Instructors' Orientation on Mathematical Meaning

Ahsan Habib Chowdhury

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Estrella Johnson (Chair)
Anderson Norton
Megan Wawro
Jesse Wilkins

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ABSTRACT

Students often ask "when is this ever going to be useful?", "why are we doing this?", etc. when speaking about mathematics. If we take this as a question about 'meaningfulness', how can instructors respond and how do they even understand the terms 'meaningful' and 'meaning'? My dissertation looked at how college instructors see their instruction as meaningful or not. Drawing on social and cognitive perspectives of learning, I define four ways to think of what's 'meaningful' about mathematics. From a cognitive perspective, instructors can understand 'meaningful' as mathematical understanding versus understanding the significance of mathematics. From a social perspective where meaning is taken as the experiences of everyday life within communities, teachers can understand 'meaningful' as anything that engages students in practices the mathematics community engage in versus practices non-mathematics communities engage in (e.g. pushing computation or critical thinking as a means for maintaining social hierarchies). Using these four conceptions to categorize instructors' goals, this work focuses on how four undergraduate mathematics instructors thought of their instruction as meaningful and contextual and background factors that influenced those views.

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GENERAL AUDIENCE ABSTRACT

Students often ask "when is this ever going to be useful?" when speaking about mathematics. If we interpret this as seeking the meaning or purpose of their education, how can teachers respond and how do they even understand the terms 'meaningful' and 'meaning'? I wanted to look at how college instructors thought of this and how they addressed such a question in their classrooms. Drawing on different theories of learning, I outlined four ways to think of what's 'meaningful' about mathematics and then used these four ways to categorize how instructors think of their instruction as meaningful.

To meet this end, I looked at some accounts of instructors' goals. My data came from college instructors of different mathematics classes: math for elementary education, math for liberal arts, statistics, and calculus. One important thing I found was that experiences with underserved communities or of not being 'a math person' corresponded with instructors' ability to attend to different kinds of 'meaningful' goals. What this might suggest is that educators may not feel prepared to respond to students' pursuit of meaning in diverse ways unless they have also personally struggled with it growing up or have personally experienced the consequences of disenfranchisement.

Acknowledgements

The questions in this work started from my own experiences as an undergraduate student struggling to see the point of pursuing mathematics in light of interests I was much more passionate about. Given those struggles, I am grateful for all the people who have helped me get to where I am today in completing this work. These include but are not limited to:

- My family and friends who supported me, even if I could be a pain to deal with.
- My advisor, Estrella Johnson, who was the first person in my life to actually be a mentor to me. Having a mentor figure for the first time has really highlighted how those without mentors have fewer opportunities. In the past, I've mostly relied on myself to be my own advocate. As a result, there were so many things I just couldn't know and missed because I didn't have the knowledge or connections. Comparing my opportunities now to my perceived opportunities in life before I had a mentor has shown me all the ways in which mentorship can expand one's horizons. It really has taught me the value of mentorship.
- My undergraduate Jewish studies professor at the University of Virginia, Peter Ochs. His discussions with me almost a decade ago, in which he pushed me to connect my mathematics and religious studies majors, got me to appreciate mathematics. Even more so, he got me to question what my mathematics education lacked that got me to dislike mathematics so much before. Those questions eventually developed into the research questions at the center of this study.

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- All my teachers, even the ones who didn't believe in me. In fact, it was those teachers who didn't believe in me that taught me grit and resilience because I had to believe in my own ability to pull myself back up even when others did not. For those who did believe in me, I hope I've lived up to the potential you all believed I could achieve.

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Chapter 1: Motivation and Introduction

Students in mathematics have often voiced complaints to the effect of "when is this ever going to be useful?" (Gough, 1998). Prior work has suggested that usefulness and applicability are important in learning and teaching (Jones & Wilkins, 2013; Saxe & Brady, 2015). I interpret this question and others like it, such as 'why are we learning this?', 'why are we doing things this way?' as students asking for the purpose, usefulness, and/or relevance of what they are learning. Under the assumption that all these terms are closely related, if not synonymous in an instructor's and students' eyes, I group these questions under an umbrella term I call the 'meaningfulness question(s).' Such questions can put an onus on instructors to make mathematics meaningful to their students. However, how does an instructor interpret what is 'meaningful'? And how do instructors, if at all, tailor their instruction to what they consider meaningful? This study aims to better understand how instructors view their instructional goals (as a subset of instructors' beliefs) as meaningful for students and how those goals influence instructors' instructional practices.

Beliefs and Belief Systems

Since my work focuses on what instructors believe are 'meaningful' instructional goals and how those specific beliefs then do or do not influence their instruction, I will first review some general literature on the role of beliefs in mathematics education. Much research literature has been devoted to looking at the beliefs of mathematics instructors (Calderhead, 1996; Ernest, 1989; Leder, Pehkonen, & Törner, 2006; Leatham, 2007; Lew, Fukawa-Connelly, Mejía-Ramos, & Weber, 2016; Pajares, 1992; Schoenfeld, 1998;

Speer, 2008; Thompson, 1992; Weber, 2004; Wilkins, 2008). Some studies have looked at how specific beliefs play into instruction (Speer, 2008; Sztajn, 2003), such as how beliefs about what students need can lead to either pushing procedural student engagement or student sense-making (Sztajn, 2003). The mathematics education research community's focus on instructors' beliefs was slow to develop, initially not account for them to then focus on the inconsistencies between beliefs and practice (Hoyles, 1992). More recently, many have come to realize the situated nature of instructors' beliefs in an instructor's surrounding context (Hayward, Kogan, & Laursen, 2016; Hora 2012; Hoyles, 1992; Johnson, Caughman, Fredericks, & Gibson, 2013; McDuffie & Graeber, 2003; Speer, 2005; Turpen, Dancy, & Henderson, 2016). In turn, some researchers have pushed back against the cliché of inconsistency between beliefs and practice (Hoyles, 1992; Leatham; 2006; Speer, 2005).

As a way to consider context and move beyond the tired notion of inconsistency, Leatham (2006) synthesized a belief systems framework for conceptualizing instructors' beliefs as coherent and sensible (as opposed to inconsistent). Beliefs are organized with other beliefs in constellations called belief systems. In these systems, beliefs vary on three dimensions:

- The psychological strength factor describes the relative strength of a belief, ranging from peripheral to central.
- The quasi-logical relationships factor pertains to how beliefs organize as implications or precedents of one another.

- The extent to which beliefs cluster in isolation from other beliefs (3rd dimension) is determined by context, and thus individuals can have seemingly contradictory beliefs because contexts vary.

Under the assumption that the terms ‘meaningful’, ‘relevant’, and ‘important’ can be closely related if not synonymous, I hold that what instructors consider ‘meaningful’ is itself a belief which is then related to other beliefs within belief systems. However, there are various ways the term ‘meaningful’ can be thought of.

Conceptions of Mathematical Meaning

Some in the mathematics community have consciously or unconsciously aligned the term ‘meaningful’ with a sense of conceptual understanding of mathematics (Brownell, 1947; Thompson, 2013, 2015; Wawro, Sweeney, & Rabin, 2011, p. 17; Weber & Alcock, 2004, p. 227). ‘Meaningful’ can also be understood from the standpoint of relevance or ‘real-life’ applicability (Brownell, 1947). This view of meaning is important due to the motivating power of usefulness for learning (Jones & Wilkins, 2013) and the calls for instructional reform to include applications of mathematics in instruction (Saxe & Brady, 2015).

Brownell (1947) acknowledges both views of meaning and in turn, distinguishes “the *meaning of* a thing and the *meaning of* a thing *for* something else; for the sake of brevity, between the *meaning of* and *meaning for*” (emphases in the original, p. 236). In the context of meaningful arithmetic, Brownell takes the use of arithmetic to meet real-life needs as related to the *meaning for* arithmetic. By contrast, just as the *meaning of* the

atomic bomb refers to the sciences behind the bomb's workings, the *meaning of* arithmetic comes from mathematics,

> They are not to be found in the life-settings in which they are normally imbedded, except by him who already possesses them. They must be sought in the mathematical relationships of the subject itself, in its concepts, generalizations, and principles. In this sense, a child has a meaningful arithmetic when the situation with which he deals 'makes sense' mathematically. (p. 257)

Thus, Brownell acknowledges that 'meaningful' can be understood either in terms of mathematical understandings or as the relevance of mathematics. For Brownell then, the "meaning of" mathematics (MoM) is synonymous with mathematical understanding in the sense of understanding the connections between and within mathematical concepts. In Brownell's conceptions, the "meaning for" mathematics (MfM) is understanding mathematics' relevance for some non-mathematical purpose. From these two conceptions of mathematical meaning, I will begin to frame research and my work around meaning.

Past research has focused on the differing senses of meaning in mathematics, though not always from the standpoint of Brownell's framework. Those who cite Brownell have done so in their discussions on meaning for mathematical understandings, the implications of those mathematical understandings for non-mathematical purposes, or both (Coxbill, Chamberlin, & Weatherford, 2013; Grouws et al., 2013; Hiebert & Grouws, 2007; Kanes, 2002, April; Lappan & Even, 1989; Lo, Grant, & Flowers, 2008; Lortie-Forgues, Tian, & Siegler, 2015; National Research Council, 1988; Newton, 2008; Newton & Sands, 2012; Noddings, 2013; Reys et al., 1999; Siegler & Braithwaite, 2017; Silver, Shapiro, & Deutsch, 1993; Sriraman, 2003; Thomas & Harkness, 2013). However,

none of the articles I found have focused their study on comparing how both meanings can manifest and interact in teachers' instruction (though Sullivan (1989) does to some extent). Going beyond Brownell, by focusing on students understanding of mathematical concepts themselves, Brownell (1947) and Thompson (2013, 2015) focus on the MoM from a cognitive perspective, with Thompson even taking a Piagetian perspective towards meaning. Butler (2013) and Niss (1996) focus on relevance in the form of individual civic engagement or broader national policy goals for instance and thus align with Brownell's definition of MfM.

Both conceptions of meaning thus far focus on how an *individual* understands meaning. However, Bruner (1996) and Wenger (1998) look at meaning from a social perspective. For Wenger specifically, *meaning is the experience of everyday life* as determined by ones' practices within broader communities and social configurations. If one is situated within a community of mathematicians, one's day-to-day experiences (i.e., meaning) would then entail the mathematical practices that the mathematics community engages in to meet its goals. By contrast, if one is situated in a community composed of non-mathematicians, one's day-to-day experiences of using mathematics (if mathematics is used) would be geared towards the non-mathematical goals of that community. Using Wenger's views of meaning to expand on Brownell's framework, I classify the practices that the mathematics community engages in to meet its goals as the *social meaning of mathematics* (sMfM). Such practices could include symbolizing, algorithmatizing, or defining through progressive mathematization (Rasmussen, Zandieh, King, & Teppo, 2005). Similarly, I define *the social meaning for mathematics* (sMoM) as practices that are transferrable to other communities, such as procedural computations in physics

(Danielak, Gupta, & Elby, 2014) or using mathematics to foster civic engagement on the local level (Gregson, 2007, April; Hadlock, 2013).

Synthesizing the prior thinking, I can delineate four orientations towards mathematical meaning: a social "meaning of" (sMoM), a social "meaning for" (sMfM), an individual "meaning of" (iMoM), and an individual "meaning for" (iMfM).

The Role of Meaning in Instruction

The different orientations on mathematical meaning are just one aspect of an instructor's beliefs and goals. However, as prior literature has found, beliefs can influence instruction, and thus we have reason to suspect the orientations towards mathematical meaning may influence instruction as well. For instance, Jaworski, Treffert-Thomas, and Bartsch (2009) found a lecturer wanted to foster students' conceptual understanding but was caught up when students could only engage computationally. The teacher acknowledged that computations had to be focused on to help students pass exams, but they wanted to move students beyond computations to understand mathematics abstractly. From the standpoint of meaning outlined above, trying to promote conceptual understanding can be understood as an iMoM-oriented goal (aiming to build understanding of mathematics) while focusing on computations may be understood as an iMfM-oriented goal because of the goal to pass exams. Butler (2013) noted how she implemented service-learning to generate more interest in mathematics itself (iMoM-oriented). For her then, the focus on the iMoM led her to incorporate certain instructional practices which engaged students in real-world problems, which would seem sMfM- or iMfM-oriented at first glance. One could even frame Butler's desire to promote interest in mathematics through practices compatible with the MfM as aiming to

foster long-term sMoM-oriented goals since she was trying to further her students' progress towards completing a degree in mathematics. Such a degree would (hopefully) enable students to function as part of the community of mathematicians.

Research Question

Within this complex interrelationship of beliefs, meanings, context, and instruction, I pose the following research question: how does an instructor's navigation of mathematical meanings manifest in instruction? This entails numerous sub-questions:

1. What, in terms of background, affective factors, etc., leads instructors to prioritize certain meanings over other meanings?
2. How do instructors' orientations of mathematical meaning translate into specific pedagogical decisions?
3. How does context (e.g., student population, lack or prevalence of student interactions, course, etc.) influence the meanings instructors emphasize?
4. How do backgrounds, contexts, and orientations come together in pedagogical decisions with respect to how instructors respond to students' meaningfulness questions (e.g., "when is this ever going to be useful?", "why are we learning this?").

Methodology

I suspect various instructional practices could align with each of the four understandings of meaning I set out. Using a case study methodology, I observed how four instructors of different courses navigate mathematical meaning in their instruction.

My choice for picking multiple classes came from my suspicion that different contexts scaffold or constrain the mathematical meaning-related goals instructors may pursue. Level of course control, student populations, and course content all vary with different courses. For these reasons, I looked at instructors in courses ranging from calculus to mathematics for preservice teachers at a research university in the Western United States. I observed and interviewed each instructor for three separate weeks throughout the quarter.

Case studies examine one or more cases in detail and can be used to research questions focusing on exploring, describing, and explaining (Johnson and Christensen, 2014). Case studies focus on understanding how a system operates holistically. Numerous parts make up the whole of a system, and understanding the whole requires drawing connections between the parts to see how they come together. Multiple-case designs involve considering multiple cases so as to predict similar results (literal replication) or predict contrasting results based on the propositions of the research study (theoretical replication; Yin, 2009). Both outcomes are meant to further develop the theoretical framework a researcher brings to bear on their topic of study.

Kuntz (2015), however, critiques common research methodologies for extracting context and relational dynamics from the objects of study to turn participants into decontextualized bits of data. To mitigate these sorts of pitfall, I used Speer's (2005) video clip interview approach, interviews in which a researcher and participating instructor watch a clip of a class session together and questions proceed towards building a shared understanding. Engaging in this sort of interview naturally required that I record class sessions and develop an interview protocol to ask instructors about their meaning-

related goals. To further account for the relational dynamics instructors experience in teaching, I collected homework problems and used White and Mesa's (2012) observation protocol to document instructors' various pedagogical decisions.

Data analysis required that I analyze each instructor individually by looking at the homework assignments they give to their students, the kinds of activities they engage in during class, and the goals they stated during interviews. I used thematic analysis (Braun & Clarke, 2006) to pull all these sources together to understand how each instructors' meaning oriented-goals affected their instruction. During the latter phases of the individual analysis, I began engaging in cross-case analysis (Yin, 2009) using thematic analysis to look across instructors for general trends and differences in how instructors approached their instruction. By doing this, I moved towards answering my research questions about how the instructors collectively see their instruction as meaningful for their students.

Chapter 2: Literature Review and Frameworks

Many mathematics teachers have heard the common student complaint to the effect of "when is this ever going to be useful?" (Gough, 1998). Jones and Wilkins (2013) suggest that usefulness is an important motivating factor in student learning, while Saxe and Brady (2015) note the importance of applications and modeling in larger instructional reform efforts. I interpret this question, and others like it such as 'why are we learning this?', 'why are we doing things this way?' as students asking for the purpose, usefulness, and/or relevance of what they are learning, and thus I group these questions under an umbrella term I call the 'meaningfulness question(s).' My study explores how instructors' goals, beliefs, and values (related to what is considered 'meaningful') influences instructors' pedagogical practices.

I take the terms pedagogy and instruction to broadly refer to the methods, assessments, and practices instructors adopt to teach, whether these are connected under a larger framework or connected simply by the instructor's choice to place them together in their instruction. I use the term pedagogical and instructional decisions and beliefs to include the choices and associated beliefs regarding activities or actions the instructor and/or students engage in, as determined by the instructor to meet instructional goals. This includes decisions and rationale regarding curriculum, assessment, classroom activities, the kinds of discourse the instructor wishes to create in the classroom, and instructors' decisions on how to deliver course content. I also assume that the terms meaningful, relevant, useful, and important can be closely related, if not synonymous, in the eyes of students and/or instructors. I group questions asking for these things together under an umbrella term I call the 'meaningfulness question(s).' I am interested in looking

at the specific pedagogical beliefs about “meaningful” instruction, but this calls for examining the broader spectrum of beliefs concerning instruction first.

Beliefs and Instruction

Many have looked at the beliefs of mathematics instructors. I will outline some which highlight the importance of beliefs on instruction. Following Ernest’s (1989) model of the beliefs, attitudes, and knowledge of mathematics instructors, Wilkins (2008) found teachers' beliefs can provide insight into specific pedagogical decisions. In building models of instruction-in-practice, Schoenfeld (1998) found that instructors’ beliefs about subject matter (generally and of specific topics), learning, teaching (and roles of various kinds of instruction), and individual students or groups of students all have important relations with instructors’ pedagogical decisions. Of particular note, instructors may have the same goals and lesson plans, yet their lessons can play out entirely differently because each instructor’s beliefs and knowledge base can differentially influence how goals are enacted.

Speer (2008) conducted an in-depth analysis of how specific beliefs can impact undergraduate mathematics instruction. In looking at a graduate student teaching calculus, she found two collections of beliefs played prominently in the instructor’s decision-making: beliefs about evidence of student understanding and beliefs about learning. Beliefs about evidence of understanding entailed beliefs related to when the instructor would ask questions and what kind of questions to ask since he held beliefs that students’ inability to respond to such questions were indications of poor understanding. His beliefs about learning focused around beliefs that students should make knowledge their own by connecting knowledge to things students already knew. This justification

was connected to the instructor's pedagogical decisions of having students solve problems (as opposed to him presenting content to them) and centering class discussions on connecting conceptual issues to one another. Others have also identified the importance of beliefs and goals on instruction (Calderhead, 1996; Leder, Pehkonen, & Törner, 2006; Leatham, 2007; Lew, Fukawa-Connelly, Mejía-Ramos, & Weber, 2016; Pajares, 1992; Thompson, 1992; Weber, 2004).

The prior work cites the importance of beliefs on instruction but does not address how beliefs have been prioritized and conceptualized in research. Hoyles (1992) conducted a meta-analysis of the attention the mathematics education community devoted towards the role of instructors and their beliefs in mathematics instruction. The process of recognizing teachers' influence was slow, starting with the community predominantly focused on students while teachers simply dispensed facts. This shifted when the research community began examining teacher's beliefs about mathematics, the goals and tasks for mathematics classrooms, and students' and teachers' responsibilities regarding motivation, discipline, and assessment. Research then began to 1) identify inconsistencies between classroom actions and instructors' professed beliefs, 2) understand the role of attitude and beliefs towards mathematics and the teaching of mathematics on teaching style, and 3) realize the importance of teaching and classroom practice. These latter realizations led to a view that teachers and their beliefs were something to be accounted for (often as an obstacle) in implementing curricula reform. However, shifts came that called for attention to teacher's beliefs within social and cultural contexts.

Hoyles (1992) notes how, in light of the supposed inconsistencies between beliefs and practice, frameworks arose to try to explain the phenomenon. Notions such as

espoused versus enacted beliefs were one such product of these research efforts. In turn, research began looking into innovation and curriculum change. Such attempts were often characterized by the perspective that teachers need to change, and such change could be brought about by engaging teachers in certain practices such as formal reflections on instruction or engaging in technology use.

In contradiction to and to explain away the kinds of inconsistencies found in past research approaches, Hoyles (1992) advocates abandoning the beliefs and beliefs-in-practice dichotomy in favor of a situated beliefs framework where beliefs are situated in complex interactions of beliefs and context. In this framework, Hoyles holds that beliefs are dialectically constructed products of activity, context, and culture, varying by context. Thus “it is self-evident that any individual can hold multiple (even contradictory) beliefs and "mismatch", "transfer" and "inconsistency" are irrelevant considerations and replaced by notions of constraint and scaffolding within settings” (p. 40). Hoyles briefly demonstrates the potential of such an approach by reflecting on and re-analyzing a prior analysis they conducted.

Speer (2005) echoes the arguments made by Hoyles in her analysis of prior research on beliefs. Past research looked at what constitutes a belief (versus knowledge), specifically focusing on views on the nature of mathematics and teaching and learning mathematics. Attempts were made to categorize beliefs, with some attempts focusing on classify as many beliefs as possible while others focused specifically on smaller sets of or singular beliefs. Spear notes how past research looked at teachers’ classroom practices and beliefs separately. Such studies found that what teachers said they did (like advocating reform) was not what the researchers perceived them to be doing in class

observations. Speer attributes this disconnect to the lack of shared understanding of terms between the researcher and teachers (like what does it mean to do group work) during data collection.

However, Speer (2005) notes that past research attributed the disconnect to things like logistical, practical, or social constraints of teaching in the moment, and improperly assumed beliefs remain constant across contexts. Such work then pushed the dichotomy between professed beliefs (those that instructors explicitly stated) versus attributed beliefs (the beliefs inferred from classroom practices and other data sources). Some methodological attempts were made to better account for the connection between beliefs and practice. For instance, the interactionist perspective held that beliefs adapt due to interactions with one's environment, priorities, and students, but still assumed the professed versus attributed beliefs dichotomy. On the other hand, discursive psychology focused entirely on what teachers said or wrote explicitly.

In light of previous methodological shortcomings, Speer (2005) poses and demonstrates her own method of examining the relationship between beliefs and practice. Compared to Hoyles (1992), Speer does acknowledge the possibility for instructors to hold beliefs that contradict their practices. However, such inconsistencies can only be determined through rigorous data collection methods in which both researcher and instructor build a shared understanding of terms. Thus, Hoyles and Speer both challenge prior mathematics education research on how beliefs have affected instructors and their instruction. They differ in that Speer challenges the notion of inconsistency on methodological grounds and thus still allows for the distinction between beliefs and practice. Hoyles, by contrast, creates a situated framework of beliefs where contradictions

are eliminated. Instead, instructors can hold contradictory beliefs because different contexts call different beliefs to the fore.

The role of context. The role of context figured tacitly in my discussions above. I will address the role of context on beliefs and instruction more directly now. Ingram and Clay (2000) outline the theory of new institutionalism, a theory focused on how individuals are actors within institutions and associated constraints. The main tenets of the theory are that 1) actors are rationally bounded (by limited knowledge and cognitive capacity), 2) institutions are defined as the rules and enforcement mechanisms that constrain actors' choices, and 3) institutions constrain actors towards goals (e.g., the collective good or the personal goals of powerful actors).

Prior research has documented ways instructors' contexts (i.e., their institutionally situated realities) constrain or scaffold their goals and instruction. Institutions and communities have been found to affect instruction. Examples of this include supportive/unsupportive administrators who advocate or detract from instructional development (Hayward, Kogan, & Laursen, 2016; Hora 2012; McDuffie & Graeber, 2003), supportive colleagues to turn to (Hayward et al. 2016), and institutional responsibilities (such as research or course load requirements). Institutional rewards could either constrain or scaffold instruction depending on whether instructional development was considered in promotion and tenure (Hayward et al., 2016; Hora 2012; McDuffie & Graeber, 2003). Aspects of teaching in institutions themselves constrain or scaffold instruction. Such aspects include class time required for non-lecture pedagogies (McDuffie & Graeber, 2003; Turpen, Dancy, & Henderson, 2016), teaching environment and available resources (McDuffie & Graeber, 2003), class size (Hayward et al., 2016),

student resistance (Hayward et al., 2016), and content coverage pressure and concerns over preparation for future courses (Hayward et al., 2016; Hora, 2012; Johnson, Caughman, Fredericks, & Gibson, 2013).

For some specific examples of how the various aspects of institutions come together and affect instruction, Sztajn (2003) looked at the instruction of different teachers teaching to students of different socio-economic status in different locations. Sztajn found that the teacher teaching students of low SES taught mathematics from the standpoint of applying rules and procedures. This was because the teacher felt that the kinds of careers such students would get into would only require the ability to follow rules. By contrast, the high SES students' teacher taught with a greater emphasis on understanding. This was done because they felt their students would go onto college, and such skills would be necessary for such contexts. Hayward et al. (2016) also found that instructors "felt pressure to expose science and engineering students to a large set of computational techniques, rather than seeing the learning goal for these students as conceptual understanding" (p. 71). In both studies, the context instructors taught in (in terms of the needs of specific student populations within different contextual realities) entailed different instructional beliefs and goals. This was paralleled in Schoenfeld (1998). Thus, as Turpen et al. (2016) state, "behavior is the result of the interplay between an individual's characteristics (e.g., beliefs, knowledge, attitudes, experience, etc.) and the environment in which that individual is located" (p. 10).

Sensible belief systems. Leatham (2006) describes a framework for conceptualizing instructors' beliefs as situated and coherent and instructors as sensible (as opposed to inconsistent). In this framework of sensible belief systems, Leatham

partially sidesteps the issue of differentiating belief from other terms, such as knowledge and conceptions,

> In the sensible system framework, the following distinction is made: Of all the things we believe, there are some things that we ‘‘just believe’’ and other things that we ‘‘more than believe – we know.’’ Those things we ‘‘more than believe’’ we refer to as knowledge and those things we ‘‘just believe’’ we refer to as beliefs. Thus beliefs and knowledge can profitably be viewed as complementary subsets of the set of things we believe. (p. 92)

Pajares (1992) similarly cites conceptions of beliefs that do away with distinctions between attitudes, beliefs, knowledge, and values. Beliefs cannot always be articulated (or even consciously held) but are assumed to influence what one does. As such, beliefs must be inferred from one's actions, statements, and intentions. As opposed to describing what one believes, Leatham focuses on how beliefs are related to other beliefs. Leatham follows others in holding that individuals organize beliefs within constellations of beliefs (called belief systems) that make sense to the individual. Beliefs become *viable* "when the beliefs make sense with respect to that individual's other beliefs" (p. 93).

Drawing on past frameworks on belief systems (particularly Green (1971) and Rokeach (1968)), Leatham (2006) holds that beliefs can vary on three dimensions. The *psychological strength* of a belief describes the relative strength of a belief, ranging from peripheral (more likely to change) to central (more likely to resist change). This is determined by how strongly a belief coheres with other beliefs in a belief system. The sense-making process of organizing beliefs coherently with other beliefs is a process we are not always aware of. *Quasi-logical relationships*, the second dimension, relates to

how some beliefs are derivatives of other (primary) beliefs, like in an if-then statement. However, these sorts of implicative relationships do not imply that derivative beliefs have to be psychologically weaker than primary beliefs. An individual can adhere more strongly to a derivative belief than its primary belief antecedent. The *extent to which beliefs cluster in isolation from other beliefs* (3rd dimension) is determined by context. In a fashion similar to how beliefs are situated for Hoyles (1992), different beliefs and belief clusters arise in different contexts, and thus individuals can hold seemingly contradictory beliefs because contexts vary. Individuals can even support contradicting beliefs with the same underlying evidence. Such evidence is simply used differently according to differing circumstances. As such, contradictions do not exist in belief systems. In turn, Leatham holds that inconsistencies are in the eyes of observers who have either misunderstood certain beliefs' implications (second dimension) or failed to account for other beliefs becoming more prominent due to context (first and third dimensions). Both Leatham (2006) and Speer (2005) hold that in cases of inconsistency, it is the observer's job to probe further.

Leatham (2006) points to the power of the synthesized belief systems framework by reanalyzing past research findings where teachers' beliefs were deemed inconsistent with their instructional practices. His framework allowed him to point out flawed assumptions and unaccounted factors, such as pointing out certain beliefs having more psychological strength depending on context, researchers and teachers not having a shared understanding of terms, and assuming teachers are able to articulate all aspects of their beliefs. Leatham notes prior attempts to get around unstated beliefs by only comparing explicitly stated beliefs with classroom practice. Inconsistencies could still

show up within such frameworks. While such frameworks allowed for attempts to probe further and realize other priorities may have come to the fore for instructors, Leatham points out such perspectives may still hold onto assumptions that are not necessarily true, such as the assumption that beliefs about mathematics would be the strongest influence on practice.

Past frameworks and research on beliefs have tended to gloss over beliefs and belief systems (if even acknowledged) and held tight to notions of universal consistency (as a theoretical assumption). As a result, prior research was devoted to examining the inconsistencies between what instructors practice and what they believe. However, Hoyles, Leatham, and Speer critique prior conceptions of instructors' beliefs on methodological and theoretical grounds. In contrast to this, Hoyles and Leatham advocate for a more nuanced view of beliefs as situated. Leatham in particular synthesizes work to present a framework to aid in explaining inconsistencies and providing researchers with a better understanding of what beliefs are the most influential amongst instructors.

Brownell's Conceptions of Mathematical Meaning

I opened with the oft-posed student question: "when is this ever going to be useful?" I interpret this question as a student cry for a 'meaningful' experience. The question for instructors is how to respond to this. Hayward et al. (2016) and Sztajn (2003) found that mathematics teachers tailored their instruction to meet what they *believed* their students would need. This then relates to what instructors believe but, more specifically, what instructors believe they should focus on instructionally. From this framing, what is it that makes an instructional goal meaningful? I take what instructors consider meaningful to be a particular belief, which would then function within the confines of

belief systems as outlined in the literature above. To begin approaching this question of what makes a goal meaningful, I need to conceptualize how I think of the term "meaning."

In light of differences in how the term "meaningful" can be used, Brownell (1947) draws a distinction between the "meaning of" a thing versus the "meaning for" a thing. For instance, he acknowledges that he does not know the *meaning of* the atomic bomb because he takes the *meaning of* the bomb to refer to the kinds of knowledge (chemistry, physics, etc.) required to understand how such a thing works in and of itself. By contrast, he notes that he does know the atomic bomb's *meaning for* other things, such as consequences for war and culture. Within the context of arithmetic then, if one states that students have meaningful experiences because they used arithmetic in relation to real-life needs, one is speaking about the *meaning for* arithmetic. By contrast, Brownell holds that just as the *meaning of* the atomic bomb is found in the scientific knowledge needed to understand how the bomb works, so too are the *meanings of* arithmetic found within the subject which makes arithmetic make sense, i.e., mathematics. Brownell holds that these meanings are not tied to the life-settings in which arithmetic is embedded (except for those who hold the meanings already) but are instead found in the "mathematical relationships of the subject itself, in its concepts, generalizations, and principles" (p. 257). From the perspective of the *meaning of* then, an individual has a meaningful mathematical experience when they can deal with a situation in a way that "makes sense" mathematically.

Thus, Brownell sets out two senses of meaning in mathematics education: the *meaning of* and the *meaning for* mathematics. The *meaning of* mathematics (MoM) is

synonymous with mathematical understanding in the sense of understanding the connections between and within mathematical concepts. The *meaning for* mathematics (MfM) can be understood as understanding mathematics' significance for some non-mathematical purpose. From these two conceptions of mathematical meaning, I will frame research and my work on meaning.

MoM. Having set out the different ways to think of meaning, Brownell (1947) states, "*meanings of* may be defined as mathematical understandings, and it is in this sense that the word will be used" (p. 257) to explicitly focus on the MoM with regard to arithmetic. This entails making arithmetic sensible to students through understanding the mathematical concepts subsumed under arithmetic and the relations between those concepts. Brownell takes arithmetic meanings as the meanings of 1) "basic concepts" of whole numbers, fractions, decimal fractions, percent, ratio, and proportion, 2) fundamental operations of addition, subtraction, etc., 3) principles, relationships, and generalizations of arithmetic (such as adding zero to a number keeps the number constant, dividing the numerator and denominator by a number, but still retaining the same fraction, etc.) and 4) our decimal number system (and its associated role in understanding our arithmetic algorithms). Thus, by setting out the understandings of various mathematical relationships, operations, and concepts, Brownell sets out his view of what is necessary to grasp the *meaning of* arithmetic.

Brownell further demonstrates his focus on the MoM by discussing the poor arithmetic performance of individuals across society in his time and countering the objections of those who would question the importance of the *meaning of* arithmetic. Brownell states the following in relation to poor arithmetic competence,

> School personnel and, to some extent, the public at large are beginning to awaken to the fallacy of treating arithmetic as a tool subject. …. In such programs, arithmetical meanings of the kinds mentioned above have little or no place. Without these meanings to hold skills and ideas together in an intelligible, unified system, pupils in our schools for too long a time have “mastered" skills which they do not understand, which they can use only in situations closely paralleling those of learning, and which they must soon forget. (p. 260)

Brownell then outlines counterarguments against an instructional focus on the *meaning of* arithmetic. The arguments against include 1) whether meaning is actually needed for learning arithmetic, 2) whether students can grasp such meanings, 3) whether attending to the MoM pulls time away from more important things, and 4) whether such meanings actually prove useful once they have been learned. Brownell counters 1) by noting how mechanical computation leads to skills that deteriorate without practice and hardly ever transfer to novel contexts. He debunks 2) for absolutist thinking (that attending to meaning entails fully grasping all understandings whereas Brownell suggests progressive understanding that develops as thinking progresses) and 3) on the grounds that teaching for understanding takes time at first, but such understandings facilitate further understanding and thus save time in the long run. He debunks 4) because students will understand the rationale for the algorithms they use.

In addition to countering skeptical voices, Brownell also notes the benefits of focusing on the *meaning of* arithmetic for teachers but more so for students (he notes only one reason for teachers but ten for students). For teachers, teaching for the MoM is more interesting than teaching rules. For students, focusing on the MoM can lead to retention

and use of learned material, avoidance of non-sensical answers, and learning by problem-solving rather than memorization and drill (to name a few examples). Brownell's focus on outlining the necessary understandings for arithmetic, the reasons why such mathematical understandings are important to better comprehend arithmetic, and counter-arguments against those who disagree with focusing students' education on those understandings all go back to Brownell's explicitly stated focus on the MoM.

Brownell's outline of the *meaning of* arithmetic sounds very much like a conceptual analysis of a mathematical concept (von Glasersfeld, 1996), and it is from this standpoint of mathematical understanding that some have approached meaning. Thompson (2013, 2015) *explicitly* links meaning and mathematical understanding. Noting how "meaning and understanding were synonymous to Piaget" (p. 5), Thompson (2015) takes a Piagetian perspective towards meaning by drawing on the notions of assimilation and accommodation in the construction of meaning. Meaning as understanding refers to the inferences and reflections drawn from things in the world but also the sign-referent-meaning relationships that are constructed as one builds their mathematical understanding of a concept. Meanings are constructed through assimilating understanding to a scheme through repeated reconstruction/application. As meaning is taken to mean mathematical understandings, meaning resides in individuals. If meaning resides in individuals, those meanings are conveyed through conversations involving the implicit negotiations of and reflexive adjustments of understandings.

Having taken the position that meaning is the same as understanding, Thompson (2015) notes that teachers who try to teach for understanding essentially constrain the meanings students build towards what the teacher considers the 'right' understanding.

Such endeavors can still result in unintended meanings because students come in with their own backgrounds that can lead to different understandings than those the teacher sanctions. Like Brownell, Thompson also argues in favor of teaching for understanding. When teachers themselves lack a clear sense of a concepts' meaning, they can resort to vague language which does not constraint students from constructing unintended meanings. Thompson notes that constructing "meaning requires repeatedly constructing and using the operations (ways of thinking) whose organizations constitute that meaning. The most usable meanings are those that are richly connected with imagery action and that tie into other meanings" (p. 25). Thompson demonstrates this via examples of a teacher who had students engage in adding functions graphically (starting with an action conception of adding functions and preparing for a process understanding) and drawing connections between multiple representations to an invariant meaning.

Thompson explicitly holds meaning in mathematics to refer to mathematical understanding, much the same way Brownell talks about how the *meaning of* arithmetic is found in the mathematical understandings requisite to understand arithmetic. From this frame of reference then, Piagetian notions of assimilation and accommodation are the ways by which meaning *as understanding* is constructed. Thompson even goes on to describe how the most usable meanings are those that are richly connected with other mathematical understandings and how instructors should be teaching for understanding. Brownell shares this view when he talks about teaching meaningfully (in the *meaning of* sense) by making arithmetic sensible through mathematical relationships. Considering the focus on mathematical understanding, the meaning Thompson speaks of aligns with the

same sense of meaning Brownell discusses. As such, I hold the meaning Thompson defines as falling under Brownell's MoM.

Some who have cited Brownell have similarly focused on meaning as understanding and thus focus on the MoM. The National Research Council (1988) cited Brownell as calling for attention to students' interaction with subject knowledge. In their discussion on the effects of mathematics teaching on student learning, Hiebert and Grouws (2007) cite a study conducted by Brownell and a colleague comparing instruction aiming for understanding and instruction focused on mechanical procedures. Based on the findings, students taught 'meaningfully' (i.e., for understanding) performed better than mechanically taught students on measures of retention, transfer and explanation. Hiebert and Grouws then cite Brownell (1947) and the aforementioned study to argue that "we can reasonably conclude that explicit attention to conceptual underpinnings resulted in higher levels of conceptual understanding" (p. 385).

Newton (2008) looks at preservice teachers in a mathematics content course aimed at deepening teachers' knowledge of fractions. After taking the course that explicitly connected fractional concepts to common algorithms, the preservice teachers had shown signs of a deeper understanding of fractions. This is important for Newton "because prospective teachers will likely continue the cycle of what Brownell (1947) called "meaningless arithmetic" if they do not gain a deeper understanding of the mathematics they will be teaching" (p. 1104). In all the previously mentioned cases, the focus on instruction was directed towards learners' mathematical understandings of the connections between algorithms, other understandings, and multiple representations. These foci on mathematical understandings and the relations between understandings are

characteristic of what Brownell takes to refer to the *meaning of* a thing. As such, all the previous work focuses on the MoM.

MfM. There are some who cite Brownell and focus on the MfM (by focusing students on non-mathematical outcomes) alongside their focus on the MoM. Grouws et al. (2013) found students' mathematical learning was greater under an integrated content curriculum where "emphasis is placed on real-world data-driven problems, with formal definitions and theory being initially delayed and then addressed later in the same course or in subsequent courses" (p. 426). They attribute this higher performance to Brownell's idea that,

> when ideas are learned with meaning, they are better retained, more easily applied in new situations, and more likely to be used. Put another way, the focus on integration of content increases the likelihood of learning with meaning by developing ideas in contexts in which the relationship between different concepts can be an integral part of the instructional focus. (p. 459)

Lappan and Even (1989) looked at the development of a mathematics content course for preservice teachers, which emphasized understanding the connections between mathematical concepts but also "problems that require mathematical thinking in their solution and that come out of a real-world situation" (p. 14). Noddings (2013) cites Brownell for his emphasis on meaning, a goal that they hold has not been met because attention has not been paid to making connections across disciplines. Silver, Shapiro, and Deutsch (1993) cite Brownell for their emphasis on making sense of procedures and concepts, but conclude "that students' failure to provide an appropriate interpretation may be due, at least in part, to their dissociation of school mathematics performance from

more natural ways of thinking and reasoning about the world" (p. 132-133). In all these works citing Brownell, attention is on mathematical understanding (thus on the MoM) but also connections to other disciplines and everyday life. In the same way Brownell talks of the atomic bomb's *meaning for* non-physics things, these studies pay attention to things (real-life circumstances, data, non-mathematical disciplines) alongside mathematical understanding itself.

Moving beyond those who cite Brownell, Butler (2013), Gregson (2007), and Niss (1996) highlight relevance in the form of individual civic engagement or broader national policy goals. Butler (2013) discusses her motivation and use of service-learning in undergraduate math courses. She specifically notes her attempts were aimed to get math majors more interested in their majors. Gregson (2007, April) looked at the lesson/curriculum planning, views of math in relation to goals of students, and difficulties of one 8th grade teacher's implementation of an equity-oriented mathematics program in a school with the explicitly stated goals of college preparation, community connection, and preparation of students to work in social justice. The teacher in the study even explicitly stated:

> We are a social justice school so I really do want to be teaching about math in the service of how to make the world a better place. You know, frankly, flashing some stuff on the board about integers or graphing inequalities, or you know, what's the area of this polygon isn't that. (p. 3)

Both Butler and Gregson focus on using mathematical understanding *for* some non-mathematical things, and thus *I hold* these efforts to be focused on the MfM. In these

cases, instructors' goals and what is valued or 'relevant' feature prominently, suggesting that meaning depends on instructor goals.

Niss (1996) looks more broadly at the historical and theoretical goals of mathematics education. They differentiate a reason from a goal in that a reason is the justification (articulated or not) for mathematics education. Reasons are derived from verbal statements and fall under three types: technological/socio-economic development (e.g., promoting computational fluency for specific careers), societies' political/ideological/cultural maintenance and development (like promoting democracy, maintaining tradition or certain power structures, etc.), and prerequisites to cope with life (encouraging critical thinking as a life skill). Niss provides a historical analysis of the international reasons for mathematics education, starting with only the elite being mathematically educated due to societal maintenance reasons. Later as the power of commoners and democracy rose, mathematics education shifted to focus on the masses as a way of coping with life. Institutionalization of mathematics education for commoners at the elementary school level started in the 19th century for technological/socio-economic reasons and later progressed to all school-age levels in the 20th (for various reasons differing by context, like colonialism). In all cases though, there always seems to be some connection to societal maintenance.

Niss (1996) defines a goal as anything between a long-term end or a short-term objective. Focusing first on the goals of primary and lower secondary mathematics education, Niss highlights the goals of job qualification, dealing with everyday calculations, being critical citizens in democracies, but also engaging mathematics for mental discipline. Goals were often to rote-memorize procedures and be able to do

computations quickly and accurately (criticality and justification were often secondary or absent). In the early 20th century, the goals of lower-level mathematics education vacillated between promoting critical thinking, construction of personality/quality of life/society, and utilitarian/pragmatic uses. These shifts continued post-WWII up to the 'new math' movement. In modern times, the prior goals have either been embedded or absorbed into larger goals (unrelated to mathematics itself, such as developing personality, being able to model extra-mathematical situations; but also the MoM such as encouraging mathematical reasoning), but a main trend has been an (aspirational) shift away from rote learning and mechanical drill and kill.

Niss (1996) also discusses upper secondary and lower tertiary education goals. Since such education was mainly for elites until recently, mathematics was valued for formative reasons (development of personality, logical thinking, etc.) and utility for those few people who went into STEM fields (even if that meant imparting disconnected skills). That began to change in the 2nd half of the 19th and early 20th century as mathematicians began trying to link all the mathematical disciplines together. Math educators wanted to show that unity to students for practical/utilitarian aims (e.g., realizing mathematics applications to everyday circumstances), disciplinary aims (e.g., promoting mathematical understanding and thus MoM oriented), and cultural aims (e.g., understanding the role mathematics has played in the development of civilization, industry, and philosophy). The following 'new math' movement continued to prioritize industrial/utilitarian goals, but also strengthened conceptual goals (which would fall under Brownell's conception of the MoM). In many cases, this focus on conceptual abstraction led to the unexpected rise of 'facts and drill' practices and fueled the 'new

math' movement's abandon. Since then, modern goals for upper secondary and lower tertiary mathematics education have varied by context and stakeholders and can range from preparation for adult life, fostering an appreciation of mathematics' usefulness for society, to proficiency with mathematical modeling.

Many of the goals outlined above on the individual, historical, or policy level have focused on mathematics' relevance towards non-mathematical outcomes occasionally alongside MoM-related goals. Jones and Wilkins (2013) and Saxe and Brady (2015) highlight the importance of 'usefulness' while Philipp (2007) cites studies where individual teachers tailored instruction to focus on skills they felt would be useful to students' future careers based on their students' current SES. I take 'useful' and 'relevant' to be equivalent. Brownell holds that the *meaning of* a thing can be thought of a thing's relevance. As such, *I would say* these foci on usefulness would all fall under the MfM. However, 'useful' for non-mathematical purposes does not always have to be construed in such grandiose forms as development of character or democratic engagement. Niss highlights that goals can be short-term and focused on simple computations (as such computations can be useful for everyday circumstances). Star (2005) in fact pushed for a reconceptualization of such procedural knowledge while Jaworski, Treffert-Thomas, and Bartsch (2009) found that a teacher could not deny the importance of procedural fluency for exams. Passing exams and gaining procedural knowledge (perhaps to pass exams) in and of themselves are not necessarily tied to mathematical understanding, and thus, I also attribute these goals as falling under Brownell's conception of the MfM.

The idea of ‘usefulness’ can further refine the MfM. Ryan and Deci (2000) expand on usefulness as a significant factor in external motivation. Extrinsic motivation is defined as doing something to attain some separable outcome, whereas internal motivation is defined as doing something for its inherent satisfaction rather than some separable consequence. Usefulness plays into the different forms of extrinsic motivation specifically as a reason for engaging in behaviors. Such reasons include meeting external pressures (e.g. social stigma or maintenance of pride), perceived usefulness towards an individual’s other goals, to realizing the instrumental/utility-value of behaviors with respect to separable outcomes, not for the rules/behaviors themselves. Niss’ historical analysis and Ryan and Deci’s work allows one to think of ‘relevance’ and ‘usefulness’ that characterizes the MfM not only in terms of such grand goals as democratic engagement, but also in terms of imparting skills for future exams since doing well on an exam may be ‘useful’ for reasons not inherently tied to mathematical activity itself. For example, procedural computations can be a factor teachers think students will be motivated by if there is a social stigma associated with appearing ‘dumb’ to peers. On the other hand, teachers may view such computations as essential skills in courses leading up to a career; one historically cited (MfM-oriented) goal of mathematics education Niss cites.

Upon reflecting on prior literature, I extend Brownell’s views on meaning. *The ‘meaning of mathematics’ (MoM) focuses on understanding mathematics for mathematics’ own sake (i.e., a non-separable outcome). By contrast, the ‘meaning for mathematics’ (MfM) is understanding mathematics for some non-mathematics-related reason (i.e., a separable outcome).* This could range from focusing students on how

mathematics is relevant to the real world to simply its capacity to endow computational skills needed for upcoming tests, future coursework, or one's career.

Social Construction of Meaning

What becomes apparent from both views of meaning thus far is that both focus on how an *individual* views meaning. Bruner (1996) and Wenger (1998) however consider meaning from a social perspective. Wenger (1998) specifically looks at the negotiation of meaning through practice within communities of practice and broader social configurations. Wenger states, "Practice is, first and foremost, a process by which we can experience the world and our engagement with it as meaningful" (p. 51) and "Practice is about *meaning as an experience of everyday life*" (p. 52, emphasis added). I interpret Wenger to be saying that meaning is how one experiences everyday circumstances within social and historical contexts. Practice includes tacit and explicit dimensions ranging from language, documents, tools, implicit relations, tacit conventions, to even shared world views. Wenger in particular takes practice to entail five main aspects: meaning, community, learning, boundaries, and locality (p. 49-50).

Five aspects of practice. With regard to *meaning* as an aspect of practice, Wenger (1998) states, "Human engagement in the world is first and foremost a process of negotiating meaning" (p. 53) and focuses on the negotiation of meaning through the processes of reification and participation. Participation refers to taking part in the social relations and practices others in a community partake in (such as engaging in the work an organization is dedicated to). Reification refers to "the process of giving form to our experience by producing objects that congeal this experience into 'thingness'" (p. 58). Some examples of reification include creating a routine get-together lunch or realizing

and creating a process to deal with a particular kind of challenge that arises in one's job, like instructors creating and following specific procedures to report cheating.

With regard to the aspect of *community*, Wenger (1998) states three dimensions by which "practice is a source of coherence of a community: 1) mutual engagement, 2) a joint enterprise, 3) a shared repertoire." (p. 72-73). Mutual engagement refers to communities forming dense relations of engagement directed towards a shared, negotiated goal called a joint enterprise. The community's shared repertoire refers to the "resources for negotiating meaning" (p. 82), such as routines, words, and tools that the community accepts as part of its practice. Closely associated with community, Wenger takes *learning* in practice to include evolving forms of mutual engagement, understanding and tuning the community enterprise, and various forms of development (p. 95). I interpret Wenger to mean learning occurs over time in community practice as practices are either continued or discontinued within the interactions of newcomers working with existing members.

Boundaries set the limits of a community of practice. Specifically, practice creates boundaries by creating close relationships between participants who have an inaccessible-to-outsiders understanding of their enterprise (Wenger, 1998, p. 113). However, boundaries also provide means, e.g., in the form of brokers and boundary objects, to connect to the rest of the world. The term 'brokers' refers to individuals with multi-membership status importing practices from one community to another, while boundary objects are artifacts shared between different communities. Lastly, Wenger uses *locality* to highlight the differences between communities of practice and larger constellations of practice since some social configurations are too broad to be considered

one community of practice. Specifically, "Practice is always located in time and space because it always exists in specific communities and arises out of mutual engagement, which is largely dependent on specific places and times." (p. 130-131)

Identity. Wenger (1998) holds that through practice, we construct identities based on participation or non-participation, and, through modes of belonging, identify with groups and negotiate meaning. How we understand our identity is influenced by what we do (participation) but also what we do not do (non-participation). Wenger demonstrates this point by referring to claims processing employees,

> Whereas claims processors for the most part maintain a distance- and are maintained at a distance- from the institutional aspects of their job, they mostly identify with their shared practice and membership in their community… In other words, the identities of non-participation that they develop with respect to the institution and to the content of their work are an integral part of their identities of participation in their own communities of practice (p. 171)

Wenger goes on to discuss how we then identify with a group and appropriate or negotiate meanings by engaging or not engaging in certain practices, imagining our place in or our trajectory through a community, and aligning in various ways to the group's authority (p. 188-189).

