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Correspondence to:
A. Rinaldo,
andrea.rinaldo@epfl.ch

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Reply to comment by Porporato and Calabrese on "Storage selection functions: A coherent framework for quantifying how catchments store and release water and solutes"

Andrea Rinaldo[1,2], Paolo Benettin[1], Ciaran J. Harman[3], Markus Hrachowitz[4], Kevin J. McGuire[5], Ype van der Velde[6], Enrico Bertuzzo[1], and Gianluca Botter[2]

[1]Laboratory of Ecohydrology ECHO/IIE/ENAC, Ecole Polytechnique Fédérale de Lausanne, Lausanne, Switzerland, [2]Dipartimento ICEA, University of Padua, Padova, Italy, [3]Department of Geography and Environmental Engineering and Department of Earth and Planetary Sciences, Johns Hopkins University, Baltimore, Maryland, USA, [4]Faculty of Civil Engineering and Geosciences, Delft University of Technology, Delft, Netherlands, [5]Virginia Water Resources Research Center, Department of Forest and Environmental Conservation, Virginia Tech, Blacksburg, Virginia, USA, [6]Faculty of Earth and Life Sciences, Department of Earth Sciences, VU University Amsterdam, Amsterdam, Netherlands

We appreciate the interest our Commentary [*Rinaldo et al.*, 2015] has generated. We also thank *Porporato and Calabrese* [2015] (hereinafter P&C) for pointing out similarities between our approach using storage selection (SAS) functions to determine water ages and the age-structured population approach. P&C make a rather technical point. Regardless of the merits of the technical issues they raise, on which we shall return below, we note that the scope of our work was aimed at explaining in simple terms involved mathematical concepts. Overall, we meant to stress the open research issues and the perspective tools our approach brings to hydrologic practice for predicting both flow and transport at catchment scales.

As per the technical issues, we had previously concluded that the wealth of existing exact solutions to the McKendrik-Von Foester equation is unlikely to apply to catchment flow and transport phenomena. We maintain our position after learning the mathematical exercises proposed by P&C. The reasons can be summarized as follows:

1. The demographic model proposed by P&C (a formulation of the transport problem perfectly equivalent to the one used by the authors) is based on assigning a loss function, $\mu(T,t)$, to obtain the distribution of individuals exiting the control volume and their total number. The hydrologic analogy, in P&C's notation, is to assign $\mu(T,t)$ to obtain the distribution of deceased individuals, $n_0(T,t)=\mu(T,t)n(T,t)$ (which is not a pdf), and their total flux, i.e.,

$$Q(t)=\int_0^\infty \mu(T,t)n(T,t)dT \quad (1)$$

   This approach is interesting from the theoretical viewpoint, but it implies that the loss function $\mu(T,t)$ must be capable of reproducing both flow and transport processes consistently. In the SAS approach, equation (1) is imposed as a nonlinear constraint (a position that per se drastically affects chances of obtaining exact solutions for the general master equation [*Botter et al.*, 2010, 2011]). This allows the direct use of values of discharge Q measured or modeled. Why remove a known term from the formulation of the problem, requiring additional assumptions be made regarding it? Moreover, by using a transformed domain for the equivalent term, as in the fractional [*van der Velde et al.*, 2012] or ranked [*Harman*, 2015] SAS functions (termed fSAS and rSAS, respectively), equation (1) is automatically verified. Enforcing the constraint (1) in the exercise proposed by P&C defies exact solutions.

2. It is easy to demonstrate that μ must contain time variability due to the input, contrary to the claim of P&C (also in cases for which fSAS or rSAS functions do not). Consider a simple flow tube of fixed volume but variable flow, the case dealt with by, e.g., *Niemi* [1977], *Zenger and Niemi* [2009], and *Botter* [2012]. The $\mu(T,t)$ function is a Dirac delta distribution located at whatever the maximum age in the system is at a particular time. This will naturally change in time depending on the sequence of "recent" fluxes through the stream tube. Thus, the value of $\mu(T,t)$ cannot be specified unless those fluxes are known. However, fSAS and rSAS functions are invariant in this case. The former is a Dirac delta distribution centered at a unit value of the rescaling parameter, the latter is a Dirac delta centered at the volume of the stream tube. Clearly, the claim that μ is less likely to contain input and output variability is not justified.

