

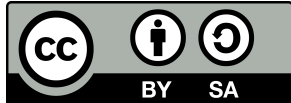
Problems in *Electromagnetics*, Vol. 2  
Version 1.0

Steven W. Ellingson  
ellingson.1@vt.edu  
Virginia Tech

December 17, 2019

This manual accompanies *Electromagnetics Vol. 2*, an open textbook freely available at <https://doi.org/10.21061/electromagnetics-vol-2>

© 2019 Steven W. Ellingson CC BY SA 4.0.



<https://creativecommons.org/licenses/by-sa/4.0/>

# Change History

- Version 1.0: First publicly-available version.

# Contents

2	Magnetostatics Redux	4
3	Wave Propagation in General Media	8
4	Current Flow in Imperfect Conductors	11
5	Wave Reflection and Transmission	13
6	Waveguides	18
7	Transmission Lines Redux	20
8	Optical Fiber	23
9	Radiation	25
10	Antennas	27

## Chapter 2

# Magnetostatics Redux

[m0017]

[1]

**2.2-1** A thin straight wire is aligned along the the  $z$  axis. A current of 2 A flows along the wire in the  $-\hat{\mathbf{z}}$  direction. Over a particular 25 cm long segment of this wire, the magnetic field is equal to  $B_0(\hat{\mathbf{x}} - \hat{\mathbf{y}})$ , where  $B_0 = 3$  mT. What is the force experienced by this segment?

[2]

**2.2-2** Current  $I$  flows in the  $+\hat{\phi}$  direction around a loop of radius  $a$  centered on the  $z$  axis. This loop lies in the  $z = z_0$  plane. The magnetic field in this region is given by

$$\mathbf{B} = \frac{\hat{\rho}B_{\rho}\rho z + \hat{\mathbf{z}}(B_{zz}z^2 - B_{z\rho}\rho^2)}{(\rho^2 + z^2)^{5/2}} \quad (2.1)$$

where  $B_{\rho}$ ,  $B_{zz}$ , and  $B_{z\rho}$  are arbitrary constants.

- What is the  $\hat{\mathbf{z}}$  component of the force experienced by the loop?
- What are the SI base units of the constants  $B_{\rho}$ ,  $B_{zz}$ , and  $B_{z\rho}$ ?
- Show that your answer to part (a) is dimensionally correct.

[3]

**2.2-3** Figure 2.1 shows a semi-circular wire loop carrying current  $I = 3$  A in the direction shown in the figure. The radius of the semi-circular part of the loop is  $a = 20$  cm. The loop exists in the presence of a static spatially-uniform magnetic field  $\mathbf{B} = \hat{\mathbf{x}}B_0$  where  $B_0 = 0.7$  T.

- What is the force on the straight part of the loop (i.e., the segment lying along the  $z$  axis)? Do this part of the problem by evaluating an integral over this part of the loop.
- What is the force on the semi-circular part of the loop? Do this part of the problem by evaluating an integral over this part of the loop.
- What is the net force on the *entire* loop?

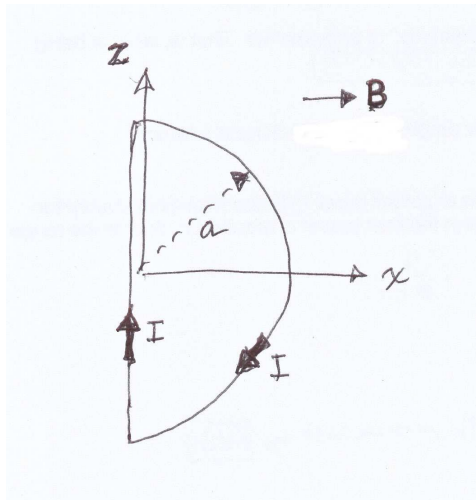


Figure 2.1: Semi-circular current loop in a magnetic field.

[3]

**2.2-4** A segment of rigid wire in free space lies along the  $x$  axis, is 5 cm long, and carries 300 mA of current in the  $+\hat{\mathbf{x}}$  direction. A spatially-uniform magnetic field of  $+\hat{\mathbf{z}}800$  kA/m

is applied. What is the force on this segment of wire?

[m0024]

[1]

**2.3-1** The rectangular loop shown in Figure 2.2 is attached to a frictionless shaft which lies along the  $z$  axis. The loop width  $W$ , height  $h$ , and current  $I$  are as indicated in the figure. The loop is immersed in a spatially-uniform magnetic field  $\hat{y}B_0$ .

