

CHAPTER VI. Parameters

The two-dimensional model is complete with the exception of the values of the physical properties. The material properties, cavity length, rod velocity, electric field, and ambient and cavity temperature values need to be assigned before any calculations can be performed.

1. Properties of Mullite

Any ceramic or material may be used in the model as long as its properties are known. Mullite ($3\text{Al}_2\text{O}_3\text{-}2\text{SiO}_2$) was selected as the material used in this study for comparison purposes. In March of 1997, experiments were carried out at LANL [12] in which mullite rods were heated in a microwave applicator at various electric fields to determine if high temperatures could be achieved with a steady electric field while avoiding thermal runaway. Detailed data for temperature vs. time, forward and reflected cavity power, and rod speed are available for comparison as described in section VIII, subsection 6.

1.1 Density

The density of mullite does not vary significantly with temperature; therefore, the measured value of $2500 \frac{\text{kg}}{\text{m}^3}$ is used at all temperatures.

1.2 Specific Heat

Figure 2.1 shows values of the specific heat of mullite at different temperatures obtained from Schneider [15] and Russell et al. [16]. The data obtained from Ref [15] are given through a temperature range of 127°C to 1727°C and those from [16] ranges from temperatures of 25 °C to 595 °C. The specific heats match well in the region where temperatures overlap, so both sets of data are plotted together in Fig. 2.1, and an overall curve fit was applied. For low temperatures, the specific heat is low but increases rapidly with temperature. For high temperatures, the specific heat varies only slightly.

For a motionless rod, the specific heat only effects how fast the ceramic rod is heated but is not a factor for the final steady state temperatures. For low values of specific heat, steady state is reached faster compared to high values. Hence, when a mullite rod is initially heated, temperatures increase quickly with time due to a low specific heat at low temperatures. As the rod becomes hotter, the rate of temperature increase diminishes for the same energy absorption.

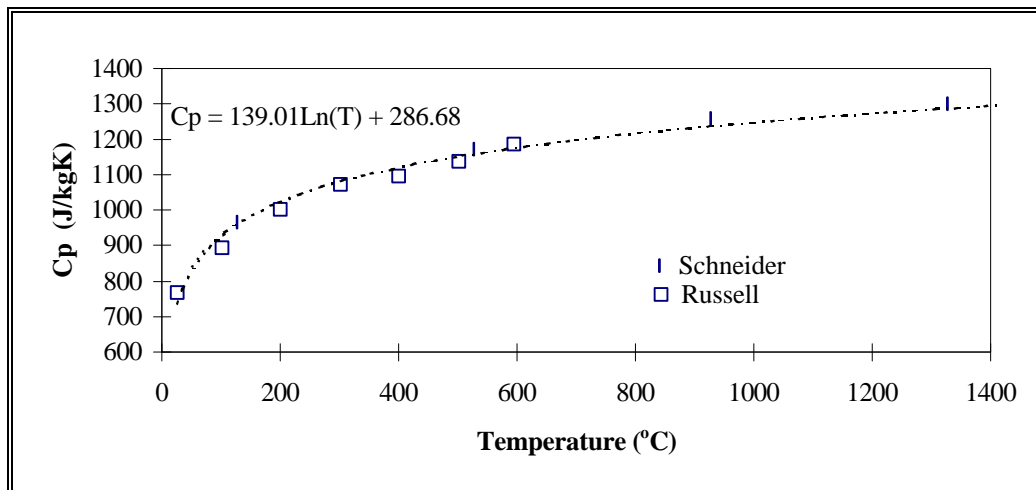


Figure 2.1 Specific Heat of Mullite with Curve Fit

1.3 Thermal Conductivity

The thermal conductivity is derived from the thermal diffusivity obtained from Russell et al. [17]. Since the specific heat is known from the previous section and the density is also known, the thermal conductivity is calculated using $k = \alpha \cdot C_p \cdot \rho$ where α is the thermal diffusivity, C_p is the specific heat, and ρ is the density. A curve fit for the specific heat is used since two sets of data are involved. The resulting thermal conductivity is plotted as a function of temperature in Fig 2.2. The data are given up to only 1500 °C, so higher values correspond to extrapolated data from the last two data points. This appears to be a valid assumption since the thermal conductivity remains fairly constant in that temperature range. Thermal conductivity values between all data points were determined by linear interpolation.