Wenger (1998) then takes learning to be the ability to negotiate and renegotiate new meanings as we engage in practices. Learning also constitutes trajectories of how our participation within a community evolves as well as that of the community writ large. Because learning involves the evolution of participation and identity is derived by what

we do or do not do, learning then also entails a transformation of our identities (Wenger, 1998, p. 226-228).

Learning. When looking in the educational context, Wenger (1998) notes that teaching does not cause learning, but rather learning is a dynamic, emerging process that may use teaching and instructional design as resources for negotiating meaning. Instead, teachers' role in learning (i.e., in the construction of meaning) is not simply their pedagogical goals, but the opportunities they provide for students to engage in central practices of various communities of practice. As practice is tied to identity, 'instruction is relevant if it allows students to experiment with their identities' (Wenger, 1998, p. 267-269).

According to Wenger (1998), 'the problem with school is that it is too disconnected from actual experience to allow for students to experiment with identity as the meanings it creates are only applicable to the practices of traditional schooling' (p. 269-270). Part of the problem leading to this lack of identity experimentation may be what Wenger notes as the tendency in school to focus "on the mechanics of learning at the cost of meanings … In many cases, when meanings of learning are properly attended to, the mechanics take care of themselves" (p. 266). For example, he cites the authentic use of language as far more effective at learning to speak than by memorizing a list of words since dialogue focuses us on meaning and mechanics simultaneously. A stark example of this tension between identity, meaning, and learning in traditional educational contexts can be found in Danielak, Gupta, and Elby (2014).

Social MoM and MfM. Wenger however, does not differentiate between the 'meaning of' or 'meaning for'. He *does not use the terms, nor does he speak specifically*

about the context of mathematics education. Nor does Brownell speak of meaning as the experience of everyday life through communal practices. I however, posit that Brownell's distinction can be extended through Wenger's conception of meaning and practice. In *my* extension of *Brownell's positions on learning* then, MoM compatible practices would be those that the mathematics community engages in, such as those identified by Rasmussen, Zandieh, King, and Teppo (2005). Framing activity specifically according to the mathematics community's practices, Rasmussen et al. talk about students' activity as a way to look at students' mathematical understanding, particularly their progressive mathematization. Such activity can be characterized as horizontal or vertical mathematizing. The former refers to "formulating a problem situation in such a way that it is amenable to further mathematical analysis" (p. 54) and thus includes drawing out the mathematics from a situation via experimenting, looking for patterns, classifying, conjecturing, and organizing. Vertical mathematizing refers to activities like reasoning with the abstract, generalizing, or formalizing, which build on horizontal activity leading to the creation of 'new mathematical realities.' Rasmussen et al. acknowledge that the distinction between vertical and horizontal activity is artificial since both reflexively lead to one another and can be difficult to parse.

Rasmussen et al. (2005) then take progressive mathematizing to refer to the progression through layers of both forms of mathematizing in all socially and culturally situated mathematical activities. The kinds of authentic mathematical activities that students engage in then include symbolizing, algorithmatizing, and defining, all of which can manifest horizontally or vertically. They argue that, "rather than primarily viewing mathematics as a set and preorganized discipline that is carefully articulated to students,

these three mathematical practices constitute a key collection of activities through which learners can create, organize, and systematize mathematics" (p. 56). This shifts the focus to the types of social activities that lead to the creation of mathematical concepts generally, not just for specific concepts (as done by Steffe, 2001). It is these sorts of sense-making practices that the mathematics community engages in. I hold that prioritizing the social meaning of mathematics (sMoM) entails having students engage in such practices. One example is having students play with the symmetries of a triangle to define the algebraic group operations if the teacher was aiming to have students grow accustomed to the practice of defining.

MfM-compatible practices are those that can be transferred to non-mathematical communities and thus enable an individual to construct an identity as a member of those communities. From the perspective of practice, focusing on the MfM entails enabling students to serve as brokers. A broker is a person on the periphery of a community (in this case, the mathematics community) who also belongs to another community. Considering Niss's discussion on goals however, there seems to be a nuance in the MfM based on the focus of instruction. Math for engineering seems geared towards computation, procedures, and efficiency (such as procedural computations in physics without really understanding formulas conceptually (Danielak et al., 2014). In other contexts like service-learning or math for social justice, calculations can serve as a means to an end in using mathematics to foster civic engagement within local communities (Gregson, 2007, April; Hadlock, 2013).

However, procedural engagement also shows up in precalculus and other classes that focus on imparting techniques that may be useful for the next course, test, etc. The

context of Danielak et al. (2014) suggests that the communities for which students are being enculturated into may value these procedural forms of engagement. In the context of preparing for future courses, procedural engagement does not seem geared towards the practices of a certain community. It appears as though the social conception of meaning is indeterminate in such a case. Because they are not focused on directing students towards a certain community, teachers may instead choose to focus on students' individual meanings, strictly in the sense of doing well on the next test, quiz, or class, just trying to make math cool, or having students enjoy mathematics as a subject in and of itself. In the former two cases, teachers focus on either 'real-world' relevance and/or 'procedural' relevance. This focus is on individual student outcomes, separable from that of mathematics itself, and thus teacher's goal would be iMfM-oriented. In the latter case, instruction is geared towards individual student outcomes, non-separable from mathematics itself, and thus I would take these under an individual MoM (iMoM). The importance here is that notions of separable and non-separable outcomes become relevant for determining the meanings teachers intend for individual students in cases where social meanings may be ambiguous.

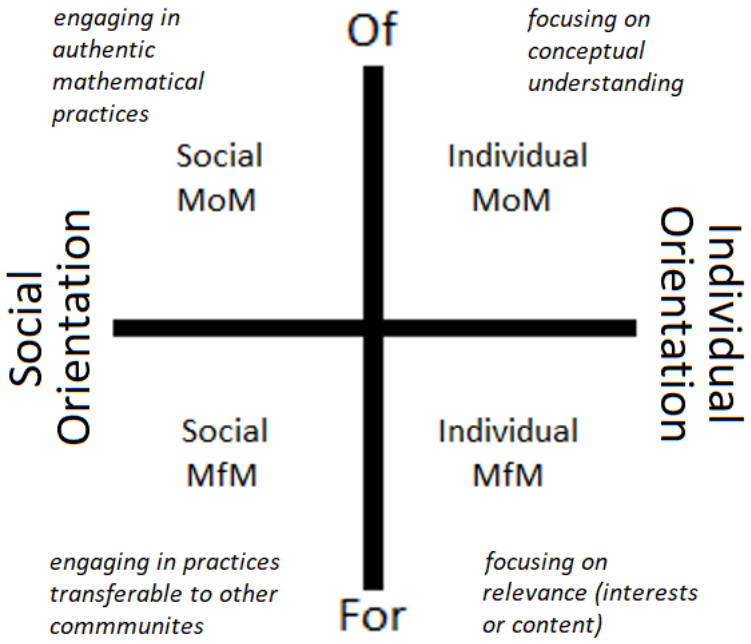
Synthesis of Mathematical Meanings

As a way to model the meanings instructors focus on, I can delineate four orientations towards mathematical meaning: a *social meaning of mathematics* (sMoM), a *social meaning for mathematics* (sMfM), an *individual meaning of mathematics* (iMoM), and an *individual meaning for mathematics* (iMfM). The iMoM follows Brownell and Thompson's view of meaning as conceptual understanding of mathematical ideas. The sMoM involves engaging in authentic activities that the mathematics community engages

in, such as symbolizing, algorithmatizing, and defining as discussed by Rasmussen et al. (2005). The assumption underlying this is that by engaging in such activities, one begins experimenting with their identity by aligning with the community of mathematicians.

Figure 1

Four Orientations of Mathematical Meaning as a Model of Instructors' Meaning Foci



The iMfM refers to understanding the relevance of mathematics for separable outcomes. These outcomes could relate to relevance in real-world applications to the simple utility of passing exams, preparing for future courses, and/or, by extension, indeterminate career plans. The sMfM would focus on engaging in practices that are transferable to other (non-mathematics) communities. These could include procedural use of formulas to be able to engage competently in a STEM community to using mathematics to engage in local community projects. In Wenger's framework, focusing on the 'meaning for' means focusing on creating brokers, individuals who shift between

multiple communities of practice because of their ability to engage in each community's respective practices.

It may seem as though there should be a distinction in MfM if we include both real-world relevance and fluency for future tests and courses under the same category, and perhaps there is. However, as was mentioned before, the notion of relevance and separable outcomes and the distinction between social and individual meanings will clarify why I hold both orientations together. On the social dimension, Danielak et al. (2014) suggest that the community in which students are being enculturated into may value procedural forms of engagement. In the context of preparing for future courses however, procedural engagement does not seem geared towards the practices of a certain community. Thus the social orientation of meaning seems indeterminate. This suggests some sort of disconnect in application relevance and relevance for exams. However, if students are not being geared towards a certain community, teachers and/or students may then choose to focus on the individual meanings, strictly in the sense of doing well on the next test, quiz, or class or just trying to make math applicable to daily life. Both exam relevance and real-world relevance however, are separable outcomes from that of mathematics itself, and thus both still fall under the iMfM orientation. I take the prior distinction in MfM then to be more an interplay in the split between the social and individual orientations of meaning. Lastly, I see these orientations as categorical because I see instructor goals as being described by these orientations (essentially, these are labels).

The Importance of Meaning in Learning and Instruction

The role of meaning in learning. There have been calls in the mathematics education community to focus on active learning, where teaching for understanding (iMoM) and including applications and modeling (i/sMfM) are included in larger instructional reform efforts (Saxe & Braddy, 2015). Jones and Wilkins (2013) highlight the role of usefulness (i.e., iMfM in the mathematics education context) in motivating student learning. However, Loveless (2006) found an interesting relationship between student achievement and relevance in an international study. Countries that prioritize significance (i/sMfM) did worse on achievement (if we take achievement to measure iMoM-oriented goals of understanding). This all highlights the importance of individual meanings.

Danielak et al. (2014) highlight the role of social meanings as they interact with individual meanings. From the vantage point of the student in the study, the physics community focused on the usefulness of mathematics (sMfM), but not understanding and sense-making as the student wanted (iMoM). The resulting conflicts in meaning led the student to numerous identity crises. He eventually switched his major to mathematics out of the belief that the mathematics community would focus on sense-making. He became disillusioned that the local mathematics community seemed just as focused on simply answering questions as the physics community had been. This alone poses questions about what sense of meaning the mathematics community was engaging students in, if not in the sense of what it does professionally. The student eventually dropped out of school for other endeavors.

More generally, Wenger (1998) talks about how school does not allow students to play with their identities. ‘If learning involves the ability to negotiate new meanings via

practice', as Wenger states (p. 226-228), the implication is that 'if the meaning teachers focus instruction towards does not align with meanings students associate with identities they want to experiment with, school loses its relevance in students' lives' (p. 269-270). The distinction in meaning I have adapted from Brownell helps to distinguish the kinds of challenges students face with different meanings they face, and all the previous highlight the importance of meaning for student learning.

The role of meaning in instruction. Before continuing, I reiterate that I take pedagogy and instruction to mean the methods, assessments, and practices instructors adopt to teach. I use the term pedagogical and instructional decisions and beliefs to mean the choices and associated beliefs regarding activities or actions the instructor and/or students engage in, as determined by the instructor to meet instructional goals. This includes decisions and rationale regarding curriculum, assessment, classroom activities, classroom discourse, and content delivery.

Based on teachers' accounts, there are reasons to suspect social and individual meanings affect instruction. Jaworski, Treffert-Thomas, and Bartsch (2009) found a lecturer who wanted to foster students' conceptual understanding (iMoM), but was caught up when students could only engage computationally. The teacher acknowledged that computations had to be focused on to help students pass exams (iMfM). However, they wanted to move students beyond computations to understand conceptually (iMoM). Butler (2013) noted how she implemented service-learning to generate more interest in mathematics itself. The focus on iMoM led her to enact certain pedagogical decisions that engaged students in real-world mathematics to make mathematics in general personally relevant (iMfM-oriented). One could even frame Butler's desire to promote interest in

mathematics through MfM-compatible practices as aiming to foster long-term sMoM-oriented goals since she was trying to further her students' progress towards completing a degree in mathematics. Such a degree would (hopefully) enable students to function as part of the community of mathematicians. In both cases, through the verbal accounts of instructors, we can see that meaning, *as I have defined above, can be attributed* to the instructors in both studies.

Thompson (2013) points to numerous studies finding a lack of meaningful mathematics instruction in the US (an iMoM-oriented focus in light of Thompson's explicitly stated connection between meaning and understanding) and the mathematics education research community's inattention to it (mistaking meaning for performance results). He even considers some conceptual analyses/genetic decomposition of concepts as lacking meaning. Thompson's claims arise from research that finds students learn little under the notions subsumed under the cited decompositions. He also points out the lack of meaning in research on teachers' mathematical knowledge for teaching. In this regard, Thompson holds that the research community's attempts to categorize specific types of knowledge used in teaching does not explain why teachers do what they do. Thompson thus calls for assessments of mathematical meanings for teaching and outlines policy goals to make meaning (iMoM) a central focus in mathematics education (e.g. benchmark meaning as better nations do, set national goals aimed at meaning/quality, focus equity efforts on students who are farthest from developing sanctioned meanings, help teachers use formative assessments).

Going beyond what individuals explicitly state, Wenger's social perspective of engaging in certain communities' practices may allow one to observe the impact of

meaning on instruction by looking at the implementation of different pedagogies and pedagogical decisions. Thus, according to the definitions given in prior research, pedagogical decisions associated with mathematics equity practice, critically relevant pedagogy, service learning, problem-based learning, and utility-value interventions (Dochy, Segers, Van den Bossche, & Gijbels, 2003; Gregson, 2007; Hadlock, 2013; Leonard, Napp, & Adeleke, 2009) *may* suggest instances in which instructors focus on the "meaning for" mathematics. Inquiry-based and inquiry-oriented instruction (Laursen, Hassi, Kogan, & Weston, 2014) *may* suggest an instructor's focus on the "meaning of" mathematics.

Taking Wenger's perspective on practice, I posit that some studies on the implementation of various pedagogies may hint at tensions in meaning and instruction. With regard to both levels of the MoM and instruction, Johnson and Larsen (2012) and Wagner et al. (2007) found that the lack of specific kinds of pedagogical knowledge to implement inquiry-oriented instruction (compatible with the sMoM) could lead to challenges managing the pacing and organization of the course, productively building off class discussions, gauging student understanding, and even maintaining the instructors' sense of autonomy. Thus, in perhaps trying to focus *students* on the sMoM, the *instructor* faced numerous hurdles.

I posit that prior research has also suggested the possibility of tensions with the MoM and instruction resulting from instructors focusing on the MfM. Instructors implementing service-learning and mathematics equity practice (sMfM-compatible) explicitly noted the lack of breadth and depth of mathematical content (iMoM) students were exposed to (Carducci, 2014; Donnay, 2014; Gregson, 2007). In mathematics equity

practice in particular, such concerns were parallel to teachers' challenges of managing unexpected complexities, data collection and logistic hurdles, having the necessary background knowledge (and making sure students have such knowledge), covering dominant curriculum, and having the pedagogical knowledge to bring out the mathematics in the problems (Gregson, 2007). Rousseau (2004) found that teachers implementing a mathematics in context curriculum (MfM compatible) faced issues gauging student understanding and pacing. These issues only exacerbated and eventually pushed teachers towards abandoning pedagogical change. Cooper (2014) found that service-learning implementing instructors faced challenges with time, planning, and their department, though Johnson, Keller, and Fukawa-Connelly (2017) and Olitsky's (2015) findings suggest these concerns may generalize to other non-lecture pedagogies.

According to the research above, I posit that the differing orientations of meaning enacted through instruction can have various instructional consequences and even conflicts between meanings can occur, such as real world uses of mathematics (which may relate to MfM on both individual and social orientations) coming into tension with goals of building mathematical understanding (iMoM-oriented) as found in Carducci (2014), Donnay (2014), and Gregson (2007). Brownell's framework is useful as an initial step to characterize the meanings instructors can focus individual students on in a more holistic way than that of Thompson. From there, Wenger's social perspective of meaning allows one to consider the meanings teachers try to foster between students, and any complications that may result from that.

Mathematical Meaning Within Sensible Belief Systems

I noted earlier that instruction could focus on conceptual understanding (iMoM), but still fall under focusing on the MfM if the aim of gaining a conceptual understanding was for some separable outcome and/or being able to competently engage in the practices of a non-mathematics community. Thus, the sense of meaning focused on by a teacher when they make pedagogical decisions is ultimately dependent on the goals behind their pedagogical decisions. If we take what instructors focus on instructionally as indicative of an instructor's goals and values, the previous studies may suggest that the senses of meaning that appear in instruction depends on instructors' goals and values in relation to their pedagogical decisions.

Niss (1996) and Wagner et al. (2007) looked at the goals of mathematics education on different levels, ranging from national policy goals (such as promoting democratic values) to specific, localized goals on one instructional task in a class session. Looking specifically at the goals behind instructors' pedagogical decisions, I take such goals as being directed towards the MoM and/or MfM (social and/or individual). I take instructional goals to be determined by what instructors *believe* they should emphasize instructionally, and thus I treat goals as a subset of beliefs. This stance would fit with the work Pajares (1992) cites that group beliefs, values, knowledge, and attitudes together as part of an instructor's belief system. One indication of how meaning may impact beliefs then is by examining the degree to which instructors sanction statements about what they should focus on in instruction. Agreeing with a statement on making math relevant by using real-world examples could demonstrate iMfM-oriented goals, while agreeing with a statement emphasizing that instruction should engage students in generalizing (or other

practices characteristic of the mathematics community) could demonstrate sMoM-oriented goals.

In light of the work of Hoyles (1992), Leatham (2006), and Speer (2005), I take teachers' goals behind their pedagogical decisions (and thus the senses of meaning focused on) to be situated and to vary sensibly as teachers navigate instruction, context, and affective factors. I take instructors' focus on meanings to vary on Leatham's (2006, 2007) three dimensions:

- *Psychological strength*- The psychological strength of a meaning goal describes the relative strength of the goal, ranging from peripheral (more likely to change) to central (more likely to resist change). This is determined by how strongly a particular meaning goal coheres with other beliefs in a belief system. The sense-making process of organizing meaning coherently with other beliefs is a process we are not always aware of.
- *Quasi-logical relationships*- The meanings instructors value relates to how differing beliefs on meaning are derivatives of or antecedents to other beliefs. These sorts of implicative relationships do not necessarily imply anything about the psychological strength of the meanings instructors value.
- *Belief clusters*- The extent to which meaning goals cluster in isolation from other beliefs is determined by context. Different configurations of various mathematical meaning goals arise in different contexts, and thus individuals can emphasize one set of mathematical meanings in one context and a different set of mathematical meanings in another. Individuals can even support these differences in emphasis with the same underlying evidence. Such evidence is simply used differently

according to differing circumstances. An example of this could be an underlying reason to cover course content. In the context of an abstract algebra course, this could lead to implementing inquiry-oriented instruction out of a belief that students need to learn content but also the ways in which mathematicians play with concepts (so a sMoM focus). In the context of a college calculus course with a set curriculum, this focus could instead manifest in lecturing so students see all the required topics for future classes (a separable outcome, and thus a focus on the iMfM).

One question that arises then when studying instructors' mathematical meaning focus is whether one is looking at instructors' meaning with regard to instruction, the meanings instructors want their students to be exposed to, or the meanings instructors intend for their courses. Following Hoyles and Leatham, I hold all these goals to interact with one another in instructors' belief systems. Thus I hold that drawing hard distinctions between these would undo the fluidity that Leatham's sensible beliefs framework affords me in approaching instructors' beliefs.

Suspicions on how Instructors Navigate Meaning

Navigating individual meanings. Having discussed the various ways meaning has shown up in instruction in prior research, it would be useful to outline how I see various meaning orientations possibly playing out in instruction. Regarding instructors' focus on individual meanings, an iMoM-orientation could be characterized by a teacher trying to help students develop a conceptual understanding of mathematics. This could occur by instructors choosing to lecture and cover how ideas develop and relate to one another. Lecture however, is not the only means by which they may attempt to have

students make connections between concepts. Instructors could engage students in group work, student presentations, or inquiry (practices I suspect are sMoM compatible) in order for students to understand material better. Instructors may choose homework problems they deem as requiring a deep knowledge of the concepts involved. Butler (2013) highlights that instructors may even use the MfM (perhaps in example problems or class projects) to motivate student interest in mathematics itself. The key thread through all this is that instructors may enact pedagogical decisions with the goal of building students' mathematical understanding.

For instructors attending to the iMfM, my framework may suggest instruction would aim for separable outcomes from mathematics itself. Content delivery could range from lecturing to having students engage in real-life problems. The kinds of problems given to students or worked out by the instructor could focus on application type problems. Applications could range from problems in STEM to problems dealing with social inequalities, all of which can be approached through the use of mathematical concepts. Since concepts are necessary to work on applications, instructors may also devote attention to the iMoM, but do so to use the concepts in later applied problems. I noted earlier that knowing content for future tests, courses, or careers all would fall under separable outcomes from mathematics itself. As such, iMfM-oriented goals could also focus on imparting procedural or conceptual knowledge to students in order for them to do well in the course.

Navigating social meanings. As was mentioned earlier, Wenger's framework on the negotiation of meaning through engagement in practices allows one to observe classroom practices for the kinds of meaning created on the social plane. But as my

review of Johnson, Caughman, Fredericks, and Gibson (2013), McDuffe and Graeber (2003), Niss (1996), Speer (2008), Sztjin (2003) and Thompson (1992) suggests, the meanings instructors intend for their students goes back to instructors' situated goals behind their pedagogical practices. From this viewpoint, one can suspect that pedagogical decisions in the spirit of mathematics equity practice, critically relevant pedagogy, service learning, and problem-based learning may indicate sMfM-oriented goals. On the other hand, pedagogical decisions geared towards inquiry-based and inquiry-oriented instruction may indicate an instructor's focus on the sMoM. The differentiating factor here is with reference to the kinds of practices students engage in. sMfM- compatible practices would be transferable to other communities, such as engaging in numerous calculations and procedural manipulations to be able to engage in routine tasks required of the physics community (which may show up in problem-based learning). Inquiry-oriented learning may have students engage in deriving relationships between the symmetries of a triangle to *generalize* and *define* the notion of algebraic groups (Larsen, 2013). This sort of task would have students engage in practices typical of that of the mathematics community more broadly and thus may be compatible with sMoM-oriented goals.

Problem-based learning highlights the complicated nature of the meaning constructs since an instructor's goals could focus on various meanings based on how PBL is defined in the literature (Dochy et al., 2003). Problems could be mathematical (like deriving group relations from the symmetries of a triangle, thus i/sMoM compatible) or applied (looking at real-world problems, thus i/sMfM compatible). Discerning meaning then requires going back to individual teachers' reasons for implementing PBL. On the

individual perspective on meaning and relying on the distinctions set out by Ryan and Deci (2000), the iMoM focuses on understanding mathematics for separable outcomes while the iMfM focuses on understanding mathematics for non-separable outcomes. Instruction might be amenable to building conceptual understanding but still fall under focusing on the iMfM if the instructor's aim was ultimately for a separable outcome (e.g., helping students do well on their next quiz and/or course). Alternatively, the reverse could occur where a teacher may choose to focus on making mathematics relevant in students' lives by implementing service-learning projects (compatible with the individual and social MfM) to generate more interest in students' mathematics itself (iMoM). This in fact showed up in Butler (2013).

An important thing to note regarding how instruction can tailor to different meanings is that instructors' meaning orientations are not mutually exclusive. Instructors could focus on building students' conceptual understanding (iMoM), want their students to do well on exams (iMfM), and want to prepare students for community engagement by focusing classroom activity towards the sMfM and/or sMoM. This in fact often showed up in the studies cited earlier. In some studies (Dochy et al., 2003; Donnay, 2013; Gregson, 2007, April), focusing on multiple meanings was exactly where tensions could arise, and that is what I am interested in exploring.

Research Question

Many studies have identified the importance of beliefs on instruction, yet the previous section highlights the possibility of tension between instructors' meaning orientations (as a subset of instructors' beliefs) and instruction. Part of this stems from the belief that beliefs are static. Hoyles (1992), Leatham (2006), and Speer (2005) would

caution or even argue against the resulting notion of inconsistency between belief and practice. Taking their perspective, I view beliefs as varying sensibly within belief systems. I want to look specifically at the belief cluster around instructors' orientation towards mathematical meaning and how the beliefs within or that feed into that belief cluster do or do not impact instruction. I am focusing on this belief cluster in particular because of my belief that this belief cluster will impact how instructors discuss their pedagogical decisions in relation to giving students a meaningful learning experience.

Within this complex interrelationship of beliefs, meanings, context, and instruction, I pose the following research question: how does instructors' navigation of mathematical meanings manifest in instruction? This entails numerous sub-questions:

1. What, in terms of influences on belief clusters (e.g., background, affective factors, other beliefs, etc.), leads instructors to prioritize certain meanings over other meanings?
2. How do instructors' orientations of mathematical meaning translate into specific pedagogical decisions?
3. How does context (e.g., student population, lack or prevalence of student interactions, course) influence the meanings instructors emphasize?
4. How do backgrounds, contexts, and orientations come together in pedagogical decisions with respect to how instructors respond to students' meaningfulness questions (e.g. 'when is this ever going to be useful?', 'why are we doing this?')

I suspect various teaching practices would show up for each type based on the typography of four understandings of meaning set out in the beginning.

Chapter 3: Methodology

The purpose of this study is to examine how instructors' beliefs about mathematical meaning are related to instruction. I used a case study methodology to investigate the beliefs and instruction of four mathematics instructors of various courses at a Western US university to explore, describe, and explain how instructors with different beliefs within different contexts navigate mathematical meanings in instruction.

Case Study

I used a case study design to approach my study because I wanted to study *how* instructors navigate different mathematical meanings. Case studies examine one or more cases in great detail (Johnson & Christensen, 2014). Case studies can be used to research questions focusing on exploring, describing, and explaining. Yin (2009) holds that research questions focusing on "how" and "why" typically seek explanatory goals and thus are better suited towards a select few methodologies, case study being one of them. In case studies, a researcher has no control over the actual events and has direct temporal access to the studied events. My study looks at *how current* instructors navigate meaning, thus falling within the characteristics Yin addresses.

Yin's description of the aims of case study research also aligns with my study's goals. Case studies aim to generalize cases "to theoretical propositions and not to populations or universes" (p. 15). Yin describes this generalization as analytic generalization, where a previously generated theory is compared to the findings from the cases under study. In my study, I had a framework of mathematical meaning informed by existing research. I intended to use this study to compare what I found to what I theorized would occur instructionally with different sorts of meaning being emphasized.

Yin (2009) defines a case study to be an "empirical inquiry that investigates contemporary phenomena in-depth in real-life contexts" (p. 18) in which 'the boundaries between phenomena and context are unclear' (p. 18). Case study inquiry copes with dynamic contexts in which many variables of interest may exist and intersect. As such, a case study inquiry relies on numerous data sources to capture these dynamics. Both Yin (2009) and Johnson and Christensen (2014) hold that despite the various uses and research questions that case studies can address, they all share a focus on the unit of analysis called a 'case' as it is situated in larger contexts. Johnson and Christensen define a case to be a bounded system, a set of interrelated elements that form an organized whole that definable boundaries can distinguish. This notion of setting apart or defining by clear boundaries is a notion that other methodologists however take issue with on the grounds that they extract phenomena from the contextual realities to which they are constantly in relation with (critiques that I will address later when I address my data collection efforts).

This notion of separation is not how Yin (2009) describes a case however. Rather, Yin holds that one's research questions and propositions should help inform what aspects of each case one wishes to collect. "The more a case study contains specific questions and propositions, the more it will stay within feasible limits" (p. 29). As my literature review suggests, the propositions guiding my study on meaning is that how instructors navigate meaning affects their instruction in numerous ways and vice versa. Differing conceptions of mathematical meaning entail different foci which may or may not manifest with particular instructional practices. It is within those questions and

propositions on meaning and instruction that I focus on instructors of the courses mentioned as my cases.

Johnson and Christensen (2014) state that case studies focus on understanding how a system operates holistically. Numerous parts make up the whole of a system, and understanding the whole requires drawing connections between the parts to see how they contribute to the whole. Johnson and Christensen's statements highlight the need to look at what makes up an individual (cognitively, emotionally, etc.). In my study of instructors then, Leatham's beliefs systems framework highlights the ways beliefs come together and relate in individual instructors. My data sources then needed to account for ways in which beliefs can interact with instruction. This was a supporting rationale for my use of video interviews (Speer, 2005), observation protocols, and secondary data sources (e.g., homework assignments).

Johnson and Christensen (2014) identify numerous types of case study designs. There are intrinsic case studies (aimed at understanding a specific case as a holistic entity), instrumental case studies (focused on understanding a case with the aim of understanding something more general), and collective case studies (when a researcher studies multiple cases simultaneously in one overall study). Collective case studies (also known as multiple-case designs) can range in the number of cases studied depending on resources and research purposes. A general benefit to this design is that one can compare and contrast across several cases at once to gain greater insight. This allows one to test a theory more effectively but also lends a greater ability to generalize than what is afforded by a single case design. Yin (2009) adds that the results of multiple-case designs are much more compelling than single-case designs.

With regard to multiple-case designs, Yin (2009) notes that by considering multiple cases, one is essentially thinking of considering multiple experiments (following a replication design in his terms). Each case is selected so as to predict similar results (literal replication) or predict contrasting results based on the propositions of the research study (theoretical replication). Both outcomes are meant to further develop the theoretical framework a researcher brings to bear on their topic of study. This logic goes back to Yin's points on the aims of case study research to generalize cases "to theoretical propositions and not to populations or universes" (p. 15) by comparing previously generated theory to the findings from the cases under study. By using a theoretical example of studying multiple presidents to understand leadership, Yin further demonstrates his point,

> Your investigation deals with the role of the presidency of the United States, and you are interested in doing a multiple-case study of a (few) presidents to test your theory about presidential leadership. However, the complexity of your topic means that your choice of a small number of cases could not adequately represent all the 44 presidents since the beginning of the Republic. Critics using sampling logic might therefore deny the acceptability of your study. In contrast, if you use replication logic, the study is eminently feasible. (p. 56)

Ultimately, Yin argues that the rationale behind multiple-case designs is for theoretical and literal replication in order to flesh out a theory. Cases are selected based on prior knowledge of research propositions on what one expects to find in different scenarios. The hope is then to shed light on how and why expectations were or were not met. Yin also notes another rationale for multiple-case designs could be that theory posits different

types of conditions and the researcher thus aims to have representatives who fall within each type.

I was interested in how instructors with various beliefs about mathematical meaning approach and tailor instruction in light of their lived realities and beliefs. This could lead to different pedagogical decisions focusing on different goals. In this way, each instructor, with their own beliefs, is like a new experiment. It is for these reasons that I used a multiple-case design. For reasons that will be discussed in my critiques of current research methods, I want to push back against viewing each instructor as simply a new experiment. I believe that each instructor was a unique individual, and no two people have exactly the same beliefs in all regards and in all contexts. Through those unique traits, I can come to know a bit more about how mathematical meaning affects instruction. What I have posed about mathematical meaning and how meaning may interact with instruction is only theoretical. By using a multiple-case design on individual instructors and their lived reality, I could further elaborate my framework of mathematical meaning. This aim fits with the focus on using multiple cases to elaborate theory as Yin asserts.

Yin (2009) sets out the critical components of a case study research design: appropriate research questions, propositions, units of analysis, the logic linking data to propositions, and the criteria for interpreting findings. I have already addressed my research questions' focus on the 'how' and 'why' characteristic of case studies, the proposition that meaning affects instructors as my units of analysis, and how I will need multiple data sources to account for ways in which beliefs can interact with instruction in a non-reductive way. That leaves me to address the issue of criteria for interpreting

findings. Yin holds that a major part of interpreting findings is identifying and addressing rival explanations. The design phase involves anticipating and enumerating rival points. In the context of my study, one connection of beliefs and practices could be challenged on the grounds of me attributing a cause (certain beliefs) to an effect (instructional practices), whereas another cause may prove more relevant. However, through the use of the data sources I've chosen for this study, I would at least gain a better picture of what instructors say causes them to enact certain pedagogical decisions over others. Within Leatham's framework however, one is not always cognizant of the beliefs that come to the fore in certain circumstances or more generally. This then allows rival explanations to arise. By using video clip interviews and non-interview data, I was able to partly address such non-vocalized beliefs.

Critiques of common methods. Kuntz (2015) takes issue with the goal to rule out all other explanations (advocated by Yin), a critique that only becomes more relevant when considering the issues with multiple case designs. Multiple cases come with a natural decrease in the amount of time one can devote to analyze each case. This results in compromising the depth of analysis for breadth of analysis. Results are then drawn from having reduced an individual down to decontextualized data snippets. If one follows Yin's advice, data analysis would then focus on curtailing counternarratives based on such data. Such an analysis is bound to be reductive of human experience.

At this point, I feel delving into Kuntz's (2016) critique bears relevance. Kuntz takes numerous issues with the use of traditional data collection methods as being extractive. Kuntz sets his critique of many methodologies (such as case studies) and associated data collection techniques (e.g., interviews) against the backdrop of

neoliberalism's influence as a governing rationality of our current age and the methodologies we use as a result of its governing power. Kuntz takes neoliberalism to be the extension of market rationality to aspects of life that were not conceived of in economic terms. This view manifests in education by shifting values towards measurable outputs and hyper-individualism (conceiving the self as a timeless representation independent of surrounding contexts and histories). Individualism focuses on taking individuals' voices and commodifying them as objects to be bartered for methodological credibility. This can be seen in the ways that some have conceived of reciprocity as an exchange, often as physical gifts in order to 'buy' the participation of individuals into research studies (McGregor & Marker, 2018).

Kuntz (2015) continues that the methodologist, in a neoliberal order, clearly presents data as objects to be presented to the rest of the world. This process of extraction of data into clear and presentable formats takes an individual's lived experiences and relations and reduces them down to segments of data and statistics devoid of context. This reduction then strips an individual of agency and denies the capacity for generating new thought because such extractive processes create a representation of a timeless individual who is consistent across contexts. This critique is particularly striking considering the tendencies of past research on beliefs to identify inconsistencies between what teachers say and do in classrooms, as noted by Hoyles (1992) and Speer (2005). The traditional methodologist then is one who engages in the logic of extraction (a rationale focused on the separation of data in order to make things clear) by fixating "on the procedures of coming to know—the incessant production of prescribed methodological

techniques aimed at producing extracted forms of data, rendered meaningful through detached mechanisms of analysis" (p. 42).

More specifically, Kuntz describes the logic of extraction as "a conservative system of rationality that privileges discrete, fully knowable entities that remain consistent across time and space, absent the immediacy of material context" (p. 44). It assumes that the most efficient and productive way to make things known is by extracting them from their surrounding contexts. Kuntz states that the logic of extraction dominates the methodological considerations of current research efforts and, by focusing on presenting clear, digestible data, is a product of the neoliberal focus of making the world marketable. Methods attending to the logic of extraction then excel at stripping away context and producing objects of analysis. With traditional means of coding and analysis aimed at efficient sense-making, the experience of 'being' can be lost (Kuntz seems to take 'being' as referring to surrounding contexts and relations directly related to an object of study).

Kuntz pushes back against extracting isolated experiences as a proxy for everyday life. Instead of accounting for the complexities of 'being', traditional research practices, like interviews, "aim to both uproot (or extract) partial meanings and then delegitimize all else as excessive or outside the scope of the inquiry project" (p. 54) to create 'moveable objects' of inquiry removed from the contexts from which they derive. Such practices also render experience as a linear development towards coherent, definable goals.

Accounting for Critiques. To counter extractive tendencies of sensemaking from a distance, Kuntz (2015) advocates for materialist methodologies because they account

for the ‘noise’ of daily life that traditional data collection methods ignore. Such methods give up aiming for certainty.

> As a result, even as the materialist methodologist relinquishes the privilege of full or complete knowledge, s/he also releases the anxiety of influence—concerns about affecting data or findings fall away as objectivity and/or neutrality cease to be justifiable goals. (p. 65)

This focus on closeness and immersive experience is done out of the realization that all sensemaking develops within relationships (not outside them). Relational ways of knowing, coming to know, and being are thus key to materialist methods. In a world that is never fixed and thus constantly makes things more than what they were, all data are treated as being in a state of becoming. Methodologists are not to seek some complete, clear meaning, but rather prod “the affective assemblages of being, maintaining the contradictions and blurred connections of being without normalizing their meaning” (p. 79). This then foregoes the attempt to achieve clarity in favor of accepting a state a flux where new interpretations, critiques, and ways of being remain open to emerging. This spirit of fluidity and openness all fall in line with Leatham’s framework. In fact, Leatham’s reanalysis of a prior study’s discovery of inconsistencies between instructor beliefs and practice demonstrates the power such openness can allow (Leatham, 2006).

The notions Kuntz expresses above are directly contrary to Yin’s arguments to rule out rival interpretations. I mentioned an earlier critique Kuntz makes of traditional methodologies creating ‘movable objects.’ I connect that critique specifically to the issue of criteria for judging findings. Through extractive processes, subjects and phenomena are removed from their material contexts in the name of efficient sensemaking. The

resulting data is then made known through researchers' efforts to provide full definitions in the absence of the extracted contexts. This however denies the relational nature of subjects and phenomena and thus constrains the possibilities of interpretation. Given the rise of neoliberalism, Kuntz notes research methods' reductive tendencies:

> As Davies (2010) notes, "Neoliberalism is founded on a strong assertion that there is no alternative, making critique redundant" (p. 65, original emphasis) … if we seek to invoke some element of change, we need to challenge our existing assumptions about how we come to know and how such knowing affects how we operate. (p. 125)

I mention this tendency to focus on specificity with regard to questions about 1) what meanings instructors hold and try to emphasize and 2) how that relates to Yin's argument that one should address rival explanations. I had disagreed with the notion of separating instructors' meanings for their course, students, or instruction on the grounds that Leatham's beliefs framework would hold these beliefs as possibly interrelated in an individual's beliefs systems. Here I want to point out the methodological downsides to drawing such distinctions. Kuntz notes the consequences of extractivist logic (such as assuming fixed ways of knowing, linear progression of experience, and loss of context). These arguments highlight that drawing such boundaries on what meanings I look at would be a form of extraction that cuts off possible relational dynamics. Instead of further dissecting something so complex as meaning, leaving the distinction in what meanings instructors focus on for what and whom leaves more room for insights and understandings I would not have considered in my framework.

More importantly, it leaves room for interpretation and critique. Kuntz notes, "Critique makes alternatives possible" (p. 126). This then connects to the issue of rival explanations of data I had mentioned. Cutting such relational dynamics leaves inquirers with only a set of privileged interpretations, and expecting to rule out rival explanations only further narrows the pool of interpretations. Kuntz (2015) argues that narrowing interpretations is a form of governance on thinking. For something so important to human experience, I oppose holding ownership of meaning (mathematical or interpretative). I stand to learn much more from allowing rival explanations, and at least I can address them through the power Leatham's framework.

While I aimed to eschew ownership of meaning, the meanings I value would inevitably color my analysis. The relational thinking of materialism holds that because I am constantly in relation, I am necessarily a part of any data collection efforts. There can be no extraction of the self, and as such, I needed to be transparent about the points from which I engage. Relational thinking holds that how researchers inquire and come to know affects what they do with knowledge. Thus, I did a case study similar to those discussed by Johnson and Christensen and Yin. However, in approaching any form of data collection, I came in with my notions of meaning derived from my own experience and past literature. As such, I needed means to ensure instructors gave their own meanings within their contextual realities. It was for this reason that I used semi-structured interviews. Kuntz however criticizes interviews for reducing individuals down to marketable objects but also for the narrowness in interpretation they create. By extracting the 'being' of researchers and those they study, inquirers are only provided a limited number of ways to interpret information. This narrowness results in an image of a static

representation of research participants that remains unchanged across time and space, of a consistent individual (the exact tendency that past research on instructors' beliefs and practices resulted in). This static representation of participants goes against the fluidity and complexity of individuals Leatham's sensible beliefs framework lends me.

To mitigate these pitfalls, I embedded video interviews (Speer, 2005) within my interviews to allow instructors to reflect on the relational dynamics of their teaching experiences. In light of the shortcomings of past methodologies which led to the researcher-imposed dichotomy of beliefs and practice, Speer poses a method of collecting belief and practice data simultaneously: video clip interviews. Before an interview, a researcher carefully selects videoclips and questions to go over with instructors. During these interviews, the researcher and participant watch recordings of classroom sessions together while the researcher asks questions. Together, the researcher and instructor come to a shared understanding of terms (and thus a more accurate attribution of beliefs becomes possible). Speer goes into the specifics of how to prepare for the interview (selecting clips to watch together, selecting questions, and opting to use participant definitions when disagreements occur), and what procedures should be followed (showing the video, having the teacher explain their thinking, then asking further questions). Speer argues that sharing videoclips helps create more accurate attributions by building a shared understanding. Schepens, Aelterman, and Van Keer (2007) used a similar technique to study student teachers and note the importance of positioning the video clip interview in relation to other questions.

To further address Kuntz's critique, I also coupled classroom recordings with structured observation protocols and collected secondary data sources in the form of

homework and exam problems. While it was preferable to fully engage in the relational approach Kuntz calls for, this study's limitations constrained my ability to be as immersive in my data collection techniques. However, I used multiple data sources focusing on instruction, beliefs, and context as outlined above. This will at least be able to account for some degree of relationality that Kuntz calls for.

Study Context and Procedures

Setting and courses. To attend to the nuances of context, I looked at instructors teaching mathematics courses for preservice teachers, calculus, mathematics for liberal arts, and statistics at a U.S. research university. The university was in an urban location and is fairly atypical in terms of ranking reasonably high on various counts of diversity. 46.2% of the student body is between the ages of 18-24, while older age groups make up the rest of the population. The faculty make-up is 55% female to 45% male; 72.4% white. The student body make-up is 53% female to 47% male; 55.8% white (based on university website demographics). I focused on instructors of various mathematics courses within the university's mathematics department because different courses can be taught in various ways, with differing foci, and serve diverse student populations, such as preservice teachers to engineering majors. Sztajn (2003) found that context and student populations could influence teachers' instruction of mathematics, while Leatham's (2006) belief system framework holds that context is a pivotal factor in instructors' beliefs. By looking at instructors of different courses with various student populations, I could observe the effects of context on different meaning-related instructional goals.

In the following paragraphs, I will describe the university course catalog descriptions for the courses I observed. The mathematics for liberal arts class was stated

to cover various modern mathematical topics such as the mathematics of voting systems, art and nature, population growth, probability, and networks. The course has a textbook set by the department, but topics to cover are chosen at the instructor's discretion. Full-time and/or adjunct instructors typically teach the course. The course is intended for undergraduate students without a strong algebra or calculus background who need to fulfill a general university mathematics course requirement. This course requires passing the university's college algebra course with a C- or above or passing a mathematics placement exam with an appropriate score as prerequisites.

The university's calculus classes consist of a sequence of courses. Calculus I was stated to cover differential calculus of functions of a single variable, including limits, the definition and computation of the derivative, and applications of the derivative. Calculus II was stated to cover integral calculus of functions of a single variable, including the Fundamental Theorem of Calculus, numerical integration, and applications. Calculus III was stated to cover infinite series, parametric equations, polar coordinates, conic sections, vector geometry, and the calculus of vector-valued functions. Calculus IV was stated to cover differential equations, integral calculus of functions of several variables, and applications. The courses have a common textbook set by the department. Course coordinators sometimes share instructional materials, course objectives, and assignments, but course coordination is mixed, and there are no common exams across most sections.

I observed an instructor for Calculus III, which is typically taught by faculty, full-time instructors, graduate teaching assistants, and adjunct faculty. The course is intended for undergraduate students who need to fulfill a general university mathematics course requirement or need calculus for a STEM major. Given the sequential nature of the

courses, Calculus II requires passing Calculus I, Calculus III requires passing Calculus II, and Calculus IV requires Calculus II with introductory linear algebra or Calculus III as prerequisites.

The Mathematics for Teachers II class I observed was the middle course in a sequence of three courses, all of which were stated to cover fundamental mathematical concepts from a constructivist approach. The first course in the sequence was stated to cover numeration, operations, number theory, and problem-solving. The middle course was stated to cover rational numbers, probability, and statistics. The final course was stated to cover algebra, geometry, and measurement. The courses have a textbook set by the department. Course coordinators sometimes share instructional materials, course objectives, and assignments, but course coordination is mixed, and there were no common exams across most sections. Faculty, full-time instructors, graduate teaching assistants, and adjunct faculty who are (or have been) in a mathematics education Ph.D. program typically teach the course. The course is intended for preservice K-8 teachers who need to fulfill a course requirement for their major and teaching certification. Given the courses are a sequence for preservice teachers, Mathematics for Teachers II required passing Mathematics for Teachers I, while Mathematics for Teachers III required passing Mathematics for Teachers II.

The Introductory Statistics class was stated to introduce statistical analysis with an emphasis on applications of statistical tools to real-world datasets in the health and social sciences. Topics include statistical analysis, theoretical foundations and tools, and application of statistics to problem-solving in certain health and social science environments. The courses have a textbook set by the department. Instructors sometimes

share instructional materials, course objectives, and assignments, but there are no common exams across most sections. Faculty, full-time instructors, graduate teaching assistants, and adjunct faculty typically teach the course. The course is intended for undergraduate students who need to fulfill a general university mathematics course requirement. This course requires passing the university's college algebra course with a C- or above or passing a mathematics placement exam with an appropriate score as prerequisites.

As can be seen from the diversity presented above, the courses I sampled instructors from focused on very different concepts, contexts, and student populations. Courses could be approached from a very abstract mathematical stance, an applied stance, or some combination of the two. The diversity of options created contexts conducive to observing different mathematical meaning-oriented goals. The range in courses allowed me to find instructors with oriented goals of multiple different meanings as opposed to focusing on a small subset of oriented goals had I just sample calculus instructors, for instance. It was precisely this diversity that I wanted to study.

Instructional units for observation. I wanted to observe class sessions for evidence of how meaning orientations could manifest. Thus, it was informative to try to observe instructional units in which I could observe how instructors with different orientations teach within contexts that are compatible or incompatible with particular meanings. Units which could be amenable to various meanings and thus more susceptible to various individual instructors' orientations would aid in my research efforts as well. Ultimately however, what units I could observe depended on instructors' schedules. I also picked units near the beginning, middle, and end of the semester for each instructor so as

to get a holistic view of how instructors pursued their instructional goals throughout the semester.

Participants. I interviewed four mathematics department faculty members from the university's mathematics department. Inclusion criteria were that they be instructors of record of any of the courses noted above during the spring 2019 quarter.

Recruitment. I connected with the mathematics department's department head to explain my study and seek approval. After receiving approval, I contacted the department with the contact information of the instructors teaching the courses outlined above during the spring of 2019. From there, I contacted the instructors by email, describing the study I planned to undertake for my dissertation, how volunteering instructors would be compensated for their participation ($10 per interview they participate in), and requested that any volunteers willing to participate in the study should fill out in an initial two to a three-minute-long survey. The initial survey consisted of four questions asking which course instructors taught that I was interested in observing, the years of experience they have teaching the course, what they hoped students would take away from their instruction (a rank choice response between four responses aligning to the differing senses of mathematical meaning) and the percentage of class time they think they devote to lecturing as opposed to engaging in other classroom activities like groupwork. The rank choice question gave insight into what forms of mathematical meaning instructors may emphasize, while the class time questions provided me with some information on instructional decisions. These latter two questions served as a way to contact instructors who I collectively thought may have goals relevant to each of the mathematical meaning quadrants (see Figure 1).

Based on the initial survey, I contacted a subset of instructors who volunteered to further describe this study. The description of the study included how participants would be asked for three interviews (at the beginning, middle, and end of the semester), copies of assignments pertaining to the instructional units I observed in the study, and how I, as the researcher, would seek to record class sessions numerous times throughout the semester. Instructors were told that interviews would last between 30 minutes to an hour and a half. They were also informed that they would be compensated for their participation ($10 per interview they participated in).

Data collection. With regard to data, Yin (2009) enumerates three key principles to data collection in case study data collection that I will try to meet. The first principle is the use of multiple data sources. The rationale for this is that triangulation to "develop converging lines of inquiry" (using multiple data sources to corroborate the same fact; p. 115) makes one's claims more convincing. The second principle is to create a case study database so that other investigators can review primary sources directly. This helps improve the reliability of the study. Case study notes (notes I take during interviews, classroom observation protocols, or protocols used while reviewing physical artifacts) were stored on a password-protected cloud account. Notes were dated and titled to know what they refer to so that I could go back and easily connect a data source with any case study notes.

The last principle of data collection is to maintain a chain of evidence. Yin takes this to mean that steps should be taken so that any reader can "follow the derivation of any evidence from initial research questions to ultimate case study conclusions" (p. 122). This requires no evidence to be lost (thus going back to the importance of the steps I take

to address the second principle). To meet this principle, Yin states that 1) final reports should sufficiently cite data, 2) the database should reveal evidence and the circumstances in which the evidence was collected, 3) data collection should follow the case study protocol procedures, and 4) the protocol should explicitly state its links to one's research questions. All this would produce a sufficiently convincing chain of evidence. As such, I aimed to meet similar standards in terms of the write-up of my conclusions, analysis of my data (to be addressed in my data analysis section), and data storage.

My aim was to see how my model of meaning orientations (discussed in the literature review) relates to instructors' instructional goals and practices and how instructors connect those goals and practices to their response to students' meaningfulness questions. To examine this, I needed data sources that would inform how oriented instructional goals manifest in instructional practices, specifically by documenting teaching practices. Some teaching practices which I anticipated to be influenced by meaning-oriented instructional goals included the inclusion of context in homework/class problems; the kinds of problems instructors assign; the presence, absence, and type (if present) of discussion between the instructor and the whole class or the instructor and individual students; the presence or absence of groupwork; and the use (or not) of technology and how it is used (considering the focus Leatham (2007) placed on technology use as noted in the literature review).

