THE EFFECT OF NON-NORMALITY ON THE POWER OF THE TEST OF A TWO-WAY CLASSIFICATION, FIXED EFFECTS MODEL--NO INTERACTION

by

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CHAPTER I

INTRODUCTION

In most experimental situations, more than one factor affects the outcome of an experiment. There are many situations in which treatments change from one experimental unit to another. For example, the amount learned by each of three teaching methods (factor A) may vary considerably for students of six different intelligence (I.Q.) groups (factor B). As another example, suppose an experimenter wants to measure the effect of different ovens and different temperatures on the strength of a certain metal. Further, the responses of different treatments may be affected remarkably by different batches of raw material.

The two-way cross classification, fixed effects model (without interaction), will be considered in this thesis. This model enables one to study the effects of two variables on a response. In the above examples, one thinks of the observations as being classified according to two criteria at once, where the effects of both variables are to be tested. In each case one variable may be considered primary and the other secondary to the purposes of the experiment. Let us call the primary variable factor A and the secondary variable factor B. In the first example, the teaching methods represent different rows of factor A and the I.Q. groups represent different columns of factor B.

In order to obtain better results, an experimenter will often replicate his experiment assuming that this is feasible.

The general two-way cross classification, fixed effects model (without interaction and with replicates) is defined by the model

$$Y_{ijk} = \mu+\alpha_i+\delta_j+\epsilon_{ijk} \qquad \begin{array}{l} i=1,2,\ldots,a \\ j=1,2,\ldots,b \\ k=1,2,\ldots,n \end{array} \qquad (1.1)$$

where Y_{ijk} is the observation in the i^{th} row, j^{th} column, and k^{th} replicate, μ is a general mean, α_i is the i^{th} effect of the factor A, δ_j is the j^{th} effect of the factor B, and ϵ_{ijk}'s are random errors which are independently distributed with mean zero and variance σ^2.

Before proceeding, it should be pointed out that for the purpose of this thesis the following information will be used concerning the above model: μ will be equal to zero, which was used in the computer program to make computations easier; a and b will be three; σ^2 will be one; n will be two. Below, in Table 1, is the experimental format that was used.

TABLE 1

Two-way Classification, Fixed Effects Model with $a=b=3$ and $n=2$

		FACTOR B		
FACTOR A	1	Y_{111} Y_{112}	Y_{121} Y_{122}	Y_{131} Y_{132}
	2	Y_{211} Y_{212}	Y_{221} Y_{222}	Y_{231} Y_{232}
	3	Y_{311} Y_{312}	Y_{321} Y_{322}	Y_{331} Y_{332}

The constraints $\sum_{i=1}^{a} \alpha_i = 0$ and $\sum_{j=1}^{b} \delta_j = 0$ were made in order to obtain estimates of the parameters in the model.

The model equation

$$Y_{ijk} = \alpha_i + \delta_j + \varepsilon_{ijk} \qquad \begin{array}{l} i=1,2,3 \\ j=1,2,3 \\ k=1,2 \end{array} \qquad (1.2)$$

is the form used most often when the main concern is in making statements about either the column effects or the row effects or both. The typical hypotheses tested by analysis of variance are:

$$H_0: \alpha_1=\alpha_2=\alpha_3=0 \quad H_1: \text{some } \alpha_i \neq 0 \qquad (1.3)$$

$$H_0: \delta_1=\delta_2=\delta_3=0 \quad H_1: \text{some } \delta_j \neq 0 \qquad (1.4)\ .$$

There are three possible sources of variation and there are three corresponding measures of variability, called sum of squares. Totals instead of averages can be used in calculating the sum of squares for rows, columns, total and error. The computing formulas used in this thesis were:

$$\left.\begin{aligned}
SS_{A(rows)} &= \frac{\sum_{i=1}^{a} Ti..^2}{bn} - \frac{T...^2}{abn} \text{ with d.f.}=a-1 \\
SS_{B(columns)} &= \frac{\sum_{j=1}^{b} T.j.^2}{an} - \frac{T...^2}{abn} \text{ with d.f.}=b-1 \\
SS_{T(total)} &= \sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{k=1}^{n} Y_{ijk}^2 - \frac{T...^2}{abn} \text{ with d.f.}=abn-1 \\
SS_{E(error)} &= SS_T - SS_A - SS_B \text{ with d.f.}=abn-a-b+1
\end{aligned}\right\} \quad (1.5)$$

where T... is the grand total, Ti.. is the i^{th} row total, and T.j. is the j^{th} column total.

The model equation (1.1) or (1.2) for the two-way classification is an additive model. Since model (1.2) was assumed to have fixed effects, the expectation of the sum of squares in (1.5) can be written:

$$\left.\begin{aligned} E(SS_A) &= (a-1)\sigma^2 + bn\sum_{i=1}^{a}\alpha_i^{\,2} \\ E(SS_B) &= (b-1)\sigma^2 + an\sum_{j=1}^{b}\delta_j^{\,2} \\ E(SS_E) &= (abn-a-b+1)\sigma^2 \\ E(SS_T) &= (abn-1)\sigma^2 + bn\sum_{i=1}^{a}\alpha_i^{\,2} + an\sum_{j=1}^{b}\delta_j^{\,2} \quad . \end{aligned}\right\} \quad (1.6)$$

Further, if the null hypotheses (1.3) and (1.4) hold, that is, if

$$\sum_{i=1}^{a}\alpha_i^{\,2} = \sum_{j=1}^{b}\delta_j^{\,2} = 0$$

then the following are true:

$$\left.\begin{aligned} \frac{SS_A}{\sigma^2} &\cap \chi^2{}_{a-1} \\ \frac{SS_B}{\sigma^2} &\cap \chi^2{}_{b-1} \quad . \end{aligned}\right\} \quad (1.7)$$

Under either the null or alternative hypothesis, it is known that:

$$\frac{SS_E}{\sigma^2} \cap \chi^2_{abn - a - b + 1}$$
(1.8)
$$\frac{SS_T}{\sigma^2} \cap \chi^2_{abn - 1} \quad .$$

In Table 2 is the analysis of variance and expected mean squares for a fixed model experiment, without interaction, in a two-way classification design with n observations per cell.

TABLE 2

Analysis of Variance for Two-way Classification

Source of Variation	Sum of Squares	Degrees of Freedom
Row Effects	$\frac{\sum_{i=1}^{a} Ti..^2}{bn} - \frac{T...^2}{abn}$	$a-1= \nu_1$
Column Effects	$\frac{\sum_{j=1}^{b} Tj..^2}{an} - \frac{T...^2}{abn}$	$b-1= \nu_1'$
Error	*Difference	$abn-a-b+1=\nu_2$
Total	$\sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{k=1}^{n} Y_{ijk}^2 - \frac{T...^2}{abn}$	abn-1

*Difference= $SS_E = SS_T - SS_A - SS_B$

TABLE 2

(Continued)

Source of Variation	Mean Squares	Expected Mean Square
Row Effects	$\frac{SS_A}{a-1} = s_1^2$	$\sigma^2 + bn \frac{\sum_{i=1}^{a} \alpha_i^2}{a-1}$
Column Effects	$\frac{SS_B}{b-1} = s_2^2$	$\sigma^2 + an \frac{\sum_{j=1}^{b} \delta_j^2}{b-1}$
Error	$\frac{SS_E}{abn-a-b+1} = s_3^2$	σ^2
Total		

If $\sum_{i=1}^{a} \alpha_i^2 = \sum_{j=1}^{b} \delta_j^2 = 0$, then s_1^2, s_2^2, s_3^2 are independent unbiased estimators of σ^2. If the null hypothesis (1.3), that the row effects are equal to zero, holds, the ratio s_1^2/s_3^2 has an F distribution with (a-1) and (abn-a-b+1) degrees of freedom. Likewise, if the null hypothesis (1.4), that the column effects are equal to zero, holds, the ratio s_2^2/s_3^2 has an F distribution with (b-1) and (abn-a-b+1) degrees of freedom. The above F ratios are valid provided the random variables ϵ_{ijk}'s are normally and independently distributed with means of zero and common variances.