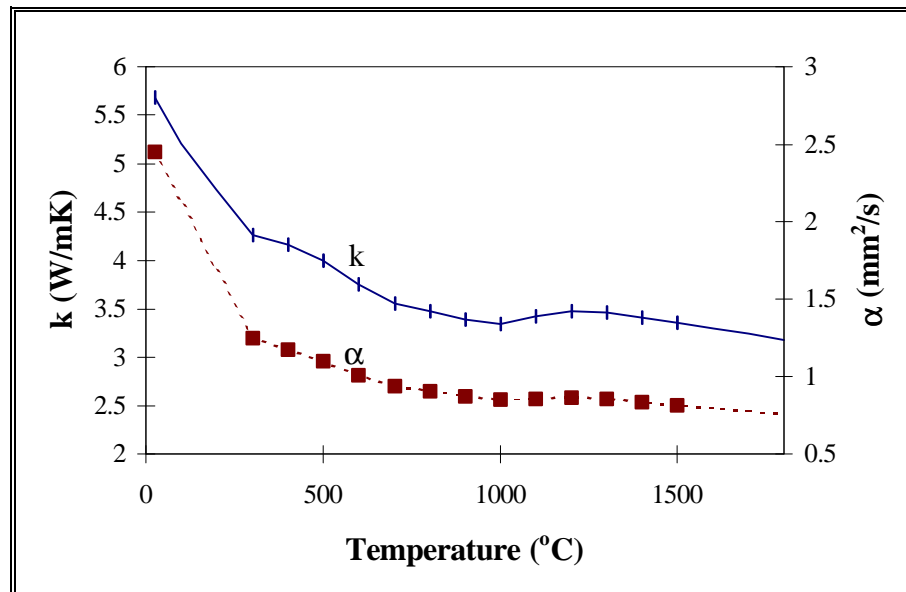


Figure 2.2 Thermal Conductivity and Thermal Diffusivity of Mullite with Interpolation

1.4 Dielectric Loss Coefficient

Two sets of data for the dielectric loss coefficient for mullite are shown in Fig. 2.3. The first set of data is from Xi and Tinga [18]. Since the dielectric loss coefficient may be the most important material property to determine if thermal runaway occurs, measurements were also made on the actual mullite rods used in the experiments. These measurements were performed by Jackson [19].

For the computer application, a linear interpolation of Tinga's dielectric loss data [18] is used to simulate the dielectric loss at temperatures greater than 25°C. This interpolation is included with the program in the appendix. Jackson's data [19] can be fit with the exponential

$$\epsilon'' = 0.0310 + 0.0263 \exp(0.00351 \cdot T) . \quad (2.1)$$

where T is in °C. The curve fit represents the data well up to 1050°C, but since no values were measured at higher temperatures, a linear extrapolation is used for higher values, and a linear interpolation is used to determine loss values between data points. Both sets of data show an exponential increase of the loss as temperature increases, meaning that mullite has the ability to absorb more microwave energy as its temperature increases. This effect is the major reason that thermal runaway occurs.

1.5 Thermal Emissivity of Mullite

Touloukian [20] gives a range of the total emissivity for alumina (Al_2O_3) as a function of temperature. The range is a result of the variance of measured data. Figure 2.4 is a replication of his plot, and an average of the range of emissivities from temperatures of 300K to 1900K is used as the modeled emissivity. Mullite is not pure