If oriented goals did not manifest instructionally, my data sources needed to account for factors that influenced instructors to shift their goals away from the meaning-oriented goals they wished to focus on. Both circumstances required my data sources to

allow me to understand enough of instructors' belief systems to posit which goals instructors wished to focus on and how other beliefs may have interacted with their meaning-oriented goals within instructors' belief systems. An example of this could be how an oriented goal may result from an antecedent belief situated in a particular context (the quasi-logical and belief cluster dimensions in Leatham (2006, 2007). Determining such a relationship would require asking instructors about anything which could affect their instruction, but given the inability to vocalize all one's beliefs as noted by Speer (2005), it was unreasonable to expect that I could capture everything. *Some* aspects I anticipated could influence instructors' beliefs were concerns related to students, self-efficacy, and departmental expectations; past experiences in work or personal life; personal views towards mathematics; and knowledge of their students (demographic and personal). Ultimately, this assumes the models of instructors' belief systems that I posed would be incomplete snapshots in time since I nor instructors would be able to bring up all the beliefs that may play a role, and even what I could elicit would only be representative of instructors at the particular time and context I asked questions. The goal then was to progressively build my understanding so that I could model enough of an instructor's belief system, specifically their goals, over the course of the semester.

In order to connect beliefs and instruction and progressively refine my understanding, data sources consisted of audio-recorded semi-structured interviews to understand instructors' beliefs; video recordings and associated observation protocols documenting classroom instruction, the nature of problems done in class, and the nature of problems assigned on homework; and the collection of problems assigned by the teacher to students on homework, quizzes, and tests during the weeks of observation. I

interviewed the instructors three times: at the beginning, middle, and end of the term. Video recording occurred during each class session for the week prior to the 2nd and 3rd interviews. During these class sessions, I took field notes on the class session by using White and Mesa's (2012) observation protocol. In the following paragraphs, I will detail these data collection methods.

Surveys. Along with a description of my study (the purposes of my study, the compensation participants will receive, etc.), an initial survey was sent out to instructors to see who would be willing to participate in the study. The survey consisted of three questions asking about the years of experience they have teaching the course they were being observed in, a rank choice response to what they wanted students to take away from their instruction and the percentage of class time they devote to lecturing as opposed to engaging in other classroom activities like groupwork. The rank choice question gave insight into what orientations of mathematical meaning instructors may emphasize, while the class time question provided me with information on instructional practices. These latter two questions helped narrow down instructors to contact to a representative example that could provide information on each of the quadrants of mathematical meaning.

Interviews. In this study, I used focused interviews (Yin, 2009). These interviews entailed speaking with an individual about an hour. Questions were asked in a conversational manner and drawn from an interview protocol. I used an interview guide approach (Johnson & Christensen, 2014) in which an interviewer goes into an interview with a set topic they are trying to investigate (mathematical meaning and instruction in my case). The research topic was investigated through open-ended questions recorded on

an interview protocol developed prior to interviews. Questions were asked in any order, and the wording of the questions could be changed to allow the interview to flow in a conversational manner.

Interviews were used to explore the views and experiences of participating instructors. Study participants were asked for an initial interview, a middle-of-the-course interview, and an end-of-course interview, always a week after observations for the second and third interviews. Instructors were emailed the interview questions before interviews so that they could begin to reflect on them. Interviews lasted between 30 minutes to 1.5 hours. Interview questions on beliefs and goals focused on asking instructors about what mathematical goals they wish to focus on and why (e.g. because of past experiences in work or personal life, personal views towards mathematics, knowledge of their students), their concerns, and how those goals, concerns, and justifications interact in their instruction. Instructors were asked about what kinds of instructional decisions they would implement and how their answers to the prior questions relate to their beliefs and instruction goals. The second and third interviews also asked how these goals may have been changing or had changed throughout the semester. Chowdhury (2018, February) used interviews to document instructors' beliefs about their mathematical meanings and concerns. I draw on and adapt some of the questions used in that study for my study.

In keeping with Speer's (2005) suggestion to talk over observations with instructors, I asked questions based on videoclips I brought to the second and third interviews. I selected clips based on moments I found interesting related to the mathematical goals instructors indicated they focused on (as suggested by the initial

survey and/or prior interview responses). I asked instructors what they were doing, what they wanted students to take away from the event, anything that may have influenced their in-the-moment decisions, and how the recorded events and any resulting insights related to the mathematical goals they said they were pursuing. If there were perceived discrepancies between the goals and practices instructors expressed and the meanings and practices I attributed, I would present my interpretation of events and ask them to confirm or disconfirm my thinking and how that related to what instructors said.

Video recorded class sessions. To mitigate the reductionist tendency of interview data and follow the interview approach outlined above, I also needed to video record class sessions. Yin (2009) notes that observation data can provide additional insight in terms of context and covering events in real-time. For my study in particular, video recordings served multiple purposes. The first justification was that video recordings of class sessions provided a means to situate instructor interviews. In addition to the reasons mentioned in the prior section, instructors would not be able to keep track of all aspects of the class during instruction, and thus video recordings could document interactions the instructor may have forgotten. I showed clips the instructors may have overlooked and questioned them about their goals in those circumstances. Second, I was interested in the kinds of instructional decisions the instructor made within the classroom. Recording class sessions allowed me to document instructional decisions such as the inclusion of context in problems; the kinds of problems instructors assigned; the presence, absence, and type (if present) of discussion between the instructor and the whole class or the instructor and individual students; the presence or absence of groupwork; and the use (or not) of technology and how it was used. Both rationales help to address Kuntz's critique of

extractive logic by providing moderate means to capture the relational aspects associated with individuals.

I will now elaborate on the recording and using video recordings of class sessions. During observations, I recorded lessons for the unit and took fieldnotes on the instructional decisions I saw instructors taking in the classroom (e.g., what kinds of problems the instructor presented, how they responded to students). After recording class sessions, I reviewed clips and fieldnotes to select clips I felt may provide insight into how meaning orientations may influence pedagogical decisions (e.g., instances of connecting a mathematical concept to another field may be manifestations of an sMfM- or iMfM-oriented goal). I choose some events based on those I thought would be associated with the meaning-oriented goals instructors expressed interest in during the initial interviews. During the second and third interviews then, I brought the clips from the prior week in and questioned instructors as described in the interview section above.

By using recordings of what occurred in class, I could better facilitate a deeper understanding of how instructors' instructional decisions related to their stated beliefs and goals and work towards the relational approach Kuntz advocates for. One important note here is that classroom observations focused on capturing instruction, not record students or their thinking. With the focus being on instruction, I could more easily mitigate any discomfort to students (e.g., not including recordings with students who opt-out).

My justifications of using video recordings (situating interviews and documenting pedagogical decisions) were further enhanced by using White and Mesa's (2012) observation protocols to structure my fieldnote taking. Particularly important for my study were the activity log and problem logs. During class observations, I used both the

previously mentioned logs as detailed by White and Mesa. On the activity log, observers record classroom activity in 5-minute intervals according to a priori codes and only assigns one code for each time slot. For instance, White and Mesa have a code 'L' to signal an instructor was lecturing, among many other codes. The problem log focuses on the examples the instructor poses in a class. Space is provided to record notes, what the problem is, the time when the problem started and when it ended, and sections to record actor (e.g., lecturer, student groups), technology (e.g., graphing calculator), representation (e.g., graphical, symbol), and features (e.g., proof/justification, contextualized). Those sections have codes for each option, and the observer circles whichever codes they feel apply to a particular problem. The only section that cannot have multiple codes assigned is the actors section. If the actor driving the problem changes, a new instance of the problem should be recorded with the new actor. All this information helped give me insight on instructors' classroom practices which I then used in the protocol's post-observation survey to reflect on what factors may have influenced certain pedagogical decisions and the meanings instructors may have been focusing on.

After the class session ended, I would complete portions of White and Mesa's (2012) post-observation survey. The survey is meant to capture aspects that the previous two logs may not capture. I used the survey specifically as a chance to reflect on which meaning-oriented goals may have manifested during instruction as seen in the classroom and any influencing factors. For example, if I coded a problem in the activity log and problem log actor sections as 'I' (students working on a problem individually), 'S/M' (skill/method) in the problem log features section, and 'S' (symbol) in the problem log representation section, and I noted the problem seemed like practicing a routine

algorithm, the problem may have been indicative of an iMfM-oriented goal (preparing students for a quiz). If I noticed most of the students were male, I could also consider asking the instructor during interviews if their decisions were influenced by the male demographic in any way. In this way, the post observations allowed me to reflect on which clips I wished to show and interview instructors on and what influences shifted their in-the-moment goals. To reach these sorts of hypotheses and questions, I modified the post-observation survey to retain questions that could inform how meaning may affect instruction, cut out questions that seemed unrelated to my study, and added questions on instructors' goals.

Physical artifacts. Another data source noted by Yin (2009) is physical artifacts. In this study, the physical artifacts I collected were any homework, quiz, or exam problems related to the units I observed. Such problems were another area of instructional decisions whichinstructors' meaning orientations may influence. I asked instructors for the homework problems they assigned and compared them to the completed observation protocols and video recordings for the associated unit. By comparing to the insights I noted in the observation, I selected homework, quiz, and exam problems as a resource to question instructors on during interviews to further inform any trends I noticed in the classroom observations.

For example, suppose an instructor had students spend a significant portion of class defining material, and I coded the class activities by various codes and suspected that the teacher wanted students to engage in the authentic mathematical practice of defining. In my notes on the problem log however, I noted that students had not used the definitions yet. Then suppose the instructor assigned a homework problem using the

definitions set out in class. I could look back at my classroom observations to compare them to the homework problem. Since students had not developed and used an algorithm in class, I could surmise the homework problem was not aimed at practicing a routine skill. Possible goals could include engaging students in proof/justification and/or developing an algorithm (depending on what the problem asked). All these activities can be seen as authentic mathematical practices, and thus there could be evidence that the teacher was focusing students on the sMoM. This would give me something to consider asking instructors about during interviews.

After I analyzed the problems and decided on clips to use for an upcoming video clip interview, I selected problems related to the clips I had chosen and took them to the video clip interview. As time permitted, after discussing the videoclips, I presented problems from assignments to instructors and asked some of the same questions as those asked about the clip (e.g., what was your goal with assigning this problem).

More broadly then, comparing homework problems with my observations in class gave me insight into the kinds of pedagogical decisions instructors enacted. By asking instructors about the goals behind the problems they select, homework and assessment problems also provided further opportunities to question instructors on the possible meanings instructors may have been emphasizing.

Data Analyses

I describe how I used the various data sources to first analyze each instructor as an individual case in the hopes of answering my research questions regarding: 1) What, in terms of background, affective factors, other beliefs, etc., leads instructors to prioritize

certain meanings over other meanings, 2) how and why can context influence instructors meaning orientations, 3) how do instructors' orientations of mathematical meaning translate into specific pedagogical decisions and 4) how do backgrounds, contexts, and orientations come together in pedagogical decisions concerning how instructors respond to students' meaningfulness questions. Once I have described how each individual is analyzed, I will describe my use of cross-case analysis in order to address my research questions more generally, particularly my final research question: how do backgrounds, contexts, and orientations come together in pedagogical decisions with respect to how instructors respond to the students' meaningfulness questions.

Individual case analysis. To analyze individual instructors, I relied on the techniques of thematic analysis (Braun & Clarke, 2006) to understand instructors' belief systems to answer my research questions. I will describe how I analyzed my research questions using pertinent data sources and how the analyses were brought together in a narrative summary for each individual instructor. In terms of the actual data, I looked at individual instructors' interview responses and homework problems.

Braun and Clarke (2006) define thematic analysis as "a method for identifying, analyzing, and reporting patterns (themes) within data" (p. 79). The themes that are generated in this analysis draw from reviewing data and are things that stand out according to the researcher's interests which seem to follow patterns in some form (showing up multiple times, for instance). Themes can focus on describing an entire data set or describing particular aspects of the data. Braun and Clarke (2006) note how thematic analysis can be inductive (building themes up from the data) or theoretical (themes derive from prior research and are fitted to the data). Themes can also be

semantic (being identified directly by what participants say) or latent (where themes are based on underlying ideas, assumptions, or characterizations that do not have to be stated). My study is concerned with theoretical thematic analysis because I am coming in with an a priori framework of mathematical meaning and could be focused on latent or semantic themes because I am looking at what instructors say (in an attempt to lend instructors a voice, thus semantic) but also mining the reasons for their statements (latent).

Braun and Clarke (2006) set out six recursive processes for conducting a thematic analysis. They are:

1) *Familiarizing yourself with the data* by transcribing, re-reading data, and taking notes of initial ideas
2) *Generating initial codes* by coding interesting portions across the data in a systematic fashion
3) *Searching for themes* by collating themes and relevant data
4) *Reviewing themes* by checking how themes hold up with initial codes, cutting or combining themes, and then creating thematic maps connecting themes
5) *Defining and naming themes* by refining the specifics of themes, reflecting on what a theme contains and the stories they contain. Then generating definitions and names for the themes
6) *Producing a report* of the final analysis with examples and tying back to the research questions

Familiarizing. In my study, the four orientations on meaning are taken as a way to categorize or type instructors' goals and I took instructor goals (e.g., showing the

beauty of mathematics, preparing students for particular fields) to be a subset of their beliefs situated within belief systems as Leatham (2006) outlines. I saw various types of beliefs as being included in belief systems (e.g. beliefs on the nature of mathematics, role as an instructor, beliefs about their prior life experiences, other goals, values), where I avoid any distinctions between beliefs, knowledge, or anything else many have debated before. In turn, initial codes focused on identifying goals and types of goals (i.e., the meaning orientations of instructors' goals) and background themes in all interviews, instructional decisions in the second and third interviews, and any characteristics (contextual or otherwise) of those instructional decisions in all interviews and my fieldnotes. Because of the need to compare instructional goals and decisions, particularly during video clip interviews, familiarizing and initial coding was done concurrently with data collection and afterwards. Being able to bring instructional decisions and meaning orientations into subsequent interviews required that I had already familiarized myself with the data and posited some initial ideas regarding underlying orientations of goals (and thus the need for some initial coding during data collection as I will describe in the following paragraph).

In terms of the first interview, comments instructors made on their background or instructional goals were coded to represent those aspects. For subsequent interviews, I compared instructional decisions, and goals instructors noted during prior interviews with the instructional decisions and orientations posited in my fieldnotes and observation protocols so that I could question instructors about connections during upcoming interviews. After asking about those connections during the subsequent interview, I attributed codes to note what instructors voiced as their goals and how those connected to

instructional decisions. I heavily weighted instructors' interpretation regarding goals and decisions over my interpretations. However, I retained my interpretation of events as recorded in my observation protocols when our interpretations were not mutually exclusive. Leatham notes that one is not always cognizant of one's beliefs. In relation to the difference between what instructors stated and what I hypothesized in my observation protocols, such differences may not indicate inconsistency noted in prior literature. Rather, such discrepancies could indicate additional effects of the ways instructors navigate mathematical meaning that they may not be fully cognizant of. However, in cases where teachers explicitly ruled out my interpretation during interviews, I took the instructors' interpretation regarding goals and decisions over my interpretation. However, the retention of my hypotheses served as rival conjectures that were disproven.

Homework assignments were analyzed against observation protocols as mentioned in the physical artifacts section above. Comparing the codes and notes recorded during classroom observations with homework and assessment problems, I coded for meaning orientations and/or goals instructors may have been focusing on. For instance, if a problem had students using an algorithm, and looking back at the class notes revealed students had developed a similar algorithm but modifications needed to be made in order to solve the homework problem, I would suspect this to be a case of generalizing, and thus a possible manifestation of an instructor's sMoM-oriented goal. If students had done a similar example, the homework problem would serve as routine practice, possibly for the aim of preparing students for later work. In this case, I would suspect the instructor was using the problem to focus on the iMfM or sMfM (since procedural engagement may be required for STEM fields). I carried out this sort of

analysis for the homework problems from units I observed. I then asked instructors during interviews about their goals behind a problem to check and/or amend my interpretation.

Initial coding. I organized events and codes using MS Excel in a tabular approach to list out codes for each instructor in their own excel file. To start this process, I coded the first interview for both the suspected orientations on meaning but also instructional goals, background, and contextual factors, with a thinking that followed the representation shown in table 1 below (where the rows represent different instructional goals and decisions, background, or contextual factors while the columns represented orientations). While I was not explicitly using this table, this table was how I was thinking when I was coding for both orientation and instructional goals and decisions, background, and context simultaneously. The columns' categories were mainly fixed as the orientations were the only things I perceived as orienting teacher goals related to students. However, I came to think of a fifth column of 'General' or 'Facilitating teaching', a catch-all category for things that inform all goals and teaching more generally.

Table 1

Mental Organization of Initial Coding per Interview

	sMoM	sMfM	iMoM	iMfM	General
Definitions	Showing students definitions to them get used to math people language				

Proof					
Groupwork			Having students engage in groupwork	Having students engage in groupwork	
Contextualized problems		Introducing new material by showing applications of concepts first	Introducing new material by showing applications of concepts first		
Test skills		Showing calculator shortcuts to focus exam on interpretation of data		Showing calculator shortcuts to focus exam on interpretation of data	
Supportive Cast					supportive father (encouraging academics, involved in/helping with school)

Instructional goals and decisions, background, and context were captured by the rows and were variable. Based on my literature review, there were instructional goals and decisions that I had hypothesized could be oriented in certain ways so there was some foundation of actions and goals I started to look out for. However, I realized those would not be all the instructional decisions and goals, background, and context factors instructors may have, and thus I was open to coding for things I had overlooked. Another important thing to note was that an instructional decision or goal was connected with an orientation, not by me (or my literature review driven hypotheses) but according to the instructor's own statements. For instance, I had posited that showing contextualized

problems may indicate an sMfM- or iMfM-oriented goal. However, with the example drawn from my data above, contextualized problems was put under iMoM because the instructor (in this case, Jordan) had said that she used contextualized problems to make concepts more accessible to students and thus further understanding of the mathematical concepts in the abstract, which fits with the iMoM definition set out earlier for understanding mathematical concepts themselves. In this way, the rows above indicated what the instructor did while the columns indicated why the instructor did those actions according to the instructor's own statements.

When coding for background and context, orientations were not necessarily attributed, but I could posit some orientations that instructors might have valued based on my own sense-making hypotheses of their life experiences or by the ways instructors explicitly spoke about their own experiences that clearly seemed to push them towards one of the orientations as I defined them in the literature review. For instance, Benjamin said he was always curious about why mathematical rules were what they were and asked his family about them. He was really interested in how concepts connected to lead to mathematical results, and this fit very well with how the iMfM orientation was defined. That does not mean that experience of wondering about mathematical concepts was iMfM oriented, but rather that I suspected Benjamin would have iMfM-oriented goals based on that experience. Other times, background and context factors would not have a clear implication for any orientation on meaning, and so I would not associate an orientation to that (e.g., 'supportive cast' above).

However, I recorded instructional goals and decisions, background and contextual factors, and orientation as per Table 2 in the 3rd (orientations and concise codes), 4th

(instructional goals, background, and context; including timestamps to facilitate ease of locating), and 5th columns (instructional decisions; including timestamps to facilitate ease of locating). I chose to follow the organization for Table 2 instead of the organization for Table 1 because I was anticipating generalizing instructional goals, background, and context themes and then separating background, instruction, and contextual factors (as separate excel files) in later steps of the data analysis process so as to answer each of my research questions. For codes related to instruction for instance, I could then move and/or copy those themes and initial codes into separate excel tabs for each orientation and thus focus my analysis and discussion to my second research question (about how orientations manifest in instruction decisions and goals) around the orientations. So while my thinking was not nested when I was coding and looked more like the organization in Table 1, I chose to organize the thinking from table 1 into a nested structure as in Table 2 to facilitate later data analysis and coding so that I could then focus the answers to my research questions around each meaning orientation and how they are influenced by background and context. Table 2 below represents a clip from my excel file for Jordan.

Table 2

Organizing Themes Table

Broader Theme	Background & Beliefs, Instructional Goal, or Context Themes	Concise Code &/or Orientation	1st interview initial goal codes/descriptions	2nd and 3rd interview instructional decisions
Conceptual understanding	Understanding	Use of tech, imom	23.37> started interest in using tech in	2a2. worksheet had multiple kinds of graphs/diagrams/methods

			learning/teaching. Graphing calc anecdote> she wants to move past calculations (26.15) but rather to help bigger understanding, help to visualize things (26.45)	>to point out errors she saw
		Imom, imfm, smfm	intro concept via ex similar to hw/real life ex	1a-3. introduces new notation/terminology w/ real life ex (26.00)
		Imfm, imom	32:50- get students to problem solve and multiple solutions (v. students who want recipe/one way)	5. did she hit goals? 1.23.25 (extra credit as a chance to do this)
	Concerns	ambitious teaching, imom->imfm	1.0.30-less concern after having taught so much. Teaching for 18 years, but still trying to learn/implement new techniques (via conferences, colleagues,)- knows where students will get stuck	2c1. she went around checking people's work, pointing out the need for hypoth as symbols but emphasized sentences, 1.16.00 (use of table)

At the stage of coding the first interview, nothing was noted in the 1st or 2nd or from the 5th column on (Table 2) as those were used in later steps. I coded the first interview for statements on instructional goals, background, and contextual factors and (if possible to determine or conjecture) associated orientations on meaning with those statements, with a thinking that followed the representation shown in table 1 above, but organized according to Table 2 above. As I listened to and read transcripts, I first made entries in the 4th column for any goal, background, context initial codes, and then noted

orientations (if possible to posit based on the instructor's statements) and/or concise codes for the initial codes in the third column.

I then coded the second and third interviews by making tentative associations between the first interview codes (4th column, Table 2) with instructional decisions in the second and third interviews (5th column and beyond, Table 2) I suspected may be related to those first interview codes. To do this, I used cells in the 5th column onwards to record timestamps (with a brief summary of context, the instructional practice question being asked, etc.) for instances from the interviews. 5th column cells in green text indicate being discussed in the third interview, instances in orange indicate coming from the second interview. The number and letter sequence starting each event in column 5 and onward (e.g., "1a-3") referred to the question being asked in the interview, and for the purpose of analysis, was only used to recall the question I asked as recorded in my interview protocol. The color-coding in the fourth column was only important in that text in gray ("intro concept via ex similar to hw/real life ex") was meant to signal that that initial goal code had not shown up in the first interview but instead emerged during the second and third interviews. While coding the second and third interviews for instructional decisions, I progressively updated orientations (column 3, Table 2) to reflect which orientations (as I defined them in my literature review) instructors themselves connected to each instructional goal. Sometimes, I could not associate a code on orientation until later. In those cases, I did not note an orientation in the 3rd column.

Searching for and reviewing themes. Searching for and assigning themes (1st and 2nd column, Table 2) involved drawing similarities between groups of codes to arrive at a broader characterization of the instructional goal, context factor, or instructor

background factor underlying all the codes in the group. To facilitate this process, I first looked at each row starting at column 4 onward (Table 2) and rearranged the rows into groups according to similarities I was seeing connecting the instructional goal codes (4th column) and or decisions (5th column onward). Once they were rearranged, I assigned a theme in column 2 signifying those similarities. This theme I assigned to the group of instructional goal codes was meant to signify a more general instructional goal, background, or contextual factor.

So using the table above as an example, I reorganized "intro concept via ex similar to hw/real life ex" and "32:50- get students to problem solve and multiple solutions (v. students who want recipe/one way)" together as instructional goal codes because during the second and third interviews respectively, Jordan had referred to associated instructional decisions (5^{th} column onward) as related to her lesson planning to foster understanding of mathematical concepts. Because of this underlying thread then, I assigned the theme 'Understanding' as the instructional goal theme capturing both instructional goals more broadly. Thus in Table 2, the 2nd column with 'Understanding' is a theme, with both "intro concept via ex similar to hw/real life ex" and "32:50- get students to problem solve and multiple solutions (v. students who want recipe/one way)" under it.

The process noted above was carried out across all codes. Once themes were assigned in column 2, I then began reviewing themes by looking across the themes and their entries in column 3 (Concise Code & Orientation) for broader themes related to the orientations as I defined them in my literature review. To start this process, I set up separate tabs in each instructor's excel file for each orientation and a tab for 'facilitating

teaching'. From there, I copied themes according to my 3rd column (Concise Code & Orientation) into each respective orientation tab. For instance, Table 2 shows entries that were moved from a general tab that held all my initial codes into an iMoM tab because they all had iMoM listed in column 3. However, that does not mean that the codes under the 'understanding' theme only showed up in the iMoM tab for Jordan. Based on the entries in column 3, some codes under 'Understanding' would appear in both the iMfM and sMfM tabs with only the rows from that theme that the instructor connected with those orientations (so in this case 'intro concept via ex similar to hw/real life ex' with the theme 'Understanding' would appear in both tabs). In cases where I had not noted an orientation in the 3rd column, I reflected on the concise and initial codes (3rd and 4th columns) and the associated pedagogical decisions (5th column on) and copied those rows with their associated themes to the tab(s) I felt best fit where they belonged.

Once themes were placed into separate tabs that I coded them for in column 3, I then regrouped rows according to the themes present in a tab when necessary, specifically if parts of a theme were moved but not the whole theme. I then looked at all the themes in each tab to form even broader themes on background, instructional goals, and contextual factors. This involved looking at the collection of themes (column 2, Table 2) and the second and third interview instructional decisions associated to the initial goal codes under that theme (column 5 on, Table 2) to see if themes had a broader underlying pattern to them. So in table 2, for instance, the theme of 'Understanding' and 'Concerns' both had an underlying theme of aiming for understanding mathematical concepts. Thus, I created and assigned the broader theme 'Conceptual Understanding' in column 1.

In some cases, once the themes were moved, I would find there would be no larger theme or that some themes now seemed to be nested in other themes. In these cases, the themes I assigned in column 2 were then moved to become the broader themes in column 1. In the case of nesting, I would copy the larger theme to column 1 and keep the same themes in column 2. This was done to keep the subtheme code separate from the theme, which became the larger theme. The nesting situation shows up in table 2 in that 'Understanding' and 'Conceptual Understanding' were the same and 'Concerns' fell under understanding. I only changed the wording of 'Understanding' to 'Conceptual Understanding' in the transition to column 1 because I was looking ahead to the cross-case analysis while trying to balance the fact that 'Understanding' was closer to the instructor's own terminology.

Connected to this last point, an important note here about the initial codes and themes was that I tried to use themes that fit the instructor's own terms when possible. My reasoning for this was informed by the fact I was going to be representing an individual's story and felt strongly about respecting that story in their own terms. This was not an issue all the time. Something like 'joy in math' and 'love of math' can be synonymous to many people, but in other cases, terms do matter for people and their personal experience, and if I was writing representative narratives, instructors might feel I am no longer describing their experience if I was not using their terms. Such a dismissal of the individual messiness within each instructors' story would be a textbook example of Kuntz's (2015) critiques of modern research objectifying ideas to create marketable research products.

Another important thing to note here is that once I had established a theme consistently to mean something for an instructor in their own terms, I applied the theme to other cases which had language that connected to how the theme showed up for an instructor. For instance, Jordan had routinely brought up a theme of ‘Student success’, and for that instructor, this theme meant students passing their course. I coded student success under the iMfM theme because students passing fell under a student achieving academic success, which is not practice-related (ruling out social orientations), nor is it necessarily related to conceptual understanding (ruling out the iMoM). Once the participant has consistently used a term, such as "student success", in a way that is consistent with an orientation code, all subsequent turns of talk that use that term received that code. I also then applied that established theme as a code to situations that did not necessarily state the term explicitly, but the instructor’s consistent use of language surrounding that theme implied they were referring to the same thing. For instance, Jordan stated, “Well, it's always a challenge when you have, I have 41 students, and I think that could be a big group and so making sure students don't fall to- through the crack.” This instance was under one of the context subthemes (‘Dept/Colleagues’ specifically) because of statements directly after it about having department resources. However, in determining the context theme's orientation, I assigned ‘iMfM’ because of her statement about ‘students falling through the cracks.’ Jordan consistently used this language around talking about student success which I had coded as iMfM for the reasons described before, and thus the same orientation code was carried over here.

Defining and naming themes. In defining all themes, I first refined and defined the themes (under the broader themes) by reflecting on what instructional goals (column

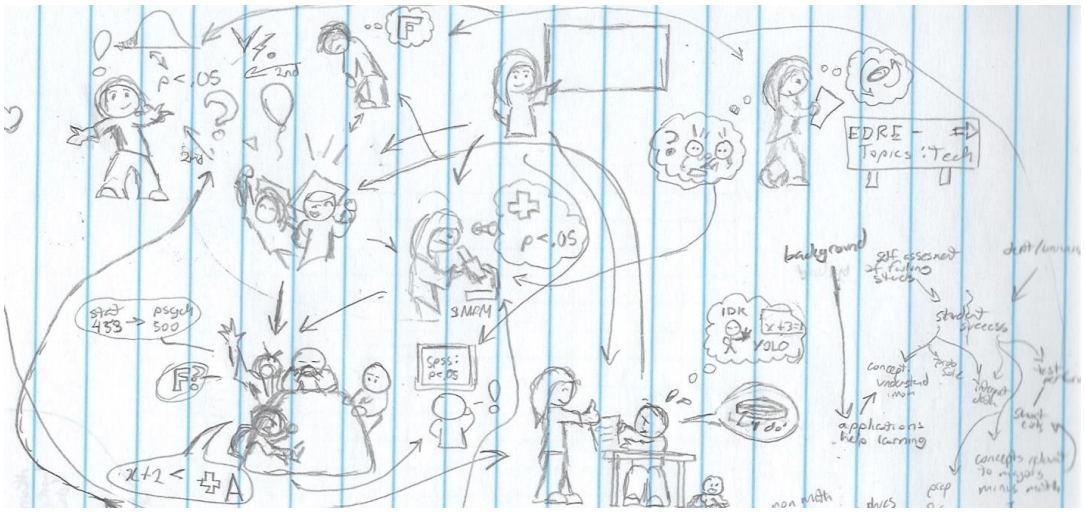
4, Table 2) and associated second and third interview coded instructional decisions (5th column on) a theme contained. This involved looking at which instructional goals were described by the instructor and how that connected with second and third interview pedagogical decisions (class activities, assigned problems, etc.). For instance, by looking under the iMoM tab for Jordan above (Table 2), I would use the 4th column instructional goal codes and reference the associated second and third interview instructional decisions (column 5 on, Table 2) to define what 'Understanding' meant for Jordan. Part of this also involved defining themes in contrast to other themes by comparing what instructional goals and decisions one theme had or did not have that other themes had/did not have, and if two themes had similar instructional goals, how the instructional goal manifested differently in one theme versus others (something which could be derived by reflecting on the associated second and third interview instructional decisions (column 5 on, Table 2)).

I did this using thematic maps to connect themes and instructional decisions (a portion of the map created for Jordan is depicted in Figure 2 below). In terms of understanding my thematic maps, each individual picture in the map below depicts a theme (column 2) synthesized *with* the instructional decisions coded from the second and third interviews. I came up with an individual image based on my reflections on the theme and associated instructional decisions. For instance, there were other rows in the instructional goal theme for 'Understanding' and the image I drew in the bottom center of the page (the instructor giving a thumbs up to a student giving her a piece of paper) was drawn after reflecting on all the instructional goal codes and decisions associated with that theme. For instance, the parts of the image described previously depicts '32:50- get

students to problem solve and multiple solutions (v. students who want recipe/one way)' that was accomplished by '5. did she hit goals? 1.23.25 (***extra credit*** as a chance to do this)'. More succinctly, the image shows the instructor pushing students to problem solve and think in different ways by assigning extra credit opportunities. The other aspects of that particular image (the child on the ground, the thought and speech bubbles) represent other initial background and instructional goal and decision codes associated with that instructional goal theme. However, I will avoid talking about those for the sake of clarity. I used these synthesized images and the connections I drew (as represented by the arrows in Figure 2) to write up a preliminary summary defining themes and how they connect in an instructor's belief system. I then sent the preliminary summary to the instructor for checking. Aside from a few minor adjustments, they agreed that I had represented their perspectives accurately.

Figure 2

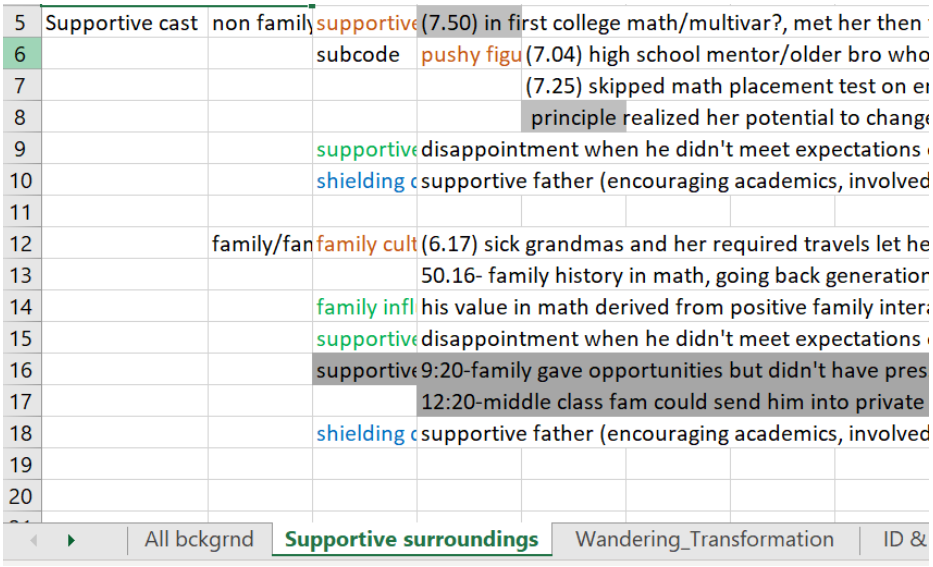
Portion of Thematic Map of Jordan's Belief System



However, the process of defining themes was not completed after getting instructor approval. One important thing to note here was that my later interviews were already starting to be informed by my previous interviews with all instructors. As such, some cross-case thinking was already going to manifest even if I were to try to bracket my thinking during the creation of the thematic maps and preliminary summary of themes. For instance, 'supportive cast' was a broader theme related to background that showed up across instructors. If I blinded the cases and went back, would I still call 'supportive cast' in cases I have looked at for the first instructor after having looked at the third and fourth instructors (since I already had some cross-case thinking at the time of interviewing the third and fourth instructors but did not for the first instructor)? As such, after receiving the approval of themes from instructors, I started reconciling my excel files into a code book by looking at the excel documents across instructors to collapse, nuance (regrouping broader themes but keeping themes, because themes across instructors may be related but qualitatively different), or tease apart themes that were similarly named. This was done to use the same codes across instructors for both my individual case analysis (chapter 4) and cross-case analysis (chapter 5).

Figure 3

Background-Related Themes Excel File Organization



To facilitate this process, I looked across excel files at similarly worded themes and broader themes in groups (e.g., putting all background related broader themes, and their associated themes and instructional manifestations, across instructors in an excel file then making separate sheets for each group of similar or related broader themes to compare the similarities/differences between themes as shown in Figure 3 above). As a way to review themes after making the previously stated moves, I looked at *initial codes* (black colored text in the 4th or 5th columns, Figure 3) again individually to see if there were ideas that could be teased apart into separate themes (as a way to be critical of my coding), and then looked across themes (colored text in the 3rd or 4th columns, Figure 3) within each tab of an excel file to collapse similarly phrased themes under the same theme or group them into the same broader theme but retain and or rename themes. The different text colors were used to differentiate which themes came from which instructor (e.g., green text signaled coming from Archy). For instance, after putting all background related themes into an excel file (the 'All backgrnd' tab as shown in Figure 3 above), I moved all themes related to supportive families or environments from all instructors into

the ‘Supportive surroundings’ sheet, using different colors to keep track of what theme came from which instructor. Within that tab, I nuanced variations of the theme ‘supportive cast’ that showed up across instructors by making ‘supportive cast’ a broader theme (1st column, Figure 3) and creating family/fam culture and non-family themes within it (2nd column, Figure 3).

What is important to note here is that the original themes (colored text in the 3rd and 4th columns, Figure 3) were essentially being renamed with my own terms (1st and 2nd columns, Figure 3). So as a way to have narratives stick close to participants' own terms, I collapsed and renamed themes under the same name (subsuming 'happenstance' and 'wandering' into a common term like 'wandering' in the 1st or 2nd columns). However, I then retained the themes in instructors own terms (colored text in 3rd or 4th columns) so that in each individual’s narrative, whenever I used the new themes from the codebook, I would name a theme by whatever the participant would name it by (so if I used the 'wandering' code for Natalie, I would call it 'happenstance' in her narrative to match her own terms, but if I used it in Benjamin’s narrative, I would call it 'wandering' to fit his terms). By keeping the instructor’s own terminology for themes delineated by colors, I would have the themes that stuck closer to the participants’ own word in one column but then still have the synthesized theme names of what the old themes were now subsumed under.

For themes on instructional goals, I grouped themes as before, putting all instructional goal themes and broader themes into an excel file and making separate sheets for iMoM, sMoM, iMfM, sMfM, and facilitating teaching. Then within each sheet, similar themes were grouped under new synthesized theme names. The nesting could

have been reversed so that each instructional goal theme could be looked at from the perspective of having multiple orientations of meaning possible within them. This would follow a flow of analysis in terms of first describing a theme and then associating a meaning orientation to the theme based on evidence. However, as was mentioned in the discussion of Table 1, this choice in nesting instructional goal themes in orientations was made so that analysis could focus on answering each research question with what meaning orientations came to the fore and then have particular manifestations of each orientation.

After synthesizing all themes related to background, context, and instruction into a code book, I then set about formally writing definitions of the synthesized themes by reflecting on initial background, goal, and context codes and associated second and third interview coded instructional decisions (each row, Figure 3) each synthesized theme contained. By reflecting on how instructional goal, background, and context initial codes under the synthesized theme connected with second and third interview pedagogical decisions, I wrote up an appropriate definition to capture what the synthesized theme represented. Part of this also involved defining themes in contrast to other themes by comparing what instructional goals and associated decisions one theme had or did not have that other themes had/did not have, and if two themes had similar initial codes, how those initial codes manifested differently or similarly as instructional decisions in one theme versus others.

Once definitions were assigned to each theme in my codebook, I first wanted to check the reliability of my themes by selecting interview transcript clips with the codes as I assigned and checking if another individual would similarly code the data using the

definitions set out in the codebook. To this end, I had another individual code a collection of 42 transcript clips along the dimensions of background and background beliefs, instruction, and context. Agreement on my codes and the other coder on each dimension were: 77% on background beliefs, 82% on background, 90% on context, and 88% on instruction. We then discussed any significant differences in our interpretation of theme definitions and amended them accordingly to make sure definitions were concise and understandable. From there, I began recoding my data using the new themes as codes. My excel files already had initial codes documented with timestamps. So I went back to those points in the transcripts as per the excel files' timestamps, and recoded them with my new codebook. After referencing the timestamps to recode, I went over the transcripts completely to code any moments I may have overlooked in my excel file timestamps.

Producing a report to answer my research questions. The last step for the individual analysis involved writing up a narrative summary *for each instructor individually* of how themes connect to answer my four research questions. As mentioned in the previous step, I had already written up preliminary summaries for each instructor using thematic maps. After a member check, the creation of my synthesized codebook, and recoding, I went back to edit and restructure these preliminary summaries into formal narratives. Part of this involved restructuring the discussion of old themes with an eye towards my new synthesized themes. As I mentioned before, I wanted to stick close to participants' own terms, and as such, I retained the themes I had originally derived from an instructor's terms under each synthesized theme. Going back over the summaries then meant that the way themes were referred to by instructors was not changed but that the use of the themes in instructors' own terms was checked for consistency with the newly

synthesized themes. Each instructor's narrative was broken up into four portions, one for each research question.

My first research question asked: What, in terms of background, affective factors, other beliefs, etc., leads instructors to prioritize certain meanings over other meanings. Answering this question required me to look back at the themes under background and how they do or do not connect to other themes. This had already been done in the thematic map creation, so my task here was to summarize and frame those connections into a complete narrative using Leatham's framework. My third research question asked: how can context influence instructors' meaning orientations. Similar to my write-up for my first research question, I would collect themes associated with context and write up a narrative summary, framed within Leatham's belief clusters dimension, of how they connect to other themes, particularly goals and orientations. These questions focused on the orientations instructors focus on due to affective and contextual factors.

My second and fourth research questions and their dedicated responses dealt more with how orientations manifest for an instructor. My second research question asked: how do instructors' orientations of mathematical meaning translate into specific pedagogical decisions? Like the previous questions, I would look at how goals, orientations, and pedagogical decision themes connect (in the thematic maps) and summarize those relationships. My fourth research question asked: How do backgrounds, contexts, and orientations come together in pedagogical decisions with respect to how instructors respond to students' meaningfulness questions, such as "when is this ever going to be useful?" This, in particular, would require looking back at the previous research question subsections for that instructor and themes that arose in reference to how

they responded to the interview question: In this course, how would you respond if someone asks you, "when is the stuff ever going to be useful?". In looking back at the previous subsections, I would look at why instructors voice certain oriented goals in *responding to student meaningfulness questions* (informed by question one: background and question three: context) and how those *specific oriented goals instructors connect to students' meaningfulness questions* manifest in an instructor's pedagogical decisions (question two).

With these four-part narratives written for each instructor, I sent my summaries to instructors to have them member check my interpretations and make adjustments per their suggestions.

Cross-case analysis. Yin (2009) notes cross-case analysis as a technique specific to multiple case study designs. Engaging in cross-case analysis first requires that each individual has been analyzed as a separate study. Only thereafter can the researcher engage in compare findings across each case. Here, I used thematic analysis again, starting from the phase of reviewing themes since I already had themes based on the individual interview analysis. To facilitate this step, I used pattern matching as defined by Yin. Using thematic analysis with an eye to pattern matching, I wrote up themes that appeared across instructors and then started to answer my research questions using these cross-instructor themes.

Pattern matching and reviewing themes. Pattern matching can take numerous forms and entails comparing observed patterns in the data with theoretical patterns posed by one's theory (Yin, 2009). In this study, this would entail comparing instructors' stated and observed patterns of instruction with the kinds of instructional practices I had posited

to be associated with instructors' mathematical meaning goals. My analysis focused on what Yin defines as *nonequivalent dependent variables as a pattern*. According to this pattern, there are numerous relevant outcomes (in my study, these are the various pedagogical decisions and oriented goals instructors may have). Yin describes the following,

> Pattern matching occurs in the following manner: If, for each outcome, the initially predicted values have been found, and at the same time alternative "patterns" of predicted values (including those deriving from methodological artifacts, or "threats" to validity) have not been found, strong causal inferences can be made. (p. 137)

Although I am not as concerned with causation in my study (I am more concerned with description and exploration), pattern matching in my study entailed looking at instructors' overall to compare different themes related to pedagogical decisions, oriented goals, instructor background, and/or context. Different backgrounds, instructional goals, and context themes and their associated orientations represent different outcomes.

Yin suggests using word tables as a starting point for such an analysis. These tables organize and collect aspects of the individual cases uniformly. Multiple, complementary word tables can be used to reflect other relevant outcomes. Whether looking at a single word table or a collection of tables, one can go beyond a single case's insights to start looking for trends of differences. If one sets out tables for each individual in the study, one can start to make case-to-case comparisons. More specifically,

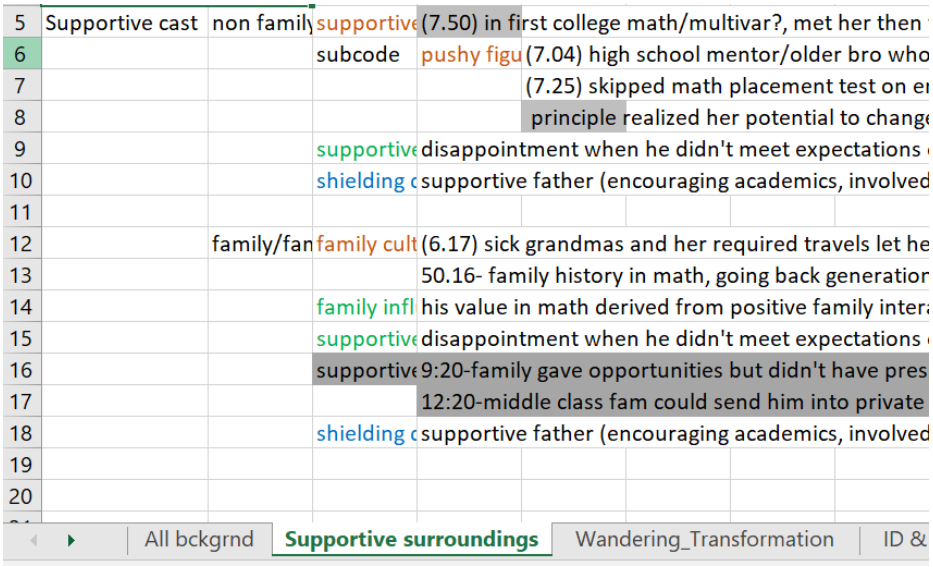
> The analysis can start to probe whether different groups of cases appear to share some similarity and deserve to be considered instances of the same "type" of general case. Such an observation can further lead to analyzing whether the arrayed case studies reflect subgroups or categories of general cases-raising the possibility of a typology of individual cases that can be highly insightful. (p. 160)

In my study, I already had a table for each individual (the excel files discussed in the individual analysis section), which I then used as a starting point for my cross-case pattern matching.

As noted before, one important thing to note was that my later interviews were already starting to be informed by my previous interviews with all instructors. As such, some cross-case thinking was already going to manifest even if I were to try to bracket my thinking during the creation of the thematic maps and preliminary summary of themes. As such, after receiving the approval of themes from instructors, I started reconciling my excel files into a codebook by looking at the excel documents across instructors to collapse, nuance (regrouping broader themes but keeping themes, because themes across instructors may be related but qualitatively different) or tease apart themes that were similarly named. This was done so that I would be using the same codes across instructors for both my individual case analysis (chapter 4) and cross-case analysis (chapter 5).

Figure 4

Background-Related Themes Excel File Organization



To facilitate this process, I looked across excel files at similarly worded themes and broader themes in groups (e.g., putting all background related broader themes as a group in an excel file then making separate sheets for each group of similar or related broader themes to compare the similarities/differences between themes as shown in Figure 4 above). As a way to review themes after making the previously stated moves, I looked at *initial codes* (black colored text in the 4th or 5th columns, Figure 4) again individually to see if there were ideas that could be teased apart into separate themes (as a way to be critical of my coding). I then looked across themes (colored text in columns 3 or 4, Figure 4) within each tab of an excel file to collapse similarly phrased themes under the same theme or group them into the same broader theme but retain and/or rename themes. The different text colors were used to differentiate which themes came from which instructor (e.g., green text signaled coming from Archy). For instance, after putting all background related themes into an excel file (the 'All backgrnd' tab as shown in Figure 4 above), I moved all themes related to supportive families or environments from all instructors into the 'Supportive surroundings' tab, using different colors to keep track

of what theme came from which instructor. Then within that tab, I nuanced variations of the theme ‘supportive cast’ that showed up across instructors by making ‘supportive cast’ a broader theme (column 1, Figure 4) and creating family/fam culture and non-family themes within it (column 2, Figure 4).

What is important to note here is that the original themes (colored text in columns 3 and 4, Figure 4) were essentially being renamed with my own terms (columns 1 and 2, Figure 4). As a way to have *individual narratives* stick close to participants' own terms, I collapsed and renamed themes under the same name (subsuming 'happenstance' and 'wandering' into a common term like 'wandering' in columns 1 and 2). However, I then retained the themes in instructors own terms (colored text in columns 3 and 4) so that in each *individual’s narrati*ve, whenever I used the new themes from the codebook, I would name a theme by whatever the participant would name it by (so if I used the 'wandering' code for Natalie, I would call it 'happenstance' in her narrative to match her own terms, but if I used it in Benjamin’s narrative, I would call it 'wandering' to fit his terms). In my *cross-case analysis* write-up however, I presented the themes by the overarching name I attributed and then discussed how that theme looked across instructors. With the prior example, I wrote up a section titled ‘Wandering & Transformation’ where I have a discussion dedicated to the ‘wandering’ theme. Both instances of the 'wandering' code for Natalie and Benjamin would be noted in this section. Both would be referred to as instances of the ‘wandering’ code instead of being treated as separate ‘happenstance’ and ‘wandering’ themes.

As noted in my individual analysis, I grouped instructional goal themes as before, putting all instructional goal themes and broader themes into an excel file and making

separate sheets for iMoM, sMoM, iMfM, sMfM, and facilitating teaching. Then within each sheet, similar themes were grouped under new synthesized theme names. The nesting could have been reversed so that each instructional goal theme could be looked at from the perspective of having multiple orientations of meaning possible within them. However, as was mentioned in the discussion of Table 1, this choice in nesting instructional goal themes in orientations was made so that analysis could focus on answering each research question with what meaning orientations came to the fore and then have particular manifestations of each orientation.

After synthesizing all themes related to background, context, and instruction into a codebook, I then set about formally writing definitions of the synthesized themes by reflecting on initial background, goal, and context codes and associated second and third interview coded instructional decisions (each row, Figure 4) each synthesized theme contained. By reflecting on how instructional goal, background, and context initial codes under the synthesized theme connected with second and third interview pedagogical decisions, I wrote up an appropriate definition to capture what the synthesized theme represented. Part of this also involved defining themes in contrast to other themes by comparing what instructional goals and associated decisions one theme had or did not have that other themes had/did not have, and if two themes had similar initial codes, how those initial codes manifested differently or similarly as instructional decisions in one theme versus others.

