The Accuracy of River Bed Sediment Samples

by

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(Abstract)

One of the most important factors that influences a stream's hydraulic and ecological health is the streambed's sediment size distribution. This distribution affects streambed stability, sediment transport rates, and flood levels by defining the roughness of the stream channel. Adverse effects on water quality and wildlife can be expected when excessive fine sediments enter a stream. Many chemicals and toxic materials are transported through streams by binding to fine sediments. Increases in fine sediments also seriously impact the survival of fish species present in the stream. Fine sediments fill tiny spaces between larger particles thereby denying fish embryos the necessary fresh water to survive. Reforestation, constructed wetlands, and slope stabilization are a few management practices typically utilized to reduce the amount of sediment entering a stream. To effectively gauge the success of these techniques, the sediment size distribution of the stream must be monitored.

Gravel bed streams are typically stratified vertically, in terms of particle size, in three layers, with each layer having its own distinct grain size distribution. The top two layers of the stream bed, the pavement and subpavement, are the most significant in determining the characteristics of the stream. These top two layers are only as thick as the largest particle size contained within each layer. This vertical stratification by particle size makes it difficult to characterize the grain size distribution of the surface layer. The traditional bulk or volume sampling procedure removes a specified volume of material from the stream bed. However, if the bed exhibits vertical stratification, the volume sample will mix different populations, resulting in inaccurate sample results. To obtain accurate results for the pavement size distribution, a surface oriented sampling technique must be employed. The most common types of surface oriented sampling are grid and areal sampling. Due to limitations in the sampling techniques, grid samples typically truncate the sample at the finer grain sizes, while areal samples typically truncate the sample at the coarser grain sizes. When combined with an analysis technique, either frequency-by-number or frequency-by-weight, the sample results can be represented in terms of a cumulative grain size distribution. However, the results of different sampling and analysis procedures can lead to biased results, which are not equivalent to traditional volume sampling results. Different conversions, dependent on both the sampling and analysis technique, are employed to remove the bias from surface sample results.

The topic of the present study is to determine the accuracy of sediment samples obtained by the different sampling techniques. Knowing the accuracy of a sample is imperative if the sample results are to be meaningful. Different methods are discussed for placing confidence intervals on grid sample results based on statistical distributions. The binomial distribution and its approximation with the normal distribution have been suggested for these confidence intervals in previous studies. In this study, the use of the multinomial distribution for these confidence intervals is also explored. The multinomial distribution seems to best represent the grid sampling process. Based on analyses of the different distributions, recommendations are made. Additionally, figures are given to estimate the grid sample size necessary to achieve a required accuracy for each distribution. This type of sample size determination figure is extremely useful when preparing for grid sampling in the field.

Accuracy and sample size determination for areal and volume samples present difficulties not encountered with grid sampling. The variability in number of particles contained in the sample coupled with the wide range of particle sizes present make direct statistical analysis impossible. Limited studies have been reported on the necessary volume to sample for gravel deposits. The majority of these studies make recommendations based on empirical results that may not be applicable to different size distributions. Even fewer studies have been published that address the issue of areal sample size. However, using grid sample results as a basis, a technique is presented to estimate the necessary sizes for areal and volume samples. These areal and volume sample sizes are designed to match the accuracy of the original grid sample for a specified grain size percentile of interest. Obtaining grid and areal results with the same accuracy can be useful when considering hybrid samples. A hybrid sample represents a combination of grid and areal sample results that give a final grain size distribution curve that is not truncated. Laboratory experiments were performed on synthetic stream beds to test these theories. The synthetic stream beds were created using both glass beads and natural sediments. Reducing sampling errors and obtaining accurate samples in the field are also briefly discussed. Additionally, recommendations are also made for using the most efficient sampling technique to achieve the required accuracy.

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List of Symbols

α =	confidence coefficient
α_s =	particle shape factor for area
β_s =	particle shape factor for volume
b(x: n, p) =	probability distribution function of a binomial random variable X
B(x: n, p) =	cumulative distribution function of a binomial random variable X
$\chi^2_{\alpha,1}$ =	the upper (1 - α)100 percentage point of the chi-square distribution with one degree of freedom
D_i =	sieve diameter of size class *i*
E[x] =	expected value of x
n =	total number of trials in a sample
N_A =	total number of stones in an areal sample
N_G =	total number of stones in a grid sample
N_V =	total number of stones in a volume sample
η =	porosity
p_i =	known probability of success for the i^{th} parameter
$\hat{p}_i$ =	estimated probability of success for the i^{th} parameter
p_l =	lower confidence interval proportion
p_u =	upper confidence interval proportion
$p(A\text{-}N)_i$ =	frequency-by-number proportion of the i^{th} class of an areal sample
$p(A\text{-}W)_i$ =	frequency-by-weight proportion of the i^{th} class of an areal sample
$p(G\text{-}N)_i$ =	frequency-by-number proportion of the i^{th} class of a grid sample
$p(V\text{-}N)_i$ =	frequency-by-number proportion of the i^{th} class of a volume sample
$p(V\text{-}W)_i$ =	frequency-by-weight proportion of the i^{th} class of an volume sample
σ_g =	geometric standard deviation
Var[x] =	variance of x

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Chapter 1: Introduction to Fluvial Sediment Sampling

1.1 Introduction

One of the most important factors that influences a stream's hydraulic and ecological health is the stream bed's grain size distribution. This distribution affects stream bed stability, sediment transport rates, and flood levels by defining the roughness of the stream channel. Adverse effects on water quality and wildlife can be expected when excessive fine sediments deposit in a stream. Increases in fine sediments can seriously impact the survival of fish species present in the stream. Fine sediments fill spaces between larger particles, thereby denying fish embryos the necessary fresh water to survive (Waters, 1995). Many chemicals and toxic materials are also transported through streams by binding to fine sediments. Reforestation, constructed wetlands, vegetation strips, and slope stabilization are a few techniques typically utilized to reduce the amount of fine sediment entering a stream. To effectively gauge the success of these techniques, the sediment size distribution of the stream must be monitored.

Riverbeds typically exist in two forms: gravel and sand beds. Sand beds are comprised of sand and smaller sized particles. Gravel beds consist of a wide range of particles, from fine sediments to gravel or coarser particles. Typically, gravel streambeds are vertically stratified in three layers, in terms of particle size (Diplas and Fripp, 1992). The top two layers of the streambed, the pavement and subpavement, are the most significant in determining the hydraulic and ecological characteristics of the stream (Diplas and Fripp, 1992). Typically, the pavement is only as thick as the largest particle size contained in the layer (Kellerhals and Bray, 1971). To accurately sample a gravel streambed, surface sampling techniques that only sample the top layer of sediment must be employed.

From the above discussion, the importance of accurately representing the grain size distribution is evident. The present study will discuss many of the issues involved with accurately sampling river bed sediment. Sampling techniques are reviewed along with their respective advantages and disadvantages. New techniques are introduced to aid in arriving at more accurate sample results.

1.2 Sediment Properties

Understanding the basic properties of sediment is necessary to study sediment sampling. Sediment can be classified as cohesive and non-cohesive. Cohesive sediments are very small particles where electrochemical forces dominate. Clay is an example of cohesive sediment. Non-cohesive sediments are larger than cohesive sediments and mechanical forces are dominant. Sand, gravel, and cobbles are examples of non-cohesive sediments. The specific gravity of sediment is a convenient way of representing the density of the sediment. The following is the definition of the specific gravity of sediments:

$$s = \rho_s / \rho_w \tag{1.1}$$

where, s = specific gravity of the sediment

ρ_s = density of the sediment
ρ_w = density of water at 4°C

Porosity is another important parameter for fluvial sediment deposits. Porosity is represented as the ratio of the total volume of voids to the total volume of voids and sediment.

Size first and then shape are the most important properties of an individual particle. The shape of a particle is represented by three axes, all perpendicular to each other. The a-axis is the longest axis on the particle, the b-axis is the intermediate axis, and the c-axis is the shortest axis on the particle. A commonly used parameter to represent shape is the shape factor:

$$\text{Shape factor} = c / (ab)^{0.5} \tag{1.2}$$

The shape factor for a sphere is 1.0 and approximately 0.7 for natural sediments (Diplas, 1996).

Size directly affects the mobility of the particle; hence, it is of primary importance. Particle size is typically measured using a sieve diameter, D. The sieve diameter represents the smallest square sieve opening that the particle will pass through. The units used for particle size are usually millimeters or phi-units. The relationship between the two is as follows, with D in millimeters:

$$D = 2^{-\phi} \qquad \text{or} \qquad \phi = -\log_2 D$$

Various techniques can be used to measure particle size in the field, depending on the relative size of the particle. For particles larger than 256-mm, a measuring tape or calipers are used to measure the intermediate b-axis. A gravelometer is used to measure particles between about 10-mm and 256-mm. The gravelometer, shown in Figure 1.1, is a template with square openings corresponding to various different sieve sizes. For 0.0625-mm < D < 10-mm, a sieve analysis is performed. For very small particles (D<0.0625), hydraulic settling is used (Vanoni, 1975). It should be noted that only the gravelometer and sieve analyses are equivalent measuring techniques.

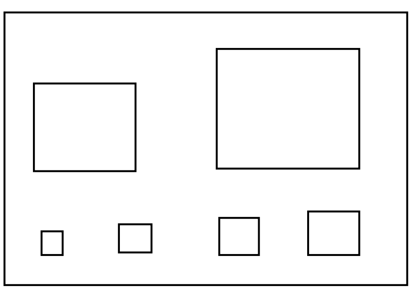

Figure 1.1 Gravelometer

Estimates of sediment size distributions are best represented using a cumulative grain size distribution curve. The process of obtaining this curve will be discussed in section 1.4.1, Representation of Sample Results. From the grain size distribution, the mean diameter can be obtained. The mean diameter represents the average or expected grain size for the entire distribution. The standard deviation represents the sorting by size of the distribution and is commonly referred to as the sorting coefficient. Distributions with a sorting coefficient less than 1.3 are considered well sorted. Poorly sorted distributions have a sorting coefficient greater than 1.3. The probability distribution curve, which represents the individual probabilities for each particle size, can have one, two, and rarely more maxima. A distribution with one maximum is referred as unimodal. A bimodal distribution has two maxima. Figure 1.2 illustrates the difference between unimodal and bimodal distributions. Typically, sandy streams are unimodal. About 40% of gravel streams are bimodal.

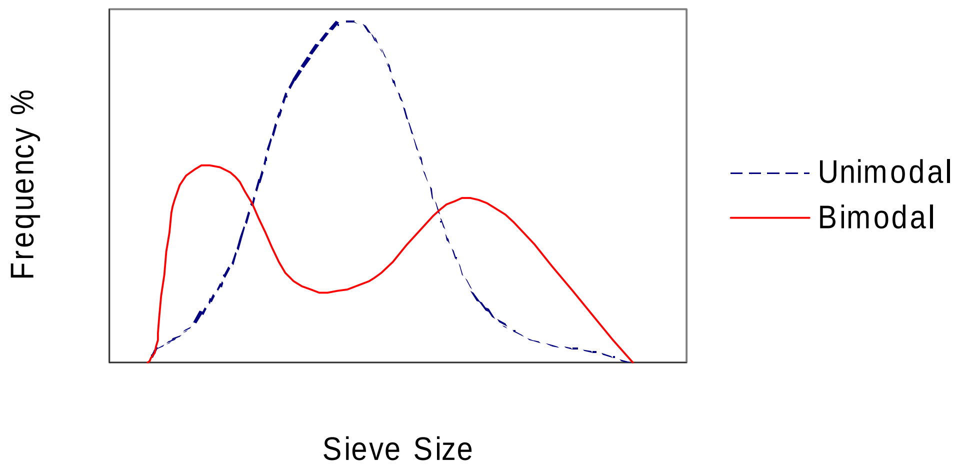

Figure 1.2 Comparison of unimodal and bimodal distributions

1.3 Sample Collection Techniques

The results of sediment samples depend on the techniques used to both collect and analyze the sample. A variety of techniques have been proposed to collect sediment samples. These techniques are explained in this section. Advantages and disadvantages for each technique are discussed. Additionally, previously made recommendations concerning sample size and accuracy are presented. Volume, grid, and areal sampling are the main types of sediment sampling techniques.

1.3.1 Volume Samples

Volume, or bulk, sampling is the first step in the common sieve analysis. A predetermined volume or weight of material is removed, usually with a shovel, from the riverbed. This volume should be large enough so that it becomes independent of the volume of the individual particles that comprise the sample. The volume sampling technique cannot be used to sample the pavement of a gravel bed river or any layer that exhibits size stratification. The reasoning is that the pavement is typically one grain size thick, therefore the volume of the sample is dependent on the individual grain sizes. A volume sample can be taken of the substrate of a gravel bed river by first removing the surface particles (Kellerhals and Bray, 1971).

The problem of determining the necessary sample volume to represent the grain size distribution with certain degree of accuracy is a difficult one. A technique to determine the standard deviation of a volume sample is not straight forward. Difficulty arises when attempting to assign the number of degrees of freedom to a sample weight, say 10 Kg, as opposed to a count. Using approximations for number of grains retained by each sieve (Sahu, 1964), De Vries (1970) made sample volume recommendations based on three levels of accuracy (high, normal, and low) and the grain size corresponding to the 84th percentile, D_{84}. Sample parameters were empirically determined from sieve analyses of natural sands. However, the stochastic variation of the number of particles in each sieve is neglected and the resulting volumes tend to be conservative (Ferguson and Paola, 1997). By sieving many subsamples of a large sample volume, Church et al. (1987) make empirical recommendations for appropriate sample volumes. The largest grain size present in the sample is used as a basis for the sample volume following the reasoning that the largest particles will be the fewest in number and, therefore, least well represented (Church et al., 1987). The sample volumes recommended by Church et al. (1987) can be excessively large for certain applications (Ferguson and Paola, 1997). Gale and Hoare (1992) use many samples of actual gravel for their sample size criterion. Recommendations are made based on the type of gravel being sampled (tills, fluvial gravel, and beach gravel). The basis of this approach is not clear and may not be applicable to many situations (Dunkerley, 1994). Ferguson and Paola (1997) use computer simulations to analyze a variety of sample sizes for log-normal distributions. Based on the results, sample size recommendations are made using a dimensionless volume. This dimensionless volume is defined as the ratio between the sample volume and the volume occupied by a specific grain size, say D_{50}. Despite the recommendations made in the works discussed above, no technique has been developed to place a confidence level on a volume sample grain size distribution.

1.3.2 Grid Samples

The basis for grid sampling was first presented in Wolman (1954). The motivation behind the development of this technique was to derive a sampling method that could accommodate the wide range of grain sizes present in a river bed, represent an entire river reach, and only sample surface particles. To obtain the sample, a grid is established over the river reach of interest. The particles below each grid point are removed, typically, by hand. The total number of particles sampled becomes the grid sample for the river reach.

The common methods of applying the grid are placing a wire grid on top of the bed, placing survey tape marked at regular intervals across the bed, and pacing equal steps across the bed (Church et al., 1987). Recommendations for grid spacing have been made from a few grain diameters (Church et al., 1987) to seven feet (Bevenger and King, 1995). However, the grid spacing should not affect the final results assuming that the sediment deposit is isotropic in the horizontal directions. The grid spacing should be selected so that the river reach in question can be sampled conveniently. To avoid bias in selecting particles, Wolman (1954) recommends that the sampler not look at the riverbed as the hand descends to touch a particle and as the particle is removed. Additionally, Wolman (1954) recommends consistently selecting the particle directly beneath the big toe of the sampler. Difficulties arise if the hand comes in contact with more than one particle. Kondolf (1996) states that by selecting a specific point on the hand as a reference point (i.e. the right corner of the fingernail on the index finger) this problem can be overcome.

The advantages of the grid sample are summarized as follows (Wolman, 1954 and Kondolf, 1996):

1. Procedure is simple to perform.
2. The sample is representative of the entire river reach.
3. The sample is reproducible.

The principle disadvantage of the grid sampling procedure as noted by Wolman (1954) and Fripp and Diplas (1993) is the inability of the technique to adequately represent fine particles. The limiting factor has to do with the sampler's ability to distinguish among fine particles. Using the approximate width of a typical index finger, Fripp and Diplas reason that 15-mm is about the smallest particle that can be sampled with the grid method when hands are used for sampling. If tweezers are used, this size can be reduced. Several reports however have stated sampling particles below 8-mm (Wolman, 1954 and Kondolf, 1996). Kondolf (1997a) again proposes selecting a reference point on the sampler's hand to help sample small particles. Even employing this technique, the fact that a lower limit exists must be recognized. To avoid truncating the sample at 15-mm, or some other cutoff diameter, when a particle smaller than the cutoff diameter is sampled, it can be recorded as simply being below the cutoff diameter. This practice allows the proportion of particles below the cutoff diameter to be determined, even though the exact distribution below the cutoff diameter can't be determined.

The sample size required to obtain an accurate sample has been discussed frequently in the literature. Originally, Wolman (1954) suggested 100-stones as an adequate sample size. This recommendation is based on applying the Student's t-test to field samples as well as considering simplicity of computations. Kondolf (1997) supports this recommendation. Hey and Thorne (1983) found that 40-stones were sufficient statistically by analyzing logarithmically transformed data. Using a normal approximation of the binomial distribution, Fripp and Diplas (1993) applied confidence intervals to grid sample results. Then, the sample size, given the desired level of accuracy, can be obtained. By considering the effort involved in grid sampling, they concluded that a 400-stone sample would provide the best return. Rice and Church (1995) find additional evidence to support a 400-stone sample size. The statistical technique of bootstrapping was employed to generate accuracy levels without making any assumptions about the grain size distribution. However, to apply the bootstrap technique, sample sizes in excess of 1200-stones were required.

1.3.3 Areal Samples

Another surface oriented sampling technique that has been purposed is areal sampling. This technique is performed by removing and measuring all the surface particles that cover a specified area. The surface particles can be removed by a variety of techniques. The sample area can be spray painted and the marked stones removed by hand and measured. A photograph can be taken of the sample area and used to determine the sizes of the surface particles. Using this photographic technique, difficulties arise because the b-axis of some particles may not be exposed in the photograph and, therefore, cannot be accurately measured. The most effective areal sampling method appears to be with the use of adhesives. Adhesives are applied to the sample area and then removed. The stones caught by the adhesive are measured with a sieve analysis and become the areal sample. Among the adhesives that have been used are wax, tape, and clay. Clay can be used to sample underwater, which makes it the preferable adhesive. The inability to sample large particles is the limitation of the adhesive areal sample. Fripp and Diplas (1993) found that a 40-mm diameter particle was about the largest that an adhesive could remove consistently.

No information had been presented on the area necessary to obtain an areal sample with a desired degree of accuracy, until recently. The issue of degrees of freedom, similar to that of the volume sample, again appears. Diplas and Fripp (1992) present a technique that gives a minimum sample area based on the largest particle size. It is not apparent how to quantify the accuracy of this sample. Again, as with the volume sample, no method has been derived to place a confidence statement on the areal sample results.

1.4 Sample Analysis Techniques

Once a sample has been collected, the particles in the sample are placed into sizes classes based on particle sieve diameters. Three analysis techniques can be used for analyzing the sample, frequency-by-number, frequency-by-volume, and frequency-by-area. Frequency-by-number analyses are performed by calculating the ratio of the number of particles in each size class to the total number of particles in the sample. Frequency-by-volume analyses are performed by calculating the ratio of the volume of particles in each size class to the total volume of particles in the sample. For simplicity, weight is typically substituted for volume in frequency-by-volume calculations. This substitution can be performed without any additional adjustments for particles with the same specific gravity. A frequency-by-area analysis is performed by taking the ratio between the total projected area of each size class and the total projected area of the sample. The results for these analyses can be thought of as probabilities of occurrence for each size class.

Once samples are analyzed, combining the sampling and analysis techniques yields a descriptive name. For example, an area-by-number sample is an areal sample that has been analyzed with the frequency-by-number technique. Due to the large number of particles present in volume and areal samples, frequency-by-weight analyses are typically used. Volume-by-weight samples are often referred to as volumetric samples. Grid samples are typically analyzed using frequency-by-number.

1.4.1 Representation of Sample Results

Once the sample has been analyzed, the results are most often presented in the form of a cumulative grain size distribution curve. The curve is graphed on a semi-log plot, with the sieve sizes corresponding to the x-axis and cumulative percent finer on the y-axis. The cumulative distribution is calculated by consecutively adding the individual probabilities of occurrence. The cumulative distribution can also be thought of as the ratio between the particles passing through each sieve size and the total number of particles in the sample. Table 1.1 and Figure 1.3 demonstrate the concept of the cumulative grain size distribution curve as applied to a grid-by-number sample.

Table 1.1 Example calculating cumulative distribution from a grid-by-number sample

Sieve Size (mm)	No. Particles in Size Class	No. Particles Passing	Cumulative Probability
32.0	0	100	1.00
22.6	18	82	0.82
16.0	28	54	0.54
11.3	23	31	0.31
8.00	17	14	0.14
5.66	10	4	0.04
4.00	4	0	0

Sample Size = 100

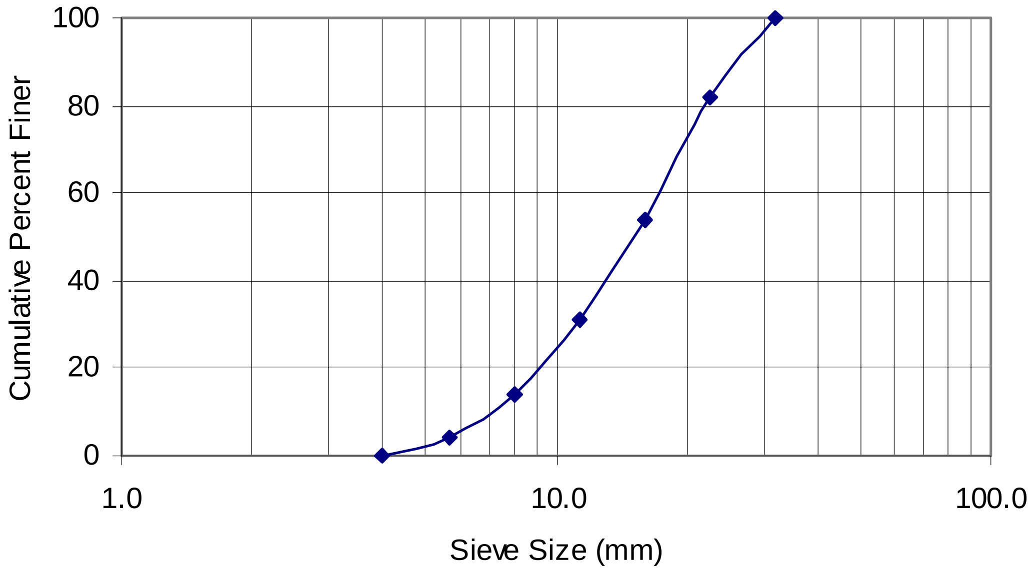

Figure 1.3 Cumulative grain size distribution curve from Table 1.1

1.4.2 Equivalence of Sample Results

Due to differences in the various sampling and analysis techniques, the results from different techniques may not be directly comparable. The only exception to this rule is the case of grid-by-number, volume-by-weight, and area-by-area samples, which will be discussed later. In order to make meaningful comparisons between different types of samples, a standard representation is necessary. The volume-by-weight representation is used as this standard due to the fact that it represents an unbiased sample.

Converting between different analysis types can be done by representing the sample data for the new analysis type. However, there is no direct way to convert between the different sample types. Once data is taken using a specific sample type, the resulting data will always be of that specific type and cannot directly be changed. To illustrate this point, consider a grid sample. If this grid sample is to be converted into a volume sample, would the number of particles be multiplied by the cube of the sieve diameters? This would be incorrect because it would simply correspond to a frequency-by-volume analysis on the grid data. To overcome this difficulty, Kellerhals and Bray (1971) derived a method to convert different types of samples using a cube model. The cube is made up of three different sized cubes randomly packed together. By analyzing the cube, conversion factors were proposed based on the particle size, D. Table 1.2 shows the recommendations of Kellerhals and Bray (1971) with some additional conversions. Diplas and Sutherland (1988) recommend an alteration of the conversion from area-by-weight to volume-by-weight samples when wax is used as the adhesive. The recommendation made is to raise the diameter to the negative one half power. This recommendation is supported empirically, with areal sampling experiments, and theoretically, by modifying the Kellerhals and Bray (1971) cube to consider voids. The conversion from area-by-weight obtained using wax to volume-by-weight as proposed by Diplas and Sutherland (1988) can be stated as follows:

$$p(\text{V-W})_i = Cp(\text{A-W})_i D_i^{-0.5} \qquad (1.3)$$

where $p(\text{V-W})_i$ and $p(\text{A-W})_i$ are the proportions of size fraction i for the volume-by-weight and area-by-weight samples, respectively. D_i is the mean grain for size fraction i, and C is a proportionality constant to insure the range of percents finer is from 0 to 100. The exponent on the diameter is not exact, but rather varies with the porosity of the surface layer. An exponent of –0.42 corresponds to a porosity of 33.3% and the exponent decreases until it reaches –1.00 which corresponds to a porosity of zero, as found by Kellerhals and Bray (1971). A general formula is useful to convert various samples to volume-by-weight proportions. The conversion formula is the following:

$$p(\text{V-W})_i = C \cdot p(\text{S})_i D_i^{x} \qquad (1.4)$$

where, $p(\text{S})_i$ = proportion of sample to be converted for size fraction *i*.

D_i = sieve diameter for size fraction *i*.

x = conversion constant, see Table 1.3.

$$C = \frac{1}{\sum p(\text{S})_i D_i^{x}}$$

Table 1.3 shows the appropriate constant to use with Equation 1.4 for the various sampling techniques. Figure 1.4 illustrates the importance of proper conversions using data obtained from lab samples.

Table 1.2 Conversions based on the recommendations of Kellerhals and Bray (1971)

	Conversion to								
Conversion From	volume-by-weight	volume-by-area	volume-by-number	grid-by-weight	grid-by-area	grid-by-number	area-by-weight	area-by-area	area-by-number
volume-by-weight	1	1/D	$1/D^3$	D^3	D^2	1	D	1	$1/D^2$
volume-by-area	D	1	$1/D^2$	D^4	D^3	D	D^2	D	1/D
volume-by-number	D^3	D^2	1	D^6	D^5	D^3	D^4	D^3	D
grid-by-weight	$1/D^3$	$1/D^4$	$1/D^6$	1	1/D	$1/D^3$	$1/D^2$	$1/D^3$	$1/D^5$
grid-by-area	$1/D^2$	$1/D^3$	$1/D^5$	D	1	$1/D^2$	1/D	$1/D^2$	$1/D^4$
grid-by-number	1	1/D	$1/D^3$	D^3	D^2	1	D	1	$1/D^2$
area-by-weight	1/D	$1/D^2$	$1/D^4$	D^2	D	1/D	1	1/D	$1/D^3$
area-by-area	1	1/D	$1/D^3$	D^3	D^2	1	D	1	$1/D^2$
area-by-number	D^2	D	1/D	D^5	D^4	D^2	D^3	D^2	1

Table 1.3 Conversion exponents for Equation 1.4

p(S)	x
Volumetric	1
Grid-by-number	1
Grid-by-Weight	-3
Area-by-number	2
Area-by-Weight	-1 or -0.5

Previously, the fact that grid-by-number and volume-by-weight sample results are equivalent was presented. This important statement was first presented by Kellerhals and Bray (1971) and deserves a brief discussion here. When the conversions are derived both the sampling method and analysis technique must be considered. To convert from a frequency-by-number to a frequency-by-volume representation, a factor of the diameter cubed (D^3) is applied to the frequency-by-number sample. When considering the sampling method used, the conversion factor is not as straightforward. A volume sample is designed so that the sample is independent of the individual particle sizes. This is not true for a grid sample, however. The total volume of a grid sample is determined by all three dimensions of the sampled particles. In order to convert a grid sample into a volume sample, this particle volume dependence must be removed. To render a grid sample's volume dependent from the individual particles, a factor of the inverse of the diameter cubed ($1/D^3$) is applied to the grid sample. When the effects of the sampling method and analysis technique are combined the factors cancel. By following a similar line of reasoning, it can be shown that an area-by-area sample is equivalent to both grid-by-number and volume-by-weight samples. Underwood (1970) uses dimensional analysis to show that dimensionless quantities, such as grid-by-number and area-by-area, will give equivalent and unbiased results. Empirical proof of this equivalence will be presented in the discussion of the laboratory experiments.

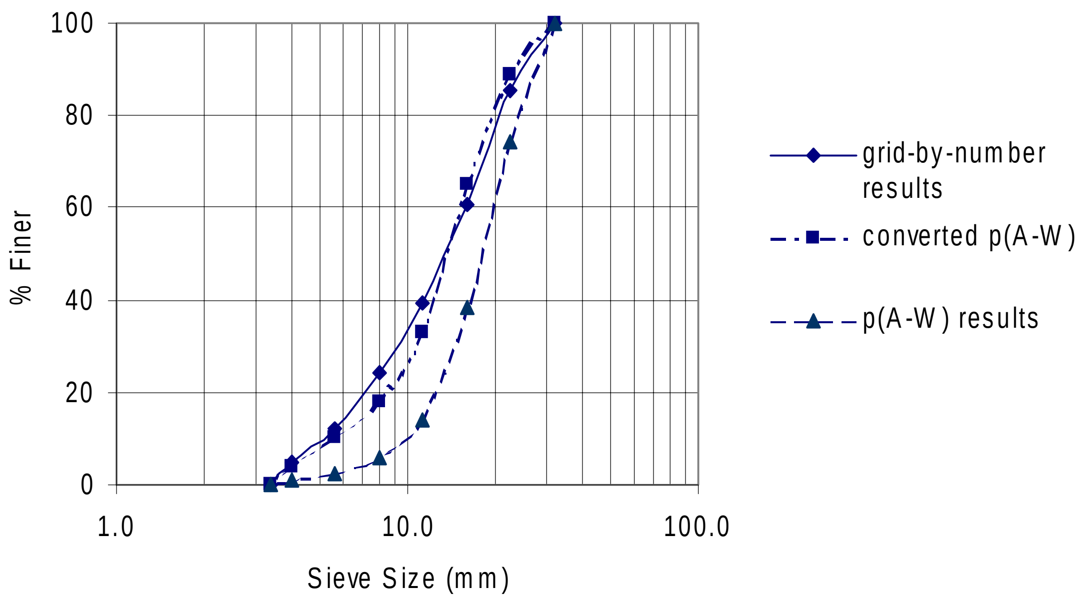

Figure 1.4 Converted vs. unconverted area-by-weight samples

1.5 Overview of Present Study

The present study analyses the accuracy of sediment samples for the sampling techniques discussed in this introductory chapter. A new method is presented for quantifying the accuracy of grid samples using multinomial confidence intervals. This technique can also be extended to determine the sample size of a grid sample for a required accuracy level. Another new method is proposed for quantifying the accuracy of areal and volume samples. This technique calculates the necessary area or volume to sample to match the accuracy of a previously performed grid sample. To test these new techniques, laboratory sampling tests were performed. By creating a synthetic streambed, grid, areal, and volume samples were able to be taken. The new techniques were then applied to the laboratory sampling results. The complete laboratory data and analyses are included in the appendices. The benefit of combining grid and areal sample results in to a hybrid sample is discussed. An example determining a hybrid sample is presented. A brief discussion is also included on field sampling techniques. This discussion primarily addresses the topic of reducing sampling and analysis errors.

Chapter 2: The Accuracy of Grid Sampling

2.1 Introduction

When making a statistical inference from sample data, the accuracy of the sample must be considered. A convenient way to represent sample accuracy is with confidence intervals. If p represents a population parameter of interest, the confidence interval around p is determined by finding p_l and p_u so that,

$$P(p_l < p < p_u) = 1 - \alpha$$

The (1-α)100 % confidence interval around p becomes (p_l , p_u). α is referred to as the confidence coefficient. The interpretation of this interval is that there is a probability of (1-α) that the interval (p_l , p_u) contains the true population value for p.

Actual grain size distributions are nearly continuous functions. Through the sieve analysis process, a discrete function is employed to approximate the grain size distribution. If the grain size distribution were to be represented as a continuous distribution, close to an infinite number of sieves would be necessary. Due to the integer constraint on discrete functions, confidence intervals cannot be found with a confidence coefficient exactly equal to (1-α). Intervals can be determined, however, for the case that the confidence coefficient is not less than (1-α). The following is the mathematical representation of a discrete confidence interval:

$$P(p_l < p < p_u) \geq 1 - \alpha$$

Another common technique for statistical inference is the hypothesis test. Hypothesis testing utilizes statistical distributions to determine whether sample data is consistent to a specified degree of accuracy with a parameter value of interest. Burdick and Graybill (1992) state that confidence intervals are almost always “uniformly more informative” than hypothesis tests. Hypothesis tests are also rarely necessary in situations where confidence intervals are available (Burdick and Graybill, 1992). The present study will use confidence intervals whenever possible.

Confidence intervals give a range of proportions that a particular grain size occupies for a given level of accuracy. These intervals are useful when considering both a specific grain size and the entire grain size distribution curve. This chapter will look at applying confidence intervals to grid-by-number samples using different distributions. The distributions considered are the binomial, normal approximation for the binomial, and multinomial.

The equations used to calculate confidence intervals can also determine sample sizes. The sample size necessary to obtain accurate results is an important issue. Logically, large samples will yield the most accurate results. Large samples may not always be practical, however. Time, manpower, and the size of the river reach are a few items that may limit the sample size. The question becomes "what is the smallest sample size possible to achieve a desired level of accuracy?" Techniques for calculating minimum sample sizes will be discussed for each type of distribution.

2.2 Binomial Confidence Intervals

Binomial confidence intervals can be applied to any situation involving a binomial distribution. A binomial distribution is identified by the following properties (Walpole and Myers, 1993):

1. The experiment consists of *n* repeated trials.
2. Each trial results in an outcome that may be classified as either a success or failure.
3. The probability of success, denoted by *p*, remains constant for each trial.
4. The repeated trials are independent.

The grid-by-number approach is consistent with all these properties. The removal and grain size determination of a stone represents a trial, with a total of *n* trials. Each stone is either within a grain size class, a success, or not, a failure. The probability for a success remains constant as long as the grain size distribution remains constant over the sample space of the *n* trials. The stone removed in each trial is not at all affected by any of the previous stones removed and therefore each trial is independent. The number of stones within a specified size range, represented by *X*, is then a binomial random variable. The probability distribution function of a binomial random variable is represented by the following:

$$b(x;n,p)=\binom{n}{x}p^{x}(1-p)^{n-x}\text{, for } n = 0, 1, 2, 3, \ldots, n \qquad (2.1)$$

where, $\binom{n}{x}=\dfrac{n!}{x!\,(n-x)!}$

x = an observation of X

The cumulative distribution function is represented by the following:

$$B\left(x;n,p\right)=\sum_{X=0}^{X=x} b\left(x;n,p\right) \qquad (2.2)$$

Using the probability and cumulative distribution functions, as well as the notation described above, the formulas necessary to compute the binomial confidence intervals are derived.

2.2.1 Derivation of Binomial Confidence Intervals

To develop the confidence interval for a binomial random variable *X* as found in Tables of Cumulative Binomial Probability Distribution (1955), the probabilities of unlikely or extreme events are quantified. First, the event that the observed number of successes *x* has a larger value than expected is examined. The probability that the random variable is greater than or equal to the observed successes is represented as follows:

$$P[X \geq x] = \frac{\alpha}{2} \tag{2.3}$$

where, $\frac{\alpha}{2}$ = confidence coefficient for (1-α)100 percent confidence

Because x is large, the probability of a success, X equaling or exceeding x, is much smaller than expected. This probability of success becomes the lower confidence limit for x, and is denoted $p_l(x)$. The formula for the lower confidence limit is found by expanding Equation 2.3 as follows:

$$P[X \geq x] = \sum_{X=x}^{X=n} b\left(x; n, p_l(x)\right) = \frac{\alpha}{2}$$

Rewriting the above using the cumulative binomial distribution:

$$\sum_{X=x}^{X=n} b(x;n,p_l(x)) = 1 - \sum_{X=0}^{X=x-1} b(x;n,p_l(x)) = 1 - B(x-1;n,p_l(x)) = \frac{\alpha}{2}$$

$$B\left(x-1; n, p_l(x)\right) = 1 - \frac{\alpha}{2} \tag{2.4}$$

Equation 2.4 represents the final equation used to determine the lower confidence limit around the observed number of successes *x*.

To derive an equation for the upper confidence limit, the case of a smaller than expected number of observed successes is considered. The probability of the random variable being less than or equal to the observed successes is represented as follows:

$$P[X \leq x] = \frac{\alpha}{2} \tag{2.5}$$

Because the number of observed successes is much smaller than expected, the probability of a success, *X* exceeding *x*, is much greater than expected. This probability of success becomes the upper confidence limit on the observed successes *x*. The equation is derived from Equation 2.5 as follows:

$$P[X \le x] = \sum_{X=0}^{X=x} b(x;n,p_u(x)) = B(x;n,p_l(x)) = \frac{\alpha}{2}$$

$$B(x;n,p_u(x)) = \frac{\alpha}{2} \tag{2.6}$$

Equation 2.4 and Equation 2.6 are used to apply a confidence interval to a binomial random variable *X*. This confidence interval is exact with no approximations and can be applied to any binomial random variable. Binomial confidence intervals using this exact method are usually not symmetric. It can easily be seen that there is not a direct solution to either of these equations. However, iterative computer programs are available to perform these calculations.

The confidence coefficient $\alpha/2$ is selected based on a $(1-\alpha)100$ level of confidence. The one half coefficient on α represents the possibility of error at both the upper and lower levels of confidence. For example, a 95 % level of confidence would correspond to $\alpha = 0.05$ and $\alpha/2 = 0.025$. It should be noted that at the two extreme cases of $x = 0$ and $x = n$, the $\alpha/2$ confidence coefficient does not apply. For the case of $x = 0$, the lower confidence limit is also zero because the lower limit cannot have a negative value. The upper confidence limit at $x = 0$, therefore, has a confidence coefficient of α in order to maintain the same level of confidence over the entire confidence interval. The opposite is the case for $x = n$, where the upper confidence limit becomes one. The lower confidence limit now has a confidence coefficient of α. The following example applies the binomial confidence interval to a grid by number sample.

2.2.2 Example using Binomial Confidence Intervals

The information given below was obtained from a fifty stone grid-by-number sample taken from a gravel bed stream. Use this information to determine the following:

(a) the 95 % confidence interval on the 12.5 mm sieve.
(b) the 95 % confidence interval on the 37.5 mm sieve.

Sieve Size (mm)	Number of Particles Retained
37.5	0
25	10
15.9	14
12.5	10
9.5	6
6.3	5
4.75	3
3.35	2

Solution:

(a) The 12.5 mm sieve allowed a total of 16 pebbles to pass through, or, in other words, retained 34 stones. The cumulative percent finer than 12.5 mm would then be estimated from our data to be:

$$\hat{p} = \frac{particles \quad not \quad retained}{total \quad particles} = \frac{x}{n} = \frac{16}{50} = 0.32$$

For 95 % confidence, α = 0.05. To find the lower confidence limit, rewrite Equation 2.3 and then Equation 2.4 for our specific problem.

$$P[X \geq 16] = 1 - B(x-1; n, p_l(x)) = \frac{\alpha}{2} = 0.025$$
$$B(15;50, p_l(16)) = 0.975$$

The equation is solved for $p_l(16)$ using an iterative spreadsheet program shown in Figure 2.1. The lower 95 % confidence limit is shown to be 0.195.

To calculate the upper confidence limit, Equation 2.5 and Equation 2.6 are rewritten.

$$P[X \leq 16] = \frac{\alpha}{2} = 0.025$$
$$B(16;50, p_u(16)) = 0.025$$

This $p_u(16)$ is solved using the spreadsheet program, as shown in Figure 2.1. The upper 95 % confidence limit is shown to be 0.467. From these results, it can be said with 95 % confidence that between 19.5 % and 46.7 % of the pebbles in the sampled gravel bed are finer than 12.5 mm.

(b) All 50 stones pass through the 37.5 mm sieve. The upper confidence limit is then 1.0. To find the lower confidence limit, rewrite Equation 2.3 and then Equation 2.4 for our specific problem. Remember to use α as the confidence coefficient.

$$P[X \geq 50] = 1 - B(x-1; n, p_l(x)) = \alpha = 0.05$$
$$B(49;50, p_l(50)) = 0.95$$

Figure 2.2 shows the iterative solution for the above equation. It is shown that the lower 95 % confidence limit is 0.942. These results state that with 95 % confidence between 100 % and 94.2 % of the pebbles in the sampled stream are finer than 37.5 mm.

Lower Limit , Goal of 0.975 for B(15;50,pl(16))

pl(16)	B(15;50,pl(16))
0.10	0.99998
0.11	0.99994
0.12	0.99984
0.13	0.99960
0.14	0.99907
0.15	0.99805
0.16	0.99619
0.17	0.99303
0.18	0.98798
0.19	0.98030
0.20	0.96920

pl(16)	B(15;50,pl(16))
0.190	0.98030
0.191	0.97936
0.192	0.97838
0.193	0.97736
0.194	0.97631
0.195	0.97523
0.196	0.97410
0.197	0.97294
0.198	0.97173
0.199	0.97048
0.200	0.96920

pl(16)	B(15;50,pl(16))
0.1950	0.97523
0.1951	0.97512
0.1952	**0.97500**
0.1953	0.97489
0.1954	0.97478
0.1955	0.97467
0.1956	0.97456
0.1957	0.97444
0.1958	0.97433
0.1959	0.97422
0.1960	0.97410

Upper Limit , Goal of 0.025 for B(16;50,pu(16))

pu(16)	B(16;50,pu(16))
0.40	0.15609
0.41	0.12431
0.42	0.09746
0.43	0.07520
0.44	0.05710
0.45	0.04265
0.46	0.03133
0.47	0.02263
0.48	0.01606
0.49	0.01120
0.50	0.00767

pu(16)	B(16;50,pu(16))
0.460	0.03133
0.461	0.03035
0.462	0.02940
0.463	0.02847
0.464	0.02757
0.465	0.02668
0.466	0.02583
0.467	0.02499
0.468	0.02418
0.469	0.02340
0.470	0.02263

pu(16)	B(16;50,pu(16))
0.4660	0.02583
0.4661	0.02574
0.4662	0.02566
0.4663	0.02558
0.4664	0.02549
0.4665	0.02541
0.4666	0.02533
0.4667	0.02524
0.4668	0.02516
0.4669	0.02508
0.4670	**0.02499**

Figure 2.1 Iterative calculations for 95 % binomial confidence intervals for part (a)

Lower Limit (Goal of 0.950 for B(49;50,pl(50)))

pl(50)	B(49;50,pl(50))
0.90	0.99485
0.91	0.99104
0.92	0.98453
0.93	0.97344
0.94	0.95467
0.95	0.92306
0.96	0.87011
0.97	0.78193
0.98	0.63583
0.99	0.39499
1.00	0.00000

pl(50)	B(49;50,pl(50))
0.940	0.95467
0.941	0.95219
0.942	0.94959
0.943	0.94684
0.944	0.94395
0.945	0.94090
0.946	0.93769
0.947	0.93431
0.948	0.93075
0.949	0.92700
0.950	0.92306

pl(50)	B(49;50,pl(50))
0.9410	0.95219
0.9411	0.95194
0.9412	0.95168
0.9413	0.95143
0.9414	0.95117
0.9415	0.95091
0.9416	0.95065
0.9417	0.95038
0.9418	**0.95012**
0.9419	0.94985
0.9420	0.94959

Figure 2.2 Iterative calculation for lower 95 % binomial confidence interval for part (b)

2.3 Normal Approximation for Binomial Confidence Intervals

The normal distribution can be used to approximate the binomial distribution with a sufficient degree of accuracy for many situations. This approximation can be extended to the concept of confidence intervals (Walpole and Myers, 1993). The following notation will be employed in the derivation of the normal confidence interval:

p = unknown actual binomial proportion we are interested in.
n = sample size

$\hat{p} =$ estimate of p = $\dfrac{total \quad x_i \le X}{n}$

$z_{\alpha/2} = z$-value leaving an area of $\dfrac{\alpha}{2}$ to the right of the normal curve

The following also applies because p is binomial:

$E[\hat{p}] = p =$ expected value or mean of $\hat{p}$

$Var[\hat{p}] = \dfrac{p(1-p)}{n} =$ variance of $\hat{p}$

$$Z = \frac{X - E[X]}{\sqrt{Var[X]}} = \frac{\hat{p} - E[\hat{p}]}{\sqrt{Var[\hat{p}]}} = \frac{\hat{p} - p}{\sqrt{\dfrac{p(1-p)}{n}}} = \text{standard normal variate for } \hat{p}$$

2.3.1 Derivation of the Normal Approximation

If a confidence level is set at (1-α)100 % with a corresponding $z_{\alpha/2}$ from the normal curve, the following confidence interval can be set:

$$P\left[-z_{\alpha/2} < Z < z_{\alpha/2}\right] = 1-\alpha \tag{2.7}$$

Rewriting Equation 2.7, substituting for *Z*, the following is obtained:

$$P\left[-z_{\alpha/2} < \frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}} < z_{\alpha/2}\right] = 1-\alpha \tag{2.8}$$

Using elementary algebra, the confidence interval is placed around the unknown proportion *p*:

$$P\left[\hat{p} - z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}} < p < \hat{p} + z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}}\right] = 1-\alpha \tag{2.9}$$

The unknown *p* must be approximated by our estimate $\hat{p}$ in the far left and right equations. The final (1-α)100 % confidence interval for *p* is then:

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} < p < \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \tag{2.10}$$

The normal approximation of the binomial distribution is accurate when *n* is large and *p* is not close to either 0 or 1, and when *n* is small and *p* is close to ½ (Walpole and Myers, 1993). A rule-of-thumb often applied to this approximation is that $n \cdot p$ and $n \cdot (1 - p)$ should be larger than around 5 (Walpole and Myers, 1993). It can be seen from the above statements that the normal confidence intervals will begin to differ noticeably from the binomial confidence intervals at the extreme high and low percentages of the grain size distribution curve. An additional drawback of the normal approximation is that the required confidence coefficient may not always be met (Agresti and Coull, 1998). This failure to meet the required confidence level becomes more important as the proportions move further away from one half.

2.3.2 Example using the Normal Approximation for Binomial Confidence Intervals

Rework the example from section 2.1.2 using normal confidence intervals.

Solution:

The cumulative percent finer than 12.5 mm would then be estimated from our data to be:

$$\hat{p} = \frac{particles \quad not \quad retained}{total \quad particles} = \frac{x}{n} = \frac{16}{50} = 0.32$$

For 95 % confidence, α = 0.05. $z_{\alpha/2} = z_{0.025} = 1.96$ from the normal distribution curve. Equation 2.10 is now written as:

$$0.32 - 1.96\sqrt{\frac{0.32(1-0.32)}{50}} < p < 0.32 + 1.96\sqrt{\frac{0.32(1-0.32)}{50}}$$

The normal confidence interval simply becomes:

$$0.191 < p < 0.449$$

This answer compares relatively well with the results of 0.195 and 0.467 found using the binomial distribution. With $n \cdot p = 50 \cdot 0.32 = 16$, this case is well above the approximate cutoff point of 5. Because the estimated proportion at the 37.5-mm sieve is 1.0, both the upper and lower confidence intervals are also equal to 1.0, as can been seen from Equation 2.10.

2.4 Simultaneous Confidence Intervals for Multinomial Proportions

The multinomial distribution applies to situations where more than two outcomes are possible for each trial. In other words, for each of *n* independent trial, there are *k* possible outcomes, each with an individual probability of occurrence. For the multinomial distribution, the following notation is employed:

k = the number of possible outcomes for each trial
x_i = the number of occurrences for the i^{th} outcome
n = the total number of trials
p_i = the probability of a success for the i^{th} outcome = $\frac{x_i}{n}$

The following must then be true:

$$\sum_{i=1}^{k} x_i = n \qquad \text{and} \qquad \sum_{i=1}^{k} p_i = 1$$

The probability distribution function for the multinomial distribution is:

$$f(x_1, x_s, \ldots, x_k; n, p_1, p_s, \ldots, p_k) = \begin{pmatrix} & n & & \\ x_1 & x_2 & \ldots & x_k \end{pmatrix} p_1^{x_1} p_2^{x_2} \ldots p_k^{x_k}$$

From the previous equation it can be seen that the binomial distribution is actually a special case of the multinomial distribution, where $k = 2$. The multinomial distribution can be applied to the grid-by-number sampling technique; where k is the number of size classes, x_i is the number of stones in the sample falling into the i^{th} size class, and n is the total number of stones in the sample.

2.4.1 Simultaneous and One-at-a-Time Confidence Intervals

The concept of a simultaneous confidence interval is fundamentally different from the binomial approach to confidence intervals. The binomial confidence limits are calculated for point estimates, a type of one-at-a-time confidence intervals as labeled by Burdick and Graybill (1992). For a grain size distribution, a one-at-a-time confidence interval with a confidence level of α at a specific grain size states that there is a probability of $(1-2\alpha)$ that the percent finer of the population is within the confidence interval for this grain size. However, when calculated simultaneously, confidence intervals consider multiple parameters from the same sample together. In the case of a grain size distribution, the percent finer estimates for each size class become these parameters. This simultaneous consideration generates a confidence interval around the entire grain size distribution curve. Therefore, a simultaneous confidence interval with a confidence level of α around a grain size curve states that there is a probability of $(1-2\alpha)$ that the population grain size curve is within the confidence interval at each size class. It can be seen that simultaneous confidence intervals are wider than one-at-a-time intervals.

When is it appropriate to use simultaneous confidence intervals? Burdick and Graybill (1992) state that if the statistical inference is desired for several functions of parameter values simultaneously, then simultaneous confidence intervals apply. Considering this criterion, simultaneous confidence intervals seem to hold a high degree of relevance to the issue of accuracy in sediment sampling, where we are typically dealing with several particle sizes. The distinction between these two types of confidence intervals is important and must always be considered when applying and interpreting confidence intervals.

The following example further illustrates the difference between one-at-a-time and simultaneous confidence intervals. Consider ten sediment samples, all taken from the same population. Due to the size range of particles in the samples, ten sieve diameters are necessary to describe the size distributions. Once the sieve analysis is performed on each sample, a total of ten sample grain size distribution curves are obtained. Binomial confidence intervals are then applied to each sample curve with a confidence level of $\alpha = 0.05$. Because the binomial confidence intervals are applied at each sieve diameter, each grain size curve consists of ten points were confidence intervals are calculated. Based on the given confidence level, the actual percent finer values would be expected to be within the binomial confidence intervals 90% of the time. From the given data, out the 100 points for all the grain size curves, the confidence intervals at ninety points would be expected to contain the actual percent finer values. Now, simultaneous multinomial confidence intervals are calculated for each sample grain size curve, again with a confidence level of $\alpha = 0.05$. Because the simultaneous intervals consider all points in the sample grain size curve together, the actual grain size curve is expected to be within the entire confidence interval for each sample curve 90% of the time. This translates to the actual grain size curve falling outside the simultaneous confidence interval for one out of the ten sample grain size curves. It should be noted that the actual distribution can fall outside of this confidence interval more than once unlike the binomial confidence intervals. This example reinforces the importance of correctly interpreting confidence intervals.

2.4.2 Calculating Simultaneous Confidence Intervals

A confidence ellipsoid from considering multinomial proportions simultaneously can be obtained from the chi-square statistic. However, since this ellipsoid is difficult to interpret, confidence intervals are preferred for making statistical inferences. Methods for calculating simultaneous confidence intervals for multinomial proportions have been proposed by Quesenberry and Hurst (1964), Goodman (1965), Fitzpatrick and Scott (1987), and Sison and Glaz (1995). May and Johnson (1997) compared these techniques based on empirical coverage probabilities and enclosed volume. Based on their recommendations, the Goodman (1965) technique is employed here due to simplicity of the calculations and the technique's ability to meet the required confidence coefficient.

Before discussing the exact derivation of the Goodman (1965) technique, a useful tool employed in this derivation is explained. This tool, known as the Bonferroni inequality, enables a transformation to consider many sample parameters simultaneously. Following from Wilks (1962), the Bonferroni inequality is derived. For each unknown parameter, p_i, for $i = 1, 2, \ldots, k$, there exists the intervals (p_{li} , p_{ui}), each with a confidence coefficient of $\frac{\alpha}{k}$. Consider the event, E_i , that (p_{li} , p_{ui}) contains p_i. $\overline{E}_i$ is then the complement of E_i , or the event where (p_{li} , p_{ui}) does not contain p_i. The probability of $\overline{E}_i$ then becomes:

$$P\left[\overline{E}_i\right] = \frac{\alpha}{k} \qquad \text{for } i = 1, 2, \ldots, k \tag{2.11}$$

To apply a simultaneous confidence interval, all the events, E_i , must occur simultaneously. Relating the simultaneous probability of the events and their complements, we arrive at the following:

$$P\left[E_1 \cap \ldots \cap E_k\right] = 1 - P\left[\overline{E}_1 \cup \ldots \cup \overline{E}_k\right] \tag{2.12}$$

It can also be seen that this must be true:

$$P\left[\overline{E}_1 \cup \ldots \cup \overline{E}_k\right] \le P\left[\overline{E}_1\right] + \ldots + P\left[\overline{E}_k\right] \tag{2.13}$$

The following can now be stated from Equations 2.12 and 2.13:

$$P\left[E_1 \cap \ldots \cap E_k\right] \geq 1-\left(P\left[\overline{E}_1\right]+\ldots+P\left[\overline{E}_k\right]\right) \tag{2.14}$$

Substituting Equation 2.11 into Equation 2.14, the conclusion is stated as:

$$P\left[E_1 \cap \ldots \cap E_k\right] \geq 1-\alpha \tag{2.15}$$

This result states that for unknown parameters $p_i, \ldots, p_k$, if (p_{li} , p_{ui}), . . . , (p_{lk} , p_{uk}) are 100[1 - α/k] % confidence intervals for $p_i, \ldots, p_k$, respectively, then the probability is at least (1-α) that these confidence intervals simultaneously contain $p_i, \ldots, p_k$. For a description of applications for the Bonferroni inequality, Miller (1981) is recommended.

To derive confidence intervals, Goodman (1965) first treated each parameter $i = 1, 2, \ldots, k$ as a binomial random variable. Using the normal approximation for a binomial random variable, the following is stated:

$$Z = \frac{\left(\hat{p}_i - p_i\right)}{\sqrt{p_i\left(1-p_i\right)/n}}$$

The square of a normal variate is equivalent to a chi-square variate with one degree of freedom (Walpole and Myers, 1993). By squaring the normal approximation for a binomial random variable, the following is true:

$$Z^2_{\alpha/2} = \frac{n\left(\hat{p}_i - p_i\right)^2}{p_i\left(1-p_i\right)} = \chi^2_{\alpha,1} \tag{2.16}$$

where, $\chi^2_{\alpha,1}$ = the upper (1 - α)100 percentage point of the chi-square distribution with one degree of freedom.

The Bonferroni inequality is now applied to the chi-square variate to consider all parameters simultaneously. Equation 2.16 is now rewritten as the final equation for Goodman (1965) simultaneous confidence intervals:

$$n\left(\hat{p}_i - p_i\right)^2 = \chi^2_{\alpha/k,1}\, p_i\left(1-p_i\right) \qquad \text{for all } i = 1, 2, \ldots, k \tag{2.17}$$

This equation can also be written in the form of a quadratic equation as shown below:

$$p_i = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{2.18}$$

where, $a = n + \chi^2_{\alpha/k,1}$

$b = -2n\hat{p}_i - \chi^2_{\alpha/k,1}$

$c = n\hat{p}_i^2$

To calculate the simultaneous confidence intervals, either Equation 2.17 or 2.18 is written and solved for all p_i's. Each equation will have two solutions corresponding to a maximum and minimum. The maximum solution will become the upper confidence interval and, likewise, the minimum solution will become the lower confidence interval. As with the binomial confidence intervals, the lower confidence is set to zero at $x = 0$ and the upper bound is set to one at $x = n$.

2.4.3 Example Calculating Multinomial Confidence Intervals

Apply simultaneous multinomial confidence intervals at the 95 % confidence level to the grid by number data given in the example in section 2.1.2 using the Goodman (1965) method.

Solution:

Sieve Size (mm)	i	Particles Retained	Particles Passing, x_i	$\hat{p}_i = \frac{x_i}{n}$
37.5	8	0	50	1.00
25	7	10	40	0.80
15.9	6	14	26	0.52
12.5	5	10	16	0.32
9.5	4	6	10	0.20
6.3	3	5	5	0.10
4.75	2	3	2	0.04
3.35	1	2	0	0

From the given data, it can be seen that the possible number of occurrences is eight or, $k = 8$. Using the chi-square function in MS-excel, we find that $\chi^2_{0.05/8,1} = 7.4768$.

Writing Equation 2.17 for each i, the following series of quadratic equations is obtained:

$$50(0 - p_1)^2 = 7.4768 p_1 (1 - p_1)$$

$$50(0.04 - p_2)^2 = 7.4768 p_2 (1 - p_2)$$
$$50(0.10 - p_3)^2 = 7.4768 p_3 (1 - p_3)$$
$$50(0.20 - p_4)^2 = 7.4768 p_4 (1 - p_4)$$
$$50(0.32 - p_5)^2 = 7.4768 p_5 (1 - p_5)$$
$$50(0.52 - p_6)^2 = 7.4768 p_6 (1 - p_6)$$
$$50(0.80 - p_7)^2 = 7.476 p_7 (1 - p_7)$$
$$50(1.00 - p_8)^2 = 7.4768 p_8 (1 - p_8)$$

The solutions to these equations are shown below in Table 2.1. Figure 2.3 shows the grid results, the known population size distribution, and compares 95 % multinomial and binomial confidence intervals.

Table 2.1 Results of Goodman (1965) confidence interval calculations

Sieve Size (mm)	$\hat{p}_i$	p_l	p_u
37.5	100	92.9	100
25	80	61.1	91.0
15.9	52	33.7	69.8
12.5	32	17.4	51.3
9.5	20	8.98	38.8
6.3	10	3.20	27.2
4.75	4	0.72	19.2
3.35	0	0	13.0

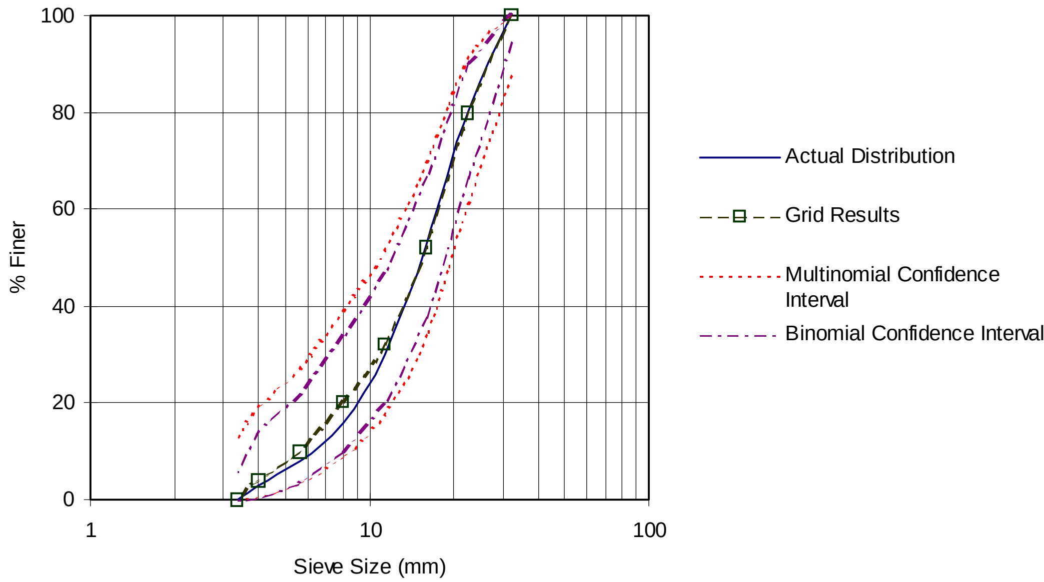

Figure 2.3 50-stone grid sample results and actual grain size distribution with 95 % multinomial and binomial confidence intervals

2.5 Sample Sizes for a Desirable Accuracy

The sample size of a grid sample is the total number of individual stones removed from the riverbed. To determine the minimum sample size that will achieve the desired accuracy, the techniques used to apply confidence intervals to grid samples are modified. Some representative results are shown here in Figures 2.4, 2.5, and 2.7, while more complete results are shown in Appendix A, B, and C. These graphs are based on grain sizes D_{10}, D_{16}, D_{30}, D_{50}, D_{70}, D_{84}, D_{90} and accuracy levels of 95%, 90%, and 80%. The grain size used will depend on the whether the median, coarse, or fine grains are of primary interest. The 95% and 90% accuracy levels are most commonly used, however the 80% level was included for cases were lower accuracy is acceptable.

2.5.1 Binomial Distribution

By solving Equations 2.4 and 2.6 for the sample size, *n*, graphs can be generated that compare the confidence bands at various grain sizes for different sample sizes. The confidence coefficient, α, also needs to be specified. The confidence intervals cannot be solved for directly. However, computer programs are available for these types of iterative calculations. The figures for the binomial distribution in Appendix A demonstrate the results of these calculations. Figure 2.4 is an example of the figures found in Appendix A. To use these figures, first select the confidence coefficient, based on the required accuracy level. The grain size of interest is the next item to be determined. The grain size is dependent on the intended use of the sample results. Once the grain size and accuracy level have been determined, the maximum allowable size of the error band is selected. Based on these three steps the necessary sample size can be determined.

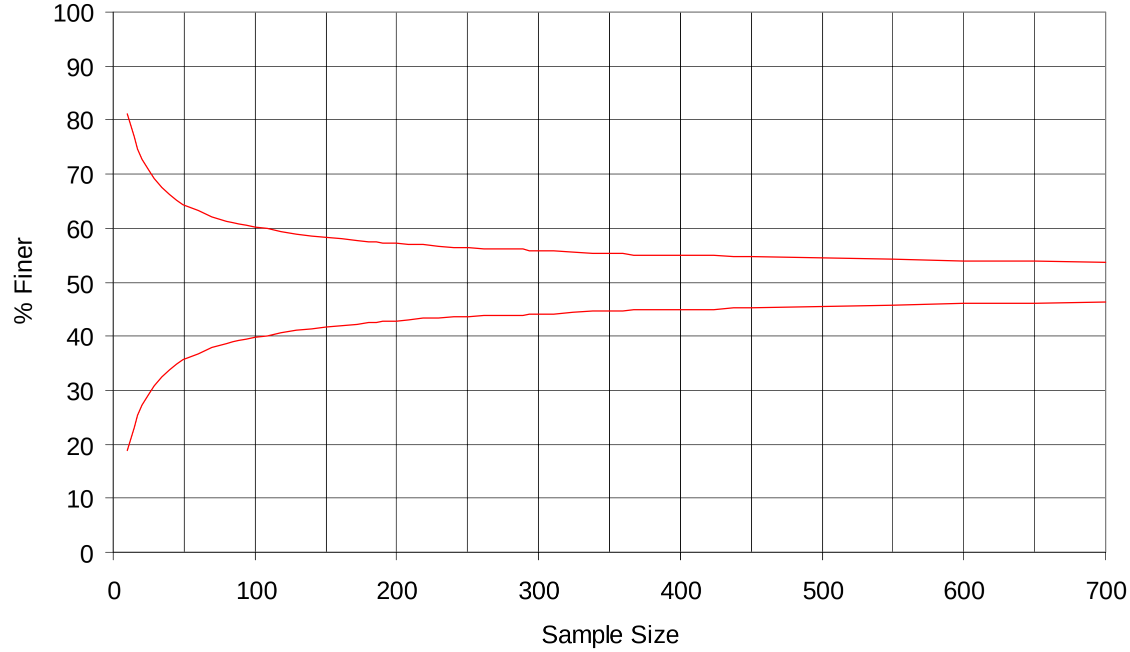

Figure 2.4 Sample size graph for D_{50} using the exact binomial distribution (α = 0.025)

To demonstrate the steps to determining the sample size using the binomial distribution, the following example is solved. Determine the necessary sample size to determine the median grain size with a 95 % accuracy. 10 % confidence bands around the median grain size will be allowed. For an accuracy level of 95 %, α = 0.025. The first step is to determine the figure dealing with the appropriate grain size, in this case D_{50}. This figure corresponds to Figure 2.4 above. Then, following the steps listed above, it can be seen that a sample size of 100-stones is necessary to keep the confidence intervals around D_{50} within ±10 %. If the required accuracy level is decreased to 80 %, what is the sample size now? For the same 10 % confidence interval, the sample size decreases to 50-stones as seen in Figure A.5. This decrease in sample size is consistent with what one would expect. As the required accuracy is decreased from 95 % to 80 %, the number of stones necessary to meet the accuracy requirements will decrease.

2.5.2 Normal Approximation for the Binomial Distribution

The normal approximation can be used to determine the sample size in a similar manner as the exact binomial distribution. Equation 2.10 is rearranged to solve for the confidence interval around the grain size of interest based on the required accuracy and sample size. The equations arrived at are as follows:

$$p_l = p - z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}} \tag{2.19}$$

$$p_u = p + z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}} \tag{2.20}$$

where, p = grain size percentage of interest
p_l = lower confidence interval
p_u = upper confidence interval

The figures for the normal approximation in Appendix B show the results of these equations. Figure 2.5 gives an example of the figures found in Appendix B. The figures are employed in a similar manner as those for the binomial distribution. As expected, the normal distribution approximates the binomial distribution fairly well. The exceptions are at small sample sizes and extremely fine and coarse grain sizes. Figure 2.6 compares the binomial distribution and the normal approximation at several percentages to illustrate where the approximation behaves well.

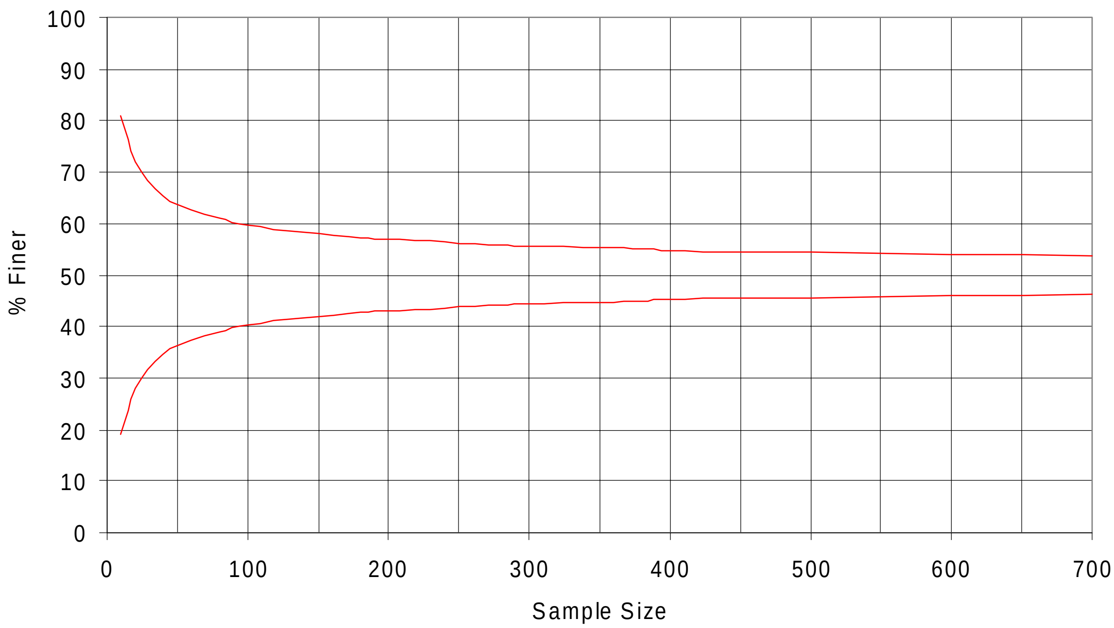

Figure 2.5 Sample size graph for D_{50} using the normal approximation of the binomial distribution (α = 0.025)

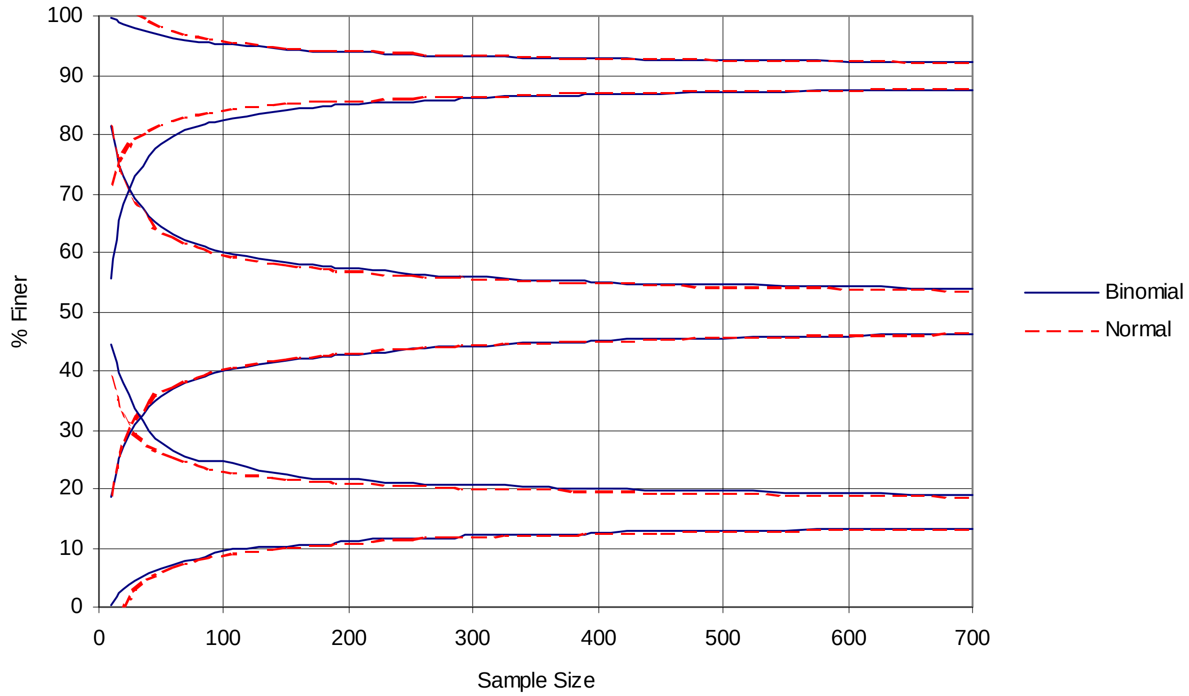

Figure 2.6 Comparison of the binomial distribution and the normal approximation for D_{16}, D_{50}, and D_{90} ($\alpha = 0.025$)

2.5.3 Multinomial Distribution

When determining a sample size using the multinomial distribution, an additional variable is introduced. This new variable is the number of sieves used in the analysis. The number of sieves can be solved for by rearranging Equation 2.17 and specifying the sample size, grain size of interest as a percentile, required accuracy, and size of the confidence interval. This equation is found to be:

$$\chi^2_{\alpha/k,1} = \frac{n \cdot E^2}{(E+p)(1-E-p)} \tag{2.21}$$

where, E = the size of the confidence interval or error
p = grain size percentage of interest
k = the number of sieves used in the analysis
n = the total number of stones in the sample
$\chi^2_{\alpha/k,1}$ = the upper (1 - α/k)100 percentage point of the chi-square distribution with one degree of freedom.

The results of this equation are shown in Appendix C, with Figure 2.7 serving as an example. An alternate representation of Figure 2.7 is shown in Figure 2.8. This alternate representation is similar to the sample size graphs for the binomial and normal distributions, while considering the additional variability of the number of sieves. The representation of Figure 2.7 is preferred for designing experiments. The grain size of interest, accuracy level, and width of the confidence interval are determined to use Figure 2.7. Additionally, the number of sieves necessary for the analysis must be estimated. Even though the particle size range is not known before the sample is taken, the number of sieves necessary can be estimated by surveying the site and making a visual approximation of the largest and smallest particles present. This approximate size range can be used to estimate the number of sieves necessary for the analysis.

To illustrate the use of the multinomial sample size determination graphs, determine the necessary sample size to determine the median grain size with a 95 % accuracy (α = 0.05). 10 % confidence bands around the median grain size will be allowed. From a brief survey of the site to be sampled, it is estimated that 10 sieves will be necessary during the analysis. Figure 2.7 or Figure C.1 can be used. Starting on the line for 10 sieves, find the point that corresponds to a confidence interval width of 10% (E = 0.10). Where the line and the curve intersect is the point on the x-axis that corresponds to the necessary sample size. From Figure 2.7, the sample size appears to be about 190-stones. For the same problem, a sample size of 100-stones was determined using the binomial distribution in section 2.5.1. This increase in sample size is expected due to the simultaneous coverage of the multinomial distribution.

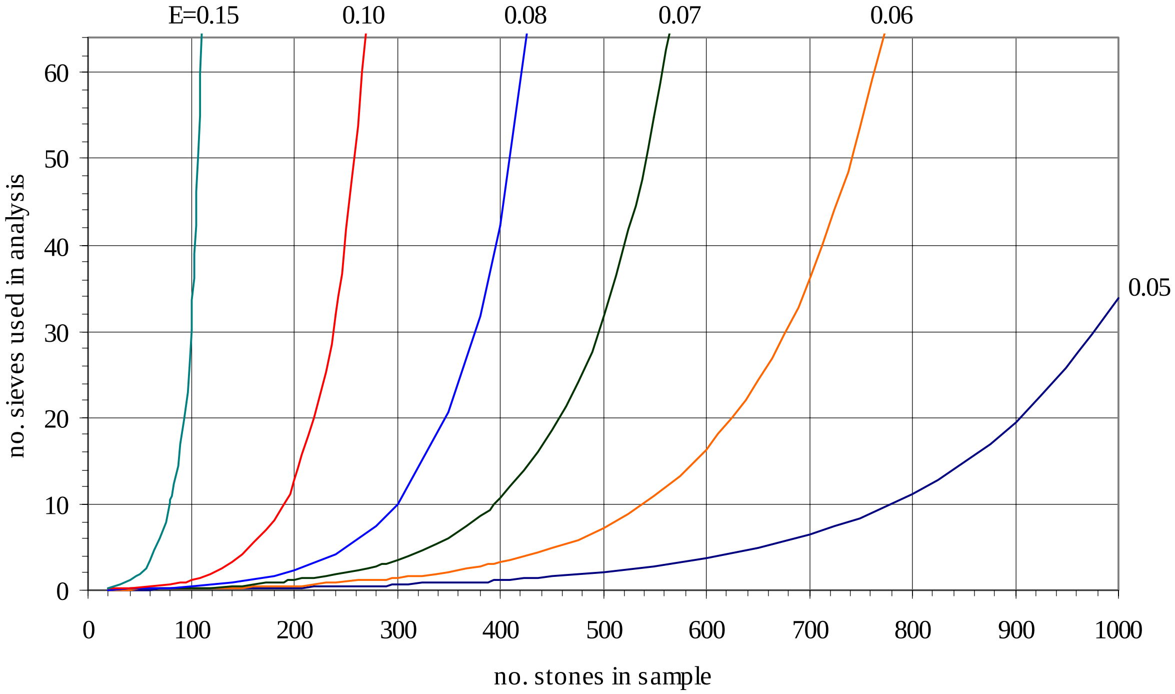

Figure 2.7 Sample size graph for D_{50} using the multinomial distribution (α = 0.025)

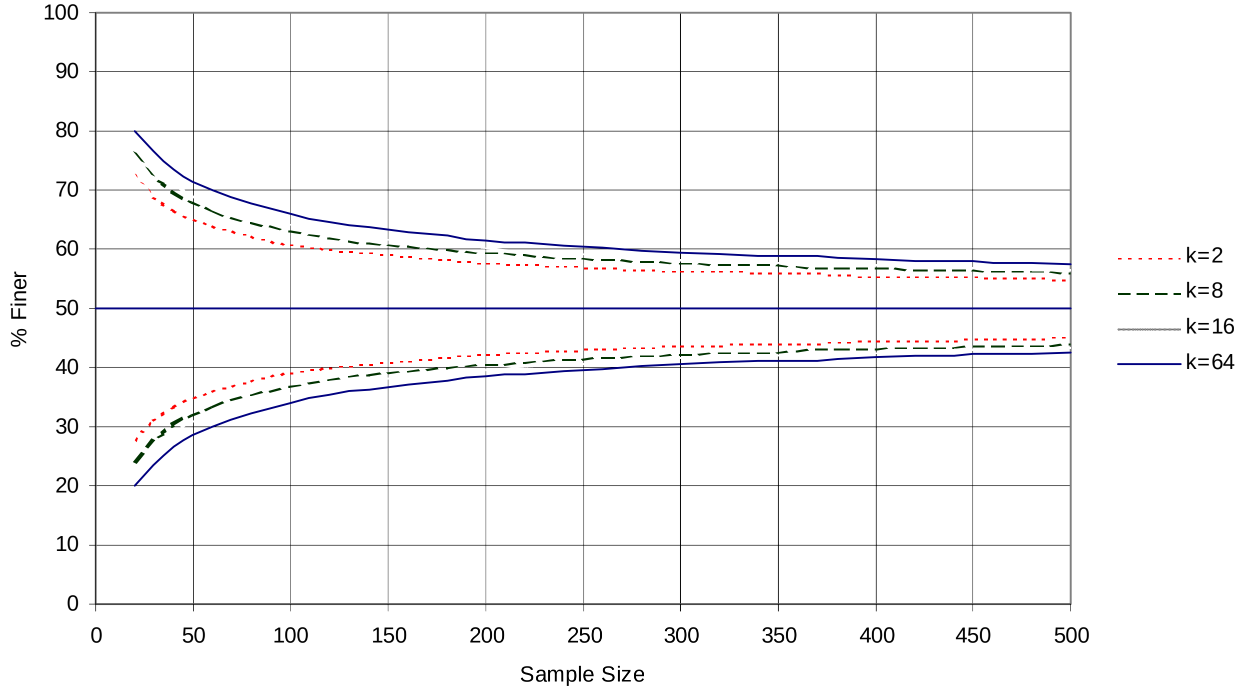

Figure 2.8 Alternate sample size graph for D_{50} using the multinomial distribution (α = 0.025)

2.6 Accuracy Based on Percentiles

All accuracy statements made in this chapter are based on percentages rather than actual grain size measurements. Confidence intervals calculated in terms of percentages provide a range around the estimated parameter in terms of a percentage of the parameter. When calculating confidence intervals based on the grain size, a particle size range is placed around the estimated grain size. When interpreting confidence intervals, it is important to identify clearly the type of confidence interval used.