3. We note that, in P&C's framework leading to closed-form solutions, i.e., $\mu(T)$, flow and transport must necessarily have the same time scale. In fact, if H is the Kernel of the relationship linking J and Q, one can show that the analog linking input and output concentrations of a tracer is instead $H(T)\frac{J(t-T)}{Q(t)}$, hardly modifying the basic time scales. Empirically measured time scales of tracer concentration in the runoff, however, are orders of magnitude larger [e.g., *Kirchner et al*., 2000; *Kirchner and Neal*, 2013], entailing such concepts as residual and dynamic storages much discussed in the literature [e.g., *Zuber*, 1996; *Kirchner*, 2009; *Birkel et al*., 2015]. In fact, major differences between flow velocities in the system (that control the tracer response) and the celerity with which hydrologic perturbations are transmitted (which control the hydrograph) are to be expected since they are driven by different mechanisms [e.g., *Beven*, 2012; *McDonnell and Beven*, 2014]. Therefore, if one is interested in both flow and transport at catchment scales, the exact solutions proposed by P&C ignore the effects of the so-called old-water paradox [e.g., *McDonnell*, 1990] exposed empirically by the forceful growth of isotope hydrology and high-frequency tracer measurements. Also, P&C's claim that a "reasonable first-order approximation is to assume that the basin behaves in a time-invariant way, so that the specific loss function only depends on age," i.e., $\mu(T,t) \approx \mu(T)$. As such, the catchment behaves in a linear manner corresponding to a time-invariant instantaneous unit hydrograph $H(T)$. Such claim is at odds with the above results and much empirical and theoretical evidence (omitted here for brevity). P&C's solutions suggest a return to the state of affairs we found before SAS functions were proposed. The authors firmly believe, on the contrary, that understanding of time variant and mixing behaviors of the basic kernels transforming inputs into outputs of both flow and transport are crucial to catchment hydrology and find misleading the commenters' focus on the need to find linear solutions.
4. Neglecting for the sake of the mathematical exercise the partitioning of losses between Q and other fluxes, chiefly evapotranspiration, is misleading as a number of technical issues arises (discussed in the outlook of *Rinaldo et al*. [2015]).
5. The statement "Unfortunately, an always-useful loss function is not available" is unclear. Why should there be one? SAS functions can be computed moving from observational data. For instance, Plynlimon data [*Harman*, 2015; *Benettin et al*., 2015a] suggest that the instantaneous ratio of travel and residence time distributions (the SAS function) displays a preference for younger waters during wet conditions. Such tendency is enhanced at increasing degrees of wetness. Conversely, when the catchment becomes dry, older water tends to be preferentially discharged from the catchment because the shallow system becomes almost inactive. The link between age-selection and storage dynamics, however, is not one-to-one because the same storage can correspond to different catchment conditions depending on whether the catchment is wetting or drying [*Hrachowitz et al*., 2013] and similar age-selection functions may correspond to different shallow storage states (say, during peaks or recessions). Thus, SAS functions are known to provide useful insights for the characterization of the hydrologic state of a catchment highlighting the key differences between flow and transport that linearized solutions miss. If a parsimonious nonlinear, yet insightful and accurate, SAS function can be formulated, then important problems are being solved and progress is being made.

In conclusion, we note that P&C's solutions have not yet confronted the simultaneous description of both flow and transport contrasting catchment hydrochemical data, which is precisely the scope of the SAS function development. If they did so, they would have realized that their approach, however prone to exact solutions, is analogous to earlier ones [e.g., *Rinaldo and Marani*, 1987; *Rinaldo et al*., 1989] whose appealing theoretical angles proved incapable of reproducing real life tracer data [e.g., *Kirchner*, 2003; *McGuire and McDonnell*, 2006; *McGuire et al*., 2007; *Birkel et al*., 2012; *Kirchner and Neal*, 2013; *Aubert et al*., 2013], prompting us to foster the development of the approach summarized in *Rinaldo et al*. [2015] and applied to hydrologic transport in a number of diverse catchments where intensive field data had been gathered [e.g., *Hrachowitz et al*., 2010; *van der Velde et al*., 2010; *Harman et al*., 2011; *Rinaldo et al*., 2011; *van der Velde et al*., 2012; *Hrachowitz et al*., 2013; *Benettin et al*., 2013; *Harman and Kim*, 2014; *van der Velde et al*., 2014; *Queloz et al*., 2015; *Harman*, 2015; *Birkel et al*., 2015; *Benettin et al*., 2015a; *Hrachowitz et al*., 2015; *Benettin et al*., 2015b].

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