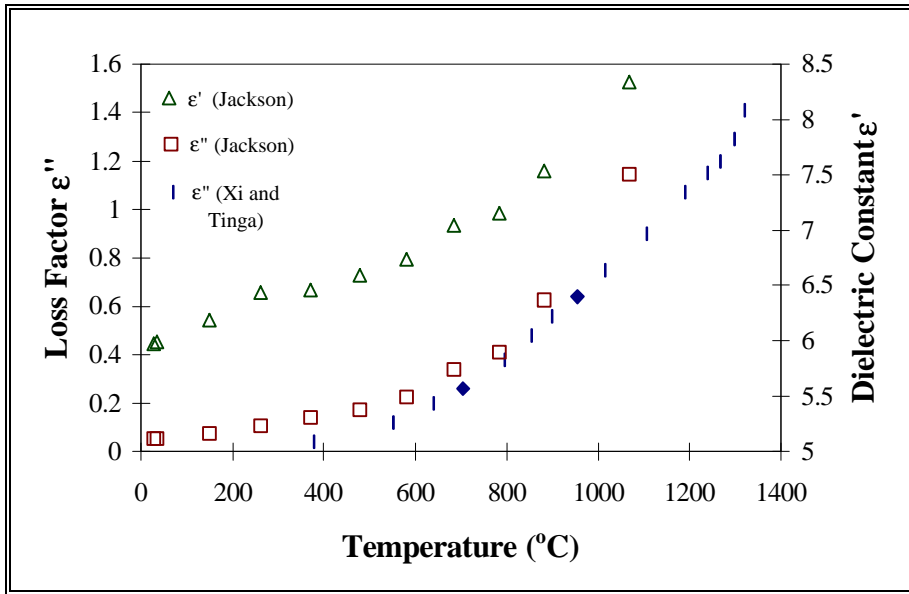


Figure 2.3 Dielectric Loss and Dielectric Constant of Mullite

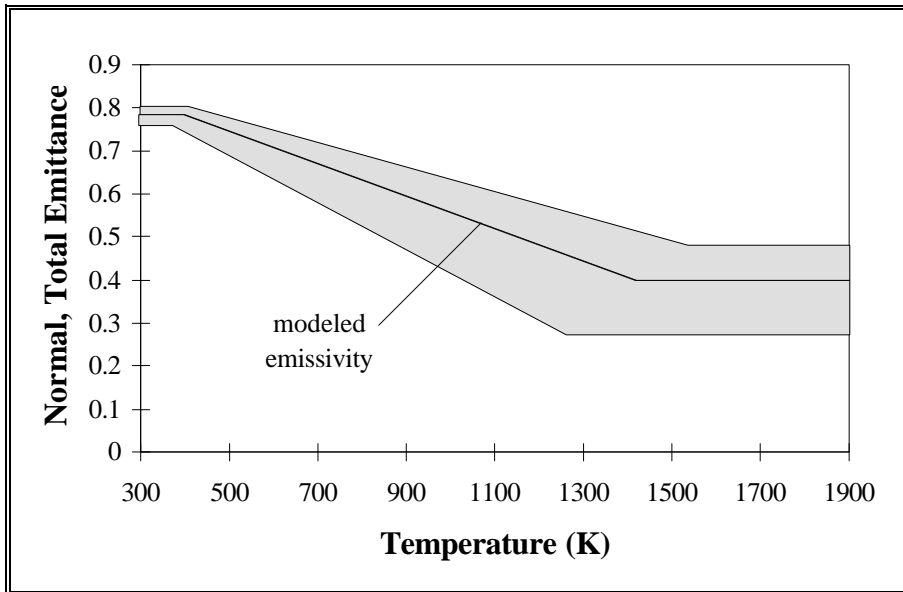


Figure 2.4 Normal, Total Emittance of Aluminum Oxide

alumina, but alumina makes up 60% of mullite on a molar basis and serves as a satisfactory substitute to estimate the emissivity. Constant values of 0.78 and 0.4 are used for temperatures less than 400K and greater than 1500K, respectively. Linear interpolation between the data points at 400K and 1500K produce a good estimate at intermediate temperatures. It is noted that as mullite increases in temperature, its efficiency to emit heat decreases. Due to this decrease, less heat is emitted and the temperature of the sample increases. This effect causes the material to be more susceptible to thermal runaway.

2. Heat Transfer Coefficient

The ceramic rod that is being heated loses energy in four ways:

- Conduction
- Radiation
- Convection
- Energy carried by the moving rod

This section describes the radiation and convection heat transfer coefficients used in the model calculations.

