OPTIMAL ONE-WAY TRAFFIC CONTROL STRATEGY

for

UNDER-SATURATED TWO-LANE HIGHWAY WORK ZONE OPERATION

by

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Traffic operation and characteristic on two-lane highway work zone combine traffic characteristics on two-lane highways and signalize intersections. A literature review was conducted to understand traffic characteristics on two-lane highways and at signalized intersections. Delay models and theoretical methods of calculating delay were also reviewed for two-lane highways, unsignalized and signalized intersections. Particularly, research methods of developing delay models for signalized intersections provide a fundamental base in developing delay models and optimal control strategies for two-lane highway work zone operation.

The main framework contained in this thesis identifies the operational problems existing in two-lane highway work zone. Development of mathematical delay models is then performed for all operational aspects in the work zone. The analysis of traffic delay, the optimal selection of timing and the optimal delay control strategies are also detailed in the thesis.

This theoretical attempt to develop optimal control strategy for two-lane highway work zone is preliminary, yet valuable. The promising results from this research effort open the door to more exciting topics in this field.

Dedicated to my beloved wife

Qun(Sonya) Ma

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1. CHARACTERISTICS OF TRAFFIC FACILITIES

1.1 INTRODUCTION

The facility types range from travel roadway for moving vehicles such as freeways, two-lane and multilane highways, intersections, both signalized and unsignalized, urban arterials, to terminals to transfer passengers such as bus stations and parking facilities to storage vehicles.

The principal focus in this study is on two-lane highways and signalized intersections of which characteristics are directly related to this study. Study of characteristics of two-lane work zone under operation and development of optimal control strategy to minimize average total vehicle delay are based on understanding characteristics of the above two types of traffic facilities. This section provides brief review of basic characteristics of two-lane highways and signalized intersections and the quantitative elements reflecting their operation.

1.2 CHARACTERISTICS OF TWO-LANE HIGHWAYS

Two-lane highways is the type of facility with one travel lane in each direction and passing of slow-moving vehicles requires the use of a lane used by opposing traffic. Thus, traffic characteristics in each direction interact and both direction of travel must by analyzed as a unit. The basic elements in analysis of two-lane highways are the two-way volume, the average travel speed of all vehicles, and the percent time delay. Figure 1.1 shows the relationship between average speed and flow on two-lane highways, while the relationship between percent time delay and flow on two-lane highways is illustrated in Figure 1.2.

The basic traffic flow relationship to reflect operational characteristics is

$$SF_i = 2{,}800 \times (v/c)_i \times f_d \times f_w \times f_{HV} \qquad (1.1)$$

where:

SF_i - total service in both directions

$(v/c)_i$ - ratio of low rate to capacity

f_d - adjustment factor for directional distribution of traffic

f_{HV} - adjustment factor for the heavy vehicles

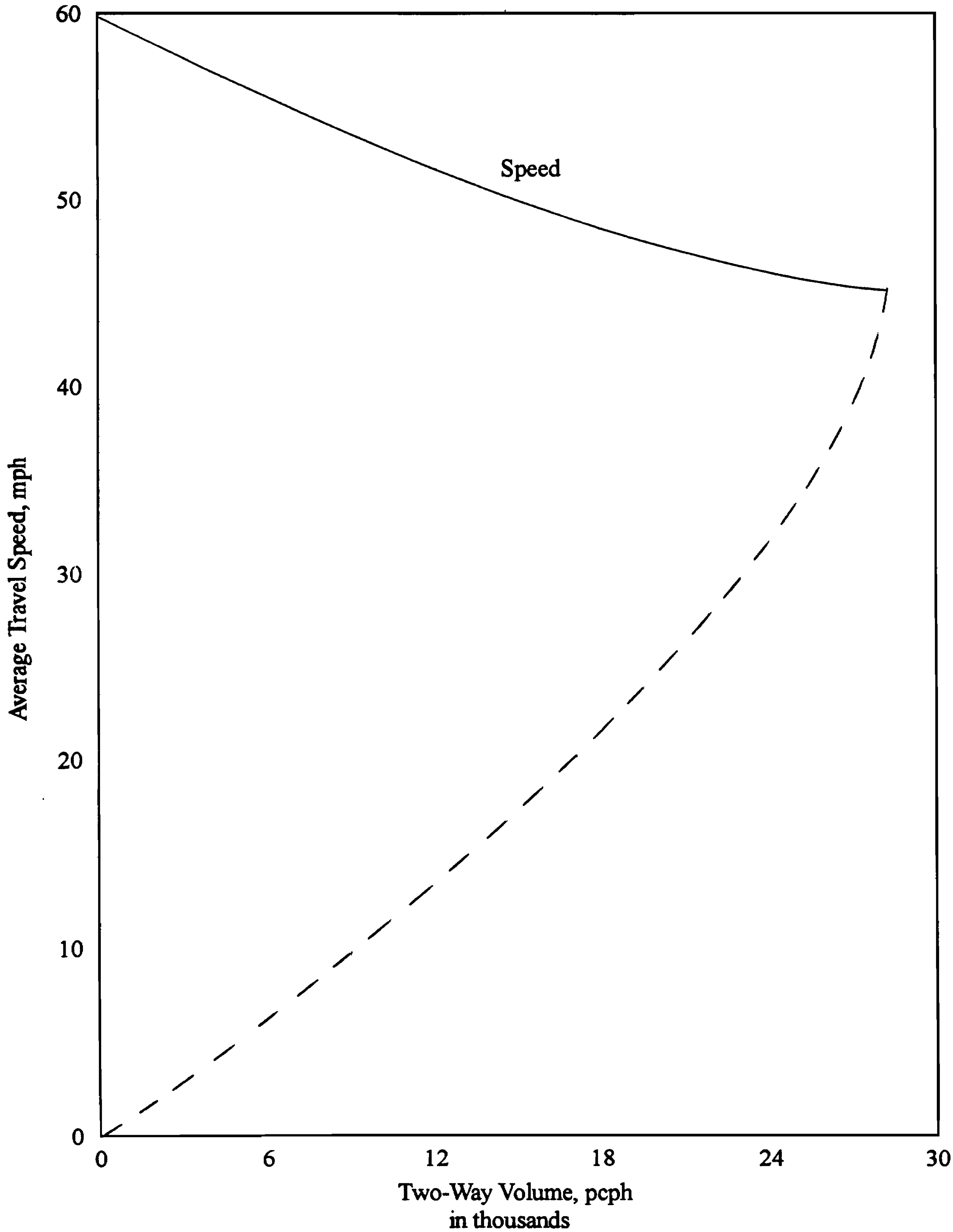


Figure 1.1 - Relationship Between Average Speed And Flow On Two-Lane Highways

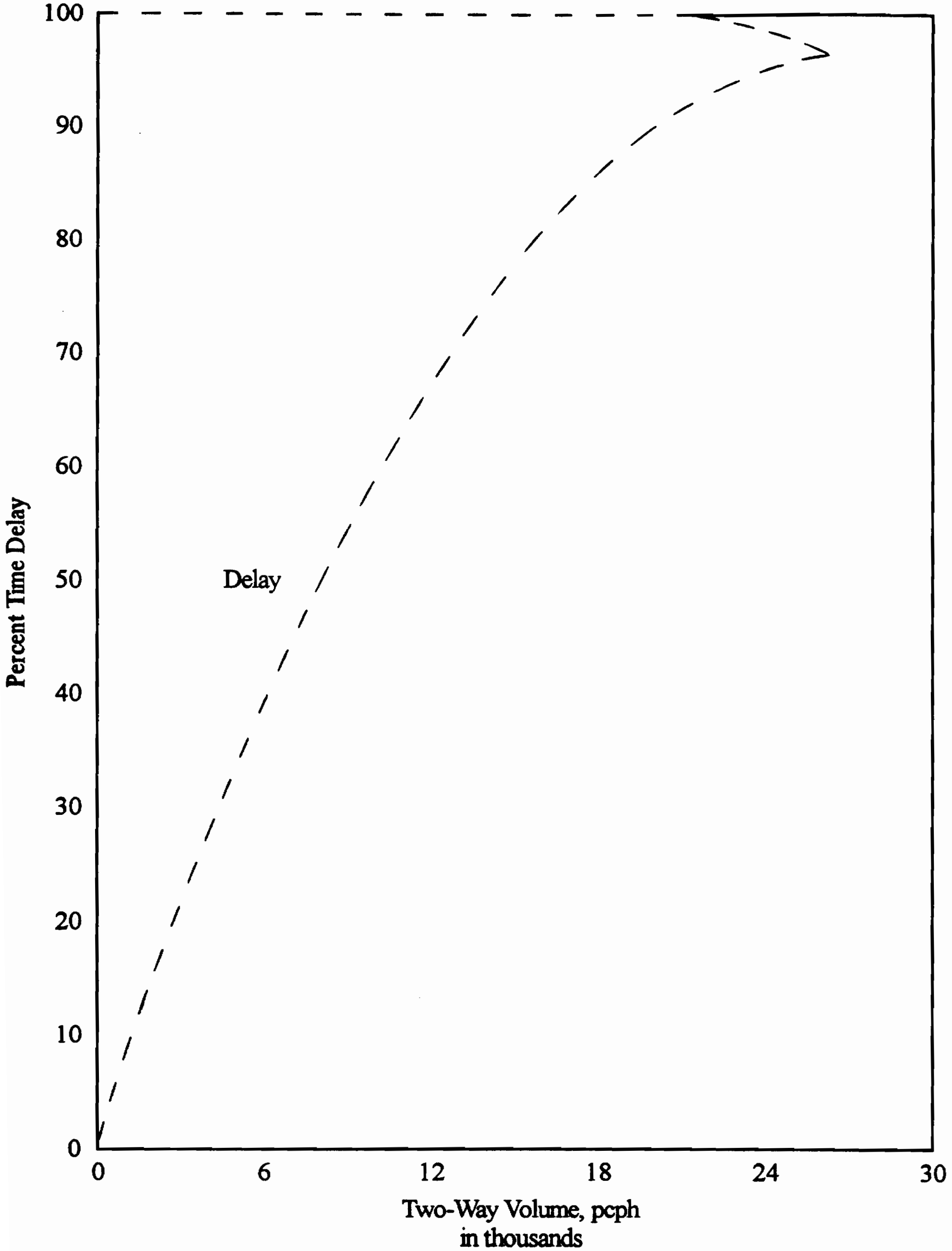


Figure 1.2 - Relationship Between Percent Time Delay And Flow On Two-Lane Highways

There are other elements such as terrain, grade and percentage of no passing zones which are discussed here. Due to the nature of theoretical study, only ideal conditions are assumed and essential characteristics are discussed thereafter.

1.3 CHARACTERISTICS OF SIGNALIZED INTERSECTIONS

The signalized intersection is one of the most complex locations in a traffic system, and traffic signals provide a complex type of operation for traffic flow. Installation of traffic control signal is necessary at locations where increased traffic demand causes excessive delay and unsafe operation. Traffic signal controls a intersection by allocating green time between conflicting movements in order to provide for an orderly movement of traffic and increase the operational efficiency of the intersection. Other advantages of installation of a traffic control signal is that it can reduce the frequency of certain types of traffic accidents, and also under favorable conditions, several signals can be coordinated to provide for continuous movement of traffic at a definite speed along a given route.

