

**Täuber, Howard, and Hinrichsen Reply:** The preceding Comment [1] draws attention to the important fact that there is an additional  $O(\epsilon = 4 - d)$  fluctuation contribution to the density exponent  $\beta^{(2)}$  at the second hierarchy level of coupled directed percolation (coupled DP) [2], which originates from a renormalization-group (RG) analysis of the scaling function for the equation of state. This additional term was neglected in our recent Letter [2], and renders the scaling relation (16) there obsolete for  $d \leq d_c = 4$ .

In order to notice and verify this point, one has to consider the active phase explicitly, which was not done in Ref. [2]. We have now computed the equation of state, i.e., the function  $n_B(r)$ , to one-loop order as well, albeit with a slightly different approach to the one reported in the preceding Comment [1]. To simplify the calculation, we perform our analysis on a special line in parameter space, where except for  $s_0$  all the nonlinear couplings generated on the tree level vanish, i.e.,  $s'_0 = \tilde{s}_0 = \tilde{s}'_0 = 0$  in Eq. (8) of Ref. [2]. Of course, this choice breaks the special symmetry mentioned in Ref. [1], which holds precisely at the multicritical point. However, the advantage is that one may describe the entire crossover from ordinary DP to the multicritical behavior of unidirectionally coupled DP in terms of the single additional three-vertex  $-s_0 \bar{\varphi}_0 \psi_0$  (in the notation of Ref. [2]).

Furthermore, we have consistently evaluated all one-loop diagrams contributing to the equation  $\langle \varphi_0 \rangle = 0$ . This condition then yields the desired equation of state  $n_B(r)$ . A straightforward calculation shows that, in fact, one may equivalently demand that the vertex function  $\Gamma_{\bar{\varphi}} = 0$ . Consequently, one needs to take into account one-particle irreducible Feynman graphs only, namely, diagrams (a), (e), and (c) in Fig. 1 of Ref. [1] [notice that diagram (b) there is one-particle reducible]. After evaluating the corresponding momentum integrals, and replacing the bare parameters with the renormalized ones (using the renormalization constants computed in the inactive phase), one then inserts the fixed-point values  $(u/D)^* = (u'/D)^* = 2(\epsilon/3)^{1/2} = (s/D)^*$  at the multicritical point. Finally, after exponentiating the emerging logarithms of  $|r|$  and  $n_B$  appropriately, we find as the leading contribution (for  $|r| \rightarrow 0$ ), and to first order in  $\epsilon = 4 - d$ ,

$$n_B^{2+\epsilon/6} \sim |r|^{1-\epsilon/6}, \quad (1)$$

or  $n_B(r) \sim |r|^{\beta^{(2)}}$  with the (multi-) critical exponent

$$\beta^{(2)} = 1/2 - \epsilon/8 + O(\epsilon^2). \quad (2)$$

This confirms Eq. (1) of the preceding Comment [1].

The scaling function contribution to the density exponent (2) is of conceptual interest, but does, of course, not invalidate the main findings in Ref. [2], namely, the identification of a *multicritical* regime in unidirectionally

coupled DP processes, which is characterized by the critical exponents  $\eta_{\perp}$ ,  $\nu_{\perp}$ , and  $z$  of conventional DP, yet by considerably lower values of  $\beta^{(k)}$  on each hierarchy level  $k$ . The basic feature may be understood on the mean-field level already; a field-theoretic RG analysis leads to a further downward renormalization of these exponents.

Besides the observation that the correct value (2) fits our simulation data better than our original result, Eq. (16) in [2], it also resolves an apparent physical problem for high hierarchy levels  $k$  near four dimensions. Namely, based on our previous calculation one would conclude  $\beta^{(k)} = 1/2^{k-1} - \epsilon/6 + O(\epsilon^2)$ . As the  $O(\epsilon)$  contribution here is independent of  $k$ , this would predict that for some fixed  $\epsilon \ll 1$  and sufficiently large  $k$ ,  $\beta^{(k)}$  would become negative. The correct result (2), however, shows that the  $O(\epsilon)$  corrections to the mean-field value become successively *smaller* for larger  $k$ , which is likely to keep the density exponents positive at each hierarchy level.

Finally, the preceding Comment questions the validity of renormalized perturbation theory for coupled DP, based on the infrared behavior of diagram (c) in Fig. 1 of Ref. [1]. This diagram is, in fact, ultraviolet *convergent*, and therefore does not contribute to the renormalization constants in the inactive phase. We feel that the apparent problems with this diagram are of a technical rather than of a physical nature, and can probably be cured, e.g., by carefully retaining the (nonanalytic)  $p_c$  shifts in the corresponding integral. A more thorough discussion of these conceptual issues, as well as explicit RG calculations for coupled DP both in the inactive and the active phase will be presented in a future joint publication [3].

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