- (a) What is the torque experienced by the shaft? (i.e., give an expression in terms of the relevant variables of the problem)
- (b) Given  $W = 20$  cm,  $h = 40$  cm,  $I = +500$  mA,  $B_0 = +1.2$  T, and  $\phi = 60^\circ$ , what is the torque experienced by the shaft? (i.e., give the value of this vector)
- (c) Following up part (b): If the loop is held and then released, will structure rotate *toward* or *away from* the observer of Figure 2.2?

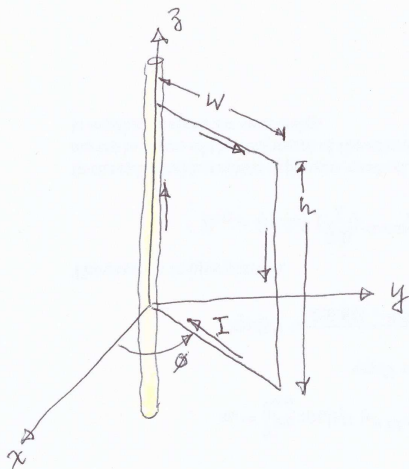


Figure 2.2: Current loop attached to a shaft.

[2]

**2.3-2** A square loop having area  $7$  mm<sup>2</sup> is mounted symmetrically on a shaft that is fixed in space but free to rotate around its axis. A current of  $3$  mA flows through the loop. The loop is immersed in a spatially-uniform magnetic field having magnitude  $0.3$  T. What is the minimum and maximum torque that can be expected?

[m0066]

[1]

**2.4-1** Determine the magnetic flux density at  $(x = 0.5$  m,  $y = z = 0)$  due to currents flowing in two wires of infinite length as follows: The first wire is aligned along the  $z$ -axis with current  $\hat{z}I_1$  with  $I_1 = 6$  A. The second wire is parallel to the first wire, intersects  $(x = 2.0$  m,  $y = z = 0)$ , with current  $-\hat{z}I_2$  with  $I_2 = 3$  A. Assume free space.

[2]

**2.4-2** Ampere's law,  $\nabla \times \mathbf{H} = \mathbf{J}$ , is a partial differential equation. What is the solution to this differential equation for the special case that the current has the form of a line current  $I\hat{\mathbf{l}}(\mathbf{r})$  in free space?

[m0059]

[1]

**2.5-1** A segment of rigid wire lies along the  $x$  axis, is 5 cm long, and moves in the  $+\hat{y}$  direction at 3 m/s in the presence of a spatially-uniform magnetic field of  $+\hat{z}2$  T. What is the potential difference generated across this segment of wire?



## Chapter 3

# Wave Propagation in General Media

[m0073]

[1]

**3.1-1** An energy-harvesting device captures incident electromagnetic wave energy and stores it for future release into an electrical circuit. Name the two possible ways that the energy can be stored (plain English, please). Separately, name a third internal process (plain English, please) that should be considered to determine whether the device is practical.

[m0122]

[1]

**3.2-1** A plane wave with sinusoidal time variation propagates through free space. At some point in space, the electric field is  $\hat{\mathbf{z}}(4 \mu\text{V/m})$  (peak). At the same point, the magnetic field points in the  $\hat{\mathbf{y}}$  direction. Write the simplest possible expressions for (a) the magnitude of the power density and (b) the direction of power flow.

[m0128]

[1]

**3.3-1** Write the *simplest* possible *exact* expression for the phase propagation constant in the case that the loss tangent is equal to  $\sqrt{8}$ . Your answer should be in terms of the frequency  $\omega$  and the constitutive parameters  $\mu$ ,  $\epsilon$ , and  $\sigma$ .

[m0132]

[1]

**3.5-1** A particular material has loss tangent  $2 \times 10^{-6}$  and conductivity  $5 \text{ nS/m}$  at  $2 \text{ MHz}$ . What is the relative permittivity? (Note this will be a real-valued quantity.)

[2]

**3.5-2** The loss tangent of a material is  $0.05$  at  $2 \text{ GHz}$ . The conductivity of this material is approximately constant from  $0.1 \text{ GHz}$  to  $10 \text{ GHz}$ . Estimate the loss tangent at  $2.5 \text{ GHz}$ .