The important characteristics of a signalized intersection are that a portion of approaching traffic is stopped at the intersection during the red signal phase and then departs during the green signal phase. The relationship between traffic demand and intersection capacity for one directional approach at a fixed-time signalize intersection is illustrated in Figure 1-3. As shown in Figure 1-3, traffic approaches the inspection at a constant flow rate(Demand q) either during a green phase or during a red phase, then traffic departs the intersection at either maximum departure rate (Capacity Q) or the constant flow rate q during the green phase. It is obvious that the number of arrival vehicles per signal cycle are less than the capacity for departures, or

$$(R+G)q < GQ \tag{1.2}$$

where:

Q - departure rate

q - arrival rate

G - green phase

R - red phase

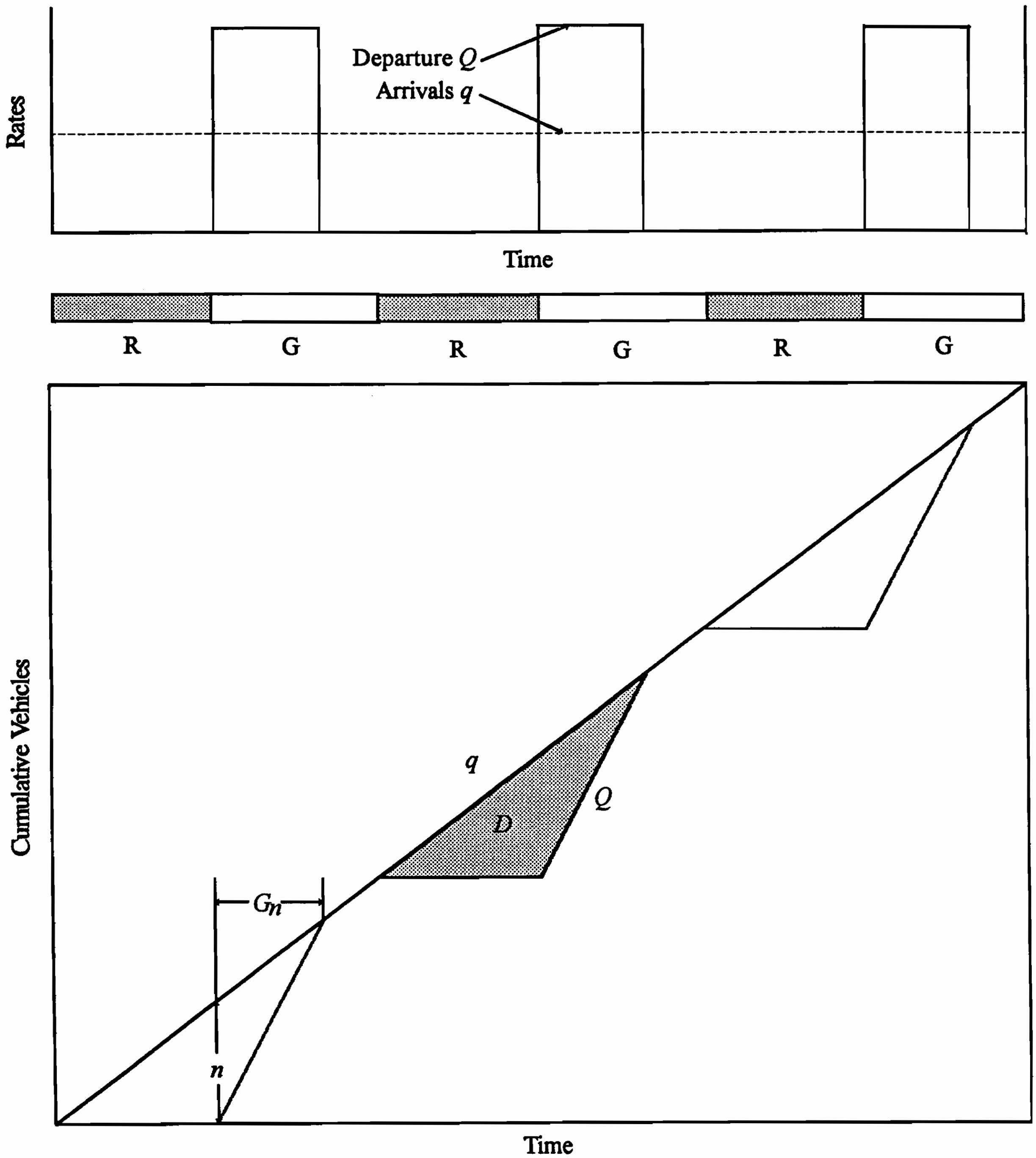


Figure 1.3 - Relationship Between Demand And Capacity At A Signalized Intersection[7]

The maximum number of vehicles approach the intersection during a red phase is given by

$$n = qR \tag{1.3}$$

The effective green time G_n necessary to dissipate the n vehicles is less than the green phase G, or

$$G_n = \frac{\rho}{1-\rho} R \tag{1.4}$$

where:

ρ is the factor for the degree of saturation q/Q

The shaded area shown if Figure 1.3 stands for the total delay, D, and can be expressed as

$$D = \frac{1}{2} R \cdot (G_n Q) \tag{1.5}$$

Substituting equation (1.4) into (1.5), equation (1.5) becomes

$$D = \frac{\rho R^2 Q}{2(1-\rho)} \tag{1.6}$$

The average delay per vehicle, d, is the total delay divided by the total number of vehicles n:

$$d = \frac{R^2}{2(R+G)(1-\rho)} \tag{1.7}$$

The delay studies for signalized intersections have played a important role in understanding traffic characteristics at signalized intersections and providing theoretical support for efficient operation of signalized intersections. Research efforts in delay studies were reviewed and are presented in Chapter 2.

2. TRAFFIC DELAY STUDIES

2.1 INTRODUCTION

An important consideration encountered by traffic professionals is traffic delay. An understanding of the delay problem is important because the economic losses involved by vehicular delays can be enormous. Delay is also the measure of effectiveness for traffic facilities. Study of traffic delay is needed to provide effective measurements and develop effective procedures to minimize delay.

Research efforts in the field of delay study have been intensive and various mathematical models and procedures were developed by researchers. This chapter presents delay studies for two-lane highways, unsignalized and signalized intersections. The principal focus is on delay studies for signalized intersections which are the theoretical sources to study the traffic delay on under-saturated two-lane highway work zone.

2.2 DELAY ON TWO-LANE HIGHWAYS

Traffic on two-lane highways requires the use of a lane used by opposing traffic in order to pass slow-moving vehicles. This is the main factor contributed to traffic delay on two-lane highways. The average delay, caused by a vehicle in the traffic stream to make an overtaking and passing maneuver, can be expressed as follows[16]:

$$d = \frac{1}{2}\frac{3{,}600TS_2}{g(S_1 + S_2)} \tag{2.1}$$

where:

d - average vehicle delay

T - percent of blocking time

g - number of acceptable gaps

S_1 - overtaken speed

S_2 - oncoming speed

2.3 DELAY AT UNSIGNALIZED INTERSECTIONS

This section presents traffic delay study at unsignalized intersections, where moving priority is assigned to main-street traffic and side-street is cautioned to slow or halt. The same situation can be found when traffic on a ramp waits for a suitable gap to join the main stream of traffic.

A systematic study of the delay problem at unsignalized intersections was conducted by *Adams* in 1936 [1]. It is assumed that the arrival pattern of vehicles is *Poisson* distribution which is random, thus headways between vehicles are exponentially distributed and the probability of a headway being equal to or greater than t sec, is:

$$P(h \geq t) = e^{-\lambda t} \qquad (2.2)$$

where:

h - headway between vehicles

t - time interval

λ - mean arrival rate or V/T

V - Volume of arrival vehicles

T - duration of time for V to pass

The proportion of time occupied by intervals greater than t seconds is given by:

$$P_t = e^{-\lambda t}(\lambda t + 1) \tag{2.3}$$

Thus, the proportion of time occupied by gaps less than t can be expressed as:

$$1 - P_t = 1 - e^{-\lambda t}(\lambda t + 1) \tag{2.4}$$

In general, the probability of a side-street vehicle having to wait for n main-street intervals, each less than t, is

$$\bar{n} = \frac{1 - e^{-\lambda t}}{e^{-\lambda t}} \tag{2.5}$$

The average length of intervals through which the side-street vehicles have to wait is

$$\frac{1}{\lambda} - \frac{te^{-\lambda t}}{1 - e^{-\lambda t}} \tag{2.6}$$

Therefore, the expected mean time-in-service, $\bar{d}_s$, is equal to the product of Equation (2.5) and expression (2.6):

$$\bar{d}_s = \frac{1 - e^{-\lambda t}}{e^{-\lambda t}} \left(\frac{1}{\lambda} - \frac{te^{-\lambda t}}{1 - e^{-\lambda t}} \right)$$

$$= \frac{1 - e^{-\lambda t}}{\lambda e^{-\lambda t}} - t \tag{2.7}$$

Expression (2.7) is a useful tool for computing delays at priority intersections.

2.4 DELAY AT SIGNALIZED INTERSECTIONS

2.4.1 Webster's Delay Model

In 1958, English researcher Webster conducted a comprehensive study for signalized-intersection performance. The importance of his research is that it dealt with the random nature of traffic flow and developed mathematical models from which the average delay per vehicle can be computed as a function of the cycle length, the proportion of the cycle length which is green, the approach volume or actual flow per lane, and the degree of saturation. In addition, Webster developed models for computing the approximate cycle length that will minimize the total intersection delay, and for calculating average queue lengths.

In general, the total effective green time available for vehicular movement is given by

$$G_E = C - \sum_{i=1}^{p} K_i \qquad (2.8)$$

where:

G_E - -total effective green time

K_i - total lost time during i th phase

p - number of phases

C - cycle length

Also, $G_{E,i}$, the effective green time during the i th phase, is the subtraction of the lost time K_i from the total green time G_i, or

$$G_{E,i} = G_i - K_i \tag{2.9}$$

The average delay experienced per vehicle by th flow on the j th particular approach during the i th phase can be expressed as [27]

$$dj = \frac{C(1-\lambda i)^2}{2(1-\lambda_i \chi_j)} + \frac{\chi_j^2}{2V_j(1-\chi_j)} - 0.65\left(\frac{C}{V_j^2}\right)^{\frac{1}{3}} \chi_j^{2+5\lambda_i} \tag{2.10}$$

where:

dj - average delay per vehicle for low on j th approach during i th phase

λ_i - proportion of cycle length effectively green during i th phase

V_j - actual volume on jth phase

x_j - degree of saturation for jth approach $(= V_j/\lambda_i S)$

S - saturation flow rate $(= 1/h)$

h - minimum vehicle headway at maximum flow

For Equation (2.10), it was necessary to make a distinction between each of the approaches whose traffic moves during the ith phase, The reason for this is that the effective green time (and, in turn, λ_i) for the ith phase is determined on the basis of the design volume or the volume of the most saturated approach lane of all approaches whose traffic moves during the ith phase, while the delay for the jth approach is based on the actual volumes for that approach (which may be different from the design volume for the ith phase).

Once d_j has been calculated form Equation (2.10), the total hourly delay per lane for the jth approach can be determined simply by multiplying d_j by the hourly lane volume. Further, if the jth approach has multiple lanes and different volumes on each lane, the delay for each lane of the jth approach can be computed separately. One the other hand, if the volumes on each lane of the jth approach are equal or if approximate results are sufficient (where the lane volumes differ somewhat), the volumes on the jth-approach lanes can be averaged in order to compute V_j and χ_j, and , in turn, d_j.

Finally, the total intersection delay can be computed by summing the delays for all lanes of all approaches at the intersection (or by summing the delays for all approaches if the approximate method is used).

The general framework of analysis outlined in the previous section represents a useful approach for selecting a cycle length with minimum delay. The analysis should be conducted for a series of different cycle lengths and for other special conditions where appropriate, such as biasing the green split toward the heavy-volume direction. In many cases it may be sufficient to implicitly value these different aspects and select an appropriate cycle length. In other cases a more complex economic analysis of the alternatives may be necessary.

2.4.2 Greenshields' Theoretical Method of Calculating Delay

In 1947, Research Scientist Greenshields developed a theoretical method for determination of delay at signalized intersections. His research work was reported in *Traffic Performance at Urban Street Intersections*(see Reference 11). Similar to Webster's delay model, Greenshields' theoretical method also dealt with the random nature of traffic flow at signalized intersections. However, this theoretical method was developed to calculate discrete vehicle delay as a function of actual flow rate, headways, red interval, time after change to red that vehicle arrives and time after change to red that first vehicle enters.

In general, it can be assumed that the arrival pattern of traffic to a signalized intersection follows a *Poisson* distribution. Thus, the probability of number of vehicles arriving after the signal changes to red phase is

$$P(\lambda) = e^{-\chi u} \frac{(\mu\chi)^{\lambda}}{\lambda!} \qquad (2.11)$$

where:

χ - Time after change to red that vehicle arrives

μ - actual traffic flow rate

λ - number of arrival vehicles

Therefore, the probability of the first vehicle arriving at 'χ' is the product of the probability of zero vehicles arriving during 'χ' and the probability of one vehicle arriving in '$d\chi$', or expressed as

$$P(1) = e^{-\chi\mu}\mu d\chi \tag{2.12}$$

where

$d\chi$ is when 'χ' approaches zero.

