1. PROBLEM DESCRIPTION

1.1 Introduction

The principal focus of traffic management and control systems is to ensure the safe and efficient movement of traffic on roadways. This is indeed a challenging task since incidents, of varying degrees of magnitude, can routinely impact the flow of traffic. These incidents need to be identified, and responded to in a timely fashion. This constitutes towards the bulk of the work in traffic management. To manage traffic under such conditions as well as in normal situations, specific information must be known about the flow of traffic. This requires surveillance, detection, and monitoring of freeways, arterial roads, and intersections. An effective surveillance/detection system provides a foundation of information on which Advanced Transportation Management Systems (ATMS) and other Intelligent Transportation Systems (ITS) depend. Surveillance technologies can be used in several traffic situations concerning disabled vehicles, accidents, and road constructions. These technologies can also be used to solve problems pertaining to growing traffic, environmental, and economic issues.

As mentioned above, traffic conditions must be observed and monitored, and real-time information collected in order to keep the traffic moving safely and efficiently. This basic traffic data is essential for developing traffic management and traffic adaptive control strategies, as well as determining highway improvement projects and their priorities. The more accurate the information is, the better these strategies and decisions will be.

Surveillance and monitoring also allow for the detection of any event or incident (such as accidents, disabled vehicles, snow/severe weather conditions, etc.) that might reduce the traffic capacity of the transportation system.

Traditionally, traffic data has been collected using inductive loop detectors that are cut into the pavement, or road tubes that are laid on the road. Both these methods can provide fairly accurate data, but the installation and repair can be burdensome, and costly. The emergence of surveillance technologies has eased the number of disruptions caused due to the maintenance requirements of the equipment involved. Several technologies that provide traffic data without causing traffic disruptions have been developed. These devices are usually mounted overhead or on the side of the roadway (mostly on pre-existing structures) instead of on the roadway.

The Virginia Department of Transportation (VDOT) currently operates a significant number of surveillance technologies that include loop detectors, surveillance cameras, Weigh-in-Motion (WIM) scales, and vehicle classifiers. Other emerging surveillance devices that may be of use to VDOT include Automatic Vehicle Identification (AVI) systems, and emerging cellular location devices.

While some of these devices are used primarily to support traffic management functions, others such as AVI and WIM are intended to support specialized applications. However, it is clear that all of these technologies work together to provide a comprehensive sample of the state of the surface transportation system. This knowledge of state is the key ingredient to effective traffic management and provision of traveler information services. These technologies could also be used to provide information in real-time to transportation system operators, or to provide information off-line to transportation planning agencies, and/or for Advanced Traffic Management System applications.

While the current state-of-the-art surveillance technologies do provide localized information on traffic flow, occupancy, and in some instances speed, they suffer from a number of shortcomings. First, these detection technologies only provide localized spot information. Second, such devices are generally unreliable and are prone to failure. Third, they have inherent errors in them. For example, loop detectors provide accurate speed estimates higher than 20km/hr, however, they are less accurate at lower speeds.

Consequently, there is a need to enhance the usability of current surveillance technologies. Specifically, algorithms need to be developed to estimate spatial travel time estimates from localized speed measurements for the provision of real-time traffic information. These algorithms should address accuracy issues with the current detection devices and investigate the potential for emerging technologies. In addition, algorithms need to be developed to extrapolate network-wide traffic parameters using available sporadic localized traffic information that is provided by the current surveillance infrastructure. Finally, algorithms are required to assist VDOT in designing future surveillance endeavors in order to meet specific project objectives.

1.2 Research Objectives and Purpose

The proposed research is concerned with the development and implementation of Intelligent Transportation Systems (ITS) in urban areas. Within the ITS umbrella, the proposed research is more particularly concerned with the use of Advanced Vehicle Detection technology for gathering real-time information on traffic conditions within urban transportation networks. Specifically, this research attempts to develop methodologies that minimize the cost of the data collection efforts, maximize the utilization of the data, and apply the best data collection technology to address the unique needs of Advanced Traffic Information Systems Applications.

The specific objectives of the overall ongoing research effort are threefold. These three major research objectives are tailored to benefit VDOT, and are defined below. However, the principal focus of this thesis will be on the third research objective.

(a) **To develop robust algorithms for estimating link travel times using alternative surveillance technologies.**

VDOT's customers expect a higher level of information from VDOT's traffic management systems. Some regions, such as Atlanta and San Antonio post expected travel times to key network locations on system variable message signs. Researchers and professionals have reported only moderate satisfaction with the algorithms currently used by these regions. The purpose of this aspect of the proposed research is to develop effective travel time algorithms that may be applied in VDOT traffic control systems using data collected by various technologies, including standard single and dual loop detectors, Automatic Vehicle Identification (AVI) systems, and other emerging detection technologies.

(b) To develop algorithms for enhancing the quality of information provided by infield surveillance technologies.

Surveillance technologies currently used by VDOT have a number of shortcomings. First, they provide location-specific information only. Second, they frequently malfunction due to the extreme environment in which they operate. This aspect of the research will attempt to find better ways of deriving network-level information from location-specific detectors. In addition, this phase will include investigation into effective "data-replacement" techniques to use for malfunctioning detectors. Using historical and spatially adjacent information, it may be possible to estimate the measurements that should have been collected by the sensors that malfunctioned in a given situation.

(c) To develop algorithms for enhancing the design of future surveillance technologies.

As stated above, the installation of sensor systems is difficult and maintenance is quite problematic. There is a need to develop design guidelines to ensure that sensors are only placed at critical locations. Current guidelines are typically driven by the needs of incident detection algorithms that are rarely used in practice. This phase of research will attempt to develop more comprehensive guidelines and methodologies for the design of the sensor systems. In particular, the third objective is not only concerned with the need to locate the detection stations to best cover the network under surveillance but is also concerned with the need to locate the detection technology in such a way as to obtain maximum information about traffic variability within the network.

To attain the third objective, the following tasks are identified.

Task1: Determination of typical variability in traffic conditions within urban transportation networks using simulation. The simulation analysis could consider conditions that were not necessarily observed in the field, and would allow for the direct comparison of alternative technologies under identical conditions.

Task 2: Development of algorithms and other criteria for determining an optimal number of readers, and their prescribed locations (vehicle detection stations) in urban transportation networks, so as to maximize some suitable measure of total benefits accruing from their usage.

1.3 Current Problem under Consideration

The goal of the Metropolitan Model Deployment Initiative (MMDI) Operation time saver was to reduce travel times in major US cities by 15% by the year 2006 (Riley, Rakha, and Van Aerde, 1999). These cities were New York, San Antonio, Phoenix, and Seattle. As part of the San Antonio MMDI, over 70,000 AVI tags were distributed among volunteer drivers free of charge. In addition, 53 tag reader locations were identified and readers were installed at selected points on freeways and major arterials and spaced a mile or several miles apart. Travel times were computed by matching the tag readings taken by the readers at various locations.

Description of the San Antonio AVI System

The specifics of the San Antonio system is described in terms of the type of tags that were utilized, the specifics of the tag readers, the location of the tag readers, and the methodology used for computing the travel times. Another aspect of the system is to determine whether the AVI technology provides accurate results that correspond to the actual link travel times. Riley et al. (1999) describe in their paper the steps taken to establish the reliability of the San Antonio system. The two related hypotheses are as follows:

- AVI probes provide accurate measurements of segment travel times, and
- AVI probes are a reliable mechanism for estimating travel times.

These two assumptions come to the fore only when the following conditions are satisfied. First, it is assumed that the AVI tags distributed and the reader locations are adequate and provide a sufficiently large data/information base to decide the reliability of the system. Second, in certain cases, although the number of AVI tags distributed may be sufficient to accurately assess the system, it is possible that the location of the readers induces an inaccurate picture of the system. For example, if the readers are located on paths along which the traffic is not a true representation of the entire area, then the computed travel times might not accurately represent the true or the observed travel times. Also, reader locations that are not placed along paths used by many O-D pairs, or are placed in areas such as residential zones (typically, zones that can be classified as either production zones or attraction zones alone; home is always taken to be a production zone), are not time sensitive and might yield errors in travel time measurements.

The San Antonio AVI system currently collects travel information from 70,000 vehicles voluntarily equipped with unique vehicle identification tags through a network of 53 tag readers placed at selected points on freeways and major, high-volume arterials within the city's transportation network. This system was implemented as part of the San Antonio Metropolitan Model Deployment Initiative (MMDI) to evaluate the benefits and costs of AVI technology and how the use of such technology can supplement standard loop detector information within Advanced Traveler Information Systems (ATIS).

Figure 1 illustrates a conceptual overview of the San Antonio AVI system. As indicated, the system relies on a combination of passive tags and electronic interrogators mounted on overhead structures to detect the passage of vehicles at each reading location within the network. The system detects the passage of a vehicle by monitoring the signal sent by the interrogator's antenna. A vehicle is detected each time an altered signal is reflected back to the antenna. Since each vehicle tag returns a unique signal, specific vehicles can then be identified. Upon detection of a vehicle, the system transmits the detection information, via a modem, to the AVI data processing system center, where an algorithm matches new tag readings to previous readings from the same vehicle and calculates the vehicle's travel time on the roadway segment between the current and previous reading station.

Taking the above factors into consideration, the problem of locating the readers can be viewed as an optimization problem that is formulated based on the underlying transportation network as described next.

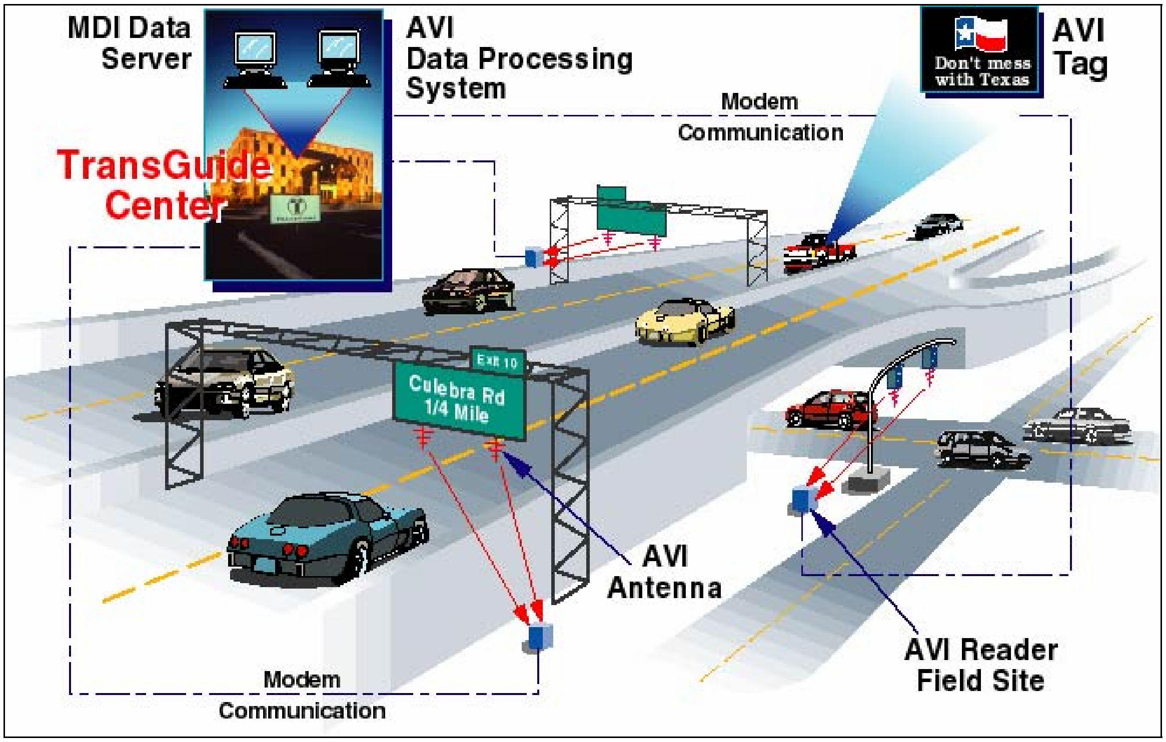

Figure 1: Conceptual Overview of the San Antonio Automatic Vehicle Identification System.

1.4 Reader Location Problem

Any given transportation network consists of cities and towns that are connected by a set of freeways and arterial roads. However, a detailed representation of such a network is not congenial for use as the graph on which the reader location problem is to be effectively solved. The steps taken for the conversion of the transportation network to a graph on which the reader location problem is to be solved is an important ingredient of the current problem under consideration. This section discusses how to derive the graph on which the reader location problem is to be solved.

Consider the following conceptualization of the problem. Suppose that we are given a certain transportation road network that identifies the various freeways, major and minor arterials, and different streets of interest in some region under consideration. Furthermore, on this network, suppose that we identify various *Origin-Destination* (O-D) pairs (i, j) belonging to some set A for which we might be interested in measuring travel times, if economically feasible to do so. We assume that for each O-D pair, there exists a unique corresponding route. Furthermore, the routes connecting these O-D pairs may or may not share common sections of roadways (links). For example, in Figure 2(a), the O-D pairs (1, 2) and (3, 6) share the link AB, whereas O-D pairs (2, 3) and (4, 5) have no links in common. Similarly O-D pairs (1, 6) and (2, 6) have link BE in common. Naturally, we would be able to measure travel times for any $(i, j) \in A$ if and only if we locate a reader at each of the upstream and downstream end-points i and j.

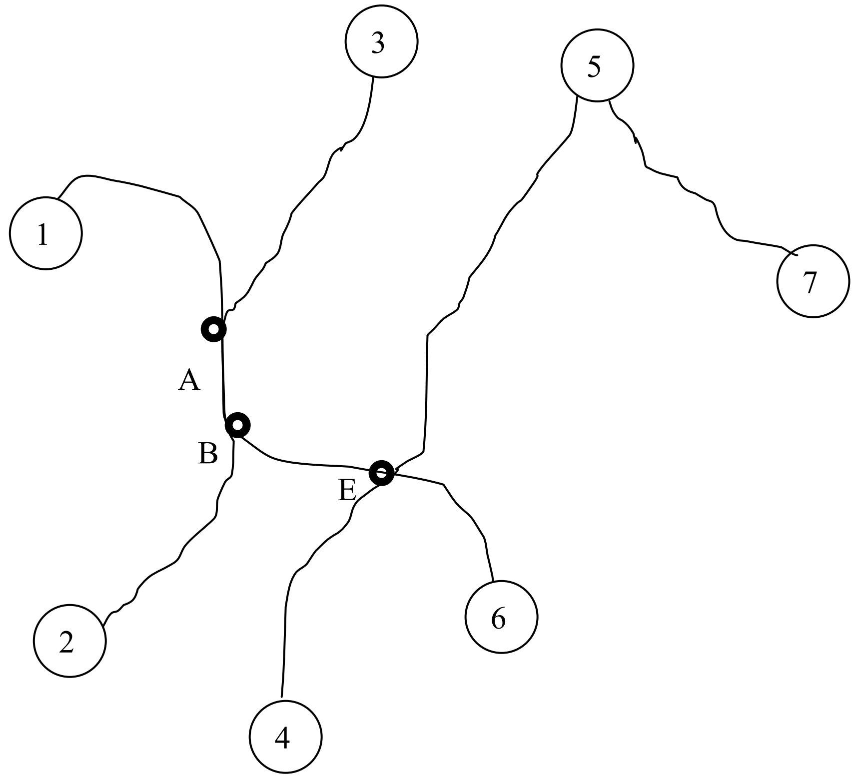

Figure 2(a): Sample Transportation Network

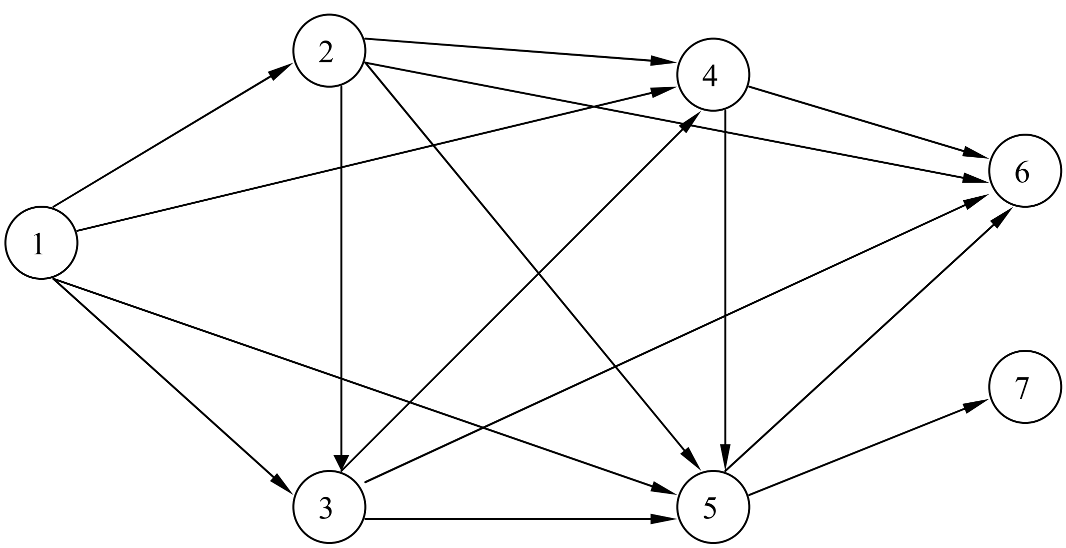

Figure 2(b): Graph *G*(*N*, *A*)

Accordingly, given an urban road network for a particular region, of the kind described above, we next describe the construction of a graph on which the reader location problem is to be solved.

This graph denoted $G(N, A)$, has a node set N given by the union of the end-points of the various O-D pairs, and has a set of directed arcs $(i, j) \in A$, each of which represents a particular O-D pair and its corresponding route as identified above. Note that if we are interested in measuring travel times on O-D pairs (i, j) as well as (j, i) for any pair of nodes i and j, we would include both (i, j) and (j, i) in A. With respect to the transportation network of Figure 2(a), the corresponding graph G would be as depicted in Figure 2(b). Each O-D pair (i, j) in Figure 2(a), that is connected by a set of roadways, and for which we are interested in measuring travel times if economically feasible, is represented in our graph G as a corresponding link (i, j) joining the two nodes i and j. Note that, even though certain O-D pairs have links in common in the transportation network, this is not represented in our graph G, and they appear to be independent of each other in G. However, the actual nature of these O-D routes in the original transportation network will figure into the benefit factors that will be ascribed to each link in G as described below. Also, in the urban transportation network, there might physically exist more than one route connecting a given O-D pair (i, j). However, as noted above, the measurement of travel time would assume that only one route is being used, and that if a reader is installed at each of the end nodes i and j, then the corresponding travel time data obtained would pertain to this route. The benefit factor derived for this corresponding arc in graph G, would be a representation of this route.

As before, with respect to this graph G, a link $(i, j) \in A$ will be *covered* with respect to being able to obtain a measurement of the travel time on the corresponding roadway segment if and only if a reader is located at each of the end-point nodes i and j in N.

Accordingly, define a binary variable

$$y_j = \begin{cases} 1, \text{ if a reader is located at node } j, \\ 0, \text{ otherwise}, \forall\ j \in N. \end{cases} \tag{1}$$

Note that in the following model description, it might be that we already have certain readers in place (or we might wish to pre-select certain specific locations at which readers must be placed), and then we might wish to determine where best to locate an additional set of readers. In such a case, we might simply augment the proposed model below by fixing the variables y_j equal to one for $j \in N$ corresponding to fixed reader locations.

Let C_j denote the site-specific cost of installing a reader at location $j \in N$, and let R denote the total number of available readers, or the number of readers to be installed as the case might be, depending on the following two problem scenarios. Note that the parameter C_j includes the (amortized) cost of the reader itself, beside the cost of installation. In the first scenario (**Problem RL1**), we assume that we have a certain (maximum) number of readers R that we can install on the network. In the second scenario (**Problem RL2**), we relax the foregoing explicit constraint, and let the model determine the number of readers to install that would yield the best compromise between benefits and costs, by instead imposing a total budgetary limit B. The actual reader location model **RL** formulated below accommodates both these restrictions simultaneously as a third alternative. The models **RL1** and **RL2** alluded above can be derived from **RL**, and could be computationally investigated in a similar manner using our proposed approach.

As far as the coverage on the links $(i, j) \in A$ is concerned, it is relatively more beneficial to cover a link that might represent an O-D pair that enjoys significant usage, as well as that which exhibits a greater variability in travel times. In order to identify such O-D pairs that would be of particular interest from a measurement viewpoint, one could develop and run a pre-processing micro-simulation, or macro-simulation model, or perform some preliminary analysis on the roadway network. Accordingly, suppose that we develop a mechanism for prescribing a benefit factor b_{ij} for covering link $(i, j) \in A$**.** A suitable means for deriving these benefit factors is as described in Section 3.2 of this thesis.

The proposed *Reader Location* problem (**RL**) can then be formulated as follows, based on the binary decision variables defined earlier in (1).

RL: Maximize $$\sum\sum_{(i,j)\in A} b_{ij} y_i y_j \quad (2a)$$

subject to : $$\sum_{j\in N} y_j \le R \quad (2b)$$

$$\sum_{j\in N} C_j y_j \le B \quad (2c)$$

$$y \text{ binary.} \quad (2d)$$

The objective function (2a) seeks to maximize the total coverage benefit. Constraint (2b) asserts that the total number of readers should not exceed the available maximum number *R*, and constraint (2c) imposes a budgetary restriction on the total acquisition plus installation cost. Finally, constraint (2d) represents the binary restrictions on the decision variables *y*.

In the alternative formulations **RL1** and **RL2** alluded above, we would *delete* the constraint (2c) or (2b), respectively. Observe that if the maximum possible number of readers that could be installed subject to the budget restrictions (2c) is itself less than or equal to *R*, (i.e., the sum of the $R + 1$ cheapest readers exceeds *B*), then constraint (2b) is redundant and can be omitted. On the other hand, if the maximum total installation cost for *R* readers is no more than *B* (i.e., the sum of the *R* most expensive readers is less than or equal to *B*), then (2c) is redundant and can be deleted. In this latter case, because of the nature of the objective function, we can impose (2b) as an equality constraint. Note that problems **RL1** and **RL2** are 0-1 quadratic knapsack problems, while **RL** imposes a pair of constraints. Sherali and Adams (1999) describe various reformulation-linearization techniques that can be gainfully applied to effectively solve these problems. The underlying linearized polyhedral structure of these problems, and related valid inequalities to enhance their solvability, have also been variously investigated by Adams and Hadavas (1999), Deza and Laurent (1989), Padberg (1989), and Sherali, Lee and Adams (1995). We propose to prescribe, implement and extensively test a suitable effective solution approach for solving this class of problems.

1.5 Overview and Organization of this Thesis

The current section is an overview of the methodology adopted in this thesis, and the organization of the different components. This thesis primarily focuses on two key tasks. The first of these involves the determination of the benefit factors that appear in the objective function of the reader location problem. The second task is to solve the corresponding optimization problem that incorporates these benefit factors in its objective function.

In practice, typical travel time variability is measured using loop detector data. Loop detectors are used to measure the travel times of vehicles at 15-minute intervals. Note that for data points collected at a certain time, there is coefficient of variation[1] (COV) of travel time associated with these data points. For a link *j*, denote COV_w^i, as the coefficient of variation of travel time *within* the data points at time *i*. Using the data collected over an entire day for link j, the COV of travel time *between* 15 minute measurements can be computed, and is denoted as COV_b. The COV for link j, over the period of an entire day could be taken as $COV^j = \max\{COV_w^i, COV_b\}$.

However, in this thesis, we deal only with COV_w^i , and this value is computed by performing simulation runs for the various types of basic freeway sections. To achieve this purpose, the following basic freeway sections are identified: on-ramps, off-ramps, weaving sections, and bottleneck sections. For each of these sections, we perform

[1] The coefficient of variation is defined as the standard deviation divided by the mean .Refer Section 3.2 of this thesis for further details.

simulation runs for a wide range of traffic demand distributions, and the COV values of travel time corresponding to these demand distributions are presented as generic look-up tables. Knowing the characteristics of the traffic on the freeway, and that for the basic section, the corresponding COV of travel time can be obtained from these generic look-up tables.

Given an underlying transportation network, the construction of the graph on which the optimization problem is to be solved is the next step. The various O-D paths on this network, for which the measurement of travel time is deemed necessary, are identified and the corresponding graph *G* is obtained as described in Section 1.4. The benefit factors for the various O-D paths are obtained by using a suitable composite benefit factor function.

Once the graph *G* with the associated benefit factors for each edge is known, the reader location problem is solved on this graph *G*, and the results obtained can be used to obtain the location sites on the actual transportation network. The entire methodology as described, is depicted in pictorial format as a flow chart in Figure 3.

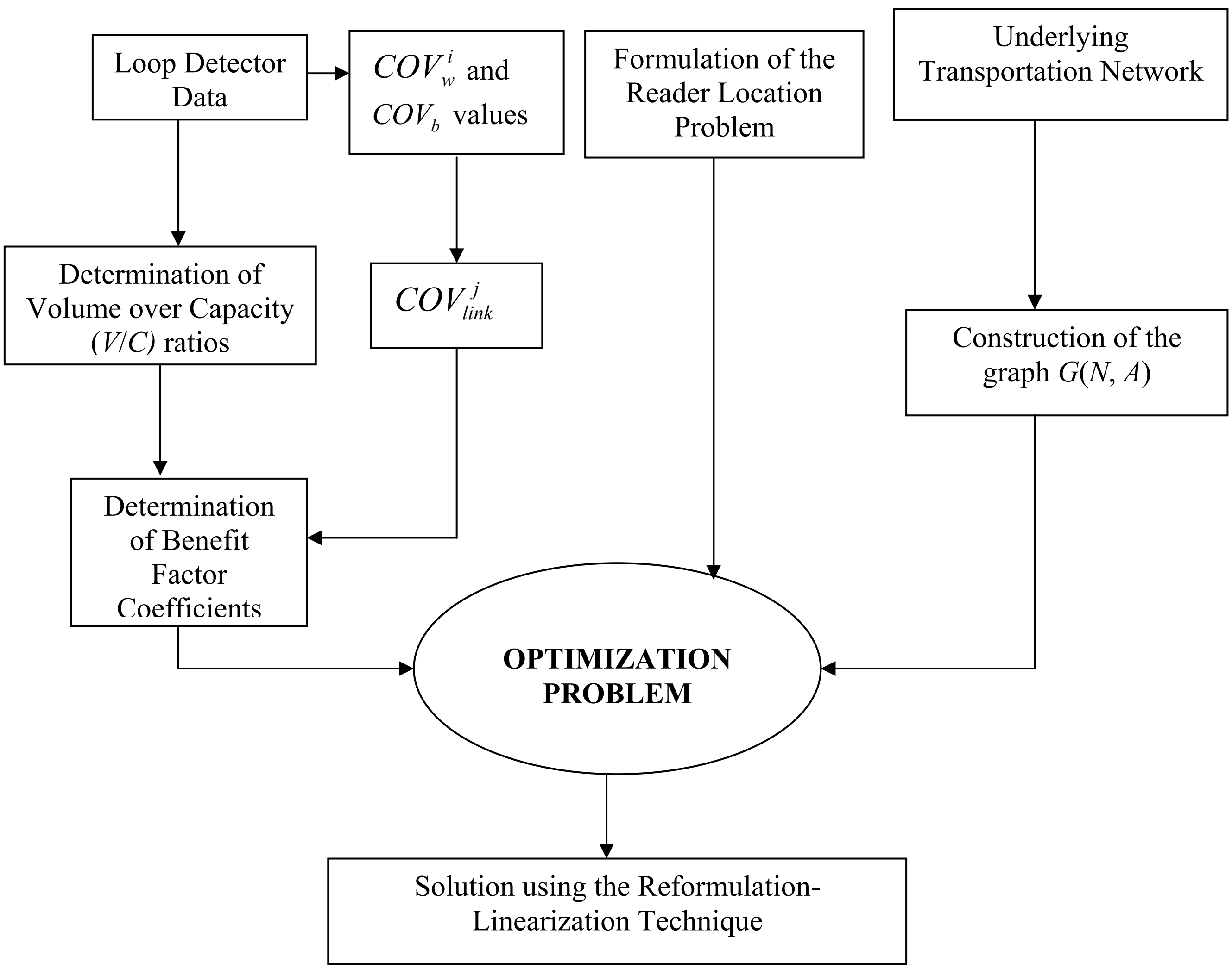

Figure 3: Flowchart Representation of the Current Research Effort

The remainder of this thesis is organized as follows. Chapter 2 contains a summary of the existing literature describing the usage of advanced technologies for measuring travel times. The latter part of the chapter reviews the literature concerning the optimization problem, and the effectiveness of relevant solution methodologies. Chapter 3 discusses in detail the modeling and analysis of the research problem under consideration. The determination of the benefit factor coefficients, the derivation of a composite benefit factor function, as well as the reformulation of the problem to produce the equivalent linearized problem are described. In Chapter 4, a sample problem is examined and we summarize the results obtained for different model variations. Computational results as and sensitivity analysis results are also presented for a variety of test cases. Chapter 5 summarizes the entire research effort, and provides some additional insights as well as directions for future research.

2. LITERATURE REVIEW

This literature review presented in this chapter is comprised of two parts. One part is concerned with the review of literature on methodologies for computing travel times, and the other is a review of the literature pertaining to the mathematical structure of the optimization problem involved.

Literature Review - Part 1

2.1 Overview

Urban transportation planning evolved from highway and transit planning activities in the 1930s and 1940s. These efforts were primarily intended to improve the design and operation of individual transportation facilities. The focus was on upgrading and expanding facilities. Early urban transportation studies were primarily systems-oriented based on a twenty-year time horizon and being region-wide in scope.

Gradually, starting in the 1970s, planning processes turned their attention to shorter-term time horizons and the corridor-level scale. This came about as the result of a realization that long-range planning had been dominated by concern for major regional highway and transit facilities with only minor attention being paid to lesser facilities and the opportunity to improve the efficiency of the existing system. The increasing difficulties and costs in constructing new facilities, as well as growing environmental concerns, reinforced this shift.

By the early 1990s, there were major changes underway that would have significant effects on urban transportation. With only limited highway expansions and with growth of traffic continuing unabated, new approaches were needed to serve this travel demand. One of the key inputs into the existing Transportation Management Systems (TMS) is travel time. In order to reduce the congestion on the links, and to render the transportation system performance more efficient, a good estimate of the travel times along the various links is required.

The evaluation of travel times has been extensively reviewed in the literature. This is an important feature of transportation research because it helps address several problems encountered on freeways, arterials, etc. Travel time measures are compatible with multi-modal analyses and are understood by non-technical audiences, and yet are rigorous enough for technical analyses by transportation engineers and planners (Benz and Ogden, Transportation Research Record, 1996). Hence, this has been a subject of discussion for more than twenty years. Transportation researchers have realized the importance of estimating travel times and have developed several related methods. One important step in the estimation of the travel time on any path from an origin to a destination (the travel time between any given O-D pair) is to identify the links that comprise the path from the origin to the destination and to estimate the individual link travel times. These component travel times are then summed to yield the estimated travel time between the given O-D pair.

Link travel times are one of the most widely used and valuable measures of congestion. Several techniques are used to measure travel times and these methods have been fairly successful in estimating travel times. Each technique has certain advantages and disadvantages in terms of the work involved, accuracy of information obtained, and cost of obtaining the data (Benz and Ogden, Transportation Research Record, 1996). The conventional methods of estimating travel times were labor intensive, prone to manual error, costly, and time consuming processes. The most widely used technique was the floating car technique in which the test car travels along a predefined route and the times were recorded at predefined check points. This method involved a great deal of manual observations and was susceptible to considerable error. The level of accuracy also differed from observer to observer.

The use of electronic equipments to eliminate manual errors came to the fore in the mid-nineteen-eighties. In 1985, Reid published results of a study conducted to develop a travel time program and to derive certain travel related results. From then onward, there have been several methods to compute travel times using various electronic technologies, and numerous related papers have been published in the literature.

Advanced techniques for travel time data collection are becoming more important for applications ranging from congestion measurement to real-time travel information. With the emphasis switching from evaluation of travel times to obtaining estimates of travel times in real-time and informing the public of any hazards instantaneously, the usage of advanced techniques is becoming imperative. Some of the methods used are Distance Measuring Instruments (DMIs), Automatic Vehicle Identification (AVI), and Automatic Video Location.

The following section provides an overview of the various technologies that are currently used to measure travel times along the links. A comparison between the different methodologies is also discussed.

2.2 Surveillance Technologies

From the defined need, research, and discussions with traffic experts, a set of general functional and performance requirements have been established for a detector technology to be used in traffic management applications (Earnesty. Lisa H, Evaluation of Detection Technologies for use in Traffic Management and Control, M.S. Systems, Virgina Tech, 1996). These requirements are based on:

(1) the needs of current general traffic management applications (i.e. operational and functional characteristics, accuracy, etc.);
(2) the types of environments that the system operates in, and
(3) the type of installation and maintenance required.

Each of the surveillance technologies available can be described in regard to these requirements, once such requirements are identified. Some of these technologies are mentioned and the ones that are relevant to this research are discussed in detail below.

1. Microwave Radars

2.Ultrasonic Detectors

3. Infrared Detectors

4. Loop Detector Technology

The inductive loop detector has been the most popular and widely used form of traffic detection system for years. These detectors provide information on vehicle passage, presence, occupancy, and count. Loop detectors can be used as either passage or presence type of detectors. As a passage detector, a loop can be used for traffic counting, in individual lanes, in all lanes, and in both directions. The current research includes data collected from presence loop detectors.

The most common traffic detector used today is a presence-type detector, which records the presence and passage of vehicles over a short segment of roadway. When a vehicle enters the detection zone, the sensor is activated and remains activated until the vehicle leaves the detection zone. The detector remains on for a distance of travel equivalent to the length of the vehicle plus the length of the detection zone. The detection zone, which does not necessarily equal the physical length of the detector, is numerically determined through calibration. The inductance loop detectors that are currently installed in the Seattle area are examples of presence-type detectors. Inductance loop detectors act as an inductor in an oscillating inductor-capacitor (L-C) circuit. L-C circuits oscillate at a resonant frequency that depends on the value of the capacitance and inductance. A large metallic object that travels over the detector (within the detection zone) changes the value of the inductance, resulting in a change in the resonant frequency. The change in resonant frequency produces an electric current giving what may be thought of as a “1” signal. Alternatively, when there is no vehicle over the detection zone, no signal is produced and it may be thought of as giving a “0” signal. The loop detector is scanned at regular intervals (60 times per second in the case of the Seattle loop detectors) generating a pictorial output depicted in Figure 4(a).

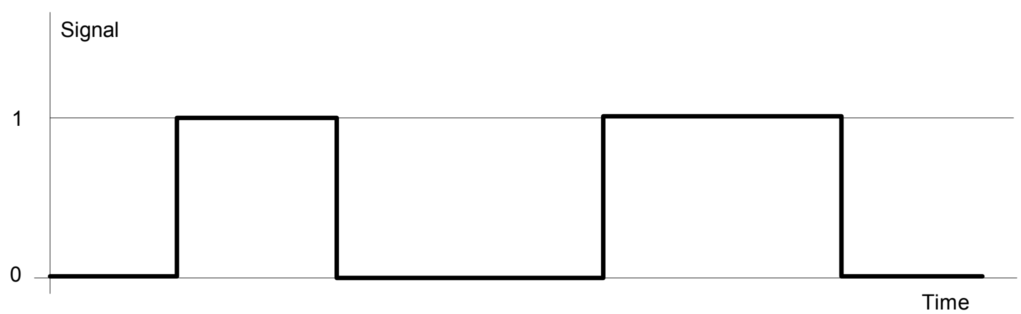

Figure 4(a): Output Signals for a Loop Detector.

The classical steady-state traffic flow relationship states that the traffic flow rate equals the product of the traffic density and the traffic space-mean-speed, as demonstrated in Equation 1 (Lighthill and Witham, 1955).

$$q = k \times u \qquad (1)$$

here:

q = Hourly flow rate (veh/h/lane)

k = Traffic density (veh/km/lane)

u = Traffic space-mean-speed (km/h).

Single loop detectors measure occupancy, which is defined as the percentage of time a detector is occupied over a specified time interval. Occupancy ranges from 0, meaning that the detection zone was never occupied during the time interval, to 1, meaning that the detection zone was occupied 100 percent of the time interval. Single loop detectors also measure the traffic volume that passes the detection zone, which is defined as the number of activations for a specified time interval. The measurement time interval is typically 20 seconds, or in some instances can be 30 seconds.

The flow rate over a 20-second time interval can be easily computed using Equation (2). The computation of traffic density from occupancy measurements is not straightforward because it depends on a variable parameter (average vehicle length), as demonstrated in Equation (3). The detection length is a constant parameter that is calibrated, however, the average vehicle length over a time interval is a function of the traffic composition (percentage trucks, buses, etc.). Furthermore, the average vehicle length can vary from one time interval to another.

$$q_i = \frac{3600}{T} \times N_i \qquad (2)$$

$$k_i = \frac{1000}{l_v + l_D} \times O_i \qquad (3)$$

where,

q_i = Hourly flow rate for time interval i (km/h/lane)

T = Time interval duration (20 seconds for WSDOT loop detectors)

N_i = Number of activations for time interval i (unitless)

k_i = Traffic density for time interval i (veh/h/lane)

l_v = Average vehicle length (meters)

l_D = Length of detection zone (meters)

O_i = Occupancy for time interval i (unitless and ranges from 0 to 1)

Using the steady-state traffic flow relationship (Equation 1) the speed over a time interval can be computed. Hall and Persaud (1988) have assumed the average vehicle length to be constant allowing the vehicle speed to be equal to a constant ("G" factor) multiplied by the volume to occupancy ratio. The constant is a parameter that is derived through calibration. This approach is referred to as the G-factor technique.

Figure 4(b) illustrates a scatter plot of dual loop detector speed measurements (x-axis) versus speed estimates using the G-factor technique (y-axis) for an uncalibrated single loop detector. The line of perfect correlation (45° line) is also drawn in order to visually demonstrate how the two speed estimates compare. The figure clearly demonstrates that calibration of the "G" factor is critical for the accurate estimation of vehicle speeds.

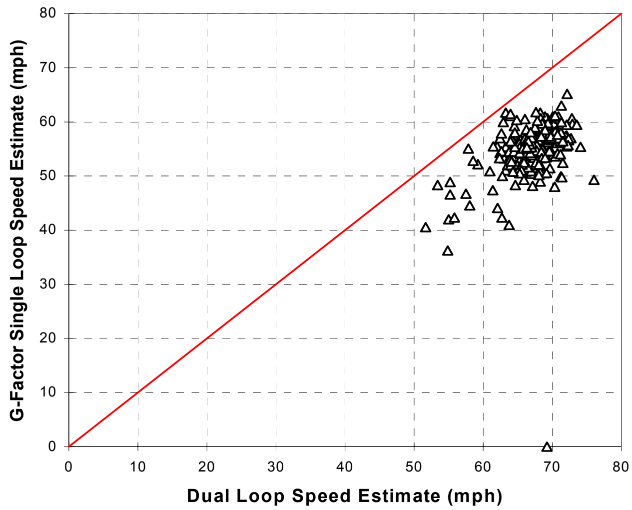

Figure 4(b): Speed Estimate Comparison between Single and Dual Loop Detectors.

2.3 Data Sources

The increased public usage of the World Wide Web (WWW) has resulted in the development of numerous web-based Advanced Traveler Information Systems (ATIS). A key component to these ATIS systems is the provision of real-time travel time information. The introduction of vehicle tags for purposes of toll collection provides a unique opportunity to estimate roadway segment travel times by installing tag readers along major freeway and arterial sections. In addition, real-time travel time information can be estimated from the many inductance loop detectors that are currently installed in urban areas. Furthermore, the use of cellular phones and the development of cellular location systems could be utilized to estimate travel times by tracking vehicles that are equipped with cellular technology.

These three sources of travel time data cover a wide spectrum of data resolution. The highest level of resolution includes GPS and cellular location technology that provide continuous real-time data. However, these data are provided for a sample of the traffic fleet as opposed to the entire fleet. The second source of real-time data includes AVI travel time estimates, which by matching vehicle tags, provides a spatial estimate of roadway travel times for a sample of the vehicle population. The difference in resolution between the first and second data sources is that the first data source captures the speed variability along the roadway segment while the second data source does not. The third and final level of resolution in detection technology uses spot measurements of vehicle speeds to compute spatial travel time estimates. This data source, while of lower resolution, is available for the entire population that traverses the detection location.

Automatic Vehicle Identification Technology:

In the current problem under consideration, we address the use of loop detector technology (as described above), and AVI technology to estimate travel times. Automatic Vehicle Identification is a technology that has emerged recently in various traffic management on-toll collection procedures (Turner, TRB 1996). An AVI system consists of an in-vehicle tag, a roadside reader unit, and a centralized computer system. Whenever a vehicle having a tag passes across a reader unit, the tag sends a signal, which is recorded by the central computer. Each vehicle has a unique tag number and for the purpose of computing travel times, the central computer monitors several consecutive reader units and matches the corresponding tag numbers.

An issue regarding AVI travel time data collection is the required number of vehicles that must be equipped with AVI tags. AVI travel times are usually collected and computed in real-time, and the accuracy and reliability of the data is directly related to the number of AVI tags distributed. In the case study under consideration, i.e., the San Antonio system, over 70,000 AVI tags were distributed among volunteer drivers free of charge. In addition, 53 tag reader locations were identified and readers were installed at selected points on freeways and major arterials spaced a mile or sometimes several miles apart. The study indicated that on average, the level of AVI tag market penetration on the roadways in San Antonio during the Metropolitan Model Deployment Initiative (MMDI) evaluation was approximately 1%, while varying from 8% during the early morning to 0.1% during other parts of the day.

The second source of field data includes vehicle AVI data. Instead of collecting data from test probe vehicles in costly field experiments, it is proposed in this research to analyze available data from the growing number of automatic vehicle identification (AVI) systems that have been implemented in urban transportation areas. For this research in particular, it is specifically proposed to analyze data collected by the San Antonio AVI system. If any other data sources are available at the time of the study, these data sources will be considered.

<u>6. Other Advanced Technologies</u>

Other technologies that can provide real-time travel time information are emerging. These data collection devices have the advantage that they do not necessarily require investment in extensive infrastructure and can provide information on detailed vehicle speed profiles. The speed profile data can be utilized to compute a vehicle's travel time, in addition to a vehicle's fuel consumption, emissions, and crash risk. Specifically, Rakha *et al.*, (2000a, b) have developed statistical models that compute a vehicle's fuel consumption, emissions, and crash risk based on its instantaneous speed and acceleration levels. These models have been incorporated within the INTEGRATION microscopic traffic assignment and simulation model (Rakha *et al*., 2000a) and have been applied directly to field data that were collected using GPS technology (Rakha *et al*., 2000b).

7. Traffic Simulation Data

The study of traffic by using simulation tools, instead of field data, has come to the fore only in recent times. Gary B. Thomas has investigated in detail the relationship between detector location and the ability of a system to monitor traffic characteristics and estimate link travel characteristics (Thomas G.B., ITE, 1999). This analysis is for the case of arterials, whereas this thesis focuses on freeway volumes. The use of CORSIM (a micro-simulation program), to estimate travel characteristics is one of the pioneering works in the area of using simulation tools for detector location. In our analysis, we use a program known as INTEGRATION, for the study of detector location techniques.

In addition to field data, simulated data from the INTEGRATION microscopic traffic assignment and simulation model will be utilized as part of the research effort (Van Aerde *et al*., 1996). The INTEGRATION model (which is currently being upgraded), is a trip-based microscopic traffic assignment, simulation, and optimization model that is capable of modeling networks having up to 10,000 links and 500,000 vehicle departures. The model is designed to trace individual vehicle movements from a vehicle's origin to its final destination at a level of resolution of one deci-second. The model computes a number of measures of performance including vehicle delay, stops, fuel consumption, hydrocarbon emissions, carbon monoxide emissions, nitrous oxides emissions, and the crash risk for 14 crash types. In addition, the model allows the user to locate loop detection and AVI technology. Furthermore, the model allows the user to gather second-by-second vehicle statistics similar to GPS data. Consequently, the model will provide an ideal test-bed for a systematic and direct comparison and evaluation of the different detection technologies that are available in the field.

The specifics of the San Antonio system are not only dependent upon the number of tags but also dependent on the location of the readers. The following section of the literature review concerns the optimization problem formulated for determining optimal locations for the readers.

Literature Review - Part 2

2.4 Preface:

Linear programming (LP) deals with the problem of optimizing (maximizing or minimizing) a linear objective function in the presence of linear equality and/or inequality constraints. Since the development of the Simplex Method in 1947, linear programming has been extensively used in several, and varied fields. The popularity of linear programming can be attributed to many factors including its ability to model large and complex problems, and the ability of the users to solve large problems in a reasonable amount of time by the use of effective algorithms and modern computers. Since the development of the Simplex method (by George B. Dantzig, by the end of the Summer of 1947), many people have contributed to the growth of linear programming by developing its mathematical theory, devising efficient computational methods and codes, exploring new algorithms and applications, and by their use of linear programming as a tool for solving more complex problems such as discrete programs, non-linear programs etc. (for example, see Bazaara, Jarvis and Sherali, 1990).

Integer optimization and its importance in solving practical problems came to the fore as a result of the impressive developments in the arena of linear programming. Any decision problem in which a subset of the decision variables is restricted to take on specified discrete values may be classified as an integer or discrete optimization problem. In general, an integer program can be constrained or unconstrained, and the functions representing the objective and constraints can be linear or nonlinear. In the strict sense, every integer program must be regarded as a nonlinear program, since its functions are defined only at discrete values. However, this technical detail may be overlooked for the sake of classification from the viewpoint of developing solution methodologies for integer programming problems. More commonly, an integer program is said to be a linear integer program if by relaxing the integer restriction on the variables, the resulting objective function and constraints are linear. Otherwise, the problem is classified as a nonlinear integer program (Taha, 1975).

2.5 Review of Optimization Techniques

Integer programming, as defined above, is not a new mathematical subject, but until the application of Operations Research became recognized in the late 1940s and early 1950s, most of the problems tackled were primarily of a mathematical nature. It was only after this period, that several important practical problems were formulated as integer programs. General solution methods for integer programs were hitherto unknown, until in 1958, Ralph Gomory developed the first finite (non-enumerative) integer programming (cutting plane) technique for solving integer programs. Since the pioneering work of Gomory, integer programming has been one of the most exciting and rapidly developing areas of Operations Research. Several real-world applications of integer programming problems have been developed, and algorithms have been devised for general, and specific models. These algorithms vary widely in spirit and mathematical sophistication (for example, see Nemhauser and Wolsey, 1998). Nemhauser and Wolsey (1998) and Parker and Rardin (1998) provide comprehensive descriptions of the general subject matter, and Sherali and Driscoll (2000) provide a state-of-the-art review.

The specific techniques of integer programming that are used have been reviewed as necessary, based on the development of the proposed algorithm. In particular, to solve the Reader Location problem (**RL**), as defined in Section 1.4, we use the *Reformulation-Linearization/Convexification* (RLT) approach of Sherali and Adams (1990, 1994, 2000) and the subsequent part of this review is dedicated to this technique and its application to problems of the type relevant to this thesis.

Reformulation-Linearization / Convexification Technique (RLT)

The Reformulation-Linearization Technique is a unifying approach for solving discrete as well as continuous nonconvex optimization problems. In a very broad sense, the RLT methodology reduces a given nonconvex program to a corresponding equivalent or sometimes lower bounding (for minimization problems) linear/convex program, and solves this resultant problem. The representation, thus derived, of the nonconvex program must be tight to facilitate the solvability of the problem, and it has been amply demonstrated that the RLT methodology gives very tight, underlying linear programming relaxations. As mentioned above, a unified treatment of both discrete and continuous nonconvex programs can be done using this approach. In essence, representing discrete variables as polynomial constraints can bridge the gap between the two types of nonconvexities. For example, the 0-1 restriction on a binary variable x_j can be represented as the continuous nonconvex constraint $x_j(1- x_j) = 0$. The main thrust of the RLT is to formulate a tight linear/convex programming representation that has a useful structure, and then to further strengthen the relaxation through reformulation and constraint generation techniques (Sherali and Adams, 1998). In problem **RL**, as referred to in this thesis, we generate additional constraints using concepts from semidefinite programming. These concepts are reviewed in a subsequent section of this chapter.

The RLT, in general, operates in two phases. In the *Reformulation Phase*, certain additional types of implied polynomial constraints are appended to the problem. In the subsequent *Linearization/Convexification Phase*, the resulting program is linearized by performing appropriate variable substitutions (sometimes certain simple, convex bounding constraints are retained), that involve defining new variables to replace each variable-product term. The higher dimensional representation yields a linear/convex programming relaxation. In fact, the use of higher order polynomial constraints can be used to generate a hierarchy of relaxations. It must be noted that in several applications such as location-allocation, and various engineering design problems, it has been demonstrated that even the first level of relaxation in this hierarchy generates very tight relaxations. As a result, the RLT has been frequently employed to solve nonconvex programs to near optimality via a single LP relaxation. The theory concerning RLT can be extended to provide a unifying mechanism for generating valid inequalities and constructing convex hull representations in certain cases. The approach can also be shown to extend to the class of multilinear polynomial programming problems (see Sherali and Tuncbilek (1992, 1995, 1997)).

The importance of having tight linear programming representations to enhance the effectiveness of an algorithm for solving integer programming problems has been extensively dealt with in optimization literature. Most of the attention was focused on pure and mixed zero-one programming problems because of the wide variety of applications that these models represent. The emphasis is on constructing a formulation whose continuous relaxation is "tight" in that it closely represents the convex hull of integer feasible solutions, at least in the vicinity of optimal solutions. While several other strategies exist for constructing good underlying representations of the problem, the RLT in particular has been shown to subsume and dominate these representations and also has the capability of generating suitable, valid implied inequalities.

Consider a mixed-integer zero-one linear programming problem whose feasible region X is defined in terms of some equalities and inequalities in binary variables $x = (x_1, x_2, \ldots, x_n)$. Given a value of $d \in \{1, 2, \ldots, n\}$, the RLT methodology constructs various polynomial factors of degree d comprised of the product of d binary variables x_j or their complements $(1- x_j)$. These factors are then used to multiply each of the constraints defining X, to create a nonlinear polynomial mixed-integer programming problem. Using the relationship $x_j^{\,2} = x_j$ for each binary variable x_j, and substituting $w_{ij} = x_i x_j \quad \forall\, i < j$, and relaxing integrality, the nonlinear programming problem is re-linearized into a higher dimensional polyhedral set X_d defined in terms of the original variables x and the new variables w. For X_d to be equivalent to X, the only constraint we need is that x needs to be binary valued, with the remaining variables being treated as continuous. Viewing the projection of X_d in the space of the original variables, and denoting this projection as X_{Pd}, it is shown that as d varies from 1 to n, we get,

$$X_{P0} \supseteq X_{P1} \supseteq X_{P2} \supseteq \ldots \supseteq X_{Pn} \equiv conv(X),$$

where X_{P0} is the ordinary linear programming relaxation, and *conv*(X) represents the convex hull of X. The projection process also produces an algebraic representation of $X_{Pn} \equiv conv(X)$, which has a structure that can be exploited to derive facets for various special classes of combinatorial optimization problems. For problem **RL**, we employ the first level representation. Hence, we have, $d = 1$, and the product factors are y_j and $(1- y_j)$ $\forall j$.

As evident from the foregoing discussion, higher order representations can be derived and used for solving problem **RL**, beyond the first level ($d = 1$). However, from the computational viewpoint, it has been found that the first level representation works well, and is quite beneficial. Moreover, incorporating other constraints into the RLT framework can further tighten the corresponding continuous relaxation. In our case, we utilize concepts from semidefinite programming to provide such an enhancement of the reformulated problem. The remainder of this literature review will describe the importance and applicability of semidefinite programming for solving mixed-integer zero-one programming problems, and its use in augmenting the relaxation obtained from the RLT framework.

Semidefinite Programming

In semidefinite programming, we seek to minimize a linear objective function subject to the constraint that an affine combination of symmetric matrices is positive semidefinite. Such a constraint is convex, and hence semidefinite programs are convex optimization problems. Semidefinite programming unifies several classes of problems (e.g., linear and quadratic programming) and is in vogue in many areas of optimization. Although semidefinite programs are much more general than linear programs, they are not much harder to solve. Most interior point methods have been generalized for solving semidefinite programs. As in LP, these methods have polynomial worst-case complexity (for a fixed optimality tolerance ε). Due to its many applications in control theory, robust optimization, combinatorial optimization, and eigenvalue optimization, semidefinite programming has been in widespread use, even before the era of interior point algorithms. In practice, it is one of the widespread tools of optimization (see Vandenberghe and Boyd, 1996 and Helmberg, 2002).

A semidefinite program (SDP) can be stated as follows.

$$\begin{aligned} \mathbf{P}: \quad & \text{Minimize} \quad C \cdot X \\ & \text{subject to} \quad A_i \cdot X = b_i \quad \forall\, i = 1, 2, \ldots, m. \qquad (1) \\ & \qquad X \succeq 0 \end{aligned}$$

where, C, A_1, A_2, …, A_m, and X are real symmetric $n \times n$ matrices, $b_i \in R^m \ \forall\, i$, and where the product ($\cdot$) denotes the inner product of the matrix components, and $X \succeq 0$ denotes that X should be symmetric and positive semidefinite (PSD). In general, semidefinite constraints (imposed on some affine sum of matrices) are also referred to as *linear matrix inequalities*.

Hence, semidefinite programming can be regarded as an extension to linear programming where the non-negativities on the variables are replaced by positive semidefinite restrictions on matrices. This is equivalently a replacement of the first orthant (or non-negativity constraints) by a cone defined by the PSD matrices. Alternatively, we can also view problem **P** as a *semi-infinite linear program* (SILP), since the matrix inequality $X \succeq 0$, is equivalent to a set of infinite linear constraints of the type $\alpha^t X \alpha \geq 0, \forall \alpha \in R^n$.