[m0130]

[1]

**3.6-1** Consider an electromagnetic wave propagating through ice. The conductivity of ice tends to increase as it melts. What happens to the difference in phase between the electric and magnetic fields in ice as it melts?

[2]

**3.6-2** A wave exhibits a electric field magnitude of  $100 \mu\text{V/m}$  at a reference point in some medium. The attenuation constant is  $0.2 \text{ m}^{-1}$ . What is the electric field intensity after propagation over an additional  $7 \text{ cm}$ ?

[m0133]

[1]

**3.7-1** If the attenuation resulting from propagation through a lossy medium is  $3 \text{ dB}$ , then what is the corresponding reduction in the electric field intensity, in percent?

[m0155]

[1]

**3.9-1** An electric field propagates with an attenuation constant of  $0.03 \text{ m}^{-1}$ . What is the attenuation after 20 m? Give your answer in dB.

[2]

**3.9-2** A wave exhibits a electric field magnitude of  $100 \mu\text{V}/\text{m}$  at a reference point in some medium. The attenuation constant is  $0.2 \text{ m}^{-1}$ . By how much is the wave's power density reduced 7 cm beyond this reference point, in the direction of propagation? Give your answer in dB.

[m0156]

[1]

**3.10-1** A particular dielectric material used for printed circuit board substrate exhibits relative permittivity 3.0 and loss tangent 0.02. Estimate the attenuation constant  $\alpha$  for a wave propagating in this material at a frequency of 2.4 GHz.

[m0157]

[1]

**3.11-1** The phase velocity of a plane wave propagating through a certain material is found to increase by a factor of 2 when the frequency is increased by a factor of 4. The permittivity, permeability, and conductivity of the material are approximately constant with frequency over this range. Would you categorize this material as (a) lossless, (b) a poor conductor, or (c) a good conductor?

[m0158]

[1]

**3.12-1** The skin depth of a particular good conductor is determined to be  $2.00 \mu\text{m}$ . Write a phasor expression for the electric field intensity of a plane wave propagating in the  $+\hat{z}$  direction through this medium. The electric field intensity at  $z = 0$  is  $\hat{y}E_0$ . Be sure to identify the value of any constants in your expression.

[2]

**3.12-2** The skin depth in aluminum (loss tangent  $\gg 1$ ) is found to be  $2.5 \mu\text{m}$ . Estimate the skin depth if the frequency is decreased by a factor of 4, assuming negligible changes in the constitutive parameters over that frequency range.

[3]

**3.12-3** A metal enclosure is used to keep undesired electromagnetic radiation from radiating away from a device. The wall of the enclosure is 2 skin depths thick. By how much does the enclosure reduce the power density? Give your answer in dB.

## Chapter 4

# Current Flow in Imperfect Conductors

[m0159]

[1]

**4.2-1** A resistor is made from a block of aluminum 1 cm long. The cross-section is square with dimensions  $0.1 \text{ cm} \times 0.1 \text{ cm}$ . The terminals are attached to the square ends. Assume the aluminum is non-magnetic with conductivity  $5 \times 10^6 \text{ S/m}$  at 10 MHz. What is the resistance at this frequency?

[2]

**4.2-2** For the resistor in the previous problem, what is:

- (a) the reactance?
- (b) the equivalent series inductance?

[3]

**4.2-3** For a particular cylindrical wire, the skin depth is much less than the minimum cross-section. The wire has uniform conductivity and the radius of the wire is  $a$ . The magnitude of the electric field intensity slightly below surface of the wire is  $E_0$ .

- (a) What is the magnitude of the current density slightly below the surface of the wire?
- (b) Estimate the current through the wire in terms of  $E_0$ .
- (c) Show that your answer to part (b) is dimensionally correct.

[4]

**4.2-4** A pin of an RF integrated circuit (RFIC) having the following characteristics: The pin is 2 mm long and has rectangular cross-section  $0.2 \text{ mm} \times 0.1 \text{ mm}$ . The pin is plated with non-magnetic gold with thickness much greater than the skin depth. The conductivity of the plating is about  $10^8 \text{ S/m}$ . The signal on this pin is an RF signal at a center frequency of 10 GHz.

- (a) What is the resistance of the pin?
- (b) What is the equivalent inductance of the pin?

[5]

**4.2-5** A wire having circular cross-section is found to have resistance equal to  $9 \text{ m}\Omega$  at 10 MHz. Estimate the impedance of the wire at 5 MHz. You may assume that the skin depth remains much less than the radius of the wire.