Also assume

T_1 - potential blocking period for the first vehicle

T_2 - potential blocking period for the second vehicle

t_1 - time after change to red that first vehicle enters intersection

t_2 - time after change to red that second vehicle enters intersection

L_1 - average time loss of first vehicle

L_2 - average time loss of second vehicle

$\varnothing$ - headway between vehicles

The loss of time of the first vehicle arriving at 'χ' is

$$t_i - \chi \tag{2.13}$$

Therefore, the average time loss of first vehicle, L_1, can derived by combining (2.12) and (2.13) as

$$L_1 = \int_0^{T_1} (t_1 - \chi) e^{-\chi\mu} \mu d\chi$$

$$= t_1 - \varnothing + [\varnothing - (t_1 - T_1)] e^{-\mu T_1} \tag{2.14}$$

For the second vehicle, the average time loss is computed in a similar manner. The expression for the probability of the second vehicle arriving at 'χ' is

$$P(2) = e^{-\chi u} \frac{(\mu\chi)^1}{1!} \times \mu d\chi$$

or

$$P(2) = e^{-\chi u}\chi\mu^2 d\chi \tag{2.15}$$

and the average corresponding delay, d_{21}, for the second vehicle during T_1 is

$$d_{21} = \int_0^{T_1}(t_2 - \chi)e^{-\chi\mu}\chi\mu^2 d\chi \tag{2.16}$$

The probability of the second vehicle arriving at T_1 is

$$P(2T_1) = e^{-T_1 u}\frac{(T_1\mu)^1}{1!} \times e^{-(\chi - T_1)\mu} \times \mu d\chi \tag{2.17}$$

and the corresponding delay is

$$d_{22} = \int_{T_1}^{T_2}(t_2 - \chi)e^{-T_1\mu}T_1\mu \cdot e^{-(\chi - T_1)\mu} \cdot \mu d\chi \tag{2.18}$$

Therefore, the average loss of time to the second vehicle is the sum of (2.17) and (2.18) or

$$L_2 = \int_0^{T_1} (t_2 - \chi)e^{-\chi\mu}\chi\mu^2 d\chi + \int_{T_1}^{T_2} (t_2 - \chi)e^{-T_1\mu}T_1\mu \cdot e^{-(\chi - T_1)\mu} \cdot \mu d\chi$$

$$= t_2 - 2\varnothing + T_1[1 - \mu(t_2 - T_2)]e^{-T_2\mu} + [2\varnothing - (t_2 - T_1)]e^{-T_1\mu} \qquad (2.19)$$

In the above calculations, time loss due to acceleration is neglected but a correction may be made by assuming that the first-in-line vehicle loses an additional two seconds due to acceleration and the second and third vehicles each lose one second. By the time the fourth-in-line and subsequent vehicles have entered the intersection, they are assumed to be traveling at normal speed.

In order to compute the total delay caused be the signal, it would be necessary to calculate the average delay for the third, fourth, fifth, etc., vehicles and then add them all together. As the number of vehicles increases, the mathematics become too involved to be practical. No simplification has yet been found which would make this theoretically correct method workable enough for ordinary computation. However, this theoretical method represents calculation of traffic delay at signalized intersection in a clear mathematical manner.

3. OPTIMAL CONTROL STRATEGY FOR WORK ZONE OPERATION

3.1 INTRODUCTION

It is common that state DOTs or utility companies have to block down a section of one travel lane on two-lane highway in order to perform roadway improvement work or roadside construction. This may be called tow-lane highway work zone operation and results in creating a bottleneck where traffic from both directions has to use only one travel lane within the work zone to pass the area. The distance of block-down travel lane may varies, it certainly causes the increase in traffic delay due to the decrease in capacity.

The current practice of traffic control on two-lane highway work zone relies on manual operation with crew members standing on both ends of the work zone and flag passing signals to traffic waiting to pass. Even though two-way communication devices are used, this manual-control method results in inefficient operation of the work zone. The unnecessary increase in delay causes inconvenience to motorists.

The operational characteristics of two-lane highway work zone needs to be investigated in order to develop control strategy to improve the efficiency of the work zone. The following sections presents feasible solutions to solve the delay problems. Mathematical delay models was first developed to reflect traffic flow characteristics within the work zone, optimal control strategies were then developed to minimize the average vehicle delay through optimal selections of timing.

3.2 DEVELOPMENT OF DELAY MODEL

It is assumed that normal traffic conditions occur in the two-lane highway work zone area and the two-lane highway is under-saturated. The following theory may not apply to high volume two-lane highways. It is also assumed that the pattern of approaching traffic is random and corresponds to a *Poisson* distribution process.

For an under-saturated two-lane highway system, assume

$$S > (q_i + q_o) > q_o \geq q_i \tag{3.1}$$

where:

S - saturation flow rate of the work zone area;

q_i - demand at direction i ;

q_o - demand at direction o.

For convenience, it is assumed that the work zone is controlled by a traffic signal which is similar to traffic control at signalized intersections. Traffic signals allocate green times for passing traffic in both directions. The terms of signal operation such as cycle length, green time, red time and lost time are used to define the work zone operation.

Based on the above assumptions, one can select proper effective green durations, t_i and t_o, for both directions to determine the appropriate size for the cycle length, C, in order to accommodate the demands; that is, as long as the inequality in Equation (3.1) holds. The given parameters for this analysis include:

t_L - effective lost time for signal change ($= t_t + L_S$);

t_t - average vehicle travel time through the work zone;

L_s - starting lost time.

The travel time through the work zone t_t and the starting lost time L_s may take different values for each direction. However, t_t and L_s are fixed for the given conditions at the work zone, thus the lost time per cycle, $2t_L$, is fixed. The traffic flow relationships is depicted in Figure 3.1. As shown if Figure 3.1, the total vehicle delay, D, can be derived as

$$D = W - X + Y - Z$$

where:

$$W = \frac{1}{2}\left(t_o + 2t_L + \frac{q_i}{S - q_i}(t_o + 2t_L)\right)^2 q_i$$

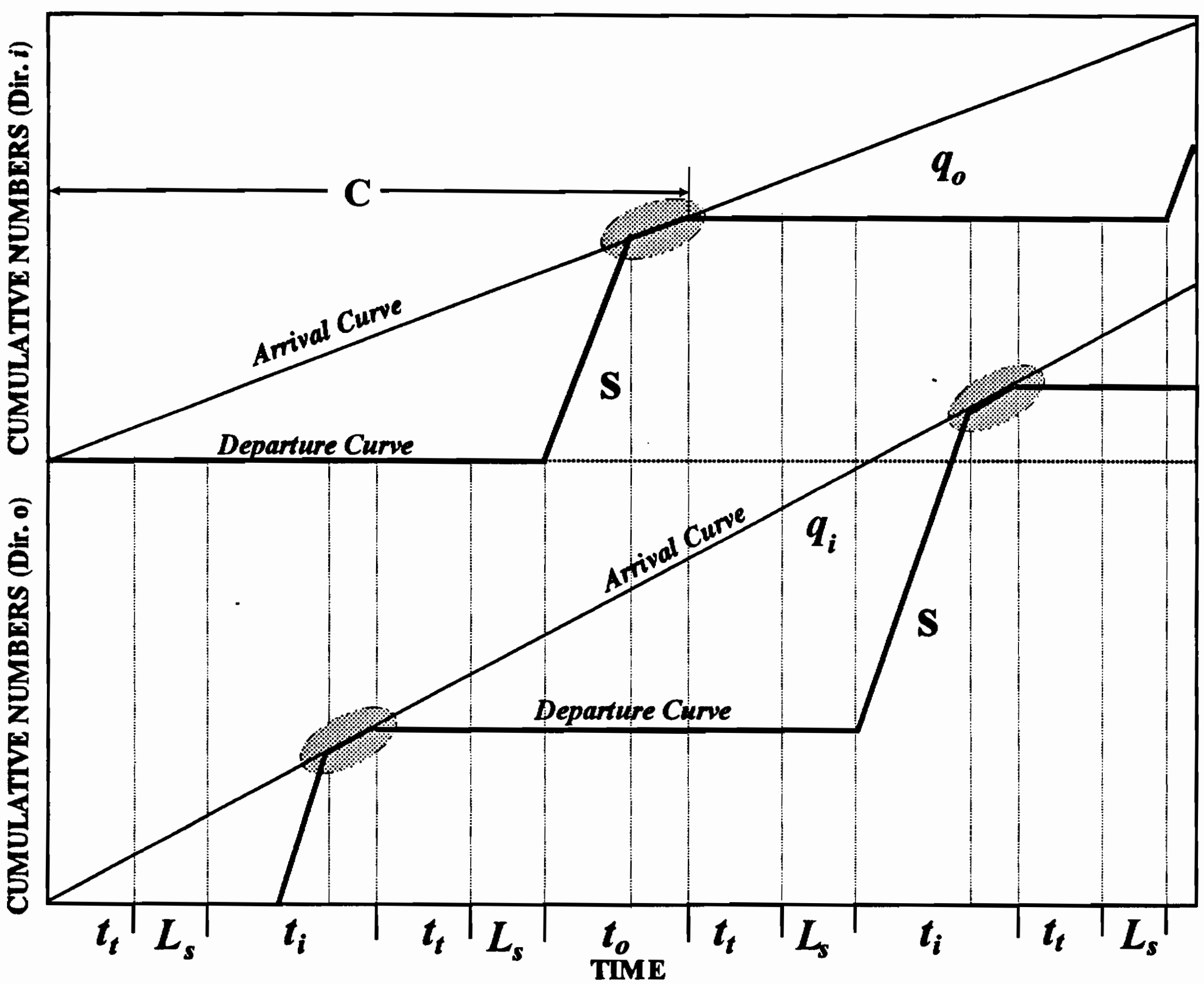


Figure 3.1 - Cumulative Flow Diagram For Case 1

$$X = \frac{1}{2}\left(t_o + 2t_L + \frac{q_i}{S - q_i}(t_o + 2t_L)\right)\frac{q_i^2}{S - q_i}(t_o + 2t_L)$$

$$Y = \frac{1}{2}\left(t_i + 2t_L + \frac{q_o}{S - q_o}(t_i + 2t_L)\right)^2 q_o$$

$$Z = \frac{1}{2}\left(t_i + 2t_L + \frac{q_o}{S - q_o}(t_o + 2t_L)\right)\frac{q_o^2}{S - q_o}(t_i + 2t_L)$$

Simplifying the above equation, it becomes

$$D = \frac{S}{2}\left(\frac{q_i}{S - q_i}(t_o + 2t_L)^2 + \frac{q_o}{S - q_o}(t_i + 2t_L)^2\right)$$

Thus, the average vehicle delay per cycle is the total vehicle delay divided by the number of vehicles arrived during the cycle, or

$$d = \frac{D}{(q_i + q_o)(t_i + t_o + 2t_L)}$$

$$= \frac{S}{2} \frac{1}{(q_i + q_o)(t_i + t_o + 2t_L)} \left(\frac{q_i}{S - q_i} (t_o + 2t_L)^2 + \frac{q_o}{S - q_o} (t_i + 2t_L)^2 \right) \quad (3.2)$$

where:

d - average delay per vehicle;

t_i - effective green time for direction i ;

t_o - effective green time for direction o.

The objective is to minimize the average delay per vehicle, d, by seeking the optimum set-up for t_i and t_o which ought to strictly satisfy various restricted conditions to accommodate the demands at both directions. Once a section of rural highway is blocked for the maintenance or reconstruction purpose, the dynamic conditions of traffic flow vary from time to time such as existing peak-hour or non peak-hour traffic. In order to simplify that for the convenience of discussion, one may assume:

$$q_i : q_o = k \quad (3.3)$$

where:

k - ratio

3.3 ANALYSIS AND OPTIMIZATION

3.3.1 DEFINITION OF HIGHWAY OPERATING CONDITIONS

The entire average delay minimization process and the optimal strategy seeking method can be classified into four cases:

Case 1 - Continue for a while at both directions after queue ends;

Case 2 - Switch right after queue ends at both directions;

Case 3 - Continue for a while at direction i , switch right after queue ends at direction o;

Case 4 - Continue for a while at direction o, switch right after queue ends at direction i .

Case 1 is based on the assumption that the duration of effective green time at both directions is designed longer than what it needs to terminate queue, that is, the green phase at direction i will provide a certain amount of time to allow the traffic pass through the work zone with the saturation flow rate, S, and then the traffic light at green phase will not switch to the opposite direction o after the queue ends but continue to maintain green state for a while to let the traffic pass through the work zone with the arrival rate q_i and this also is the same as that at direction o. This situation is illustrated in Figure 3.1.