The purpose of this thesis is to study the power of the test of a two-way classification (fixed effects model) when the errors are distributed non-normally. If no significant difference in power is noted for the errors being distributed non-normally with the errors being distributed normally, the two-way classification will be considered robust.

CHAPTER II

DISCUSSION AND FORMULATION OF PROBLEM

In a fixed model experiment, without interaction, in a two-way classification with n observations per cell, let us consider the two factors A and B, each with three levels and two observations per cell. That is, consider the model in equation (1.2) with a=3, b=3, and n=2. However, assume for our purpose that ε_{ijk} is distributed non-normally with mean zero, common variance $\sigma^2=1$, and standardized third and fourth cumulants $\lambda_1=\sqrt{\beta_1}$ and $\lambda_4= \beta_2-3$. As is well known, for a normal distribution $\beta_1=0$ and $\beta_2=3$.

The power must first be determined under normal theory; and as a check, a computer program was written, which will be explained later, to verify the power found under normal theory. Secondly, the power must be determined when skewness and kurtosis are given values other than their normal values. With these changes in β_1 and β_2, three questions arise which need to be given special consideration.

The power, the probability of rejecting the null hypothesis given that the alternative hypothesis is true, must be determined for the normal case, $\beta_1=0$ and $\beta_2=3$. If the hypothesis in (1.3) and/or (1.4) is false, then the expected values of $s_1{}^2$ and $s_2{}^2$ are equal to

$$\left.\begin{array}{l} \sigma^2 + \dfrac{bn}{a-1}\sum_{i=1}^{a}\alpha_i{}^2 \\ \text{and} \\ \sigma^2 + \dfrac{an}{b-1}\sum_{j=1}^{b}\delta_j{}^2 \quad . \end{array}\right\} \qquad (2.2)$$

Thus $s_1{}^2$ and $s_2{}^2$ are not distributed as $\sigma^2\chi^2/(a-1)$ and $\sigma^2\chi^2/(b-1)$, but as non-central chi squares, and the ratios $s_1{}^2/s_3{}^2$ and $s_2{}^2/s_3{}^2$ are not distributed as F with (a-1) and (abn-a-b+1), and (b-1) and (abn-a-b-1) degrees of freedom respectively, but as a non-central F's. The non-centrality parameters (λ_α and λ_δ) for a two-way classification are

$$\left.\begin{array}{ll} \lambda_\alpha = \dfrac{bn_i\sum_{i=1}^{a}\alpha_i{}^2}{\sigma^2} & \text{for rows} \\ \lambda_\delta = \dfrac{an\sum_{j=1}^{b}\delta_j{}^2}{\sigma^2} & \text{for columns} \quad . \end{array}\right\} \qquad (2.3)$$

For finding the power of the test, TABLE VIII from R. Wine (1964, page 647) was used. Two levels of significance $\alpha=0.01$ and $\alpha=0.05$, were used with $\nu_1=\nu_1{}'=2$

and $\nu_2=13$. The operating characteristic curve for the desired degrees of freedom was not present. One was approximated for $\nu_1=\nu_1'=2$ and $\nu_2=13$, where the ϕ used in his table corresponds to the present ϕ's being equal to

$$\phi_1 = \frac{\lambda_\alpha}{a} \qquad \phi_2 = \frac{\lambda_\delta}{b} \qquad (2.4)$$

where a=b=3, indicating the number of rows and columns. Now the power of the test of the hypothesis in (1.3) and (1.4) can be expressed as a function of ϕ_1 and ϕ_2 respectively.

Four different sets of α's and δ's were used in this thesis, and these are given in Table 3. They were preselected so that they would sum to zero. Throughout the rest of this thesis, only Set 1, Set 2, Set 3, and Set 4 will be used when refering to a particular set of α's and δ's.

TABLE 3

Row and Column Effects

Set No.	α's for Rows			δ's for Columns		
1	α_1= .8	α_2=-.4	α_3= -.4	δ_1= .8	δ_2=-.4	δ_3= -.4
2	α_1=1.0	α_2=-.5	α_3= -.5	δ_1=1.0	δ_2=-.5	δ_3= -.5
3	α_1=1.2	α_2=-.6	α_3= -.6	δ_1=1.2	δ_2=-.6	δ_3= -.6
4	α_1=1.0	α_2=0.0	α_3=-1.0	δ_1=1.0	δ_2=0.0	δ_3=-1.0

Table 4 gives the power for the four different sets of α's and δ's for α=0.01 and α=0.05 levels of significance, using the above procedure and Wine's TABLE VIII. The power had to be approximated because the correct curve was not present. Since $\lambda_\alpha=\lambda_\delta$, the power in TABLE 4 is shown only for rows with non-centrality parameter λ_α. The power would be the same for columns with non-centrality parameter λ_δ.

TABLE 4

Power for Different Sets of α's and δ's for α=0.01 and α=0.05 Levels of Significance

Set No.	λ_α	Power-Rows	
		α=0.05	α=0.01
1	5.76	.47	.21
2	9.0	.67	.36
3	12.96	.81	.55
4	12.0	.79	.50

A program was written, called "Normal Generator" and generating normal random numbers, to see if the same results, in terms of power, were obtained corresponding to the results in Table 4. This program is explained later.

Now we turn to the question, what will happen to the power if the ε_{ijk}'s are distributed non-normally? That is, what is the power if the ε_{ijk}'s have means of zero, variance $\sigma^2=1$, $\beta_1\neq 0$ and $\beta_2\neq 3$?

To do this, a new program was written, called "PURGE2 GENERATOR". This program contains a sub-routine called "PURGE2" which was written in FORTRAN IV machine language by Donald Thomas (1966). With the aid of this sub-routine and by using different values of β_1 and β_2 in the "PURGE2 GENERATOR" program (explained later) the power of the test was computed in the case of non-normal populations.

Several questions which arise as a result of changing β_1 and β_2 will be exploited. They include:

1. What will happen to the power of the test if β_1 is allowed to change while β_2 remains fixed?

2. What will happen to the power of the test if β_2 is allowed to change while β_1 remains fixed?

3. What will happen to the power of the test if β_1 and β_2 are both increased by using several particular Pearson curves?

Two levels of significance, $\alpha=0.01$ and $\alpha=0.05$, will be used in attempting to answer the above questions. It should be remembered that β_1 is a measure of skewness and β_2 is a measure of kurtosis as shown in (2.5). The assumption that will be made for the above questions will be that $\beta_1=0$ and $\beta_2=3$ will not occur together at any one time because this is the normal case. Since the moments will be read into the program, it will be helpful to note that the moments (CM_i is the ith central moment) and β_1 and β_2 are related as:

$$\left.\begin{aligned} \beta_1 &= \frac{CM3^2}{(CM2)^3} = CM3^2 \quad (CM2=1) \\ \beta_2 &= \frac{CM4}{(CM2)^2} = CM4 \quad (CM2=1) \end{aligned}\right\} . \tag{2.5}$$

The relationship between β_1 and β_2 must also be known in order to know what type of Pearson Curve is being used. This relationship is given in Table 5 along with some remarks. Let kappa = k be defined as

$$k = \frac{\beta_1(\beta_2+3)^2}{4(4\beta_2-3\beta_1)(2\beta_2-3\beta_1-6)} , \tag{2.6}$$

TABLE 5

Frequency - Curves

No. of Type (Pearson)	Criterion	Remarks
I	k negative	Limited range; skew; usually bell-shaped, but may be U-shaped, J-shaped or twisted J-shaped, L-shaped.
III	$2\beta_2=6+3\beta_1$	Unlimited range in one direction; usually bell-shaped, but may be J-shaped.
IV	$0<k<1$	Unlimited range;skew; bell-shaped.
VI	$k>1$	Unlimited range in one direction, skew; bell-shaped, but may be J-shaped.