# Chapter 5

## Wave Reflection and Transmission

[m0161]

[1]

**5.1-1** A sinusoidally-varying plane wave is incident from air onto a planar interface with a lossless homogeneous dielectric medium exhibiting relative permittivity equal to 36. The direction of propagation is perpendicular to the planar interface. The peak electric field of the plane wave is 30 V/m. The frequency of the plane wave is 50 MHz.

- (a) What is the reflection coefficient?
- (b) What is the time-average power density of the incident wave?
- (c) What is the time-average power density of the reflected wave?
- (d) What fraction of incident power is transmitted into the dielectric?

[2]

**5.1-2** A uniform plane wave in air is normally-incident on a half-space consisting of a non-magnetic material having a relative permittivity of 16. What fraction of the incident power is transmitted? Give your answer in percent.

[3]

**5.1-3** A plane wave propagating in free space encounters a planar interface. The material in the second region has twice the permittivity and twice the permeability as the material in the first region. The direction of incidence is perpendicular to the interface. What fraction of power is reflected?

[m0163]

[1]

**5.3-1** An X-band radar operates at 10 GHz and uses a planar phased array antenna. The antenna is embedded deep within a dielectric material, which itself is enclosed by a planar radome. The purpose of the radome is to protect the antenna from the outdoor environment while facilitating 100% transmission of power to and from the antenna. The radome is comprised of a dielectric material having relative permittivity equal to 6. You may assume planar wavefronts at both material boundaries.

- (a) What is the relative permittivity of the material within which the phased array antenna is embedded?
- (b) What is the minimum thickness of the radome?
- (c) What is the next-smallest possible thickness of the radome?

[2]

**5.3-2** A well-designed radome is made from a lossless dielectric having relative permittivity equal to 8. It is positioned 30 cm in front of a planar antenna array operating at 77 GHz, with an air gap between the array and the radome. What is the minimum possible thickness of the radome?

[m0165]

[1]

**5.4-1** A uniform plane wave has the form

$$\tilde{\mathbf{E}} = \hat{\mathbf{e}}E_0e^{-j\mathbf{k}\cdot\mathbf{r}} \quad (5.1)$$

The wave is propagating in free space at a frequency of 600 MHz. At a particular point in space, the wave is propagating directly away from the origin in the direction  $\theta = \theta_0$  and

$\phi = \phi_0$ . What is  $\tilde{\mathbf{E}}(x, y, z)$ ? In other words, rewrite the expression for the wave as a function of  $x$ ,  $y$ , and  $z$ . Give values for any constants other than  $\hat{\mathbf{e}}$ ,  $E_0$ ,  $\theta_0$ , and  $\phi_0$ .

[m0166]

[1]

**5.5-1** A plane wave is incident on the planar boundary between two semi-infinite media. The plane wave is 100% TM. In which plane does the electric field vector lie? (Multiple choice:)

- (a) In the plane of incidence
- (b) Perpendicular to the plane of incidence
- (c) Could be in either plane; i.e., could be either (a) or (b)
- (d) Not necessarily limited to be in either plane

[2]

**5.5-2** A plane wave is incident on the planar boundary between two semi-infinite media. The angle of incidence is  $30^\circ$ . The boundary lies in the  $z = 0$  plane, and the magnetic field vector lies in the plane of incidence. Is this situation (a) TE, (b) TM, (c) TEM? (Indicate all that apply.)

[m0167]

[1]

**5.6-1** A transverse electric plane wave is obliquely incident on the planar boundary between two semi-infinite media. The angles of incidence and transmission are  $\psi^i$  and  $\psi^t$ , respectively. The wave impedances in Regions 1 and 2 are  $\eta_1$  and  $\eta_2$ , respectively. If the incident electric field intensity vector at some point on the boundary is  $\hat{\mathbf{e}}E_0$ , then what is the reflected field intensity vector at the same point? *Use only the variables given in this problem statement.*

[m0164]

[1]

**5.7-1** It is shown in the textbook that when a TM-polarized wave is obliquely-incident on the planar boundary between two media, the magnitude and phase of the transmitted electric field intensity is related to the magnitude and phase of the incident electric field intensity by the factor

$$(1 - \Gamma_{TM}) \frac{\eta_2}{\eta_1}$$

Show that this factor can also be expressed the product of  $1 + \Gamma_{TM}$  (note the change of sign) times something that depends only on geometry.