It is therefore not surprising to note that the theory of semidefinite programming closely parallels the theory of linear programming, or that interior point algorithms for solving LPs have been generalized to solve SDPs. There are several reasons for studying semidefinite programming. Particularly, discrete and continuous nonconvex programs (or their relaxations) can be cast as semidefinite programs in order to derive tight bounding problems. Sherali and Fraticelli (2002) have also shown that semi-infinite linear programs can be formulated for nonconvex problems by using the RLT methodology and concepts from semidefinite programming in tandem, in order to enhance model representation for such challenging classes of problems.

We propose to use the above-mentioned concepts in the modeling and analysis of problem **RL** in this thesis. The forthcoming chapter, Chapter 3, comprises the modeling and analysis of the reader location problem, as formulated in Chapter 1. A detailed description of the various modules of Figure 3 can also be found.

3. MODELING AND ANALYSIS

As described in Chapter 1, the measurement of typical travel time variability is part of Transguide, which is an ongoing project in San Antonio, Texas. Transguide is meant to develop an area wide real-time traffic database. To achieve this purpose, 70,000 volunteers have had small transceiver tags installed on their windshields. A reader located at a certain specific point in the city records the time when a particular car (with a specific tag number) crosses its location. When this tagged car passes another reader located downstream, at a certain distance away from the first reader, a computer at Transguide headquarters compares the distance between the reader locations and the time it took for that car to travel the corresponding distance. Using this information, the computer estimates the speed of the vehicle, and compares this to its database to ascertain whether there is congestion along that route.

There are several useful purposes of Transguide. It provides real-time traffic flow information to commuters traveling along congested areas. These congestion warnings and reports are particularly useful to people who are not familiar with these routes (e.g. truckers and tourists passing along that route).

The modeling and analysis of the problem under consideration involves several procedural steps and issues. In Section 1.4, we modeled the reader location problem as a 0-1 integer-program, with the objective function having certain benefit coefficients (b_{ij}) as inputs. The first and foremost issue of concern is the determination of the associated benefit factor matrix. This critical step turns out to be quite a demanding task and a considerable portion of this chapter is devoted to it. Second, the reader location problem is a pure 0-1 problem that requires the development of suitable, effective solution techniques in order to facilitate its implementation in practice.

3. 1 Quantification of Typical Travel Time Variability for Recurring Conditions

Section 3.1 deals with investigating the variability in travel times along a roadway section under recurring conditions. This task, as described later in this chapter, will involve selecting a 5 to 10 mile freeway section in order to assess typical travel time variability during recurring conditions. In addition, the simulation of four basic network components is considered, including a straight bottleneck section, a merge section, a diverge section, and a weaving section. These four sections are tested for different demand levels and demand distributions in order to identify the sensitivity of travel times to these factors. The results from this analysis are then used to estimate the benefit factors. Comparisons with field data are typically used to validate the simulation findings. However, this is beyond the scope of this thesis.

3.2 Determination of Benefit Factors (b_{ij}).

In the reader location problem, the objective function parameters are the benefit factors for the corresponding arcs in the constructed graph *G*. The determination of these benefit factors is one of the key components in solving the reader location problem. Since we have a limited budget and/or restricted number of readers available, the objective is then to allocate the readers in such a way that the maximum benefit is derived subject to these constraints. In our case, the benefit accrues from an effective measurement of travel times and garnering as much information about the variability in travel times as possible. Measuring travel time along an arc (i, j) is considered more beneficial than measuring it on an arc (p, q) whenever the variability in travel time is more along the arc (i, j) than along (p, q). If the travel time on an arc is nearly the same for all time periods on a given day (this assumes that no seasonal variations exist; in certain cases, the travel time on a given arc might be without variation on a given day but might significantly vary on a yearly basis), then it is of little value to install special devices to measure the travel time along such an arc. Other low technology sampling techniques could be used to estimate the expected travel time in this case. On the other hand, for a link having a considerable variation in travel time during different periods of the same day, it is far more important to capture information about the movement of traffic on that particular arc as a function of time.

Below, we derive benefit factors for different types of freeway sections, assuming that the transportation network that is being studied consists predominantly of freeways. A node is placed in this network wherever there is a change in the freeway section due to an entering ramp, or an exit ramp, and so on, and the section of the roadway between any two nodes is modeled as a link. Each segment of this transportation network between some origin-destination pair for which we are interested in measuring travel times might be composed of several such links. These segments effectively translate to the arcs in the model graph G, with the union of the end-points of these segments translating to the nodes of G, as described above.

Another important aspect of the problem is that the type of section that is being considered on the roadway affects the travel time. Any particular link can be assumed to be composed of the following basic types of sections.

1. On-Ramp or Merge section
2. Off-Ramp or Exit section.
3. Weaving section
4. Bottleneck section

These four sections are as described below. The specifications for the sections are in accordance with the Highway Capacity Manual requirements (Highway Capacity Manual, Transportation Research Board, Washington D.C.).

1. **On-Ramp or Merge section:** The segment of the roadway that lies within a distance of 450m from an entering ramp is considered to be an on-ramp section.

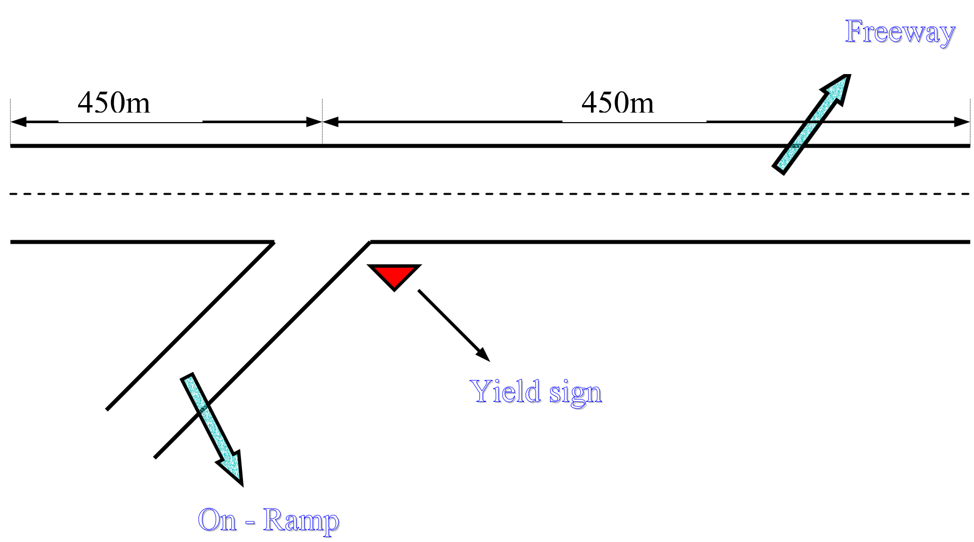

Figure 5: On-Ramp Section

2. **Off-Ramp or Exit section:** The section of the roadway that lies within a distance of 450m from an exit ramp is considered to be an off-ramp or an exit section.

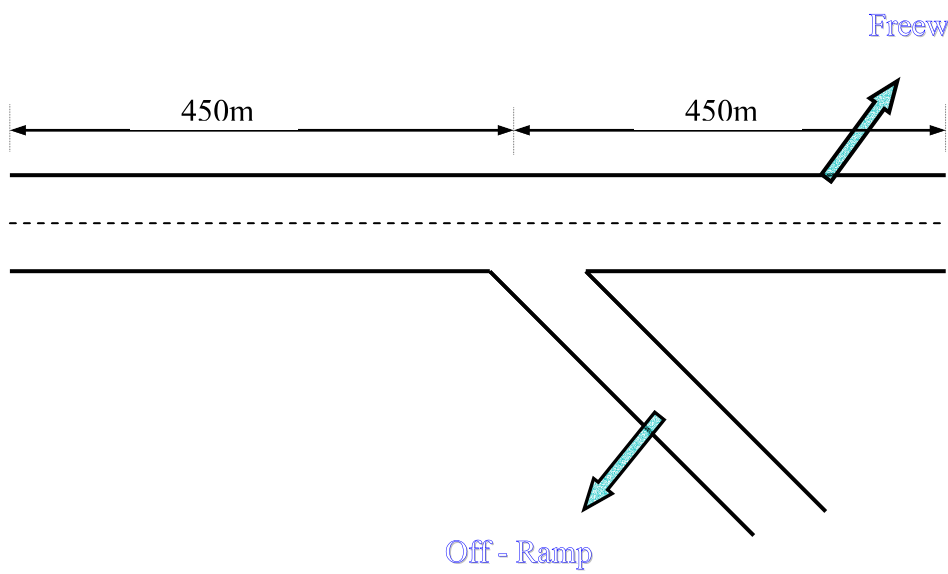

Figure 6: Off-ramp section

3. **Weaving section:** A section of the roadway in which both an entering ramp and an exit ramp occur close to each other, within 450m, is called a weaving section.

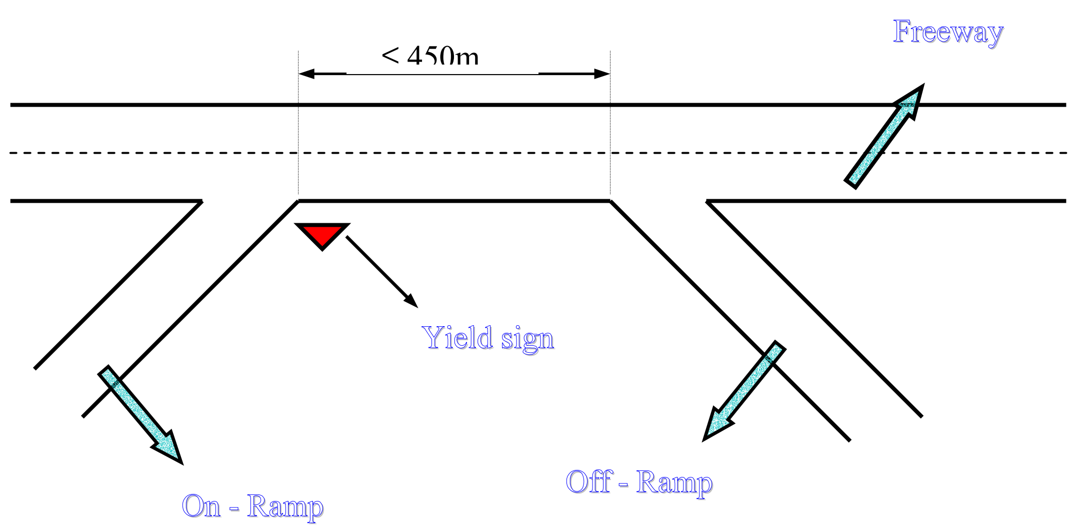

Figure 7: Weaving section

4. **Bottleneck section:** A section of the roadway in which the number of lanes is suddenly reduced due to various causes such as accidents, road construction, repair work, or because of geometric conditions, is called a bottleneck section.

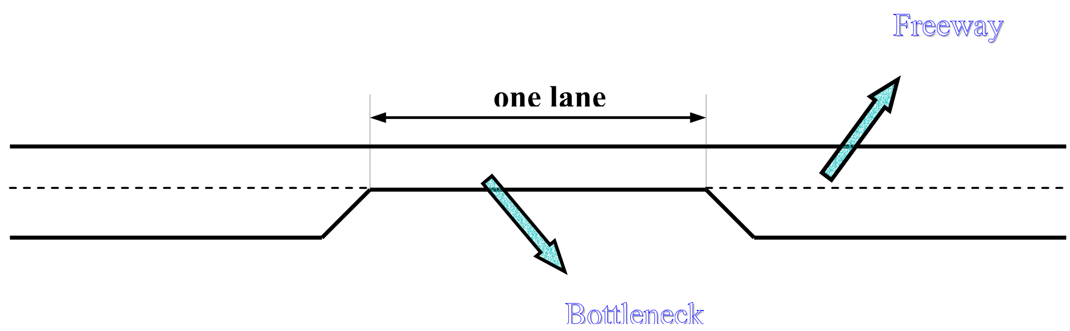

Figure 8: Bottleneck section

Any link in the network can be assumed to contain one or more of the above types of sections, depending on the delineations of the nodes. If the benefit factors can be estimated for these sections, then the benefit factors for the links can be determined by using a suitable combination of these basic factors (for example, a suitable weighted sum of corresponding benefits).

In addition to the foregoing considerations, the following principal attributes pertaining to a link, or a section of the roadway, which contribute toward, or affect travel time on that link, are identified.

1. Expected demand on the link (vehicles/hr or V/C ratio);
2. Demand distribution (ratio of vehicles on the freeway to vehicles on the corresponding on-ramp, or exit, or weaving, or bottleneck sections);
3. Number of lanes on the freeway;

4. Number of lanes on the on-ramp, or exit section, or weaving, or bottleneck sections;
5. Auxiliary (Acceleration/Deceleration/Weaving) lane length (m).

Each of the above attributes is varied, and the travel time characteristics are determined for various combinations. For example, for the on-ramp section, the total demand on the freeway is varied from 1000 veh/hr to 4000 veh/hr, using increments of 500 vehicles for each run, leading to a total of seven different expected freeway demands. Similar to the freeway demand, the on-ramp demand is varied from 200 veh/hr to 1800 veh/hr using increments of 400 vehicles for each run. This gives us a total of 35 simulation runs for each (*freeway demand, on-ramp demand*) pair. This computation is repeated for each of the five foregoing attributes using the INTEGRATION[2] simulation package (Appendix A). For each resulting combination, this will yield a series of travel time values using the simulation model. The mean and standard deviation of the travel time can thus be calculated for each combination. The coefficient of variation is defined as:

***COV* = Standard Deviation / Mean**

This statistic provides a measure of the variability of travel time for a particular section of roadway under specified traffic demand characteristics. Every roadway in the network can be assumed to be composed of the four basic sections, and hence, the corresponding benefit factors for the links can be determined by either using the COV values obtained

[2] The INTEGRATION simulation package was developed by Dr. Van Aerde and Dr. Rakha, at the Virginia Tech Transportation Institute.

directly for some cases, or by using a weighted combination of the COV values for other cases.

Traditionally, loop detector data is used to measure the traffic volumes across these freeway sections. Typically, the volumes are measured every 15 minutes, and these measurements are done for the period of an entire day (24 hours). For a given link *i*, there exists a COV for measurements taken *within* a given time *j*. This is denoted as COV_w^{ij}. Moreover, we can determine another COV *between* measurements taken over an entire day for the same link *i*. This is denoted as COV_b^i. The COV for that link *i* is taken as COV^i = maximum $\{COV_w^{ij}, COV_b^i\}$. However, we only compute the COV_w^{ij} values, based on our simulation runs, at a particular time, and the average of these values is taken to be the COV for that link.

Once the benefit factors for the links are determined, we can solve the optimization problem for determining the reader locations. Note that the weighting scheme used to compute the benefit factors could be altered to derive several alternative solutions. Based on the insights gleaned from such sensitivity analyses, the concerned management can then select a best solution for implementation.

Network Analysis

The modeling of each of the basic freeway sections, using the INTEGRATION package is described below.

1. *On-Ramp or Merge section*

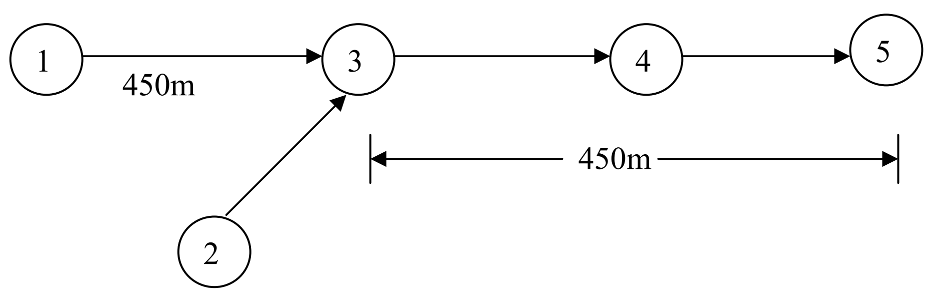

Figure 9: Model of On-ramp section

Network Description: As shown in Figure 5, the section of the freeway that lies within 450m of a merge is characterized as an on-ramp or merge section. Figure 9 depicts the representation of an on-ramp section within INTEGRATION. While performing the simulation runs, we model the on-ramp in INTEGRATION using nodes 1 and 2 as origin nodes and node 5 as the destination node. The freeway is modeled as consisting of two lanes, whereas the on-ramp consists of just one lane. That is to say, links (1, 3) and (4, 5) have two lanes each but link (2, 3) has just one lane. It is to be noted that link (3, 4) will have three lanes due to an acceleration lane being provided for traffic coming in from the on-ramp. Since acceleration lane length is an input parameter for the simulation model, the length of link (3, 4) varies based on the acceleration lane length chosen. Corresponding to the length of link (3, 4), the length of link (4, 5) is varied so as to keep their sum constant at 450m. The freeway capacity is assumed to be 2,000 veh/lane/hr and the same holds true for the on-ramp capacity as well. For the purpose of this simulation, the acceleration lane length was varied from 50m to 200m in steps of 50m to obtain four distinct on-ramp sections. The following analysis is done for each of these four independent on-ramp sections.

Traffic Demand Description: The *V/C* ratio for the freeway is varied from 0.25 to 1 in steps of 0.125 – this leads to seven different demands on the freeway. Note that the demand originates at node 1, (since, as mentioned, node 1 is an origin node, and these vehicles are denoted as vehicle class-1 by the simulation logic), and has its destination node as node 5. Similarly, the on-ramp *V*/*C* is varied from 0.1 to 0.9 in steps of 0.2 – this leads to five different demands on the on-ramp, with the origin node being node 2 and the destination node being node 5, and these vehicles being denoted as vehicle class-2. This leads to 35 different (*freeway V*/*C* ratio, *on-ramp V*/*C* ratio) pairs. Also, the simulation logic generates the traffic demand with the *inter-arrival times* of vehicles having a *negative exponential* distribution, and hence, the arrivals themselves will then be correspondingly *Poisson* arrivals. The random number seed used to generate realizations from these distributions can be varied in INTEGRATION, and to eliminate a bias factor, the random number seed was varied from 1 to 5 in steps of 1. This, when coupled with the already existing 35 pairs of demands, gave rise to a total of 175 simulation runs for the on-ramp section with a fixed acceleration lane length. The resulting data obtained on the travel times for vehicle class-1, vehicle class-2, and the *total travel time* (sum of the travel times of vehicle class-1 and vehicle class-2), over the runs corresponding to the five random number seeds are used to compute the required average and standard deviation of travel time.

For the purpose of illustration, the surface plots of the Freeway Average Travel Time and the Freeway COV for an on-ramp section having an acceleration lane length of 50m are shown in Figures 10 and 11, respectively.

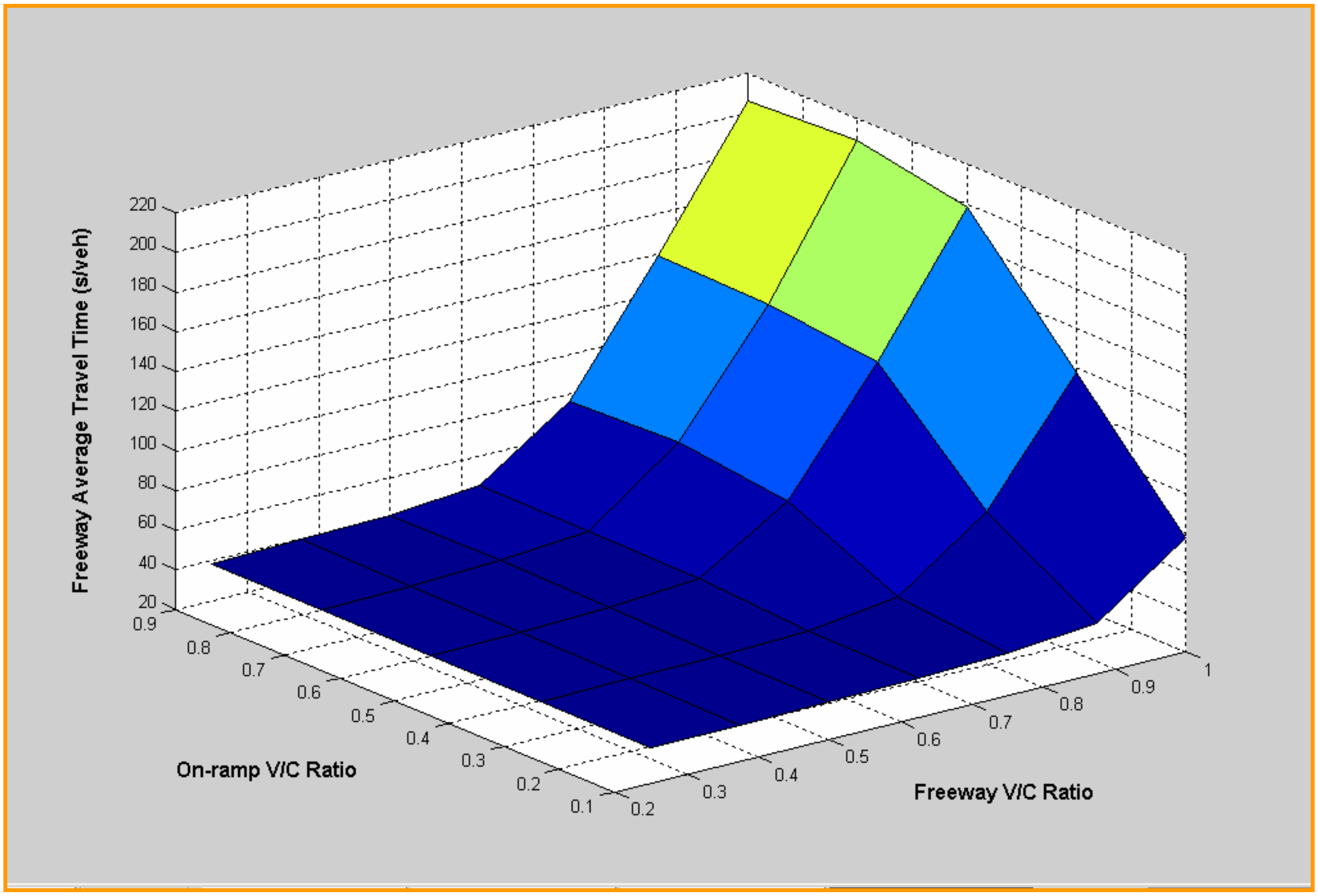

Figure 10: Freeway Average Travel Time for an On-Ramp Section with an Acceleration Lane Length of 50m

From the figure it can be seen that, as the $V/C_{freeway}$ ratio of the freeway increases keeping the V/C_{ramp} constant at 0.2, the average travel time for vehicles on the freeway remains the same until the capacity of the freeway is nearly reached, i.e., $V/C_{freeway}$ is close to unity. However, as the on-ramp volume begins to increase, the average travel time of traffic on the freeway begins to increase, as expected, due to the slowing down of vehicles at the merge, in order to accommodate the on-ramp traffic volume. It can be seen that the travel time is the highest when both the freeway as well as the on-ramp are close to their respective capacities. Hence, Figure 10 is consistent with the actual traffic characteristics observed in practice.

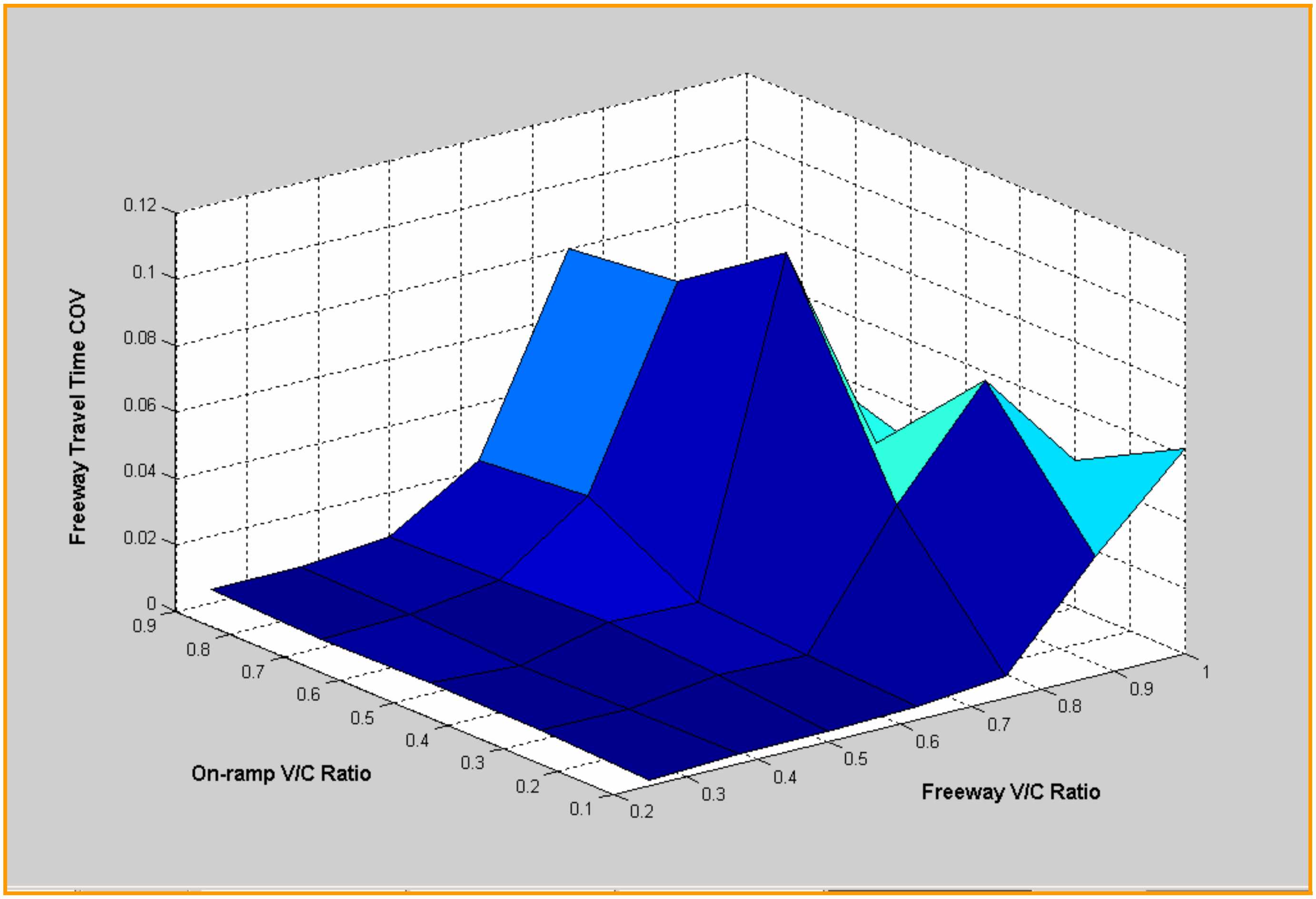

Figure 11: **Freeway Travel Time COV for an On-Ramp Section with an Acceleration Lane Length of 50m**

Next consider Figure 11. It can be seen that as the $V/C_{freeway}$ ratio of the freeway increases keeping the V/C_{ramp} constant at 0.2, the travel time COV for vehicles on the freeway remains the same until the capacity of the freeway is nearly reached i.e., $V/C_{freeway}$ is close to unity. However, as the on-ramp volume begins to increase, not only does the average travel time of traffic on the freeway increase, but so does variability, to the extent that this leads to a corresponding increase in the COV. This can be explained by the fact that a certain volume of traffic on the freeway faces an increase in travel time due to the merge, while the remaining traffic does not. Hence, this difference in the travel time characteristics leads to a higher COV value. It can be seen that the travel time COV is the greatest when the on-ramp is close to its capacity and the freeway volume is close to a $V/C_{freeway}$ ratio of 0.5. When the freeway is close to capacity, the on-ramp volume no longer plays a significant role due to congestion already being existent. Hence, although the average travel time is at its peak, the COV for freeway traffic is relatively low since the travel time is nearly the same for all vehicles. This is also in accordance with actual traffic characteristics observed in practice.

From the above discussion, it can be claimed that the figures shown are generic and can be applied to any given freeway merge section. The remaining results for the on-ramp section are tabulated and are shown in Appendix A.1.

2. *Off-Ramp or Diverge Section*

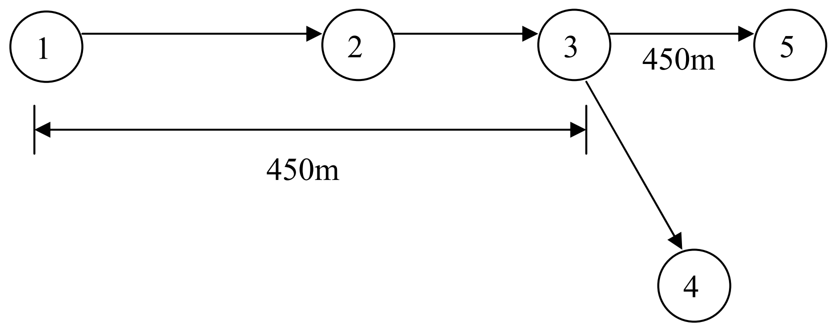

Figure 12: Model of an Off-Ramp Section

Network Description: As shown in Figure 6, the section of the freeway that lies within 450m of an exit is characterized as an off-ramp or diverge section. Figure 10 depicts the representation of an off-ramp section within INTEGRATION. While performing the simulation runs, we model the off-ramp in INTEGRATION using node 1 as the origin node and nodes 4 and 5 as the destination nodes. The freeway is modeled as consisting of two lanes whereas the off-ramp consists of just one lane. That is to say, links (1, 2) and (3, 5) have two lanes each but link (3, 4) has just one lane. It is to be noted that link (2, 3) will have three lanes due to a deceleration lane being provided for traffic exiting onto the off-ramp. Since the deceleration lane length is an input parameter for the simulation model, the length of link (2, 3) varies based on the deceleration lane length chosen. Corresponding to the length of link (2, 3), the length of link (1, 2) is varied so as to keep the sum of their lengths constant at 450m. The freeway capacity is assumed to be 2,000 veh/lane/hr and the same holds true for the off-ramp capacity as well. For the purpose of this simulation, the deceleration lane length was varied from 50m to

200m in steps of 50m to obtain four distinct off-ramp sections. The following analysis is done for each of these four independent off-ramp sections.

Traffic Demand Description: The *V/C* ratio for the freeway is varied from 0.25 to 1 in steps of 0.125 – this leads to seven different demands on the freeway. Note that this demand originates at node 1, (since, as mentioned, node 1 is an origin node, and these vehicles are denoted as vehicle class-1 by the simulation logic), and has its destination node as node 5. Similarly, the off-ramp *V/C* is varied from 0.1 to 0.9 in steps of 0.2 – this leads to five different demands on the off-ramp, with the origin node as node 1 and the destination node as node 4, and these vehicles being denoted as vehicle class-2. This leads to 35 different pairs of (*freeway V/C* ratio, *off-ramp V/C* ratio) pairs. Also, the simulation logic generates the traffic demand with the *inter-arrival times* of vehicles having a *negative exponential* distribution, and hence, the arrivals themselves will then be correspondingly *Poisson* arrivals. The random number seed used to generate realizations from these distributions can be varied in INTEGRATION, and to eliminate a bias factor, the random number seed was varied from 1 to 5 in steps of 1. This, when coupled with the already existing 35 pairs of demands, gave rise to a total of 175 simulation runs for the off-ramp section with a fixed acceleration lane length. The data obtained on the travel times of vehicle class-1, vehicle class-2, and the *total travel time* (sum of the travel times of vehicle class-1 and vehicle class-2), over the runs corresponding to the five random number seeds are used to compute the required average and standard deviation of travel time information.

For the purpose of illustration, the surface plots of the Freeway Average Travel Time and the Freeway COV for an off-ramp section having a deceleration lane length of 50m are shown in Figures 13 and 14, respectively.

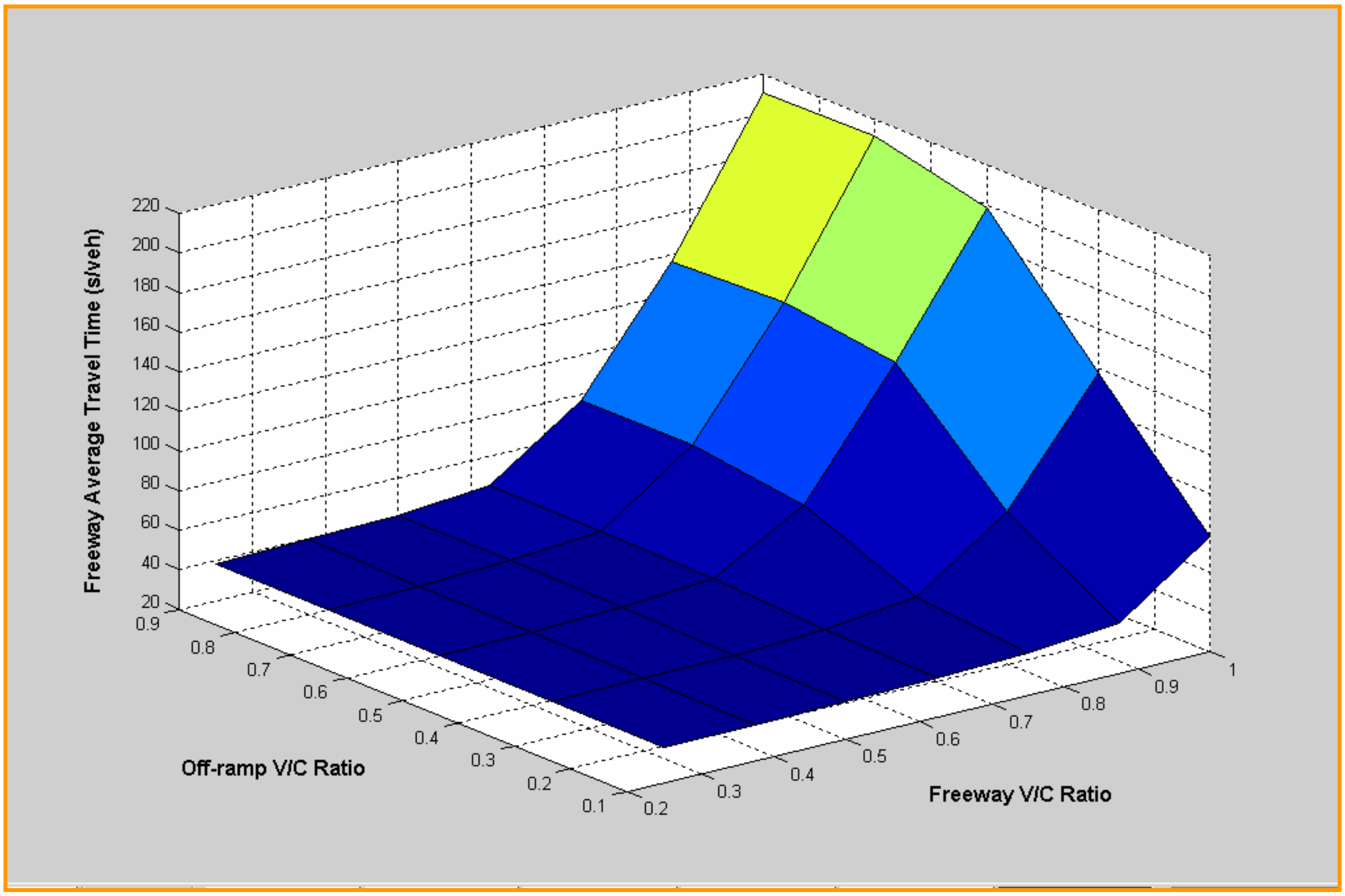

Figure 13: Freeway Average Travel Time for Off-Ramp Section with Deceleration Lane Length of 50m

From Figure 13, it can be seen that as the $V/C_{freeway}$ ratio of the freeway increases keeping the $V/C_{off\text{-}ramp}$ constant at 0.2, the average travel time for vehicles on the freeway remains the same until the capacity of the freeway is nearly reached, i.e., $V/C_{freeway}$ is close to unity. However, as the off-ramp volume begins to increase, the average travel time of traffic on the freeway begins to increase, as expected, due to the slowing down of vehicles at the diverge, in response to the slowing down of the off-ramp traffic. It can be seen that the average travel time is the highest when both the freeway as well as the off-ramp are close to their respective capacities. Hence, Figure 13 is consistent with the actual traffic characteristics observed in practice.

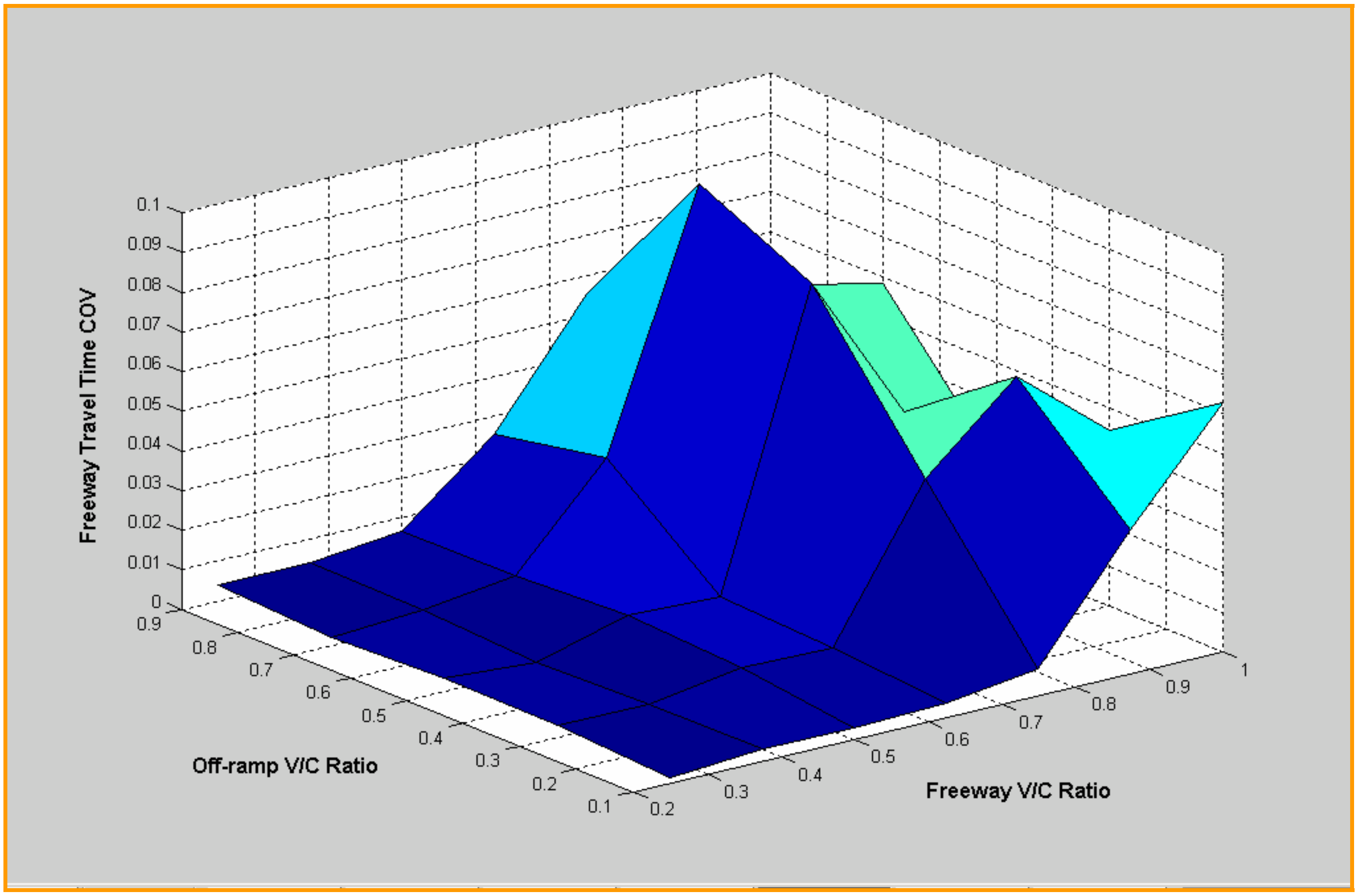

Figure 14: Freeway Travel Time COV for an Off-Ramp Section with a Deceleration Lane Length of 50m

Considering Figure 14, it can be seen that as the $V/C_{freeway}$ ratio of the freeway increases keeping the $V/C_{off\text{-}ramp}$ constant at 0.2, the travel time COV for vehicles on the freeway remains the same until the capacity of the freeway is nearly reached, i.e., $V/C_{freeway}$ is close to unity. However, as the off-ramp volume begins to increase, not only does the average travel time of traffic on the freeway increase, but so does the variability, to the extent that this leads to a corresponding increase in the COV. This can be explained by the fact that a certain volume of traffic on the freeway faces an increase in travel time due to the diverge, while the remaining traffic does not. Hence, this difference in the travel time characteristics leads to a higher COV value. It can be seen that the travel time COV is the greatest when the off-ramp is close to its capacity and the freeway volume is close to a $V/C_{freeway}$ of 0.5. When the freeway is close to capacity, the off-ramp volume no longer plays a significant role due to congestion already being existent. Hence, although average travel time is at its peak, COV for freeway traffic is relatively low since the travel time is nearly the same for all vehicles. This is also in accordance with the actual traffic characteristics observed.

From the above discussion, it can be claimed that the figures shown are generic and can be applied to any given freeway diverge section. The remaining results for the off-ramp section are tabulated and are shown in Appendix A.2.

3. *Weaving Section*

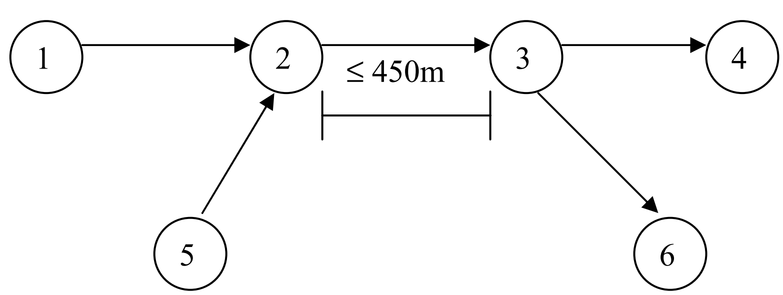

Figure 15: Model of a Weaving Section

Network Description: As shown in Figure 7, when a merge and diverge section of the freeway lie within 450m of one another, that section is characterized as a weaving section. In essence, a weaving section can be viewed as a combination of an on-ramp and an off-ramp section. Figure 11 depicts the representation of a weaving section within INTEGRATION. While performing the simulation runs, we model the weave in INTEGRATION using nodes 1 and 5 as origin nodes and nodes 4 and 6 as the destination nodes. The freeway is modeled as consisting of two lanes whereas the on-ramp and off-ramp each consist of just one lane. That is to say, links (1, 2) and (3, 4) have two lanes each but links (5, 2) and (3, 6) have just one lane each. It is to be noted that link (2, 3) will have three lanes due to an auxiliary weaving lane being provided for traffic exiting onto the off-ramp. Since the weave lane length is an input parameter for the simulation model, the length of link (2, 3) varies based on the weave lane length chosen. Given the length of link (2, 3), there exists a corresponding weave lane length. The freeway capacity is assumed to be 2,000 veh/lane/hr, and the same holds true for the on-ramp and off-ramp capacities as well. For the purpose of this simulation, the weave lane length was varied from 100m to 400m in steps of 100m to obtain four distinct weaving sections. The following analysis is done for each of these four independent weaving sections.

Traffic Demand Description: The traffic demand description for the weaving section is a combination of the traffic demands for the on-ramp and the off-ramp sections. Also, instead of the two different types of vehicle classes as in the above cases, there exist four vehicle classes for the weaving section. This is because traffic originating at nodes 1 and 5 can end up at either of the nodes 4 or 6, and this leads to traffic demand being categorized as four distinct vehicle classes. Just as in the above two cases, the demands are specified for each of the traffic flows in terms of their *V/C* ratio. For the sake of simplicity, it is assumed that there is no traffic originating at node 5 that has its destination as node 6. Hence, this eliminates one particular vehicle class. The *V/C* ratio for the weaving section is varied from 0.2 to 1 in steps of 0.2 – this leads to five different demands on the weave. Note that this is the total demand that originates at nodes 1 and 5 and has its destination at nodes 4 or 6, and at node 4 respectively.

Now, given the total demand, the proportion of the total demand that belongs to either vehicle class-1, or vehicle class-2, or vehicle class-3, can be specified. Obviously, the sum total of all the three vehicle class demands must equal the total demand of the weave section. Hence, if the proportion of total demand for two of these vehicle classes is given, then the third vehicle class proportion is also determined. Just as in the on-ramp and off-ramp demand cases, the proportion of vehicles originating from node 1 and ending in node 6 is varied from 0.1 to 0.5 in steps of 0.1 – this leads to five different demands of vehicle class-1. Similarly, with the origin node as node 5 and the destination node as node 4, the proportion of vehicle class-2 is varied from 0.1 to 0.5 in steps of 0.1 - this leads to five different demands for vehicle class-2. This results in a total of 625 different pairs of demands for each weave length. Also, the simulation logic generates the traffic demand with the *inter-arrival times* of vehicles having a *negative exponential* distribution, and hence, the arrivals themselves will then be correspondingly *Poisson* arrivals. The random number seed used to generate realizations from these distributions can be varied in INTEGRATION, and to eliminate a bias factor, the random number seed was varied from 1 to 5 in steps of 1. This, when coupled with the already existing 625 pairs of demands, gave rise to a total of 3125 simulation runs for the weaving section with a fixed weave lane length. The resulting data obtained on the travel times for vehicle class-1, vehicle class-2, and the *total travel time* (sum of the travel times of vehicle class-1 and vehicle class-2), over the runs corresponding to the five random number seeds are used to compute the required average and standard deviation of travel time.

For the purpose of illustration, the surface plots of the Total Average Travel Time and the Total Travel Time COV for a weave section with a weave lane length of 100m are shown in Figures 16 and 17, respectively.

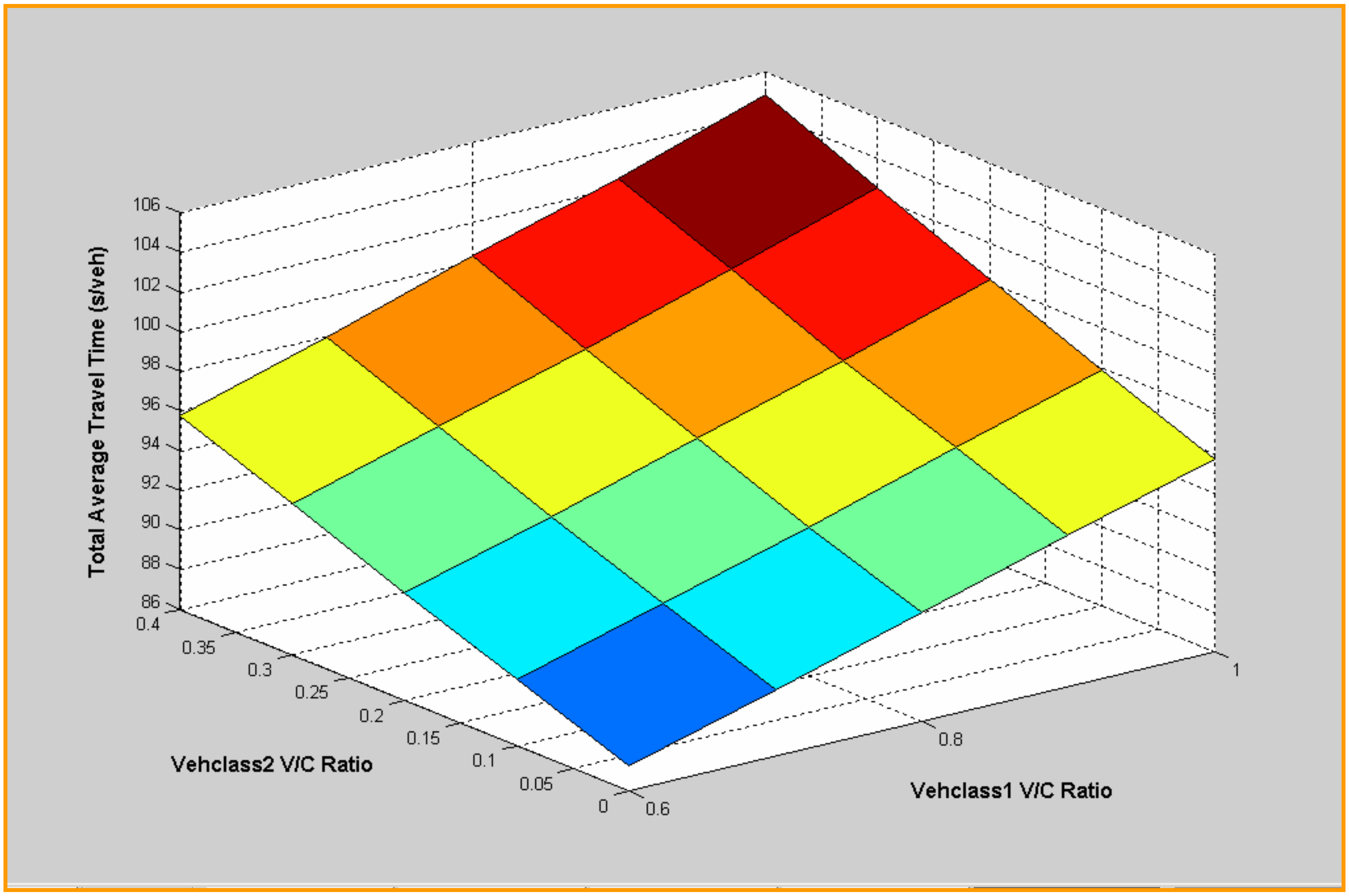

Figure 16: Total Average Travel Time for a Weaving Section having a Weave Length of 100m and $V/C_{freeway}$ = 0.2

From the figure it can be seen that as the $V/C_{veh\text{-}class1}$ increases keeping the $V/C_{veh\text{-}class2}$ constant at 0, the average travel time for vehicles increases gradually until the capacity of the freeway is nearly reached, i.e., $V/C_{veh\text{-}class1}$ is close to unity. Since traffic originating at node 1 can have destinations at either nodes 4 or 6, the $V/C_{veh\text{-}class1}$ incorporates vehicles diverging onto the off-ramp and hence the corresponding gradual increase in travel time due to the slowing down of these vehicles. However, as the volume originating at the on-ramp begins to increase, the average travel time of traffic on the freeway begins to increase, as expected, due to the slowing down of vehicles at the merge, in order to accommodate the on-ramp traffic volume. It can be seen that the travel time is the highest when both the freeway volumes as well as the on-ramp are close to their respective capacities. Hence, Figure 16 is consistent with the traffic characteristics observed in practice. Note that Figure 16 is symmetric about the plane $V/C_{freeway} = V/C_{on\text{-}ramp}$ and because of the symmetric distribution of vehicles emerging from the on-ramp and diverting from the off-ramp.

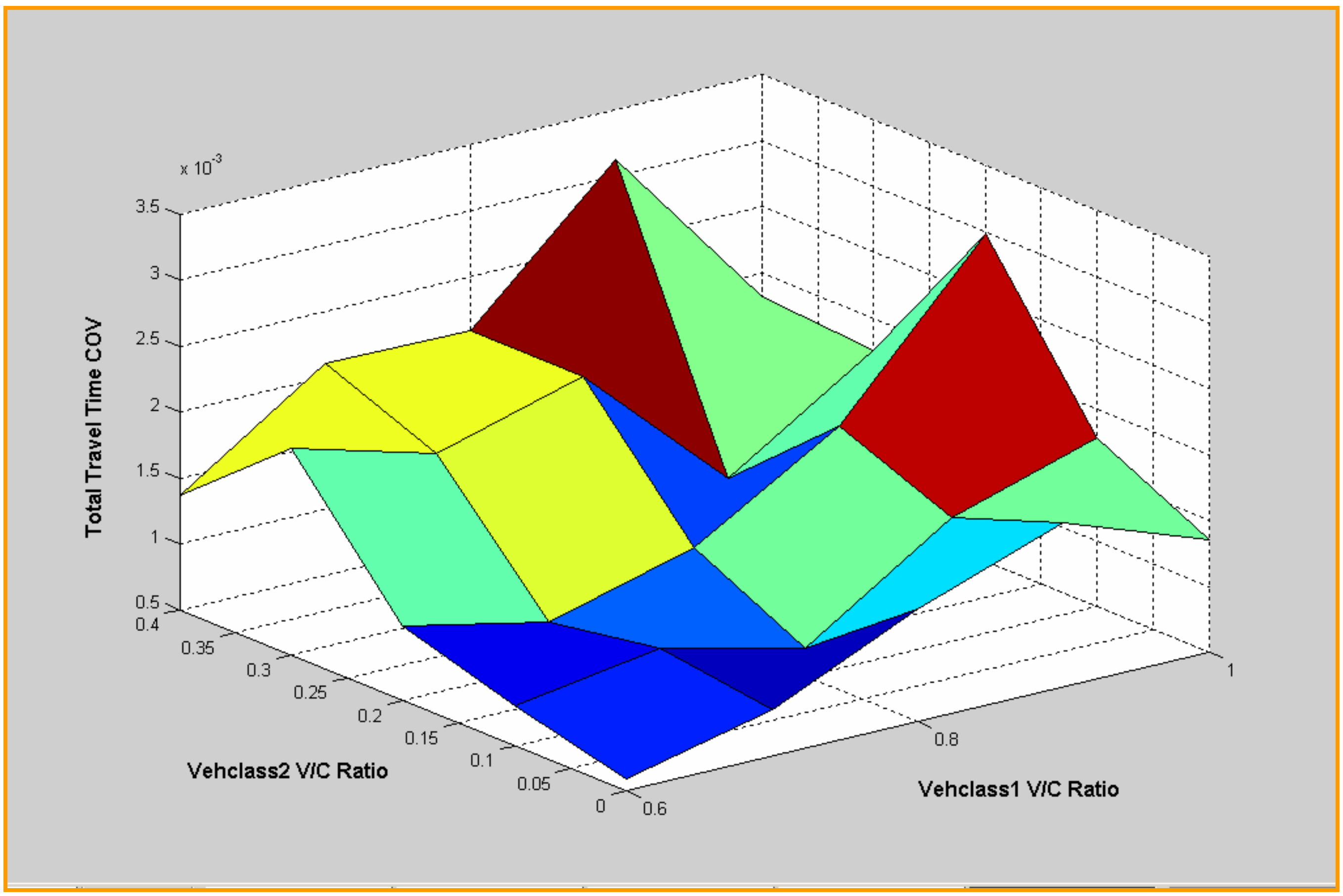

Figure 17: Total Travel Time COV for a Weaving Section having a Weave Lane Length of 100m and $V/C_{freeway} = 0.2$

Next consider Figure 17. It can be seen that, as the $V/C_{veh\text{-}class1}$ ratio of the freeway increases keeping the $V/C_{veh\text{-}class2}$ constant at 0, the travel time COV for vehicles on the freeway increases gradually till the capacity of the freeway is nearly reached, i.e., $V/C_{veh\text{-}class1}$ is close to unity. However, as the on-ramp volume begins to increase, not only does the average travel time of traffic on the freeway increase, but so does the variability, to the extent that this leads to a corresponding increase in the COV. It can be seen that the travel time COV is the highest when either the on-ramp is close to its capacity and the freeway volume is close to a $V/C_{freeway}$ of 0.5, or the on-ramp capacity is close to 0.25 and $V/C_{freeway}$ is close to unity. These two situations lead to the two peaks in the figure. The COV surface plot displays a similar symmetry as for the Total Average Travel Time plot by the same logic. This is in accordance with the actual traffic characteristics observed in practice.