Once definitions were assigned to each theme in my codebook, I first wanted to check the reliability of my themes by selecting interview transcript clips with the codes as I assigned and checking if another individual would code the data in a similar fashion

using the definitions set out in the codebook. To this end, I had another individual code a collection of 42 transcript clips along the dimensions of background and background beliefs, instruction, and context. Agreement on my codes and the other coder on each dimension were: 77% on background beliefs, 82% on background, 90% on context, and 88% on instruction. We then discussed any significant differences in our interpretation of theme definitions and amended them accordingly to make sure definitions were concise and understandable. From there, I began recoding my data using the new themes as codes. My excel files already had initial codes documented with timestamps. So I went back to those points in the transcripts as per the excel files' timestamps, and recoded them with my new codebook. After referencing the timestamps to recode, I went over the transcripts completely to code any moments I may have overlooked in my excel file timestamps.

Producing a report to answer my research questions. The last step for the cross-case analysis involved writing up a narrative summary of how synthesized themes appeared across instructors and how those themes connect to answer my four research questions. I first wrote up descriptions of the background and instructional themes that appeared across instructors. Within the instructional goal themes, I dedicated subsections detailing how each theme fit within certain orientations by citing examples from the instructors. Once the previous sections were done, I then wrote up summaries about how those themes fit together in trying to answer my research questions.

My first research question asked: What, in terms of background, affective factors, other beliefs, etc., leads instructors to prioritize certain meanings over other meanings. Answering this question required me to look back at the themes under background and

how they do or do not connect to instructional goal themes. This would have already been done in the thematic map creation in my individual analysis, so my task here was to summarize and frame general connections that showed up across instructors. My third research question asked: how can context influence instructors' meaning orientations. Similar to my write-up for my first research question, I would collect themes associated with context and write up a narrative summary about how context shifted instructors' goals generally. These questions focused on the orientations instructors focused on as a result of affective and contextual factors.

My second and fourth research questions and their dedicated responses dealt more with how orientations manifest for an instructor. My second research question asked: how do instructors' orientations of mathematical meaning translate into specific pedagogical decisions? Like the previous questions, I would look for generalities across instructors in how goals, orientations, and pedagogical decision themes connect (in the excel files) and summarize those relationships. My fourth research question asked: How do backgrounds, contexts, and orientations come together in pedagogical decisions with respect to how instructors respond to students' meaningfulness questions, such as "when is this ever going to be useful?" This required looking back at how instructors responded to the interview question: In this course, how would you respond if someone asks you, "when is the stuff ever going to be useful?" from their individual interviews. I looked at why instructors voiced certain oriented goals in *responding to student meaningfulness questions* (informed by question one: background and question three: context) and how those *specific oriented goals instructors connect to students' meaningfulness questions* manifest in an instructor's pedagogical decisions (question two). I found trends in how

instructors responded to such questions personally, how they did or did not attend to students' meaningfulness questions in their instructional practice, and how background and context influenced how instructors responded to students' meaningfulness questions. I wrote up summaries of each of these overall trends as separate subsections of the section devoted to my fourth research question.

I acknowledge that the whole effort I described above are all cases of separation of individual aspects from human individuals themselves, and thus I fall victim to the extraction logic Kuntz (2015) critiques. The use of tables generally violates Kuntz's extraction critique of research methodologies today. However, as the sole researcher on this project and the level of immersion Kuntz's alternatives demand, I could only take minor steps towards such goals. Part of that effort has been through using Leatham's (2006) framework (that sees instructors as complex individuals) and video clip interviews to build a shared understanding. From there, I aimed to document complexities that arose across instructors' perspectives in drawing general trends. By respecting such complexities, I hoped to provide a more nuanced view about the generalizations I drew and thus open up my findings to critique and discussion.

Chapter 4: Individual Analysis

My individual analysis section will be broken up into four sections, one for each instructor. Within each section, I will talk about the themes that showed up regarding the instructor's background and instructional goals, with subsections dedicated to how different instructional goals were oriented. I will then answer my research questions for that particular instructor.

Jordan's Narrative, Instructional Goals, and Analysis

In the following sections, I will review Jordan's background in a narrative format. Next, I will describe Jordan's instructional goals and how those goals may be characterized using my meaning framework. Afterwards, I will overview my model of Jordan's belief system and answer my research questions with regard to Jordan's instruction.

Background Narrative

Jordan grew up experiencing math as a challenge "from middle school till I'd say my sophomore year of college because I moved around a lot." She jumped from one topic to another, often unrelated topic in another school. As a result, she had a lot of difficulty learning mathematics. When she got to college, she started as a business major with no intention of majoring in mathematics. She had taken on this major out of being unsure what to major in, but that soon changed when she went to take a business calculus course. Stripped of some of the more complicated topics that typical calculus courses include, Jordan "could see where calculus could actually be applied ... So I decided to take a regular calculus section." This involved struggling to learn all the precalculus knowledge

that was a challenge for her growing up and eventually switching majors to mathematics. She noted, “what drew me to math is that calculus course where I could finally see math used in real-life scenarios.” In line with this thinking, she noted calculus is “like this hourglass… a bunch of foundational building blocks that kind of come to a peak in calculus and then it branches back out again” to many different branches of mathematics with the kinds of applications she liked to see.

However, her challenges with mathematics did not end with her newfound appreciation of mathematical applications and major as she then began to confront many prevalent gender stereotypes in STEM fields, a phenomenon she had been shielded from due to the supportive roles her father and past teachers had played in her education up to that point. Her father was a “computer engineer, and he kind of expected us to know about math,” and he was very supportive of Jordan academically. Jordan’s prior teachers were predominantly women. She felt they took a very equitable stance towards women in STEM, so much so that “I just was ignorant of the sexism that was in the hard sciences at that point.”

Once in college though, things changed. An emblematic moment of the kind of gender stereotyping she recalled was a comment one of her STEM professors made to the effect of “yeah, we actually have a girl in here. Don't worry; she won't be here by the end.” This was one instance among many, but they in fact spurred Jordan to push back against the stereotypes by trying harder to pursue a major in mathematics. Jordan went on to pursue graduate school, and due to work-life balance considerations (e.g., finding a program offering classes around her childcare and work responsibilities), she decided to study statistics. However, she soon came to love the major because of how she could see

the applications of theoretical mathematics. She felt seeing applications was the best way for her to learn theory, and statistics gave her that. After graduating, she pursued a career as a statistician in the corporate world.

However, she soon became disillusioned with this corporate life and started missing her time in graduate school in which she experienced what it felt like to teach mathematics as a graduate teaching assistant. As a GTA, she had anticipated working under another professor but was thrown into teaching a class on her own. Despite the struggles of receiving minimal training leading her courses, she came to look back on these teaching experiences fondly. “I really liked the helping people, the social interactions with people. Going to an isolated office experience on a computer all day from academia was kind of a shock. and so I missed that, that atmosphere.” One day during that business phase of her life, a contact from her graduate school unexpectedly reached out with an offer for a teaching position. After a bit of negotiating, Jordan decided to return to a teaching position at her previous university. As she transitioned back into a teaching profession, she initially taught large lecture-based courses for several years. Over those years, she became disturbed by the number of students she saw falling through the cracks. She noted, “If they don't show up to class, they just don't do well on the quiz. And if they dropped, a lot of times I don't even know that until I've put in grades at the end.” She began collecting her own internal numbers of students’ pass-fail rates and presented them to her sympathetic dean to shift towards smaller classes. She simultaneously began making steps towards implementing active learning into her classes and implementing more technology “in a way that will help us understand what's going on in that bigger picture” behind the concepts students needed to learn.

All the previous leads up to Jordan's current situation. In the sections that follow, I will start to discuss Jordan's instructional goals, the orientations for those goals, and answers to my research questions in regard to Jordan's instruction.

Characterization of Instructional Goals

Jordan's main instructional goals consisted of promoting student success, building understanding and skills of various sorts, and meeting departmental expectations. In the following sections, I will discuss how I grouped Jordan's various teaching goals for introductory statistics into two overarching groups: promoting student success and understanding and building confidence, comfort with statistics, and skills. There were some connections between these which will also be mentioned. Seeing as her class was a sort of hybrid between a flipped model and a traditional course, Jordan's class routine consisted of students ideally coming in having read pre-class material, Jordan lecturing a little on the topics needed for the day's group work, and then students working in groups to solve problem sets.

Student Success and Understanding. Jordan stated, "I'm really invested in student success in trying to capture those students that are failing." In keeping with Jordan's language and ideas, I named this set of goals promoting student success and understanding. As was mentioned before, Jordan was dismayed with the number of students who 'fell through the cracks' by failing out of her course. In light of this, she began trying active learning techniques and came to believe "everybody should be using active learning at this level… and get away from the traditional lecture." She adopted a flipped model because the resources she had enabled her to decide whether more time needed to be spent on understanding a topic or not by tracking how students did on online

assessments and whether they even viewed the material to participate in class. Based on the focus on understanding in the prior and her comments about using active learning and technology to "help us understand what's going on in that bigger picture," one goal I grouped under student success was addressing student failure by promoting a greater conceptual understanding. Some manifestations of greater understanding for her include: being able to problem solve using a variety of strategies (e.g., by her highlighting the different ways students could answer a problem using either a table or a decision tree), long term recall of concepts (by having questions on past concepts randomly on current classwork), and being able to summarize, describe, and interpret a data set (e.g., having students interpret the results of statistical tests out in lay terms on assignments).

Also subsumed under the goal of promoting student success were institutional definitions of and pushes towards student success. Jordan mentioned that the course fulfills the university's quantitative course requirement where "everybody that's getting a Bachelors of Science has to have a math class, so we are touching students from all different departments in our department." Students came from mostly non-mathematics majors, including health science, psychology, and business. An additional layer of complexity arose as we discussed: Jordan noted departmental jostling between the mathematics and statistics department and the other departments the course was a service course for. For the non-mathematics departments, student success meant higher grades, as demonstrated by Jordan's comment that other departments "want us to make this class easier so students pass it more." Non-mathematics departments complained students were not achieving because the content was too mathematically theoretical and thus often inapplicable to their fields and or too difficult for students to understand. In light of this,

other departments considered having their own introductory statistics course for their majors only. As part of an effort to maintain control of the course, the mathematics department arranged to cover various topics so as to make the course relevant to non-mathematics majors. This manifested in Jordan placing an emphasis on using calculator shortcuts, use of statistical software like SPSS in classwork, and content based on what students needed for future courses and their majors (which she facilitated using the online platform's monitoring abilities in conjunction with minilectures to determine what to cover or skip). For example, on her emphasis on calculator shortcuts, "our course is set up so primarily that we expect them to do a lot of shortcuts and not do necessarily a lot of the formulas" because "a lot more departments have their own statistics course where they're not concentrating on the math class. That's what we're trying to compete with now"

***In terms of the iMoM*.** The iMoM orientation is characterized as focusing on understanding and/or promoting interest in mathematical concepts themselves (see the chapter 5, 'Research Questions' section, 'Oriented Goals and Manifestations' subsection for more details). One goal I grouped under student success was addressing student failure by promoting a greater conceptual understanding, specifically through active learning. For instance, Jordan mentioned how she had trouble understanding standard deviation "just by [talking about] difference of like fat tails versus skinny tails and things like that," and continued on from there to say,

> So now when I teach, one of the first things I do is give them a bunch of different graphs and match those up with the new numeric values and do a lot of tactile kind of learning as well as visual learning with my students.

To make sure she was building students understanding through active learning, Jordan often started the beginning of class with a mini-lecture in response to viewing students' access history. She often found students had not read up on the material. Thus, she would use the minilectures to make sure students understood the definitions and notation relevant for students to engage in the day's activities. Because of the focus on understanding statistics itself by implementing active learning to the instructional decisions that enabled active learning, I would group addressing student failure by promoting a greater conceptual understanding under the iMoM orientation.

Another interesting aside was how Jordan often introduced new concepts by first referring to real-life examples and then building up towards new concepts. Jordan stated, "if I only have theory with no applications, I never really understood it very well and so, yes in some way, I'm teaching by what not to do from how I learned it … I don't think any of my students are going to be thriving on theory alone, or theory at all." Thus, she tied this to her own background of understanding things better when she could see applications of concepts as opposed to just being given theory. Given that such examples were used to introduce mathematic concepts, using examples to introduce concepts seemed more like an instructional manifestation of promoting conceptual understanding of mathematics (iMoM). However, the possibility of other goals and associated orientations will be discussed in the following sections.

In terms of the sMoM. Instructional goals focused on enculturating students in the practices of the mathematics community indicate sMoM-oriented goals (see the chapter 5, 'Research Questions' section, 'Oriented Goals and Manifestations' subsection for more details). Jordan's focus on understanding notation was also sMoM-oriented, as

evidenced by her take on why she often asked students to interpret null hypotheses: students would often have difficulty understanding what the symbols meant.

> It's just making sure that they- they really understood it, and they just weren't copying it word for word. So a lot of math, I find the students will see a similar problem and take the numbers and put it in the same spots … sometimes I'll read back these statements, and they'll have like four or five 'nots' in it, and you're like … what does that mean?

By having students voice the meaning of such statements back to her as sentences during group work, she would make sure they understood what the notation was communicating in real-life terms. Thus, there is both a component of conceptual understanding and an aspect of understanding the language of mathematicians and thus fits the iMoM and sMoM orientations.

***In terms of the iMfM*.** iMfM-oriented goals would aim to convey the relevance of mathematics separate from mathematics itself (see the chapter 5, 'Research Questions' section, 'Oriented Goals and Manifestations' subsection for more details). There was an iMfM-oriented goal clustering under student success, specifically student academic success, which is personally relevant. This is evidenced, for instance, by Jordan including old content on assignments. She noted, "I'll even bring in stuff from like three chapters back, just to throw it in the middle of something to get them to be able to do those same type of tools on the exam." While part of her focus on the student success goals overall dealt with understanding mathematics, there was an aspect of those goals that was connected to students' academic achievement.

The institutionally defined aspects of Jordan's student success goals further highlight iMfM-oriented academic success goals. The mathematics department arranged to cover various topics to appease serviced departments. This manifested in Jordan placing an emphasis on calculator shortcuts, the use of statistical software like SPSS in classwork, and content based on what students needed for future courses. In turn, students would focus on things departments wanted to focus on, e.g., SPSS output, and concentrate on if they "know how to correctly write the hypotheses and make the decision based off the p-value method, and not even have a really good concept of what a p-value is." The focus on passing exams and the course more generally by "dumbing down the math" and focusing on calculator shortcuts related to institutional prerogatives of reducing DFW rates. This focuses on individual students' academic success, which fits an iMfM orientation.

In terms of the sMfM. sMfM-oriented goals would focus on enculturating students into practices transferable to other (non-mathematics) communities (see the chapter 5, 'Research Questions' section, 'Oriented Goals and Manifestations' subsection for more details). I noted previously that this student success goal focused on promoting understanding of mathematics (iMoM); however, there was the possibility of less central sMfM-oriented goals clustering along as well. For instance, she noted, "I have about six different articles I'm going to show them that kind of use the things that we've been using throughout the quarter like where they might see a p-value in a research article." These articles highlighted studies in different fields to show how statistics looks in practice (fitting the sMfM orientation in regards to professional practice) or where statistics were

abused as a way to have students become critical of publicly circulated information (fitting an sMfM orientation of fostering informed citizenship).

The institutionally defined aspects of Jordan's student success goals further highlighted the presence of sMfM-oriented goals. The mathematics department arranged to cover various topics to make the course relevant to non-mathematics majors. This manifested in Jordan placing an emphasis on calculator shortcuts, use of statistical software like SPSS in classwork, and content based on what students needed for future courses and their majors (which she facilitated using the online platform's monitoring abilities in conjunction with her minilectures to determine what to cover or skip). For instance, Jordan noted how she would have rather avoided using SPSS, but "we are required to use that software because of the partnership that we have with the psychology and other social science departments, and they want their students to have experience on SPSS." Because of the focus on preparing students for their majors, which then supposedly enables them to function as competent members within their professional communities, this subset of derivative goals of student success fits the sMfM orientation.

Confidence, Comfort, and Skills. Jordan had mentioned that students often do not succeed, not for lack of ability, but rather because life can get in the way for many legitimate reasons. Similarly, she believed that no one was really bad at math, but rather, such students simply did not have the right opportunities to learn mathematics well. These unsuccessful attempts could then snowball into issues of lacking confidence, test anxiety, etc., which would be all the more expected in the non-mathematics majoring student population the class serves. For these reasons, Jordan often emphasized goals related to building student's personal skills and attributes such as confidence, comfort

with mathematics and statistics, and the associated struggles of learning and study skills. She noted that her goals in this regard were aimed at building confidence and tools so students have good study habits and can realize they are not bad at mathematics.

One of her goals then was to promote a classroom filled with group work and communication. By assigning students to groups based on their individualized strengths and weaknesses, Jordan had students use their strongpoints to complement their peers in completing the routine groupwork class time was spent on. Additionally, she mentioned that group work could also serve to have students work on skills they are not great at. Both goals aimed at students' personal growth (e.g., confidence). Interestingly, Jordan connected her empathy towards students who fail with her own disconnected experience of struggling to learn mathematics. Thus such an experience and the observations of student failure both may contribute to why building students up was one of Jordan's goals.

In terms of the iMfM. iMfM-oriented goals would aim to convey the relevance of mathematics separate from mathematics itself (see the chapter 5, 'Research Questions' section, 'Oriented Goals and Manifestations' subsection for more details). I viewed the confidence, comfort, and skills goal as a focus on personal development. For instance, Jordan stated, "it's the very first stat class they've ever [had] … And so just getting them used to statistics and not being afraid of it" and

> Usually, they either have poor management skills or study skills, or they just are second-guessing themselves, and they just don't have the confidence. So it's like, 'yeah, no, you're doing great. That's exactly it.' They don't feel comfortable

> struggling as well. They want that-, it's uncomfortable to not know the answer. And, but I think that is helpful for them to learn that process.

Throughout these, Jordan emphasizes promoting personal growth in terms of confidence and study habits. For this reason, I saw this focus on personal growth as iMfM-oriented. At the same time, Jordan also mentioned that she prioritized group work because "working with people is a good life skill. In a job and a career, you're going to be having to work usually with other people." Because she mentioned groupwork as pertinent to *any* job, this was classified as a personal skill (iMfM) and not a practice geared at a particular community (sMoM or sMfM).

In terms of the iMoM. The iMoM orientation is characterized as focusing on understanding and promoting interest in mathematical concepts themselves. Another reason Jordan aimed to incorporate groupwork was that she believed that students learn material better from one another. She noted, "if somebody knows it really well and the other person doesn't, the students teaching it learn it better, and the students that are learning it seem to trust the student's response more than mine." Additionally, her whole reason for shifting to a hybrid model with a lot of group work was motivated by having students succeed by gaining greater understanding. This focus on understanding ideas through groupwork then would fall under the iMoM orientation.

Analysis

In this section, I will present my model of Jordan's belief system based on the prior narrative and characterization of instructional goals. From there, I will begin

answering each of my research questions with regard to Jordan's instruction. In first describing Jordan's belief system (see Appendix B, Figure B1), It seemed as though Jordan's belief system had two clustering goals of what student success entailed. One form of her student success goals derived from Jordan's prior teaching experiences of watching students who failed and coupled with her own experiences of learning through applications. These experiences led to the goal of promoting a better understanding of statistics (iMoM-oriented) with derivative, less central goals with a sMoM orientation (by focusing on notation) and iMfM and sMfM orientations (through real-life examples). On the other hand, the institutionally defined goal of student success derived from departments the course served and the university. This version of student success goals then led to derivative goals of prioritizing content relevant to student majors, exposure to statistical software that Jordan would have rather avoided, and test performance (which then had derivative goals of showing calculator shortcuts). This second branch of student success fits the iMfM and sMfM orientations as mentioned earlier. In regard to her goals on student confidence, comfort, and skills, it seems Jordan's own experience as a learner of mathematics and her concern for students who fell through the cracks led to an understanding that students do not succeed because life can get in the way and/or students may not have had the right opportunities and habits to learn mathematics. Her first priority then was to foster a greater understanding (iMoM) through groupwork. Fostering groupwork then clustered with two iMfM-oriented goals: 1) a focus on personal development and 2) being better equipped for work and life generally. These latter iMfM goals seemed to be clustering or derivative goals of groupwork. Promoting understanding (iMoM) seems to be the antecedent to groupwork (since student success vis-à-vis

understanding was the driving force behind her whole shift to a hybrid learning format). In this way, an iMoM-oriented goal then seems to have associated derivative goals associated with the iMfM (personal skills).

The first research question asked: what, in terms of background, affective factors, etc., leads instructors to prioritize certain meanings over other meanings? One form of Jordan's student success goals derived from Jordan's prior teaching experiences of watching students who failed and coupled with her own experiences of learning through applications. These experiences led to her to take actions signaling a central iMoM-oriented goal (understanding focus through active learning) and less central, derivative goals with orientations fitting the sMoM (notation focus in minilectures), iMfM (applied examples in daily life), and sMfM (applied professional and citizenship examples).

The second research question asked: How do instructors' orientations of mathematical meaning translate into specific pedagogical decisions? Jordan acted on her student success-conceptual understanding goal (iMoM) by implementing a hybrid instructional format. The hybrid format required students to work in groups to complete class time activities. In turn, Jordan needed to make sure students knew enough notation and content knowledge (iMoM, sMoM) to be able to engage. So she would do minilectures at the beginning of class to go over notation, concepts, and definitions, some of which she introduced through real-life examples (subsequently hitting iMfM and sMfM goals of personal-, professional-, or citizenship-related relevance). Outside of fostering conceptual understanding, group work was also a manifestation of Jordan's confidence, comfort, and skills goal (iMfM). Jordan acted on her institutionally defined student success goals (iMfM, sMfM) by bringing in articles and applied examples of

statistics, having students work with SPSS, using the online learning platform and minilectures to keep up with content coverage expectations, highlighting when things were important for exams, and sometimes ‘dumbing down’ the mathematics in favor of calculator shortcuts.

The third research question asked: how does context influence the meanings instructors emphasize? Part of how Jordan’s student success goal manifested went back to her student population, which she knew was not comfortable with mathematics. She knew they would have trouble understanding notation and mathematical concepts abstractly and often had confidence issues in approaching mathematics. Thus with her more central iMoM-oriented goal of understanding, she took actions signaling less central, derivative goals with orientations fitting the sMoM (notation focus in minilectures), iMfM (introducing topics with everyday examples, using group work to build skills), and sMfM (citing applied examples in professions/society). There was also an aspect to Jordan’s student success goals influenced by departments the course served and the university. This version of student success goals manifested through actions signaling derivative goals with orientations fitting the sMfM (prioritizing material according to majors, exposure to SPSS software Jordan would rather avoid) and iMfM (academic success/test performance through calculator shortcuts and ‘dumbing down the mathematics’). What is notable here is that the departmental push towards student success (iMfM, sMfM) served as a detractor from Jordan’s student success-understanding goals (iMoM). Despite this, Jordan made concessions both ways to hit both goals (e.g., emphasizing calculator shortcuts during class, but at the same time, pushing students to interpret their results to clarify their understanding).

The fourth research question asked: How do backgrounds, contexts, and orientations come together in pedagogical decisions with respect to how instructors respond to the students' meaningfulness questions (e.g., "when is this ever going to be useful?", "why are we learning this?"). The following except introduced my conversation on this topic with Jordan.

> Me: how would you respond in this class, if someone asked you, 'when is this stuff ever gonna be useful?'
>
> Jordan: I mean I think with, you know, each little tiny equation I can always-, not always, but I can usually come up with a very specific example of where it might be useful.

Finding personal relevance in decision making (iMfM), and more broadly as ways to engage in future professions and as informed citizens (both sMfM) were Jordan's responses to how she would answer student questions for meaningfulness. Continuing on from the last except, she stated,

> but I think in the broader scheme at the end of the class, I always, on the last day, emphasize to them that they may not use this specific statistic in their life going forward, but with the comfort of visually looking at graphs and looking at data that we hope that they become more educated consumers and not get fooled by statistics. And that's my goal, is that they can understand that that statistic was manipulated by a politician. Or if it was an ad from, you know, from some type of company … So I try to bring in articles where those statistics are used.

She had taken measures during her instruction to meet those forms of meaning, primarily though showing where the concepts covered during class can show up in research, society, and daily life. However, Jordan's main goals centered on promoting understanding and student success (iMoM, iMfM), an interesting difference from her prior experiences as a learner where the iMfM and sMfM orientations appealed to her (in the form of being able to see applications in personal or professional life). Her focus on these arose because of departmental influences and her own background experiences of seeing students fall through the cracks.

Archy's Narrative, Instructional Goals, and Analysis

In the following sections, I will review Archy's background in a narrative format. Next, I will describe Archy's instructional goals and how those goals may be characterized using my meaning framework. Afterwards, I will overview my model of Archy's belief system and answer my research questions with regard to Archy's instruction.

Background Narrative

Archy's experiences with mathematics in school could be characterized as favorable, to say the least. Archy identifies as a 'math person' and ties it back to his school and his family experiences. He stated, "I don't think I've ever viewed myself as having been born a math person. I think I view it as the-, my math personiness as a consequence of my upbringing." He had a family culture that valued math: his mother was a computer scientist, and his brother was also a 'math person', often speaking with

Archy about math. Archy had shown promise in school from a young age, particularly in mathematics, where he did the best with the least amount of effort (compared to other school subjects). In turn, he was placed in more advanced mathematics courses and received quite a bit of praise from those around him, like his family and teachers. Because of this image, he even mentioned how he would be bothered if he did not live up to expectations, like an instance where his mom "was not willing to teach me how to do long division. She was kinda like you need to be able to figure this out" and he subsequently felt like he was letting her down for not understanding it. Archy's successes in mathematics also fostered a means for him to connect with others in making friends. He noted, "like how I even like made any friends in school was because they were trying to do math, … and I was like, well I know how to do that like, somewhat, or I'm happy to help you." These interactions of peer tutoring then served as the foundation for friendships. Together, all these prior school experiences led Archy to major in mathematics as it seemed like a natural choice for him when he went to college.

Archy' personal views towards mathematics when he was first entering college can be summed up in his own words when he stated, "I developed a perspective of math as being so pure that it was- I'd idealized it at that point." He believed other subjects fell behind it in terms of grasping at **T**ruth and that "people will find you employable because I had at least convinced myself that like, you can do anything with math." In a similar vein, Archy also mentioned that by studying math, one practices critical thinking, which helps one better navigate their own life (a view he still holds). As he went on though, things began to change when he began to see the value in other subjects and his own self-described shift away from idealism to pragmatism. In regard to other subjects for

instance, “I've certainly had flashes of like... I was not paying attention to this prior, and now I am because now I realize it's, it's been important all along I just haven't been looking at it”. He started to notice many injustices and inequalities in our society by watching current events, associating with activist friends, and realizing his own responsibility to use the privileges our society afforded him to do good. His resulting pragmatist world view led to re-evaluating math, not as the Truth, but as a way of upliftment in our society, as demonstrated by his comment that “what would be the best thing to do if you were like poor and trying to get out of your situation, like study some math …that's my pragmatism like shining stronger than my idealism.”

This understanding carried for into Archy’s motivations in teaching. Specifically, Archy noted the status our society places on STEM majors, stating,

> In a different society, history could be the most important thing, and that would be the thing that I would be like, dude, you just need to go study history …now, you get the benefit of the doubt if you studied math. So you should study math because you need to take care of yourself and like, you need to improve your station.

He connected this to his own experience of seeing how studying math afforded him economic opportunities in our society. Archy acknowledged that he still wants people to see the beauty and fun of math for its own sake (due to his own experiences with mathematics), but his more pragmatist stance now is to not have people suffer by improving their socioeconomic status in a society that values mathematics. He felt he could contribute to that upliftment by teaching mathematics.

All the previous leads up to Archy's current situation. In the sections that follow, I will start to discuss Archy's instructional goals, the orientations for those goals, and answers to my research questions in regard to his instruction.

Characterization of Instructional Goals

I grouped Archy's various teaching goals in mathematics for liberal arts into three overarching groups: personal relevance, societal relevance, and mathematical fun. I will delve further into what each of these meant for Archy in the following discussion on Archy's teaching goals. What is important to note here is that Archy himself mentioned the three ways mathematics is relevant and how his goals were tailored to those, stating, "the way I'm imagining structuring it [the course] is like kind of in, in three bits." He mentioned additional goals during our conversations that I then grouped with the three course goals if I felt there were relevant connections. His class routine consisted of introducing the initial concepts and definitions needed for the topic of the day through notes sheets, connecting those basics to the main topic through a few examples (worked either by him lecturing or students in groups and then as a class), and then going into a stretch problem (time permitting).

Personal Relevance. In regard to the first group of goals, Archy stated, "one section is just like the literally, this pertains to you cuz this is how taxes work, or this is how finances work" This group of Archy's goals dealt with having students see the personal relevance of mathematics. One way this took form was him devoting nearly a third of the course content to things he included in this category, primarily personal finance. Here, students covered things like percent change and interest. Regarding the latter, Archy focused on showing the different ways interest can be calculated and drew

ties more broadly to how credit cards, loans, and other interest-like financial matters students would bring up work (e.g., Roth IRAs and stocks). A natural consequence of this content taking a third of the course was Archy giving notes, homework, and exam questions and spending class time to introduce, talk about, and work out examples on these topics. One of the homework assignments that Archy considered in our first interview was having his students track their spending for a week to get an idea of how they manage their finances. He felt this was really personal to each individual student.

An aside that should be addressed is Archy's focus on student involvement and contribution. Archy noted that most of his students were female, and many were students of color. He noted,

> That class is like, I don't know, like 80% female, just from looking. I don't know how they identify. So I mean I've like made that recognition. And like of the men in there, like more than half of them, I think are people of color. And so like I think the main thing that's gonna affect my presentation is like having an expectation, again just like running off a probability, that a lot of these people … that their experiences in math classes have probably involved some like, what's it called, like diminishing experiences.

Due to the unfortunate realities of our society, Archy believed his students were mostly non-mathematics majors who probably took the course because mathematics was one of their least favorite subjects, yet they needed the course as part of university requirements. He believed these students likely had negative experiences learning mathematics from teachers who they could not relate to. In turn, he stated, "I'm trying to be a compassionate ambassador of math" by being affirming of students and making space for them to

contribute ideas and contradict him. This manifested in his seeking student involvement as involvement during discussions could allow students to voice their ideas and be heard. However, he mentioned that student involvement would be something he would strive for in any class. As a result of this generality to all classes and being more directed at class facilitation, aiming for student involvement was a general goal supporting all other goals. His goals of being a compassionate ambassador and involving students brought up interesting scenarios in class however when students brought up real-world situations in which Archy could not answer due to a lack of knowledge. One of Archy's concerns was that he did not have many resources to draw from to find applications. This issue will show up in his goals on societal relevance as well.

In terms of the iMfM. iMfM-oriented goals would aim to convey the relevance of mathematics separate from mathematics itself (see the chapter 5, 'Research Questions' section, 'Oriented Goals and Manifestations' subsection for more details). I grouped Archy's personal relevance goals generally under the iMfM orientation because he characterized the goals as directly impactful on students' lives. In reference to this set of goals for instance, Archy stated mathematics is "gonna be just like directly impactive upon what you are doing because like you have a job and you need to know like what fraction of the people will show up late or something to make a decision." He gave other examples, for example knowing how to save if one anticipates facing unemployment at some point. This idea of relevance to students individually, without reference to understanding mathematics or functioning within different professional communities fits the iMfM orientation.

One interesting case of personal relevance appeared alongside a societal relevance goal. Archy had students engage in a data collection project because “I just want people to think constantly when people tell them stuff. How do you know that? I mean that just would affect everybody's life on a daily basis if they did that more.” While he mentioned this critical thinking was societally relevant, he held it to be personally relevant because by helping individual students navigate society, it was also helping them in their personal lives. Based on this line of reasoning, the goal of fostering critical thinking of publicly circulated information was iMfM-oriented (but also sMfM-oriented, as will be discussed below)

I classified Archy’s compassionate ambassador and student involvement goals as non-oriented. For instance, when asked whether making space for student involvement and explaining their thinking was related to his student population, Archy stated, “I think it is. I don't think it's specific to that though. It's like any class benefits.” As a result of this generality, Archy’s compassionate ambassador and student involvement goals were directed at class facilitation of all his other goals. They thus were not oriented towards any specific meaning orientations. Perhaps related to this involvement goal however, Archy wanted his students to have “greater confidence in dealing with numbers in their life, whether they be finance or just otherwise. So yeah, so greater comfort and confidence” at the end of the semester. This was a personal skill-related goal and thus fit the iMfM orientation. My reasons for drawing this connection between involvement and confidence goals are that Archy may have hoped his students would grow comfortable with numbers by getting his students involved, which subsequently has students dealing with numbers and explaining their thinking. Considering the negative experiences he

suspected his students were used to and the way he wanted to push against that by involving students and being a compassionate ambassador, it would not be unreasonable that this goal of confidence and comfort with numbers (iMfM) related to his student demographics and clustered with his compassionate ambassador and student involvement goals (non-oriented). The first quote in this paragraph suggests such a connection as well.

In terms of the sMfM. sMfM-oriented goals would focus on enculturating students into practices transferable to other (non-mathematics) communities (see the chapter 5, 'Research Questions' section, 'Oriented Goals and Manifestations' subsection for more details). As I mentioned previously, Archy had students engage in a data collection project with the intention of making them critical of statements circulated around public media. When asked, he confirmed that this was personal for the reasons mentioned above, but "societally as well. Like people on the radio are like, 'you need to go do this because this is how this works.' It's like, 'how do you know that?'... that's both personal and societal." This statement focuses on getting students to critically assess publicly circulated information and thus function as critical members of society. Based on this line of reasoning, the goal of fostering critical thinking of publicly circulated information was sMfM-oriented (in addition to iMfM-oriented, as mentioned above).

Societal Relevance. One of the goals Archy had noted was how mathematics could be relevant on a societal level, specifically on how "math is incorporated in society. You're reading a news article and it's telling you this and it's influencing your opinion. Or like this is how voting works." He viewed mathematics as societally relevant when mathematics is used as a tool to evaluate what is said around our society or how aspects of our society worked. He devoted a third of the course towards such material. Topics

Archy included in the societal relevance sections consisted of voting and data analysis. Archy gave notes, homework, and exam questions on these topics and spent class time introducing, talking about, and working out examples.

With regard to voting, Archy wanted students to get an idea of the different kinds of voting methods that exist and how methods compare in state and federal elections. For instance, he had a class discussion on "what are the pros and cons of this voting method." During discussions, he wanted to maintain an open dialogue, again, partly due to the negative prior experiences he suspected his students endured but also because he had more freedom with the course content. There were no department-mandated topics to cover, so he had more freedom to hold such discussions. He stated, "my concern is that like I don't have any material for this course and that like I'm just making it all up." Pushing the concern aside for the moment, this open space allowed students to ask about, and students from underserved backgrounds to challenge and criticize, the problems they perceived in our current political system. For instance, Archy allowed space for students to bring up gerrymandering, in which "a women was like, 'it's a really [expletive] racist system that keeps people down… She was a woman of color who said that, and so I wanted to ensure that space was held for that thought."

The section on data collection and analysis aimed at examine the claims of studies and surveys. Archy wanted to foster critical thinking with regard to data collection and methods through a project of determining how many books in the school library were red. Archy stated,

> I think people will get an idea that it's like hard to answer a question. And if you can come away [like], 'wow, research is hard and like we should be skeptical

> when people like give us information and we should like check their methods and stuff like that.' I'm like, that's a success.

The class devoted a few weeks to this section and project, using class time to start exploring the problem, conjecturing data collection methods, and physically going to the library to carry out a determined sampling technique. Archy's aim was to give students insight into the decision-making process and logistical complications behind seemingly simple tasks and then connect that to more significant cases when people make claims publicly. In this regard, He stated, "people on the radio are like, 'you need to go do this because this is how this works.' it's like, how do you know that?" If students could see all the complications behind determining answers to simple questions, they would have all the more reason to be critical of statements made by political figures, studies, and other beacons of power in society.

The lack of departmental resources however, left Archy in a bind with regard to both societal relevance goals. Archy stated, "nobody has a Google Drive full of like a bunch of activities … It's just like totally embarrassing to me that there's not just like-, somebody has not organized all this stuff." Students brought up real-world situations Archy was unfamiliar with due to his own human limitations on knowing things. On this limitation, He noted,

> I've never done anything useful. So, and what I mean by that is like applied. So the aesthetic side of it [math] is the most important thing to me. So I'm sure that, that is my interest and focus. And I wish I had more examples ... Like if I had more examples, like on my head about stuff, that would be better.

Regarding the data collection and statistics section, the project's simplicity was a source of concern. Archy noted, “it is a contrived scenario, no doubt about it. And it's like, I'm sorry. It was like semi-realistic.” Archy was concerned that if students saw the task as irrelevant, they may be less inclined to buy into the project and remain apathetic (a concern he did see realized to some extent as students engaged in the task). This in some ways was only furthered by his concern for students’ negative experience and him wanting to give students a voice in the class. For instance, a student began questioning the premise of the project, on which Archy commented, “I wanted to just accept that we had to do this task and let's figure out how to do the task and she wanted to be like, let's question why we're doing the task in the first place.” By giving them a voice, students could (and did) question the entire project itself, thus challenging the societal relevance goal of the task.

The lack of departmental resources meant Archy had little scaffolding in attending to his societal goals, and thus he felt he could have improved from a facilitation standpoint. He noted many issues that arose with the project as it was being undertaken from student apathy, logistical challenges with data collection (students not collecting enough data or in inconsistent sampling methods), to better facilitation of the class reflections to wrap up (“I didn't have a good plan for what to say”). As a result of all these issues, he noted, “I have developed a concern that people are just not gonna believe anything that they hear. And like, it's like, no, no, no, that's not the point.” While his goal may have been to make students critically informed citizens, he had not wanted them to fall into that kind of cynicism of society. While Archy had a resource to draw the data collection project from, that resource did not come from the university, and he had to

adjust the project to fit the time he had. Thus, Archy had to go outside his institution to find resources to attend to this critical citizenship goal.

In terms of the sMfM. sMfM-oriented goals would focus on enculturating students into practices transferable to other (non-mathematics) communities. I viewed Archy's societal goals as largely sMfM-oriented because of how such goals were directed at critically evaluating society. This was noted numerous times above, such as when Archy mentioned, "we should be skeptical when people like give us information and we should like check their methods" in reference to the data collection project. Because of the focus on helping Archy's students understand our political system, the goals behind the voting section fit with the idea of having students learn how to participate in a non-mathematics community (in this case, democratic society) and thus fit the sMfM orientation.

In terms of the iMfM. iMfM-oriented goals would aim to convey the relevance of mathematics separate from mathematics itself (see the chapter 5, 'Research Questions' section, 'Oriented Goals and Manifestations' subsection for more details). I previously noted how personal relevance appeared alongside societal relevance in data collection. Archy had students engage in a data collection project because "I just want people to think constantly when people tell them stuff. How do you know that? I mean that just would affect everybody's life on a daily basis if they did that more... that's both personal and societal" This is a boundary example of something that impacts a person's immediate quality of life but also enables them to function in a democratic society. Because of the former and Archy's comments, these societal goals could also have iMfM-oriented goals subsumed under them.

Mathematical Fun. Tied to his own enjoyment of mathematics for its own sake, Archy's third goal for the course was to have students see mathematics as fun in and of itself. For instance, he stated, "the third thing I'm thinking is like, math is just fun … like the geometry of stuff is interesting. And so like, sequences are cool" and "there's just a recreational side that pertains directly to your quality of life because you don't want your life to be boring and this is an easy way to make it more…interesting." It was hard to pinpoint what exactly made mathematics fun, but for Archy, part of that meant having students see how connections can be made across mathematical topics and following the reasoning process. For instance, he had a goal to talk about arithmetic sequences, then their partial sums, and then "relating all problems that surround the sequence of partial sums coming from an arithmetic sequence, … to this one problem [sum of the 1^{st} n numbers], so there's like this sweet connection." Archy devoted a third of the course content to mathematics that Archy just found fun, like geometric constructions and sequences and series. As with the previous two goals, Archy gave notes, homework, and exam questions on these topics and spent time to introduce, talk about, and work out examples. On an exam for instance, he had a question on arithmetic sequences where,

> I picked B_{13} cuz I was like, well, you could just like jam it out if you needed to, or you could be like, 'I don't want to go all the way to B_{13}. I'm gonna find an explicit formula and plug in 13' ... somebody's gonna not want to add 5 to something 13 times. And there's gonna be like, Oh I just need to do five times thirteen and add it to seven.

Archy had devoted a third of the course content to these topics but was only able to touch sequences and series because of falling behind schedule, and even then, he was not able

to touch on everything he had hoped. After relating to the sum of the 1st n numbers, he hoped to build up to talking about the properties of the Koch snowflake. However, he could not do so because students took longer than he had anticipated getting the basics of sequence and series.

In terms of the iMOM. The iMoM orientation is characterized as focusing on understanding and/or promoting interest in mathematical concepts themselves (see the chapter 5, 'Research Questions' section, 'Oriented Goals and Manifestations' subsection for more details). There were moments where Archy seemed to characterize mathematical fun in ways that fit the iMoM-oriented. For instance, with his trajectory through sequences and series, "I wanted to be like, cool, not only do you know the formula, you know where the formula came from." This certainly fits the idea of understanding how ideas develop and relate to one another, thus focusing on mathematical concepts and ideas themselves.

In terms of the sMOM. Instructional goals focused on enculturating students in the practices of the mathematics community indicate sMoM-oriented goals (see the chapter 5, 'Research Questions' section, 'Oriented Goals and Manifestations' subsection for more details). I also attributed a sMoM orientation to some aspects of Archy's mathematical fun goals. This was because of Archy's prior comment about "relating all problems that surround the sequence of partial sums … so there's like this sweet connection." He later went on to say, "that seems like a cool demonstration of how solving one problem in math like generalizes to the other problems and … here's like a theme in math." This sounds like generalizing, as discussed by Rasmussen, Zandieh,

King, and Teppo (2005). In regard to these same goals, he framed the goals around 'doing' mathematics.

> Math is fun. And so you can just enjoy it the way you enjoy playing basketball. It's just like, basketball's meaningless too, but it's fun. So like, of course you should do it. Coloring a map is fun because it's like a challenge and you got to do it.

Because the focus here is on having individuals *doing* mathematics and finding enjoyment in that activity for its own sake, these goals seemed to center around engaging in authentic mathematical practices. As such, I considered the goals he stated under mathematical fun as sMoM-oriented.

Analysis

In this section, I will present my model of Archy's belief system based on the prior narrative and characterization of instructional goals. From there, I will begin answering each of my research questions with regard to Archy's instruction. In first describing Archy's belief system (see Appendix B, Figure B2), Archy's personal relevance goals related to how mathematics could be used for an individual's life (iMfM). Some of these were direct connections Archy made in how mathematics can be relevant personally, like finding out interest for car loans and managing finances. Others were related to building up students' personal skills (e.g., building confidence in dealing with numbers), which could relate to the specifics of the students he was dealing with, though he had not explicitly stated so. Archy's beliefs about his student population led to his derivative goal of being a compassionate ambassador of mathematics by listening to

students and giving them space to be involved. The fact that pauses and uncertainty often preceded some skill goals before coming to a justification (which was not the case when speaking of ambassador goals) may suggest that skill goals had a lower psychological strength than the ambassador goals. In that way, being a compassionate ambassador and fostering personal skills seemed like separate goals of different psychological strength in his belief system, though they may partly derive from the same antecedent belief about his students. Both goals seemed separate from his goal of showing direct mathematics applications in one's personal life. The applications to personal life seemed to cluster with Archy's statements of personal upliftment by studying mathematics. Across these goals then, the common thread of using mathematics as an opportunity to focus on non-mathematical topics relevant to individuals fits the iMfM orientation.

I saw the instances of societal relevance as sMfM-oriented because of the focus on providing students with experiences that help them function as critical members of society. However, the concerns of student buy-in did seem to connect with statements Archy made of trying to make the class fun on a personal level. Additionally, Archy had mentioned how learning to navigate society also helps individuals in their personal lives. Both these suggest iMfM-oriented goals of personal enjoyment and relevance may have also clustered with the sMfM-oriented goals of making critically informed citizens. As a separate strand of beliefs deriving from his beliefs about his students and the circumstances of having fewer departmental constraints, Archy's focus on giving voice to students meant that both the sMfM- and iMfM-oriented goals could be challenged by students questioning the relevance of tasks or bringing up circumstances Archy did not have enough knowledge to comment on.

Archy characterized his goal of mathematics as fun in and of itself by the fact that such fun comes from tying concepts together and following reasoning. A lot of what showed up in this group of goals focused on acts of *doing* mathematics and on the *process* of understanding connections and thus fell under the sMoM orientation. This goal connects well with Archy's personal enjoyment of mathematics growing up and thus may be why the goal had the psychological strength to manifest in our first interview (as opposed to being voiced only after some video clips highlighted the goal).

In any of the three sections, Archy noted, "I want to like get some basic facts about a thing. Then I want like a broader idea to think about, and then I want like a stretch problem or a stretch." He further explained this three-step progression as first building up the initial concepts needed for the topic, only going over the basic mathematical vocabulary necessary to understand the material for a given section. He would then connect the basics to the main topic, where he made a point to try to make sense of why things were what they were mathematically, such as when he made a connection between building interest and tying that progression to sequences. Another instance of this was when he would routinely show how terms could be factored in calculating interest plus principle to show why a 1 shows up in the formula: $A=P(1+rt)$. Last in his lesson progression, Archy would plan a stretch problem that he knew he might not be able to cover. This flexibility with the schedule was because students could get stuck on notation, conventions, and basic concepts that are part of the initial foundation, e.g., when students got stuck on notation and thus could not discuss the Koch snowflake stretch problem or

> Like where we were supposed to be talking about the derivative of a line, now it's just like, okay no, if I give you an equation, why is that equation look like a picture? And that is the now, that's the new lesson.

The need for flexibility in light of possible confusions may be tied to Archy's student demographics, particularly the fact that most of his students were non-mathematics majors who did not like mathematics and experienced discomfort with the subject. Archy noted, "we need to talk about notation and talk about how math people like make up notation so they don't have to speak so much." This focus on understanding how mathematicians communicate suggests a sMoM-orient to the notation-related aspects of Archy's lesson planning and facilitation goals. Understanding basic concepts and definitions focuses on understanding the *connections and concepts* themselves and thus fits the iMoM orientation. This use of foundations for all Archy's content goals suggests the goal of building up basic understanding (sMoM- and iMoM-oriented) clustered with all other meaning-oriented goals.

The first research question asked: what, in terms of background, affective factors, etc., leads instructors to prioritize certain meanings over other meanings? Archy's mathematical fun goals derived from Archy's childhood experiences leading up to his view of mathematics as an ideal. While he no longer espouses such beliefs, his appreciation for math still holds and manifested through focusing on sMoM-oriented goals (devoted purely mathematical sections), and perhaps to a lesser extent in sMoM- and iMoM-oriented goals (lesson planning to make sense of basic concepts and notation). However, this background in mathematics led to teaching self-efficacy concerns in that Archy felt like he did not have enough background knowledge or preparation to address

his MfM-oriented goals. Archy's background of making a pragmatist shift due to societal inequalities manifested through his goals focusing on the sMfM (promoting critical citizenship as societal relevance) and iMfM (finances and economic upliftment as personal relevance).

The second research question asked: How do instructors' orientations of mathematical meaning translate into specific pedagogical decisions? Archy acted on his personal relevance (iMfM), societal relevance (iMfM,sMfM), and mathematical fun goals (sMoM) by devoting sections of course content to each of these three goals. All of these goals required students to first understand the basic mathematical foundations of a topic, such as notation (sMoM) and why the mathematics makes sense (iMoM), and so Archy would talk about those basic foundations through the notes and devoted class time on lecture and group work activities.

The third research question asked: how does context influence the meanings instructors emphasize? Student demographics associated with those taking the course seemed to suggest that the initial focus on building foundational mathematics for a section could be an area of trouble if students had trouble moving past that foundation. What is notable here is that the focus on understanding foundations (iMoM, sMoM) could then hinder Archy' main instructional goals from being fully realized, such as when Archy's larger math as fun goal goals (e.g., talking about the Koch snowflake, sMoM- and iMoM-oriented) had to be cut out of the schedule. Additionally, student demographics also played into Archy's compassionate ambassador goals (unoriented) in that his focus on giving students a voice let them challenge the tasks Archy posed for each oriented goal (e.g., students challenging the data collection task). In such cases,

Archy either cut material because of the time he devoted to open discussion or pressed on with a task (e.g., continuing with the data collection project). At the same time, student demographics, that allowance of student's voice, and the inequalities of our society more largely could sometimes bolster Archy's oriented goals (e.g., when underserved students were able to comment on issues of gerrymandering, thus furthering Archy' sMfM-oriented goals of critical citizenship). Departmental influences also played a role in that the lack of support regarding resources meant Archy felt unprepared to meet his oriented goals, particularly his societal and personal relevance goals (iMfM, sMfM). This compounded with his background, leading him to feel chronically unprepared and search for his own resources to meet his MfM-oriented goals. Archy also noted content coverage concerns (iMfM) resulting from this under preparations, specifically that others in the department may not look favorably on his course compared to another course where an instructor can cover more material.

The fourth research question asked: How do backgrounds, contexts, and orientations come together in pedagogical decisions with respect to how instructors respond to the students' meaningfulness questions (e.g., "when is this ever going to be useful?", "why are we learning this?"). I posed the following question to introduce this conversation with Archy,

> So you started kind of getting into this when you started talking about like, you know, bringing people up and improving their situation, but in this particular course, how would you respond if someone asks you 'When is this stuff ever gonna be useful?' You kinda already started getting into it.