The values of β_1 and β_2 are, however, restricted to a certain domain for distribution functions. The domain must be

$$\begin{aligned} 8\beta_2-15\beta_1-36 &\leq 0 \\ \beta_2-\beta_1-1 &\geq 0 \end{aligned} \qquad (2.7)$$

Frequency types for low and high values of β_1 and β_2 are given in TABLE XXXV and TABLE XXXVI, which were reproduced from Tables for Statisticians and Biometricians, pages 66-67, and are listed as Table 21 and Table 22 in the List of Tables of this thesis. These tables were used in determining what values of β_1 and β_2 to use.

One would like to know if the power of the test is insensitive to the condition that the ε_{ijk}'s are non-normally distributed. That is, is the power obtained from the ε_{ijk}'s that are distributed non-normally the same as the power for the ε_{ijk}'s that are normally distributed? If so, the test will be said to be robust, which implies that we can ignore the fact that the ε_{ijk}'s are not normally distributed and proceed as if they were.

CHAPTER III

EXPLANATION OF COMPUTER PROGRAMS USED

The use of the IBM 7040-1401 system, located at Virginia Polytechnic Institute's Computer Center, made possible a speedy study concerning the power of the analysis of variance of a two-way cross classification, fixed effects model. Two programs utilizing the FORTRAN IV language were written. They were called "Normal Generator" and "PURGE2 GENERATOR" located in Appendix A and B respectively. These two programs were used in the generation of pseudo-random numbers by means of Monte Carlo methods. The "Normal Generator" program generated normal pseudo-random numbers. This routine of generation was called by the use of the word RNOR(X) in the main line program. The "PURGE2 GENERATOR" program makes use of a subroutine located in the mainline program which in turn generates random numbers from Pearson distributions. By using the call statement

CALL PURGE2 (N1,N2) (3.1)

data cards are read and random numbers are generated according to the type of Pearson Distribution that is to be fitted.

The "Normal Generator" program was used in verifying that the results of Table 4, page 16, were correct. Each time RNOR(X) was used, a new normal pseudo-random number was generated.

The model of (1.2) was written for this program as

$$Y(I,J,K) = RNOR(X) + AA(I) + BB(J) \quad (3.2)$$

where Y(I,J,K) was the response, AA(I) and BB(J) was one set of effects of the α's and δ's respectively, and RNOR(X) was the normal random generator which produced normal pseudo-random numbers. Eighteen numbers were generated from which an analysis was run. The analysis was run in the usual manner of a two-way classification, fixed effects model (with two replicates and no interaction) as described in Table 2. If either or both of the hypotheses $\alpha_1=\alpha_2=\alpha_3=0$ and/or $\delta_1=\delta_2=\delta_3=0$ were rejected, a count of it was made. Eighteen more numbers were generated; and the rejections, if any, were added to the previous number of rejections, depending on whether it belonged to the α's or δ's. This procedure was continued until one hundred such analyses were performed. The percentage of rejections out of one hundred was taken to be the power. The above procedure was done twenty times, each time obtaining a percentage of rejections out of one hundred. Using these twenty percentages, an average and a standard deviation were calculated. A standard error was then calculated by taking the standard deviation and dividing it by the square root of twenty. This average was taken to be the power of the analysis of variance test of the two-way classification for a specific set of effects. The last normal random number generated was printed, along with the average of percentages out of one hundred, the standard deviation, and the standard error. This last number was then read into the next program insuring that the same set of random numbers was not used over again. This same procedure was followed for the remaining three sets of α's and δ's.

The "PURGE2 GENERATOR" program made use of the subroutine PURGE2 (N1,N2), which was a revision made by Donald Thomas (1966) of PURGE (Pearson Universal Random Distribution Generator, written especially for the IBM 7090 system in FORTRAN II). The subroutine was called in by the use of the call statement in (3.1) where N1 and N2 are arguments which specify the options desired. Table 6 will tell what the options of N1 and N2 are. The option that was first used was CALL PURGE2

(3,2). The next call to PURGE 2 was accomplished by the statement CALL PURGE2 (2,5). This last option was used throughout the remainder of the program. The model used in this program was still different from (1.2) and (3.1) and was indicated in the program by

Y(I,J,K) = RDS(IJK) + AA(I) + BB(J) (3.3)

where RDS(IJK) means an array used to store the generated random numbers. The first four moments were used to describe the Pearson distribution to be fitted under the desired option. Except for the new model equation of (3.3) and the call statement (3.2), the program was run exactly like the one described in the "Normal Generator" program.

TABLE 6

Values of the Arguments for PURGE2

Value of N1	Action Taken
1	Read OPTION CARD and take action prescribed by it, i.e. read data cards, fit Pearson distribution, and generate 0 to 100 random numbers.
2	Generate up to 100 random numbers from a distribution previously described in the same program under the N1=1 or N1=3 option.
3	Fit a new Pearson curve from moments supplied by calling program via the common statement and generate 100 random numbers from this distribution
Value of N2	**Action Taken**
1	Summations of first four powers of all numbers generated from this distribution are kept for later calculation of sample moments.
2	Summations of first four powers of all numbers generated from this distribution are used to calculate sample moments which are then printed.
3	The c.d.f. is punched on cards in binary along with necessary parameters for generating random numbers from this distribution. Then the same action is taken as for N2=2.

TABLE 6 (Continued)

Value of N2	Action Taken
4	Same action is taken as for N2=2, then a graph of the distribution is printed. (Must be last option taken for a given distribution.)
5	Random numbers are generated without calculating summations for computations of moments. (Moments should not be printed for a given distribution after this option is used.)

CHAPTER IV

LITERATURE REVIEW

Several studies have been made concerning the effect of non-normality on test functions used in the analysis of variance, but little literature was found to be available on the power of the analysis of variance for the two-way classification, fixed effects model (with replicates and no interaction).

David and Johnson (1951) investigated the effect of various forms of departure from the theoretical models used in the analysis of variance. They studied the systematic (or parametric) form of model. One such model was

$$X_i = a_{i1}\theta_1 + a_{i2}\theta_2 + - - - + a_{is}\theta_s + Z_i \quad (4.1)$$

where the a's were known constants, the matrix $A=(aij)$ was non-singular, the θ's were unknown parameters, and the Z's were independent normal random variables each with expected value zero and variance σ^2. Their aim was to provide a method and detailed formulae for investigating the significance level and the Type II error, i.e. the power function of the test when the Z's were, in fact, not normally distributed. It was found that the method which was described in this article provided a useful means of investigating the effect of various forms of departure from the theoretical model used in the analysis of variance. They discussed the systematic form of model. A similar approach has been found useful in the case of the random model and also in investigations of the distributions arising in randomization theory. There is some uncertainty about the accuracy of the probabilities obtained from the curves fitted to the moments they used. At any rate, for the systematic model, it was concluded that the method used provided an adequate mode of approximation.

Correction factors to the probabilities that the two-sample t and F statistics shall exceed fixed positive values t_0 and F_0 either numerically or arithmetically and to the probabilities that t shall be exceeded by fixed negative values t_0 were derived geometrically by Bradley (1952). He investigated the effect of non-normality of the two-sample t and the F distribution. The two-sample t-statistic was expressed as a simple multiple of the cotangent of the angle between two lines in space of dimensionality one less than the total of the sample sizes. The F statistic for k samples was expressed as a multiple of the cotangent of the angle between a line and a plane of (k-1) dimensions in space. He concluded that these geometrical approximations were exact in the special case where the parent population was normal, but these approximation procedures were found to be valid for distributions under both the null and non-null hypotheses. These correction factors permitted the use of existing tables and the results were asymptotically correct for numerically large values of the test statistics. For small sample sizes, he found it possible to evaluate the correction factors exactly; but for large sample sizes, he found it to be tedious.