From the above discussion, it can be claimed that the figures shown are generic and can be applied to any given weaving section. The remaining results for the weaving section are tabulated and are shown in Appendix A.3.

4. Bottleneck Section

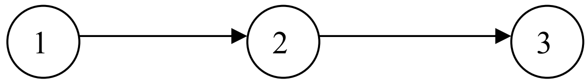

Figure 18: Model of Bottleneck Section

Network Description: As shown in Figure 8, the section of the freeway in which the number of lanes is suddenly reduced due to various causes such as accidents, road construction, repair work, or because of geometric conditions, is called a bottleneck section. Figure 18 depicts the representation of a bottleneck section within INTEGRATION. While performing the simulation runs, we model the bottleneck section in INTEGRATION using node 1 as the origin node and node 3 as the destination node. The freeway is modeled as consisting of two lanes whereas the bottleneck section consists of just one lane. That is to say, link (1, 2) has two lanes but link (2, 3) has just one lane. The freeway capacity is assumed to be 2,000 veh/lane/hr and the same holds true for the bottleneck capacity as well.

Traffic Demand Description: The $V/C_{bottleneck}$ ratio for the bottleneck is varied from 0.1 to 2 in steps of 0.1 – this leads to twenty different demands on the freeway. Note that this demand originates at node 1 and has its destination node as node 3. Moreover, the traffic demand on the freeway is measured in terms of the capacity of the bottleneck ($C_{bottleneck}$) instead of the capacity of the freeway, since the critical section is the bottleneck. The simulation logic generates the traffic demand with the *inter-arrival times* of vehicles having a *negative* *exponential* distribution and hence the arrivals themselves will then be correspondingly *Poisson* arrivals. The random number seed used to generate realizations from these distributions can be varied in INTEGRATION, and to eliminate a bias factor, the random number seed was varied from 1 to 5 in steps of 1. This, when coupled with the traffic demand, results in a total of 100 simulation runs for the bottleneck section. The resulting data obtained on the travel times over the runs corresponding to the five random number seeds are used to compute the required average and standard deviation of travel time.

For the purpose of illustration, the surface plots of the Average Travel Time and Travel Time COV for a bottleneck section are shown in Figures 19 and 20, respectively. It can be clearly seen that as the V/C ratio increases, the average travel time for vehicles increases and the same holds true for the travel time COV as well.

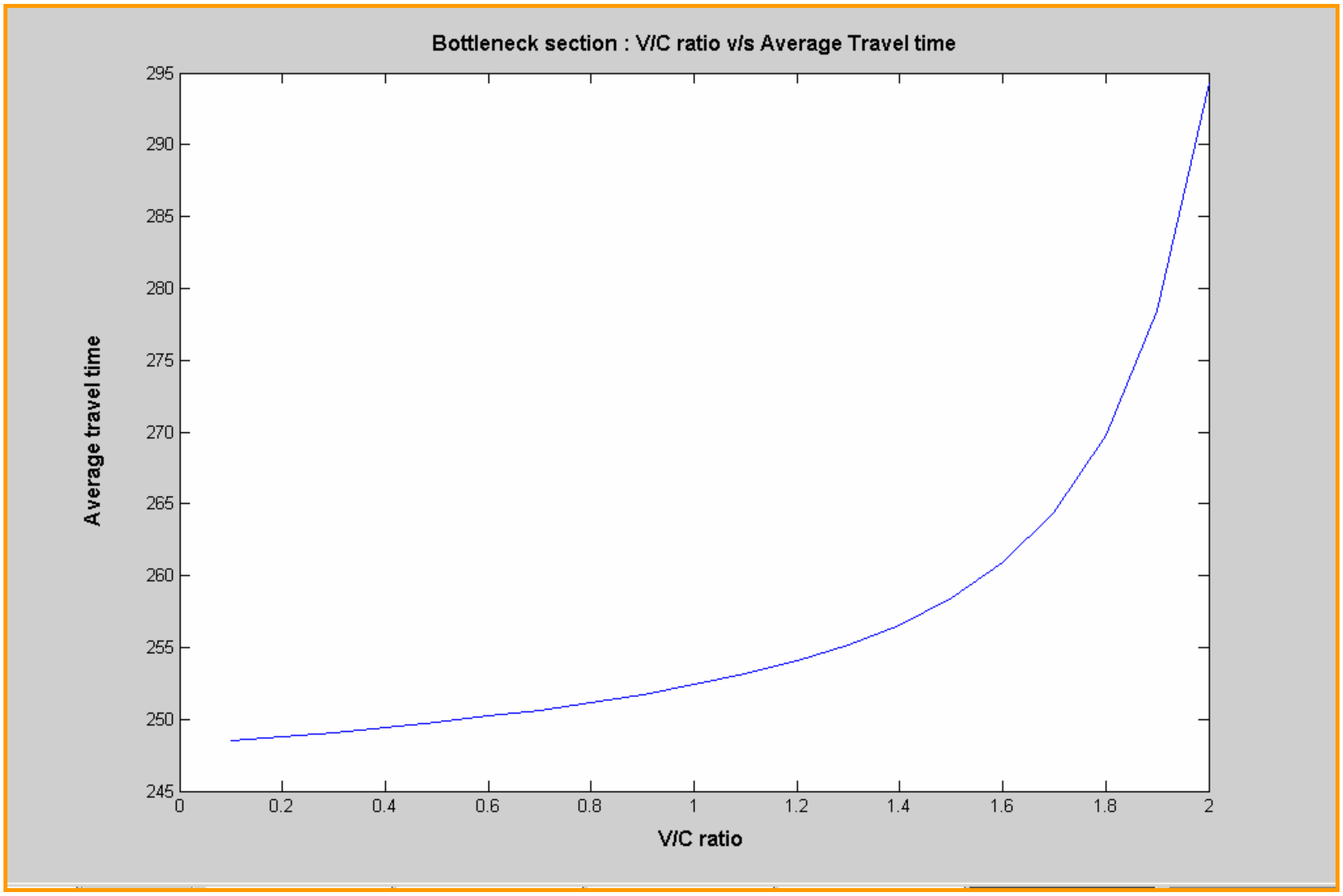

Figure 19: Average Travel Time for Bottleneck Section

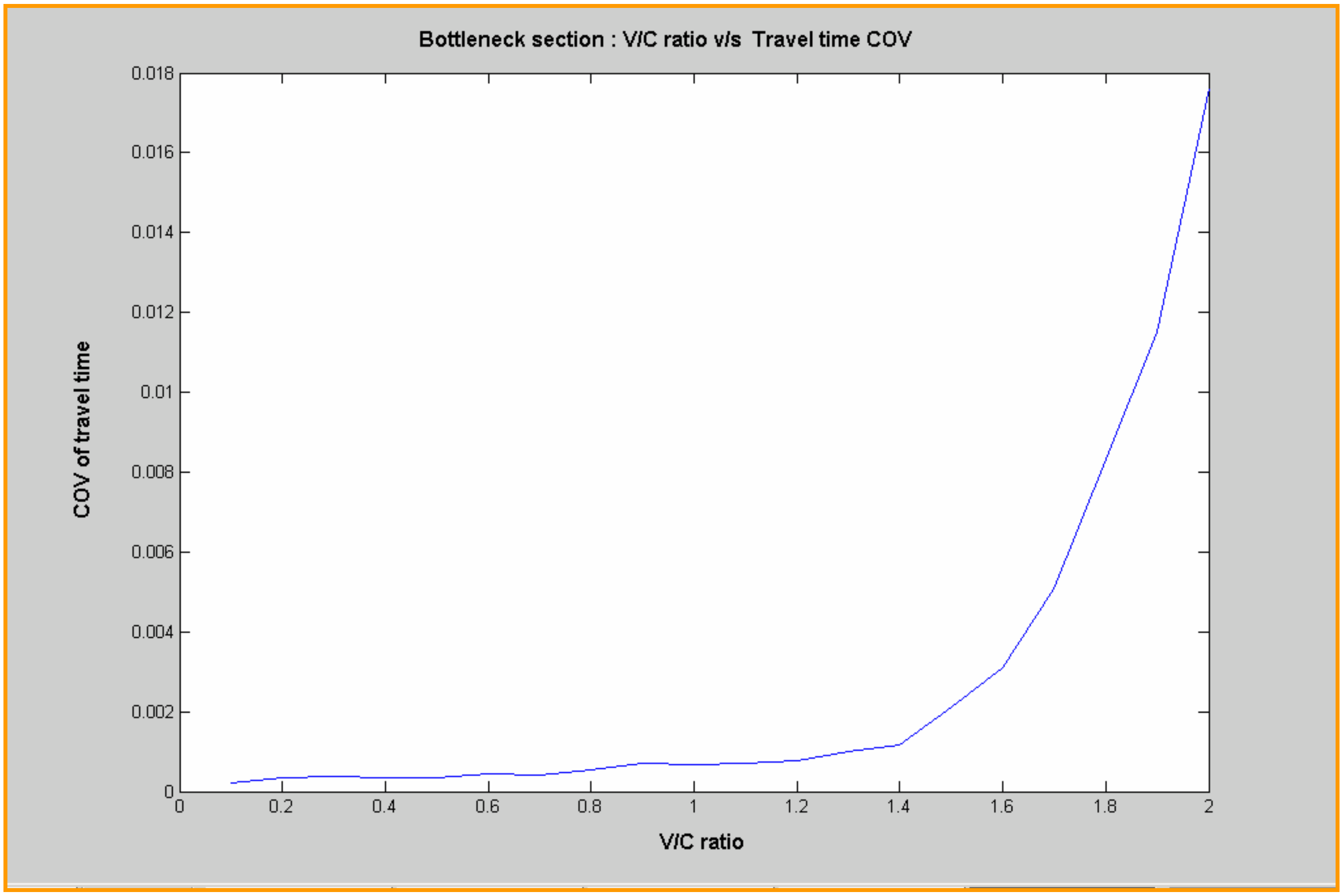

Figure 20: Travel Time COV for Bottleneck Section

Once the required travel time variability is obtained from the simulation runs, this information is used to derive the benefit factors for an entire freeway section that comprises several such basic freeway sections. We next describe the derivation of a composite benefit factor function.

3.3 Derivation of a composite benefit factor function

As described above, the section of roadway between an O-D pair, say (*p, q*), can be composed of several links. For the sake of illustration, assume that there are two links along the route joining *p* and *q* and we know the corresponding coefficients of variation for each of these links.

We now derive an expression for the composite COV for the entire route between *p* and *q*. Let μ_1 and μ_2 be the expected values of the travel times and σ_1 and σ_2 be the standard deviations for each of the two links, respectively (all assumed to be non-zero). Then, define:

$$COV_i = \sigma_i / \mu_i \ , \qquad i = 1, 2$$

$$COV_{max} = \max \{ COV_1 , COV_2 \} \text{ and}$$

$$COV_{min} = \min \{ COV_1 , COV_2 \};$$

Therefore,

$$\mu_i^2 = \sigma_i^2 / COV_i^2 \quad i = 1, 2$$

$$\Rightarrow \quad \mu_1^2 + \mu_2^2 = (\sigma_1^2 / COV_1^2) + (\sigma_2^2 / COV_2^2)$$

$$\Rightarrow \quad \mu_1^2 + \mu_2^2 \geq (\sigma_1^2 / COV_{max}^2) + (\sigma_2^2 / COV_{max}^2)$$

$$\Rightarrow \quad COV_{max} \geq [(\sigma_1^2 + \sigma_2^2) / (\mu_1^2 + \mu_2^2)]^{1/2} .$$

Similarly,

$$COV_{min} \leq [(\sigma_1^2 + \sigma_2^2) / (\mu_1^2 + \mu_2^2)]^{1/2} .$$

Hence, motivated by these bounding inequalities, one composite COV function that we propose is as follows:

$$b_{p-q}^{(1)} \quad = \quad [\,(\sigma_1^{\,2} + \sigma_2^{\,2}\,) \,/\, (\,\mu_1^{\,2} + \,\mu_2^{\,2}\,)\,]^{1/2} \,. \tag{3a}$$

This is equivalent to saying that

$$b_{p-q}^{(1)} \quad = \quad [\,\lambda_1(\mathrm{COV}_1)^{\,2} + \,\lambda_2\,(\mathrm{COV}_2)^{\,2}\,]^{\,½}, \tag{3b}$$

where, $\lambda_i \;=\; \mu_i^{\,2} \,/\, (\,\mu_1^{\,2} \,+\, \mu_2^{\,2}\,)$ for $i = 1, 2$ serve as convex combination weights.

In general, for an O-D pair (*p, q*) comprising *m* such links, this composite O-D function can be written as:

$$b_{p-q}^{(1)} \;=\; \Big\{\sum_{i=1}^{m} \sigma_i^{\,2} \,/\, \sum_{i=1}^{m} \mu_i^2 \,\Big\}^{1/2} \;=\; \sum_{i=1}^{m} [\lambda_i (COV_i)^2]^{1/2} \tag{4a}$$

$$\text{where, } \lambda_i \;=\; \mu_i^{\,2} \,/\, \sum_{k=1}^{m} \mu_k^2 \qquad \forall\; i \;= 1, \ldots, m. \tag{4b}$$

As another alternative, suppose that we assume the independence of travel time behavior on the individual links (i.e., the travel time along a particular link is independent of the travel time on previous links). This assumption might not be completely valid since travel times on adjacent links might typically bear some correlation in practice. Nevertheless, for the sake of deriving benefit factor coefficients that tend to reflect variability in travel times, we might adopt this simplifying assumption.

This leads to an alternative composite COV function as:

$$b_{p-q}^{(2)} = \left\{ \sum_{i=1}^{m} \sigma_i^{\,2} \right\}^{1/2} / \sum_{i=1}^{m} \mu_i \ . \qquad (5)$$

Observe from (4a) and (5) that, in general,

$$b_{p-q}^{(1)} \geq b_{p-q}^{(2)} \ ,$$

i.e., alternative (5) is more conservative than (4a). In our computations, we will examine both these composite functions and study the sensitivity of the prescribed reader locations to the choice of using b_{p-q} as given by either of these two functions.

3.4 Reformulation of the Reader Location Problem

The Reader Location Problem (**RL**) (as defined in Section 1.4) belongs to a class of problems known as linearly constrained mixed-integer zero-one quadratic programming problems. Problems of this type arise in the context of various economics and facility location problems under interactions between components (see Adams and Sherali (1986), for example), and several methodologies have been developed for tackling such problems. Here, we adopt the *Reformulation-Linearization Technique* (RLT) of Sherali and Adams (1990, 1994, 2000), to solve Problem **RL**, which has been demonstrated in the aforementioned references to be a competitive procedure for this class of problems.

The RLT method transforms a given mixed-integer quadratic programming problem into an equivalent zero-one mixed-integer linear programming problem. The emphasis on deriving this linearization is to achieve as tight a linear programming relaxation as possible in order to enhance problem solvability. For this purpose, consider the problem **RL** and its conversion to **RL'** by applying the RLT methodology as prescribed below. (We assume here that neither (2b) nor (2c) is implied by the other. Whenever this assumption does not hold, then the redundant constraint would be deleted from the analysis below.)

Recall that Problem **RL** is defined as follows.

$$\textbf{RL:} \quad \text{Maximize} \quad \sum_{(i,\ j)\in A} \sum b_{ij}\, y_i\, y_j \tag{2a}$$

$$\text{subject to:} \quad \sum_{j\in N} y_j \leq R \tag{2b}$$

$$\sum_{j\in N} C_j\, y_j \leq B \tag{2c}$$

$$y \text{ binary.} \tag{2d}$$

We prescribe the following set of steps to derive the equivalent linearized problem.

Step 1: Multiply the constraint (2b) by y_i, and (1- y_i), $\forall\, i\in N$.

$$\sum_{j\in N} y_j\, y_i \leq R\, y_i \qquad \forall\, i\in N, \tag{3a}$$

$$\sum_{j\in N} y_j\,(1-y_i) \leq R(1-y_i) \quad \forall\, i\in N. \tag{3b}$$

Remark: Note that when (2b) is imposed as an equality constraint (as for example in the absence of (2c)), then we simply need ti multiply this equality with each variable y_i, $\forall\, i\in N$, at this step, and also retain this original constraint in the reformulated problem (see Sherali and Adams (1990) for such treatment of equality constraints).

Step2: Multiply the constraint (2c) by y_i and (1- y_i) , $\forall\, i\in N$.

$$\sum_{j\in N} C_j\, y_j\, y_i \leq B\, y_i \qquad \forall\, i\in N, \tag{3c}$$

$$\sum_{j\in N} C_j\, y_j\,(1-y_i) \leq B\,(1-y_i) \qquad \forall\, i\in N. \tag{3d}$$

Step 3: Multiply the $0 \le y_j \le 1$ constraints by y_i and $(1 - y_i)$ $\forall\, i \in N,\ i < j,$ for each $j \in N$:

$$0 \le y_i y_j \le y_i \text{ , and} \tag{3d}$$

$$0 \le y_j - y_i y_j \le 1 - y_i, \quad \forall\, i < j\ ,\ \{i\ ,\ j\} \in N. \tag{3e}$$

Step 4: In the system (3), set $y_j^{\ 2} = y_j \quad \forall \quad j \in N$ (since y is supposed to be binary valued), and substitute $w_{ij} = y_i y_j \quad \forall\, \{i,\ j\} \in\ N,\ i <\ j.$ This yields the following reformulated linear mixed-integer program, which we call **RL'**.

$$\textbf{RL'}: \qquad \text{Maximize} \qquad \sum_{\substack{(i,j)\in A \\ i<j}}\sum b_{ij} w_{ij} + \sum_{\substack{(i,j)\in A \\ i>j}}\sum b_{ij} w_{ji} \tag{4a}$$

$$\text{subject to:} \qquad \sum_{j\in N} y_j - R \le \sum_{\substack{j\in N \\ j>i}} w_{ij} + \sum_{\substack{j\in N \\ j<i}} w_{ji} + y_i(1 - R) \le 0 \quad \forall\, i \in N \tag{4b}$$

$$\sum_{j\in N} C_j y_j - B \le \sum_{\substack{j\in N \\ j>i}} C_j w_{ij} + \sum_{\substack{j\in N \\ j<i}} C_j w_{ji} + y_i(C_i - B) \le 0 \quad \forall\, i \in N \tag{4c}$$

$$y_j - w_{ij} \ge 0 \qquad \forall\, \{i,\ j\} \in N,\ \ i < j \tag{4d}$$

$$y_i - w_{ij} \ge 0 \qquad \forall\, \{i,\ j\} \in N,\ \ i < j \tag{4e}$$

$$- y_i - y_j + w_{ij} \ge -1 \qquad \forall\, \{i,\ j\} \in N,\ \ i < j \tag{4f}$$

$$w_{ij} \ge 0\ ,\ \ \forall\, \{i,\ j\} \in N,\ \ i < j \tag{4g}$$

$$y_j \ \text{ binary, } \forall\ j \in N. \tag{4h}$$

Observe that if we denote $|N| = n$, then Problem **RL'**, has $n(n-1)/2$ continuous variables, n binary variables, and $n(3n+5)/2$ structural constraints in addition to the logical restrictions (4g) and (4h). In **RL'**, we have replaced the products of the original variables, $y_i y_j$, by a new variable, w_{ij}, and this operation has derived a higher dimensional equivalent linear mixed-integer programming representation of **RL**. Note that for any binary y, constraints (4d) - (4g), enforce that $w_{ij} = y_i y_j \quad \forall \{i, j\} \in N, \ i < j$, and hence the equivalence. However, from a continuous relaxation point of view, incorporating certain additional valid inequalities that further tighten this relaxation and help derive stronger (smaller) upper bounds is an avenue worth exploring. With this motivation, we propose the generation of a class of valid inequalities or cutting planes derived using *semidefinite programming* concepts. These cuts are referred to as *semidefinite cuts* (Sherali and Fraticelli, 2002). Furthermore, following the reduced RLT representation guidelines recommended by Sherali, Smith and Adams (2000), we will also explore the use of an alternative representation **RL"**, in which the left-hand inequalities in (4b) and (4c), as well as the inequalities in (4f), are deleted. In this case, the original constraints (2b) and (2c) are no longer implied, and are therefore added back into the model. The motivation for this is that the objective function is trying to drive the w_{ij} variables to as high values as possible, and hence the constraints that restrict this tendency are most relevant. This would reduce the number of structural constraints in **RL"** to $n(n+1)$. Likewise, we will also experiment with using the most basic linearized form of the proposed model, referred to as $\mathbf{RL_{base}}$, comprised of (4a), (2b), (2c) and (4d), (4e), (4g) and (4h). (Note that the constraints (4f) can be deleted in this case because of the nature of the objective function, while ensuring that $w_{ij} = y_i y_j$ holds true $\forall$ $i < j$, for any binary solution y.)

To present the semidefinite cuts, consider the $n \times n$ symmetric matrix $[y y^T]_L$, where $[\cdot]_L$ denotes linearization of $[\cdot]$ under the substitution defined in *Step 4*. Note that,

$$[y y^T] = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ \\ \\ \vdots \\ y_n \end{bmatrix} \begin{bmatrix} y_1 & y_2 & \cdots & \cdots & y_n \end{bmatrix}$$

$$= \begin{bmatrix} {y_1}^2 & y_1 y_2 & y_1 y_3 & \cdots & \cdots & \cdots & y_1 y_n \\ y_1 y_2 & {y_2}^2 & \cdots & \cdots & \cdots & \cdots & y_2 y_n \\ \vdots & & \ddots & & & & \\ \vdots & & & \ddots & & & \vdots \\ \vdots & & & & & & \\ \vdots & & & & & & \vdots \\ y_1 y_n & \cdots & \cdots & \cdots & \cdots & \cdots & {y_n}^2 \end{bmatrix}_{n \times n} .$$

Hence,

$$[y y^T]_L = \begin{bmatrix} y_1 & w_{12} & w_{13} & \cdots & \cdots & \cdots & \cdots & w_{1n} \\ w_{12} & y_2 & \cdots & \cdots & \cdots & \cdots & & w_{2n} \\ \vdots & & \ddots & & & & & \\ \vdots & & & \ddots & & & & \vdots \\ \vdots & & & & & & & \\ \vdots & & & & & & & \vdots \\ w_{1n} & \cdots & \cdots & \cdots & \cdots & \cdots & & y_n \end{bmatrix}_{n \times n}$$

Observe that $[y\,y^T]$ is positive semidefinite (PSD) since $\alpha^T [y\,y^T]\,\alpha = [(\alpha^T y)^2] \geq 0$ $\forall$ $\alpha \in R^n$. However, if $(\bar{y}, \bar{w})$ is an optimal solution to the continuous relaxation of **RL'**, (denoted $\overline{RL}$, say), and if $\bar{y}$ has fractional components so that $\bar{w}_{ij}$ does not necessarily equal $\bar{y}_i \bar{y}_j$, $\forall\, i < j$, the matrix $[y\,y^T]_L$ need not be PSD for $(y, w) = (\bar{y}, \bar{w})$. But since feasibility to **RL'** requires $[y\,y^T]_L$ to equal $[y\,y^T]$, and therefore be PSD, we can validly impose the constraints

$$\alpha^T [y\,y^T]_L \alpha \;\geq\; 0 \quad \forall\; \alpha \in R^n,\ \|\alpha\| = 1. \tag{5}$$

Observe that,

$\alpha^T\ [y\,y^T]_L\ \alpha \;=\; [\alpha^T\ y\,y^T \alpha\ ]_L \;=\; [(\alpha^T y)^2\ ]_L$, and so, the class of semidefinite inequalities (5) is equivalent to the class of RLT inequalities:

$$[(\alpha^T y)^2]_L \;\geq\; 0 \qquad \forall\; \alpha \in R^n,\ \|\alpha\| = 1. \tag{6}$$

Given a fractional solution $(\bar{y}, \bar{w})$ to **RL'**, we will now adopt the polynomial-time scheme developed by Sherali and Fraticelli (2002) to generate violated members of (5) (or equivalently (6)), in order to further tighten the continuous relaxation of **RL'**. This scheme is described next.

3.5 Semidefinite Cut Generation Scheme:

Instead of simply imposing nonnegativity restrictions on the variables (y, w) in **RL'**, we will additionally impose positive semidefiniteness on $[y\,y^T]_L$, as embodied by constraint (6). However, this yields a *semi-infinite linear programming* representation, and instead of trying to solve this directly, we will employ a relaxation approach as prescribed below, which in essence, results in a recursive cutting plane strategy that leads to optimality (for further details, refer Sherali and Fraticelli, 2002). We first solve problem **RL'** without any of the constraints (6) being incorporated. Let the solution obtained be denoted $(\bar{y}, \bar{w})$ and let $\overline{W}$ represent the matrix $[y y^T]_{\mathrm{L}}$ evaluated at $(\bar{y}, \bar{w})$. If $\overline{W}$ is PSD, then we are done with the model enhancement phase. Otherwise, the current solution violates some constraints in (6) (or equivalently (5)), and we seek to generate an $\alpha \in \mathrm{R}^n$, $\|\alpha\|$ = 1, such that $[(\alpha^{\mathrm{T}} y)^2]_L \geq 0$, is violated by $(\bar{y}, \bar{w})$, to yield a cutting plane.

We shall now describe a stepwise methodology for generating valid inequalities of the type (6). Note that this concept of generating cutting planes based on semidefinite relaxations can be used to augment any RLT relaxation for discrete or continuous nonconvex programs.

Toward this end, consider the application of the Superdiagonalization Algorithm for checking definiteness (see Bazaara et al, 1993) on the symmetric matrix, $\overline{W}$, proceeding in the order of rows i = 1,…, n, so long as the diagonal elements encountered are positive. Starting with $G^1 \equiv \overline{W}$, corresponding to iteration 1, suppose that we are presently examining the reduced submatrix, $G^i \in R^{(n-i+1)\times(n-i+1)}$, appearing in rows and columns i, i+1,…, n of the partially diagonalized matrix. Let us view G^i in the following partitioned form:

$$G^i \equiv \begin{bmatrix} G_{11}^i & (g^i)^T \\ g^i & G \end{bmatrix} \tag{7}$$

where the first row and column are explicitly shown. We will now state the cutting plane generation strategy.

Main Step:

Solve the first-level RLT relaxation along with any generated constraints of the type (6). Let $(\bar{y}, \bar{w})$ be the solution obtained. If $\bar{y}$ is binary valued, stop; this is an optimal solution to **RL**. If a prescribed limit on the number of cuts to be generated has been attained, stop. The resulting enhanced model is recommended for solution by CPLEX-MIP. Else, delete previous inactive SDP cuts, put i = 1, and continue.

Step 1: Given G^i as in (7), if $G_{11}^i < 0$, then set $\alpha^i = (\alpha_i, ..., \alpha_n)^T = (1, 0, 0, ..., 0)^T \in R^{n-i+1}$, and proceed to Step 8. Else, proceed to *Step 2*.

Step 2: If $i = n$, then $[yy^T]_L$ evaluated at $(\bar{y}, \bar{w})$ is PSD; stop, and prescribe the resulting model to be solved by CPLEX-MIP. Otherwise, proceed to *Step 3*.

Step 3: If $G_{11}^i = 0$, proceed to *Step 4*. Else, proceed to *Step 6*.

Step 4: If $G_{1j}^i = 0 \; \forall j$, proceed to *Step 5*. Else, select that $j \in \{2, \ldots, n\text{-}i+1\}$ for which

$G_{1j}^i = G_{j1}^i = \theta \neq 0$, with $\phi \equiv G_{jj}^i$, and which is such that we obtain the smallest

resulting value of $\lambda = \dfrac{\phi - \sqrt{\phi^2 + 4\theta^2}}{2}$.

Accordingly, then, set

$\alpha^i = (\alpha_i, 0, 0, \ldots, \alpha_{i+j-1}, 0, \ldots, 0)^T \in \mathrm{R}^{n\text{-}i+1}$ where,

$\alpha_i = \dfrac{1}{\sqrt{1 + \dfrac{\lambda^2}{\theta^2}}}$, $\alpha_{i+j-1} = \dfrac{\alpha_i \lambda}{\theta}$, and where $\lambda = \dfrac{\phi - \sqrt{\phi^2 + 4\theta^2}}{2}$.

Proceed to *Step 8*.

Step 5: Store $G_{11}^i = 0$, and let G^{i+1} be obtained by deleting the first row and column of G^i. Proceed to *Step 7*.

Step 6: Store $\begin{pmatrix} G_{11}^i \\ g^i \end{pmatrix}$ and compute $G^{i+1} = G - \dfrac{g^i (g^i)^T}{G_{11}^i}$. Proceed to *Step 7*.

Step 7: Increment *i* by 1 and return to *Step 1*.

Step 8: Check if $i > 1$. If yes, proceed to *Step 9*. Else, proceed to *Step 11*.

Step 9: Recursively for $\gamma = i - 1$ to 1, compute α_γ and α^γ as follows:

If $G_{11}^\gamma = 0$, then set $\alpha_\gamma = 0$; otherwise, compute $\alpha_\gamma = \dfrac{-(\alpha^{\gamma+1})^T g^\gamma}{G_{11}^\gamma}$

where, $\alpha^{\gamma+1} \equiv (\alpha_{\gamma+1}, \ldots\ldots, \alpha_n)^T$. Set $\alpha^\gamma \equiv \begin{bmatrix} \alpha_\gamma \\ \alpha^{\gamma+1} \end{bmatrix}$, and proceed to *Step 10*.

Step 10: Normalize α and proceed to *Step 11*.

Step 11: Generate a new SDP cut (6), using $\alpha \equiv \alpha^1$, append this to the current relaxation, and return to the ***Main Step***.

The foregoing approach establishes an inductive polynomial-time process for generating valid inequalities for the first RLT relaxation. The implementation of this algorithm on a sample problem and the corresponding results are presented in the subsequent chapter.

4. RESULTS

4.1 Sample Problem Description

This chapter considers the implementation of the proposed approach to solve some sample realistic test cases. The test cases are based on an existing freeway section for which the different O-D paths are identified for possible travel time measurement, and this transportation network is modeled as a graph *G* as described in Section 1.4. The benefit factors for all the arcs in the graph are computed by using the tabulated results and generic graphs that have been derived from the analysis of Chapter 3. The corresponding reader location optimization problem is then solved on this corresponding graph *G* using the various proposed alternative model representations, and the results obtained are reported.

For the purpose of illustrating the methodology, consider the *Interstate*-35 *North* Freeway, also known as the North Panam Freeway, in San Antonio, Texas. This freeway serves the Northeast Corridor and provides access to several industrial outlets in San Antonio. The total length of this freeway in the San Antonio jurisdiction is 20 miles. The route is entirely urban and suburban, and the majority of the adjacent land-use consists of warehouse, light industry, and heavy commercial development. Several new major projects are proposed or are under construction in this corridor. This route is also the southern continuation of the San Antonio-Austin Corridor, and is part of the “ NAFTA Superhighway ”. An 8-mile section of this freeway is chosen, as shown in Figure 21(a), and we focus our attention on this particular stretch of the freeway. This portion of the freeway lies between Shin Oak Drive and Pine Street.

Figure 21(a) depicts a typical freeway section. The legend provided identifies the various basic freeway sections found on the freeway, along with other structures and their locations.

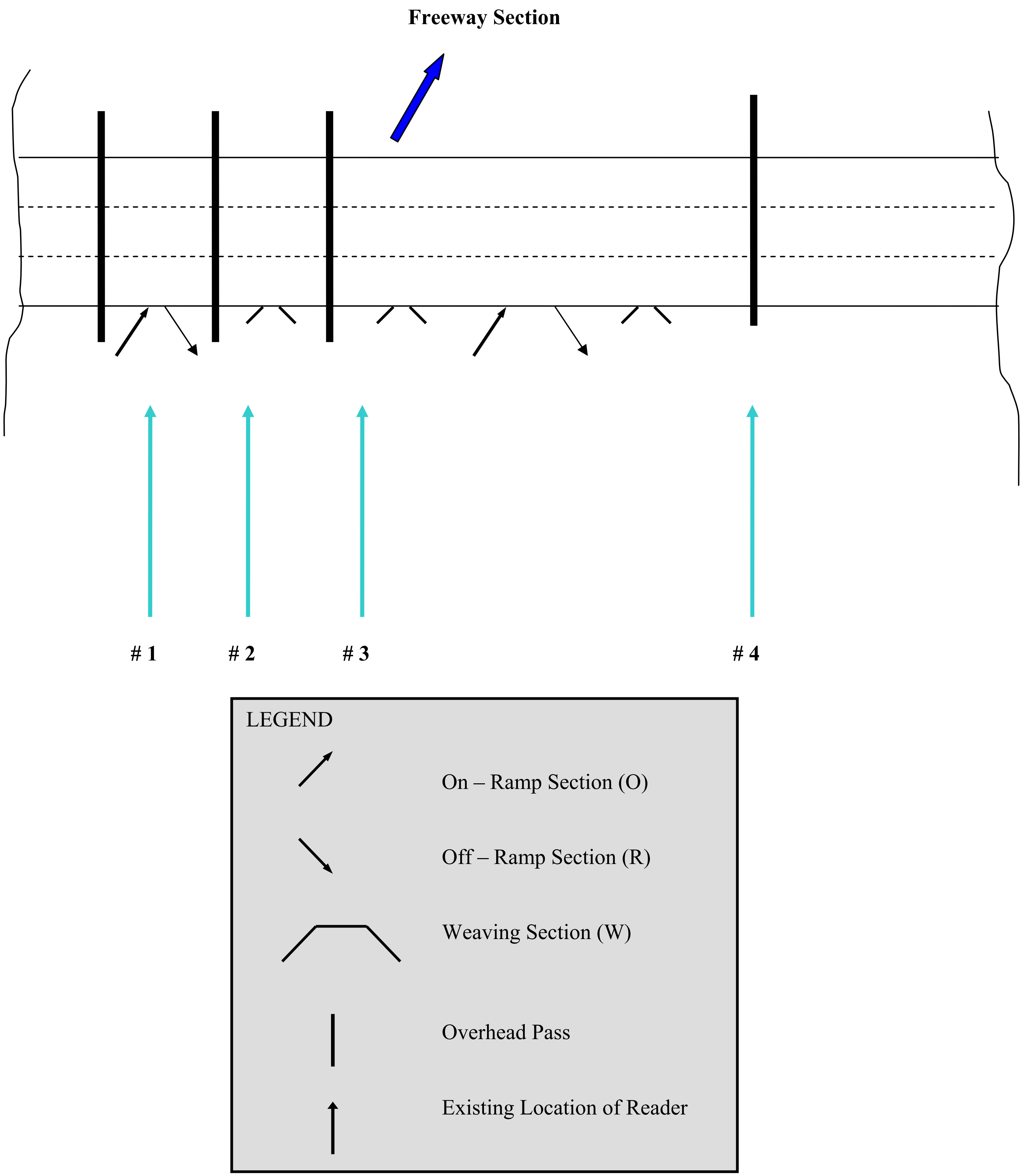

Figure 21(a): Schematic Diagram of a Typical Freeway Section

A detailed examination of the lane schematic diagram for Interstate – 35 reveals that the portion of the Interstate under consideration is comprised of the following basic sections: 7 on-ramps, 9 off-ramps and 7 weaving sections. Currently, there are 7 readers located along this route at the different numbered station points as indicated in Figure 21(b). Figure 21(b) also indicates the basic freeway sections between each overhead pass. The notation showing the locations of the various sections is based on the legend provided in Figure 21(a).

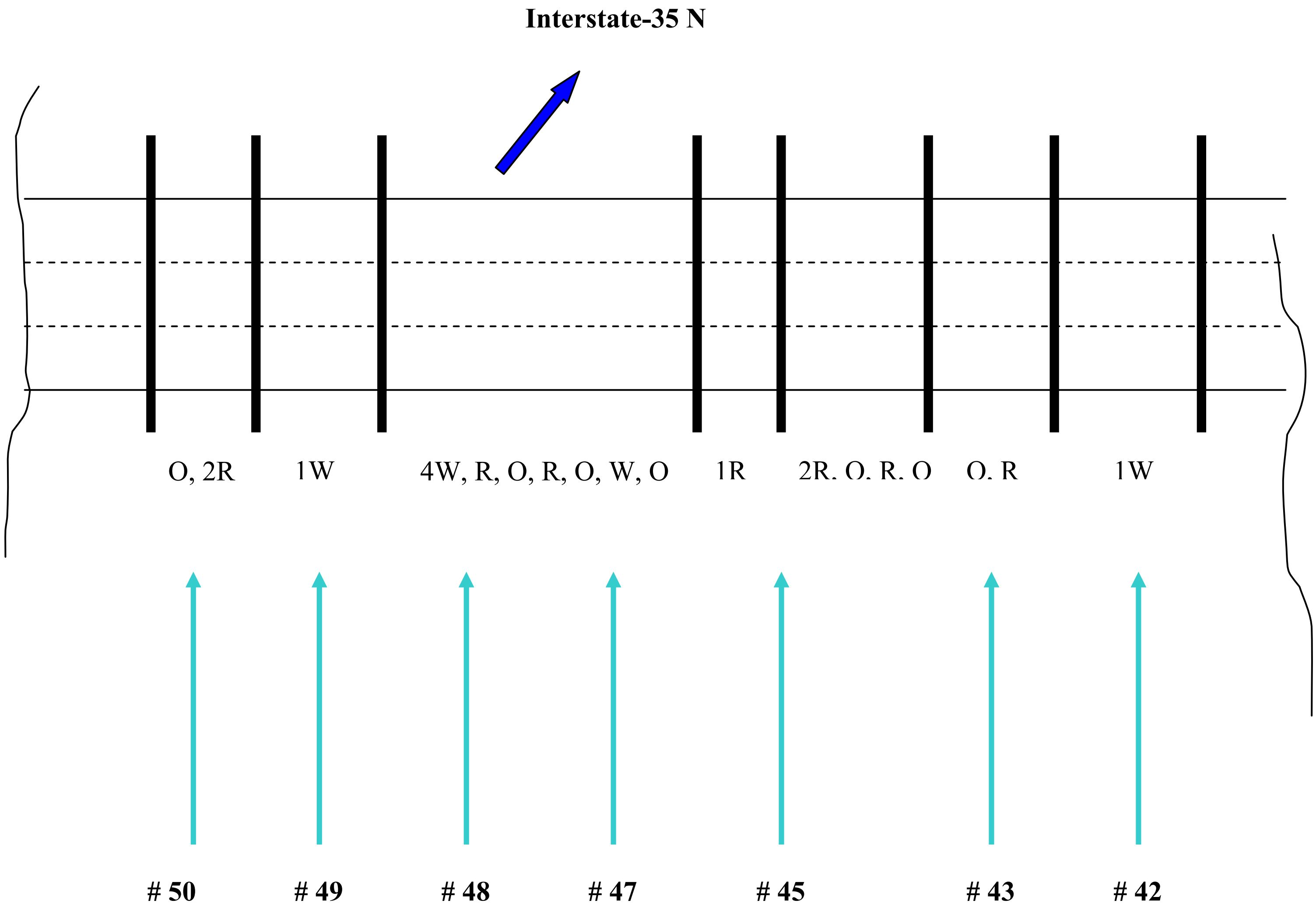

Figure 21(b): Interstate-35 North

4.2 Computations

<table>
<tr><th>Reader node</th><th>Name of Overhead Pass</th><th>Average Annual Daily Traffic (veh/day)</th><th>Coefficient of Variation</th></tr>
<tr><td rowspan="2">1</td><td rowspan="2">Shin Oak Drive</td><td rowspan="3">107,000</td><td rowspan="3">0.3341</td></tr>
<tr></tr>
<tr><td rowspan="2">2</td><td rowspan="2">Judson</td></tr>
<tr><td rowspan="2">107,000</td><td rowspan="2">0.2400</td></tr>
<tr><td rowspan="2">3</td><td rowspan="2">O’ Connor</td></tr>
<tr><td rowspan="2">162,000</td><td rowspan="2">1.55</td></tr>
<tr><td rowspan="2">4</td><td rowspan="2">George Beach</td></tr>
<tr><td rowspan="2">108,000</td><td rowspan="2">0.1187</td></tr>
<tr><td rowspan="2">5</td><td rowspan="2">Loop 410</td></tr>
<tr><td rowspan="2">120,000</td><td rowspan="2">0.5354</td></tr>
<tr><td rowspan="2">6</td><td rowspan="2">Walters</td></tr>
<tr><td rowspan="2">131,000</td><td rowspan="2">0.2230</td></tr>
<tr><td rowspan="2">7</td><td rowspan="2">New Braunfels</td></tr>
<tr><td rowspan="3">190,000</td><td rowspan="3">0.2768</td></tr>
<tr><td rowspan="2">8</td><td rowspan="2">Pine</td></tr>
<tr></tr>
</table>

Table 1: Average Traffic Data for Interstate -35

Table 1 displays the COV values for each of the links along the entire section of the 8-mile freeway, defined for consecutive node pairs, as designated by the rows in Table 1. Note that each such individual link is comprised of several basic freeway sections that include the aforementioned on-ramp sections, off-ramp sections, or weaving sections (this freeway currently has no bottleneck sections). The COV values for these basic sections are derived from the analysis performed in Chapter 3 (the relevant computations are shown in Appendix C). Given the individual COV values for each link as shown in Table 1, Table 2 presents the composite benefit factors, $b^{(1)}_{p-q}$, for all the possible O-D pairs (p, q), using the formula (3) of Chapter 3, for $p < q$, where p represents the origin and q the destination. This is a valid form of displaying the computations in the present case, since we are considering a freeway section, and the traffic has a unidirectional flow. Hence, the indices defining the origin nodes will always be less than those for the destination nodes. For 8 nodes, this produces a total of $\binom{8}{2}$ = 28 O-D pairs.

O-D nodes	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	-	0.3341	0.2871	0.9615	0.7998	0.7492	0.6514	0.5462
2	-	-	0.2400	1.1521	0.9079	0.8238	0.6963	0.5669
3	-	-	-	1.5500	1.1096	0.9515	0.7716	0.6024
4	-	-	-	-	0.1187	0.3489	0.2989	0.2889
5	-	-	-	-	-	0.5354	0.3655	0.3182
6	-	-	-	-	-	-	0.2230	0.2595
7	-	-	-	-	-	-	-	0.2768
8	-	-	-	-	-	-	-	-

Table 2: Benefit Factor Matrix for the Freeway Section

For the purpose of illustration, a sample computation is shown below to derive the corresponding benefit factor $b_{1-3}^{(1)} \equiv b_{13}$, based on Equation (3) of Chapter 3. Consider the O-D pair (1, 3), which consists of two links (1, 2) and (2, 3).

μ_1 = 107,000 ; this is the average demand of traffic from node 1 to node 2.

μ_2 = 107,000 ; this is the average demand of traffic from node 2 to node 3.

Therefore, the corresponding standard deviations computed via the COV values taken from Table 1 are:

$\sigma_1 = \mu_1(COV_1) = 107{,}000 * 0.3341 = 35748.7.$

$\sigma_2 = \mu_2(COV_2) = 107{,}000 * 0.2400 = 25680.$

Hence, $b_{13} = [(\sigma_1^{\,2} + \sigma_2^{\,2}) / (\mu_1^2 + \mu_2^{\,2})]^{1/2} = 0.2871.$

The remaining b_{ij} values can be computed similarly.

The site-specific cost C_j for establishing a reader at location j is specified in Table 3.

Location *j*	1	2	3	4	5	6	7	8
Cost C_j	6.32	9.16	7	3.63	9.11	1.24	3.68	5.15

Table 3: Site-Specific Cost Matrix for the Reader Locations

We also assume that:

Total available budget = 30, and

Number of readers available for installation = 5.

Note that the parameters used ensure that neither constraint (2b) nor (2c) are implied by the other because the sum of the six cheapest readers is 27.01 < 30, and also the sum of the five most expensive readers exceeds 30. Using the above parameters, problem **RL'** is solved by using the mathematical programming software, *AMPL – Student Version* 6.1.

AMPL is equipped with the optimization solver CPLEX 6.5.3, which is adjudged to be one of the best solvers available for solving large-scale linear and mixed integer optimization problems. In the first run, we solve the *continuous relaxation* of **RL'** i.e., taking (y, w) as continuous variables. Since, this is a maximization problem, the optimal objective function value of the continuous relaxation of **RL'** will yield an upper bound on the optimal value of the objective function for problem **RL'**. Also, at optimality, it is possible that some of the y_j variables will have fractional components, and hence, that $y_i y_j \neq w_{ij}$ for some (i, j) pair. It is to be noted that in our computations, we have scaled the benefit coefficients by a factor of 10^4 for numerical purposes.

Results for Run 1:

Continuous relaxation objective function value = 80950.5874, with solution, (y^1, w^1), where

y^1 = [0.9530 0.0947 1 1 0.9521 1 0 0]^T.

Note that the remainder of the solution w^1 can be obtained from the matrix $[yy^T]_L$ evaluated at (y^1, w^1), as shown below.

$$[yy^T]_{L|(y^1,w^1)} = \begin{bmatrix} 0.9530 & 0.0478 & 0.9530 & 0.9530 & 0.9530 & 0.9530 & 0 & 0 \\ 0.0478 & 0.0947 & 0.0947 & 0.0947 & 0.9052 & 0.0947 & 0 & 0 \\ 0.9530 & 0.9530 & 1 & 1 & 0.9521 & 1 & 0 & 0 \\ 0.9530 & 0.9052 & 0.9521 & 0.9521 & 0.9521 & 0.9521 & 0 & 0 \\ 0.9530 & 0.0947 & 1 & 1 & 0.9521 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Here, $[yy^T]_{L|(y^1,w^1)}$ denotes $[yy^T]_L$ evaluated at the current solution (y^1, w^1). For the current solution, (y^1, w^1), we examine $[yy^T]_L$ at (y^1, w^1) and check the positive semidefiniteness of this matrix, by using the step-wise algorithm of Section 3.4. In this case, we obtain $[yy^T]_L$ to be PSD and hence we send the problem to CPLEX-MIP to obtain an integer solution. If $[yy^T]_L$ had not turned out to be PSD at the current solution (y^1, w^1), then the algorithm would have produced the required α vector, such that $[(\alpha^T y)^2]_L < 0$ at (y^1, w^1). The cut $[(\alpha^T y)^2]_L \geq 0$ would then have been introduced into Problem **RL'** to delete the old solution (y^1, w^1), and we would have resolved the updated relaxation.

CPLEX-MIP solution:

Optimal objective function value = 74117, with solution:

$y^* = [\,1\quad 0\quad 1\quad 1\quad 1\quad 0\quad 1\quad 0\,]$.

From this solution it is evident that a maximum benefit is achieved for measuring travel time variability by placing the readers should be placed at nodes 1, 3, 4, 5, and 7.

4.3 Comparison of results for variations of the reader location problem

Problem	Constraints included	Number of nodes enumerated	Number of (MIP) Simplex Iterations	% gap between LP and IP	Optimum
RL' (*without cuts*)	(4a) – (4h)	9	144	9.22	74117
RL_{base}	(4a), (2b), (2c), (4d), (4e), (4g) and (4h)	13	108	35.17	74117
RL' (with cuts)	(4a)-(4h) and semidefinite cuts	9	124	9.22	74117
RL"	(4a)-(4h) w/o the LHS inequalities in (4b) and (4c), and w/o (4f), but with (2b) and (2c)	4	93	19.83	74117

A comparative study of the different model formulation variations of the reader location problem reveals that in terms of the tightness of the relaxations, the enhanced models **RL'**, with and without the cuts, yield significantly tighter representations, leading to the enumeration of fewer nodes to solve the problem. In this case, we notice that there isn't any significant impact of the semidefinite cuts on reducing the number of nodes taken by CPLEX-MIP to solve the optimization problem. This is attributed to the fact that the first order RLT relaxation produces an extremely tight linear underlying representation of the problem, and the solution obtained by solving this relaxation is very close to optimality. Hence, $[yy^T]_L$ is nearly PSD and this results in the infeasibility produced by the semidefinite cuts having a negligible effect on the solution of this instance of problem **RL'** .

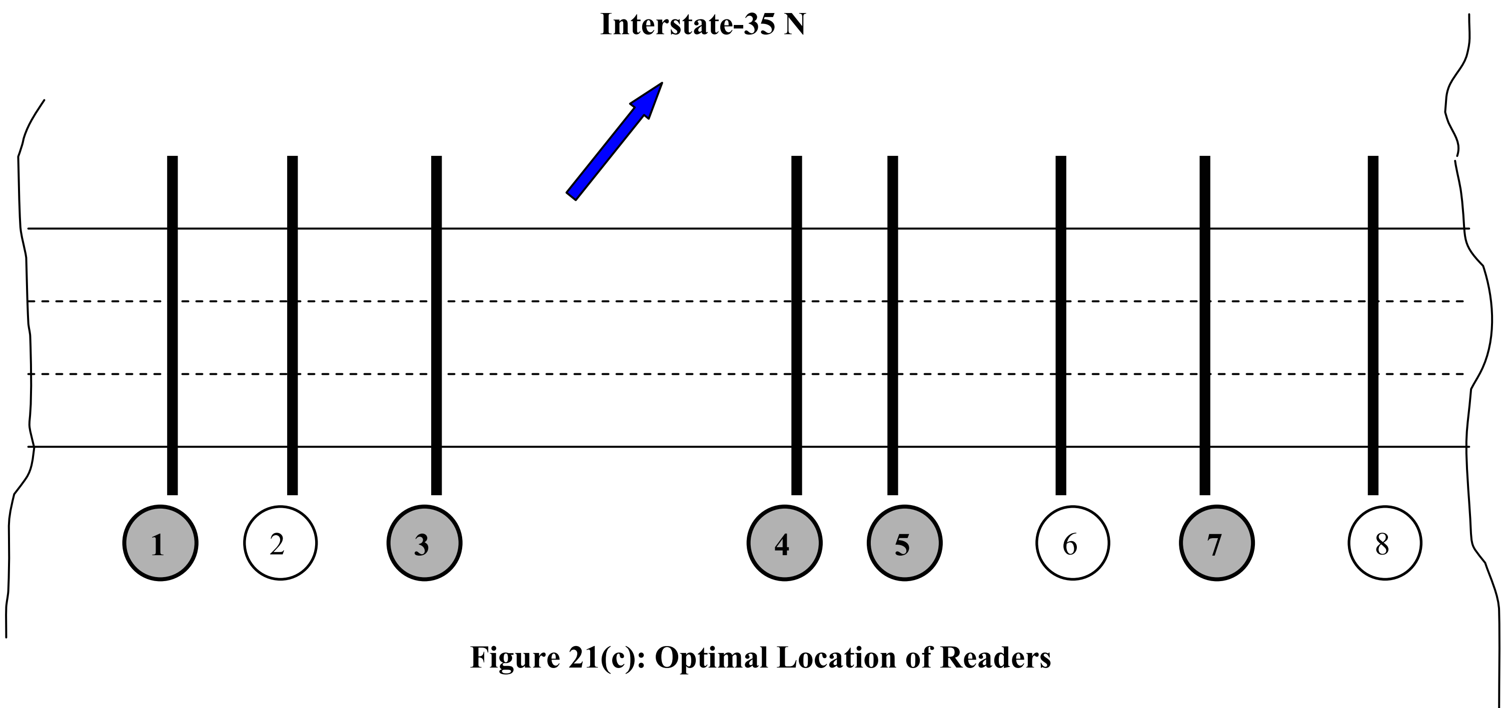

Figure 21(c): Optimal Location of Readers

Figure 21(c) displays the optimal locations (shaded nodes) of the readers on the Interstate-35 North freeway. Figure 22 depicts the graph $G(N, A)$ on which the reader location problem was solved and the optimal reader locations, along with the corresponding optimal links that contribute to the objective function value.