Archy noted that mathematics is important on three points: it's useful for its own sake, to society, and on a personal level (e.g., personal upliftment). In response to my question above, he stated,

> This is my working theory, is that like, everything we're doing in math is gonna sort out in one of these three ways. So it's gonna be just like directly impactive upon what you are doing…and now I'm like, society also directly impacts you and society's using math. And then it's like, well look, if you want to distance yourself from society, … you want to have some fun, so like math is fun.

What stood out to me then was that these already started to sound like the iMoM or sMoM, sMfM, and iMfM, respectively. Archy went on to structure the course goals around these three points, specifically through the selection of topics to cover that fell under mathematics' relevance for its own sake (math as fun, sMoM- and iMoM-oriented), to society (sMfM, slight iMfM clustering), and on a personal level (iMfM). However, Archy noted many concerns around his course goals deriving from the lack of departmental support and resources and his own background in mathematics precluding resources to attend to his sMfM- and iMfM-oriented goals. His priorities of being a compassionate ambassador (informed by his realizations of an unequal society) could help his sMfM- and iMfM- oriented goals when underserved students could further critical discussions. However, that same criticality could also mean students could challenge the tasks themselves, which only compounded Archy's teaching self-efficacy concerns from being unprepared.

Natalie's Narrative, Instructional Goals, and Analysis

In the following sections, I will review Natalie's background in a narrative format. Next, I will describe Natalie's instructional goals and how those goals may be characterized using my meaning framework. Afterwards, I will overview my model of Natalie's belief system and answer my research questions with regard to Natalie's instruction.

Background Narrative

Natalie came from a family culture rooted in mathematics, but despite this, she described having a "love-hate relationship" with mathematics. For instance, Natalie recalled fond memories of how "I used to line up my dolls and teach them math." At the same time, she would go and create unreasonably long math tests for her parents. She stated, "I'd put like 50 addition problems and tell them they have 12 seconds to do it, and then they'd say, 'that's not possible.' I'd say, 'yes it is! That's what make us do at school." In middle school, Natalie had tied for best performance in the state on a state exam. The student she had tied with was also from her school and was known to stick out from the rest of the students. Natalie did not know if it was being associated with that student, being labeled as good at math, or something else, but "my mom remembers me being horrified to have to go up and get this math award." She recalled other negative experiences, such as when she had received low grades in algebra, not because she could not do the math, but rather because she did not state 'let x=…' in her solutions. Another negative recollection was her experience of making up a test outside her trailer classroom in the cold because she had gotten sick and missed the scheduled day for the test. In terms of other positive experiences, Natalie recalled how a mentor and older brother figure

recommended her for a tutoring job, to which she stated, “I loved it, I loved teaching math.”

When she went off to college, Natalie had no intention of becoming a math major, despite her academic success in the subject. However, numerous obligations and whom I will call supportively pushy people, led Natalie down a chain of happenstance events leading her to major in math, become a teacher, and now work as a teacher educator. First, she was pushed to take the mathematics placement exam by the mathematics department head when Natalie went to accompany a friend taking the exam. “She like made me take the placement test. I missed the placement test; I had to take a special one.” The department head then encouraged her to enroll in multivariable calculus due to her good performance. Taking this course also fit in with Natalie's promise to her older brother figure to take multivariable calculus and linear algebra. She went on to take linear algebra to keep her promise, but that was where things began to change. There, she met her now-husband, and they studied for the class together. The teacher for that course left a notable impact on her. “I had this just amazing math teacher who taught us linear algebra like all the algebra we learned up to then was a special case of this bigger thing. And it was just a fun, fantastic class, and after then, I was kind of in love with my math classes.” This shift to seeing the bigger picture was where Natalie attributed a shift from a love-hate relationship with mathematics to a pure love relationship. As a related aside, while she first cared for mathematics applications in undergrad, she gradually shifted her interest into just pure mathematics and enjoyed it for its own sake in graduate school.

After undergrad, Natalie sought a teaching credential to help her with tutoring clients and was quickly encouraged to apply to her graduate program. She noted that

those at the teacher education department she contacted about credentialling “were like, 'you should start our program. It starts in a month. It doesn't matter that you missed the application deadline and didn't take the test’… And they just kind of made things happen.” As part of that happenstance, the program included an emergency teaching position due to a shortage of teachers in her state. She ended up teaching in underserved communities, where she began to see the applied value of active learning but also the importance of just caring for students. She recalled very capable individuals who were removed from her purview due to behavioral issues in other parts of their school life. There was also a student who had a complicated home life who just wanted to be near Natalie because “she once told me I was the older sister she always wanted.” It was also common for students who would be otherwise out doing questionable things to just hang out in her class, even if she was not their teacher.

She loved her job, and all this was not lost on Natalie’s school principal, whom I will call another pushy individual. He suggested Natalie pursue a PhD in mathematics, stating, “you're gonna change math education in this country, and I think you should think about getting a degree in math,” because he believed that people listen more to the people with PhDs in math. After some tangents around that suggestion in which she planned (but abandoned) to teach math and art students to foster interest and appreciation in both, Natalie went to graduate school while intending to keep teaching public school. However, “I start getting offered these research opportunities here and … all of a sudden, I was no longer a middle school teacher; I was now a professor” teaching preservice mathematics teachers. While she had not intended to teach preservice teachers, she came to see the difference she could make in their lives too by building their confidence and

changing their relationship to mathematics. She stated, “I am starting to see like the difference I am making in these teachers that…they came in hating math, thinking they're bad at it. Now they're excited about it. They see how creative it is.”

While Natalie came to love mathematics for its own sake eventually, she always loved teaching it, a trait that carried through from her childhood up to her time teaching. She mentioned wanting to initially be a psychiatrist as a kid, stating,

> One of the reasons I wanted to be a child psychologist was to be able to help people be more confident, and I realized that people have this picture of math as hard and that when people do-, can-, tend to either get demoralized by it or they can get boosted up by it, and it's a great way to help people feel confident and change their opinion [of themselves].

Teaching would then be her way to address both concerns and hit her lifelong love. Her experiences teaching in underserved communities highlighted for her the gatekeeper role mathematics plays in our society (“my teaching credential was focusing on title one schools, low-income kids, English language learners. And so that gatekeeper was really important”), how important active learning is in reconstructing mathematical knowledge, and how teaching is about caring for students. These would play into Natalie’s goals for her mathematics for teachers course.

All the previous leads up to Natalie’s current situation. In the sections that follow, I will start to discuss Natalie’s instructional goals, the orientations for those goals, and answers to my research questions in regard to her instruction.

Characterization of Instructional Goals

Natalie talked about how her overarching priority for the class was to prepare preservice teachers to teach mathematics and entailed numerous subgoals within it. I grouped Natalie's goals into the following groups: mathematical knowledge and consideration for teaching, personal development, and showing a new kind of classroom. Her class routine consisted of going over highlights from prior classes and assessments, moving onto the main material of the day through a progression of problems facilitated by having students ponder individually, then in their groups, then as a class, and then back in their groups to make sense of the different approaches. Class often ended with Natalie making connections between the topics discussed and the common core standards.

Mathematical knowledge and Consideration for Teaching. Natalie emphasized one of the main goals for this course was that "the whole point is to teach them not math but mathematical knowledge for teaching." Given her research background, it was not surprising that Natalie described this in terms of how literature has talked about knowledge of mathematics specific to teaching, such as knowing "the different ways a kid could reason about it … following kids reasoning," since this sort of knowledge would help a teacher teach mathematics in a classroom. In our interview, Natalie mentioned this in relation to how "we all had that teacher that said … 'You didn't do it my way, no credit.' And none of us went in to teaching to be that teacher." (a situation she had to deal with when she was penalized for not including 'let x=…' in her own educational experience). Additionally, Natalie wanted the preservice teachers to see what struggles kids will run into while learning mathematics and how to deal with them. She

stated, “I try to help them struggle with the disequilibrium their students are gonna struggle with and see what it is and how to follow through… I carefully structure problems to guide them to the issue.” This was the reasoning she provided for how she structured and sequenced problems, choosing numbers intentionally to follow the developmental progression researchers like Steffe outline (2001).

The goal of having teachers follow different (student) ways of thinking is also connected with Natalie’s goal of getting preservice teachers to support all their students. By learning to follow a student’s thinking, the teachers would also realize “that all their students can do math.” Being able to support all students by realizing their capacity to do mathematics connected with Natalie’s goal of helping the preservice teachers keep their students from getting trapped in the ‘mathematics as a gatekeeper’ problem. She stated, “I think being able to help people succeed and not get trapped by that gatekeeper is important.” By imparting mathematical knowledge for teaching, she was “hoping that they'll (the preservice teachers will) open up that gate for sure.” It was for this and similar connections that I expanded this group of goals beyond just mathematical knowledge for teaching as described in the literature to also include mathematical considerations for teaching.

In terms of the sMfM. sMfM-oriented goals would focus on enculturating students into practices transferable to other (non-mathematics) communities (see the chapter 5, ‘Research Questions’ section, ‘Oriented Goals and Manifestations’ subsection for more details). Given Natalie’s statements, mathematical knowledge and considerations for teaching had many subgoals under it. Theses ranged from avoiding the gatekeeping role of mathematics (consideration), learning how students will develop an

understanding of a concept (knowledge), to understanding different ways of thinking (knowledge). Because of the focus on preparing preservice teachers for their profession, these goals often fit the sMfM orientation.

As an example of this sMfM-orientation from the knowledge-related goals, I had brought up how Natalie structured her progression of tasks according to guidelines set out by researchers like Steffe (2001). During group work phases of class, Natalie would walk around and note students to call on based on the progression she wanted to convey to students. She stated, “I choose the answer based on the mathematical goal I'm trying to get up there.” Following the guidelines and reasoning set out by the research she relied on, Natalie’s progression of tasks was meant to show preservice teachers how they can get their future students to reconstruct mathematics. Other actions she aligned with this focus on mathematical knowledge for teaching included tasks of following other’s ways of thinking, such as preservice teachers making sense of Natalie’s own son’s thinking on such tasks, reading articles on student thinking for homework, and the portions of her class after discussion in which students made sense of presented approaches. Considerations for teaching included, for instance, understanding and unpacking the common core standards (thus related to enabling her students to act as competent teachers).

All of these had an underlying focus on preparing teachers to teach mathematics. With regard to this whole group of goals for instance, Natalie had mentioned, “It's not just about doing math. It's about how to teach math. And you know, if you're gonna take this class, that's what you're gonna learn.” Because of the focus on preparing teachers for

their professional practice, the goals of mathematical knowledge and considerations for teaching fit the sMfM orientation.

Personal Development. The matter of having preservice teachers support all students connected to another set of goals I broadly classified as personal development goals. For Natalie, teachers also need to have "confidence in following kids' reasoning" by having the preservice teachers share and make sense of each other's answers. This would expose an individual to ways of thinking not their own, and by making sense of the approach, "you have their way of thinking as an option …you assimilate it based on your understanding, so it's not necessarily exactly their way of thinking, it's your own new way of thinking that's based off of their way of thinking." This instance points to Natalie's goal to promote preservice teachers' confidence in their own mathematical abilities. She wanted the preservice teachers "to have the confidence to think through problems. I want them to have the grit to think through problems. To realize … they need to help kids to feel perseverance and grit." If preservice teachers felt that they could do mathematics through perseverance and grit and realized the importance of that perseverance and grit, such realizations could help them help their students realize that as well.

***In terms of the iMfM*.** iMfM-oriented goals would aim to convey the relevance of mathematics separate from mathematics itself (see the chapter 5, 'Research Questions' section, 'Oriented Goals and Manifestations' subsection for more details). Developing preservice teachers' confidence and skills in handling mathematics is a personal matter, and thus I categorized this goal as iMfM-oriented. As an example, Natalie had students working on a task one day. After walking around student groups, she held a class

discussion pointing out where the preservice teachers were beginning to fall back on procedural ways of thinking during problems. She stated, “it was me trying to tell them what they were doing that was making the discomfort worse.” Something about the uncertainty the students faced triggered them to abandon sense-making and instead “fall into a trap there of: 'I don't know what to do, I can't do this. Let me find a procedure and throw it in here.'” Natalie made a point of noting what was causing the preservice teachers’ discomfort. To the preservice teachers during class, she stated, “Your schooling was that you were taught one procedure, and you didn't think about when that procedure applied. You just applied that procedure…And that's different from what we're doing here.” Her stated reason for making it clear to them where they were experiencing discomfort was so that the preservice teachers could then monitor and check that behavior of throwing algorithms at problems. Regarding this instance, Natalie confirmed my question that she was also trying to foster metacognition amongst the preservice teachers. Developing preservice teachers’ confidence and skills in handling mathematics is a personal matter, and thus fits the iMfM orientation.

In terms of the sMfM. sMfM-oriented goals would focus on enculturating students into practices transferable to other (non-mathematics) communities. The previously mentioned instance of promoting metacognitive practices also demonstrated an sMfM-oriented goal. Natalie noted that having the preservice teachers engage in metacognitive practices and “realizing what was helpful and what was unhelpful will help you [the preservice teachers] help your students.” Natalie had made similar statements about how fostering teachers’ confidence would help them help their students. Since these personal development goals were intended to help preservice teachers then help

their students, these goals focused on preparing preservice teachers for their professional practice and thus also had an sMfM orientation.

Different Kind of Math Class. Natalie mentioned all of what she does (from the routine order of class to the sequencing of problems) "are for them (the preservice teachers) to see what a different type of math class would look like and to have that buy-in." There were goals though, that seemed specific to this group, particularly with regard to ownership of knowledge creation. Natalie mentioned she wanted her students to "understand that math is something that could be kind of reconstructed ... for them to realize the power of learning, I guess themselves rather than somebody just transmitting knowledge." Here she wanted to promote the idea of teaching centered around student construction of mathematics and getting buy-in through the preservice teachers' own reconstruction of mathematics. In this regard, this different kind of class is explicitly constructivist, an unsurprising stance given Natalie's research background in mathematics education. She noted how the constructivist approach could be a difficult position to adopt for students who had success in very procedural mathematics classes because they feel like, "why do I have to do it different ways? why do I have to explain it' like. 'I can do it, it doesn't matter.'"

However, Natalie's view of what a different classroom looks like went beyond just focusing on constructivism. She noted, "I also try to model what they should be doing," and what she meant by that was being a different kind of teacher. For instance, there were instances where Natalie would interact with her students, and they would bring up things like mathematical conventions that Natalie would not always be aware of. In that regard, she would admit not knowing. When asked about this, she noted that such

actions reframe what it means to be a teacher. “it's important for them to know that they don't have to know every answer… they feel like if they're gonna show weakness, you know, if they don't know.” For Natalie, a teacher does not have to be all-knowing, and she tries to show what this different kind of teacher looks like through her own modeling of the behavior.

Showing a different kind of mathematics classroom subsumed another set of goals that corresponded to Natalie’s interest in mathematics for its own sake: showing math as connected. She stated,

> I really liked the way my linear algebra teacher taught us that things were a special case of bigger ideas, and I think that impacted how I see and teach math that like, there aren't all these different individual concepts to learn. That there's bigger ideas and if you really take time to get to know them, then it's easier to down the line.

As was mentioned in her background, Natalie came to appreciate mathematics when she learned how much of what she had learned prior to linear algebra was a specific case of the broader ideas contained within linear algebra. Because of this, one of her goals was to have students see mathematics not as a collection of individual concepts, but that there are larger ideas to understand. She stated that she wanted students to see that “math is not about memorizing procedures. It's about thinking. that these operations or procedures we've learned came as a way to organize people's mathematical thinking, but it's not the mathematical thinking in themselves.” As can be seen from the prior statements, the focus is on understanding concepts by thinking through them and reasoning.

There was another way in which Natalie wanted to expose students to a different kind of mathematics class. She stated that just as she wanted her students to realize that mathematics is not a bunch of individual concepts, her students should also not isolate mathematics from other subjects. This was in regards to the preservice teacher's students because "math is going to be more meaningful for them if it's connected. And, you know, kids ask this 'when am I ever going to use this' question because math is just this weird like busywork thing." This goal of showing how mathematics connects to external subjects was notable because of the fact that Natalie personally lost interest in the applicability of mathematics. She realized not all teachers and students would be motivated by mathematics itself. She stated, "I care about them (preservice teachers) engaging our students, and that's a way to engage," and thus, teachers need to know how to appeal to relevance. Furthermore, she stated that a teacher's most important jobs are to 1) make data literate students and 2) promote students' critical thinking skills because no one knows what kind of jobs students would get into.

In terms of the sMfM. sMfM-oriented goals would focus on enculturating students into practices transferable to other (non-mathematics) communities. Both of Natalie's goals of showing preservice teachers a constructivist classroom and a different kind of teacher had sMfM orientations to them. Take, for instance, her comments that her goal was "to change their idea teaching as being about transmitting knowledge to being about the students are always constructing the world around them …my goal's for them to change their view of what it means to know and to teach." The focus behind promoting a constructivist mindset towards mathematics here is in preparation for the preservice teachers' professional practice. In regard to modeling a different kind of teacher, Natalie

stated further that by willing to admit one does not know something, “I also do think that we're modeling for kids, that it's okay for them not to know everything, you know, and how to handle it.” Because of the focus on preparing preservice teachers to teach, these goals fit the sMfM orientation.

The goals of showing how mathematics is connected externally to other subjects was a bit complicated because it can seem iMfM-oriented. For instance, in her section on data collection, she collected data each day on her student’s emotions on a Likert scale. She intended to then present the data to her students as part of the course content. Similarly, she had gathered real-life data on geysers and had students graph the data. When asked about why she would use things such as students’ emotional well-being as one source of data, she brought up how

> Teachers do a great job of say integrating social studies and English …they come up with these great creative lessons for social studies and English and how they're incorporated. And then math is this boring thing off on the side.

She highlighted that teaching math could be an opportunity to touch on other subjects like teaching emotional well-being, geography, and even social justice (due to the current political climate in the U.S.) While these goals would seem to fit the iMfM orientation (appealing to personal interests), it is not the preservice teachers themselves who seem to be Natalie’s first focus. Natalie realized that iMfM-oriented goals could appeal to students and thus wanted to show preservice teachers how to incorporate such goals into their instruction. She stated, “I care about them (preservice teachers) engaging our students, and that's a way to engage.” The focus is still on preparing teachers for their

practice, and thus these also, and perhaps more predominantly, fit the sMfM orientation from a professional preparation standpoint.

In terms of the iMoM. The iMoM orientation is characterized as focusing on understanding and/or promoting interest in mathematical concepts themselves (see the chapter 5, 'Research Questions' section, 'Oriented Goals and Manifestations' subsection for more details). Natalie's goals of showing how mathematics is connected fit the iMoM orientation because of how Natalie centered these goals around understanding the "bigger ideas" in mathematics, as mentioned in a previous quote. One instance she associated with this was her introduction of different kinds of graphs during the data collection section. She started by having students make pictographs of data. The emphasis she placed on lining things up aimed to foster the leap from pictographs to dot plots and then to bar graphs and histograms, thus showing students how these forms of graphs built off each other. When I asked her about what she was trying to convey in this sequencing, she stated, "that's a logical progression … the logical flow of the field of math." This was a specific instance of her sequencing of tasks, but the whole practice of sequencing and her choosing students to call up during class discussions also had undertones of showing math as connected. For instance, she stated, "I carefully go around and think about what story I want to tell… so my first priority is getting the correct mathematical idea out there," to which she later clarified over correspondence, "what I meant was more about moving towards a learning goal (so correct in terms of the idea I intended to focus on and not a side tangent)." She went on to state in her correspondence,

> In general, and with this pictograph example in particular, it is less about math as connected (at least explicitly) and is more about how their students transition from

one idea to another- how each idea can grow out of a previous one (so more about student thinking than about math- though those ideas aren't entirely disconnected).

Because of this undertone of understanding mathematical concepts and how they connect, the mathematics as connected goals fit the iMoM orientation.

In terms of the sMoM. Instructional goals focused on enculturating students in the practices of the mathematics community indicate sMoM-oriented goals (see the chapter 5, 'Research Questions' section, 'Oriented Goals and Manifestations' subsection for more details). There may be parts of Natalie's math as connected goals that fit the sMoM-orientation because she was focused on having the preservice teachers realize mathematics is about reasoning. For instance, she stated, "I want them to realize that math can be about reasoning," a statement she later commented should have used 'is' instead of 'can be'. As a possible example of this reasoning from a homework assignment, preservice teachers were reasoning with fractions by drawing base ten blocks. As they went on, problems had them dealing with larger and larger fractional quantities, like dividing each block up to talk about how many hundredths they had. Natalie noted to the preservice teachers that "You don't have to actually draw those ten lines. We can describe what we happened to do. You could say, well, if I took my 10/10ths and cut each of them into 10 pieces, I would have 100 pieces." Natalie went on to tell me that she structured this sort of progression because "one of the ways that you push people to make those generalizations is to give them opportunities where they don't want to draw it." There was an idea of generalizing here, and this could be one form of

reasoning Natalie aimed to convey. This is a practice that Natalie associated with mathematics and could thus signal a sMoM-orientation.

Analysis

In this section, I will present my model of Natalie's belief system based on the prior narrative and characterization of instructional goals. From there, I will begin answering each of my research questions with regard to Natalie's instruction. In first describing Natalie's belief system (see Appendix B, Figure B3), Natalie's goal of promoting mathematical knowledge for teaching focuses on preparing the students in her course to function as adept mathematics teachers in schools (sMfM-oriented). That then clustered with supporting all students (sMfM). Together, that cluster of beliefs (which I named mathematical knowledge and considerations for teaching) seemed to lead to the derivative goals being able to follow students thinking and keeping preservice teachers from furthering the gatekeeper role of mathematics. This cluster of beliefs manifested through actions like Natalie's class structure centered on discussion and interpretation of others' thinking, assigned homework readings on how students learn mathematics, and her progression of tasks in class. Her assigned homework problems also aimed at having students dig more into the student ways of reasoning on particular concepts talked about during class because class time has sometimes to progress to keep up with all the information they need to cover from the common core. Because all of this involved learning mathematics to function as a member of a professional community (mathematics teachers), the mathematical knowledge and considerations for teaching goal would fall under the sMfM. However, the goal of imparting mathematical knowledge for teaching requires individuals to understand mathematical concepts in particular ways, and so an

iMoM orientation may also be present, but perhaps with less psychological strength than the sMfM goal.

With regard to her personal development goals, the immediate goal of promoting confidence in mathematics follows what I understood to be the iMfM in terms of developing personal skills. This confidence goal was however, subsumed under a larger cluster of goals, including promoting confidence in following student thinking and other skills (e.g., building metacognitive skills), all of which serve an individual. In turn, the personal development cluster of goals seemed to fall under the iMfM-orientation. However, because of the nature of the course being for preservice teachers, Natalie then connected these goals to teachers functioning in the classroom and many of the same goals that derived from Natalie's mathematical knowledge and consideration for teaching goals (predominantly sMfM). Thus for Natalie, it seems iMfM-oriented goals led to derivative sMfM-oriented goals due to the nature of the course being all about mathematics that is useful for teachers. It made sense then to see these goals manifest in many of the same actions as the mathematical knowledge and considerations for teaching goals (e.g., the progression of tasks, class structure of communication, and sense-making).

In looking at goals under a different kind of class, the focus on having preservice teachers understand mathematics as a broad set of interrelated ideas certainly focuses on how an individual understands mathematics. Natalie's focus on constructivism seems to build on this iMoM orientation since the idea behind constructivism would be to set up and build those connections. But the process of building connections is then part of the different kind of classroom Natalie wants her students to see and hopefully implement. In

that way, it seems like the iMoM-oriented goal of understanding mathematics as a broad set of ideas then serves to enable teachers to operate in a particular way (sMfM). On the other hand, Natalie also realized that some students and teachers are not interested in mathematics itself and do care for applications. Add to that, Natalie's belief in promoting data literate students and critical thinking (focusing on students' skill because of the uncertainty of what career they may go into) sounds as though part of being a teacher of mathematics (sMfM) means knowing how to use the iMfM and sMfM orientations. Perhaps then, Natalie's sMfM-oriented goal of showing preservice teachers a different kind of math class clustered with her constructivist view on teaching to lead to the derivative goal of seeing math as an interconnected web of ideas. Mathematics as a connected set of ideas then clustered with the beliefs that 1) teachers need to make data literate students and critical thinkers and 2) just as math should not be isolated ideas, mathematics should not be isolated from other subjects. In that way, it seems as though Natalie's sMfM-oriented goals of teacher preparation has sMfM-, iMoM-, and iMfM-oriented goals as derivative goals.

The first research question asked: what, in terms of background, affective factors, etc., leads instructors to prioritize certain meanings over other meanings? Natalie's experience research background and her experiences implementing active learning with underserved students (and the subsequent gatekeeper role mathematics can play) influenced her mathematical knowledge and considerations for teaching and different kind of math class goals (sMfM). Mathematical knowledge and considerations for teaching had derivative goals fitting the iMoM orientation (understanding mathematics as part of mathematical knowledge for teaching) and sMfM orientation (mathematical

knowledge for teaching, supporting all students). Natalie's different kind of math class goals had derivative goals fitting the sMfM orientation (constructivist classroom, show how to teach mathematics as connected internally and externally to other topics), and perhaps less central iMfM and sMfM orientations (the preservice teachers' students' skills, citizenship, and interests). Her beliefs about a rapidly progressing society also played into the less central iMfM-oriented goals (preservice teachers' students' skills).

The second research question asked: How do instructors' orientations of mathematical meaning translate into specific pedagogical decisions? Natalie enacted her oriented goals through her course's structure and flow. The routine cycles of individual-group-class-group-class discussions around carefully sequenced tasks and homework on student learning had the preservice teachers routinely seeing other ways of thinking to support all students (sMfM) and learning mathematics in a holistic way needed to teach (sMfM, iMoM). Natalie's use of homework may have also signaled the same goals around content coverage (iMfM) because of how homework was used. It gave students more practice with the material covered in the common core standards; material they could not spend more class time on due to the amount of content the common core touches. This same cycle of discussion and group work, along with Natalie's remark on student triggers, would also build students' personal skills (iMfM, sMfM), e.g., their confidence with math and following others' thinking. The discussion and group work cycles, devoted sections to data collection, and appeals to how mathematics could be connected to other subjects would model for preservice teachers how to run a different kind of mathematics classroom (sMfM) which could motivate the preservice teachers' students who are motivated by sMfM- or iMfM-oriented goals.

The third research question asked: how does context influence the meanings instructors emphasize? All of Natalie's goals were informed by the course's student demographics: preservice teachers. Each cluster of goals, and the actions that stemmed from those, had a central goal with an sMfM orientation of preparation for professional life as mathematics teachers. These same goals may have also been influenced by society to a degree because of the material students need to know according to the common core standards (signaling perhaps a less central content coverage goal, iMfM). Natalie's department and colleagues also had a scaffolding influence on her mathematical knowledge and considerations for teaching and different kind of math class goals and derivative math as connected and mathematical knowledge for teaching goals. She had colleagues engaged in education research whom she could engage with, especially a senior colleague who worked with Natalie in ways ranging from structuring the constructivist course (sMfM, iMoM) to giving ideas on how to show a different kind of math classroom through connections to other topics, like social justice (sMfM, iMfM). What is notable here is how I did not find any cases in which context shifted Natalie towards certain oriented goals to the detriment of her other oriented goals. Everything seemed to flow smoothly together.

The fourth research question asked: How do backgrounds, contexts, and orientations come together in pedagogical decisions with respect to how instructors respond to the students' meaningfulness questions (e.g., "when is this ever going to be useful?", "why are we learning this?"). The following except introduced my conversation on this topic with Natalie.

> Me: If in this course like, how if a student were to ask you like, when is this stuff ever going to be useful? You're kind of in a particularly interesting situation teaching teachers.
>
> Natalie: Yeah, well in this class, it's funny because the whole point is to teach them useful math.

Natalie was in a bit of a strange position when I asked this question because she stated the whole point of the class was to talk about mathematics which is useful to teachers (sMfM). Continuing on from the last except, She stated,

> It's not just about being able to do math, and you know it's part of the buy-in of why we're not just doing math… I would less justify when are we ever gonna use the math more as why- I think of myself more as answering the question, 'why are we doing it this way? Why aren't you just telling us to do it?' And I guess part of my, one of my answers would be you know, this is how people learn. Like research has shown us this to how people learn.

Based on this context and student demographic, everything she did in her instruction, from the group-class discussion cycles to interpretation of others' work, had an sMfM-oriented central clustering or antecedent belief. On a more micro level then, Natalie took actions signaling goals such as personal development (iMfM), showing mathematics as connected internally (iMoM) and externally to other topics (iMfM, sMfM). However, all these goals were either derivative or clustering goals with her sMfM-oriented goals of preparing preservice mathematics teachers. Her background as a teacher to underserved students and as a researcher were driving forces in informing her oriented goals,

specifically her goals of preparing the preservice teachers to teach using active learning according to constructivist learning theories. Her institutional context further bolstered her efforts: those who were not involved in mathematics education in the department generally did not interfere with her goals, while those involved served as a support group for ideas and collaboration. Given this supportive environment, prior experiences teaching, and having a specific student demographic to attend to; it seemed as though all Natalie's less central oriented goals and more central sMfM oriented goals seamlessly played into one another.

Benjamin's Narrative, Instructional Goals, and Analysis

In the following sections, I will review Benjamin's background in a narrative format. Next, I will describe Benjamin's instructional goals and how those goals may be characterized using my meaning framework. Afterwards, I will overview my model of Benjamin's belief system and answer my research questions with regard to Benjamin's instruction.

Background Narrative

Benjamin stated that math "was something that even from a young age I found that I was intrigued by" and how he would be "asking questions as to why the answer was what it was even from a young age." He remembers going to his father at a young age to ask why a divisibility rule was what it was, and his father was only able to show that such things worked but could not explain why. Benjamin stated, "the proof for that is like a one- or two-page long proof depending on your conciseness, that you don't learn

until like the 300-level math, you know. But that was the kind of question that I was asking." In a way, he already had a practicing mathematician's curiosity as a kid. While Benjamin's family could not meet that mathematical curiosity with answers, they were a middle-class family who could afford to send him to private school. Teachers in school could see Benjamin's knack for mathematics. In 5th grade, he was with a teacher who taught both 5th- and 6th-grade mathematics. She realized his talent and "allowed me to do both the 5th- and 6th-grade math together. And so I got a year ahead of all of my peers at that age, and after that, I was always a year ahead." His success in mathematics continued on into high school when Benjamin was one of the only two students taking calculus as a junior and was doing well at it. Despite his successes however, Benjamin had no intention of continuing on, thinking instead that once he was done with calculus, "I was like, 'Cool, done with that, don't have to think about math classes anymore.'"

Instead, Benjamin went to art school but even then, "when I was going to art school, I found myself using math for the things that I wanted to create… I just knew I had an appreciation for it." He eventually decided to leave art school and spent time working odd jobs, taking a few college courses here and there, and traveling. One day when he was working at a coffee shop, a coworker asked him what he really wanted to do for a living. Still unsure of where to take his life, Benjamin mentioned how "I kind of looked at her, and I'm like, 'I don't know. I've always been good at math. I could see myself just tutoring people math, and that would be satisfying." However, he went off to see more of the world before coming back to this thought when he returned to school and considered majoring in physics or mathematics.

During his time working odd jobs, traveling, and taking a few classes, he was also recommended to work at the mathematics tutoring center. Once he finished up his travels, he took up this job and "like really fell in love with tutoring more than anything else. Like really fell in love with the teaching math as much as the doing." Specifically, he realized that teaching mathematics afforded him a chance to talk about mathematical details in a natural way. He stated,

> When you're in a classroom and people are excited to learn something, and I get to talk about something that I'm passionate about and get to break it down into its elements, that is very fulfilling, you know, to talk about something that I've spent so much time learning and that I don't get an opportunity to talk about in conversations.

He went on to major in mathematics in the hope of continuing the chance to be able to teach the subject.

All the previous leads up to Benjamin's current situation. In the sections that follow, I will start to discuss Benjamin's instructional goals, the orientations for those goals, and answers to my research questions in regard to his instruction.

Characterization of Instructional Goals

Benjamin talked about how his main focus for the course was going over the main storylines in calculus III. I grouped Benjamin's goals for his course into three categories: "storyline" of material, focus on algebra, and mathematics generally. Based on the details above, Benjamin's focus on mathematical details sounds like an iMoM orientation may influence his goals. His comments on society and the role mathematics plays may suggest

iMfM or sMfM orientations also play a role. Benjamin's class routine followed a typical lecture format; he introduced the initial concepts and definitions needed for the day's topic and then tackled a few examples, worked either by him or students in groups, and then Benjamin working them out.

Storyline of Material. Having taught calculus III numerous times before, but this being only the second time with the department's new textbook, the first goal Benjamin mentioned was "really looking at restructuring the course in a way where the overarching storyline of the material is more apparent than it was last term." His aim was to streamline and organize the concepts covered so students could compartmentalize the course's most important aspects. Benjamin stated then, "there's two storylines in calc III: there's the sequences and series, and then there's the 3d coordinate systems." Sequences and series were the first storyline which would be the first half of the course. The second half storyline was 3D coordinate systems; better understanding trigonometry, parametric equations, and vector functions; and giving an appreciation for the resulting geometry. Related to this goal was the goal of preparing students for the follow-up course, multivariable calculus, because part of his job was "to make sure students are really prepared for fourth term calculus where they're actually doing a lot of calculus in three dimensions."

These storyline goals manifested in how he set up note sheets, the examples he picked, and the questions he would choose for exams. He stated,

> I actually structured my lectures and content first. So I planned the overall arc without looking at the textbook at all. So I knew, I knew the content that … I'm supposed to teach over the course of the term. I broke that up into individual

> lessons that contain any of that, just lesson titles without specifics. And then as I went through the term, I would essentially write the lesson first and then look at the book last and say, okay, I would try and scour the book, not just on one section but multiple sections that would have questions that gave practice to the ideas that I'd give in class.

What was notable in his structuring of lessons and notes were moments when Benjamin chose to go over proofs or not. If a proof involved things students could follow and would be necessary to understand other concepts in a storyline, he would go into them. For instance, he went into the proof for the arc length formula, which required Riemann sums because "in calc IV, we build, you know, the volumes under a surface and the Riemann sum there… So that I think has a lot more applications that- I think it's good to go into a deeper explanation." As can be seen in the prior, sometimes those storylines may not be the storylines in his course but in following mathematics classes like multivariable calculus.

However, these main storylines were not the only things Benjamin mentioned wanting to cover. He wanted to get into ways "you can actually utilize the specific subject as opposed to having the specific subject just be part of a bigger overarching story but realizing that there are subplots within the main plot." Concepts related to the main storyline elements could show up and be relevant to certain STEM fields. These tangents or subplots were picked based on what Benjamin thought would be cool, what students could digest, and sometimes what the department mandated he cover. One thing Benjamin mentioned regarding subplots was that students could get lost with trying to memorize them and lose sight of the main storylines. While he felt that such subplots

were important, he emphasized exercising discretion in including and how in-depth he would go into them because the main plotlines could be lost in those details.

In terms of the iMoM. The iMoM orientation is characterized as focusing on understanding and/or promoting interest in mathematical concepts themselves (see the chapter 5, 'Research Questions' section, 'Oriented Goals and Manifestations' subsection for more details). I would classify Benjamin's main storyline goals as iMoM-oriented because of how he focused on showing how concepts coalesced into two main storylines, as opposed to saying he was hitting topics as separate stories that were not connected. This could be found in his comments on how he was trying to structure lessons so the "overarching storyline of the material is more apparent than it was last term" to how he thought "it's good to go into a deeper explanation" of a proof using Riemann sums because of how prevalent Riemann sums are in the main storyline. His focus throughout is connecting concepts and fostering understanding of two wholistic 'stories.' The broader aim of showing two overarching stories of connected ideas and having students understand fits Brownell's initial conceptualization of the meaning of mathematics and thus fits the iMoM orientation.

In terms of the iMfM. iMfM-oriented goals would aim to convey the relevance of mathematics separate from mathematics itself (see the chapter 5, 'Research Questions' section, 'Oriented Goals and Manifestations' subsection for more details). Benjamin's storyline of content also had an iMfM-orientation because of the presence of content coverage goals (pushed by institutional influences). For instance, a notable aspect of his lesson structure was the amount of time he gave students to work examples out before showing them how to work through them. He noted, "a bigger concern, I guess as far as

that time management is really when or how am I going to give students the opportunity to practice in the classroom." One of the challenges he dealt with was how to balance being a sage lecturer or a guide while students work in groups. While he attributed that partly to his own belief that we're "standing on the shoulders of giants where you're not going to recreate every single piece of math on your own," he also attributed this to the amount of content he had to cover according to department guidelines. For instance, this was demonstrated when he said he would want more student involvement but could not "because of the amount of content that I'm looking at covering over the course of the term." The associated content coverage goal relates to information students need to have covered and not necessarily related to understanding mathematics. This fits the iMfM orientation.

In terms of the sMfM. sMfM-oriented goals would focus on enculturating students into practices transferable to other (non-mathematics) communities (see the chapter 5, 'Research Questions' section, 'Oriented Goals and Manifestations' subsection for more details). The subplots within the two main overarching plotlines were often sMfM-oriented because these subplots were often directed at specific professions. For instance, one subplot Benjamin covered (and included exam questions on) was using Taylor series for higher degree approximations of functions. The use of higher degree approximations was something "that's really important in physics," and thus, he wanted to emphasize that to an extent. One thing he was mandated to cover by the department was curvature, about which Benjamin noted in class and in our interviews, "apparently curvature is important to electronics. And again, I think that when you take an electronics class, you see that formula, and you just learn to use that formula more." Because of his

statements of such subplots showing up for particular majors, like physics or electronics in the previous examples, these subplot goals had an sMfM orientation related to professional practice.

Focus on Algebra. While Benjamin was primarily concerned with covering the storyline of material, he also emphasized the need to be aware of the learning process, specifically "being understanding that my students don't know something the first time they see it, they don't remember everything that they see …and understand how humans learn is not a one-and-done, we've mastered it right away." Additionally, students come in and "maybe they don't have as much prerequisite knowledge or aren't as strong in that prerequisite knowledge as they probably need to be," thus making new content difficult to learn if one has not come in with the appropriate prerequisite concepts. He noted this was particularly the case for those in our society who have been historically underprivileged like African Americans. In his view then, a lack of success isn't because a student is incapable of understanding mathematics, but rather a myriad of other reasons may get in the way: "are they working full time outside of this? How's their family situation and their own personal willpower and drive, right? Those things are going to tend to be bigger obstacles than simply saying, does someone have the capacity." All the previous does not mean however, that he felt he should make the content easy by giving easy problems. To him, patronizing his students with easy examples "would be an awful thing for me to think about my students." At the same time, Benjamin realized that many of his students would be taking this class as their last mathematics class, and the material learned may not stick as a result.

Thus Benjamin's focus on algebra. Benjamin noted how many instructors would cut algebraically challenging problems out of a belief that many students "are never going to be good at algebra." However, Benjamin held that

> But as you take more and more of something, then the earliest stuff you took sticks with you because that stuff had to be used over the course of those years after. So if I'm teaching someone their last math class and that last math class is a calculus class, I'm like, chances are, you're never going to take this forward and use it in your life because it's something that you only saw once in school.

However, regardless of whether the course material is retained or not, "maybe they will be a lot better at algebra… maybe they'll have spent this much extra time being forced to do hard algebra that that will be a skill that they can carry on and actually utilize in their lives." To this end, Benjamin had devoted a lot of time to reviewing old algebra skills when they came up in the context of the course material. Part of this also had to do with Benjamin's animus towards patronizing his students. He believed that all his students have the capacity to do well, and so he taught as if all his students tested into a good school. To cut out algebraically intense problems would run counter to that belief.

In terms of the iMfM. iMfM-oriented goals would aim to convey the relevance of mathematics separate from mathematics itself. Benjamin's goal of imparting algebra skills fits the iMfM orientation since he wanted to have students strengthen their algebra skills to use in whatever way each individual student wishes. He noted that by going through his class, "maybe now you're gonna have those algebra skills down to a point where those will move forward with you, and you will have those skills available to you

whenever you want them." Benjamin believed that to have mathematics be useful to an individual, they have to be fluent with the material to see its uses. Benjamin stated,

> If you're only 80% comfortable with a certain topic, you may or may not realize that it could be used in the situation that you're looking at. And even if you do notice that maybe it will be, you might not feel comfortable enough with it to want to go down that avenue. And students don't gain that 100% comfortability, 100% fluency, humans, I shouldn't say students, like anyone, until we've been exposed to it for quite some time, you know.

Furthermore, that understanding comfort only comes with routine exposure, and thus the reason why the new material learned in this course may not stick, but the algebra work it was routinely based on would. This algebra skill is entirely centered on an individual's personal use, as demonstrated by Benjamin's comments that "It could be this skill that they only tap into on occasion or it could be something that they apply in their lives on a regular basis, and that is up to the student." As such, Benjamin's focus on algebra fits the iMfM-orientation.

Mathematics Generally. Connected to his storyline goals for the course, Benjamin wanted to talk to students generally about the overarching storylines in *mathematics generally*, how to approach math, and why math is beautiful for its own sake. He stated,

> As much time is spent in the classroom lecturing and talking just about the math, there is also just talking to your students, not, not on how to do a specific problem but just talking to them about math and what concepts are important, how to

> compartmentalize these things, trying to talk to them about what the overarching storyline looks like and why it's cool and why they could be interested in it.

He made similar statements about how mathematics itself has its own value. When I asked him what that value was, he talked about how the beauty of math comes from the fact that it is not just arbitrary rules that "by some random happenstance- that they fit together some way. The beauty about math is that all of this stuff is built from things. It's built from ideas. It's constructed from the ground up." While there may be multiple ways to go about a problem, he held that that structure and foundation still direct you towards one answer. It was not surprising then that I observed cases of Benjamin going over the details of a proof in class. Sometimes these proofs were done if they related to the storyline of material (such as showing the derivation of arc length using Reimann sums in the previously noted example). Sometimes they related to common student errors. For instance, he stated to his class, "You guys love proofs right? The question was: prove or disprove that you can take the square root of a sum by square rooting the individual terms. And I proved that you cannot by contradiction." Here, he used proof by contradiction and explicitly referred to it as a technique for disproving things when he corrected a student's mistake of distributing a square root through terms being added.

***In terms of the iMoM*.** The iMoM orientation is characterized as focusing on understanding and/or promoting interest in mathematical concepts themselves. I saw Benjamin's mathematics generally goals as having an iMoM orientation because of how the goals focused on ways of understanding mathematics generally. Take the previously mentioned case of intentionally highlighting proof by contradiction as a technique to his students. When asked about the situation, Benjamin stated, "I think that talking about

how proofs work is never a bad idea, I mean especially the basics of proofs." More specifically, he stated,

> I remember like not knowing how something worked, so just trying a few examples with numbers…then that would give me some insight on whether or not I could say that there was a rule. And that basic strategy for trying to understand algebra, I think, is a really handy one to have, even if you're not technically proving something.

He went on to state how algebra work itself is a way of proving things and that having these ways of thinking is useful to assess mathematical rules and further understanding. He tied this to his own childhood experience of when he was unsure of something; he would check a few examples to see if he could find a counterexample. So for Benjamin, this focus on proofs under his mathematics generally goals was about checking parts of the structure of mathematics and furthering one's understanding. As such, this fits an iMoM orientation.

In terms of the sMoM. Instructional goals focused on enculturating students in the practices of the mathematics community indicate sMoM-oriented goals (see the chapter 5, 'Research Questions' section, 'Oriented Goals and Manifestations' subsection for more details). While I saw Benjamin's mathematics generally goals as predominantly iMoM-oriented, one might suspect there could be sMoM-oriented goals here as well because of how the goals included aspects of proof and proving practices. Benjamin stated that "talking about how proofs work is never a bad idea," and so there could be subconscious aims to foster dispositions characteristic of those in the mathematics

community. However, Benjamin only spoke of using proofs to *understand* mathematical concepts and rules, and thus I could not attribute an sMoM orientation in that regard.

On another count however, I was able to attribute a sMoM orientation to a subgoal of mathematics more generally. When asked why he felt it was important to correct students on notational errors (which he was observed doing in class), he stated how he believed,

> Math is a language, so I think notation is important, like conventional notation. I mean, even if we're teaching writing, we don't teach people how to do poetry and break the grammar rules when we're first teaching them how to write. We teach them how to follow the conventions that have been established. And then as people become fluent in writing, they learn how to break those rules, and the language evolves as people learn to break them in creative ways, and that's really cool. And here, same thing, you know. I want to give students like the insight on like this is convention, and honestly, you're probably not going to be allowed to break these conventions and come up with your own until you hit a three or four hundred level math class.

The way Benjamin views mathematics as a language focuses on ways of communicating and following conventions of communication, especially given how Benjamin connects these ideas to how people communicate by writing prose. The goals of proper notation focus on communicating properly in the language of mathematics, and thus by extension, the language and practices of mathematicians. It was for this reason, some of Benjamin's mathematics more generally goals fit the sMoM orientation.

Analysis

In this section, I will present my model of Benjamin's belief system based on the prior narrative and characterization of instructional goals. From there, I will begin answering each of my research questions with regard to Benjamin's instruction. In first describing Benjamin's belief system (see Appendix B, Figure B4), Benjamin's focus on storylines could be summed up in showing the main concepts to be learned in the course, which he accomplished through structured notes and selected problems to highlight main aspects of the story. I would classify his main storyline goals as iMoM-oriented because of the focus on how concepts in each storyline came together into two main storylines, as opposed to saying he was hitting topics as separate stories that were not connected. However, focusing on the 3D geometry storyline because it prepares students for future courses can be ambiguous. Students could use that content for mathematics, some other STEM field, or neither. Thus, it might seem preparation for future courses was a derivative, iMfM-oriented goal of passing on skills that students then use in whatever way they wish, paralleling Benjamin's belief that 'it's up to students to make mathematics useful to themselves.' However, because of this usefulness belief, he viewed his job as teaching the concepts within the storyline of course content, which may then connect to concepts in other courses. This still prioritized looking at how mathematics connects together (iMoM). That is not to say there could be iMfM or other oriented goals which cluster, precede or proceed from Benjamin's iMoM-oriented goals. In fact, Benjamin's debate over student involvement versus him lecturing due to the amount of material he needs to cover suggests an associated content coverage goal. This entails

information individual students need to have seen and thus would fit an iMfM orientation.

Benjamin's focus on tangents also suggests that he also would take chances to go into applications when they would be digestible and useful to students and were interesting, in his own opinion. Benjamin's comments on touching on interesting subplots might lend the subplots goal to an sMfM orientation (appealing to personal relevance/interest). However, his interests were heavily aligned to physics, and subplots often focused on how certain topics applied to physics and engineering fields. Thus, it seemed more like his subplot goals fit an sMfM orientation since knowing those tangents would help particular students become competent members in their fields. Additionally, these tangents were not derivatives to the two main storyline goals but rather arose alongside the concepts within each story. This suggests these sMfM-oriented goals clustered with the main storyline iMoM-oriented goals but were less central because Benjamin did not want students to be lost in these subplots.

I viewed Benjamin's focus on algebra as an iMfM oriented goal because it clustered with Benjamin's view that the usefulness of any skill one has is up to an individual. His focus on algebra entailed a lot going on between the second and third belief systems dimensions. His understanding that students take time to learn led to the derivative belief that the material learned may not stick. His views on gaining proficiency with a concept through routine use clustered with his view of not wanting to patronize his students even if they come in at different levels of background knowledge. Both then led to the derivative of focusing on algebra skills. I viewed that goal as iMfM oriented goal

because it clustered with Benjamin's view that the usefulness of any skill one has is up to an individual.

With regard to Benjamin's mathematics generally goals, the idea of an individual using the structure of mathematics to check other parts of that structure only relates to clarifying an individual's understanding of mathematics itself, thus fitting an iMoM orientation. However, engaging in proofs is a part of the mathematical communities' practice while correcting notation out of the belief that mathematics is a language focuses on the mathematical community's language. As such, there may be sMoM-orient goals under mathematics more generally, a fact supported by Benjamin's comments on how some of the things he covers would be useful for math majors (e.g., dealing with inequalities in determining the radius of convergence for a series). Given that Benjamin talked about the storylines of mathematics generally and how the course goals he mentioned earlier were also storylines, it seems as though the storyline of material goals clustered with the mathematics generally goals. Given that Benjamin initial interview statements that he eschews complicated proofs and the instances where he just gave students formulas without proof, it may be that the storyline of content goals have a greater psychological strength than the mathematics generally goals because of the course context and subsequent demands from departments and STEM majoring student demographics who may not follow or need such proofs. However, perhaps because of his own appreciation of talking about mathematical details, this goal may still have quite a bit of psychological strength behind it, at least enough that it had him talking about proof by contradiction with an audience including many non-mathematics majors.

The first research question asked: what, in terms of background, affective factors, etc., leads instructors to prioritize certain meanings over other meanings? It appeared as though Benjamin's general mathematics goals were influenced by Benjamin's background experiences of talking about mathematical details. Benjamin had mentioned always being curious about mathematics, even from childhood. Later, he came to love tutoring and chose to go into teaching because he enjoyed talking about mathematical details. Teaching was the context in which that naturally occurred. These experiences led him to take actions signaling less central, derivative goals under this overarching goal with orientations fitting the sMoM (discussing notation) and iMoM (using proofs to clarify understanding). His physics background also played a role in elevating sMfM-oriented goals (profession-related tangents in the storyline of material). Interestingly, I will discuss how this appreciation of mathematics itself served as a driving force away from particular types of meaning-oriented goals.