For *Case* 2, it is considered with another condition with exact amount of time for the green phase to end queue, that is, the green phase at direction i will end and switch to direction o as soon as the queue at direction i ends rather than be extended to let traffic pass through work zone with arrival rate q_i, this is also same for direction o. As a matter of fact, the traffic will pass through the work zone during the green phase with departure rate S which is the saturation flow rate of the work zone area and it is the special case of the preceding condition in which the traffic continue to pass through the work zone with arrival rate q_i or q_o after the queue that ends will not exist in this condition. A diagram illustrating this condition is shown in Figure 3.2.

Case 3 is based on the assumption that the green phase will first let the traffic pass through the work zone with the saturation flow rate,

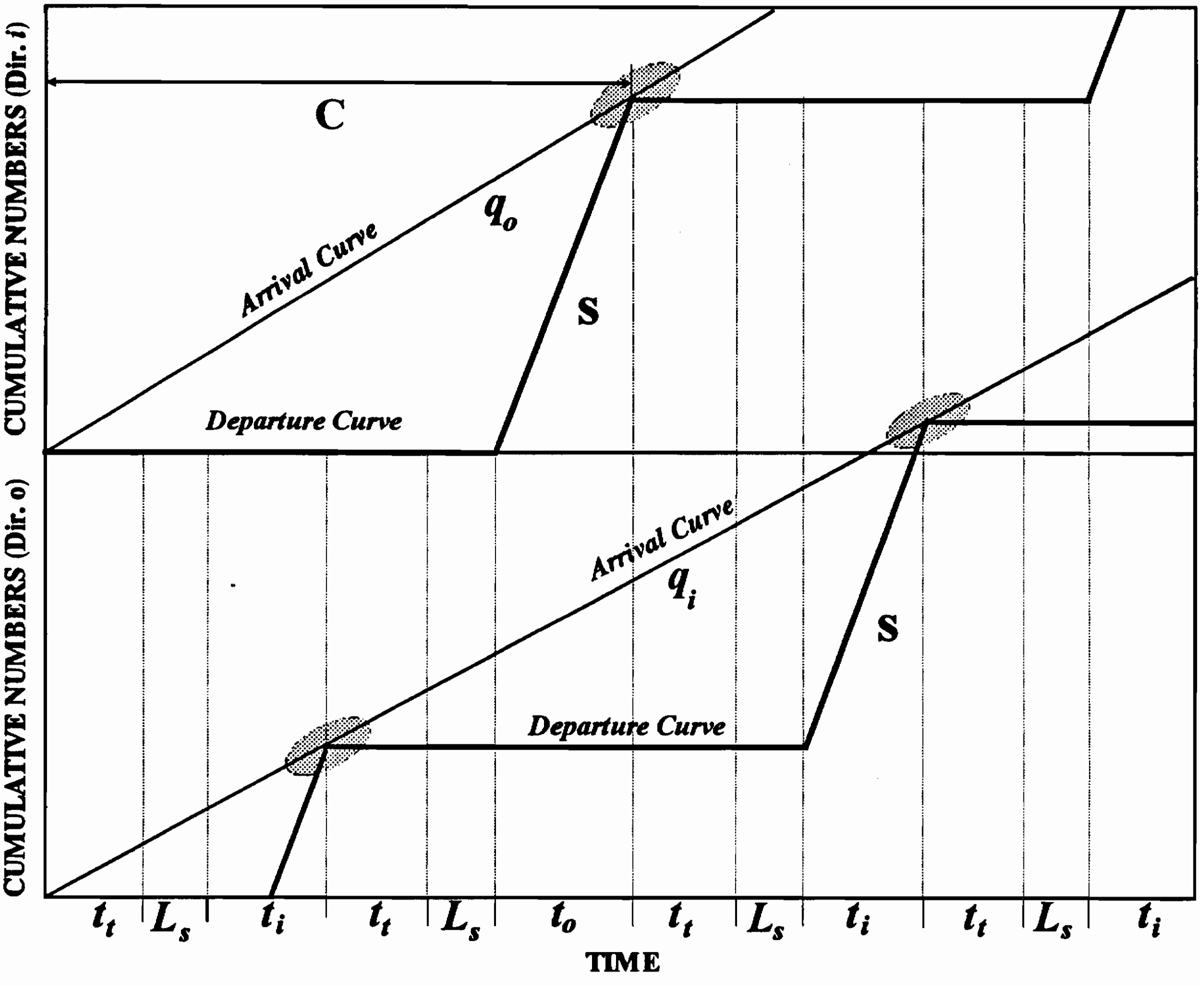


Figure 3.2 - Cumulative Flow Diagram For Case 2

C, to end the queue which has been cumulated during the red phase and then still maintain the green phase stage to allow the traffic pass through the work zone with the uniform arrival rate q_i. After the completion of the green duration, plus the effective lost time, at direction i, it will switch to the red phase which it is the green phase at direction o. During the green phase at direction o, it will just let the traffic pass through the work zone with the saturation flow rate S, then end the queue. And then the traffic light will switch back to direction i. A diagram illustrating this situation is shown in Figure 3.3.

Conversely, the operation of the traffic light for *Case 4* is the same as that for *Case 3* except direction i exchanges with direction o and vice versa. This situation is shown in Figure 3.4.

It is apparent that *Cases 3* and *Case 4* are just the special cases of *Case 1*. In order to seek the optimal strategy for minimizing the average delay, it is essential to thoroughly analyze all the conditions such as boundary conditions in *Case 1*. Then *Cases 2* through *4* are discussed.

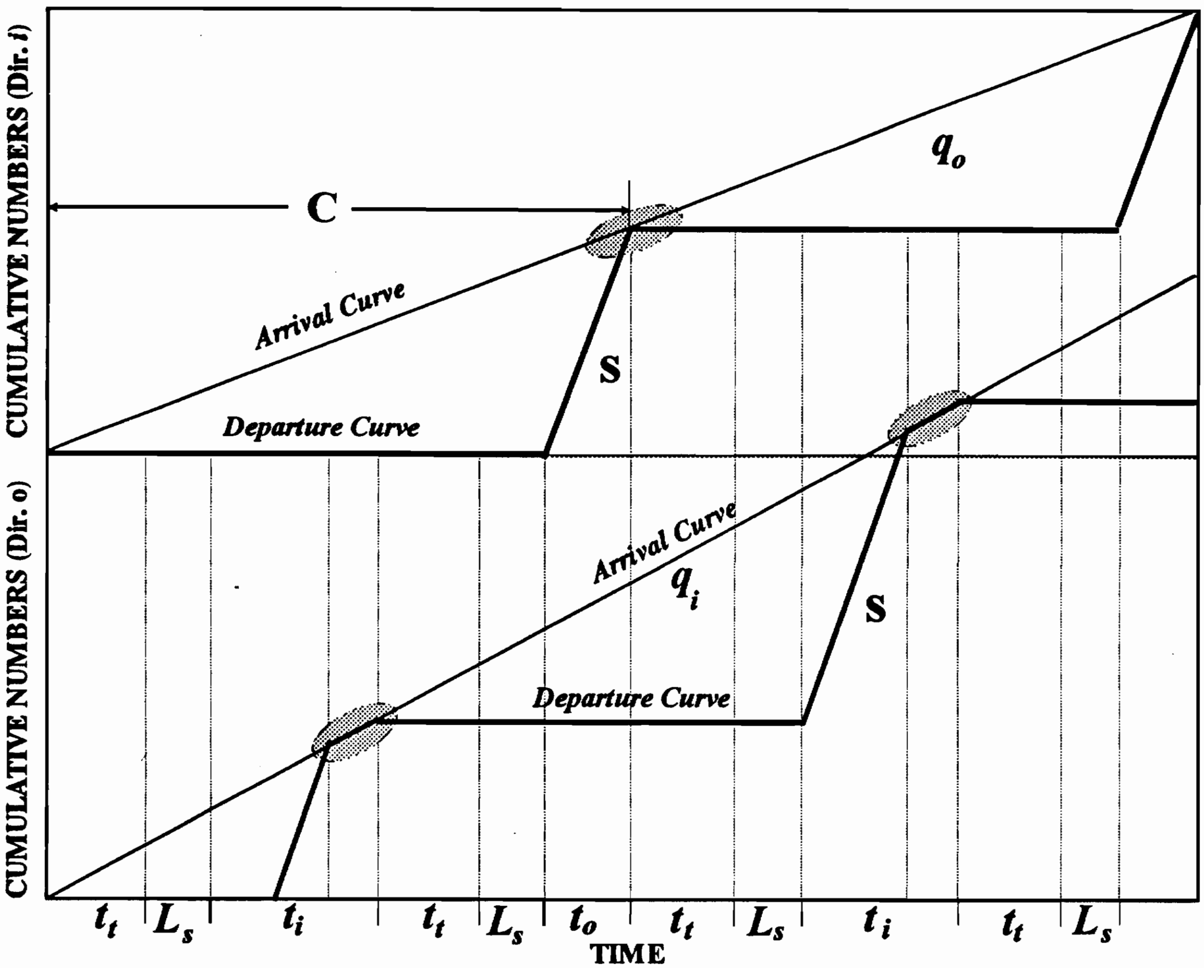


Figure 3.3 - Cumulative Flow Diagram For Case 3

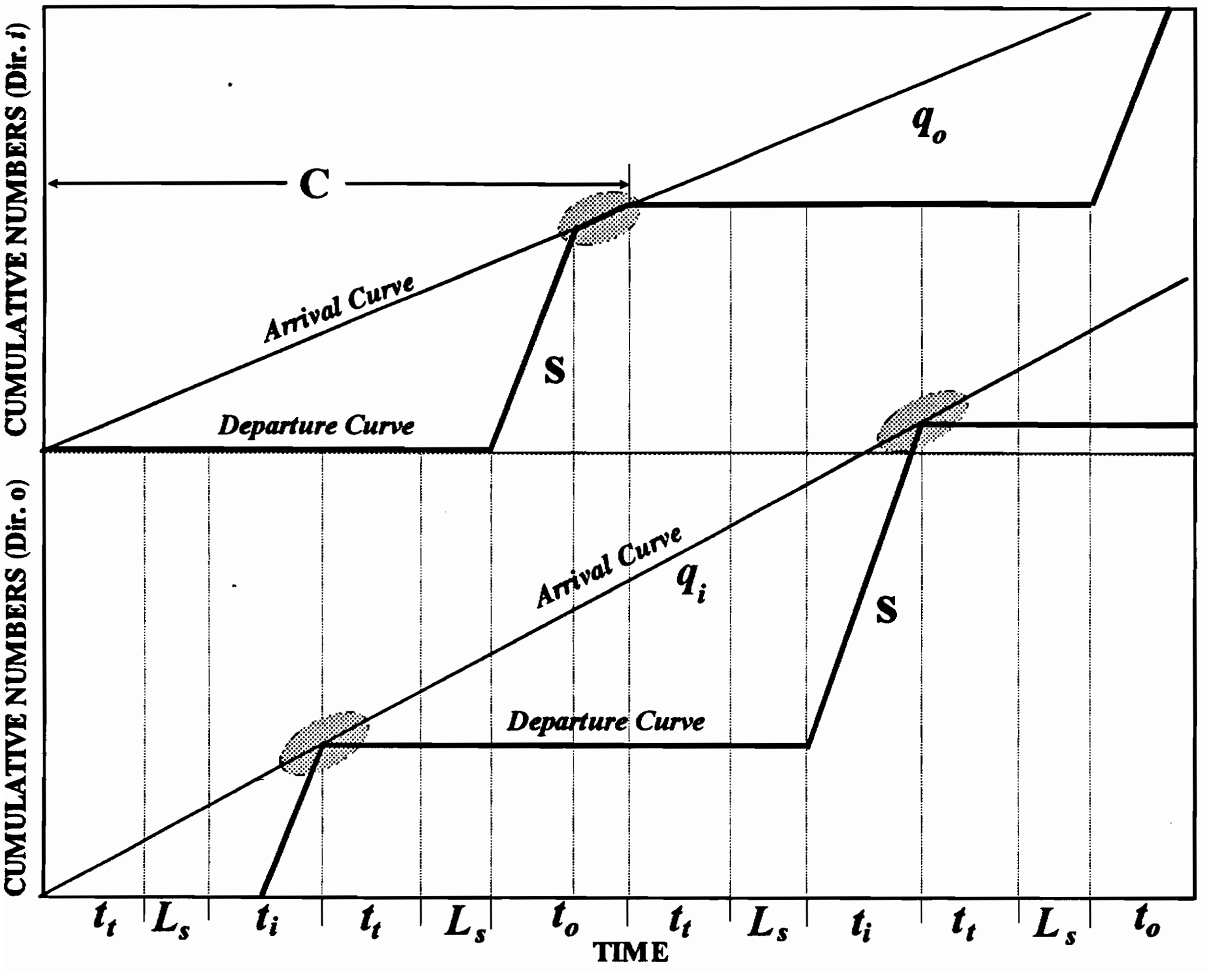


Figure 3.4 - Cumulative Flow Diagram For Case 4

3.3.2 ANALYSIS OF CASE 1

In *Case 1*, the objective is to minimize d, the average delay per vehicle, which is expressed as

$$d = \frac{S}{2} \frac{1}{(t_i + t_o + 2t_L)(q_i + q_o)} \left(\frac{q_o}{S - q_o} (t_i + 2t_L)^2 + \frac{q_i}{S - q_i} (t_o + 2t_L)^2 \right)$$

where:

d - average delay per vehicle;

t_i - effective green time for direction i ;

t_o - effective green time for direction o.