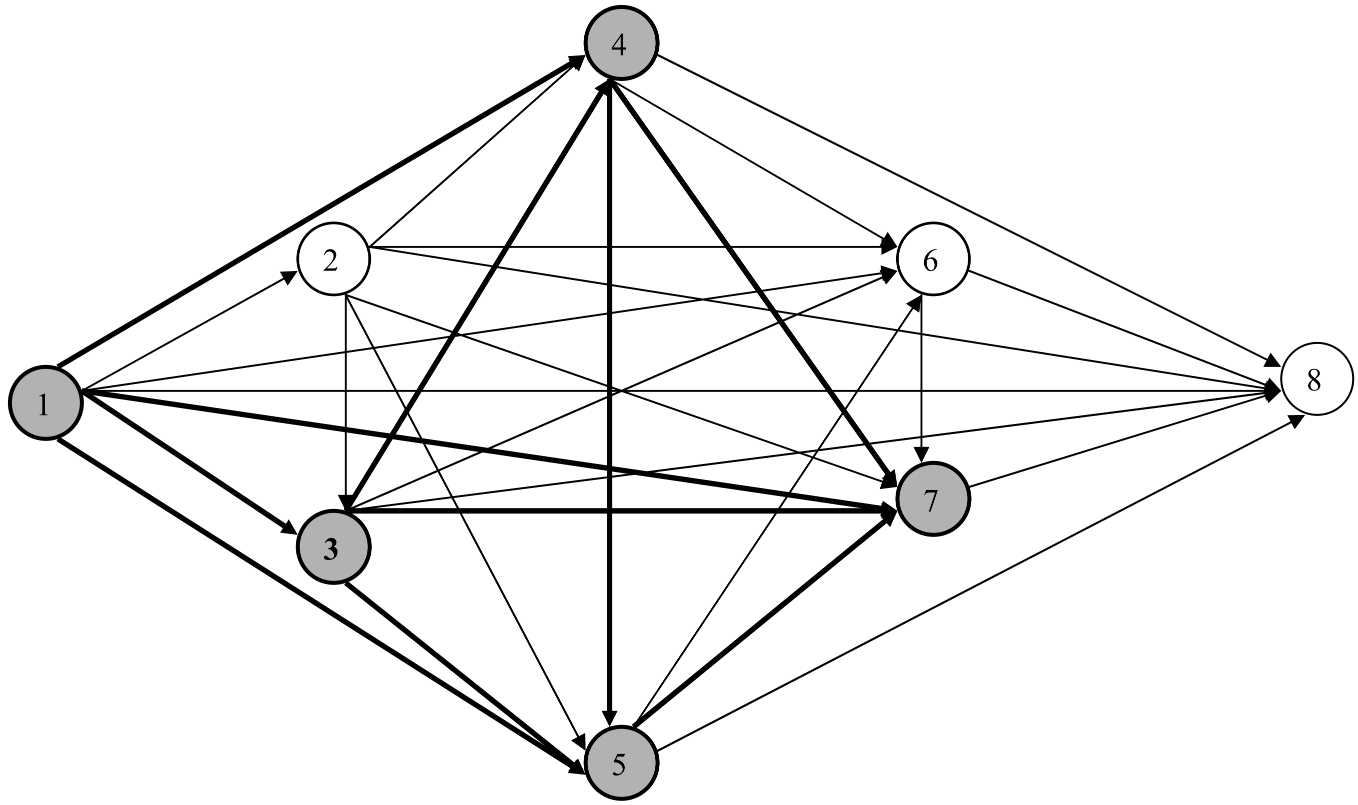

Figure 22: Optimal Locations of Readers on *G*(*N*, *A*) (shaded nodes) and the corresponding arcs (*i*, *j*) for which $y_i y_j = 1$

4.4 Problem results

The Interstate – 35 North, as mentioned above, currently has 7 readers located along the 8- mile section. Considering the existing reader location nodes and the 8 overhead passes as possible reader location sites, the reader location problem was solved. Three synthetic traffic demands were generated, and these traffic data sets were used to solve the reader location problem for various cases. Also, the density of the b_{ij} matrix can be varied to determine the sensitivity of the reader locations to the number of O-D paths. In this analysis, we used 1 and 0.75 as the two different densities of the b_{ij} matrix. The sensitivity of the reader locations was also tested for the two composite benefit factor functions proposed in Section 3.3. For all the test cases, we ensure that the parameters R and B are such that neither of the constraints (2b) or (2c) imply the other. Accordingly, the results reported below have been obtained for three different traffic loading patterns, each tested with the two types of benefit factor functions, and two densities of the b_{ij} matrix. This yields a total of $3 \times 2 \times 2 = 12$ different test cases. All runs have been made on an 833 MHz, 128 MB RAM, Pentium 3 computer. The results for the various cases are shown.

1. Benefit Factor Type 1, Loading 1, Density = 1

Problem	Constraints included	Number of nodes enumerated	Number of (MIP) Simplex Iterations	% gap between LP and IP	Optimum
RL' **(without cuts)**	(4a) – (4h)	12	412	7.86	68165
$\mathbf{RL_{base}}$	(4a), (2b), (2c), (4d), (4e), (4g) and (4h)	300	2357	39.88	68165
RL' **(with cuts)**	(4a)-(4h) and semidefinite cuts	9	356	3.26	68165
RL"	(4a)-(4h) w/o the LHS inequalities in (4b) and (4c), and w/o (4f), but with (2b) and (2c)	11	319	19.78	68165

Optimal reader locations : [3 4 6 7 8 10 11]

2. Benefit Factor Type 1, Loading 1, Density = 0.75

Problem	Constraints included	Number of nodes enumerated	Number of (MIP) Simplex Iterations	% gap between LP and IP	Optimum
RL' **(without cuts)**	(4a) – (4h)	28	545	7.85	48774
$\mathbf{RL_{base}}$	(4a), (2b), (2c), (4d), (4e), (4g) and (4h)	76	714	31.94	48774
RL' **(with cuts)**	(4a)-(4h) and semidefinite cuts	18	412	3.68	48774
RL"	(4a)-(4h) w/o the LHS inequalities in (4b) and (4c), and w/o (4f), but with (2b) and (2c)	12	256	21.48	48774

Optimal reader locations : [3 4 6 7 8 10 11]

3. Benefit Factor Type 2, Loading 1, Density = 1

Problem	Constraints included	Number of nodes enumerated	Number of (MIP) Simplex Iterations	% gap between LP and IP	Optimum
RL' **(without cuts)**	(4a) – (4h)	14	610	2.04	44122
RL_{base}	(4a), (2b), (2c), (4d), (4e), (4g) and (4h)	48	605	23.61	44122
RL' **(with cuts)**	(4a)-(4h) and semidefinite cuts	10	500	1.34	44122
RL"	(4a)-(4h) w/o the LHS inequalities in (4b) and (4c), and w/o (4f), but with (2b) and (2c)	2	120	9.70	44122

Optimal reader locations : [3 4 6 7 8 10 11]

4. Benefit Factor Type 2, Loading 1, Density = 0.75

Problem	Constraints included	Number of nodes enumerated	Number of (MIP) Simplex Iterations	% gap between LP and IP	Optimum
RL' **(without cuts)**	(4a) – (4h)	13	522	5.36	36344
RL_{base}	(4a), (2b), (2c), (4d), (4e), (4g) and (4h)	24	290	27.86	36344
RL' **(with cuts)**	(4a)-(4h) and semidefinite cuts	6	233	3.33	36344
RL"	(4a)-(4h) w/o the LHS inequalities in (4b) and (4c), and w/o (4f), but with (2b) and (2c)	12	289	15.97	36344

Optimal reader locations : [3 4 5 6 7 8]

5. Benefit Factor Type 1, Loading 2, Density = 1

Problem	Constraints included	Number of nodes enumerated	Number of (MIP) Simplex Iterations	% gap between LP and IP	Optimum
RL' (without cuts)	(4a) – (4h)	16	456	3.13	71404
RL_{base}	(4a), (2b), (2c), (4d), (4e), (4g) and (4h)	202	1795	10.62	71404
RL' (with cuts)	(4a)-(4h) and semidefinite cuts	9	123	2.67	71404
RL"	(4a)-(4h) w/o the LHS inequalities in (4b) and (4c), and w/o (4f), but with (2b) and (2c)	5	148	16.87	71404

Optimal reader locations : [3 4 6 7 8 10 11]

6. Benefit Factor Type 1, Loading 2, Density = 0.75

Problem	Constraints included	Number of nodes enumerated	Number of (MIP) Simplex Iterations	% gap between LP and IP	Optimum
RL' (without cuts)	(4a) – (4h)	29	662	8.134	50881
RL_{base}	(4a), (2b), (2c), (4d), (4e), (4g) and (4h)	86	731	29.55	50881
RL' (with cuts)	(4a)-(4h) and semidefinite cuts	24	596	3.76	50881
RL"	(4a)-(4h) w/o the LHS inequalities in (4b) and (4c), and w/o (4f), but with (2b) and (2c)	15	221	20.92	50881

Optimal reader locations : [3 4 6 7 8 10 11]

7. Benefit Factor Type 2, Loading 2, Density = 1

Problem	Constraints included	Number of nodes enumerated	Number of (MIP) Simplex Iterations	% gap between LP and IP	Optimum
RL' **(without cuts)**	(4a) – (4h)	15	433	2.417	47687
RL_{base}	(4a), (2b), (2c), (4d), (4e), (4g) and (4h)	34	504	22.26	47687
RL' **(with cuts)**	(4a)-(4h) and semidefinite cuts	14	400	1.76	47687
RL"	(4a)-(4h) w/o the LHS inequalities in (4b) and (4c), and w/o (4f), but with (2b) and (2c)	2	89	9.314	47687

Optimal reader locations : [3 4 5 6 7 8]

8. Benefit Factor Type 2, Loading 2, Density = 0.75

Problem	Constraints included	Number of nodes enumerated	Number of (MIP) Simplex Iterations	% gap between LP and IP	Optimum
RL' **(without cuts)**	(4a) – (4h)	11	455	3.10	39727
RL_{base}	(4a), (2b), (2c), (4d), (4e), (4g) and (4h)	26	310	22.65	39727
RL' **(with cuts)**	(4a)-(4h) and semidefinite cuts	6	197	2.01	39727
RL"	(4a)-(4h) w/o the LHS inequalities in (4b) and (4c), and w/o (4f), but with (2b) and (2c)	6	197	14.08	39727

Optimal reader locations : [3 4 5 6 7 8]

9. Benefit Factor Type 1, Loading 3, Density = 1

Problem	Constraints included	Number of nodes enumerated	Number of (MIP) Simplex Iterations	% gap between LP and IP	Optimum
RL' (without cuts)	(4a) – (4h)	11	413	0	71377
RL_{base}	(4a), (2b), (2c), (4d), (4e), (4g) and (4h)	229	1872	8.68	71377
RL' (with cuts)	(4a)-(4h) and semidefinite cuts	6	197	0	71377
RL"	(4a)-(4h) w/o the LHS inequalities in (4b) and (4c), and w/o (4f), but with (2b) and (2c)	5	163	0	71377

Optimal reader locations : [3 4 6 7 8 10 11]

10. Benefit Factor Type 1, Loading 3, Density = 0.75

Problem	Constraints included	Number of nodes enumerated	Number of (MIP) Simplex Iterations	% gap between LP and IP	Optimum
RL' (without cuts)	(4a) – (4h)	22	485	5.621	50979
RL_{base}	(4a), (2b), (2c), (4d), (4e), (4g) and (4h)	101	818	25.985	50979
RL' (with cuts)	(4a)-(4h) and semidefinite cuts	15	300	3.90	50979
RL"	(4a)-(4h) w/o the LHS inequalities in (4b) and (4c), and w/o (4f), but with (2b) and (2c)	15	208	18.61	50979

Optimal reader locations : [3 4 6 7 8 10 11]

11. Benefit Factor Type 2, Loading 3, Density = 1

Problem	Constraints included	Number of nodes enumerated	Number of (MIP) Simplex Iterations	% gap between LP and IP	Optimum
RL' (without cuts)	(4a) – (4h)	20	518	3.37	46402
RL_{base}	(4a), (2b), (2c), (4d), (4e), (4g) and (4h)	42	552	20.10	46402
RL' (with cuts)	(4a)-(4h) and semidefinite cuts	15	446	2. 76	46402
RL"	(4a)-(4h) w/o the LHS inequalities in (4b) and (4c), and w/o (4f), but with (2b) and (2c)	2	83	8.012	46402

Optimal reader locations : [3 4 6 7 8 10 11]

12. Benefit Factor Type 2, Loading 3, Density = 0.75

Problem	Constraints included	Number of nodes enumerated	Number of (MIP) Simplex Iterations	% gap between LP and IP	Optimum
RL' (without cuts)	(4a) – (4h)	19	467	4.068	38027
RL_{base}	(4a), (2b), (2c), (4d), (4e), (4g) and (4h)	20	302	22.09	38027
RL' (with cuts)	(4a)-(4h) and semidefinite cuts	10	446	3. 16	38027
RL"	(4a)-(4h) w/o the LHS inequalities in (4b) and (4c), and w/o (4f), but with (2b) and (2c)	9	244	15.097	38027

Optimal reader locations : [3 4 6 7 8 10 11]

From the results, the following conclusions can be drawn.

1. The locations of the readers are insensitive to either the traffic demand on the network, or the density of the graph, or the type of benefit factor used. However, the relative gap between the LP and the IP solution values is somewhat greater than the lower density graphs, usually lower for the type 2 benefit factor, and also varies with the demand pattern.
2. It can be seen that for most of the cases, the optimal reader locations turn out to be [3 4 6 7 8 10 11]. That is, all the seven readers are utilized. The only exception is for the case of the second type of traffic loading, and using benefit factor type 2. In this case, the optimal reader locations turn out to be [3 4 5 6 7 8]. Observe that only 6 readers are used, because the budget constraint turns out to be restrictive in this case.
3. The number of nodes enumerated by **RL'**, is significantly lower than the other model variations. This is because the linear programming relaxation is tight, and provides a good underlying representation to the problem.
4. The computational results obtained validated the tightness of the RLT relaxation, and an optimal solution was found for all the test problems in a fraction of a second (in most cases, it was found within 0.02 seconds of CPU time).

5. SUMMARY AND CONCLUSIONS

In this chapter, we summarize the research performed in this thesis, while providing insights into the logical thought process that led to the development of this work, and the conclusions derived therefrom. The latter part of this chapter also throws some light on some future research directions and possible extensions.

5.1 Summary

It has only been in recent times that a due emphasis has been placed on travel time related issues in transportation planning. It was nearly a decade ago when it became evident that congestion had to be viewed as a major problem to be overcome. In the initial stages, the onus on transportation researchers was to develop algorithms that effectively measured travel times. However, it was soon realized that measuring travel time effectively was not sufficient but one further step was necessary: To inform people of their expected travel times under real-time conditions. With the advent of AVI technology, and the installation of Variable Message Sign (VMS) boards, it has become possible to inform the traveling public of traffic conditions, and the ensuing expected travel times. The accuracy of this travel time prediction vastly depends upon the nature and analysis of historical data of traffic conditions along that route. Indeed, the accuracy of travel time prediction is highly data intensive. Hence, we need to accumulate a large amount of traffic flow information along those routes on which we intend to predict travel times.

In this thesis, we have addressed the issue of optimally locating AVI tag readers to capture as much travel time information as possible, and in particular, information about travel time variability. This problem contextually becomes very important in light of the above discussion. To gather the required traffic information and to build a sufficiently large database of travel time information, we measure the travel times of cars that are equipped with a specific tag number. The task of optimally allocating these readers in such a way as to maximize an objective function that suitably measures the benefits accruing from their usage can be modeled as an optimization problem. Finding such an optimal location of readers is a crucial step to gather the right information about traffic flow in a network. For example, if the readers are placed on streets that have a very low traffic volume, or variability in travel times about an expected value, the resulting information would be of little consequence to the general public.

Hence, the readers ought to be located at the "right" sites. It is to be noted that even such an optimal set of locations will vary, based on the definition of the objective criterion. In problem **RL**, the objective function parameters are the benefit factors that capture travel time variability in a network. The logic behind choosing travel time variability is that if a certain section of a roadway has a constant travel time over the duration of an entire day, there is no benefit derived from measuring travel time across this link from the point of view of providing useful travel time information. On the other hand, if a certain link has a large variation based on the time of day, then a user needs to be constantly updated on the current travel time across that particular link. This motivates the location of readers based on maximizing a measure that reflects the capturing of travel time variability information.

In an ideal situation, we would like to measure travel times across all links, but considering that there inevitably exist restrictions on the number of readers available, as well as the cost allotted to installing these readers at specific sites, the problem of allocating these readers can be viewed as an optimization problem to maximize the capture of travel time variability information, subject to such resource based constraints.

Thus far, we have discussed why it was more beneficial to measure travel time across a particular link that had greater travel time variability than an alternate link. The next issue of concern was the quantification of travel time variability, and the associated design of an appropriate objective function for the optimization problem. One typical measure of travel time variability is the standard deviation, but the use of this alone was discounted due to the following reason. Consider two links (a, b) and (p, q) that have average respective traffic volumes μ_{ab} and μ_{pq}, and standard deviations σ_{ab} and σ_{pq}. If $\sigma_{ab} << \sigma_{pq}$, but $\mu_{ab} << \mu_{pq}$, then by just considering standard deviations alone, we would prefer measuring travel time across the link (p, q). However, it is desirable to consider measuring travel times along links that not only have a high standard deviation, but also such links where this variation occurs on a proportionately lower average traffic volume, i.e., the travel time variation increases proportionately as the link is correspondingly used more (as an average) by commuters. To incorporate the effect of traffic volume, the coefficient of variation is taken as a measure of travel time variability. The use of COV (which is a normalized, dimensionless measure) ensures that instead of travel time variability being the only parameter, the average traffic volume is considered as well.

To derive the corresponding benefit factors for various sections of the freeway we used simulation. Simulation proves to be a very useful tool in the analysis of traffic conditions because of the large number of scenarios that can be evaluated. In our case, simulation studies were performed for the different types of basic sections, namely for on-ramps, off-ramps, weaving sections, and bottleneck sections, under a wide range of traffic conditions. The expected travel time, and the corresponding coefficients of variation were computed for each of the scenarios examined by the simulation model. Since these simulation results are generic in nature, they can potentially be used for any given freeway.

Having performed the analysis to determine the benefit factors for the basic freeway sections, we were able to design a mechanism for composing the objective function of the optimization of the reader location problem. This reader location problem turns out to be a quadratic 0-1 integer program, which, in practice, can be difficult to solve for large scale problem instances. We therefore adopted the Reformulation-Linearization Technique to derive a tight equivalent linear mixed-integer programming representation of the problem, and further augmented it with certain additional constraints called semidefinite cuts, and subsequently solved it using a branch-and-bound methodology implemented within CPLEX. The reformulation of the problem tends to produce a tight underlying linear relaxation. In our case, it was found that the first order RLT relaxation itself produced bounds that were close to optimality, and the generation of the cuts improved this representation only slightly further. The computational results obtained validated the tightness of the RLT relaxation, and an optimal solution was found for all the test problems in a fraction of a second (in most cases, it was found within 0.01 seconds of CPU time).

5.2 Future Research

There are several avenues that merit further exploration. These are delineated below.

1. One of the main steps involved in this overall approach, is the construction of the graph $G(N, A)$ on which the reader location problem is to be solved. Our solution depends upon the nature of the graph and the identified O-D paths. One area for future research is to develop an automated scheme for determining the graph $G(N, A)$, given the underlying transportation network.

2. The determination of the b_{ij} coefficients is a data intensive task, since it deals with a large number of data points for each type of section. The V/C ratios for different sections on the roadway need to be determined by using loop detector data; this leads to the corresponding COV values for each section, and hence the benefit factor, b_{ij}, for an O-D pair (i, j). This methodology could be made less data intensive perhaps, by composing alternative means for composing these coefficients in a more efficient manner.

3. An alternative for determining optimal (or desirable) reader locations is to employ a simulation model. Given a transportation network, we can model the entire graph in a simulation environment, run it for various sets of solutions that

correspond to different locations of readers and thereby pick the alternative that best captures the simulated variations by some defined measure.

4. For the optimization model **RL** itself, we could explore alternative solution strategies, e.g., enumeration methods that do not rely on a linearization of the objective function.

As a final word, it is beneficial to note that physically establishing these readers is an extremely costly venture, since each reader costs $15,000/lane to install when an overhead pass is already existent, and $70,000/lane if an overhead structure needs to be erected along with the reader itself. With such large monetary investments being involved, it is imperative to identify the critical locations for these readers. Optimization techniques can therefore be gainfully employed to determine such effective allocations of scarce physical, and monetary resources. An application of the methodology prescribed in this thesis to solve such a reader location problem provides an impetus for the effective deployment of surveillance technologies in Advanced Traffic Information Systems.

APPENDIX A

INTEGRATION

Overview of Simulation Features

The INTEGRATION package, developed by Dr. Van Aerde and Dr. H. Rakha, at the Virginia Tech Transportation Institute, implements a trip-based microscopic traffic simulation method. This method is designed to trace individual vehicle movements from a vehicle's origin to its destination at a level of resolution of one status every deci-second. In order to appreciate the domain of application of INTEGRATION, it is useful to view travel activities within urban areas as an inter-related sequence of six decisions that travelers must typically make in order to complete a particular trip. Three of these decisions must be made prior to the trip and cannot be revisited during the same trip. The others, however, need to be visited repeatedly even after a trip has been initiated.

At the highest level of the trip making process, the traveler must determine how many trips to make towards each potential destination during each particular departure time-window. Once the decision to make a particular trip to a given destination has been made, the traveler must then decide on the mode of transportation to be utilized, and the final decision relates to the particular time at which the traveler may elect to start the trip in a given time-window. In contrast, the next three types of trip decisions are made once a trip has started, and is usually invoked more than once in any given trip. These decisions include route selection, speed at which one must drive, and which lane to utilize along a link. Finally, upon arriving at the end of the link, the driver may be required to cross an opposing traffic stream, and must then decide whether to accept or reject any available gaps and/or how to merge with a converging traffic stream.

Microscopic Modeling Approach

INTEGRATION is a fully microscopic simulation model, as it tracks both the lateral and longitudinal movements of individual vehicles at a resolution of one deci-second. The microscopic approach permits the analysis of many dynamic traffic phenomena such as shock waves, gap acceptance, and weaving. These attributes are usually very difficult, or infeasible, to capture under non-steady-state conditions. The INTEGRATION model can consider virtually continuous time varying traffic demands, routings, link capacities, and traffic controls without the need to pre-define explicit time-slice durations between these processes. Consequently, instead of treating each of the above model attributes as a sequence of steady-state conditions, as necessary in most rate-based models, all of these attributes can be changed, virtually, on a continuous basis over time. The microscopic approach also permits considerable flexibility in terms of representing spatial variations in traffic conditions. For example, while most rate-based models consider traffic conditions to be uniform along a given link, INTEGRATION permits the density of traffic to vary continuously along the link.

General Traffic Flow Fundamentals

The manner in which INTEGRATION represents traffic flows can be best presented by discussing how a vehicle initiates its trip, selects its speed, changes lanes, transitions from link to link, and selects its route.

Prior to initiating the actual simulation logic, the individual vehicles that are to be loaded onto the network need to be generated. As most available O-D information is macroscopic in nature, INTEGRATION permits the traffic demand to be specified as a time series histogram of O-D departure rates for each possible O-D pair within the entire network. The actual generation of individual vehicles occurs in such a fashion as to satisfy the time-varying macroscopic departure rates that were specified by the modeler within the input files. It should be noted that as the externally specified O-D traffic demand file is disaggregated, each of the individual vehicle departures is tagged with its desired time, trip origin and trip destination, as well as a unique identification number. This unique vehicle number can be subsequently utilized to trace a particular vehicle along the entire path towards its ultimate destination. When the simulation clock reaches a particular vehicle's scheduled departure time, an attempt is made to enter that vehicle into the network at its origin zone. Upon entering its first link, the vehicle selects a particular lane, which is usually the lane having the greatest available distance headway at the point of entry. From this point, the vehicle proceeds, in a link-by-link fashion towards its final destination.

Once the vehicle has selected which lane to enter, the vehicle computes its desired speed on the basis of the distance headway between it and the vehicle immediately downstream of it, but within the same lane. This computation is based on a link specific microscopic car-following relationship that is calibrated macroscopically to yield the appropriate target aggregate speed-flow attributes for that particular link (Van Aerde, 1995; Van Aerde and Rakha, 1995). The macroscopic calibration of the microscopic car-following relationship ensures that vehicles will traverse each link in a manner that is consistent with the link's desired free-speed, speed-at-capacity, capacity, and jam density. Qualitatively, it can be noted from the speed-headway relationship that vehicles will only attain their desired free speeds when the distance headway in front of them is very large. In contrast, when the distance headway becomes sufficiently small, so as to approach the link's jam density headway, vehicles will decelerate until they eventually come to a complete stop.

Freeway Sections

The INTEGRATION simulation logic, while facility independent, is designed to deal with a number of conditions or situations that are usually perceived to be unique to freeway sections, such as merging, diverging, weaving and bottleneck sections.

Modeling merges*:*

In general, when two traffic streams merge, all available merge capacity is allocated using entitlements in proportion to the non-queued capacities of the two merging links. Therefore, at an on-ramp merge, queues may form downstream of the ramp, upstream of the ramp on the freeway, on the on-ramp, or on both, depending upon the prevailing demands. For example, when an acceleration lane is present, following the ramp merge, the queue will automatically be modeled as occurring immediately upstream of the lane drop. When the queue subsequently grows to fill the entire merge area, the queue may spill back further onto the on-ramp or onto the upstream section of the mainline freeway, depending upon the split in the vehicle arrival rates. However, if there is no acceleration lane provided, the queue will form upstream of the on-ramp merge. The split of the queues on the on-ramp is again a function of the relative vehicle arrival rates on the mainline and on the on-ramp. It is important to note that the allocation of queues to different upstream arms at a freeway merge is critical to estimating the relative capacities and travel times on each of these links. Errors in estimating these relative travel times not only affect the resulting Measures of Effectiveness (MOEs) in isolation, but also have a significant impact on any dynamic traffic assignments or diversions that consider these MOEs within their routing objective functions.

Modeling Diverges

At diverging sections, queues may also form when one of the discharge arms receives more demand than there is available capacity. Alternatively, the mainline section may spill back at diverges due to downstream mainline congestion, even though there is sufficient capacity available both upstream of the diverging section and on the off-ramp. When one of the diverge arms, say an off-ramp, possesses insufficient capacity, vehicles destined for this bottleneck will eventually queue on the upstream section of the diverge. Consequently, this queue may ultimately constrain the flow of through vehicles, where these through vehicles may not even be utilizing the off-ramp in question. The

INTEGRATION model computes the resulting queue spill-back as a function of the prevailing off-ramp over-saturation, as well as the extent to which vehicles utilizing the off-ramp tend to congregate in the lane feeding the off-ramp.

An important impact of such queue spill-back at diverges is that the link as a whole no longer complies with standard FIFO (First-In-First-Out) queuing logic. Instead, considerable differences in link travel times may arise, depending upon the destinations of the vehicles. Such differences complicate any routing logic, which may need to consider that different link users will arrive at the next downstream link after a different lag time, even though they may have entered the link concurrently. In addition, while the two traffic streams do not share the same travel times, they do interact considerably, as the split in off-ramp, versus through vehicles, will significantly alter the travel times experienced by both groups. This is the case even if the total arrival flow on the link remains relatively constant.

Modeling Weaving

Within INTEGRATION, the final impact of a weaving section is a direct function of the interaction between the prevailing car-following and lane-changing behavior (Van Aerde *et al*., 1996; Stewart *et al*., 1996). Specifically, vehicles that engage in a lane-changing maneuver briefly occupy space in both the lane they are leaving from and the lane into which they are changing. The fact that a single lane-changing vehicle consumes capacity in both lanes makes a single weaving vehicle temporarily have the equivalent impact of two vehicles. The weaving logic is also sensitive to the type of weave that takes place, as different weave types require different numbers of lane changes per vehicle. The model is also sensitive to the length of the weave, because a longer weaving section permits the impact of the lane changes to be spread out over a longer length of the road segment.

Measures of Effectiveness (MOEs)

The INTEGRATION model does not contain an explicit link travel time function in a fashion similar to most macroscopic or planning-oriented traffic assignment models, but rather the travel times are implicitly determined by the use of speed-flow and car-following relationships. In effect, link travel times emerge as a consequence of the weighted sum of the speeds that vehicles experience as they traverse each link segment. This distinction introduces both a level of complexity and a level of accuracy.

Specifically, the dynamic temporal and spatial interactions of shock waves, which form upstream of a traffic signal or along a freeway link that is congested, are such that the final link travel time is neither a simple function of the inflow nor the outflow of the link. Instead, the travel time is a complex product of the traffic flow time-series and associated dynamics along the entire link. The strength of a microscopic approach is that, beyond the basic car-following/lane-changing/gap-acceptance logic, there is no need for any further analytical expressions to estimate either uniform, over-saturation, coordination, random, left-turn, or queue spill-back delay. While such complexity precludes the simplicity of a functional relationship, such as the Bureau of Public Roads relationship, it does permit two distinct travel times to be properly considered for the same flow level, depending on whether forced or free-flow conditions prevail.

Estimation of link travel times

The model determines the link travel time for any given vehicle by providing that vehicle with a *time card* upon its entry to any link. Subsequently, this *time card* is retrieved when the vehicle leaves the link. The difference between these entry and exit times provides a direct measure of the link travel time experienced by each vehicle. Further, each time a vehicle decelerates, the drop in speed is recorded as a partial stop. The sum of these partial stops is also recorded on the above *time card.* This sum, in turn, provides a very accurate explicit estimate of the total number of stops that were encountered along that particular link. Multiple stops arise from the fact that a vehicle may have to stop several times before ultimately clearing the link stop line. This finding, while seldom recorded by or even permitted within macroscopic models, is a common observation within actual field data for links on which considerable overly saturated queues occur.

Aggregation of Statistics by Link and O-D pair

The same *time card* concept used for recording a vehicle's travel time and number of stops on a particular link is also utilized to track the fuel consumption and emissions for each vehicle on each link. Internal to the model, these statistics are further aggregated, both for all links traversed by a particular vehicle and for all the vehicles that have traversed a particular link. The former statistics can be aggregated at the O-D level by time period or vehicle class. Alternatively, they can be aggregated by time period for each link, or by cell within a latitude/longitude grid.

Loop detectors and vehicle probes

The microscopic approach used by INTEGRATION also permits the generation of surveillance data that is representative of the data obtained from loop detectors and/or vehicle probes. Specifically, when a vehicle traverses any location that is considered to be a loop detector site, the model records a vehicle count, the vehicle's speed, and the estimated vehicle detector occupancy. These data are then accumulated into 20-, 30- or 60-second readings, which can be provided as an additional model output. The ability to locate vehicle detectors anywhere on a link, and/or to locate multiple vehicle detectors on a given link, permits considerable flexibility in the collection of various model statistics. In addition, this ability permits the evaluation of alternative surveillance levels in support of real-time control and/or various incident detection schemes. Detectors can also be coded as trapping only a certain vehicle class. This could represent, for example, a special bus or truck sensor.

Certain vehicles can also be flagged as being vehicle probes. In the most basic setup case, a separate record is generated each time a probe vehicle starts or ends a trip. In a more intermediate level of analysis, a separate record can be generated each time a probe vehicle completes traversing a link. Finally, in its most detailed form, a separate record can be created for each probe vehicle at a user-specified time frequency. The latter detailed statistics are most useful in tracking the speed profiles of vehicles along a targeted set of links.

Fundamental Model Input Requirements

The input data that are required to run the model are divided into fundamental data and advanced data categories. The fundamental data are essential to run the model, while the advanced data allow optional model features to be activated.

The names of all input data files are to be entered as ASCII characters in a tabular format. Spaces, commas, or tabs are used to separate various numeric fields. However, the use of any other special visible or hidden characters are prohibited. The input data files can be generated/modified using any standard editor, spreadsheet or word processor (in non-document mode), given that the above guidelines are complied with. However, the inclusion of special formatting characters and the insertion of blank lines are again prohibited. The input data files may also be generated through the use of special purpose translation programs that convert the input data files that were initially generated for some other traffic simulation or transportation planning model into an INTEGRATION format.

APPENDIX A.1

On-ramp section with acceleration lane length = 50m

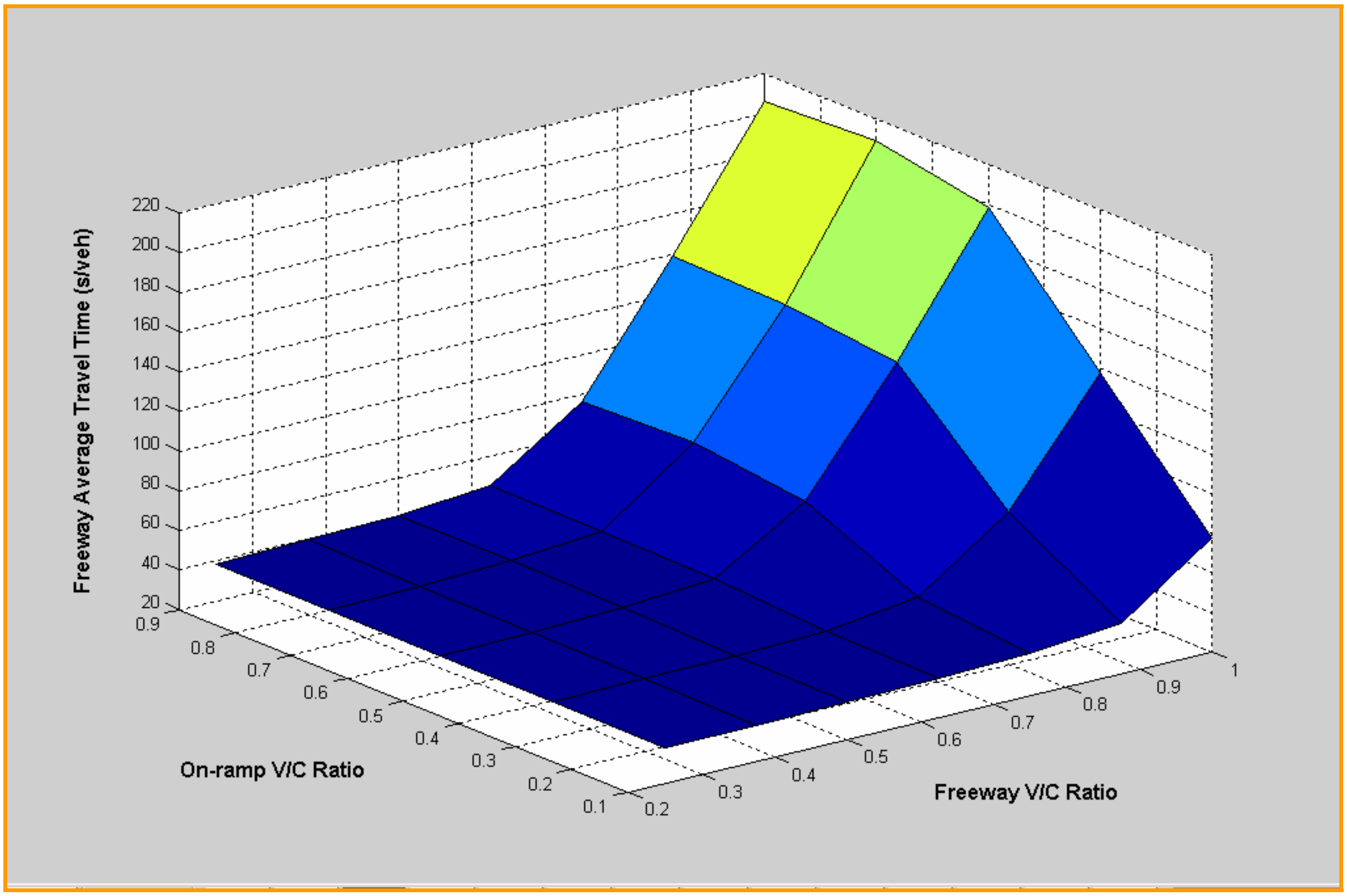

Figure 1: Freeway Average Travel Time v/s $V/C_{freeway}$ and $V/C_{on\text{-}ramp}$

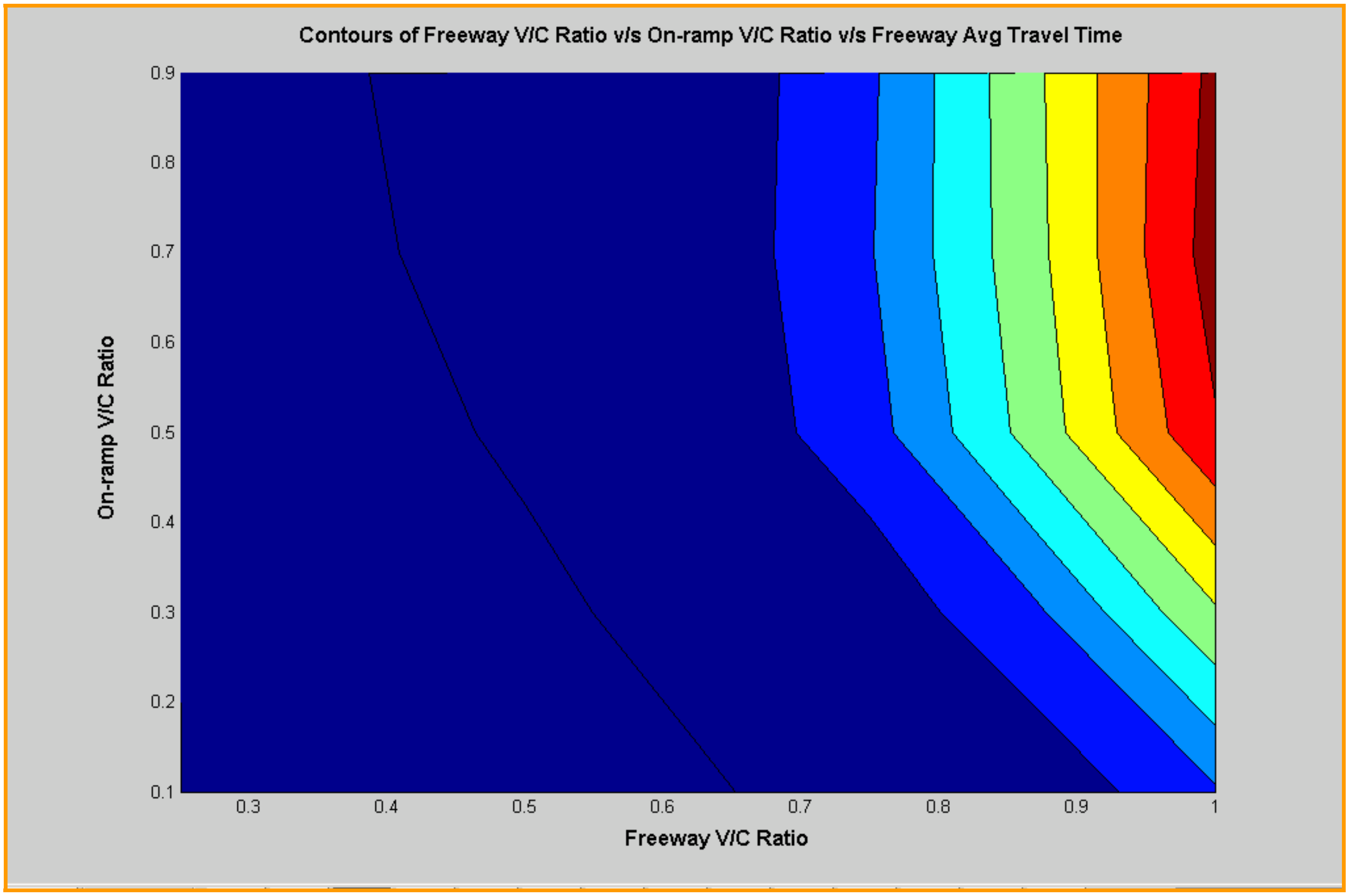

Figure 2: Freeway Average Travel Time Contours v/s $V/C_{freeway}$ and $V/C_{on\text{-}ramp}$

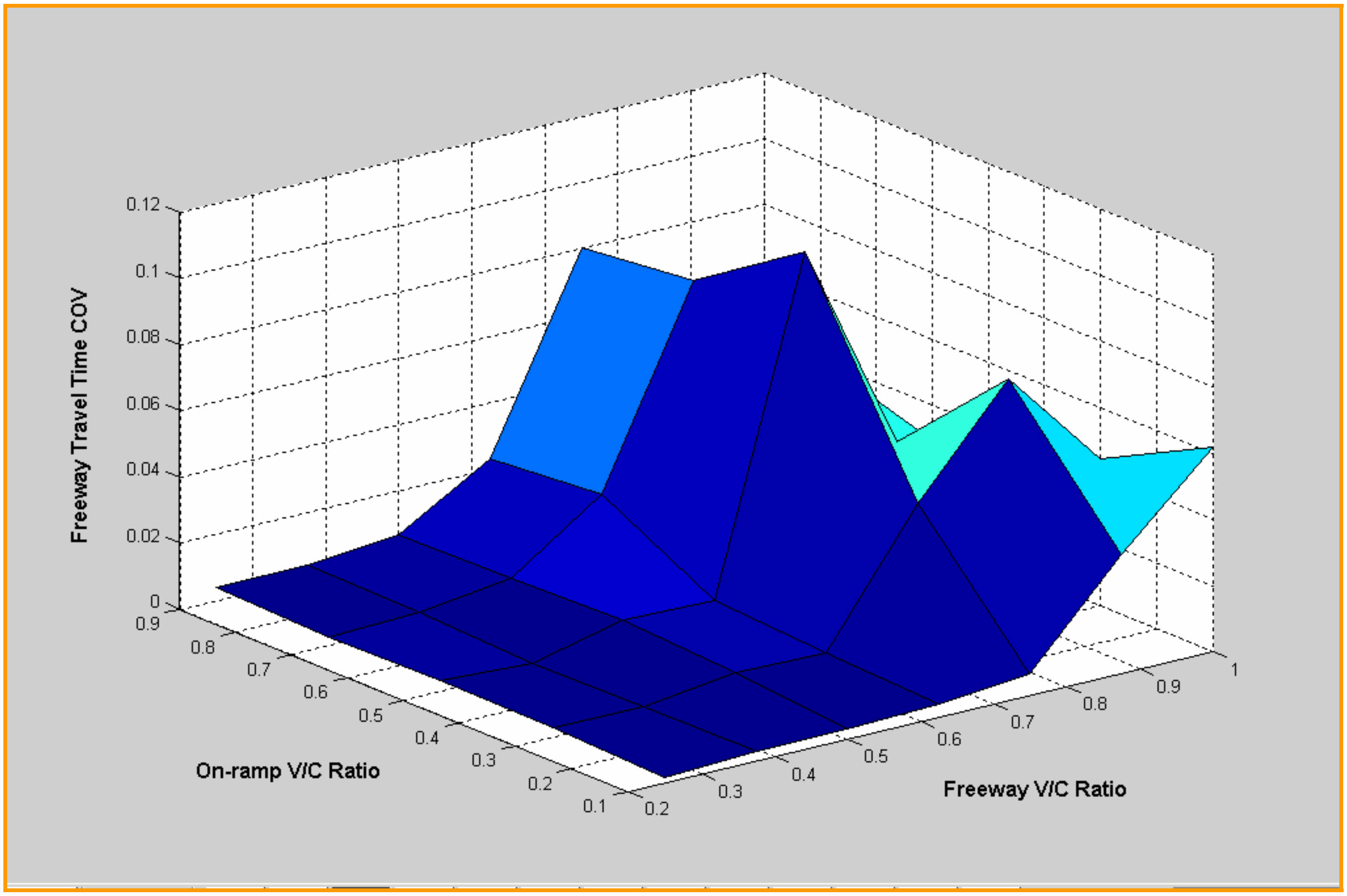

Figure 3: Freeway Travel Time COV v/s $V/C_{freeway}$ and $V/C_{on\text{-}ramp}$

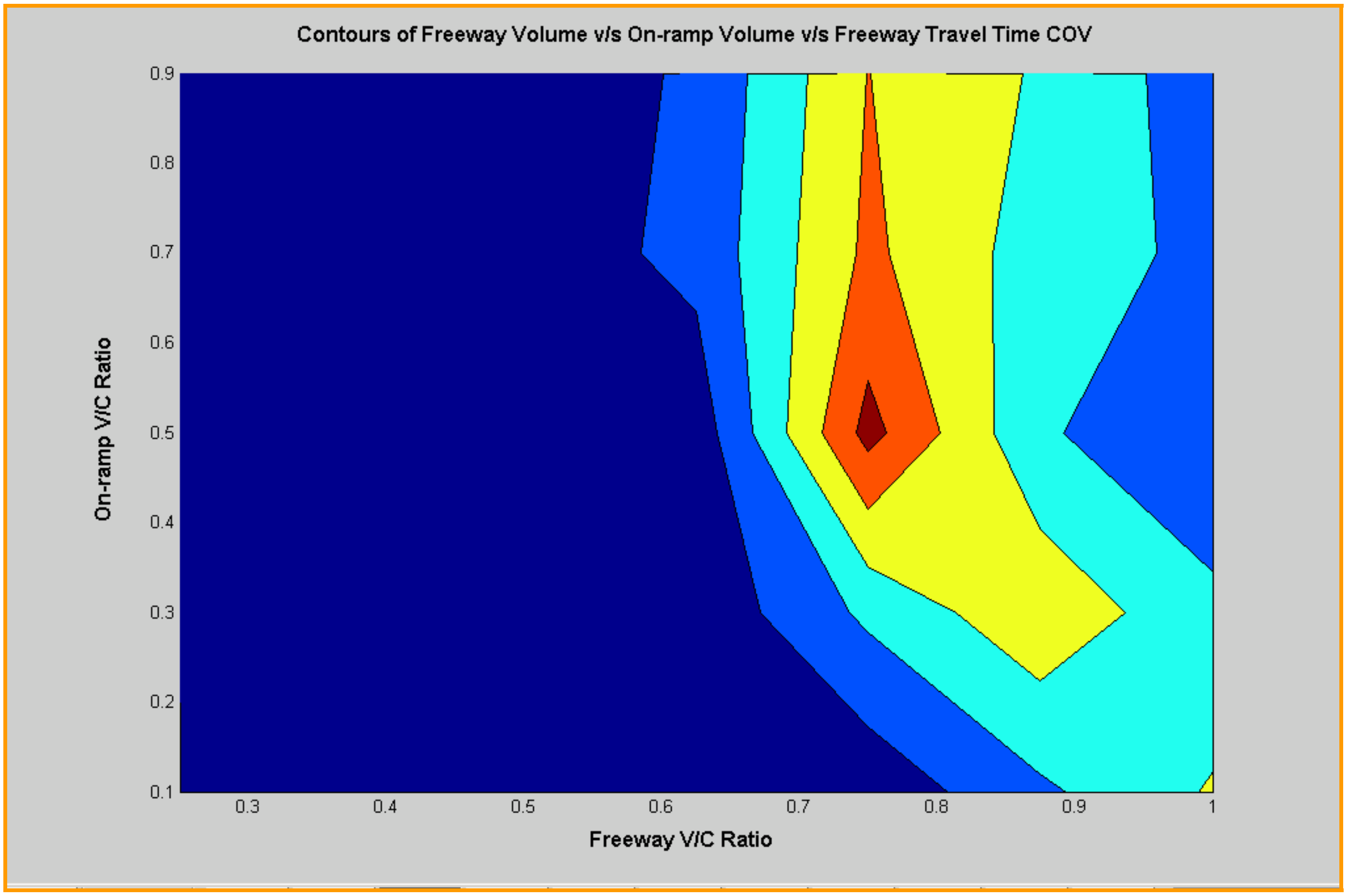

Figure 4: Freeway Travel Time COV contours v/s $V/C_{freeway}$ and $V/C_{on\text{-}ramp}$

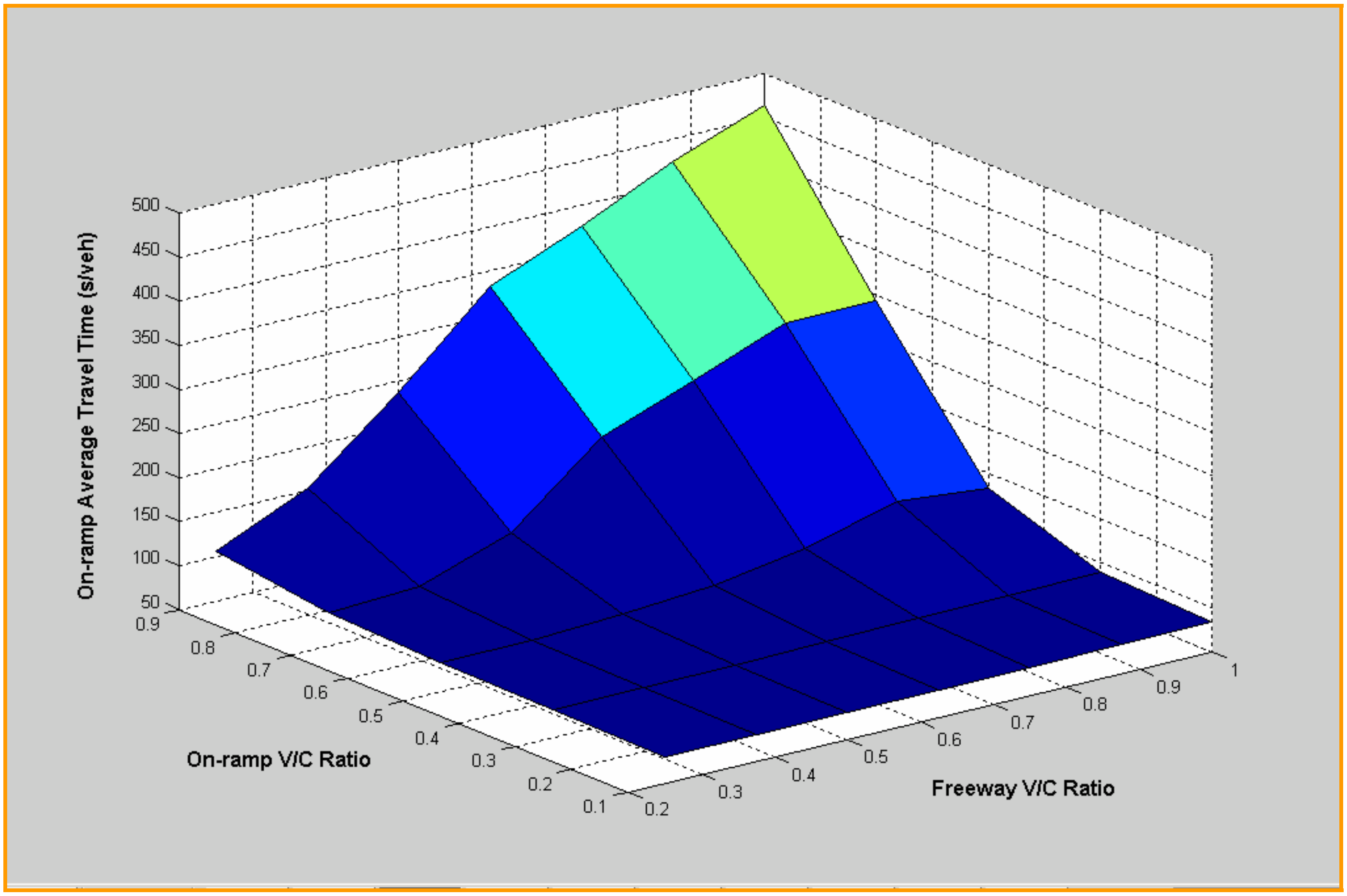

Figure 5: On-Ramp Average Travel Time v/s V/C$_{freeway}$ and V/C$_{on\text{-}ramp}$

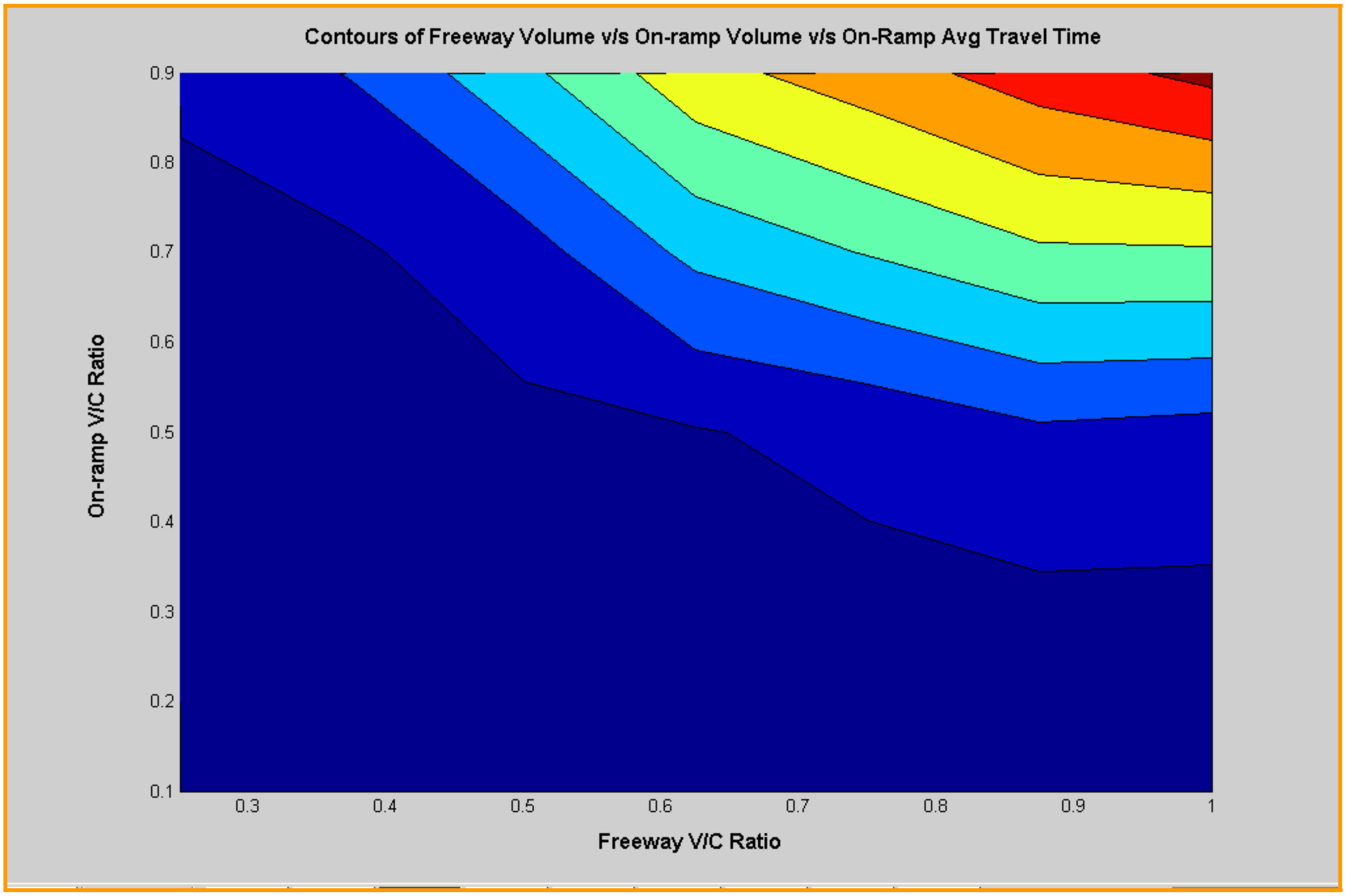

Figure 6: On-Ramp Average Travel Time Contours v/s $V/C_{freeway}$ and $V/C_{on-ramp}$