The relationship between the confidence intervals in terms of percentage and actual grain size depends on the percentage of interest and is not constant. To illustrate this relationship, consider the grain size distribution curve and confidence interval shown in Figure 2.3. First look at the confidence interval around D_{50}. In terms of percentage, the width of the interval is about 36 percent. In terms of actual grain sizes, the confidence interval ranges from about 11-mm to 20-mm. Now consider the confidence interval around D_{84}. The interval width is about 28 percentage points and ranges from around 20-mm to 31-mm. Even though the confidence interval around D_{84} is smaller in terms of percentage points, the confidence interval in terms of actual grain size is larger. This issue becomes more relevant as the percentile of interest moves farther from the mean grain size. The local slope of the grain size distribution in addition to the width of the confidence interval in terms of percentage also affect the range in terms of grain size.

Chapter 3: Sample Size Estimation for Areal and Volume Sampling

3.1 Introduction

An important factor for determining the accuracy of a sample is the sample size. For statistical distributions, the sample size is the total number of trials, or counts, performed during the sampling procedure. This sample size requirement causes difficulty when placing an accuracy on either an areal or volume sample. For areal and volume samples, the sample size is measured as an area or volume, respectively. The large amounts of fine sediment contained in the collected area and volume samples make it extremely difficult to count every particle present in the sample. For this reason, sieves are used to separate the different size classes and a frequency-by-weight analysis is performed. The frequency-by-weight analysis is not well conditioned for statistical analysis.

An additional difficulty is that areal and volume samples containing n-stones are not expected to be as accurate as a grid sample containing the same number of stones. To illustrate this concept, consider a stream bed where particles with a sieve diameter of 10-cm occupy 10% of the population and particles with a sieve diameter of 1-cm also occupy 10% of the population. If a 100-stone grid sample were performed, one would expect to see 10-stones for both the 1-cm and 10-cm sieve diameters. Now, if an areal sample taken from the population contains only one particle with a 10-cm sieve diameter, this particle will project an area of 100-cm^2. The projected areas of the 1-cm and 10-cm sieve diameters must be equal to accurately represent the population in terms of area-by-area. The area projected by each 1-cm particle is 1-cm^2. Therefore, 100-particles with a diameter of 1-cm must be present in the areal sample. The areal sample could not contain all 100-particles with a 1-cm sieve diameter and still represent the other particle sizes contained in the stream bed. From this example it can be seen that for an areal and a grid sample to have the same accuracy, the areal sample typically must contain many more stones. When considering volumetric sampling, even larger numbers of stones are necessary. To overcome the difficulties of directly placing an accuracy on areal and volume samples, a procedure is presented to estimate the number of particles required in an areal or volume sample to match the accuracy of a previously performed grid sample. The procedure will first be presented for the case of areal sampling. The procedure for determining a volume sample size follows directly from the area procedure.

3.2 Theory to Estimate Sample Areas

Because confidence intervals can be readily assigned to grid-by-number sample curves, an attempt is made here to use the accuracy of the grid sample to make statistical inference about an areal sample. The conversion function between grid-by-number proportions and area-by-number proportions is known (see Table 1.2). Using this conversion, it is possible to represent a grid-by-number sample curve and the corresponding confidence intervals in terms of an area-by-number representation. The confidence intervals around the area-by-number sample curve represent the same accuracy level as for the grid-by-number sample curve. The total number of stones for an areal sample to contain can be calculated based on the width of the confidence interval in the area-by-number representation. This number of stones is the necessary number of stones for an areal sample to contain to match the accuracy level of the original grid sample. Using an approximation based on the projected area of a particle, an areal sample size can be obtained from the number of stones the areal sample should contain. This procedure is useful when combining grid and areal sample results to avoid truncated grain size distribution curves. By matching the accuracies of the two samples, a single confidence interval can be placed on the entire grain size curve after the two samples are combined.

3.3 Procedure to Estimate Sample Areas

The goal of this procedure, as described above, is to estimate the size of an areal sample necessary to match the accuracy of a previously performed grid sample from the same population. The first step is to perform a grid sample on the population of interest. The number of stones taken in this sample is designated as N_G. Once the sample is taken, the data is analyzed in terms of frequency-by-number and the results are plotted as a grain size distribution curve. Simultaneous multinomial confidence intervals are then placed on the grid-by-number curve based on N_G and a required confidence coefficient, as described in section 2.4.2.

Next, the number of stones that an area sample must contain to match the confidence interval width, and therefore the accuracy, of the grid sample is estimated. This number is denoted as N_A. To accomplish this, the grid-by-number sample curve and confidence interval curves are converted to an area-by-number sample curve with corresponding confidence intervals following the Kellerhals and Bray (1971) model (see Table 1.2):

$$p(\mathrm{A\text{-}N})_i = \mathrm{C}\, p(\mathrm{G\text{-}N})_i\, \mathrm{D}_i^{-2} \tag{3.1}$$

where, $p(\mathrm{A\text{-}N})_i$ = the proportion of all the particles included in an areal sample that belong to the size fraction *i*.

$p(\mathrm{G\text{-}N})_i$ = the proportion of all the particles included in a grid sample that belong to the size fraction *i*.

D_i = sieve diameter for size fraction *i*.

$$\mathrm{C} = \frac{1}{\sum p(\mathrm{G - N})_i \mathrm{D}_i^{-2}}$$

By transforming the confidence intervals from the grid-by-number curve into area-by-number, N_A can be calculated based on the width of either the upper or lower confidence interval at each point in the area-by-number curve. This calculation is performed by modifying Equation 2.17 for the Goodman (1965) confidence intervals, as shown below:

$$n = \mathrm{N_A} = \frac{\chi^2_{\alpha/k,1}\, p_i (1 - p_i)}{(\hat{p}_i - p_i)^2} \tag{3.2}$$

Two difficulties arise when transforming the confidence intervals. The first is that the confidence interval cumulative proportions do not vary exactly between zero and one on the grid-by-number graph. This occurs because multinomial confidence intervals allow for the possibility that some particles in the population may be smaller than the smallest particles or larger than the largest particles obtained in the grid sample. The scaling constant, C, in Equation 3.1 is based on the assumption that both the original and transformed cumulative proportions vary between zero and one. This assumption makes it impossible to determine the exact ranges of the upper and lower confidence intervals for the area-by-number curve. The second problem that arises from transforming the confidence intervals to area-by-number is that when the number of stones for the areal sample to contain is calculated, the number will change depending on the sieve size considered. This problem occurs due to the bias within the area-by-number representation of the grain size distribution. As discussed earlier, areal samples contain much more fine sediment than larger particle sizes. Therefore, when represented as area-by-number, finer particles occupy much larger proportions than in an unbiased sample. As a result, the larger particle sizes occupy smaller proportions in terms of area-by-number than in grid-by-number. To adequately represent the larger particle sizes, the area sample must be larger and contain considerably more particles than is necessary to adequately represent the smaller particles. The following section proposes solutions to overcome these difficulties.

3.3.1 Recommendations to Transform Confidence Intervals

When transforming the confidence intervals from a grid-by-number curve to an area-by-number curve, the first problem to overcome is to handle the fact that the grid-by-number confidence intervals do not vary between zero and one, as discussed above. This difficulty arises from the multinomial confidence intervals allowing for the possibility of both smaller and larger particles than those sampled to exist. It would then seem reasonable to allow for the same possibility with the confidence intervals once they have been transformed to the area-by-number representation. To perform this task, either an upper or lower cutoff diameter must be determined. This cutoff diameter serves to force the confidence interval to either zero or one at a diameter where it is unlikely to find a particle beyond this diameter. This diameter can be estimated from a visual survey of the site. Due to the visibility of larger particles, it is recommended to use an upper cutoff diameter to force the lower confidence interval to vary between one and zero. This solution allows the basic form of the confidence interval for the grid-by-number distribution to be maintained in the transformation to an area-by-number representation.

Once the grid-by-number sample curve and confidence intervals have been transformed using Equation 3.1, the transformed confidence interval width and Equation 3.2 can be used to calculate the number of stones, N_A, for the areal sample to contain to match the original grid sample accuracy. However, this number will be different at each sieve diameter. The sieve diameter used for the calculation will depend on the percentile of interest. The grain sizes below the percentile selected will be represented at the appropriate accuracy level, the sizes above this percentile may not be, however. For this reason, if the areal sample is to adequately represent the entire particle size range, N_A should be calculated at the coarsest particle size from the grid sample. A smaller grain size can be used if only the finer grain sizes are of interest, as is the case when using a hybrid sample. An additional consideration is the method used to obtain the areal sample. Because adhesives cannot remove very coarse particles, it would seem unreasonable to calculate N_A for a grain size that will not be represented in the areal sample. Fripp and Diplas (1993) found that a 40-mm particle was the largest particle that clay would remove consistently. Then for clay, N_A would be calculated at 40-mm. If spray paint is used to perform an areal sample on a dry deposit, the largest particle size in the grid sample could be used for this calculation. To calculate the number of stones in an areal sample to match the accuracy of a grid sample, both the goal of the areal sampling and the method of removing the areal sample must be considered.

3.3.2 Calculating the Sample Area

Once the number of stones that the area sample must contain, N_A, has been estimated, the final step is to represent this number as an area. To begin with, consider that the results of the areal sample should match the area-by-number curve obtained from the original grid sample when analyzed using frequency-by-number. If the total number of stones in the area sample is to be N_A, then the number of stones one would expect to see in size class *i*, would be:

$$N_A \, p(\text{A-N})_i$$

where, $p(\text{A-N})_i$ = the proportion occupied by size class *i* obtained from transforming the original grid-by-number percentages to area-by-number.

The area projected by a particle in size class *i* is equal to be the sieve diameter squared multiplied by a shape factor. The shape factor would be equal to $\pi/4$ for a spherical particle, for example. The total projected area of all the particles in size class *i* can be found:

$$(\text{Projected Area})_i = N_A \, p(\text{A-N})_i \, \alpha_s \, D_i^2$$

where, D_i = the sieve diameter for size class *i*.
α_s = particle shape factor

Summing these individual projected areas over all sieve diameters represents the expected total area projected by all the particles in the areal sample. The sample area that will give equivalent accuracy to the original grid sample can be calculated with the following equation:

$$\text{Total Area} = \sum_i N_A \, p(\text{A-N})_i \, \alpha_s \, D_i^2 \qquad (3.3)$$

This equation gives the final sample size for the areal sample. This total area may be a slightly larger than necessary due to the fact that some overlapping may be present among adjacent particles. A factor of (1 + porosity) can be used to correct the total sample area for the effects of porosity. The following example verifies that this formula does result in the proper proportions when the areal sample is analyzed in terms of area-by-area.

3.3.3 Example using Equation 3.3

The results of a 100-stone grid sample are given in the following table:

D (mm)	x_i	$p(G\text{-}N)_i$
38	15	0.15
25	24	0.24
19	22	0.22
12.5	17	0.17
9.5	14	0.14
6.3	8	0.08
N_G =	100	

The grid-by-number percentages are then converted to area-by-number percentages using Equation 3.1 as shown:

D (mm)	$p(G\text{-}N)_i$	$p(G\text{-}N)_i\ D_i^{-2}$	$p(A\text{-}N)_i$
38	0.15	0.000104	0.01806
25	0.24	0.000384	0.06676
19	0.22	0.000609	0.10595
12.5	0.17	0.001088	0.18915
9.5	0.14	0.001551	0.26968
6.3	0.08	0.002016	0.35041
	1/C =	0.005752	
	C =	173.847616	

To determine the necessary area to sample, Equation 3.3 is applied to the previous results. To simplify this example, it is assumed that following the recommendations presented in section 3.3.1 results in N_A = 200-stones. Additionally, it is assumed that for all the particles, $\alpha_s = 1$. Based on the values for N_A and α_s, the final area sample size from Equation 3.3 is estimated to be about 347.7-mm^2, as shown below:

D (mm)	$p(A\text{-}N)_i$	$(Area)_i$ (cm^2)	$p(A\text{-}A)_i$
38	0.018059	52.15	0.15
25	0.066757	83.45	0.24
19	0.105946	76.49	0.22
12.5	0.189146	59.11	0.17
9.5	0.269681	48.68	0.14
6.3	0.350411	27.82	0.08
	Total Area =	347.695	cm^2

The ratio of the area occupied by each different size class to the total sample area represents an area-by-area proportion, which should be equivalent to the original grid-by-number proportions. As the last column shows, these proportions are equal, as expected. While it is unlikely that in reality these two proportions will be exactly equal, this example verifies that the sample area obtained from Equation 3.3 will theoretically represent the proper proportions for each size class.

3.4 Example Estimating an Areal Sample Size of Desired Accuracy

This example combines the different techniques discussed in this chapter to determine an areal sample size for a desired accuracy. The data used in this example was obtained from laboratory experiments performed on a synthetic stream bed of natural sediment. The specific details of these laboratory experiments are discussed in Chapter 4.

A 100-stone grid sample was preformed on the population of interest. Based on the results of the grid sample, an areal sample is desired to match the accuracy level of the entire grid sample. To obtain the areal sample, the total area to sample must be determined. The results of the grid sample are shown below:

D_i (mm)	x_i	$p(\text{G-N})_i$
38.1	15	0.15
25	23	0.23
12.5	18	0.18
9.5	14	0.14
6.3	11	0.11
4.75	12	0.12
3.35	7	0.07
N_G =	100.00	

From the grid-by-number percentages, the cumulative percent finer is determined. Simultaneous multinomial confidence intervals, with $\alpha = 0.05$, are then placed on the grid results using the Goodman (1965) technique discussed in section 2.4.2. The grid results and the confidence interval are plotted as grain size distribution curves in Figure 3.1. The results of these calculations are shown in the following table:

D_i (mm)	Grid % Finer	Confidence Interval	
50.8	100	93.04	100
38.1	85	72.84	92.29
25	62	48.34	73.99
12.5	44	31.32	57.52
9.5	30	19.22	43.56
6.3	19	10.59	31.73
4.75	7	2.63	17.36
3.35	0	0	6.96

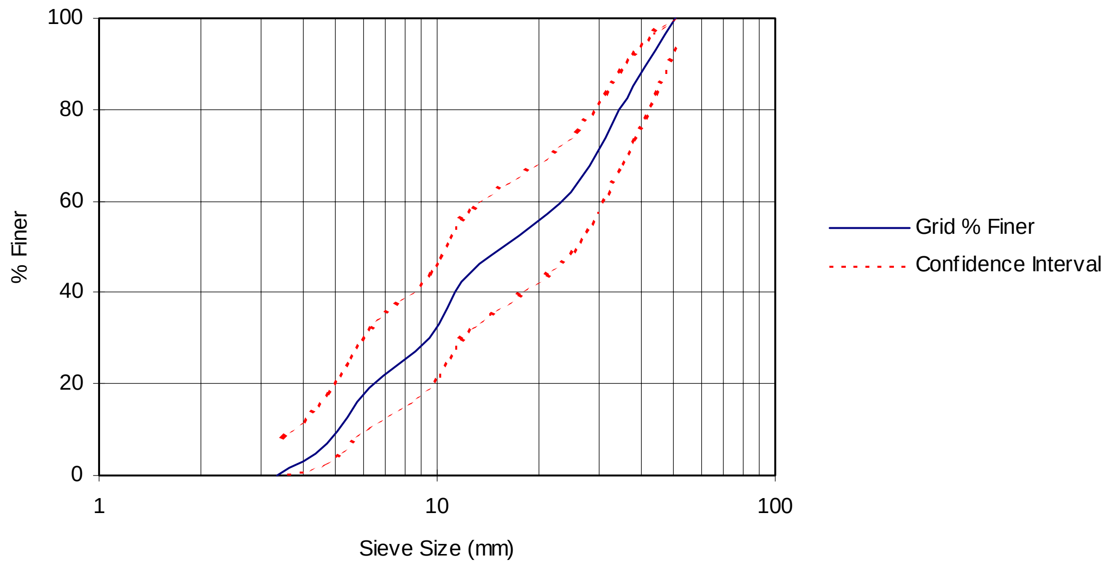

Figure 3.1 Results from the grid sampling with a 95% multinomial confidence interval

The results of the grid-by-number calculations must now be transformed to the area-by-number representation. The table below illustrates how Equation 3.1 is used to transform the sample curve results:

D_i (mm)	$p(G\text{-}N)_i$	$p(G\text{-}N)_i\, D_i^{-2}$	$p(A\text{-}N)_i$	Areal Results
50.8	0	0	0	100
38.1	0.15	0.000103	0.00590407	99.41
25	0.23	0.000368	0.02102606	97.31
12.5	0.18	0.001152	0.06582072	90.72
9.5	0.14	0.001551	0.08863208	81.86
6.3	0.11	0.002771	0.15835133	66.03
4.75	0.12	0.005319	0.30388143	35.64
3.35	0.07	0.006237	0.3563843	0
	1/C =	0.0175021		
	C =	57.1360401		

Using Equation 3.1 for the confidence intervals is not as straight forward as for the sample curve case as discussed in section 3.3.1. For comparative purposes, both the upper and lower confidence interval will be transformed. A visual survey of the sample population is performed to estimate the largest and smallest particle sizes present in the population. Based on sieve sizes that vary with √2, it is determined that it would be unlikely to sample a particle that is more than one sieve size larger than the largest sieve diameter and more than one sieve size smaller than the smallest sieve diameter found in the grid sample. This statement corresponds to the lower confidence interval approaching one at the 72-mm sieve size and the upper confidence interval approaching zero at the 2.83-mm sieve size. The individual probabilities for the confidence intervals are denoted by $p_l(x)_i$ or $p_u(x)_i$ for the lower and upper intervals, respectively. The table below displays the transformation of the confidence intervals to an area-by-number representation. Figure 3.2 shows the results of the sample grain size curve and confidence interval transformations.

Lower Confidence Interval Transformation

D_i (mm)	Cumulative $p_l(G\text{-}N)_i$	$p_l(G\text{-}N)_i$	$p_l(G\text{-}N)_i D_i^{-2}$	$p_l(A\text{-}N)_i$	Cumulative $p_l(A\text{-}N)_i$
72	1	0	0	0	100
50.8	0.93043	0.06957	0.00002696	0.002443	99.7556541
38.1	0.72838	0.20206	0.00013920	0.012617	98.4939532
25	0.48336	0.24502	0.00039203	0.035535	94.940499
12.5	0.31318	0.17017	0.00108911	0.09872	85.0685079
9.5	0.19225	0.12094	0.00134001	0.121461	72.9223763
6.3	0.10587	0.08638	0.00217628	0.197263	53.1960418
4.75	0.02627	0.07960	0.00352809	0.319794	21.2166273
3.35	0	0.02627	0.00234070	0.212166	0

1/C = 0.011032365
C = 90.642396750

Upper Confidence Interval Transformation

D_i (mm)	Cumulative $p_u(G\text{-}N)_i$	$p_u(G\text{-}N)_i$	$p_u(G\text{-}N)_i D_i^{-2}$	$p_u(A\text{-}N)_i$	Cumulative $p_u(A\text{-}N)_i$
50.8	1	0	0	0	100
38.1	0.9229	0.07707	0.0000531	0.001755	99.82448
25	0.7399	0.18298	0.0002928	0.009678	98.856643
12.5	0.5752	0.16478	0.0010546	0.034863	95.370326
9.5	0.4356	0.13959	0.0015467	0.051129	90.257377
6.3	0.3173	0.11832	0.0029811	0.098548	80.402533
4.75	0.1736	0.14370	0.0063690	0.210548	59.347708
3.35	0.0696	0.10399	0.0092664	0.306329	28.71478
2.83	0	0.06957	0.0086862	0.287148	0

1/C = 0.0302498
C = 33.0580567

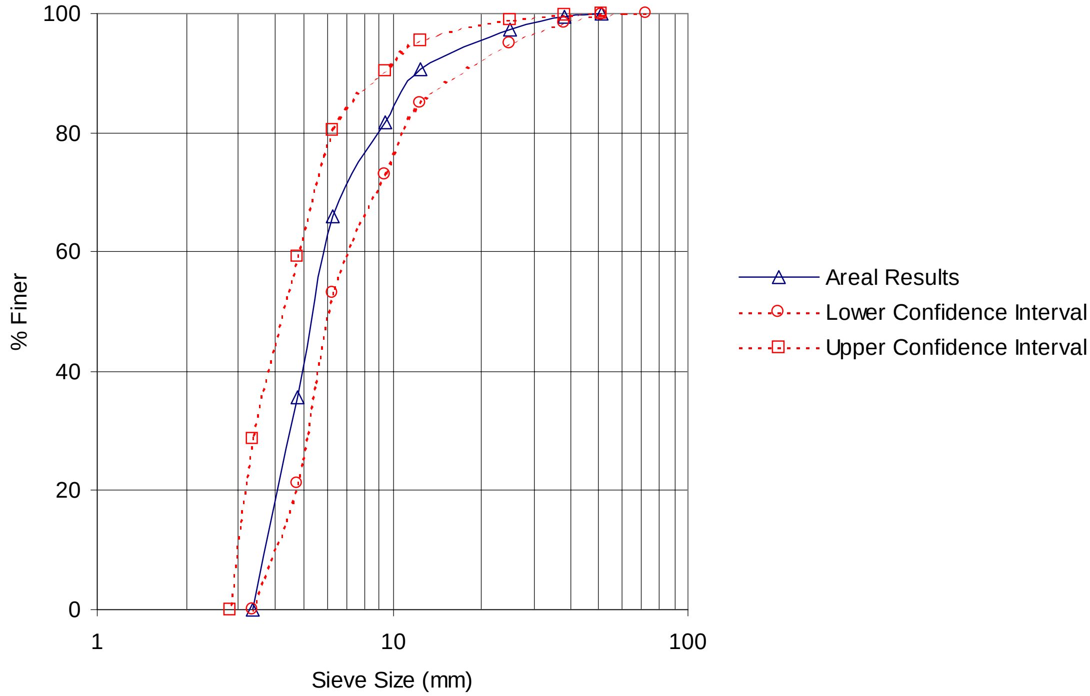

Figure 3.2 Sample grid-by-number curve and confidence intervals in area-by-number representation

The number of stones for the areal sample to contain, N_A, must now be estimated. Because the goal is to match the accuracy of the entire grain size curve obtained by the grid sample, the coarsest sieve, 38.1-mm, is selected. Equation 3.2 is now used to estimate N_A. The following results are taken from the previous calculations at the 38.1-mm sieve:

$\hat{p}_i = 0.9941$ for the upper and lower interval

$p_i = 0.9849$ for the lower interval, 0.9982 for the upper interval

The chi-square value for Equation 3.2 was determined using a statistical computer program.

$\chi^2_{0.05/9,1} = 7.689$

Plugging these values into Equation 3.2 yields N_A = 1360-stones for the lower interval, and N_A = 783-stones for the upper interval. The area sample size calculations using Equation 3.3 are shown as follows:

Lower Confidence Interval (N_A = 1360)

D_i (mm)	p(A-N)$_i$	(Area)$_i$ (cm2)
38.1	0.005904	116.56
25	0.021026	178.72
12.5	0.065821	139.87
9.5	0.088632	108.79
6.3	0.158351	85.48
4.75	0.303881	93.25
3.35	0.356384	54.39

Total Sample Area = 777.05015 cm^2

Upper Confidence Interval (N_A = 783)

D_i (mm)	p(A-N)$_i$	(Area)$_i$ (cm2)
38.1	0.005904	67.11
25	0.021026	102.90
12.5	0.065821	80.53
9.5	0.088632	62.63
6.3	0.158351	49.21
4.75	0.303881	53.69
3.35	0.356384	31.32

Total Sample Area = 447.37519 cm^2

As expected, the upper and lower confidence intervals give different sample sizes. It is recommended to use the sample size from the lower confidence interval due to difficulty of estimating the lower limit to the particle size. The total area that must be sampled with an areal technique, then, is about 777-cm^2. It is interesting to note that the projected area of the original grid sample is about 410-cm^2. Additionally, an area of about 97-cm^2 would need to be sampled to obtain one 38.1-mm particle and still represent the other grain sizes. Using spray paint, areal samples were taken in the laboratory on the population of interest. Due to sampling constraints discussed in Chapter 4, four samples, two with an area of 700-cm^2 and two with an area 875-cm^2, could be obtained. The tables that follow display the results of these areal samples:

700-cm^2 Sample (No.1)

D_i (mm)	Weight (g)	p(A-W)$_i$	p(A-W)$_i$ D_i^{-1}	p(V-W)$_i$	Areal Result
50.8	0	0	0	0	100
38.1	611.6	0.2154	0.00565	0.0990	90.10
25	1494.4	0.5264	0.02106	0.3688	53.21
12.5	334.8	0.1179	0.00943	0.1653	36.69
9.5	196.5	0.0692	0.00729	0.1276	23.93
6.3	115.0	0.0405	0.00643	0.1126	12.66
4.75	60.2	0.0212	0.00446	0.0782	4.84
3.35	26.3	0.0093	0.00277	0.0484	0

Total = 2838.800 1/C = 0.0570928

C = 17.5153367

700-cm^2 Sample (No. 2)

D_i (mm)	Weight (g)	p(A-W)$_i$	p(A-W)$_i$ D_i^{-1}	p(V-W)$_i$	Areal Result
50.8	0	0	0	0	100
38.1	897.5	0.3162	0.00830	0.1614	83.86
25	959.9	0.3381	0.01353	0.2631	57.54
12.5	335.2	0.1181	0.00945	0.1838	39.16
9.5	228.0	0.0803	0.00845	0.1645	22.72
6.3	100.6	0.0354	0.00563	0.1094	11.77
4.75	49.4	0.0174	0.00366	0.0713	4.64
3.35	22.7	0.0080	0.00239	0.0464	0
Total =	2593.300	1/C =	0.0513995		
		C =	19.4554551		

875-cm^2 Sample (No. 3)

D_i (mm)	Weight (g)	p(A-W)$_i$	p(A-W)$_i$ D_i^{-1}	p(V-W)$_i$	Areal Result
50.8	0	0	0	0	100
38.1	1039.9	0.2861	0.00751	0.1371	86.29
25	1720.3	0.4733	0.01893	0.3456	51.73
12.5	389.1	0.1071	0.00856	0.1564	36.09
9.5	253.0	0.0696	0.00733	0.1338	22.71
6.3	128.2	0.0353	0.00560	0.1022	12.49
4.75	70.2	0.0193	0.00407	0.0742	5.07
3.35	33.8	0.0093	0.00278	0.0507	0
Total =	3634.500	1/C =	0.0547760		
		C =	18.2561819		

875-cm^2 Sample (No. 4)

D_i (mm)	Weight (g)	p(A-W)$_i$	p(A-W)$_i$ D_i^{-1}	p(V-W)$_i$	Areal Result
50.8	0	0	0	0	100
38.1	1101.6	0.3031	0.00796	0.1570	84.30
25	1205.9	0.3318	0.01327	0.2620	58.09
12.5	402.4	0.1107	0.00886	0.1749	40.61
9.5	281.7	0.0775	0.00816	0.1611	24.50
6.3	137.1	0.0377	0.00599	0.1182	12.68
4.75	64.4	0.0177	0.00373	0.0736	5.32
3.35	32.8	0.0090	0.00269	0.0532	0
Total =	3225.900	1/C =	0.0506548		
		C =	19.7414667		

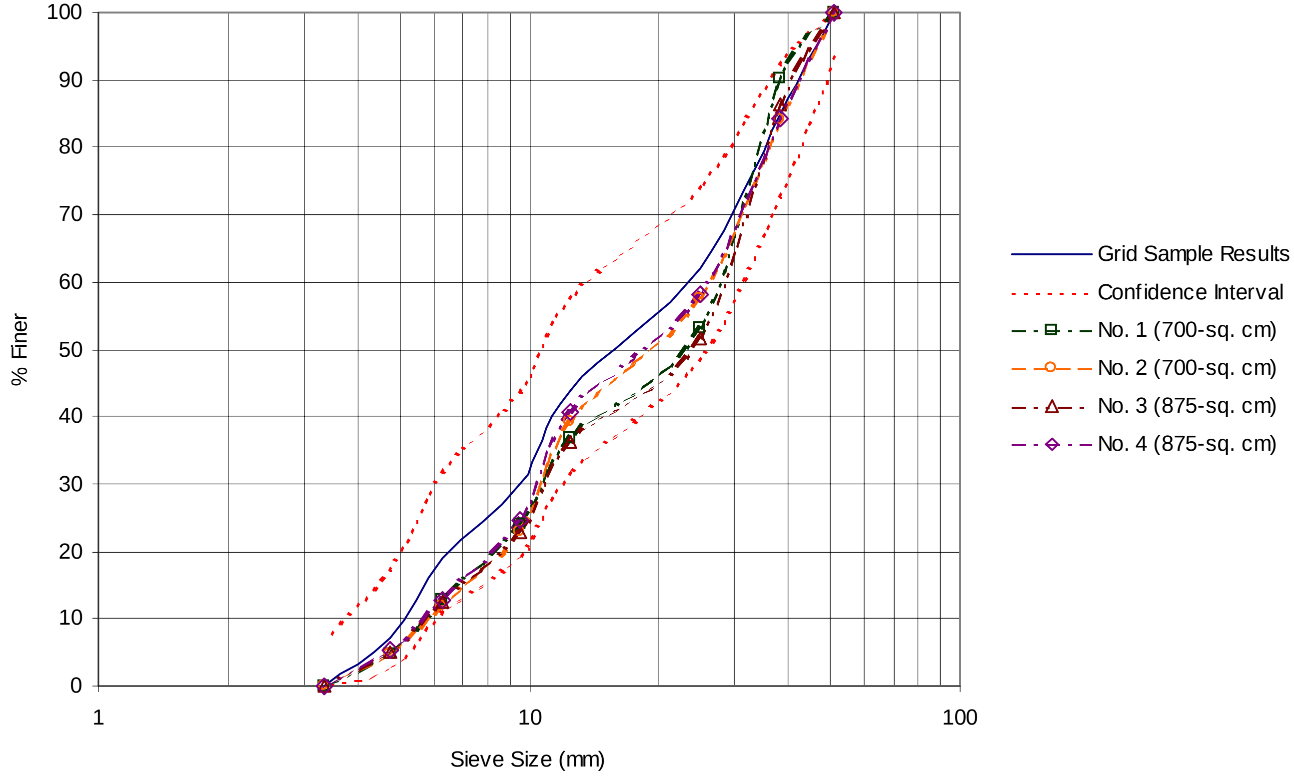

Figure 3.3 Comparison of results of the grid and areal samples with the 95% confidence intervals on the grid sample

Figure 3.3 compares the results of the four areal samples with the original grid sample results. It can be seen that the areal sample results for both sample sizes are within the multinomial confidence intervals of the grid sample. It should be noted that the areal sample curves that are closest to the confidence intervals are from the smaller areal sample size of 700-cm^2. These areal sample results suggest that the areal sample sizes give results of similar accuracy levels with the original grid sample. This example illustrates the steps that are necessary to obtain an areal sample size based on the results of a grid sample on the same population. Next, the effect on the sample size of changing the particle size where the confidence intervals converge to zero and one is demonstrated. Then, it is shown how to modify the area sample size procedure to predict a volume sample size based on the results of a grid sample.

3.5 Example Moving the Confidence Intervals

The data from the previous example is now used to illustrate the effect of moving the extreme sieve sizes of the areal confidence intervals on the sample size. Suppose that the site survey had resulted in estimating that the sieve size passing the largest particles was 100-mm. Additionally, it was determined that no particles would be finer than the 1.00-mm sieve. The confidence interval transformations from grid-by-number to area-by-number are shown in the tables below. Figure 3.4 shows the transformed confidence intervals as well as the transformed sample curve.

Lower Confidence Interval Transformation

D_i (mm)	Cumulative $p_l(G\text{-}N)_i$	$p_l(G\text{-}N)_i$	$p_l(G\text{-}N)_i\, D_i^{-2}$	$p_l(A\text{-}N)_i$	Cumulative $p_l(A\text{-}N)_i$
100	1	0	0	0	100
50.8	0.93043	0.06957	0.00002696	0.00244	99.7557
38.1	0.72838	0.20206	0.00013920	0.01262	98.4940
25	0.48336	0.24502	0.00039203	0.03553	94.9405
12.5	0.31318	0.17017	0.00108911	0.09872	85.0685
9.5	0.19225	0.12094	0.00134001	0.12146	72.9224
6.3	0.10587	0.08638	0.00217628	0.19726	53.1960
4.75	0.02627	0.07960	0.00352809	0.31979	21.2166
3.35	0	0.02627	0.00234070	0.21217	0
		1/C =	0.01103236		
		C =	90.6423968		

Upper Confidence Interval Transformation

D_i (mm)	Cumulative $p_u(\text{G-N})_i$	$p_u(\text{G-N})_i$	$p_u(\text{G-N})_i\, D_i^{-2}$	$p_u(\text{A-N})_i$	Cumulative $p_u(\text{A-N})_i$
50.8	1	0	0	0	100
38.1	0.9229	0.07707	0.0000531	0.0006	99.9417
25	0.7399	0.18298	0.0002928	0.0032	99.6205
12.5	0.5752	0.16478	0.0010546	0.0116	98.4632
9.5	0.4356	0.13959	0.0015467	0.0170	96.7660
6.3	0.3173	0.11832	0.0029811	0.0327	93.4948
4.75	0.1736	0.14370	0.0063690	0.0699	86.5059
3.35	0.0696	0.10399	0.0092664	0.1017	76.3376
1.00	0	0.06957	0.0695666	0.7634	0

1/C = 0.09113029

C = 10.97330029

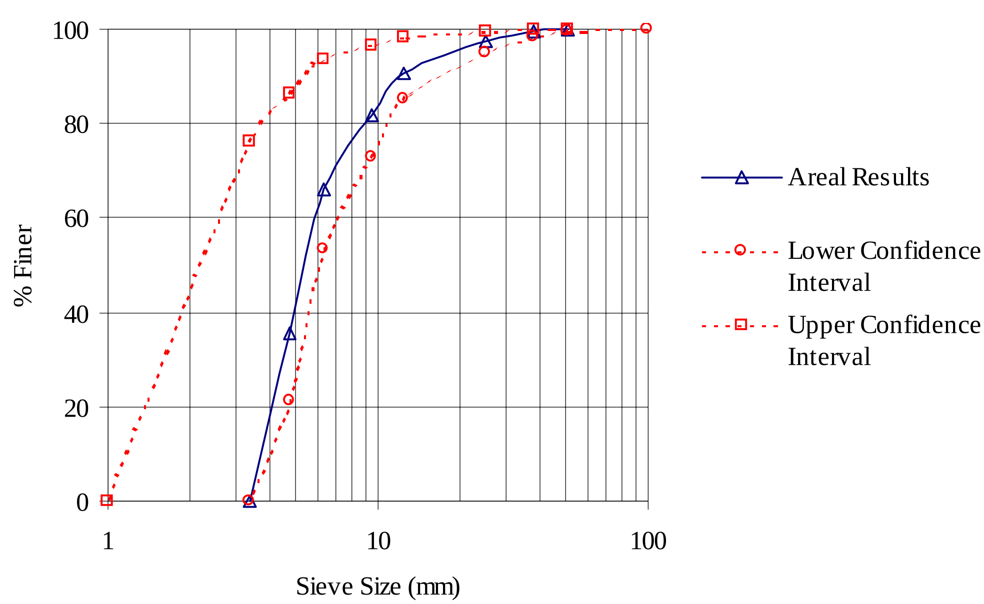

Figure 3.4 Results of confidence interval transformations

Again, the 38.1-mm sieve is selected to calculate N_A and represent the entire grain size distribution. When Equation 3.2 is applied to the upper and lower confidence intervals, N_A is found to be 158-stones and 1360-stones, respectively. Comparing Figure 3.2 and Figure 3.4, N_A for the upper confidence interval decreases from 783 to 158-stones. The upper confidence interval becomes wider in Figure 3.4, which indicates a smaller sample size. The N_A-value for the lower confidence interval does not appear to change between the two different upper limits**.** When comparing the transformation tables for the lower confidence interval in sections 3.4 and 3.5, it can be seen that in both tables the percent retained by each of the largest sieves is zero. This fact appears to cause the largest sieve selected to not affect the transformation of the lower confidence interval. From analyzing the changes in N_A, the upper interval is more sensitive to changes in the selected grain size than the lower interval. This example reinforces the recommendation to apply Equation 3.2 using the transformed lower confidence interval.

3.6 Procedure to Estimate Sample Volumes

Sample volumes necessary to characterize gravel deposits can be very large. The reasons for this are that the distributions are typically poorly sorted, which leads to the possibility of sampling an unusually large particle, and a given mass may contain few particles of a certain size, due to the increased mass of the coarser particles (Ferguson and Paola, 1997). Most of the procedures presented to determine a sample volume use a coarse grain size such as D_{84} or D_{max} to determine the necessary sample volume (De Vries, 1970 and Church et al., 1987). Ferguson and Paola (1997) present a technique to determine a sample volume based on a grain size percentile of interest. That is, the sample volume selected will adequately characterize the grain size curve up to a specified percentile of interest. The procedure presented here to determine a volume sample size to match the accuracy of a grid sample is similar to the procedure just described for areal samples. The equations used for the areal case need to be modified to consider sample volumes. After the grid sample has been performed and analyzed, the grid-by-number sample curve and confidence intervals are converted to a volume-by-number representation. Utilizing the information from Table 1.2, Equation 3.1 is modified as follows:

$$p(\text{V-N})_i = \text{C}\, p(\text{G-N})_i\ \text{D}_i^{-3} \tag{3.4}$$

where, $p(\text{V-N})_i$ = the proportion of all the particles included in a volume sample that belong to the size fraction *i*.

$p(\text{G-N})_i$ = the proportion of all the particles included in a grid sample that belong to the size fraction *i*.

D_i = sieve diameter for size fraction i.

$$\text{C} = \frac{1}{\sum p(\text{G - N})_i \text{D}_i^{-3}}$$

Once Equation 3.4 has been used to transform the grid-by-number sample curve, the confidence interval must be transformed as well. The technique discussed in section 3.3.1 can be applied, considering that the coarsest particle must be estimated for the whole volume and not just the surface area. The number of stones that the volume sample should contain to match the grid sample's accuracy, N_V, can be estimated from Equation 3.2 without modification. Keeping in mind, however, the percentages are in terms of volume-by-number. Typically, volume samples are used to represent the entire grain size distribution. Considering this fact, the confidence interval width at the largest grain size contained in the grid sample should be used to estimate N_V. By employing this grain size, it is ensured that all grain size classes will be adequately represented. Equation 3.3 is altered, as shown below, to convert the number of stones, N_V, into a volume sample:

$$\text{Total Volume} = \sum_i N_V\, p(V\text{-}N)_i\, \beta_s\, D_i^3 \qquad (3.5)$$

Where, β_s = particle shape factor for volume

Following this procedure will estimate the necessary volume sample size to match the accuracy of a grid sample. This technique can be used in populations where the grain size distribution is isotropic. If this is not the case, volume sampling is not appropriate to describe the grain size distribution due to the mixing of different populations.

3.7 Example Estimating a Volume Sample Size of Desired Accuracy

The techniques discussed in the previous section to estimate the sample size of a volume sample are applied to the example presented in section 3.4. The table below illustrates how Equation 3.4 is used to change the grid-by-number sample proportions into volume-by-number proportions:

D_i (mm)	$p(G\text{-}N)_i$	$p(G\text{-}N)_i\, D_i^{-3}$	$p(V\text{-}N)_i$	Volume Results
50.8	0	0	0	100
38.1	0.15	0.00000271	0.000734	99.93
25	0.23	0.00001472	0.003984	99.53
12.5	0.18	0.00009216	0.024946	97.03
9.5	0.14	0.000163289	0.044199	92.61
6.3	0.11	0.000439917	0.119076	80.71
4.75	0.12	0.001119697	0.303077	50.40
3.35	0.07	0.001861931	0.503984	0
	1/C =	0.00369443		
	C =	270.6780083		

To transform the confidence intervals, it is assumed that the estimates of the smallest and largest particle sizes made in section 3.4 apply to the entire population and not just the surface layer. The confidence interval transformations are shown below. Figure 3.5 show the results of the sample curve and confidence interval transformations.

Lower Confidence Interval Transformation

D_i (mm)	Cumulative $p_l(\text{G-N})_i$	$p_l(\text{G-N})_i$	$p_l(\text{G-N})_i\ D_i^{-3}$	$p_l(\text{V-N})_i$	Cumulative $p_l(\text{V-N})_i$
72	1	0	0	0	100
50.8	0.93043	0.06957	5.30652E-07	0.000261	99.9739232
38.1	0.72838	0.20206	3.65342E-06	0.001795	99.7943902
25	0.48336	0.24502	1.56812E-05	0.007706	99.0237997
12.5	0.31318	0.17017	8.71291E-05	0.042816	94.7421841
9.5	0.19225	0.12094	0.000141053	0.069315	87.8106828
6.3	0.10587	0.08638	0.000345441	0.169754	70.8353314
4.75	0.02627	0.07960	0.000742755	0.364997	34.3355865
3.35	0	0.02627	0.000698715	0.343356	0

1/C = 0.00203496

C = 491.4103632

Upper Confidence Interval Transformation

D_i (mm)	Cumulative $p_u(\text{G-N})_i$	$p_u(\text{G-N})_i$	$p_u(\text{G-N})_i\ D_i^{-3}$	$p_u(\text{V-N})_i$	Cumulative $p_u(\text{V-N})_i$
50.8	1	0	0	0	100
38.1	0.9229	0.07707	1.39356E-06	0.00018	99.9823817
25	0.7399	0.18298	1.17108E-05	0.00148	99.8343265
12.5	0.5752	0.16478	8.43684E-05	0.01067	98.7676856
9.5	0.4356	0.13959	0.000162806	0.02058	96.709383
6.3	0.3173	0.11832	0.000473186	0.05982	90.7270502
4.75	0.1736	0.14370	0.001340852	0.16952	73.7751113
3.35	0.0696	0.10399	0.002766091	0.34971	38.8043504
2.83	0	0.06957	0.003069317	0.38804	0

1/C = 0.007909724

C = 126.4266666

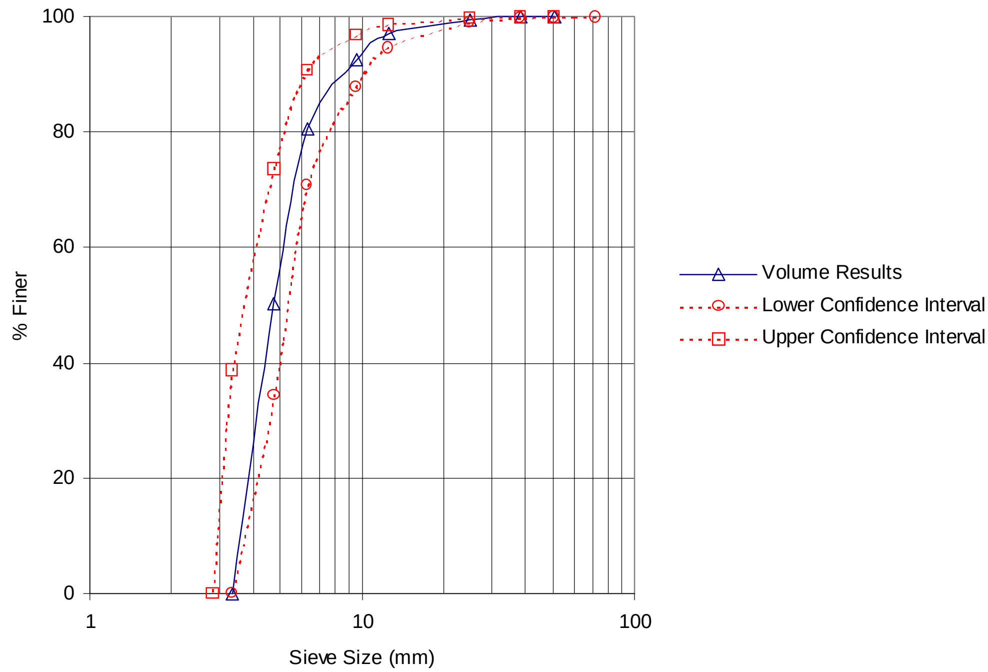

Figure 3.5 Sample curve and confidence intervals in volume-by-number representation

The 38.1-mm sieve size is again selected for the calculation of N_V to represent the entire grain size curve. The following parameters for Equation 3.2 were taken from the transformation calculation:

$\hat{p}_i = 0.9993$ for the upper and lower interval

p_i = 0.9979 for the lower interval, 0.9998 for the upper interval

The chi-square value from section 3.4 will not change. Using Equation 3.2 yields N_V = 9028-stones for the lower interval, and N_V = 4351-stones for the upper interval. The area sample size calculation using Equation 3.5 is shown below:

Lower Confidence Interval (N_V = 9028)

D_i (mm)	p(V-N)$_i$	(Volume)$_i$ (cm^3)
38.1	0.005904	2947.94
25	0.021026	2965.99
12.5	0.065821	1160.60
9.5	0.088632	686.05
6.3	0.158351	357.47
4.75	0.303881	294.02
3.35	0.356384	120.96

Total Sample Volume = 8533.02133 cm^3

Upper Confidence Interval (N_V = 4351)

D_i (mm)	p(V-N)$_i$	(Volume)$_i$ (cm^3)
38.1	0.005904	1420.74
25	0.021026	1429.44
12.5	0.065821	559.35
9.5	0.088632	330.64
6.3	0.158351	172.28
4.75	0.303881	141.70
3.35	0.356384	58.30

Total Sample Volume = 4112.4475 cm^3

The lower confidence interval value is again recommended to use as the final sample size. The volume occupied by the original grid sample is about 1240-cm^3. Also, a volume of about 370-cm^3 would be necessary to sample one 38.1-mm particle and still sample the other grain sizes sufficiently. If the density of the gravel is assumed to be 2650-Kg/m^3, the volume sample will contain about 23-Kg of particles. This 23-Kg for the volume sample compares well against the 80-Kg recommendation for low accuracy from De Vries (1970) for this sample. This low accuracy, as defined by De Vries, corresponds to a relative error of 10% for the probabilities by mass. The recommendations from Church et al. (1987) are shown in Table 3.1. The percentages shown in Table 3.1 represent the proportion of the total sample weight occupied by the largest particle in the sample. The result from the method presented in section 3.5 falls between 0.5% and 0.1%.

Table 3.1 Sample size recommendations from Church et al. (1987)

Percentage for Largest Particle	Sample Size (Kg)
5.0%	1.5
2.0%	4.0
1.0%	7.6
0.5%	10.5
0.1%	74.0

Based on a minimum sample depth and area, a method to determine a minimum weight to is presented by Diplas and Fripp (1992). Using this method, a sample size of 29.3-Kg is calculated. This is close to the 23-Kg found calculated above. Ferguson and Paola (1997) present an empirical formula for estimating volume sample sizes based on a dimensionless volume. The equations are based on extensive computer sampling of log-normally distributed data with various standard deviations. The sample size recommendations are based on two cases. The first recommends the necessary sample size to avoid bias from the particle sizes. The second case recommends a sample size for achieving accurate precision between samples. Using the empirical equations on the results presented here, for an unbiased sample, 11.9-Kg must be sampled. However, to achieve good precision, 714-Kg must be sampled. Ferguson and Paola (1997) define good precision as a 10% error around the diameter. This good precision estimate was, by far, the largest volume sample size calculated. Additionally, only Ferguson and Paola

(1997) went beyond the issue of unbiased samples and specifically addressed the issue of precision. However, it does seem reasonable that for poorly sorted distributions, extremely large sample sizes are necessary to obtain accurate replicate samples.

3.8 Conclusion

The techniques discussed in this chapter provide a statistically sound method to determine the accuracy of volume and areal samples. The individual steps reinforce the findings of Ferguson and Paola (1997) that no simple guideline for volume sample size is possible. Because both volume and areal samples are biased towards the finer particles, it is reasonable to extend this statement to areal sample size as well. Ferguson and Paola (1997) state that in order to determine a sample size, the following must be considered: the distribution percentile of interest, what the distribution is like, and the desired level of accuracy. The technique presented here addresses the percentile of interest issue through selecting a grain size at which to calculate the number of stones necessary to match the accuracy of the original grid sample . Even though a size, not a percentile, is selected, the grain size corresponds to a percentile below which the desired accuracy level is to be achieved. The characteristics of the population grain size curve are estimated from the initial grid sample. This estimate allows the volume or areal sample size to consider the such parameters as the mean size and the standard deviation. Additionally, the initial grid sample with confidence intervals prescribes the accuracy that is to be achieved with the volume or areal sample. The sample size estimation techniques presented in this chapter account for the necessary factors when considering grain size distributions that are poorly sorted and contain a wide range of particle sizes.

Chapter 4: Laboratory Experiments and Procedures

4.1 Introduction

To examine the techniques presented in the previous chapters, laboratory data was collected from experiments performed on simulated sediment deposits. Two deposits were created using, first, glass beads and then, natural sediments. A predetermined size distribution was assigned, then the particles, either glass beads or natural sediment, were randomly placed into a sample box. Seven sizes of glass beads with diameters 25.4, 19.05, 14.28, 10, 7, 5, and 4-mm, shown in Figure 4.1, make up the grain sizes present in the first particle size distribution. The different color for each size of glass beads allowed the particle size to be determined by inspection. The natural sediments ranged in sieve diameter from 3.35-mm to 38.1-mm. The sample box is made of plexiglass with a length of 91-cm, a width of 45-cm and a depth of 23-cm. Grid, areal, and volume samples were obtained from the synthetic sediment deposits.

Figure 4.1 Glass beads used in laboratory experiments

4.2 Experiments with Glass Beads

To simulate the grain size distribution of an actual river, a unimodal distribution, skewed towards the coarse grains, was selected for the glass beads. The distribution is poorly sorted, with σ_g = 1.65-mm. To create this distribution, an average density of 2.5 g/cm^3 was used for all particle sizes. The void spaces were accounted for by considering 30 % of the volume as voids. A total volume of particles was obtained by considering the depth of particles in the sample box to be close to 8 cm. Volume percentages were then assigned to each grain size corresponding to a unimodal distribution.

To achieve a random, homogeneous mixture and to facilitate the placement of the glass beads in the sample box, the total sample volume is broken up into forty subgroups with a size distribution equal to that of the total volume. The approximate number of particles in each subgroup was determined using the individual particle volumes. Each number of particles was rounded to the nearest integer, because fractions of glass beads could not be obtained. Using these rounded numbers of particles, the final grain size distribution was determined. Table 4.1 shows the process used to obtain the size distribution, including the subgroup rounding process and the final grain size distribution. The probability distribution function for the final grain size distribution is shown in Figure 4.2. The cumulative grain size distribution curve is shown in Figure 4.3. The grain size distribution has the following population statistics:

mean, $\mu = \sum_i x_i f(x_i) = \sum_i D_i (\%Volume)$=15.85 mm

median, D_{50} = 16.35 cm

standard deviation,

$$\sigma = \sqrt{\sum_i (x_i - \mu)^2 f(x_i)} = \sqrt{\sum_i (D_i - \mu)^2 (\%Volume)} = 6.53 \text{ mm}$$

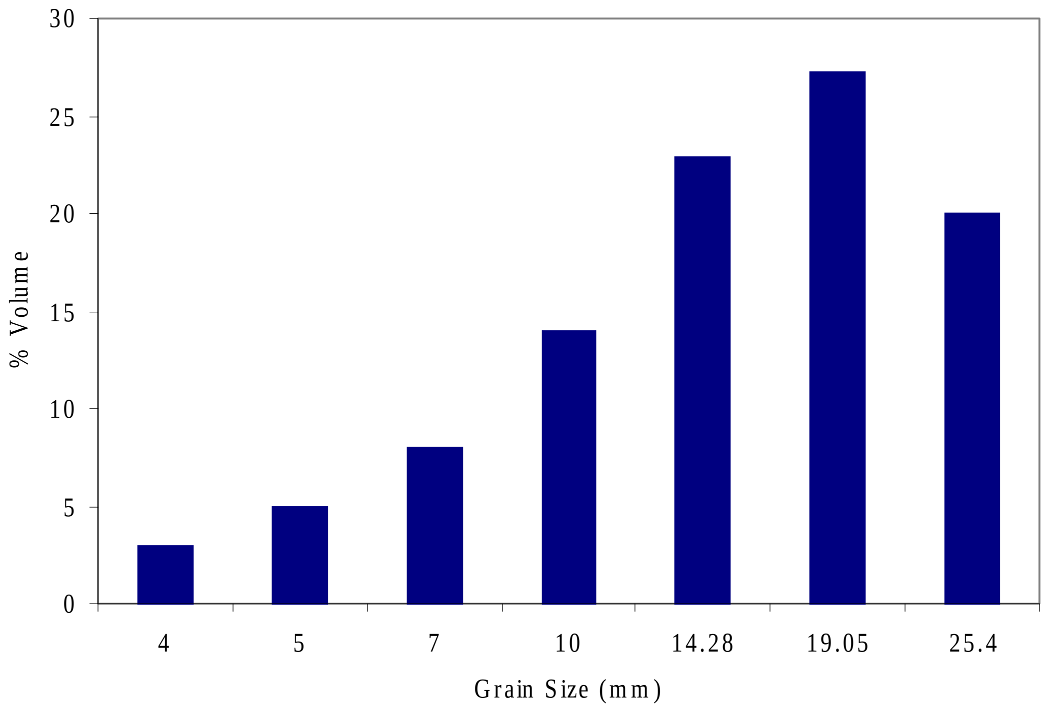

Figure 4.2 Laboratory particle size probability distribution function for glass beads

Table 4.1 Laboratory particle size distribution table for glass beads

Particle Size, D (mm)	Initial Estimate of % Volume	Vol. per Particle (cm^3)	Volume (cm^3)	Approx. No. of Particles	Approx. No. Particles per Subgroup (40)	Particles Per Subgroup	Volume of Particles per Subgroup (cm^3)	Final % Volume	Final Cumulative Distribution
25.4	20	8.5802	4459.9033	519.7873	12.9947	13	4461.7283	19.9609	100
19.05	27	3.6198	6020.8695	1663.3194	41.5830	42	6081.2498	27.2064	80.04
14.28	23	1.5247	5128.8888	3363.8765	84.0969	84	5122.9784	22.9192	52.83
10	14	0.5236	3121.9323	5962.4516	149.0613	149	3120.6487	13.9612	29.91
7	8	0.1796	1783.9613	9933.2804	248.3320	248	1781.5762	7.9704	15.95
5	5	0.0654	1114.9758	17035.5759	425.8894	426	1115.2654	4.9895	7.98
4	3	0.0335	668.9855	19963.5655	499.0891	499	668.8660	2.9924	2.99

Total Volume = 22299.5167 cm^3 Actual Total Volume = 22352.3128 cm^3

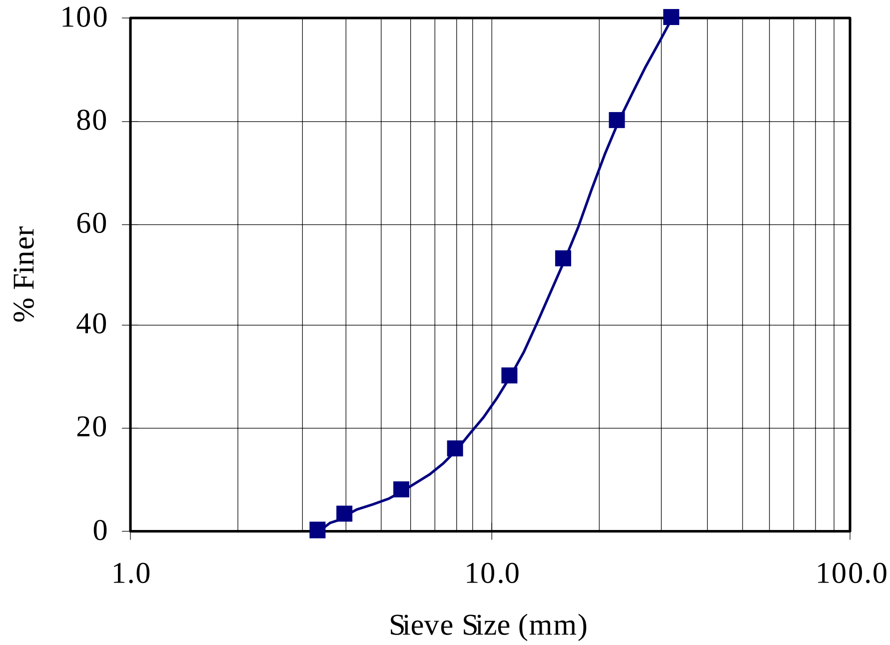

Figure 4.3 Laboratory cumulative grain size distribution curve for glass beads

Each subgroup was placed into the sample box separately to ensure a good mix. The particles in each subgroup were randomly tossed into the sample box starting with all the largest particles in the subgroup then going consecutively down to the smallest particles. A cardboard grid approximately 6 cm deep, shown in Figure 4.4 was placed on the bottom of the plexiglass box while the first few subgroups were applied. The purpose of this grid is help hold the initial particles in place until a solid bed of particles is present. This solid particle bed helps to hold the newly placed particles in place by minimizing the initial motion of the new particles. The final approximate depth of particles in the plexiglass box is about 10 cm. This depth is large enough that the use of the cardboard grid will not affect the surface particles. Figure 4.5 and Figure 4.6 show the sample box filled with the simulated sediment deposit.

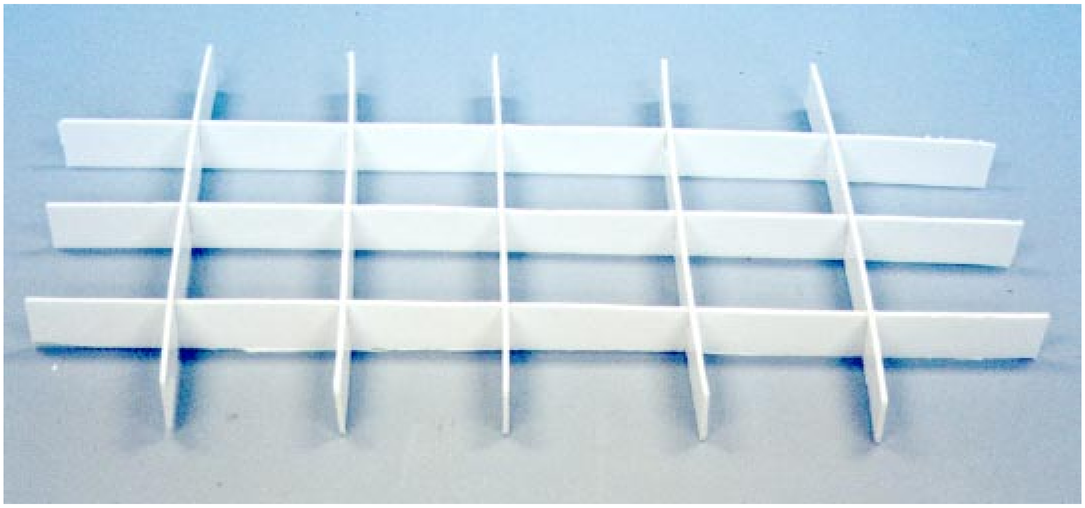

Figure 4.4 Cardboard grid used for initial placement of glass beads

Figure 4.5 Front angle view of final deposit

4.2.1 Grid Samples

Grid-by-number samples were taken first from the glass bead deposit. The sampling was performed by dropping a surveying plumb bob from a level straight edge above the deposit surface. The plumb bob technique, which is shown in Figure 4.7, allows samples to be obtained without disturbing the particles. The size as well as location of the surface

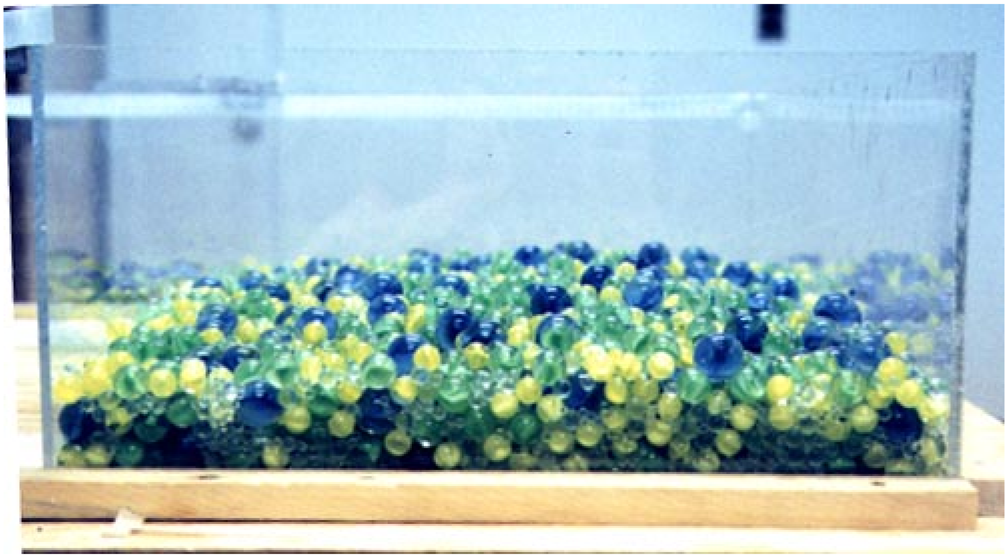

Figure 4.6 Side view of final deposit

particle the plumb bob touched was recorded. The particle size could be determined without removing the particle due to the individual bead color for each size class. By recording the particle locations, the order in which individual particles were sampled could be easily determined. A spacing of 3 cm was maintained between each consecutive sample point as well as the boundaries of the plexiglass box. This grid spacing was selected to avoid sampling the same particle twice. A sample size of 420 stones was obtained by sampling fourteen rows of thirty stones each.

The data compiled from the grid sample can be found in Appendix D. The different grid points correspond to the locations on the bed surface where the individual samples were taken. Each different particle size was numbered, starting with one for the smallest particle (4-mm) and going to seven for the largest particle (25.4-mm).

The grid data is analyzed using the frequency-by-number technique. In order to obtain the most information from the 420-stone sample, the data is separated into different sample sizes. The sizes used for analysis were one 400-stone sample, four 100-stone samples, and eight 50-stone samples. Within each different sample size, there is no overlap of data points. In other words, with the four 100-stone samples, the first 100 particles sampled becomes the first 100-stone sample, the second 100 particles sampled makes up the second 100-stone sample, and so on. The resulting grain size distribution curves from each analysis are shown with the actual grain size distribution in Figures 4.8, 4.9, and 4.10. For a complete analysis of each of the individual samples and individual comparisons with the actual distribution see Appendix D. Table 4.2 summarizes the sample statistics for each grid-by-number sample.


Figure 4.7 Laboratory grid sampling technique

As Figures 4.8, 4.9, and 4.10 show, the grid-by-number sample was successful. The 100 and 400-stone samples match the actual distribution well. The 50-stone samples match nicely with a few exceptions. The overall shape of the actual distribution is followed by all the sample sizes. These results empirically prove the equivalence of grid-by-number and volumetric samples as stated in Kellerhals and Bray (1971).

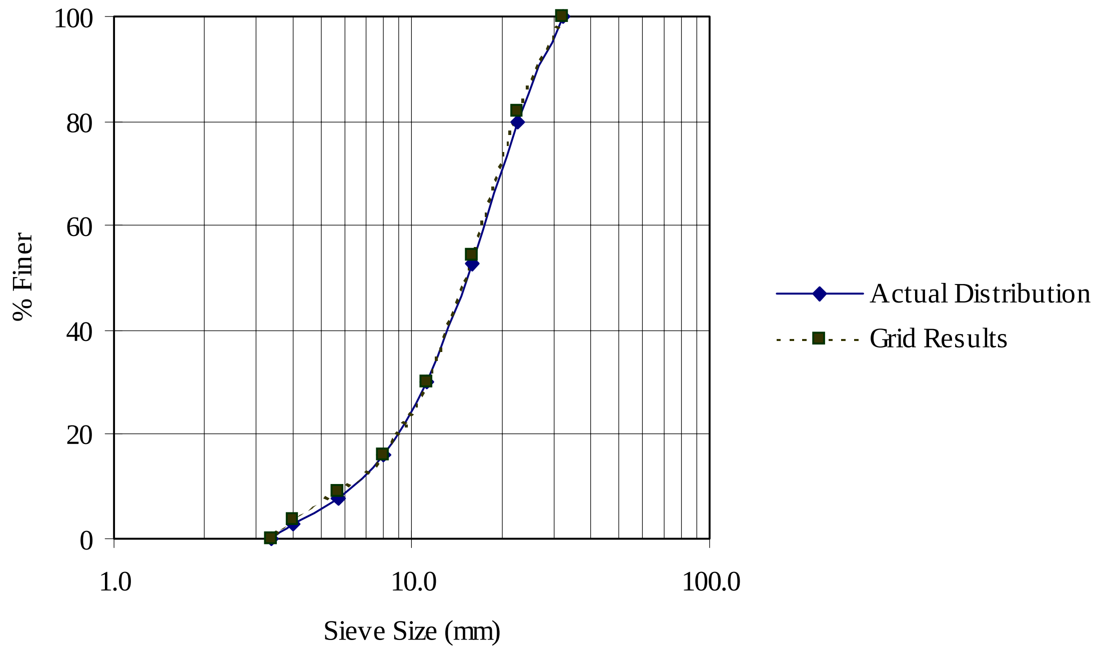

Figure 4.8 Results of 400-stone laboratory grid sample with the actual size distribution

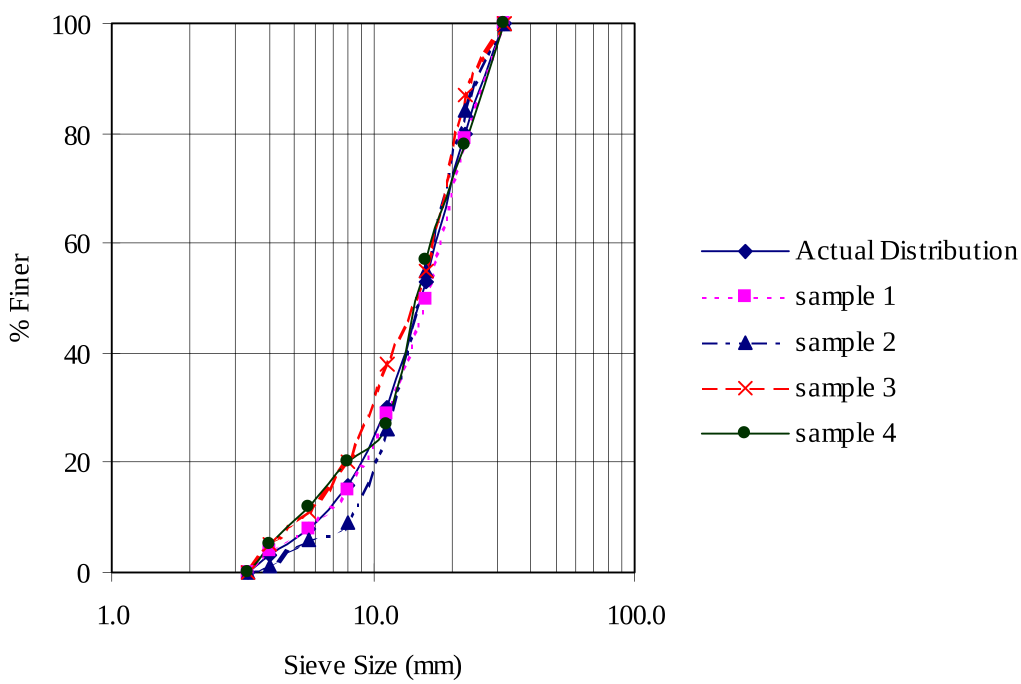

Figure 4.9 Results of 100-stone laboratory grid samples with the actual size distribution

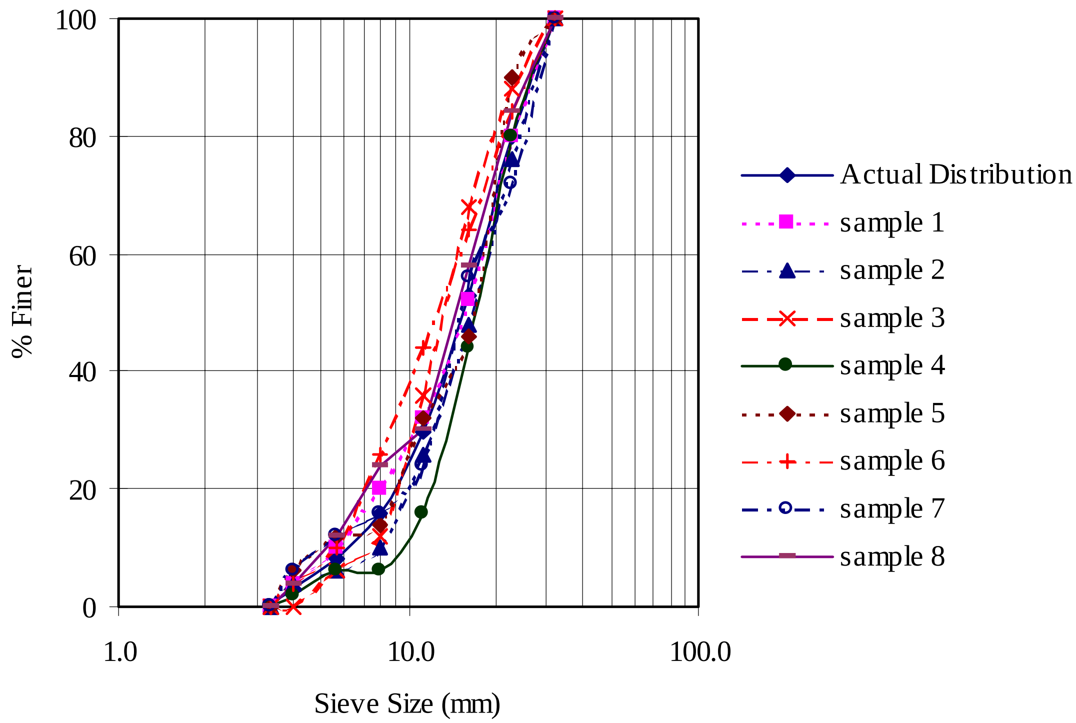

Figure 4.10 Results of 50-stone laboratory grid samples with the actual size distribution

Table 4.2 Laboratory grid sample statistics (mm)

400 stone samples			100 stone samples			50 stone samples		
Sample	Mean	Std. Dev.	Sample	Mean	Std. Dev.	Sample	Mean	Std. Dev.
1	15.62	6.47	1	16.11	6.57	1	15.63	6.79496
			2	15.93	5.81	2	16.71	6.413702
			3	14.76	6.45	3	14.55	5.662237
			4	15.68	6.84	4	17.22	5.629354
						5	15.4	6.081506
						6	14.11	6.743714
						7	16.35	7.00526
						8	15.02	6.613257

4.2.2 Areal Samples

To continue to use the sample distribution for future experiments, an areal sampling technique is needed that does not disturb the glass beads. To begin with, the sample surface was partitioned off into seven-by-seven centimeter squares, referred to as cells. This was accomplished by running small threads across the area at seven centimeter intervals and taping the ends of the thread to the plexiglass, as shown Figure 4.11. After leaving about five centimeters space between the sample box boundaries and the first cells, sixty cells were created. Each cell was given a row and column number to identify its location on the sample area. The areal samples were taken by counting the total number of surface particles within each cell. Each particle was marked with a dry-erase marker, see Figure 4.12, to insure that it was not counted twice. Difficulties were encountered in some cases determining if particles were part of the surface layer. A consistent criterion was necessary to determine if a particle was to be counted. The adopted criterion was that if it appeared that the particle would be removed with an adhesive, the particle was counted.

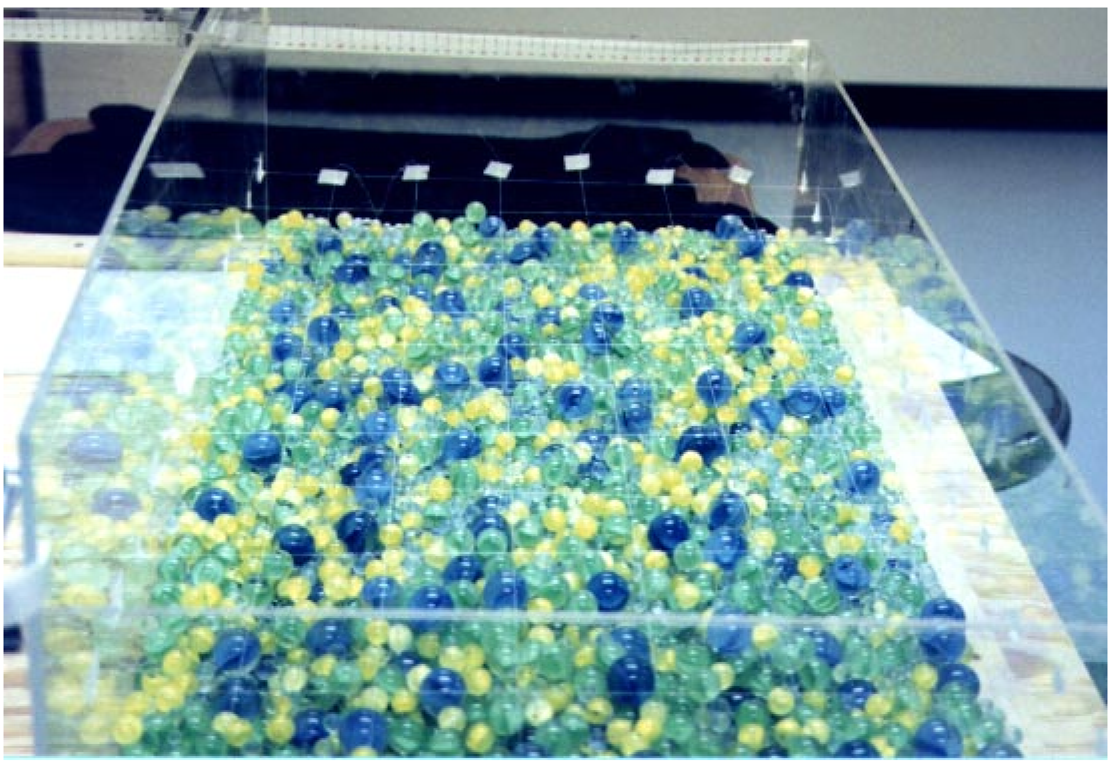

Figure 4.11 Simulated sediment deposit broken up into cells for areal sampling

The data from the areal sampling is shown in Appendix E. There were a total of five rows and twelve columns in the sample area. Cell 1-1 corresponds to the upper right hand corner of the sample area. The analysis was performed by taking the ratio of the projected area of particles of each grain size to the total projected area in each cell. This technique is equivalent to the volume-by-weight approach. In order to gain a larger number of samples, in addition to analyzing the whole areal sample, an analysis was also performed on area of cells five columns wide and five rows long. A total of eight samples were obtained from breaking up the area. Figure 4.13 shows the results of the total areal sample compared to the actual distribution. Figure 4.14 shows the actual distribution with the eight smaller samples. It can been seen that the areal samples consistently underestimate the finer particles. A possible explanation is that many of the finer particles were obstructed from the individual's view by the larger particles. If an actual adhesive, such as clay, had been used, these hidden particles may have been removed. Additionally, counting the finer particles is difficult when clusters of fine particles were present. The differences between consecutive sizes classes was also large due to the exact sizes of the glass beads. This difference increased the likelihood that the smaller particles would fall below the top layer. In nature the difference between consecutive size classes is much smaller, thereby reducing the possibility of smaller particles falling below the top layer.