The second research question asked: How do instructors' orientations of mathematical meaning translate into specific pedagogical decisions? Benjamin acted on his storyline of content goals by structuring his notes and selecting problems to highlight the main storylines (iMoM) and tangents (sMfM as professional practice). He also included test questions on these storylines (iMoM, sMfM). The two storylines of content however, had a lot of material to cover (iMfM). Thus, Benjamin routinely debated how much time he spent working out problems for students versus having students engage with problems first. Benjamin would also attend to his general mathematics goals (sMoM, iMoM) and his algebra skills goals (iMfM) throughout the course of his storyline of content goals, from emphasizing proper notation, to picking problems requiring a lot

of algebra techniques, to showing proofs when it related to future parts of storylines or when trying to highlight how things are done by mathematicians to clarify understanding.

The third research question asked: how does context influence the meanings instructors emphasize? There was an aspect to Benjamin's storyline of material goals that derived from the mathematics department and perhaps other departments the course served. Specifically, the mathematics department listed a breadth of content in each of the storylines to cover, and some of the things Benjamin was mandated to cover were in relation to specific fields (suggesting the influence of being a service course). These versions of the storyline of material goals manifested through actions signaling derivative goals with orientations fitting the sMfM (subplots prioritizing material relevant for particular majors) and iMfM (debating student involvement versus content coverage). There was also an interesting relationship between student demographic and departmental influences on Benjamin's iMoM- and sMoM-oriented goals. When asked if racial demographics play into his goals, Benjamin stated,

> The idea that a large percentage of black Americans, because I mean that's really the largest demographic that we talk about as far as underprivileged and tend to have some of the worst success within mathematics of any demographic, and how can we get more people of African-American heritage to take math classes and be successful in math classes, and how can we make math relevant to them? I don't have an answer for that question.

He realized how making mathematics culturally relevant was important for underserved populations because of systemic inequalities, but he did not feel like he had the knowledge or the resources to talk about mathematics in that way. He wished the

department had some mandated material or resources to help in this regard. But absent that kind of support, he noted that he was only hired ‘to teach the mathematics’ and that that mathematics can be relevant for its own sake and beauty (going back to his iMoM- and sMoM-oriented storyline of material goals). In that way, Benjamin’s realizations around race and disenfranchisement led him to believe other forms of meaning can be important for underserved students, but the lack of departmental support or resources left him to focus on what he knew how to prioritize.

The fourth research question asked: How do backgrounds, contexts, and orientations come together in pedagogical decisions with respect to how instructors respond to the students’ meaningfulness questions (e.g., “when is this ever going to be useful?”, “why are we learning this?”). The following except introduced my conversation on this topic with Benjamin.

> Me: So if a student in your class came up to you and asked, 'when is this stuff ever gonna be useful?' How would you respond to them?
>
> Benjamin: I'm just gonna project here. Well Ahsan, it's not.

The focus on mathematics itself (as part of his sMoM-oriented mathematics generally and iMoM-oriented storyline of content goals) showed up in how Benjamin said he would respond to this question. Benjamin held that mathematics has no importance or relevance from a philosophical or existential point of view and does not have to be relevant, but is important for advancing society and technology. Continuing on from the last excerpt, he stated,

> It's not [relevant] … it doesn't have to be. It's going to be as useful as you want it to be. Whatever tool, whatever skill you learn how to use is only useful in the context of how you want to try and utilize it. Like you, I mean, you don't need to learn math. Nobody needs to learn math. I mean, we need a certain percentage of the population needs to learn math, but no single individual is required to learn this stuff. It's not necessarily relevant unless you want it to be. You can take these skills and do whatever you want with them or you can do nothing with them. It's really up to you and where you want to go with this.

Some portion of the population would need to know mathematics to help advance society, but that responsibility does not fall on any one individual. In that sense then, mathematics does not have to be relevant to any one person individually. Furthermore, he stated that students end up relearning mathematical concepts in their own fields. Coupled with the fact that it is impossible for a mathematics teacher to know all the applications of the mathematics they teach, he wondered why it would be on him to make mathematics relevant.

To paraphrase some of his statements, He stated however ‘that that does not mean mathematics isn’t relevant. It’s highly applicable. Studying it would grant skills which open up opportunities, even if *all* those mathematical skills aren’t used. It’s up to an individual then to make mathematics relevant for themselves and furthermore become proficient enough in their mathematical skills to see how they can make mathematics relevant.’ He went further and argued that mathematics can be important regardless of whether one uses their mathematical skillset or not; they do not have to be relevant or applicable to one’s life. Instead, mathematics itself, along with its details, can have value

in and of itself. Benjamin compared this perspective of mathematics to playing the piano. When one learns the skills to play the piano, they can use those skills however which way they wish, from playing at a church, to trying to become a famous pianist, or do nothing at all. Whether someone does something with those skills or not however, does not take away from the beauty of playing the piano. Benjamin held that one could say the same thing about mathematics and the kinds of discussions on mathematical details he got into teaching for. It was for these reasons, the lack of resources to address underserved students, and perhaps his own childhood to adult life appreciation of mathematics itself that Benjamin stated, “I wish I could make my classes 100% relevant to everyone but what I end up focusing on more is simply that this subject is fascinating”, specifically the storyline of content (iMoM, sMfM) and mathematics more generally (iMoM, sMoM). He went to state that mathematics being fascinating “doesn’t have to have anything to do with the people around you, that it can just be its own safe haven of just looking at this subject.” His views on realizing how to make mathematics relevant after gaining proficiency connected with his focus on algebra (iMfM).

Chapter 5: Cross-Case Analysis

In the following sections, I will review the background and instructional themes that I noticed across instructors. I will first start by noting themes across instructors' backgrounds and foreground a few ways in which those background themes influenced instructors' goals. I will go into those connections more deeply then within the instructional goal themes I noticed across the instructors. From there, I will address each one of my research questions.

Background

I will first discuss the general themes I noticed across the instructors' backgrounds. These themes were identity & success or nonsuccess, supportive surroundings, wandering & transformation, and teaching interest. From there, I will discuss possible connections to the kinds of goals instructors had based on similarities and differences in their backgrounds.

Supportive Surroundings

All the instructors had made comments I coded as noting the role of supportive environments and/or people at various points in their experiences learning mathematics. First and foremost were the roles of supportive individuals. These individuals could be family members. For instance, Archy mentioned how his "mom was a computer scientist. So she like valued math. And my older brother was also like a math person… because of their interest and influence was big on me and they valued math, it transferred to me,"

while Jordan talked about how her dad "was a computer engineer and he kind of expected us to know about math" so much so that "I just was ignorant of the sexism that was in the hard sciences at that point." Other times, these supportive individuals were not family, but mentors, teachers, or other impactful individuals. For instance, Benjamin talked about how his 5th-grade math teacher "saw that I just had a knack for mathematics and so she allowed me to do both the 5th- and 6th-grade math together. And so I got a year ahead" while Natalie talked about a principal she worked for who told her, "you're gonna change math education in this country, and I think you should think about getting a degree in math instead of a degree in education because people listen more to people with degrees in math.'" These supportive individuals all played a role in supporting the instructors in their path to learning mathematics and/or becoming mathematics instructors.

A less common trend in this theme was the presence of supportive learning environments. Not all the instructors had made comments which I coded in this way, but the instructors who were men did seem to indicate they were placed in learning environments in which more was expected of them. For instance, Archy mentioned,

> In middle school, the way my math class was set up was like the same, I don't know, maybe 10 or 15 people for all three years of middle school, cuz we were in like the 'special math' or whatever. So that was very collaborative, and just by virtue of the way that they had set it up for all of us, or for us, whatever, you know, privileged folk.

Similarly, Benjamin had mentioned how his parents "were doing well enough, like in middle class, that they were able to put me through private school. And... I-, so I had a small class size" in which he could get support and get ahead. These supportive learning

environments came together with supportive individuals in supporting these instructors' mathematical development, as seen in Benjamin's teacher having the time and ability to have Benjamin learn both 5th- and 6th-grade material. It is noteworthy however, that some instructors dealt with unsupportive environments. These will be noted in the ID & success or nonsuccess section.

Identity & Success or Nonsuccess

Instructors noted both positive and contentious (or barrier) factors that I coded as relating to their mathematical identities and/or experiences. In terms of positive identity factors, I found the men positively identified with mathematics throughout their childhood (though the women did later in life or in specific cases or ways). For instance, Benjamin talked about how "I was always applauded as being good at it [math] growing up," while Archy stated, "some component of my identity is...whatever you know how people are. They're like, that's a math person. And I'm like, 'well I guess I am.'" These statements, particularly the notion of being a 'math person', center around identifying oneself through mathematics or as mathematically competent. Closely related to this idea were instances I coded as instructors talking about having positive life experiences associated with mathematics. For instance, Natalie recalled how

> I would also like sit in the backseat of the car with coins for the toll, and my parents would be like, 'oh the next toll's a dollar, how many ways can you make a dollar?' And I fondly remember playing with my parents' coins and finding all the ways to make a dollar.

Positive life experiences with mathematics were typically activities or events involving mathematics that instructors looked positively on. This is different but closely related to the amenable identity codes in that the focus is not on identifying oneself through mathematics but just positive experiences and recollections about life events involving the use of mathematics.

Not all identity-forming and experience codes came from a place of fondness or positively construed factors. Some instructors in this study, particularly the women, noted less than favorable circumstances that I coded as negative or barrier identity factors and/or experiences. In terms of contentious identity factors, I found instances in which some instructors talked about their wanting to avoid being identified with mathematics, routinely struggling with mathematics, and/or failing to meet other's mathematical expectations of themselves. Natalie talked about how she was recognized for her mathematical achievement but how "my mom remembers me being horrified to have to go up and get this math award. I don't know if it was being labeled as good at math as embarrassing. I don't know what it was." This demonstrates wanting to avoid identifying with mathematics. As an example of failing to meet expectations, Archy recalled how,

> There was a situation with my mom and older brother and she like was not willing to teach me how to do long division. She was kinda like, 'you need to be able to figure this out.' And so I was like, disappointed to not know how to do it because I was like letting her down in a way… that's an example of a motivation coming from self-disappointment or like discouragement. Like this, like it's like a negative influence more or less. Like a 'you can't do this, and that's bad.'

These sorts of recollections were also closely related to experiences with mathematics that instructors described in negative terms. For instance, Jordan talked about how she faced a lot of gender-based discrimination: she talked about a teacher for a math modeling course where students were to pick their own projects and "I picked one of them, and he goes, 'oh honey, that's too hard for you. You should pick something easier.' But none of those comments ever came out to any of the men." The trend through all of these was a general sense of negativity associated with either being identified with mathematics or experiences connected to mathematics.

Wandering & Transformation

Two notable trends I noticed across the instructors were their unintentional paths to majoring in mathematics and becoming mathematics instructors at the college level (wandering) and/or a transformation in their perspective towards mathematics as they grew up (transformation).

Archy, Jordan, and Natalie all had experienced some transformation in their view towards mathematics. For instance, Archy talked about how when he was younger, he wanted to "find the perfect thing. And uh it's changed in that I've become a bit maybe more of a pragmatist" in that "yeah, I want you to like see beauty in geometry. But like also, I just like don't want you to suffer. So like get your statistics done and like you'll be in a better situation." This was a transformation from seeing mathematics as a way to grasp Truth to a view of mathematics as a practical means of upliftment in a STEM-centered society. Jordan talked about how she "really didn't like math from middle school till I'd say my sophomore year of college" when she took a business calculus class, then more and more mathematics classes until "I decided that I actually really enjoyed the

challenge, and it was so interesting, and so I switched majors in my sophomore year to math." Natalie talked about how she "did kind of have a love-hate relationship with it. I mean I went to college knowing the only thing I wasn't gonna major in was math, and then I ended up as a math major," with her relationship to math becoming a pure love relationship after going through her linear algebra course. Here, Jordan and Natalie talked about having a conflicted view towards mathematics which then shifted to one of being interested in the subject.

Natalie's last quote starts to touch on the wandering life path that I noticed across some of the instructors, specifically Jordan, Benjamin, and Natalie. Natalie's prior quote starts to highlight this theme, but it continued on in her unexpected path to getting an emergency teaching credential in underserved communities, to her pursuing a graduate degree and then becoming a teacher educator, all of which she described as "all one big happenstance" with the comment that "I've always firmly believed on life telling you what you're meant to do and life just led me down this path." Jordan's wandering life path similarly occurred alongside a shift in her perspective towards mathematics and involved her initially being unsure of what to major in, switching majors to mathematics after her business calculus class, pursuing a career in the corporate sector, and then coming back to her university to teach mathematics. Benjamin talked about how "I was always applauded as being good at it [math] growing up, but I didn't really have any intention of doing anything with it." Instead, he went traveling, taking college classes sporadically in different subjects, working odd jobs, all until he came back to school and "started tutoring in the tutoring center, and like really fell in love with tutoring more than anything else. Like really fell in love with the teaching math as much as the doing." In all

of these, I noticed an underlying theme of not having any intentions to pursue a professional career in mathematics but then different life circumstances or occurrences playing a role in causing a change to pursuing the mathematics careers they wound up in.

Teaching Interest

The last theme I noticed across the instructors was an interest in teaching. The previously mentioned theme of wandering life path connected well with the teaching interest theme; all these wandering paths ended with an interest in teaching math and thus the instructors' current careers. For Jordan and Benjamin, an initial event served as a 'tug' or pull towards teaching. For instance, Benjamin talked about how he "really fell in love with the teaching math as much as the doing" after working at a tutoring center. After entering the corporate world and looking back at her unexpected teaching career in graduate school, Jordan noted how "I kind of was missing the teaching, which I was surprised at. Like I really liked the helping people, the social interactions with people." For Archy and Natalie, their initial draw towards teaching was less so a series of consolidated events. For instance, Natalie did not have an initial event that pulled her towards her teaching career because she "always loved teaching math. That was my job in high school, that kind of job, I've always been teaching math," and multiple events throughout her 'happenstance' life reinforced that interest. Archy similarly was drawn to teaching as a result of a myriad of factors spread throughout his life.

Often connected to the initial draw towards teaching, the instructors also commented on their reasons for why they liked to teach. I grouped these reasons into two trends: understanding & beauty and care for others. Instructors who gave reasons in the understanding & beauty category talked about how their teaching was motivated by

wanting to foster understanding of mathematics or an appreciation of mathematics aesthetically. For instance, Jordan talked about how the rapid pace of technology "has awakened like a, 'okay, we need to not concentrate on this' numerical process, old-school paper pencil, but how we can use technology in a way that will help us understand what's going on in that bigger picture.'" This focus on a bigger picture was an example of teaching motivations around fostering understanding. Benjamin talked about how "when you're in a classroom and people are excited to learn something, and I get to talk about something that I'm passionate about and get to break it down into its elements, that is very fulfilling." This was an example of aesthetic appreciation because Benjamin held that those details are part of the beauty of mathematics.

Instructors also gave reasons for why they liked to teach centered around caring for others. For instance, Archy talked about how "yeah, I want you to like see beauty in geometry. But like also, I just like don't want you to suffer. So like get your statistics done and like you'll be in a better situation." Natalie talked about how she wanted to change "how people feel about math in general. And it comes back, I guess, all to this whole like being a child psychologist and wanting kids to have confidence and knowing they can do it." Natalie also talked about how gatekeeper issues around mathematics education have motivated her to convey a new kind of mathematics classroom to her preservice teachers. In these instances, the instructors talked about caring for students' quality of life or their confidence. Overall then, the instructors were all drawn to teaching due to various life circumstances (expected and unexpected), and they were motivated to teach for greater understanding and/or out of caring for others.

Possible Connections to Instruction

Considering everyone's background, what stood out to me was that Jordan was the only instructor to respond to my question about how they respond to meaningfulness questions without mentioning mathematics for its own sake. In response to my posing of the meaningfulness question, for example, she responded,

> I always, on the last day, emphasize to them that they may not use this specific statistic in their life going forward, but with the comfort of visually looking at graphs and looking at data that we hope that they become more educated consumers and not get fooled by statistics. And that's my goal, is that they can understand that that statistic was manipulated by a politician. Or if it was an ad from, you know, from some type of company … So I try to bring in articles where those statistics are used.

The comments about 'not being fooled by politicians' fit the idea of preparing students for life in a democratic society (sMfM-compatible) while buying something from an ad appears to relate to one's individual quality of life (iMfM-compatible). All the other instructors either did not care about when math is useful or mentioned uses alongside appreciating mathematics for its own sake (sMoM- and/or iMoM-oriented). For example, one of Archy's responses to students' meaningfulness questions was to talk about how "math is just fun" (sMoM- and iMoM-compatible). Looking at this difference then, it was notable that all the other instructors had childhood experiences of academic success with mathematics. Often, that success was accompanied by an interest in the subject, though that could be complicated with Natalie having a love-hate relationship with it. Jordan did not have that sort of experience growing up. What that may suggest then is that experiences of early academic success in mathematics, and perhaps personal enjoyment

during childhood, may lead to a more central belief in doing mathematics or promoting conceptual understanding (beliefs which would imply an iMoM- and sMoM-orientation). That does not necessarily mean that individuals who experience less success in mathematics growing up do not value iMoM and/or sMoM orientations if they go on to teach though (Jordan had goals of fostering understanding, iMoM-oriented). However, the fact that she did not state anything sounding like either MoM-orientation in response to the meaningfulness question may indicate that that appreciation may simply not have much psychological strength in relation to that question.

Despite this difference in childhood success in mathematics and how they subsequently responded to the usefulness question, all the instructors had mentioned iMfM-, sMfM-, sMoM- and iMoM-oriented goals. There are a few noteworthy points to make here that will be expanded on along with the following discussion on instructional themes. First, there was a difference in how the instructors approached iMoM-oriented goals that did break down along the difference in childhood success. Out of all the instructors, Jordan was the only instructor who had mentioned that she usually tries to introduce new concepts through direct applications (something that would seemingly imply an sMfM- or iMfM-orientation).

The second point to note is that there seemed to be a spectrum in how instructors approached goals that derived from a care for students who were ‘falling through the cracks’ in some sense and possibly instructors’ own experiences of educational disenfranchisement. Instructors with background experiences that led them to care for students who were ‘falling through the cracks’ all had that care manifest in their instruction through different iMfM- and sMfM-oriented goals, e.g., enabling engagement

in a democratic society or professional community or promoting students' personal confidence and skills. For instance, Natalie talked about how her teacher certification program played a role in how "I think being able to help people succeed and not get trapped by that gatekeeper [of mathematics] is important." By imparting mathematical knowledge for teaching, she was "hoping that they'll [the preservice teachers will] open up that gate" (thus preparing teachers for their profession and having an sMoM-orientation). Similarly, Archy talked about how his realization around injustices and inequalities in our society resulted in a pragmatist worldview where mathematics was a way of upliftment in our society. He stated, "you get the benefit of the doubt if you studied math. So you should study math because you need to take care of yourself and like, you need to improve your station." This then led him to see the importance of the societal and personal relevance of mathematics, which included financial competence as an example of personal relevance (and thus iMfM-compatible). He then went on to have a section devoted to developing students' financial skills. These goals were similar in that they all fell under the sMfM- and/or iMfM- orientations.

Characterization of Instructional Goals

In this section, I will talk about the general instructional goal themes I noticed across instructors. I will then have subsections arguing how each theme fits different orientations according to instructors' reasons for their goals. These themes and subthemes are outlined in Table 2 below.

Table 2

Themes Across Instructors

Theme	Subtheme	Instructors
Conceptual Understanding	Conceptual understanding	All
	Base concepts	Archy, Jordan
Doing Math		Archy, Benjamin, Natalie
Community Engagement	Professional engagement	Jordan, Benjamin, Natalie
	Civic engagement	Archy, Jordan, Natalie
Personal Relevance	Personal 'objects'	All
	Non-STEM specific relevance	Archy, Jordan, Natalie

Conceptual Understanding

I classified all the instructors as having goals focused on fostering understanding and/or appreciation of mathematical concepts and their connections. Jordan implemented active learning to "help us understand what's going on in that bigger picture," while Natalie had students engage in reconstructing mathematical knowledge partly because she wanted to show "there aren't all these different individual concepts to learn. that there's bigger ideas." In trying to show how mathematics is 'fun,' Archy wanted students to go through and see how there was a 'sweet connection' between sequences of partial sums to the sum of the first n integers. Benjamin's aims of conveying two overarching 'storylines' in his class also fit within these prior examples as his goal here was counter to just teaching a bunch of individual concepts. In all the prior, there was a focus on having students understand mathematical concepts themselves and how they connect to other mathematical concepts. For Jordan and Natalie, these goals manifested in the use of more active learning techniques. For Archy and Benjamin, these goals manifested in the

types of content they presented to their students. In terms of reasons, Jordan, Benjamin, and Natalie all made statements about how understanding mathematics from this conceptual standpoint makes learning the material easier. For instance, Jordan mentioned how she implemented technology and active learning because "I wanted to … help students learn better."

There was a strand of fostering understanding that was a bit different from the notions pushed earlier. Archy and Jordan both talked about how they would need to build an understanding of base concepts to continue onto the content they wanted to teach. For instance, Archy's stated, "I want to like get some basic facts about a thing" as the first step in his lessons, and these basic facts could be a basic understanding of notation ("we need to talk about notation") or of concepts ("we were supposed to be talking about the derivative of a line, now it's just like, okay no, if I give you an equation, why is that equation look like a picture?") Similarly, Jordan mentioned how she would do minilectures on base concepts, formulas, and notation at the beginning of class so students could engage in the activities of the day. Both instructors often mentioned such goals because their students were non-STEM majoring students who may not be comfortable with or have a good grasp of mathematics. The notion of building understanding of concepts remains the same here as in the previous paragraph, but the nuance here is in building a basic understanding so that the instructors can move on to hit other goals.

The focus on notation did show up for Benjamin however for a different reason. For him, "Math is a language, so I think notation is important. Like conventional notation. I mean, even if we're teaching writing, …when we're first teaching them how to

write. We teach them how to follow the conventions." Here, Benjamin focused on notation because he sees mathematics as a language, and he wants students to understand the basic conventions of that language. This is different from building understanding as I have outlined more generally in this theme. However, I grouped this here because it shares the same focus on notation that showed up under Archy and Jordan's focus on building a basic understanding of concepts to be used elsewhere. This similarity is furthered in that both Archy and Benjamin's focus on notation centered around conventions of the mathematics community. This trait has implications for ways the goals on notation can be oriented in the meaning framework.

In terms of the iMoM. The iMoM orientation was characterized as focusing on understanding and promoting interest in mathematical concepts themselves. All the instructors made statements about fostering understanding of mathematical concepts. Sometimes the focus was understanding mathematics that formed the content of the course. Other times, it could be understanding base concepts and notation so that instructors could get their students to understand the material that they wanted to teach. Because of all the prior statements about understanding mathematical concepts and how they connect, I classified the goals around building conceptual understanding as iMoM-oriented.

There was a difference in how the instructors approached iMoM-oriented goals that did break down along the difference in childhood success. Out of all the instructors, Jordan was the only instructor who had mentioned that she usually tries to introduce new concepts through direct applications (something that would seemingly imply an sMfM- or iMfM-orientation). For instance, she stated, "I think introducing it as an example first

and then giving that formula... when I give a formula with nothing else, no examples at all, there's- they don't make the connection to how they apply it" and went on to state

> I found that if I only have theory with no applications that I never really understood it very well, and so, yes in some way, I'm teaching by what not to do from how I learned it. Like going through, I could finally, I think, understand it and grasp the theory a lot more once I had those applications, and at this level of mathematics, I don't think any of my students are gonna be thriving on theory alone.

Jordan made it a routine priority to introduce new concepts through direct applications, stating that this goes back to her own experience of learning mathematics through applications.

That does not mean however, that the other instructors did not introduce concepts through applications from time to time. Natalie had done so with data collection, and Archy had when he brought up voting theory and finance. However, Natalie and Archy's instances of using direct applications to lead into iMoM-oriented goals may have resulted from the topics they were teaching lending themselves to the MfM orientations. I say this because, in other instances, they and Benjamin approached iMoM-oriented goals by following mathematical reasoning to make sense of a concept without referencing applications. In terms of belief systems then, experiences of struggling to learn concepts through theory alone may play a role in dictating short-term goals and beliefs regarding how mathematics is learned, in particular, raising the psychological strength of learning concepts generally through applications. This idea of using something that would seem

oriented one way to promote another oriented goal will be discussed in the Oriented Goals and Manifestations section later.

In terms of the sMoM. Instructional goals focused on enculturating students in the practices of the mathematics community indicate sMoM-oriented goals. While the prior statements often focused on understanding concepts, there was the aspect of understanding notation for Archy and Jordan that did stick out as a bit different. Archy had stated, “we need to talk about notation and talk about how math people like make up notation so they don't have to speak so much.” Benjamin talked about ‘mathematics as a language’ and understanding ‘conventional’ ways of communicating in that language. Jordan had her students interpret null and alternate hypotheses out in words to understand what kinds of decisions would be made based on a statistical test but also just to understand what the symbols were communicating. Here, wanting her students to understand mathematical notation from a conceptual standpoint to understand what symbols convey certainly had an iMoM-orientation. However, within this and more prominently in Archy and Benjamin’s goals around notation, there was a focus on understanding how the mathematics community speaks. Wenger (1998) talks about how communities of practice have their own jargon that community members use to communicate within their community. Taking this perspective then, while the instructors may not always be preparing their students to function within the mathematics community, they were trying to get their students to understand the language of the mathematics community. Thus, I took the focus on notation for the reason of understanding the language of the mathematics community as sMoM-oriented. This does

not negate such goals also having an iMoM-orientation when there was a focus on understanding notation conceptually.

Doing Math

One theme that showed up across Archy, Benjamin, and Natalie was the idea of 'doing mathematics'. For instance, Archy talked about how one of his goals was to convey how "Math is fun. And so you can just enjoy it the way you enjoy playing basketball. It's just like, basketball's meaningless too, but it's fun. So like, of course you should do it." Here, the focus is on *doing* mathematics as an activity, the same way one plays basketball as Archy stated or "someone learning piano," a comparison Benjamin made. In terms of how this goal could play out, the instructional practice manifestations ranged from practices aimed at getting students to engaging in things instructors believe mathematicians do that I had not previously considered (e.g., categorizing, or reasoning) to engaging in proof. For instance, Archy was commenting on a homework problem he had given his students and stated,

> The next one is about like the different sampling techniques … I think it's good because it's like, well, we just spent like three class periods trying to develop one sampling technique. And it's like, okay, there are a lot of different things that you could have done, let's just kind of categorize them. *That seems like a typical task in math* [emphasis added].

Here, the act of categorizing techniques is something Archy considers a typical thing that is *done* in mathematics. In my individual analysis, I had talked about how Natalie wanted

her preservice teachers "to realize that math can be about reasoning" and how she structured her progression of a class discussion to get students to generalize. Whether generalizing fit within what Natalie considered reasoning or not, generalizing and reasoning are something one *does in mathematics,* according to Natalie.

Benjamin was a bit of an anomaly here. Fitting with the idea of *doing things that are typical in mathematics*, Benjamin stated how "I think that talking about how proofs work is never a bad idea … there are general proof strategies and one of the most common by far is by contradiction, proving something is false by contradiction." Here, proving and general proof strategies are things that are enacted. However, the difference for Benjamin compared to the prior cases was that his focus was not on the action of proving itself but more so on what a proof proves or disproves ("proving *something* is false by contradiction"). This distinction from the prior instructors is a difference that has implications for how goals under the doing math theme could have different orientations, and I will address this difference in the following sections.

In terms of the sMoM. Instructional goals focused on enculturating students in the practices of the mathematics community indicate sMoM-oriented goals (see the chapter 5, 'Research Questions' section, 'Oriented Goals and Manifestations' subsection for more details). As I was looking across the instructors, I found that the scope of the sMoM may have been too narrowly defined and/or how the instructors conceptualized practices of the mathematics community were too expansive. The latter may result from beliefs about what the goals of mathematics education are for the instructors' student populations (none of the classes were specifically for mathematics majors, most in fact were for non-STEM majors). Given the more expansive views instructors communicated,

it became useful to broaden the scope of the sMoM-orientation to look for goals that focused on *things that mathematicians do*. Such goals having an sMoM-orientation is still justified because this affordance still carried an underlying theme about the practices the mathematics community engages in, though not as fine-grained as initially understood.

From this perspective of *things that mathematicians do*, Archy and Natalie's doing math themed goals fit the sMoM-orientation. For instance, Archy had a section on sequences and series and another planned (but never enacted) section on geometric constructions and noted how such lessons were so "you can just enjoy it [mathematics] the way you enjoy playing basketball ... So like, of course you should do it." For Archy, just as playing basketball is something one does for its own sake, mathematics then is something one does as well. There were instances where my more fine-grained definition did help orient some of the doing math themed goals. I had talked about how Natalie wanted her preservice teachers "to realize that math can be about reasoning" and how she structured her progression of a class discussion to get students to generalize because "one of the ways that you push people to make those generalizations is to give them opportunities where they don't want to draw it." There was an idea of generalizing here that I posited as one manifestation of what reasoning entailed for Natalie. Whether generalizing fit within what Natalie considered reasoning or not, generalizing fits with the ways Larsen (2013) typifies the practices of mathematicians while 'reasoning' is something one *does* as a characteristic of mathematics, according to Natalie. The former fits the more fine-grained perspective on the sMoM orientation, while the latter fits the broader perspective on the sMoM orientation.

In terms of the iMoM. The iMoM orientation was characterized as focusing on understanding and promoting interest in mathematical concepts themselves. I had mentioned how Benjamin was an anomaly in how his focus on doing mathematics, specifically engaging in the proof process, was not on the action of proving itself but more so on what a proof proves or disproves. In my individual analysis of his mathematics generally goals, I cited his statement about his own use of proof by contradiction and what he used it for: "I remember like not knowing how something worked, so just trying a few examples with numbers…then that would give me some insight on whether or not I could say that there was a rule." He also showed proofs for theorems and results in his overarching stories when the proofs were useful for building understanding of the storyline of content, as opposed to when he would skip over a proof because "it's a very kind of left-field proof that's not... that I don't think is something that's really helpful in building a conceptual understanding of what's going on." So for Benjamin, the process of proving was about building conceptual understanding and thus fit the iMoM-orientation.

Community Engagement

A theme that showed up across all the instructors was goals aimed at getting students ready for life within different social and/or professional settings, which I called community engagement. I broadly grouped these into two subgroups: Professional and Civic engagement. The subgroup of Professional engagement consisted of goals where instructors aimed to have their students function within different professional communities related to student majors. Benjamin, for instance, talked about how he

wanted to touch on how concepts related to elements of the main storyline could show up and be relevant to certain STEM fields, such as Taylor series approximations of functions because “that's really important in physics.” Jordan would cover certain topics and things like understanding SPSS because serviced departments wanted their student majors to be exposed to those things to prepare for their major. Natalie’s whole class was directed at preservice teachers, and so for instance, she talked about how “this class, the whole point is to teach them not math but mathematical knowledge for teaching.” Each of these examples highlights a subset of the instructors’ goals of preparing students for particular majors and, by extension, to function within specific professions.

The subgroup of Civic engagement consisted of goals where instructors aimed to have their students function as informed members of a democratic society. Jordan, for example, hoped that by going through their course, students would gain general quantitative literacy skills and result in “just being a better consumer in general. I don't know about better but just uh… yeah, more informed.” She had used this same term (informed consumer) when talking about how she went about some examples she had on worksheets talking about statistics with voting and political parties. In our discussion on that, I asked her,

> Interviewer: When you had mentioned informed consumer, I'm curious, does that, does that term, is that the same term to you as informed citizen?
>
> Instructor: oh, yeah, I guess so…but they are different, right. If you're actually buying something versus if you're involved within your community, yeah.

Here, Jordan indicated she had been using the term ‘informed consumer’ in the literal meaning of consumer but had also been thinking about informed citizenship and engagement in society. The focus in this theme then is the latter perspective on ‘informed consumer’ as informed citizenship and engagement in society. Archy had a section on voting and another on data collection because “math is incorporated in society. You're reading a news article, and it's telling you this, and it's influencing your opinion. Or like this is how voting works.” Specifically on the data collection section, he hoped his students would “come away [like], ‘wow, research is hard and like we should be skeptical when people like give us information and we should like, check their methods and stuff like that.’” His goals behind both his sections on voting and on data collection were aimed at enabling students to navigate our democratic society as critically informed citizens.

In terms of the sMfM. sMfM-oriented goals would focus on enculturating students into practices transferable to other (non-mathematics) communities. One set of goals that fell under community engagement focused on preparing students for specific professions. These were STEM professions for Benjamin, teaching professions for Natalie, and health and social science professions for Jordan. For instance, Jordan motivated the content students were learning by routinely introducing concepts using applications. While more of the focus seemed to be about getting students to understand mathematics, there was a degree of appealing to students’ majors. She noted for instance, “I have about six different articles I'm going to show them that kind of use the things that we've been using throughout the quarter like where they might see a p-value in a research article.” These articles highlighted studies in different fields to show how statistics looks

in practice. By focusing on goals directed at specific student majors, the instructors were by extension preparing their students for life within specific professions, and thus such goals fit the sMfM orientation. The other set of goals that fell under community engagement focused on preparing students for life within a democratic society. Such goals ranged from getting students to be generally informed for Jordan by focusing on quantitative literacy to understand how our electoral system functions for Archy through his voting section. Such goals focused on enabling students to function as citizens within our society. This still fits Wenger's (1998) notions of being a practicing member of a community, though perhaps on a larger scale than Wenger may have conceptualized. In turn, I felt this focus on civic community engagement fit the sMfM orientation.

An interesting connection to some of the instructors' background comes in here. Both Archy and Natalie's community engagement goals seemed to connect to the instructors' care towards those who 'fell through the cracks.' Natalie talked about how her teacher certification program played a role in how "I think being able to help people succeed and not get trapped by that gatekeeper [of mathematics] is important." By imparting mathematical knowledge for teaching, she was "hoping that they'll [the preservice teachers will] open up that gate." Similarly, Archy talked about how his realization around injustices and inequalities in our society resulted in a pragmatist worldview where mathematics was a way of upliftment in our society. He stated, "you get the benefit of the doubt if you studied math. So you should study math because you need to take care of yourself and like, you need to improve your station." This then led him to see the importance of the societal and personal relevance of mathematics. Archy went on to state that he had students engage in a data collection project because "I just

want people to think constantly when people tell them stuff. How do you know that? I mean that just would affect everybody's life on a daily basis if they did that more... that's both personal and societal." However, Archy expressed a lot of self-efficacy concerns in meeting these goals because of his limited knowledge and available resources. Notably, part of Benjamin's reasons for the absence of goals like Archy's personal and societal relevance goals was that he also did not have the knowledge or resources to do so. This focus on an MfM-oriented goal because of a care for those who 'fell through the cracks' will be discussed further in a later section.

In terms of the iMfM. iMfM-oriented goals would aim to convey the relevance of mathematics separate from mathematics itself. Archy had students engage in a data collection project because "I just want people to think constantly when people tell them stuff. How do you know that? I mean that just would affect everybody's life on a daily basis if they did that more... that's both personal and societal." His focus on the 'personal' related back to statements about how math directly impacts one's personal life and thus fits the iMoM-orientation.

Personal Relevance

A theme that showed up across all the instructors was goals aimed at things that directly impact students as individuals, which I called 'personal relevance' goals. I had two subgroups within this theme: personal 'objects' and non-STEM specific relevance.

The subgroup of personal 'objects' consisted of goals where instructors focused on getting students immaterial deliverables. These could be skills, such as when

Benjamin talked about building students' algebra skills within his calculus storyline goals because "maybe they will be a lot better at algebra… maybe they'll have spent this much extra time being forced to do hard algebra that that will be a skill that they can carry on and actually utilize in their lives." Similarly, Archy, Jordan, and Natalie talked about hoping to build their students confidence with numbers, such as when Natalie wanted her preservice teachers "to have the confidence to think through problems. I want them to have the grit to think through problems. To realize … they need to help kids to feel perseverance and grit" (this also highlights building resilience skills). Other examples include Natalie hoping to build her preservice teachers' metacognitive skills and their ability to focus on critical thinking in their teaching and Jordan trying to build students' collaborative skills partly because "I think working with people is a good life skill. In a job, in a career, you're going to be having to work usually with other people, so I put a lot more emphasis on group work." These prior examples focus on particular skills, from algebraic computation skills to personal confidence, to cooperation skills, that the instructors want their individual students to gain.

Sometimes these deliverables could be academic deliverables ranging from content coverage (as material students need to have seen) to students doing well in the course or on assessments in terms of their grades. For example, Jordan talked about how serviced departments had put pressure on the mathematics department to "I don't want to say dumb down the course even more, but we are. We're looking at like how we can improve student success by not teaching any of the math concepts within the probability chapters." Other departments were exerting these sorts of pressures to cut mathematical concepts and focus on shortcuts so that more students could pass and graduate. Content

coverage goals figured prominently for Jordan and Benjamin. This was demonstrated for instance, when Benjamin said he would want more student involvement but could not "because of the amount of content that I'm looking at covering over the course of the term." Part of the reason for Jordan's minilectures at the beginning of class was to catch students up. While Archy did not express any content coverage goals, he had expressed concerns over the amount of material he covered being viewed negatively if the department compared his instruction to that of another instructor. These examples highlight a subset of the instructors' goals focused on academic 'objects' the instructors wanted their students to have (e.g., good grades).

The subgroup of non-STEM specific relevance consisted of goals where instructors aimed to convey how mathematics could apply to aspects of students' personal lives. For instance, Archy had a section on finance because "literally, this pertains to you cuz this is how taxes work, or this is how finances work." In her data collection section, Natalie talked about

> Math is more than just- I mean teaching is more than just teaching the topics but also another example of how you can connect to the other types of thinking…I think the same way that we have metacognition about our math, kids also need metacognition about their emotions and realizing ways to do it.

The prior instances involved instructors aiming to show how mathematics can be applied to other aspects of individuals' lives, from their finances to their emotional well-being. Other times, such goals focused on how mathematics can be extended to certain topics that could be personally motivating to students. For instance, Natalie wanted to show preservice teachers how to connect mathematics to other subjects for the preservice

teachers' own students: "math is going to be more meaningful for them if it's connected. And, you know, kids ask this 'when am I ever going to use this' question because math is just this weird like busywork thing." My reason for putting these topical relevance goals here as opposed to under the community engagement theme was that instructors seemed to be trying to show relevance broadly as an appeal to students' personal interests and motivation, not necessarily towards particular professions. Overall, I called these non-STEM specific relevance because goals that fell into this category tended to be non-STEM field related, though sometimes connections could be made to STEM fields.

A notable aside to the non-STEM specific relevance was how some instructors had difficulty attending to such goals when they dealt with non-STEM topics, specifically for Archy and Benjamin. Benjamin had no goals in this category and partly because he felt he did not have the knowledge to attend such goals. He stated, "People come up and show me things that-, where math is used … and I'm always fascinated. I'm like, 'that's so cool. I wish I had the time and energy to go and like study that specific thing more.'" He also talked about

> The idea that a large percentage of black Americans, because I mean that's really the largest demographic that we talk about as far as underprivileged and tend to have some of the worst success within mathematics of any demographic, and how can we get more people of African-American heritage to take math classes and be successful in math classes, and how can we make math relevant to them? I don't have an answer for that question.

In the above quotes, Benjamin noted the importance of non-STEM specific relevance goals, partly because of deep-seated racial issues. However, he felt more comfortable

focusing “just on the mathematics” because he did not have the resources or knowledge to attend to such goals. Similarly, Archy noted many preparation struggles to meet his personal relevance goals (as well as his community engagement goals) because his “concern is that like I don't have any material for this course and that like I'm just making it all up.”

In terms of the iMfM. iMfM-oriented goals would aim to convey the personal relevance of mathematics separate from mathematics itself. Much of the personal ‘objects’ goals had an iMfM-orientation to them since they focused on ‘things’ the instructors wanted their students to have that would impact students on a personal level, not necessarily connected to understanding mathematics itself better. Archy, Jordan, and Natalie wanted their students to have confidence in mathematics and other cognitive skills (e.g., metacognitive skills, collaborative skills, critical thinking). Benjamin wanted his students to have algebra skills “that they can carry on and actually utilize in their lives” however which way the individual student chooses. Other times, the instructors wanted students to ‘have’ certain academic deliverables such as good grades or content knowledge. From academic success to skillsets, all these are things instructors aimed for students to have for their personal lives and thus fit the iMfM orientation.

Of note is the inclusion of content coverage and collaborative skills as iMfM-oriented. Instructors' content coverage goals focused on ‘material students need to have seen’ (paraphrasing Benjamin). This only related to mathematical concepts (iMoM) in that the content to cover was mathematical. But saying you have content to cover itself is not related to mathematical concepts. On collaborative skills, Jordan mentioned, “I think working with people is a good life skill. In a job, in a career, you're going to be having to

work usually with other people," Such goals are directed at preparing students for general careers. They are not specific to certain professions and instead skills that help an individual succeed wherever they may go. It was for this generality that I saw this as focusing on how mathematics adds to an individual's life, and thus as iMfM-oriented.

In terms of the sMfM. sMfM-oriented goals would focus on enculturating students into practices transferable to other (non-mathematics) communities. While I noted how many personal 'object' goals were iMfM-oriented, it was also possible for such goals to have an sMfM-orientation. For instance, Natalie talked about building her preservice teachers' confidence and metacognitive skills because such skills "will help you [the preservice teachers] help your students." I had also mentioned how Natalie wanted to get her preservice teachers to focus on building students' critical thinking skills. Natalie talked about the fast pace of advancement in society and how "You're trying to train- prepare kids to have jobs that don't even exist but who knows what skills they're gonna need. But critical thinking is obviously something they're going to need." Here, the focus is not on building the preservice teachers' critical thinking skills but rather on getting the preservice teachers to build their students' critical thinking skills. Similarly, non-STEM-specific relevance could also be sMfM-oriented for Natalie. She justified her non-STEM specific relevance goals because "I care about them (preservice teachers) engaging our students and that's a way to engage." Across all these instances, Natalie had focused on such goals because they would help the preservice teachers in their professional practice. It was for this reason that personal relevance goals can have an sMfM-orientation.

In terms of the iMoM. The iMoM orientation was characterized as focusing on understanding and promoting interest in mathematical concepts themselves. For Benjamin, content coverage was also iMoM-oriented because while the department had mandated topics for Benjamin to teach, he was also thinking about what students need to have to understand other concepts. For instance, Benjamin talked about how his second storyline goals of understanding 3D coordinate systems; better understanding trigonometry, parametric equations, and vector functions; and giving an appreciation for the resulting geometry aimed to "make sure students are really prepared for fourth term calculus" by having them understand the mathematics. While being prepared for a follow-up course is a personal academic deliverable, building understanding of mathematical concepts so that they can understand other concepts in the future fits the iMoM-orientation.

Research Questions

Having described the themes I noticed across the instructors, I will begin addressing my research questions. I will start by addressing how instructors' backgrounds influenced the orientations of their goals. I will then compare how instructors' oriented goals actually looked like in their teaching in comparison to the initially theorized scope of orientations. I will then talk about how context influenced instructors' oriented goals. Lastly, I will discuss how all the previous came together in how instructors said they would respond to students' meaningfulness questions.

Background and Orientations

The first research question asked: what, in terms of background, affective factors, etc., leads instructors to prioritize certain meanings over other meanings? I will first talk about this research question in relation to the meaning **of** mathematics, both individual and social. From there, I will talk about this research question in relation to both meanings **for** mathematics.

A significant part of everyone's background was their mathematical identities and experiences learning mathematics growing up. The men had childhood experiences of academic success with mathematics with an accompanying interest in the subject. This situation was more complicated for the women in this study. Natalie had a love-hate relationship with mathematics despite her success, while Jordan was not interested in, and experienced difficulties learning mathematics before college. Despite these differences, all the instructors had mentioned iMfM-, sMfM-, sMoM- and iMoM-oriented goals, though there were notable differences that showed up that did break down along these gender lines.

In Relation to the MoM. One thing that three out of the four instructors shared as they were growing up was that they had not intended to pursue a career in mathematics. Jordan, Benjamin, and Natalie grew up without any plans to pursue mathematics, but then went on to do so. It was interesting then to find that all the instructors had goals falling under the conceptual understanding theme focused on building understanding of mathematical concepts. Often, these goals were iMoM- or sMoM-oriented without any notable patterns or differences in comparison to Archy. There was also a lack of any noticeable differences along these lines within the doing mathematics theme (sMoM- and iMoM-oriented). As a result, this intended career difference from Archy does not seem to

have impacted instructors' MoM-oriented goals to pursue. This may be because the instructors had all come to some appreciation of mathematics itself at some point in their background. It could also be due to contextual influences that make such goals manifest in similar ways, e.g., how society construes 'success' in mathematics.

Rather, there was a difference in how the instructors approached iMoM-oriented goals that did break down along the difference in childhood mathematics achievement. Out of all the instructors, only Jordan did not mention iMoM-compatible beliefs in response to students' meaningfulness questions that I posed to her. In response to my question on this, she stated views that aligned with the personal relevance and community engagement themes:

> I always, on the last day, emphasize to them that they may not use this specific statistic in their life going forward, but with the comfort of visually looking at graphs and looking at data that we hope that they become more educated consumers and not get fooled by statistics. And that's my goal, is that they can understand that that statistic was manipulated by a politician. Or if it was an ad from, you know, from some type of company … So I try to bring in articles where those statistics are used.

The comments about 'not being fooled by politicians' fit the civic community engagement theme (sMfM-compatible) while buying something from an ad fits the personal relevance theme (iMfM-compatible). This is not to say that other instructors had not mentioned iMfM- or sMfM-compatible goals of community engagement or personal relevance in response to my question, but those instructors had mentioned iMoM- and sMoM-compatible goals and/or beliefs alongside these MfM-compatible goals. For

example, Archy talked about goals of showing how math is fun (doing math theme, sMoM- and iMoM-oriented) alongside the personal and societal relevance goals (personal relevance and community engagement themes, sMfM- and iMfM-oriented). Benjamin talked about algebra skill goals (personal relevance theme, iMfM-oriented) emphasized within his larger storyline of content goals (conceptual understanding theme, iMfM- and iMoM-oriented). However, Jordan was the only instructor to make no mention of MoM-oriented goals in response to my question.

Furthermore, Jordan was the only instructor who had mentioned that she usually tries to introduce new concepts through direct applications (sMfM- or iMfM-oriented). However, that does not mean that the other instructors did not introduce concepts from time to time through applications. Natalie had done so with her data collection section, and Archy had in his voting and finance sections. Jordan however, made this a routine priority, stating that this goes back to her own experience of learning mathematics through applications. Natalie's and Archy's instances of using direct applications to lead into iMoM-oriented goals may have resulted from the topics they were teaching lending themselves to MfM orientations. In other instances, they and Benjamin approached iMoM-oriented goals by following mathematical reasoning to make sense of a concept. In terms of belief systems then, experiences of struggling to learn concepts through theory alone may play a role in dictating short-term goals and beliefs regarding how mathematics is learned, in particular, raising the psychological strength of learning concepts generally through applications. This could then provide important implications for tailoring professional development efforts in different forms of active learning. For

instance, instructors with beliefs and experiences like Jordan may be more open to professional development efforts in project-based learning.

In Relation to the MfM. There was a notable difference regarding iMfM-oriented goals focused on the personal relevance of mathematics and sMfM-oriented critical citizenship goals that broke down along gender and identity. However, my discussion here will bring in Battey and Leyva's (2016) framework for talking about whiteness in mathematics education, and so I will first briefly overview their framework. From there, I will address the gender difference in how the previously mentioned MfM-oriented goals were approached.

Battey and Leyva (2016) focus on setting out a framework of understanding whiteness in mathematics education as a way to make whiteness transparent. They hold that by whiteness remaining historically invisible, it became the standard by which things are normatively considered neutral or objective. They go into literature on how white supremacy has developed in the US through law, sociologically, historically, and educationally (e.g., enslavement, disenfranchisement, minorities trying to become white by disassociating from blackness) such that white people, ideas, and values are valued institutionally and in the mathematics classroom as objective and neutral. Battey and Leyva argue that this institutionally sanctioned normalization makes whiteness invisible "while oppressing blackness through deficit frames, poor treatment, and lower quality instruction" (Battey & Leyva, 2016; p.55). Furthermore, they argue that colorblind policies in mathematics education may seem neutral but actually benefit whites because of past racist structures and the invisible whiteness such colorblind policies unknowingly adhere to.

Battey and Leyva's (2016) framework for discussing whiteness in mathematics education as a white institutional space consists of three factors: institutions, labor, and mathematical identity. Institutions can be examined on four dimensions for whiteness:

- *ideological discourses* at institutions such as discourses around racial hierarchies on achievement or mathematics as a neutral domain,
- *physical space* aspects of institutions such as imagery representing institutional values,
- *history* of an institution in building curricula around white elite norms and its resulting investment impacts, and
- *organizational logic* of an institution in determining power distribution (which may have roots in segregation and slavery).

Labor refers to the work minorities engage in to fit within neutral, colorblind educational norms and discourses. Labor can be examined on three dimensions for whiteness:

- *cognitive* work students are asked to take on by engaging in mathematical work (e.g., following procedural work robs minorities of cognitive depth),
- *emotional* work of managing resulting emotions from discrimination and deficit views in ways that appease white norms, and
- *behavioral* work of imitating white behaviors taken as the ideal or norm (while other behaviors are devalued).

Mathematical identity is taken as dispositions and beliefs about one's mathematical abilities and the use of those abilities to impact one's life. Mathematical identity can be examined on three dimensions for whiteness:

- *academic delegitimization* of underserved students who need to prove themselves or fit in to succeed while white students are assumed intelligent,
- institutional and peer *co-construction of meaning* regarding what it means to be good at math, and
- the *agency and resistance* underserved students exert to respond to co-constructed meanings of success (through stereotype management, disassociating from their race, etc.)