Srivastava (1959) studied the effect of non-normality on the power of the analysis of variance test by investigating the non-central distribution of the variance ratio test, onthe assumption the distribution of the error term could be represented by the first four terms of the Edgeworth series. He calculated the effect of non-normality on the probability of the TYPE II error, and hence on the power, when the standard F-table significance levels were used. His model was a one-way classification where the sampled population was represented by the first four terms of the Edgeworth series. The populations considered were only moderately non-normal, as the terms higher than the fourth in the Edgeworth series were assumed to be zero. He found that the effect of skewness did not significantly effect the power of the analysis of variance. In the presence of a fair degree of kurtosis, he found a noticable change in the power curve in the case of small samples. But a small departure from normality with respect to kurtosis did not cause any significant deviation in power. In practice, kurtosis was more likely to affect power than skewness.

Box and Anderson (1954) considered the case of robust tests. They concluded that a large variety of tests for comparing means were highly robust to both non-normality and inequality of variances. The exception to this rule was the sensitivity of the analysis of variance to variance heterogeniety when the groups are of unequal size. Student's t-tests for testing a sample mean against a hypothesized value, in contrast to tests for comparing two or more means, were quite sensitive to skewness in the parent population.

Geary (1947), considering the effect of kurtosis only, gave an approximate formula for the probability correction for w, based on the large sample assumption. He defined w as

$$w = \frac{(n'' - 1) \sum_{i=1}^{n'} (x_i - \bar{x})^2}{(n' - 1) \sum_{i=1}^{n''} (y_i - \bar{y})^2} \qquad (4.2)$$

with $x_1, x_2, \ldots, x_{n'}$ and $y_1, y_2, \ldots, y_{n''}$ being two independent samples drawn at random from the same universe (normal or non-normal). He pointed out that the actual probability of difference between means and variances derived from random samples on the null hypothesis would differ considerably from the probability derived from the standardized tables, when, in fact, the universal distribution is not normal. It was shown that in the simplest case of analysis of variance, when two sample numbers were of the same order of magnitude, the variance was proportional to $(\beta_2 - 1)$, that is, a small measure of universal kurtosis materially changes the probability.

It is interesting to consider the quotation from R. C. Geary (1947) that "Normality is a myth; there never was, and never will be, a normal distribution."

CHAPTER V

DISCUSSION OF RESULTS

A. Power Obtained by the "Normal Generator" Program

The two null hypotheses, (1.3) and (1.4) assume that the α's and δ's, deviation of rows and columns from the true mean, are zero. If differences do exist, the α's and δ's will not all be zero; and their set of values will define an alternative to the two null hypotheses.

In practice, the true values of these differences will not be known, and one would depend on the non-centrality parameters λ_α and λ_δ to find the power.

The "Normal Generator" program, generating normal random numbers, made it possible to obtain the power of the test in a two-way classification. Given a certain set of α's and δ's, and by using the technique of Monte Carlo simulation, the power was obtained as explained in Chapter III. The power of the different sets of α's and δ's was found to be about the same as the power obtained through the use of non-centrality parameters λ_α and λ_δ. Table 7 gives the power, using the four different sets of α's and δ's, obtained through the use of this program. In comparing Table 7 with Table 4, page 16, it can be clearly seen that the powers are in close agreement. (The standard error found in using this program never exceeded .012 and was as low as .007 in some instances.)

TABLE 7

Power Obtained Through the Use of the "Normal Generator" Program for Different Sets of α's and δ's at the α=0.01 and α=0.05 Significance Levels

Set No.	Power-Rows		Power-Columns	
	α = 0.05	α = 0.01	α = 0.05	α = 0.01
1	.47	.22	.47	.22
2	.68	.37	.66	.36
3	.83	.57	.82	.54
4	.80	.52	.81	.52

B. Effect on Power if β_1 is Allowed to Change While β_2 Remains Fixed

The question under consideration is: What will happen to the power of the test if β_1, a measure of skewness, is allowed to change while β_2, a measure of kurtosis, remains fixed? Tables XXXV and XXXVI, pages 51-52, were used in deciding what values of β_1 and β_2 were to be used. However, the choice of β_1 for a fixed β_2 was restricted to the domain as given in (2.7), page 19.

> <u>Example</u>: If β_2 was equal to four, in what region would β_1 be confined?
>
> From (2.7), page 19, it is easy to verify that
>
> $$-4/15 \leq \beta_1 \leq 3 \ .$$

In Tables 8-11, different β_1's were chosen, and for each of these, different β_2's were selected. This was done for each set of α's and δ's. By picking out a particular β_2 in each table, it is clearly seen that there are practically no differences between the values of power associated with the β_1's for that particular β_2. In fact, the different powers are in close agreement with that of the normal case, as given at the bottom of each table. If anything, the power seems to increase slightly as β_2 increases.

It can be said then, that skewness, β_1, has little effect on the power of the test when β_2 remains fixed.

C. Effect on Power if β_2 is Allowed to Change While β_1 Remains Fixed

The question under consideration is: What will happen to the power of the test if β_2 is allowed to change while β_1 remains fixed? Again Tables XXXV and XXXVI were used. As before, if β_1 is allowed to be fixed, then β_2 will be restricted to a certain range which can be figured out by the use of (2.7), page 19.

β_1 of 1.0 was chosen and was used in all four sets of α's and δ's. For β_1=1.0, β_2 was given the values 3.0, 4.0, 4.8, 5.0, and 6.0. If one compares the normal power found at the bottom of Tables 12-15 with the powers obtained for each row and column at the α = 0.05 and α = 0.01 significance levels and for β_1 = 1.0 with varying β_2's, it can be seen that kurtosis has little effect on the power of the test, when a curve has a predescribed skewness. With additional β_1's, some of which are seen in the tables, the same conclusion is reached. This implies that for any curve that is skewed, the power obtained for a change in kurtosis will not be far off from the power of the normal case.

D. Effect on Power for an Increase in β_1 and β_2 for Several Pearson Curves

The question under consideration is: What will happen to the power of the test if β_1 and β_2 are both increased in using a particular Pearson Curve?

As before, Tables XXXV and XXXVI were used, but most of the values came from Table XXXVI because β_1 and β_2 are both being increased at the same time. Since only one curve is to be considered at a time, the particular curve being used limits the values that β_1 and β_2 can have. Pearson Curves that will be considered are Type I-J shaped, Type I-L shaped, and Type I-bell shaped.

In Tables 16 (α=0.05) and 17 (α=0.01), Pearson Type I-L shaped curves are considered. Since the TYPE I-L is a left tail curve, one may produce a mirror image, that is, a right tail curve, merely by placing a negative sign in front of CM3, the third central moment. Everything else remains the same resulting in a Type I-J curve.

The Type I-J curves are in Tables 18 (α=0.05) and 19 (α=0.01). The object is to see if a change in power for these two particular Pearson Curves is noticed when β_1 and β_2 are both increased. At the 5 o/o level of significance, Tables 16 and 18, it is noticed that there is a definite increase in power for both Types of curves and for all sets of α's and δ's except for the possibility of Set 3. At the 1 o/o level of significance, Tables 17 and 19, there is definitely an increase in power, even in the case of Set 3. In all cases for both types of Pearson Curves, the power seems to converge to some value between .65 and .85.

Since the probability of committing a Type II Error is measured in terms of power, one finds that an increase in power, as indicated above, results in a decrease in this probability.

Since both types of curves are mirror images of each other, one would expect that the power would be about the same for each curve with regard to the same β_1's and β_2's. An examination of the tables will verify that this is the case. In comparison with the normal power, Table 7, one can observe that the power of the two Pearson Curves is much higher.

With regard to the Pearson Type I-bell shaped curve, Sets 1, 2, and 3 were considered. From Table XXXVI, it is seen that the Type I-bell shaped curve is restricted to a certain region. For Sets 1, 2, 3 at the 5 o/o and 1 o/o significance level, the deviation of power from that of the normal case was very small. This indicates that for Type I-bell shaped curves and for increases in β_1 and β_2 at the same time that there is no effect on the power of the test.