2.1 Natural Convection Heat Transfer Coefficient, h , for a Horizontal Cylinder

A hot rod convects heat at a rate proportional to the temperature difference between its surface and the ambient; the proportionality "constant" is called the convection heat transfer coefficient. This heat transfer coefficient depends upon the surrounding fluid properties, geometry, and temperature difference. For the present model, it is assumed that the surrounding fluid is 25°C air, and the geometry is either a thin, horizontal cylinder

or a thin vertical cylinder. A description of the horizontal case is presented in this section, and the vertical case will be discussed in the following section.

The convection heat transfer coefficient for a horizontal cylinder can be calculated from

$$h_c = \frac{\text{Nu} \cdot k}{D}, \quad (2.2)$$

where

Nu = Nusselt Number,

k = thermal conductivity of air,

and

D = Diameter of rod.

The two unknowns in Eq. 2.2 are the Nusselt number and the thermal conductivity. Kreith [21] tabulated the thermal conductivity of air with respect to temperature, and Fig. 2.5 shows these data. The temperature corresponds to the mean film temperature or the average temperature of the ambient and surface temperatures.

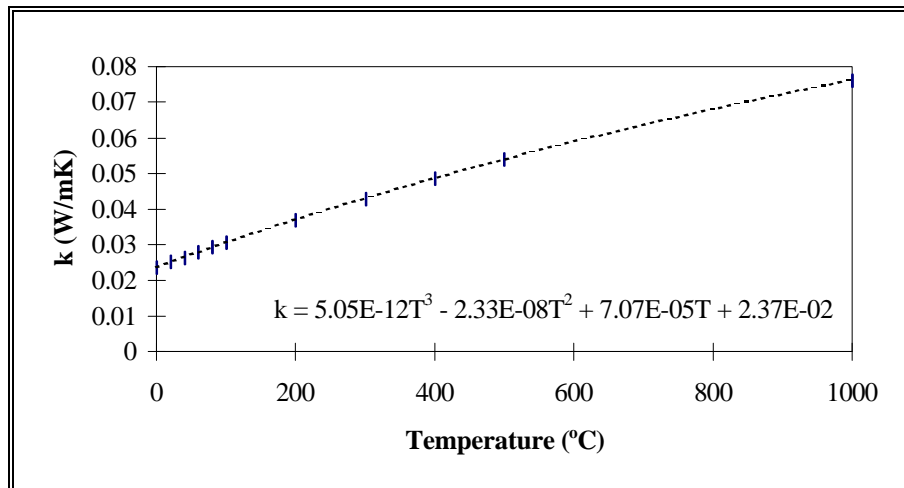


Figure 2.5 Thermal Conductivity of Air with Curve Fit

The Nusselt number for free convection from horizontal cylinders in gases is also taken from Kreith [22]. Figure 2.6 displays the logarithm of the Nusselt number with respect to the logarithm of the Rayleigh number. With a known Rayleigh number, Ra, the Nusselt number, Nu, can be easily calculated using the curve fit equation given in Fig. 2.6.

The Rayleigh number is defined by Kreith [23] as follows:

$$Ra_D = \frac{g\beta \Delta T D^3}{\nu^3} Pr, \quad (2.3)$$

where

g = acceleration of gravity,

β = coefficient of expansion $\cong \frac{1}{T(K)}$ for gases,

ΔT = temperature difference between surface and ambient,

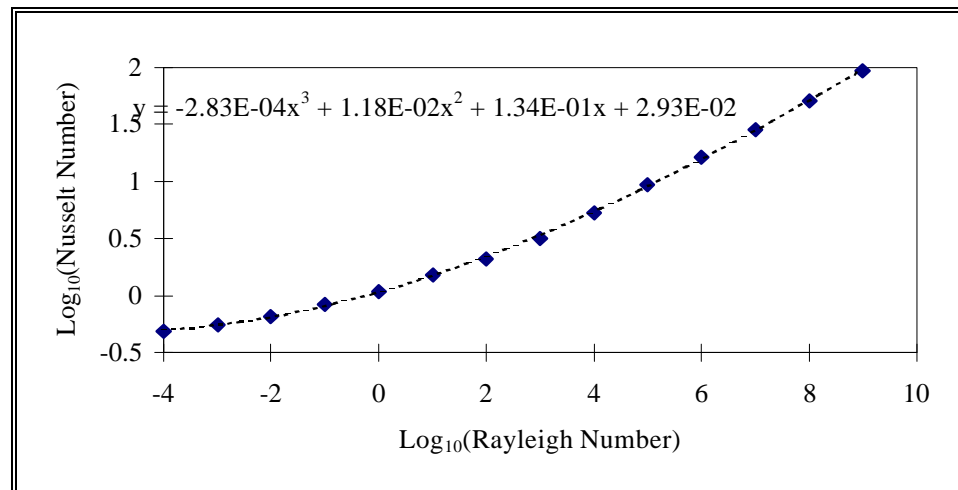


Figure 2.6 Correlation Between Nusselt Number and Rayleigh Number for Natural Convection from Horizontal Cylinders in Gases

D = diameter,

Pr = Prandtl number,

and

ν = kinematic viscosity.

Data for ν again come from Kreith [24] and is plotted in Fig. 2.7. The kinematic viscosity is calculated using the curve fit equation in Fig. 2.7 using the mean film temperature defined as the mean temperature halfway between the surface temperature and the free-stream temperature.

Using these equations and parameter values, the heat transfer coefficient for horizontal cylinders is calculated. These coefficients are depicted as a function of temperature for three diameters in Fig 2.8. We see that the heat transfer coefficient

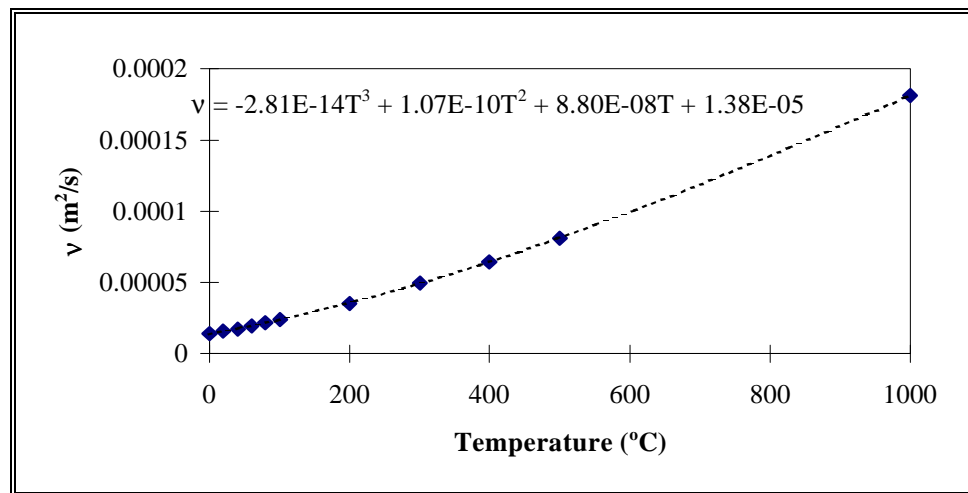


Figure 2.7 Kinematic Viscosity of Air with Curve Fit

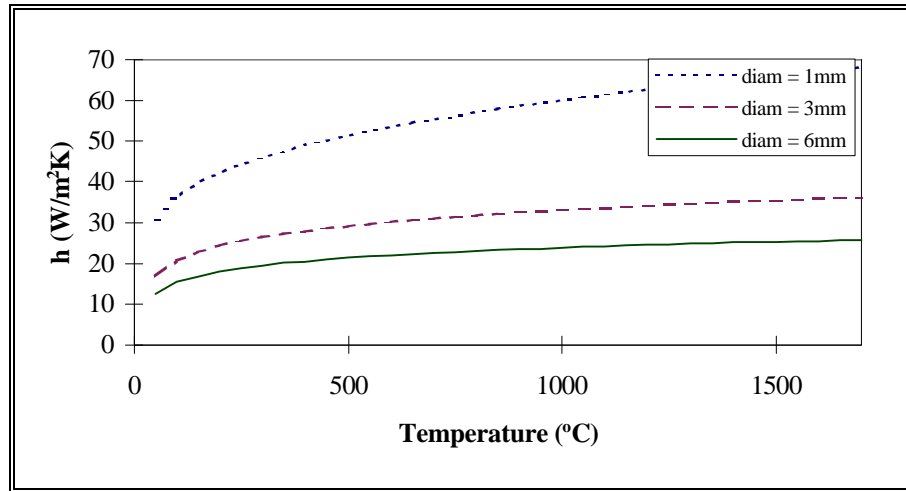


Figure 2.8 Natural Convection Heat Transfer Coefficient for Horizontal Cylinders of Different Diameters

increases with temperature. As the ceramic rod becomes hotter, more convection heat transfer occurs at the surface. Smaller diameter rods also produce higher heat transfer values. It is noted from Fig. 2.8 that a decrease from a 6mm rod to a 1 mm rod produces a heat transfer coefficient three times as great.