The cycle length C can be expressed as

$$C = (t_i + t_L) + (t_o + t_L) = t_i + t_o + 2t_L$$

If there were no boundary conditions, i.e., t_i and t_o can take any real number values, d will also have a wide range of values (which is of course not realistic). One can derive from equation (3.2) that d is always positive when $t_i + t_o < -2t_L$; and d is always negative when $t_i + t_o > -2t_L$. In fact, the only situation where d is equal to zero happens when $t_i + t_o = -2t_L$. It is also clear that d goes to either positive or negative infinity near the line representing $t_i + t_o = -2t_L$.

However, we do know that there exist several sets of boundary conditions:

1. The effective green time t_i and t_o are both positive numbers. That is:

$$t_i > 0 \tag{3.4}$$

and

$$t_o > 0 \tag{3.5}$$

2. In order to guarantee enough effective green time to accommodate the demands, one would like to have a cycle length which satisfies:

$$C \geq \frac{2t_L S}{S - q_i - q_o} \tag{3.6}$$

where:

$$C = t_i + t_o + 2t_L \tag{3.7}$$

Therefore, one can obtain from (3.6) and (3.7):

$$t_i + t_o \geq 2t_L \frac{q_i + q_o}{S - q_i - q_o} \tag{3.8}$$

3. For any given value of t_o, there is a range that t_i is bounded in order to meet the demand of q_i; that is:

$$\frac{(S - q_o)t_o}{q_o} - 2t_L \geq t_i \geq \frac{q_i}{S - q_i}(t_o + 2t_L) \tag{3.9}$$

Similarly, for any given t_i, there is a fixed range for t_o:

$$\frac{(S - q_i)t_i}{q_i} - 2t_L \geq t_o \geq \frac{q_o}{S - q_o}(t_i + 2t_L) \tag{3.10}$$

Equations (3.9) and (3.10) describe exactly the same following boundary conditions:

$$t_i \geq \frac{q_i}{S - q_i}(t_o + 2t_L) \tag{3.11}$$

and

$$t_o \geq \frac{q_o}{S - q_o}(t_i + 2t_L) \tag{3.12}$$

In fact, the feasible region bounded by equations (3.11). The two boundaries of equations (3.11) and (3.12) intersect at $(t_i, t_o) = \left(\frac{2t_L q_o}{S - q_i - q_o}, \frac{2t_L q_i}{S - q_i - q_o} \right)$, where equation (3.8) also goes through. The boundary conditions and feasible region are illustrated in Figure 3-5.

Although all the boundary conditions are linear conditions, the average delay function, (3.2), is non-linear. Therefore, the average delay minimization process is actually a non-linear programming problem which can be expressed as:

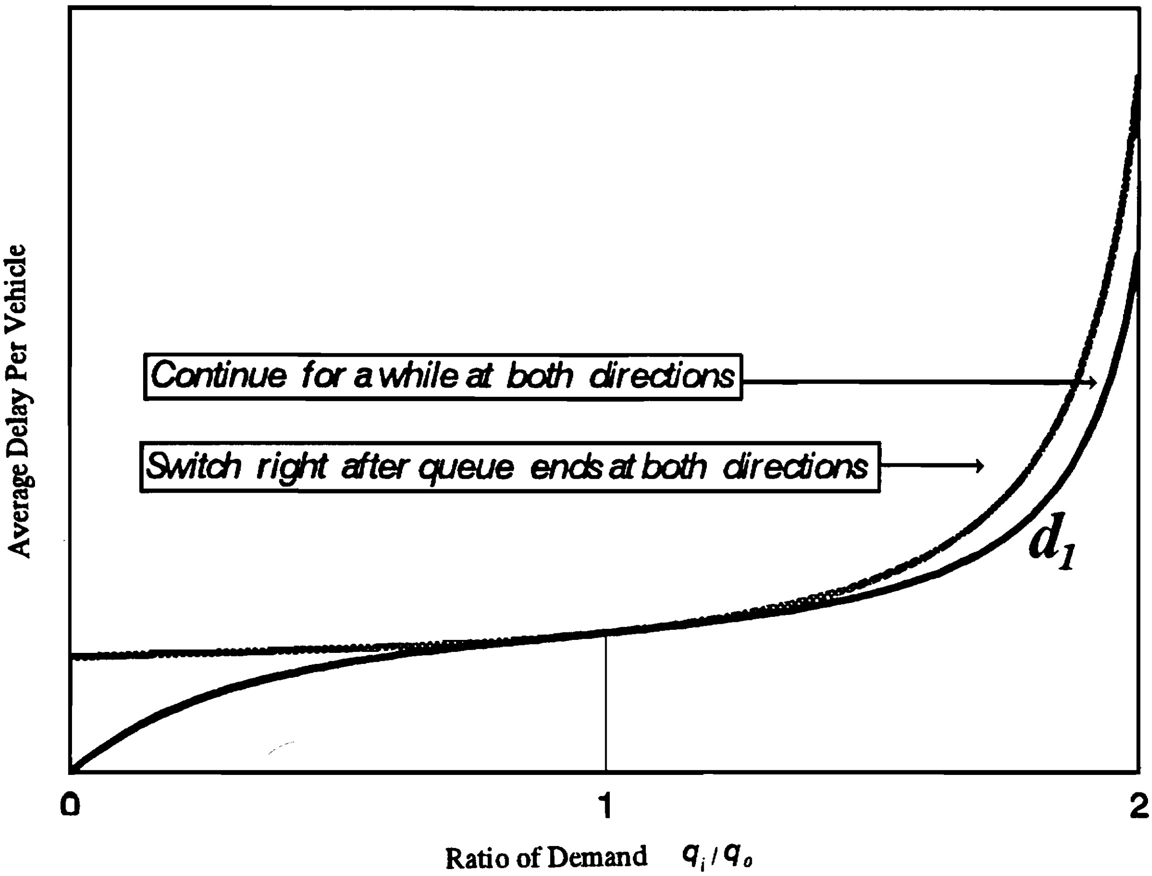


Figure 3.5 - Boundary Conditions And Feasible Region

OBJECTIVE:

$$\text{Min. } d = \frac{S}{2}\frac{1}{(t_i + t_o + 2t_L)(q_i + q_o)}\left(\frac{q_o}{S - q_o}(t_i + 2t_L)^2 + \frac{q_i}{S - q_i}(t_o + 2t_L)^2\right)$$

SUBJECT TO:

$$t_i \geq \frac{q_i}{S - q_i}(t_o + 2t_L)$$

$$t_o \geq \frac{q_o}{S - q_o}(t_i + 2t_L)$$

S, q_i, q_o, and t_L are given.

To simplify the optimization calculation for equation (3.2), we let:

$$K_1 = \frac{S}{2(q_i + q_o)}, \qquad K_2 = 2t_L, \qquad K_3 = \frac{q_o}{S - q_o}, \qquad K_4 = \frac{q_i}{S - q_i}$$

where K_1, K_2, K_3, and K_4 are all positive real numbers. Hence, equation (3.2) becomes:

$$d = \frac{K_1}{(t_i + t_o + K_2)}\left[K_3(t_i + K_2)^2 + K_4(t_o + K_2)^2\right] \tag{3.13}$$

The above function is a non-linear problem with constraints. Let's first consider a situation with the cycle length C given. One would be restricted to have:

$$t_i + t_o = C - 2t_L \tag{3.14}$$

By using the *Lagrange* algorithm, another term is applied to equation (3.13) in order to transformed equation (3.13) to a non-linear problem without constrains, such as:

$$d = \frac{K_1}{(t_i + t_o + K_2)}\left[K_3(t_i + K_2)^2 + K_4(t_o + K_2)^2\right] - \lambda(t_i + t_o - C + 2t_L) \tag{3.15}$$

where:

λ - *Lagrange* multiplier

Taking partial derivatives of d over t_i,t_o and λ, and one can obtain:

$$\frac{\partial d}{\partial t_i} = \frac{2K_1K_3(t_i + K_2)}{t_i + t_o + K_2} - \frac{K_1\left[K_3(t_i + K_2)^2 + K_4(t_o + K_2)^2\right]}{(t_i + t_o + K_2)^2} - \lambda \tag{3.16}$$

$$\frac{\partial d}{\partial t_o} = \frac{2K_1K_4(t_o + K_2)}{t_i + t_o + K_2} - \frac{K_1\left[K_3(t_i + K_2)^2 + K_4(t_o + K_2)^2\right]}{(t_i + t_o + K_2)^2} - \lambda \qquad (3.17)$$

$$\frac{\partial d}{\partial \lambda} = -(t_i + t_o - C + 2t_L) \qquad (3.18)$$

The optimal t_i (or t_o) value, for a given C, that minimizes d can be obtained by letting (3.16) and (3.17) equal zero:

$$\frac{2K_1K_3(t_i + K_2)}{t_i + t_o + K_2} - \frac{K_1\left[K_3(t_i + K_2)^2 + K_4(t_o + K_2)^2\right]}{(t_i + t_o + K_2)^2} - \lambda = 0 \qquad (3.19)$$

$$\frac{2K_1K_4(t_o + K_2)}{t_i + t_o + K_2} - \frac{K_1\left[K_3(t_i + K_2)^2 + K_4(t_o + K_2)^2\right]}{(t_i + t_o + K_2)^2} - \lambda = 0 \qquad (3.20)$$

Since $(2K\ )$ and $(t_i + t_o + K_2)$ both have positive values, one can obtain from equations (3.19) and (3.20) that:

$$K_3 t_i - K_4 t_o + K_3 K_2 - K_4 K_2 = 0 \tag{3.21}$$

Let (3.18) equal zero, it can be transformed into equation (3.14):

$$t_i + t_o = C - 2t_L$$

One can solve $\hat{t}_i$, the extreme value for t_i, from equations (3.14) and (3.21):

$$\hat{t}_i = \frac{CSq_i - Cq_i q_o - 2t_L Sq_o + 2t_L q_i q_o}{Sq_i + Sq_o - 2q_i q_o} \tag{3.22}$$

Since $\hat{t}_o = C - 2t_L - \hat{t}_i$, one can also obtain the $\hat{t}_o$, extreme value for t_o, as:

$$\hat{t}_o = \frac{CSq_o - Cq_i q_o - 2t_L Sq_i + 2t_L q_i q_o}{Sq_i + Sq_o - 2q_i q_o} \tag{3.23}$$

The boundary conditions for t_i and t_o, for a given C, can be obtained from equations (3.9), (3.10) and (3.16) which are:

$$\frac{(S-q_o)C}{S}-2t_L \geq t_i \geq \frac{q_i}{S}C \tag{3.24}$$

$$\frac{(S-q_i)C}{S}-2t_L \geq t_o \geq \frac{q_o}{S}C \tag{3.25}$$

Therefore, the optimal t_i^* and t_o^* that yields minimal average delay, d, for a given cycle length C, can be solved:

$$t_i^* = \frac{q_i}{S}C, \qquad \textit{if} \quad \hat{t}_i > \frac{q_i}{S}C$$

or

$$t_i^* = \frac{(S-q_o)C}{S}-2t_L, \qquad \textit{if} \quad \hat{t}_i < \frac{(S-q_o)C}{S}-2t_L$$

or

$$t_i^* = \hat{t}_i, \qquad \text{otherwise}$$

and

$$t_o^* = \frac{q_o}{S} C, \qquad if \quad \hat{t}_o > \frac{q_o}{S} C$$

or

$$t_o^* = \frac{(S - q_i)C}{S} - 2t_L, \qquad if \quad \hat{t}_o < \frac{(S - q_i)C}{S} - 2t_L$$

or

$$t_o^* = \hat{t}_o, \qquad \text{otherwise}$$

Once the optimal effective green time(t_i^* and t_o^*) at both directions (i and o) obtained, one can utilize equation (3.2) to obtain the minimal average delay, d^*, for the given cycle length, C. Therefore, if one knows the optimal cycle length, C^*, one would be able to set t_i and t_o to such that the minimum of d can be reached.