TABLE 8

Effect on Power of an Increase in β_1 with β_2 Fixed

(Set 1)

β_2	β_1	α = .05		α = .01	
		Rows	Columns	Rows	Columns
4.0	0.0	.48	.50	.23	.23
	0.4	.47	.47	.23	.21
	0.6	.47	.48	.22	.21
	1.0	.49	.49	.23	.23
	1.6	.48	.48	.23	.23
	2.0	.48	.48	.23	.23
3.0	0.4	.46	.48	.22	.21
	1.0	.47	.47	.20	.20
5.0	0.6	.48	.49	.22	.23
	1.0	.48	.48	.22	.23
	1.2	.51	.50	.23	.23
	1.8	.48	.50	.22	.24
	2.6	.50	.50	.24	.25
6.0	1.2	.50	.50	.25	.23
	1.8	.50	.49	.24	.23
	2.4	.51	.51	.25	.25
	2.8	.50	.51	.25	.25
Normal		.47	.47	.22	.22

TABLE 9

Effect on Power of an Increase in β_1 with β_2 Fixed

(Set 2)

β_2	β_1	α = .05		α = .01	
		Rows	Columns	Rows	Columns
4.0	0.0	.67	.66	.38	.39
	0.4	.66	.68	.38	.39
	0.6	.67	.69	.39	.38
	1.0	.67	.67	.38	.37
	1.6	.67	.67	.40	.40
	2.0	.69	.68	.40	.40
3.0	0.5	.66	.65	.37	.35
	1.0	.64	.66	.37	.35
5.0	1.0	.67	.69	.39	.39
	1.5	.69	.69	.41	.42
6.0	1.0	.68	.69	.40	.41
	1.5	.69	.69	.42	.41
	3.0	.71	.70	.44	.43
Normal		.68	.66	.37	.36

TABLE 10

Effect on Power of an Increase in β_1 with β_2 Fixed

(Set 3)

β_2	β_1	α = .05 Rows	α = .05 Columns	α = .01 Rows	α = .01 Columns
4.0	0.0	.82	.82	.57	.56
	0.4	.82	.82	.56	.56
	0.6	.83	.83	.58	.54
	1.0	.83	.83	.56	.57
	1.6	.81	.81	.56	.56
	2.0	.82	.84	.57	.58
3.0	0.4	.82	.83	.53	.54
	0.9	.82	.84	.54	.56
5.0	0.6	.83	.82	.57	.57
	1.0	.83	.83	.60	.59
	1.2	.81	.82	.58	.57
	1.8	.81	.83	.57	.58
	2.5	.82	.82	.58	.58
Normal		.83	.82	.57	.54

TABLE 11

Effect on Power of an Increase in β_1 with β_2 Fixed

(Set 4)

β_2	β_1	α = .05		α = .01	
		Rows	Columns	Rows	Columns
4.0	0.0	.79	.80	.51	.53
	0.4	.80	.80	.53	.52
	0.6	.81	.80	.53	.54
	1.0	.79	.79	.52	.53
	1.6	.78	.80	.52	.53
	2.0	.76	.79	.53	.52
Normal		.80	.79	.53	.52

TABLE 12

Effect on Power of an Increase in β_2 with β_1 Fixed

(Set 1)

β_1	β_2	$\alpha = .05$		$\alpha = .01$	
		Rows	Columns	Rows	Columns
1.0	3.0	.47	.47	.20	.20
	4.0	.50	.48	.25	.22
	4.8	.49	.47	.23	.21
	5.0	.50	.48	.23	.24
	6.0	.50	.50	.25	.23
1.4	4.0	.47	.49	.22	.21
	5.0	.48	.50	.23	.24
	6.0	.50	.52	.23	.26
1.8	4.0	.49	.48	.23	.22
	5.0	.48	.50	.22	.24
	6.0	.50	.49	.24	.23
2.2	5.0	.50	.51	.25	.26
	6.0	.51	.51	.25	.25
2.8	5.0	.52	.52	.27	.26
	6.0	.50	.51	.25	.25
Normal		.47	.47	.22	.22

TABLE 13

Effect on Power of an Increase in β_2 with β_1 Fixed

(Set 2)

β_1	β_2	α = .05 Rows	α = .05 Columns	α = .01 Rows	α = .01 Columns
1.0	3.0	.64	.66	.37	.35
	4.0	.67	.67	.38	.37
	4.8	.68	.68	.40	.40
	5.0	.67	.69	.39	.39
	6.0	.68	.69	.40	.41
1.5	4.0	.68	.67	.39	.40
	5.0	.69	.69	.41	.42
	6.0	.69	.69	.42	.41
	7.0	.71	.71	.42	.44
3.0	5.0	.67	.68	.40	.40
	6.0	.71	.70	.44	.43
	7.0	.69	.67	.42	.41
	8.0	.70	.70	.42	.43
	9.0	.72	.70	.46	.44
	10.0	.70	.70	.42	.43
Normal		.68	.66	.37	.36

TABLE 14

Effect on Power of an Increase in β_2 with β_1 Fixed

(Set 3)

β_1	β_2	α = .05		α = .01	
		Rows	Columns	Rows	Columns
1.0	3.0	.84	.83	.55	.55
	4.0	.83	.82	.58	.57
	4.8	.84	.82	.58	.58
	5.0	.83	.83	.60	.59
	6.0	.84	.85	.60	.60
0.6	4.0	.83	.83	.56	.54
	5.0	.83	.82	.57	.57
1.3	4.0	.82	.81	.58	.57
	5.0	.82	.83	.57	.56
Normal		.83	.82	.57	.54

TABLE 15

Effect on Power of an Increase in β_2 with β_1 Fixed

(Set 4)

β_1	β_2	α = .05 Rows	α = .05 Columns	α = .01 Rows	α = .01 Columns
	3.0	.79	.80	.51	.52
	4.0	.79	.80	.53	.53
1.0	4.8	.81	.81	.56	.54
	5.0	.79	.80	.55	.54
	6.0	.80	.79	.55	.56
Normal		.80	.81	.52	.52

TABLE 16

Effect on Power of an Increase in β_1 and β_2 for a Pearson Type I-L Shaped Curve at the 5% Significance Level

β_1	β_2	Rows			
		Set 1	Set 2	Set 3	Set 4
0.2	2.0	.44	.65	.83	.79
1.2	3.0	.47	.66	.82	.77
2.0	4.0	.48	.69	.82	.78
3.0	6.0	.54	.71	.87	.81
10.0	17.0	.60	.74	.83	.81
20.0	30.0	.66	.76	.85	.82
30.0	40.0	.68	.77	.84	.81
40.0	60.0	.71	.79	.84	.83
50.0	70.0	.72	.80	.86	.85
60.0	80.0	.74	.80	.85	.86

β_1	β_2	Columns			
		Set 1	Set 2	Set 3	Set 4
0.2	2.0	.46	.64	.83	.77
1.2	3.0	.47	.66	.84	.78
2.0	4.0	.48	.68	.84	.78
3.0	6.0	.52	.70	.83	.77
10.0	17.0	.61	.74	.82	.82
20.0	30.0	.65	.76	.85	.82
30.0	40.0	.69	.77	.83	.81
40.0	60.0	.70	.80	.84	.83
50.0	70.0	.72	.80	.86	.84
60.0	80.0	.75	.80	.85	.86

TABLE 17

Effect on Power of an Increase in β_1 and β_2 for a Pearson Type I-L Shaped Curve at the 1 o/o Significance Level

β_1	β_2	Rows			
		Set 1	Set 2	Set 3	Set 4
0.2	2.0	.18	.33	.54	.48
1.2	3.0	.23	.35	.54	.50
2.0	4.0	.23	.40	.57	.51
3.0	6.0	.26	.44	.59	.58
10.0	17.0	.40	.53	.65	.64
20.0	30.0	.49	.59	.72	.68
30.0	40.0	.54	.65	.72	.69
40.0	60.0	.57	.66	.73	.73
50.0	70.0	.62	.69	.75	.76
60.0	80.0	.65	.71	.75	.77

