\theta_i\left(D_i' - X_i'/\tau_i\right) + \theta_i^*\left(D_i'^* - X_i'^*/\tau_i^*\right)} = -\frac{M_i' p_i^w}{\theta_i M_i' + \theta_i^* M_i'^*} < 0\cdot$$

Substituting (C8) and a similar condition for $\frac{\partial p_i^w}{\partial \theta_i^*}$ into (C7) and collecting terms yields

$$(C9)\quad \begin{aligned} \frac{dp_i^s}{p_i^s} &= \frac{-M_i'^*}{\theta_i M_i' + \theta_i^* M_i'^*}d\theta_i^* + \frac{\theta_i^* M_i'^*}{\theta_i M_i' + \theta_i^* M_i'^*}d\theta_i \\ &= \frac{-M_i'^*(\theta_i d\theta_i^* + \theta_i^* d\theta_i)}{\theta_i(\theta_i M_i' + \theta_i^* M_i'^*)}. \end{aligned}$$

The denominator of (C9) is negative, and the sign of (C9) depends on the sign of the numerator. In particular, the following holds

$$(C10)\quad Z = \theta_i d\theta_i^* + \theta_i^* d\theta_i > 0 \Rightarrow \frac{dp_i^s}{p_i^s} < 0$$

Under free trade, which is consistent with a negotiated outcome (C4), the following holds

$\theta_i = \theta_i^* = 1.$

Thus, using equation (C2) together with the expression for the foreign trade policy Z can be expressed as

$$(C11) \quad Z = -\frac{M_i}{p_i^w M_i'} - \frac{M_i^*}{p_i^w M_i'^*}$$

Suppose the home (foreign) country is importing (exporting) good i. Then Z is positive if and only if in the noncooperative equilibrium the home import demand is less elastic than the foreign import demand. Q.E.D.