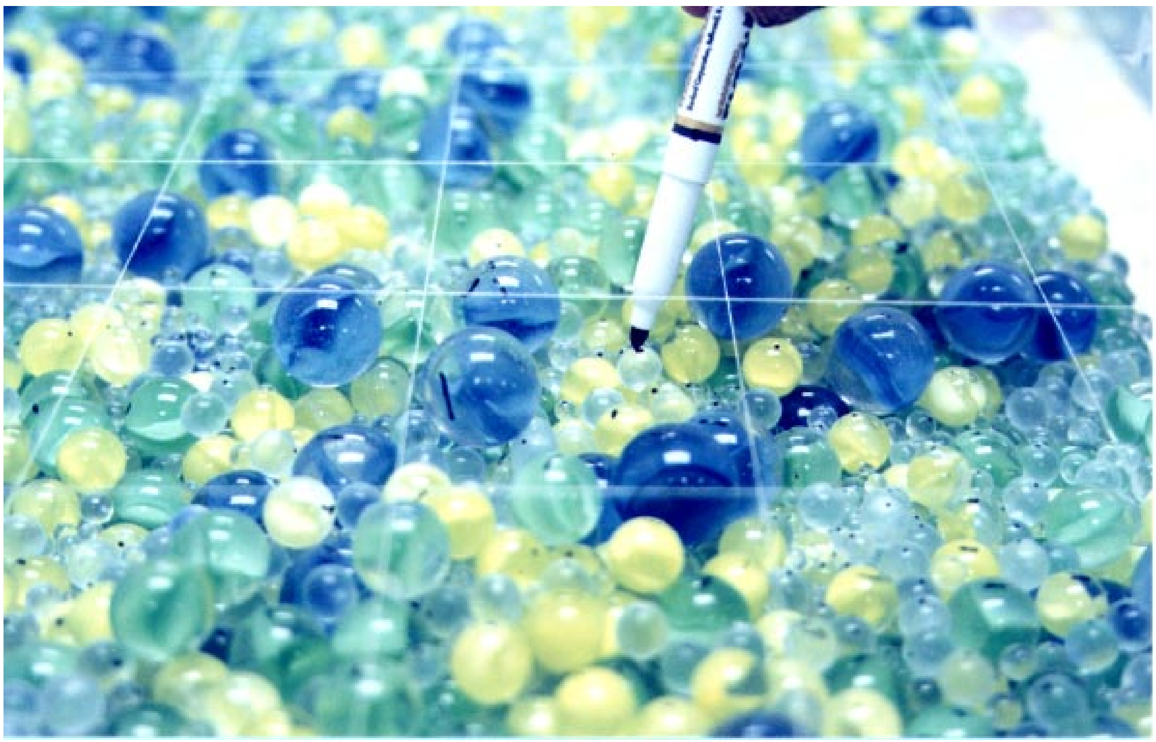

Figure 4.12 Laboratory areal sampling technique

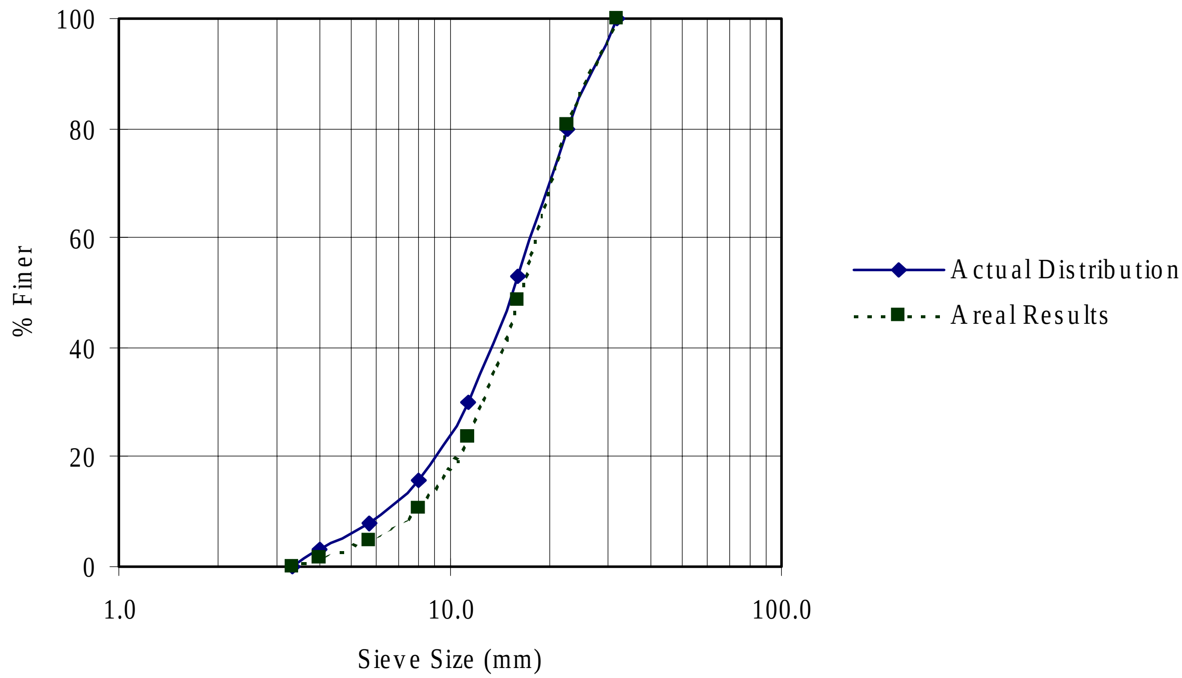

Figure 4.13 Results of total areal laboratory sample (A = 9800-cm^2) with the actual size distribution

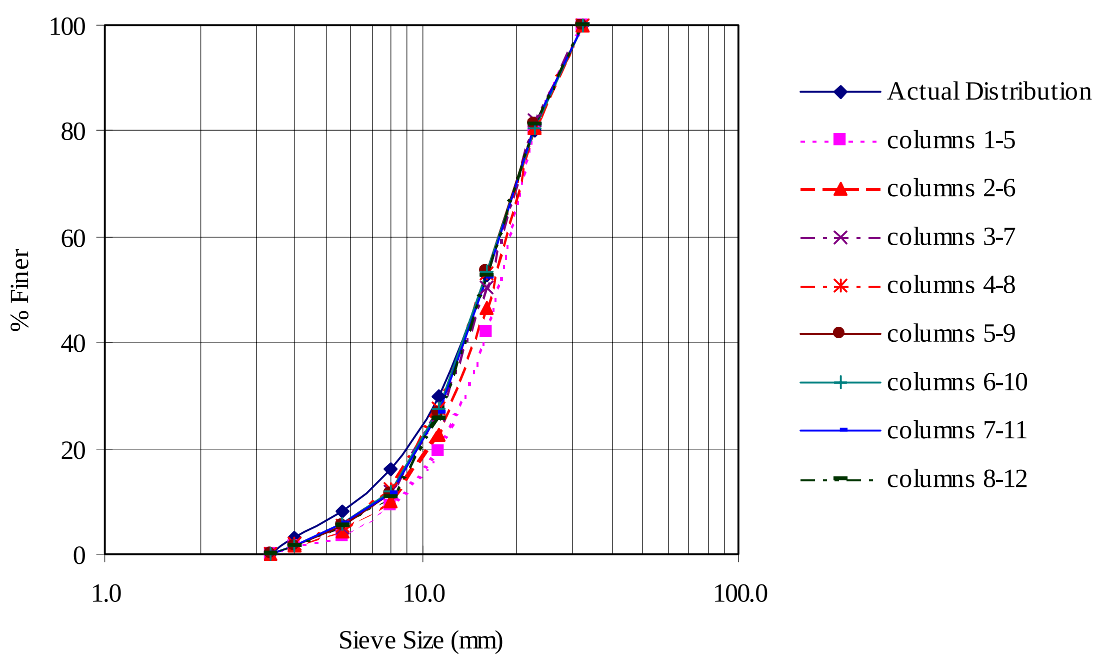

Figure 4.14 Results of laboratory areal (A = 1225-cm^2) samples with the actual size distribution

4.2.3 Volumetric Samples

The volumetric sample was the last laboratory experiment performed on the glass bead distribution. The primary reason is that particles had to be removed to obtain the samples. To obtain the samples, a circular tin with a diameter of approximately 15.3-cm was pushed down through the particles until the tin was fully in contact with the base of the sample box. All the glass beads within the tin were then removed. These particles became the volumetric sample. Once the particles were removed, the tin was left in place to minimize shifting among the particles outside the tin. Figures 4.15,4.16, and 4.17 show the volume sampling procedure. A total of six volumetric samples were taken from the sample box. The sample locations are shown in Figure 4.18.

Figure 4.15 Circular tin used for volume samples

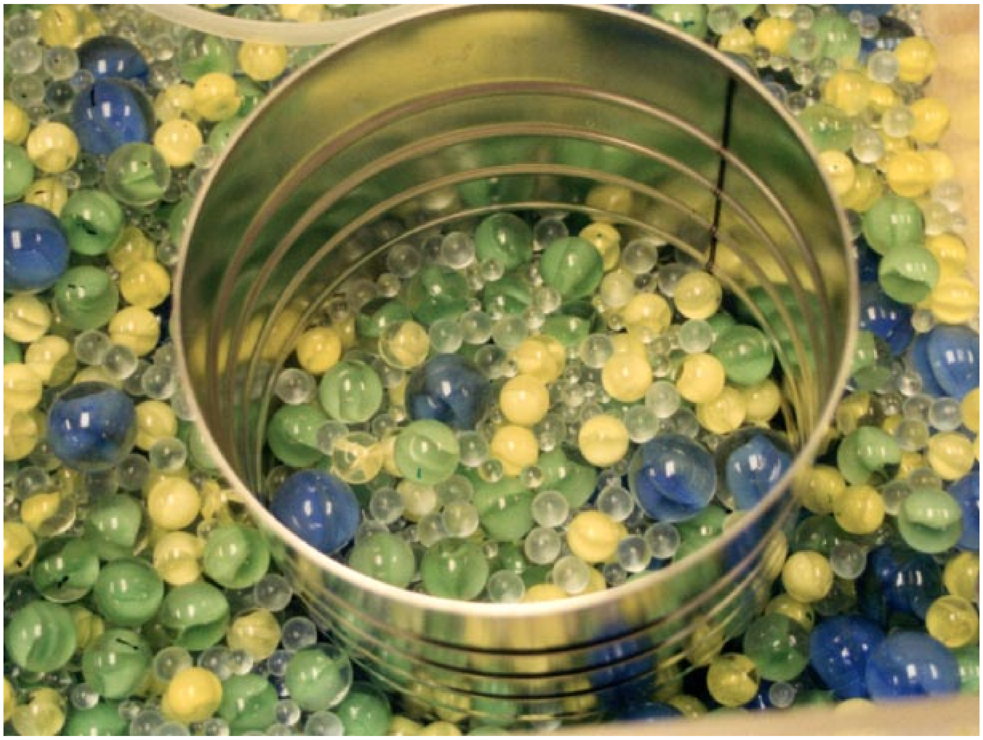

Figure 4.16 Tin before volume sample is removed

Figure 4.17 Tin after volume sample is removed

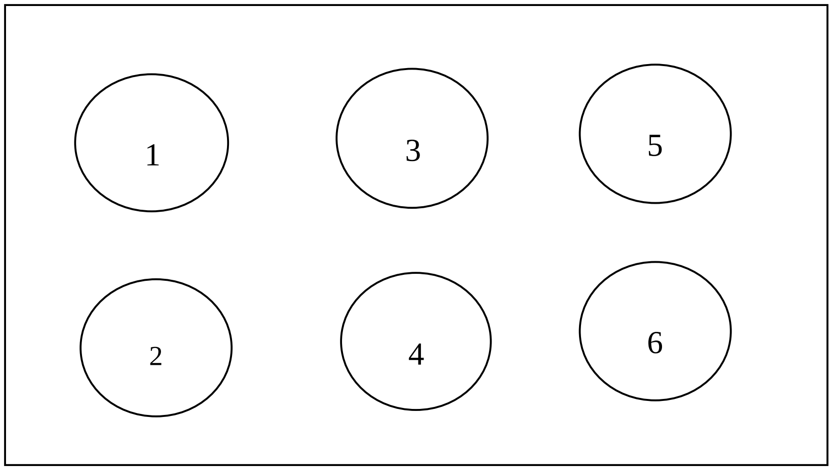

Figure 4.18 Locations of the six volume samples

The data from the volumetric samples can be found in Appendix F. The data was analyzed using the frequency by weight technique. Figure 4.19 shows the results of the total combined volumetric sample with the actual distribution. Figure 4.20 shows the six individual volumetric sample results with the actual size distribution. The volumetric results match up well with the actual distribution. As expected, the combined sample is much more accurate than the individual samples.

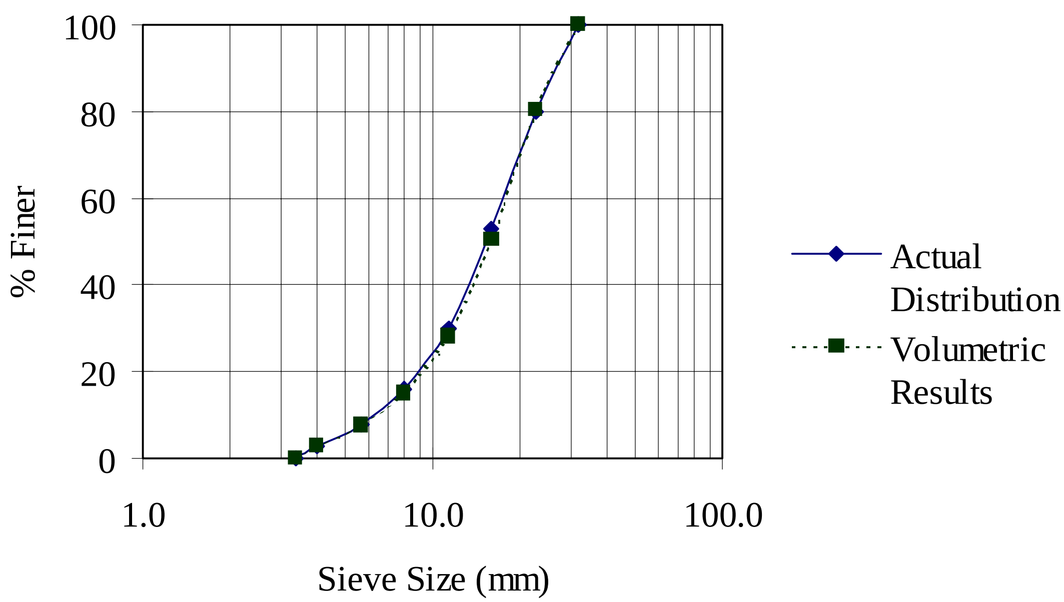

Figure 4.19 Results of the six combined volume samples and the actual distribution

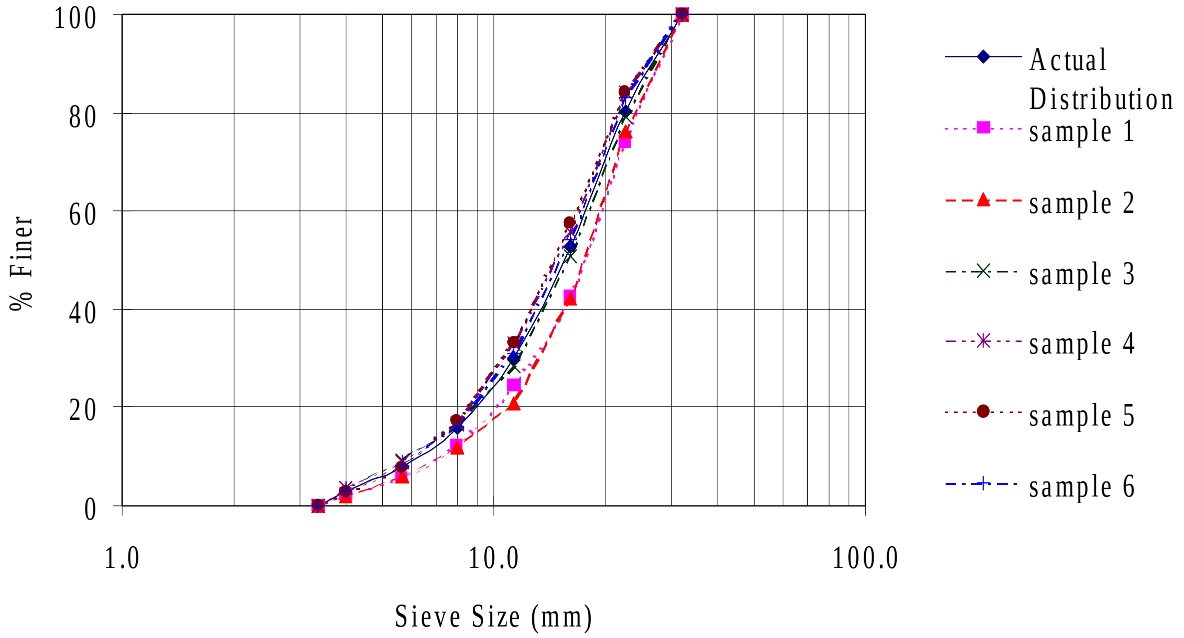

Figure 4.20 Six individual volume sample results with the actual distribution

4.3 Experiments with Natural Sediments

The particle size distribution for the natural sediments was created similarly to that of the glass beads. This distribution was also poorly sorted, with $\sigma_g = 2.05$. The depth of particles was assumed to be 7.75-cm and the porosity was assumed to be 30 %. A specific weight of 2.65 was assumed for all the particles. The same plexiglass box was utilized. A grain size distribution was arbitrarily assigned for the sieve sizes present, 3.35-mm to 38.1-mm. It should be noted that due to a lack of materials, no particles retained by the 19.1-mm sieve are present in the sample.

The concept of subgroups that was employed for the glass beads is used again with the natural sediment to gain a random, homogeneous mixture. Once the initial volumes for each sieve size were assigned, the total weight of sediment was determined. This total weight was then divided into fifteen subgroups, each with the same size distribution. As each subgroup was placed in the sample, the actual weights of each sieve size were recorded. An attempt was made in each subgroup to match the calculated weight for each sieve size. However, due to the variability of the natural sediment, it was impossible to match these weights exactly. The final grain size distribution was calculated from the weights used in each subgroup. Table 4.3 shows the assumed distribution, the weights of each subgroup, and the final grain size distribution. Again, as with the glass beads, the largest grain sizes in each subgroup were placed first followed by the next largest, consecutively down to the smallest grain size present. The cardboard grid that was used to help place the glass beads was not necessary for the natural sediments. The flat edges

Table 4.3 Subgroups used to determine the actual grain size distribution for the natural sediments deposit

			Actual weight per Subgroup (g)																
Sieve Size (mm)	Total Weight retained (kg)	Theoretical Weight per Subgroup (g)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Final Total Weight (kg)	Final % of Volume
3.35	3	200	200.2	200.1	200.1	200.0	200.1	199.9	200.1	199.8	200.0	199.9	199.9	200.0	200.1	200.1	200.1	3.0004	0.04839
4.75	4.5	300	300.0	300.1	300.7	299.8	300.0	300.1	300.0	300.3	300.2	300.0	299.9	299.9	300.3	300.1	300.1	4.5015	0.07260
6.3	7.5	500	500.1	500.4	500.0	499.4	500.1	500.1	500.3	500.2	499.6	500.0	500.6	500.5	499.8	500.2	499.9	7.5012	0.12097
9.5	9.5	633.3333	634.7	634.3	635.6	634.2	633.4	633.2	634.1	633.3	634.2	633.4	632.9	634.0	633.1	633.3	633.9	9.5076	0.15333
12.5	13	866.6667	867.0	866.5	865.8	864.8	867.8	866.0	867.2	868.6	866.4	866.9	866.1	868.3	867.3	866.7	866.4	13.0018	0.20968
25	15.5	1033.3333	1033.8	1031.8	1025.3	1035.0	1032.0	1034.1	1035.6	1038.7	1028.6	1030.3	1033.0	1032.6	1031.4	1033.3	1032.9	15.4884	0.24979
38.1	9	600	590.9	600.5	592.3	611.0	599.0	614.7	593.0	580.8	594.9	600.0	602.9	612.0	595.4	616.4	601.7	9.0055	0.14524

total weight = 62.0064 Kg

present on the natural sediments helped hold them in place easier than the glass beads. Figure 4.21 shows the final probability grain size distribution and Figure 4.22 shows the final cumulative grain size distribution curve.

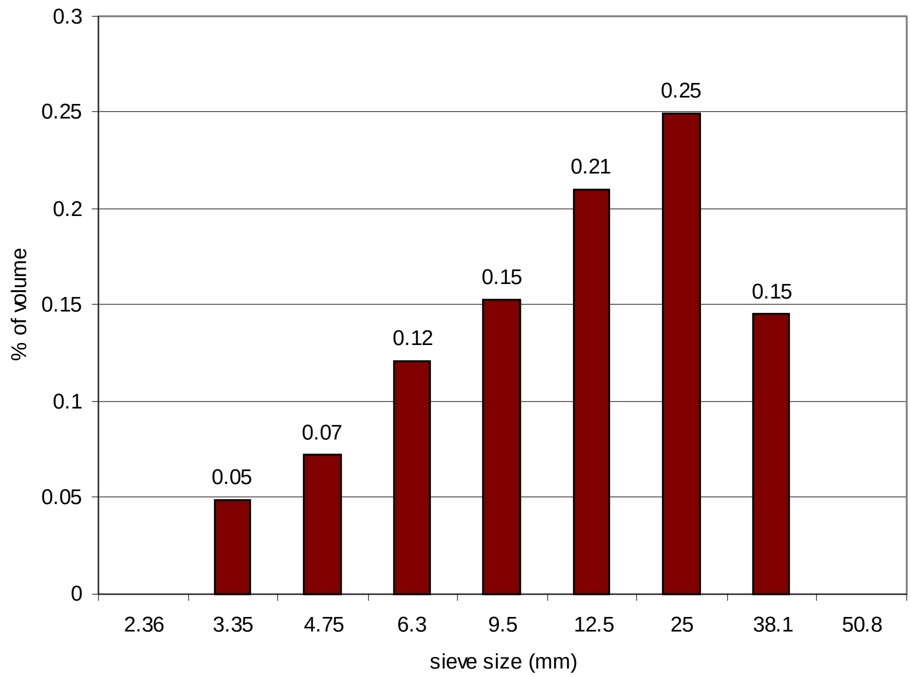

Figure 4.21 Probability distribution function for the natural sediment

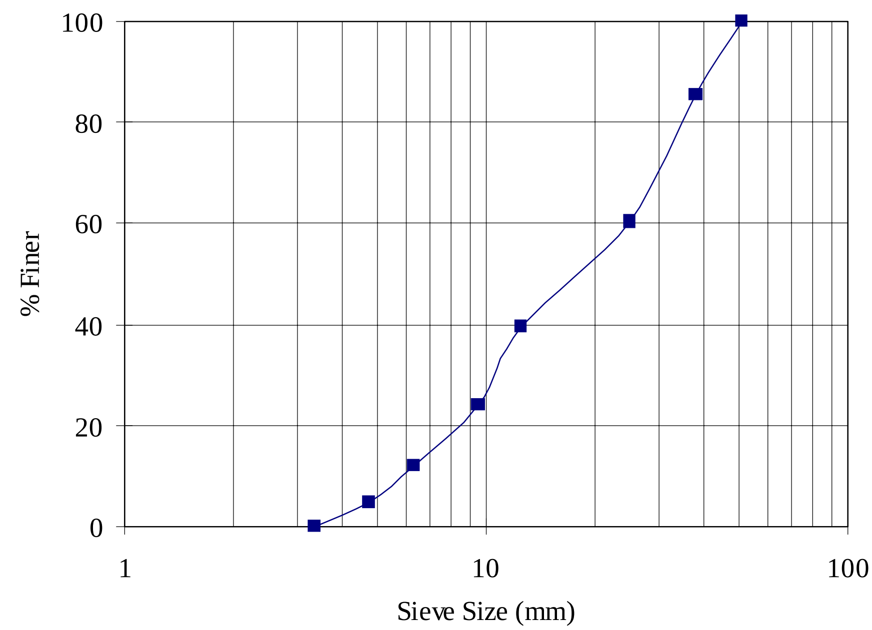

Figure 4.22 Cumulative grain size distribution curve for the natural sediment deposit

The population statistics based on the final grain size distribution are as follows:

mean, μ = 17.12-mm
median, D_{50} = 18.44-mm
standard deviation, σ = 11.28-mm

Figures 4.23 and 4.24 show a plane view and a side view, respectively, of the final natural sediment deposit before any sampling was performed.

Figure 4.23 Plane view of natural sediment deposit

Figure 4.24 Side view of natural sediment deposit

4.3.1 Grid Samples

The grid samples taken on the natural sediment deposit employed the same plumb bob technique as with the glass beads. The particle touched by the plumb bob was removed from the bed and measured with a sieve. Once the particle was measured, an attempt was made to return the particle as close to its original position as possible. Figure 4.25 shows the plumb bob technique as it was applied to natural sediments. Many particles were too small to be removed by hand. When a smaller particle was sampled, it was removed with tweezers, as shown in Figure 4.26. Occasionally a particle would be so surrounded or covered by smaller particle that removing the particle would alter the bed particles. When this occurred, the sampled particles were clearly marked with a question mark. After all experiments were performed on the sediment deposit, these particles were measured and included in the final grid sample.

Figure 4.25 Grid sampling the natural sediment deposit with a plumb bob

As with the glass beads, a total of 420-stones were sampled with a grid spacing of three centimeters. Due to the small grid spacing, some of the larger particles were sampled twice or even three times. When a particle was sampled multiple times, the particle size was recorded each time. The sizes were recorded on a grid that specified the location of each particle. Recording the particle locations enables the consideration of spatial variability over the sample area.

As with the glass beads, the 420-stone grid sample was divided into different sized samples. One 400-stone, four 100-stone, and eight 50-stone samples were extracted from the total grid sample. The 400-stone sample was obtained by selecting the first 400-grain sizes that were sampled. The 100-stone samples were obtained by selecting a particle, starting with the first particle sampled, then skipping three grid spaces. This process was repeated until 100-stones were counted. Then the second 100-stones were obtained in the same manner, starting with the second stone sampled. A similar technique was employed for the 50-stone samples, except a spacing of seven grid spaces was used after each sampled particle. This technique of skipping grid spaces allows for each grid sample to contain particles from the entire sample area. Spatial variability is then accounted for by considering the entire area in each sample. The results of the grid samples are shown in Figures 4.27, 4.28, and 4.29. Table 4.4 gives the sample statistics for the grid samples. Appendix G gives the grid sample data and results for each 400, 100, and 50-stone sample.

Figure 4.26 Removing fine particles with tweezers

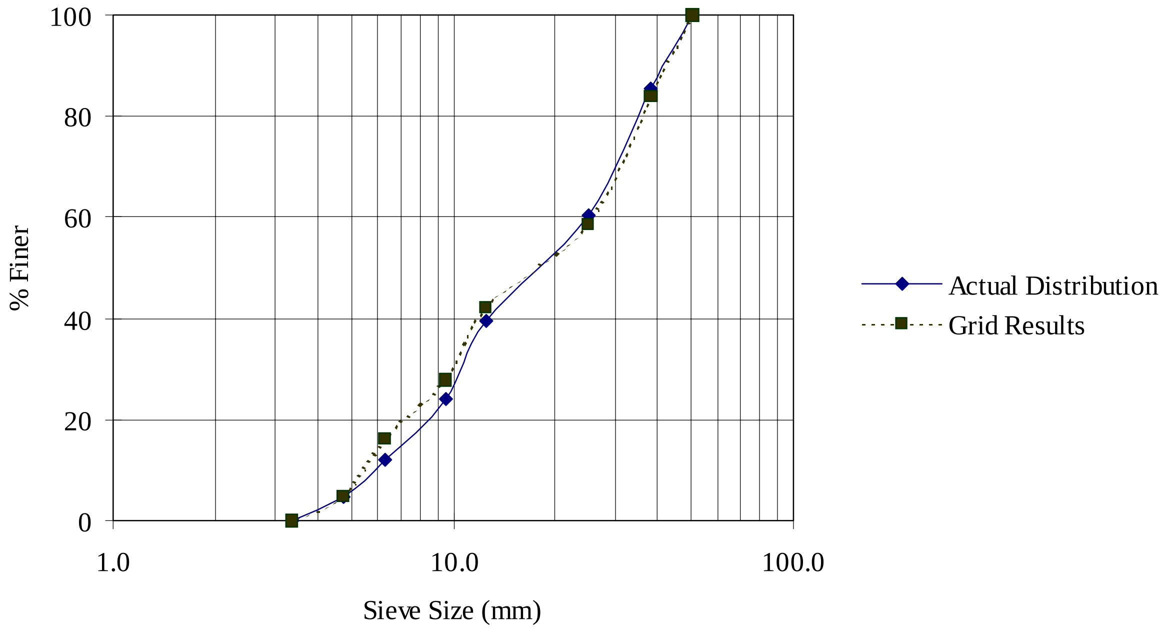

Figure 4.27 Results of the 400-stone grid sample performed on the natural sediment deposit with the actual grain size distribution curve

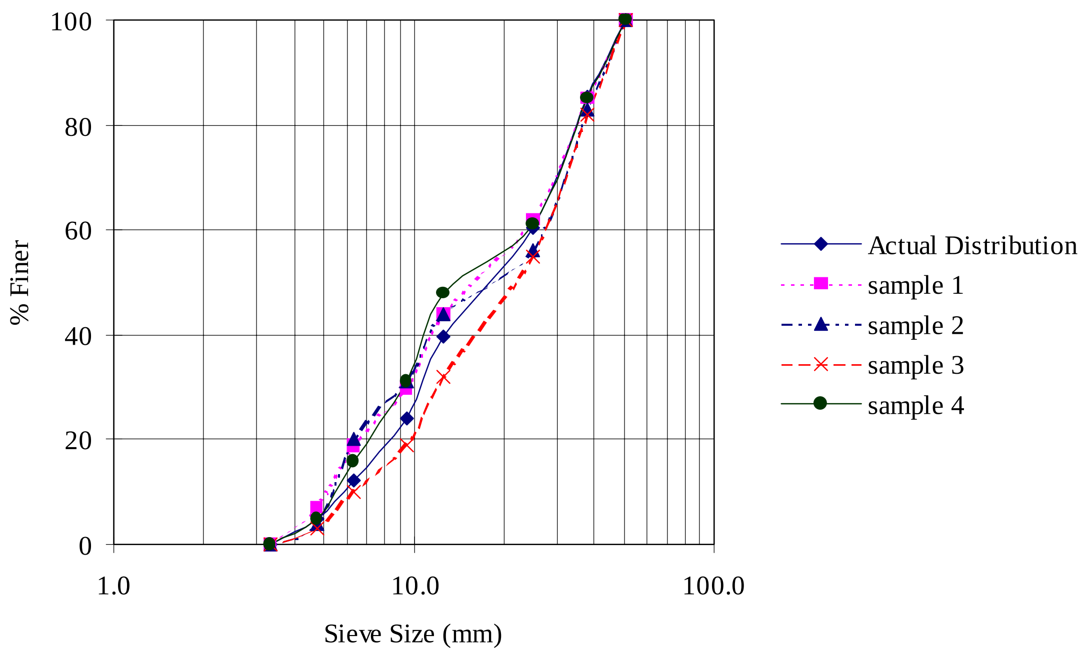

Figure 4.28 Results of the 100-stone grid samples for the natural sediment

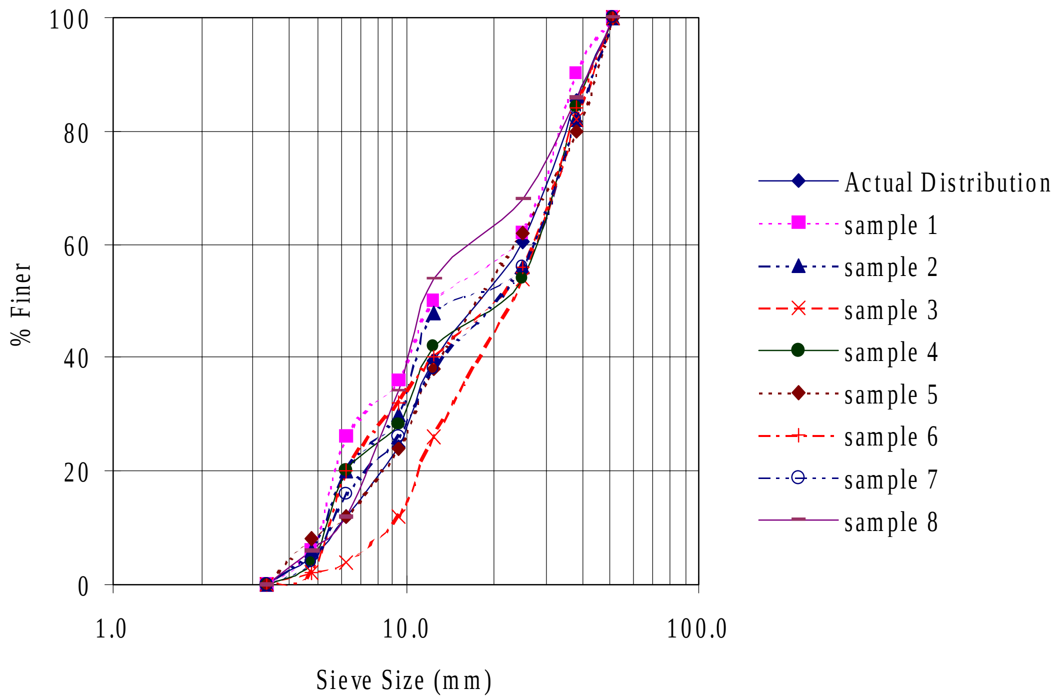

Figure 4.29 Results of the eight 50-stone grid samples for the natural sediment

Table 4.4 Sample statistics for the natural sediment grid samples (mm)

400 stone samples			100 stone samples			50 stone samples		
Sample	Mean	Std. Dev.	Sample	Mean	Std. Dev.	Sample	Mean	Std. Dev.
1	17.35	11.79	1	16.54	11.68	1	15.42	11.09
			2	17.55	12.10	2	17.56	12.32
			3	18.72	11.53	3	19.35	11.04
			4	16.59	11.71	4	17.82	11.86
						5	17.66	12.15
						6	17.53	11.88
						7	18.08	11.95
						8	15.36	11.43

4.3.2 Areal Samples

To obtain areal samples representative of the whole area, eight areal samples were taken. The samples cover a projected area of about 175-cm^2 each. To mark the area of each sample, a template, shown in Figure 4.30, was held over the sample area. The area was then marked with spray paint, as shown in Figure 4.31. After all eight areas were marked, the painted stones were removed from each marked area. A stone that lay on the border of the area was removed only if more than half the projected area of the stone was painted. Figure 4.32 shows the final locations of the samples.

Figure 4.30 Template used for areal sampling

Each areal sample was then analyzed using the frequency-by-weight technique. The results were then converted to an equivalent volume sample using Equation 1.4. An exponent of –1 was used in the conversion. The results of the eight combined areal samples are shown in Figure 4.33. Results for the eight individual areal samples are shown along with the actual distribution in Figure 4.34. Appendix H contains more complete results for the areal samples.

Figure 4.31 Applying spray paint to the sample area

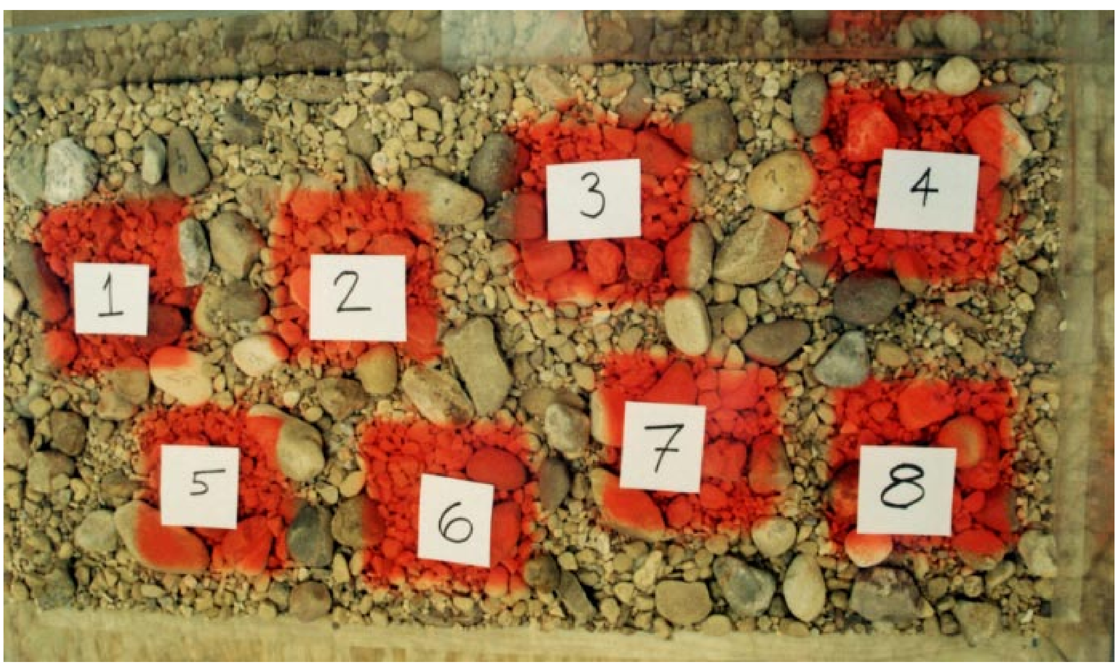

Figure 4.32 Final layout the of eight areal samples

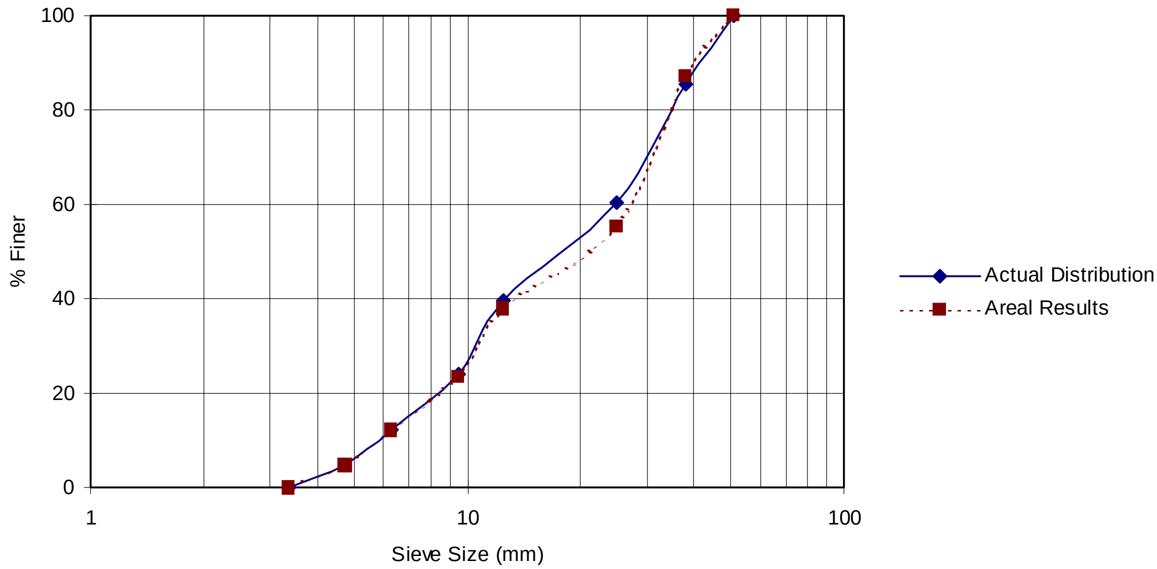

Figure 4.33 Results from the combined areal samples (A = 1400-cm^2) and the actual grain size distribution curve

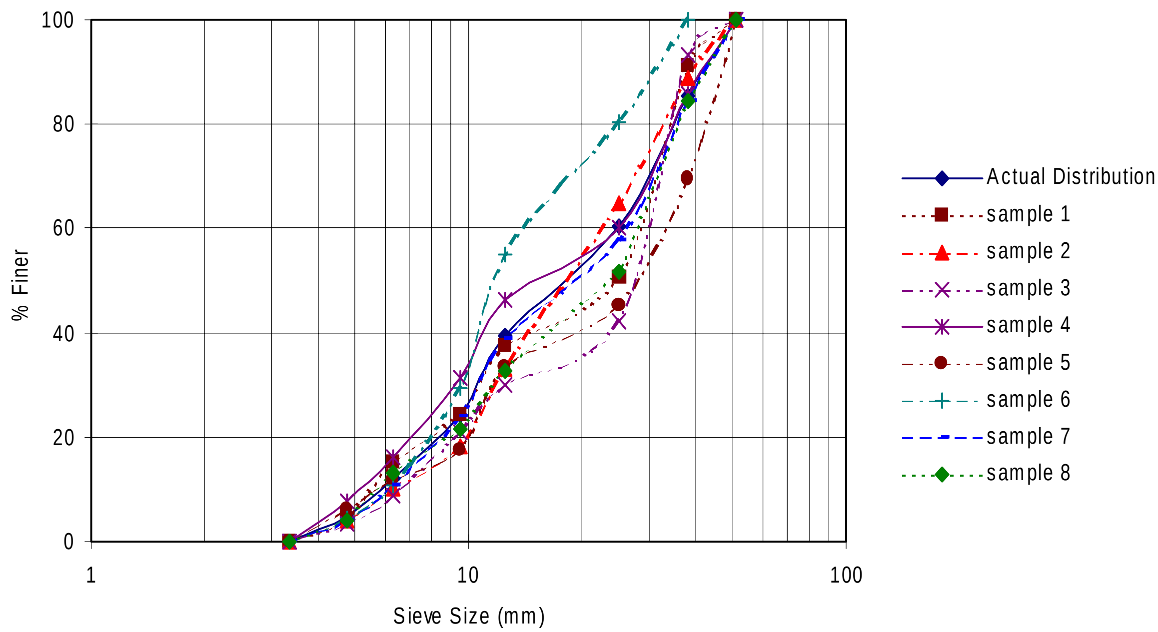

Figure 4.34 Eight individual areal sample results (A = 175-cm^2) for the natural sediment

4.4 Summary and Conclusions

The results from the laboratory experiments are used to test the theories and procedures presented in Chapters 2 and 3. Favorable results from these analyses suggest that these techniques can be successfully applied to field situations. This is provided that the basic assumptions used in the laboratory are also carried out in the field experiments. For example, when sampling a stream bed, all the samples should be random and taken from a single grain size population.

4.4.1 Confidence Intervals

The binomial confidence intervals discussed in section 2.2.1 are applied to the grid sampling results from both the glass bead and natural sediment experiments. The results of the confidence interval calculations are shown in graphic and tabular form in Appendices I and J, for the glass beads and natural sediment, respectively. Each grain size distribution curve has a total of six points were the actual distribution curve can fall outside the confidence interval. The 50-stone sample curves contain a total of forty-eight sample points, the 100-stone sample curves contain twenty-four sample points, and the 400-stone sample curve has six sample points. With the 95% confidence intervals, one point out of twenty would be expected to fall outside the confidence interval.

From Appendix I, the binomial confidence intervals perform well for the 400 and 100-stone sample sizes of the glass bead grid samples. The actual distribution does fall outside the confidence interval for three of the 50-stone sample curves. Statistically, it would be expected for about two points to fall outside the confidence interval for the 50-stone sample curves. Three points out of the total forty-eight are not unreasonable. For the natural sediment samples, one point falls outside of the confidence interval for each of the three sample sizes. For the 50 and 100-stone sample sizes, these results are consistent with what would be expected. For the 400-stone case, however, one point beyond the confidence interval out of six for 95% confidence intervals would not be expected.

Ninety-five percent simultaneous multinomial confidence intervals using the technique discussed in section 2.4.2 were applied to the grid sample data for the glass bead and natural sediment experiments, as shown in Appendices J and L. Because these confidence intervals are simultaneous, the entire grain size curve is considered as a whole and not as individual points, as was the case for the binomial confidence intervals. Subsequently, there are fewer trials to test the multinomial confidence intervals than with the binomial case. The number of trials for the multinomial confidence intervals corresponds to the number of grain size curves available for each sample size.

Appendix J shows the results of the glass bead grid samples with multinomial confidence intervals. The confidence intervals contain the entire actual distribution curve for each curve of the glass bead samples. These results would be expected for the limited amount of sample curves available. However, for the natural sediment grid samples, one actual distribution curve does fall outside the confidence interval. The actual distribution curve falls just below the lower confidence interval at the 6.3-mm sieve for the first 50-stone sample. While one curve out of twenty would be expected to fall outside of the confidence interval, one curve out of eight actually falls out for the natural sediment grid samples. Despite the limited data for the multinomial distribution, the confidence intervals appear to perform well for the data available from both the glass bead and natural sediment laboratory grid samples.

Both types of confidence intervals appear to perform consistently with the underlying statistical theory. However, because the entire grain size distribution curve is generally of interest, the multinomial confidence intervals are recommended. When interpreting binomial confidence intervals, the number of sieves used in the analysis represent the number of points on each curve where the actual curve could fall outside the confidence interval. It can become difficult when considering multiple sample curves to account for the total number of points where the binomial confidence interval could fail. However, with the simultaneous multinomial confidence intervals, the entire sample curve represents a trial. When considering multiple samples, then, the multinomial confidence intervals make the sample results easier to interpret.

4.4.2 Areal and Volume Sample Accuracy

A new technique was presented in Chapter 3 for determining the accuracy of areal and volume samples. The technique proposed allows a confidence statement to be made concerning the results of sampling a specific area or volume. The results obtained for sample volumes seemed to agree with the previously published studies by Church et al. (1987) and Ferguson and Paola (1997). The results of the examples in this chapter also support the use of this technique.

Chapter 5: Discussion of Field Procedures

5.1 Introduction

The goal of the procedures presented in Chapters 2 and 3 is to obtain sediment samples that accurately represent the true grain size distribution of the stream bed. This section briefly discusses some aspects of sampling that should be considered when obtaining field samples. The main discussion is focused on reducing sampling errors and combining grid and areal samples to obtain a hybrid grain size distribution. This hybrid distribution can accurately represent a wide range of particle sizes. The truncation problem associated with both grid and areal samples can be avoided with the hybrid sampling technique.

5.2 Sampling Errors

When performing sediment samples in the field, it is important to minimize the errors involved. Underwood (1970) states that the three general types of sampling errors are:

1. Errors from experimental limitations.
2. Errors from improper sampling technique.
3. Errors from nonrepresentative samples.

Errors from experimental limitations arise from human error factors, such as incorrect counting of a grid sample. These errors are rarely the limiting factor in the overall accuracy of the analysis and can be reduced by careful sampling (Underwood, 1970). Errors from improper sampling technique occur in situations where random samples are not obtained in a truly random way. Anytime the sampler makes a subjective decision, bias is introduced into the sample. To overcome these types of errors, it is imperative that selecting grid, areal, and volume samples be done in a truly random fashion. To accomplish random sampling, it is recommended that the sampler not look at the stream bed before and during the sample removal. The third type of error is due to nonrepresentative samples. These errors occur when population characteristics are determined from samples that are not large enough to make any meaningful inference. Techniques are presented in Chapters 2 and 3 to overcome these errors by obtaining statistically meaningful samples. Additionally, nonrepresentative samples may be obtained if sample results from different populations are combined. Samples should only be obtained from a single population. Recognizing and addressing these three general sampling errors is important when planning field sampling procedures.

An additional use of sampling errors is measuring the relative efficiencies of sampling techniques. Different techniques are available for determining the grain size distribution of surfaces and volumes. Grid and areal samples can be used for surface samples and grid, areal, and volume samples can be used to sample volumes. It is useful to know which method will match a required accuracy with the least amount of effort. Underwood (1970) shows that the grid sampling technique, in particular systematic point counts, will require the least amount of effort for a given, constant accuracy. When performing grid sampling in the field, it is recommended that a systematic grid spacing be used consistently throughout the sampling effort. Given the extremely large weights required for representative volume samples in Chapter 3 and confirmed by Church et al. (1987) and Ferguson and Paola (1997), among others, grid samples may be preferable for characterizing homogeneous gravel deposits. Grid samples are simple to perform and does not require samples to be removed from the site for analysis. However, to avoid obtaining truncated sample results, areal samples may need to be taken if particles finer than about 10-mm are present in the deposit. The areal and grid sample results can be combined into a hybrid sample grain size distribution.

5.3 Analysis and Interpretation Errors

Misunderstandings behind the theory of sediment sample analyses can cause errors in interpreting sampling results. To avoid errors in sample interpretation, it is recommended that all sample results, regardless of sampling technique, be represented as volume-by-weight proportions. Converting sample results into unbiased proportions enables comparison between different samples. When a biased technique is used to represent the sample proportions, different sample results cannot be compared, even if the samples were removed and analyzed with the same techniques (Diplas, 1992). Considering this fact, all sample results should be presented in unbiased volume-by-weight proportions.

5.4 Hybrid Sampling Technique

As mentioned in previous chapters, by combining the benefits of grid and areal samples, the entire grain size distribution of a stream bed can be represented without truncating the sample results at a particular grain size. Fripp and Diplas (1993) present a procedure for combining grid and areal sample results to obtain a hybrid grain size distribution curve. This procedure is applied to data from the natural sediment laboratory experiments.

Using Appendix C., a sample size of 100-stones is selected. This sample size was selected to provide an error between 0.10 and 0.15 around D_{50} for 95% accuracy. The results of the first 100-stone grid sample are used for this example. To simulate an actual field sample, it is assumed that the particles finer than the 12.5-mm sieve are too small to be removed by hand. Therefore, these particles are counted, but not measured. The percentage finer than the 12.5-mm sieve is estimated, but the shape of the curve is not. If this sample were performed in the field, an estimate of the finest particle size present in the population would be necessary. However, 3.35-mm will be used for the lower limit in this example to be consistent with the laboratory experiment. The results of the grid sample are shown in Table 5.1and Figure 5.1. The different shading on the lower end of the sample curve in Figure 5.1 represents the portion between the 3.35-mm sieve and the 12.5-mm sieve where the sample distribution curve shape is not known. The results for the grid and areal samples in this section are written in terms of volume-by-weight to emphasize the importance of converting sample results to an unbiased representation.

Table 5.1 100-stone grid sample and multinomial confidence interval

D_i (mm)	p (V-W)$_i$ grid	cumulative grid results	95% Confidence Interval	
50.8	0	100	93.04	100
38.1	0.15	85	72.84	92.3
25	0.23	62	48.34	74.0
12.5	0.18	44	31.32	57.5
< 12.5	0.44	0	0	7.0

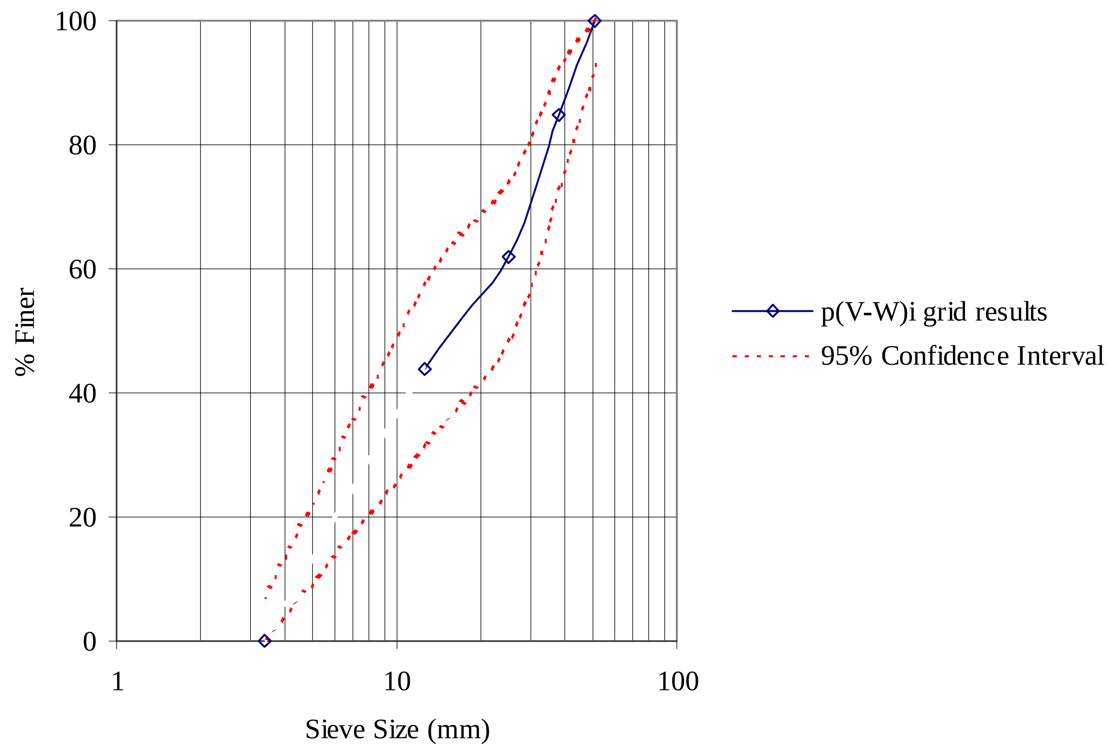

Figure 5.1 100-stone grid sample results and multinomial confidence intervals

Following the procedures presented in section 3.3, the area to be sampled is estimated to be about 1070-cm^2. This area was calculated based on transforming the lower confidence interval and calculating N_A at the 38.1-mm sieve, which represents the largest size that clay areal samples will consistently remove. The laboratory areal samples one through six are combined to obtain a sample size of about 1050-cm^2.

Table 5.2 shows the results of the areal sample as well as the procedure for combining the grid and areal samples to create a hybrid sample. First, a match point is selected. This point should occur in the overlap region where both the areal and grid samples can consistently represent the grain sizes. The match point should also occur at a point were the proportions for the areal and grid samples agree well. Table 5.1 shows that the match point was selected to be the 12.5-mm sieve. Once the match point is selected, the grid proportions are scaled so there is agreement at the match point with the areal sample. The two proportions are then combined in a total sample, with the grid sample representing the grain sizes coarser than the match point and the areal sample representing the grain sizes finer than the match point. These combined results must then be scaled so that the final cumulative proportions vary between zero and one exactly. Figure 5.2 show the final hybrid grain size distribution curve along with the grid and areal sample proportions.

Table 5.2 Hybrid sample procedure

D_i (mm)	p (V-W)$_i$ areal	p (V-W)$_i$ grid	Adjusted p (V-W)$_i$ grid	Combined Results	p (V-W)$_i$ total	Hybrid Percent Finer
50.8	0	0	0	0	0	100
38.1	0.1191	0.15	0.1410	0.1410	0.1546	84.5
25	0.3260	0.23	0.2162	0.2162	0.2370	60.8
12.5	0.1692	0.18	0.1692	0.1692	0.1855	42.3
9.5	0.1498	-	-	0.1498	0.1643	25.9
6.3	0.1130	-	-	0.1130	0.1239	13.5
4.75	0.0730	-	-	0.0730	0.0800	5.5
3.35	0	-	-	0.0499	0.0547	0

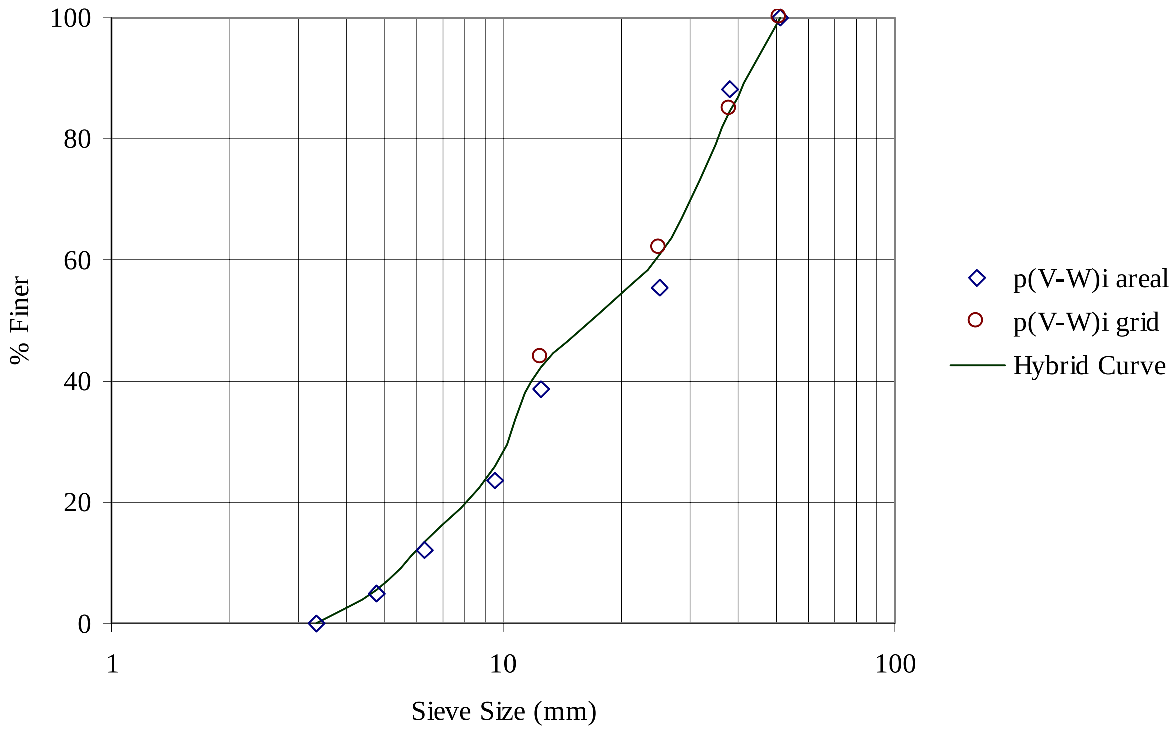

Figure 5.2 Hybrid sample curves along with individual grid and areal sample proportions

An important feature of the hybrid sample curve that should be addressed deals with applying confidence intervals to the sample. Because the areal sample size was determined in a way that the areal results are at least as accurate as the results of the original grid sample, confidence intervals can be applied to the entire hybrid sample using the multinomial technique present in Chapter 2. The accuracy of this confidence interval is based on the original grid sample. Figure 5.3 compares the hybrid sample curve given in Table 5.2 with 95% multinomial confidence intervals and the actual grain size distribution of the synthetic stream bed. The match of the hybrid curve and the actual grain size distribution curve appears to be good for this example.

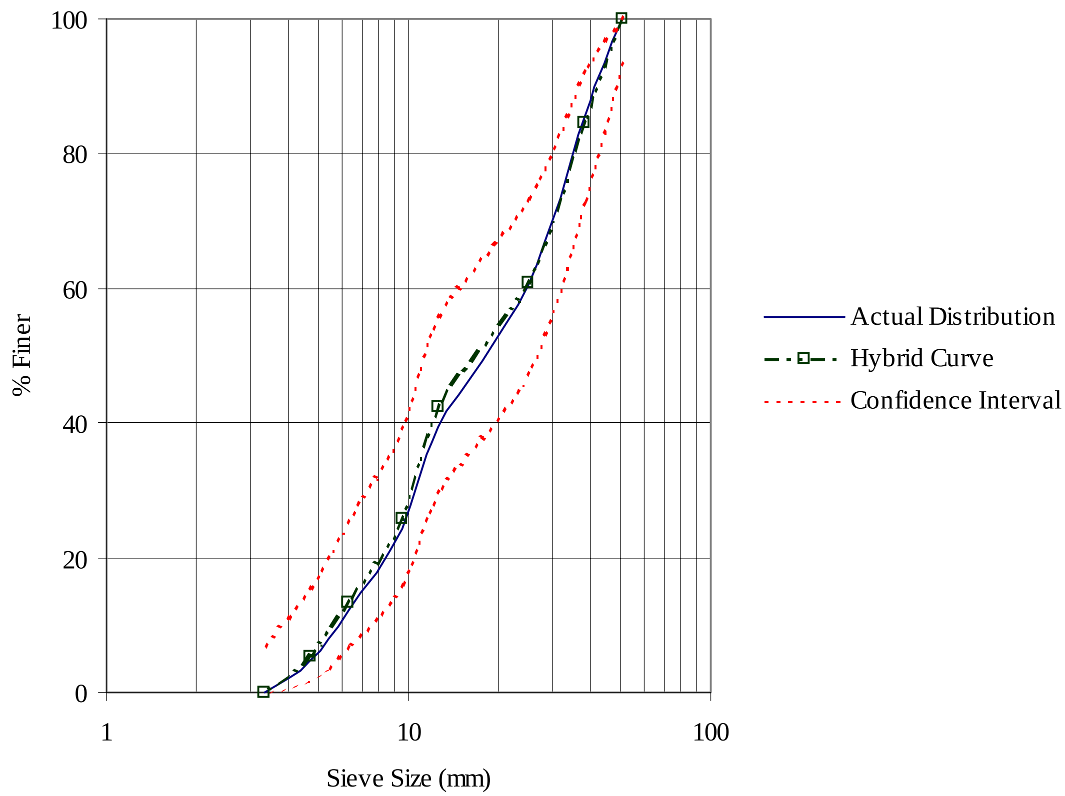

Figure 5.3 Hybrid sample curve with 95% multinomial confidence intervals for 100-stone grid sample

5.5 Conclusions

To successfully extend sampling techniques to field applications, the fundamental assumptions used in the techniques must be maintained. By considering these assumptions, accurate results with little effort can be obtained. By utilizing the grid sampling technique on coarse, homogeneous, gravel deposits, accurate results can be obtained with out removing large volumes of material. By employing a hybrid of the grid and areal sampling techniques, the entire grain size distribution curve of a stream bed can be characterized without truncation.

Chapter 6: Conclusions

This study presents new techniques for determining the accuracy of grid, areal, and volume sediment samples. The grain size distribution of a river bed provides valuable information about the hydraulic and ecological behavior of the river. For this reason, it is important to obtain accurate estimates of the river bed's grain size distribution. The accuracy of the sample results is represented in the form of confidence intervals. Confidence intervals are derived through sound statistical formulation and provide a simple way to interpret the accuracy of sample results.

A technique is presented in Chapter 2 for placing confidence intervals on grid sample results. This technique introduces the multinomial distribution to applications in sediment sampling. The parameters of the multinomial distribution seem to exactly represent the process of grid sampling. The multinomial confidence intervals are a type of simultaneous interval that considers the entire grain size distribution. This simultaneous representation is preferred because the results are easy to interpret. Based on a required accuracy, sample sizes can also be estimated for grid samples using the multinomial distribution. Sample size estimates are particularly useful when developing field guidelines or planning for a sampling routine.

The multinomial distribution can also be applied to areal and volume samples. Difficulties are encountered though, because the areal and volume sample sizes are not trials, but rather areas and volumes, respectively. To overcome this problem, the technique proposed in Chapter 3 uses the results of a grid sample on the population of interest to estimate the necessary area or volume to sample. The estimated areal or volume sample size is designed to give results with an equivalent accuracy to the grid sample. Confidence intervals can then be placed on the areal or volume sample results using the original grid sample size. The results of grid and areal samples can also be combined to form a single sample curve with confidence intervals based on the hybrid sampling method. This final hybrid grain size distribution curve avoids truncated results. The technique proposed here is the first that specifically addresses the issue of areal and volume sample accuracy with confidence intervals.

Laboratory experiments were performed that support the new techniques, as explained in Chapter 4. Combining the new sample size estimation and confidence interval techniques, the grain size distribution of a river bed can be accurately represented. The results of sediment samples can be used to assess the health of a river as well as monitor changes in the hydraulics and ecology of a river. The applications of the various techniques presented extend beyond river bed sediment sampling. Similar problems exist in fields as diverse as biology, metallurgy, and volcanology.

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Appendix A. Sample Size Determination Graphs for the Binomial Distribution

Using a desired accuracy and an error band around a percentile of interest, the graphs in this appendices use the binomial distribution to estimate a grid sample size to achieve this desired accuracy. Further discussion and an example using these graphs is provided in section 2.5.1. The grain percentiles shown are D_{10}, D_{16}, D_{30}, D_{50}, D_{70}, D_{84}, and D_{90}. Additionally, accuracies of 95%, 90%, and 80% are shown for each grain percentile.

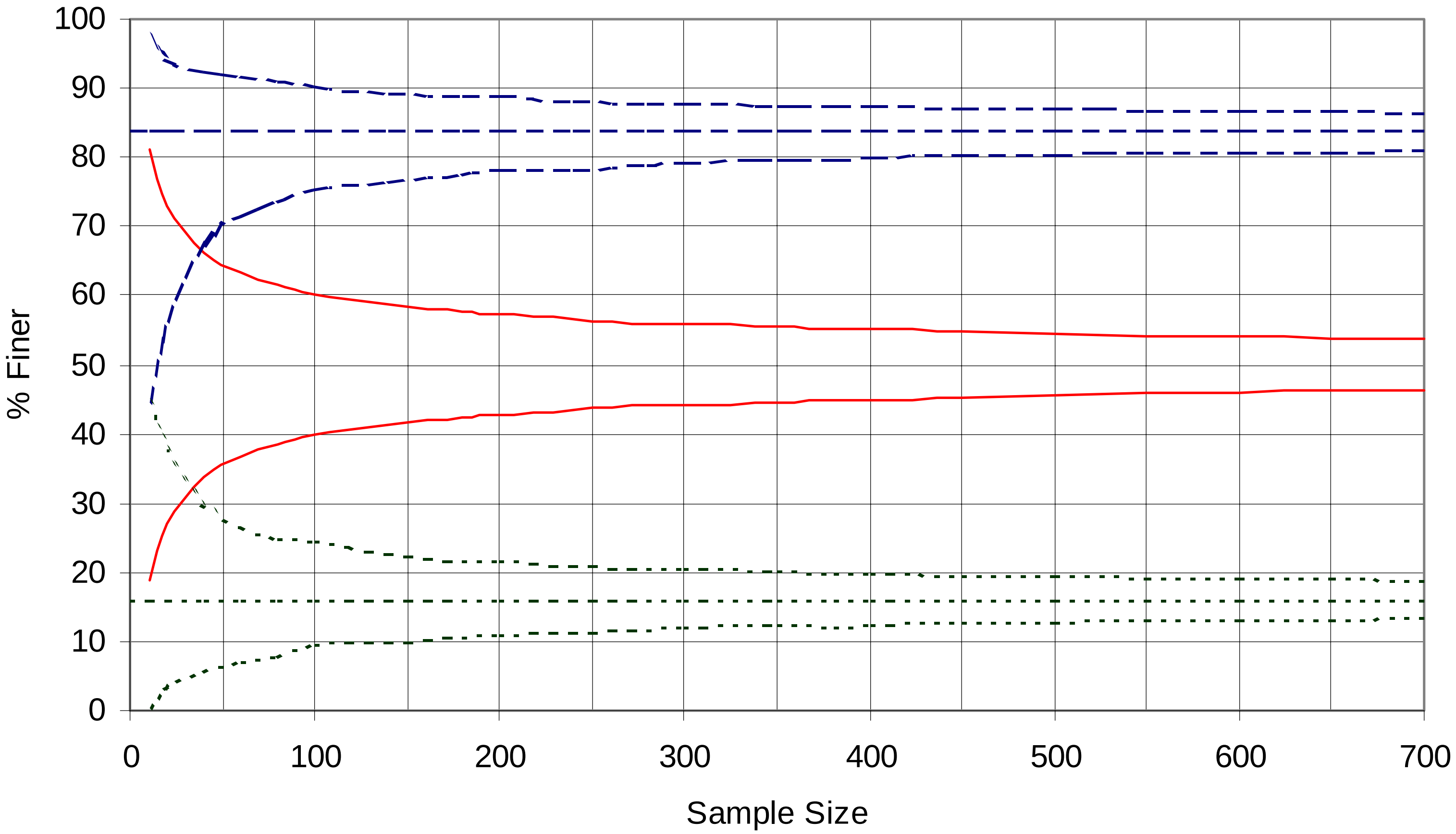


Figure A.1 Binomial sample size determination graph for D_{16}, D_{50}, and D_{84} for $\alpha = 0.025$

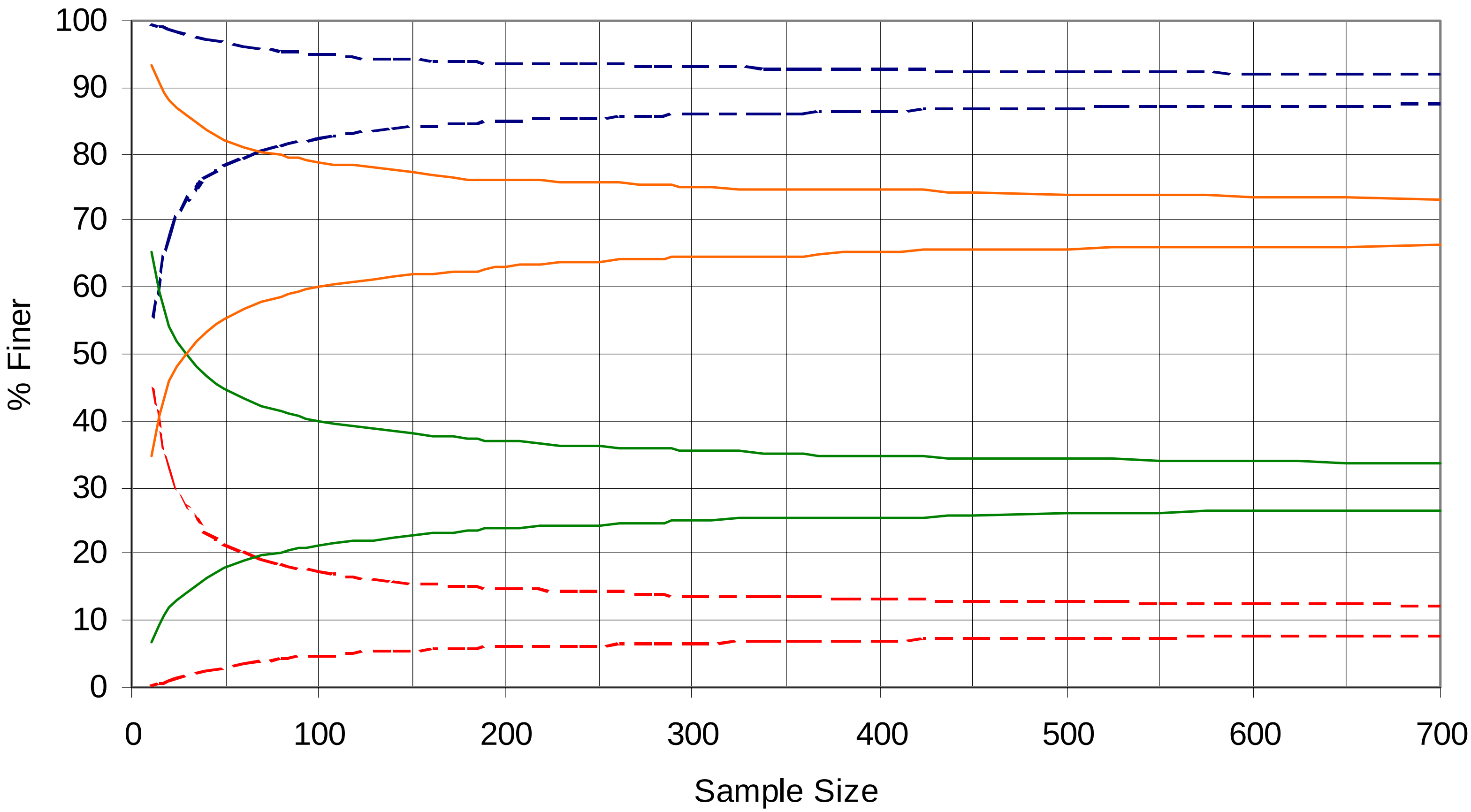


Figure A.2 Binomial sample size determination graph for D_{10}, D_{30}, D_{70} and D_{90} for $\alpha = 0.025$

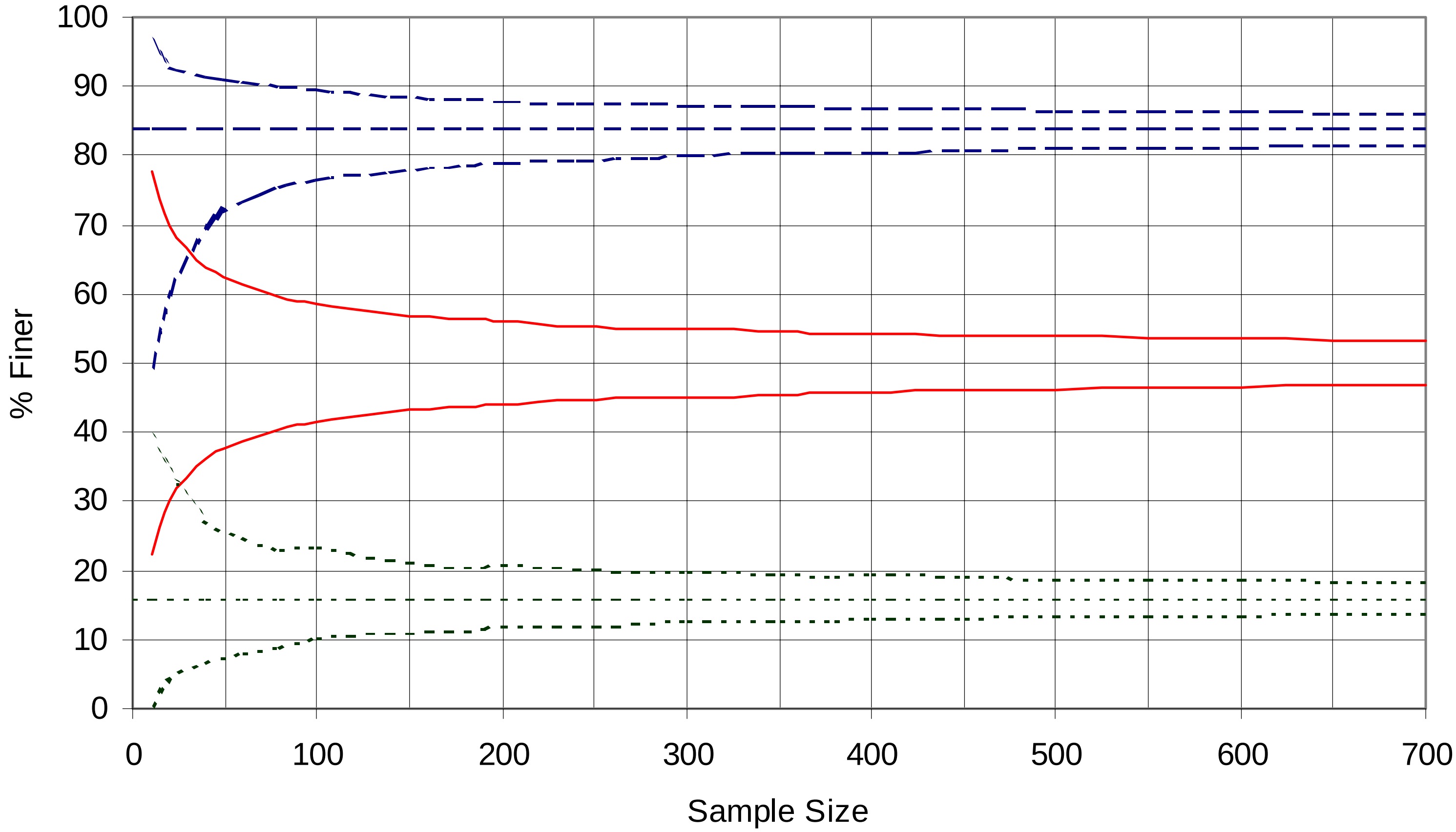


Figure A.3 Binomial sample size determination graph for D_{16}, D_{50}, and D_{84} for $\alpha = 0.05$

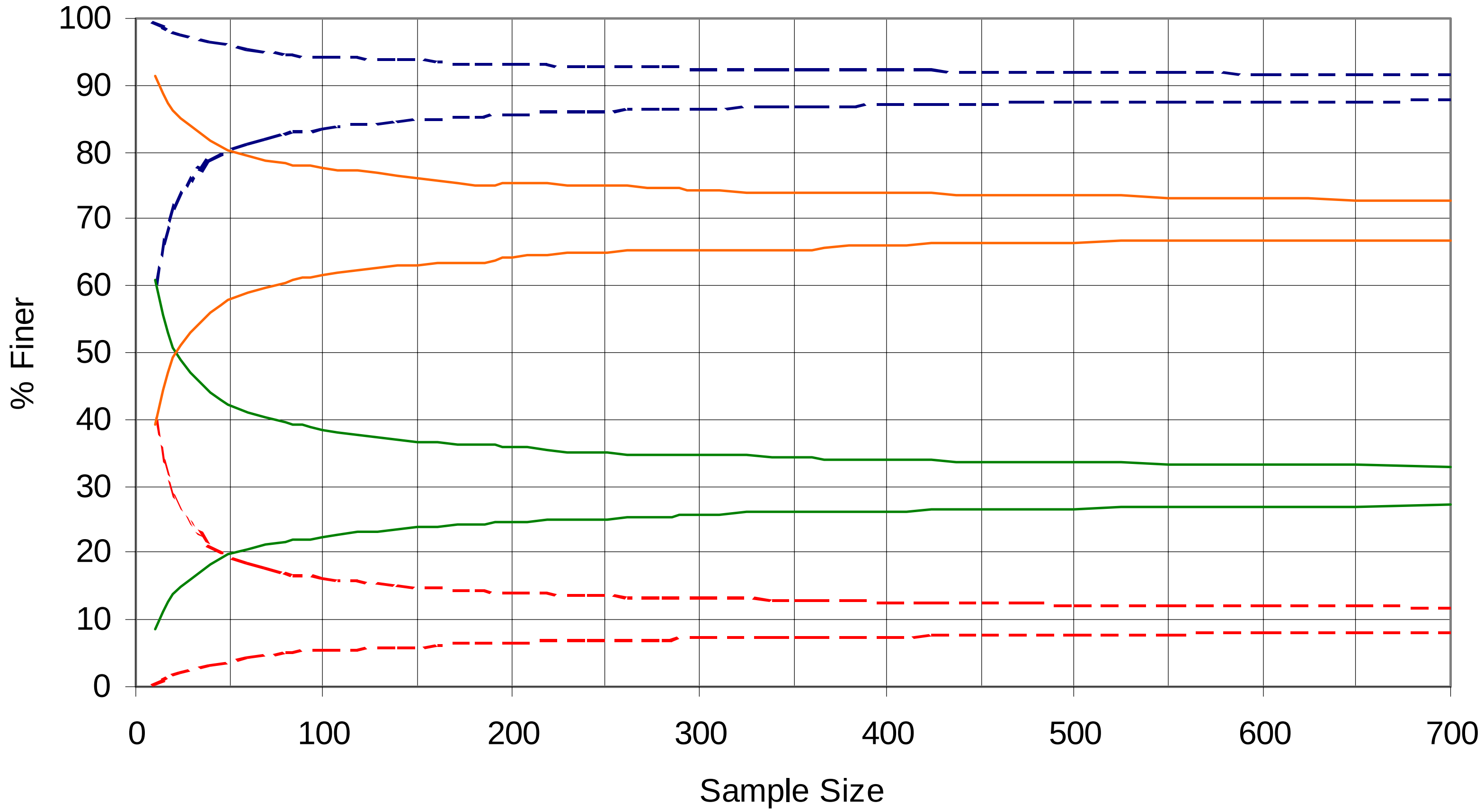


Figure A.4 Binomial sample size determination graph for D_{10}, D_{30}, D_{70} and D_{90} for $\alpha = 0.05$

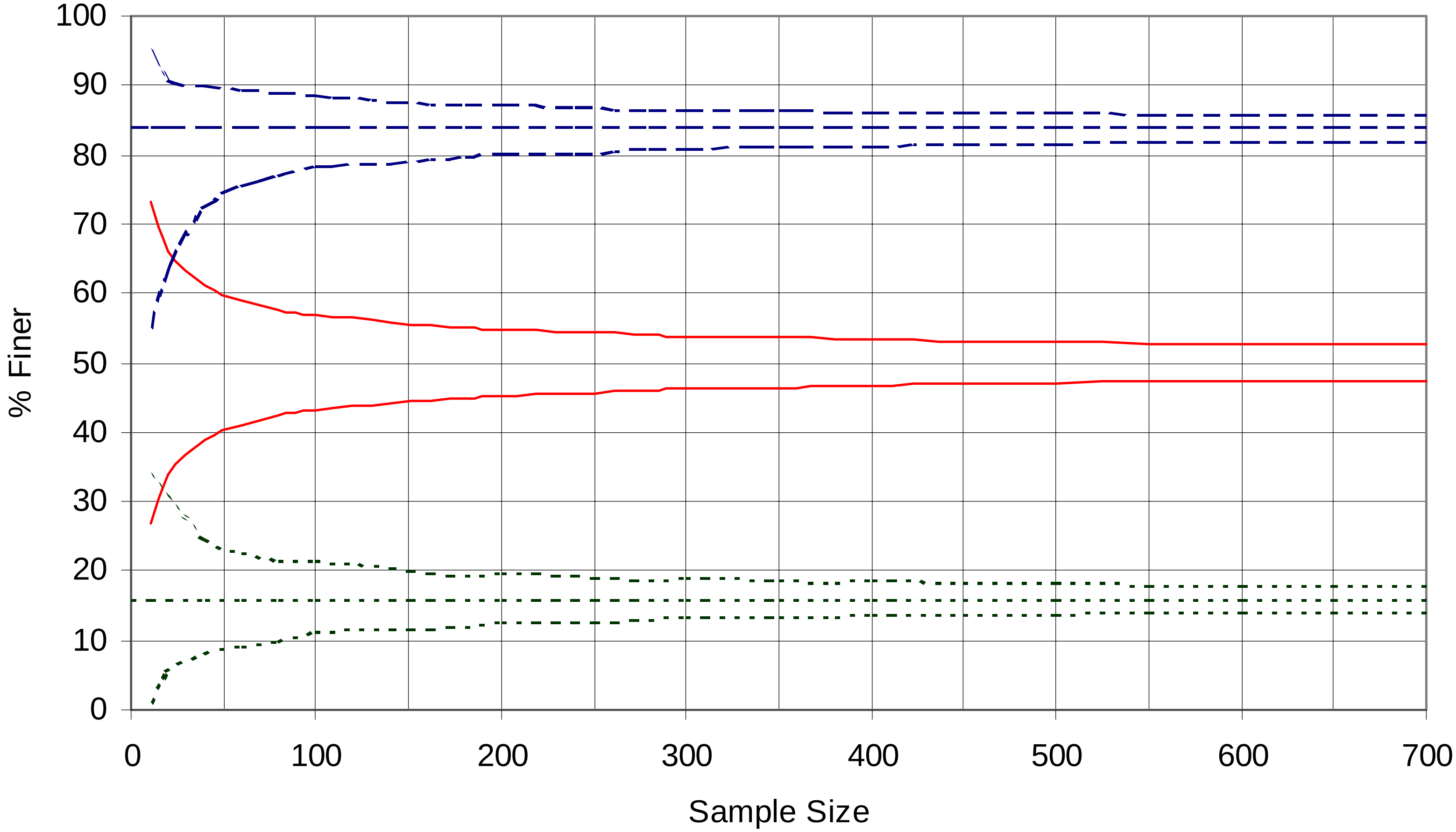


Figure A.5 Binomial sample size determination graph for D_{16}, D_{50}, and D_{84} for $\alpha = 0.10$

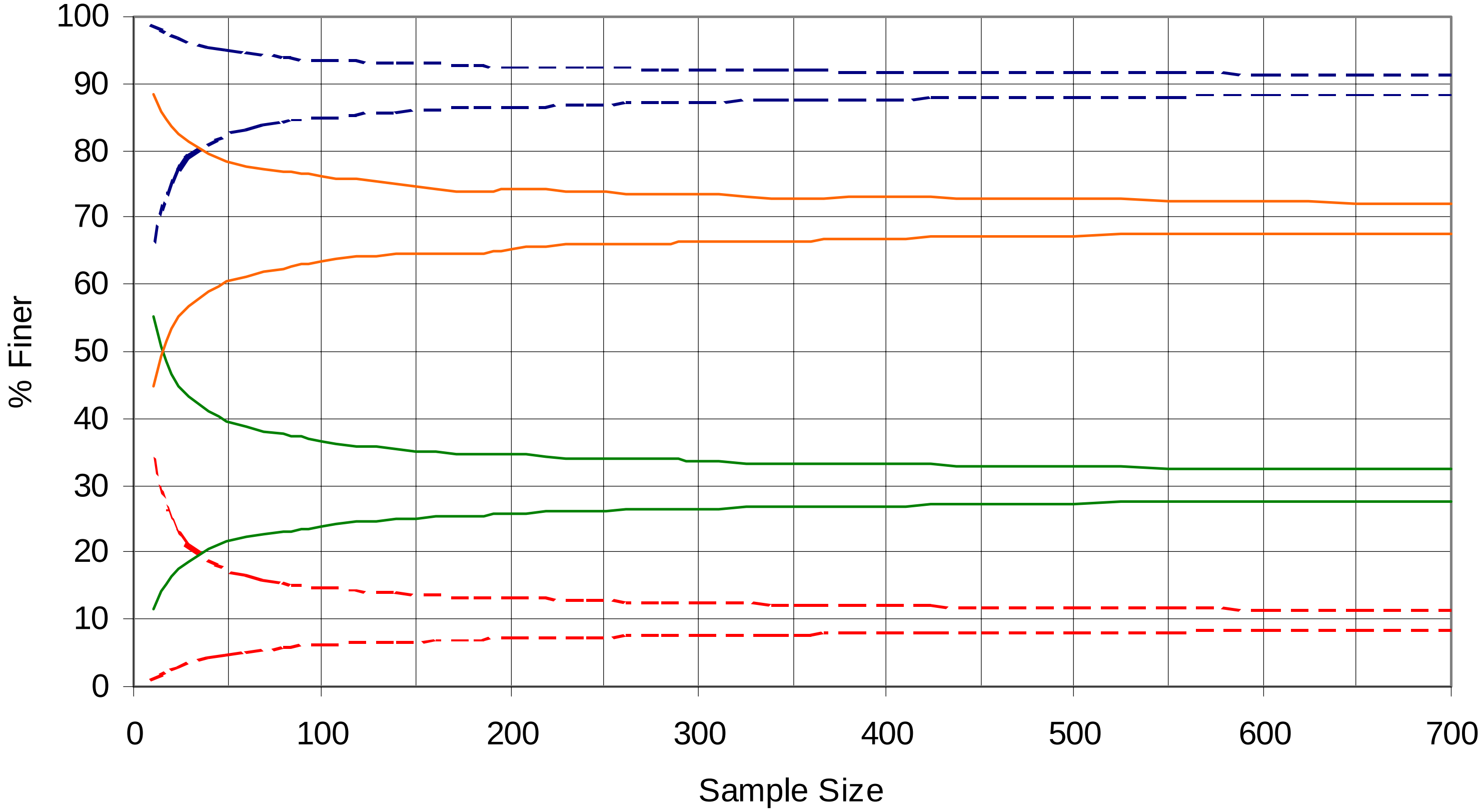


Figure A.6 Binomial sample size determination graph for D_{10}, D_{30}, D_{70} and D_{90} for $\alpha = 0.10$

Appendix B. Sample Size Determination Graphs for the Normal Distribution

Using a desired accuracy and an error band around a percentile of interest, the graphs in this appendices use the normal distribution to approximate the binomial distribution and estimate a grid sample size to achieve this desired accuracy. Further discussion is provided in section 2.5.2. The grain percentiles shown are D_{10}, D_{16}, D_{30}, D_{50}, D_{70}, D_{84}, and D_{90}. Additionally, accuracies of 95%, 90%, and 80% are shown for each grain percentile.