[m0168]

[1]

**5.8-1** A laser beam in free space is incident on a planar boundary with a semi-infinite, lossless, and non-magnetic material. The index of refraction of the second material region is 1.7. The beam arrives at an angle of  $23^\circ$  measured from the normal to the surface. At what angle does the beam depart the planar boundary into the second material region, measured from the normal to the surface pointing into this material region?

[2]



**5.8-2** A plane wave is obliquely incident on the planar boundary between two semi-infinite media, both lossless and non-magnetic. The wave impedance of the material on the transmit side is greater than the wave impedance of the material from which the wave is incident. Relative to the angle of incidence, what happens to the direction of the wave on the transmit side? (Multiple choice:)

- (a) The direction of the transmitted wave is unchanged.
- (b) The transmitted wave bends away from the surface normal pointing into the transmit side.
- (c) The transmitted wave bends toward the surface normal pointing into the transmit side.

[3]

**5.8-3** A plane wave is incident from Region 1 onto a planar boundary with Region 2. The media in both regions are non-magnetic. The angle of incidence is  $30^\circ$  and the angle of transmission is  $20^\circ$  (both angles measured from the associated surface normals). The relative permittivity in Region 2 is 3. What is the relative permittivity in Region 1?

[m0172]

[1]

**5.10-1** A uniform plane wave in free space is incident on a planar boundary with a non-magnetic material having relative permittivity 5.0. For what angle(s) of incidence – if any – will there be 100% transmission? Answer separately for the following two cases:

- (a) The incident wave is 100% transverse electric.
- (b) The incident wave is 100% transverse magnetic.

[2]

**5.10-2** A uniform plane wave is incident on the planar boundary between two material regions. The wave is incident from a dielectric having relative permittivity equal to 1.5. The other material region is a dielectric having relative permittivity equal to 6.3. The wave is TM-polarized.

- (a) Plot the reflection coefficient as a function of angle of incidence, over the full range of possible values.
- (b) Compute the Brewster's angle directly using the relevant formula from the textbook, and confirm that the result is consistent with your plot.

[3]

**5.10-3** A plane wave is incident from free space onto a planar boundary with a non-magnetic dielectric material. It is noted that the sign of the reflection coefficient changes as the angle of incidence is varied from  $0^\circ$  to  $60^\circ$ . Which of the possible cases could we have (multiple choice:) (a) TE, (b) TM, (c) TEM.

[m0169]

[1]

**5.11-1** A uniform plane wave is obliquely incident on the planar boundary between teflon (non-magnetic, relative permittivity 2.1) and air. The wave is incident from the teflon side. Over what range of angles of incidence can total internal reflection occur?

[2]

**5.11-2** A uniform plane wave in a non-magnetic material having relative permittivity 2.9 is incident on a planar boundary with air on the opposite side. For what *range* of angles of

incidence (acute angles measured from the surface normal) will there be non-zero transmission of power across the interface?

<sup>[3]</sup>

**5.11-3** A plane wave is obliquely incident on a planar interface between two non-magnetic dielectric media. The plane wave arrives from a material having permittivity  $\epsilon_1$  and the other material has permittivity  $\epsilon_2$ . There is no transmission of power across the interface. Assuming nothing else, is  $\epsilon_2$  less than, equal to, or greater than  $\epsilon_1$ ?

# Chapter 6

## Waveguides

[m0174]

[1]

**6.3-1** A parallel plate waveguide is being used as a filter to suppress all TE modes having frequencies below 4 GHz. The spacing between the plates is 2 cm. The material between the plates may be assumed to be a lossless dielectric. Determine the relative permittivity of the material between the plates.

[2]

**6.3-2** Consider an air-filled parallel plate waveguide. The spacing between the plates is 15 mm. The  $m = 2$  TE mode exists and propagates in the waveguide at frequency  $f$  which is 1.5 times the cutoff frequency.

(a) Determine the phase velocity of this mode.

(b) Determine the group velocity of this mode.

(c) Compare (a) and (b) to the phase velocity and group velocity in unbounded free space.

[3]

**6.3-3** A parallel plate waveguide consists of plates separated by 3 cm and filled with lossless dielectric having relative permittivity 2.4. What is the range of frequencies for which *exactly one* TE mode can propagate in this waveguide?

[4]

**6.3-4** In an air-filled parallel plate waveguide, what is the maximum spacing between plates that guarantees propagation free of mode dispersion for frequencies less than 94 GHz? Give your answer in millimeters.

