

# **Multi-Objective Optimization: Riccati Iteration and the Lotfi Manufacturing Problem**

by

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Thesis submitted to the Faculty of  
Virginia Polytechnic Institute and State University  
in partial fulfillment of the requirements for the degree of

Master of Arts in Economics

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June 17, 2002

Falls Church, Virginia

**Key Words: Optimization, Simulation, Modeling, Riccati Iteration**

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**BENJAMIN CONAWAY MULL**

## **(ABSTRACT)**

In current economic research, there are many problems that are difficult to solve without powerful computers, unique software, or novel approaches. I wrote this thesis because I believe that a powerful solution technique known as the Riccati Iteration is such a novel approach, and can be applied to complex problems that would otherwise be infeasible to solve. This thesis will demonstrate the power of the Riccati iteration by employing the Riccati iteration with spreadsheet software to solve a difficult dynamic optimization problem – a capital replacement problem posed by Lotfi where multiple objectives have been identified. The Riccati iteration will be shown to be the most practicable method for solving this problem, especially when compared to the Lagrange and Least-Squares solution methods. It is hoped that the demonstration in this thesis is so compelling that others may consider using the Riccati approach in their own research.

To my Wife and Family, without whom I would never have completed this work.

## Acknowledgements

I would like to thank Dr. Thomas Lutton, for his continuous support, guidance, and enthusiasm throughout the life of this project. I would also like to thank Dr. Roger Waud for his thoughts, vision, and his insights, which really added depth to my work. I would also like to thank Dr. Nancy Wentzler and Dr. Richard Theroux for their time and efforts on my thesis.

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*June, 2002*

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