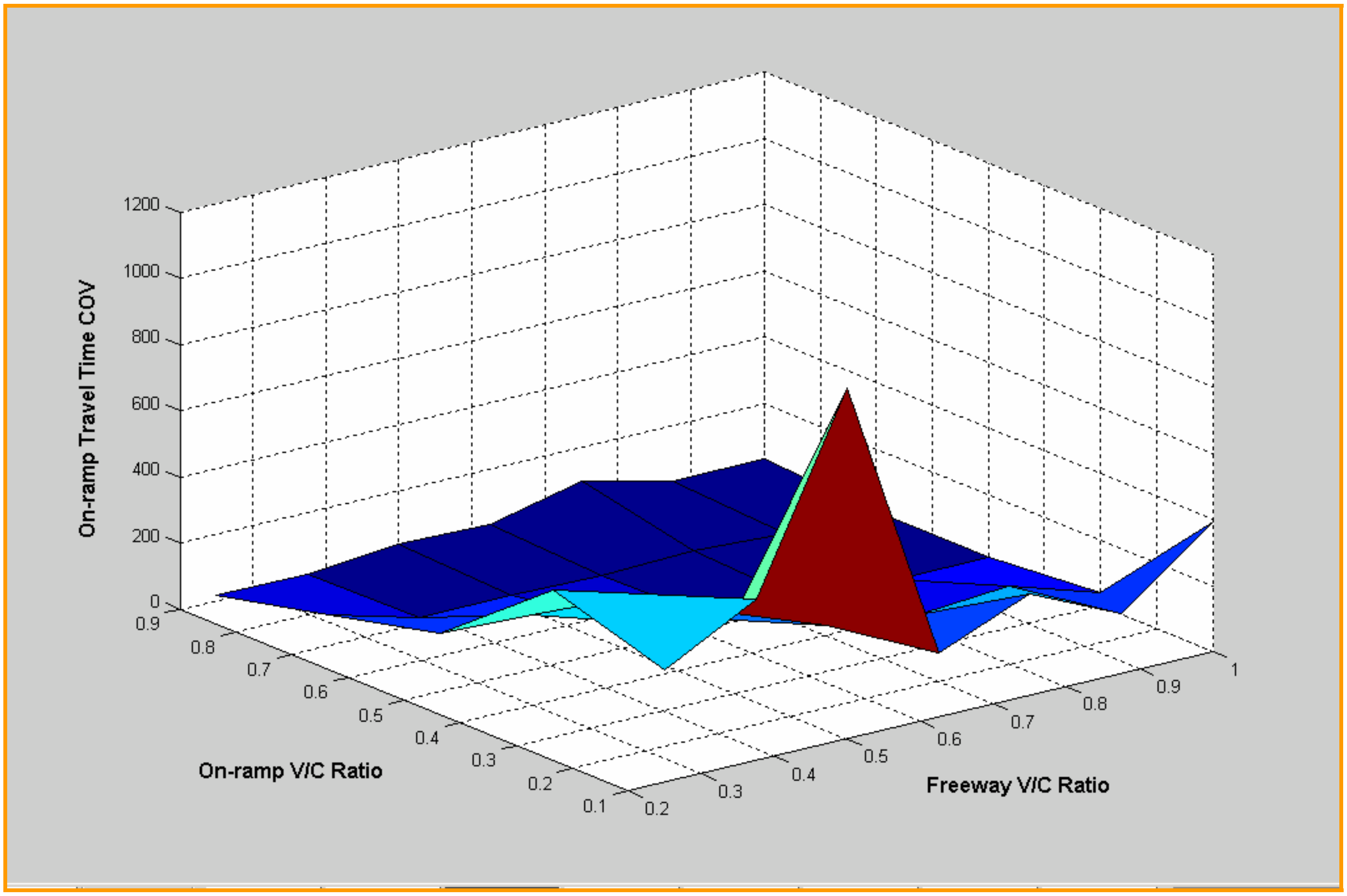

Figure 7: On-Ramp Travel Time COV v/s V/C$_{freeway}$ and V/C$_{on-ramp}$

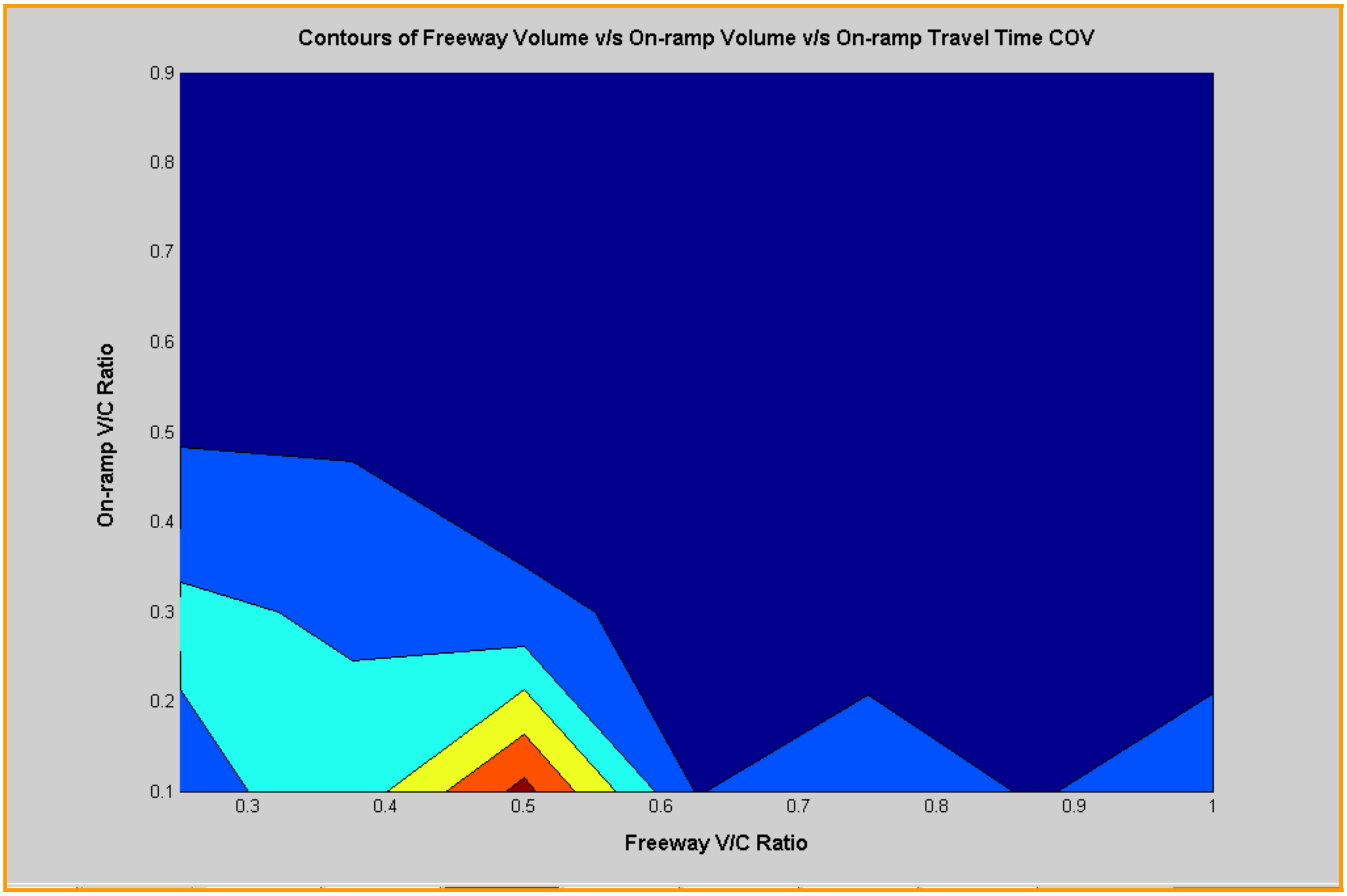

Figure 8: On-Ramp Travel Time COV Contours v/s $V/C_{freeway}$ and $V/C_{on\text{-}ramp}$

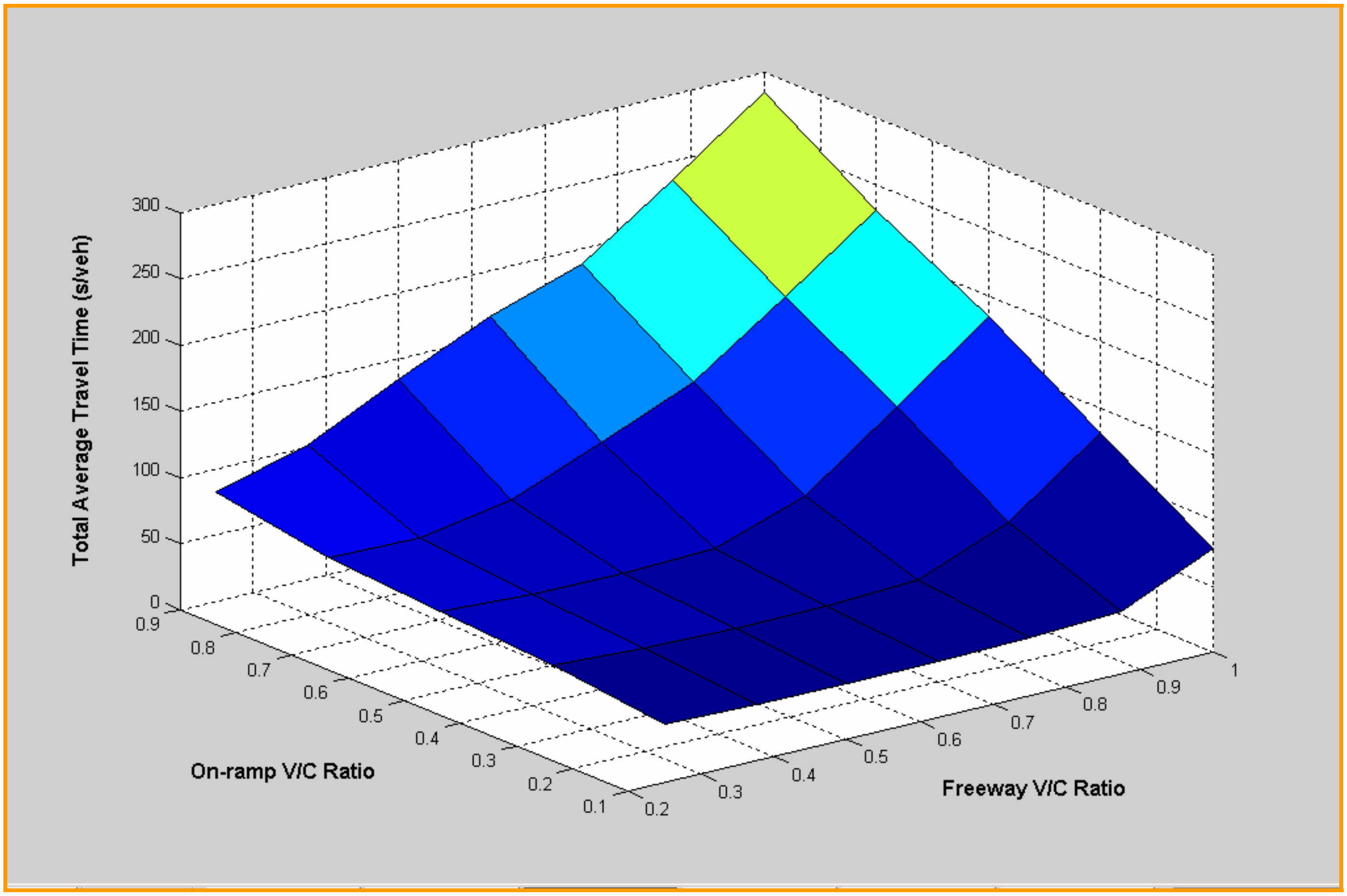

Figure 9: Total Average Travel Time v/s V/C$_{freeway}$ and V/C$_{on\text{-}ramp}$

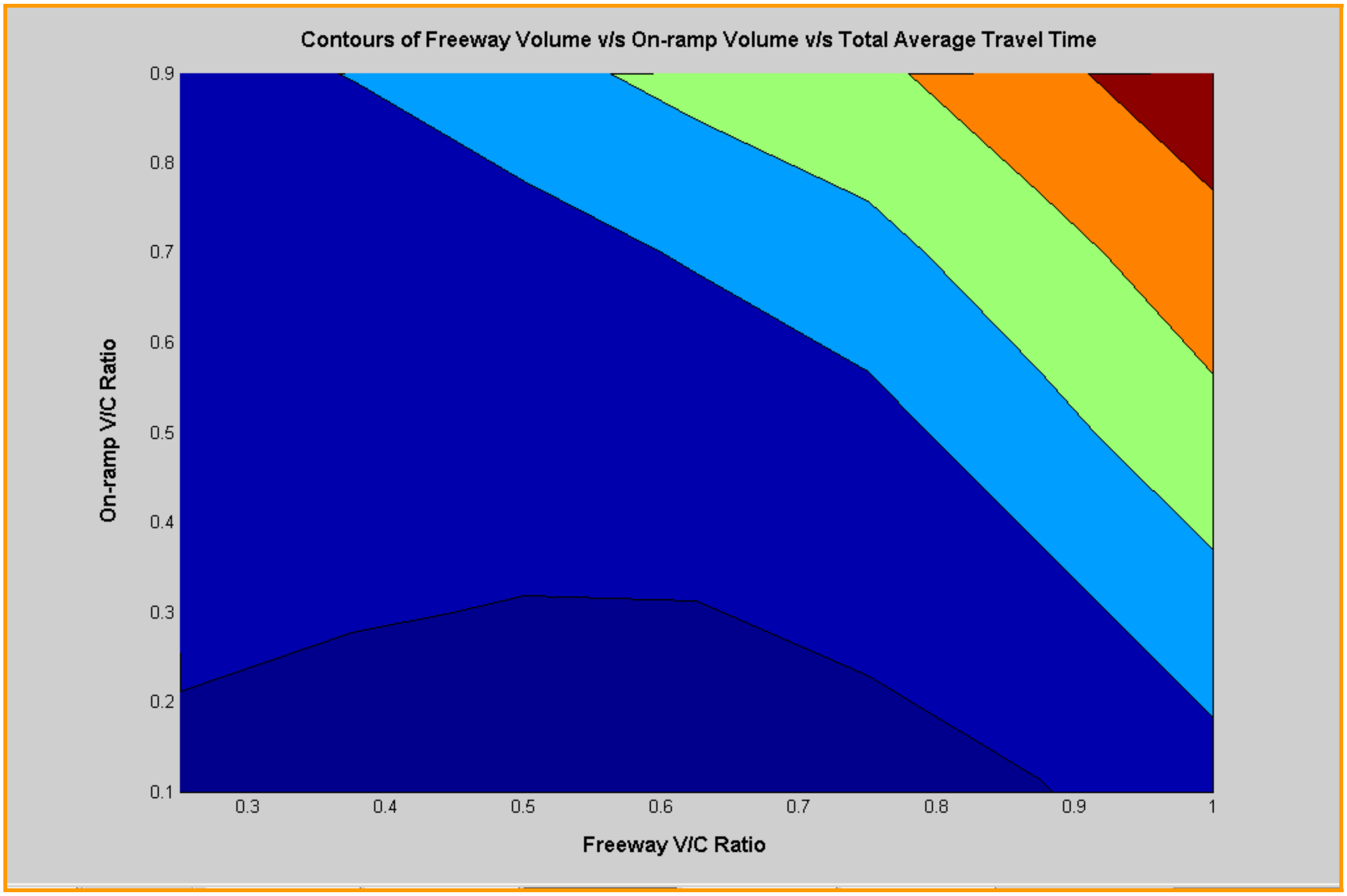

Figure 10: Total Average Travel Time Contours v/s $V/C_{freeway}$ and $V/C_{on\text{-}ramp}$

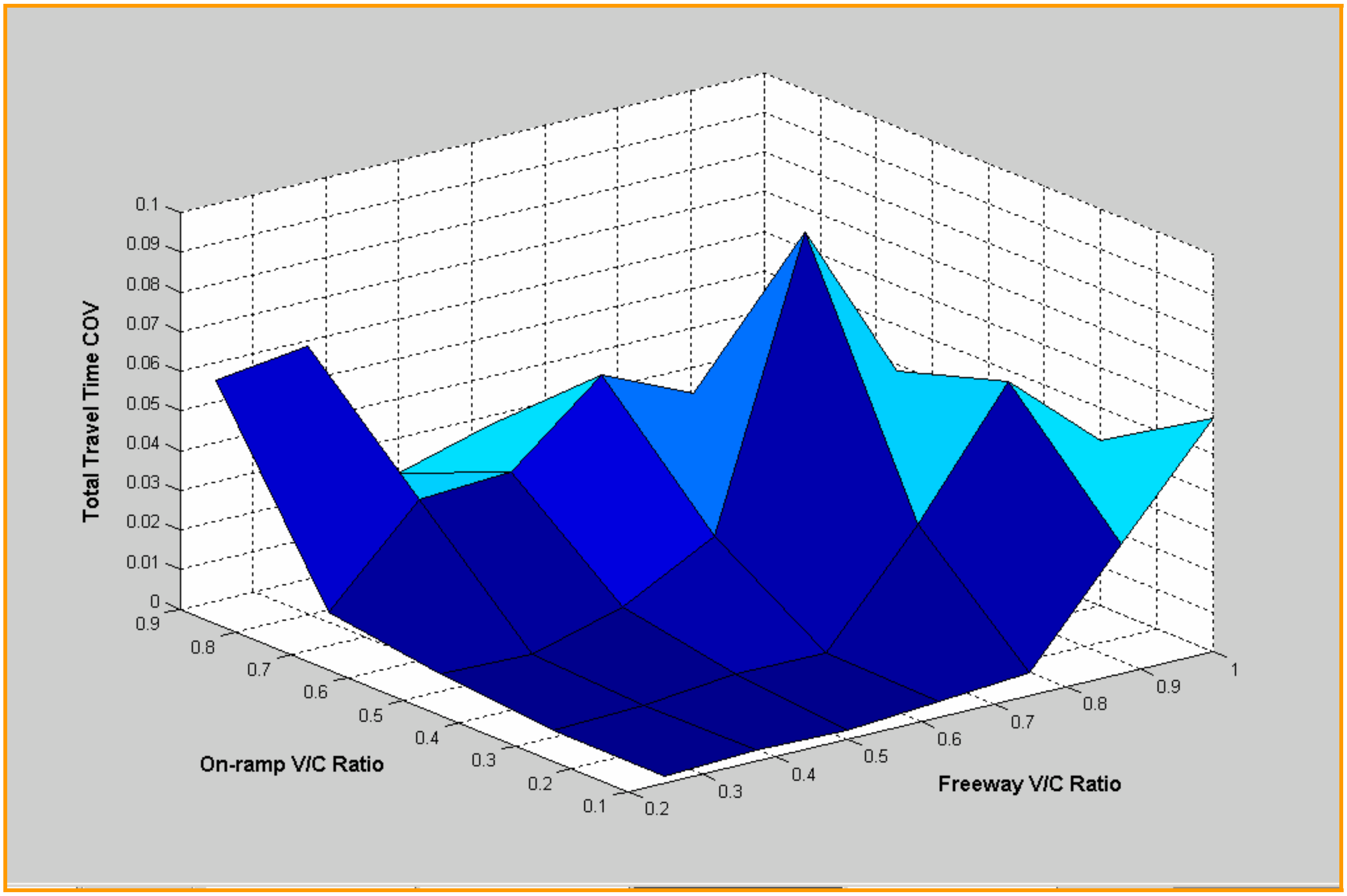

Figure 11: Total Travel Time COV v/s V/C$_{freeway}$ and V/C$_{on\text{-}ramp}$

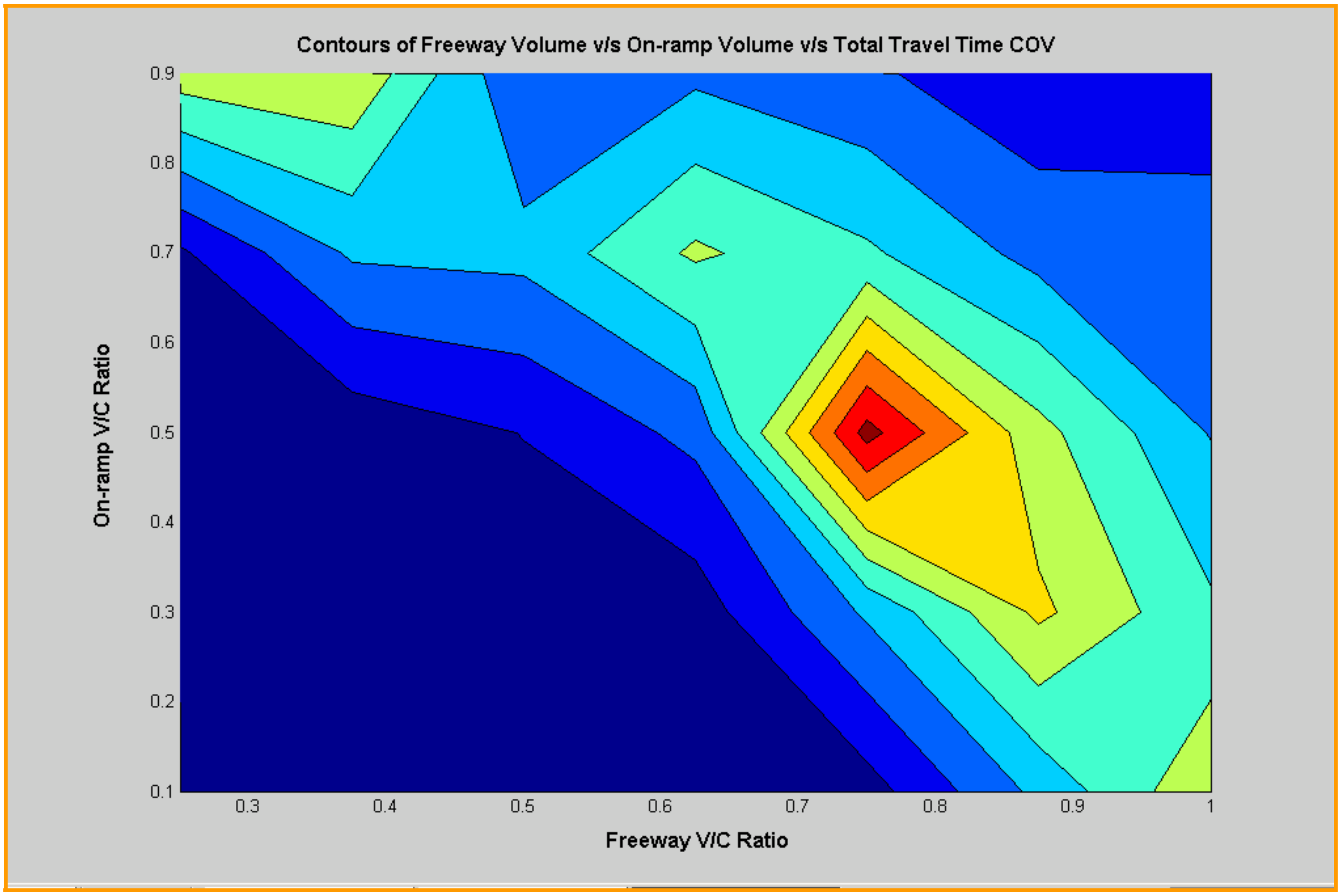

Figure 12: Total Travel Time COV Contours v/s $V/C_{freeway}$ and $V/C_{on\text{-}ramp}$

APPENDIX A.2

Off-Ramp Section with Deceleration Lane Length = 50m

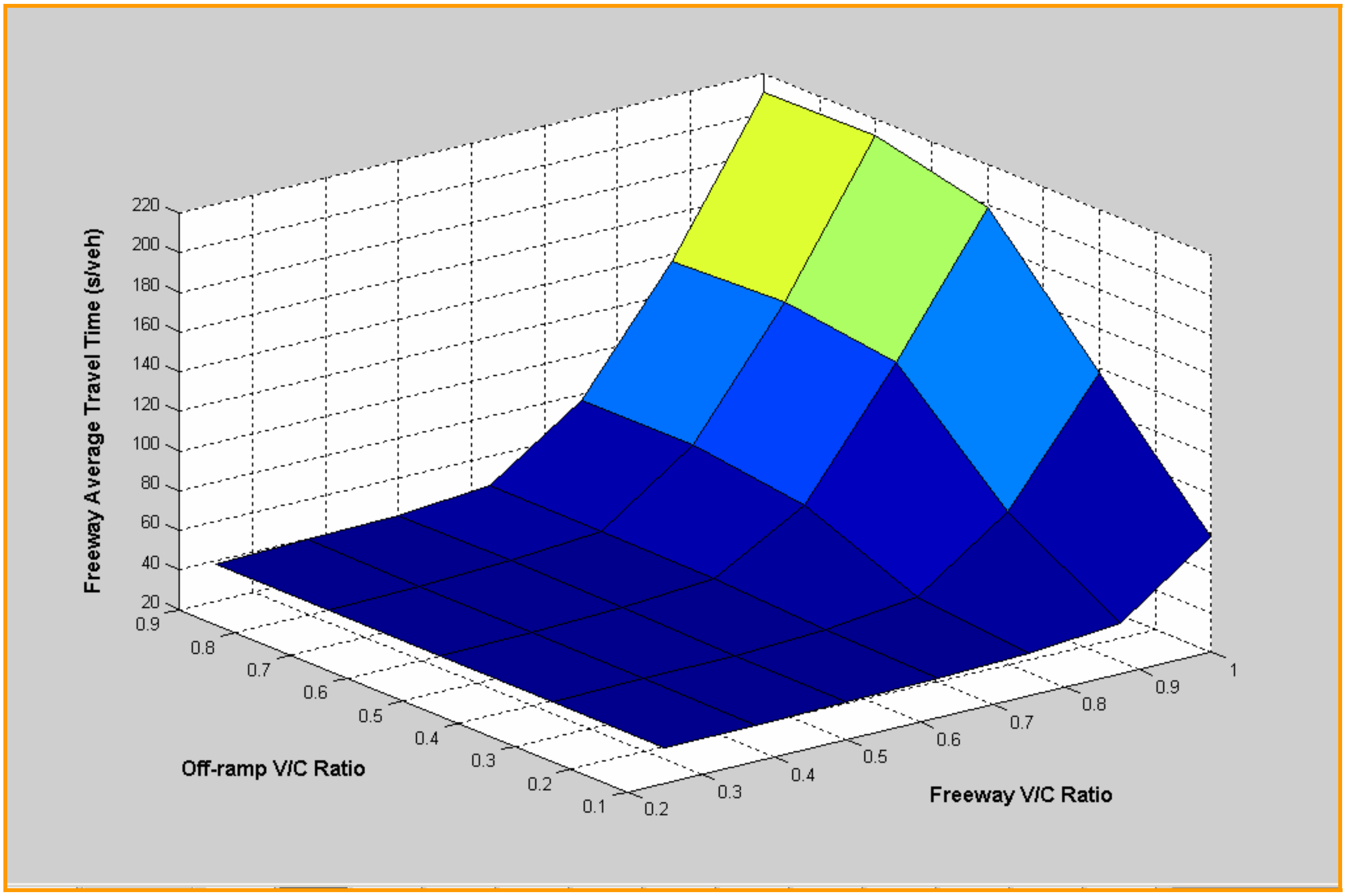

Figure 1: Freeway Average Travel Time v/s $V/C_{freeway}$ and $V/C_{off\text{-}ramp}$

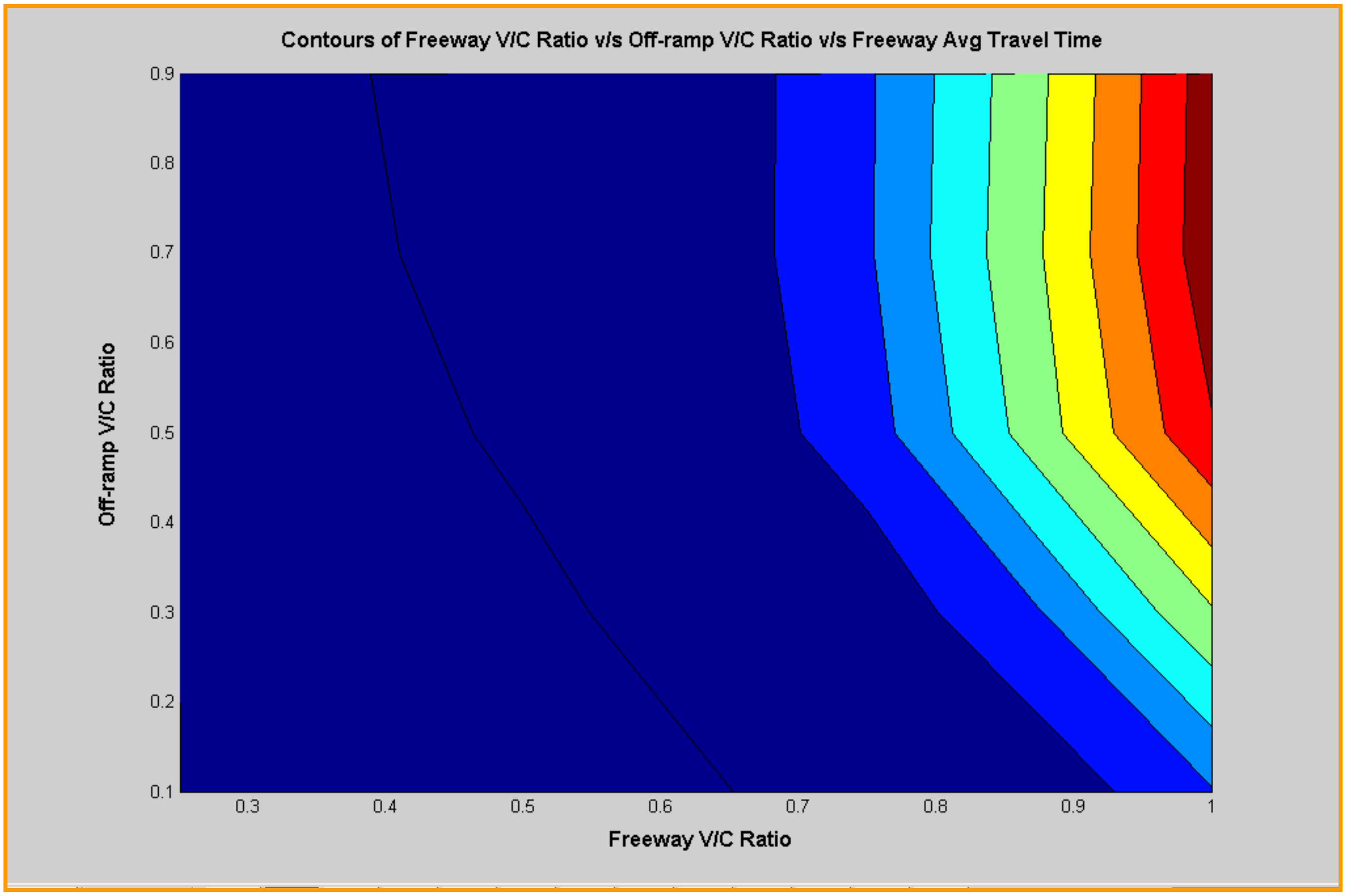

Figure 2: Freeway Average Travel Time Contours v/s $V/C_{freeway}$ and $V/C_{off\text{-}ramp}$

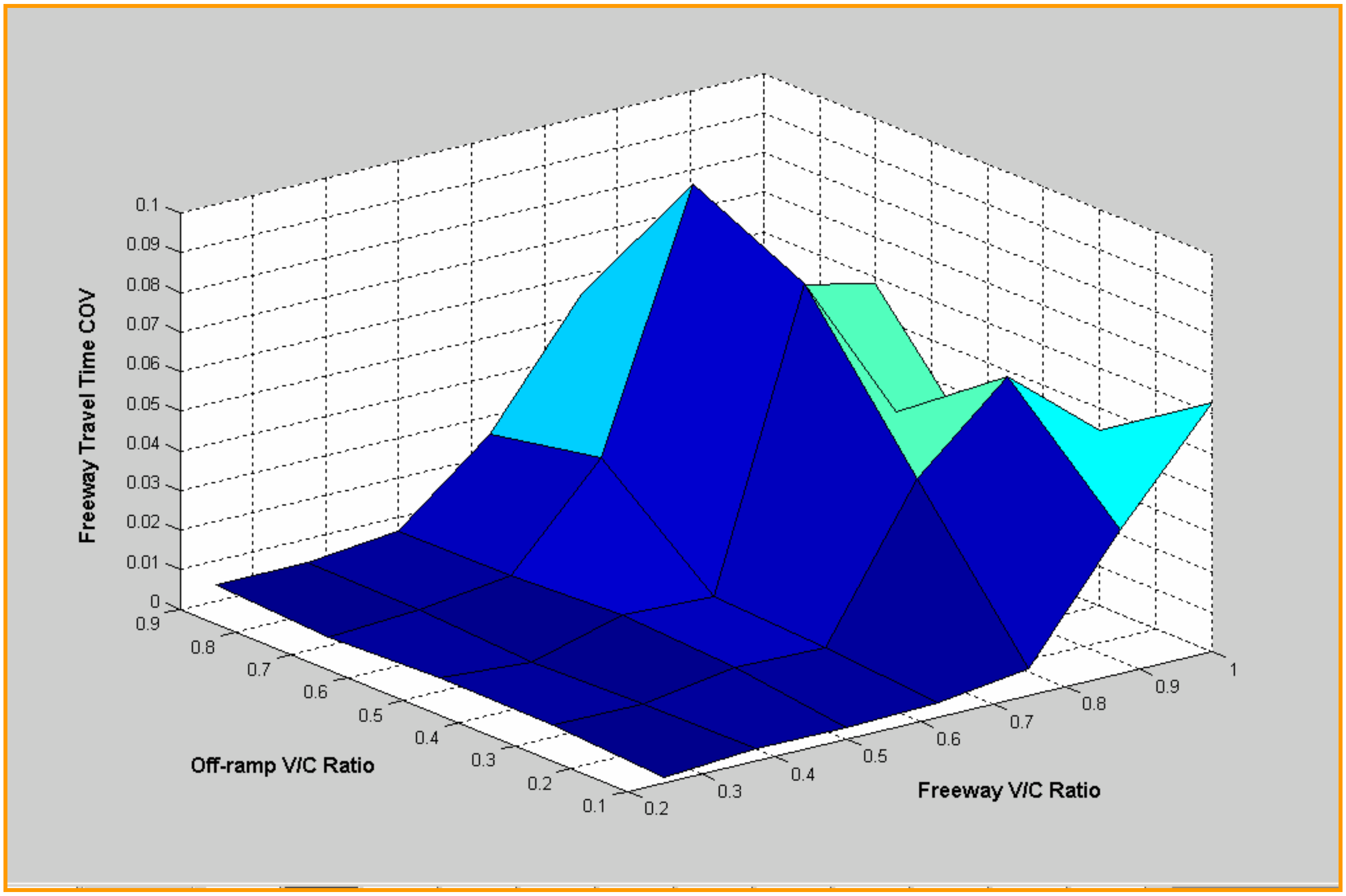

Figure 3: Freeway Travel Time COV v/s $V/C_{freeway}$ and $V/C_{off\text{-}ramp}$

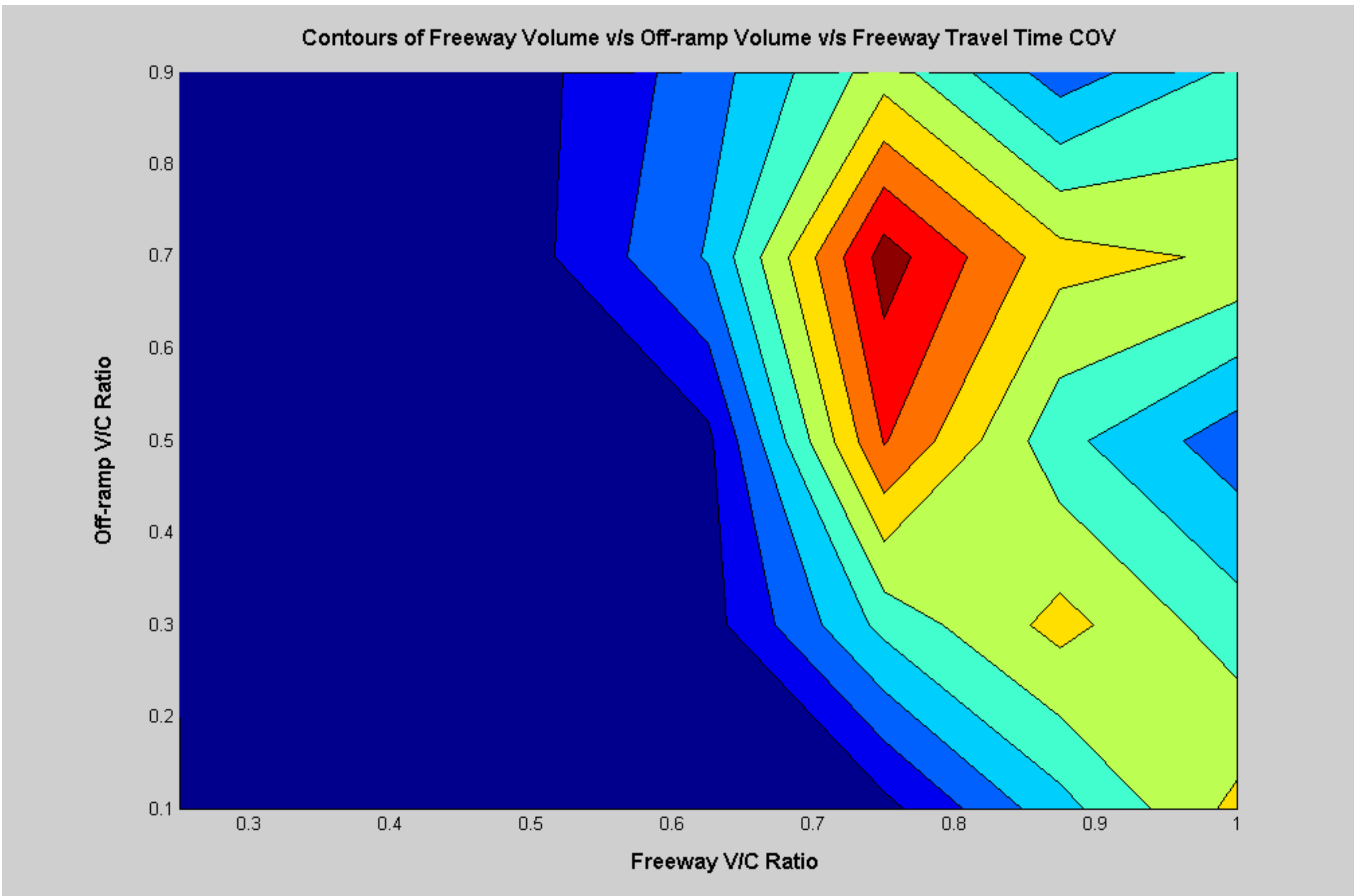

Figure 4: Freeway Travel Time COV Contours v/s $V/C_{freeway}$ and $V/C_{off\text{-}ramp}$

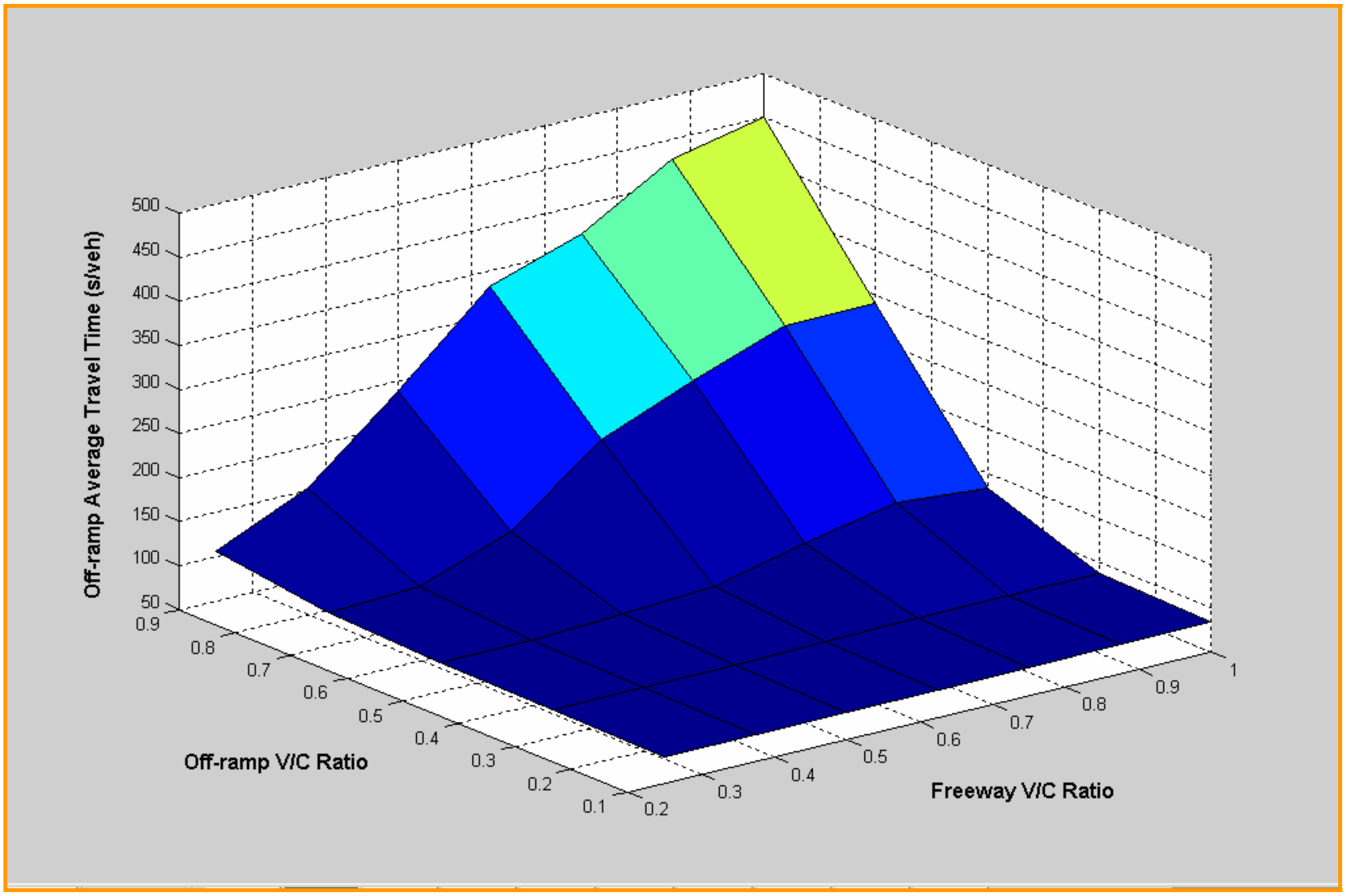

Figure 5: Off-Ramp Average Travel Time v/s V/C$_{freeway}$ and V/C$_{off\text{-}ramp}$

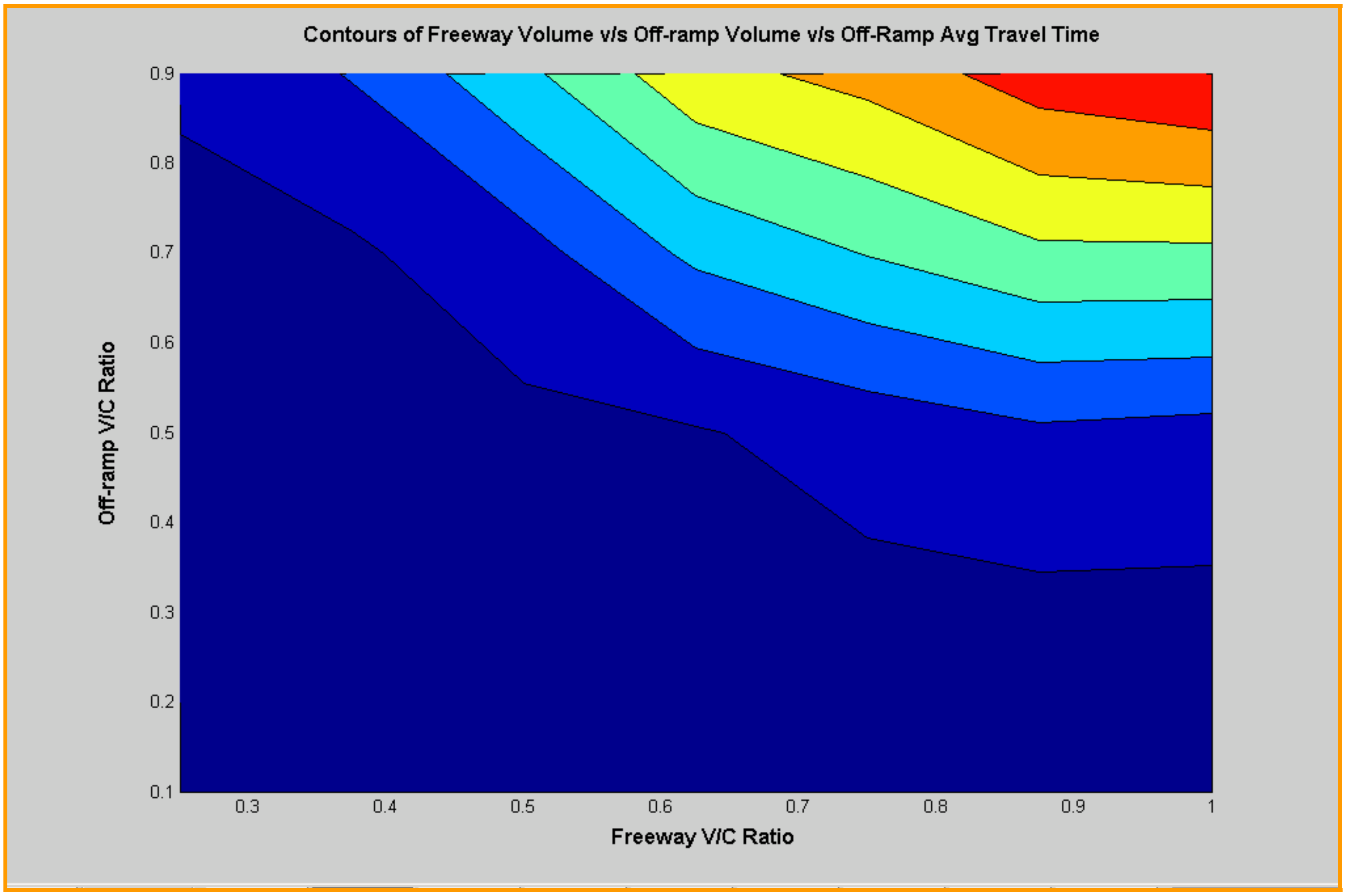

Figure 6: Off-Ramp Average Travel Time Contours v/s $V/C_{freeway}$ and $V/C_{off\text{-}ramp}$

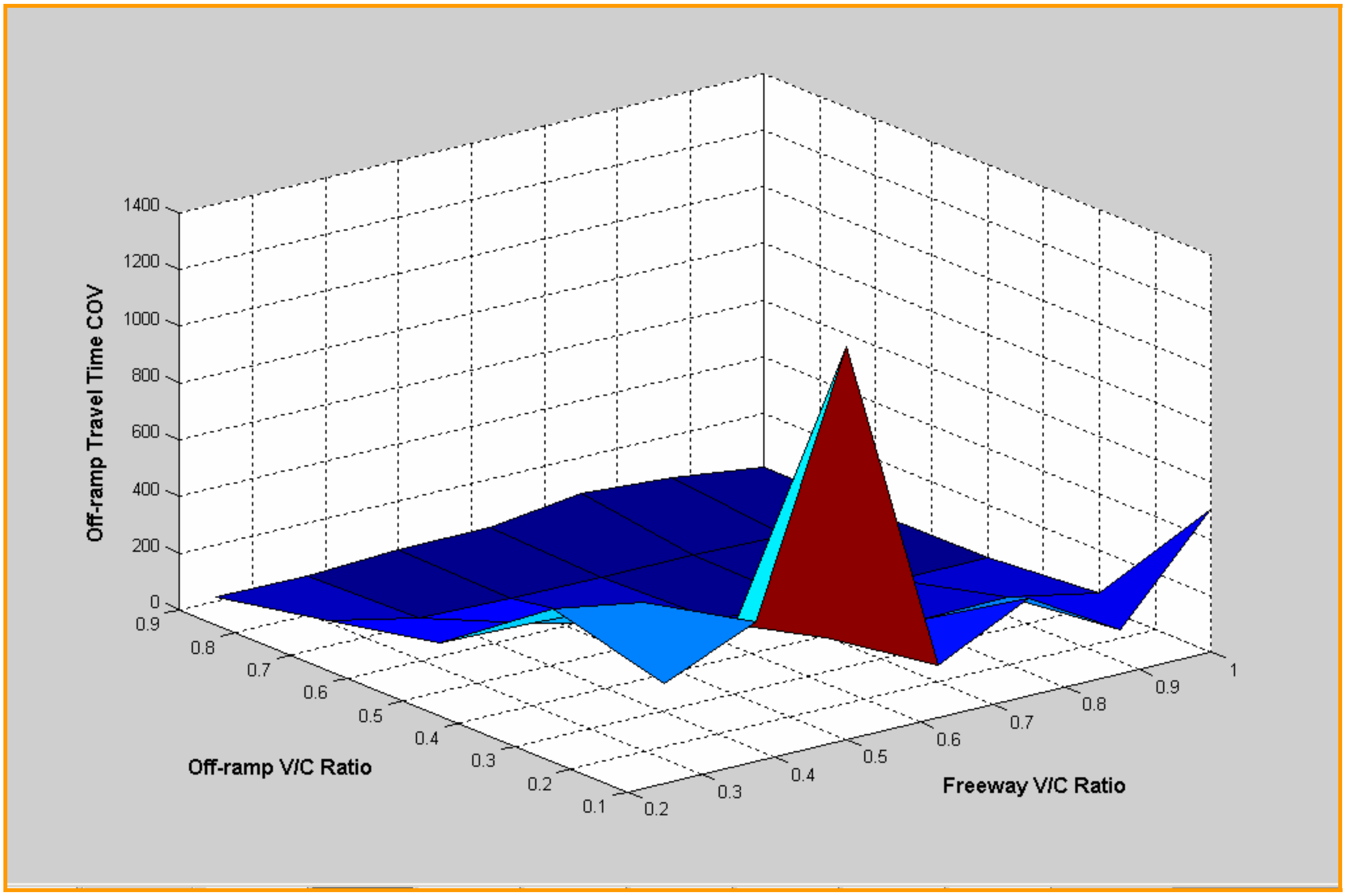

Figure 7: Off-Ramp Travel Time COV v/s V/C$_{freeway}$ and V/C$_{off\text{-}ramp}$

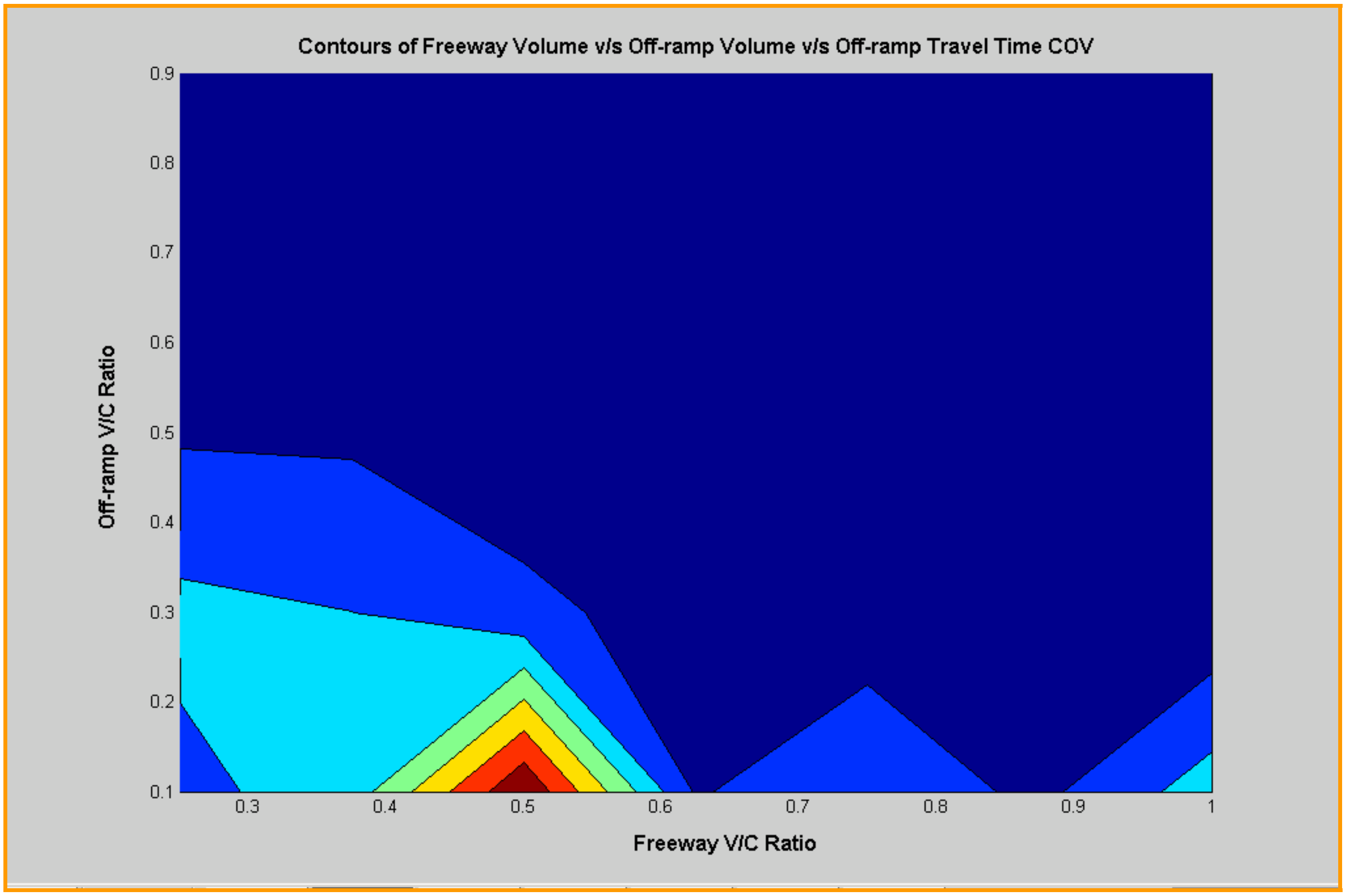

Figure 8: Off-Ramp Travel Time COV Contours v/s $V/C_{freeway}$ and $V/C_{off\text{-}ramp}$