However, since the optimal cycle length, C^*, is seldom known in practical applications. Without knowing the cycle length, how to obtain the optimum t_i and t_o to such that the minimal average delay, d^*, can be reached? From the experience in setting traffic signals, one knows that a short cycle length usually yields less delay and, therefore, is more desirable than longer ones. Yet, whether this would work for the problem at hand needs to be further investigated.

Let's first consider a simplified case with the same amount of demand at each direction, that is,

$$q_i = q_o = q \qquad (3.26)$$

Substitute (3.26) into objective function (3.2), one can obtain:

$$d = \frac{S}{4(S-q)}\left(\frac{1}{(t_i + t_o + 2t_L)}\right)\left((t_i + 2t_L)^2 + (t_o + 2t_L)^2\right) \qquad (3.27)$$

Substitute (3.26) into (3.22) and (3.23), one would be able to obtain the extreme value of t_i and t_o:

$$\hat{t}_i = \frac{C - 2t_L}{2} \tag{3.28}$$

$$\hat{t}_o = \frac{C - 2t_L}{2} \tag{3.29}$$

It is clear that the extreme value of t_i and t_o are equal to each other and only associated with the cycle length C and the effective lost time t_L.

One can obtain the extreme value of d, the average delay per vehicle, by substituting (3.28) and (3.29) into (3.27):

$$\hat{d} = \frac{S}{8(S-q)} \frac{1}{C}(C + 2t_L)^2 \tag{3.30}$$

Taking derivative of d over C:

$$\frac{\partial \hat{d}}{\partial C} = \frac{S}{8(S-q)}\left(-\frac{1}{C^2}(C+2t_L)^2 + \frac{2}{C}(C+2t_L)\right) \tag{3.31}$$

Let (3.31) = 0 and one can solve it for $\hat{C}$:

$$\hat{C} = \pm 2t_L$$

Since the cycle length must have a positive value, one can obtain a extreme value of C to such that the extreme value of delay d can be reached:

$$\hat{C} = 2t_L \tag{3.32}$$

One can obtain the extreme point of t_i and t_o by substituting (3.32) into (3.28) and (3.29):

$$\hat{t}_i = 0, \qquad \hat{t}_o = 0$$

It is obvious that these two extreme points violate the boundary conditions expressed in (3.24) and (3.25). And the extreme point of d, the average delay per vehicle, can be obtained which equals zero ($\hat{d} = 0$).

In this case, the green phase at both direction (i or o) will not exist for the calculated extreme C value which is $2t_L$ and the average delay per vehicle, d, will be totally eliminated. In fact, we would not be able to set this amount of time for the cycle length so as to entirely eliminate the average delay per vehicle. From the experience on setting cycle length, this condition will not happen in reality. The reason why we arrived at a contradictory result is that the calculated C value has violated its own boundary condition described in equation (3.6). One would be able to imagine that there will be a certain critical value for the cycle length C in order to minimize the average delay. The boundary condition for C can be obtained from equation (3.24) and (3.25):

$$C \geq \frac{S}{S - q_i - q_o} 2t_L \tag{3.33}$$

where:

t_L = Starting lost time L_s + Travel time through work zone t_t

Suppose $2t_L$ is a variable, equation (3.33) can be considered that C is a element function of $2t_L$ which actually depends upon the travel time through the work zone (t_t). The selection of optimal C is determined by parameters S, q_i, q_o and t_L as discussed below.

1) If S, q_i and q_o are given, one can observe from (3.33) that the cycle length C has a linear relationship with the effective lost time for signal change t_L and obviously the variation of C will be determined by t_L. And it is also easy to find out that the effective green time t_i and t_o have linear relationship with t_L as well. This will lead to a consequence from the objective function (3.2) that the average delay d is a linear function of t_L. Since the effective lost time t_L is composed of the travel time through the work zone t_t and the starting lost time L_s, it would be convenient to assume that the starting lost time L_s is fixed which is almost true in practice. Then the entire traffic control procedure will be uniquely determined by the travel time through the work zone t_t. When the work zone area increase with which require longer travel time, it will result in the increase of the effective green time at both direction. The average delay per vehicle d will then be increased along with the increase of t_i and t_o. So it is clear that the optimal control strategy will be resolved by the selection of the length of the work zone. Once the work zone is blocked down, one would be able to decide the minimal cycle length C under this specific condition from equation (3.33) and substitute it into equation (3.22) and (3.23) to set up the effective green time to such that the relatively minimal average delay can be reached.

2) If the work zone is chosen and the effective lost time t_L is fixed, the minimal delay control procedure will be determined by the demand q_i and q_o at direction i and o, respectively. Let's first consider some exceptional condition. If there is no demand at both directions in which q_i and q_o are both equal to zero, one can derive from (3.33) that $C = 2t_L$. Since $C = t_i + t_o + 2t_L$, the effective green time t_i and t_o will reach zero which means that there is no need to set up green phase for the nonexistent traffic. However, this condition will of course not happen in practice.

If the sum of demand q_i and q_o approaches to maximum capacity S which is the saturation flow rate of the work zone, one can derive from (3.33) that the cycle length C will approach to infinity. Then the green time from (3.22) and (3.23) will take very large value in which the green phases at both directions will infinitely extend so as to infinitely increase the average delay. But this will not likely happen in reality because the maximum capacity is relatively much larger than the demand on the rural highway even during the peak hour.

3) If the demand q_i and q_o both take reasonable values which the sum of them is much smaller than the capacity S, Let's first consider a situation with q_i and q_o equal ($q_i = q_o = q$), one can take a minimal value of C from (3.33) which is:

$$\hat{C} = \frac{S}{S - 2q} 2t_L \tag{3.34}$$

where

$S >> (q_i + q_o)$

Substituting (3.34) into (3.22) and (3.23), one can obtain two equal values of t_i and t_o which will also satisfy the boundary conditions in (3.24) and (3.25):

$$t_i^* = \frac{2t_L q}{S-2q} \quad (3.35)$$

$$t_o^* = \frac{2t_L q}{S-2q} \quad (3.36)$$

One can substitute (3.35) and (3.36) into the objective function (3.2) to obtain a optimal value of the average delay per vehicle d:

$$d^* = t_L(\frac{S-q}{S-2q})$$

4) If the demands at both directions are not equal to each other ($q_i \neq q_o$), one can substitute (3.2) into (3.33) and take the minimum value of C from (3.33), hence, (3.33) becomes:

$$C^* = \frac{S}{S-kq_O-q_O}2t_L \quad (3.37)$$

where:

$$k << \frac{S - q_o}{q_o} \tag{3.38}$$

Substituting (3.37) into (3.22) and (3.23), one can obtain the optimal values of t_i and t_o:

$$t_i^* = \frac{2t_L S^2(k-1) + 2t_L q_o S(k+1) - 2t_L k q_o^2 (k+1)}{(kS + S - 2kq_o)(S - kq_o - q_o)} \tag{3.39}$$

$$t_o^* = \frac{2t_L S^2(1-k) + 2t_L k q_O S(k+1) - 2t_L k q_o^2 (k+1)}{(kS + S - 2kq_o)(S - kq_o - q_o)} \tag{3.40}$$

If $k = 1$, (3.39) and (3.40) will become (3.35) and (3.36) which we have already discussed previously. However, the ratio of demands k may take some values without satisfying its boundary condition in (3.38).

One can substitute (3.39) and (3.40) into (3.2) to obtain the optimal minimum average delay d^*:

$$d^{*} = \frac{t_L}{(S - kq_o - q_o)(k+1)}\left[w_1 + w_2\right] \tag{3.41}$$

where:

$$w_1 = \frac{k^2}{S - q_o}\left(\frac{2S^2 - q_o S(3+k) + q_o^2(k+1)}{Sk + S - 2kq_o}\right)^2$$

$$w_2 = \frac{k}{S - kq_o}\left(\frac{2S^2 - Sq_o(3k+1) + kq_o^2(k+1)}{Sk + S - 2kq_o}\right)^2$$

From the equation shown above, it will not be so easy to find out the variable tendency of d corresponding to the different value of k. A diagram illustrating their relationship with each other is shown in Figure 3.6.

3.3.3 ANALYSIS OF CASE 2

In the condition of *Case 2*, all the parameters such as S, q_i, q_o, etc. maintain the same as those in preceding *Case 1* except the control procedure is different. In *Case 1*, the strategy of selecting t_i and t_o is to allow the traffic pass through the work zone with arrival rate q_i (or q_o)

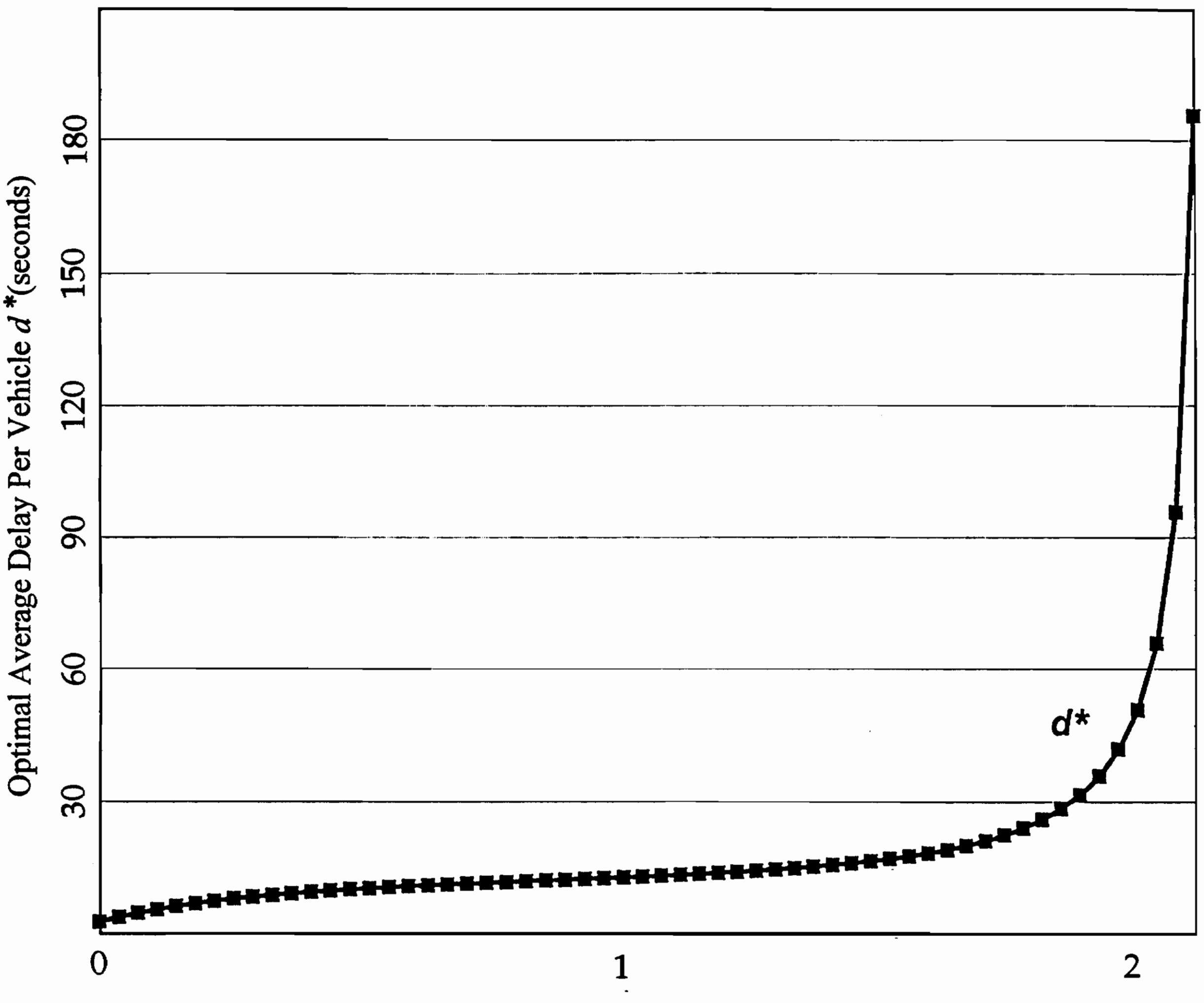


Figure 3.6 - Relationship Of Minimum Delay And Demand Ratio

for a while at both directions after queue departs the work zone with the saturation flow rate S, while *Case 2* deals with switching the green phase to the opposite direction right after queue departs the work zone with the saturation flow rate S.