β_1	β_2	Columns			
		Set 1	Set 2	Set 3	Set 4
0.2	2.0	.18	.33	.53	.47
1.2	3.0	.21	.37	.56	.50
2.0	4.0	.23	.40	.58	.52
3.0	6.0	.26	.43	.59	.56
10.0	17.0	.39	.53	.66	.65
20.0	30.0	.47	.60	.70	.69
30.0	40.0	.56	.64	.73	.67
40.0	60.0	.57	.68	.73	.73
50.0	70.0	.61	.69	.77	.75
60.0	80.0	.65	.71	.76	.77

TABLE 18

Effect on Power of an Increase in β_1 and β_2 for a Pearson Type I-J Shaped Curve at the 5 o/o Significance Level

β_1	β_2	Rows			
		Set 1	Set 2	Set 3	Set 4
0.2	2.0	.44	.65	.83	.79
1.2	3.0	.47	.67	.82	.77
2.0	4.0	.48	.66	.80	.78
3.0	5.0	.51	.69	.79	.77
3.0	6.0	.56	.65	.81	.80
20.0	30.0	.65	.76	.84	.81
30.0	40.0	.71	.78	.84	.83
40.0	60.0	.73	.79	.84	.85
50.0	70.0	.74	.80	.86	.84
60.0	80.0	.77	.82	.86	.83

β_1	β_2	Columns			
		Set 1	Set 2	Set 3	Set 4
0.2	2.0	.44	.66	.82	.80
1.2	3.0	.46	.67	.81	.79
2.0	4.0	.50	.67	.80	.79
3.0	5.0	.49	.68	.80	.78
3.0	6.0	.53	.66	.82	.80
20.0	30.0	.65	.75	.86	.80
30.0	40.0	.71	.78	.84	.84
40.0	60.0	.71	.78	.85	.84
50.0	70.0	.74	.80	.87	.85
60.0	80.0	.77	.82	.85	.85

TABLE 19

Effect on Power of an Increase in β_1 and β_2 for a Pearson Type I-J Shaped Curve at the 1 o/o Significance Level

β_1	β_2	Rows			
		Set 1	Set 2	Set 3	Set 4
0.2	2.0	.18	.35	.54	.48
1.2	3.0	.22	.40	.55	.50
2.0	4.0	.24	.41	.55	.52
3.0	5.0	.28	.43	.55	.53
3.0	6.0	.29	.42	.60	.57
20.0	30.0	.46	.59	.71	.67
30.0	40.0	.57	.65	.73	.70
40.0	60.0	.58	.66	.74	.73
50.0	70.0	.63	.70	.76	.74
60.0	80.0	.67	.73	.77	.75

β_1	β_2	Columns			
		Set 1	Set 2	Set 3	Set 4
0.2	2.0	.19	.35	.51	.49
1.2	3.0	.22	.37	.54	.50
2.0	4.0	.25	.40	.56	.52
3.0	5.0	.27	.44	.59	.53
3.0	6.0	.27	.42	.62	.55
20.0	30.0	.48	.59	.71	.64
30.0	40.0	.57	.65	.73	.72
40.0	60.0	.58	.66	.75	.74
50.0	70.0	.64	.69	.76	.75
60.0	80.0	.66	.72	.76	.77

TABLE 20

Effect on Power of an Increase in β_1 and β_2 for a Pearson Type I-Bell Shaped Curve

(Set 1)

β_1	β_2	α = .05		α = .01	
		Rows	Columns	Rows	Columns
0.04	2.0	.45	.44	.18	.19
0.4	3.0	.46	.48	.22	.21
1.0	4.0	.49	.49	.23	.23
1.6	5.0	.52	.51	.24	.24
2.2	6.0	.51	.51	.25	.25

(Set 2)

β_1	β_2	α = .05		α = .01	
		Rows	Columns	Rows	Columns
0.25	2.0	.67	.67	.35	.35
0.5	3.0	.66	.64	.36	.35
1.0	4.0	.67	.67	.38	.37
1.5	5.0	.69	.69	.41	.42
3.0	6.0	.71	.70	.44	.43
3.0	7.0	.69	.67	.42	.41

(Set 3)

β_1	β_2	α = .05		α = .01	
		Rows	Columns	Rows	Columns
0.04	2.0	.82	.82	.53	.54
0.4	3.0	.82	.83	.53	.54
1.0	4.0	.83	.83	.56	.56
1.6	5.0	.83	.84	.59	.59

TABLE 21

XXXV. *Diagram to determine the type of a Frequency Distribution from a knowledge of the Constants β_1 and β_2. Customary Values of β_1 and β_2.*