'Deficits' Challenging Mathematics as a White Space. The women in this study had contentious childhood identities and experiences with mathematics, while the men had cordial to appreciative childhood identities and experiences with mathematics. It was interesting then to find a gender difference in how instructors approached iMfM-oriented goals focused on the personal relevance of mathematics and sMfM-oriented community engagement goals. All the instructors described a need to focus on MfM-oriented goals of non-STEM specific relevance and/or civic community engagement. Often, these beliefs grew out of the instructors' realizations around how societal issues of race, gender, and class played a role in keeping underserved minorities and women from achieving in STEM. For instance, Benjamin stated,

> The idea that a large percentage of black Americans, because I mean that's really the largest demographic that we talk about as far as underprivileged and tend to have some of the worst success within mathematics of any demographic, and how can we get more people of African American heritage to take math classes and be successful in math classes, and how can we make math relevant to them?

As a slight aside before continuing, I took Benjamin's view of math as "relevant" as either iMfM- or sMfM-compatible since he went on to talk about cultural relevance, and it was not clear if this meant relevant to an individual (iMfM-oriented) or relevant to functioning in the African American community (sMfM-oriented). In the above quote, Benjamin explicitly connected the importance of community engagement and personal relevance-themed goals (sMfM- and iMfM-oriented) because of deep-seated issues around race. Similarly, Archy talked about realizing the importance of mathematics because of his pragmatist shift that resulted from seeing societal inequities. Natalie stated how her teacher credential program centered around "kids that hadn't succeeded in our school system, and we're kind of pushed down by our society. And so the whole focus was on you know things like gatekeepers … so I think that really informed how I teach," thus influencing her professional community engagement goals (sMfM-oriented). Additionally, Archy, Jordan, and Natalie all noted their students likely had troubled relationships with mathematics growing up, as evidenced by their focus on personal 'object' goals (personal relevance, iMfM-oriented) of building students' confidence.

Taking Battey and Leyva's (2016) institutional and identity dimensions, these instructors realized the organizational logic and history behind why underserved students fell behind and believed MfM-oriented goals would help underserved students to be successful. In this way, the instructors believed that MfM-oriented goals would provide opportunities for students to exert their *agency* in positive ways to succeed in mathematics, ways that perhaps undermine the racial hierarchies co-constructed by the organizational logic and history they recognize.

There was a noteworthy demarcation in translating these realizations into practice however. Within this study, the men instructors with historical identities rooted in mathematics expressed discomfort attending to the MfM-oriented goals, while the women instructors who had more contentious historical identities associated with mathematics felt confident attending to these MfM-oriented goals. For instance, Benjamin continued on his previous quote, "how can we get more people of African American heritage to take math classes and be successful in math classes, and how can we make math relevant to them? *I don't have an answer for that question* [emphasis added]." Benjamin noted the importance of MfM-oriented goals because of deep-seated racial issues, "but what I end up focusing on more, is simply that this subject [mathematics] is fascinating, … it doesn't have to have anything to do with the people around you, that it can just be its own safe haven" (tying to his MoM-oriented goals) because he did not have the resources to attend to such goals. Similarly, Archy noted many preparation struggles to meet MfM-oriented goals because his "concern is that like I don't have any material for this course and that like I'm just making it all up." Notably, Benjamin always had a curiosity around mathematics and Archy referred to himself as a "math person," and those around both men recognized and supported their mathematical interest. In contrast to the men in this study, the women instructors felt comfortable implementing MfM-oriented goals ranging from highlighting how mathematics connects to other topics (non-STEM specific relevance, Natalie) to showing how statistics play into other fields and broader culture (community engagement, Jordan). Notably, Jordan grew up struggling to learn mathematics in her childhood and later in a patriarchal college environment devaluing women. Natalie had a love-hate relationship with the

subject and went on to teach underrepresented students as part of a teacher certification program centered around "gatekeeper" issues and underserved students.

This distinction is stark: the instructors with positive identity-shaping experiences in mathematics during their childhood expressed discomfort attending to meaning orientations they believed to play an important role for underserved and marginalized students' learning. The instructors with contentious historical mathematical identities and experience with disenfranchisement felt comfortable attending to such goals. From the perspective of Battey and Leyva's (2016) identity component, the men come from backgrounds of academic legitimization. In contrast, Jordan experienced academic delegitimization due to her college environment perpetuating the notion that women are unsuited for STEM, a type of ideological discourse stemming from historical aims of education geared towards elite white men. Despite this, she exerted agency and resistance when she pushed back to attain her degree, even stating, "it pissed me off in a lot of ways and I wanted to change those views … I still am up against sexism constantly in my committees and in the work I do." Having taught in underserved communities for her teacher certification program, Natalie had seen her own students' experiences of academic delegitimization and how they exerted their agency and resistance in response.

Together, these findings suggest that if instructors lack personal experiences of academic delegitimization, they may come to identify with mathematics as the white institutional space it has been argued to be (Battey & Leyva, 2016). Personal experiences of academic delegitimization in this study range from experiencing long-term difficulty learning mathematics themselves to directly interacting with those who are delegitimized in sustained, immersive ways. This lack of personal experiences of academic

delegitimization then may lead instructors to internalize ideological discourses around the neutrality of mathematics and struggle to translate beliefs around helping underserved students into practice. This discomfort in attending to meaning orientations believed to be important for underserved student populations could imply that such instructors will struggle to decenter whiteness in mathematics education.

Summary. My first research question asked: what, in terms of background, affective factors, etc., leads instructors to prioritize certain meanings over other meanings? All the instructors had iMoM- and sMoM-oriented goals, despite any difference in their background. However, how instructors went about implementing those goals did break down along childhood success in mathematics. Jordan was the only instructor to have experienced significant hurdles learning mathematics growing up until she was able to see how mathematics could be applied in her undergraduate years. This then partly informed how she went about trying to spur students' understanding of mathematics (iMoM-oriented) through the use of applications (something seemingly sMfM- or iMfM- oriented). iMfM- and sMfM-oriented goals showed up across the instructors as well. Interestingly, the men instructors with historical experiences of academic legitimization struggled or avoided specific iMfM- and sMfM-oriented goals of community engagement and personal relevance because of an admitted lack of knowledge and resources to attend to such goals. The women instructors in this study, who had historical experiences with academic delegitimization, did not voice any self-efficacy issues in attending such sMfM- and iMfM-oriented goals. This might suggest that these experiences of academic delegitimization may have provided the women in this

study resources or a strong enough motivation to seek out resources to attend to such goals.

Oriented Goals and Manifestations

The second research question asked: How do instructors' orientations on mathematical meaning translate into specific pedagogical decisions? In the following sections, I will discuss how orientations were theorized to manifest in the literature review versus how they actually manifested across the instructors by going into each orientation individually.

Theorized sMoM. Instructional goals focused on enculturating students in the practices of the mathematics community indicate sMoM-oriented goals. Using Rasmussen et al.'s (2005) notions of mathematical activity, such practices could include symbolizing, generalizing, and defining. Additionally, symbolizing is heavily tied to notation, and Wenger (1998) talks about how communities of practice have their own jargon that community members use to communicate within their community. Notation is part of how the mathematics community communicates, and the associated practice of notating could thus also be an example of mathematical practice. Taking this perspective then, instructional decisions aiming to enculturate students into such practices or understand notation as a means of communication could signal sMoM-oriented goals.

Empirical sMoM. Initially, there were few explicitly stated instances that I classified under the sMoM orientation. Regarding Benjamin's conceptual understanding goals, Benjamin gave students a note sheet with various formulas and series tests (e.g.,

direct comparison) on an exam and a test question asked whether a given series converged. Determining this required working with series given in the note sheet and understanding domain restrictions. Benjamin stated, "setting up these inequalities and recognizing the difference between the direct comparison test and the squeeze theorem … if you can get comfortable with … these situations at this level, then anyone that's looking at a math minor and needs to take a 300-level analysis course is going be a little bit more prepared." In this, the conceptual understanding goal of highlighting the main parts of his storyline of content goals also had an associated side goal of preparing math majoring students for doing proofs. Proving is a practice theoretical mathematicians engage in and thus fit the sMoM orientation. However, as I mentioned in my aside about Benjamin's focus on proof in the doing math theme, Benjamin more often viewed proof as a way to check the structure of mathematics and thus proving tended to be iMoM-oriented more often than sMoM-oriented for him.

While the instructors' goals I placed under the conceptual understanding theme often focused on understanding concepts, there was the aspect of understanding notation for Archy, Jordan, and Benjamin that did stick out as a bit different (as mentioned in the Conceptual Understanding theme, 'In terms of the sMoM' subsection). Archy had stated, "we need to talk about notation and talk about how math people like make up notation so they don't have to speak so much." I observed Benjamin writing the equation of a tangent vector at a point and talking about the subscripts and postscripts with his students because for him, "math is a language, so I think notation is important. Like conventional notation." Jordan had her students interpret null and alternate hypotheses out in words to understand what the symbols were communicating. Within this and more prominently in

Archy and Benjamin's goals around notation, there was a focus on understanding how the mathematics community speaks. This can be seen in Archy's and Benjamin's prior comments where they want students to understand the basic conventions of that language. Taking Wenger's (1998) perspective on jargon in a community of practice, while the instructors were trying to get their students to understand the language of the mathematics community. Thus, I took the focus on notation for the reason of understanding the language of the mathematics community as sMoM-oriented. This does not negate such goals also having an iMoM-orientation when there was a focus on understanding notation conceptually.

Both understanding notation and preparing for proofs seemed to relate to how mathematicians operate and thus fit the sMoM orientation in some cases as initially defined. As I was looking across the instructors however, I found that the initial scope of the sMoM may have been too narrowly defined and/or how the instructors conceptualized practices of the mathematics community were too expansive. The latter may result from beliefs about what the goals of mathematics education are for the instructors' student populations (none of the classes were specifically for mathematics majors, most in fact were for non-STEM majors). Given the more expansive views instructors communicated, it was more accommodating of instructors' views to broaden the scope of the sMoM to look for goals that focused on *things that mathematicians do*. The modified conception still carried an underlying theme about the practices the mathematics community engages in, but it is not as fine-grained as initially theorized.

From this perspective of *things that mathematicians do,* Archy and Natalie's doing math themed goals fit the sMoM-orientation, as was mentioned in the Doing Math

section, ‘In terms of the sMoM’ subsection. For Archy, just as playing basketball is something one does for its own sake, mathematics then is something one does as well. There were instances where my more fine-grained definition did help orient some of the doing math themed goals. I had talked about how Natalie wanted her preservice teachers to make generalizations, which I posited as one manifestation perhaps of what reasoning entailed for Natalie. Generalizing fits with the ways Larsen (2013) typifies the practices of mathematicians while ‘reasoning’ is something one *does* as a characteristic of mathematics, according to Natalie. The former fits the more fine-grained definition of the sMoM orientation, while the latter fit the broader definition of the sMoM orientation. The research literature had suggested a limited scope to the sMoM-orientation. My study had refined that into a more nuanced construct.

Theorized sMfM. sMfM-oriented goals would focus on practices transferable to other (non-mathematics) communities, such as using mathematics to solve community problems (as in service-learning). Thus, one may suspect instructional decisions in the spirit of mathematics equity practice, critically relevant pedagogy, service learning, and problem-based learning to indicate sMfM-oriented goals. Doing routine calculations could also be a part of STEM community practices (Danielak, Gupta, & Elby, 2014). Thus, focusing on calculating work, or other applied calculations using mathematics could also indicate sMfM-oriented goals.

Empirical sMfM. There were many areas of alignment with the previously outlined scope and instructors’ community engagement goals in that Jordan, Benjamin, and Natalie had goals directed towards certain fields or professions. For example, Natalie often had students discuss and interpret each other’s solution strategies or her own sons’

mathematical thinking. Reasoning out her actions, Natalie stated that "making sense of student thinking is an important part of your work as a teacher," connecting these actions with the goal of having preservice teachers see how students learn and struggle with different concepts. Benjamin had exam questions using Taylor polynomials to approximate integrals. He stated his emphasis on this and other application tangents (from the main course content) was because of their use in specific STEM fields, like with Taylor polynomials being something "that's really important in physics." Jordan had students use SPSS to interpret data and often demonstrated calculator shortcuts (e.g., to calculate descriptive statistics). Jordan stated she made these instructional decisions because non-mathematics departments complained that theoretical content was inapplicable to their fields, and they instead wanted students to be exposed to other things, like statistical software. In turn, Jordan focused on looking at SPSS output and concentrating on if students "know how to correctly write the hypotheses and make the decision based off the p-value method, and not even have a really good concept of what a p-value is." In all the prior cases, instructors' goals were aimed at specific (non-mathematics) majors. By extension, this focuses on preparing students for life within certain non-mathematics professional communities and thus fit the sMfM orientation.

Other community engagement goals fit the sMfM orientation in that instructors' goals aimed to have their students function as informed members of a democratic society. Jordan for example, hoped that by going through their course, students would gain general quantitative literacy skills and result in "just being a better consumer in general. I don't know about better but just uh… yeah, more informed." As noted in a previous section, Jordan indicated she had been using the term 'informed consumer' in the literal

meaning of consumer but had also been thinking about informed citizenship and engagement in society. Archy had a section on voting and another on data collection because “math is incorporated in society. You're reading a news article, and it's telling you this, and it's influencing your opinion. Or like this is how voting works.” Specifically on the data collection section, he hoped his students would “come away [like], ‘wow, research is hard and like we should be skeptical when people like give us information and we should like, check their methods and stuff like that’” and thus promote critical thinking about things circulated in society. To this latter point, Archy had a follow-up assignment where students evaluated claims in a podcast. Both instructors’ community engagement goals aimed to enable students to navigate our democratic society as critically informed citizens. This still fits Wenger’s (1998) notions of being a practicing member of a community, though perhaps on a larger scale than Wenger may have conceptualized. In turn, I felt this focus on citizenship fit the sMfM orientation. Notably, Archy had students who did not need specialized mathematical knowledge. It may be for this reason that his sMfM-oriented goals were solely aimed at citizenship in contrast to the other instructors.

However, there were unforeseen differences, and these highlight the utility of the belief systems framework. During their data collection section, Natalie collected data on her students’ emotional well-being, had students engage data related to geology and geysers, and noted connections to social justice. Natalie also held a class discussion to explicitly call attention to how she noticed discomfort with reasoning triggered the preservice teachers to fall back on procedures and that they need to be self-aware of that. She explained that she did this to foster metacognition. Both these seem iMfM-oriented

(promoting personal interest and skills). Regarding the personal interests example however, she noted, “I care about them [preservice teachers] engaging our students and that's a way to engage.” Regarding metacognition, she noted, “as a teacher, you know, realizing what was helpful and what was unhelpful will help you help your students.” Because of the antecedent goals centered on functioning as teachers, I would classify these as sMfM-oriented. That is not to say there are no clustering, antecedent, or preceding iMfM-oriented goals, but it may suggest that the sMfM-oriented goals have more psychological strength behind them.

Theorized iMfM. For the iMfM, my framework suggests goals and decisions would aim for individual outcomes separable from mathematics itself. Content delivery could range from lecturing to problem-based learning. Problems could focus on real-world problems (e.g., in STEM, social inequalities) all to convey personal relevance. Since some level of understanding is necessary to work on applications, thus instructors may devote attention to conceptual understanding for later use in applied problems. Knowing content or imparting skills for future tests or courses or for skills in and of themselves are also separable goals from mathematics, and thus could be iMfM-oriented as well.

Empirical iMfM. Goals related to real-world problems did occur to an extent within the non-STEM specific relevance subtheme of the personal relevance theme. Archy included a section on personal finance because “this pertains to you because this is how taxes work or this is how finances work.” He even spent class time asking students where interest applied in their lives and included a personal expenditure tracking activity. In her data collection section, Natalie stated,

> Math is more than just- I mean teaching is more than just teaching the topics but also another example of how you can connect to the other types of thinking…I think the same way that we have metacognition about our math, kids also need metacognition about their emotions and realizing ways to do it.

The prior instances involved instructors aiming to show how mathematics can be applied to other aspects of individuals' lives, from their finances to their emotional well-being.

As theorized from the literature, there were also goals centered around skills and students' academics, as was shown with the personal 'objects' subtheme of the personal relevance theme. Archy, Jordan, and Natalie wanted their students to have confidence in mathematics and other cognitive skills (e.g., metacognitive, collaborative, resilience skills), while Benjamin wanted his students to have algebra skills "that they can carry on and actually utilize in their lives" however which way the individual student chooses. Benjamin stated that such skills open up opportunities, but students need to make those skills useful for themselves. For similar skill-related reasons, Jordan implemented group work because "working with people is a good life skill. In a job and a career, you're going to be having to work usually with other people." All these are skills instructors aimed for students to have for their personal lives and thus fit the iMfM orientation. Other times, the instructors wanted students to 'have' certain academic deliverables such as good grades (e.g., Jordan's focus on 'dumbing down' the mathematics for student success) or knowledge of content (e.g., Jordan doing minilectures and Benjamin cutting down on student engagement to cover content). From academic deliverables to skillsets, all these are things instructors aimed for students to have for their own personal lives and thus fit the iMfM orientation.

However, there were some unforeseen differences. Non-STEM specific relevance goals focusing on how mathematics can be personally motivating to students would seem to be iMfM-oriented but sometimes were not. For instance, Natalie wanted to show preservice teachers how to connect mathematics to other subjects for the preservice teachers' own students: "math is going to be more meaningful for them if it's connected. And, you know, kids ask this 'when am I ever going to use this' question because math is just this weird like busywork thing." Here, Natalie was not trying to motivate preservice teachers per se, but rather enable the preservice teachers to motivate their students and thus better function as teachers. Thus, Natalie was using non-STEM specific relevance goals to help the preservice teachers in their professional practice (sMfM-oriented). Similarly, I did not hold Natalie's instance of critical thinking skills as iMfM-oriented. Natalie talked about the fast pace of advancement in society and how "You're trying to train- prepare kids to have jobs that don't even exist but who knows what skills they're gonna need. but critical thinking is obviously something they're going to need." Here, Natalie's goals are not directed at preparing her students for general careers; she was focused on getting her preservice teachers to focus on preparing their students for general careers. Both these instances then aimed to prepare preservice teachers for the teaching profession; thus, as opposed to being iMfM-oriented, I classified these as sMfM-oriented. These examples highlight the utility of the 2nd dimension of the belief systems framework since it allows one goal to have multiple antecedent goals (and thus multiple orientations) behind them.

Theorized iMoM. An iMoM orientation was characterized as focusing on understanding and interest in mathematical concepts themselves. This orientation may

occur by instructors choosing to lecture and cover how ideas develop and relate to one another. Instructors could also engage students in group work, student presentations, or inquiry if they believe students will make connections better through such decisions. Instructors may even use real-life examples (seemingly iMfM-oriented) to motivate interest in mathematics (Butler, 2013). The defining theme is the goal of building understanding or appreciation of mathematics itself.

Empirical iMoM. Many of the conceptual understanding themed goals were iMoM-oriented, as would be suspected from how the iMoM orientation was defined. This was specifically because of all the statements about understanding mathematical concepts and how concepts connect that showed up across the instructors. For instance, Jordan implemented active learning through a hybrid model of instruction to "help us [her students] understand what's going on in that bigger picture." Natalie wanted to show that math is not just a bunch of individual concepts, but rather "bigger ideas." One instance she associated with this was her introduction of different kinds of graphs during the data collection section. She started by having students make pictographs of data. The emphasis she placed on lining things up aimed to foster the leap from pictographs to dot plots and then to bar graphs and histograms, thus showing students how these forms of graphs built off each other. She stated, "that's a logical progression … the logical flow of the field of math". In trying to show how mathematics is 'fun', Archy wanted students to go through arithmetic sequences, then their partial sums, and then "relating all problems that surround the sequence of partial sums coming from an arithmetic sequence, … to this one problem [sum of the 1st n numbers], so there's like this *sweet connection* [emphasis added]." Benjamin's aims of conveying two overarching 'storylines' in his class also fit

within these prior examples as his goal here was counter to just teaching a bunch of individual concepts. In all the prior, there was a focus on having students understand mathematical concepts themselves and how they connect to other mathematical concepts. This fits with how the iMoM orientation was hypotheses to look.

As a slight aside, for Jordan and Natalie, these iMoM-oriented goals manifested in the use of more active learning techniques. For instance, Jordan mentioned how she implemented technology and active learning because "I wanted to … help students learn better" and see the "bigger picture." Natalie had students engage in reconstructing mathematical knowledge through routine think-pair-share cycles partly because she wanted to show "there aren't all these different individual concepts to learn. that there's bigger ideas." Across both these instructors, they are resorting to active learning techniques as a way to build understanding. For Archy and Benjamin, these goals manifested in the types of content they presented to their students. For Archy, this involved devoting a section to mathematical fun in which students could make connections between mathematical concepts. For Benjamin, this manifested in his push to convey two overarching storylines of content in the course. This also showed up for Jordan in how she routinely used real-life examples to introduce concepts when she stated,

> If I only have theory with no applications, I never really understood it very well and so, yes, in some way, I'm teaching by what not to do from how I learned it … I don't think any of my students are going to be thriving on theory alone, or theory at all.

Across these examples, instructors focused on presenting material to have students see connections and/or understand the mathematics being discussed.

There was a strand within the conceptual understanding theme that was a bit different from the notions pushed earlier but still iMoM-oriented. Archy and Jordan both talked about how they would need to build an understanding of base concepts so that they could then continue onto the content they wanted to teach. For instance, Archy's stated, "I want to like get some basic facts about a thing" as the first step in his lessons. These basic facts could be a basic understanding of notation ("we need to talk about notation") or of concepts ("we were supposed to be talking about the derivative of a line, now it's just like, okay no, if I give you an equation, why is that equation look like a picture?"). Similarly, Jordan mentioned how she would do minilectures on base concepts, formulas, and notation at the beginning of class so students could engage in the day's activities. Both instructors often mentioned such goals because their students were non-STEM majoring students who may not be comfortable with or have a good grasp of mathematics. The notion of building understanding of concepts remains the same here as in the previous paragraph, so there was still an iMoM-orientation to these goals. However, the nuance here is that in building a basic understanding, instructors can move on to hit other goals. In this way, these iMoM-oriented goals are derivative goals of other oriented goals or have other orientations in addition to an iMoM orientation.

Within the conceptual understanding theme, there were larger differences from what was hypothesized that highlighting the importance of belief systems. I had noted how Benjamin aimed to convey two overarching storylines. One storyline was 3d geometry, trigonometry, parametric equations, and vectors because part of his job was "to

make sure students are really prepared for fourth term calculus where they're actually doing a lot of calculus in three dimensions." Based on the literature review, this would seem like an iMfM orientation (individual preparation for doing well). However, Benjamin firmly believed that students need to make mathematics relevant for themselves. His job was to teach the concepts within the storyline of course content, which may then connect to concepts in other courses. This still prioritized looking at how mathematics connects together, and thus I felt it still had an iMoM orientation. I had also noted Archy, Jordan, and Benjamin had subgoals of focusing on notation. Oftentimes, this fits the sMoM-orientation as hypothesized since it could be connected to understanding notation as a form of communication. However, I had not addressed understanding notation 'conceptually'. For instance, Jordan had her students interpret null and alternate hypotheses out in words to understand what kinds of decisions would be made based on a statistical test. While there is some focus on understanding how mathematicians communicate, Jordan wanted her students to understand mathematical notation to understand what symbols convey and interpret their implications and thus focused on making connections. As such, this would also have an iMoM-orientation, something I had not initially considered.

Another difference was in Benjamin's doing mathematics goals focused on proofs. I had mentioned how Benjamin's focus on the proof process was not on the action of proving itself but more so on what a proof proves or disproves. I cited his statement about his own use of proof by contradiction: "I remember like not knowing how something worked, so just trying a few examples with numbers…then that would give me some insight on whether or not I could say that there was a rule." He also showed proofs

for theorems and results in his overarching stories when the proofs were useful for building understanding of the storyline of content, as opposed to when he would skip over a proof because "it's a very kind of left-field proof that's not... that I don't think is something that's really helpful in building a conceptual understanding of what's going on." So for Benjamin, the process of proving was about building understanding of concepts and thus fit the iMoM-orientation.

Summary. My second research question asked: How do instructors' orientations on mathematical meaning translate into specific pedagogical decisions? Many of the ways orientations were theorized to manifest instructionally did hold. Some examples from the instructors in my study included using active learning to promote greater understanding of mathematical concepts (iMoM-oriented), trying to promote the acquisition of skills and/or academic deliverables for students' personal use (iMfM-oriented), and engaging students in exercises relevant to non-mathematics communities (sMfM-oriented). However, there were instances in which the orientation constructs were nuanced or re-examined in light of instructors' justifications for their actions. These included moments of instructors using proof as a means to understand the structure of mathematical concepts (iMoM-oriented) and instructors trying to get students to do mathematics as an activity (sMoM-oriented).

Contextual Influences

The third research question asked: how does context influence the meanings instructors emphasize? In the discussion that follows, I will first overview Ingram and Clay's (2000) framework of new institutionalism to frame my discussion around how context influences the meanings instructors emphasize. Instructors had commented about

how student demographics, departments, the university, and society more broadly impacted their goals. So after summarizing the framework, I will fit the contextual influences instructors mentioned into aspects of Ingram and Clay's framework. I will then use the framework to discuss the research question around context and meaning orientations.

New Institutionalism. Ingram and Clay's (2000) theory of new institutionalism is a framework focused on how individuals are actors within institutions (i.e., constraints). They outline various tenets of their theory. The first tenet is that actors are rationally bounded by limited knowledge and cognitive capacity. The 2nd tenet is that institutions are defined as the rules and enforcement mechanisms that constrain actors' choices. It is important to note that institutions can be formal rules, regulations, and law enforcement (falling within what Ingram and Clay define as public-centralized institutions) to norms and expectations enforced by inclusionary and exclusionary behaviors by individual people (falling within what Ingram and Clay define as private-decentralized institutions). Ingram and Clay hold that this would, by extension, include beliefs and shared understandings as constraints individuals or collectives may impose on others. The last tenet is that institutions constrain actors towards goals (e.g., the collective good or the personal goals of powerful actors).

Ingram and Clay's framework assumes some basic axioms. The first is that "actors pursue their interests by making choices within constraints" (p. 525-526). They define three types of actors, individuals, organizations, and states, each of which produces their own kind of institutional constraint on other actors. Ingram and Clay state that "actors lead a double life in the new institutionalism, pursuing their own interests

within constraints, while producing constraints for other actors" (p. 527). I mention this to highlight that actors *make* institutions, the rules that constrain other actors. In this, I interpret Ingram and Clay to be taking constraints and institutions to be the same thing. Lastly, they assume that actors make choices that further their interests, limited by the amount of information actors can know and process (this being their rationally bounded assumption).

My Base Assumptions and Lines of Inquiry. My understanding of 'context' is broader than just the institutions Ingram and Clay (2000) identify. Instructors had commented about how student demographics, departments, the university, and society more broadly have impacted their goals. Academic departments, the university, and society more broadly fit Ingram and Clay's framework well in that they are all organizational actors that produce their own constraints (i.e., institutions) on other actors or are institutions themselves created by other actors (e.g., society being the institution of the U.S. state actor). In this way, actors' policies and initiatives are part of the resulting institutions. However, student demographics are not rules or constraints; they are an outcome of policies the university and departmental actors pursue. In this way, the people who come into a class are just a characteristic or outcome of institutions. Students are part of the university organization (as well as individual actors), but student demographics are a trait then of the university (or state) actor.

For the purposes of this discussion then, I define context as the actors (besides the instructors themselves), the institutions the non-instructor actors create, and the characteristics of non-instructor actors and their institutions. From this standpoint, I take academic departments (e.g., the mathematics department or other serviced departments)

and the university as organizational actors and U.S. society as an institution resulting from state and organizational actors (e.g., the U.S. government). Given how the instructors talked about student demographics in reference to the students within their classroom and/or within larger U.S. society more generally, I take student demographics as a characteristic of the university and departmental actors and/or state actors, resulting from the institutions those actors created. In the following sections then, I will separate my discussion into how student demographic characteristics, departments and the university actors, and the U.S. societal institution influenced the meanings instructors emphasized. While it may be odd to compare a student demographic characteristic to actual actors to institutions, I follow this organization because this division of influences was mentioned enough by instructors as themes that they deserved their own sections. In each section, I will have subsections for each orientation discussing how that orientation came to be focused on.

Student Demographics. Institutions have put students in instructors' classes which creates part of the context instructors work within. The institutions here are the norms, expectations, and rules the university, departmental, and/or societal actors create regarding different student populations. These include expectations about what instructors are to achieve given their student populations to what classes different student populations should enroll in and opens questions about how instructors deal with and meet those expectations. In this section, I will talk about how student demographics have influenced instructors to prioritize or not prioritize certain orientations on meaning. As an important aside given my data, I only have instructors' perceived constraints.

In terms of the sMfM. sMfM-oriented goals would focus on enculturating students into practices transferable to other (non-mathematics) communities. It was interesting to find that all the instructors besides Archy had sMfM-oriented goals largely directed towards their students' majors or careers, while Archy's sMfM-oriented goals were solely directed at civic engagement. For instance, Natalie had goals of showing how mathematics is connected (conceptual understanding theme) to prepare preservice teachers to teach better, Jordan had goals of exposing students to statistical software they would use in their careers, and Benjamin aimed to show subplots of the overarching storylines that were relevant to particular STEM fields (community engagement theme). By contrast, Archy did not have any goals directed at specific professions, but he did have goals of getting students critical of publicly circulated information that served as his motivation for his data collection section (community engagement theme).

This difference between Archy and the other instructors may be because the other courses had been tailored to specific student populations: Natalie's to preservice teachers, Jordan's to health science and psychology majors, and Benjamin to STEM majors. Archy's course aimed more towards students who simply needed to meet general college mathematics course requirements and thus may have attracted students who would rather avoid mathematics if they could. Such students would likely not need specialized mathematical knowledge or skills for any specific careers. Combined with Archy's own concern for student buy-in, it seems very sensible that Archy's sMfM goals would be directed more broadly towards civic engagement. Considering sMfM-oriented goals broadly though, it seems that institutions (in the form of expectations) around student populations nuanced how instructors focus on sMfM-oriented goals. Having a student

demographic with relatively homogeneous majors to deal with may create institutions that steer instructors' sMfM-oriented goals towards a particular profession. On the other hand, instructors perceived institutions (in the form of expectations) around students in general quantitative courses fulfilling university requirements may push instructors more towards civic engagement type sMfM-oriented goals.

In terms of the iMfM. iMfM-oriented goals would aim to convey the relevance of mathematics separate from mathematics itself. Another type of goal that was prominently voiced in classes with non-STEM majoring students was personal 'objects' focused on building confidence. For instance, Natalie hoped her preservice teachers would "have the confidence to think through problems. I want them to have the grit to think through problems. To realize … they need to help kids to feel perseverance and grit." Jordan talked about how she wanted to get her students "used to statistics and not being afraid of it" because "they just are second-guessing themselves and they just don't have the confidence. So it's like, 'yeah, no, you're doing great. That's exactly it.'" Archy had hoped that by the end of his course, his students "got greater confidence in dealing with numbers in their life, whether they be finance or just otherwise. So yeah, so greater comfort and confidence." In all of these, there is a theme of getting students the personal 'object' of comfort and confidence with mathematics.

Notably, these instructors had made statements about their student populations having an engrained discomfort with mathematics that instructors connected to the confidence-building goals mentioned before. For instance, Natalie stated,

> I just think everybody can do math and people just got the bad shake of it, and I want to show people they can, and I want to help them help their students realize

> that, you know. I just want to change … how people feel about math in general. And it comes back, I guess, all to this whole like, being a child psychologist and wanting kids to have confidence and knowing they can do it.

Natalie had also stated this in connection to a comment that "there's something wrong with our society right? Like if you ask a class, if you ask- provoke people, how many of you can do math? People proudly say they can't do math." When I asked Jordan about her focus on addressing students' math anxiety and comfort, she noted that some of her colleagues who have received their PhD and had a lot of success in mathematics "don't have the empathy, maybe, to understand those students that are struggling. And so when they teach a service course for non-math-stat majors, it's really hard for them to connect and understand the level." Throughout these examples, the instructors focus on these confidence and comfort personal 'object' iMfM-oriented goals because they expect their non-STEM majoring students to have a history of struggling with mathematics. These expectations and beliefs seemed to be about non-STEM majoring students in society more broadly, not just the students at their university, as demonstrated by Natalie's comment that "there's something wrong with our society … people proudly say they can't do math." In this way, instructors focused on confidence and comfort iMfM-oriented goals for non-STEM majoring students because of the U.S. society institution's characteristics.

In terms of the iMoM. The iMoM orientation was characterized as focusing on understanding and promoting interest in mathematical concepts themselves. There were instances in which student demographics pushed instructors towards conceptual understanding iMoM-oriented goals, and these again center around student demographics

of non-mathematics majors. Both Archy and Jordan had goals of building understanding of base concepts (iMoM-oriented to a degree) within their larger goals. Archy talked about having flexible lesson plans, partly because students could get stuck on notation, conventions, and base concepts that are part of the initial foundation for any lesson. This occurred for instance, when students got stuck on notation and thus could not discuss the Koch snowflake stretch problem Archy had planned or when

> Like where we were supposed to be talking about the derivative of a line, now it's just like, okay no, if I give you an equation, why is that equation look like a picture? And that is the now, that's the new lesson.

Similarly, Jordan talked about going over foundational knowledge through giving handouts and minilectures to having students interpret null and alternative hypotheses out loud because "the only prerequisite for this course is second-year high school algebra, but the problem is that it's not time enforced... it's not that they don't know, it's just that they haven't recalled it [the prerequisite knowledge] in so long." Archy's need for flexibility in light of possible confusions tied to the fact that most of his students were non-mathematics majors who did not like mathematics and experienced discomfort with the subject. Jordan had been going over building understanding of base concepts because her student population had often not seen the knowledge in so long. In both cases, the instructors focused on goals previously discussed as iMoM-oriented because the university actor created expectations and norms (institutions) of who enrolls in such courses.

In terms of the sMoM. Instructional goals focused on enculturating students into the mathematics community's practices indicate sMoM-oriented goals. There were

instances in which student demographics pushed instructors towards conceptual understanding or doing math sMoM-oriented goals. For instance, I had noted how Archy and Jordan had goals of understanding notation (partly sMoM-oriented) within their building understanding of base concepts goals. Archy talked about how sometimes "we need to talk about notation and talk about how math people like make up notation so they don't have to speak so much." As mentioned in the last paragraph, these understanding goals were due to their non-mathematics majoring student demographics who did not have comfort with or recent experience with mathematics. In these sorts of instances, the instructors focused on goals previously discussed as sMoM-oriented because the university and departmental actors created expectations and norms (institutions) of who enrolls in such courses.

Departmental and University Actors. Academic departments within a university would constitute organizational actors. So in this study, the mathematics department was one organizational actor, as were other serviced departments (such as the psychology department for Jordan). These organizational actors are within a larger organizational actor, the university. In this section, I will talk about how these different actors have influenced instructors to prioritize or not prioritize certain orientations on meaning. As an important aside given my data, I only have instructors' perceived constraints (i.e., beliefs about institutions) that result from these actors' influence.

In terms of the sMfM. sMfM-oriented goals would focus on enculturating students into practices transferable to other (non-mathematics) communities. Departmental and university actors often played a role in influencing Jordan, Benjamin, and Natalie to focus on community engagement sMfM-oriented goals. The most

prominent example of this was Natalie in that all of what she did (from the routine order of class to the sequencing of problems) “are for them (the preservice teachers) to see what a different type of math class would look like and to have that buy-in.” Jordan talked about how non-mathematics departments “want us to make this class easier so students pass it more” and how she would have rather avoided using SPSS, but “we are required to use that software because of the partnership that we have with the psychology and other social science departments, and they want their students to have experience on SPSS.” Benjamin talked about curvature and its connections to electronics in class with his students, though he did not go into it deeply. When asked why he touched on it when it was such a small part of just one class period, he stated, “I'm required to do curvature” by the mathematics department.

As discussed in prior analyses, these community engagement goals were all directed at preparing students for life within specific professions and thus sMfM-oriented. Jordan and Benjamin both highlighted how serviced and mathematics department actors have pushed them to focus on these sMfM-oriented goals. In Natalie's case, she had not explicitly noted any actors influencing her focus, but the mathematics department and university actors had set up institutions around the creation of the course and what the course was intended for: the preparation of preservice teachers for their careers (sMfM-oriented). In this way, the instructors focused on goals previously discussed as sMfM-oriented because the university and departmental actors created expectations and norms (institutions) of who enrolls in such courses. A lot of this overlaps with the previous discussion on student demographics as a characteristic of the university and departmental actors.

Notably, some instructors (Archy and Benjamin) felt like departmental and university actors did not provide scaffolding for community engagement goals, specifically those of the civic engagement variety. Benjamin had no goals in this category and explicitly stated this when he talked about how he does not know how to "get more people of African-American heritage to take math classes and be successful in math classes, and how can we make math relevant to them." He went on to state,

> That's not actually in the curriculum anywhere as far as what I'm supposed to teach. I almost wish it was. … as much as I want to answer the question, I don't have an answer, and I don't think I'm qualified to answer that question. No matter what answer I do come up with, not everyone's going to agree with whatever conclusion I try and come up with. And I almost don't want to take responsibility for trying to answer that question correctly. and I kind of want, like higher-ups within the school to make those decisions for me and be like, 'this is what we need to do, this is what you don't need to do.'

It was unclear how Benjamin meant relevance here as he went on to talk about relevance in terms of "cultural relevance", which could be iMfM-oriented if culturally relevant meant relevant on an individual level or sMfM-oriented if culturally relevant meant relevant on a community engagement level for Benjamin. In turn, I will take this as iMfM- and sMfM-compatible (sMfM compatible is relevant to the conversation here). Similarly, Archy noted many preparation struggles to meet his civic community engagement goals because his "concern is that like I don't have any material for this course and that like I'm just making it all up." In the above quotes, both instructors noted the importance of non-STEM-specific relevance goals. However, they struggled or

avoided attending to such goals because they did not have departmental or university-provided resources or support to meet such goals. Altogether then, the lack of institutions from the departmental and university actors left Archy and Benjamin unable to confidently meet iMfM-oriented goals when they lacked any resources themselves to attend to such goals. This is notable because many instructors talked about the importance of such goals. However, Archy and Benjamin's statements suggest that coming from a certain background and lacking supportive institutions makes meeting these recognized needs difficult to translate into actionable goals.

In terms of the iMfM. iMfM-oriented goals would aim to convey the relevance of mathematics separate from mathematics itself. Departmental and university actors often played a role in influencing Jordan, Benjamin, and Natalie to focus on personal relevance iMfM-oriented goals, specifically those of the academic personal 'objects' variety. For instance, Jordan talked about how service departments and the mathematics departments had been negotiating changes in the course to improve DFW rates (iMfM-oriented):

> We've even talked about a syllabus change for coming up year where we take out a lot of the probability because that's not as applicable to a lot of these majors anymore… so we're looking at moving forward by even, I don't want to say dumb down the course even more, but we are. We're looking at like how we can improve student success by not teaching any of the math concepts within the probability chapters.

Content coverage goals (often iMfM-oriented) also figured prominently for Jordan and Benjamin. This was demonstrated for instance, when Benjamin said he would want more student involvement but could not "because of the amount of content that I'm looking at

covering over the course of the term" as dictated by the mathematics department. While Archy did not express any content coverage goals, he had expressed concerns over the amount of material he covered being viewed negatively if the mathematics department compared his instruction to that of another instructor. In these examples, the instructors focused on personal 'object' goals previously discussed as iMfM-oriented because the university and departmental actors had created institutions about material students need to have seen. This constraint was significant enough to even impact an instructor when iMfM-oriented content coverage goals were not present, as was the case with Archy.

Notably, some instructors felt like departmental and university actors did not provide scaffolding for personal relevance goals, specifically the subgroup of non-STEM specific relevance. Archy and Benjamin had difficulty attending to such goals when they dealt with non-STEM topics. Benjamin had no goals in this category and explicitly stated this when he talked about how he does not know how to "get more people of African-American heritage to take math classes and be successful in math classes, and how can we make math relevant to them." As mentioned in the Departmental and University actors section above, I will take this as iMfM- and sMfM-compatible (iMfM-compatible is relevant to the conversation here). Similarly, Archy noted many preparation struggles to meet his personal relevance goals (as well as his societal relevance goals). Both instructors noted the importance of non-STEM-specific relevance goals. However, the lack of institutions from the departmental and university actors left Archy and Benjamin unable to confidently meet iMfM-oriented goals when they lacked any resources themselves to attend to such goals. This is notable because many instructors talked about the importance of such goals, but Archy and Benjamin's statements suggest that coming

from a certain background and lacking supportive institutions makes meeting these recognized needs difficult to translate into actionable goals.

Societal Institution. I previously talked about how I would take the U.S. society as an institution resulting from state and organizational actors. Since there are so many organizational and state actors that have contributed to the creation of U.S. society (from the government to political organizations), this conversation only focuses on how instructors were influenced by the U.S. society institution as a whole. In this section, I will talk about how different parts of this societal institution have influenced instructors to prioritize or not prioritize certain orientations on meaning. As an important aside given my data, I only have instructors' perceived constraints (i.e., beliefs about institutions).

STEM, Inequality, the iMfM and the sMfM. A notable theme was how instructors' beliefs and views about the underserved in society played into their orientations. Archy and Natalie in particular, noted the need to focus on personal relevance and community engagement iMfM- and sMfM-oriented goals due to factors in a STEM-centered, often unjust society. For instance, Natalie talked about how her teacher certification program played a role in how "I think being able to help people succeed and not get trapped by that gatekeeper [of mathematics] is important." By imparting mathematical knowledge for teaching (community engagement, sMfM-oriented), she was "hoping that they'll [the preservice teachers will] open up that gate." Similarly, Archy talked about how his realization around injustices and inequalities in our society resulted in a pragmatist worldview where mathematics was a way of upliftment in our society. This led him to see the importance of mathematics's societal and personal relevance, which he then tried to meet through his personal relevance and community

engagement iMfM- and sMfM-oriented goals (like his data collection section). As an aside, he had expressed many self-efficacy concerns in meeting such goals because of his limited knowledge and available resources. Notably, Benjamin voiced similar concerns,

> The idea that a large percentage of black Americans, because I mean that's really the largest demographic that we talk about as far as underprivileged and tend to have some of the worst success within mathematics of any demographic, and how can we get more people of African-American heritage to take math classes and be successful in math classes, and how can we make math relevant to them? I don't have an answer for that question.

Part of Benjamin's reason for the absence of goals like Archy's personal relevance and civic community engagement goals was that he also felt like he did not have the knowledge or resources to do so.

In terms of addressing a STEM-centered society, Natalie talked about the fast pace of advancement in society and how "You're trying to train- prepare kids to have jobs that don't even exist but who knows what skills they're gonna need. But critical thinking is obviously something they're going to need." Natalie noted this need for her preservice teachers to impart critical thinking to their students (community engagement, sMfM-oriented) because of the rapid pace of advancement in this STEM-centered society. Similarly, Archy expanded on his pragmatist motivated personal relevance and community engagement goals (iMfM- and sMfM-oriented) when he stated that in this society, "you get the benefit of the doubt if you studied math. So you should study math because you need to take care of yourself and like, you need to improve your station." Across these examples, instructors recognize the STEM-centered focus of the U.S.

society institution as well as its history of inequality and disenfranchisement. These recognitions then pushed instructors towards iMfM- and sMfM- oriented goals.

Expectations of Confidence and the iMfM. One notable belief Archy, Jordan, and Natalie voiced that they attributed to the societal institution was their expectation of widespread math anxiety and confidence issues amongst many students in this society. In turn, they aimed to promote confidence within their personal relevance goals (iMfM-oriented). I noted how Natalie hoped her preservice teachers would "have the confidence to think through problems," Jordan wanting to get her students "used to statistics and not being afraid of it," and Archy hoping that his students "got greater confidence in dealing with numbers in their life." These statements about their student populations having an engrained discomfort with mathematics were often the justification for the confidence-building goals mentioned before. For instance, Natalie stated,

> I just think everybody can do math and people just got the bad shake of it, and I want to show people they can, and I want to help them help their students realize that, you know. I just want to change … how people feel about math in general. And it comes back, I guess, all to this whole like being a child psychologist and wanting kids to have confidence and knowing they can do it.

Natalie had also stated this in connection to a comment that "there's something wrong with our society right? Like if you ask a class, if you ask- provoke people, how many of you can do math? People proudly say they can't do math." When I asked Jordan about her focus on addressing students' math anxiety and comfort, she noted that some of her colleagues who have received their Ph.D. and have had a lot of success in mathematics "don't have the empathy, maybe, to understand those students that are struggling. And so

when they teach a service course for non-math-stat majors, it's really hard for them to connect and understand the level." In these examples, the instructors focus on these confidence and comfort personal 'object', iMfM-oriented goals because they expect non-STEM majoring students to have a history of struggling with mathematics. These expectations and beliefs seemed to be about non-STEM majoring students in society more broadly, not just the students at their university, as demonstrated by Natalie's comment that "there's something wrong with our society … people proudly say they can't do math." In this way, instructors focused on confidence and comfort iMfM-oriented goals because of the U.S. society institution's characteristics.

Summary. My third research question asked: how does context influence the meanings instructors emphasize? For some instructors, non-mathematics majoring student demographics as a characteristic of the U.S. society institution and/or university and departmental actors influenced instructors' conceptual understanding goals of understanding notation and base concepts (iMoM- and sMoM-oriented). However, these same characteristic, actor, and institutional influences were more often associated with instructors' personal relevance and professional community engagement sMfM- and iMfM-oriented goals.

Responding to Students

The fourth research question asked: How do backgrounds, contexts, and/orientations come together in pedagogical decisions with respect to how instructors respond to students' meaningfulness questions (e.g., "when is this ever going to be useful?", "why are we learning this?"; refer to Appendix A, question 2 for the question as I posed it to instructors). Generally, the instructors had interpreted these sorts of

questions from the standpoint of students asking for the purpose, usefulness and/or relevance of what they were learning, all terms that I have taken to signal students asking for something 'meaningful'. For instance, when I asked, "if a student were to ask you like, when is this stuff ever going to be useful? You're kind of in a particularly interesting situation teaching teachers", Natalie talked about "in this class, it's funny because the whole point is to teach them useful math. It's not just about being able to do math," because the point of the class was to teach preservice teachers how to teach mathematics. When I posed the meaningfulness question (refer to Appendix A, question 2 for the question as I posed it to instructors), Archy immediately jumped to "everything we're doing in math is gonna sort out in one of these three ways" (his personal relevance, societal relevance, and math as fun instructional goals) in response to the question while Jordan stated, "I always, on the last day, emphasize to them that they may not use this specific statistic in their life going forward but … we hope that they become more educated consumers and not get fooled by statistics." In response to my question, Benjamin talked about how mathematics does not have to be relevant because 'whatever tool, whatever skill you learn how to use is only useful in the context of how you want to try and utilize it… It's not necessarily relevant unless you want it to be." Given the prevalence of how instructors responded with talking about what is useful or relevant across their responses, the instructors construed questions like "when is this ever going to be useful?", "why are we learning this?", etc. as students asking for the purpose, usefulness and/or relevance of what they were learning, and thus their interpretation of these sorts of questions fits the meaningfulness question as I have outlined in my introduction.

However, there was a division in how instructors responded to these questions for meaningfulness. Some instructors talked about answering questions for meaningfulness and then how their instructional goals aligned with how they would respond. Other instructors noted that they did not personally care about answering students' meaningfulness questions and went on to talk about what they focused on instead (which could occasionally still address these meaningfulness questions). The discussion that follows will first focus on those instructors who tried to answer these meaningfulness questions within their instructional goals (in the 'personally responsive' section below) versus those who did not personally care about answering these meaningfulness questions (in the 'impersonally responsive' section below). From there, I will talk about themes in both groups and then how background and context impacted how instructors responded to meaningfulness questions.

Personally Responsive. The first group of instructors responded to the meaningfulness question in the ways they interpreted students to be asking the question and then talked about how their instructional goals tried to address such questions. I called this group personally responsive because instructors in this group were different from those who did not personally care about the meaningfulness question in how they interpreted students to be asking the question.

The instructors who fell into this category were Archy and Jordan. In response to students' meaningfulness questions, Archy immediately jumped to how his instructional goals fit into this question when he stated, "everything we're doing in math is gonna sort out in one of these three ways" of personal relevance (personal relevance theme, iMfM- and sMfM-oriented), societal relevance (community engagement theme, iMfM- and

sMfM-oriented), and math as fun (conceptual understanding and doing math themes, sMoM- and iMoM-oriented). For Archy then, there was a direct correspondence he drew between the ways to answer the meaningfulness question and his instructional goals. In response to the meaningfulness question, Jordan stated that by getting students

> the comfort of visually looking at graphs and looking at data that we hope that they become more educated consumers and not get fooled by statistics. And that's my goal, is that they can understand that that statistic was manipulated by a politician. Or if it was an ad from, you know, from some type of company … So I try to bring in articles where those statistics are used.

Not being "manipulated by a politician" fit the civic engagement theme (community engagement, sMfM-oriented), "comfort" fit the personal 'objects' theme (personal relevance, iMfM-oriented), and buying from an ad fit the non-STEM specific relevance theme (personal relevance, iMfM-oriented). Jordan did attend to these as goals, manifesting through the kinds of examples she would pick like introducing topics by referring to research, politics, and society (action associated with community engagement goals, sMfM-oriented) to her focus on group work partly to build students' confidence (action associate with personal relevance goals, iMfM-oriented). Jordan did not feel like she had attended to civic engagement themed goals as well as she could have because "I could have pulled in more media, current media. I didn't do that so much as just rely on some old sources just cuz I was focused on other projects." However, Jordan's main goals fell within the conceptual understanding (iMfM- and iMoM-oriented), professional engagement (community engagement, sMfM-oriented), and personal 'objects' themes (personal relevance; iMoM, iMfM). So unlike Archy, there was not always a direct

correspondence between how Jordan answered the meaningfulness question and her main goals. However, she did make efforts to attend to the meaningfulness question within her larger goals.