The objective is also to minimize d, the average delay per vehicle, where d can be expressed in terms of the given parameters:

$$d = \frac{1}{2(q_i + q_o)}\left[q_i(t_o + 2t_L) + q_o(t_i + 2t_L)\right] \tag{3.42}$$

It is clear that equation (3.42) represents d as a linear function of parameters t_i and t_o. According the conditions in (3.4) and (3.5) for t_i and t_o, in this case, the average delay per vehicle, d, only take positive numbers. One can derive from it that the only situation where d is equal to zero happens when $t_i + 2t_L = t_o + 2t_L = 0$ (which means there is no demand at either direction.

One may go through all the derivative procedures discussed in *Case 1* to find out the boundary conditions. In fact, *Case 2* is a special case of *Case 1*, so it would not be difficult to obtain the boundary conditions for this case. First, in order to guarantee exact effective green time to accommodate the demand, one would like to have a minimal C value in (3.6) for the cycle length which is :

$$C = \frac{2t_L S}{S - q_i - q_o} \tag{3.43}$$

where C also satisfies the condition in (3.7) which is

$$C = t_i + t_o + 2t_L$$

Therefore, one can obtain from (3.43) and (3.44):

$$t_i + t_o = 2t_L \frac{q_i + q_o}{S - q_i - q_o} \tag{3.44}$$

One would also be able to have the minimal t_i and t_o values in equations (3.11) and (3.12) to represent all the boundary conditions:

$$t_i = \frac{q_i}{S - q_i}(t_o + 2t_L) \tag{3.45}$$

$$t_o = \frac{q_o}{S - q_o}(t_i + 2t_L) \tag{3.46}$$

The average delay minimization process is actually a simple linear programming problem for this particular condition which can be expressed as:

OBJECTIVE:

Min. $d = \frac{1}{2(q_i + q_o)}\left[q_i(t_o + 2t_L) + q_o(t_i + 2t_L)\right]$

SUBJECT TO:

$$t_i = \frac{q_i}{S - q_i}(t_o + 2t_L)$$

$$t_o = \frac{q_o}{S - q_o}(t_i + 2t_L)$$

S, q_i, q_o, and t_L are given

One can solve (3.45) and (3.46) for the optimal t_i and t_o:

$$t_i^* = \frac{2t_L q_i}{S - q_i - q_o} \tag{3.47}$$

$$t_o^* = \frac{2t_L q_o}{S - q_i - q_o} \tag{3.48}$$

In fact, t_i^* and t_o^* are the intersecting point of the two lines represented by (3.45) and (3.46) and the extreme point on which yields the minimal average delay.

The optimal cycle length can be obtained as

$$C = \frac{S}{S - q_i - q_o} 2t_L$$

This also is the minimum value of C in Equation (3.33)

Substituting (3.47) and (3.48) into (3.42), the minimal average delay can be obtained as:

$$d^* = \frac{t_L}{q_i + q_o}\left[q_i(\frac{S - q_i}{S - q_i - q_o}) + q_o(\frac{S - q_o}{S - q_i - q_o})\right] \qquad (3.49)$$

If the cycle length C is given, the optimal effective green time, t_i and t_o, can be obtained by solving (3.45) and (3.46):

$$t_i = \frac{q_i}{S} C \qquad (3.50)$$

$$t_o = \frac{q_o}{S} C \qquad (3.51)$$

Substituting (3.50) and (3.51) into (3.42), the minimal average delay, d, can be obtained as

$$d^* = \frac{1}{S(q_i + q_o)}(q_i q_o C + t_L q_i S + t_L q_o S) \qquad (3.52)$$

3.3.4 ANALYSIS OF CASE 3

The analysis and control process in *Case 3* combines the characteristics in *Case 1* and *Case 2*. *Case 3* may utilize the setup process for the effective green time, t_i at direction i in *Case 1* and the setup process for the effective green time, t_o at direction o in *Case 2*.

In *Case 3*, the objective function of minimizing the average delay per vehicle, d, can be expressed in terms of given parameters:

$$d = \frac{1}{2} \frac{1}{(t_i + t_o + 2t_L)(q_i + q_o)} \left[\frac{Sq_i}{S - q_i} (t_o + 2t_L)^2 + q_o (t_i + t_o + 2t_L)(t_i + 2t_L) \right] \quad (3.53)$$

Equation (3.53) presents d as a non-linear function of parameters t_i and t_o. Even though the average delay per vehicle, d, can't take any negative numbers in reality, we might as well theoretically study all possible obtain range for d by analyzing the boundary conditions of t_i and t_o. One can see from equation (3.53) that d goes to either positive or negative infinity near the line representing $t_i + t_o = 2t_L$, and approaches zero when $t_i + t_o$ approaches $-2t_L$.

On the point of view of field application, d will not take neither any negative values nor zero due to the restrictions in (3.4) and (3.5) for fixed boundary conditions of t_i and t_o. It is known that the cycle length, C, must satisfy the condition (3.6) which is $C \geq \frac{2t_L}{S - q_i - q_o}$.

In the light of the method we analyze the boundary conditions for t_i and t_o in *Case 1* and *Case 2*, the boundary conditions for t_i and t_o in this case can be obtained as:

$$t_i \geq \frac{q_i}{S - q_i}(t_o + 2t_L) \tag{3.54}$$

$$t_o = \frac{q_o}{S - q_o}(t_i + 2t_L) \tag{3.55}$$

One may notice that Equation (3.55) is the same as Equation (3.12) in *Case 1*. The effective green time at direction o, t_o, is always a constant associated with t_i whenever t_i is selected.

Therefore, the average delay minimization process can expressed as:

OBJECTIVE:

$$\text{Min. } d = \frac{1}{2}\frac{1}{(t_i + t_o + 2t_L)(q_i + q_o)}\left[\frac{Sq_i}{S - q_i}(t_o + 2t_L)^2 + q_o(t_i + t_o + 2t_L)(t_i + 2t_L)\right]$$

SUBJECT TO:

$$t_i \geq \frac{q_i}{S - q_i}(t_o + 2t_L)$$

$$t_o = \frac{q_o}{S - q_o}(t_i + 2t_L)$$

S, q_i, q_o, and t_L are given

This minimization process of non-linear programming can be solved by the means of linear programming method similar to those discussed in *Case 1* and *Case 2*.

After taking derivation of d over t_i in Equation (3.53), the optimum t_i can obtained as

$$t_i^* = \frac{2q_i q_o t_L - 2q_o S t_L + 4q_i S t_L}{q_o S - 3q_i q_o} \tag{3.56}$$

Since $t_o = \frac{q_o}{S - q_o}(t_i + 2t_L)$, one can also obtain t_o^* as:

$$t_o^* = \frac{4q_i t_L}{S - 3q_i} \tag{3.57}$$

Therefore, the optimum d^* can be solved by substituting (3.56) and (3.57) into (3.53):

$$d^* = \frac{1}{2}\frac{q_o S - 5q_i q_o + 4q_i S}{S - 3q_i} \tag{3.58}$$

3.3.5 ANALYSIS OF CASE 4

In fact, the situation of traffic control described in *Case 4* is actually the same as that in *Case 3* except that the directions are switched with each other. Therefore, we can simply follow the analysis discussed in *Case 3* and pursue the average delay minimization process in *Case 4*.

The objective function for the average delay, d, can expressed as:

$$d = \frac{1}{2}\frac{1}{(t_i + t_o + 2t_L)(q_i + q_o)}\left[q_i(t_i + t_o + 2t_L)(t_o + 2t_L) + \frac{Sq_o}{S - q_o}(t_i + 2t_L)^2\right] \quad (3.59)$$

Similarly, satisfying all the conditions provided in previous parts, the boundary conditions of the effective green time, t_i and t , can be obtained as:

$$t_o \geq \frac{q_o}{S - q_o}(t_i + 2t_L) \quad 3.60)$$

$$t_i = \frac{q_i}{S - q_i}(t_o + 2t_L) \tag{3.61}$$

Under the consideration of applying the traffic control strategies in this study to work site, we can list the essential equations instead of repeat the derivative calculations.

We might as well summarize equation (3.59), (3.60) and (3.61) to the average delay minimization process which is:

OBJECTIVE:

$$\text{Min. } d = \frac{1}{2}\frac{1}{(t_i + t_o + 2t_L)(q_i + q_o)}\left[q_i(t_i + t_o + 2t_L)(t_o + 2t_L) + \frac{Sq_o}{S - q_o}(t_i + 2t_L)^2\right]$$

SUBJECT TO:

$$t_o \geq \frac{q_o}{S - q_o}(t_i + 2t_L)$$

$$t_i = \frac{q_i}{S - q_i}(t_o + 2t_L)$$

S, q_i, q_o, and t_L are given

The boundary conditions of t_i and t , are expressed as:

$$t_i \geq \frac{q_i}{S - q_i}(t_o + 2t_L) \tag{3.62}$$

$$t_o = \frac{q_o}{S - q_o}(t_i + 2t_L) \tag{3.63}$$

The optimal values of t_i and t can be obtained through partial derivatives:

$$t_o^* = \frac{2q_i q_o t_L - 2q_i S t_L + 4q_o S t_L}{q_i S - 3q_i q_o} \tag{3.63}$$

$$t_i^* = \frac{4q_o t_L}{S - 3q_o} \tag{3.64}$$

Substitute (3.63) and (3.64) into (3.59), one can obtain the optimal d^*:

$$d^* = \frac{1}{2}\frac{q_o S - 5q_i q_o + 4q_i S}{S - 3q_i} \tag{3.65}$$

4. APPLICATION

4.1 INTRODUCTION

This section presents applications of the foregoing research on optima traffic control strategy for under-saturated two-lane highway work zone operation. Due to utility work or roadway improvement work, one travel lane on a segment of a two-lane highway is blocked down, and traffic from both directions has to use the other travel lane to pass the work zone area. This research attempts to develop optimal control strategies to optimize the operating conditions of the work zone under disturbance. Through seeking the optimal passing time for traffic at both directions, the average delay per vehicle can be minimized. Therefore, the overall traffic delay within the work zone area can be reduced and the traffic operation can be enhanced.

This research is conducted for under-saturated two-lane highways work zone operation. The saturated or over-saturated traffic conditions on two-lane highways are beyond the scope of this study. The results contained in this thesis cannot be applied to work zone areas with traffic conditions approaching saturation or exceeding saturation.

It is assumed that the two-lane highway work zone operation is similar to the operation of a signalized intersection. The applications of optimal control strategies are based on the following given parameters,

S - saturation flow rate of the work zone area;

q_i - demand at direction i ;

q_o - demand at direction o.

t_L - effective lost time for signal change ($= t_t + L_S$);

t_t - average vehicle travel time through the work zone;

L_s - starting lost time.

and the following assumptions,

$$S >> (q_i + q_o)$$

$$C = t_i + t_o + 2t_L.$$

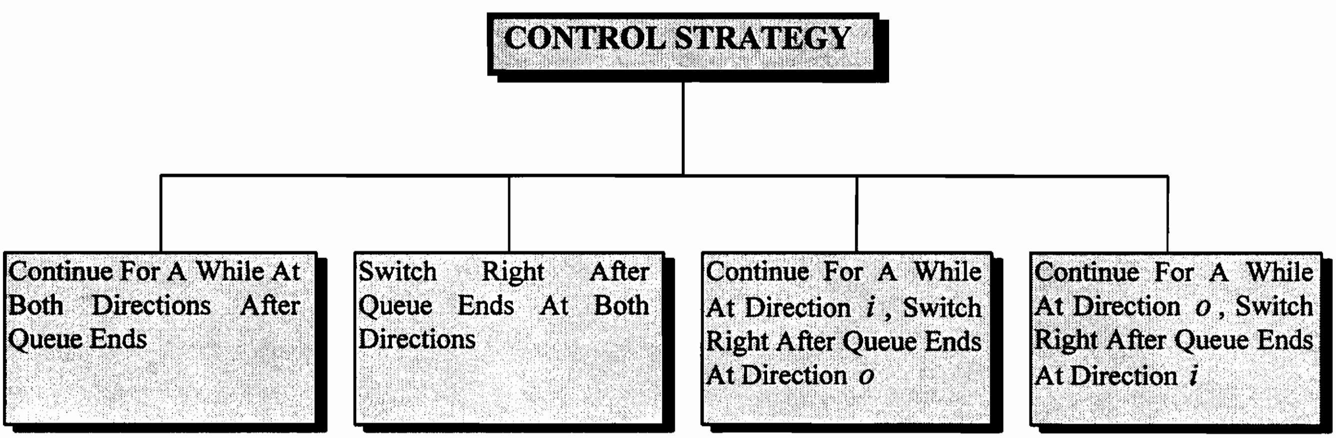
The following are used to describe the optimal control parameters:

t_i^* - optimal, effective green time for direction i,

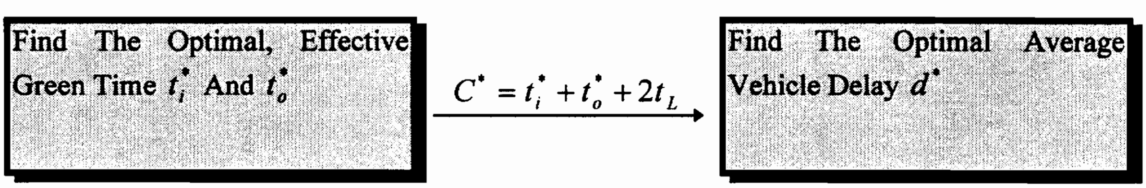
t_o^* - optimal, effective green time for direction o,

d^* - optimal, average delay per vehicle.