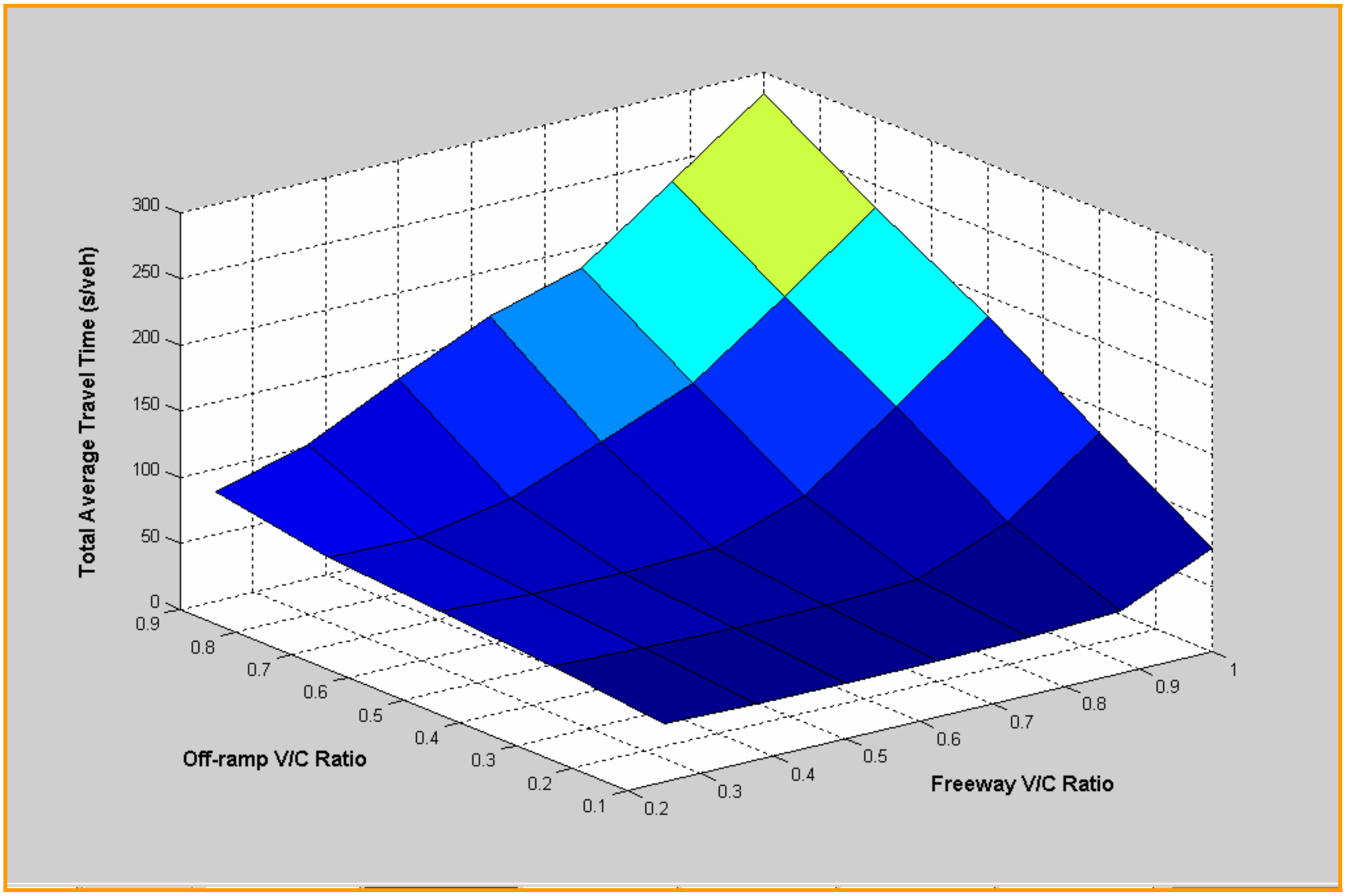

Figure 9: Total Average Travel Time v/s V/C$_{freeway}$ and V/C$_{off\text{-}ramp}$

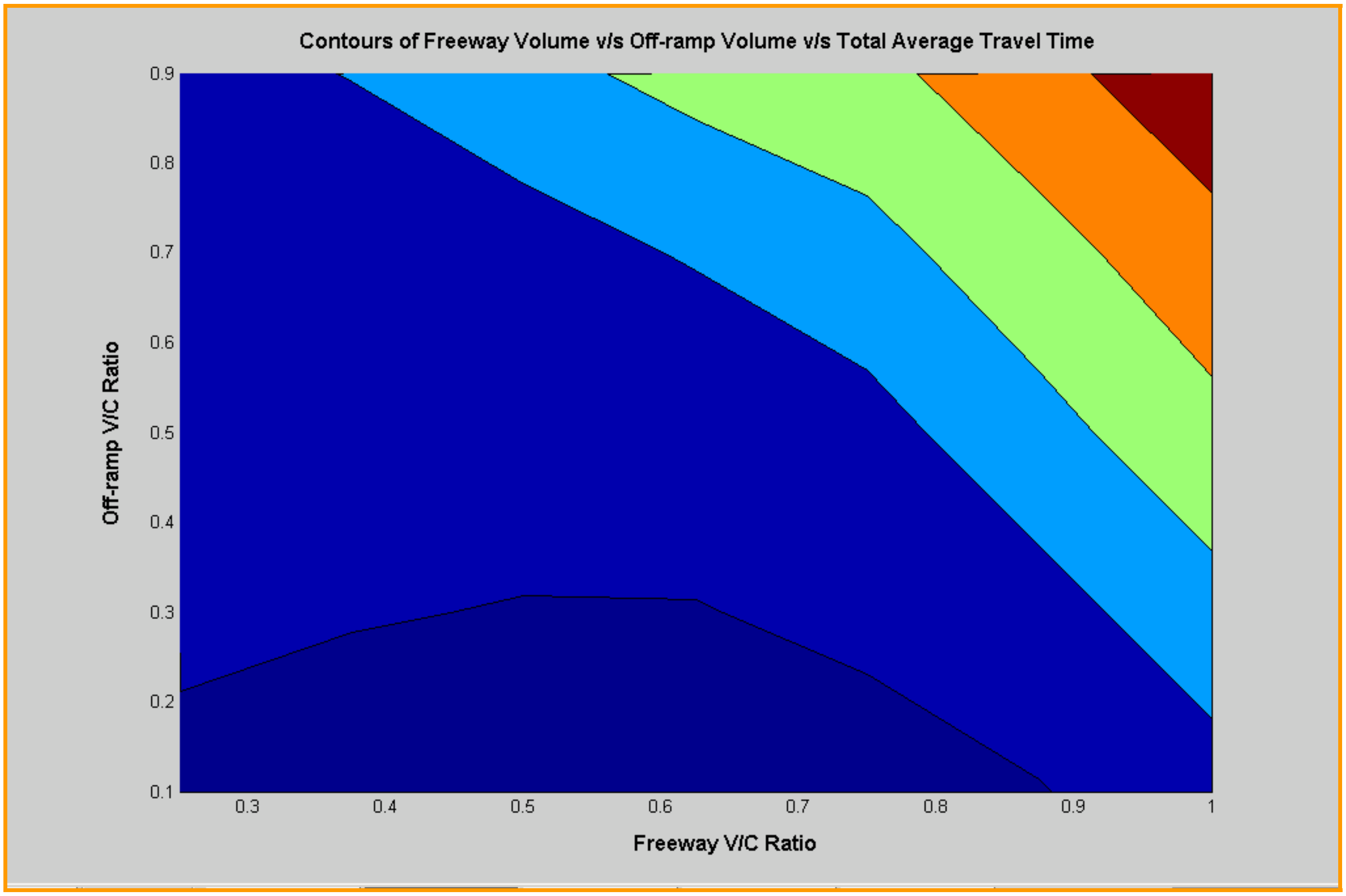

Figure 10: Total Average Travel Time Contours v/s $V/C_{freeway}$ and $V/C_{off\text{-}ramp}$

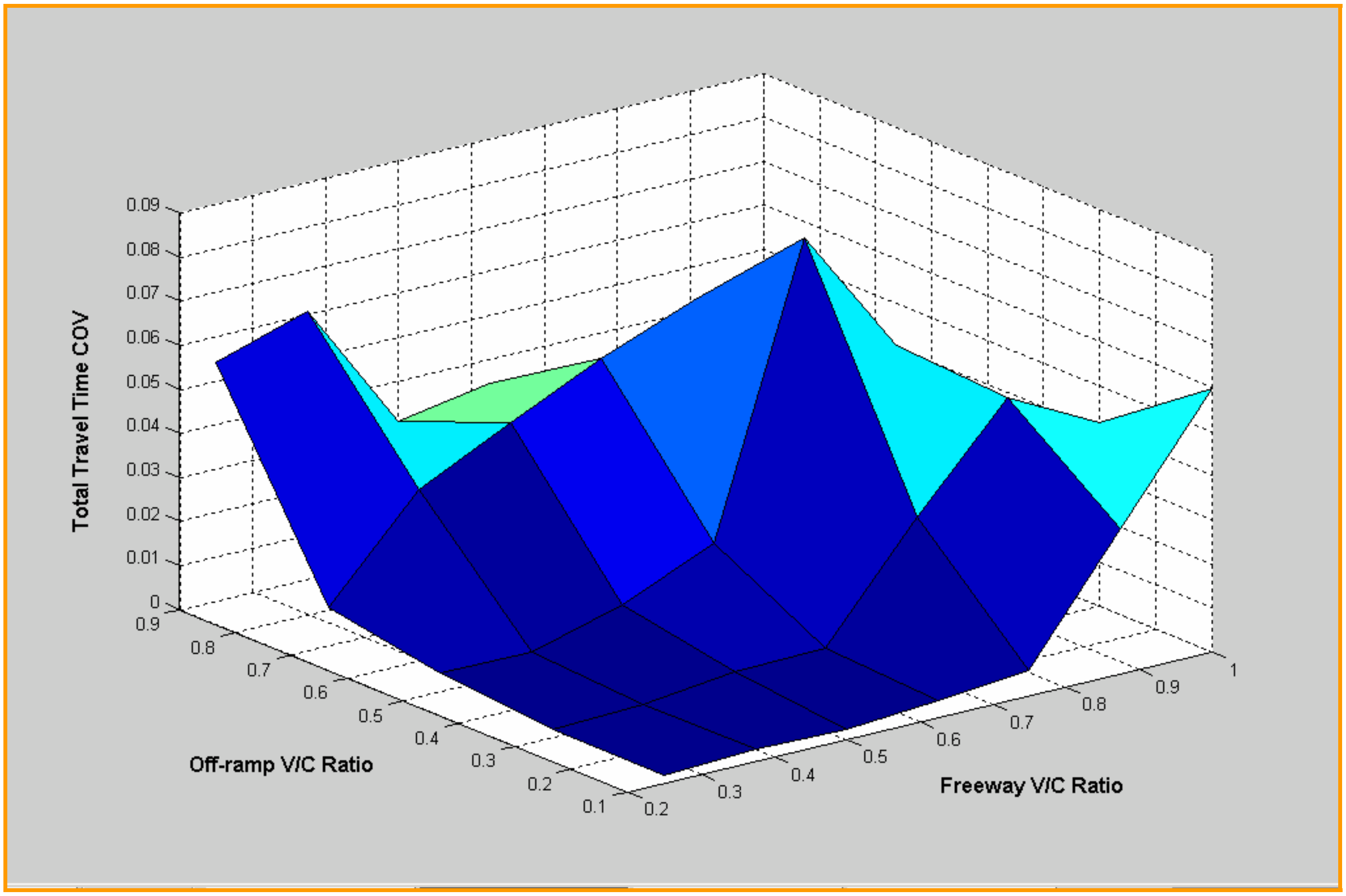

Figure 11: Total Travel Time COV v/s $V/C_{freeway}$ and $V/C_{off\text{-}ramp}$

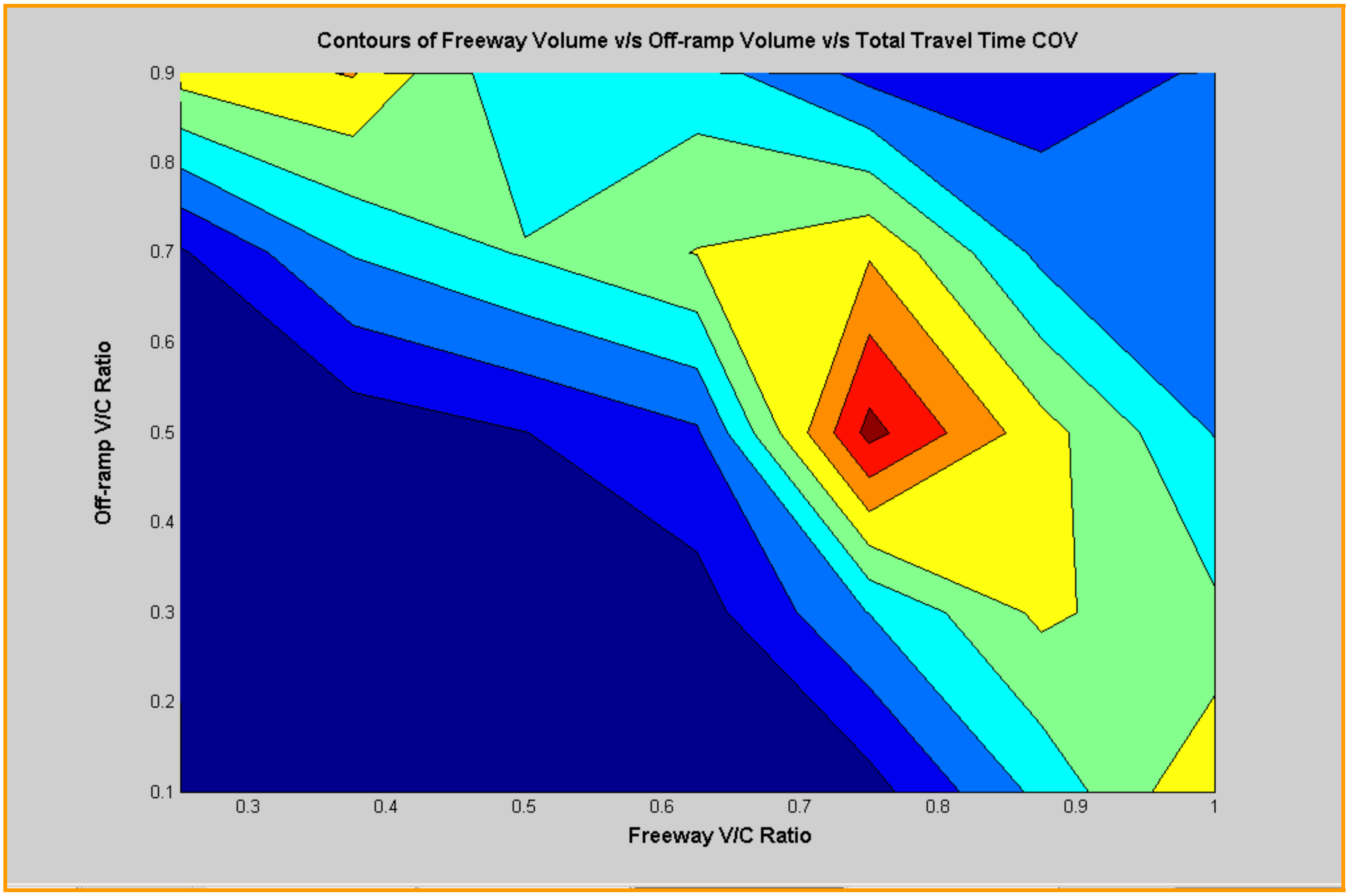

Figure 12: Total Travel Time COV Contours v/s $V/C_{freeway}$ and $V/C_{off\text{-}ramp}$

APPENDIX A.3

Weaving Section with Weave Lane Length = 100m

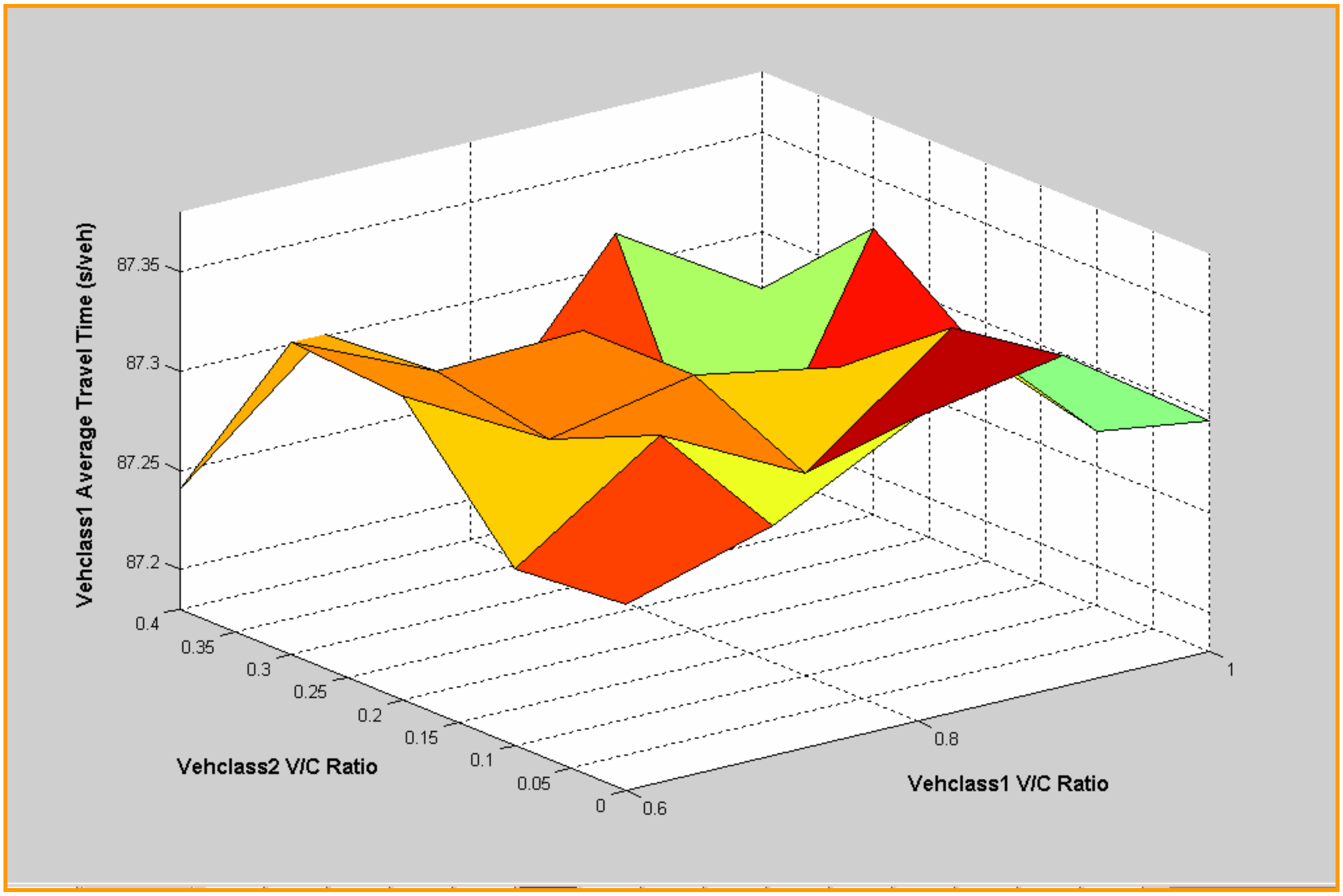

Figure 1: Vehicle Class 1 Average Travel Time v/s V/C$_{\text{veh class 1}}$ and V/C$_{\text{veh class 2}}$

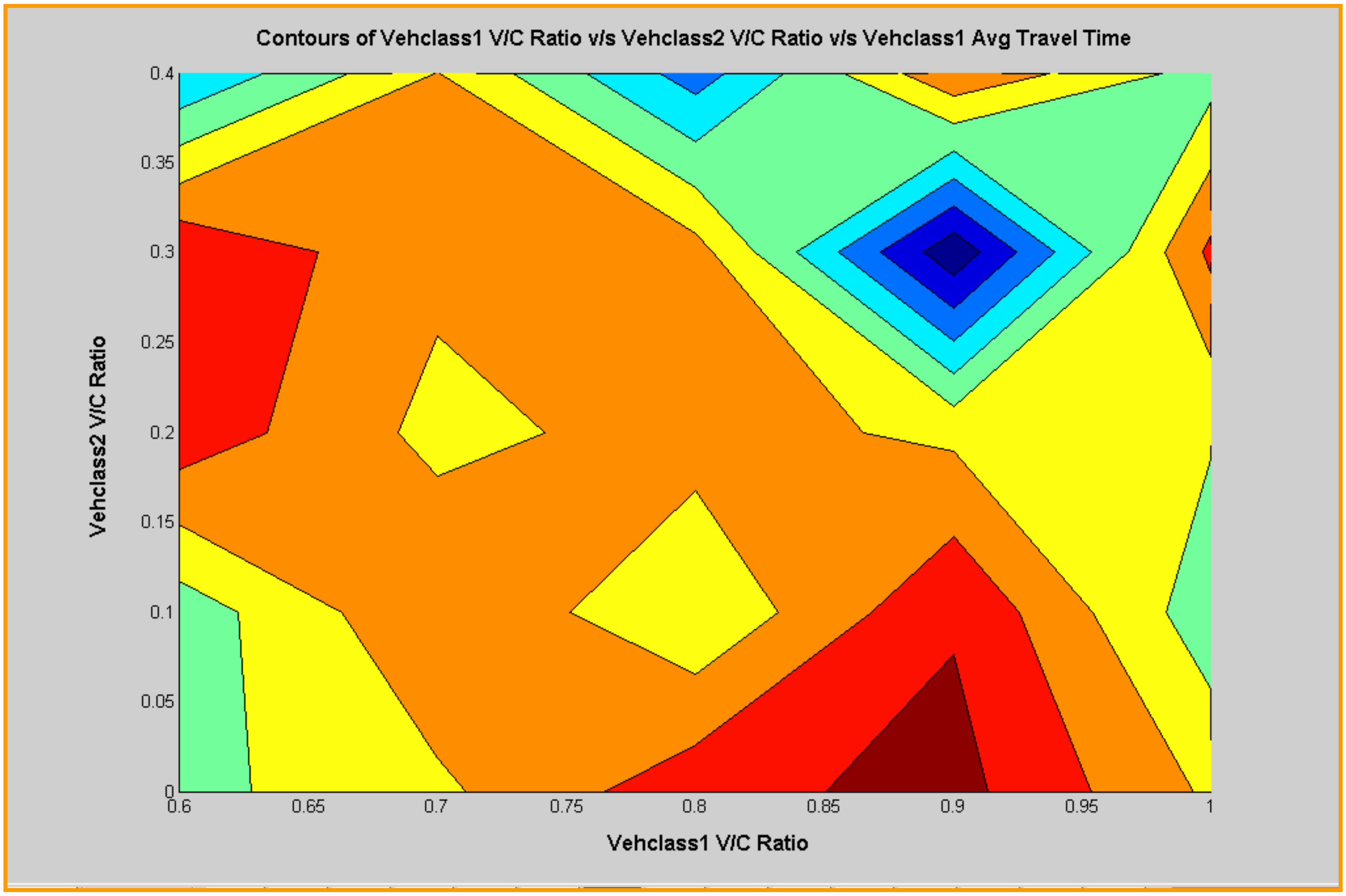

Figure 2: Vehicle Class 1 Average Travel Time Contours v/s $V/C_{veh\ class\ 1}$ and $V/C_{veh\ class\ 2}$

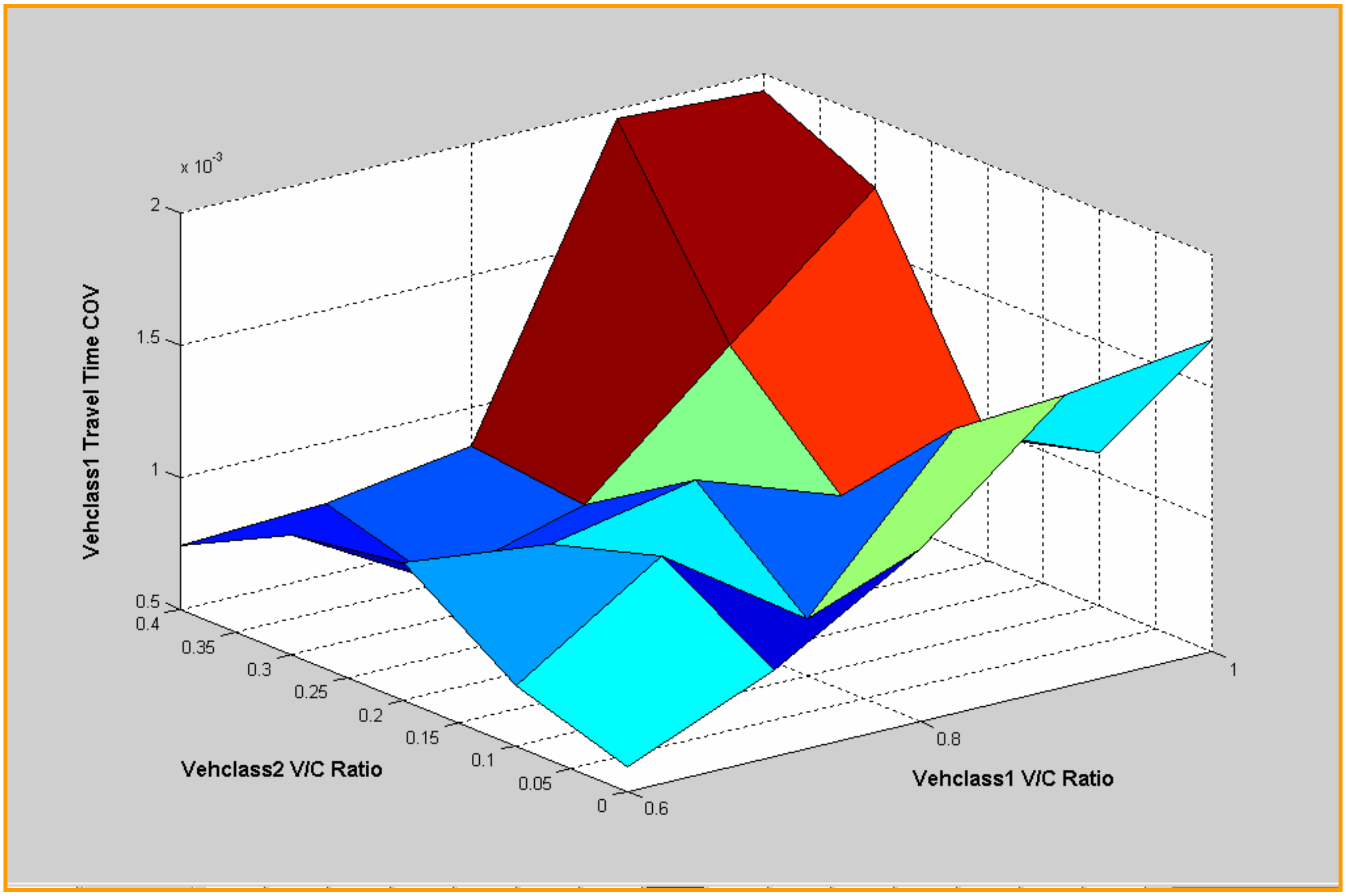

Figure 3: Vehicle Class 1 Travel Time COV v/s V/C$_{\text{veh class 1}}$ and V/C$_{\text{veh class 2}}$

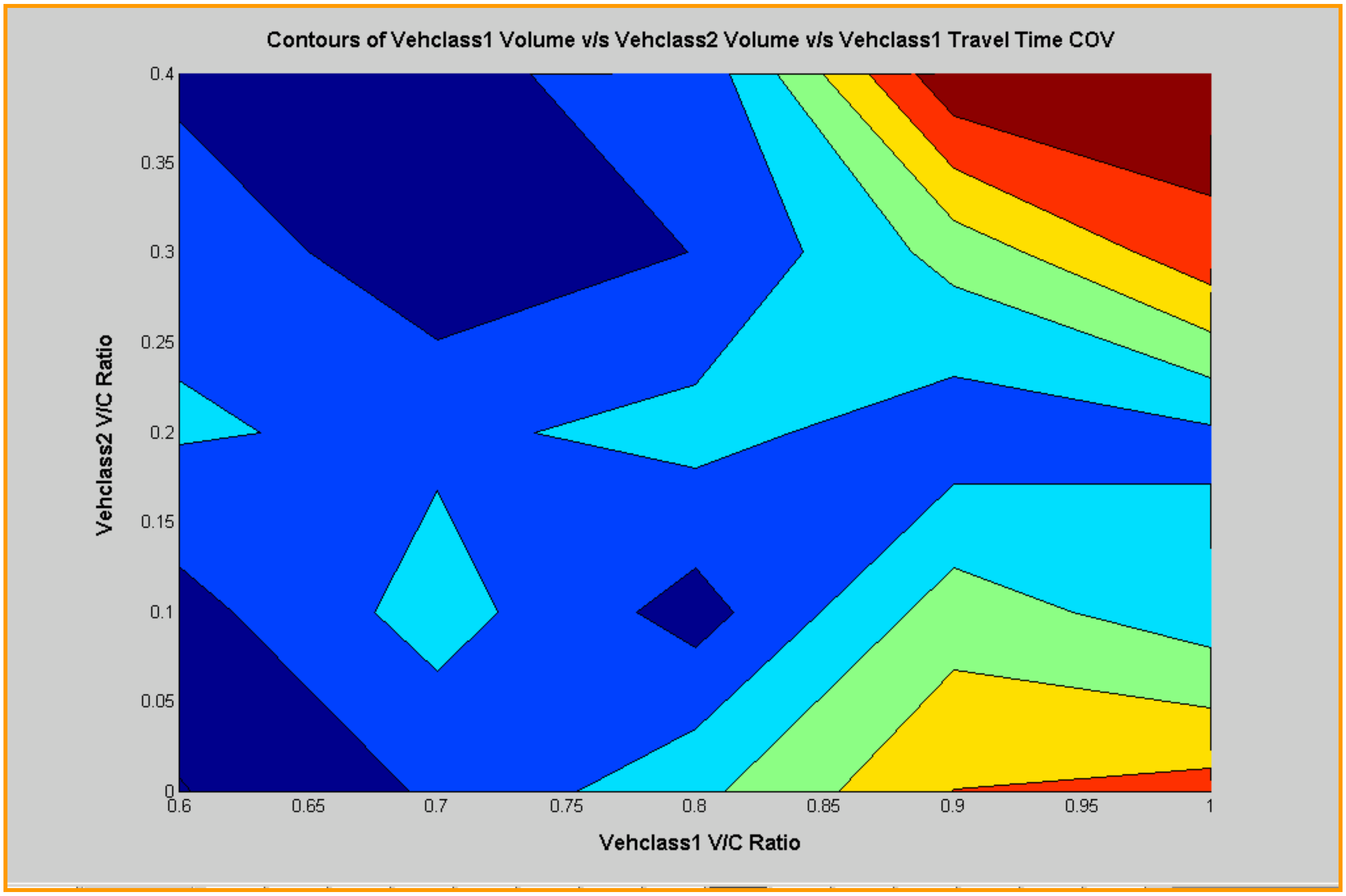

Figure 4: Vehicle Class 1 Travel Time COV Contours v/s $V/C_{veh\ class\ 1}$ and $V/C_{veh\ class\ 2}$

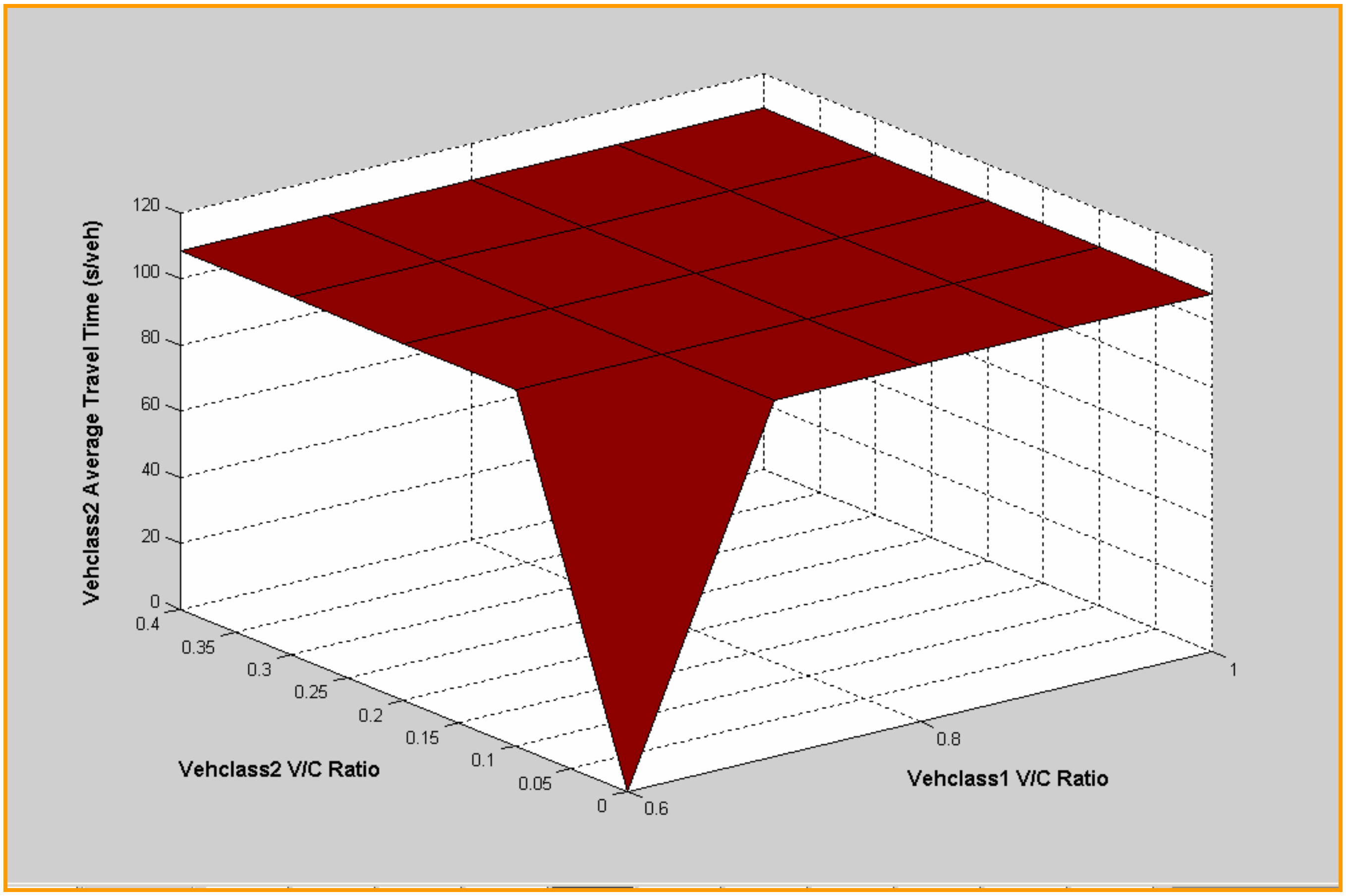

Figure 5: Vehicle Class 2 Average Travel Time v/s $V/C_{veh\ class\ 1}$ and $V/C_{veh\ class\ 2}$

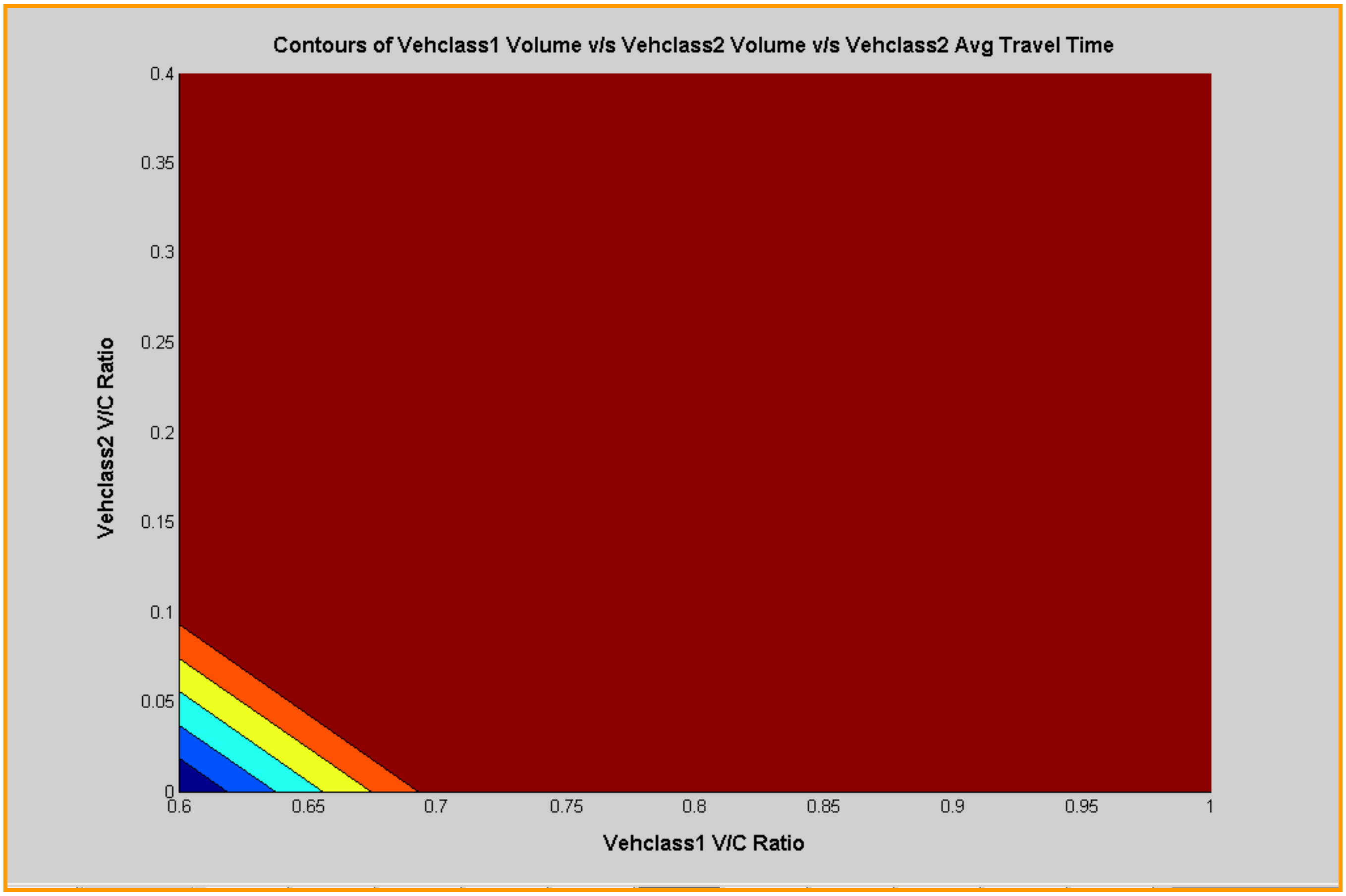

Figure 6: Vehicle Class 2 Average Travel Time Contours v/s $V/C_{veh\ class\ 1}$ and $V/C_{veh\ class\ 2}$

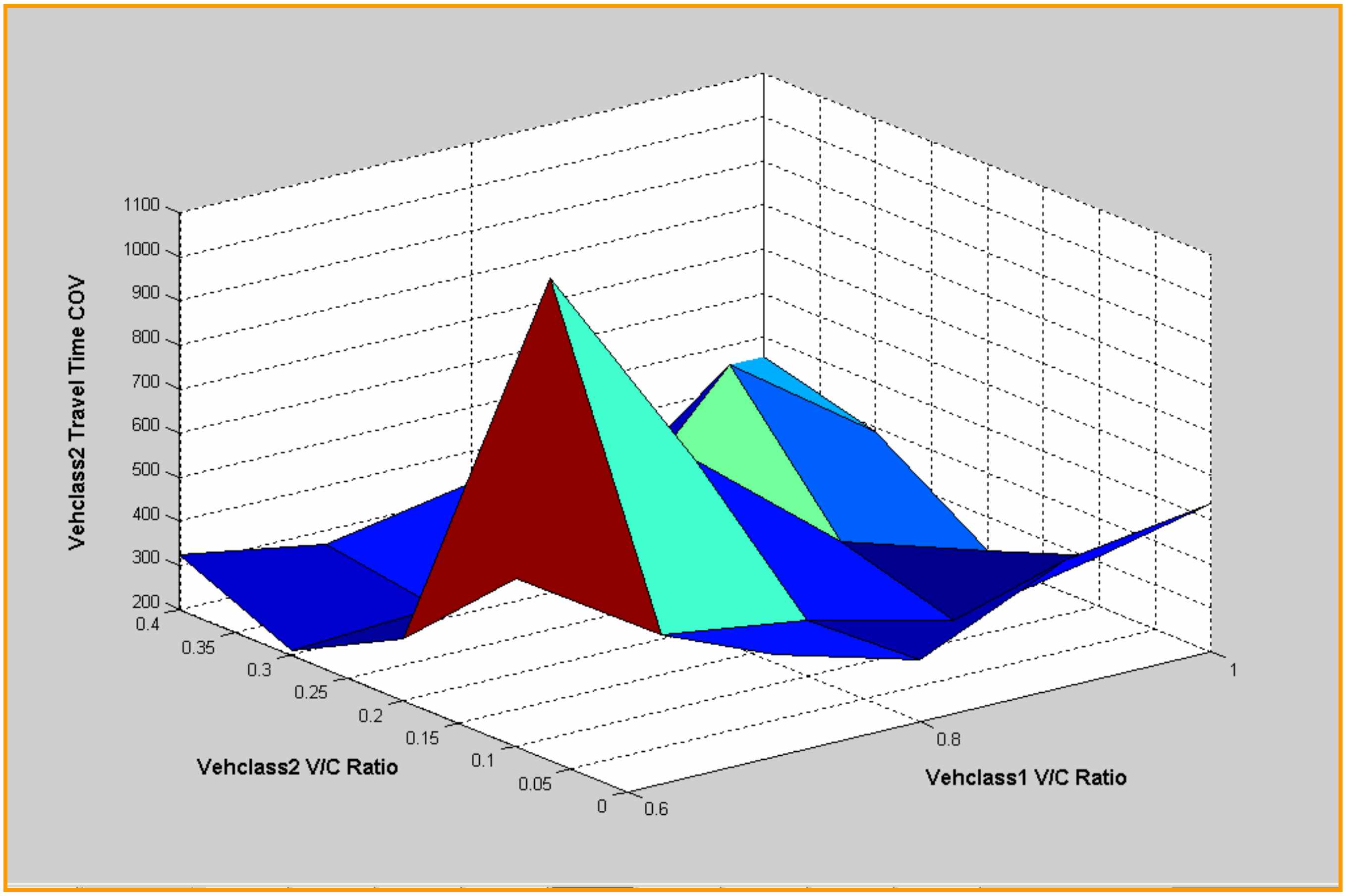

Figure 7: Vehicle Class 2 Travel Time COV v/s $V/C_{veh\ class\ 1}$ and $V/C_{veh\ class\ 2}$

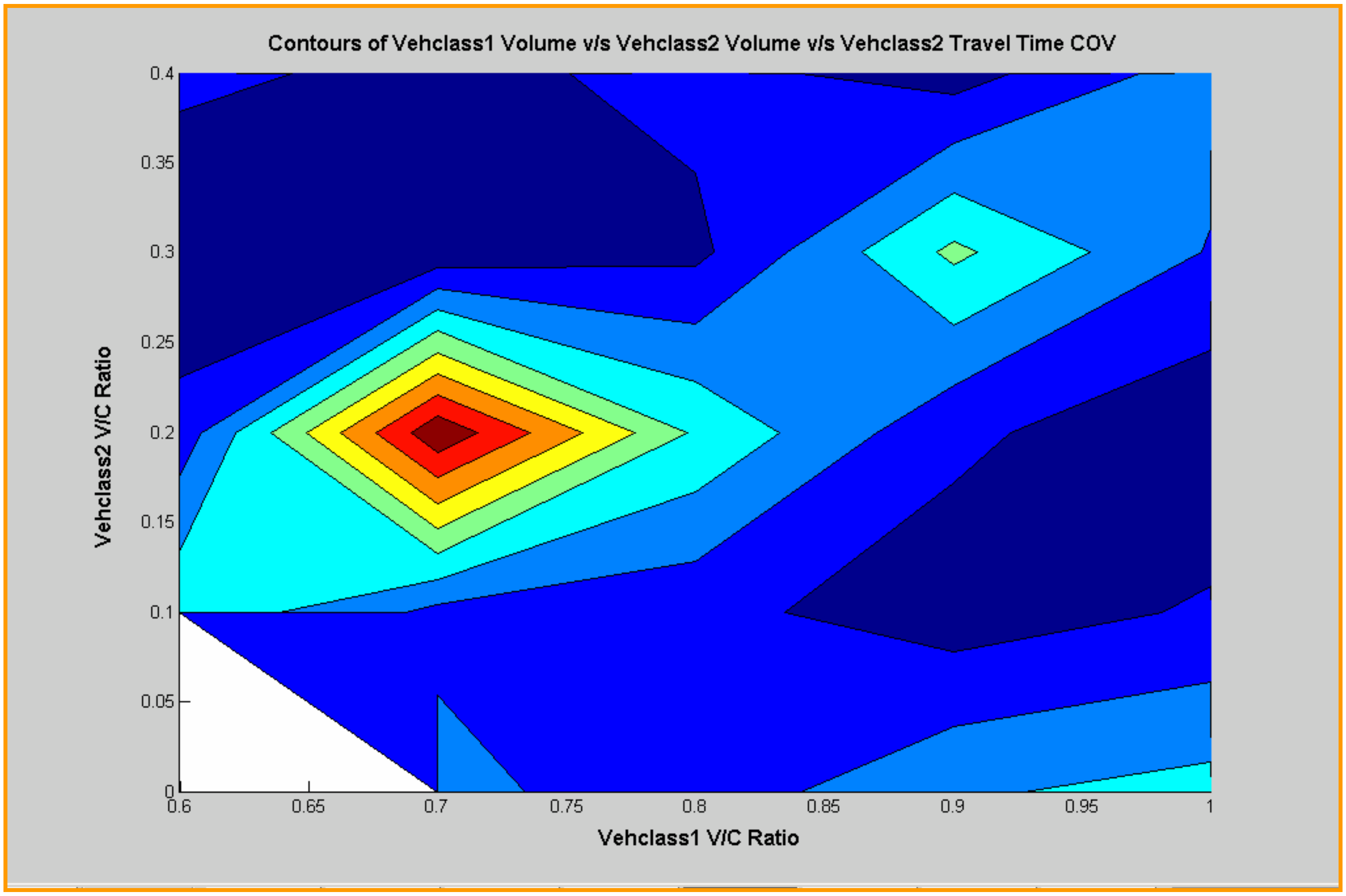

Figure 8: Vehicle Class 2 Travel Time COV Contours v/s $V/C_{veh\ class\ 1}$ and $V/C_{veh\ class\ 2}$

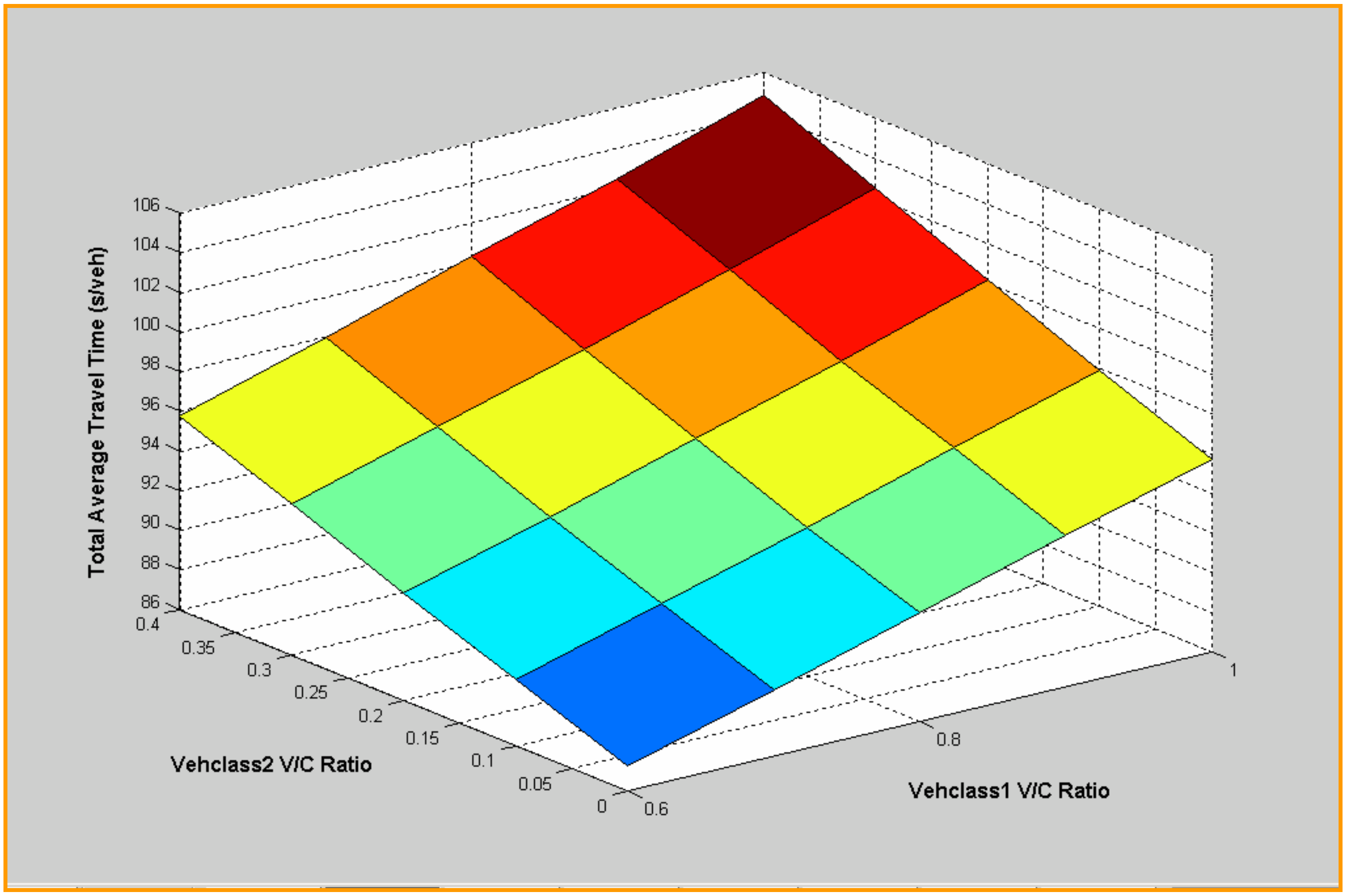

Figure 9: Total Average Travel Time v/s $V/C_{veh\ class\ 1}$ and $V/C_{veh\ class\ 2}$

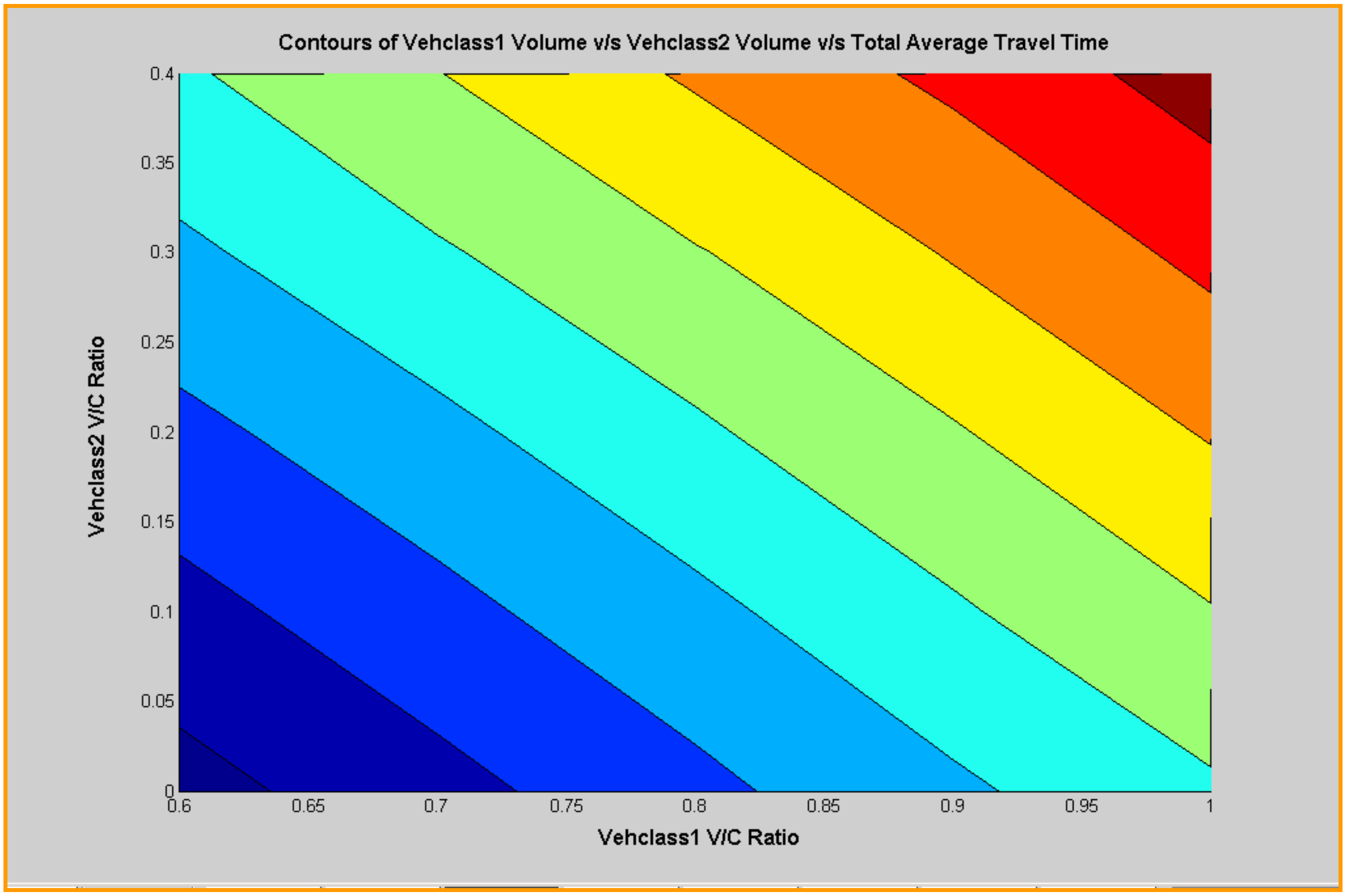

Figure 10: Total Average Travel Time Contours v/s $V/C_{\text{veh class 1}}$ and $V/C_{\text{veh class 2}}$