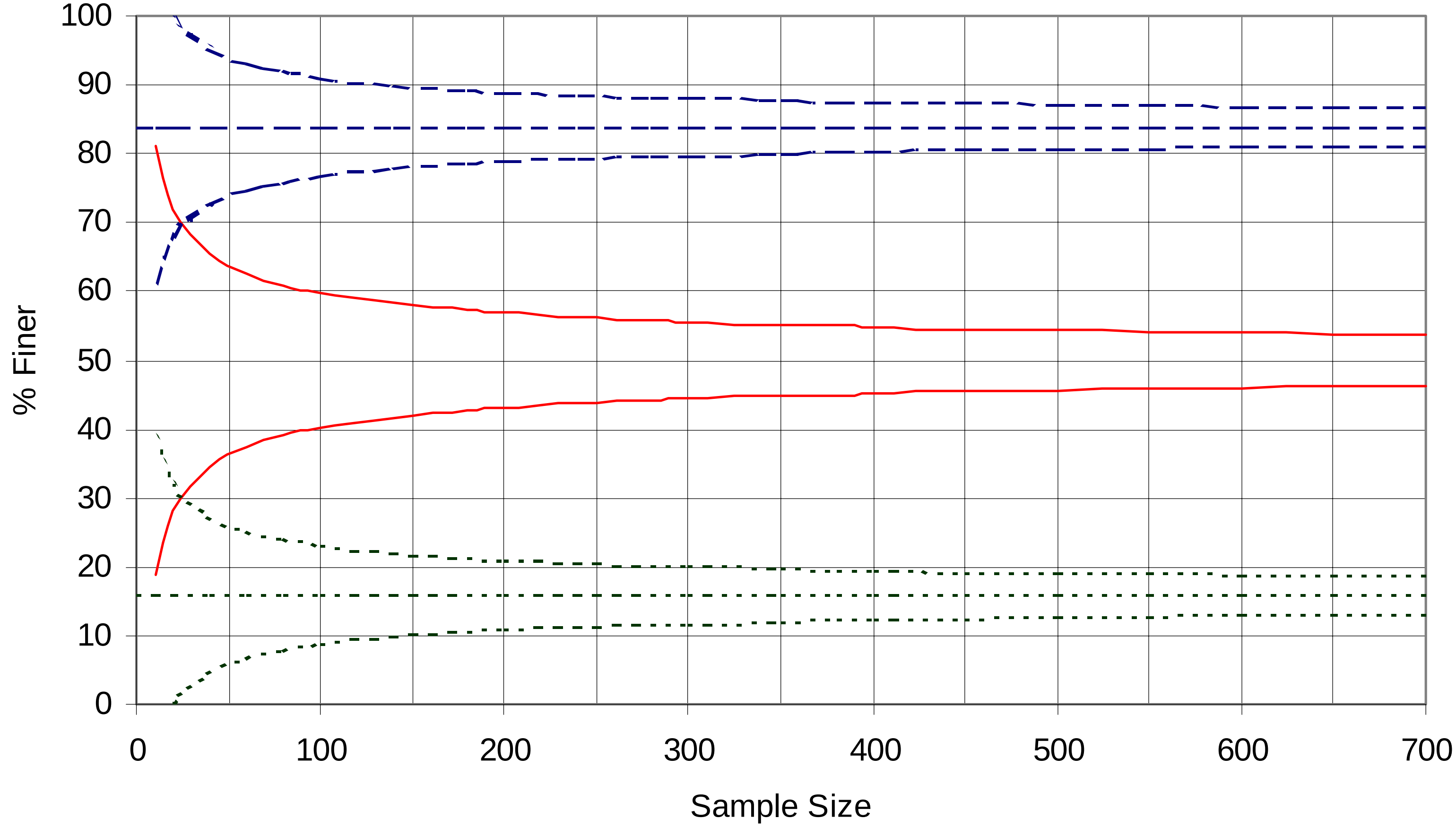


Figure B.1 Normal sample size determination graph for D_{16}, D_{50}, and D_{84} for α = 0.025

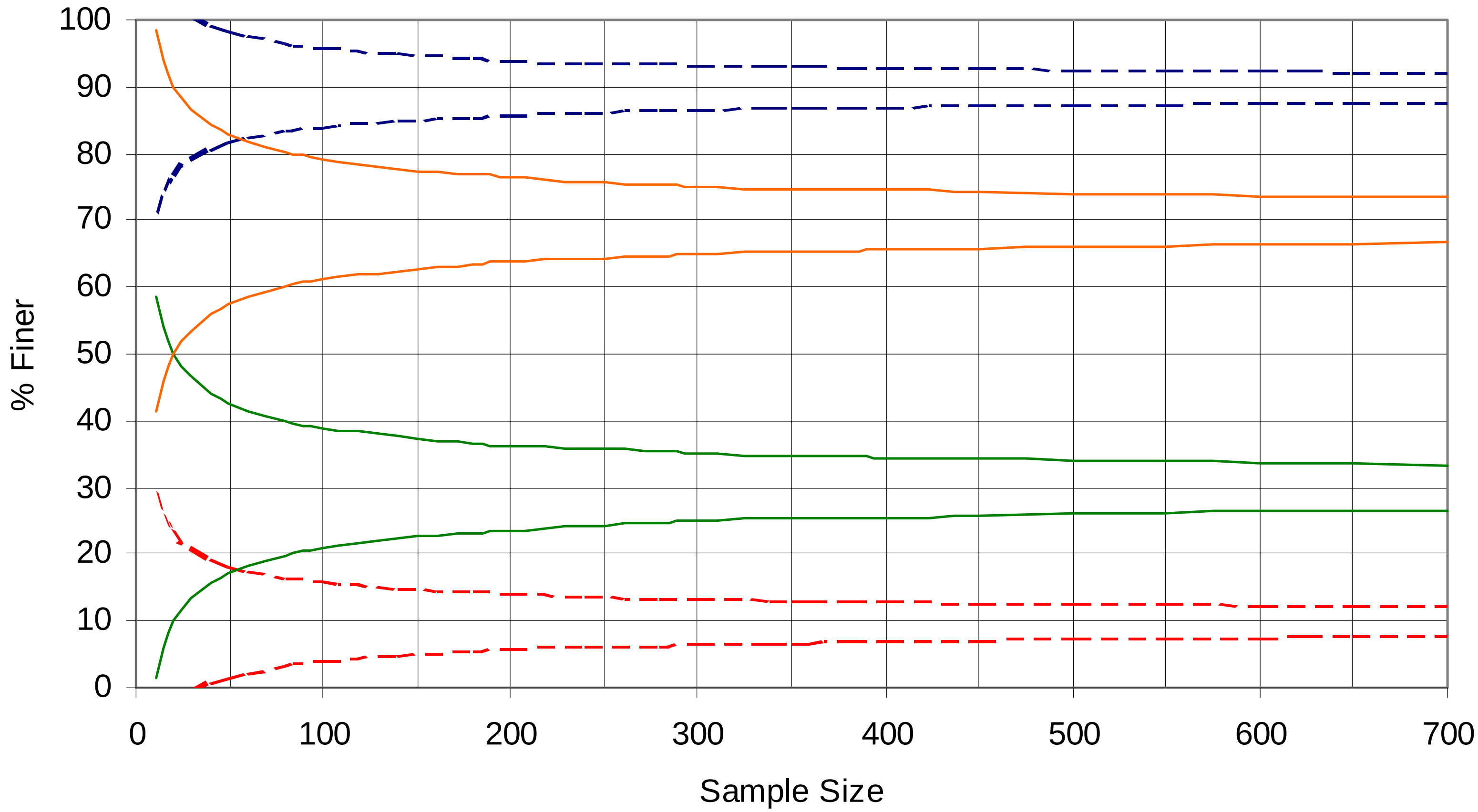


Figure B.2 Normal sample size determination graph for D_{10}, D_{30}, D_{70} and D_{90} for $\alpha = 0.025$

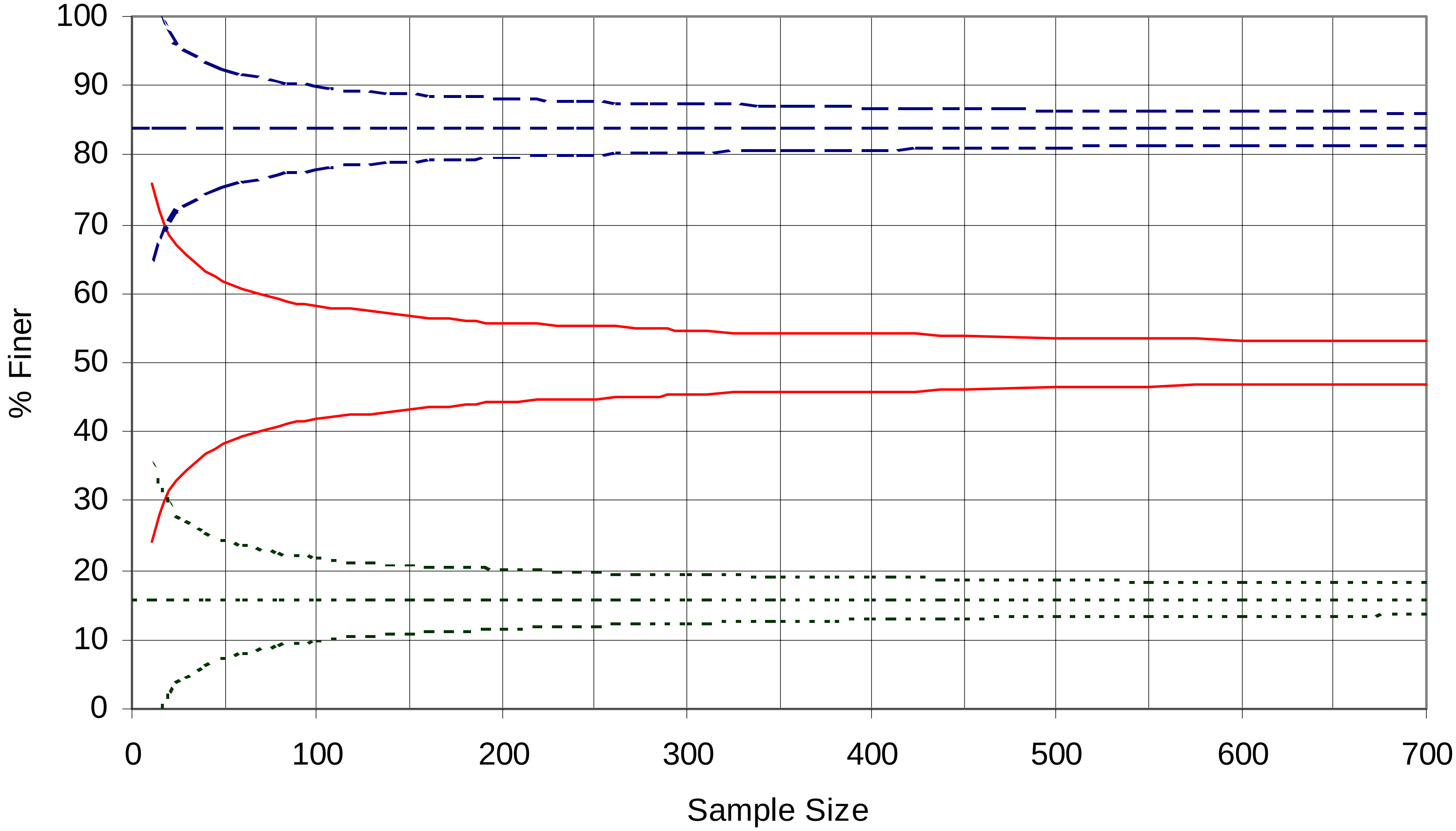


Figure B.3 Normal sample size determination graph for D_{16}, D_{50}, and D_{84} for $\alpha = 0.05$

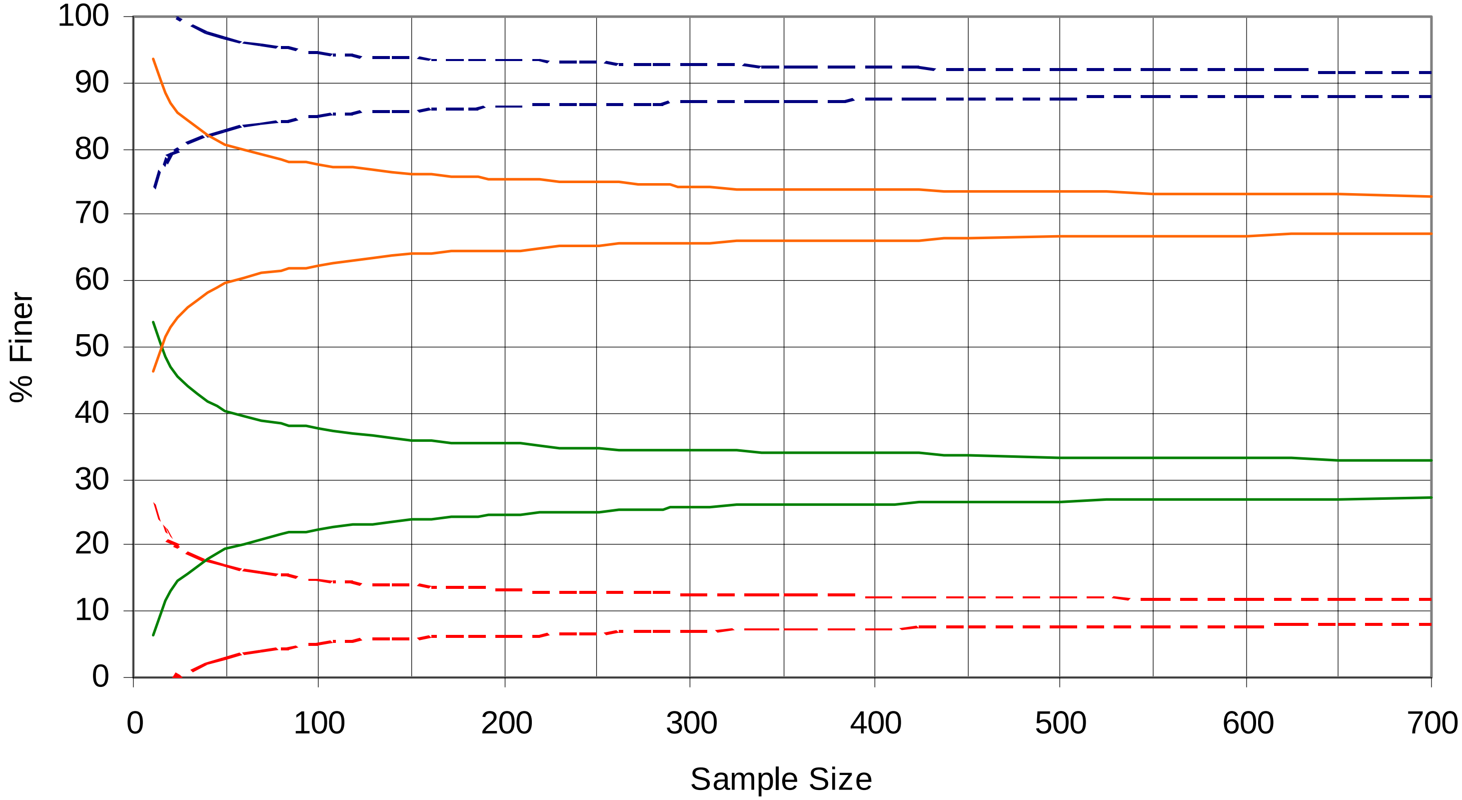


Figure B.4 Normal sample size determination graph for D_{10}, D_{30}, D_{70} and D_{90} for $\alpha = 0.05$

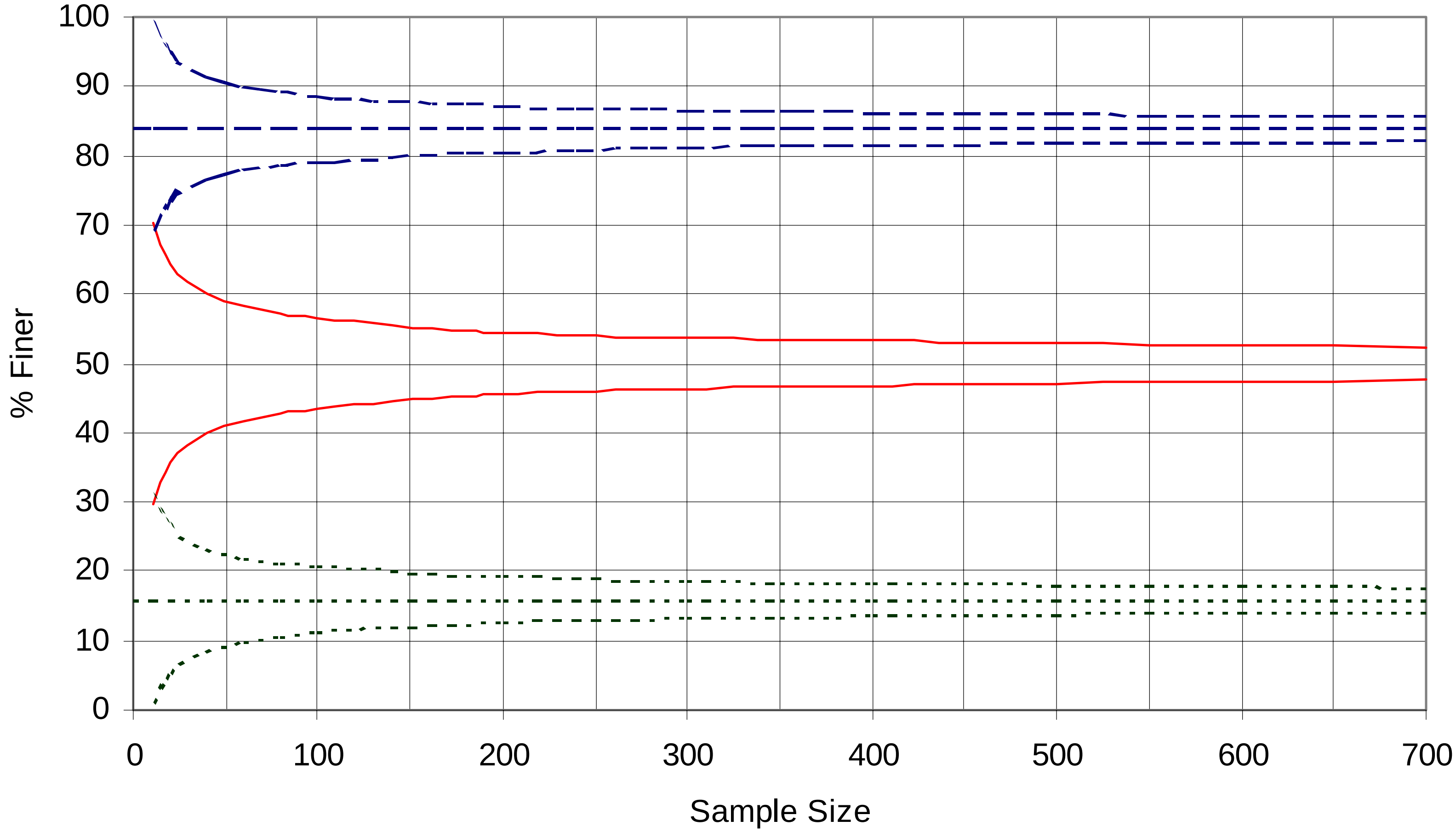


Figure B.5 Normal sample size determination graph for D_{16}, D_{50}, and D_{84} for $\alpha = 0.10$

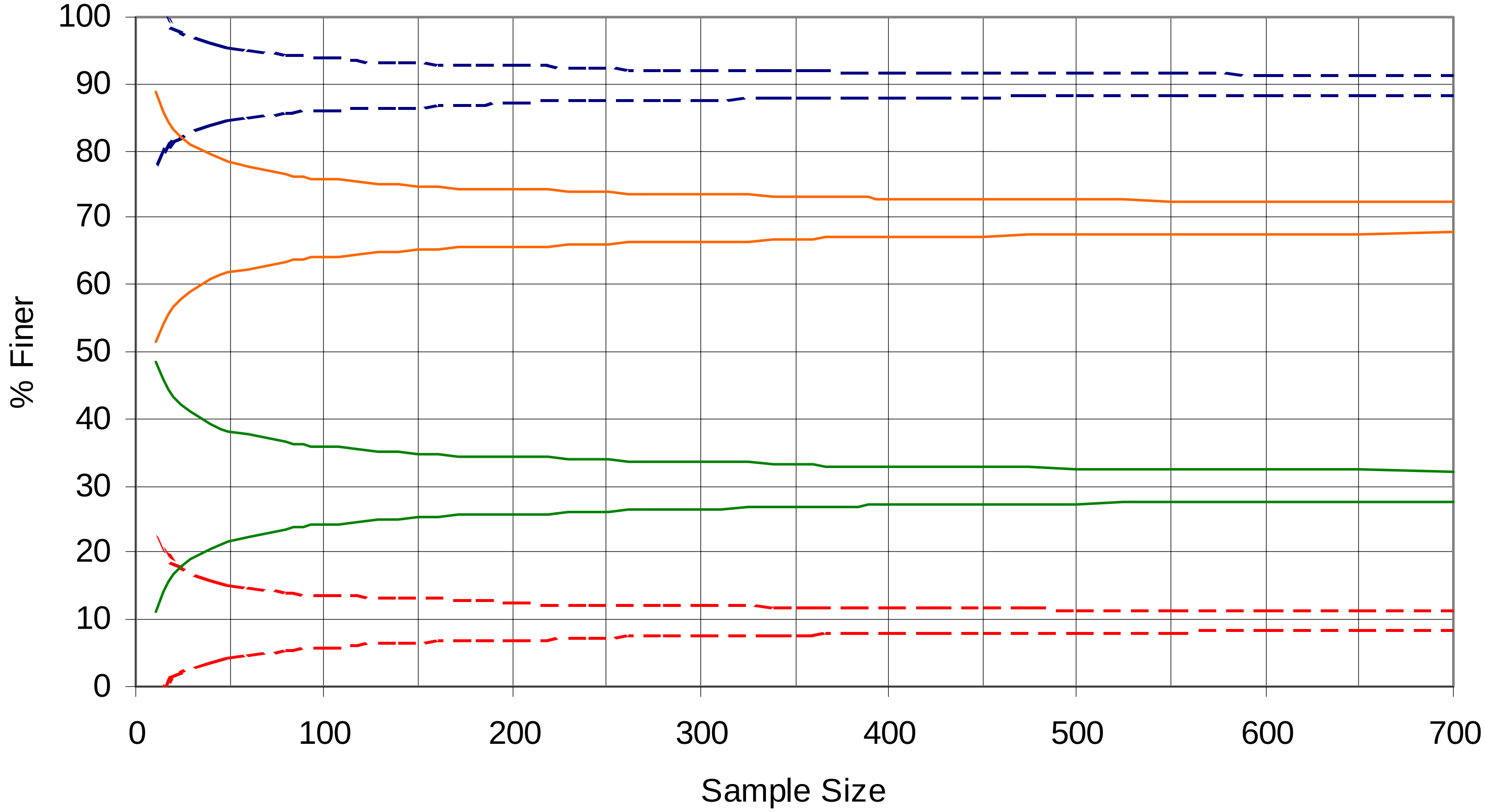


Figure B.6 Normal sample size determination graph for D_{10}, D_{30}, D_{70} and D_{90} for α = 0.10

Appendix C. Sample Size Determination Graphs for the Multinomial Distribution

Graphs using the multinomial distribution to estimate a grid sample size are shown here. The desired accuracy, error band around a percentile of interest, and the number of sieves needed for the analysis are needed to employ these graphs. A quick estimate of the site can provide an estimate of the number of sieves necessary for the analysis. Further discussion and an example using these graphs is provided in section 2.5.3. The grain percentiles shown are D_{10}, D_{16}, D_{30}, D_{50}, D_{70}, D_{84}, and D_{90}. The graphs for grain sizes that are symmetric about D_{50} are equal and are shown on the same graph. Additionally, accuracies of 95%, 90%, and 80% are shown for each grain percentile.

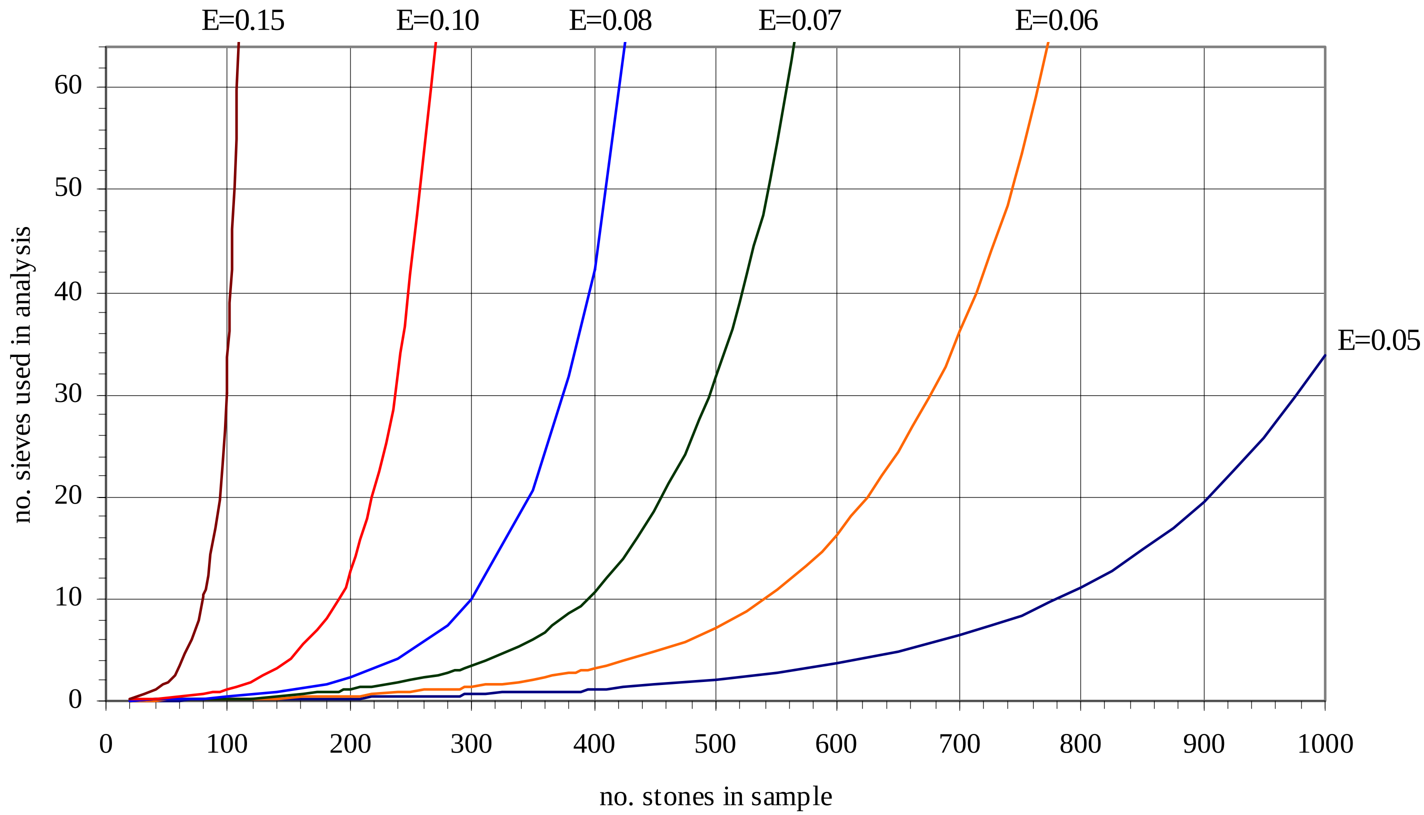


Figure C.1 Multinomial sample size determination graph for D_{50} for $\alpha = 0.05$

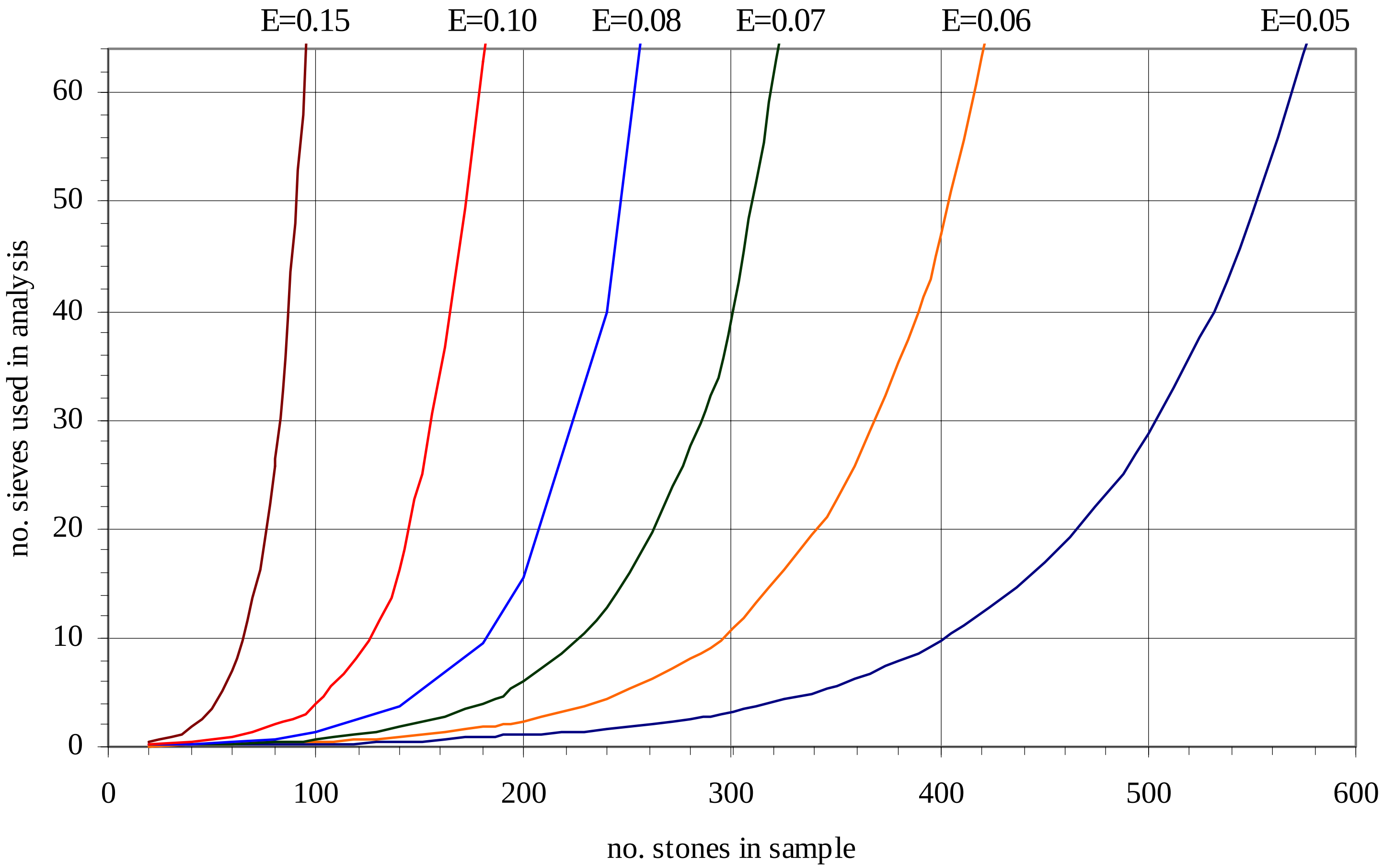


Figure C.2 Multinomial sample size determination graph for D_{10} and D_{90} for $\alpha = 0.05$

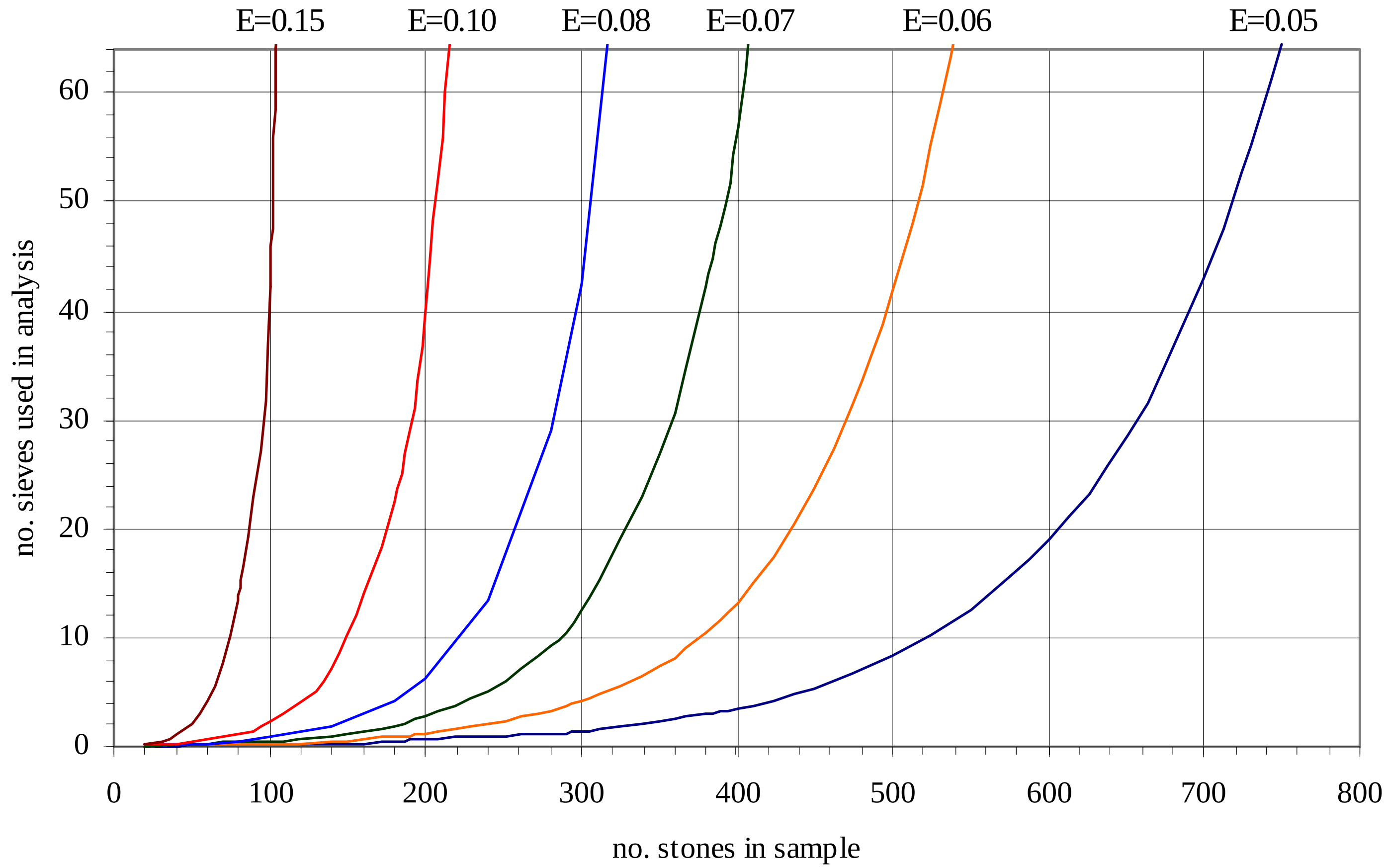


Figure C.3 Multinomial sample size determination graph for D_{16} and D_{84} for $\alpha = 0.05$

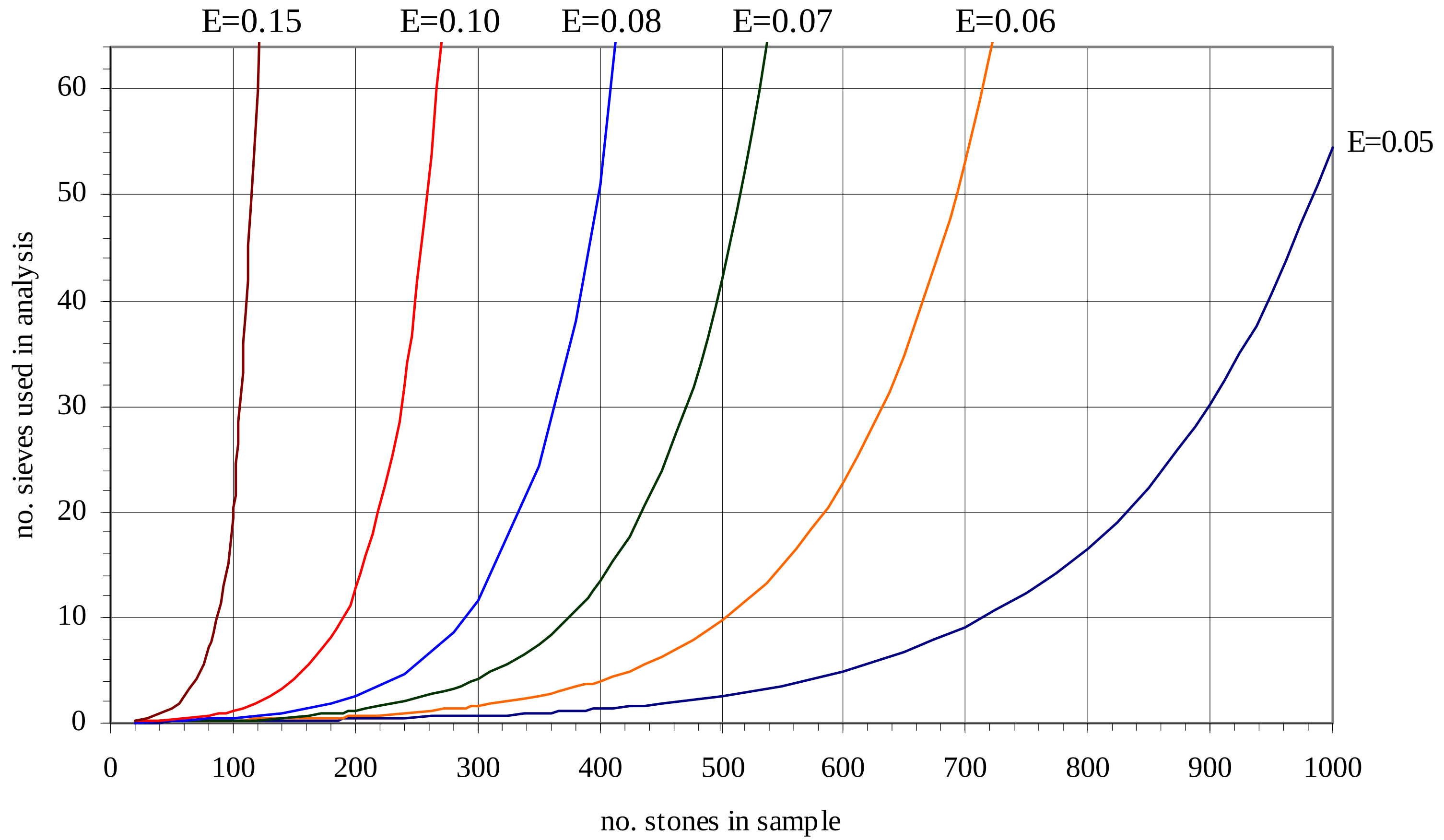


Figure C.4 Multinomial sample size determination graph for D_{30} and D_{70} for $\alpha = 0.05$

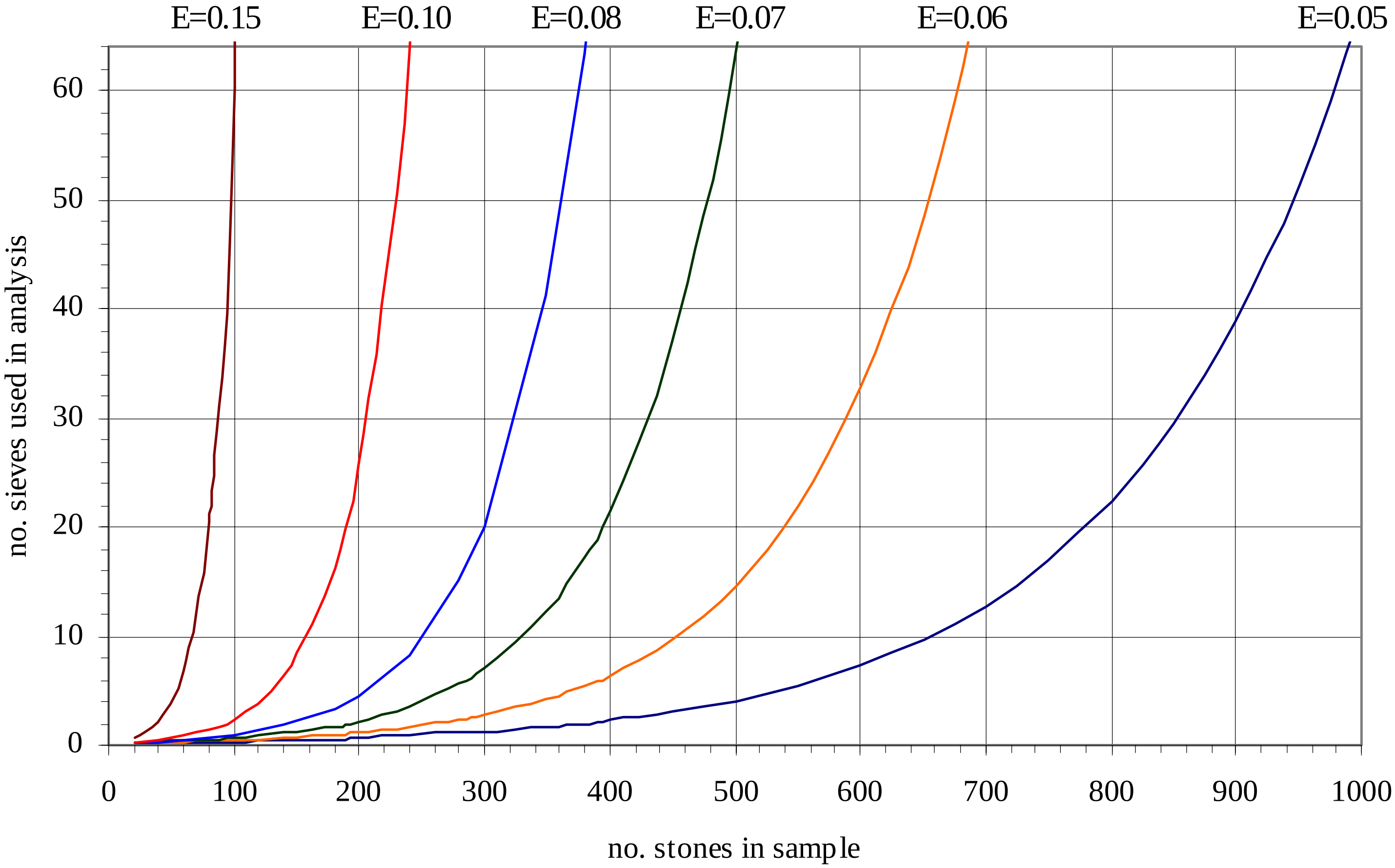


Figure C.5 Multinomial sample size determination graph for D_{50} for $\alpha = 0.10$

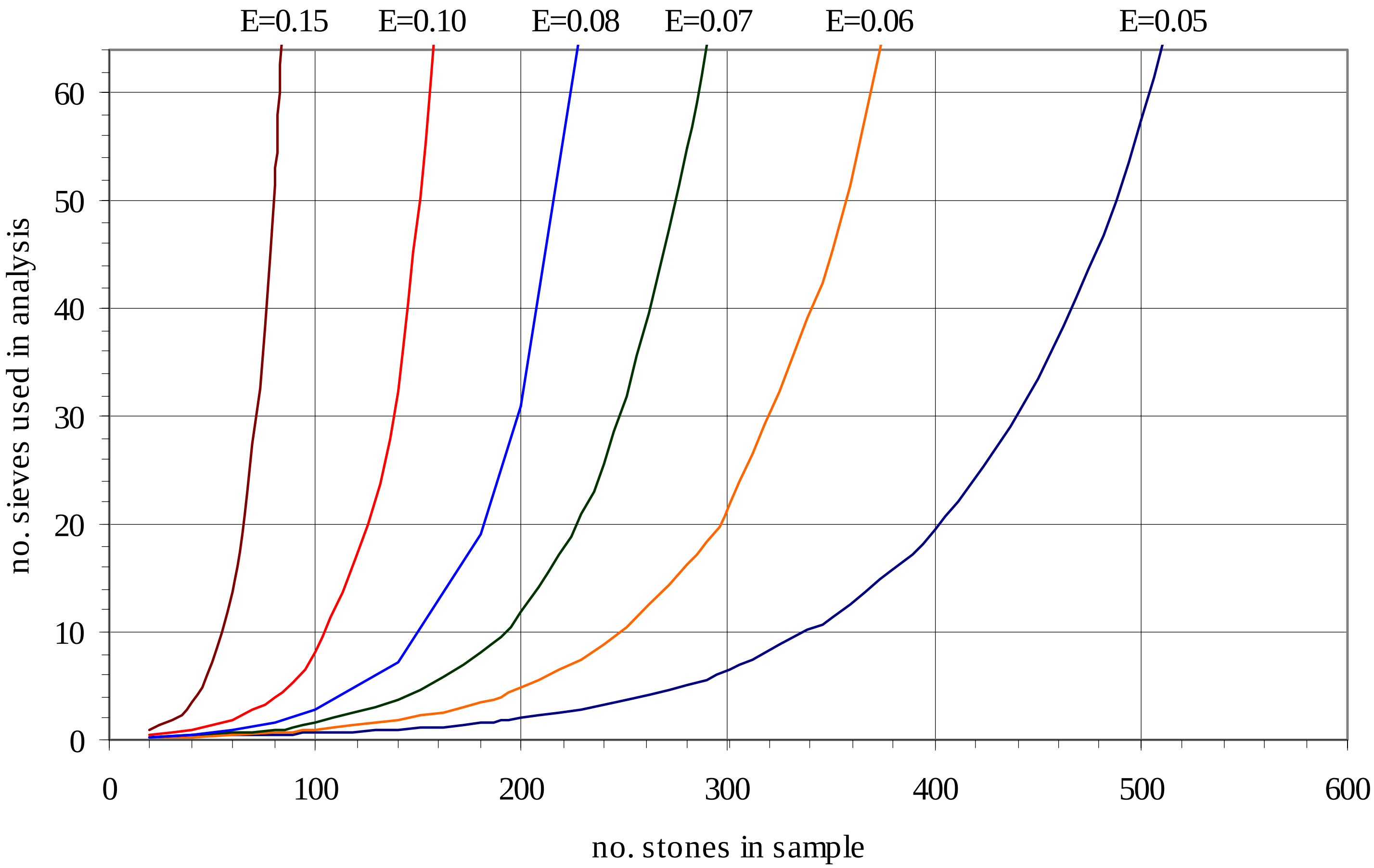


Figure C.6 Multinomial sample size determination graph for D_{10} and D_{90} for $\alpha = 0.10$

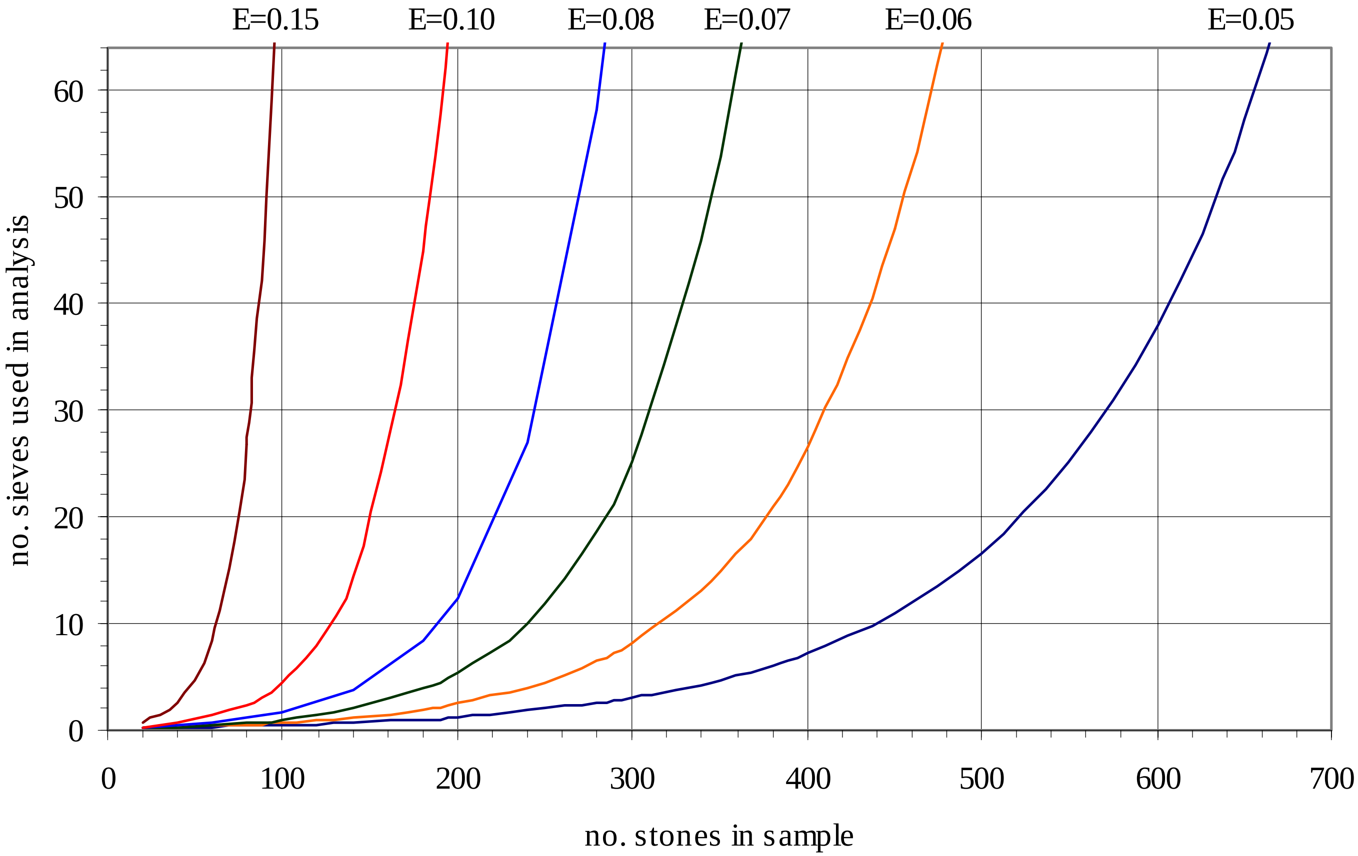


Figure C.7 Multinomial sample size determination graph for D_{16} and D_{84} for $\alpha = 0.10$

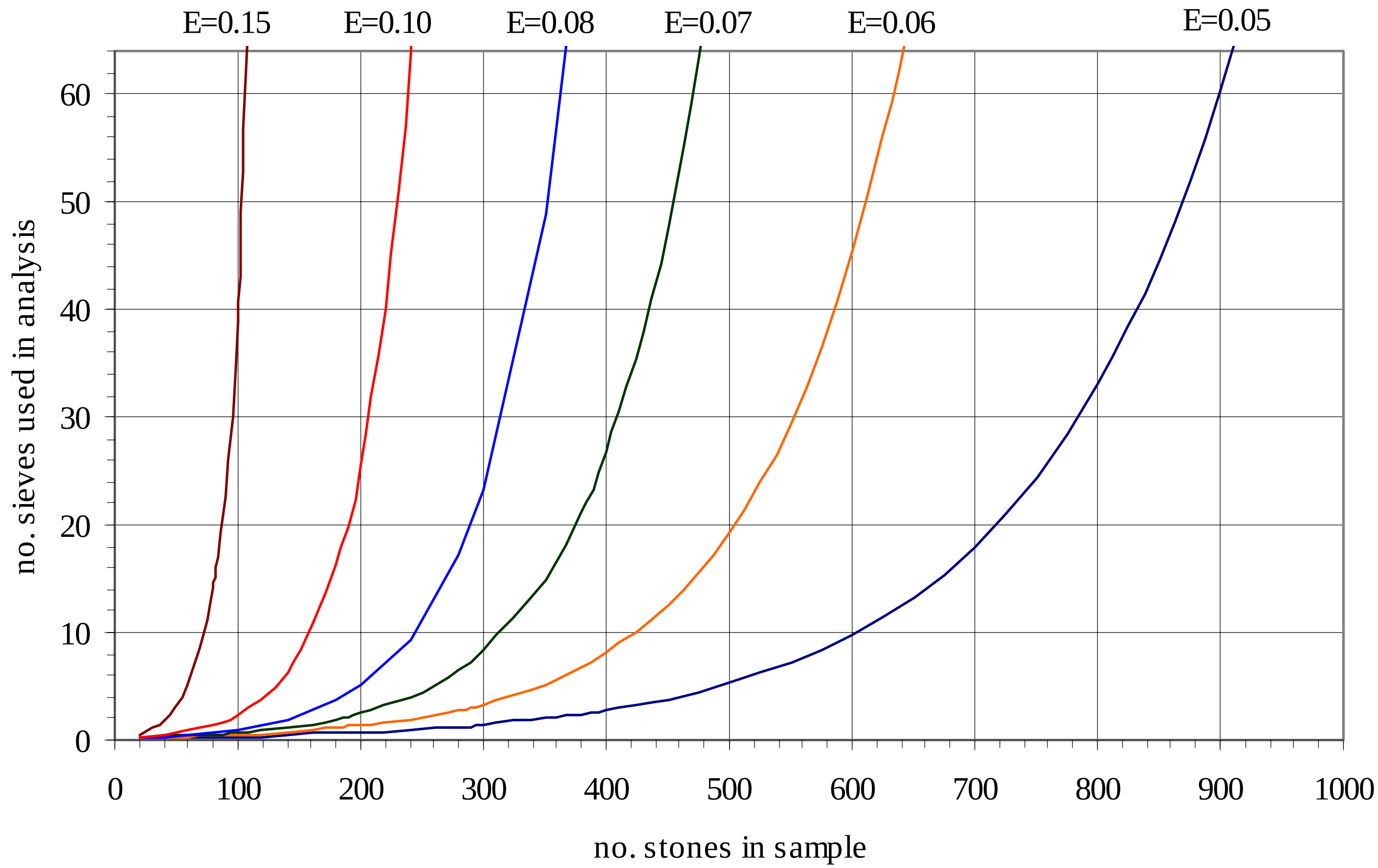


Figure C.8 Multinomial sample size determination graph for D_{30} and D_{70} for $\alpha = 0.10$

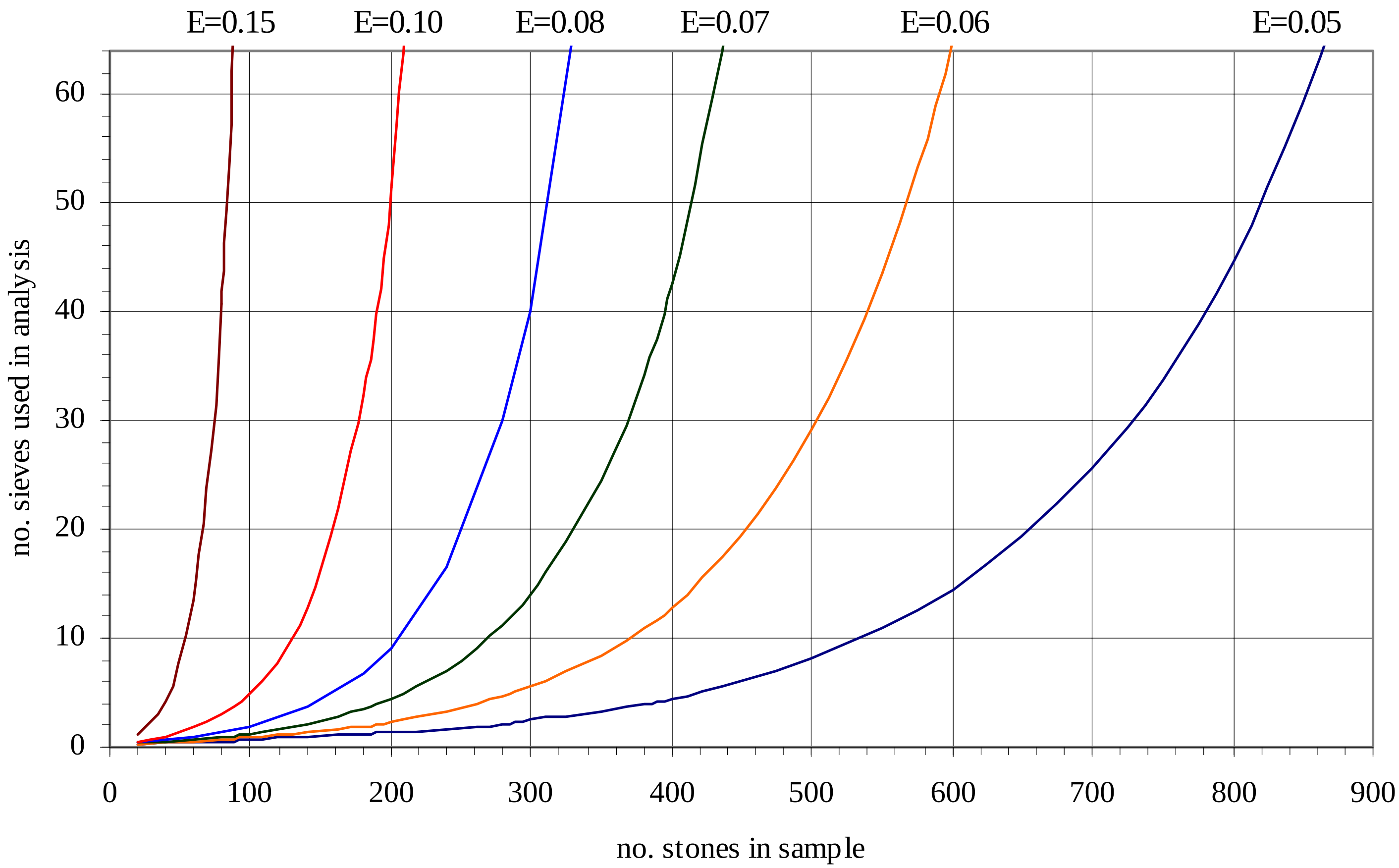


Figure C.9 Multinomial sample size determination graph for D_{50} for α = 0.20

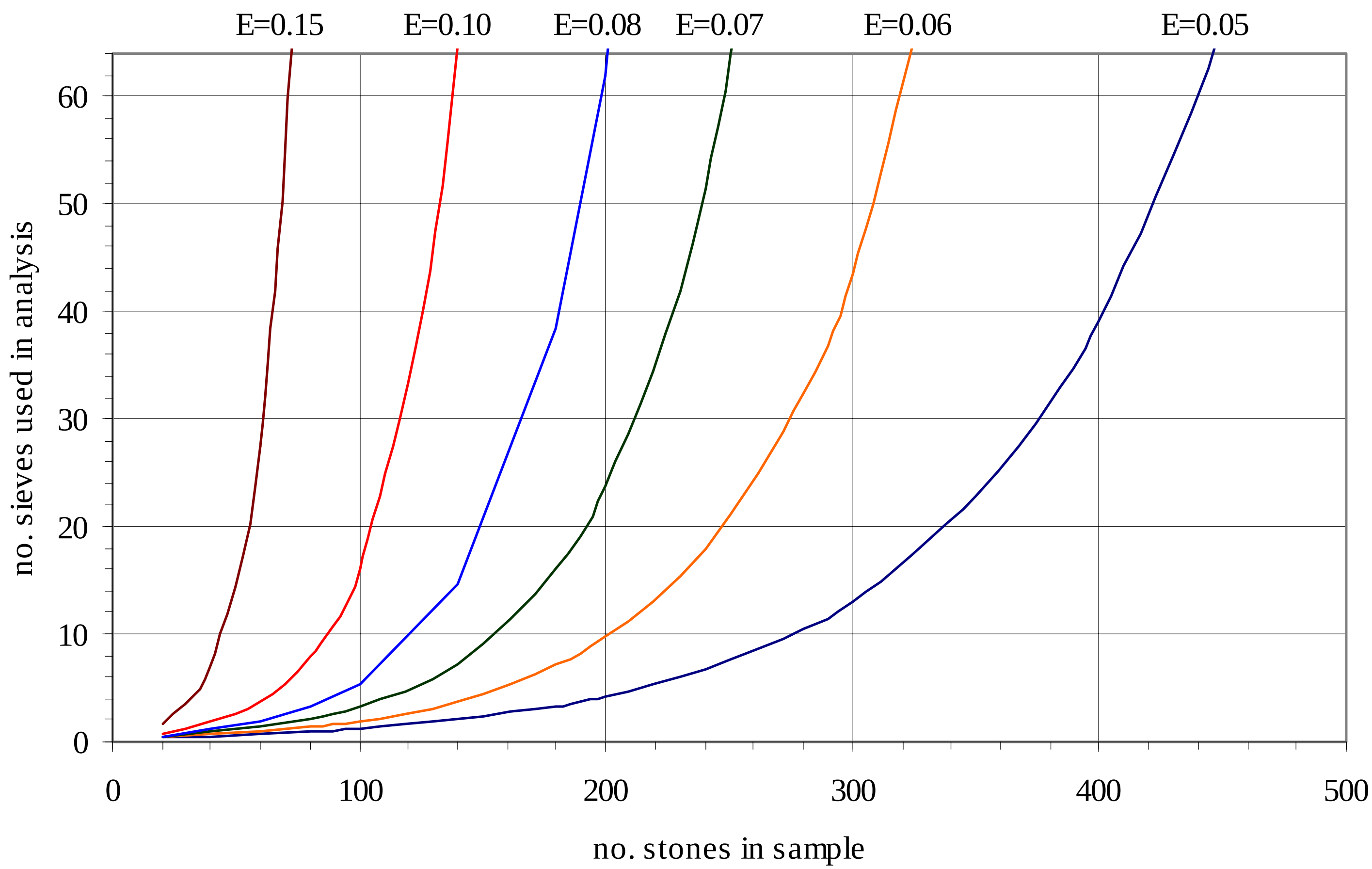


Figure C.10 Multinomial sample size determination graph for D_{10} and D_{90} for $\alpha = 0.20$

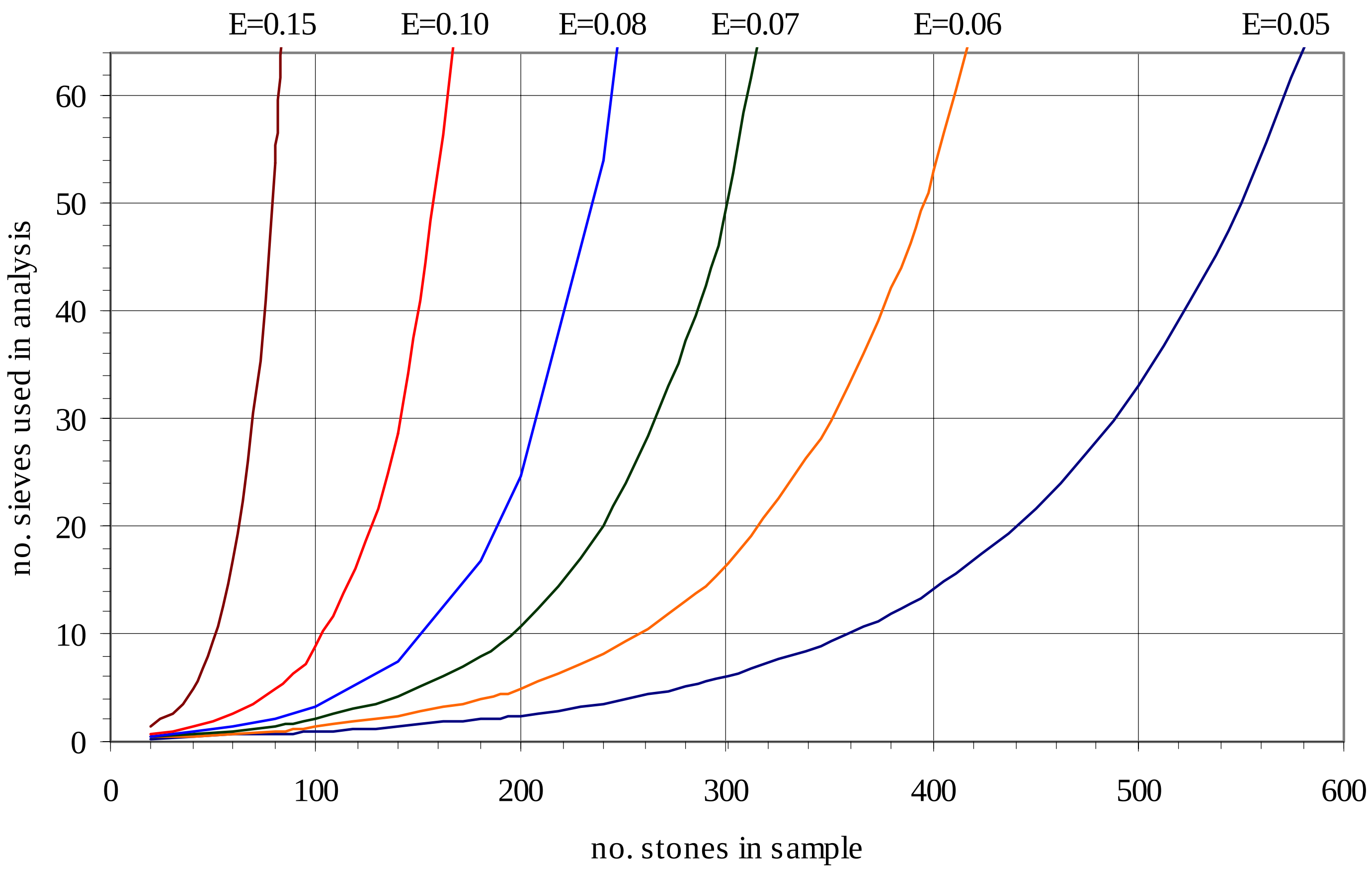


Figure C.11 Multinomial sample size determination graph for D_{16} and D_{84} for $\alpha = 0.20$

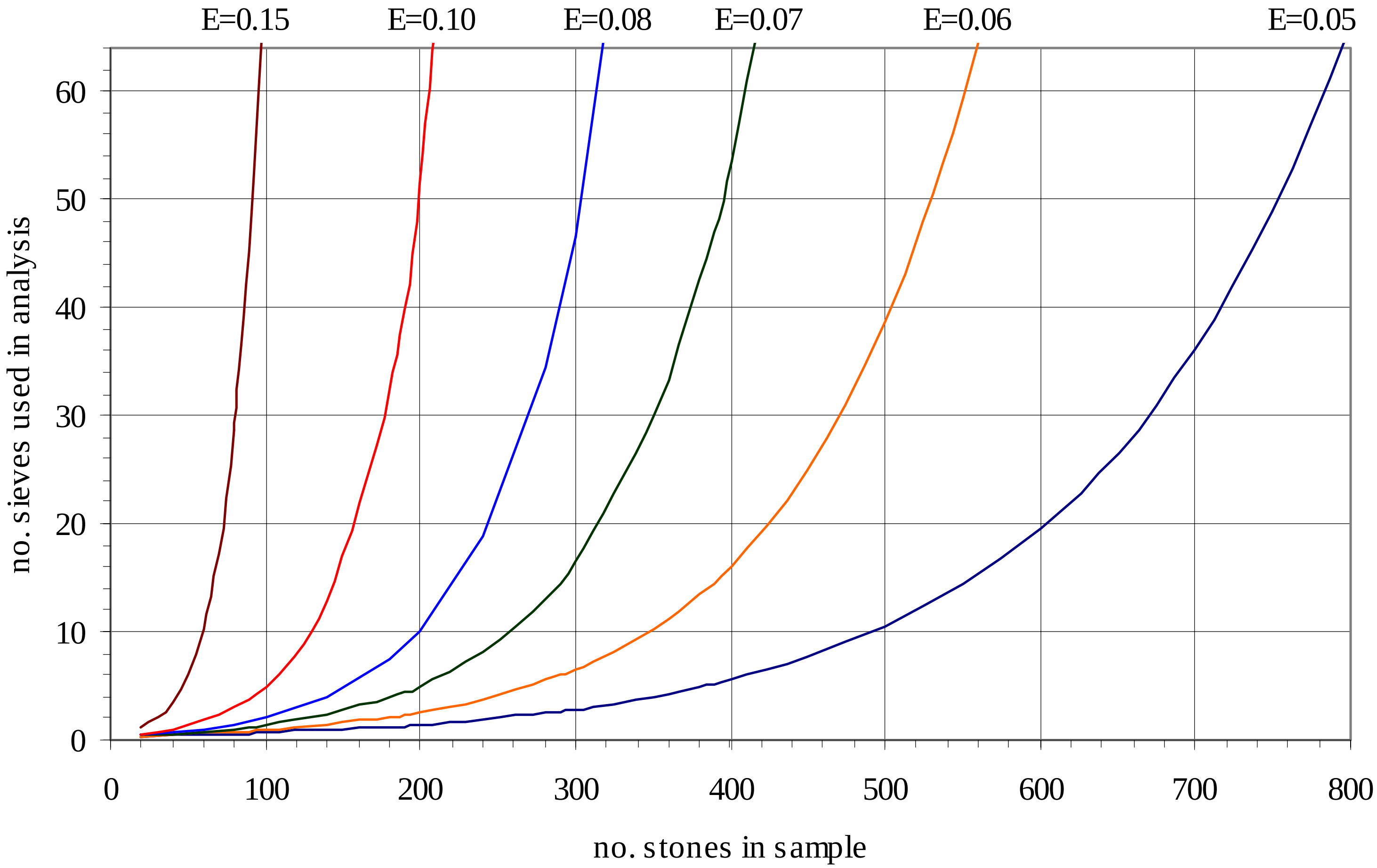


Figure C.12 Multinomial sample size determination graph for D_{30} and D_{70} for $\alpha = 0.20$

Appendix D. Laboratory Grid Data and Results for Glass Bead Experiments

The data obtained by grid sampling on the distribution of glass beads is shown here. Additionally, grid-by-number analyses for a variety of different samples sizes are included. The results of one 400-stone sample, four 100-stone samples, and eight 50-stone samples are shown. For each individual sample within the different sample sizes, no particle belongs to more than one sample. A comparison is also made between the sample and actual grain size distribution curves. A discussion of the grid sampling on the glass bead distribution is provided in section 4.2.1.