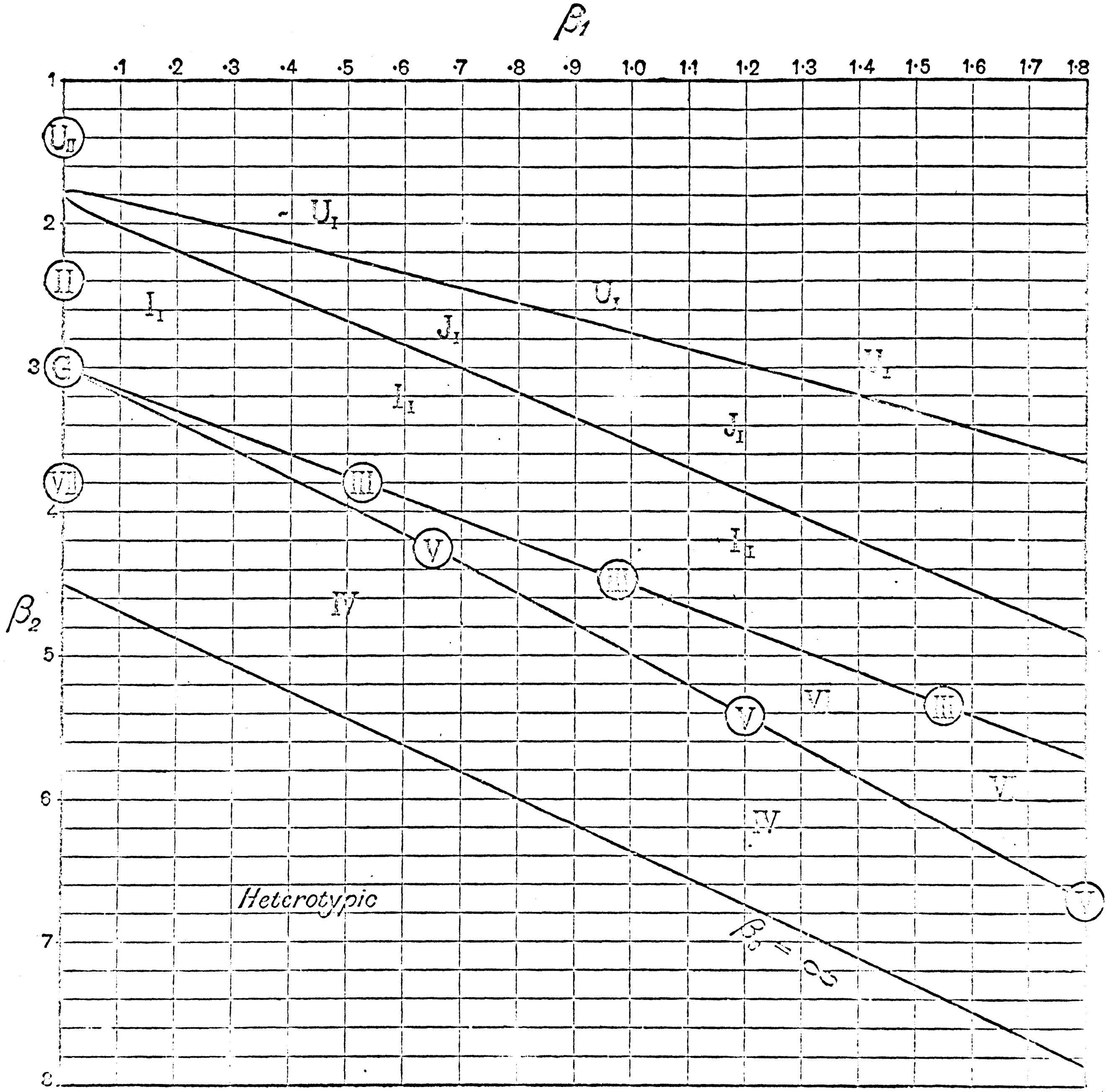
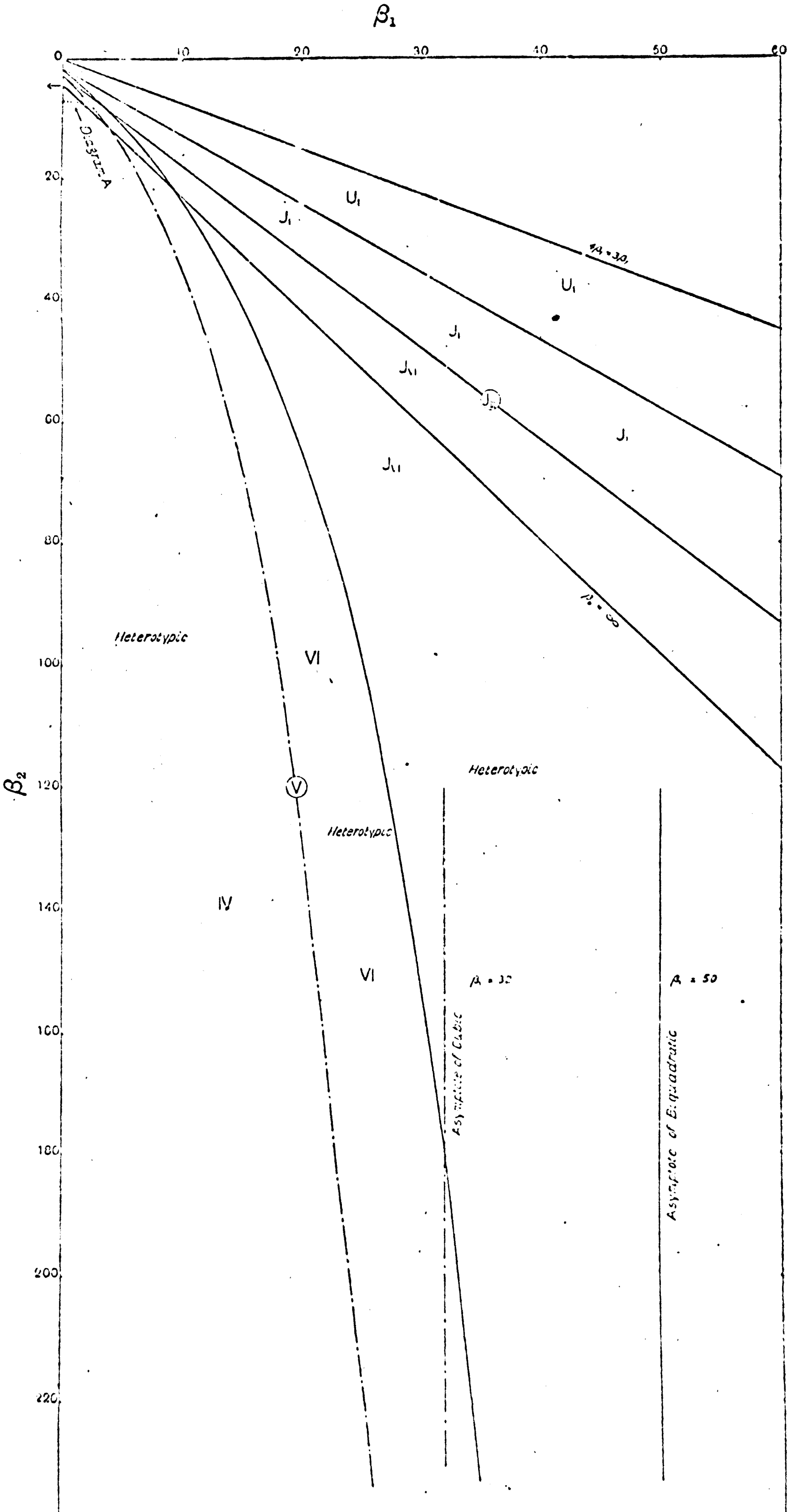


TABLE 22

XXXVI. *Diagram showing Distribution of Frequency Types for High Values for β_1 and β_2.*



CHAPTER VI

CONCLUSION

Let us consider the power of the analysis of variance test for a two-way classification, fixed effects model (with two replicates and no interaction). In the case where the sampled population was represented by the first four moments of a Pearson Curve, one has the opportunity to study the effects of non-normality on the power in terms of skewness and kurtosis. Most of the populations under consideration are only moderately non-normal, that is, only slightly skewed or peaked. However, some values of β_1 and β_2 were high (β_1 and β_2 taking on values from one to seventy) in studying what happens to certain Pearson Curves for an increase in both β_1 and β_2.

The results obtained enables one to calculate the effect of non-normality on the power, hence on the probability of the error of the second kind. It was found that the probability of the error of the second kind was not much different from that found under the normal assumption, except for the case when both skewness and kurtosis are increased for a particular Pearson Curve. The probability of the error of the second kind for large departures from normality was found to decrease with increases in power, (Tables 16-19).

It was found that the effect of skewness on the power of the analysis of variance test was not significant for a fixed kurtosis. However, with the presence of a fair degree of kurtosis, as is not uncommon, skewness seemed to increase the power. For a small departure from normality with respect to kurtosis, one can conclude that no significant deviation in the power was found.

The above conclusions are expected to be valid in general. For moderate departures from normality, β_1 as high as 3.0 and β_2 as high as 7.0, the two-way classification, fixed effects model, is robust.