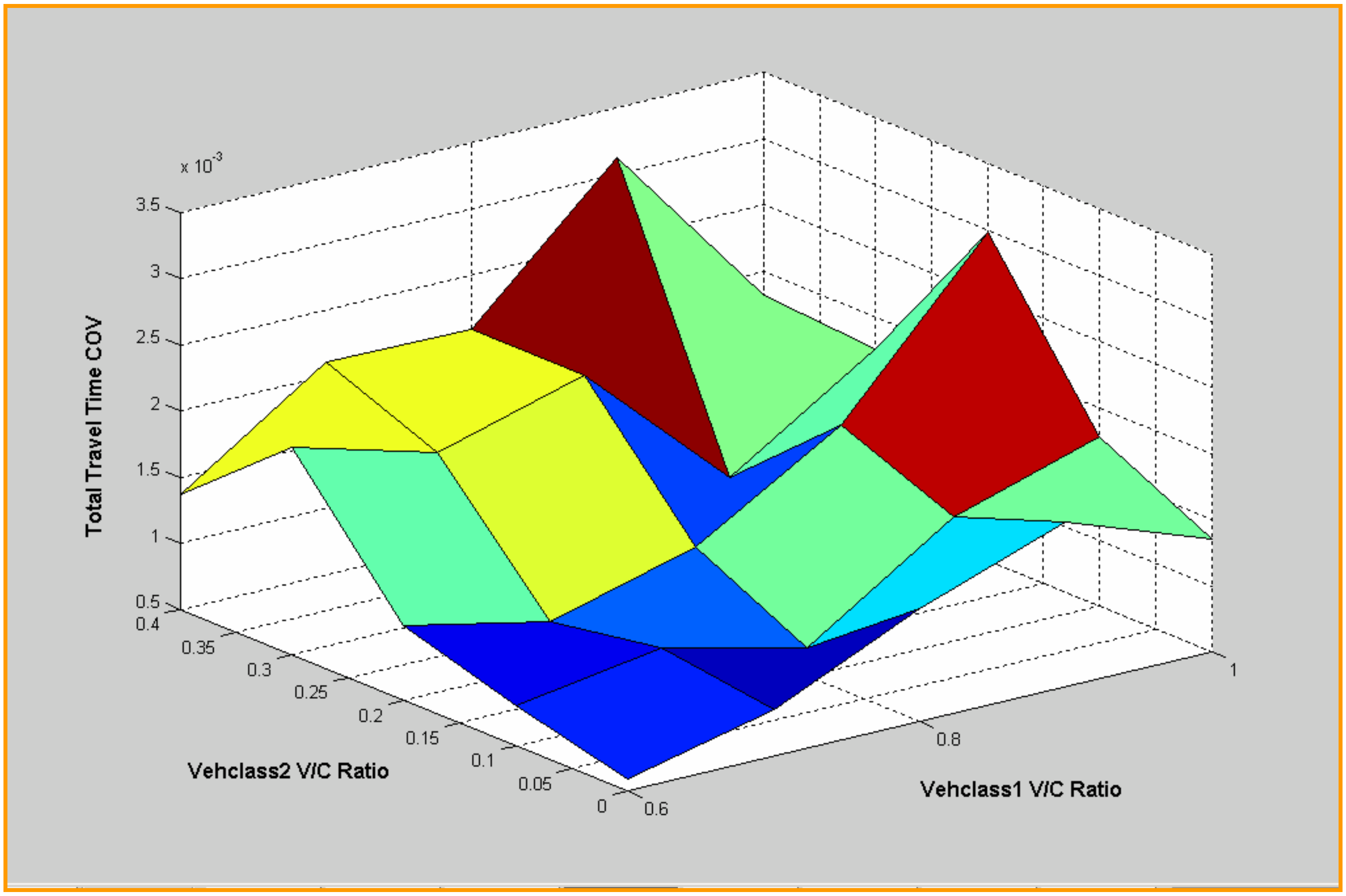

Figure 11: Total Travel Time COV v/s $V/C_{veh\ class\ 1}$ and $V/C_{veh\ class\ 2}$

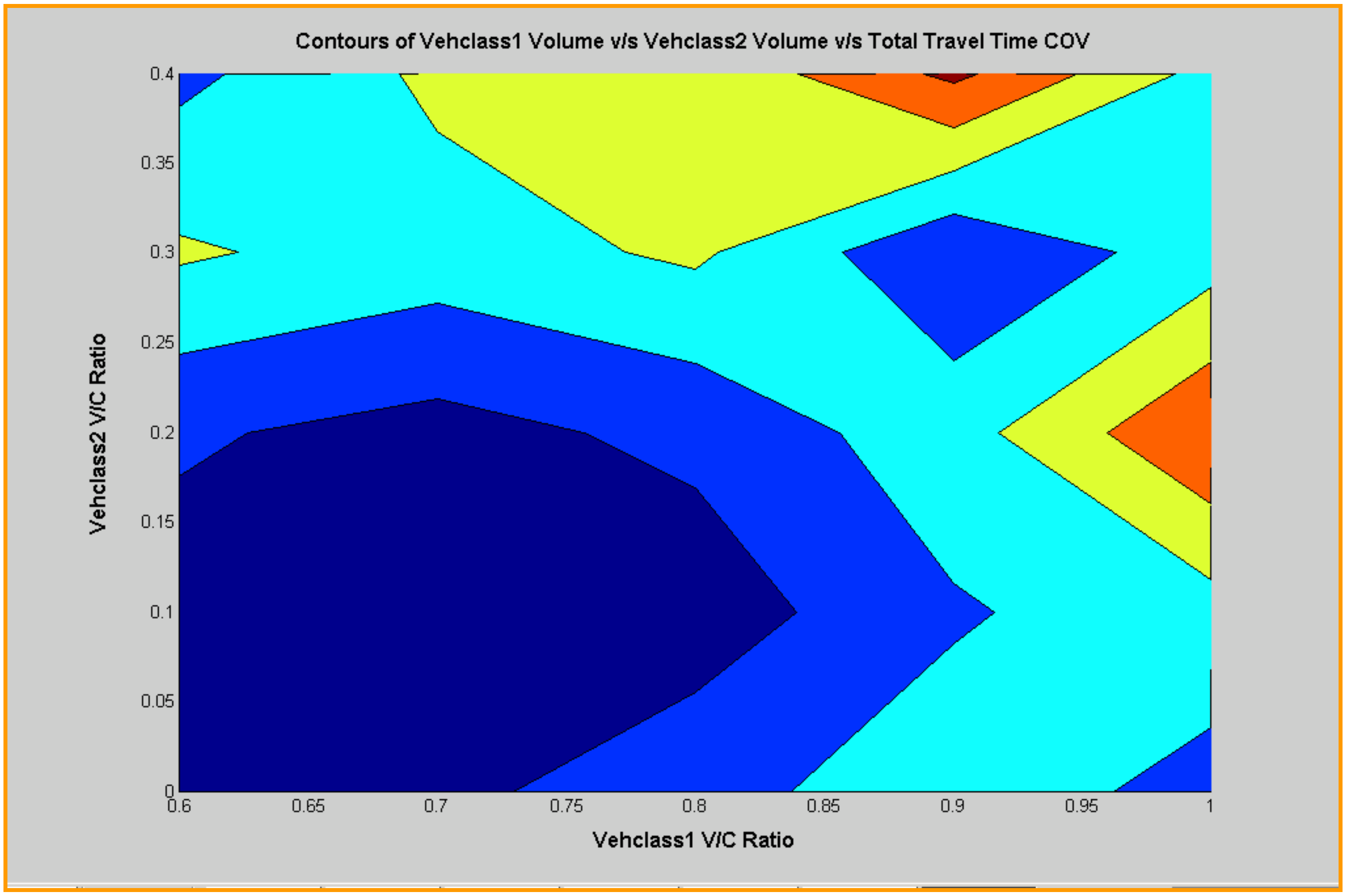

Figure 12: Total Travel Time COV Contours v/s $V/C_{veh\ class\ 1}$ and $V/C_{veh\ class\ 2}$

APPENDIX B – TABLES

ON-RAMP SECTION

Freeway demand	OR demand	Travel time-Veh 1(s)		Travel time-Veh2(s)		Total Travel time(s)		
($V/C_{freeway}$)	(V/C_{ramp})	Mean	Std dev	Mean	Std.dev	Mean	Std.dev	COV
0.25	0.1	37.7628	0.054684	79.7636	0.233487	44.7629	0.07659	0.07396
0.25	0.3	37.9767	0.104526	81.1578	0.182687	54.1696	0.0976	0.076
0.25	0.5	38.1794	0.125235	83.2931	0.471741	60.7362	0.26781	0.07734
0.25	0.7	38.4759	0.100042	87.8945	0.946377	67.3034	0.57395	0.07942
0.25	0.9	38.7783	0.159606	106.897	7.093794	82.569	4.57246	0.08056
0.375	0.1	38.1742	0.116717	79.9164	0.164182	43.0851	0.1174	0.07658
0.375	0.3	38.5195	0.10125	81.8043	0.222594	50.8866	0.13272	0.07962
0.375	0.5	38.9719	0.074072	84.3144	0.506558	57.1089	0.2271	0.08248
0.375	0.7	39.5072	0.140609	92.6487	4.163669	65.1617	2.04686	0.08498
0.375	0.9	39.8618	0.165829	152.978	10.894876	101.562	5.94008	0.08606
0.5	0.1	38.8005	0.109311	80.5816	0.075946	42.5987	0.09947	0.07982
0.5	0.3	39.3854	0.25035	82.6424	0.342224	49.3678	0.25076	0.08434
0.5	0.5	40.4194	0.335422	88.5333	1.131827	56.4574	0.57881	0.089
0.5	0.7	41.3181	0.296771	129.371	5.802413	77.575	2.55237	0.09232
0.5	0.9	41.3546	0.272267	237.504	5.825224	134.268	2.84936	0.09246
0.625	0.1	39.6472	0.149585	81.0909	0.417744	42.7171	0.16845	0.08302
0.625	0.3	40.9715	0.223932	84.2909	0.597261	49.3559	0.23738	0.0901
0.625	0.5	44.087	0.334647	96.972	4.627105	59.197	1.347	0.0992
0.625	0.7	45.013	1.168808	212.653	15.341942	105.191	5.43504	0.10136
0.625	0.9	45.0289	1.039744	333.01	10.25204	165.579	4.61926	0.10148
0.75	0.1	41.2518	0.261484	81.7688	0.26653	43.7842	0.24884	0.08654
0.75	0.3	46.8093	2.07123	86.0729	0.806023	53.3532	1.6913	0.10006
0.75	0.5	71.8821	7.659881	113.396	14.055064	82.2606	7.70666	0.123
0.75	0.7	78.6546	6.611882	252.638	10.994545	134.013	5.56818	0.12814
0.75	0.9	76.6379	6.157445	375.776	3.848286	188.815	4.09835	0.1248
0.875	0.1	45.6621	1.644192	83.2204	0.46152	47.6923	1.54919	0.09066
0.875	0.3	78.709	5.911837	87.8769	0.69968	80.0507	4.97369	0.105
0.875	0.5	131.012	5.570702	141.958	15.67659	133.445	7.09621	0.1075
0.875	0.7	137.348	6.971762	292.911	26.591534	181.794	4.85502	0.1117
0.875	0.9	138.992	8.013048	424.92	13.149114	236.1	2.87437	0.1119
1	0.1	77.7691	4.823198	84.9341	0.216799	78.1103	4.58931	0.09354
1	0.3	137.548	6.132816	88.5212	2.165075	131.153	5.46995	0.10236
1	0.5	198.2	4.795547	133.101	10.828554	185.18	5.44574	0.10672
1	0.7	208.991	7.266149	294.04	29.226647	231.041	5.32144	0.1119
1	0.9	205.502	5.955268	464.229	14.446296	285.796	4.60256	0.10714

Table 1: Travel Time Variability for Acceleration Lane Length of 50m

Freeway demand	OR demand	Travel time-Veh 1(s)		Travel time-Veh2(s)		Total Travel time(s)		
($V/C_{freeway}$)	(V/C_{ramp})	Mean	Std dev	Mean	Std.dev	Mean	Std.dev	COV
0.25	0.1	37.7473	0.053028	79.7284	0.228166	44.7441	0.07416	0.07382
0.25	0.3	37.9675	0.109411	81.1325	0.17947	54.1543	0.09626	0.07586
0.25	0.5	38.1694	0.126107	83.2613	0.478982	60.7153	0.26868	0.07718
0.25	0.7	38.4724	0.08564	87.898	0.984579	67.304	0.58784	0.07934
0.25	0.9	38.7467	0.150138	106.29	6.890507	82.1677	4.43718	0.08052
0.375	0.1	38.1737	0.11883	79.9036	0.162257	43.0831	0.1189	0.07646
0.375	0.3	38.5191	0.105889	81.7645	0.203783	50.8749	0.12866	0.07942
0.375	0.5	38.9709	0.071411	84.268	0.512072	57.0898	0.23205	0.08232
0.375	0.7	39.4942	0.144355	92.7884	4.081729	65.2224	2.0063	0.08506
0.375	0.9	39.8306	0.161088	153.217	11.30065	101.678	6.16849	0.0861
0.5	0.1	38.8069	0.111758	80.5544	0.058164	42.6021	0.10161	0.07968
0.5	0.3	39.3895	0.256673	82.6006	0.337879	49.3613	0.25429	0.08424
0.5	0.5	40.4267	0.338192	88.4246	1.055178	56.426	0.55304	0.08894
0.5	0.7	41.3187	0.285706	131.171	7.416616	78.3167	3.19951	0.09224
0.5	0.9	41.3531	0.273547	238.584	8.56684	134.778	4.16478	0.09242
0.625	0.1	39.6475	0.143799	81.0529	0.438945	42.7146	0.16508	0.083
0.625	0.3	40.9966	0.241968	84.2302	0.704872	49.3644	0.27482	0.09018
0.625	0.5	44.1246	0.333823	95.9819	3.668062	58.941	1.10909	0.0992
0.625	0.7	45.1288	1.398603	210.72	15.394401	104.572	5.27581	0.10192
0.625	0.9	45.1093	1.147791	333.23	12.437426	165.718	5.7286	0.10156
0.75	0.1	41.2665	0.27039	81.7056	0.239206	43.7939	0.25712	0.08666
0.75	0.3	46.7041	2.017502	85.9851	0.827044	53.251	1.61551	0.10058
0.75	0.5	70.1382	5.674913	119.904	12.369963	82.5797	6.8813	0.12012
0.75	0.7	77.6967	7.352699	251.942	15.539769	133.138	7.82442	0.12698
0.75	0.9	77.2922	4.276735	366.884	4.999996	185.889	3.14643	0.12632
0.875	0.1	45.7078	1.661796	83.1208	0.539091	47.7302	1.56523	0.09108
0.875	0.3	78.9082	5.011365	87.8526	0.789761	80.2171	4.1859	0.10504
0.875	0.5	130.986	5.630945	141.852	15.756862	133.4	7.17209	0.10714
0.875	0.7	138.479	8.827932	290.623	31.952655	181.949	4.97701	0.1115
0.875	0.9	136.124	3.351104	426.692	8.488488	234.807	3.31346	0.10706
1	0.1	78.3824	4.928799	84.7193	0.168642	78.6842	4.69101	0.09404
1	0.3	138.036	6.146952	88.3769	1.90142	131.559	5.4785	0.10186
1	0.5	198.414	4.822012	133.071	11.091638	185.345	5.48024	0.10644
1	0.7	211.749	12.380966	291.322	35.706493	232.379	4.7076	0.11162
1	0.9	210.366	8.912799	450.692	36.698975	284.95	6.12899	0.1117

Table 2: Travel Time Variability for Acceleration Lane Length of 100m

Freeway demand	OR demand	Travel time-Veh 1(s)		Travel time-Veh2(s)		Total Travel time(s)		
($V/C_{freeway}$)	(V/C_{ramp})	Mean	Std dev	Mean	Std.dev	Mean	Std.dev	COV
0.25	0.1	37.6392	0.060829	81.21	0.300423	44.901	0.07529	0.07396
0.25	0.3	38.1427	0.121574	82.9418	0.253766	54.9424	0.06424	0.07858
0.25	0.5	38.7508	0.223962	87.512	1.099395	63.1314	0.6041	0.08318
0.25	0.7	39.3295	0.218355	121.7	20.802948	87.3788	12.1746	0.08706
0.25	0.9	39.3682	0.176286	226.109	17.948839	159.416	11.5738	0.08718
0.375	0.1	38.0125	0.091632	81.3824	0.290519	43.1149	0.08688	0.07644
0.375	0.3	38.7607	0.038106	83.5871	0.312427	51.5682	0.09333	0.0829
0.375	0.5	39.6909	0.152389	89.0192	2.174888	59.4222	0.92827	0.08904
0.375	0.7	40.3247	0.150386	155.966	12.208369	96.1514	5.88624	0.09224
0.375	0.9	40.3566	0.154507	266.064	11.129312	163.47	6.09162	0.09248
0.5	0.1	38.6684	0.112346	81.4188	0.283103	42.5548	0.10101	0.0802
0.5	0.3	39.7637	0.245792	84.4243	0.617417	50.07	0.28389	0.08842
0.5	0.5	41.3751	0.275323	97.7137	3.570032	60.1546	1.33548	0.09606
0.5	0.7	41.8287	0.290139	211.457	9.167742	111.676	3.85944	0.0979
0.5	0.9	41.8435	0.274	328.342	7.245369	177.553	3.50805	0.09802
0.625	0.1	39.6423	0.191283	81.8997	0.151541	42.7725	0.18365	0.08414
0.625	0.3	41.592	0.360298	85.3692	0.499068	50.065	0.30174	0.09456
0.625	0.5	45.3121	0.708796	113.317	8.899294	64.742	2.15281	0.10496
0.625	0.7	45.757	1.120586	250.971	10.413391	119.424	3.8309	0.1055
0.625	0.9	45.6612	0.883088	374.964	11.828031	183.509	5.29125	0.10562
0.75	0.1	41.2711	0.240644	82.2136	0.268189	43.83	0.22083	0.08774
0.75	0.3	47.2108	1.707404	86.9722	0.933145	53.8377	1.3524	0.10376
0.75	0.5	69.1885	5.164209	131.327	13.343055	84.7231	6.5008	0.11856
0.75	0.7	70.1521	5.042845	279.451	12.766196	136.747	7.17236	0.11856
0.75	0.9	70.9143	4.611151	399.727	7.301464	194.219	5.40994	0.11844
0.875	0.1	46.0357	1.797358	83.45	0.392042	48.0581	1.68715	0.09248
0.875	0.3	79.3164	5.437858	87.8409	0.812061	80.5639	4.54252	0.10542
0.875	0.5	131.358	5.429189	142.911	15.839875	133.925	6.91309	0.10796
0.875	0.7	136.025	4.377857	303.929	12.457048	183.997	4.43915	0.108
0.875	0.9	136.784	3.967456	433.192	6.521744	237.451	3.20981	0.1081
1	0.1	78.659	5.021695	84.7197	0.12325	78.9476	4.78457	0.09456
1	0.3	138.26	6.062843	88.4396	2.033853	131.762	5.42538	0.10246
1	0.5	197.177	5.550935	133.319	10.537512	184.406	5.9839	0.10452
1	0.7	205.608	5.647601	305.473	12.899548	231.499	5.99211	0.1072
1	0.9	205.824	5.440016	467.862	14.038664	287.146	4.02167	0.10714

Table 3: Travel Time Variability for Acceleration Lane Length of 150m

Freeway demand	OR demand	Travel time-Veh 1(s)		Travel time-Veh2(s)		Total Travel time(s)		
(V/C$_{freeway}$)	**(V/C$_{ramp}$)**	**Mean**	**Std dev**	**Mean**	**Std.dev**	**Mean**	**Std.dev**	**COV**
0.25	0.1	37.6407	0.059246	81.2028	0.313136	44.9011	0.0791	0.0739
0.25	0.3	38.1482	0.111764	82.9479	0.265204	54.9481	0.06819	0.07858
0.25	0.5	38.7552	0.227067	87.5465	1.130036	63.1509	0.62224	0.0832
0.25	0.7	39.3322	0.215917	121.498	20.873218	87.262	12.2148	0.08706
0.25	0.9	39.369	0.176599	224.958	18.607556	158.676	11.995	0.0872
0.375	0.1	38.0098	0.089752	81.3768	0.291525	43.1119	0.08638	0.07644
0.375	0.3	38.7572	0.039399	83.5858	0.290747	51.5654	0.08667	0.08286
0.375	0.5	39.6842	0.149548	89.032	2.168982	59.4233	0.92563	0.08902
0.375	0.7	40.3231	0.150816	155.895	12.130113	96.1163	5.85007	0.09222
0.375	0.9	40.3496	0.150855	266.47	8.816341	163.688	4.81879	0.09248
0.5	0.1	38.6639	0.11291	81.4156	0.284735	42.5505	0.10087	0.08016
0.5	0.3	39.7591	0.247907	84.4232	0.613238	50.0662	0.28503	0.0884
0.5	0.5	41.3815	0.275814	97.7265	3.571902	60.1632	1.33718	0.09606
0.5	0.7	41.8316	0.292991	211.595	9.421958	111.734	3.96203	0.09792
0.5	0.9	41.8465	0.274843	328.346	7.528727	177.557	3.63741	0.098
0.625	0.1	39.6393	0.189115	81.8901	0.153503	42.769	0.18181	0.08416
0.625	0.3	41.5753	0.344653	85.343	0.523159	50.0465	0.28716	0.0945
0.625	0.5	45.2584	0.568871	113.366	9.221867	64.7176	2.36657	0.10486
0.625	0.7	45.6562	0.913635	250.387	10.352637	119.149	3.71546	0.10542
0.625	0.9	45.7735	1.084046	374.771	11.817424	183.494	5.37715	0.10562
0.75	0.1	41.2764	0.240998	82.1988	0.254037	43.834	0.22131	0.08772
0.75	0.3	47.1251	1.621682	86.8875	1.033797	53.7521	1.25717	0.10384
0.75	0.5	69.1047	5.023692	131.489	13.536296	84.7008	6.47302	0.11886
0.75	0.7	70.3182	4.91521	279.929	12.756442	137.013	7.1246	0.11836
0.75	0.9	71.1511	4.46878	399.73	8.194996	194.368	5.58426	0.11862
0.875	0.1	46.0603	1.757804	83.4932	0.40699	48.0837	1.64982	0.09266
0.875	0.3	79.4349	5.328425	87.8606	0.808848	80.668	4.45853	0.10566
0.875	0.5	131.266	5.372868	142.752	15.704428	133.818	6.81541	0.10778
0.875	0.7	135.919	4.32907	303.733	12.995291	183.866	4.42741	0.1079
0.875	0.9	136.757	3.93042	432.203	6.847383	237.097	3.50069	0.10796
1	0.1	78.8775	4.970246	84.5841	0.171184	79.1492	4.73407	0.09478
1	0.3	138.334	6.016217	88.4027	2.058201	131.821	5.37808	0.10246
1	0.5	197.255	5.466058	133.466	10.66047	184.497	5.92874	0.1044
1	0.7	205.493	5.623066	305.307	12.324925	231.37	5.79012	0.10706
1	0.9	205.712	5.642795	467.764	14.773422	287.039	4.35791	0.10696

Table 4: Travel Time Variability for Acceleration Lane Length of 200m

OFF-RAMP SECTION

Freeway demand	Off-R demand	Travel time-Veh 1(s)		Travel time-Veh2(s)		Total Travel time (s)		
(V/C$_{freeway}$)	(V/C$_{off\text{-}ramp}$)	Mean	Std dev	Mean	Std.dev	Mean	Std.dev	COV
0.25	0.1	37.6456	0.043933	47.8912	0.171146	39.3532	0.05289	0.07514
0.25	0.3	38.3332	0.216478	48.9162	0.235136	42.3018	0.18042	0.082
0.25	0.5	39.5067	0.415924	51.1247	0.672243	45.3157	0.53204	0.08962
0.25	0.7	43.8401	1.097612	56.502	1.223088	51.2262	1.1584	0.10654
0.25	0.9	65.9987	7.322736	79.3267	7.085727	74.5667	7.14791	0.13184
0.375	0.1	38.0839	0.029322	48.2472	0.326252	39.2796	0.04164	0.07726
0.375	0.3	39.2382	0.488258	50.1525	0.953862	42.3566	0.61789	0.08526
0.375	0.5	41.619	1.028107	52.9236	1.006039	46.1408	1.0116	0.09694
0.375	0.7	50.6143	3.881384	62.6358	4.587716	56.4178	4.16889	0.1164
0.375	0.9	99.0935	12.335675	111.616	12.244001	105.924	12.2088	0.13102
0.5	0.1	38.99	0.342337	49.3564	0.43955	39.9324	0.34752	0.08104
0.5	0.3	40.6719	0.678957	51.3338	0.763937	43.1324	0.69078	0.08998
0.5	0.5	46.4956	2.172469	57.0673	2.715424	50.0195	2.34807	0.10678
0.5	0.7	80.9828	7.191439	91.9698	6.311386	85.5069	6.79547	0.124
0.5	0.9	153.735	11.381461	167.429	11.892155	160.222	11.5771	0.12962
0.625	0.1	39.9028	0.243415	50.0865	0.455275	40.6572	0.25446	0.08324
0.625	0.3	44.2142	1.820864	53.9609	2.058913	46.1007	1.86547	0.0972
0.625	0.5	66.2689	12.065067	76.6805	12.671814	69.2436	12.2353	0.11564
0.625	0.7	119.842	7.661909	131.574	7.743777	124.053	7.67309	0.11948
0.625	0.9	203.222	10.93562	215.192	15.229144	208.232	12.5903	0.12524
0.75	0.1	41.7753	0.312674	51.578	0.918747	42.388	0.30821	0.08504
0.75	0.3	53.3141	4.883026	62.8192	5.038131	54.8983	4.90711	0.10132
0.75	0.5	103.617	8.539348	113.946	7.960185	106.199	8.38268	0.10998
0.75	0.7	175.594	9.173281	186.159	6.006486	178.955	7.99572	0.11706
0.75	0.9	244.938	7.416725	258.858	6.103633	250.158	6.6098	0.12172
0.875	0.1	47.8231	2.00711	57.096	2.421182	48.3243	2.01286	0.08904
0.875	0.3	91.8163	3.320775	102.11	2.980591	93.3228	3.25875	0.10052
0.875	0.5	159.84	10.38242	170.942	5.520507	162.307	9.22524	0.10914
0.875	0.7	226.251	7.994954	239.493	7.078277	230.034	7.51631	0.11448
0.875	0.9	289.852	9.108204	306.665	9.465309	295.562	9.10757	0.11792
1	0.1	85.1637	5.05808	92.5181	6.355213	85.5139	5.11789	0.09054
1	0.3	149.672	10.220232	160.363	9.695359	151.067	10.0449	0.10112
1	0.5	205.758	11.99596	219.946	11.166746	208.595	11.4563	0.1048
1	0.7	263.686	8.668566	282.675	3.655558	268.609	6.97131	0.11012
1	0.9	348.894	16.886149	363.874	9.554114	353.543	13.9427	0.1164

Table 5: Travel Time Variability for Deceleration Lane Length of 50m

Freeway demand	Off-R demand	Travel time-Veh 1(s)		Travel time-Veh2(s)		Total Travel time (s)		
($V/C_{freeway}$)	($V/C_{off\text{-}ramp}$)	Mean	Std dev	Mean	Std.dev	Mean	Std.dev	COV
0.25	0.1	37.6482	0.045706	47.88	0.221224	39.3535	0.06773	0.0749
0.25	0.3	38.4369	0.312398	49.0077	0.315702	42.4009	0.24898	0.08226
0.25	0.5	39.9727	0.391351	51.6811	0.408211	45.8269	0.39032	0.09272
0.25	0.7	44.9799	1.416247	57.6179	1.800011	52.3521	1.63363	0.11258
0.25	0.9	82.1776	9.010839	95.3751	8.194898	90.6618	8.4604	0.13902
0.375	0.1	38.1224	0.058963	48.3252	0.265959	39.3227	0.05467	0.0774
0.375	0.3	39.3321	0.304043	50.0947	0.515749	42.4071	0.35876	0.08608
0.375	0.5	42.218	0.488844	53.6902	0.779023	46.8069	0.58204	0.10088
0.375	0.7	55.6301	9.301677	67.5486	10.544538	61.3838	9.88657	0.12352
0.375	0.9	115.077	9.418538	127.925	10.070444	122.085	9.65906	0.1354
0.5	0.1	38.9329	0.293638	49.3956	0.304576	39.884	0.2898	0.08102
0.5	0.3	41.0085	0.584149	51.7485	0.683644	43.487	0.6012	0.092
0.5	0.5	47.7054	2.412507	58.253	2.53671	51.2212	2.43711	0.11002
0.5	0.7	90.7297	4.899474	102.517	4.784692	95.5832	4.76011	0.12942
0.5	0.9	161.81	5.441094	175.971	5.517735	168.518	5.3034	0.13256
0.625	0.1	39.7003	0.150161	49.7361	0.322103	40.4436	0.15744	0.0824
0.625	0.3	43.7732	0.721809	53.5266	0.693816	45.6609	0.70072	0.09718
0.625	0.5	67.8966	7.485755	78.4907	7.699283	70.9235	7.5383	0.11848
0.625	0.7	132.528	9.738765	145.062	10.975489	137.027	10.1705	0.12472
0.625	0.9	216.135	6.88679	228.134	2.982801	221.158	4.474	0.13016
0.75	0.1	42.0605	0.54067	51.934	1.014291	42.6776	0.52665	0.08634
0.75	0.3	55.3623	6.676724	65.0717	7.115483	56.9805	6.74848	0.10482
0.75	0.5	113.293	10.050846	124.082	9.551293	115.99	9.91266	0.11506
0.75	0.7	188.505	9.01403	199.553	6.435898	192.02	7.94347	0.12254
0.75	0.9	259.924	9.80954	274.297	11.725585	265.314	10.3225	0.12638
0.875	0.1	48.75	3.066178	58.1852	3.480237	49.26	3.06711	0.0905
0.875	0.3	97.2432	2.807617	107.635	2.980647	98.7639	2.80211	0.10344
0.875	0.5	166.889	9.673915	178.438	6.734159	169.455	8.67843	0.11262
0.875	0.7	238.764	8.655034	252.196	5.503281	242.601	7.6689	0.1194
0.875	0.9	313.639	7.410924	331.456	6.005681	319.69	6.77804	0.12486
1	0.1	85.3001	5.501456	92.7437	6.509719	85.6546	5.54615	0.09072
1	0.3	150.637	9.015319	161.01	6.131986	151.99	8.56114	0.10234
1	0.5	216.06	14.384386	230.768	8.830819	219.002	13.1225	0.10996
1	0.7	284.153	11.406693	304.202	7.267919	289.351	9.90297	0.11644
1	0.9	364.208	13.813996	379.867	8.388895	369.067	10.869	0.12168

Table 6: Travel Time Variability for Deceleration Lane Length of 100m

Freeway demand	Off-R demand	Travel time-Veh 1(s)		Travel time-Veh2(s)		Total Travel time (s)		
($V/C_{freeway}$)	($V/C_{off\text{-}ramp}$)	Mean	Std dev	Mean	Std.dev	Mean	Std.dev	COV
0.25	0.1	37.6482	0.068393	47.924	0.241878	39.3608	0.07749	0.0749
0.25	0.3	38.4606	0.222962	49.3303	0.322201	42.5367	0.22612	0.0827
0.25	0.5	40.031	0.252222	51.8822	0.336016	45.9566	0.27446	0.09306
0.25	0.7	43.6723	0.966568	56.5236	1.442751	51.1689	1.19877	0.10888
0.25	0.9	66.4716	7.831464	80.1065	7.012159	75.2369	7.2799	0.13374
0.375	0.1	38.1844	0.089289	48.6535	0.490546	39.4161	0.12561	0.07782
0.375	0.3	39.5197	0.291706	50.4676	0.376817	42.6477	0.30709	0.08742
0.375	0.5	42.1789	0.427926	53.6327	0.743878	46.7604	0.55082	0.10186
0.375	0.7	50.3786	4.024014	62.4342	4.644263	56.1985	4.3046	0.11904
0.375	0.9	98.2184	7.031715	111.451	8.040957	105.436	7.50909	0.13224
0.5	0.1	39.0486	0.322854	49.9844	0.500555	40.0428	0.32844	0.08176
0.5	0.3	41.3248	0.478418	52.2724	0.679614	43.8512	0.5209	0.09402
0.5	0.5	47.7772	2.287781	58.5858	2.037932	51.3801	2.18313	0.11256
0.5	0.7	83.37	6.899759	94.8872	7.849752	88.1124	7.24657	0.12746
0.5	0.9	159.195	4.718042	173.188	5.426482	165.824	4.90101	0.1328
0.625	0.1	39.9821	0.165064	50.7225	0.440238	40.7777	0.17588	0.0841
0.625	0.3	44.7498	0.715285	54.6496	0.821133	46.6659	0.7142	0.10076
0.625	0.5	68.825	8.567208	79.6748	8.893797	71.925	8.65057	0.1201
0.625	0.7	136.703	2.866993	149.131	3.139865	141.164	2.86263	0.12664
0.625	0.9	208.779	6.247369	220.857	3.169751	213.835	4.14128	0.1298
0.75	0.1	42.3816	0.515064	52.7764	1.503984	43.0313	0.54648	0.08796
0.75	0.3	58.2416	6.319011	68.5267	6.645425	59.9558	6.37238	0.10764
0.75	0.5	118.215	6.313937	129.283	5.270106	120.982	6.05109	0.11774
0.75	0.7	183.169	9.68435	194.502	8.010642	186.775	8.91743	0.12232
0.75	0.9	259.473	7.17466	273.952	5.603856	264.902	6.25096	0.12738
0.875	0.1	49.0392	2.981336	58.4752	3.279333	49.5492	2.98058	0.09198
0.875	0.3	101.379	4.074471	112.239	3.008389	102.968	3.90821	0.10598
0.875	0.5	168.684	9.024621	180.301	3.758645	171.266	7.71418	0.11418
0.875	0.7	238.28	8.175846	251.961	6.367748	242.189	7.4585	0.12018
0.875	0.9	306.794	7.160623	324.801	6.198002	312.91	6.65819	0.12466
1	0.1	88.0099	5.535075	95.8377	6.836232	88.3827	5.59644	0.09268
1	0.3	153.129	8.000783	163.971	5.786968	154.543	7.59869	0.1047
1	0.5	222.037	9.078559	237.176	4.826743	225.065	7.80679	0.11286
1	0.7	284.715	3.605336	305.209	6.674381	290.029	2.66872	0.11786
1	0.9	369.573	11.691333	386.094	2.407783	374.7	7.82806	0.12294

Table 7: Travel Time Variability for Deceleration Lane Length of 150m

Freeway demand	Off-R demand	Travel time-Veh 1(s)		Travel time-Veh2(s)		Total Travel time (s)		
($V/C_{freeway}$)	($V/C_{off\text{-}ramp}$)	Mean	Std dev	Mean	Std.dev	Mean	Std.dev	COV
0.25	0.1	37.6732	0.087214	48.0964	0.357839	39.4104	0.10946	0.07516
0.25	0.3	38.5871	0.332431	49.5879	0.290807	42.7124	0.28701	0.08384
0.25	0.5	39.9791	0.336291	51.9755	0.330929	45.9772	0.32384	0.09316
0.25	0.7	42.9666	0.737221	55.9137	1.170002	50.5191	0.9704	0.10706
0.25	0.9	57.1466	5.103173	71.7834	5.216046	66.556	5.16328	0.12792
0.375	0.1	38.2667	0.039835	49.1028	0.457905	39.5415	0.08004	0.07854
0.375	0.3	39.6	0.278974	50.6999	0.475555	42.7713	0.31996	0.08796
0.375	0.5	42.0074	0.320874	53.5874	0.594734	46.6394	0.40708	0.10114
0.375	0.7	48.9209	3.512053	61.023	4.043259	54.7633	3.74787	0.11624
0.375	0.9	87.009	9.922294	100.598	10.440342	94.4213	10.1683	0.1287
0.5	0.1	39.1678	0.282218	50.3604	0.289606	40.1853	0.2777	0.08254
0.5	0.3	41.262	0.441738	52.0872	0.497872	43.7601	0.43962	0.09376
0.5	0.5	46.3618	1.257121	57.2383	1.064092	49.9873	1.16468	0.10984
0.5	0.7	79.3827	7.931684	90.9815	7.822564	84.1586	7.85891	0.12582
0.5	0.9	144.116	3.551508	158.288	4.247931	150.829	3.72729	0.1288
0.625	0.1	40.1522	0.204836	51.1681	0.589332	40.9682	0.22881	0.08518
0.625	0.3	44.5825	0.714815	54.3948	0.754479	46.4817	0.68904	0.10108
0.625	0.5	64.6901	5.295574	75.4751	5.542709	67.7716	5.35604	0.11848
0.625	0.7	129.459	5.536415	141.715	5.838834	133.858	5.59918	0.1241
0.625	0.9	202.249	8.031411	214.501	7.911869	207.378	7.3859	0.12664
0.75	0.1	42.5861	0.522468	53.3044	0.988859	43.256	0.51239	0.08906
0.75	0.3	59.2254	6.052569	69.4066	6.636266	60.9222	6.14972	0.1086
0.75	0.5	114.868	9.246255	125.993	8.601333	117.65	9.07162	0.11652
0.75	0.7	184.124	8.125024	195.175	4.462113	187.64	6.79547	0.12148
0.75	0.9	251.912	6.002014	266.33	8.251374	257.319	6.62032	0.12462
0.875	0.1	48.8922	2.480745	58.4452	3.250564	49.4086	2.49431	0.09174
0.875	0.3	103.476	1.950758	114.528	1.925186	105.093	1.89893	0.10728
0.875	0.5	173.648	5.966464	185.593	1.802211	176.302	4.52917	0.11562
0.875	0.7	238.885	6.214764	252.663	3.344015	242.822	5.19541	0.1199
0.875	0.9	303.377	8.924792	321.129	7.184563	309.406	8.24504	0.12322
1	0.1	89.2327	6.016846	97.0193	7.210386	89.6035	6.07315	0.09314
1	0.3	155.408	6.018233	166.228	3.699102	156.819	5.56749	0.10518
1	0.5	223.46	9.645464	238.891	6.526971	226.546	8.51567	0.11248
1	0.7	288.209	10.037843	308.478	6.564326	293.464	8.60224	0.11786
1	0.9	362.683	12.179796	378.916	12.08929	367.721	10.6377	0.1212

Table 8: Travel Time Variability for Deceleration Lane Length of 200m

WEAVE SECTION

Weave Length = 100m

Freeway demand	Off-R demand	Travel time-Veh 1(s)		Travel time-Veh2(s)		Total Travel time (s)		
($V/C_{freeway}$)	($V/C_{off\text{-}ramp}$)	Mean	Std dev	Mean	Std.dev	Mean	Std.dev	COV
0.2	0.4	87.2713	0.168608	109.357	0.24016	104.94	0.19101	0.16792
0.3	0.3	87.3248	0.151795	109.033	0.27859	102.52	0.18046	0.16766
0.4	0.2	87.2821	0.084469	108.743	0.485937	100.159	0.2985	0.16692
0.5	0.1	87.2678	0.094186	108.525	0.347142	97.8962	0.17446	0.16702
0.6	0	87.2966	0.146398	108.336	0.201951	95.7124	0.12899	0.16704
0.3	0.4	87.3168	0.17135	108.966	0.424445	102.471	0.3197	0.16734
0.4	0.3	87.1856	0.11118	108.817	0.174964	100.164	0.10542	0.1663
0.5	0.2	87.2954	0.076553	108.366	0.337244	97.8305	0.17524	0.16732
0.6	0.1	87.3379	0.113891	108.36	0.43981	95.7466	0.13844	0.16722
0.7	0	87.3468	0.139951	108.296	0.222649	93.6316	0.16363	0.16684
0.4	0.4	87.2304	0.074418	108.976	0.330476	100.278	0.21005	0.16714
0.5	0.3	87.308	0.070247	108.447	0.392651	97.8774	0.20451	0.16712
0.6	0.2	87.3086	0.093453	108.281	0.184711	95.6974	0.10824	0.16662
0.7	0.1	87.2822	0.0623	108.002	0.329265	93.4981	0.06657	0.1663
0.8	0	87.3332	0.100411	108.084	0.316728	91.4834	0.12376	0.16648
0.5	0.4	87.3008	0.067246	108.586	0.404399	97.9432	0.20677	0.16686
0.6	0.3	87.3052	0.056695	108.496	0.472193	95.7817	0.16909	0.16666
0.7	0.2	87.2939	0.083584	107.899	0.100149	93.4753	0.07748	0.16652
0.8	0.1	87.3186	0.09499	107.785	0.288241	91.412	0.08924	0.16642
0.9	0	87.2958	0.072107	107.733	0.250429	89.3395	0.07634	0.16584
0.6	0.4	87.2406	0.064891	108.463	0.334639	95.7294	0.13098	0.16606
0.7	0.3	87.3372	0.083132	108.244	0.514277	93.6092	0.19362	0.1668
0.8	0.2	87.3332	0.089029	107.941	0.318522	91.4547	0.09706	0.16656
0.9	0.1	87.2688	0.063397	107.828	0.186243	89.3247	0.0722	0.16564
1	0	87.2739	0.051493	0	0	87.2739	0.05149	0.16542

Table 9: Travel Time Variability for Weave Length of 100m and V/C_{weave} = 0.2

Freeway demand	Off-R demand	Travel time-Veh 1(s)		Travel time-Veh2(s)		Total Travel time (s)		
($V/C_{freeway}$)	($V/C_{off\text{-}ramp}$)	Mean	Std dev	Mean	Std.dev	Mean	Std.dev	COV
0.2	0.4	88.4586	0.174034	119.516	4.822948	113.305	3.85716	0.17518
0.3	0.3	88.4527	0.327201	113.71	0.631719	106.133	0.48966	0.17402
0.4	0.2	88.6156	0.150166	111.195	0.090775	102.163	0.07695	0.17528
0.5	0.1	88.4704	0.15755	110.704	0.518693	99.5873	0.30705	0.17368
0.6	0	88.6338	0.180761	110.049	0.293574	97.1997	0.21962	0.17372
0.3	0.4	88.2806	0.206024	114.321	1.668417	106.509	1.13305	0.17378
0.4	0.3	88.4156	0.240206	112.028	0.639201	102.583	0.3386	0.17424
0.5	0.2	88.4125	0.138872	110.698	0.330076	99.5553	0.21896	0.17374
0.6	0.1	88.4824	0.10329	109.953	0.456986	97.0705	0.21409	0.17358
0.7	0	88.4041	0.222548	109.582	0.362644	94.7574	0.25383	0.17182
0.4	0.4	88.3951	0.241496	113.152	1.091505	103.249	0.69368	0.17384
0.5	0.3	88.4945	0.129865	110.927	0.113293	99.7107	0.08485	0.17402
0.6	0.2	88.3024	0.096511	110.308	0.20752	97.1044	0.12193	0.17238
0.7	0.1	88.4351	0.098616	109.269	0.157972	94.6852	0.07362	0.17266
0.8	0	88.3232	0.102672	109.131	0.272311	92.4848	0.10953	0.1708
0.5	0.4	88.4425	0.160782	112.024	1.049455	100.233	0.51423	0.17364
0.6	0.3	88.3616	0.2568	110.676	0.261383	97.2872	0.15456	0.17308
0.7	0.2	88.2793	0.097341	109.5	0.303603	94.6453	0.14977	0.17132
0.8	0.1	88.2854	0.102001	108.961	0.192621	92.4205	0.11167	0.17094
0.9	0	88.2217	0.058563	108.996	0.197445	90.2992	0.06453	0.16962
0.6	0.4	88.3098	0.097759	110.946	0.625842	97.3644	0.23696	0.17214
0.7	0.3	88.3356	0.08074	109.858	0.2743	94.7922	0.12519	0.17202
0.8	0.2	88.3191	0.057421	109.49	0.32854	92.5532	0.08636	0.1714
0.9	0.1	88.2055	0.031702	108.709	0.511835	90.2558	0.06567	0.1699
1	0	88.1106	0.087576	0	0	88.1106	0.08758	0.16838

Table 10: Travel Time Variability for Weave Length of 100m and $V/C_{weave} = 0.4$

Freeway demand	Off-R demand	Travel time-Veh 1(s)		Travel time-Veh2(s)		Total Travel time (s)		
($V/C_{freeway}$)	($V/C_{off\text{-}ramp}$)	Mean	Std dev	Mean	Std.dev	Mean	Std.dev	COV
0.2	0.4	90.3156	1.103748	207.558	10.527139	184.11	8.3813	0.18248
0.3	0.3	92.079	1.840291	136.013	6.240446	122.833	4.26729	0.19016
0.4	0.2	91.7173	0.891592	117.682	1.722489	107.296	1.35806	0.18886
0.5	0.1	91.2937	0.924053	113.852	0.738585	102.573	0.81503	0.18698
0.6	0	91.0464	0.311323	112.874	0.585372	99.7775	0.37379	0.18334
0.3	0.4	89.6621	0.255264	167.936	15.742631	144.454	11.021	0.18152
0.4	0.3	90.8801	0.682144	120.823	2.198342	108.846	1.46158	0.18582
0.5	0.2	90.7074	0.800504	114.491	0.810175	102.599	0.78768	0.18568
0.6	0.1	90.7982	0.311075	112.777	0.394885	99.5896	0.32712	0.18428
0.7	0	90.6347	0.322501	112.15	0.333461	97.0891	0.3084	0.18062
0.4	0.4	89.8656	0.114821	133.642	5.692447	116.132	3.37178	0.182
0.5	0.3	90.1541	0.141539	115.938	1.16973	103.046	0.61072	0.18358
0.6	0.2	90.4474	0.453922	112.587	0.758818	99.3033	0.5248	0.18376
0.7	0.1	90.2658	0.28249	111.165	0.367902	96.5357	0.25353	0.18074
0.8	0	90.5121	0.3173	111.45	0.350823	94.6997	0.26903	0.17876
0.5	0.4	90.1527	0.160196	121.74	1.827237	105.947	0.95824	0.18284
0.6	0.3	90.0545	0.266415	114.159	0.748206	99.6961	0.38813	0.18186
0.7	0.2	90.1257	0.185624	111.681	0.421932	96.5924	0.17862	0.18106
0.8	0.1	90.0462	0.266154	110.318	0.267939	94.1005	0.18768	0.17878
0.9	0	89.9842	0.090023	110.436	0.372031	92.0293	0.0861	0.1748
0.6	0.4	90.1955	0.213997	116.664	1.872873	100.783	0.75295	0.18242
0.7	0.3	90.0757	0.188181	112.785	0.69457	96.8886	0.29051	0.18108
0.8	0.2	90.0821	0.23115	111.05	0.384443	94.2756	0.14246	0.17984
0.9	0.1	89.7757	0.214258	109.372	0.327528	91.7354	0.16777	0.17612
1	0	89.5243	0.148276	0	0	89.5243	0.14828	0.17184

Table 11: Travel Time Variability for Weave Length of 100m and V/C_{weave} = 0.6

Freeway demand	Off-R demand	Travel time-Veh 1(s)		Travel time-Veh2(s)		Total Travel time (s)		
($V/C_{freeway}$)	($V/C_{off\text{-}ramp}$)	Mean	Std dev	Mean	Std.dev	Mean	Std.dev	COV
0.2	0.4	116.094	14.589662	298.168	10.830881	261.753	11.0584	0.22786
0.3	0.3	137.849	12.252945	226.055	13.06636	199.594	11.3127	0.24746
0.4	0.2	134.425	14.040215	155.533	7.845754	147.09	10.2384	0.24322
0.5	0.1	105.806	6.778513	127.825	6.744034	116.816	6.725	0.22174
0.6	0	103.766	2.149937	125.701	2.494912	112.54	2.22681	0.21174
0.3	0.4	97.1142	3.95897	258.268	6.773952	209.922	5.01047	0.20234
0.4	0.3	106.991	3.141571	170.453	10.416881	145.068	6.43904	0.21826
0.5	0.2	105.019	8.923965	128.737	8.582522	116.878	8.74772	0.21754
0.6	0.1	98.7865	3.271159	119.661	2.763109	107.136	3.06496	0.20642
0.7	0	97.8375	1.889048	118.614	1.724955	104.07	1.82759	0.19874
0.4	0.4	92.6723	0.501826	233.793	18.485464	177.345	11.054	0.19306
0.5	0.3	98.5941	3.625728	146.533	10.266359	122.564	5.10352	0.20404
0.6	0.2	96.3063	1.435669	118.267	1.170498	105.091	1.25962	0.20242
0.7	0.1	94.9496	0.716011	114.957	0.377394	100.952	0.55601	0.19592
0.8	0	95.8177	0.521023	115.67	0.435099	99.7881	0.45385	0.19076
0.5	0.4	92.3483	0.359025	213.735	8.63343	153.042	4.31933	0.19174
0.6	0.3	94.4543	1.082061	137.849	10.98665	111.812	4.87257	0.19674
0.7	0.2	94.0666	0.390313	115.485	0.473411	100.492	0.37209	0.19544
0.8	0.1	93.9941	0.638888	112.781	0.35802	97.7514	0.56486	0.19026
0.9	0	94.9805	1.200913	114.574	0.752818	96.9398	1.14303	0.18258
0.6	0.4	92.8927	0.339615	202.146	15.83926	136.594	6.31919	0.19362
0.7	0.3	94.2809	0.84793	128.901	11.59274	104.667	3.83276	0.19552
0.8	0.2	94.0928	0.668126	114.432	0.761138	98.1606	0.57475	0.19168
0.9	0.1	93.6249	0.714686	111.632	0.327505	95.4256	0.62822	0.18536
1	0	92.878	0.422049	0	0	92.878	0.42205	0.17438

Table 12: Travel Time Variability for Weave Length of 50m and V/C_{weave} = 0.8

Freeway demand	Off-R demand	Travel time-Veh 1(s)		Travel time-Veh2(s)		Total Travel time (s)		
(V/C$_{freeway}$)	(V/C$_{off-ramp}$)	Mean	Std dev	Mean	Std.dev	Mean	Std.dev	COV
0.2	0.4	194.369	17.871505	431.209	6.593018	383.841	8.64827	0.27602
0.3	0.3	221.141	17.465862	347.527	10.771665	309.611	11.5502	0.2761
0.4	0.2	245.713	11.713684	266.866	5.569344	258.405	7.47304	0.26756
0.5	0.1	207.731	9.795155	213.667	8.896731	210.699	9.27459	0.2527
0.6	0	180.212	6.347285	203.895	6.036888	189.685	6.1456	0.2253
0.3	0.4	127.017	10.212564	364.537	13.402901	293.281	12.0909	0.23932
0.4	0.3	157.55	13.005637	274.202	8.633873	227.541	10.2236	0.25096
0.5	0.2	187.446	5.400683	199.402	2.98774	193.424	3.41053	0.25534
0.6	0.1	169.31	7.387162	176.876	4.420669	172.336	6.07725	0.24376
0.7	0	151.793	7.331039	173.39	6.592291	158.272	7.09749	0.21476
0.4	0.4	101.872	4.676017	311.845	6.243925	227.856	4.35977	0.20904
0.5	0.3	139.77	17.968719	232.784	18.443176	186.277	17.9121	0.24356
0.6	0.2	148.943	4.392323	157.61	5.389068	152.41	4.0174	0.24384
0.7	0.1	130.636	7.26653	142.108	4.215189	134.078	6.33656	0.22882
0.8	0	139.802	6.076522	159.824	4.960868	143.806	5.84172	0.20436
0.5	0.4	96.9915	0.812872	315.398	5.583241	206.195	2.62988	0.20238
0.6	0.3	119.873	11.623536	215.481	12.681721	158.116	11.5162	0.22762
0.7	0.2	128.916	8.221344	135.651	4.371236	130.936	7.0138	0.22892
0.8	0.1	121.282	5.701571	130.352	3.485813	123.096	5.17886	0.21612
0.9	0	126.047	2.870131	144.79	2.606414	127.921	2.84112	0.1937
0.6	0.4	97.2887	1.006437	344.164	7.327274	196.039	3.34079	0.20226
0.7	0.3	108.292	3.098714	228.628	13.531264	144.393	4.43266	0.21496
0.8	0.2	131.792	7.779777	122.82	1.059902	129.997	6.11981	0.22374
0.9	0.1	121.007	2.335685	115.388	0.446173	120.446	2.09462	0.20606
1	0	112.877	5.19021	0	0	112.877	5.19021	0.17578