[m0177]

[1]

**6.5-1** Consider a parallel plate waveguide. For a given frequency, which polarization (i.e., TE or TM) supports the greater number of modes?

[m0220]

[1]

**6.6-1** The electric field of the  $TM_0$  mode half way between the plates of a parallel plate waveguide is 1.5 mV/m. The waveguide is filled with non-magnetic material having relative permittivity equal to 2.7. What is the magnetic flux density close to the center of the waveguide?

[2]

**6.6-2** In a parallel plate waveguide, the  $TE_0$  mode cannot exist, and the  $TM_0$  mode propagates just like a uniform plane wave in unbounded space. Now consider a rectangular waveguide. Such a waveguide may be viewed as a parallel plate waveguide with additional perpendicular conducting walls, such that the cross-section is rectangular. In a rectangular waveguide, can there be a mode analogous to  $TM_0$  that propagates like a uniform plane wave? Justify your answer.

[3]

**6.6-3** Consider a propagating  $m = 0$  mode in a parallel plate waveguide with a plastic (dielectric) material between the plates. (a) Is the phase velocity  $v_p$  less than, equal to, or greater than  $c$  (the speed of electromagnetic radiation in unbounded free space)? (b) Is the group velocity  $v_g$  less than, equal to, or greater than  $v_p$ ?

# Chapter 7

## Transmission Lines Redux

[m0188]

[1]

**7.1-1** You buy some parallel wire line which is advertised as having characteristic impedance equal to  $300 \Omega$ . You then bury it deep underground in dry desert sand that is non-magnetic with relative permittivity equal to 3 and negligible conductivity. Estimate the characteristic impedance as installed.

[m0186]

[1]

**7.2-1** A microstrip transmission line has characteristic impedance equal to  $50 \Omega$ . The ratio of dielectric thickness to trace width is 0.001. If this ratio is doubled, what is the new characteristic impedance?

[2]

**7.2-2** A microstrip transmission line is implemented on dielectric with relative permittivity 3. The thickness of the dielectric is 2 mm and the width of the trace is 0.1 mm. The thickness of the trace is negligible.

- Use the Wheeler 1977 formula to estimate the characteristic impedance.
- Given the choice of the “wide” (parallel plates) or “narrow” (parallel wires) approximation, which is more appropriate in this case?
- Use the approximation you determined in part (b) to estimate the characteristic impedance.
- Again using the approximation you determined in part (b), determine the trace width required to reduce the characteristic impedance by 20%.
- Use the Wheeler 1977 formula to estimate the characteristic impedance for the new trace width obtained in part (d).

[3]

**7.2-3** Estimate the width of a  $22 \Omega$  transmission line implemented on printed circuit board material that has relative permittivity equal to 3 and height 1.52 mm.

[4]

**7.2-4** A microstrip line has height 2 mm and width 0.10 mm. The characteristic impedance is found to be a particular value  $K$ . A second microstrip line on the same printed circuit board has width 0.05 mm. What is the characteristic impedance of this line?

[m0189]

[1]

**7.3-1** A coaxial cable has a characteristic impedance of  $75 \Omega$ . The cable has outer radius 1 mm, inner radius 0.1 mm, and exhibits resistance of  $0.1 \Omega/\text{m}$ . The dielectric material separating the inner and outer conductors has become waterlogged, and now exhibits a conductivity of  $0.1 \text{ mS}/\text{m}$ .

- Estimate the attenuation constant for this cable.
- Attenuation in this cable is due to a combination of conductor resistance and current leaking across the dielectric spacer. Which effect is most important in this case? Justify your answer.

[2]

**7.3-2** A 100 m length of coaxial cable having characteristic impedance equal to  $50 \Omega$  exhibits a resistance of  $125 \text{ m}\Omega/\text{m}$ . The conductivity of the spacer material is negligible. How much

power is lost in a signal passing through this cable, in dB?

# Chapter 8

## Optical Fiber



[m0192]

[1]

**8.2-1** A typical value of the numerical aperture (NA) of multimode optical fiber is 0.25. What is the (maximum) acceptance angle for this fiber?

[2]

**8.2-2** A laser used to inject a signal into an optical fiber is mounted such that its beam is aligned along the axis of the fiber. I wish to minimize the ability of stray light to enter the fiber. Given the choice, should numerical aperture be (a) minimized or (b) maximized?