Data Set For Laboratory Grid Sample

cm	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45
42	5	6	4	4	3	6	5	5	5	6	5	7	5	5	7
39	6	3	7	7	5	7	6	5	3	6	7	2	5	5	1
36	7	4	7	3	2	1	5	7	5	5	4	4	5	7	5
33	5	6	6	5	7	5	5	6	7	5	1	3	7	7	5
30	5	4	3	4	6	5	7	1	4	4	5	6	3	3	5
27	7	6	6	6	6	4	6	6	1	6	7	3	6	5	7
24	6	1	5	7	5	4	6	6	6	4	6	6	3	6	7
21	6	5	6	5	5	5	5	6	7	2	4	6	5	7	7
18	5	6	6	6	6	6	7	5	6	5	5	4	6	5	4
15	5	4	4	6	5	6	4	3	7	7	5	6	5	5	7
12	7	4	7	7	5	3	4	6	6	6	2	7	4	5	7
9	6	7	5	7	6	5	6	7	6	5	7	4	6	5	4
6	5	5	5	7	6	5	7	6	6	7	1	2	6	3	5
3	6	7	6	7	7	6	5	1	4	6	2	2	4	7	5

cm	48	51	54	57	60	63	66	69	72	75	78	81	84	87	90
42	6	6	5	6	4	4	2	7	3	4	4	1	5	6	5
39	6	6	7	2	7	3	5	5	5	3	1	6	2	7	2
36	6	5	6	7	5	5	6	6	5	5	4	7	6	3	6
33	7	6	7	7	2	5	5	4	7	6	6	7	2	5	1
30	4	5	6	3	2	6	5	7	3	4	7	7	5	6	6
27	1	5	5	7	3	6	4	2	6	6	4	2	4	3	7
24	6	2	6	4	7	6	4	6	4	5	4	7	6	5	6
21	7	6	7	5	2	5	1	2	4	2	5	6	6	4	5
18	6	7	4	7	6	7	6	6	7	5	6	5	4	6	1
15	4	2	6	6	5	4	4	4	2	4	5	6	7	3	6
12	5	5	5	6	4	6	5	4	5	5	4	5	6	5	3
9	5	7	5	4	4	6	3	6	7	6	5	1	5	5	7
6	6	5	7	6	4	7	4	6	4	5	2	6	6	1	7
3	3	3	7	6	4	5	4	4	3	6	6	6	3	7	5

400 Stone Grid Sample

(1) Grain Size Analysis

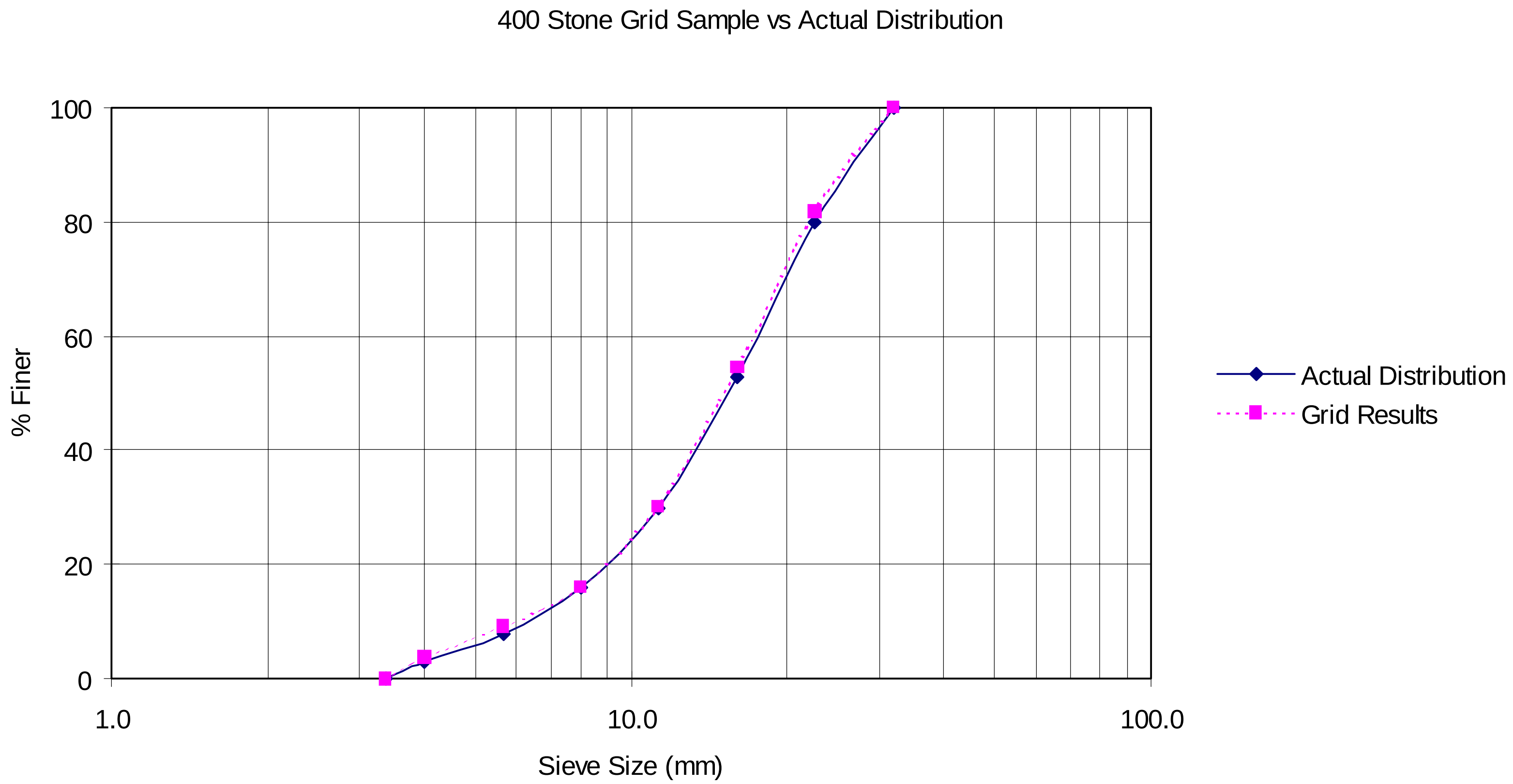
Sieve Size (mm)	Particle No.	No. of Particles	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
25.4	7	73	18.25	100	1.00
19.05	6	109	27.25	81.75	0.8175
14.28	5	98	24.5	54.5	0.5450
10	4	56	14	30	0.3000
7	3	27	6.75	16	0.1600
5	2	22	5.5	9.25	0.0925
4	1	15	3.75	3.75	0.0375
	total =	400			

Sieve Size	Actual Distribution	Grid Results
32.0	100	100
22.6	80.039	81.75
16.0	52.833	54.5
11.3	29.913	30
8.00	15.952	16
5.66	7.982	9.25
4.00	2.992	3.75
3.36	0	0

(2) Sample Statistics

Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*(D-M)^2
25.4	0.1825	4.6355	17.4461
19.05	0.2725	5.1911	3.2008
14.28	0.245	3.4986	0.4417
10	0.14	1.4000	4.4261
7	0.0675	0.4725	5.0187
5	0.055	0.2750	6.2063
4	0.0375	0.1500	5.0658

Sample Mean = 15.6227 mm
Sample Variance = 41.8056 mm^2
Sample Standard Deviation = 6.4657 mm


400 Stone Grid Sample vs Actual Distribution
100
80
60
40
20
0
% Finer
1.0
10.0
100.0
Sieve Size (mm)
Actual Distribution
Grid Results

First 100 Stone Grid Sample

(1) Grain Size Analysis

Sieve Size (mm)	Particle No.	No. of Particles	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
25.4	7	21	21	100	1.00
19.05	6	29	29	79	0.7900
14.28	5	21	21	50	0.5000
10	4	14	14	29	0.2900
7	3	7	7	15	0.1500
5	2	4	4	8	0.0800
4	1	4	4	4	0.0400
	total =	100			

Sieve Size	Actual Distribution	Grid Results
32.0	100	100
22.6	80.039	79
16.0	52.833	50
11.3	29.913	29
8.00	15.952	15
5.66	7.982	8
4.00	2.992	4
3.36	0	0

(2) Sample Statistics

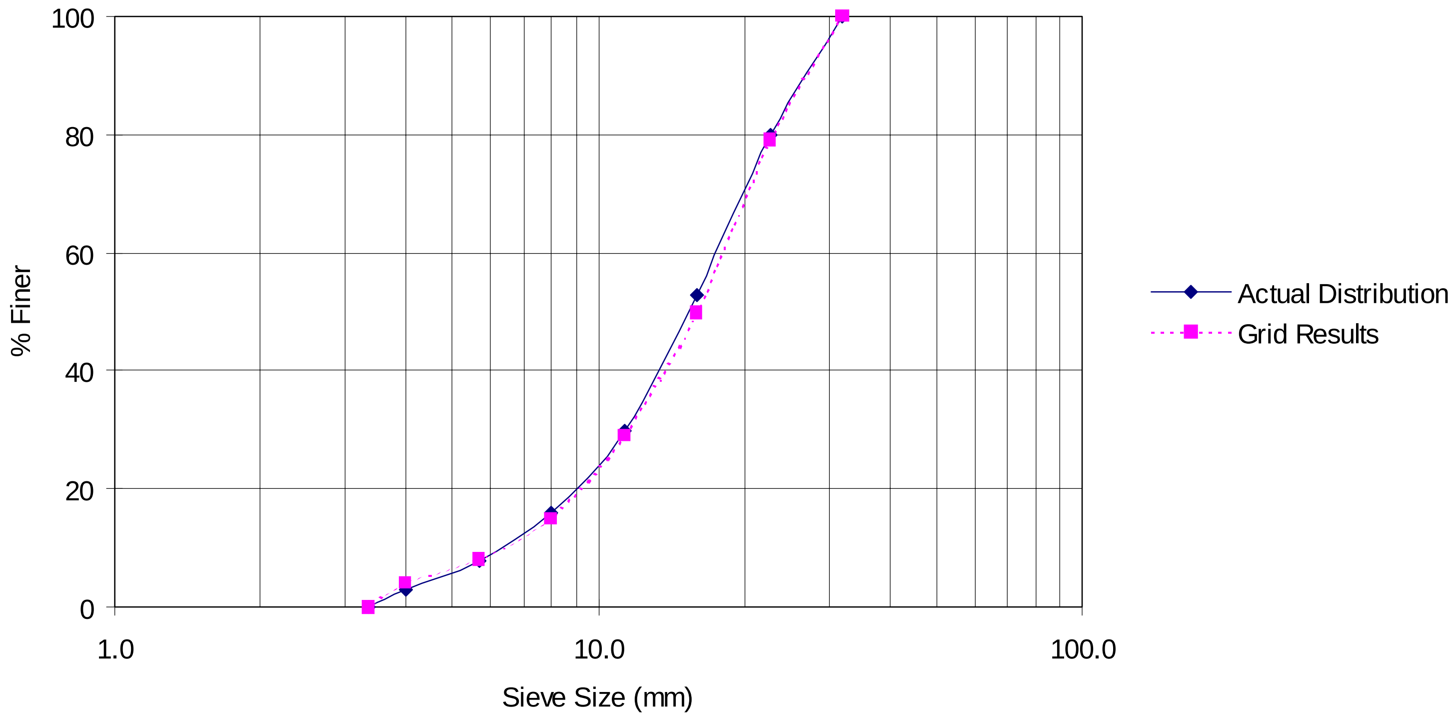
Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*(D-M)^2
25.4	0.21	5.3340	18.1344
19.05	0.29	5.5245	2.5113
14.28	0.21	2.9988	0.7012
10	0.14	1.4000	5.2219
7	0.07	0.4900	5.8060
5	0.04	0.2000	4.9349
4	0.04	0.1600	5.8635

Sample Mean = 16.1073 mm

Sample Variance = 43.1731 mm^2

Sample Standard Deviation = 6.5706 mm

First 100 Stone Grid Sample vs Actual Distribution



Second 100 Stone Grid Sample

(1) Grain Size Analysis

Sieve Size (mm)	Particle No.	No. of Particles	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
25.4	7	16	16	100	1.00
19.05	6	29	29	84	0.8400
14.28	5	29	29	55	0.5500
10	4	17	17	26	0.2600
7	3	3	3	9	0.0900
5	2	5	5	6	0.0600
4	1	1	1	1	0.0100
	total =	100			

Sieve Size	Actual Distribution	Grid Results
32.0	100	100
22.6	80.039	84
16.0	52.833	55
11.3	29.913	26
8.00	15.952	9
5.66	7.982	6
4.00	2.992	1
3.36	0	0

(2) Sample Statistics

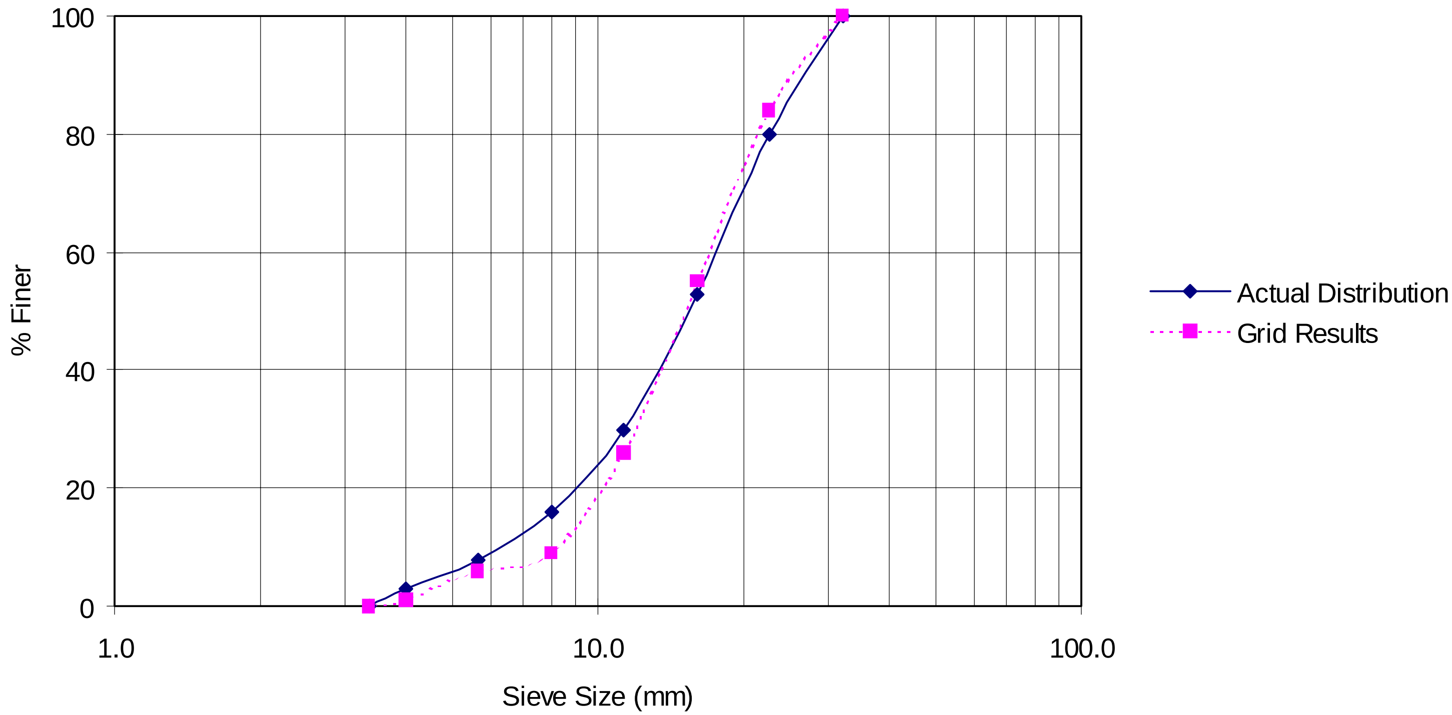
Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*(D-M)^2
25.4	0.16	4.0640	14.3499
19.05	0.29	5.5245	2.8235
14.28	0.29	4.1412	0.7892
10	0.17	1.7000	5.9774
7	0.03	0.2100	2.3922
5	0.05	0.2500	5.9729
4	0.01	0.0400	1.4232

Sample Mean = 15.9297 mm

Sample Variance = 33.7283 mm^2

Sample Standard Deviation = 5.8076 mm

Second 100 Stone Grid Sample vs Actual Distribution



Third 100 Stone Grid Sample

(1) Grain Size Analysis

Sieve Size (mm)	Particle No.	No. of Particles	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
25.4	7	13	13	100	1.00
19.05	6	32	32	87	0.8700
14.28	5	17	17	55	0.5500
10	4	18	18	38	0.3800
7	3	9	9	20	0.2000
5	2	6	6	11	0.1100
4	1	5	5	5	0.0500
	total =	100			

Sieve Size	Actual Distribution	Grid Results
32.0	100	100
22.6	80.039	87
16.0	52.833	55
11.3	29.913	38
8.00	15.952	20
5.66	7.982	11
4.00	2.992	5
3.36	0	0

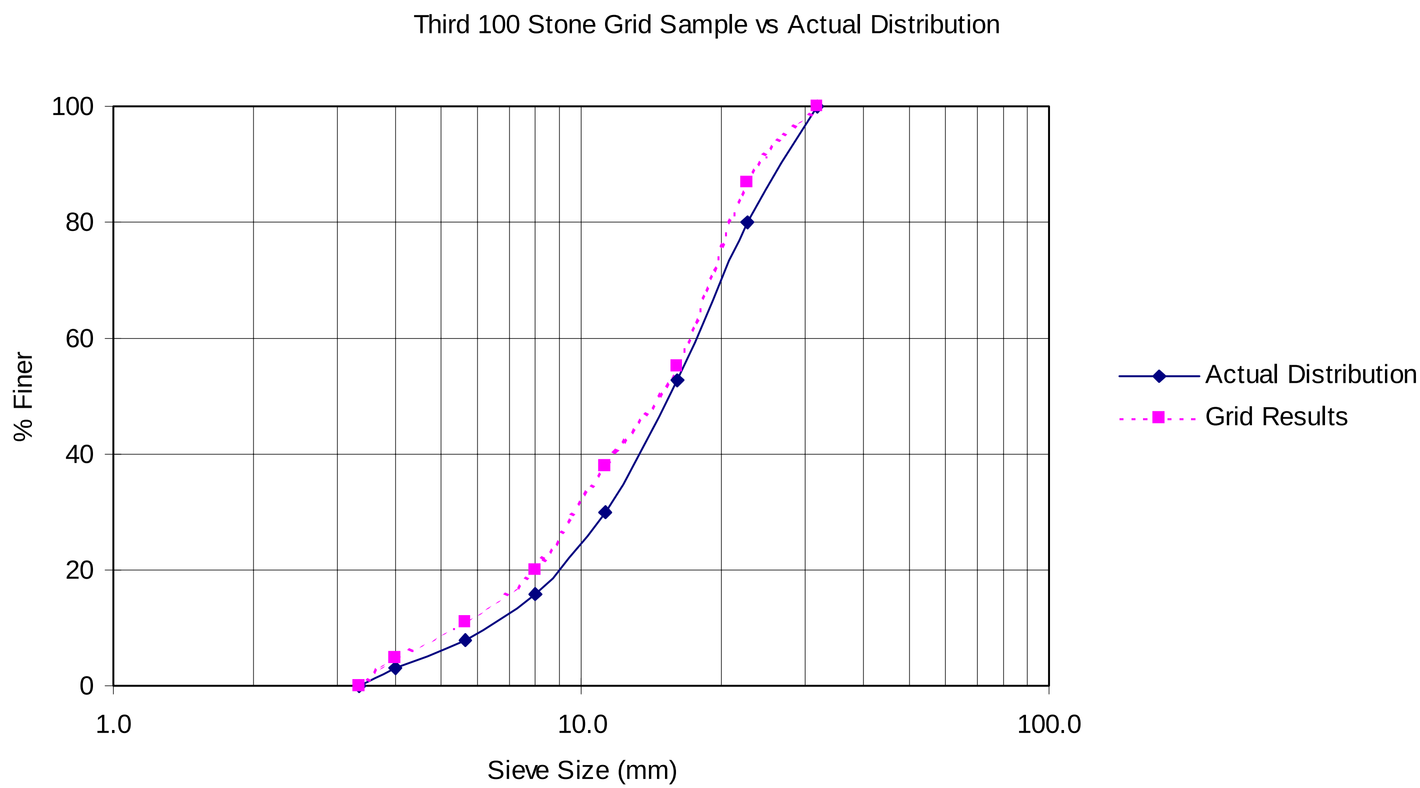
(2) Sample Statistics

Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*(D-M)^2
25.4	0.13	3.3020	14.7294
19.05	0.32	6.0960	5.9014
14.28	0.17	2.4276	0.0385
10	0.18	1.8000	4.0708
7	0.09	0.6300	5.4134
5	0.06	0.3000	5.7103
4	0.05	0.2000	5.7841

Sample Mean = 14.7556 mm

Sample Variance = 41.6480 mm^2

Sample Standard Deviation = 6.4535 mm



Third 100 Stone Grid Sample vs Actual Distribution
100
80
60
40
20
0
% Finer
1.0
10.0
100.0
Sieve Size (mm)
Actual Distribution
Grid Results

Fourth 100 Stone Grid Sample

(1) Grain Size Analysis

Sieve Size (mm)	Particle No.	No. of Particles	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
25.4	7	22	22	100	1.00
19.05	6	21	21	78	0.7800
14.28	5	30	30	57	0.5700
10	4	7	7	27	0.2700
7	3	8	8	20	0.2000
5	2	7	7	12	0.1200
4	1	5	5	5	0.0500
	total =	100			

Sieve Size	Actual Distribution	Grid Results
32.0	100	100
22.6	80.039	78
16.0	52.833	57
11.3	29.913	27
8.00	15.952	20
5.66	7.982	12
4.00	2.992	5
3.36	0	0

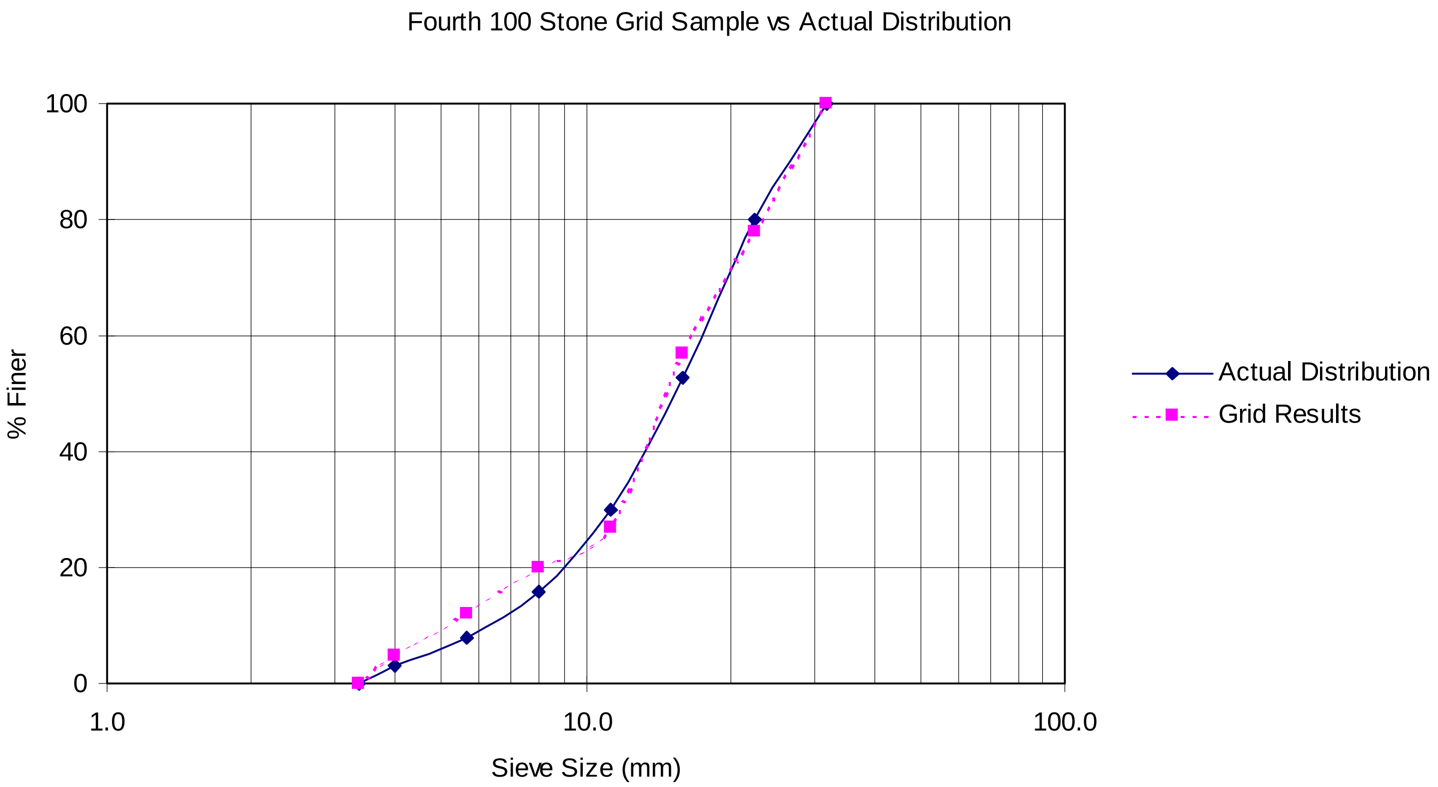
(2) Sample Statistics

Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*$(D-M)^2$
25.4	0.22	5.5880	20.7746
19.05	0.21	4.0005	2.3814
14.28	0.3	4.2840	0.5901
10	0.07	0.7000	2.2604
7	0.08	0.5600	6.0309
5	0.07	0.3500	7.9881
4	0.05	0.2000	6.8240

Sample Mean = 15.6825 mm

Sample Variance = 46.8494 mm^2

Sample Standard Deviation = 6.8447 mm



Fourth 100 Stone Grid Sample vs Actual Distribution
100
80
60
40
20
0
% Finer
1.0
10.0
100.0
Sieve Size (mm)
Actual Distribution
Grid Results

First 50 Stone Grid Sample

(1) Grain Size Analysis

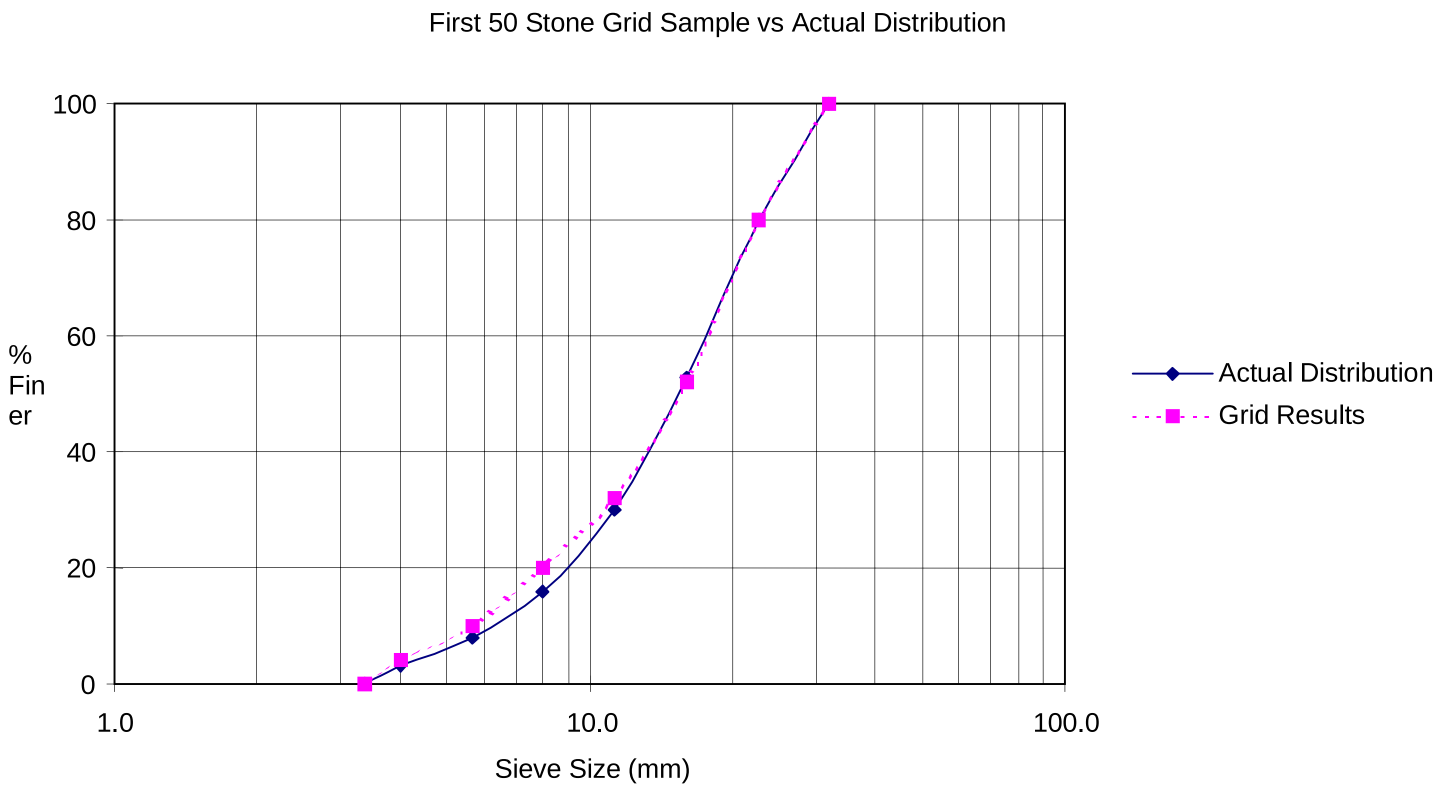
Sieve Size (mm)	Particle No.	No. of Particles	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
25.4	7	10	20	100	1.00
19.05	6	14	28	80	0.8000
14.28	5	10	20	52	0.5200
10	4	6	12	32	0.3200
7	3	5	10	20	0.2000
5	2	3	6	10	0.1000
4	1	2	4	4	0.0400
	total =	50			

Sieve Size	Actual Distribution	Grid Results
32.0	100	100
22.6	80.039	80
16.0	52.833	52
11.3	29.913	32
8.00	15.952	20
5.66	7.982	10
4.00	2.992	4
3.36	0	0

(2) Sample Statistics

Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*(D-M)^2
25.4	0.1	2.5400	30.9232
19.05	0.14	2.6670	17.6715
14.28	0.1	1.4280	4.1796
10	0.06	0.6000	0.2865
7	0.05	0.3500	0.0332
5	0.03	0.1500	0.2377
4	0.02	0.0800	0.2911

Sample Mean =	7.8150	mm
Sample Variance =	53.6229	mm^2
Sample Standard Deviation =	7.3228	mm


First 50 Stone Grid Sample vs Actual Distribution
100
80
60
40
20
0
% Fin er
1.0
10.0
100.0
Sieve Size (mm)
Actual Distribution
Grid Results

Second 50 Stone Grid Sample

(1) Grain Size Analysis

Sieve Size (mm)	Particle No.	No. of Particles	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
25.4	7	12	24	100	1.00
19.05	6	14	28	76	0.7600
14.28	5	11	22	48	0.4800
10	4	8	16	26	0.2600
7	3	2	4	10	0.1000
5	2	1	2	6	0.0600
4	1	2	4	4	0.0400
	total =	50			

Sieve Size	Actual Distribution	Grid Results
32.0	100	100
22.6	80.039	76
16.0	52.833	48
11.3	29.913	26
8.00	15.952	10
5.66	7.982	6
4.00	2.992	4
3.36	0	0

(2) Sample Statistics

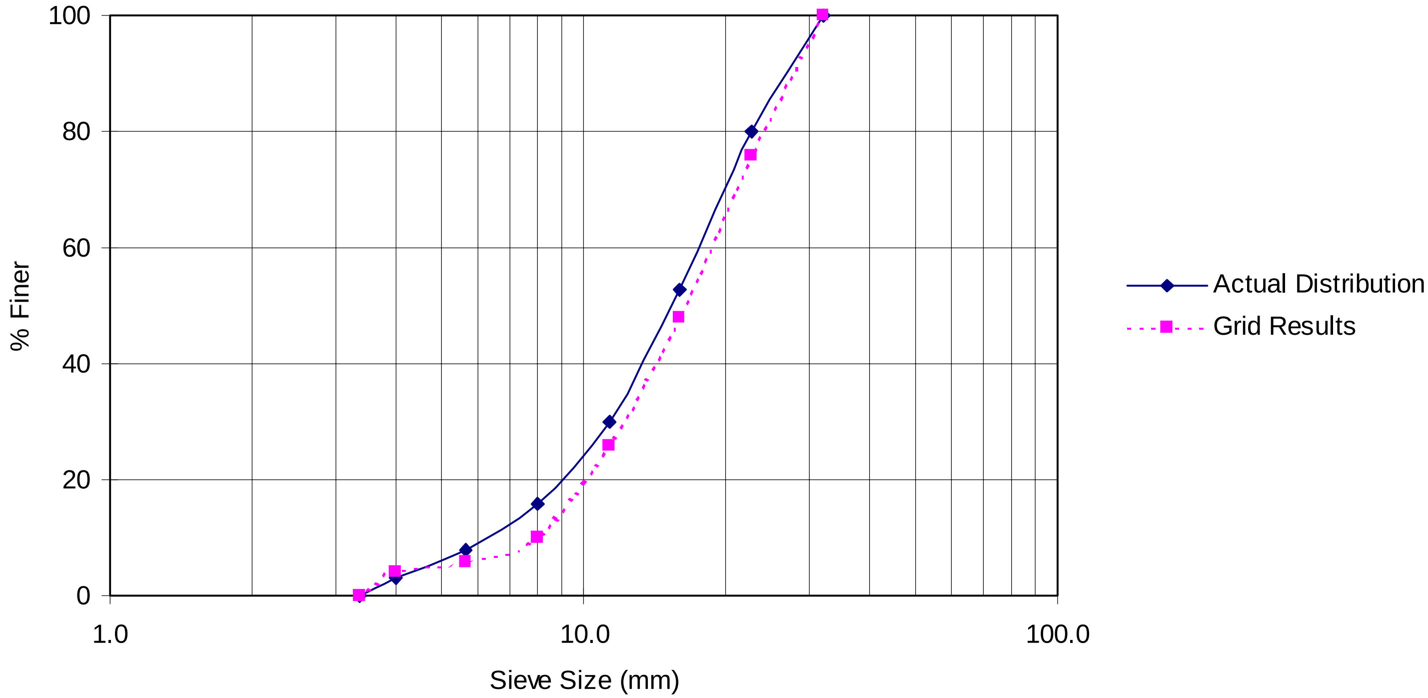
Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*(D-M)^2
25.4	0.12	3.0480	34.8606
19.05	0.14	2.6670	16.0112
14.28	0.11	1.5708	3.8606
10	0.08	0.8000	0.2163
7	0.02	0.1400	0.0368
5	0.01	0.0500	0.1126
4	0.02	0.0800	0.3795

Sample Mean = 8.3558 mm

Sample Variance = 55.4775 mm^2

Sample Standard Deviation = 7.4483 mm

Second 50 Stone Grid Sample vs Actual Distribution



Thrid 50 Stone Grid Sample

(1) Grain Size Analysis

Sieve Size (mm)	Particle No.	No. of Particles	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
25.4	7	6	12	100	1.00
19.05	6	10	20	88	0.8800
14.28	5	16	32	68	0.6800
10	4	12	24	36	0.3600
7	3	3	6	12	0.1200
5	2	3	6	6	0.0600
4	1	0	0	0	0.0000
	total =	50			

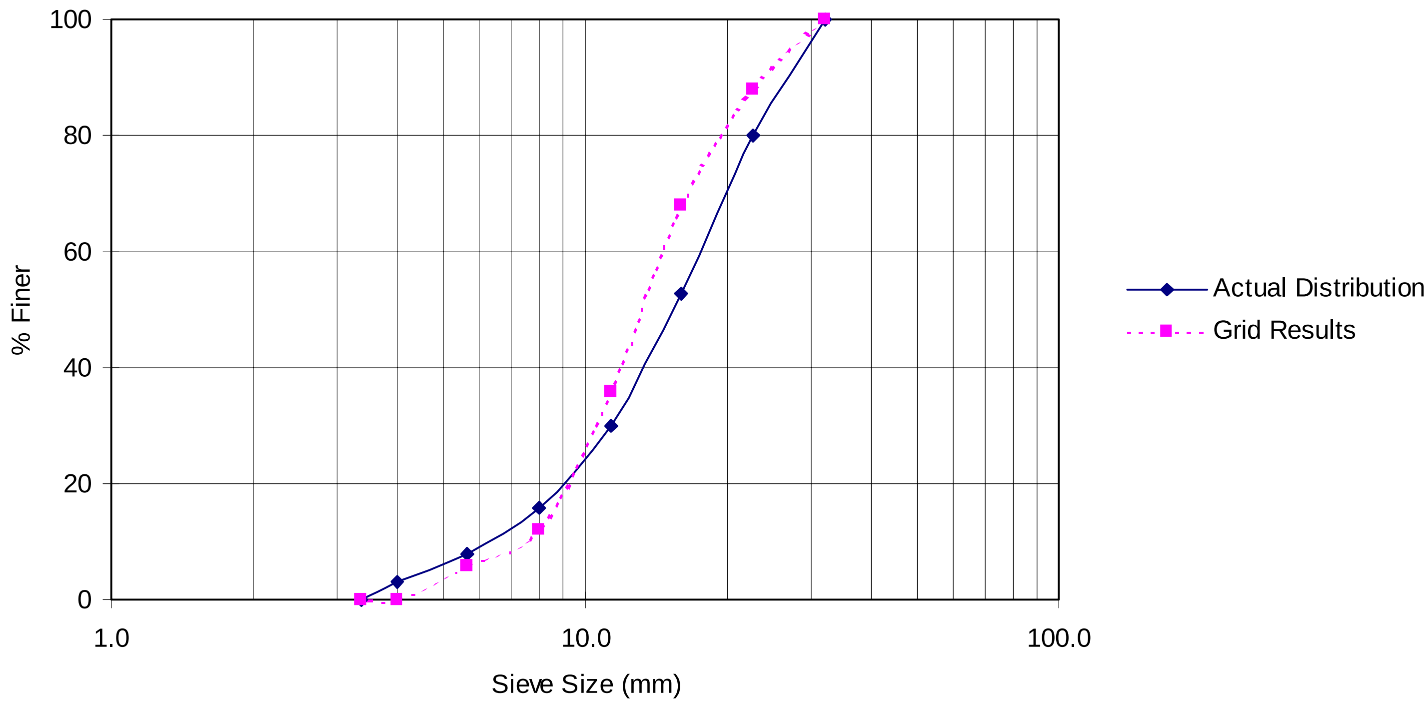
Sieve Size	Actual Distribution	Grid Results
32.0	100	100
22.6	80.039	88
16.0	52.833	68
11.3	29.913	36
8.00	15.952	12
5.66	7.982	6
4.00	2.992	0
3.36	0	0

(2) Sample Statistics

Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*(D-M)^2
25.4	0.06	1.5240	19.7135
19.05	0.1	1.9050	13.8679
14.28	0.16	2.2848	7.8539
10	0.12	1.2000	0.8919
7	0.03	0.2100	0.0022
5	0.03	0.1500	0.1551
4	0	0.0000	0.0000

Sample Mean =	7.2738	mm
Sample Variance =	42.4845	mm^2
Sample Standard Deviation =	6.5180	mm

Third 50 Stone Grid Sample vs Actual Distribution



Fourth 50 Stone Grid Sample

(1) Grain Size Analysis

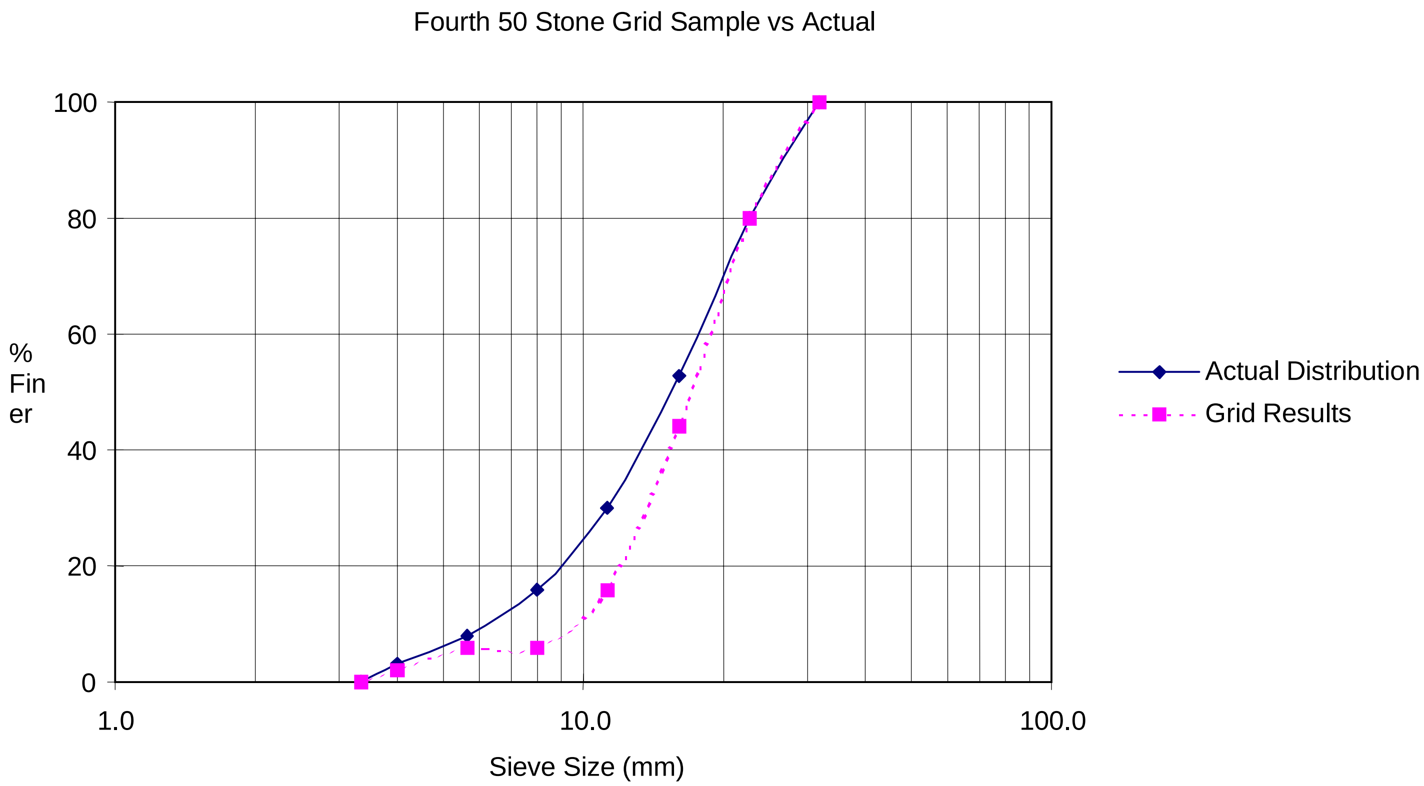
Sieve Size (mm)	Particle No.	No. of Particles	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
25.4	7	10	20	100	1.00
19.05	6	18	36	80	0.8000
14.28	5	14	28	44	0.4400
10	4	5	10	16	0.1600
7	3	0	0	6	0.0600
5	2	2	4	6	0.0600
4	1	1	2	2	0.0200
	total =	50			

Sieve Size	Actual Distribution	Grid Results
32.0	100	100
22.6	80.039	80
16.0	52.833	44
11.3	29.913	16
8.00	15.952	6
5.66	7.982	6
4.00	2.992	2
3.36	0	0

(2) Sample Statistics

Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*$(D-M)^2$
25.4	0.1	2.5400	28.1965
19.05	0.18	3.4290	19.6256
14.28	0.14	1.9992	4.5037
10	0.05	0.5000	0.0969
7	0	0.0000	0.0000
5	0.02	0.1000	0.2604
4	0.01	0.0400	0.2124

Sample Mean = 8.6082 mm
Sample Variance = 52.8954 mm^2
Sample Standard Deviation = 7.2729 mm



Fourth 50 Stone Grid Sample vs Actual
100
80
60
40
20
0
% Finer
1.0
10.0
100.0
Sieve Size (mm)
Actual Distribution
Grid Results

Fifth 50 Stone Grid Sample

(1) Grain Size Analysis

Sieve Size (mm)	Particle No.	No. of Particles	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
25.4	7	5	10	100	1.00
19.05	6	22	44	90	0.9000
14.28	5	7	14	46	0.4600
10	4	9	18	32	0.3200
7	3	1	2	14	0.1400
5	2	3	6	12	0.1200
4	1	3	6	6	0.0600
	total =	50			

Sieve Size	Actual Distribution	Grid Results
32.0	100	100
22.6	80.039	90
16.0	52.833	46
11.3	29.913	32
8.00	15.952	14
5.66	7.982	12
4.00	2.992	6
3.36	0	0

(2) Sample Statistics

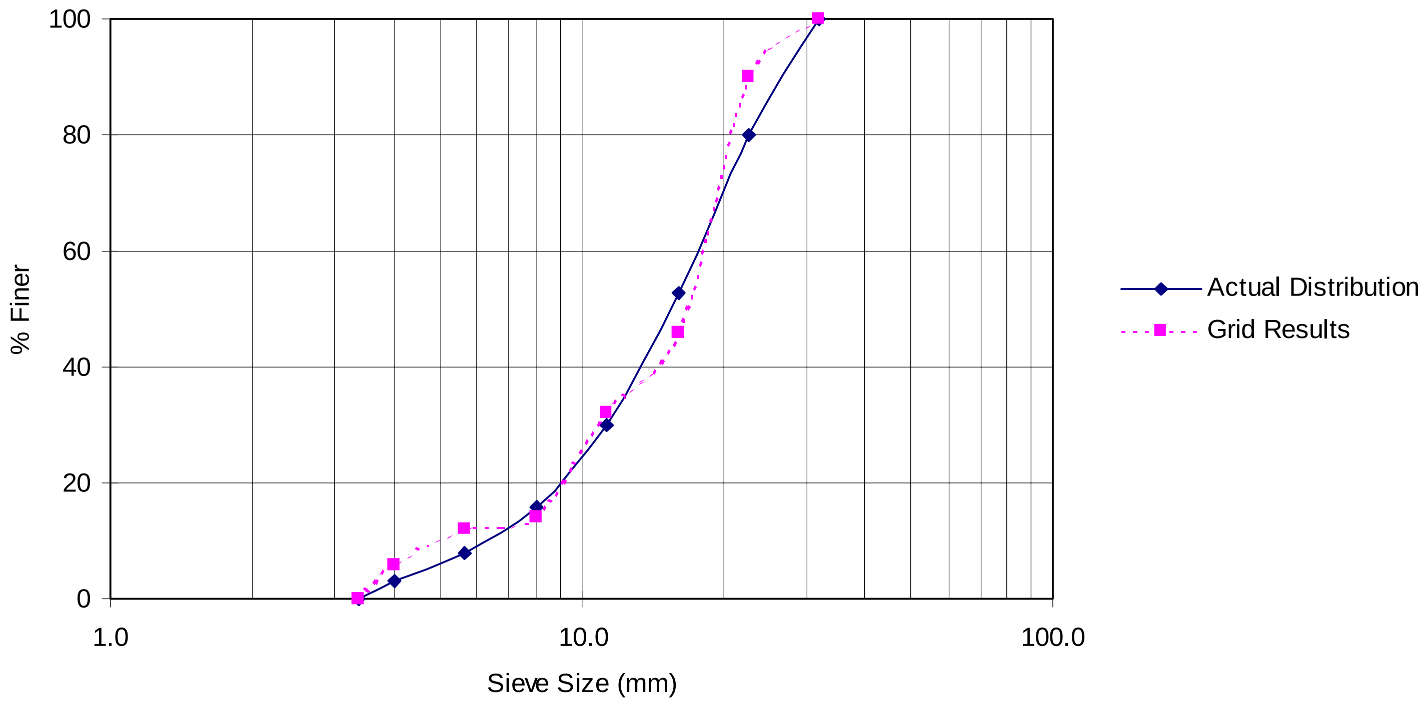
Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*(D-M)^2
25.4	0.05	1.2700	15.6634
19.05	0.22	4.1910	28.3380
14.28	0.07	0.9996	3.0302
10	0.09	0.9000	0.4759
7	0.01	0.0700	0.0049
5	0.03	0.1500	0.2188
4	0.03	0.1200	0.4108

Sample Mean = 7.7006 mm

Sample Variance = 48.1420 mm^2

Sample Standard Deviation = 6.9384 mm

Fifth 50 Stone Grid Sample vs Actual Distribution



Sixth 50 Stone Grid Sample

(1) Grain Size Analysis

Sieve Size (mm)	Particle No.	No. of Particles	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
25.4	7	8	16	100	1.00
19.05	6	10	20	84	0.8400
14.28	5	10	20	64	0.6400
10	4	9	18	44	0.4400
7	3	8	16	26	0.2600
5	2	3	6	10	0.1000
4	1	2	4	4	0.0400
	total =	50			

Sieve Size	Actual Distribution	Grid Results
32.0	100	100
22.6	80.039	84
16.0	52.833	64
11.3	29.913	44
8.00	15.952	26
5.66	7.982	10
4.00	2.992	4
3.36	0	0

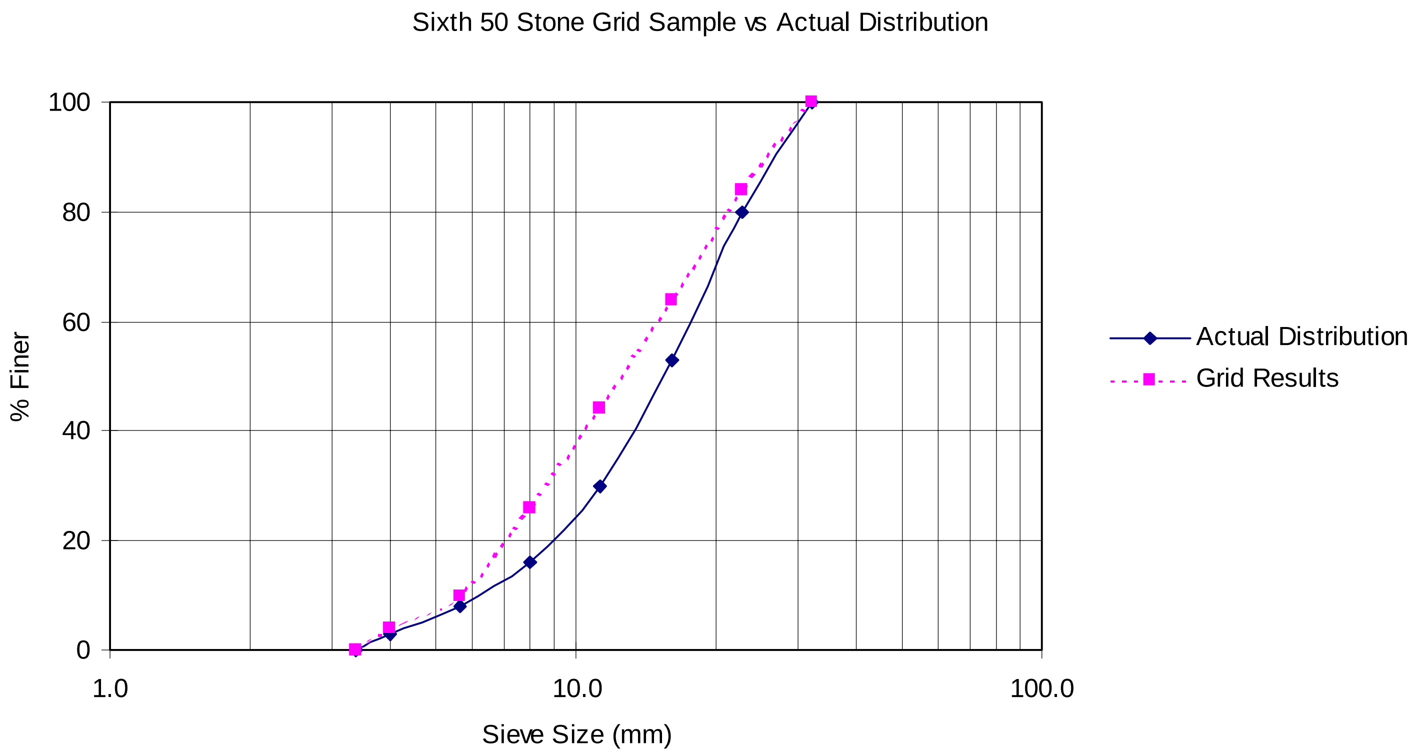
(2) Sample Statistics

Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*(D-M)^2
25.4	0.08	2.0320	26.9231
19.05	0.1	1.9050	14.3880
14.28	0.1	1.4280	5.2201
10	0.09	0.9000	0.7806
7	0.08	0.5600	0.0002
5	0.03	0.1500	0.1267
4	0.02	0.0800	0.1867

Sample Mean = 7.0550 mm

Sample Variance = 47.6254 mm^2

Sample Standard Deviation = 6.9011 mm



Sixth 50 Stone Grid Sample vs Actual Distribution
100
80
60
40
20
0
% Finer
1.0
10.0
100.0
Sieve Size (mm)
Actual Distribution
Grid Results

Seventh 50 Stone Grid Sample

(1) Grain Size Analysis

Sieve Size (mm)	Particle No.	No. of Particles	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
25.4	7	14	28	100	1.00
19.05	6	8	16	72	0.7200
14.28	5	16	32	56	0.5600
10	4	4	8	24	0.2400
7	3	2	4	16	0.1600
5	2	3	6	12	0.1200
4	1	3	6	6	0.0600
	total =	50			

Sieve Size	Actual Distribution	Grid Results
32.0	100	100
22.6	80.039	72
16.0	52.833	56
11.3	29.913	24
8.00	15.952	16
5.66	7.982	12
4.00	2.992	6
3.36	0	0

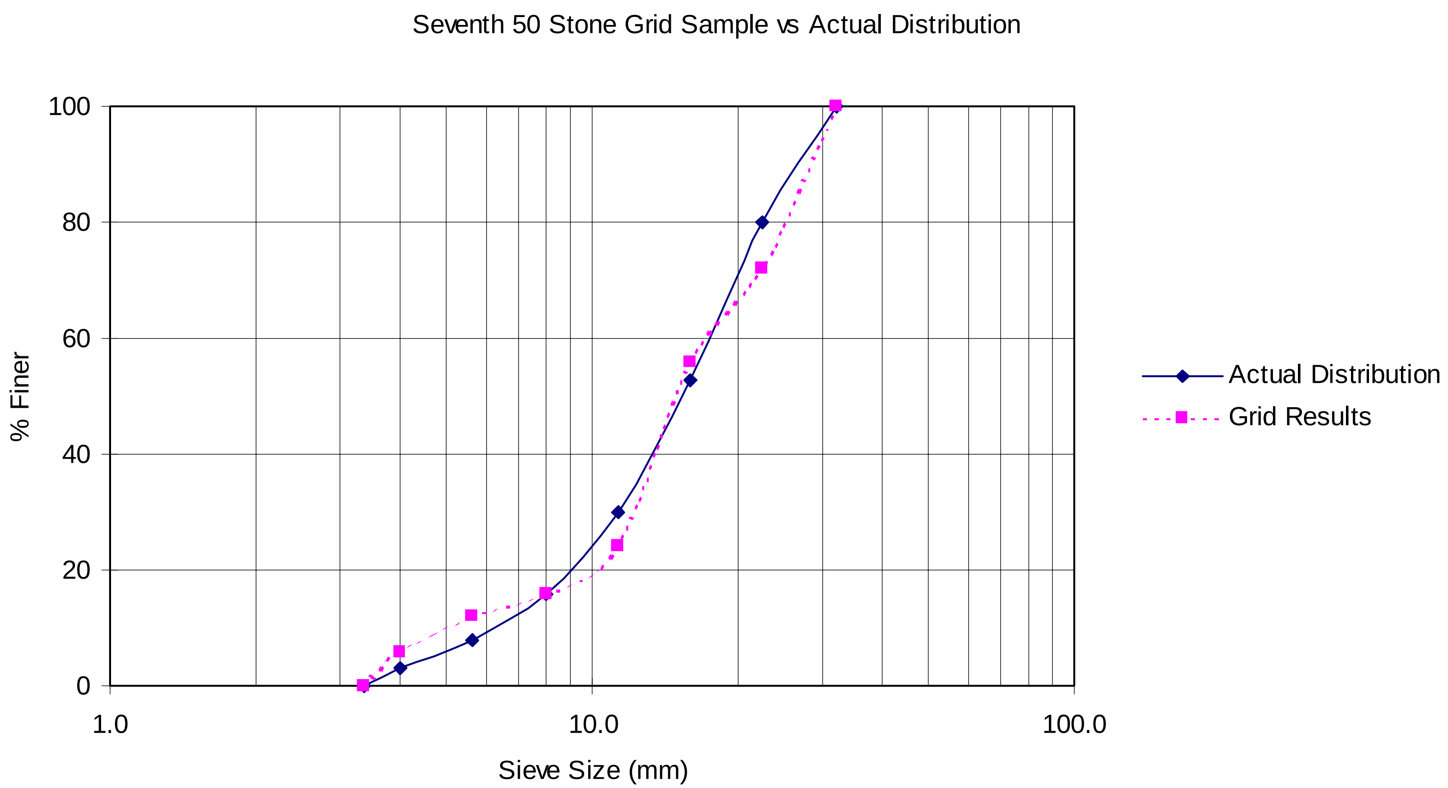
(2) Sample Statistics

Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*(D-M)^2
25.4	0.14	3.5560	41.5391
19.05	0.08	1.5240	9.4616
14.28	0.16	2.2848	5.9638
10	0.04	0.4000	0.1333
7	0.02	0.1400	0.0276
5	0.03	0.1500	0.3024
4	0.03	0.1200	0.5229

Sample Mean = 8.1748 mm

Sample Variance = 57.9505 mm^2

Sample Standard Deviation = 7.6125 mm



Seventh 50 Stone Grid Sample vs Actual Distribution
100
80
60
40
20
0
% Finer
1.0
10.0
100.0
Sieve Size (mm)
Actual Distribution
Grid Results

Eighth 50 Stone Grid Sample

(1) Grain Size Analysis

Sieve Size (mm)	Particle No.	No. of Particles	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
25.4	7	8	16	100	1.00
19.05	6	13	26	84	0.8400
14.28	5	14	28	58	0.5800
10	4	3	6	30	0.3000
7	3	6	12	24	0.2400
5	2	4	8	12	0.1200
4	1	2	4	4	0.0400
	total =	50			

Sieve Size	Actual Distribution	Grid Results
32.0	100	100
22.6	80.039	84
16.0	52.833	58
11.3	29.913	30
8.00	15.952	24
5.66	7.982	12
4.00	2.992	4
3.36	0	0

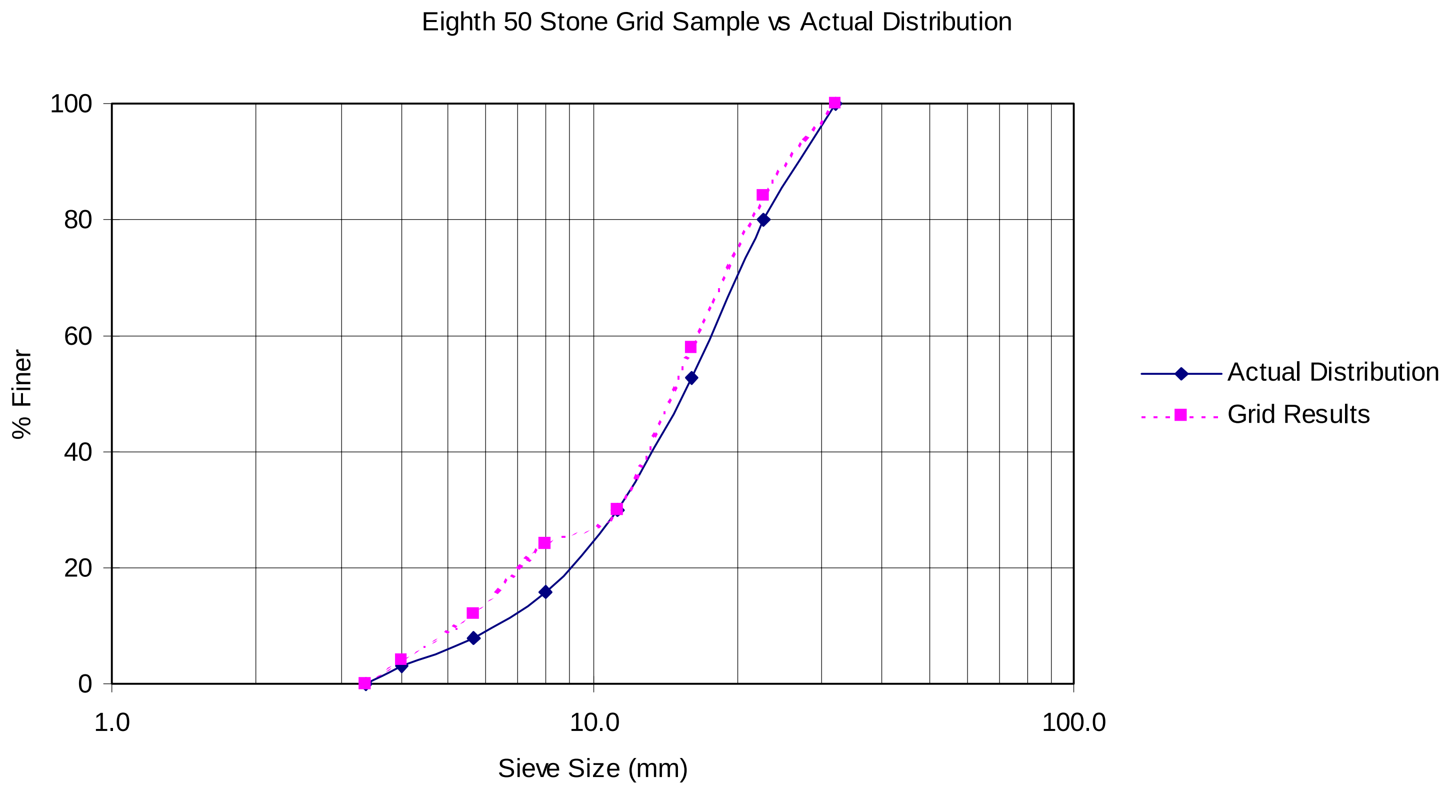
(2) Sample Statistics

Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*(D-M)^2
25.4	0.08	2.0320	25.6108
19.05	0.13	2.4765	17.3192
14.28	0.14	1.9992	6.4210
10	0.03	0.3000	0.1863
7	0.06	0.4200	0.0155
5	0.04	0.2000	0.2515
4	0.02	0.0800	0.2461

Sample Mean = 7.5077 mm

Sample Variance = 50.0504 mm^2

Sample Standard Deviation = 7.0746 mm



Eighth 50 Stone Grid Sample vs Actual Distribution
100
80
60
40
20
0
% Finer
1.0
10.0
100.0
Sieve Size (mm)
Actual Distribution
Grid Results

Appendix E. Laboratory Areal Data and Results for Glass Bead Experiments

The data obtained by areal sampling on the distribution of glass beads is shown here. Additionally, the analysis results in terms of volume-by-weight for a variety of different samples sizes are included. The results are shown for the entire areal sample as well as smaller samples made up of five sample rows. A comparison is made between the sample and actual grain size distribution curves. A discussion of the areal sampling on the glass bead distribution is provided in section 4.2.2.