Table 13: Travel Time Variability for Weave Length of 100m and V/C$_{weave}$ = 1.0

Weave Length = 200m

Freeway demand	Off-R demand	Travel time-Veh 1(s)		Travel time-Veh2(s)		Total Travel time (s)		
($V/C_{freeway}$)	($V/C_{off-ramp}$)	Mean	Std dev	Mean	Std.dev	Mean	Std.dev	COV
0.2	0.4	91.3774	0.142427	113.306	0.294798	108.92	0.252	0.17488
0.3	0.3	91.4321	0.193346	113.263	0.316199	106.714	0.21808	0.1752
0.4	0.2	91.4081	0.071128	112.939	0.560953	104.327	0.33431	0.1747
0.5	0.1	91.4194	0.115211	112.634	0.408146	102.027	0.21755	0.17464
0.6	0	91.4582	0.137255	112.498	0.179787	99.8741	0.09703	0.17474
0.3	0.4	91.4248	0.127562	113.102	0.398596	106.599	0.28654	0.1747
0.4	0.3	91.3081	0.11159	112.967	0.188913	104.304	0.11529	0.17398
0.5	0.2	91.4222	0.100058	112.499	0.354571	101.961	0.18005	0.17486
0.6	0.1	91.4701	0.101078	112.625	0.460234	99.9322	0.14545	0.1749
0.7	0	91.5032	0.132346	112.408	0.184251	97.7746	0.14695	0.17456
0.4	0.4	91.3461	0.128317	112.988	0.305355	104.331	0.21476	0.17452
0.5	0.3	91.461	0.078867	112.549	0.346012	102.005	0.1852	0.17474
0.6	0.2	91.4712	0.110783	112.434	0.144421	99.8563	0.10388	0.17464
0.7	0.1	91.4533	0.101835	112.114	0.352305	97.6516	0.05481	0.17422
0.8	0	91.4623	0.086347	112.164	0.189713	95.6026	0.09591	0.17396
0.5	0.4	91.4872	0.053132	112.68	0.428275	102.084	0.23077	0.17506
0.6	0.3	91.4171	0.061572	112.508	0.424795	99.8536	0.14437	0.17414
0.7	0.2	91.4036	0.047604	112.113	0.103312	97.6165	0.06107	0.17394
0.8	0.1	91.456	0.07863	111.923	0.413514	95.5495	0.09856	0.1741
0.9	0	91.4322	0.074523	111.87	0.191302	93.476	0.07363	0.1735
0.6	0.4	91.3831	0.049331	112.512	0.110415	99.8346	0.05078	0.1739
0.7	0.3	91.4735	0.094263	112.369	0.424337	97.742	0.18096	0.17454
0.8	0.2	91.495	0.098641	112.047	0.252894	95.6053	0.11576	0.1744
0.9	0.1	91.4075	0.063588	111.987	0.083234	93.4654	0.06125	0.1734
1	0	91.4114	0.052183	0	0	91.4114	0.05218	0.1732

Table 14: Travel Time Variability for Weave Length of 200m and V/C_{weave} = 0.2

Freeway demand	Off-R demand	Travel time-Veh 1(s)		Travel time-Veh2(s)		Total Travel time (s)		
($V/C_{freeway}$)	($V/C_{off-ramp}$)	Mean	Std dev	Mean	Std.dev	Mean	Std.dev	COV
0.2	0.4	92.5314	0.243925	122.623	5.160114	116.605	4.10443	0.18218
0.3	0.3	92.7242	0.302502	117.303	0.364831	109.929	0.3308	0.18282
0.4	0.2	92.8212	0.215851	115.557	0.204458	106.463	0.12087	0.18262
0.5	0.1	92.7197	0.241968	114.944	0.572298	103.832	0.37912	0.18164
0.6	0	92.8453	0.18935	114.298	0.394841	101.426	0.23828	0.18134
0.3	0.4	92.6519	0.414755	119.151	1.597237	111.202	1.05329	0.18272
0.4	0.3	92.6786	0.212295	116.1	0.838535	106.732	0.48525	0.1824
0.5	0.2	92.6464	0.176087	114.545	0.274284	103.596	0.18602	0.18176
0.6	0.1	92.6999	0.136882	114.308	0.621723	101.343	0.32519	0.1814
0.7	0	92.6066	0.206462	113.758	0.445222	98.9521	0.27525	0.17938
0.4	0.4	92.4941	0.225998	116.871	0.911061	107.12	0.55495	0.18154
0.5	0.3	92.713	0.09553	115.11	0.392214	103.911	0.19465	0.18186
0.6	0.2	92.4537	0.109375	114.38	0.386352	101.224	0.1691	0.18002
0.7	0.1	92.6434	0.148218	113.572	0.250997	98.9219	0.11631	0.18042
0.8	0	92.5973	0.087428	113.466	0.396964	96.771	0.13541	0.17898
0.5	0.4	92.5393	0.136254	115.728	0.731454	104.134	0.35442	0.18092
0.6	0.3	92.4928	0.214166	114.689	0.576599	101.371	0.26068	0.1805
0.7	0.2	92.4733	0.092575	113.807	0.293619	98.8734	0.13852	0.17944
0.8	0.1	92.4736	0.089689	113.249	0.233284	96.6286	0.09026	0.179
0.9	0	92.4276	0.072146	113.327	0.268056	94.5174	0.06706	0.17752
0.6	0.4	92.4489	0.102485	114.831	0.598587	101.402	0.21301	0.1802
0.7	0.3	92.4773	0.074342	113.879	0.325724	98.8977	0.12367	0.1797
0.8	0.2	92.4778	0.065311	113.519	0.273387	96.686	0.0892	0.17924
0.9	0.1	92.3656	0.066704	112.824	0.452413	94.4113	0.0631	0.17778
1	0	92.2817	0.094135	0	0	92.2817	0.09414	0.17618

Table 15: Travel Time Variability for Weave Length of 200m and V/C_{weave} = 0.4

Freeway demand	Off-R demand	Travel time-Veh 1(s)		Travel time-Veh2(s)		Total Travel time (s)		
($V/C_{freeway}$)	($V/C_{off\text{-}ramp}$)	Mean	Std dev	Mean	Std.dev	Mean	Std.dev	COV
0.2	0.4	96.6384	4.166427	198.678	16.154119	178.27	13.1102	0.19572
0.3	0.3	98.5917	2.785159	134.672	3.191615	123.848	2.51778	0.20482
0.4	0.2	96.3994	1.016828	121.655	1.317019	111.552	1.1572	0.19874
0.5	0.1	95.7437	0.735698	118.453	0.693161	107.098	0.70843	0.19608
0.6	0	95.4759	0.458168	117.225	0.629654	104.176	0.46688	0.19126
0.3	0.4	94.1613	0.467907	173.875	17.570468	149.961	12.3121	0.191
0.4	0.3	95.7524	0.743524	124.902	2.284757	113.242	1.57544	0.19672
0.5	0.2	95.4873	1.447805	118.441	0.829159	106.964	1.12464	0.19468
0.6	0.1	95.1791	0.320824	117.374	0.54556	104.057	0.38112	0.19228
0.7	0	94.8828	0.327806	116.485	0.502002	101.363	0.37626	0.1883
0.4	0.4	94.4719	0.145047	141.251	9.746033	122.539	5.88035	0.19222
0.5	0.3	94.5765	0.420337	119.951	1.438104	107.264	0.81508	0.19238
0.6	0.2	94.9285	0.395382	117.313	0.421023	103.882	0.3702	0.19322
0.7	0.1	94.8148	0.314439	115.81	0.465645	101.113	0.33884	0.19
0.8	0	94.7011	0.370247	115.604	0.324851	98.8816	0.29403	0.1858
0.5	0.4	94.4387	0.127647	124.295	1.246606	109.367	0.61593	0.1917
0.6	0.3	94.4154	0.33152	118.029	0.606216	103.861	0.38224	0.191
0.7	0.2	94.4575	0.171116	115.955	0.395925	100.907	0.127	0.18942
0.8	0.1	94.2841	0.139035	114.582	0.482581	98.3436	0.04794	0.18636
0.9	0	94.3146	0.057736	114.937	0.501929	96.3768	0.10031	0.18294
0.6	0.4	94.3567	0.211525	120.525	1.475906	104.824	0.58551	0.1908
0.7	0.3	94.2526	0.223448	116.771	0.661986	101.008	0.3141	0.1892
0.8	0.2	94.2451	0.245117	115.108	0.366238	98.4177	0.16446	0.18738
0.9	0.1	93.9852	0.227383	113.564	0.308617	95.9431	0.18473	0.184
1	0	93.7439	0.148952	0	0	93.7439	0.14895	0.17966

Table 16: Travel Time Variability for Weave Length of 200m and V/C_{weave} = 0.6

Freeway demand	Off-R demand	Travel time-Veh 1(s)		Travel time-Veh2(s)		Total Travel time (s)		
($V/C_{freeway}$)	($V/C_{off-ramp}$)	Mean	Std dev	Mean	Std.dev	Mean	Std.dev	COV
0.2	0.4	126.641	6.682419	291.709	12.201185	258.696	10.2141	0.24478
0.3	0.3	153.011	21.608585	229.045	14.351774	206.235	15.5447	0.2609
0.4	0.2	148.773	13.433805	164.797	8.522008	158.387	10.3458	0.25902
0.5	0.1	113.066	9.269388	134.38	8.754539	123.723	8.9981	0.23336
0.6	0	107.905	3.097252	130.196	3.694552	116.822	3.32735	0.22104
0.3	0.4	105.673	5.426513	258.5	7.050249	212.652	5.20849	0.21868
0.4	0.3	111.663	3.702831	171.658	7.222564	147.66	2.99701	0.22842
0.5	0.2	115.032	6.716626	135.258	5.849847	125.145	6.2571	0.23294
0.6	0.1	102.996	2.272075	123.996	2.225751	111.396	2.24467	0.21326
0.7	0	102.213	2.1224	122.715	1.772604	108.364	2.01006	0.20508
0.4	0.4	97.3203	0.66626	234.71	17.096159	179.754	10.1685	0.20242
0.5	0.3	102.435	1.744101	149.124	11.928121	125.779	6.64026	0.21348
0.6	0.2	102.16	2.166641	123.694	2.265099	110.774	2.18303	0.21204
0.7	0.1	100.106	1.366757	120.053	1.034509	106.09	1.2394	0.20544
0.8	0	100.274	0.708063	120.245	0.534967	104.268	0.65622	0.19814
0.5	0.4	97.3055	0.990549	218.919	6.22694	158.112	2.74678	0.20212
0.6	0.3	99.707	0.935718	147.37	10.192642	118.772	4.33479	0.20818
0.7	0.2	98.9558	0.381392	119.936	0.756376	105.25	0.48054	0.2052
0.8	0.1	98.7582	0.706286	117.587	0.348917	102.524	0.6085	0.19934
0.9	0	99.1384	0.775137	118.816	0.199021	101.106	0.70726	0.19004
0.6	0.4	96.9876	0.223482	201.589	18.901971	138.828	7.59175	0.20078
0.7	0.3	98.5889	0.968222	133.742	8.815427	109.135	3.10565	0.2038
0.8	0.2	98.4099	0.643923	118.57	0.734238	102.442	0.46361	0.20056
0.9	0.1	97.8836	0.723053	115.872	0.378194	99.6825	0.6396	0.1938
1	0	97.175	0.420112	0	0	97.175	0.42011	0.18218

Table 17: Travel Time Variability for Weave Length of 200m and V/C_{weave} = 0.8

Freeway demand	Off-R demand	Travel time-Veh 1(s)		Travel time-Veh2(s)		Total Travel time (s)		
($V/C_{freeway}$)	($V/C_{off\text{-}ramp}$)	Mean	Std dev	Mean	Std.dev	Mean	Std.dev	COV
0.2	0.4	242.398	19.198581	428.455	10.593395	391.244	11.5866	0.30004
0.3	0.3	247.015	20.415677	343.626	13.914599	314.643	15.6142	0.29356
0.4	0.2	268.889	12.069314	269.211	5.237602	269.082	7.7307	0.2827
0.5	0.1	215.976	9.271036	220.204	7.998328	218.09	8.601	0.26566
0.6	0	181.327	1.119444	204.957	1.636271	190.779	1.08868	0.23684
0.3	0.4	143.936	6.726247	363.004	11.571441	297.284	9.6036	0.25464
0.4	0.3	184.475	13.681139	285.303	6.814482	244.972	9.37658	0.26754
0.5	0.2	204.579	16.913695	208.379	7.416949	206.479	11.9663	0.26726
0.6	0.1	175.047	10.216486	181.778	7.805628	177.74	9.15436	0.2525
0.7	0	155.272	7.395029	177.166	6.963606	161.84	7.25633	0.22578
0.4	0.4	116.384	6.673158	318.321	7.917805	237.546	7.13699	0.2304
0.5	0.3	146.028	14.5563	239.174	14.902759	192.601	12.9919	0.2519
0.6	0.2	162.798	14.147134	164.854	4.655141	163.62	10.0713	0.2554
0.7	0.1	142.303	5.521488	150.662	3.131119	144.811	4.77525	0.24216
0.8	0	146.022	7.70576	165.819	6.585815	149.981	7.46666	0.21374
0.5	0.4	102.927	1.992704	323.318	8.521886	213.123	4.33523	0.21262
0.6	0.3	120.391	4.699349	215.589	7.924666	158.47	4.00021	0.232
0.7	0.2	136.281	6.565805	140.915	2.504582	137.671	5.34671	0.24006
0.8	0.1	126.633	2.777146	134.641	1.227632	128.234	2.42672	0.2255
0.9	0	133.863	2.499096	152.373	2.601532	135.714	2.50365	0.20366
0.6	0.4	101.138	1.06991	340.678	8.784531	196.954	3.77377	0.2094
0.7	0.3	112.854	3.487544	229.107	15.104842	147.73	5.42286	0.22252
0.8	0.2	136.616	9.060702	127.134	1.000949	134.719	7.11461	0.23106
0.9	0.1	127.023	0.970279	119.919	0.453135	126.312	0.84673	0.21336
1	0	117.595	5.215972	0	0	117.595	5.21597	0.1841

Table 18: Travel Time Variability for Weave Length of 200m and V/C_{weave} = 1.0

Weave Length = 300m

Freeway demand	Off-R demand	Travel time-Veh 1(s)		Travel time-Veh2(s)		Total Travel time (s)		
($V/C_{freeway}$)	($V/C_{off\text{-}ramp}$)	Mean	Std dev	Mean	Std.dev	Mean	Std.dev	COV
0.2	0.4	95.6114	0.24356	117.576	0.169953	113.183	0.1737	0.1834
0.3	0.3	95.6568	0.135219	117.253	0.441491	110.774	0.28804	0.18358
0.4	0.2	95.6259	0.192929	117.058	0.586639	108.485	0.363	0.18284
0.5	0.1	95.552	0.097079	116.906	0.524859	106.229	0.26968	0.18246
0.6	0	95.6084	0.129388	116.703	0.178269	104.046	0.0985	0.18248
0.3	0.4	95.5695	0.165052	117.221	0.433608	110.726	0.31232	0.1825
0.4	0.3	95.4859	0.145895	117.028	0.164523	108.411	0.09496	0.18194
0.5	0.2	95.6106	0.093413	116.815	0.287173	106.213	0.12477	0.1827
0.6	0.1	95.6534	0.097913	116.743	0.354714	104.089	0.09557	0.1827
0.7	0	95.664	0.153066	116.594	0.27073	101.943	0.18693	0.18246
0.4	0.4	95.4989	0.112127	117.176	0.349895	108.505	0.24166	0.1824
0.5	0.3	95.631	0.084187	116.819	0.478957	106.225	0.24144	0.18268
0.6	0.2	95.6272	0.13245	116.516	0.169642	103.983	0.11976	0.18248
0.7	0.1	95.6037	0.086835	116.539	0.253708	101.884	0.08147	0.18204
0.8	0	95.6495	0.127154	116.591	0.192294	99.8377	0.13722	0.18206
0.5	0.4	95.6584	0.031436	116.848	0.451449	106.253	0.23632	0.18308
0.6	0.3	95.5977	0.048425	116.654	0.357026	104.02	0.12746	0.18212
0.7	0.2	95.5864	0.062259	116.257	0.142206	101.788	0.05562	0.182
0.8	0.1	95.6092	0.072717	116.137	0.405382	99.7148	0.09432	0.18198
0.9	0	95.5963	0.079354	116.138	0.268775	97.6504	0.08445	0.18142
0.6	0.4	95.5397	0.102487	116.59	0.205665	103.96	0.14087	0.18154
0.7	0.3	95.6317	0.085471	116.509	0.366461	101.895	0.16009	0.18242
0.8	0.2	95.6476	0.107293	116.264	0.233244	99.7709	0.11158	0.18228
0.9	0.1	95.5679	0.06694	116.278	0.435768	97.6388	0.08777	0.18124
1	0	95.5689	0.056535	0	0	95.5689	0.05654	0.18104

Table 19: Travel Time Variability for Weave Length of 300m and V/C_{weave} = 0.2

Freeway demand	Off-R demand	Travel time-Veh 1(s)		Travel time-Veh2(s)		Total Travel time (s)		
($V/C_{freeway}$)	($V/C_{off\text{-}ramp}$)	Mean	Std dev	Mean	Std.dev	Mean	Std.dev	COV
0.2	0.4	96.9286	0.326236	126.221	4.471869	120.363	3.51735	0.19166
0.3	0.3	97.0007	0.435218	121.287	0.579536	114.001	0.52682	0.19154
0.4	0.2	97.1433	0.16743	119.702	0.243287	110.679	0.20927	0.19208
0.5	0.1	96.9202	0.319278	119.234	0.482325	108.077	0.34798	0.18968
0.6	0	97.1391	0.109932	118.64	0.385562	105.74	0.19248	0.18972
0.3	0.4	96.8971	0.309026	122.595	1.810856	114.886	1.2262	0.1911
0.4	0.3	96.9787	0.163867	119.895	0.441833	110.728	0.28088	0.19112
0.5	0.2	96.8048	0.143724	118.943	0.330903	107.874	0.21832	0.18962
0.6	0.1	96.936	0.174227	118.433	0.554435	105.535	0.27491	0.18924
0.7	0	96.9393	0.301436	118.286	0.58862	103.343	0.36866	0.18806
0.4	0.4	96.7697	0.172822	121.299	1.223992	111.487	0.71016	0.18966
0.5	0.3	96.9096	0.208775	119.31	0.370938	108.11	0.18243	0.19008
0.6	0.2	96.712	0.142002	118.539	0.39665	105.443	0.1952	0.18818
0.7	0.1	96.9493	0.096888	117.925	0.25895	103.242	0.09277	0.1889
0.8	0	96.8093	0.182324	117.856	0.46857	101.019	0.22782	0.18694
0.5	0.4	96.8449	0.189438	119.925	0.877329	108.385	0.36864	0.18932
0.6	0.3	96.7336	0.266636	118.848	0.231621	105.579	0.12318	0.1888
0.7	0.2	96.6802	0.110038	117.952	0.255414	103.062	0.14173	0.18732
0.8	0.1	96.6855	0.095441	117.399	0.197427	100.828	0.07941	0.18694
0.9	0	96.6222	0.110668	117.539	0.320801	98.7139	0.12575	0.18542
0.6	0.4	96.636	0.106691	119.038	0.589671	105.597	0.20975	0.18792
0.7	0.3	96.6927	0.058209	118.134	0.235928	103.125	0.07527	0.18764
0.8	0.2	96.6575	0.05155	117.755	0.26498	100.877	0.075	0.187
0.9	0.1	96.5468	0.057508	116.981	0.388276	98.5902	0.0509	0.18552
1	0	96.4665	0.093125	0	0	96.4665	0.09313	0.18402

Table 20: Travel Time Variability for Weave Length of 300m and V/C_{weave} = 0.4

Freeway demand	Off-R demand	Travel time-Veh 1(s)		Travel time-Veh2(s)		Total Travel time (s)		
($V/C_{freeway}$)	($V/C_{off\text{-}ramp}$)	Mean	Std dev	Mean	Std.dev	Mean	Std.dev	COV
0.2	0.4	101.603	4.095952	193.956	15.169888	175.486	12.306	0.20878
0.3	0.3	104.291	3.305636	135.889	3.417716	126.41	2.58899	0.2164
0.4	0.2	101.216	0.817508	125.373	1.520969	115.71	1.23603	0.21016
0.5	0.1	100.31	0.841223	122.837	0.608977	111.574	0.70062	0.2058
0.6	0	100.046	0.507897	122.085	0.849319	108.862	0.63304	0.20108
0.3	0.4	99.2346	0.186438	168.916	17.880825	148.012	12.4847	0.20388
0.4	0.3	100.873	0.864702	128.959	2.933244	117.725	1.89759	0.2076
0.5	0.2	100.227	1.309692	123.138	0.601454	111.682	0.8987	0.20568
0.6	0.1	99.765	0.432575	121.741	0.495553	108.555	0.44413	0.20242
0.7	0	99.3495	0.317282	121.224	0.734914	105.912	0.42641	0.19702
0.4	0.4	99.1841	0.349173	141.13	4.220239	124.352	2.49847	0.20302
0.5	0.3	99.0611	0.274813	124.247	0.781023	111.654	0.39819	0.20186
0.6	0.2	99.324	0.705558	121.523	0.528839	108.204	0.54496	0.20124
0.7	0.1	99.1645	0.284997	120.298	0.596601	105.505	0.26556	0.1985
0.8	0	99.1811	0.378818	120.37	0.495446	103.419	0.34864	0.1949
0.5	0.4	98.9211	0.297599	130.599	3.325531	114.76	1.79547	0.2006
0.6	0.3	98.6899	0.261695	122.272	0.654698	108.123	0.37447	0.19874
0.7	0.2	98.769	0.254607	120.408	0.627509	105.261	0.25468	0.1979
0.8	0.1	98.6282	0.174575	118.991	0.35029	102.701	0.08474	0.19462
0.9	0	98.6147	0.115931	119.366	0.299222	100.69	0.10936	0.191
0.6	0.4	98.5885	0.256273	124.808	1.603531	109.076	0.64791	0.19814
0.7	0.3	98.494	0.268352	120.976	0.712504	105.239	0.35507	0.1969
0.8	0.2	98.4858	0.251417	119.433	0.430374	102.675	0.17224	0.1952
0.9	0.1	98.2084	0.231369	117.808	0.310296	100.168	0.19133	0.19166
1	0	97.9716	0.14447	0	0	97.9716	0.14447	0.18746

Table 21: Travel Time Variability for Weave Length of 300m and V/C_{weave} = 0.6

Freeway demand	Off-R demand	Travel time-Veh 1(s)		Travel time-Veh2(s)		Total Travel time (s)		
($V/C_{freeway}$)	($V/C_{off\text{-}ramp}$)	Mean	Std dev	Mean	Std.dev	Mean	Std.dev	COV
0.2	0.4	145.842	17.790464	292.571	12.137961	263.225	11.8506	0.26128
0.3	0.3	167.88	17.261121	226.04	10.984783	208.592	11.6327	0.27534
0.4	0.2	157.308	15.193217	167.362	9.809277	163.34	11.9263	0.2659
0.5	0.1	118.406	8.912276	139.102	8.34598	128.754	8.61631	0.2432
0.6	0	112.082	1.807597	134.412	2.270421	121.014	1.93376	0.22948
0.3	0.4	115.129	6.40666	254.96	9.643516	213.011	7.78349	0.23498
0.4	0.3	124.77	6.519272	181.091	7.761915	158.563	5.77279	0.2438
0.5	0.2	119.782	6.682191	139.78	5.537488	129.781	6.09523	0.24234
0.6	0.1	109.345	2.962373	129.615	2.347823	117.453	2.71049	0.2273
0.7	0	107.147	2.251128	127.989	1.8802	113.4	2.13549	0.2162
0.4	0.4	103.221	1.026957	236.414	17.795812	183.137	10.299	0.21446
0.5	0.3	110.946	3.15252	156.534	12.624203	133.74	7.41737	0.22686
0.6	0.2	107.169	2.496916	128.525	1.980096	115.711	2.22035	0.22272
0.7	0.1	105.418	1.138248	125.146	0.784608	111.337	1.01317	0.21676
0.8	0	104.975	0.573894	125.016	0.803704	108.983	0.60695	0.20718
0.5	0.4	102.103	1.221341	220.724	7.251465	161.414	3.689	0.21114
0.6	0.3	105.03	1.321494	151.691	11.749233	123.694	5.47699	0.21782
0.7	0.2	103.571	0.481319	124.528	0.444832	109.858	0.37913	0.2134
0.8	0.1	103.602	0.89018	122.358	0.524525	107.353	0.8015	0.20872
0.9	0	103.842	0.842242	123.623	0.487831	105.82	0.79366	0.1991
0.6	0.4	101.43	0.375339	206.914	9.34013	143.623	3.87426	0.20894
0.7	0.3	102.88	1.018675	139.067	12.769675	113.736	4.3519	0.21118
0.8	0.2	102.504	0.543741	122.908	0.574367	106.585	0.3896	0.2068
0.9	0.1	102.242	0.753938	120.303	0.355482	104.048	0.67242	0.20118
1	0	101.502	0.426224	0	0	101.502	0.42622	0.18998

Table 22: Travel Time Variability for Weave Length of 300m and V/C_{weave} = 0.8

Freeway demand	Off-R demand	Travel time-Veh 1(s)		Travel time-Veh2(s)		Total Travel time (s)		
($V/C_{freeway}$)	($V/C_{off-ramp}$)	Mean	Std dev	Mean	Std.dev	Mean	Std.dev	COV
0.2	0.4	260.864	21.933211	424.257	11.688309	391.578	13.6647	0.31286
0.3	0.3	273.696	22.835964	347.029	9.755487	325.029	13.2921	0.30768
0.4	0.2	287.076	11.919101	275.691	5.10144	280.245	6.76403	0.29518
0.5	0.1	226.677	6.898906	228.24	6.771806	227.459	6.77698	0.27486
0.6	0	183.455	2.360682	207.505	3.128878	193.075	2.53127	0.24632
0.3	0.4	164.094	20.148381	368.679	13.169599	307.304	14.6559	0.26788
0.4	0.3	197.916	6.229545	286.486	5.377209	251.058	4.19627	0.28082
0.5	0.2	226.691	14.829331	218.771	4.530522	222.731	9.43947	0.27938
0.6	0.1	180.108	10.419283	185.867	6.683867	182.411	8.80036	0.26288
0.7	0	163.13	9.20379	185.052	8.364716	169.707	8.94454	0.23564
0.4	0.4	122	7.125499	321.79	4.975117	241.874	2.16301	0.24018
0.5	0.3	148.103	9.716871	235.977	13.334792	192.04	10.9674	0.25736
0.6	0.2	169.223	10.313078	169.696	6.578871	169.412	8.67392	0.26454
0.7	0.1	147.453	6.52963	155.869	3.776784	149.978	5.67262	0.24998
0.8	0	147.909	6.585619	167.969	5.894682	151.921	6.42828	0.22404
0.5	0.4	108.281	1.962567	322.177	9.022883	215.229	4.3859	0.22292
0.6	0.3	126.48	10.186116	222.009	9.718422	164.691	9.36133	0.24132
0.7	0.2	141.426	6.867771	145.164	2.988872	142.547	5.56494	0.25086
0.8	0.1	134.903	6.530918	140.199	3.554673	135.962	5.91108	0.23754
0.9	0	138.405	3.472362	157.259	3.805326	140.291	3.49557	0.21294
0.6	0.4	105.768	1.008369	345.295	13.206334	201.579	5.61273	0.21858
0.7	0.3	117.401	3.117061	235.791	11.340785	152.918	4.13466	0.22994
0.8	0.2	141.151	8.482821	131.565	0.989882	139.234	6.66211	0.23918
0.9	0.1	131.615	1.016432	124.48	0.481177	130.902	0.87568	0.22142
1	0	122.328	5.2276	0	0	122.328	5.2276	0.19204

Table 23: Travel Time Variability for Weave Length of 300m and V/C_{weave} = 1.0

Weave Length = 400m

Freeway demand	Off-R demand	Travel time-Veh 1(s)		Travel time-Veh2(s)		Total Travel time (s)		
(V/C*freeway*)	(V/C*off-ramp*)	Mean	Std dev	Mean	Std.dev	Mean	Std.dev	COV
0.2	0.4	99.7874	0.267559	121.745	0.16029	117.353	0.16014	0.19152
0.3	0.3	99.7998	0.166154	121.373	0.441442	114.901	0.33261	0.19132
0.4	0.2	99.7279	0.049912	121.238	0.556798	112.634	0.33704	0.19026
0.5	0.1	99.753	0.103307	121.087	0.463183	110.42	0.2456	0.19046
0.6	0	99.7971	0.161236	120.876	0.137399	108.229	0.12585	0.19036
0.3	0.4	99.7371	0.12721	121.386	0.46801	114.891	0.3309	0.19042
0.4	0.3	99.6631	0.152424	121.264	0.245766	112.624	0.13831	0.18966
0.5	0.2	99.799	0.124507	120.954	0.293658	110.376	0.10897	0.19066
0.6	0.1	99.8092	0.134706	120.883	0.461997	108.239	0.15027	0.19048
0.7	0	99.842	0.161666	120.799	0.384638	106.129	0.22125	0.19036
0.4	0.4	99.7272	0.185854	121.38	0.366973	112.719	0.26849	0.19068
0.5	0.3	99.8456	0.123229	120.976	0.377593	110.411	0.21137	0.19088
0.6	0.2	99.8047	0.13526	120.747	0.356728	108.182	0.21119	0.19024
0.7	0.1	99.7769	0.051829	120.611	0.252023	106.027	0.05335	0.19008
0.8	0	99.8	0.123794	120.62	0.336901	103.964	0.15708	0.18972
0.5	0.4	99.8034	0.049209	120.924	0.305238	110.363	0.16555	0.19074
0.6	0.3	99.781	0.068895	120.783	0.312796	108.182	0.10746	0.19008
0.7	0.2	99.75	0.083481	120.251	0.292601	105.9	0.11285	0.18974
0.8	0.1	99.7727	0.07865	120.296	0.222659	103.877	0.09084	0.1897
0.9	0	99.7575	0.076143	120.259	0.19281	101.808	0.07453	0.1892
0.6	0.4	99.7037	0.05545	120.876	0.131923	108.173	0.06265	0.18954
0.7	0.3	99.7812	0.081395	120.562	0.344288	106.015	0.13751	0.19012
0.8	0.2	99.8101	0.10322	120.41	0.202379	103.93	0.10229	0.19
0.9	0.1	99.7396	0.069149	120.352	0.379988	101.801	0.08171	0.18916
1	0	99.735	0.05244	0	0	99.735	0.05244	0.18886

Table 24: Travel Time Variability for Weave Length of 400m and V/C_{weave} = 0.2

Freeway demand	Off-R demand	Travel time-Veh 1(s)		Travel time-Veh2(s)		Total Travel time (s)		
($V/C_{freeway}$)	($V/C_{off\text{-}ramp}$)	Mean	Std dev	Mean	Std.dev	Mean	Std.dev	COV
0.2	0.4	101.28	0.344414	128.661	1.594456	123.185	1.24414	0.20082
0.3	0.3	101.346	0.421903	125.092	0.701881	117.968	0.58983	0.2008
0.4	0.2	101.44	0.188981	124	0.355477	114.976	0.22479	0.20058
0.5	0.1	101.194	0.273512	123.489	0.560288	112.341	0.39501	0.19834
0.6	0	101.339	0.199898	122.954	0.388964	109.985	0.25704	0.19778
0.3	0.4	101.111	0.372912	126.95	1.218891	119.198	0.74754	0.19934
0.4	0.3	101.173	0.145725	124.193	0.589277	114.985	0.3556	0.19924
0.5	0.2	101.097	0.136089	123.003	0.370325	112.05	0.23043	0.19832
0.6	0.1	101.186	0.206464	122.769	0.411741	109.819	0.2651	0.19772
0.7	0	101.137	0.279844	122.645	0.466704	107.589	0.31678	0.19622
0.4	0.4	101.047	0.223911	125.03	0.840461	115.437	0.49863	0.1985
0.5	0.3	101.137	0.15119	123.177	0.301372	112.157	0.11376	0.19798
0.6	0.2	100.976	0.095353	122.914	0.312713	109.751	0.12448	0.19684
0.7	0.1	101.105	0.102744	122.13	0.210981	107.413	0.08215	0.19668
0.8	0	100.98	0.169987	121.997	0.452165	105.184	0.21584	0.19488
0.5	0.4	101.018	0.251022	124.089	0.823742	112.553	0.39812	0.19726
0.6	0.3	100.962	0.21938	123.044	0.498414	109.795	0.23762	0.19704
0.7	0.2	100.904	0.114499	122.252	0.368989	107.309	0.18017	0.19538
0.8	0.1	100.958	0.156268	121.665	0.102244	105.099	0.11213	0.19508
0.9	0	100.863	0.096267	121.968	0.445583	102.974	0.11825	0.1936
0.6	0.4	100.85	0.10685	123.339	0.644028	109.846	0.25276	0.1958
0.7	0.3	100.908	0.051451	122.538	0.316183	107.397	0.09832	0.19576
0.8	0.2	100.869	0.060974	122.007	0.379719	105.096	0.11428	0.1951
0.9	0.1	100.76	0.033937	121.269	0.442055	102.811	0.04911	0.19348
1	0	100.647	0.098187	0	0	100.647	0.09819	0.19182

Table 25: Travel Time Variability for Weave Length of 200m and V/C_{weave} = 0.4

Freeway demand	Off-R demand	Travel time-Veh 1(s)		Travel time-Veh2(s)		Total Travel time (s)		
($V/C_{freeway}$)	($V/C_{off-ramp}$)	Mean	Std dev	Mean	Std.dev	Mean	Std.dev	COV
0.2	0.4	106.047	2.92909	184.758	13.330876	169.016	10.7688	0.2191
0.3	0.3	108.793	2.090088	138.441	2.054057	129.546	1.63465	0.22636
0.4	0.2	105.835	0.49984	129.648	0.618405	120.122	0.46914	0.2197
0.5	0.1	104.747	0.793524	127.18	0.784361	115.963	0.77786	0.21458
0.6	0	104.407	0.492694	126.769	0.737116	113.352	0.52138	0.2099
0.3	0.4	103.946	0.585294	166.217	14.2424	147.536	10.0742	0.21422
0.4	0.3	105.53	1.02378	132.26	1.635385	121.568	1.27385	0.21878
0.5	0.2	104.291	0.843927	127.074	0.237085	115.683	0.51747	0.21344
0.6	0.1	104.262	0.512697	126.319	0.760091	113.085	0.58439	0.21162
0.7	0	103.725	0.431411	125.47	0.32575	110.249	0.3871	0.2059
0.4	0.4	103.646	0.259091	142.706	7.380456	127.082	4.43137	0.21232
0.5	0.3	103.623	0.385911	128.047	0.855793	115.835	0.56064	0.21128
0.6	0.2	103.751	0.494007	125.869	0.802028	112.598	0.53938	0.21114
0.7	0.1	103.435	0.317777	124.455	0.514714	109.741	0.31948	0.20672
0.8	0	103.471	0.318259	124.709	0.235336	107.719	0.26205	0.20342
0.5	0.4	103.115	0.185722	134.096	3.044532	118.606	1.56597	0.20862
0.6	0.3	103.062	0.340635	127.071	0.953455	112.665	0.47989	0.2074
0.7	0.2	103.262	0.333752	124.629	0.260227	109.672	0.2275	0.20692
0.8	0.1	103.027	0.311182	123.622	0.508854	107.146	0.25677	0.20324
0.9	0	102.949	0.217329	123.699	0.567866	105.024	0.23251	0.19946
0.6	0.4	102.907	0.193008	129.891	1.492977	113.701	0.55865	0.20664
0.7	0.3	102.771	0.264532	125.394	0.858223	109.558	0.40687	0.20492
0.8	0.2	102.742	0.216699	123.688	0.410824	106.931	0.14115	0.20318
0.9	0.1	102.459	0.231963	121.985	0.345011	104.411	0.17773	0.19962
1	0	102.202	0.150755	0	0	102.202	0.15076	0.19526

Table 26: Travel Time Variability for Weave Length of 400m and V/C_{weave} = 0.6

Freeway demand	Off-R demand	Travel time-Veh 1(s)		Travel time-Veh2(s)		Total Travel time (s)		
($V/C_{freeway}$)	($V/C_{off-ramp}$)	Mean	Std dev	Mean	Std.dev	Mean	Std.dev	COV
0.2	0.4	173.282	19.751618	290.567	15.441647	267.11	15.8105	0.28018
0.3	0.3	196.886	16.525711	227.711	12.21595	218.463	11.192	0.2926
0.4	0.2	161.489	19.070721	170.24	11.26783	166.74	14.33	0.27488
0.5	0.1	122.241	9.604192	142.984	8.882711	132.612	9.23882	0.25022
0.6	0	116.159	1.93416	138.746	2.365865	125.194	2.07916	0.23664
0.3	0.4	122.735	7.256812	253.049	6.919744	213.955	5.18112	0.2466
0.4	0.3	132.435	6.229692	182.144	10.374925	162.26	7.15866	0.2549
0.5	0.2	120.735	5.125806	141.405	4.598824	131.07	4.82496	0.24818
0.6	0.1	113.206	1.589575	133.684	1.598754	121.397	1.5665	0.2345
0.7	0	111.473	1.775629	132.645	1.417781	117.824	1.66016	0.22426
0.4	0.4	107.443	1.121654	240.347	11.466993	187.185	6.58715	0.22486
0.5	0.3	114.708	2.702597	158.469	12.080328	136.588	6.80952	0.23562
0.6	0.2	112.123	1.099369	132.64	1.469202	120.33	1.17343	0.23352
0.7	0.1	109.93	1.908378	129.373	1.392764	115.763	1.74028	0.22536
0.8	0	109.543	0.403601	129.629	0.807627	113.56	0.42219	0.2163
0.5	0.4	106.628	1.03335	223.711	11.307547	165.169	6.01086	0.22054
0.6	0.3	109.303	1.43464	150.136	13.982209	125.636	6.40369	0.22586
0.7	0.2	108.81	0.749767	129.27	0.523661	114.948	0.58675	0.2251
0.8	0.1	108.268	1.312741	127.07	0.732038	112.028	1.17761	0.21734
0.9	0	108.505	1.020067	128.356	0.862649	110.49	0.97486	0.2081
0.6	0.4	105.67	0.281549	214.866	6.453975	149.348	2.73972	0.2165
0.7	0.3	107.148	0.76968	143.646	9.993996	118.097	3.2965	0.21918
0.8	0.2	106.906	0.612722	127.343	0.561278	110.993	0.47443	0.2149
0.9	0.1	106.565	0.72843	124.697	0.350065	108.378	0.64976	0.2089
1	0	105.838	0.440044	0	0	105.838	0.44004	0.19774

Table 27: Travel Time Variability for Weave Length of 200m and V/C_{weave} = 0.8

Freeway demand	Off-R demand	Travel time-Veh 1(s)		Travel time-Veh2(s)		Total Travel time (s)		
($V/C_{freeway}$)	($V/C_{off\text{-}ramp}$)	Mean	Std dev	Mean	Std.dev	Mean	Std.dev	COV
0.2	0.4	281.864	17.347777	422.563	8.130621	394.423	8.95478	0.32694
0.3	0.3	316.197	18.799326	350.901	11.558185	340.49	12.9778	0.32662
0.4	0.2	305.933	19.203024	277.953	10.321706	289.145	13.5159	0.30516
0.5	0.1	229.466	14.178342	230.927	9.91976	230.196	11.982	0.28306
0.6	0	187.627	5.548441	211.803	5.488255	197.298	5.43091	0.25558
0.3	0.4	175.199	16.540553	367.955	9.254065	310.129	10.9416	0.27982
0.4	0.3	213.198	15.272692	290.628	5.300745	259.656	7.41827	0.291
0.5	0.2	230.574	18.606877	218.398	7.792473	224.486	13.0661	0.28888
0.6	0.1	192.958	7.937228	196.191	6.478688	194.251	7.22241	0.27266
0.7	0	166.309	7.514641	188.169	7.112982	172.867	7.38557	0.24542
0.4	0.4	128.045	4.42665	323.887	5.108548	245.55	2.95121	0.25028
0.5	0.3	162.004	10.901842	247.086	13.961714	204.545	11.7735	0.27018
0.6	0.2	189.994	7.19472	181.179	2.483072	186.468	4.93584	0.276
0.7	0.1	156.721	7.862707	163.124	5.263683	158.642	7.05112	0.25782
0.8	0	153.715	6.983003	173.882	6.325509	157.749	6.83751	0.23378
0.5	0.4	112.341	0.92619	328.344	5.744992	220.342	2.61293	0.2313
0.6	0.3	128.701	6.327997	224.789	9.704853	167.136	6.32458	0.2461
0.7	0.2	149.439	7.866794	150.154	3.389004	149.653	6.50049	0.25982
0.8	0.1	141.698	5.361624	146.206	3.443604	142.6	4.97533	0.2475
0.9	0	142.964	2.303558	161.857	2.192499	144.854	2.28221	0.2215
0.6	0.4	110.111	1.193954	355.11	6.870507	208.11	3.37803	0.22544
0.7	0.3	121.73	2.19686	239.724	14.140122	157.128	4.23179	0.23744
0.8	0.2	146.691	8.839367	135.762	1.104534	144.505	6.91375	0.24682
0.9	0.1	136.36	1.250355	128.963	0.462438	135.621	1.08274	0.22882
1	0	127.054	5.239208	0	0	127.054	5.23921	0.1999

Table 28: Travel Time Variability for Weave Length of 400m and V/C_{weave} = 1.0

BOTTLENECK SECTION

Freeway demand	Total Travel time (s)		
($V/C_{bottleneck}$)	Mean	Std.dev	COV
0.1	248.46966	0.055146	0.00022194
0.2	248.7568	0.088283	0.0003549
0.3	249.09728	0.091922	0.00036902
0.4	249.44144	0.089327	0.00035811
0.5	249.80048	0.086028	0.00034439
0.6	250.21976	0.109132	0.00043614
0.7	250.6319	0.103518	0.00041303
0.8	251.17576	0.138061	0.00054966
0.9	251.73674	0.177136	0.00070366
1	252.39236	0.173923	0.0006891
1.1	253.16942	0.1795	0.00070901
1.2	254.06026	0.192293	0.00075688
1.3	255.20616	0.259567	0.00101709
1.4	256.5756	0.302508	0.00117902
1.5	258.42524	0.546846	0.00211607
1.6	260.91166	0.809625	0.00310306
1.7	264.427	1.345575	0.00508864
1.8	269.75308	2.24297	0.0083149
1.9	278.41834	3.209323	0.01152698
2	294.3332	5.185176	0.01761669

Table 29: Travel Time Variability for Bottleneck section

APPENDIX C

The program shown here is for an on-ramp section. Similar codes were generated for the other sections as well.

1. MATLAB™ code for generating the input Master File

```
% Program to create data input files for Integration
% This creates input master files for the ramp section that we are considering.
for i= 1:1:7
  for j=1:1:5
      for k=1:1:5
          filenam1= ['r-',int2str(i),int2str(j), int2str(k),'.int'];
          filenam2 = odfile(i,j);
          y=1800+k*0.01;
          fid=fopen(filenam1, 'wa');
          % Write Master File
          line1 ='RAMP MASTER FILE';
          fprintf(fid,'%s\n',line1);
          line2 =[num2str(y),' 2 0 1 0 '];
          fprintf(fid,'%s\n',line2);
          line3 ='                ';
          fprintf(fid,'%s\n',line3);
          line4 ='                ';
```

```
fprintf(fid,'%s\n',line4);
line5 ='ramp1.dat       ';
fprintf(fid,'%s\n',line5);
line6 ='ramp2-level1.dat       ';
fprintf(fid,'%s\n',line6);
line7 ='ramp3.dat       ';
fprintf(fid,'%s\n',line7);
line8 = filenam2;
fprintf(fid,'%s\n',line8);
line9 ='ramp5.dat       ';
fprintf(fid,'%s\n',line9);
line10='none            ';
fprintf(fid,'%s\n',line10);
line11='none            ';
fprintf(fid,'%s\n',line11);
line12='none            ';
fprintf(fid,'%s\n',line12);
line13='none            ';
fprintf(fid,'%s\n',line13);
line14='none            ';
fprintf(fid,'%s\n',line14);
line15='none            ';
fprintf(fid,'%s\n',line15);
```

```
line16='none            ';
fprintf(fid,'%s\n',line16);
line17='none            ';
fprintf(fid,'%s\n',line17);
line18='none            ';
fprintf(fid,'%s\n',line18);
line19='none            ';
fprintf(fid,'%s\n',line19);
line20='none            ';
fprintf(fid,'%s\n',line20);
line21='none            ';
fprintf(fid,'%s\n',line21);
line22='none            ';
fprintf(fid,'%s\n',line22);
line23='none            ';
fprintf(fid,'%s\n',line23);
line24='none            ';
fprintf(fid,'%s\n',line24);
line25='none            ';
fprintf(fid,'%s\n',line25);
line26='none            ';
fprintf(fid,'%s\n',line26);
line27='none            ';
```

```
        fprintf(fid,'%s\n',line27);
        line28='none          ';
        fprintf(fid,'%s\n',line28);
        line29='none          ';
        fprintf(fid,'%s\n',line29);
        line30='none          ';
        fprintf(fid,'%s\n',line30);
        line31='none          ';
        fprintf(fid,'%s\n',line31);
        line32=['summary-',int2str(i),int2str(j),int2str(k),'.out'];
        fprintf(fid,'%s\n',line32);
        line33='none          ';
        fprintf(fid,'%s\n',line33);
        fclose(fid);

      end
    end
end
```

2. MATLAB™ code for generating the input O-D File

```
function filenam2 = odfile(i,j)

% This function generates an O-D file name and prints the O-D file

C=4200;

filenam2=['r4-',int2str(i),int2str(j),'.dat      '];

V_freeway = 1000+(i-1)*500;

V_onramp = 200+(j-1)*400;

line1='O-D Traffic demand file                              ';

line2='2 0 0 1.0                                           ';

line3=['1 1 5 ',int2str(V_freeway),' 1.0 0.0 900 1.0 0.0 0.0 0.0 0.0 0.0 1.0 '];

line4=['2 2 5 ',int2str(V_onramp),' 1.0 0.0 900 0.0 1.0 0.0 0.0 0.0 0.0 1.0 '];

fid=fopen(filenam2, 'wa');

  fprintf(fid,'%s\n',line1);

  fprintf(fid,'%s\n',line2);

  fprintf(fid,'%s\n',line3);

  fprintf(fid,'%s\n',line4);

fclose(fid);
```

3. AMPL code for Problem RL'

```
set NODES; # nodes in the graph

set ARCS within {NODES, NODES}; # arcs in the graph

param benefit {ARCS} >=0; # Benefit factor for arc (i,j)

param cost {NODES}>=0;

param R>=0;

param B>=0;

var y {j in NODES} >=0;

var w {i in NODES, j in NODES: i<j}>=0;


maximize total_benefit:

sum {(i,j) in ARCS: i<j} benefit[i,j]*w[i,j] + sum {(i,j) in ARCS :i>j} benefit[i,j]*w[j,i];


subject to RL {i in NODES} :

sum {j in NODES} y[j] - sum {j in NODES : j>i} w[i,j] - sum {j in NODES: j<i} w[j,i] -
(1-R)*y[i] <=R;


subject to RL2 {i in NODES}:

sum {j in NODES :j>i} w[i,j] + sum {j in NODES :j<i} w[j,i] + (1-R)*y[i] <=0;


subject to Cost1 {i in NODES} :
```

sum (j in NODES} cost[j]*y[j] - sum {j in NODES: j>i} cost[j]*w[i,j] - sum {j in NODES: j<i} cost[j]*w[j,i] - (cost[i]-B)*y[i]<= B;

subject to Cost2 {i in NODES} :

sum {j in NODES: j>i} cost[j]*w[i,j] + sum {j in NODES: i>j} cost[j]*w[j,i] + (cost[i]-B)*y[i] <=0;

subject to RLT1 {i in NODES, j in NODES: i<j}:

y[j] - w[i,j] >=0;

subject to RLT2 {i in NODES, j in NODES: i<j}:

y[i] - w[i,j] >=0;

subject to RLT3 {i in NODES, j in NODES: i<j}:

-y[i]-y[j] +w[i,j] >= -1;

4. MATLAB™ code for generating Semidefinite Cuts

```
% Program to compute the values of alpha to produce the semidefinite cut.
clear all;

% INITIALIZATION

G = load('matrix.txt'); % loads the y*transpose(y) matrix
fprintf(' The G matrix has been opened');
global n
global i
global store_g
global store
global alpha

n=length(G); % gives the length of the matrix i.e., the number of variables in the problem
sum=0;
i=1;
check=1;
G_current = G                    % initial solution at i=1
%alpha
    while i<=n                   % Condition for termination
     %i
            if G_current(1,1) < 0 % First condition: check the sign of the first element of
G_current
        alpha(i)=1;              % Store alpha(i)=1 and the rest of the alpha vector = 0.

        for j=i+1:1:n
           alpha(j)=0;
        end

        if i > 1
                compute_alpha(alpha, G_current); % this function produces the alpha
vector recursively from (i-1) to 1

           break;
        else
           i=n+1;
           alpha
           break;
        end
      end
```

```
if i==n                        % This condition is reached iff all G is PSD; since
    fprintf('G is PSD');         G has all positive terms along the diagonal till i=n
    i=i+1;
    break;
end

if G_current(1,1)== 0           % Checking for which loop to enter if G_current =
                                 0 - two cases arise; all of the first row elements
  j=1;                             being equal to zero, or in the other   case some
                                 element being nonzero.

  for j = 2:1:n-i+1
      if G_current(1,j)~=0
        check=0;
      end
     %j
     break;
  end

  if check ==0                  % Enter if there is any term in the first row that is
                                   non-zero
     theta = G_current(1,j)
     phi = G_current(j,j)
     lamda = 0.5*(phi-sqrt(phi^2 + 4* theta^2))
     a= 1/(sqrt(1+(lamda^2/theta^2)));
     b= a*lamda/theta;
     alpha(i)=a
     alpha(i+j-1)=b

     if i+1 < i+j-2
       for k=i+1:1:i+j-2
           alpha(k)=0;
       end
    end

    if i+j-1 < n
      for k=i+j:1:n
          alpha(k)=0;
      end
   end

   % alpha

   if i > 1
       compute_alpha(alpha, G_current);
       alpha
```

```
            i=n+1;
            break;
         else
            i=n+1;
            alpha
            break;
         end

      else                          % Enter if all terms are equal to zero
         alpha(i) = 0;
         G_current = update(G_current);
         G_current
         i=i+1;
      end
   end


   if G_current(1,1) > 0             % Simply update G_current by using the
                                     Superdiagonalization algorithm and
                                     proceed.
         G_current = update(G_current);
         i=i+1;
         G_current
   end


  end
  alpha;

  for a=1:1:n
     sum=sum+(alpha(a))^2;
  end
  alpha
  normalized_alpha = alpha./sqrt(sum)
```

Function to compute alpha recursively

```
% function to compute recursively the alpha vector
function alpha = compute_alpha(x,y)

global n
global i
global store_g
global store
global alpha
```

```
m=length(x);

% Computes recursively the alpha vector.
% Use the matrices store and store_g from update in this step.


for j=i-1:-1:1
   if store(j)~=0
      for k=1:1:n-j
         temp(j+k)=store_g(j,k);
      end

   temp
   alpha(j)= -alpha*transpose(temp)/store(j);
   else
      alpha(j) = alpha(j)
   end
end
%alpha
```

Function to update the G_current matrix

```
function y = update(x)
global i
global n
global store
global store_g
global alpha     % Note that i+m =n+1, where m=length of G_current

% Stores the first element in G_current. Note that all elements of store > 0

store(i) = x(1,1);

% Stores the first row of G_current(excluding the first elelemt) before updating

for k=1:1:n-i
   store_g(i,k) = x(1,k+1);
end
for k=n-i+1:1:n
   store_g(i,k) = 0;
end

store;
store_g;
```

```
% Updates G_current by using the Superdiagonilization algorithm. The order of
G_current
% reduces by one at every step

if x(1,1) ~= 0
    for p=1:1:n-i
        g(p)=store_g(i,p);
    end

    product = (1/x(1,1))*transpose(g)*g;

    for p=1:1:n-i
        for j=1:1:n-i
            y(p,j) = x(p+1,j+1)-product(p,j);
        end
    end
else
    for p=1:1:n-i
        for j=1:1:n-i
            y(p,j) = x(p+1,j+1);
        end
    end
end
```

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- Son of Mr. H. K. Desai and Mrs. Maitreyi Desai
- Born on the 14*th* of October, 1977
- Schooling: A major part of my schooling was done at "St. Patrick's High School", Secunderabad, India. I went on to become the Captain of my school in 10*th* grade.
- I chose Engineering to be my profession when I was in my 9*th* grade. The choice was fairly obvious considering my aptitude for math.
- I did my Undergraduate degree in the Indian Institute of Technology – Madras, which is the most prestigious engineering college in India. This is located in the city of Chennai, India. It has an acceptance rate of 1.6% (i.e., out of 100,000 students who take the entrance exam, 1600 get selected!). I earned my *Bachelor of Technology* (B.Tech) degree in Spring, 2000.
- To further my career, I decided to come to the US, and earn a Masters degree. I was accepted into Virginia Tech's ISE program, with full financial support and I arrived at Blacksburg, VA for the Fall, 2000 semester.
- This is a culmination of the pursuit of my M.S degree. I plan to delve further into the field of Operations Research, and I am now embarking on my PhD, at Virginia Tech.
- Apart from academics, I play tennis at a professional level (I won the summer Intramural tournament at VT), swim for relaxation, take long drives in my car, drink enormous amounts of tea and coffee, and also read different kinds of books voraciously.