The application of optimal control strategies for the studied four different cases is illustrated as follows:



The optimal control process to minimize average vehicle delay is illustrated as follows:



4.2 CONTROL STRATEGY FOR CASE 1

The optimal control strategy for Case 1 is given to a condition where the effective green time (t_i and t_o) continues for a while after the queue ends at both directions. The optimal, effective green time (t_i^* and t_o^*) can set as

$$t_i^* = \frac{q_i}{S}C, \qquad \text{if} \quad t_i > \frac{q_i}{S}C$$

or

$$t_i^* = \frac{(S-q_o)C}{S} - 2t_L, \qquad \text{if} \quad t_i < \frac{(S-q_o)C}{S} - 2t_L$$

or

$$t_i^* = \frac{CSq_i - Cq_iq_o - 2t_LSq_o + 2t_Lq_iq_o}{Sq_i + Sq_o - 2q_iq_o}, \qquad \text{otherwise}$$

and

$$t_o^* = \frac{q_o}{S}C, \qquad \text{if} \quad t_o > \frac{q_o}{S}C$$

or

$$t_o^* = \frac{(S-q_i)C}{S} - 2t_L, \qquad \text{if} \quad t_o < \frac{(S-q_i)C}{S} - 2t_L$$

or

$$t_o^* = \frac{CSq_o - Cq_iq_o - 2t_LSq_i + 2t_Lq_iq_o}{Sq_i + Sq_o - 2q_iq_o}, \qquad \text{otherwise}$$

The minimal average delay per vehicle can be calculated as

$$d^* = \frac{S}{2}\frac{1}{(t_i^* + t_o^* + 2t_L)(q_i + q_o)}\left(\frac{q_o}{S - q_o}(t_i^* + 2t_L)^2 + \frac{q_i}{S - q_i}(t_o^* + 2t_L)^2\right)$$

4.3 CONTROL STRATEGY FOR CASE 2

The optimal control strategy for Case 1 is given to a condition where the effective green time (t_i and t_o) switches right after the queue ends at both directions. The optimal, effective green time (t_i^* and t_o^*) can set as

$$t_i^* = \frac{2t_Lq_i}{S - q_i - q_o}$$

$$t_o^* = \frac{2t_L q_o}{S - q_i - q_o}$$

The minimal average delay per vehicle can be calculated as

$$d^* = \frac{t_L}{q_i + q_o}\left[q_i\left(\frac{S - q_i}{S - q_i - q_o}\right) + q_o\left(\frac{S - q_o}{S - q_i - q_o}\right)\right]$$

4.4 CONTROL STRATEGY FOR CASE 3

The optimal control strategy for Case 1 is given to a condition where the effective green time t_i continues for a while before it switches to direction o; for direction o, the effective green time t_o switches to direction i right after the queue ends at direction o. The optimal, effective green time (t_i^* and t_o^*) can set as

$$t_i^* = \frac{2q_i q_o t_L - 2q_o S t_L + 4q_i S t_L}{q_o S - 3q_i q_o}$$

$$t_o^* = \frac{4q_i t_L}{S - 3q_i}$$

The minimal average delay per vehicle can be calculated as

$$d^* = \frac{1}{2}\left(\frac{q_o S - 5q_i q_o + 4q_i S}{S - 3q_i}\right)$$

4.5 CONTROL STRATEGY FOR CASE 4

The optimal control strategy for Case 1 is given to a condition where the effective green time t_o continues for a while before it switches to direction i; for direction i, the effective green time t_i switches to direction o right after the queue ends at direction i. The optimal, effective green time (t_i^* and t_o^*) can set as

$$t_i^* = \frac{4q_o t_L}{S - 3q_o}$$

$$t_o^* = \frac{2q_i q_o t_L - 2q_i S t_L + 4q_o S t_L}{q_i S - 3q_i q_o}$$

The minimal average delay per vehicle can be calculated as

$$d^* = \frac{1}{2}\left(\frac{q_o S - 5q_i q_o + 4q_i S}{S - 3q_i}\right)$$

4.6 A NUMERICAL TEST OF DELAY MODELS

It is necessary to test the delay models with respect to the four cases discussed in this study. A numerical test was conducted to see whether the mathematical models generate reasonable results and how the average delays may vary in the four respective cases. Due to the focus of the theoretical approach and development of mathematical models, no real field data has been collected to conduct this test. Therefore, a sample of data has been assumed for the test in this study.

It is assumed that an approximately 1000 feet section of two-lane highway at one direction has been blocked down due to roadside work.

The travel speed within the work zone area is assumed to be 15 miles per hour. Thus the time for one vehicle to travel through the work zone is approximately 0.0125 hour(or 0.75 minutes). Other given parameters are assumed and summarized in the table below:

	S	*qi*	*qo*	*tL*	C
TEST 1	1600	500	700	0.0125	0.2
TEST 2	1600	500	500	0.0125	0.2
TEST 3	1600	700	500	0.0125	0.2

As shown in the table, for all three tests, it is assumed that the saturation flow rate is 1600 vehicles per hour, and the given cycle length is 0.2 hour(or 12 minutes). The vehicle arrival rate q_i and q_o shown in the table is $q_i<q_o$, $q_i=q_o$ and $q_i>q_o$ for test 1, test 2 and test 3, respectively.

For each test, with assumed parameters and given cycle length, the green time t_i and t_o was then selected for all the studied cases. Substitute the green time t_i and t_o into the delay functions of respective studied cases, the average delay per vehicle, d, can be obtained. This computation process is summarized in the table below:

	EST 1			EST 2			TEST 3		
	ti	*to*	*d*	*ti*	*to*	*d*	*ti*	*to*	*d*
CASE 1	0.06	0.115	0.048	0.088	0.088	0.046	0.115	0.06	0.048
CASE 2	0.081	0.094	0.057	0.088	0.088	0.056	0.094	0.081	0.057
CASE 3	0.081	0.094	0.052	0.088	0.088	0.051	0.094	0.081	0.052
CASE 4	0.094	0.081	0.052	0.088	0.088	0.051	0.081	0.094	0.052

As shown in this table, for all three tests with respect to all four studied cases, Case 1, which continues the green phase for a while before the queue ends at both directions, yields the least delay. It is also obvious that greater arrival rate demands longer green time, as shown in Test 1 and Test 2. It is also true that if the arrival rate is the same at both directions, the green time at both directions is also the same and the average delay per vehicle is the least among all tests. The results of the various delays in Test 1, Test 2 and Test 3 are plotted in Figure 4-1.

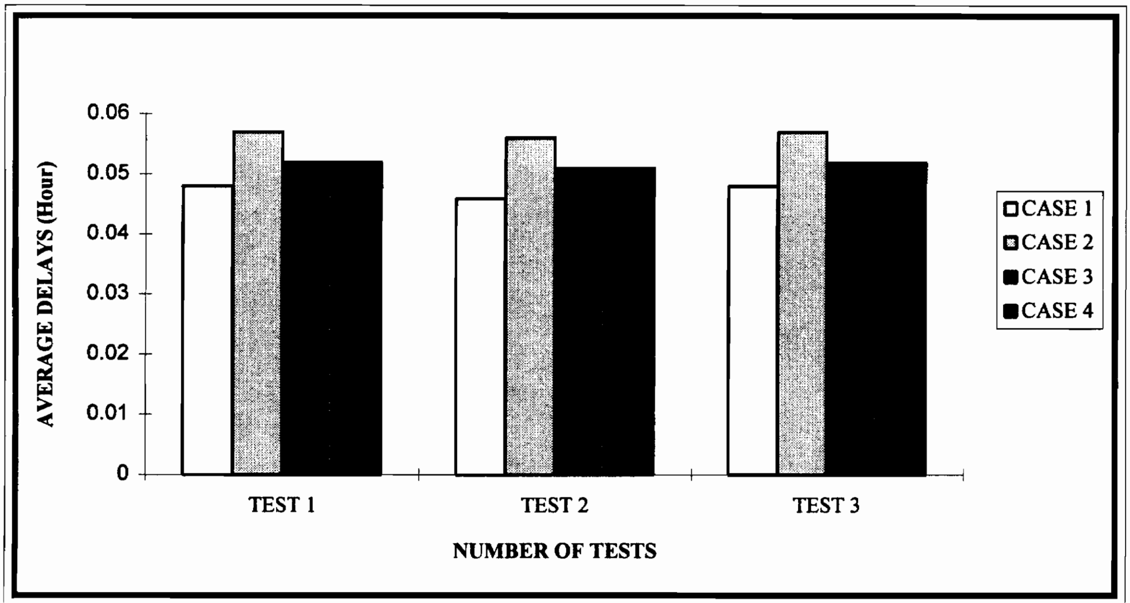


Figure 4.1 - SUMMARY OF AVERAGE DELAYS

4.7 RELEVANCE OF WORK ZONE LENGTH

The length of a work zone determines the average time that a vehicle travels through the work zone, it then determines the minimum cycle length. The average travel time through the work zone can be used to discuss the relevance of the work zone length. As shown in equation (3.6),

$$C \geq \frac{2t_L S}{S - q_i - q_o}$$

One can transform equation (3.6) into the following equation:

$$t_L \leq \frac{C(S - q_i - q_o)}{2S} \tag{4.1}$$

As shown in (4.1), the minimum cycle length and the average travel time constrains with each other. It is apparent that the longer the average travel time is, the greater the minimum cycle length. It also can be known from the optimal objective functions with respect to four studied cases that the longer average travel time through the work zone will result in the greater minimum average delay per vehicle. In practice, the appropriate cycle length can not be determined without knowing the length of the work zone, or the average travel time through the work zone.

5. CONCLUSION

Traffic operation on two-lane highway work zone has been a long-standing inconvenient experience for both motorists and traffic operational agencies. Although more research and experimental work have been accomplished at signalized intersections to mitigate congestion and improve efficiency, research efforts on two-lane highway work zone is limited. This lack of research on two-lane highway work zone may be caused by subjective factors(e.g. the perception that operational problems cause less damage on two-lane highway work zone than at signalized intersections), and objective factors(e.g. the fact that operational problems at signalized intersections have more complicated characteristics than those occurring on two-lane highway work zone). As more and more roadway improvement and construction work undergoing on urban two-lane highways, it starts to raise concern of capacity reduction and excessive delay in the system.

The research effort in this thesis seeks to describe traffic flow characteristics in a mathematical fashion. The resulting mathematical delay models reflect all operational aspects occurring on two-lane highway work zone, and present the average traffic delay as a function of measurable parameters such as saturation flow rate and actual flow rate as well as control parameters such as green and red phase. Optimal control strategies are developed to minimize the average vehicle delay through optimal timing control.

At the preliminary stage, this research deals with the development of theory and mathematical models. There is the feasibility that the theory and mathematical models developed in this research can be applied to real two-lane highway work zone operation. For further research, it is suggested to develop comprehensive computer programs utilizing equations and realistic hypothesis in this thesis, then test and finalize the mathematical models by using real data collected from the field.

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