2.2 Natural Convection Heat Transfer Coefficient for a Thin Vertical Cylinder

Since a cylinder may pass through a cavity horizontally or vertically, a second case of a convection heat transfer coefficient is discussed for vertical cylinders. When the thermal boundary layer of a vertical circular cylinder is much thinner than its radius of curvature, a flat plate solution is used in place of a vertical cylinder solution. Gebhart [25] concludes that a vertical cylinder of diameter D can be treated as a vertical flat plate of height L when

$$(Gr_L)^{1/4} \frac{D}{L} > 35 \quad (2.4)$$

where

Gr_L = Grashof number based on L = surface area/surface perimeter.

The Grashof number based on length is defined as

$$Gr_L = \frac{gb}{\nu^2} L^3 (T_s - T_i), \quad (2.5)$$

where

g = acceleration of gravity,

β = coefficient of expansion $\cong \frac{1}{T(K)}$ for gases,

T_s = temperature of the surface,

T_i = ambient temperature,

L = height of the cylinder,

and

ν = kinematic viscosity of surrounding fluid.

When a vertical cylinder can be modeled as a vertical plate, Krieth and Bohn [26] recommend using an average Nusselt number for laminar flow as

$$\overline{Nu}_L = \frac{\bar{h}_c L}{k} = 0.68 Pr^{1/2} \frac{Gr_L^{1/4}}{(0.952 + Pr)^{1/4}}. \quad (2.6)$$

Assuming that the height of the cylinder is 34mm, and the diameter ranges from 1mm to 6mm as used in the Los Alamos experiments, we find that the vertical plate solution is not valid since Eq. 2.4 is violated. In the case of a very thin cylinder, the thermal boundary layer is large in comparison to the radius. Raithby and Hollands [27] recommend correcting the Nusselt number for curvature effects according to

$$\text{Nu}_2 = \frac{2}{\ln(1 + 2/\text{Nu})} . \quad (2.7)$$

Equation 2.7 requires that the Nusselt number for a vertical flat plate be used to calculate a corrected Nusselt number. The corrected Nusselt number yields larger values than that of a flat plate, thus producing a larger heat transfer coefficient. The characteristic length is L, thus

$$h_c = \frac{\text{Nu}_2 k}{L} . \quad (2.8)$$

The kinematic viscosity of air, ν , required to calculate the Grashof number where the Grashof number is used to calculate the Nusselt number is plotted in Fig. 2.7. The thermal conductivity of air, k , used to calculate the heat transfer coefficient is plotted in Fig. 2.5.

The convection heat transfer coefficient for vertical cylinders is thus a function of the height and not the diameter. Longer cylinders yield smaller heat transfer coefficients as shown in Fig. 2.9. The heat transfer coefficient for a vertical cylinder is similar to the heat transfer coefficient for a horizontal cylinder in that they both increase gradually with temperature. The coefficient varies from 8 to 17 W/m²K for lengths ranging from 20 to 80 mm.