Data Set For Laboratory Areal Sample

Cell :	1-1
D (mm)	Quantity
25.4	4
19.05	4
14.28	5
10	8
7	6
5	8
4	5

Cell :	2-1
D (mm)	Quantity
25.4	2
19.05	7
14.28	9
10	5
7	6
5	5
4	7

Cell :	3-1
D (mm)	Quantity
25.4	1
19.05	5
14.28	8
10	7
7	7
5	1
4	1

Cell :	4-1
D (mm)	Quantity
25.4	2
19.05	4
14.28	13
10	9
7	10
5	5
4	9

Cell :	1-2
D (mm)	Quantity
25.4	0
19.05	7
14.28	9
10	6
7	10
5	4
4	7

Cell :	2-2
D (mm)	Quantity
25.4	2
19.05	13
14.28	4
10	2
7	4
5	1
4	3

Cell :	3-2
D (mm)	Quantity
25.4	2
19.05	7
14.28	5
10	2
7	4
5	3
4	14

Cell :	4-2
D (mm)	Quantity
25.4	2
19.05	7
14.28	6
10	13
7	4
5	5
4	15

Cell :	1-3
25.4	4
19.05	4
14.28	6
10	7
7	6
5	5
4	5

Cell :	2-3
25.4	4
19.05	2
14.28	9
10	5
7	5
5	0
4	2

Cell :	3-3
25.4	1
19.05	6
14.28	5
10	4
7	6
5	0
4	3

Cell :	4-3
25.4	1
19.05	3
14.28	7
10	6
7	10
5	15
4	8

Cell :	1-4
D (mm)	Quantity
25.4	0
19.05	12
14.28	6
10	6
7	5
5	1
4	4

Cell :	2-4
D (mm)	Quantity
25.4	1
19.05	10
14.28	8
10	0
7	7
5	2
4	3

Cell :	3-4
D (mm)	Quantity
25.4	1
19.05	9
14.28	3
10	7
7	10
5	7
4	3

Cell :	4-4
D (mm)	Quantity
25.4	2
19.05	6
14.28	3
10	8
7	10
5	11
4	3

Cell :	1-5
D (mm)	Quantity
25.4	3
19.05	6
14.28	7
10	2
7	11
5	3
4	1

Cell :	2-5
D (mm)	Quantity
25.4	3
19.05	8
14.28	7
10	3
7	4
5	2
4	4

Cell :	3-5
D (mm)	Quantity
25.4	2
19.05	7
14.28	6
10	5
7	8
5	3
4	3

Cell :	4-5
D (mm)	Quantity
25.4	2
19.05	7
14.28	6
10	6
7	15
5	9
4	10

Data Set For Laboratory Areal Sample

Cell :	5-1
D (mm)	Quantity
25.4	3
19.05	1
14.28	9
10	5
7	7
5	4
4	2

Cell :	6-1
D (mm)	Quantity
25.4	2
19.05	2
14.28	11
10	9
7	10
5	7
4	9

Cell :	7-1
D (mm)	Quantity
25.4	3
19.05	4
14.28	4
10	2
7	11
5	3
4	10

Cell :	8-1
D (mm)	Quantity
25.4	3
19.05	3
14.28	6
10	7
7	2
5	13
4	3

Cell :	5-2
D (mm)	Quantity
25.4	2
19.05	1
14.28	6
10	10
7	12
5	6
4	12

Cell :	6-2
D (mm)	Quantity
25.4	1
19.05	4
14.28	10
10	14
7	8
5	10
4	4

Cell :	7-2
D (mm)	Quantity
25.4	0
19.05	6
14.28	8
10	7
7	9
5	11
4	6

Cell :	8-2
D (mm)	Quantity
25.4	3
19.05	2
14.28	6
10	10
7	8
5	11
4	2

Cell :	5-3
25.4	0
19.05	8
14.28	7
10	7
7	6
5	10
4	6

Cell :	6-3
25.4	4
19.05	4
14.28	5
10	8
7	5
5	13
4	7

Cell :	7-3
25.4	1
19.05	7
14.28	2
10	9
7	10
5	13
4	4

Cell :	8-3
25.4	2
19.05	4
14.28	6
10	11
7	11
5	11
4	5

Cell :	5-4
D (mm)	Quantity
25.4	1
19.05	6
14.28	8
10	8
7	2
5	4
4	1

Cell :	6-4
D (mm)	Quantity
25.4	3
19.05	2
14.28	9
10	13
7	7
5	8
4	4

Cell :	7-4
D (mm)	Quantity
25.4	1
19.05	3
14.28	9
10	8
7	4
5	10
4	8

Cell :	8-4
D (mm)	Quantity
25.4	1
19.05	5
14.28	11
10	7
7	3
5	3
4	3

Cell :	5-5
D (mm)	Quantity
25.4	1
19.05	7
14.28	4
10	11
7	5
5	7
4	3

Cell :	6-5
D (mm)	Quantity
25.4	0
19.05	3
14.28	6
10	18
7	11
5	9
4	10

Cell :	7-5
D (mm)	Quantity
25.4	1
19.05	4
14.28	6
10	5
7	7
5	7
4	6

Cell :	8-5
D (mm)	Quantity
25.4	1
19.05	5
14.28	8
10	6
7	5
5	5
4	5

Data Set For Laboratory Areal Sample

Cell :	9-1
D (mm)	Quantity
25.4	1
19.05	3
14.28	11
10	11
7	5
5	6
4	6

Cell :	10-1
D (mm)	Quantity
25.4	2
19.05	4
14.28	8
10	8
7	5
5	6
4	4

Cell :	11-1
D (mm)	Quantity
25.4	3
19.05	4
14.28	2
10	9
7	3
5	3
4	4

Cell :	12-1
D (mm)	Quantity
25.4	0
19.05	5
14.28	6
10	3
7	5
5	5
4	7

Cell :	9-2
D (mm)	Quantity
25.4	0
19.05	7
14.28	8
10	9
7	2
5	8
4	2

Cell :	10-2
D (mm)	Quantity
25.4	2
19.05	3
14.28	3
10	10
7	8
5	11
4	2

Cell :	11-2
D (mm)	Quantity
25.4	2
19.05	3
14.28	3
10	13
7	6
5	4
4	3

Cell :	12-2
D (mm)	Quantity
25.4	1
19.05	4
14.28	10
10	10
7	5
5	3
4	14

Cell :	9-3
25.4	2
19.05	4
14.28	5
10	7
7	5
5	17
4	3

Cell :	10-3
25.4	1
19.05	4
14.28	8
10	8
7	3
5	7
4	0

Cell :	11-3
25.4	0
19.05	6
14.28	7
10	10
7	8
5	8
4	7

Cell :	12-3
25.4	2
19.05	4
14.28	4
10	10
7	7
5	9
4	7

Cell :	9-4
D (mm)	Quantity
25.4	2
19.05	5
14.28	9
10	8
7	5
5	3
4	6

Cell :	10-4
D (mm)	Quantity
25.4	1
19.05	4
14.28	4
10	12
7	4
5	20
4	6

Cell :	11-4
D (mm)	Quantity
25.4	0
19.05	4
14.28	13
10	5
7	7
5	9
4	4

Cell :	12-4
D (mm)	Quantity
25.4	1
19.05	5
14.28	9
10	5
7	6
5	13
4	8

Cell :	9-5
D (mm)	Quantity
25.4	2
19.05	5
14.28	7
10	2
7	9
5	8
4	12

Cell :	10-5
D (mm)	Quantity
25.4	2
19.05	4
14.28	2
10	7
7	10
5	6
4	2

Cell :	11-5
D (mm)	Quantity
25.4	1
19.05	1
14.28	5
10	5
7	4
5	13
4	7

Cell :	12-5
D (mm)	Quantity
25.4	3
19.05	5
14.28	11
10	4
7	2
5	0
4	3

Total Areal Sample

(1) Grain Size Analysis

Particle Size (mm)	Particle No.	No. of Particles	Projected Area	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
25.4	7	100	645.16	19.2238	100	1.00
19.05	6	299	1085.08	32.3321	80.7762	0.8078
14.28	5	408	831.99	24.7907	48.4441	0.4844
10	4	442	442.00	13.1703	23.6534	0.2365
7	3	400	196.00	5.8402	10.4832	0.1048
5	2	414	103.50	3.0840	4.6430	0.0464
4	1	327	52.32	1.5590	1.5590	0.0156
		total =	3356.0455			

Sieve Size	Actual Distribution	Areal Results
32.0	100	100
22.6	80.039	80.7762
16.0	52.833	48.4441
11.3	29.913	23.6534
8.00	15.952	10.4832
5.66	7.982	4.6430
4.00	2.992	1.5590
3.36	0	0

(2) Sample Statistics

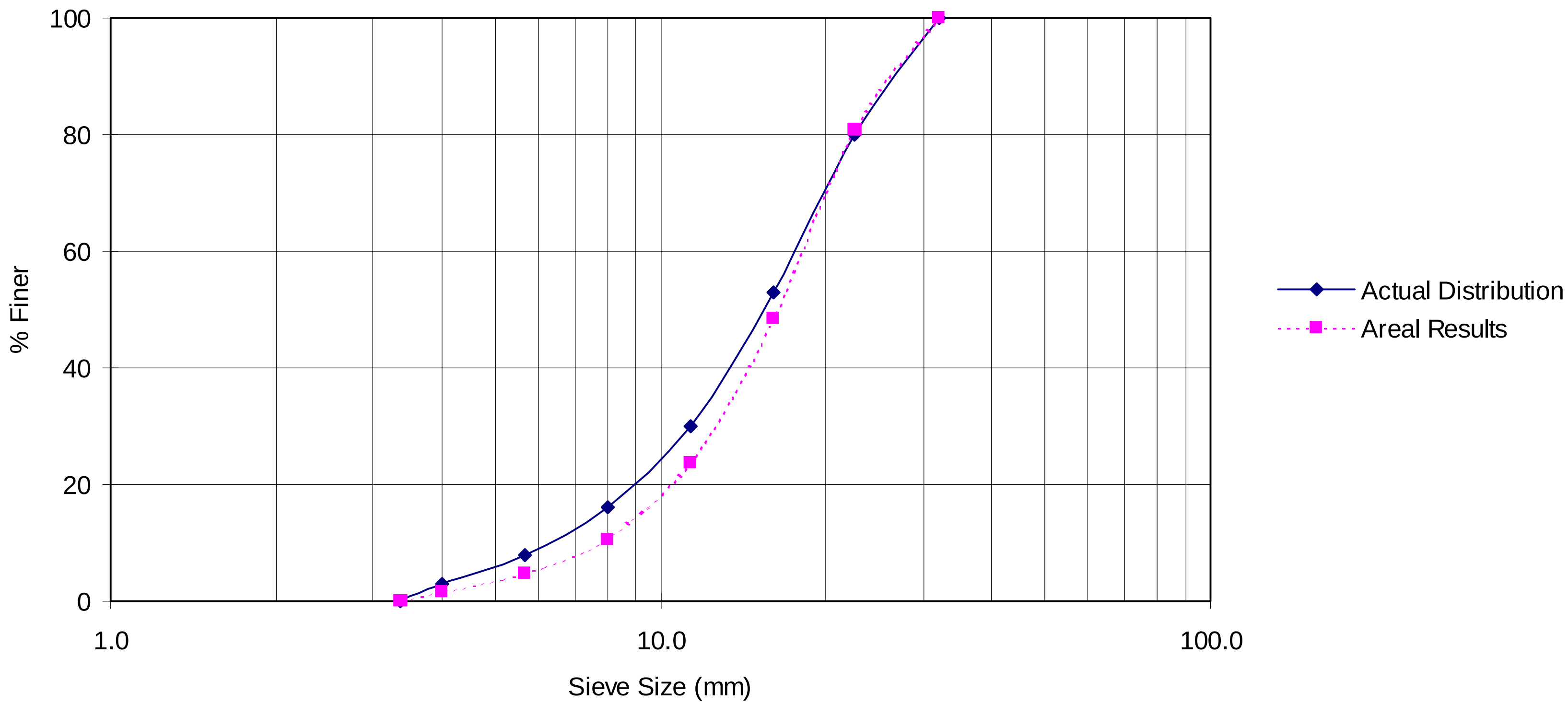
Particle Size (mm)	% Vol. (100*PDF)	%Vol.*D	%Vol.*(D-M)^2
25.4	19.2238154	4.8828	15.1431
19.05	32.3320545	6.1593	2.0620
14.28	24.7906967	3.5401	1.2490
10	13.1702623	1.3170	5.6067
7	5.84020679	0.4088	5.2981
5	3.08398675	0.1542	4.0961
4	1.55897765	0.0624	2.4455

Sample Mean = 16.5246 mm

Sample Variance = 35.9005 mm^2

Sample Standard Deviation = 5.9917 mm

Total Areal Sample vs Actual Distribution



Areal Sample for columns 1 through 5

(1) Grain Size Analysis

Particle Size (mm)	Particle No.	No. of Particles	Projected Area	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
25.4	7	46	296.77	19.8255	100	1.00
19.05	6	157	569.76	38.0618	80.1745	0.8017
14.28	5	166	338.50	22.6133	42.1126	0.4211
10	4	152	152.00	10.1541	19.4993	0.1950
7	3	180	88.20	5.8921	9.3452	0.0935
5	2	121	30.25	2.0208	3.4531	0.0345
4	1	134	21.44	1.4323	1.4323	0.0143
		total =	1496.93			

Sieve Size	Actual Distribution	Areal Results
32.0	100	100
22.6	80.039	80.1745
16.0	52.833	42.1126
11.3	29.913	19.4993
8.00	15.952	9.3452
5.66	7.982	3.4531
4.00	2.992	1.4323
3.36	0	0

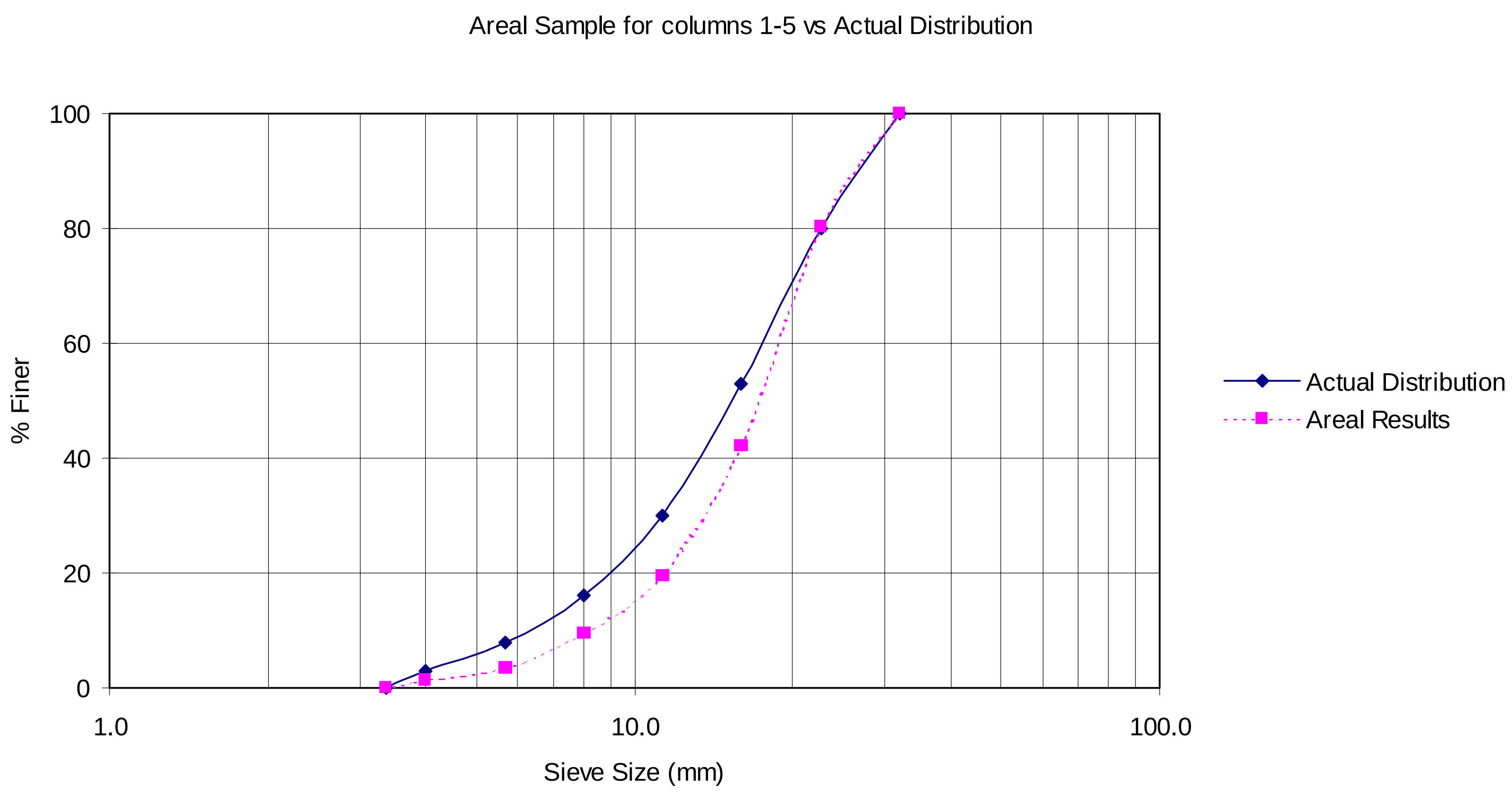
(2) Sample Statistics

Particle Size (mm)	% Vol. (100*PDF)	%Vol.*D	%Vol.*(D-M)^2
25.4	19.825548	5.0357	13.6518
19.05	38.06182	7.2508	1.4446
14.28	22.613326	3.2292	1.8007
10	10.154149	1.0154	5.1214
7	5.8920785	0.4124	6.0127
5	2.0208092	0.1010	2.9596
4	1.4322694	0.0573	2.4586

Sample Mean = 17.1018 mm

Sample Variance = 33.4492 mm^2

Sample Standard Deviation = 5.7835 mm


Areal Sample for columns 1-5 vs Actual Distribution
100
80
60
40
20
0
% Finer
1.0
10.0
100.0
Sieve Size (mm)
Actual Distribution
Areal Results

Areal Sample for columns 2 through 6

(1) Grain Size Analysis

Particle Size (mm)	Particle No.	No. of Particles	Projected Area	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
25.4	7	45	290.32	19.5588	100	1.00
19.05	6	139	504.43	33.9834	80.4412	0.8044
14.28	5	174	354.82	23.9039	46.4578	0.4646
10	4	185	185.00	12.4633	22.5539	0.2255
7	3	183	89.67	6.0410	10.0906	0.1009
5	2	147	36.75	2.4758	4.0496	0.0405
4	1	146	23.36	1.5737	1.5737	0.0157
		total =	1484.35			

Sieve Size	Actual Distribution	Areal Results
32.0	100	100
22.6	80.039	80.4412
16.0	52.833	46.4578
11.3	29.913	22.5539
8.00	15.952	10.0906
5.66	7.982	4.0496
4.00	2.992	1.5737
3.36	0	0

(2) Sample Statistics

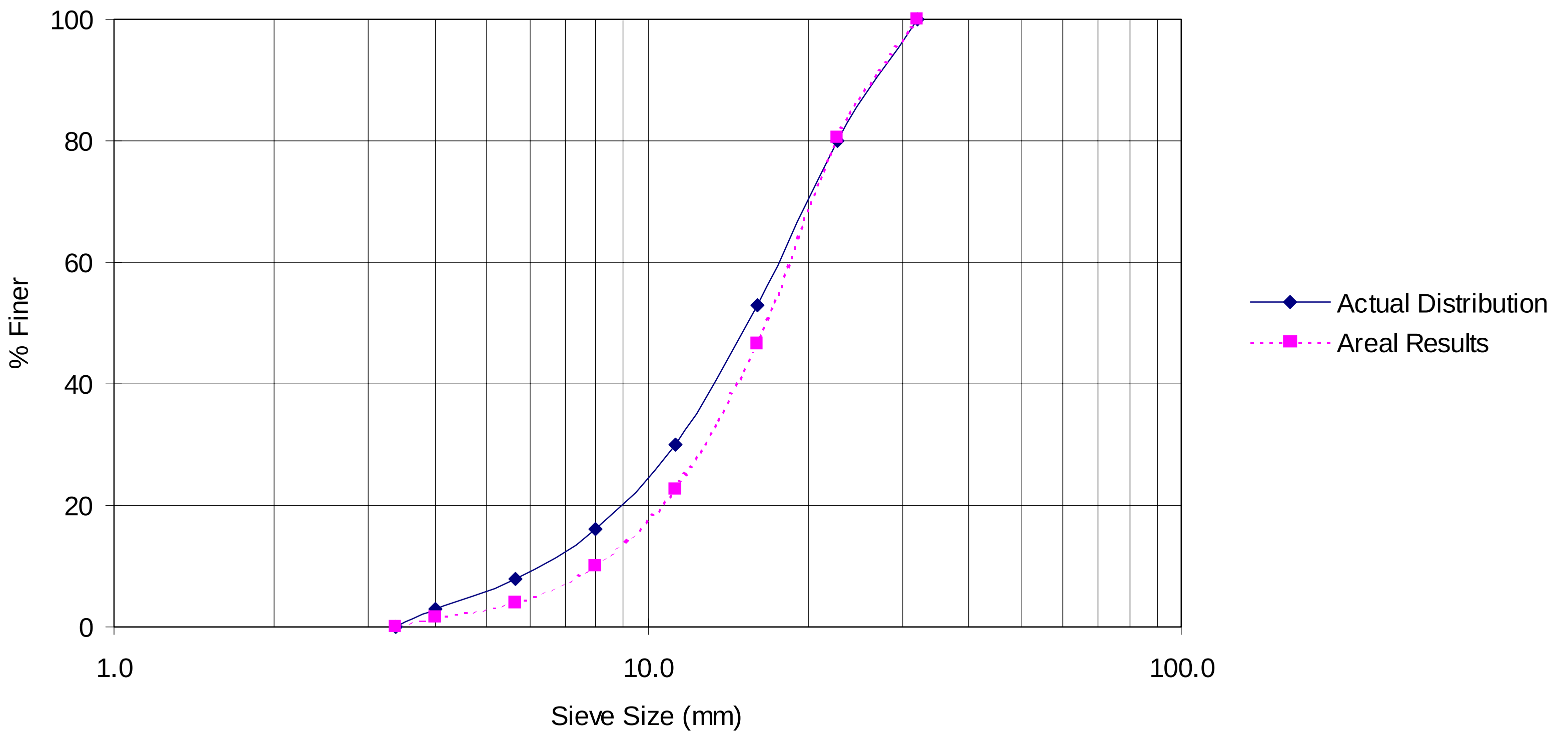
Particle Size (mm)	% Vol. (100*PDF)	%Vol.*D	%Vol.*(D-M)^2
25.4	19.558805	4.9679	14.7660
19.05	33.983424	6.4738	1.8589
14.28	23.90386	3.4135	1.4129
10	12.46333	1.2463	5.6135
7	6.0410098	0.4229	5.6971
5	2.4758237	0.1238	3.3956
4	1.5737481	0.0629	2.5428

Sample Mean = 16.7112 mm

Sample Variance = 35.2868 mm^2

Sample Standard Deviation = 5.9403 mm

Areal Sample for columns 2-6 vs Actual Distribution



Areal Sample for columns 3 through 7

(1) Grain Size Analysis

Particle Size (mm)	Particle No.	No. of Particles	Projected Area	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
25.4	7	39	251.61	17.9018	100	1.00
19.05	6	123	446.37	31.7584	82.0982	0.8210
14.28	5	166	338.50	24.0840	50.3398	0.5034
10	4	201	201.00	14.3008	26.2558	0.2626
7	3	198	97.02	6.9028	11.9550	0.1196
5	2	181	45.25	3.2195	5.0522	0.0505
4	1	161	25.76	1.8328	1.8328	0.0183
		total =	1405.52			

Sieve Size	Actual Distribution	Areal Results
32.0	100	100
22.6	80.039	82.0982
16.0	52.833	50.3398
11.3	29.913	26.2558
8.00	15.952	11.9550
5.66	7.982	5.0522
4.00	2.992	1.8328
3.36	0	0

(2) Sample Statistics

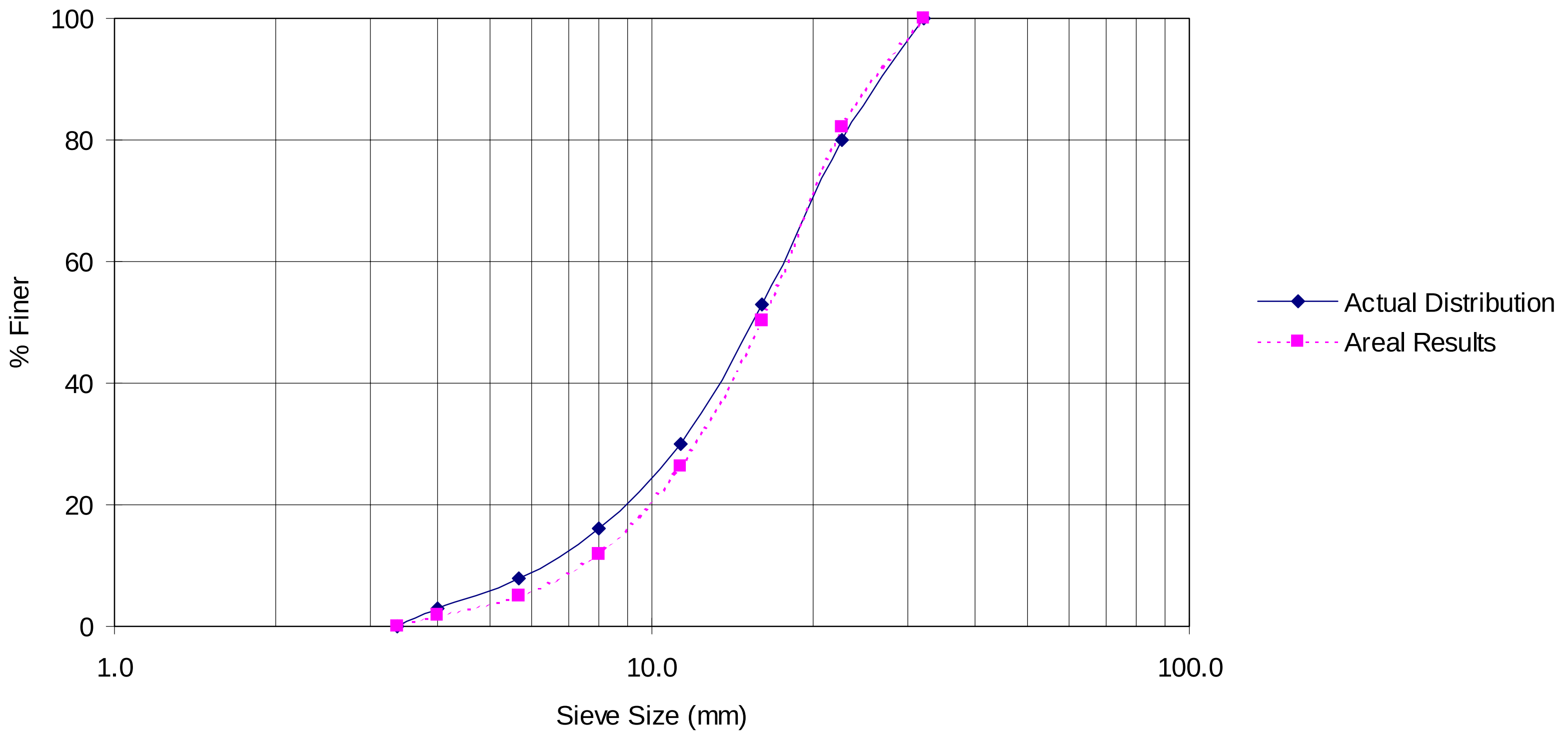
Particle Size (mm)	% Vol. (100*PDF)	%Vol.*D	%Vol.*$(D-M)^2$
25.4	17.901768	4.5470	15.2055
19.05	31.758425	6.0500	2.6090
14.28	24.083988	3.4392	0.8729
10	14.300787	1.4301	5.4685
7	6.902798	0.4832	5.8219
5	3.2194559	0.1610	4.0268
4	1.8327775	0.0733	2.7207

Sample Mean = 16.1838 mm

Sample Variance = 36.7253 mm^2

Sample Standard Deviation = 6.0601 mm

Areal Sample for columns 3-7 vs Actual Distribution



Areal Sample for columns 4 through 8

(1) Grain Size Analysis

Particle Size (mm)	Particle No.	No. of Particles	Projected Area	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
25.4	7	42	270.97	19.2151	100	1.00
19.05	6	108	391.93	27.7933	80.7849	0.8078
14.28	5	176	358.90	25.4504	52.9916	0.5299
10	4	217	217.00	15.3881	27.5412	0.2754
7	3	192	94.08	6.6715	12.1531	0.1215
5	2	210	52.50	3.7229	5.4816	0.0548
4	1	155	24.80	1.7586	1.7586	0.0176
		total =	1410.18			

Sieve Size	Actual Distribution	Areal Results
32.0	100	100
22.6	80.039	80.7849
16.0	52.833	52.9916
11.3	29.913	27.5412
8.00	15.952	12.1531
5.66	7.982	5.4816
4.00	2.992	1.7586
3.36	0	0

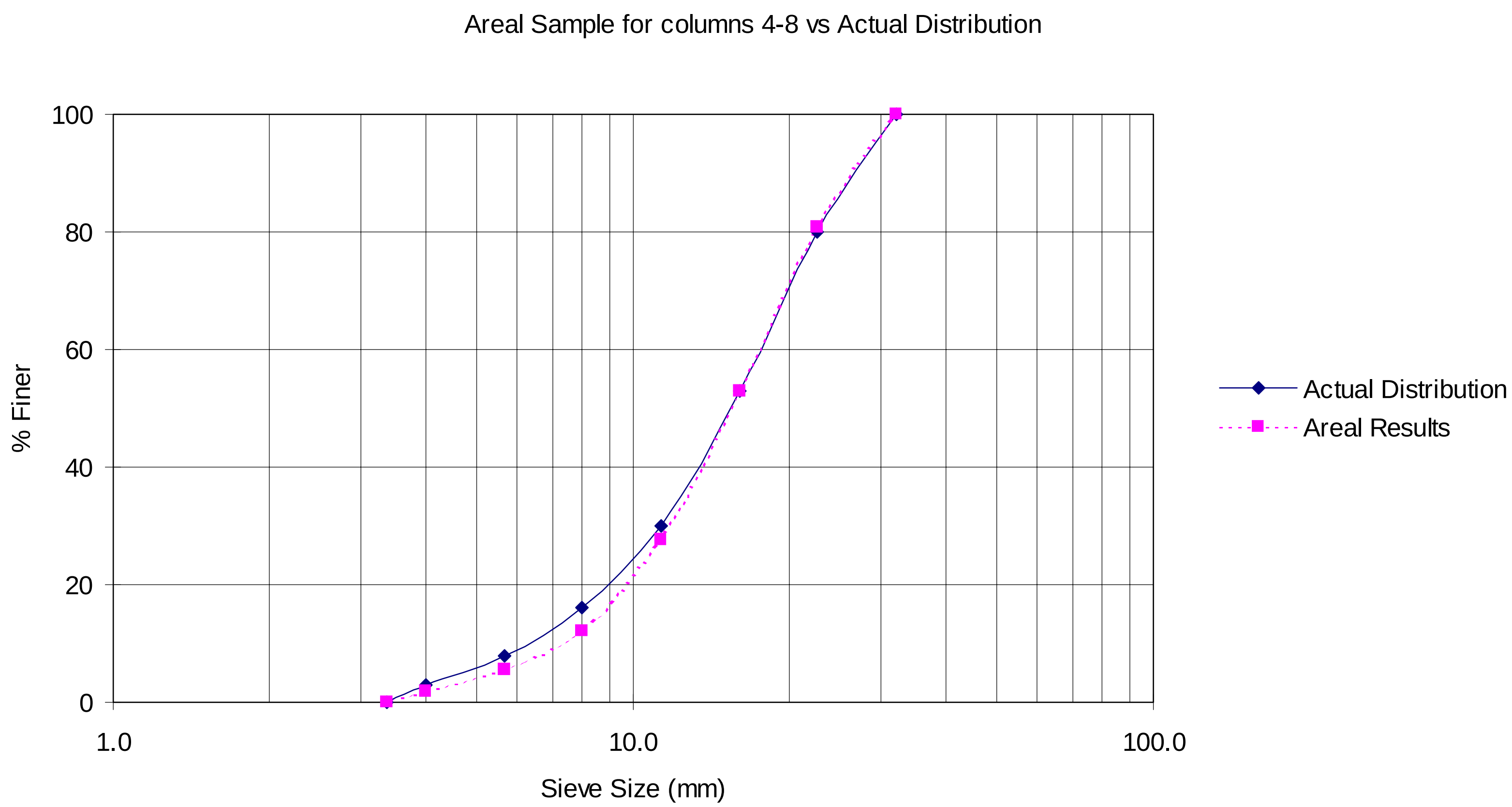
(2) Sample Statistics

Particle Size (mm)	% Vol. (100*PDF)	%Vol.*D	%Vol.*(D-M)^2
25.4	19.215102	4.8806	16.7198
19.05	27.793273	5.2946	2.4650
14.28	25.450426	3.6343	0.8172
10	15.388125	1.5388	5.6733
7	6.6714969	0.4670	5.4906
5	3.7229335	0.1861	4.5638
4	1.7586429	0.0703	2.5629

Sample Mean = 16.0719 mm

Sample Variance = 38.2925 mm^2

Sample Standard Deviation = 6.1881 mm



Areal Sample for columns 4-8 vs Actual Distribution
100
80
60
40
20
0
% Finer
1.0
10.0
100.0
Sieve Size (mm)
Actual Distribution
Areal Results

Areal Sample for columns 5 through 9

(1) Grain Size Analysis

Particle Size (mm)	Particle No.	No. of Particles	Projected Area	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
25.4	7	40	258.06	18.7410	100	1.00
19.05	6	105	381.05	27.6722	81.2590	0.8126
14.28	5	181	369.09	26.8040	53.5868	0.5359
10	4	212	212.00	15.3957	26.7828	0.2678
7	3	169	82.81	6.0138	11.3870	0.1139
5	2	207	51.75	3.7582	5.3733	0.0537
4	1	139	22.24	1.6151	1.6151	0.0162
		total =	1377.00			

Sieve Size	Actual Distribution	Areal Results
32.0	100	100
22.6	80.039	81.2590
16.0	52.833	53.5868
11.3	29.913	26.7828
8.00	15.952	11.3870
5.66	7.982	5.3733
4.00	2.992	1.6151
3.36	0	0

(2) Sample Statistics

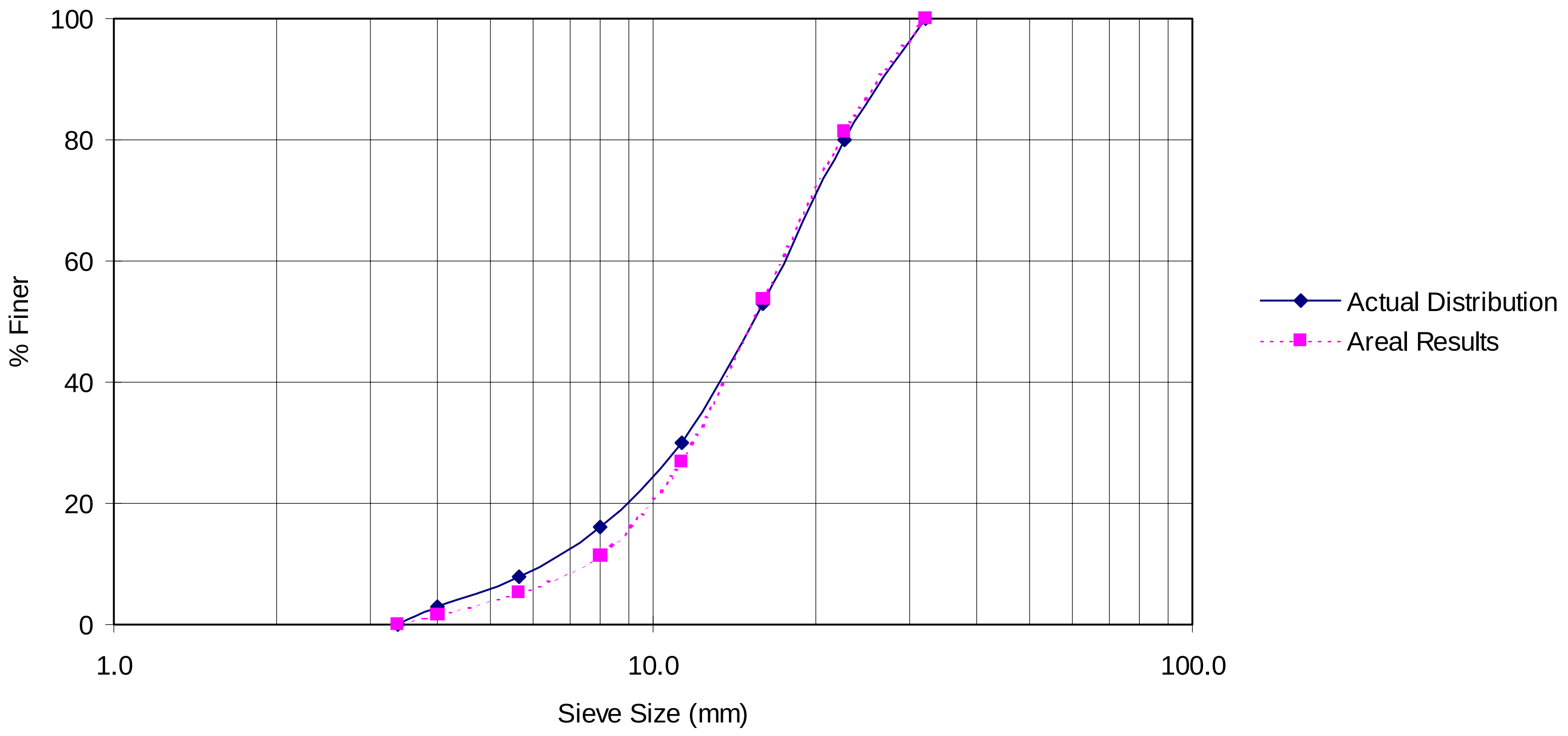
Particle Size (mm)	% Vol. (100*PDF)	%Vol.*D	%Vol.*$(D-M)^2$
25.4	18.740978	4.7602	16.3053
19.05	27.672225	5.2716	2.4534
14.28	26.804012	3.8276	0.8612
10	15.395744	1.5396	5.6771
7	6.013781	0.4210	4.9499
5	3.7581592	0.1879	4.6075
4	1.6151007	0.0646	2.3539

Sample Mean = 16.0724 mm

Sample Variance = 37.2082 mm^2

Sample Standard Deviation = 6.0999 mm

Areal Sample for columns 5-9 vs Actual Distribution



Areal Sample for columns 6 through 10

(1) Grain Size Analysis

Particle Size (mm)	Particle No.	No. of Particles	Projected Area	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
25.4	7	41	264.52	19.4962	100	1.00
19.05	6	101	366.53	27.0153	80.5038	0.8050
14.28	5	172	350.74	25.8513	53.4886	0.5349
10	4	216	216.00	15.9203	27.6372	0.2764
7	3	167	81.83	6.0313	11.7169	0.1172
5	2	226	56.50	4.1643	5.6856	0.0569
4	1	129	20.64	1.5213	1.5213	0.0152
		total =	1356.76			

Sieve Size	Actual Distribution	Areal Results
32.0	100	100
22.6	80.039	80.5038
16.0	52.833	53.4886
11.3	29.913	27.6372
8.00	15.952	11.7169
5.66	7.982	5.6856
4.00	2.992	1.5213
3.36	0	0

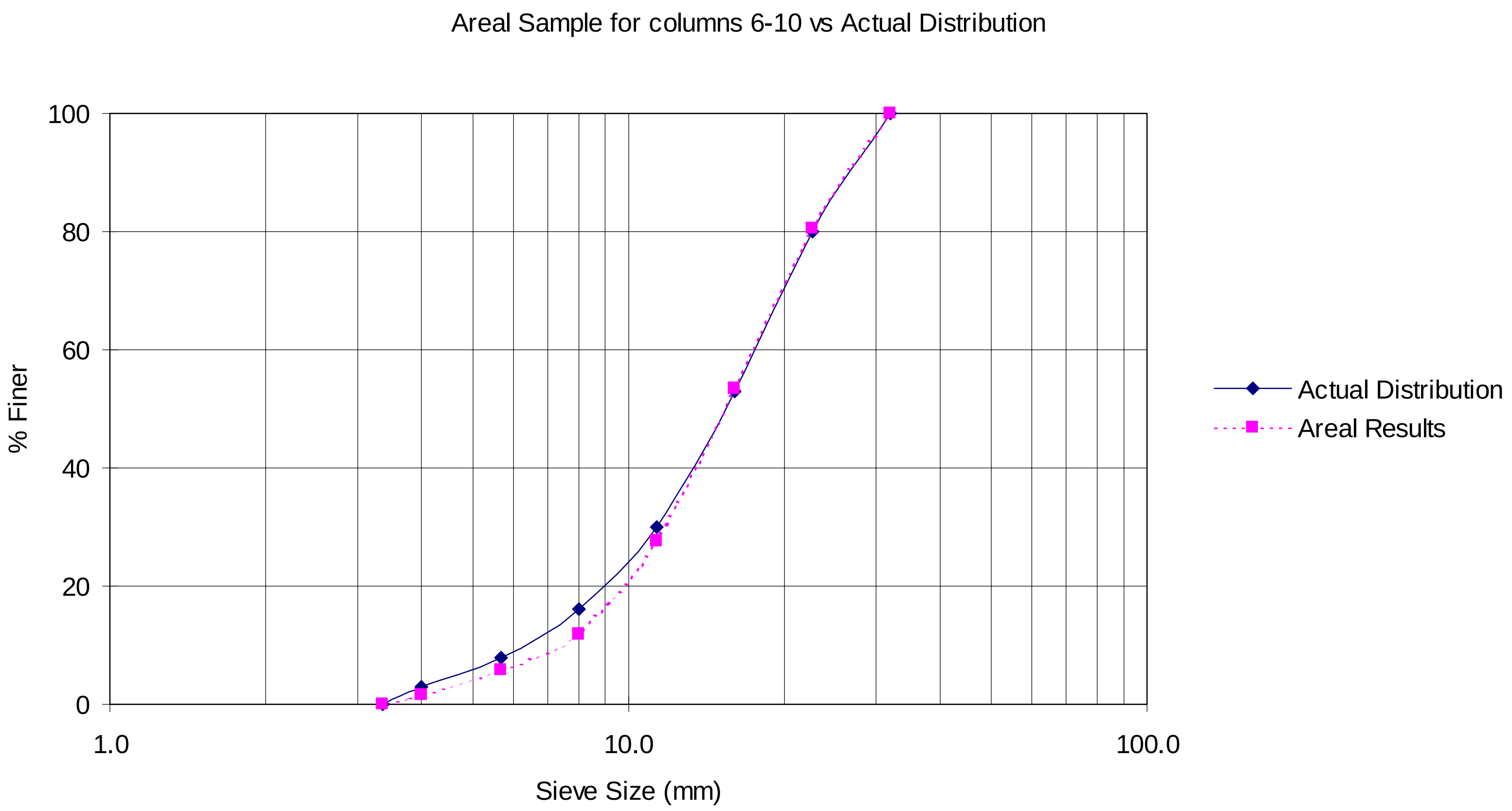
(2) Sample Statistics

Particle Size (mm)	% Vol. (100*PDF)	%Vol.*D	%Vol.*(D-M)^2
25.4	19.496169	4.9520	16.9592
19.05	27.015271	5.1464	2.3938
14.28	25.851328	3.6916	0.8314
10	15.920319	1.5920	5.8722
7	6.0312947	0.4222	4.9652
5	4.1643426	0.2082	5.1062
4	1.5212749	0.0609	2.2175

Sample Mean = 16.0733 mm

Sample Variance = 38.3455 mm^2

Sample Standard Deviation = 6.1924 mm



Areal Sample for columns 6-10 vs Actual Distribution
100
80
60
40
20
0
% Finer
1.0
10.0
100.0
Sieve Size (mm)
Actual Distribution
Areal Results

Areal Sample for columns 7 through 11

(1) Grain Size Analysis

Particle Size (mm)	Particle No.	No. of Particles	Projected Area	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
25.4	7	37	238.71	18.5176	100	1.00
19.05	6	104	377.42	29.2778	81.4824	0.8148
14.28	5	161	328.31	25.4681	52.2047	0.5220
10	4	196	196.00	15.2044	26.7366	0.2674
7	3	154	75.46	5.8537	11.5321	0.1153
5	2	216	54.00	4.1890	5.6784	0.0568
4	1	120	19.20	1.4894	1.4894	0.0149
		total =	1289.10			

Sieve Size	Actual Distribution	Areal Results
32.0	100	100
22.6	80.039	81.4824
16.0	52.833	52.2047
11.3	29.913	26.7366
8.00	15.952	11.5321
5.66	7.982	5.6784
4.00	2.992	1.4894
3.36	0	0

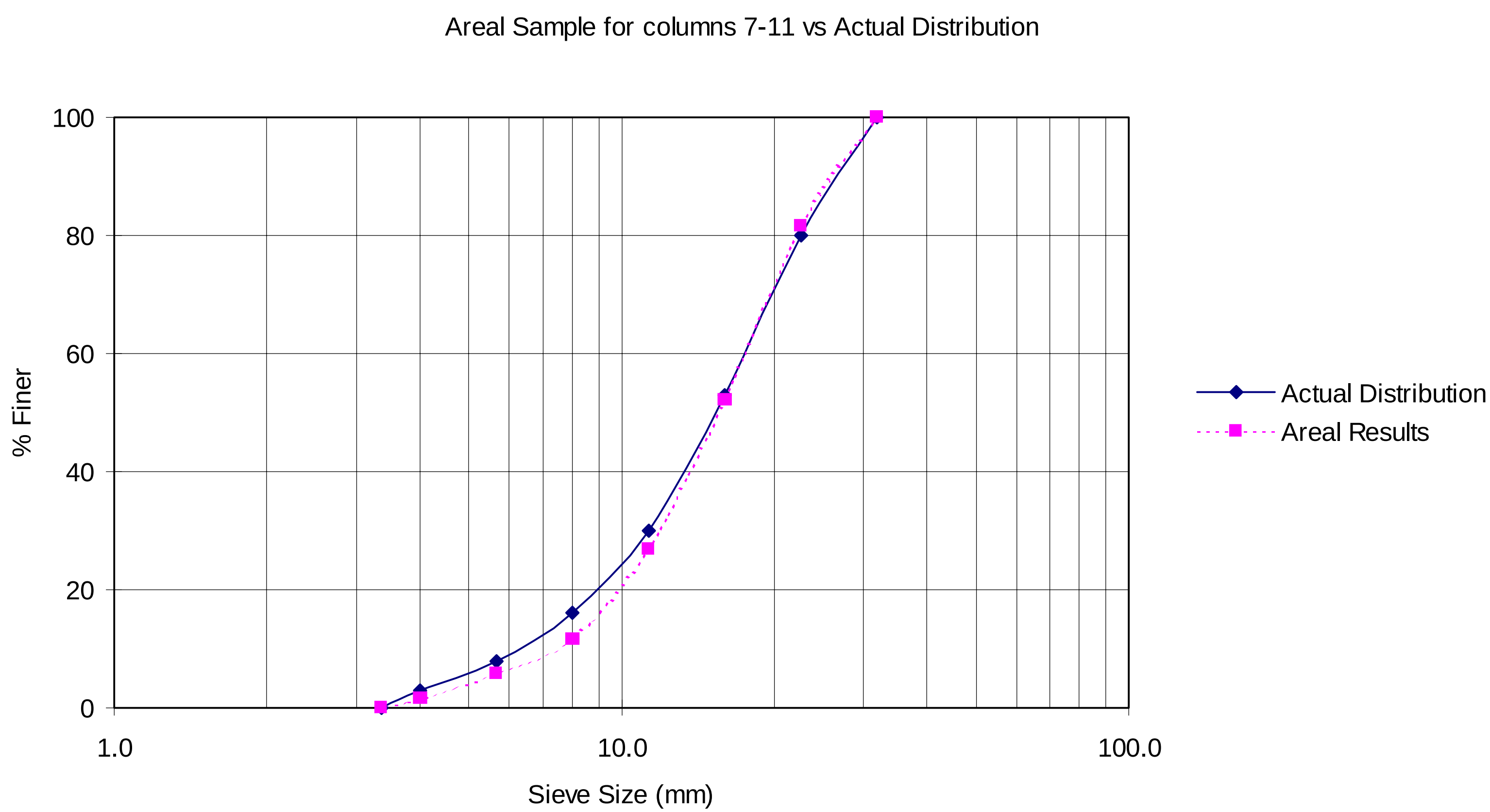
(2) Sample Statistics

Particle Size (mm)	% Vol. (100*PDF)	%Vol.*D	%Vol.*(D-M)^2
25.4	18.51756	4.7035	15.9575
19.05	29.277763	5.5774	2.5187
14.28	25.46812	3.6368	0.8594
10	15.204448	1.5204	5.6891
7	5.8537126	0.4098	4.8655
5	4.1889807	0.2094	5.1770
4	1.4894153	0.0596	2.1868

Sample Mean = 16.1170 mm

Sample Variance = 37.2540 mm^2

Sample Standard Deviation = 6.1036 mm


Areal Sample for columns 7-11 vs Actual Distribution
100
80
60
40
20
0
% Finer
1.0
10.0
100.0
Sieve Size (mm)
Actual Distribution
Areal Results

Areal Sample for columns 8 through 12

(1) Grain Size Analysis

Particle Size (mm)	Particle No.	No. of Particles	Projected Area	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
25.4	7	38	245.16	18.7890	100	1.00
19.05	6	103	373.79	28.6470	81.2110	0.8121
14.28	5	172	350.74	26.8805	52.5639	0.5256
10	4	197	197.00	15.0980	25.6834	0.2568
7	3	138	67.62	5.1824	10.5854	0.1059
5	2	202	50.50	3.8703	5.4031	0.0540
4	1	125	20.00	1.5328	1.5328	0.0153
		total =	1304.81			

Sieve Size	Actual Distribution	Areal Results
32.0	100	100
22.6	80.039	81.2110
16.0	52.833	52.5639
11.3	29.913	25.6834
8.00	15.952	10.5854
5.66	7.982	5.4031
4.00	2.992	1.5328
3.36	0	0

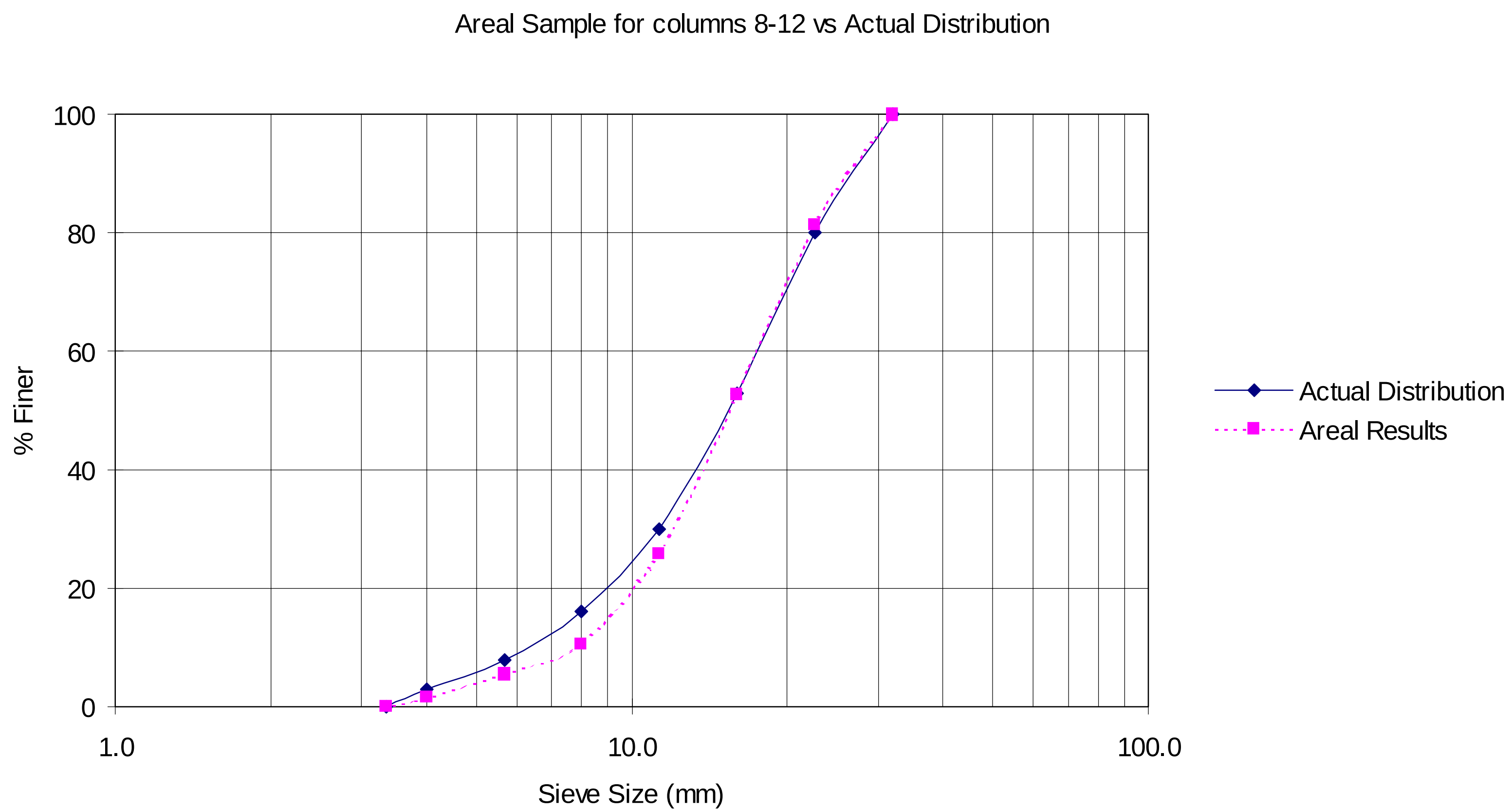
(2) Sample Statistics

Particle Size (mm)	% Vol. (100*PDF)	%Vol.*D	%Vol.*(D-M)^2
25.4	18.789003	4.7724	15.9182
19.05	28.64705	5.4573	2.3341
14.28	26.880515	3.8385	0.9864
10	15.097983	1.5098	5.7954
7	5.1823636	0.3628	4.3822
5	3.8702952	0.1935	4.8511
4	1.5327902	0.0613	2.2798

Sample Mean = 16.1956 mm

Sample Variance = 36.5471 mm^2

Sample Standard Deviation = 6.0454 mm


Areal Sample for columns 8-12 vs Actual Distribution
100
80
60
40
20
0
% Finer
1.0
10.0
100.0
Sieve Size (mm)
Actual Distribution
Areal Results

Appendix F. Laboratory Volume Data and Results for Glass Bead Experiments

The data obtained by volume sampling on the distribution of glass beads is shown here. Additionally, volume-by-weight analyses are included for the six individual samples and the total combined sample. A comparison is also made between the sample and actual grain size distribution curves. A discussion of the volume sampling on the glass bead distribution is provided in section 4.2.3.

Total Volume Sample

(1) Grain Size Analysis

Sieve Size (mm)	Mass of Particles (g)	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
25.4	3196.14	19.8617	100	1.00
19.05	4778.12	29.6926	80.1383	0.8014
14.28	3551.79	22.0718	50.4457	0.5045
10	2136.69	13.2780	28.3739	0.2837
7	1208.07	7.5073	15.0959	0.1510
5	759.58	4.7202	7.5887	0.0759
4	461.58	2.8684	2.8684	0.0287
total =	16091.99			

Sieve Size	Actual Distribution	Volume Results
32.0	100	100
22.6	80.039	80.1383
16.0	52.833	50.4457
11.3	29.913	28.3739
8.00	15.952	15.0959
5.66	7.982	7.5887
4.00	2.992	2.8684
3.36	0	0

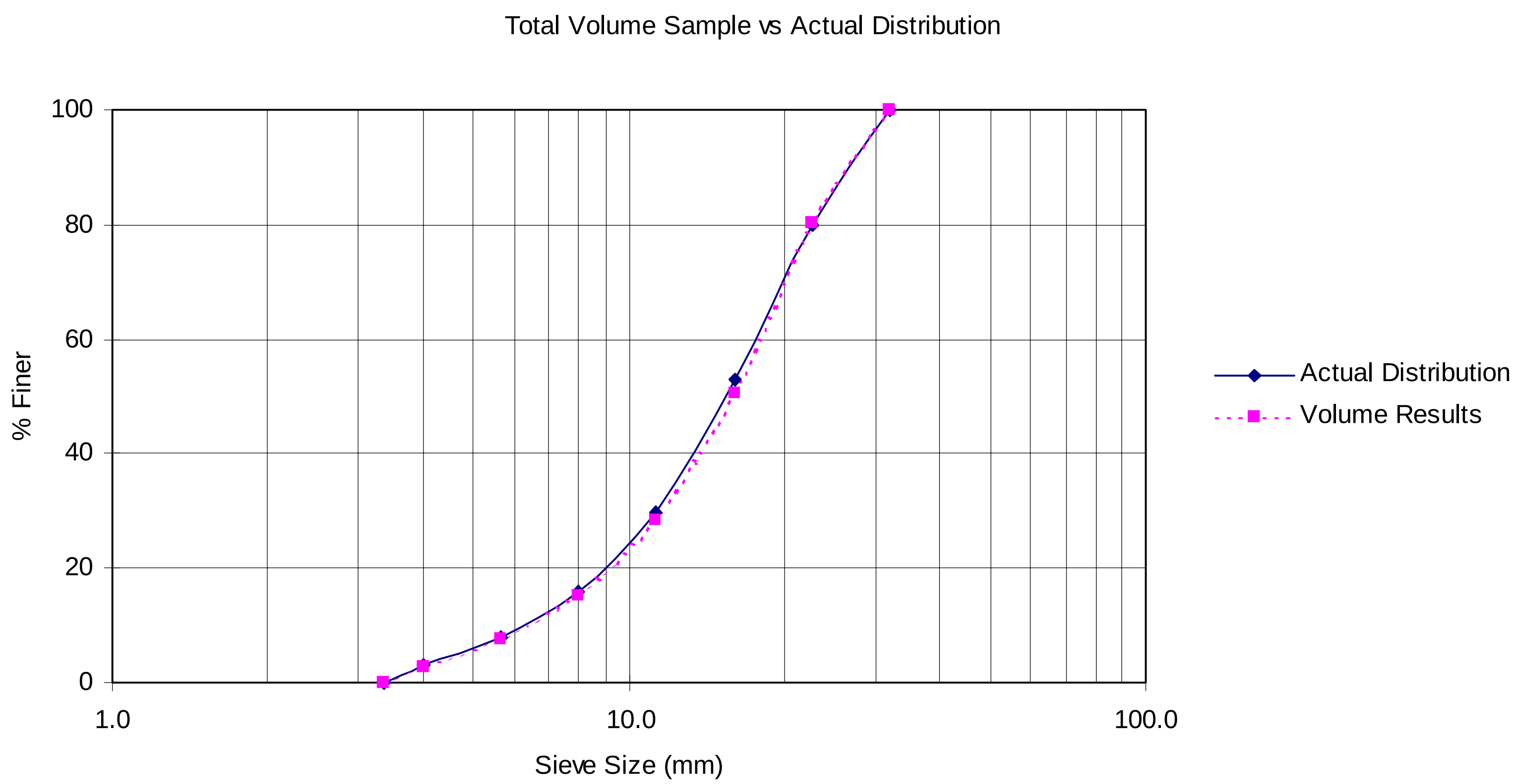
(2) Sample Statistics

Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*(D-M)^2
25.4	0.1986	5.0449	17.3368
19.05	0.2969	5.6564	2.6595
14.28	0.2207	3.1519	0.6971
10	0.1328	1.3278	4.8717
7	0.0751	0.5255	6.1585
5	0.0472	0.2360	5.7711
4	0.0287	0.1147	4.1700

Sample Mean = 16.0572 mm

Sample Variance = 41.6646 mm^2

Sample Standard Deviation = 6.4548 mm


Total Volume Sample vs Actual Distribution
100
80
60
40
20
0
% Finer
1.0
10.0
100.0
Sieve Size (mm)
Actual Distribution
Volume Results

First Volume Sample

(1) Grain Size Analysis

Sieve Size (mm)	Mass of Particles (g)	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
25.4	729.32	25.8625	100	1.00
19.05	886.85	31.4486	74.1375	0.7414
14.28	518.32	18.3801	42.6889	0.4269
10	338.88	12.0171	24.3089	0.2431
7	184.24	6.5335	12.2918	0.1229
5	96.90	3.4362	5.7583	0.0576
4	65.48	2.3221	2.3221	0.0232
total =	2820.00			

Sieve Size	Actual Distribution	Volume Results
32.0	100	100
22.6	80.039	74.1375
16.0	52.833	42.6889
11.3	29.913	24.3089
8.00	15.952	12.2918
5.66	7.982	5.7583
4.00	2.992	2.3221
3.36	0	0

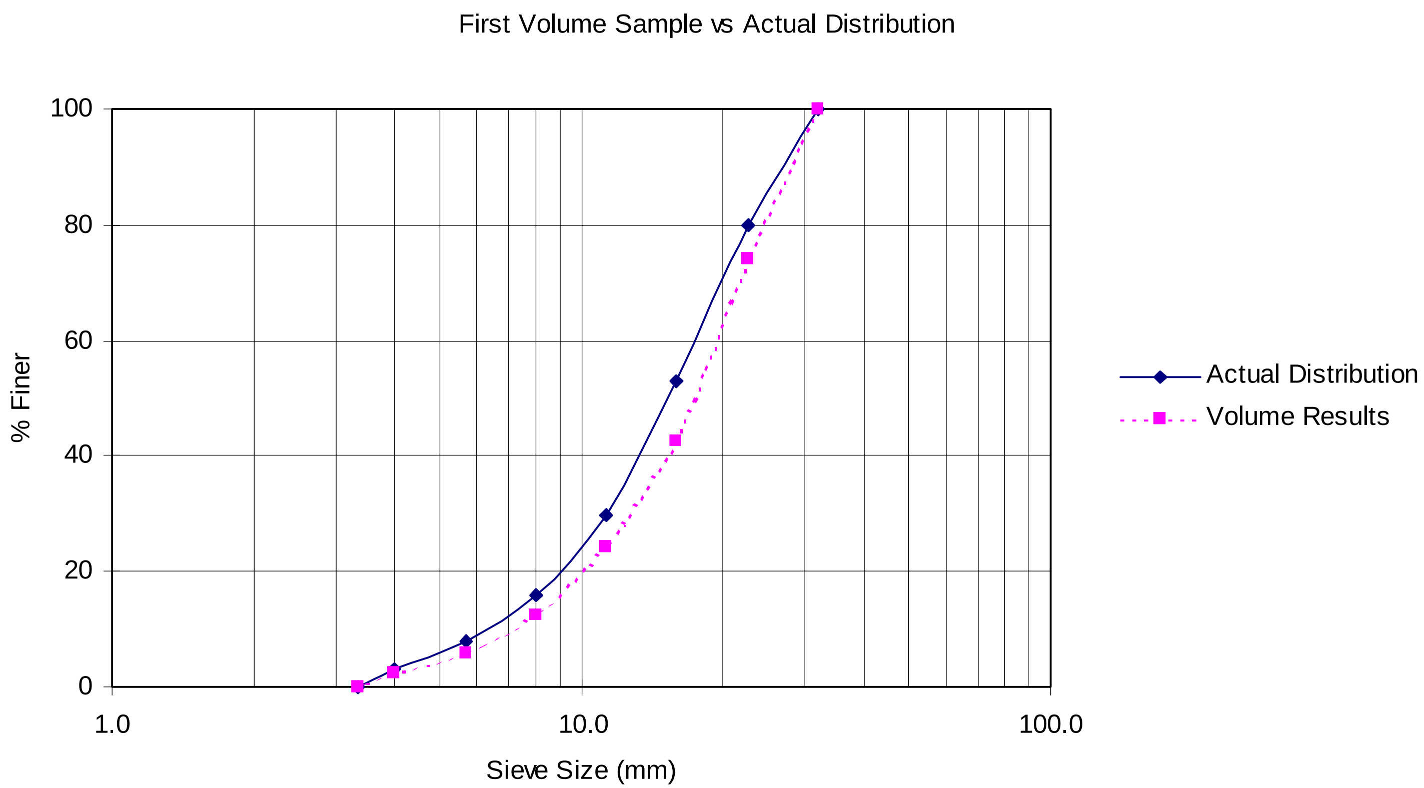
(2) Sample Statistics

Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*(D-M)^2
25.4	0.258625	6.5691	17.7804
19.05	0.314486	5.9910	1.1855
14.28	0.183801	2.6247	1.4704
10	0.120171	1.2017	6.0722
7	0.065335	0.4573	6.6760
5	0.034362	0.1718	5.0379
4	0.023221	0.0929	3.9901

Sample Mean = 17.1084 mm

Sample Variance = 42.2126 mm^2

Sample Standard Deviation = 6.4971 mm



First Volume Sample vs Actual Distribution
100
80
60
40
20
0
% Finer
1.0
10.0
100.0
Sieve Size (mm)
Actual Distribution
Volume Results

Second Volume Sample

(1) Grain Size Analysis

Sieve Size (mm)	Mass of Particles (g)	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
25.4	622.07	23.6561	100	1.00
19.05	895.90	34.0694	76.3439	0.7634
14.28	560.62	21.3193	42.2745	0.4227
10	234.74	8.9269	20.9552	0.2096
7	163.80	6.2289	12.0282	0.1203
5	99.80	3.7952	5.7993	0.0580
4	52.70	2.0041	2.0041	0.0200
total =	2629.63			

Sieve Size	Actual Distribution	Volume Results
32.0	100	100
22.6	80.039	76.3439
16.0	52.833	42.2745
11.3	29.913	20.9552
8.00	15.952	12.0282
5.66	7.982	5.7993
4.00	2.992	2.0041
3.36	0	0

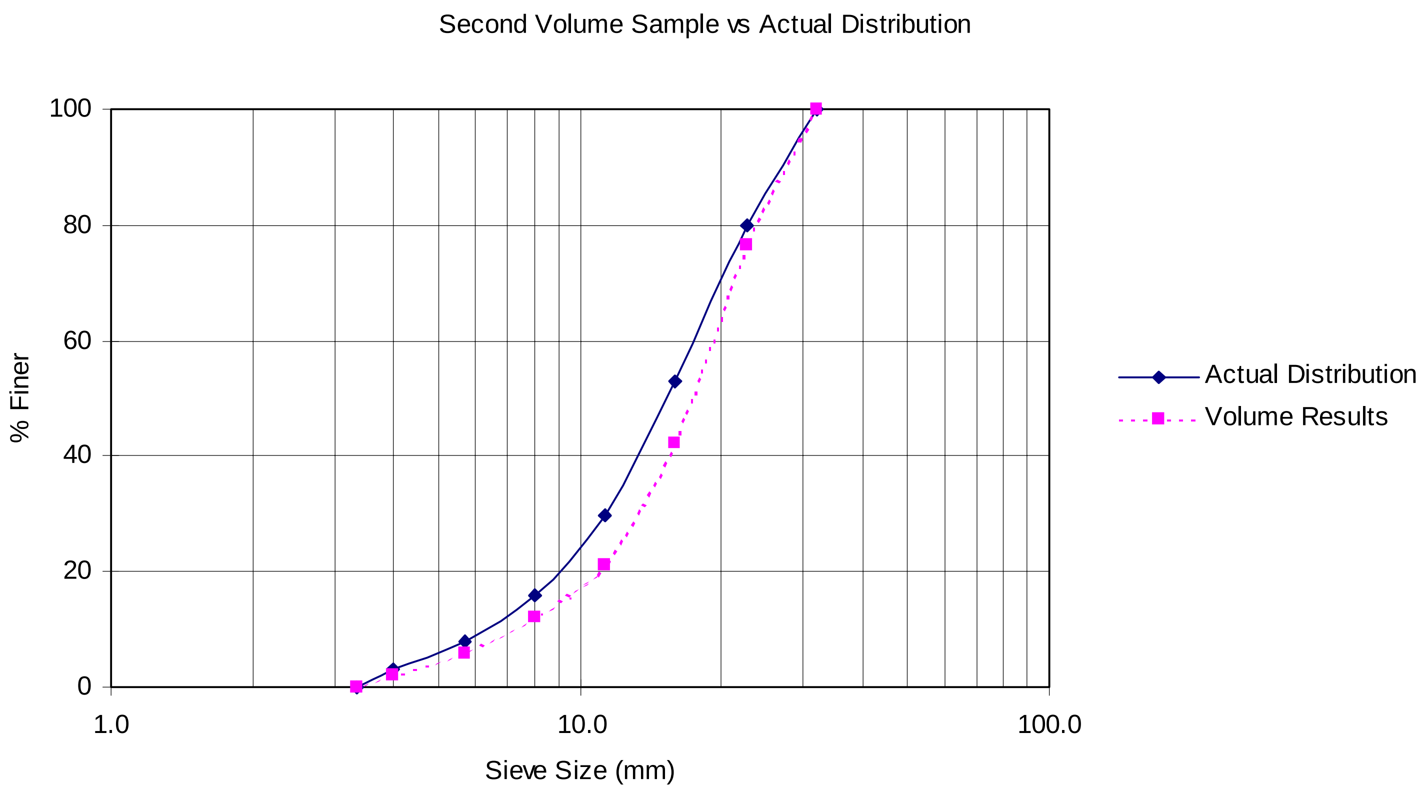
(2) Sample Statistics

Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*(D-M)^2
25.4	0.236561	6.0087	16.1325
19.05	0.340694	6.4902	1.2404
14.28	0.213193	3.0444	1.7462
10	0.089269	0.8927	4.5534
7	0.062289	0.4360	6.4070
5	0.037952	0.1898	5.5951
4	0.020041	0.0802	3.4613

Sample Mean = 17.1419 mm

Sample Variance = 39.1358 mm^2

Sample Standard Deviation = 6.2559 mm


Second Volume Sample vs Actual Distribution
100
80
60
40
20
0
% Finer
1.0
10.0
100.0
Sieve Size (mm)
Actual Distribution
Volume Results

Third Volume Sample

(1) Grain Size Analysis

Sieve Size (mm)	Mass of Particles (g)	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
25.4	579.17	20.6408	100	1.00
19.05	796.35	28.3810	79.3592	0.7936
14.28	633.06	22.5616	50.9782	0.5098
10	334.88	11.9347	28.4166	0.2842
7	206.24	7.3502	16.4818	0.1648
5	150.23	5.3539	9.1316	0.0913
4	106.00	3.7777	3.7777	0.0378
total =	2805.94			

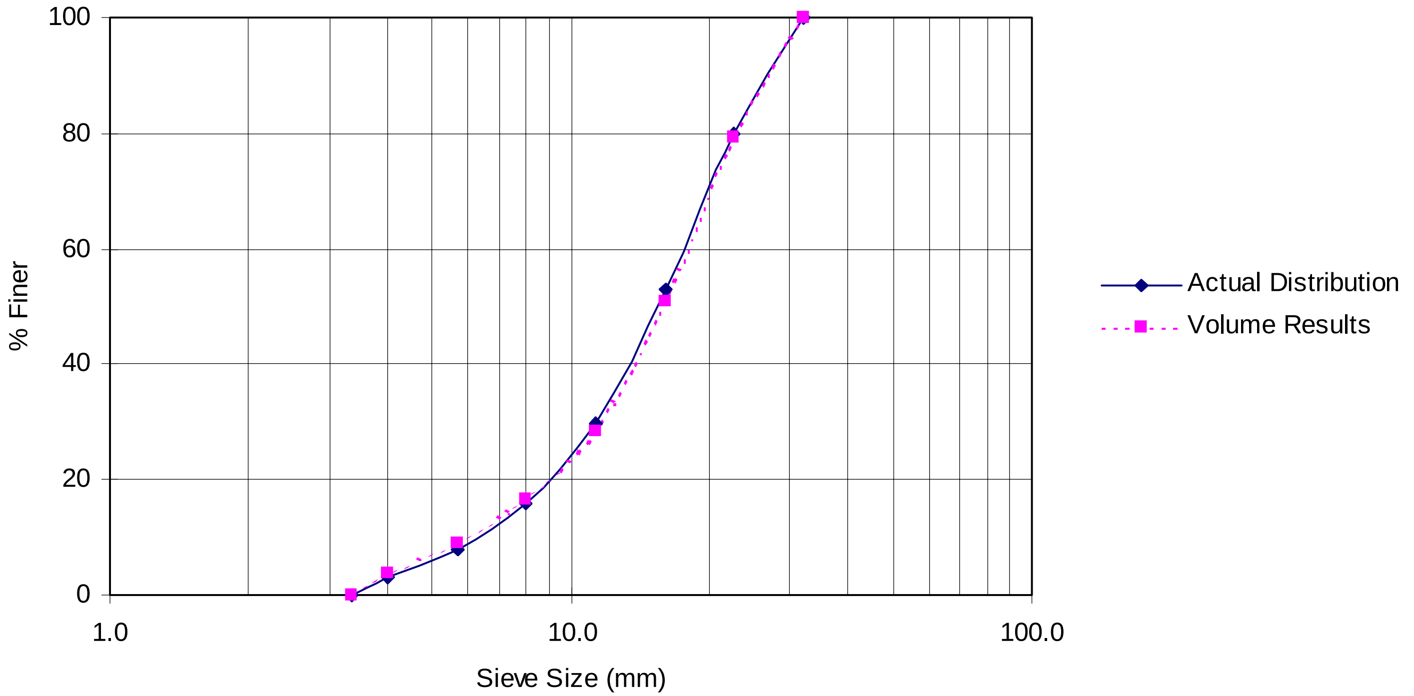
Sieve Size	Actual Distribution	Volume Results
32.0	100	100
22.6	80.039	79.3592
16.0	52.833	50.9782
11.3	29.913	28.4166
8.00	15.952	16.4818
5.66	7.982	9.1316
4.00	2.992	3.7777
3.36	0	0

(2) Sample Statistics

Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*(D-M)^2
25.4	0.206408	5.2428	18.2462
19.05	0.28381	5.4066	2.6437
14.28	0.225616	3.2218	0.6659
10	0.119347	1.1935	4.2935
7	0.073502	0.5145	5.9510
5	0.053539	0.2677	6.4758
4	0.037777	0.1511	5.4380

Sample Mean = 15.9979 mm
Sample Variance = 43.7141 mm^2
Sample Standard Deviation = 6.6117 mm

Third Volume Sample vs Actual Distribution



Fourth Volume Sample

(1) Grain Size Analysis

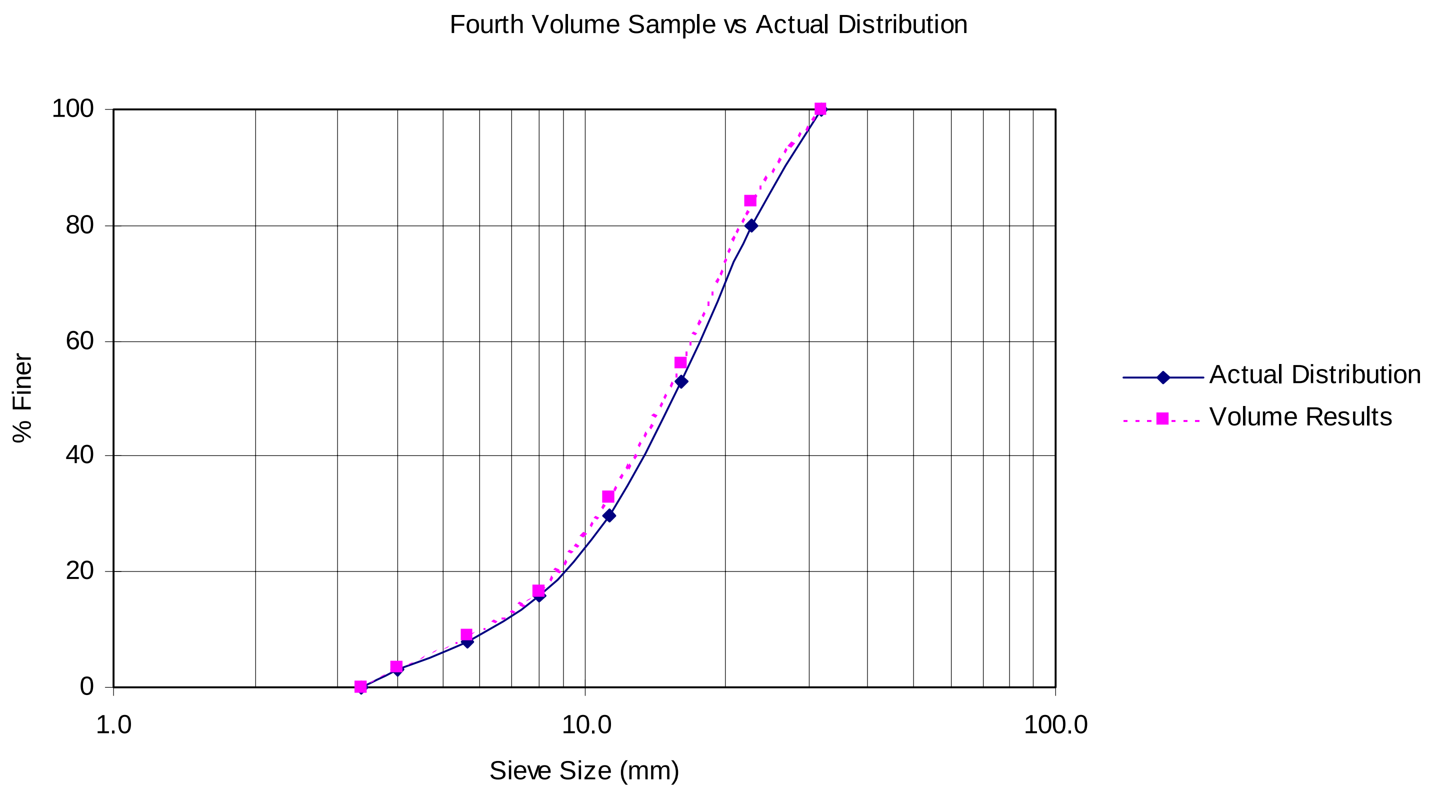
Sieve Size (mm)	Mass of Particles (g)	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
25.4	450.46	15.9264	100	1.00
19.05	796.35	28.1556	84.0736	0.8407
14.28	650.70	23.0059	55.9180	0.5592
10	456.54	16.1414	32.9121	0.3291
7	218.94	7.7409	16.7707	0.1677
5	156.10	5.5190	9.0298	0.0903
4	99.30	3.5108	3.5108	0.0351
total =	2828.40			

Sieve Size	Actual Distribution	Volume Results
32.0	100	100
22.6	80.039	84.0736
16.0	52.833	55.9180
11.3	29.913	32.9121
8.00	15.952	16.7707
5.66	7.982	9.0298
4.00	2.992	3.5108
3.36	0	0

(2) Sample Statistics

Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*(D-M)^2
25.4	0.159264	4.0453	16.3542
19.05	0.281556	5.3636	4.0303
14.28	0.230059	3.2852	0.2239
10	0.161414	1.6141	4.4771
7	0.077409	0.5419	5.2898
5	0.05519	0.2760	5.8172
4	0.035108	0.1404	4.4565

Sample Mean = 15.2666 mm
Sample Variance = 40.6490 mm^2
Sample Standard Deviation = 6.3757 mm



Fourth Volume Sample vs Actual Distribution
100
80
60
40
20
0
% Finer
1.0
10.0
100.0
Sieve Size (mm)
Actual Distribution
Volume Results

Fifth Volume Sample

(1) Grain Size Analysis

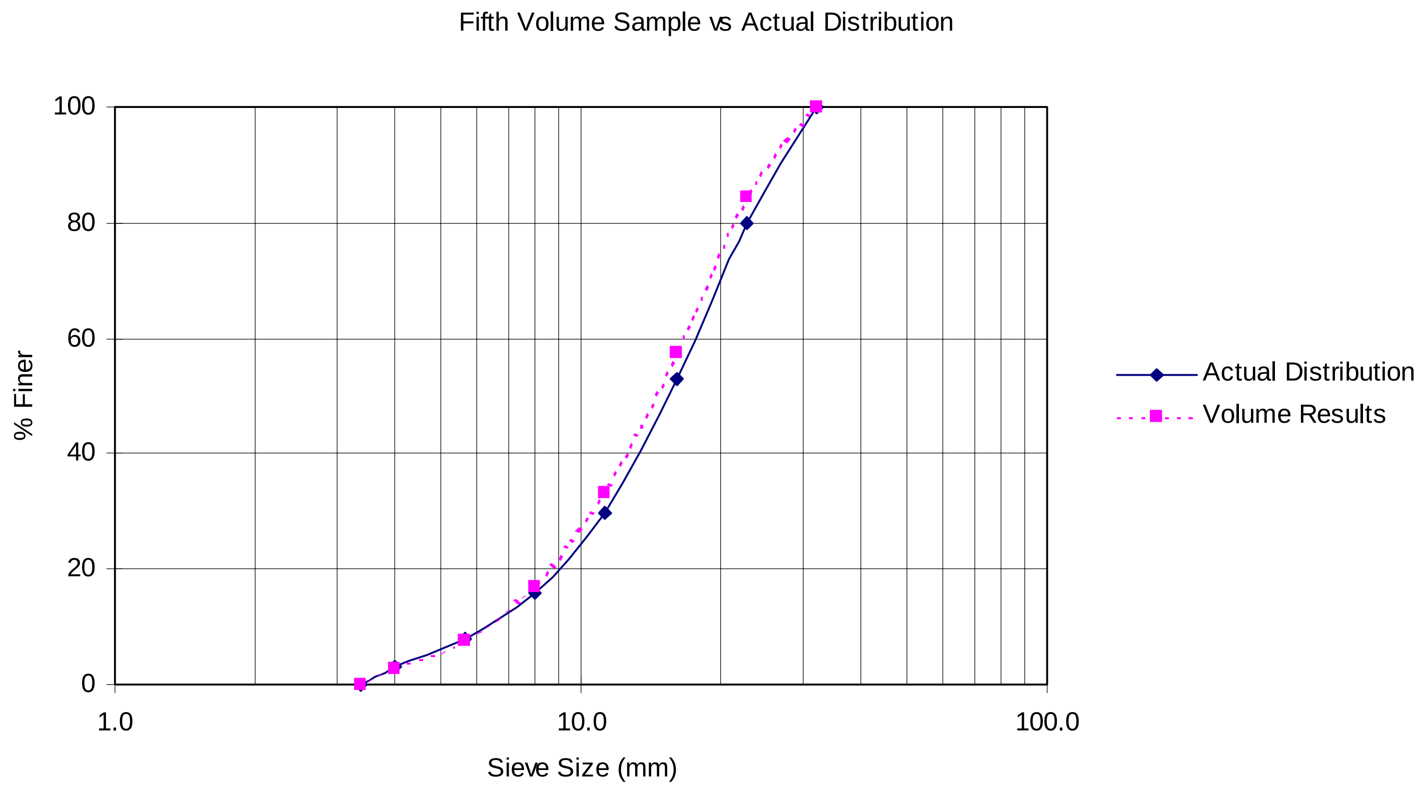
Sieve Size (mm)	Mass of Particles (g)	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
25.4	386.11	15.7404	100	1.00
19.05	660.61	26.9308	84.2596	0.8426
14.28	594.46	24.2342	57.3288	0.5733
10	392.33	15.9941	33.0945	0.3309
7	232.04	9.4597	17.1004	0.1710
5	122.73	5.0032	7.6408	0.0764
4	64.70	2.6376	2.6376	0.0264
total =	2452.99			

Sieve Size	Actual Distribution	Volume Results
32.0	100	100
22.6	80.039	84.2596
16.0	52.833	57.3288
11.3	29.913	33.0945
8.00	15.952	17.1004
5.66	7.982	7.6408
4.00	2.992	2.6376
3.36	0	0

(2) Sample Statistics

Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*(D-M)^2
25.4	0.157404	3.9981	16.3561
19.05	0.269308	5.1303	3.9788
14.28	0.242342	3.4606	0.2079
10	0.159941	1.5994	4.3353
7	0.094597	0.6622	6.3704
5	0.050032	0.2502	5.2117
4	0.026376	0.1055	3.3123

Sample Mean = 15.2063 mm
Sample Variance = 39.7726 mm^2
Sample Standard Deviation = 6.3066 mm


Fifth Volume Sample vs Actual Distribution
100
80
60
40
20
0
% Finer
1.0
10.0
100.0
Sieve Size (mm)
Actual Distribution
Volume Results

Sixth Volume Sample

(1) Grain Size Analysis

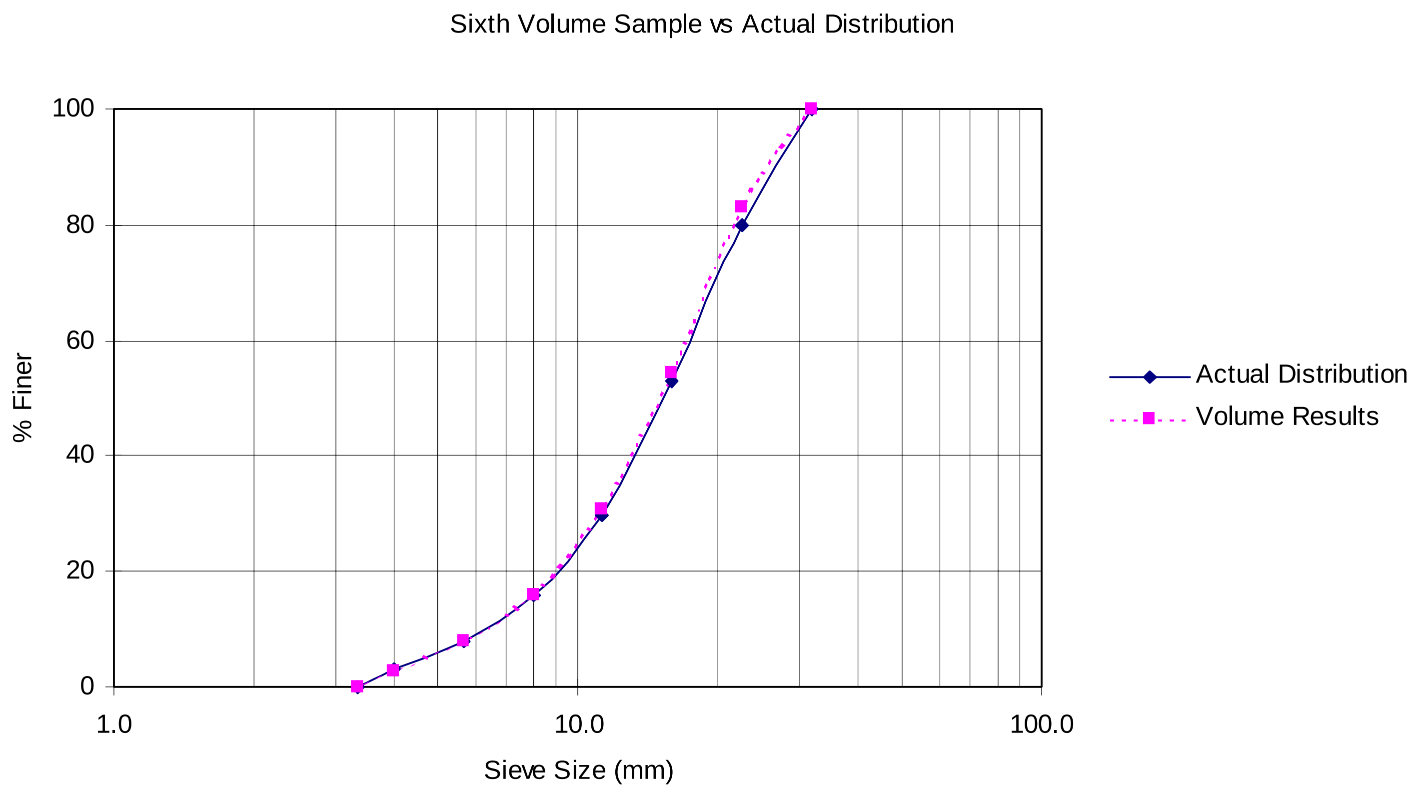
Sieve Size (mm)	Mass of Particles (g)	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
25.4	429.01	16.7909	100	1.00
19.05	742.06	29.0430	83.2091	0.8321
14.28	594.63	23.2730	54.1662	0.5417
10	379.31	14.8455	30.8932	0.3089
7	202.80	7.9371	16.0477	0.1605
5	133.83	5.2378	8.1106	0.0811
4	73.40	2.8728	2.8728	0.0287
total =	2555.03			

Sieve Size	Actual Distribution	Volume Results
32.0	100	100
22.6	80.039	83.2091
16.0	52.833	54.1662
11.3	29.913	30.8932
8.00	15.952	16.0477
5.66	7.982	8.1106
4.00	2.992	2.8728
3.36	0	0

(2) Sample Statistics

Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*(D-M)^2
25.4	0.167909	4.2649	16.3310
19.05	0.29043	5.5327	3.5824
14.28	0.23273	3.3234	0.3682
10	0.148455	1.4846	4.5529
7	0.079371	0.5556	5.7858
5	0.052378	0.2619	5.8164
4	0.028728	0.1149	3.8243

Sample Mean = 15.5379 mm
Sample Variance = 40.2611 mm^2
Sample Standard Deviation = 6.3452 mm


Sixth Volume Sample vs Actual Distribution
% Finer
100
80
60
40
20
0
1.0
10.0
100.0
Sieve Size (mm)
Actual Distribution
Volume Results

Appendix G. Laboratory Grid Data and Results for Natural Sediment Experiments

The data obtained by grid sampling on the distribution of natural sediment is shown here. Additionally, grid-by-number analyses for a variety of different samples sizes are included. The results of one 400-stone sample, four 100-stone samples, and eight 50-stone samples are shown. For each individual sample within the different sample sizes, no particle belongs to more than one sample. A comparison is also made between the sample and actual grain size distribution curves. A discussion of the grid sampling on the natural sediment distribution is provided in section 4.3.1.