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X. APPENDIX

```
$JOB    10       ROBERT W. TURNER, JR NORMAL GENERATOR


$IBJOB           NODECK
$IBFTC
      DIMENSION Y(3,3,2), RT(3), CT(3), AA(3), BB(3), G(4),
     1 F(4), NTOTAL(4), SUM(4), SUMS(4), AVG(4), SUMSQ(4),
     1 SQSUM(4), SD(4),SE(4)
      COMMON /Z1/AVE/Z9/CM2,CM3,CM4,BETA1,BETA2,SKAPPA/Z3/LI
     1MIT/Z2/RDS(100)/VPI002/NUMBER
      NUMBER = 1732067561
      REAL   NTOTAL
      DO 26 NO=1,4
      SUMSQ(NO) = 0.
   26 SUM(NO) = 0.
      KO = 1
   31 LO = 1
      DO 333 IJ=1,4
  333 NTOTAL (IJ) = 0
   22 AA(1) = .8
      AA(2) = -.4
      AA(3) = -.4
       BB(1) = .8
      BB(2) = -.4
```

```
      BB(3) = -.4
    1 DO 5 K=1,2
      DO 5 I=1,3
      DO 5 J=1,3
    5 Y(I,J,K) = RNOR(X) + AA(I) + BB(J)
      DO 10 I=1,3
      RT(I) = 0.
      DO 10 J=1,3
      DO 10 K=1,2
   10 RT(I) = RT(I) + Y(I,J,K)
      DO 15 J=1,3
      CT(J) = 0.
      DO 15 I=1,3
      DO 15 K=1,2
   15 CT(J) = CT(J) + Y(I,J,K)
      SUM1 = 0.
      DO 25 K=1,2
      DO 25 J=1,3
      DO 25 I=1,3
   25 SUM1 =SUM1 + Y(I,J,K)
      M=1
      RTS = (RT(M))**2 + (RT(M+1))**2 + (RT(M+2))**2
      CTS = (CT(M))**2 + (CT(M+1))**2 + (CT(M+2))**2
      SUMS2 = (SUM1)**2
      SUMYS = 0.
```

```
      DO 30 K=1,2
      DO 30 J=1,3
      DO 30 I=1,3
   30 SUMYS = SUMYS + (Y(I,J,K))**2
      A = 3.
      B = 3.
      C = 2.
      SSA = RTS/(B*C) - SUMS2/(A*B*C)
      SSB = CTS/(A*C) - SUMS2/(A*B*C)
      SST = SUMYS - SUMS2/(A*B*C)
      SSE = SST - SSA - SSB
      ROYA = SSA/(A-1.)
      ROYB = SSB/(B-1.)
      ROYC = SSE/(-B-A+1.+(A*B*C))
      FOA = ROYA/ROYC
      FOB = ROYB/ROYC
      F(1) = 3.81
      F(2) = 6.7
      NA = 1
      DO 7 NA=1,2
    7 G(NA) = FOA - F(NA)
      N = 1
      DO 9 N=1,2
      IF (G(N)) 9,9,35
   35 NTOTAL(N) = NTOTAL(N) + 1.
```

```
 9 CONTINUE
   NB = 1
   DO 8 NB=1,2
 8 H(NB) = FOB - F(NB)
   L = 1
   DO 80 L=1,2
   IF (H(L)) 80,80,3
 3 NTOTAL(L+2) = NTOTAL(L+2) + 1.
80 CONTINUE
   LO = LO + 1
   IF (LO - 101) 22,21,21
21 DO 24 NO=1,4
   SUM(NO) = NTOTAL(NO) + SUM(NO)
   SQSUM(NO) = (SUM(NO) / 100.) **2
24 SUMSQ(NO) = (NTOTAL(NO) / 100.) **2 + SUMSQ(NO)
   KO = KO + 1
   IF (KO - 101) 31,41,41
41 DO 42 NO=1,4
   AVG(NO) = SUM(NO) / 100.
   SD(NO) = SQRT((SUMSQ(NO) - SQSUM(NO)/ (100.))/99.)
42 SE(NO) = SD(NO) / 10.
   WRITE (6,43) SD,SE,AVG
43 FORMAT (1X F10.7,5H     ,F10.7,5H     ,F10.7,5H     ,F10.7)
   WRITE (6,97) NUMBER
97 FORMAT (I12)
```

```
      STOP
      END
$ENTRY
$IBSYS
```

```
$JOB    10       ROBERT W. TURNER, JR PURGE2 GENERATOR

$IBJOB
$IBFTC MAIN
      DIMENSION Y(3,3,2), RT(3), CT(3), AA(3), BB(3), G(4),
     1 F(4), NTOTAL(4), SUM(4), SUMS(4), AVG(4), SUMSQ(4),
     1 SQSUM(4), SD(4),SE(4)
      COMMON /Z1/AVE/Z9/CM2,CM3,CM4,BETA1,BETA2,SKAPPA/Z3/LI
     1MIT/Z2/RDS(100)/VPI001/NUMBR
   99 READ (5,98) NUMBR
   98 FORMAT (I12)
   28 READ (5,27) CM3,CM4
   27 FORMAT (2F10.8)
      CALL CHKPT(24)
      REAL   NTOTAL
      DO 26 NO=1,4
      SUMSQ(NO) = 0.
   26 SUM(NO) = 0.
      KO = 1
   31 LO = 1
      DO 333 IJ=1,4
  333 NTOTAL (IJ) = 0
   22 AA(1) = .8
      AA(2) = -.4
```

```
      AA(3) = -.4
      BB(1) = .8
      BB(2) = -.4
      BB(3) = -.4
      AVE = 0.0
      CM2 = 1.0
      LIMIT = 18
      IF (KO .EQ. 1) GO TO 56
   37 LIMIT = 18
      CALL PURGE2(2,5)
      GO TO 55
   56 CALL PURGE2(3,2)
   55 IF (LIMIT .EQ. 0) GO TO 41
    1 DO 5 K=1,2
      DO 5 I=1,3
      DO 5 J=1,3
      IJK = I + (J-1) * 3 + (K-1) * 9
    5 Y(I,J,K) = RDS(IJK) + AA(I) + BB(J)
      DO 10 I=1,3
      RT(I) = 0.
      DO 10 J=1,3
      DO 10 K=1,2
   10 RT(I) = RT(I) + Y(I,J,K)
      DO 15 J=1,3
      CT(J) = 0.
```

```
      DO 15 I=1,3
      DO 15 K=1,2
   15 CT(J) = CT(J) + Y(I,J,K)
      SUM1 = 0.
      DO 25 K=1,2
      DO 25 J=1,3
      DO 25 I=1,3
   25 SUM1 =SUM1 + Y(I,J,K)
      M=1
      RTS = (RT(M))**2 + (RT(M+1))**2 + (RT(M+2))**2
      CTS = (CT(M))**2 + (CT(M+1))**2 + (CT(M+2))**2
      SUMS2 = (SUM1)**2
      SUMYS = 0.
      DO 30 K=1,2
      DO 30 J=1,3
      DO 30 I=1,3
   30 SUMYS = SUMYS + (Y(I,J,K))**2
      A = 3.
      B = 3.
      C = 2.
      SSA = RTS/(B*C) - SUMS2/(A*B*C)
      SSB = CTS/(A*C) - SUMS2/(A*B*C)
      SST = SUMYS - SUMS2/(A*B*C)
      SSE = SST - SSA - SSB
      ROYA = SSA/(A-1.)
```

```
      ROYB = SSB/(B-1.)
      ROYC = SSE/(-B-A+1.+(A*B*C))
      FOA = ROYA/ROYC
      FOB = ROYB/ROYC
      F(1) = 3.81
      F(2) = 6.7
      NA = 1
      DO 7 NA=1,2
    7 G(NA) = FOA - F(NA)
      N = 1
      DO 9 N=1,2
      IF (G(N)) 9,9,35
   35 NTOTAL(N) = NTOTAL(N) + 1.
    9 CONTINUE
      NB = 1
      DO 8 NB=1,2
    8 H(NB) = FOB - F(NB)
      L = 1
      DO 80 L=1,2
      IF (H(L)) 80,80,3
    3 NTOTAL(L+2) = NTOTAL(L+2) + 1.
   80 CONTINUE
      LO = LO + 1
      IF (LO - 101) 37,21,21
   21 DO 24 NO=1,4
```

```
      SUM(NO) = NTOTAL(NO) + SUM(NO)
      SQSUM(NO) = (SUM(NO) / 100.) **2
   24 SUMSQ(NO) = (NTOTAL(NO) / 100.) **2 + SUMSQ(NO)
      KO = KO + 1
      IF (KO - 101) 31,41,41
   41 DO 42 NO=1,4
      AVG(NO) = SUM(NO) / 100.
      SD(NO) = SQRT((SUMSQ(NO) - SQSUM(NO)/ (100.))/99.)
   42 SE(NO) = SD(NO) / 10.
      WRITE (6,43) SD,SE,AVG
   43 FORMAT (1X F10.7,5H     ,F10.7,5H     ,F10.7,5H     ,F10.7)
      WRITE (6,97) NUMBR
   97 FORMAT (I12)
      WRITE (7,23) NUMBR
   23 FORMAT (1X I12)
      GO TO 28
      END
$ENTRY          MAIN
 30218135616
 .707107     3.0
 .707107     4.0
$IBSYS
```

THE EFFECT OF NON-NORMALITY ON THE POWER OF THE TEST OF A TWO-WAY CLASSIFICATION, FIXED EFFECTS MODEL--NO INTERACTION

by

Robert William Turner, Jr.

ABSTRACT

This thesis considers the effect of non-normality, in terms of skewness and kurtosis, on the power of the analysis of variance test for a two-way classification, fixed effects model (with two replicates and no interaction). Most of the populations considered were only moderately non-normal, that is, only slightly peaked or skewed.

We first considered the effect of a change in skewness for a fixed kurtosis. We then considered the effect of a change in kurtosis for a fixed skewness. In both cases, there was no significant deviation in power, as compared with the power in the normal case.

The effect of non-normality on the power for an increase in both skewness and kurtosis for a particular Pearson Curve was then considered. A significant deviation in the power was found.

The results indicate that for moderate departures from normality, the two-way classification, fixed effects model, is robust.