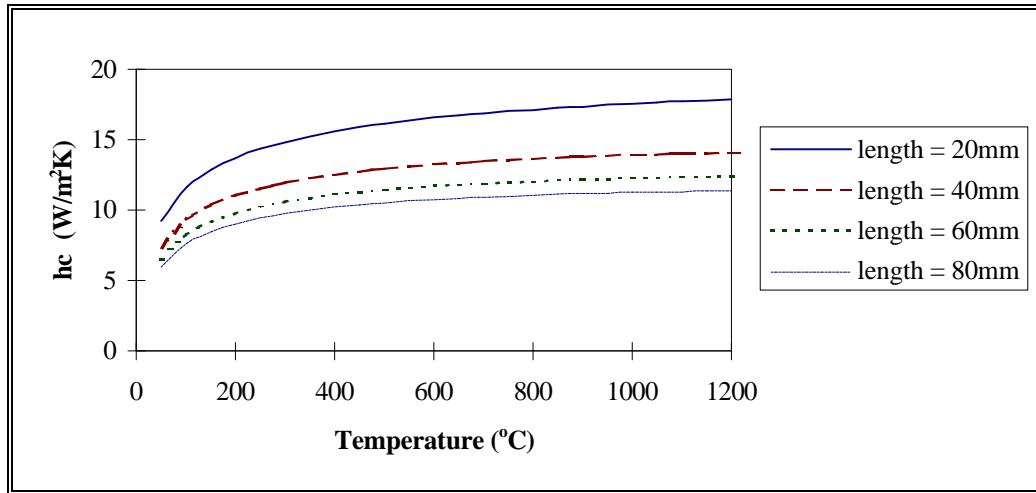


Figure 2.9 Free Convection Heat Transfer Coefficient for Vertical Cylinders of Various Lengths in 25°C Air

2.3 Radiation Heat Transfer Coefficient

Radiation is the cause of the other heat loss at the surface and does not depend on the length. Assuming that the ceramic rod is a gray body inside the microwave cavity, the heat loss can be determined from:

$$q_r = \varepsilon\sigma A_s(T^4 - T_{\text{tube}}^4) = h_r A_s(T - T_{\text{tube}}) \quad (2.9)$$

where

ε = emissivity of the material,

$\sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$,

A_s = surface area,

and

$$h_r = \varepsilon\sigma(T^2 + T_{\text{tube}}^2)(T + T_{\text{tube}}). \quad (2.10)$$

Given the emissivity, the microwave cavity temperature, and the temperature of the rod surface, the radiation heat transfer coefficient can then be easily calculated. Figure 2.10 compares h_r with $h_{c, \text{horizontal}}$ and $h_{c, \text{vertical}}$ as a function of temperature. For a 2mm diameter horizontal rod and for a 34mm long vertical rod, convective losses increase slowly with temperature, and although radiative losses are small at low temperatures, they increase much faster and are the major cause of surface heat loss at high temperatures.

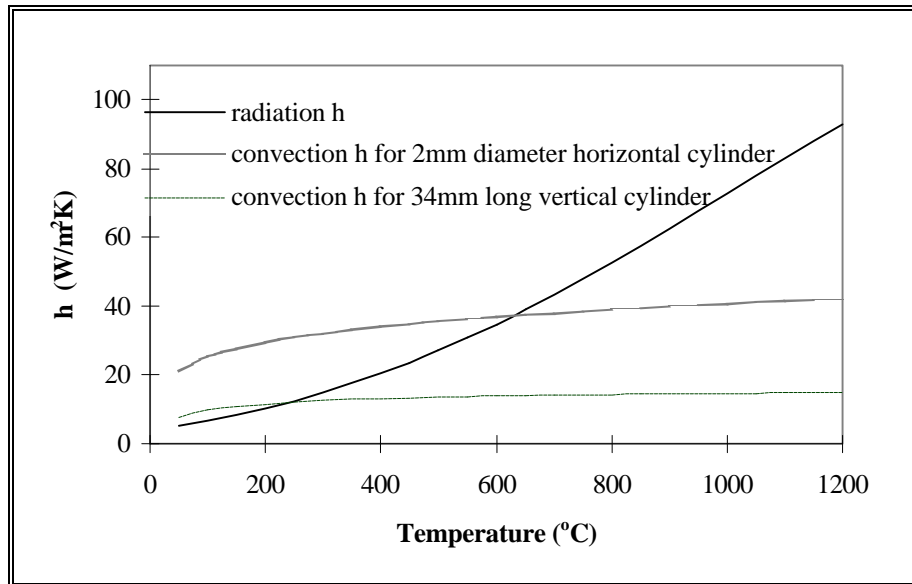


Figure 2.10 Comparison of Radiation and Convection Heat Transfer Coefficients of a 2mm Diameter, 34mm Long Cylinder in 25°C air and in a 25°C Enclosure