Data Set For Laboratory Grid Sample

cm	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45
42	9.5	9.5	4.75	3.35	4.75	9.5	25	6.3	25	3.35	4.75	12.5	9.5	12.5	9.5
39	3.35	38.1	38.1	38.1	6.3	25	25	9.5	4.75	6.3	3.35	25	9.5	12.5	6.3
36	6.3	38.1	38.1	38.1	4.75	25	25	4.75	38.1	12.5	38.1	25	12.5	38.1	38.1
33	4.75	4.75	9.5	4.75	12.5	6.3	25	38.1	38.1	12.5	12.5	12.5	4.75	12.5	6.3
30	9.5	38.1	12.5	25	3.35	12.5	25	38.1	38.1	12.5	38.1	9.5	25	12.5	4.75
27	25	38.1	38.1	25	4.75	25	4.75	25	25	9.5	38.1	38.1	9.5	25	9.5
24	9.5	25	25	9.5	9.5	25	9.5	4.75	25	12.5	4.75	12.5	25	25	38.1
21	4.75	12.5	9.5	3.35	25	25	25	9.5	25	9.5	9.5	25	25	25	38.1
18	25	12.5	25	9.5	25	3.35	4.75	3.35	38.1	38.1	9.5	25	4.75	25	25
15	25	4.75	25	6.3	4.75	9.5	25	4.75	9.5	38.1	38.1	4.75	9.5	4.75	6.3
12	12.5	25	9.5	3.35	6.3	12.5	9.5	3.35	6.3	9.5	12.5	12.5	9.5	12.5	12.5
9	12.5	6.3	12.5	25	38.1	38.1	4.75	25	25	25	25	38.1	6.3	6.3	12.5
6	4.75	12.5	3.35	25	6.3	38.1	38.1	12.5	25	12.5	25	12.5	12.5	9.5	6.3
3	4.75	12.5	25	9.5	4.75	3.35	9.5	6.3	9.5	3.35	25	3.35	3.35	4.75	9.5

cm	48	51	54	57	60	63	66	69	72	75	78	81	84	87	90
42	6.3	6.3	3.35	9.5	25	38.1	38.1	6.3	25	4.75	38.1	6.3	9.5	25	25
39	38.1	6.3	9.5	4.75	25	38.1	38.1	4.75	4.75	38.1	38.1	25	6.3	25	12.5
36	38.1	9.5	25	4.75	9.5	12.5	9.5	38.1	38.1	6.3	38.1	25	12.5	12.5	6.3
33	25	25	25	12.5	4.75	25	9.5	25	9.5	9.5	6.3	9.5	6.3	25	12.5
30	12.5	25	25	25	25	25	25	25	6.3	4.75	12.5	25	9.5	3.35	12.5
27	9.5	4.75	25	12.5	12.5	25	9.5	6.3	12.5	38.1	38.1	9.5	9.5	6.3	38.1
24	38.1	6.3	9.5	6.3	3.35	25	12.5	25	25	38.1	4.75	6.3	9.5	6.3	38.1
21	38.1	4.75	4.75	6.3	12.5	25	12.5	25	12.5	38.1	6.3	6.3	6.3	4.75	9.5
18	12.5	3.35	25	38.1	25	25	6.3	12.5	4.75	12.5	12.5	38.1	6.3	12.5	6.3
15	25	12.5	12.5	9.5	9.5	9.5	25	38.1	4.75	12.5	12.5	9.5	25	25	6.3
12	25	3.35	25	38.1	38.1	4.75	12.5	4.75	4.75	38.1	38.1	25	25	4.75	6.3
9	25	9.5	4.75	38.1	38.1	12.5	6.3	25	25	6.3	25	9.5	9.5	12.5	6.3
6	25	25	12.5	6.3	12.5	12.5	38.1	38.1	25	9.5	25	3.35	3.35	6.3	25
3	12.5	9.5	25	6.3	12.5	25	4.75	38.1	25	12.5	38.1	38.1	9.5	12.5	4.75

400 Stone Grid Sample

(1) Grain Size Analysis

Sieve Size (mm)	No. of Particles	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
38.1	65	16.25	100	1.00
25	101	25.25	83.75	0.8375
12.5	66	16.5	58.5	0.5850
9.5	57	14.25	42	0.4200
6.3	46	11.5	27.75	0.2775
4.75	46	11.5	16.25	0.1625
3.35	19	4.75	4.75	0.0475
total =	400			

Sieve Size	Actual Distribution	Grid Results
50.8	100	100
38.1	85.476499	83.75
25.0	60.497787	58.5
12.5	39.529307	42
9.50	24.196051	27.75
6.30	12.09859	16.25
4.75	4.8388553	4.75
3.35	0	0

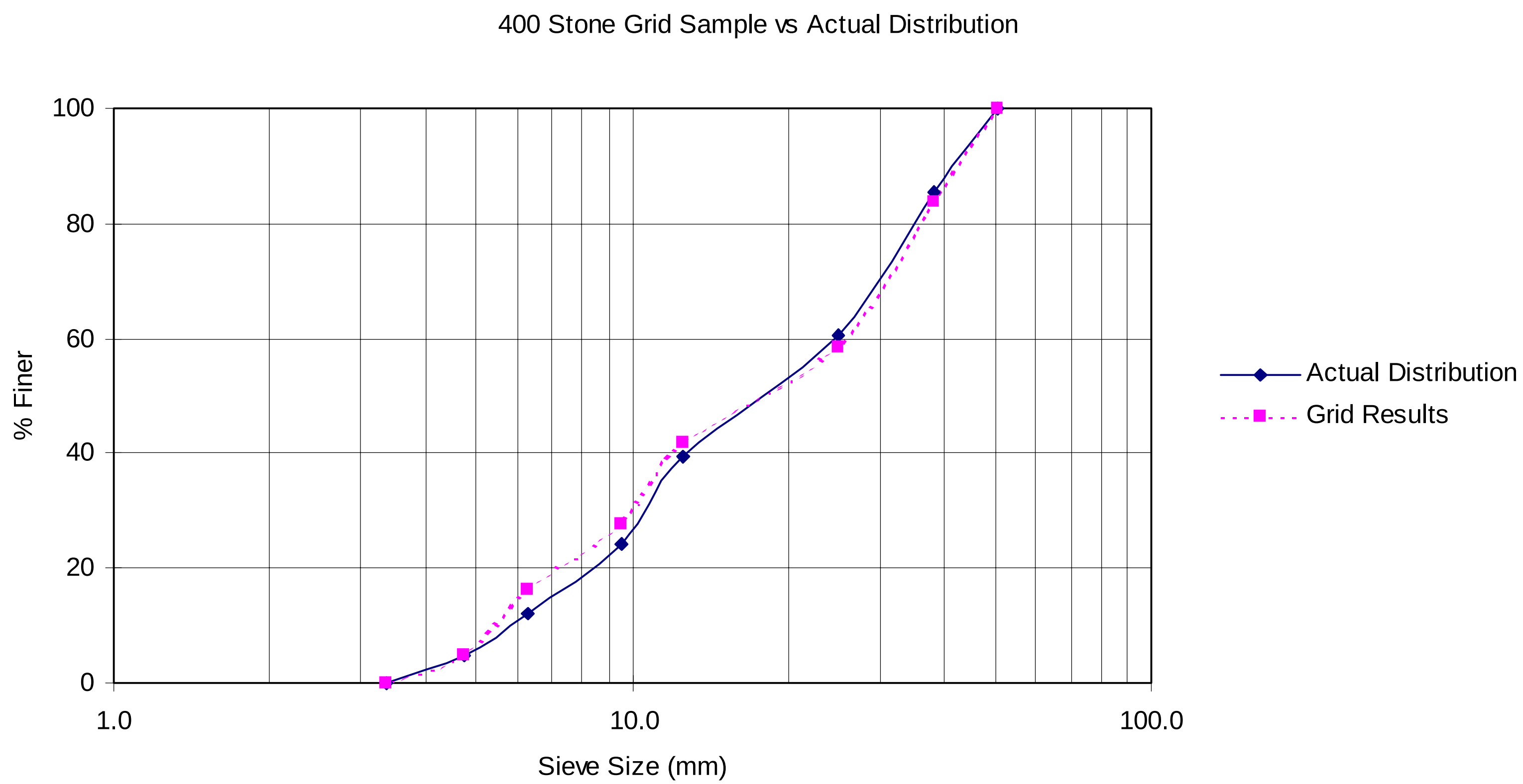
(2) Sample Statistics

Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*(D-M)^2
38.1	0.1625	6.1913	69.9672
25	0.2525	6.3125	14.7774
12.5	0.165	2.0625	3.8810
9.5	0.1425	1.3538	8.7809
6.3	0.115	0.7245	14.0415
4.75	0.115	0.5463	18.2570
3.35	0.0475	0.1591	9.3098

Sample Mean = 17.3499 mm

Sample Variance = 139.0149 mm^2

Sample Standard Deviation = 11.7905 mm


400 Stone Grid Sample vs Actual Distribution
% Finer
0
20
40
60
80
100
1.0
10.0
100.0
Sieve Size (mm)
Actual Distribution
Grid Results

First 100 Stone Grid Sample

(1) Grain Size Analysis

Sieve Size (mm)	No. of Particles	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
38.1	15	15	100	1.00
25	23	23	85	0.8500
12.5	18	18	62	0.6200
9.5	14	14	44	0.4400
6.3	11	11	30	0.3000
4.75	12	12	19	0.1900
3.35	7	7	7	0.0700
total =	100			

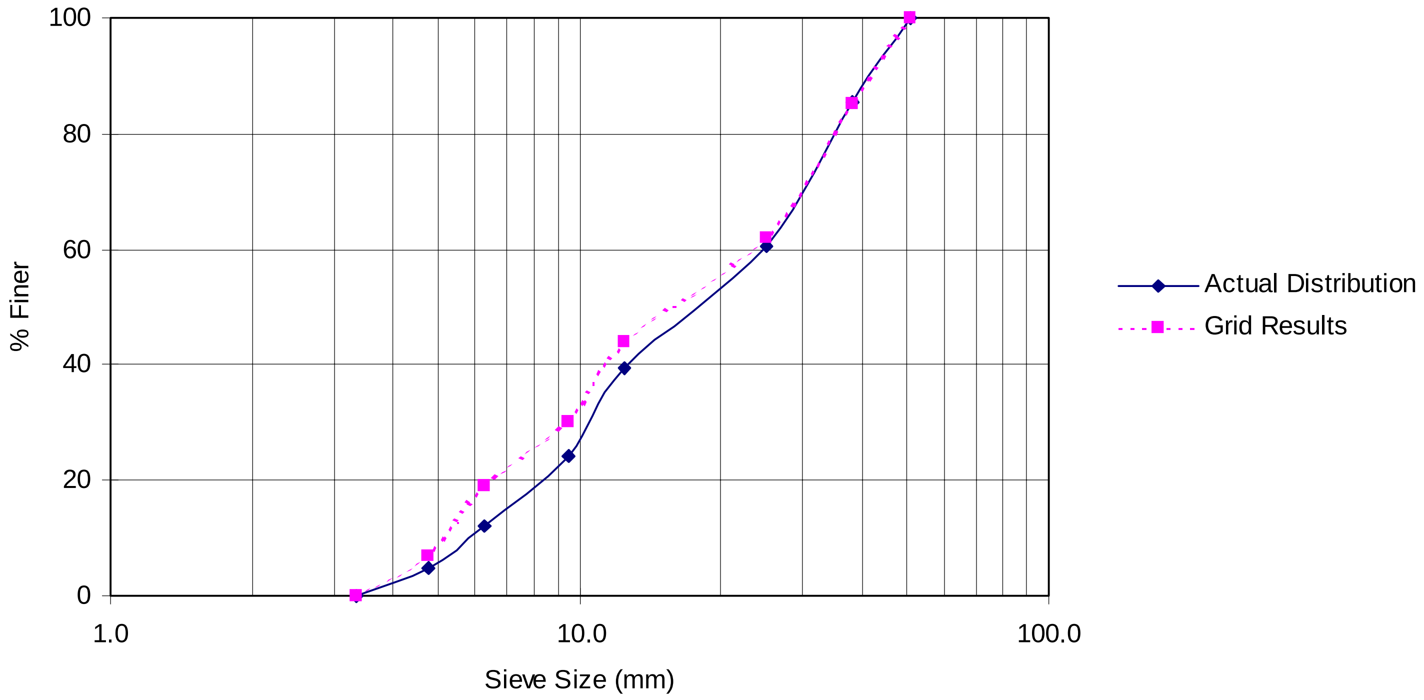
Sieve Size	Actual Distribution	Grid Results
50.8	100	100
38.1	85.476499	85
25.0	60.497787	62
12.5	39.529307	44
9.50	24.196051	30
6.30	12.09859	19
4.75	4.8388553	7
3.35	0	0

(2) Sample Statistics

Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*(D-M)^2
38.1	0.15	5.7150	69.7089
25	0.23	5.7500	16.4517
12.5	0.18	2.2500	2.9415
9.5	0.14	1.3300	6.9436
6.3	0.11	0.6930	11.5400
4.75	0.12	0.5700	16.6876
3.35	0.07	0.2345	12.1829

Sample Mean = 16.5425 mm
Sample Variance = 136.4562 mm^2
Sample Standard Deviation = 11.6814 mm

First 100 Stone Grid Sample vs Actual Distribution



Second 100 Stone Grid Sample

(1) Grain Size Analysis

Sieve Size (mm)	No. of Particles	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
38.1	17	17	100	1.00
25	27	27	83	0.8300
12.5	12	12	56	0.5600
9.5	13	13	44	0.4400
6.3	11	11	31	0.3100
4.75	16	16	20	0.2000
3.35	4	4	4	0.0400
total =	100			

Sieve Size	Actual Distribution	Grid Results
50.8	100	100
38.1	85.476499	83
25.0	60.497787	56
12.5	39.529307	44
9.50	24.196051	31
6.30	12.09859	20
4.75	4.8388553	4
3.35	0	0

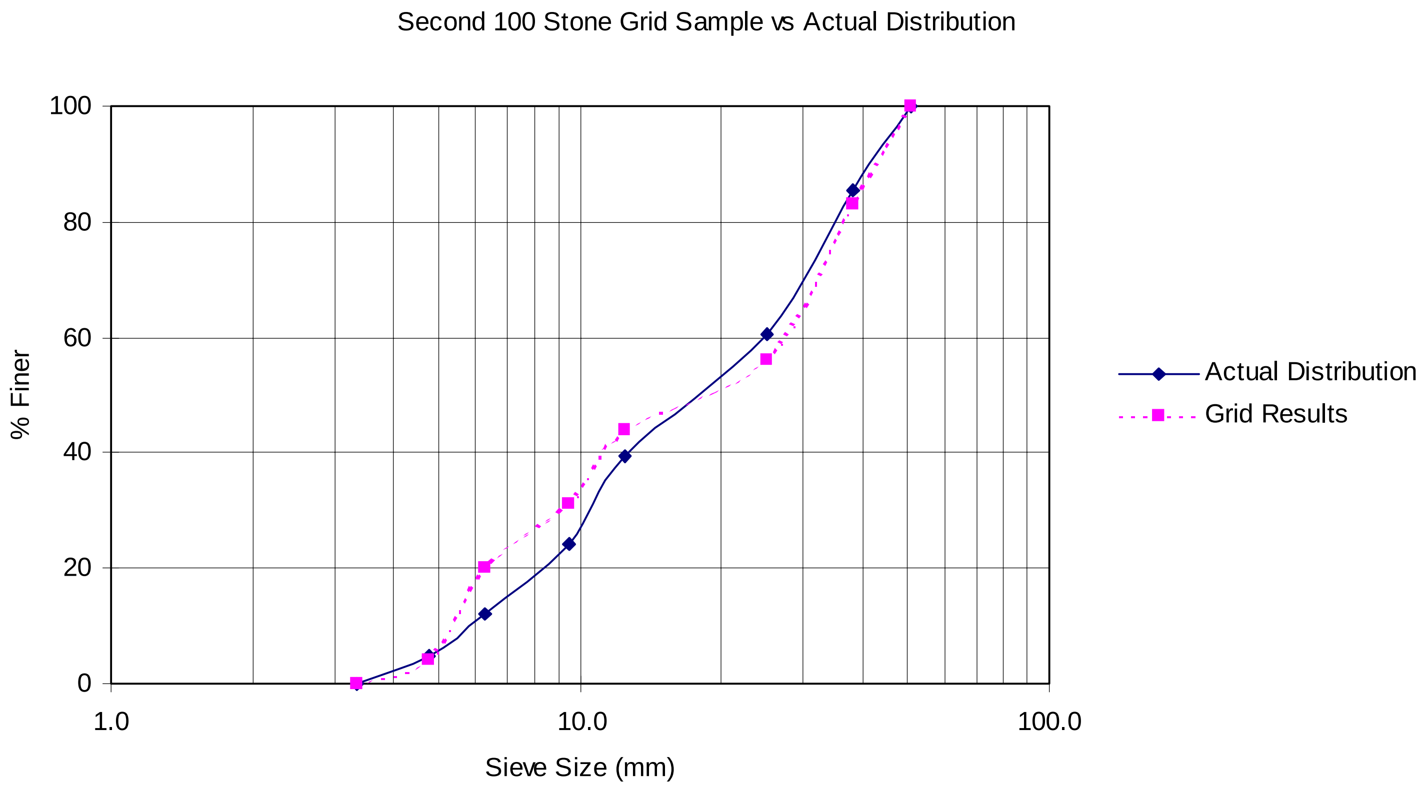
(2) Sample Statistics

Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*(D-M)^2
38.1	0.17	6.4770	71.7984
25	0.27	6.7500	14.9897
12.5	0.12	1.5000	3.0591
9.5	0.13	1.2350	8.4222
6.3	0.11	0.6930	13.9194
4.75	0.16	0.7600	26.2103
3.35	0.04	0.1340	8.0645

Sample Mean = 17.5490 mm

Sample Variance = 146.4636 mm^2

Sample Standard Deviation = 12.1022 mm



Second 100 Stone Grid Sample vs Actual Distribution
100
80
60
40
20
0
% Finer
1.0
10.0
100.0
Sieve Size (mm)
Actual Distribution
Grid Results

Third 100 Stone Grid Sample

(1) Grain Size Analysis

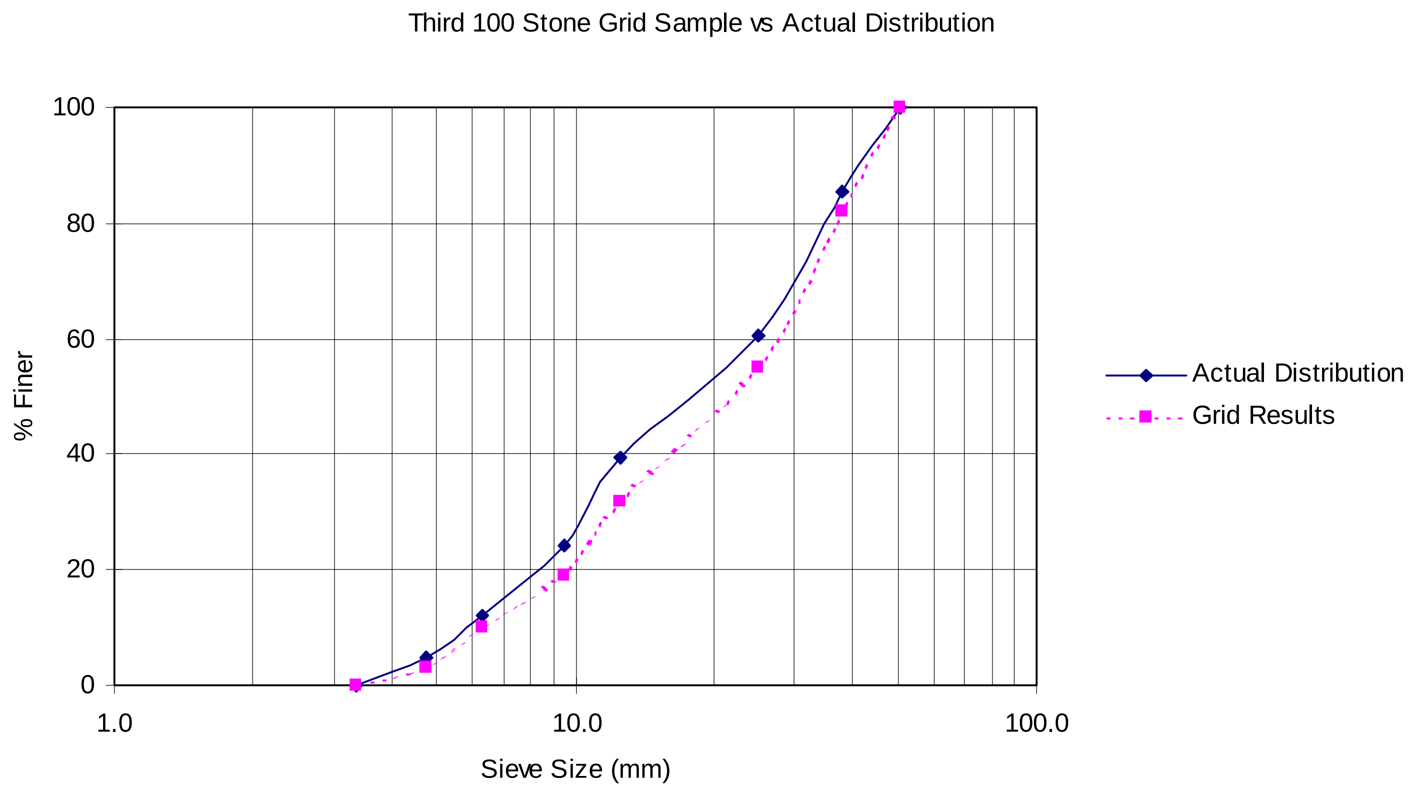
Sieve Size (mm)	No. of Particles	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
38.1	18	18	100	1.00
25	27	27	82	0.8200
12.5	23	23	55	0.5500
9.5	13	13	32	0.3200
6.3	9	9	19	0.1900
4.75	7	7	10	0.1000
3.35	3	3	3	0.0300
total =	100			

Sieve Size	Actual Distribution	Grid Results
50.8	100	100
38.1	85.476499	82
25.0	60.497787	55
12.5	39.529307	32
9.50	24.196051	19
6.30	12.09859	10
4.75	4.8388553	3
3.35	0	0

(2) Sample Statistics

Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*(D-M)^2
38.1	0.18	6.8580	67.6191
25	0.27	6.7500	10.6552
12.5	0.23	2.8750	8.8926
9.5	0.13	1.2350	11.0463
6.3	0.09	0.5670	13.8786
4.75	0.07	0.3325	13.6574
3.35	0.03	0.1005	7.0853

Sample Mean = 18.7180 mm
Sample Variance = 132.8344 mm^2
Sample Standard Deviation = 11.5254 mm


Third 100 Stone Grid Sample vs Actual Distribution
100
80
60
40
20
0
% Finer
1.0
10.0
100.0
Sieve Size (mm)
Actual Distribution
Grid Results

Fourth 100 Stone Grid Sample

(1) Grain Size Analysis

Sieve Size (mm)	No. of Particles	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
38.1	15	15	100	1.00
25	24	24	85	0.8500
12.5	13	13	61	0.6100
9.5	17	17	48	0.4800
6.3	15	15	31	0.3100
4.75	11	11	16	0.1600
3.35	5	5	5	0.0500
total =	100			

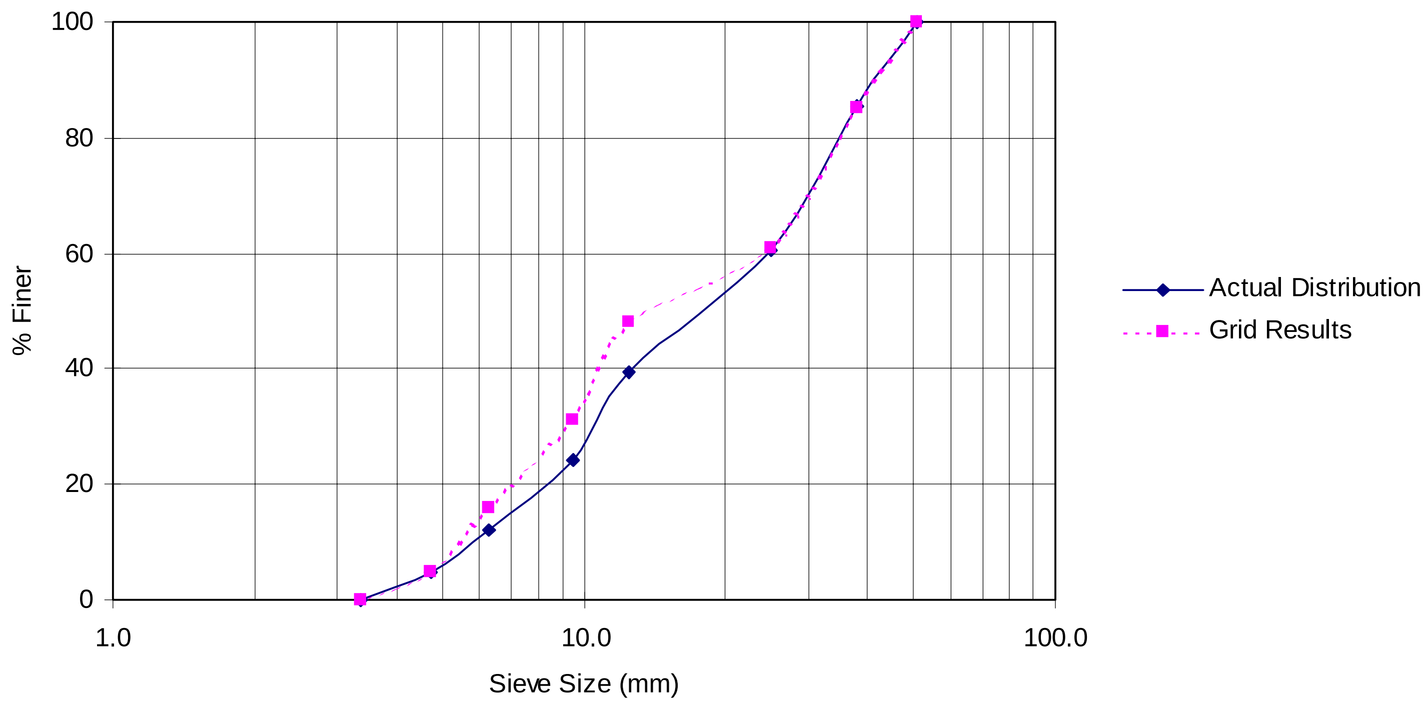
Sieve Size	Actual Distribution	Grid Results
50.8	100	100
38.1	85.476499	85
25.0	60.497787	61
12.5	39.529307	48
9.50	24.196051	31
6.30	12.09859	16
4.75	4.8388553	5
3.35	0	0

(2) Sample Statistics

Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*(D-M)^2
38.1	0.15	5.7150	69.4020
25	0.24	6.0000	16.9747
12.5	0.13	1.6250	2.1747
9.5	0.17	1.6150	8.5456
6.3	0.15	0.9450	15.8826
4.75	0.11	0.5225	15.4204
3.35	0.05	0.1675	8.7649

Sample Mean = 16.5900 mm
Sample Variance = 137.1649 mm^2
Sample Standard Deviation = 11.7117 mm

Fourth 100 Stone Grid Sample vs Actual Distribution



First 50 Stone Grid Sample

(1) Grain Size Analysis

Sieve Size (mm)	No. of Particles	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
38.1	5	10	100	1.00
25	14	28	90	0.9000
12.5	6	12	62	0.6200
9.5	7	14	50	0.5000
6.3	5	10	36	0.3600
4.75	10	20	26	0.2600
3.35	3	6	6	0.0600
total =	50			

Sieve Size	Actual Distribution	Grid Results
50.8	100	100
38.1	85.476499	90
25.0	60.497787	62
12.5	39.529307	50
9.50	24.196051	36
6.30	12.09859	26
4.75	4.8388553	6
3.35	0	0

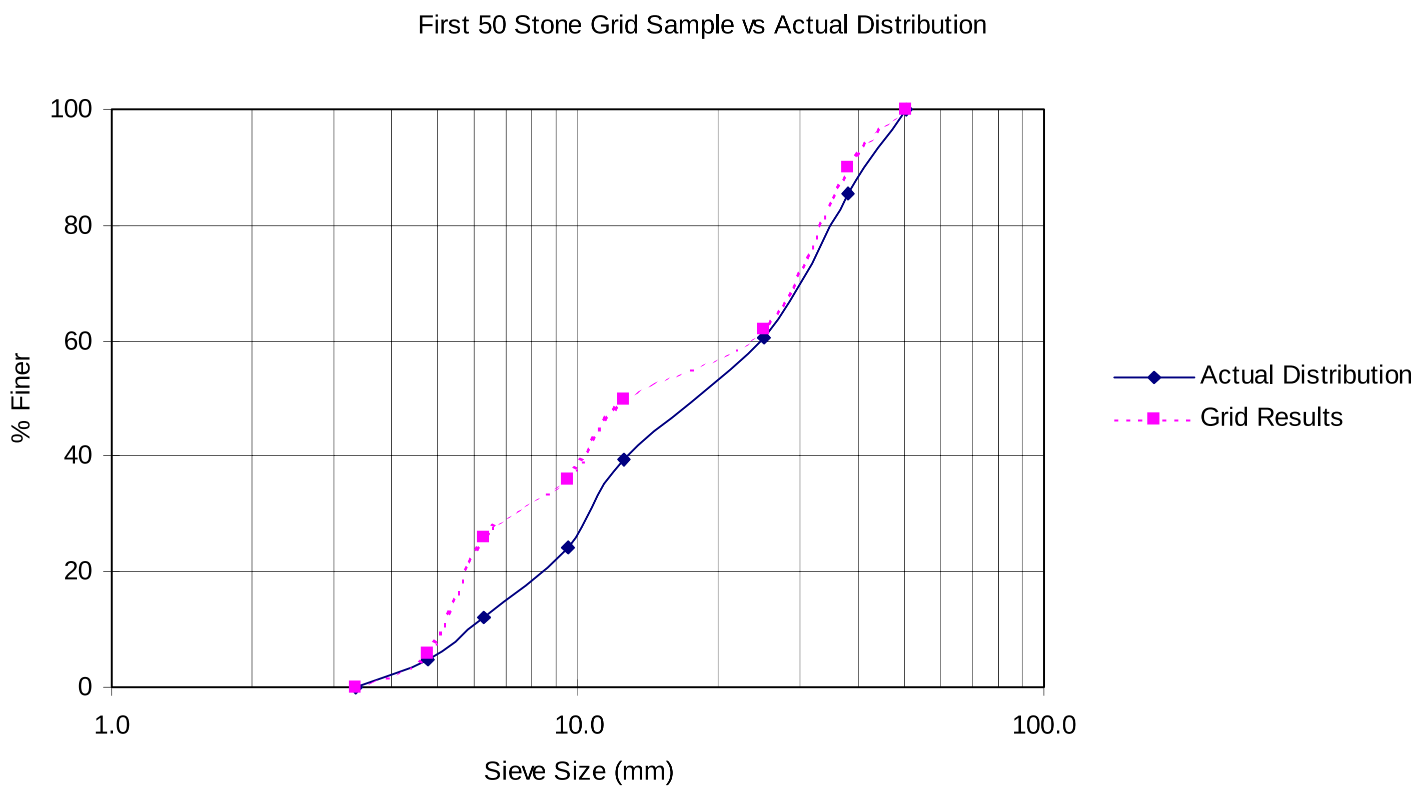
(2) Sample Statistics

Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*(D-M)^2
38.1	0.1	3.8100	51.4337
25	0.28	7.0000	25.6920
12.5	0.12	1.5000	1.0239
9.5	0.14	1.3300	4.9082
6.3	0.1	0.6300	8.3193
4.75	0.2	0.9500	22.7740
3.35	0.06	0.2010	8.7425

Sample Mean = 15.4210 mm

Sample Variance = 122.8936 mm^2

Sample Standard Deviation = 11.0857 mm


First 50 Stone Grid Sample vs Actual Distribution
100
80
60
40
20
0
% Finer
1.0
10.0
100.0
Sieve Size (mm)
Actual Distribution
Grid Results

Second 50 Stone Grid Sample

(1) Grain Size Analysis

Sieve Size (mm)	No. of Particles	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
38.1	9	18	100	1.00
25	13	26	82	0.8200
12.5	4	8	56	0.5600
9.5	9	18	48	0.4800
6.3	5	10	30	0.3000
4.75	7	14	20	0.2000
3.35	3	6	6	0.0600
total =	50			

Sieve Size	Actual Distribution	Grid Results
50.8	100	100
38.1	85.476499	82
25.0	60.497787	56
12.5	39.529307	48
9.50	24.196051	30
6.30	12.09859	20
4.75	4.8388553	6
3.35	0	0

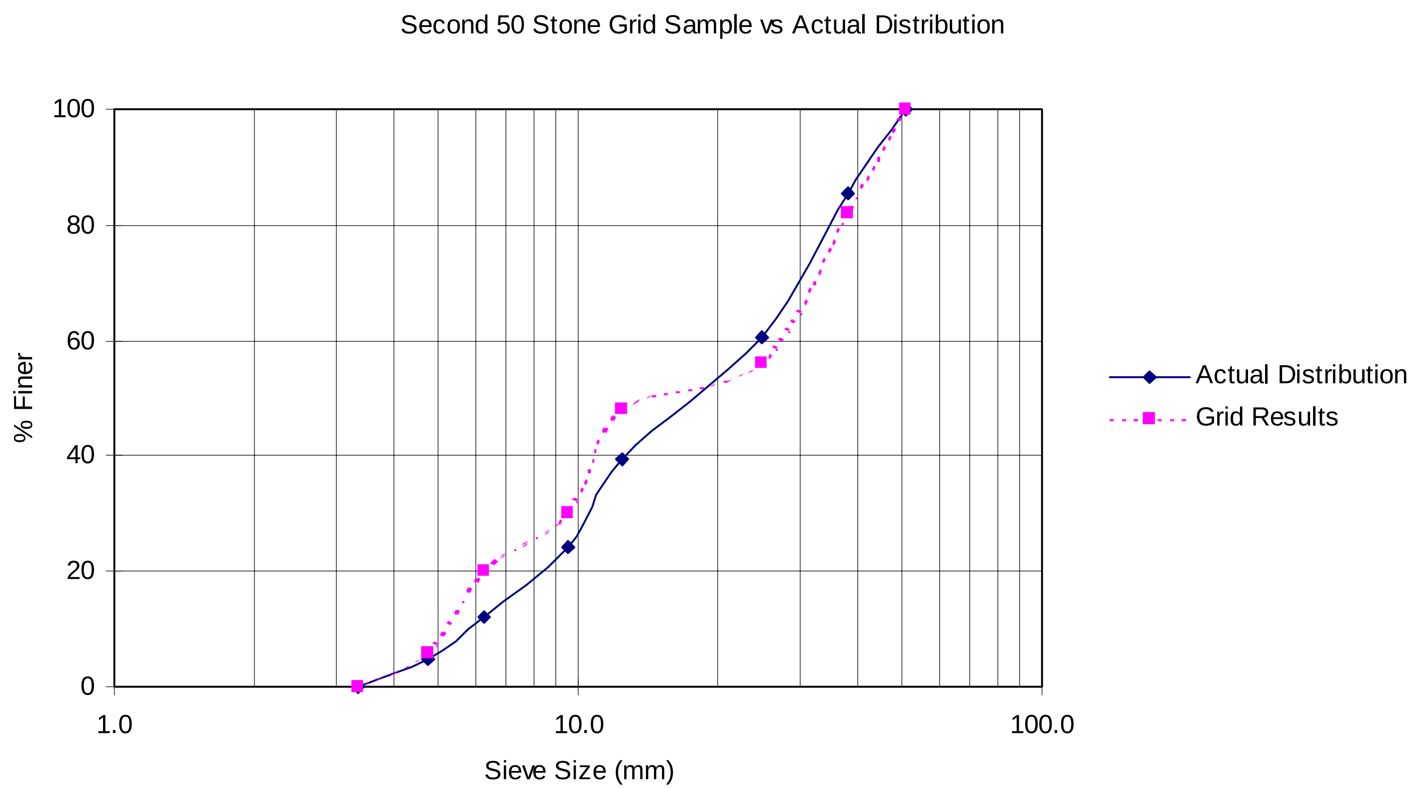
(2) Sample Statistics

Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*(D-M)^2
38.1	0.18	6.8580	75.9109
25	0.26	6.5000	14.3765
12.5	0.08	1.0000	2.0515
9.5	0.18	1.7100	11.7051
6.3	0.1	0.6300	12.6878
4.75	0.14	0.6650	22.9878
3.35	0.06	0.2010	12.1223

Sample Mean = 17.5640 mm

Sample Variance = 151.8418 mm^2

Sample Standard Deviation = 12.3224 mm


Second 50 Stone Grid Sample vs Actual Distribution
100
80
60
40
20
0
% Finer
1.0
10.0
100.0
Sieve Size (mm)
Actual Distribution
Grid Results

Thrid 50 Stone Grid Sample

(1) Grain Size Analysis

Sieve Size (mm)	No. of Particles	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
38.1	9	18	100	1.00
25	14	28	82	0.8200
12.5	14	28	54	0.5400
9.5	7	14	26	0.2600
6.3	4	8	12	0.1200
4.75	1	2	4	0.0400
3.35	1	2	2	0.0200
total =	50			

Sieve Size	Actual Distribution	Grid Results
50.8	100	100
38.1	85.4764992	82
25.0	60.4977873	54
12.5	39.5293067	26
9.50	24.1960507	12
6.30	12.0985898	4
4.75	4.83885534	2
3.35	0	0

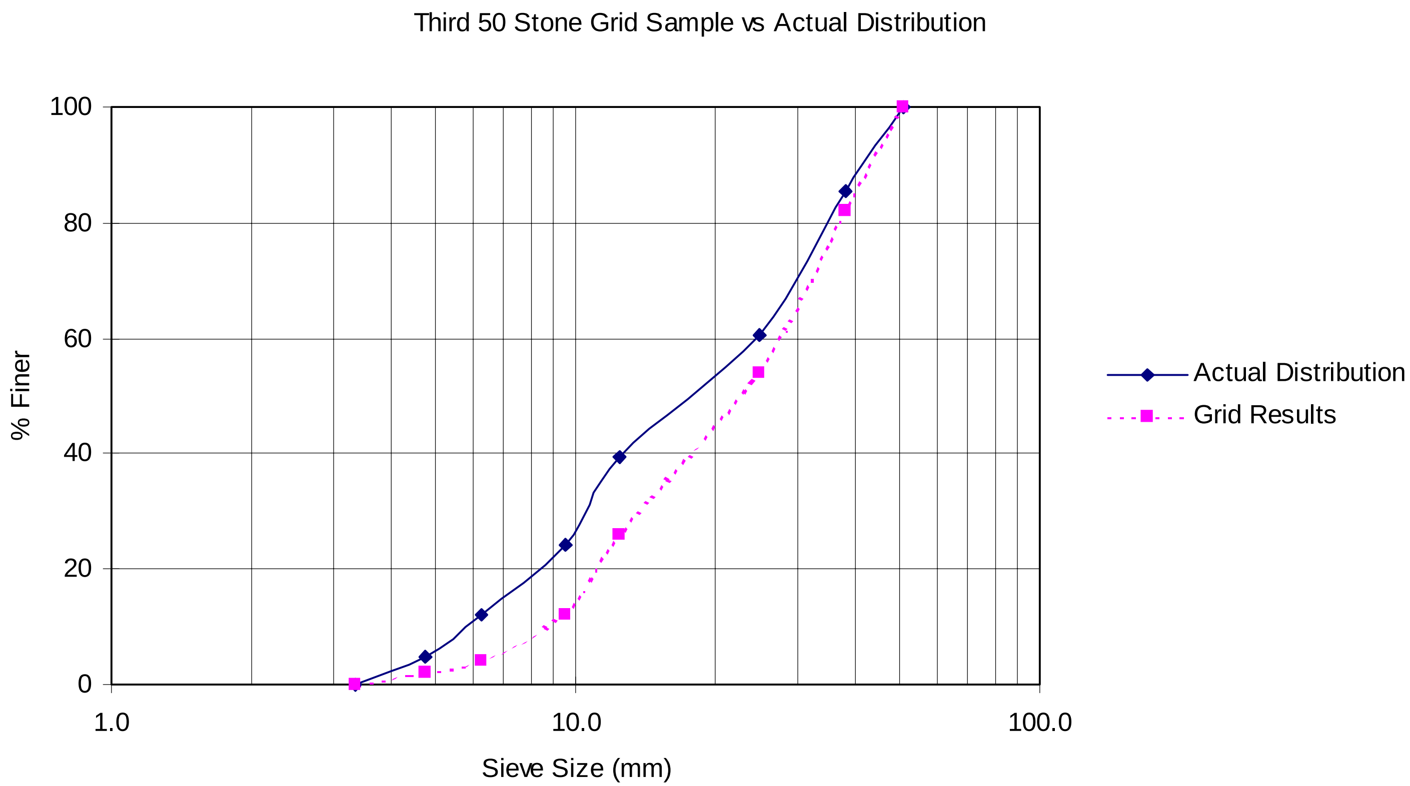
(2) Sample Statistics

Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*(D-M)^2
38.1	0.18	6.8580	63.2543
25	0.28	7.0000	8.9256
12.5	0.28	3.5000	13.1536
9.5	0.14	1.3300	13.5942
6.3	0.08	0.5040	13.6326
4.75	0.02	0.0950	4.2655
3.35	0.02	0.0670	5.1226

Sample Mean = 19.3540 mm

Sample Variance = 121.9484 mm^2

Sample Standard Deviation = 11.0430 mm


Third 50 Stone Grid Sample vs Actual Distribution
100
80
60
40
20
0
% Finer
1.0
10.0
100.0
Sieve Size (mm)
Actual Distribution
Grid Results

Fourth 50 Stone Grid Sample

(1) Grain Size Analysis

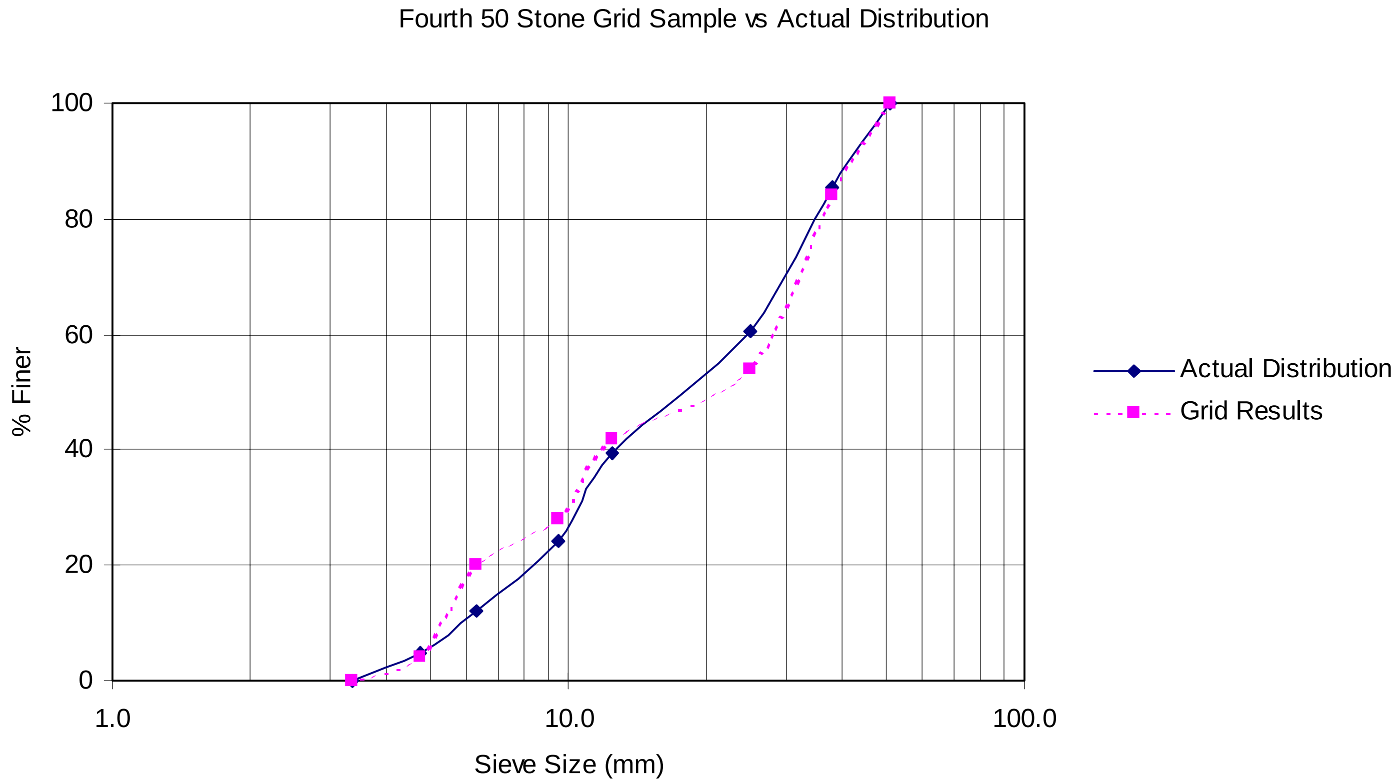
Sieve Size (mm)	No. of Particles	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
38.1	8	16	100	1.00
25	15	30	84	0.8400
12.5	6	12	54	0.5400
9.5	7	14	42	0.4200
6.3	4	8	28	0.2800
4.75	8	16	20	0.2000
3.35	2	4	4	0.0400
total =	50			

Sieve Size	Actual Distribution	Grid Results
50.8	100	100
38.1	85.476499	84
25.0	60.497787	54
12.5	39.529307	42
9.50	24.196051	28
6.30	12.09859	20
4.75	4.8388553	4
3.35	0	0

(2) Sample Statistics

Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*(D-M)^2
38.1	0.16	6.0960	65.7786
25	0.3	7.5000	15.4485
12.5	0.12	1.5000	3.4014
9.5	0.14	1.3300	9.7005
6.3	0.08	0.5040	10.6242
4.75	0.16	0.7600	27.3487
3.35	0.04	0.1340	8.3799

Sample Mean = 17.8240 mm
Sample Variance = 140.6817 mm^2
Sample Standard Deviation = 11.8609 mm


Fourth 50 Stone Grid Sample vs Actual Distribution
100
80
60
40
20
0
% Finer
1.0
10.0
100.0
Sieve Size (mm)
Actual Distribution
Grid Results

Fifth 50 Stone Grid Sample

(1) Grain Size Analysis

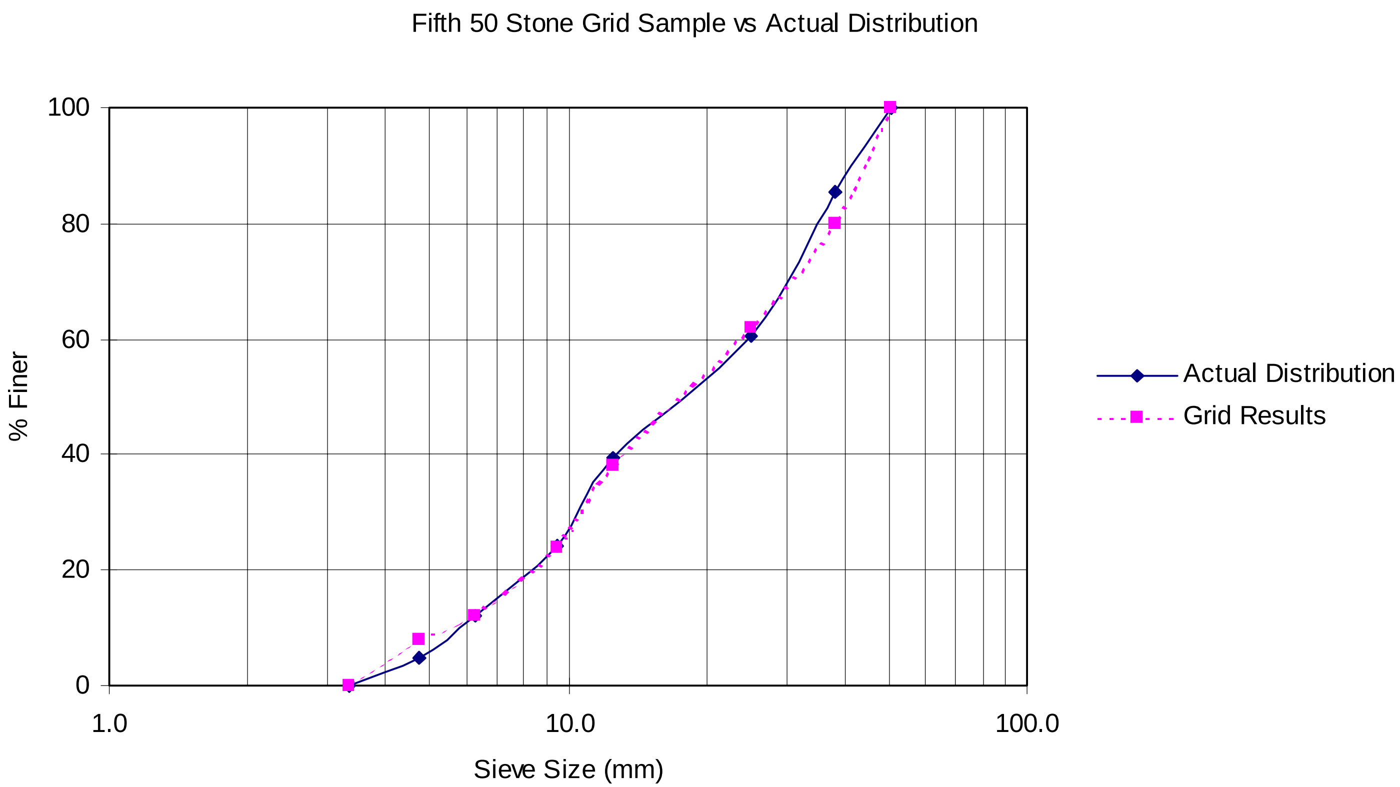
Sieve Size (mm)	No. of Particles	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
38.1	10	20	100	1.00
25	9	18	80	0.8000
12.5	12	24	62	0.6200
9.5	7	14	38	0.3800
6.3	6	12	24	0.2400
4.75	2	4	12	0.1200
3.35	4	8	8	0.0800
total =	50			

Sieve Size	Actual Distribution	Grid Results
50.8	100	100
38.1	85.476499	80
25.0	60.497787	62
12.5	39.529307	38
9.50	24.196051	24
6.30	12.09859	12
4.75	4.8388553	8
3.35	0	0

(2) Sample Statistics

Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*(D-M)^2
38.1	0.2	7.6200	83.5260
25	0.18	4.5000	9.6870
12.5	0.24	3.0000	6.4001
9.5	0.14	1.3300	9.3311
6.3	0.12	0.7560	15.4969
4.75	0.04	0.1900	6.6709
3.35	0.08	0.2680	16.3912

Sample Mean = 17.6640 mm
Sample Variance = 147.5032 mm^2
Sample Standard Deviation = 12.1451 mm



Fifth 50 Stone Grid Sample vs Actual Distribution
100
80
60
40
20
0
% Finer
1.0
10.0
100.0
Sieve Size (mm)
Actual Distribution
Grid Results

Sixth 50 Stone Grid Sample

(1) Grain Size Analysis

Sieve Size (mm)	No. of Particles	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
38.1	8	16	100	1.00
25	14	28	84	0.8400
12.5	8	16	56	0.5600
9.5	4	8	40	0.4000
6.3	6	12	32	0.3200
4.75	9	18	20	0.2000
3.35	1	2	2	0.0200
total =	50			

Sieve Size	Actual Distribution	Grid Results
50.8	100	100
38.1	85.476499	84
25.0	60.497787	56
12.5	39.529307	40
9.50	24.196051	32
6.30	12.09859	20
4.75	4.8388553	2
3.35	0	0

(2) Sample Statistics

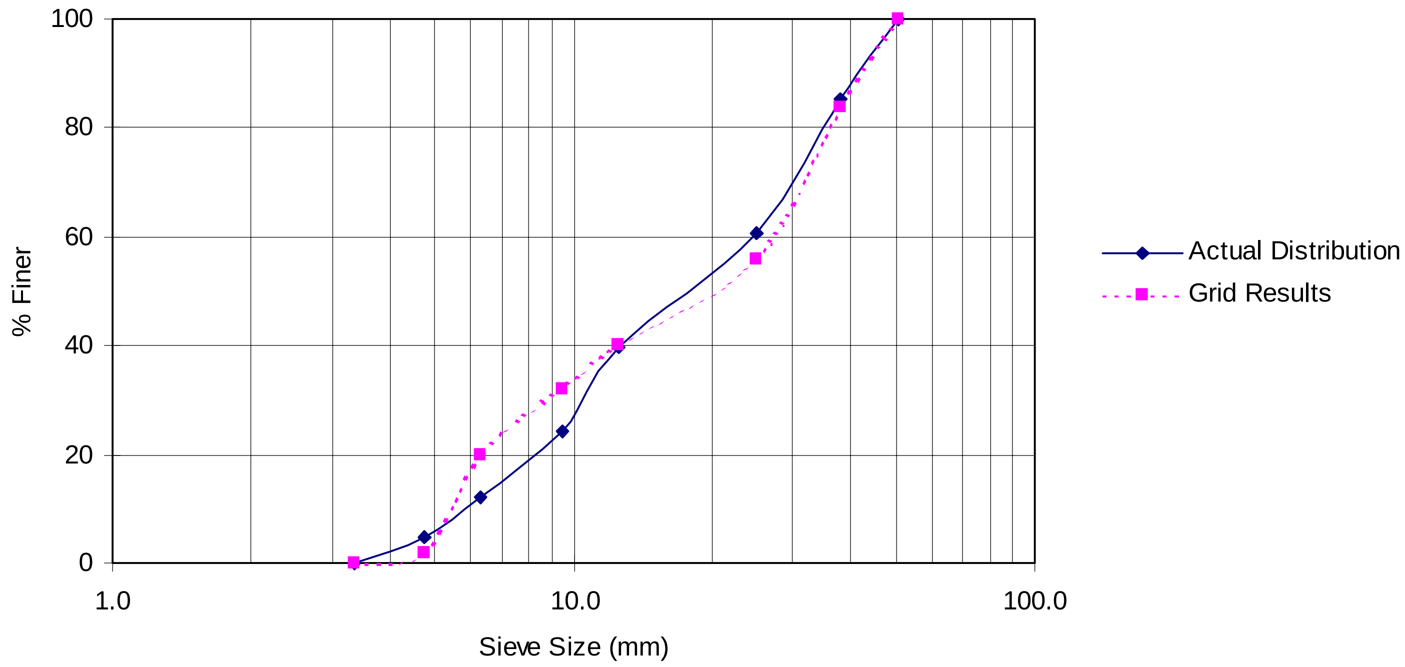
Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*(D-M)^2
38.1	0.16	6.0960	67.6737
25	0.28	7.0000	15.6075
12.5	0.16	2.0000	4.0546
9.5	0.08	0.7600	5.1636
6.3	0.12	0.7560	15.1443
4.75	0.18	0.8550	29.4175
3.35	0.02	0.0670	4.0237

Sample Mean = 17.5340 mm

Sample Variance = 141.0849 mm^2

Sample Standard Deviation = 11.8779 mm

Sixth 50 Stone Grid Sample vs Actual Distribution



Seventh 50 Stone Grid Sample

(1) Grain Size Analysis

Sieve Size (mm)	No. of Particles	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
38.1	9	18	100	1.00
25	13	26	82	0.8200
12.5	9	18	56	0.5600
9.5	6	12	38	0.3800
6.3	5	10	26	0.2600
4.75	6	12	16	0.1600
3.35	2	4	4	0.0400
total =	50			

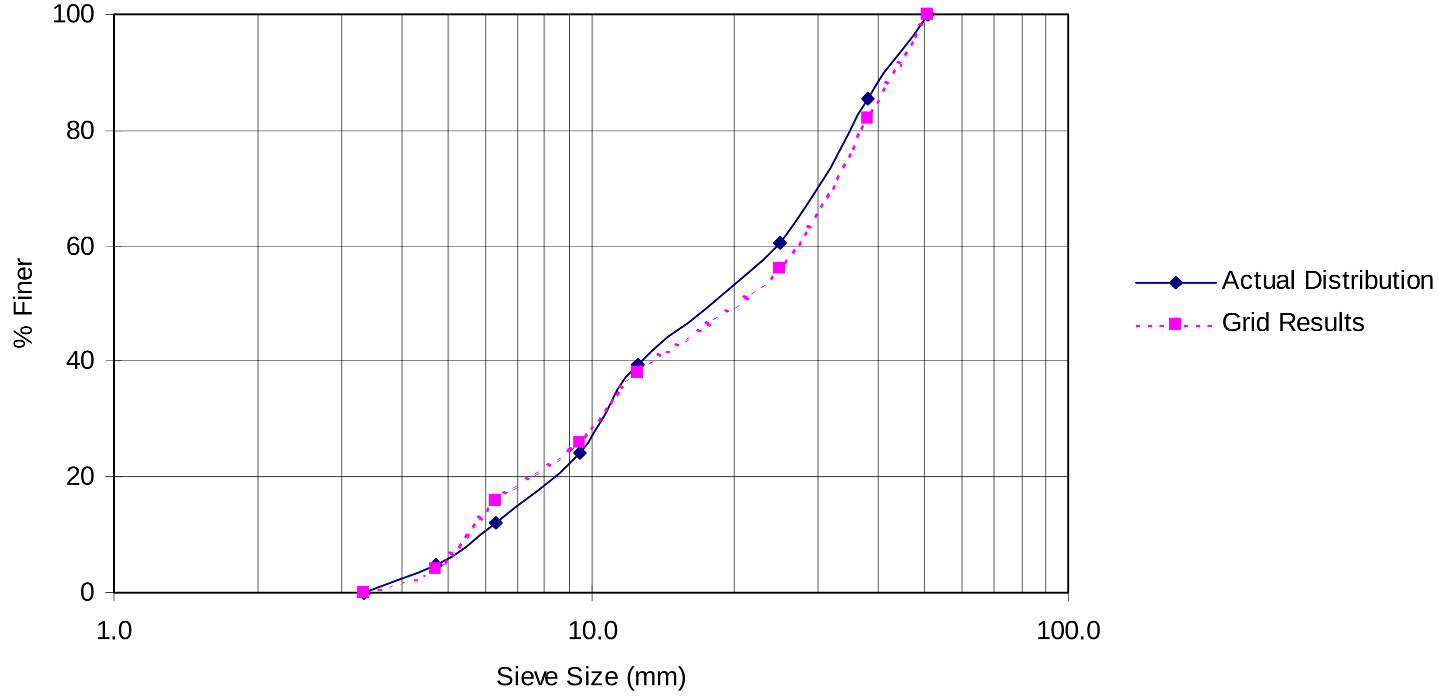
Sieve Size	Actual Distribution	Grid Results
50.8	100	100
38.1	85.4764992	82
25.0	60.4977873	56
12.5	39.5293067	38
9.50	24.1960507	26
6.30	12.0985898	16
4.75	4.83885534	4
3.35	0	0

(2) Sample Statistics

Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*(D-M)^2
38.1	0.18	6.8580	72.1297
25	0.26	6.5000	12.4433
12.5	0.18	2.2500	5.6086
9.5	0.12	1.1400	8.8381
6.3	0.1	0.6300	13.8816
4.75	0.12	0.5700	21.3291
3.35	0.04	0.1340	8.6813

Sample Mean =	18.0820	mm
Sample Variance =	142.9115	mm^2
Sample Standard Deviation =	11.9546	mm

Seventh 50 Stone Grid Sample vs Actual Distribution



Eighth 50 Stone Grid Sample

(1) Grain Size Analysis

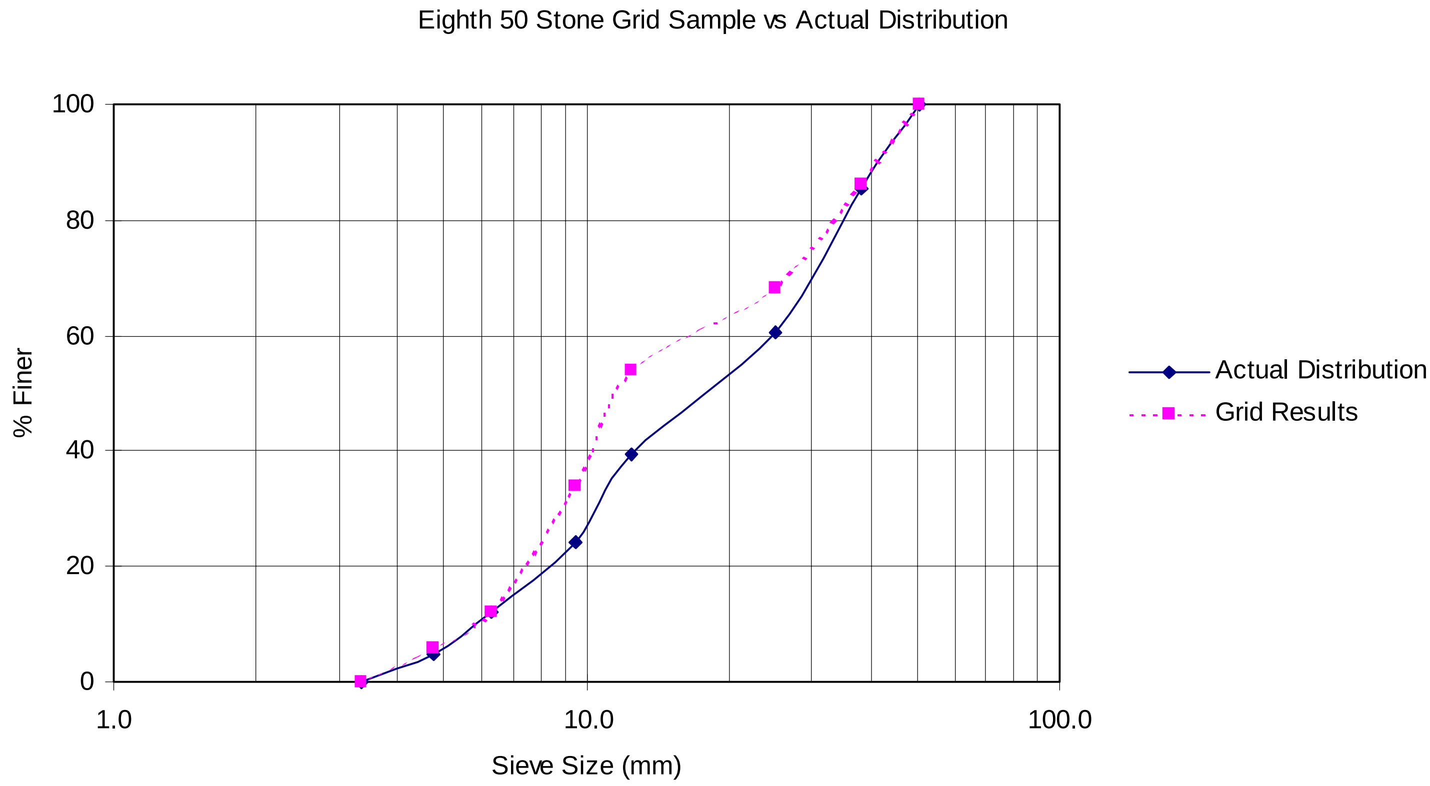
Sieve Size (mm)	No. of Particles	% Vol. (100*PDF)	% Finer (100*CDF)	CDF
38.1	7	14	100	1.00
25	9	18	86	0.8600
12.5	7	14	68	0.6800
9.5	10	20	54	0.5400
6.3	11	22	34	0.3400
4.75	3	6	12	0.1200
3.35	3	6	6	0.0600
total =	50			

Sieve Size	Actual Distribution	Grid Results
50.8	100	100
38.1	85.476499	86
25.0	60.497787	68
12.5	39.529307	54
9.50	24.196051	34
6.30	12.09859	12
4.75	4.8388553	6
3.35	0	0

(2) Sample Statistics

Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*(D-M)^2
38.1	0.14	5.3340	72.4205
25	0.18	4.5000	16.7412
12.5	0.14	1.7500	1.1419
9.5	0.2	1.9000	6.8585
6.3	0.22	1.3860	18.0424
4.75	0.06	0.2850	6.7492
3.35	0.06	0.2010	8.6486

Sample Mean =	15.3560	mm
Sample Variance =	130.6026	mm^2
Sample Standard Deviation =	11.4281	mm



Eighth 50 Stone Grid Sample vs Actual Distribution
100
80
60
40
20
0
% Finer
1.0
10.0
100.0
Sieve Size (mm)
Actual Distribution
Grid Results

Appendix H. Laboratory Areal Data and Results for Natural Sediment Experiments

The data obtained by areal sampling on the distribution of natural sediment is shown here. Additionally, the analysis results in terms of volume-by-weight for a variety of different samples sizes are included. The results are shown for the eight individual areal samples and the total combined areal sample. A comparison is made between the sample and actual grain size distribution curves. A discussion of the areal sampling on the natural sediment distribution is provided in section 4.3.2.