[1]

**8.3-1** Estimate the delay spread (i.e., the difference in the time of arrival for the slowest propagating mode with respect to the fastest propagating mode) for a 45-m length of the fiber having numerical aperture equal to 0.25. Assume the fiber (as opposed to the cladding) has a relative permittivity of 2.2, and that both fiber and cladding consist of non-magnetic materials.

[2]

**8.3-2** A square pulse of width 1 ns is injected into an optical fiber of length 10 m. The index of refraction of the fiber is 1.55. The index of refraction of the cladding is 1.50. What is the width of the pulse after travel through the fiber, including both rising and falling edges?

# Chapter 9

## Radiation

[m0194]

[1]

**9.1-1** A current moment is located at the origin and points in the  $\hat{y}$  direction. What is the reference direction of the radiated electric field vector at  $(x, y, z) = (100\lambda, 0, 100\lambda)$ ?

[m0195]

[1]

**9.2-1** What are the SI base units of magnetic vector potential? Give your answer in terms of (a) webers and (b) teslas.

[m0196]

[1]

**9.3-1** Consider the magnetic vector potential  $\tilde{\mathbf{A}}$ .

(a) Using the definition of the magnetic vector potential as the curl of magnetic flux density, determine the units of  $\tilde{\mathbf{A}}$  in SI base units.

(b) The magnetic vector potential of a  $\hat{\mathbf{i}}$ -directed current moment  $\tilde{I}\Delta l$  located at the origin is

$$\tilde{\mathbf{A}} = \hat{\mathbf{i}} \mu \tilde{I} \Delta l \frac{e^{-\gamma r}}{r} \quad (9.1)$$

where  $\gamma$  is the propagation constant. Using SI base units, show that this equation is dimensionally correct.

[m0198]

[1]

**9.5-1** Consider a dipole antenna made from a perfectly-conducting metal, having length 0.3 m and radius 0.1 mm, radiating in free space. A current of 0.5 mA (peak) having frequency 10 MHz is applied to the terminals. The resulting electric field intensity is measured over a sphere of radius 1000 m around the dipole. What are the minimum and maximum magnitudes of the electric field intensity observed on this sphere?

[2]

**9.5-2** A current of 3 mA (peak) is applied to the terminals of a thin straight dipole of length 0.01 wavelengths, in free space. The electric field intensity is measured at various points 1000 m distant from the dipole. The wavelength is  $\ll$  1000 m. What are the minimum and maximum magnitudes of the electric field intensity that would be observed in these measurement

# Chapter 10

## Antennas

[m0207]

[1]

**10.2-1** An electrically-short dipole in free space is found to radiate a total power of  $-140$  dBW (decibels relative to 1 W) at 100 MHz when a sinusoidal current source of magnitude  $1 \mu\text{A}$  (rms) is applied to the terminals. Estimate the length of the dipole in wavelengths and in meter-like units.

[2]

**10.2-2** A thin straight dipole in free space has length equal to 0.01 wavelengths. What is the magnitude of the sinusoidally-varying current at the terminals that is required to create 30 mW of radiated power? Give your answer in root-mean-square (RMS) units.

[m0202]

[1]

**10.5-1** Consider an transmitting antenna which is driven by a Thevenin equivalent circuit source consisting of an ideal voltage source in series with an impedance  $R_S + jX_S$ . It is found that radiated power is maximized when  $R_S = 10 \Omega$  and  $X_S = +2.5 \text{ k}\Omega$ . Separately, it is found that the loss resistance of the antenna is  $25 \text{ m}\Omega$ . What is the radiation resistance, the reactance, and the radiation efficiency of this antenna?

[2]

**10.5-2** You wish to improve the power delivered by a source with  $50 \Omega$  output impedance to an electrically-short dipole. You are limited to ONE of the following options: (a) add resistor(s), (b) add capacitor(s), (c) add inductor(s). Whatever component(s) you pick will be inserted between the source and the dipole in whatever arrangement (i.e., series, parallel, series and parallel, etc.) that you judge best. Which option (a, b, or c) is most likely to yield the best result?

[m0204]

[1]

**10.6-1** Estimate the radiation resistance of a thin wire dipole that is 1.5 m long and which is to be used near the center of the AM broadcast band ( $\sim 1 \text{ MHz}$ ).

[2]

**10.6-2** The radiation resistance of an electrically-short dipole is found to be  $1 \Omega$ . If the frequency is doubled, what is the radiation resistance of antenna?