Impersonally Responsive. The second group of instructors did not personally care about the meaningfulness question in the ways they interpreted students to be asking the question. Despite this lack of personal investment or interest, these instructors did occasionally have instructional goals that they connected to students' meaningfulness questions. Thus, such instructors were responsive to the meaningfulness question, but personal investment in answering the question was more variable than the former group.

The instructors who fell into this category were Benjamin and Natalie. Both instructors voiced opinions and beliefs against responding to the meaningfulness question in the ways they interpreted students to be asking the question. In response to the meaningfulness question and me describing my interest in answering that question, Natalie stated, "I just like math for its own sake. Like it's funny to me when you're having this whole conversation about it being useful because that's not even particularly important to me." Benjamin held that "It's [mathematics] not [relevant] … it doesn't have to be. It's going to be as useful as you want it to be," because he held that students relearn how to use mathematics in their intended careers and he did not have the time or knowledge to attend to such things, as demonstrated by his comments that "people come up and show me things that-, where math is used … and I'm always fascinated. I'm like, 'that's so cool. I wish I had the time and energy to go and like study that specific thing more.'" In the previous, Benjamin and Natalie each talked about how the meaningfulness question is not a priority for them personally because they were not personally interested

in the meaningfulness question in the way they interpret students to ask the question (Natalie) and/or they felt like it was not their responsibility because of other beliefs, such as leaving students to answer meaningfulness questions for themselves (Benjamin).

Despite this lack of personal investment, both instructors had sMfM- and iMfM-oriented instructional goals that they believed did answer students' meaningfulness questions. For instance, Natalie talked about the whole point of the class was to talk about mathematics which is important to teachers. She stated,

> In this class, it's funny because the whole point is to teach them useful math…I think of myself more as answering the question, 'why are we doing it this way? Why aren't you just telling us to do it?' And I guess part of my, one of my answers would be you know, this is how people learn.

All her class goals (all of which had an sMfM orientation *in addition* to other orientations) and associated instructional practice manifestations, from the group-class discussion cycles to interpretation of others' work, could thus be connected to students' meaningfulness questions. This even manifested in her preparing preservice teachers to answer *their future students*' meaningfulness questions when she focused on non-STEM specific relevance goals (personal relevance, sMfM-oriented) of how mathematics connects to other subjects,

> I don't care where math is relevant. I think it's fun. I care about them engaging their students and that's a way to engage their students. And so- I mean, I don't necessarily care about- I have fun to just doing math. But I want to make sure that

> they make whatever- they might not have fun just doing math. They might not be able to make their kids have fun just doing math.

Benjamin did not have as many goals he explicitly connected to students' meaningfulness questions because of his views about students answering the meaningfulness question for themselves but also because of his own limited knowledge, as demonstrated when he talked about not knowing how to "get more people of African-American heritage to take math classes and be successful in math classes, and how can we make math relevant to them." However, he did explicitly connect his personal 'objects' goals (personal relevance, iMfM-oriented) of promoting algebra skills to this when he stated, "maybe they'll have spent this much extra time being forced to do hard algebra that that will be a skill that they can carry on and actually utilize in their lives." Benjamin held that gaining fluency in algebra skills would be one way to enable students to make mathematics meaningful, by making algebra relevant (i.e., meaningful) to students personally.

Cross-Cutting Trends. A noteworthy aside across both groups is the realization that sMfM- and iMfM-oriented goals connect to what instructors believe students seek in meaningfulness questions. There were connections to instructors' background and context that played a role in how they approached such goals (if they did at all) and instructors teaching self-efficacy to meet such goals. These connections will be discussed in the following sections. Another point of discussion is the place of sMoM- and iMoM-oriented goals in reference to this question. On answering the meaningfulness question for themselves, the impersonally responsive instructors all redirected towards iMoM- and sMoM-compatible beliefs (e.g., appreciating mathematics for its own sake) and not caring about answering the meaningfulness question on a personal level. Add to this was

that Archy also voiced iMoM- and sMoM-compatible beliefs (e.g., mathematics is fun) in response to the meaningfulness question alongside other orientation compatible beliefs.

Only Jordan did not mention iMoM- or sMoM-compatible beliefs in response to students' meaningfulness questions that I posed to her. In response to my question on this, she stated views that aligned with the personal relevance and community engagement themes:

> I always, on the last day, emphasize to them that they may not use this specific statistic in their life going forward, but with the comfort of visually looking at graphs and looking at data that we hope that they become more educated consumers and not get fooled by statistics. And that's my goal, is that they can understand that that statistic was manipulated by a politician. Or if it was an ad from, you know, from some type of company … So I try to bring in articles where those statistics are used.

The comments about 'not being fooled by politicians' fit the civic community engagement theme (sMfM-compatible) while buying something from an ad fits the personal relevance theme (iMfM-compatible). Looking at this difference then, it was notable that all the other instructors had childhood experiences of academic success with mathematics. Often, that success was accompanied by an interest in the subject, though that could be complicated with Natalie having a love-hate relationship with it before college. Jordan did not have that sort of experience growing up. What that may suggest then is that experiences of early academic success in mathematics, and perhaps personal enjoyment during childhood, may lead to a more central belief in the importance of the iMoM and sMoM in response or contradiction to the meaningfulness question. That does

not necessarily mean that individuals who experience less success in mathematics growing up do not value MoM-oriented goals if they go on to teach though. One of Jordan's biggest goals was promoting student success (conceptual understanding theme, iMoM-oriented) after all. However, the fact that she did not state anything seemingly iMoM- and/or sMoM-compatible in response to the meaningfulness question may indicate that MoM-oriented goals may simply not have much psychological strength in relation to the meaningfulness question.

Despite this difference in childhood success in mathematics and how they responded to the meaningfulness question, it was difficult to find any overarching themes in how the MoM orientations connect to the meaningfulness question in the instructors' instructional practice. Jordan and Benjamin seemed most focused on conceptual understanding themed goals (often iMoM-oriented) which did not connect to answering the meaningfulness question. Archy had a direct correspondence between the meaningfulness question and his instructional goals, including sMoM- and iMoM-oriented goals. Natalie had conceptual understanding and doing mathematics goals (iMoM- and sMoM-oriented) aiding her larger community engagement goals (largely sMfM-oriented) which she saw as meaningful for her preservice teachers. All the instructors did have iMoM- and/or sMoM-oriented goals, but how those connected to the meaningfulness question did not break down in any clear patterns. My thoughts on this are that this lack of any patterns has to do with the student populations these courses are directed towards, and not as much with individual instructor traits. This brings up questions about how context influences the relationship between answering the

meaningfulness question and instructional goals. This will be discussed in the section on context below.

Background and Meeting Meaningfulness. As was noted in the Background and Orientations section, ‘‘Deficits’ Challenging Mathematics as a White Space’ subsection, one trend that stuck out to me was a gender gap in teaching self-efficacy to answer students’ meaningfulness questions. All the instructors believed sMfM- and iMfM-oriented goals of non-STEM specific relevance and/or civic engagement could connect to the meaningfulness question. Often, these beliefs grew out of the instructors’ realizations around how societal issues of race, gender, and class played a role in keeping underserved minorities and women from achieving in STEM. Additionally, Archy, Jordan, and Natalie all noted their students likely had troubled relationships with mathematics growing up, as evidenced by their focus on personal ‘object’ goals (personal relevance, iMfM-oriented) of building students’ confidence.

It was interesting then to find a gender difference in how instructors approached iMfM-oriented goals focused on the personal relevance of mathematics and sMfM-oriented goals of community engagement and the goals they connected to students’ meaningfulness questions. In this study, the men instructors with historical identities rooted in mathematics (e.g. identifying as ‘math people’) expressed discomfort attending to these MfM-oriented goals, while the women instructors who had more contentious historical identities associated with mathematics did not voice concerns about their ability to attend to these MfM-oriented goals, which ranged from highlighting how mathematics connects to other topics (non-STEM specific relevance, Natalie) to showing how statistics play into other fields and broader culture (community engagement, Jordan).

This distinction is stark: the instructors with positive identity-shaping experiences in mathematics during their childhood expressed discomfort attending to meaning orientations they believed connect to students' meaningfulness questions and play an important role for underserved and marginalized students' learning. The instructors with contentious historical mathematical identities and experience with disenfranchisement felt comfortable attending to such goals. From the perspective of Battey and Leyva's (2016) identity component, the men come from backgrounds of academic legitimization while the women either experienced academic delegitimization (e.g. Jordan's sexist college environment, against which she exerted her own agency and resistance) or taught in underserved communities that were academically delegitimized (e.g. Natalie's experiences with her middle school students).

Together, these findings suggest that if instructors lack personal experiences of academic delegitimization, they may come to identify with mathematics as the white institutional space it has been argued to be (Battey & Leyva, 2016). This lack of personal experiences of academic delegitimization then may lead instructors to internalize ideological discourses around the neutrality of math (as was the case with Benjamin) and struggle to translate beliefs around the meaningfulness question into practice. The implication then is that if instructors believe answering the meaningfulness question will help open space for underserved students to succeed in mathematics, being able to answer the meaningfulness question may be a way to decenter mathematics education as a white space. However, an instructor's inability or discomfort to attend to the meaningfulness question instructionally may imply that such instructors will struggle to decenter whiteness in mathematics education.

Context and Meeting Meaningfulness. Compounding on the previous discussion is the role of institutions and actors on various levels. As I mentioned before, all the instructors believed sMfM- and iMfM-oriented goals could connect to the meaningfulness question. For some instructors, these beliefs grew out of the instructors' realizations around how societal issues of race, gender, and class played a role in keeping underserved minorities and women from achieving in STEM. Benjamin not knowing how to make mathematics relevant to African American students, Natalie showing students a new kind of mathematics classroom informed by gatekeeper issues, and Archy focusing on civic engagement and personal relevance themed goals because of his pragmatist shift were all examples of instructors connecting the meaningfulness question to issues of race and social inequalities. Social inequalities and racism are part of the institution created by U.S. state and organizational actors, and so these instructors are deriving these answers to the meaningfulness question in response to a characteristic of the U.S. society institution.

From there however, the absence of institutions from the departmental and university actors came in. Notably, Archy and Benjamin felt like departmental and university actors did not provide scaffolding for non-STEM specific relevance (personal relevance, iMfM-oriented) and civic engagement (community engagement, iMfM- and sMfM-oriented) goals the instructors believed connect to the meaningfulness question. Benjamin had no goals in this category and explicitly stated this when he talked about how he does not know how to "get more people of African-American heritage to take math classes and be successful in math classes, and how can we make math relevant to them." As mentioned in the Departmental and University actors section above, I will take this as iMfM- and sMfM-compatible (iMfM-compatible is relevant to the conversation

here). Similarly, Archy noted many preparation struggles to meet his personal relevance goals (as well as his societal relevance goals). Both instructors noted the importance of non-STEM-specific relevance goals. However, the lack of institutions from the departmental and university actors left Archy and Benjamin unable to confidently meet iMfM-oriented goals when they lacked any resources themselves to attend to such goals. This is notable because many instructors talked about the importance of such goals, but Archy and Benjamin's statements suggest that coming from a certain background and lacking supportive institutions makes meeting these recognized needs difficult to translate into actionable goals.

There is a separate conversation about context regarding the sMoM- and iMoM-orientations. I had mentioned before how it was difficult to find any overarching themes in how the MoM orientations connect to the meaningfulness question in the instructors' instructional practice. Jordan and Benjamin seemed most focused on conceptual understanding themed goals (often iMoM-oriented) which did not connect to answering the meaningfulness question. There were occasional connections made through chains of derivative, less central beliefs, but that suggests there was distance between the instructors' main instructional priorities and the meaningfulness question. Archy had a direct correspondence between the meaningfulness question and his instructional goals, which included sMoM- and iMoM-oriented goals. Natalie had conceptual understanding and doing mathematics goals (iMoM- and sMoM-oriented) aiding her larger community engagement goals (largely sMfM-oriented) which she did see as meaningful for preservice teachers. All the instructors did have iMoM- and/or sMoM-oriented goals, but

how those connected to the meaningfulness question did not break down in any clear patterns.

My thoughts on this are that this lack of any patterns has to do with the student populations these courses are directed towards, and not as much with individual instructor traits. I had looked at instructors of very different courses with very different expected student populations ranging from STEM majors to non-STEM majors. None of these courses were directed specifically towards mathematics majors. As such, the context of each course I was observing was informed by institutions that were not directed towards mathematics majors. Had I observed an instructor of a course intended for mathematics majors (e.g., abstract algebra or numerical analysis), an instructor in that context might focus primarily on MoM-oriented goals and draw strong connections between those goals and students' meaningfulness questions. However, if an instructor were like the impersonally responsive instructors in my study, it's also possible that instructors in such a course would be able to enact their personal preferences against caring about the meaningfulness question and frame their goals in opposition to answering the meaningfulness question. These are just hypotheses however.

Altogether, the institutions around the courses I observed were directed at predominantly non-mathematics majoring student populations. These institutions include departmental and university decisions around the kinds of students who enter each course, expectations around follow-up courses or the lack thereof, and what students are expected to gain due to the previously noted factors. Because of these student demographic factors and institutions, it was no wonder that iMoM- and sMoM-oriented

goals did not connect to the meaningfulness question in any clear patterns for the instructors in this study.

Summary. My fourth research question asked: How do backgrounds, contexts, and/orientations come together in pedagogical decisions with respect to how instructors respond to student questions like: "when is this ever going to be useful?", "why are we learning this?", etc. Generally, the instructors had interpreted these sorts of questions from the standpoint of students asking for the purpose, usefulness and/or relevance of what they were learning, all terms that I have taken to signal students asking for something 'meaningful'. Whether instructors cared about answering these meaningfulness questions for themselves varied.

Despite this variance, all instructors drew connections between some of their oriented instructional goals and students' meaningfulness questions. One trend was that all instructors realized that sMfM- and iMfM-oriented community engagement goals and personal relevance connect to what instructors believe students seek in meaningfulness questions. These realizations often grew out of social inequities in the U.S. society institution. There was a notable demarcation however, on instructors' sense of self-efficacy to enact such goals. The men, coming from a historical experience of academic legitimization, avoided or faced difficulty in enacting these community engagement and personal relevance goals (sMfM- and iMfM-oriented) because they felt they lacked the personal knowledge and scaffolding institutions to meet such goals. The women in this study, who had experiences with academic delegitimization, did not voice any self-efficacy concerns in meeting such goals.

The relation between instructors' sMoM- and iMoM-oriented goals and their response to students' meaningfulness questions was harder to characterize. All the instructors had iMoM- and sMoM-oriented goals of promoting conceptual understanding or doing mathematics, but how those connected to the meaningfulness question did not break down in any clear patterns. I suspect this has to do with the student populations these courses were directed towards. None of these courses were directed specifically towards mathematics majors. As such, the context of each course I was observing was informed by institutions directed towards non-mathematics majors. Had I looked at courses directed towards mathematics majors, patterns could arise in how iMoM- and sMoM-oriented goals relate to instructors' response to students' meaningfulness questions.

Chapter 6: Conclusion

The purpose of this study was to see how undergraduate instructors see their instruction as meaningful or not to their students. Additionally, I was interested in what contextual and affective factors influence how instructors think of their instruction as meaningful. In the following sections, I will summarize what I set out to learn, what I learned about what is 'meaningful' in instruction, why the findings in this study are important, and where I could go next in future research.

Study Aims and Guiding Framework

This work opened with student questions such as 'why's this important?', 'when will we ever need/use this?', 'why are we doing this?', etc. I interpreted these sorts of questions as students asking about the purpose or aim of the mathematics education they receive and that students are looking for something meaningful, important, or relevant to them when they ask such questions. As such, I grouped these questions together under an umbrella term I called the meaningfulness question(s). Such questions can put an onus on instructors to make mathematics meaningful to their students. This study's focus was to better understand how instructors view their instructional goals as meaningful for students and how those goals influence instructors' pedagogical practices.

I viewed instructors' goals as being a specific category of beliefs regarding what instructors should cover in a class and that those instructional goals will vary according to instructors' background and context, as proposed by Leatham (2006). I viewed instructors' beliefs about what is meaningful or important in mathematics as antecedent

beliefs that orient instructors' goals. To frame what instructors believe to be meaningful, I used Brownell's (1947) initial distinction and later social perspectives on learning to synthesize a hypothesized framework of how the term 'meaningful' can be construed in mathematics education. This framework delineated a *social meaning of mathematics* (sMoM), a *social meaning for mathematics* (sMfM), an *individual meaning of mathematics* (iMoM), and an *individual meaning for mathematics* (iMfM). The iMoM follows Brownell and Thompson's view of meaning as conceptual understanding of mathematical ideas. The sMoM takes meaning as engaging in practices of the mathematics community. The iMfM takes meaningful as understanding the relevance of mathematics for separable outcomes for individuals (e.g., personal relevance, passing exams, indeterminate career plans). The sMfM takes meaningful as engaging in practices that are transferable to non-mathematics communities.

To then look at how instructors see their instruction as meaningful or not and whether answering the meaningfulness question relates to their instructional goals in any way, I asked the following research questions:

1. What, in terms of background, affective factors, etc., leads instructors to prioritize certain meanings over other meanings?
2. How do instructors' orientations of mathematical meaning translate into specific pedagogical decisions?
3. How does context (e.g., student population, lack or prevalence of student interactions, course, etc.) influence the meanings instructors emphasize?

4. How do backgrounds, contexts, and orientations come together in pedagogical decisions with respect to how instructors respond to students' meaningfulness questions (e.g., "when is this ever going to be useful?", "why are we learning this?")

Findings

In the following sections, I will discuss what I learned from this study. The first section will discuss things I had learned in terms of my research questions. The second section will focus on what I learned throughout the process of conducting this work.

'Meaningful' Instruction

Given the overarching aim of my work was to understand how instructors respond to the meaningfulness question, how instructors see their instruction as meaningful, and factors that play into that dynamic, concluding remarks centered around my fourth research question seem relevant here. The fourth research question asked: How do backgrounds, contexts, and/orientations come together in pedagogical decisions with respect to how instructors respond to students' meaningfulness questions and see their instruction as meaningful (if at all). Given the prevalence of how instructors responded with talking about what is useful or relevant across their responses, the instructors construed questions like "when is this ever going to be useful?", "why are we learning this?", etc. as students asking for the purpose, usefulness and/or relevance of what they

were learning. Thus, their interpretation of these sorts of questions fits the meaningfulness question as I have outlined in my introduction.

Despite differences in personal investment in the meaningfulness question, all the instructors in my study believed that community engagement and personal relevance goals I classified as sMfM- and iMfM-oriented connected to what instructors believed students sought in meaningfulness questions. Sometimes, these beliefs often grew out of the instructors' realizations around how societal issues of race, gender, and class played a role in keeping underserved minorities and women from academic achievement, and they believed MfM-oriented goals would help these students be successful.

There were connections to instructors' background and context that played a role in how they approached such goals (if they did at all) and instructors teaching self-efficacy to meet goals associated with students' meaningfulness questions. The men instructors with historical identities rooted in mathematics expressed discomfort attending to these MfM-oriented goals. For instance, Benjamin felt more comfortable focusing on mathematics for its own sake (tying to his MoM-oriented goals) because he did not have the resources to attend to such goals while Archy noted many preparation struggles to meet MfM-oriented goals because his "concern is that like, I don't have any material for this course and that like I'm just making it all up." Notably, Benjamin always had a curiosity about mathematics, Archy referred to himself as a "math person", and those around both men recognized and supported their mathematical interests. In contrast, the women instructors in this study who had more contentious historical identities associated with mathematics did not voice concerns about their ability to attend to these MfM-oriented goals.ranging from highlighting how mathematics connects to other topics

(non-STEM specific relevance, Natalie) to showing how statistics play into other fields and broader culture (community engagement, Jordan). Notably, Jordan struggled to learn mathematics in her childhood and later in a patriarchal college environment devaluing women. Natalie had a love-hate relationship with the subject and went on to teach underrepresented students as part of a teacher certification program centered around "gatekeeper" issues and underserved students.

Compounding on the previous discussion is the role of institutions and actors on various levels. As I mentioned before, all the instructors believed sMfM- and iMfM-oriented goals could connect to the meaningfulness question. For some instructors, these beliefs grew out of the instructors' realizations around how societal issues of race, gender, and class played a role in keeping underserved minorities and women from achieving in STEM. Social inequalities and racism are part of the U.S. societal institution created by U.S. state and organizational actors, and so these instructors are deriving these answers to the meaningfulness question in response to a characteristic of the U.S. society institution. From there however, the lack of institutions from the departmental and university actors left Archy and Benjamin unable to confidently meet iMfM- and sMfM-oriented goals they believed connect to students' meaningfulness questions when they lacked the resources and background themselves to attend to such goals. Archy and Benjamin's statements might suggest that coming from a certain background and lacking supportive institutions makes meeting these recognized needs difficult to translate into actionable goals.

Another point of discussion is the place of sMoM- and iMoM-oriented goals in reference to students' meaningfulness questions. Despite differences in childhood success

in mathematics and how they personally responded to the meaningfulness question, it was difficult to find any overarching themes in how the MoM orientations connect to the meaningfulness question in the instructors' instructional practice. All the instructors had iMoM- and/or sMoM-oriented goals, but how those connected to the meaningfulness question did not break down in any clear patterns. This lack of patterns might have to do with the student populations these courses are directed towards and not as much with individual instructor traits. None of these courses were directed specifically towards mathematics majors. As such, the context of each course I was observing was informed by institutions that were directed at predominantly non-mathematics majoring student populations. These institutions include departmental and university decisions around the kinds of students who enter each course, expectations around follow-up courses or the lack thereof, and what students are expected to gain due to the previously outlined factors. Because of these student demographic factors and institutions, it was no wonder that iMoM- and sMoM-oriented goals did not connect to the meaningfulness question in any clear patterns for the instructors in this study.

Process

There were two main things I learned throughout the process of conducting this work: 1) there was a need to reevaluate how I differentiate a goal; the goals, reasons, and beliefs behind that goal; and actions stemming from the goal, and 2) Leatham's (2006) framework was more of an organizational tool to facilitate my analysis process.

In terms of the first point, I realized there was a needed change in how I categorized things as either a goal; the goals, reasons, and beliefs behind a goal; and actions stemming from the goal in the course of my work. For instance, I had talked about how Gregson's (2007, April) focus on an 8th-grade teacher's implementation of an equity-oriented mathematics program was MfM-oriented because of the explicitly stated goals of college preparation, community connection, and preparation of students to work in social justice. Before data collection, I had seen the implementation of an equity-oriented mathematics program as an action and the explicitly stated goals of college preparation, community connection, and preparation of students to work in social justice as the goals behind the action. In the course of my work however, I started to see that the implementation of an equity-oriented mathematics program could be *an action as well as a goal*, and the explicitly stated goals of college preparation, community connection, and preparation of students to work in social justice as the *reasons* behind the action and goal (which could be goals themselves because Leatham's belief systems framework allows that). The reason for this double classification possibility was because this exactly happened with the goals of instructors in my study. For instance, Natalie and Jordan ran their classes in very active learning fashions. Jordan's use of active engagement was an actionable manifestation of Jordan's conceptual understanding goal, while Natalie had active engagement as part of her community engagement goals (showing a different kind of class) and an actionable manifestation of those community engagement goals.

Jordan's use of active learning is an action, while Natalie's use of active learning is a goal and an action. This had implications for how meaning orientations were attributed because, for Jordan, active learning would not be given an orientation but

would be a result of an oriented goal. For Natalie however, active learning as a goal could be oriented. This difference in applying the meaning orientations for Natalie but not Jordan was because I wanted to avoid classifying actions according to the meaning orientations framework. While one could talk about actions as having orientations, I had not conceptualized how actions fit into Leatham's (2006) framework and thus stuck to only attributing orientations to what I considered beliefs.

My other realization was how Leatham's (2006) framework was more of an organizational tool to facilitate my analysis process rather than being a central piece of my work and findings. The reasons for this go back to Kuntz's neoliberal critique of research I had brought up in my methods section. Kuntz (2015) talked about how traditional research practices, like interviews, "aim to both uproot (or extract) partial meanings and then delegitimize all else as excessive or outside the scope of the inquiry project" (p. 54) to create 'moveable objects.' Such practices also render experience as a linear development towards coherent, definable goals. This linear progression was exactly what Leatham's framework led me to do in terms of talking about goals, the reasons behind those goals, and the actions resulting from goals. The concise 'moveable object' created was my renditions of instructors' hypothesized belief systems.

In contradiction to this, Kuntz (2015) advocates that all data be treated as being in a state of becoming. Methodologists are not to seek some complete, clear meaning, but rather prod "the affective assemblages of being, maintaining the contradictions and blurred connections of being without normalizing their meaning" (p. 79). This then foregoes the attempt to achieve clarity (in my study, a solid presentable rendition of instructors' belief system) in favor of accepting a state of flux where new interpretations,

critiques, and ways of being remain open to emerging. As a result, while I had initially felt Leatham's (2006) framework was amenable to Kuntz's critique, I realized that for that to remain true, I had to not place as much focus on my rendered belief systems as products of this work. Thus, I avoided making the belief systems the main discussion points in my individual and cross-case analysis sections. I also did this by acknowledging that the renditions I made were only a hypothesized snapshot in time and that the connections and relationships I had drawn between beliefs were likely to shift over time. This idea of beliefs and justifications shifting was also something Benjamin indicated about his beliefs as well, and so I had a data-driven reason not to emphasize the hypothetical belief systems I constructed.

However, I do not mean to undermine the importance of Leatham's (2006) framework for my work. While I came to realize the creation of a hypothesized belief system fell into Kuntz's extractivist critiques, I still valued its role in helping me start my analysis. In particular, the framework's tenets were very amenable to creating concept maps of how background, beliefs, and goals connect, which served as my initial step in starting my analysis. It was also amenable to the incorporation of my meaning framework to orient beliefs since antecedent beliefs (second dimension of Leatham's framework) helped to determine the orientation of goals. Thus, while Leatham's (2006) framework may not have been the focus of my work, it was a useful analytical tool to begin my analysis process.

Importance of Findings

This work opened with student questions such as ‘why’s this important?’, ‘when will we ever need/use this?’, ‘why are we doing this?’, etc. I interpreted these sorts of questions as students asking about the purpose or aim of the mathematics education they receive and that students are looking for something meaningful, important, or relevant to them when they ask such questions. As such, I grouped these questions together under an umbrella term I called the meaningfulness question(s). It was notable that the kinds of questions I subsumed under the meaningfulness question and how instructors interpreted students’ meaningfulness questions related to aspects of student motivation as discussed by Jones and Wilkins (2013). Jones and Wilkins discuss a five-part model for student motivation: empowerment, usefulness, success (the belief that one can do well), interest (situational and/or individual), and caring (teacher care of students academically and personally). My conceptualization of the meaningfulness question as students asking about the purpose or aim of the mathematics education they receive and that students are looking for something meaningful, important, or relevant aligns with Jones and Wilkins’ usefulness and interest motivational factors.

Similarly, given how instructors responded with talking about what is useful or relevant across their responses, the instructors’ interpretation of questions like “when is this ever going to be useful?” and “why are we learning this?” fit the meaningfulness question as outlined in my introduction. By extension, instructors’ interpretation of and response to such questions relates to Jones and Wilkins’ usefulness and interest motivational factors. The importance of this is that by relating to significant aspects of student motivation as outlined in prior research, instructors’ interpretation of and ability

to address the meaningfulness question has important implications for instructors' ability to motivate students' learning through the usefulness and interest motivational factors.

My study suggests that instructors' ability to attend to these motivational factors by addressing the meaningfulness question with sMfM- and iMfM-oriented goals is partly dependent on instructors' background and surrounding context. Specifically, my work suggests that many instructors talked about the importance of such oriented goals and the meaningfulness question because of societal inequities. They realized the need to address the meaningfulness question because of the math anxiety and achievement issues underserved students face and realized the power of answering the meaningfulness question in motivating students to help them achieve, something prior research supports (Gay, 2000; Ladson-Billings 1994; Tate 1995). However, Archy and Benjamin's self-efficacy concerns and the lack thereof on the part of Natalie and Jordan might suggest that coming from a mathematically privileged background (of academic legitimization) and lacking supportive institutions makes meeting these recognized needs difficult to translate into actionable goals. This then could have consequences for instructors' ability to support students from underserved backgrounds in that instructors with a history of academic legitimization may struggle to motivate underserved students, at least in attempts to tap on student motivation through usefulness and/or interest not related to mathematical content or activity in and of themselves.

An instructor's ability to attend to usefulness and interest motivational factors by addressing the meaningfulness question with sMoM- and iMoM-oriented goals is more complicated. This will be addressed in the directions for future research section.

Another important insight from my work relates to the facilitation of my work here. In my 'Process" subsection of my findings section above, I noted the shift in how I used Leatham's (2006) belief systems framework, specifically shifting away from using such a framework to create concrete representations of instructors' belief systems. The reasons for this go back to Kuntz's (2015) neoliberal critique of research to create 'moveable objects.' The concise 'moveable object' created was my hypothesized renditions of instructors' belief systems. In contradiction to this, Kuntz (2015) advocates that all data be treated as being in a state of becoming. This then foregoes the attempt to achieve clarity (in my study, a solid presentable rendition of instructors' belief system) in favor of accepting a state of flux where new interpretations, critiques, and ways of being remain open to emerging. Benjamin's prior statements about flipping on his statements in my 'Process' subsection above demonstrated the incomplete perspective researchers have into the inner world of instructors' beliefs and thus provide narrative support to Kuntz's position on accepting a state of flux.

To try to mitigate these extractivist tendencies, I fell back on a pragmatic stance of using Leatham's (2006) framework only as an operationalizing tool to begin my analysis while fully acknowledging the subjectivity of that analysis (i.e., acknowledging that belief systems are simply organizational tools to help me structure my subjective sense-making). Afterwards, I directed attention away from the framework by decentering their emphasis in my write up. More broadly then, my work may provide ways to endorse a more critical view towards extractivist ways of thinking and conducting research that frameworks like Leatham's entail, while also positing their use in a pragmatic, subjective sense.

Future Work

As was foregrounded in prior paragraphs, there were fewer patterns in how instructors address the meaningfulness question with sMoM- and iMoM-oriented goals than with sMfM- and iMfM-oriented goals. In some cases, iMoM- and sMoM-oriented goals were framed in opposition to the meaningfulness question (e.g., Benjamin talking about mathematics having its own inherent beauty). Sometimes iMoM- and sMoM-oriented goals were framed as answers to the meaningfulness question (e.g., Archy's doing math goals) or related to other orientations that do answer the meaningfulness question (e.g., Natalie's conceptual understanding and doing math goals supporting her community engagement goals). All the instructors did have iMoM- and/or sMoM-oriented goals, but how those connected to the meaningfulness question did not break down in any clear patterns.

One important consideration in the above is that all the courses I solicited instructors from were not intended for mathematics majors. Of those, only one course was intended for STEM majors. Had I observed an instructor of a course intended for mathematics majors (e.g., abstract algebra or numerical analysis), an instructor in that context might focus primarily on MoM-oriented goals and draw strong connections between those goals and students' meaningfulness questions. However, if an instructor were like the impersonally responsive instructors in my study, it's also possible that those instructors would be able to enact their personal preferences against caring about the meaningfulness question and frame their goals in opposition to answering the meaningfulness question. However, these are just hypotheses and an area for future work

to explore. Just as my findings connecting iMfM- and sMfM-oriented goals and the meaningfulness question could have implications for instructors' ability to motivate student learning, work connecting iMoM- and sMoM-oriented goals and the meaningfulness question in the context of mathematics major-specific courses could have similar implications for instructors' ability to motivate math majoring students' learning.

Another direction for future work is how goals with different orientations compete and/or bolster one another's psychological strength as priorities instructors pursue (i.e., compete and/or bolster one another's psychological strength in an instructors' belief system). Similarly, there are open questions about how those different orientations compete and/or reconcile themselves within their shared goal for goals with multiple orientations. Some hints at these interactions were found in this study. For instance, Jordan's conceptual understanding goals and academic personal 'objects' goals (within personal relevance) seemed to have orientations that were at odds with each other. She wanted students to have a 'bigger picture' understanding of mathematics (iMoM-oriented), but this contradicted the departmental push to 'dumb down' the mathematics to promote student success (iMfM-oriented). Jordan then went about trying to attend to both goals. On the other hand, Natalie had oriented goals that seemed to go hand in hand in bolstering one another, such as her iMfM-oriented personal 'object' goals of building confidence and her sMfM-oriented community engagement goals of preparing preservice teachers for their practice.

This study had not deeply explored how such tension and synchronicity in oriented instructional priorities impacts instructors, their practice, and the meaningfulness question. For instance, I had not deeply examined how the existence of conflicting

oriented goals along with supportive and unsupportive contexts impacts instructors' ability and self-efficacy to attend to conflicting goals and the meaningfulness question. These are directions for future work building off the findings in this study.

Lastly, this study had not looked at how students view instructors' attempts to make class meaningful and whether they feel like their instructors have succeeded (if they are even aware instructors have attempted to address the meaningfulness question). This study focused solely on the instructors, their background, and context. Future work can look at how students perceive instructors' oriented goals and if they even understand instructors' goals to have the orientations instructors had intended.

Limitations

I acknowledge that the whole effort I have engaged in, despite my best efforts to respect the instructors as full, complicated individuals, still involves the separation of individual aspects from human individuals themselves, and thus I fall victim to the extraction logic Kuntz (2015) critiques. While I have taken a pragmatic stance on using Leatham's (2006) framework solely as a subjective analysis facilitation tool, the framework itself proved to very extractive. As the sole researcher on this project and the level of immersion Kuntz's alternatives demand, I could only take minor steps towards such goals. These included decentering the belief systems I posited to using video clip interviews to build a shared understanding with instructors. From there, I aimed to document complexities that arose across instructors' perspectives by retaining instructors' own ways of thinking (Chapter 4) of the cross-case themes (Chapter 5). By

respecting such complexities, I hoped to provide a more nuanced view about the generalizations I drew and thus open up my findings to critique and discussion.

Summary

In summary, I wanted to understand how instructors see their instruction as meaningful. I had set out a framework on how the term 'meaning' can be understood in mathematics education, with the perspective that when students ask for something meaningful (i.e., the meaningfulness question), instructors can respond with one of the four orientations on meaning as an answer.

What I found was that the meaning orientations showed up in both ways I did and did not expect, and all of the orientations could factor into instructors' response to students' meaningfulness questions. There were two notable subplots within this larger observation. First, the instructors in my study could frame iMoM- and sMoM-oriented goals in opposition to or in consonance with the meaningfulness question. However, it was difficult to discern patterns in how the instructors responded. I suspect this lack of patterns is due to the course context I have sampled instructors from. None of the courses in this study are directed specifically towards mathematics majoring students.

The other subplot relates to the iMfM and sMfM. I found it interesting that some sMfM- and iMfM-oriented goals arose because of the instructors' beliefs about issues of diversity, equity, and inclusions within our U.S. society. There was a notable intersection then with context and background. In terms of context, the lack of scaffolding from departmental and university actors served as an inhibitor to enacting such goals. In the cases of the men in my study, the lack of departmental and university support to meet

these sMfM- and iMfM-oriented goals left the pursuit of such goals entirely on the instructors' individual effort, which could be enough of a deterrence on its own (see Benjamin). In terms of background, those with experiences of academic delegitimization made no object of having self-efficacy concerns in meeting these sMfM- and iMfM-oriented goals the instructors realize as being important for meeting the needs of underserved student populations. This might suggest that experiences of academic delegitimization could be a resource or an indication of something else that is a resource to meet these sMfM- and iMfM-oriented goals. I by no means am advocating that everyone should experience academic delegitimization. However, I find it to be a powerful message that something we may typically assume to be a deficit could also be framed as a potential power for the women in my study to meet goals instructors in my study associated with benefitting underserved student populations.

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Appendix A

Interview Protocol

Initial interview questions (to be asked before clips, during the first interview)

1. Is math important to you? Why?
    a. (if they respond that math is important) Have any beliefs or past experiences (in work, school, personal life) influenced why math is important to you? How?
2. In this course, how would you respond if someone asks you, "when is the stuff ever going to be useful?"
3. In this course, what do you prioritize instructionally? How does this course inform that priority?
    a. Do these priorities appear in your lesson planning? How/why not?
    b. Does this priority manifest in how you teach this course? How?
    c. Does any of what you mentioned in the prior questions about math, past experiences, and usefulness influence your instructional priorities? Why or why not?
        i. (if they say why, ask) how so?
    d. Are you trying to achieve anything else instructionally?
4. What concerns do you anticipate in teaching this course?
    a. (if any mentioned) Do you see those concerns influencing your instructional priorities in any way? Why or why not?
        i. (if they say yes, ask) how so?

b. Does the student demographic impact your goals in anyway?

Videoclip interview questions (need to include ways to iteratively refine these according to past data)

1. What did you want students to get out of the class session?
    a. Describe your reasoning behind these goals? What influences these goals (e.g. past interactions with students and/or colleagues)?
    b. Do those goals relate to the goals or concerns of (insert goals/concerns derived from past data here) you mentioned initially?
        i. (if they say yes, ask) how so?
        ii. (if they say no, ask) why not?
2. Did you feel like you achieved what you wanted? Why or why not?
3. Was there another moment from that unit which you felt was more related to your initially stated goals of (insert goals from first interview here)? Why?
4. (show hw problem) how does this problem fit in with everything you mentioned so far?

Midterm interview question:

1. If you could change things about this course, what would they be?
    a. How does that relate or not to the goals you mentioned of (insert goals here based on prior interviews)

Final interview questions (to be asked after clips, during the last interview)

1. What do you feel you ended up prioritizing instructionally by the end of the semester? Why?
    a. How does your response line up with or misalign with the goals you mentioned in our first interview regarding (insert initial goals of course here)?
2. Are you satisfied with the student learning you observed in this course?
    a. Are you satisfied with your teaching in this course?
3. If a student came out of this class by the end of the semester and asked, “what did I get out of this course?” how would you respond to them?

Appendix B

Representation of Instructors' Belief Systems

Figure B1

Hypothesized Organization of Jordan's Belief System

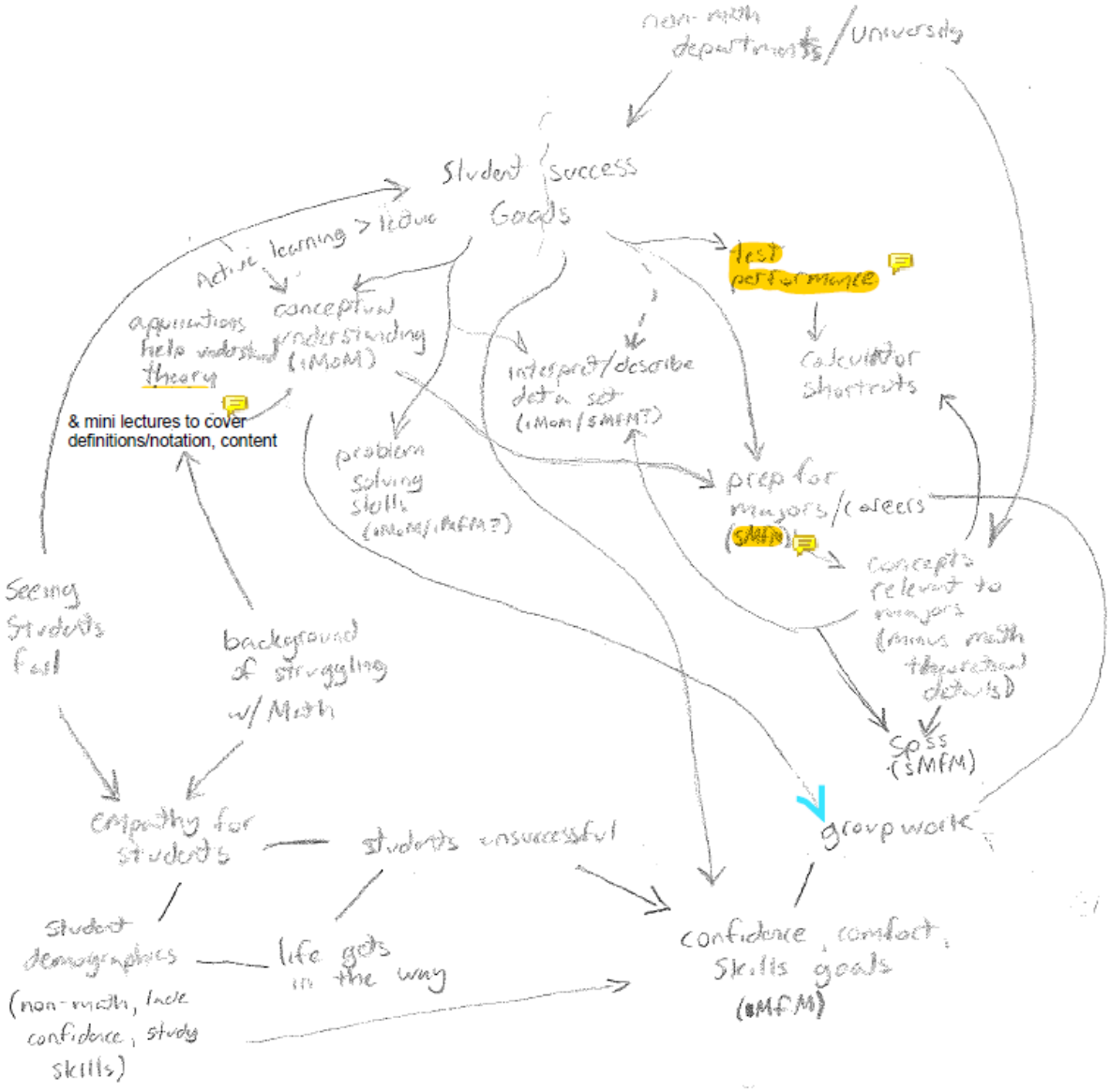

Figure B2

Hypothesized Organization of Archy's Belief System

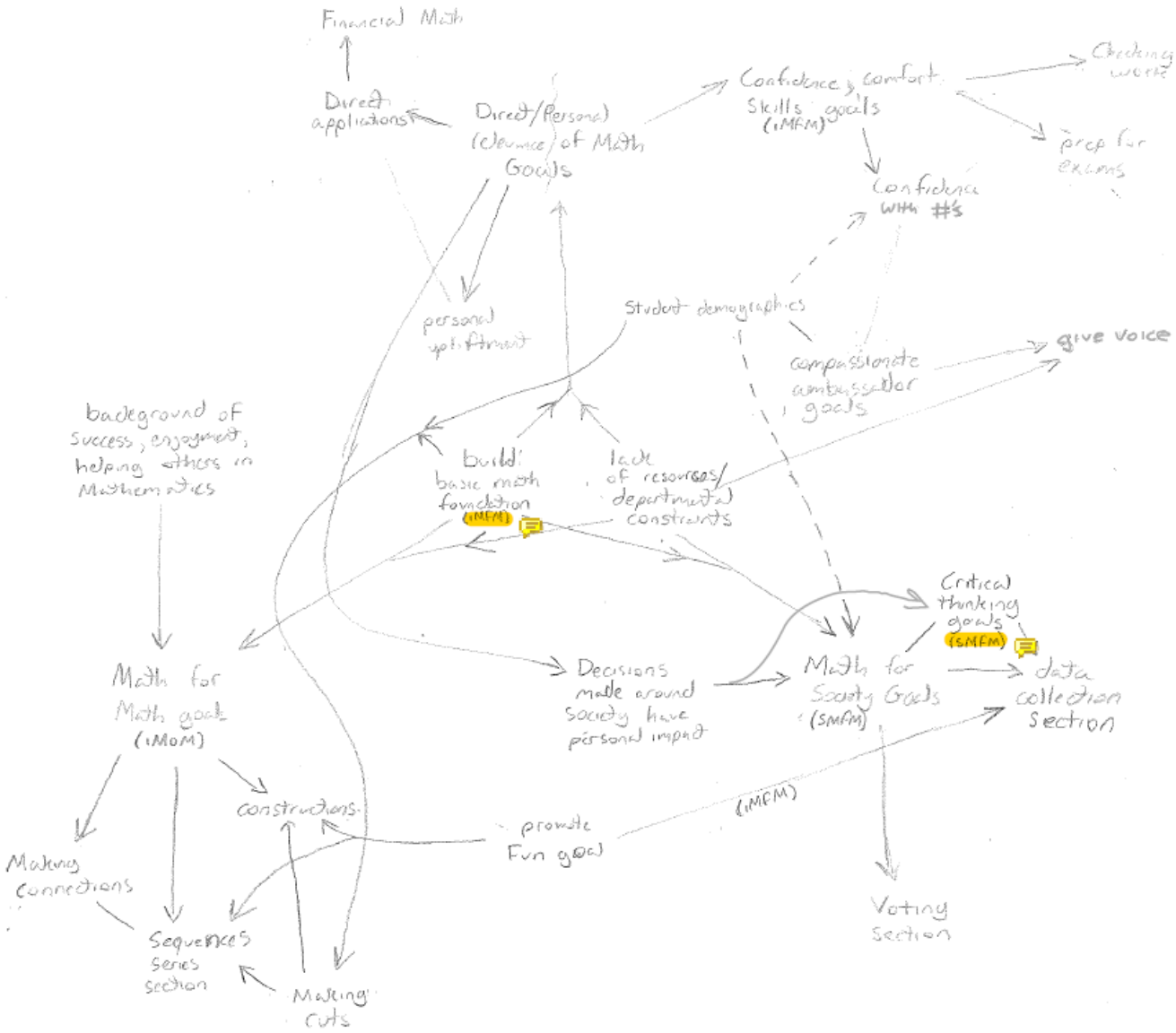

Figure B3

Hypothesized Organization of Natalie's Belief System

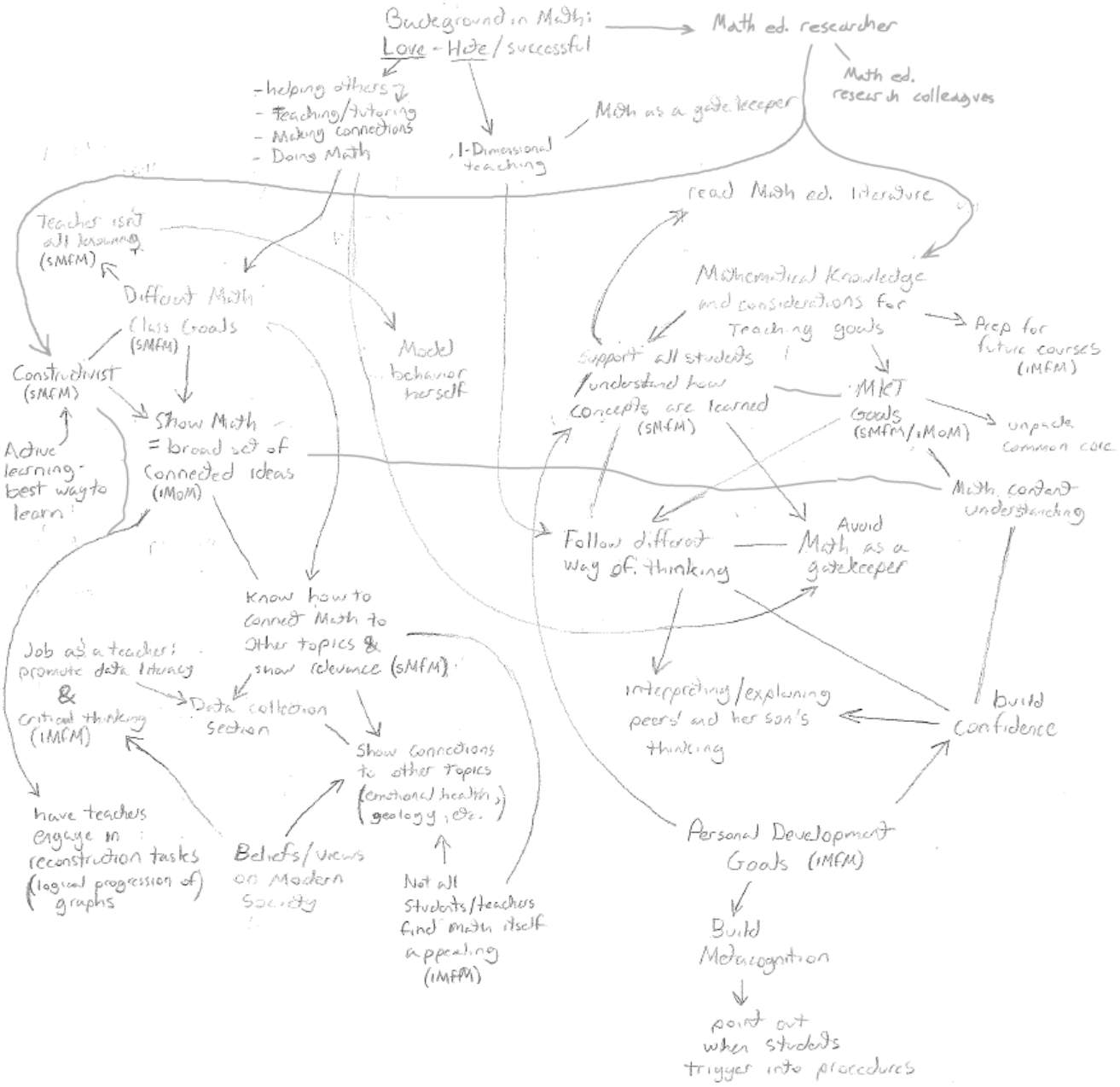

Figure B4

Hypothesized Organization of Benjamin's Belief System