Total Areal Sample

(1) Grain Size Analysis

Sieve Size (mm)	Weight (g)	p(a-w)	p(a-w)*D^{-1}	p(v-w)	CDF
38.1	1509.1	0.277812	0.007291641	0.128605452	1.00
25	2454.3	0.451814	0.018072569	0.318752775	0.8714
12.5	670.0	0.123341	0.00986727	0.174032807	0.5526
9.5	424.5	0.078147	0.008225955	0.145084302	0.3786
6.3	215.6	0.03969	0.006299999	0.111115474	0.2335
4.75	109.6	0.020176	0.004247655	0.074917501	0.1224
3.35	49.0	0.00902	0.002692672	0.047491688	0.0475
total =	5432.1	g	0.056697761		

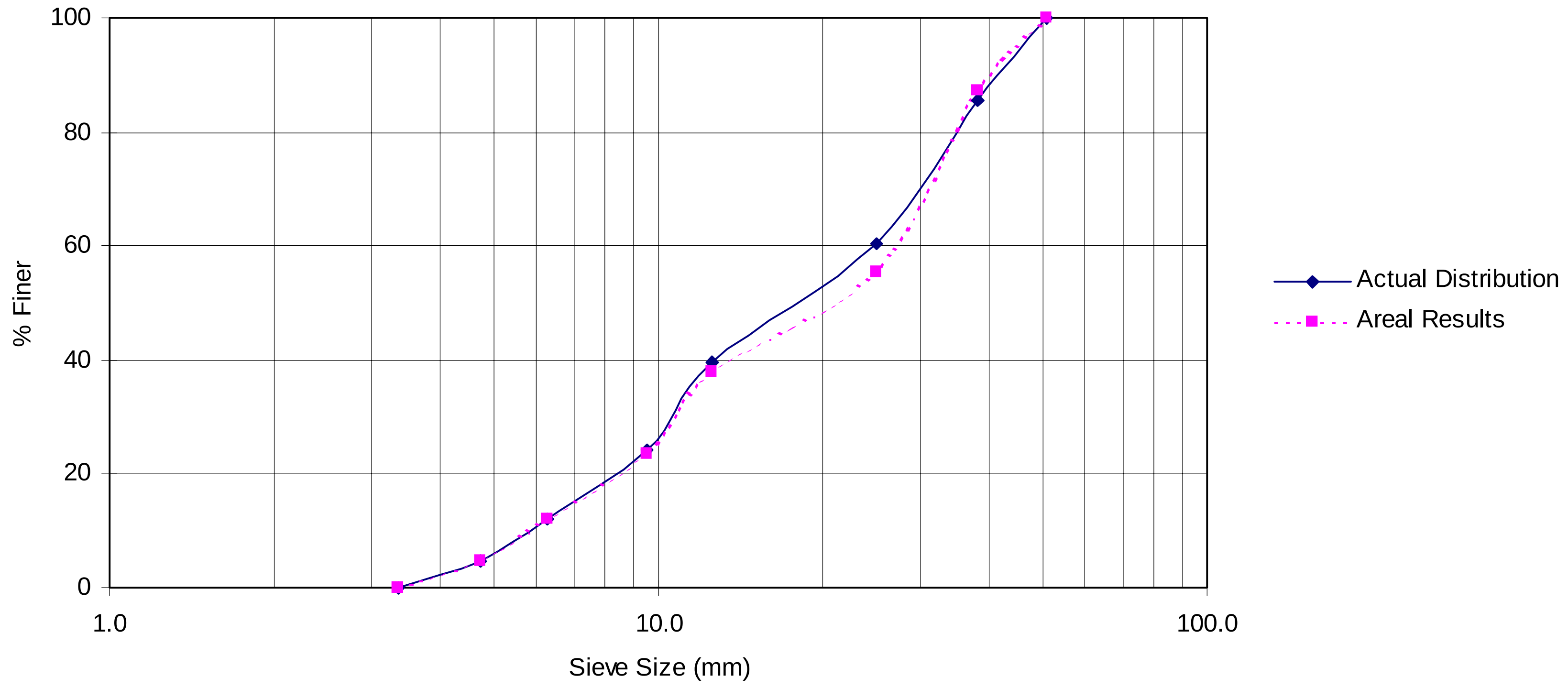
Sieve Size	Actual Distribution	Areal Results
50.8	100	100
38.1	85.4764992	87.1394548
25.0	60.4977873	55.2641773
12.5	39.5293067	37.8608965
9.50	24.1960507	23.3524663
6.30	12.0985898	12.2409189
4.75	4.83885534	4.74916883
3.35	0	0

(2) Sample Statistics

Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*(D-M)^2
38.1	0.12860545	4.8999	53.8495
25	0.31875278	7.9688	17.2790
12.5	0.17403281	2.1754	4.5932
9.5	0.1450843	1.3783	9.6070
6.3	0.11111547	0.7000	14.2824
4.75	0.0749175	0.3559	12.4426
3.35	0.04749169	0.1591	9.6944

Sample Mean = 17.6374 mm
Sample Variance = 121.7482 mm^2
Sample Standard Deviation = 11.0340 mm

Total Areal Sample vs Actual Distribution



Areal Sample 1

(1) Grain Size Analysis

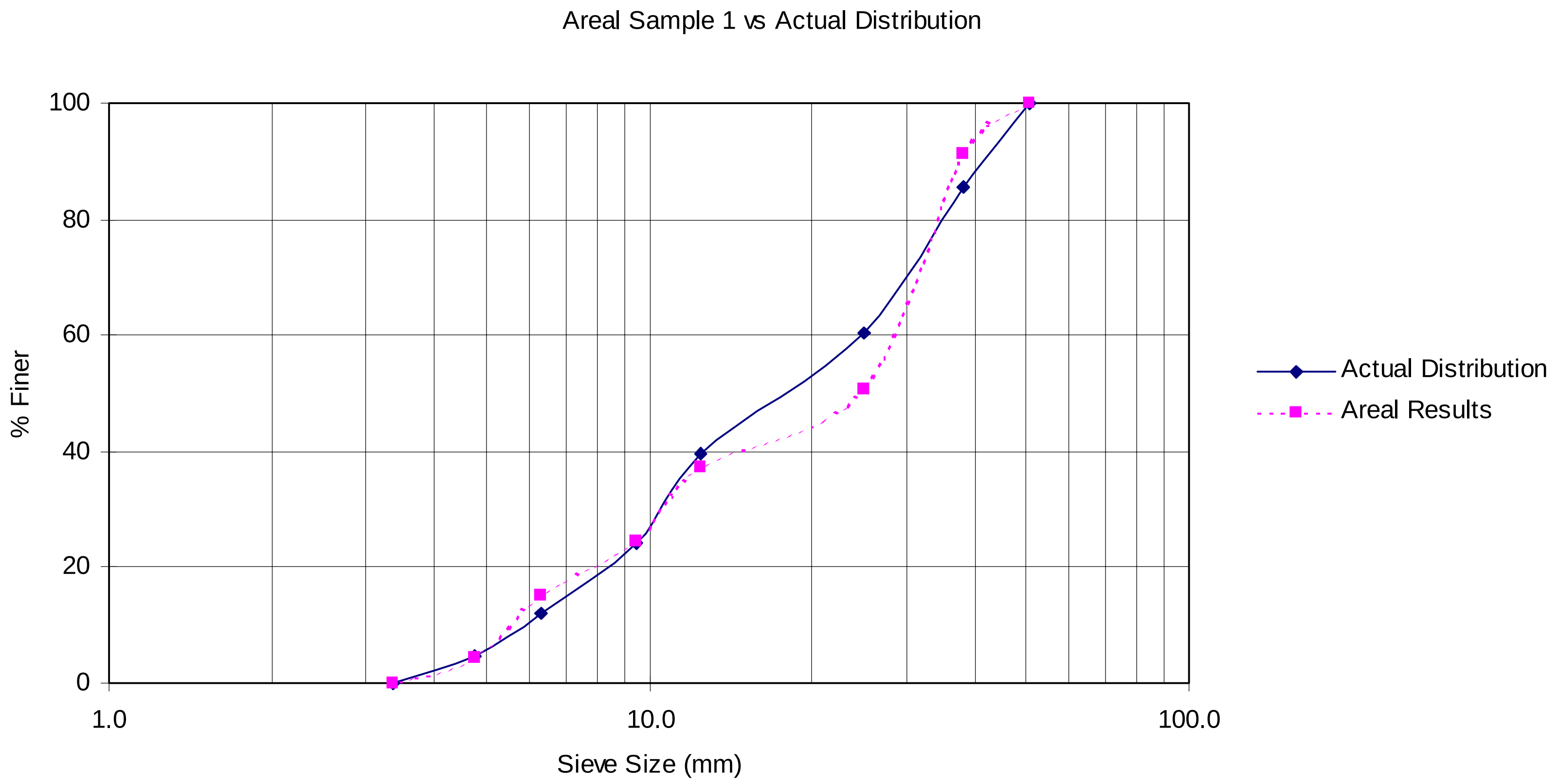
Sieve Size (mm)	Weight (g)	p(a-w)	p(a-w)*D^{-1}	p(v-w)	CDF
38.1	160.5	0.18760959	0.004924136	0.0867409	1.00
25	494.8	0.57837522	0.023135009	0.4075335	0.9133
12.5	80.0	0.09351257	0.007481005	0.1317813	0.5057
9.5	59.7	0.06978375	0.007345658	0.1293971	0.3739
6.3	28.5	0.03331385	0.005287913	0.093149	0.2445
4.75	25.0	0.02922268	0.006152142	0.1083727	0.1514
3.35	7.0	0.00818235	0.002442492	0.0430256	0.0430
total =	855.5	g	0.056768356		

Sieve Size	Actual Distribution	Areal Results
50.8	100	100
38.1	85.4764992	91.325914
25.0	60.4977873	50.57256
12.5	39.5293067	37.394435
9.50	24.1960507	24.454729
6.30	12.0985898	15.139834
4.75	4.83885534	4.3025597
3.35	0	0

(2) Sample Statistics

Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*(D-M)^2
38.1	0.0867409	3.3048	36.3979
25	0.4075335	10.1883	22.2235
12.5	0.1317813	1.6473	3.4484
9.5	0.1293971	1.2293	8.5222
6.3	0.093149	0.5868	11.9267
4.75	0.1083727	0.5148	17.9378
3.35	0.0430256	0.1441	8.7558

Sample Mean = 17.6154 mm
Sample Variance = 109.2124 mm^2
Sample Standard Deviation = 10.4505 mm


Areal Sample 1 vs Actual Distribution
% Finer
100
80
60
40
20
0
1.0
10.0
100.0
Sieve Size (mm)
Actual Distribution
Areal Results

Areal Sample 2

(1) Grain Size Analysis

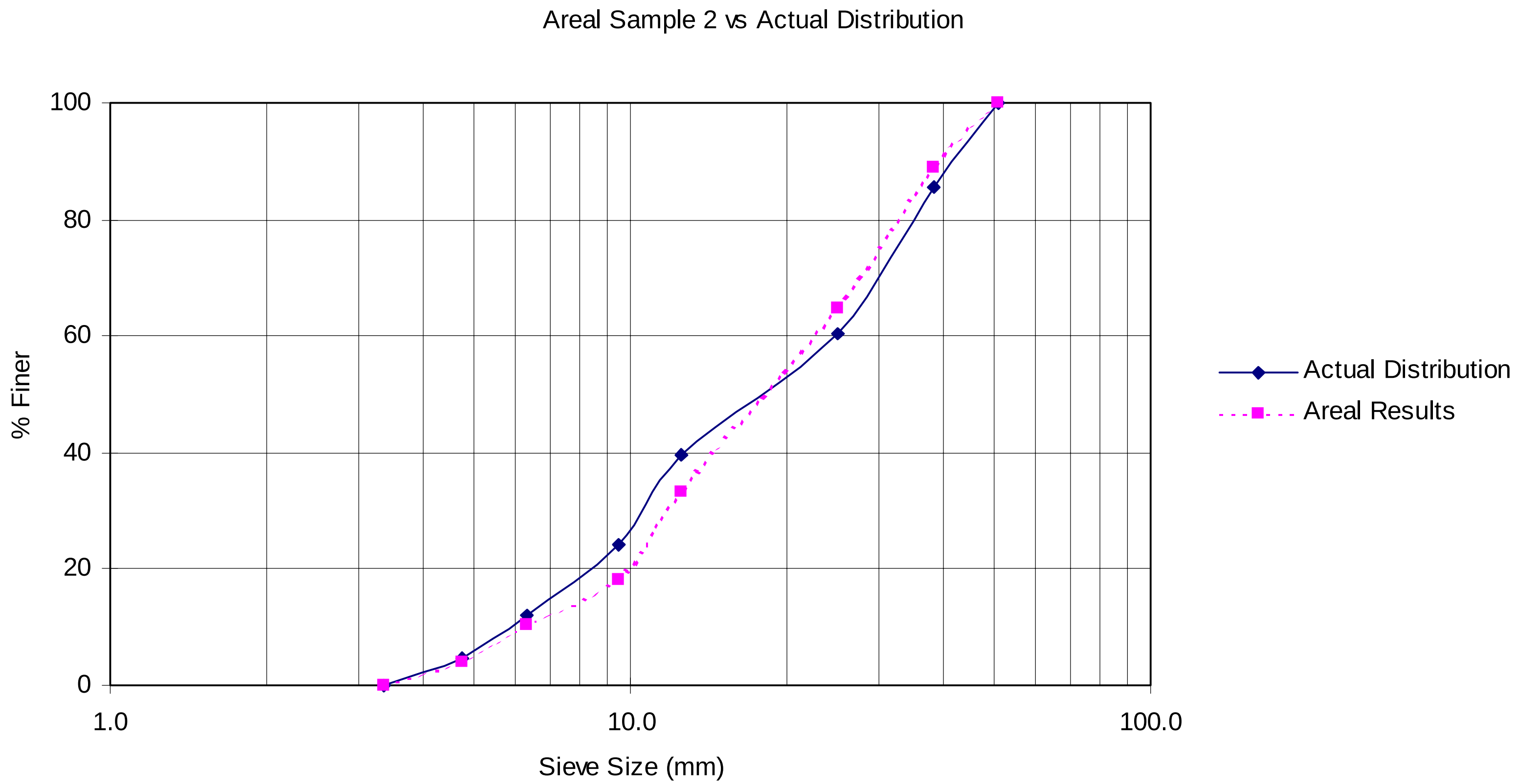
Sieve Size (mm)	Weight (g)	p(a-w)	p(a-w)*D^{-1}	p(v-w)	CDF
38.1	130.1	0.2579302	0.006769822	0.1121741	1.00
25	182.0	0.3608247	0.01443299	0.2391508	0.8878
12.5	120.3	0.2385012	0.019080095	0.3161521	0.6487
9.5	43.8	0.0868358	0.009140615	0.1514575	0.3325
6.3	15.0	0.0297383	0.004720366	0.0782152	0.1811
4.75	9.2	0.0182395	0.003839893	0.063626	0.1029
3.35	4.0	0.0079302	0.002367228	0.0392243	0.0392
total =	504.4	g	0.060351009		

Sieve Size	Actual Distribution	Areal Results
50.8	100	100
38.1	85.476499	88.782587
25.0	60.497787	64.867511
12.5	39.529307	33.252306
9.50	24.196051	18.106552
6.30	12.09859	10.285033
4.75	4.8388553	3.9224333
3.35	0	0

(2) Sample Statistics

Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*(D-M)^2
38.1	0.1121741	4.2738	51.9986
25	0.2391508	5.9788	16.9963
12.5	0.3161521	3.9519	5.2363
9.5	0.1514575	1.4388	7.5700
6.3	0.0782152	0.4928	8.2492
4.75	0.063626	0.3022	8.8889
3.35	0.0392243	0.1314	6.8549

Sample Mean = 16.5697 mm
Sample Variance = 105.7942 mm^2
Sample Standard Deviation = 10.2856 mm


Areal Sample 2 vs Actual Distribution
100
80
60
40
20
0
% Finer
1.0
10.0
100.0
Sieve Size (mm)
Actual Distribution
Areal Results

Areal Sample 3

(1) Grain Size Analysis

Sieve Size (mm)	Weight (g)	p(a-w)	p(a-w)*D^{-1}	p(v-w)	CDF
38.1	116.9	0.138131	0.003625477	0.0683705	1.00
25	571.6	0.675411	0.027016424	0.5094852	0.9316
12.5	67.3	0.079523	0.00636181	0.1199733	0.4221
9.5	39.3	0.046437	0.004888151	0.0921825	0.3022
6.3	35.0	0.041356	0.006564523	0.1237961	0.2100
4.75	11.0	0.012998	0.002736369	0.0516034	0.0862
3.35	5.2	0.006144	0.001834147	0.034589	0.0346
total =	846.3	g	0.053026902		

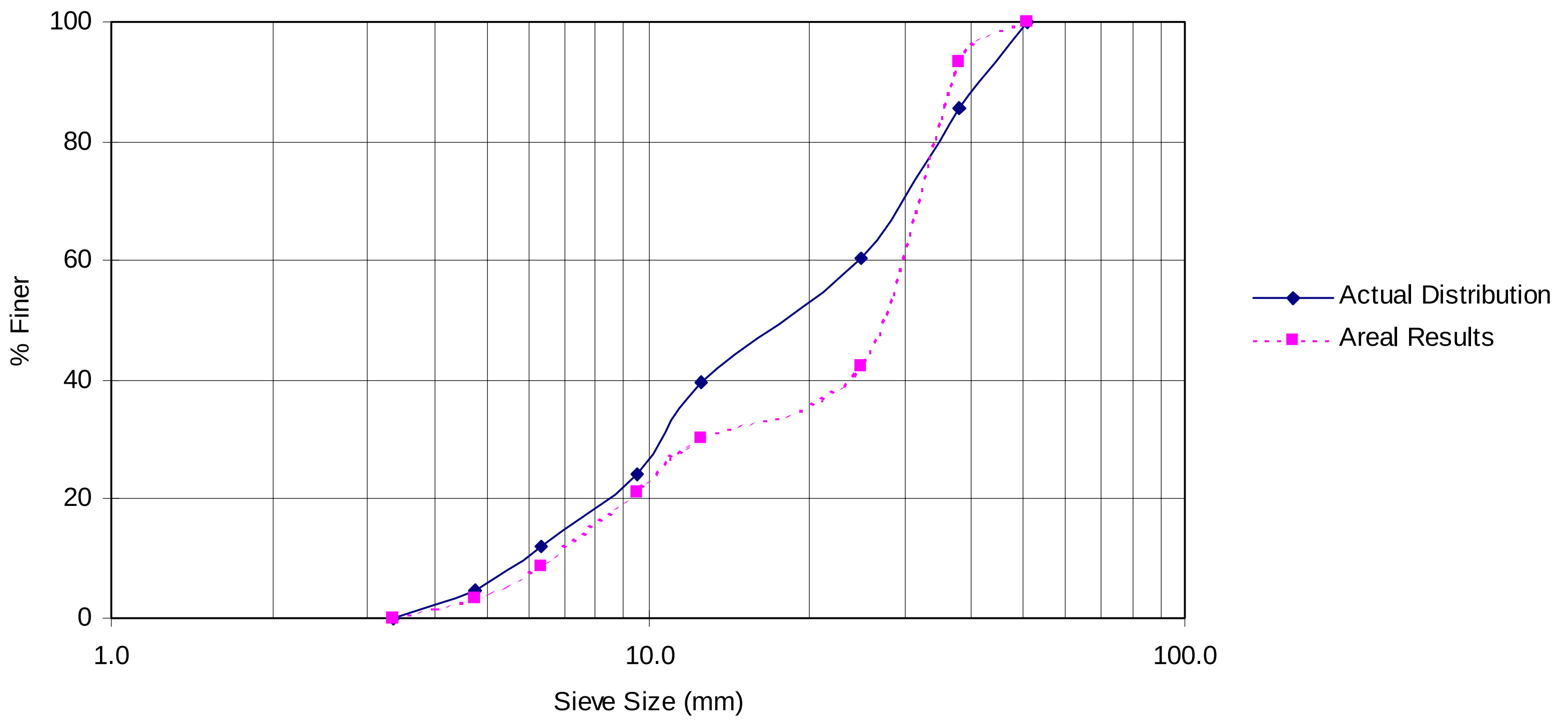
Sieve Size	Actual Distribution	Areal Results
50.8	100	100
38.1	85.476499	93.162947
25.0	60.497787	42.214422
12.5	39.529307	30.217097
9.50	24.196051	20.998849
6.30	12.09859	8.6192414
4.75	4.8388553	3.4588995
3.35	0	0

(2) Sample Statistics

Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*$(D-M)^2$
38.1	0.0683705	2.6049	25.3136
25	0.5094852	12.7371	19.2177
12.5	0.1199733	1.4997	4.8504
9.5	0.0921825	0.8757	8.0732
6.3	0.1237961	0.7799	19.5242
4.75	0.0516034	0.2451	10.2714
3.35	0.034589	0.1159	8.3190

Sample Mean = 18.8584 mm
Sample Variance = 95.5694 mm^2
Sample Standard Deviation = 9.7760 mm

Areal Sample 3 vs Actual Distribution



Areal Sample 4

(1) Grain Size Analysis

Sieve Size (mm)	Weight (g)	p(a-w)	p(a-w)*D^{-1}	p(v-w)	CDF
38.1	204.1	0.322637	0.008468156	0.1402636	1.00
25	246.0	0.388871	0.015554853	0.2576452	0.8597
12.5	67.2	0.106228	0.008498261	0.1407623	0.6021
9.5	53.7	0.084888	0.008935554	0.1480054	0.4613
6.3	36.5	0.057698	0.009158474	0.1516978	0.3133
4.75	15.0	0.023712	0.00499193	0.0826846	0.1616
3.35	10.1	0.015966	0.004765927	0.0789412	0.0789
total =	632.6	g	0.060373155		

Sieve Size	Actual Distribution	Areal Results
50.8	100	100
38.1	85.4764992	85.97364
25.0	60.4977873	60.209121
12.5	39.5293067	46.132896
9.50	24.1960507	31.332354
6.30	12.0985898	16.162575
4.75	4.83885534	7.8941161
3.35	0	0

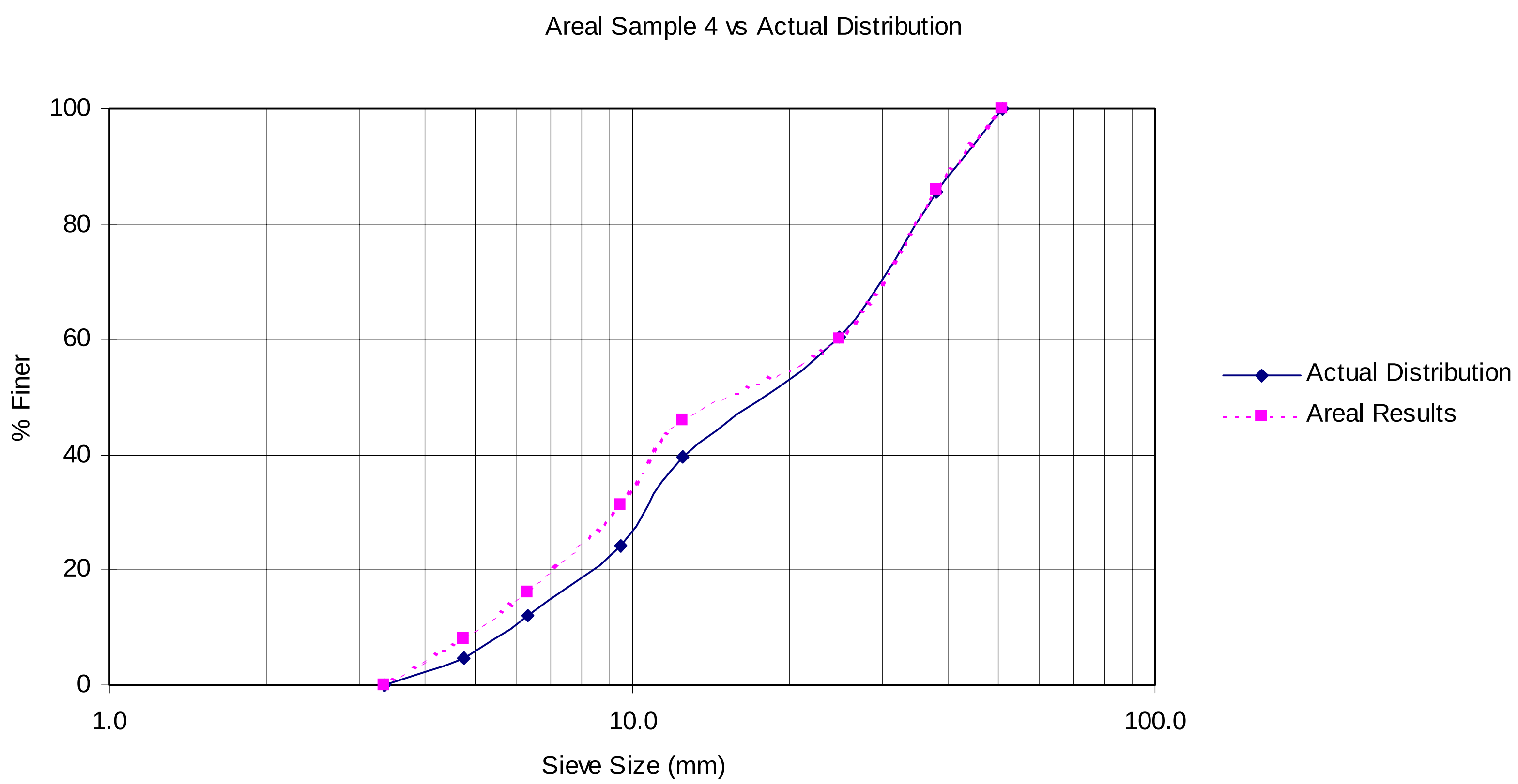
(2) Sample Statistics

Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*(D-M)^2
38.1	0.1402636	5.3440	65.0563
25	0.2576452	6.4411	18.3371
12.5	0.1407623	1.7595	2.3244
9.5	0.1480054	1.4061	7.3848
6.3	0.1516978	0.9557	15.9802
4.75	0.0826846	0.3928	11.5397
3.35	0.0789412	0.2645	13.7832

Sample Mean = 16.5637 mm

Sample Variance = 134.4056 mm^2

Sample Standard Deviation = 11.5933 mm


Areal Sample 4 vs Actual Distribution
100
80
60
40
20
0
% Finer
1.0
10.0
100.0
Sieve Size (mm)
Actual Distribution
Areal Results

Areal Sample 5

(1) Grain Size Analysis

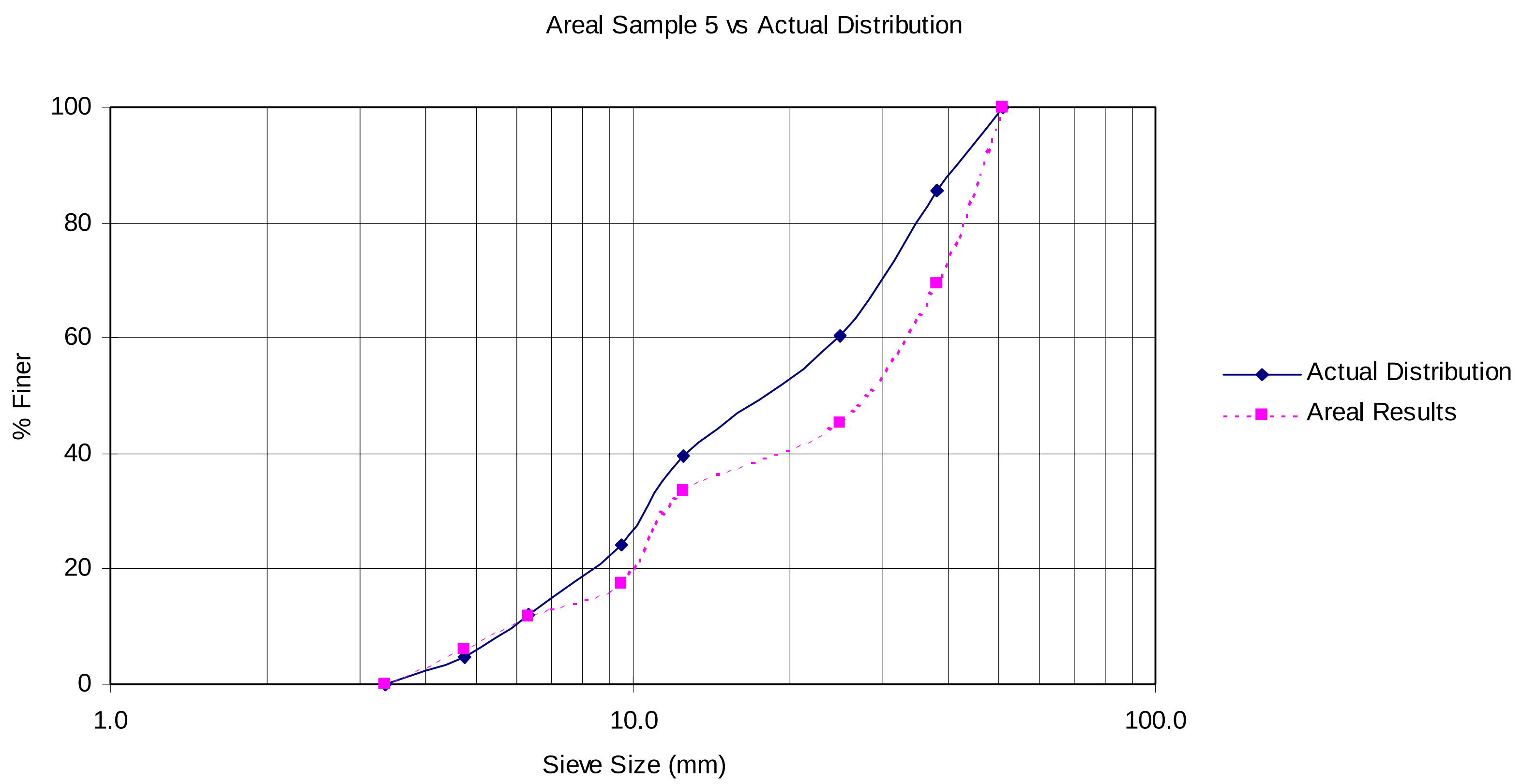
Sieve Size (mm)	Weight (g)	p(a-w)	p(a-w)*D^{-1}	p(v-w)	CDF
38.1	428.3	0.538268	0.014127774	0.3037566	1.00
25	225.9	0.283901	0.011356039	0.2441625	0.6962
12.5	54.3	0.068242	0.005459344	0.1173796	0.4521
9.5	56.5	0.071007	0.007474385	0.1607043	0.3347
6.3	13.2	0.016589	0.002633201	0.0566156	0.1740
4.75	10.0	0.012568	0.0026458	0.0568865	0.1174
3.35	7.5	0.009426	0.002813631	0.060495	0.0605
total =	795.7	g	0.046510174		

Sieve Size	Actual Distribution	Areal Results
50.8	100	100
38.1	85.476499	69.624336
25.0	60.497787	45.20809
12.5	39.529307	33.470133
9.50	24.196051	17.399703
6.30	12.09859	11.738143
4.75	4.8388553	6.0494952
3.35	0	0

(2) Sample Statistics

Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*$(D-M)^2$
38.1	0.3037566	11.5731	83.6964
25	0.2441625	6.1041	2.9898
12.5	0.1173796	1.4672	9.5092
9.5	0.1607043	1.5267	23.1440
6.3	0.0566156	0.3567	13.0816
4.75	0.0568865	0.2702	15.9615
3.35	0.060495	0.2027	19.9299

Sample Mean = 21.5007 mm
Sample Variance = 168.3124 mm^2
Sample Standard Deviation = 12.9735 mm


Areal Sample 5 vs Actual Distribution
100
80
60
40
20
0
% Finer
1.0
10.0
100.0
Sieve Size (mm)
Actual Distribution
Areal Results

Areal Sample 6

(1) Grain Size Analysis

Sieve Size (mm)	Weight (g)	p(a-w)	$p(a-w)*D^{-1}$	p(v-w)	CDF
38.1	0.0	0	0	0	1.00
25	147.3	0.404226	0.016169045	0.196148	1.0000
12.5	95.4	0.2618	0.020944018	0.2540736	0.8039
9.5	73.1	0.200604	0.021116182	0.2561621	0.5498
6.3	34.9	0.095774	0.015202202	0.1844191	0.2936
4.75	9.2	0.025247	0.005315154	0.0644786	0.1092
3.35	4.5	0.012349	0.003686289	0.0447187	0.0447
total =	364.4	g	0.08243289		

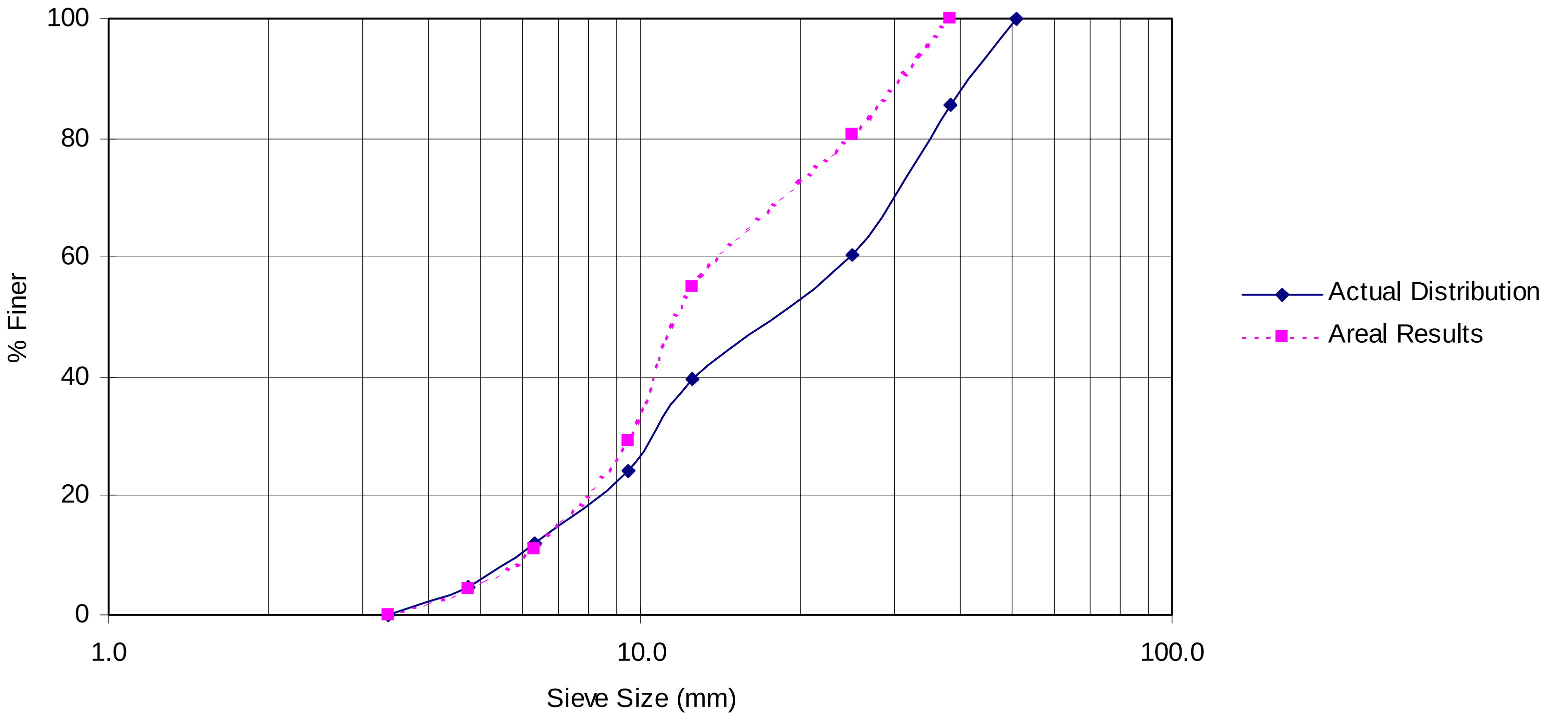
Sieve Size	Actual Distribution	Areal Results
50.8	100	100
38.1	85.476499	100
25.0	60.497787	80.385202
12.5	39.529307	54.977846
9.50	24.196051	29.361636
6.30	12.09859	10.919722
4.75	4.8388553	4.4718663
3.35	0	0

(2) Sample Statistics

Sieve Size (mm)	% Vol.	%Vol.*D	$\%Vol.*(D-M)^2$
38.1	0	0.0000	0.0000
25	0.196148	4.9037	32.4839
12.5	0.2540736	3.1759	0.0346
9.5	0.2561621	2.4335	1.7733
6.3	0.1844191	1.1618	6.2705
4.75	0.0644786	0.3063	3.5128
3.35	0.0447187	0.1498	3.4481

Sample Mean = 12.1311 mm
Sample Variance = 47.5233 mm^2
Sample Standard Deviation = 6.8937 mm

Areal Sample 6 vs Actual Distribution



Areal Sample 7

(1) Grain Size Analysis

Sieve Size (mm)	Weight (g)	p(a-w)	p(a-w)*D^{-1}	p(v-w)	CDF
38.1	232.0	0.337897	0.008868685	0.1569427	1.00
25	258.9	0.377075	0.015083018	0.2669133	0.8431
12.5	90.7	0.1321	0.010568016	0.1870146	0.5761
9.5	55.5	0.080833	0.008508746	0.1505731	0.3891
6.3	31.5	0.045878	0.00728226	0.1288689	0.2386
4.75	12.7	0.018497	0.003894093	0.068911	0.1097
3.35	5.3	0.007719	0.002304238	0.0407764	0.0408
total =	686.6	g	0.056509056		

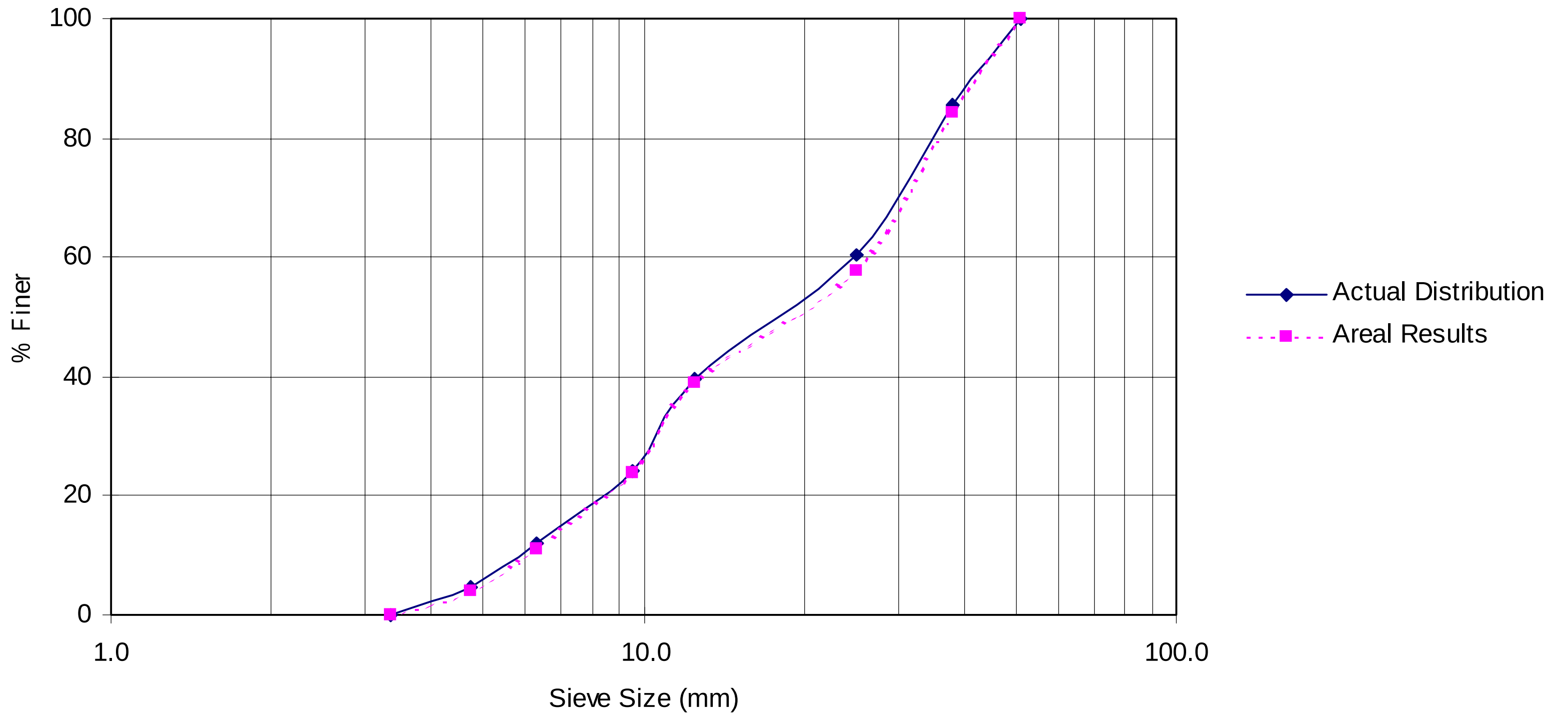
Sieve Size	Actual Distribution	Areal Results
50.8	100	100
38.1	85.4764992	84.305729
25.0	60.4977873	57.6144
12.5	39.5293067	38.912944
9.50	24.1960507	23.855629
6.30	12.0985898	10.968738
4.75	4.83885534	4.0776431
3.35	0	0

(2) Sample Statistics

Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*$(D-M)^2$
38.1	0.1569427	5.9795	65.3371
25	0.2669133	6.6728	14.2383
12.5	0.1870146	2.3377	5.0496
9.5	0.1505731	1.4304	10.1154
6.3	0.1288689	0.8119	16.7369
4.75	0.068911	0.3273	11.5499
3.35	0.0407764	0.1366	8.3924

Sample Mean = 17.6963 mm
Sample Variance = 131.4196 mm^2
Sample Standard Deviation = 11.4638 mm

Areal Sample 7 vs Actual Distribution



Areal Sample 8

(1) Grain Size Analysis

Sieve Size (mm)	Weight (g)	p(a-w)	p(a-w)*D^{-1}	p(v-w)	CDF
38.1	237.2	0.317707	0.008338765	0.155383	1.00
25	327.8	0.439057	0.017562282	0.327252	0.8446
12.5	94.8	0.126976	0.01015805	0.189283	0.5174
9.5	42.9	0.05746	0.006048472	0.112706	0.3281
6.3	21.0	0.028128	0.004464684	0.083194	0.2154
4.75	17.5	0.02344	0.004934651	0.091951	0.1322
3.35	5.4	0.007233	0.002159041	0.040231	0.0402
total =	746.6	g	0.053665946		

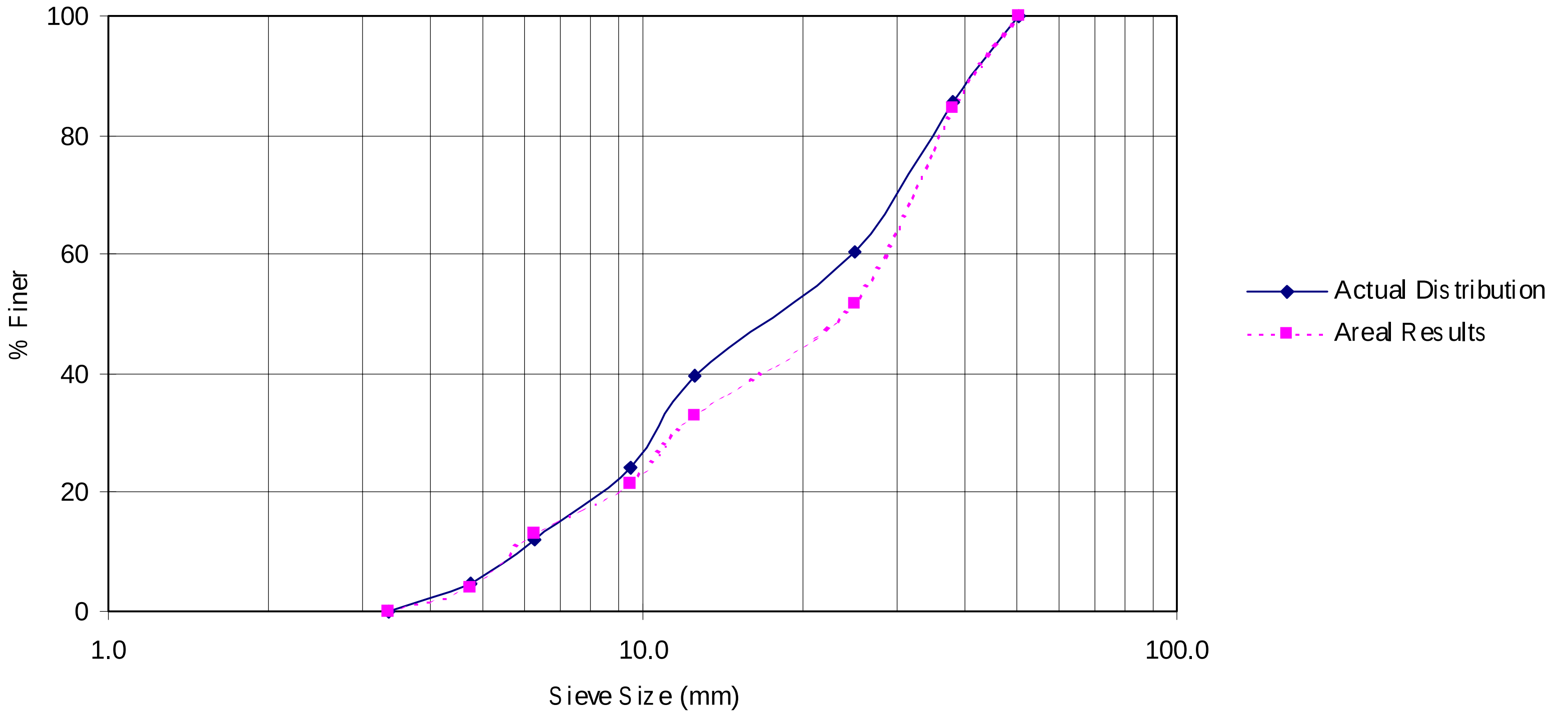
Sieve Size	Actual Distribution	Areal Results
50.8	100	100
38.1	85.476499	84.4617204
25.0	60.497787	51.7365313
12.5	39.529307	32.808234
9.50	24.196051	21.5376373
6.30	12.09859	13.218238
4.75	4.8388553	4.02311249
3.35	0	0

(2) Sample Statistics

Sieve Size (mm)	% Vol.	%Vol.*D	%Vol.*(D-M)^2
38.1	0.1553828	5.9201	58.8797
25	0.3272519	8.1813	13.2631
12.5	0.189283	2.3660	7.1215
9.5	0.112706	1.0707	9.4026
6.3	0.083194	0.5241	12.6557
4.75	0.0919513	0.4368	17.7245
3.35	0.0402311	0.1348	9.3978

Sample Mean = 18.6338 mm
Sample Variance = 128.4448 mm^2
Sample Standard Deviation = 11.3333 mm

Areal Sample 8 vs Actual Distribution

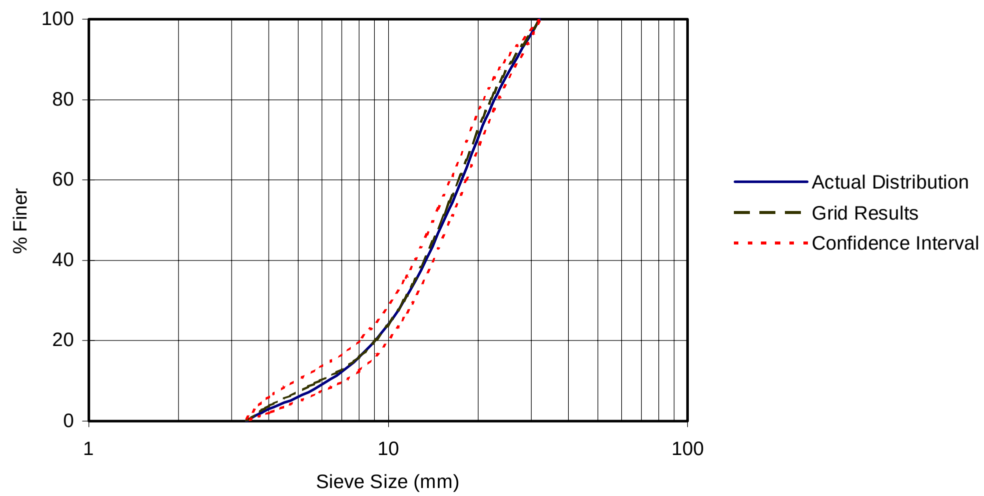


Appendix I. Binomial Confidence Interval Results for Glass Bead Grid Samples

Binomial confidence intervals are applied to the grid sampling results for the glass bead distribution as described in section 2.2. These sampling results for the different sample sizes are shown in Appendix D. The sample grain size curve and the 95% binomial confidence intervals are compared with the actual grain size distribution.

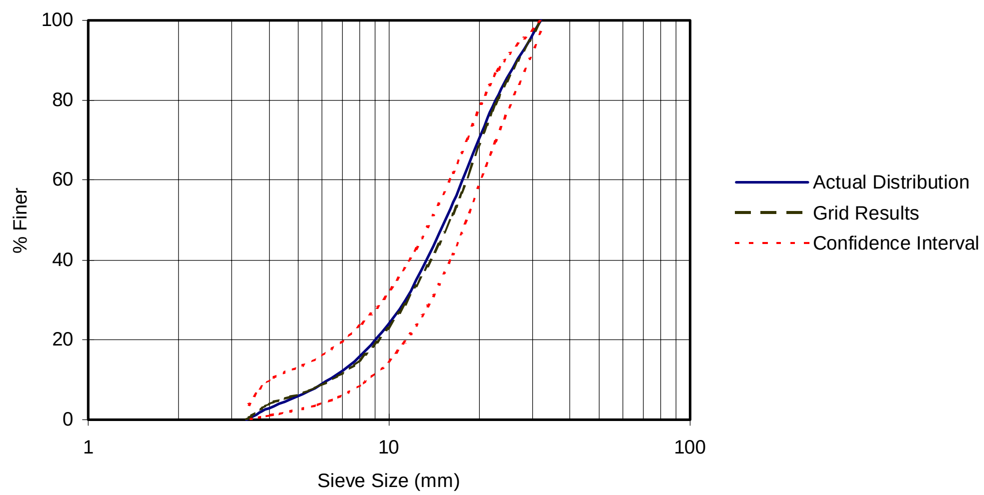
400 Stone Grid Sample with 95% Binomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
32	100	100	99.25	100
22.6	80.039	81.75	77.61	85.41
16	52.833	54.5	49.48	59.46
11.3	29.913	30	25.55	34.75
8	15.952	16	12.55	19.97
5.66	7.982	9.25	6.60	12.52
4	2.992	3.75	2.11	6.11
3.36	0	0	0	0.75



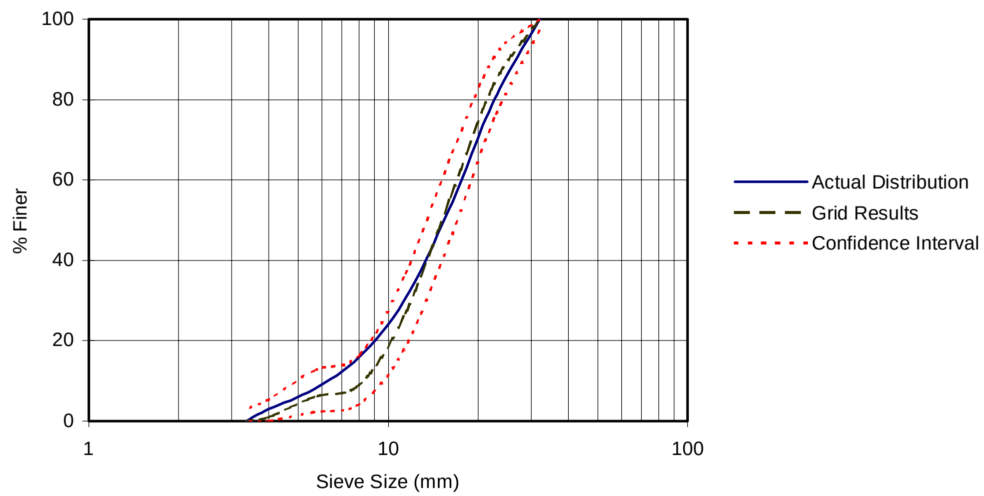
First 100 Stone Grid Sample with 95% Binomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
32	100	100	97.05	100
22.6	80.039	79	69.71	86.51
16	52.833	50	39.83	60.17
11.3	29.913	29	19.48	37.87
8	15.952	15	8.65	23.53
5.66	7.982	8	3.52	15.16
4	2.992	4	1.10	9.93
3.36	0	0	0	2.95



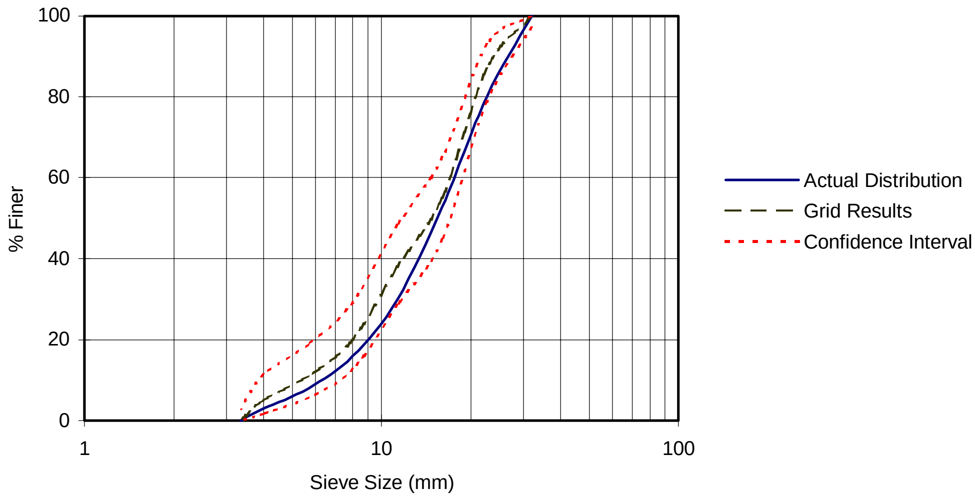
Second 100 Stone Grid Sample with 95% Binomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
32	100	100	97.05	100
22.6	80.039	84	75.32	90.57
16	52.833	55	44.73	64.97
11.3	29.913	26	17.74	35.73
8	15.952	9	4.20	16.40
5.66	7.982	6	2.23	12.60
4	2.992	1	0.03	5.45
3.36	0	0	0	2.95



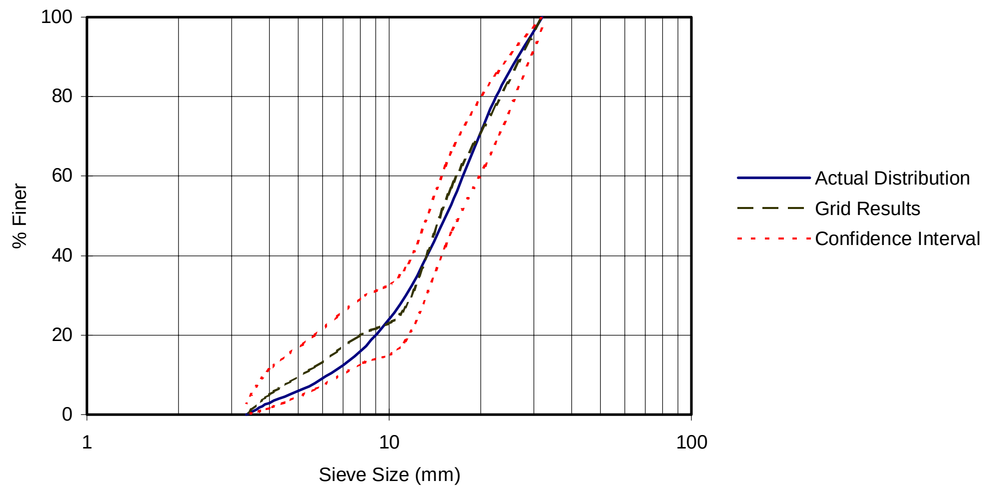
Third 100 Stone Grid Sample with 95% Binomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
32	100	100	97.05	100.00
22.6	80.039	87	78.80	92.89
16	52.833	55	44.73	64.97
11.3	29.913	38	28.48	48.25
8	15.952	20	12.67	29.18
5.66	7.982	11	5.62	18.83
4	2.992	5	1.64	11.28
3.36	0	0	0.00	2.95



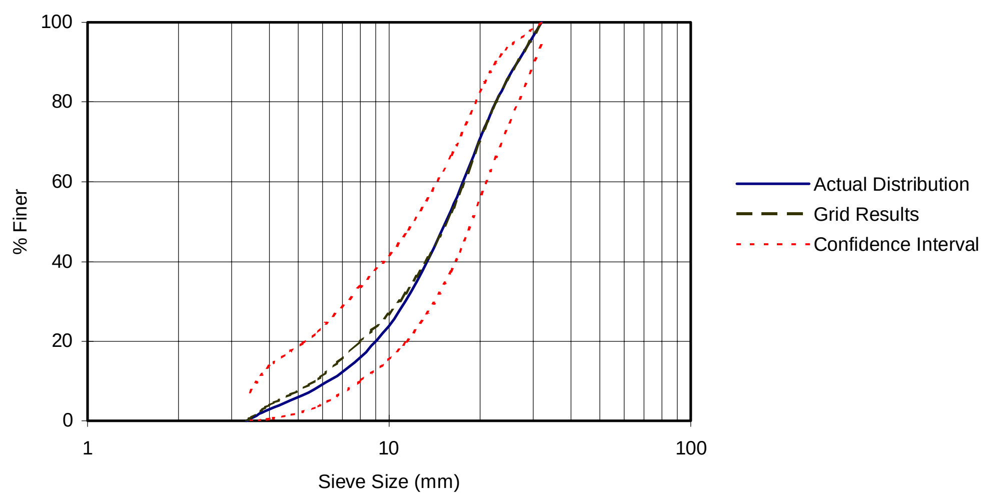
Fourth 100 Stone Grid Sample with 95% Binomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
32	100	100	97.05	100
22.6	80.039	78	68.61	85.67
16	52.833	57	45.72	65.92
11.3	29.913	27	18.61	36.80
8	15.952	20	12.67	29.18
5.66	7.982	12	6.36	20.02
4	2.992	5	1.64	11.28
3.36	0	0	0	2.95



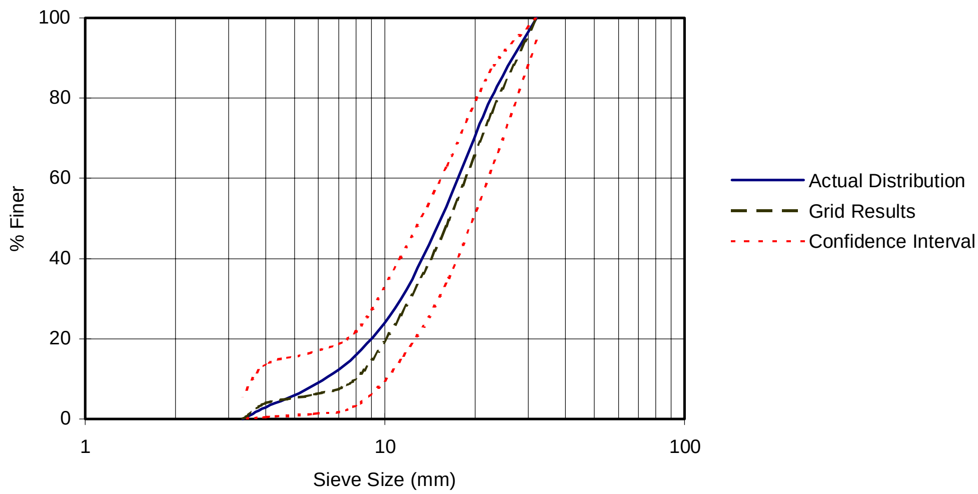
First 50 Stone Grid Sample with 95% Binomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
32	100	100	94.18	100
22.6	80.039	80	66.28	89.97
16	52.833	52	37.42	66.34
11.3	29.913	32	19.52	46.70
8	15.952	20	10.03	33.72
5.66	7.982	10	3.33	21.81
4	2.992	4	0.49	13.71
3.36	0	0	0.00	5.82



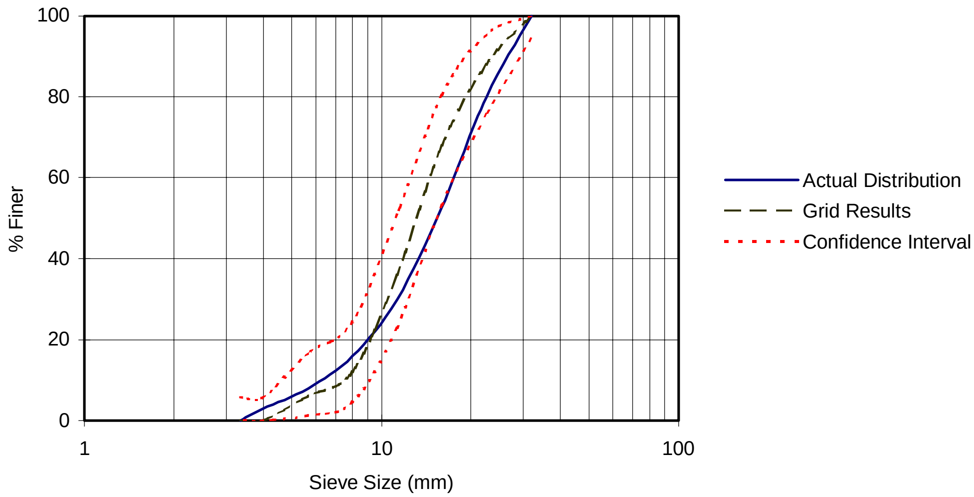
Second 50 Stone Grid Sample with 95% Binomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
32	100	100	94.18	100
22.6	80.039	76	61.83	86.94
16	52.833	48	33.66	62.58
11.3	29.913	26	14.63	40.34
8	15.952	10	3.33	21.81
5.66	7.982	6	1.25	16.55
4	2.992	4	0.49	13.71
3.36	0	0	0	5.82



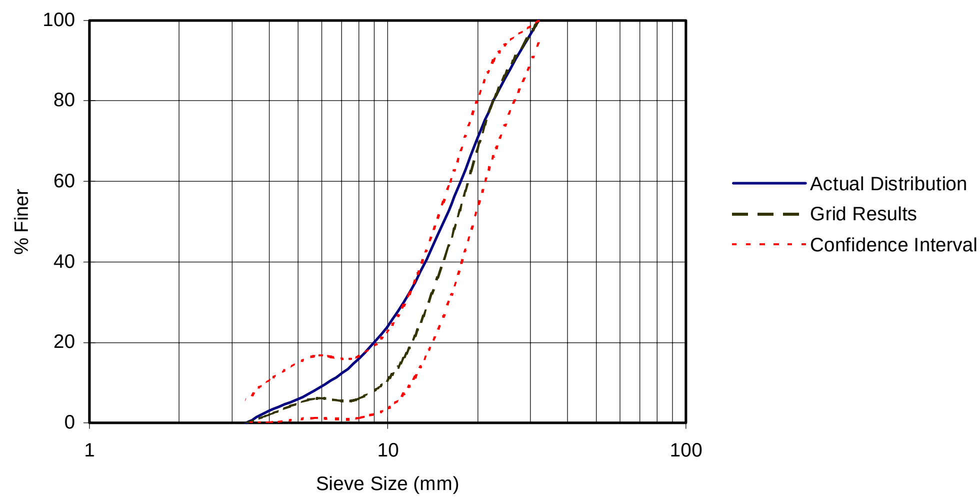
Third 50 Stone Grid Sample with 95% Binomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
32	100	100	94.18	100.00
22.6	80.039	88	75.69	95.47
16	52.833	68	53.30	80.48
11.3	29.913	36	22.92	50.81
8	15.952	12	4.53	24.31
5.66	7.982	6	1.25	16.55
4	2.992	0	0.00	5.82
3.36	0	0	0.00	5.82



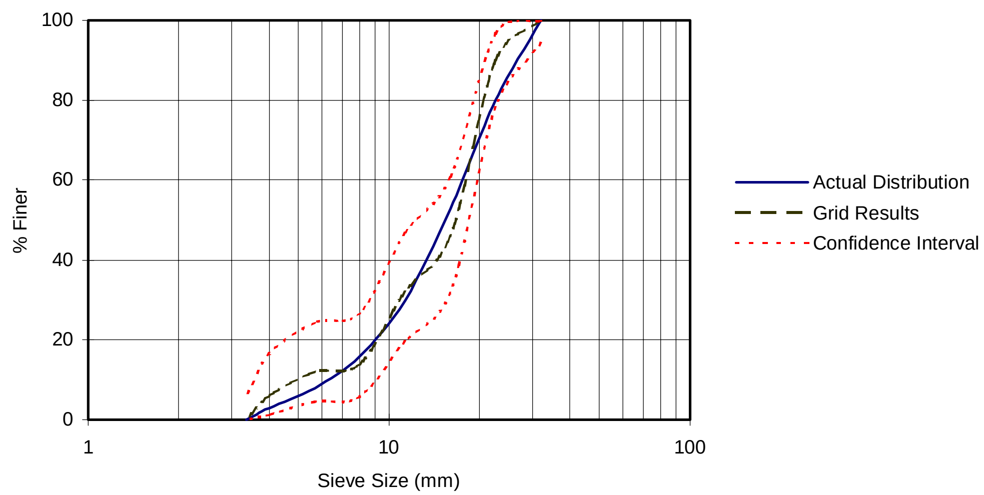
Fourth 50 Stone Grid Sample with 95% Binomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
32	100	100	94.18	100
22.6	80.039	80	66.28	89.97
16	52.833	44	29.99	58.75
11.3	29.913	16	7.17	29.11
8	15.952	6	1.25	16.55
5.66	7.982	6	1.25	16.55
4	2.992	2	0.05	10.65
3.36	0	0	0	5.82



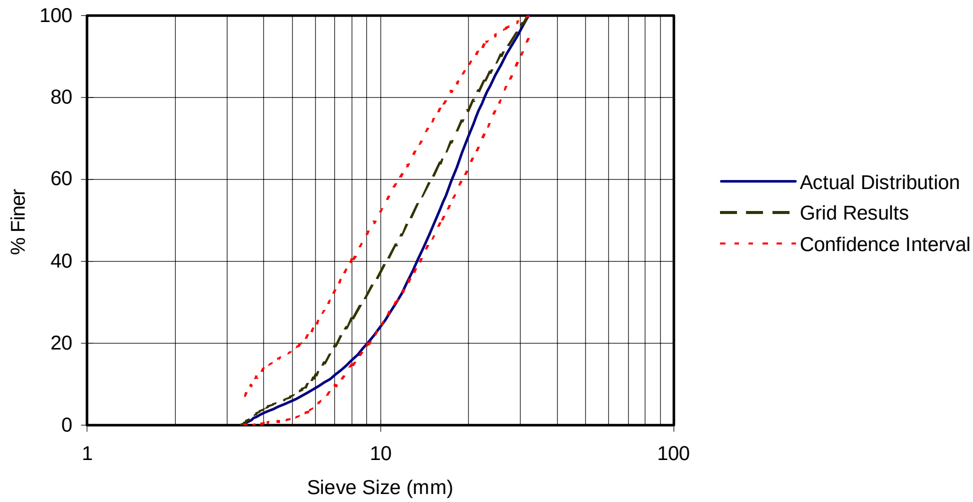
Fifth 50 Stone Grid Sample with 95% Binomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
32	100	100	94.18	100
22.6	80.039	90	78.19	96.67
16	52.833	46	31.81	60.68
11.3	29.913	32	19.52	46.70
8	15.952	14	5.82	26.74
5.66	7.982	12	4.53	24.31
4	2.992	6	1.25	16.55
3.36	0	0	0	5.82



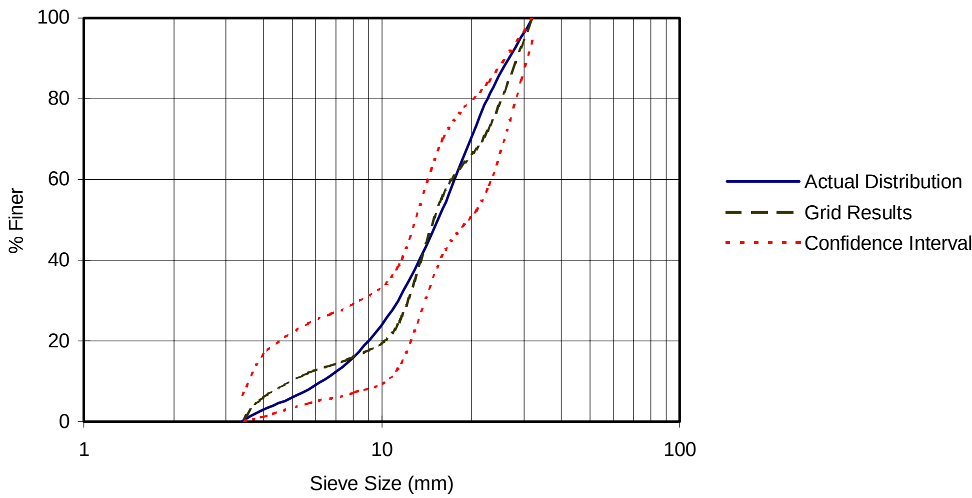
Sixth 50 Stone Grid Sample with 95% Binomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
32	100	100	94.18	100
22.6	80.039	84	70.89	92.83
16	52.833	64	49.19	77.08
11.3	29.913	44	29.99	58.75
8	15.952	26	14.63	40.34
5.66	7.982	10	3.33	21.81
4	2.992	4	0.49	13.71
3.36	0	0	0	5.82



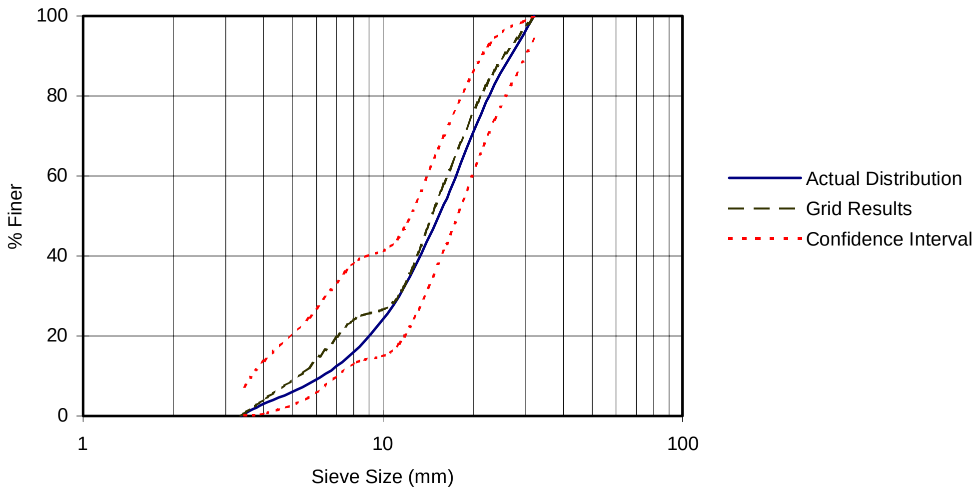
Seventh 50 Stone Grid Sample with 95% Binomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
32	100	100	94.18	100
22.6	80.039	72	57.51	83.77
16	52.833	56	41.25	70.01
11.3	29.913	24	13.06	38.17
8	15.952	16	7.17	29.11
5.66	7.982	12	4.53	24.31
4	2.992	6	1.25	16.55
3.36	0	0	0	5.82



Eighth 50 Stone Grid Sample with 95% Binomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
32	100	100	94.18	100.00
22.6	80.039	84	70.89	92.83
16	52.833	58	41.25	70.01
11.3	29.913	30	17.86	44.61
8	15.952	24	13.06	38.17
5.66	7.982	12	4.53	24.31
4	2.992	4	0.49	13.71
3.36	0	0	0.00	5.82

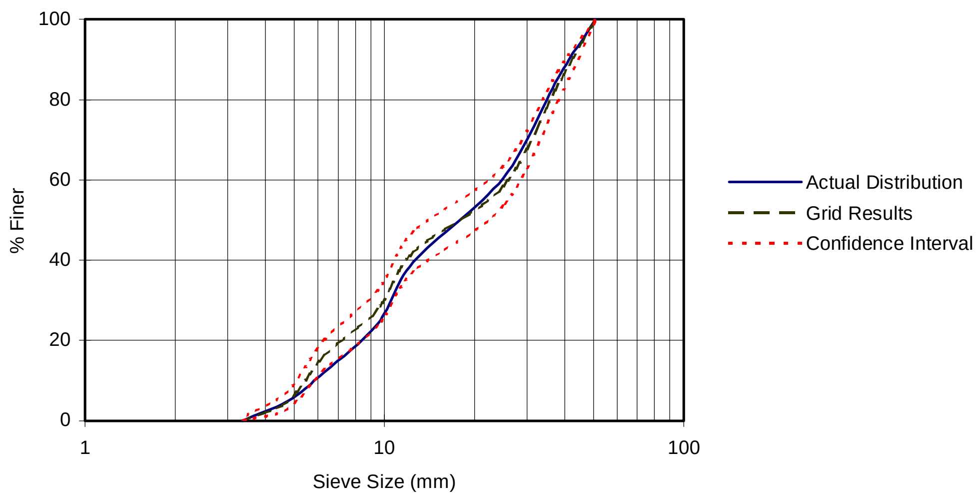


Appendix J. Binomial Confidence Interval Results for Natural Sediment Grid Samples

Binomial confidence intervals are applied to the grid sampling results for the natiral sediment distribution as described in section 2.2. These sampling results for the different sample sizes are shown in Appendix E. The sample grain size curve and the 95% binomial confidence intervals are compared with the actual grain size distribution.

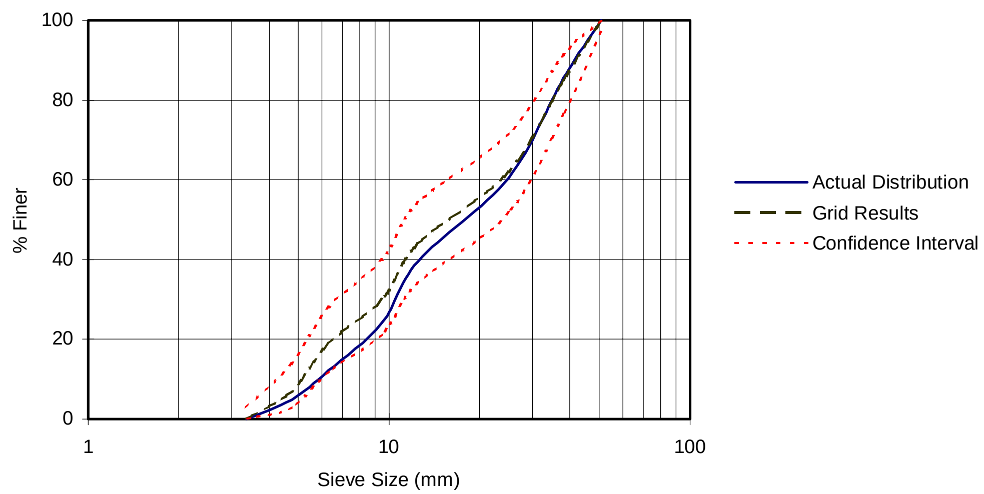
400 Stone Grid Sample with 95% Binomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
50.8	100	100	99.25	100
38.1	85.48	83.75	79.76	87.23
25	60.50	58.50	53.50	63.37
12.5	39.53	42.00	37.11	47.01
9.5	24.20	27.75	23.42	32.42
6.3	12.10	16.25	12.77	20.24
4.75	4.84	4.75	2.88	7.32
3.35	0	0	0	0.75



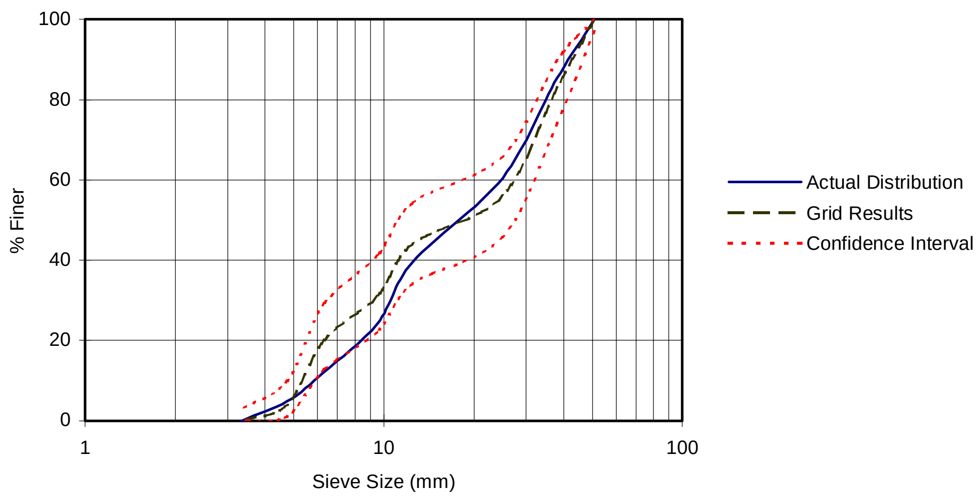
First 100 Stone Grid Sample with 95%Binomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
50.8	100	100	97.05	100
38.1	85.48	85	76.47	91.35
25	60.50	62	51.75	71.52
12.5	39.53	44	34.08	54.28
9.5	24.20	30	21.24	39.98
6.3	12.10	19	11.84	28.07
4.75	4.84	7	2.86	13.89
3.35	0	0	0	2.95



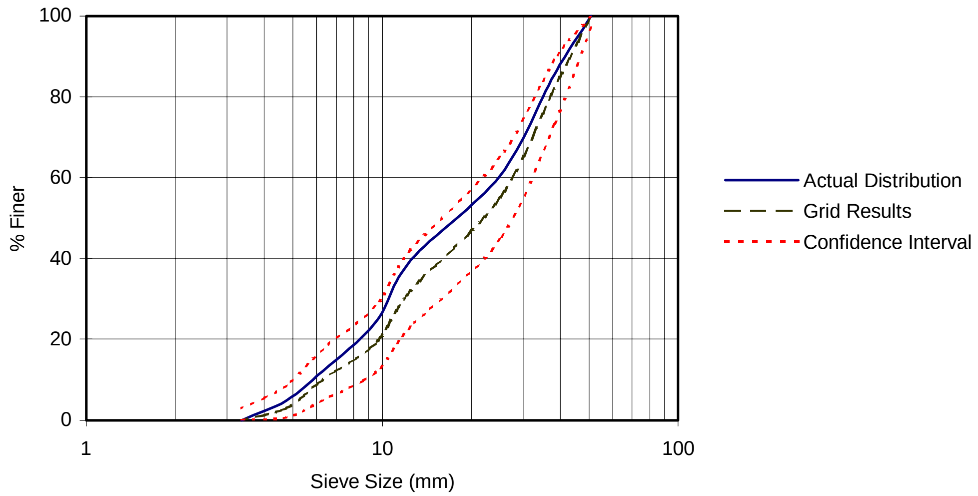
Second 100 Stone Grid Sample with 95% Binomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
50.8	100	100	97.05	100
38.1	85.48	83	74.18	89.77
25	60.50	56	45.72	65.92
12.5	39.53	44	34.08	54.28
9.5	24.20	31	22.13	41.03
6.3	12.10	20	12.67	29.18
4.75	4.84	4	1.10	9.93
3.35	0	0	0	2.95



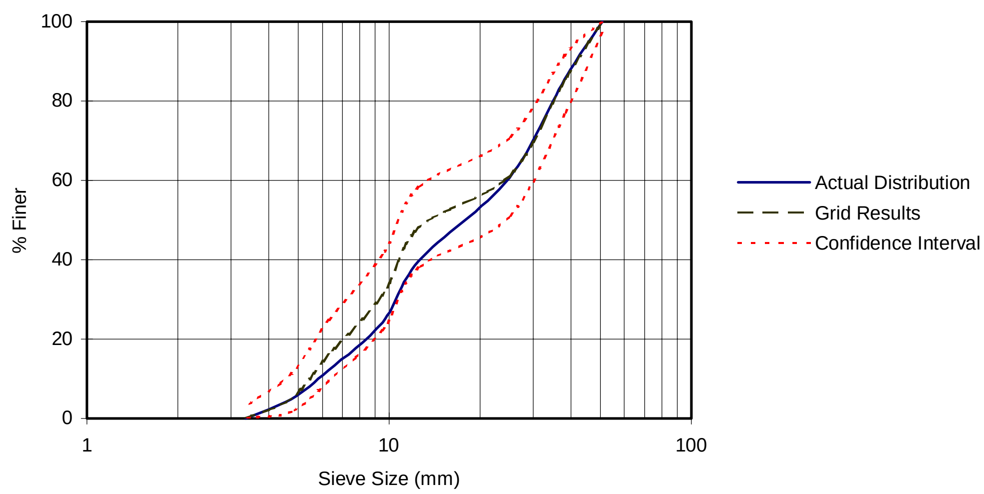
Third 100 Stone Grid Sample with 95% Binomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
50.8	100	100	97.05	100
38.1	85.48	82	73.05	88.97
25	60.50	55	44.73	64.97
12.5	39.53	32	23.02	42.08
9.5	24.20	19	11.84	28.07
6.3	12.10	10	4.90	17.62
4.75	4.84	3	0.62	8.52
3.35	0	0	0	2.95



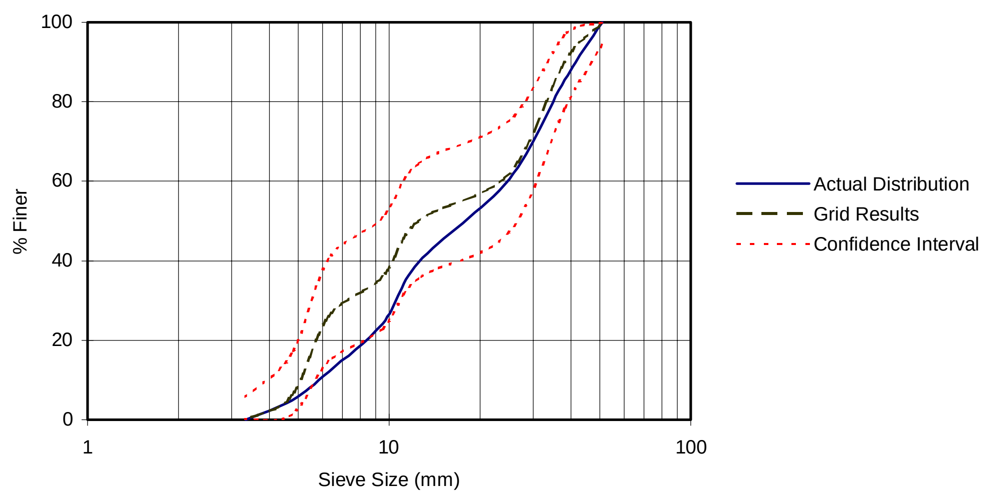
Fourth 100 Stone Grid Sample with 95% Binomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
50.8	100	100	97.05	100
38.1	85.48	85	76.47	91.35
25	60.50	61	50.73	70.60
12.5	39.53	48	37.90	58.22
9.5	24.20	31	22.13	41.03
6.3	12.10	16	9.43	24.68
4.75	4.84	5	1.64	11.28
3.35	0	0	0	2.95



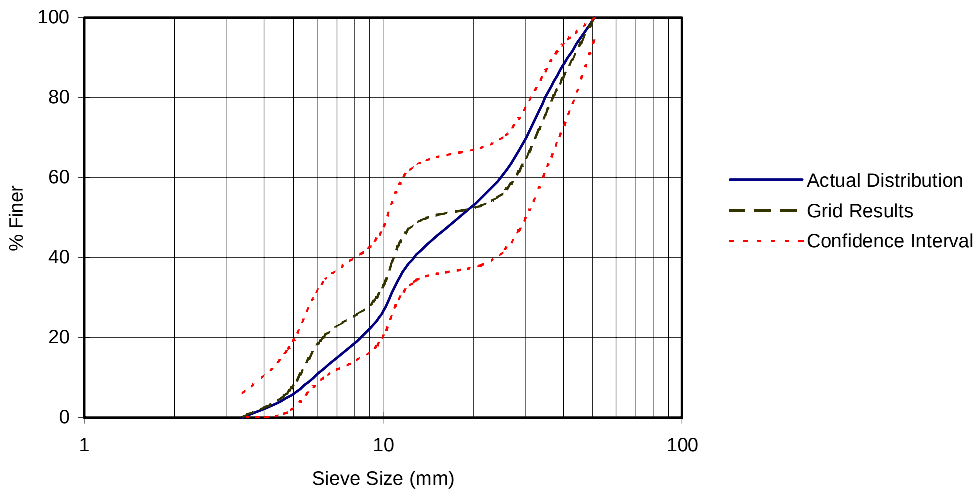
First 50 Stone Grid Sample with 95% Binomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
50.8	100	100	94.18	100
38.1	85.48	90	78.19	96.67
25	60.50	62	47.17	75.35
12.5	39.53	50	35.53	64.47
9.5	24.20	36	22.92	50.81
6.3	12.10	26	14.63	40.34
4.75	4.84	6	1.25	16.55
3.35	0	0	0	5.82



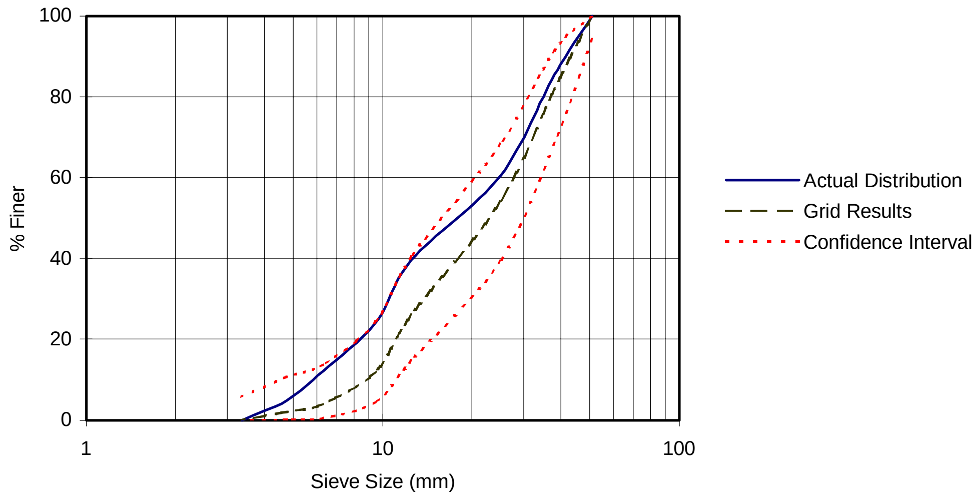
Second 50 Stone Grid Sample with 95% Binomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
50.8	100	100	94.18	100
38.1	85.48	82	68.56	91.42
25	60.50	56	41.25	70.01
12.5	39.53	48	33.66	62.58
9.5	24.20	30	17.86	44.61
6.3	12.10	20	10.03	33.72
4.75	4.84	6	1.25	16.55
3.35	0	0	0	5.82



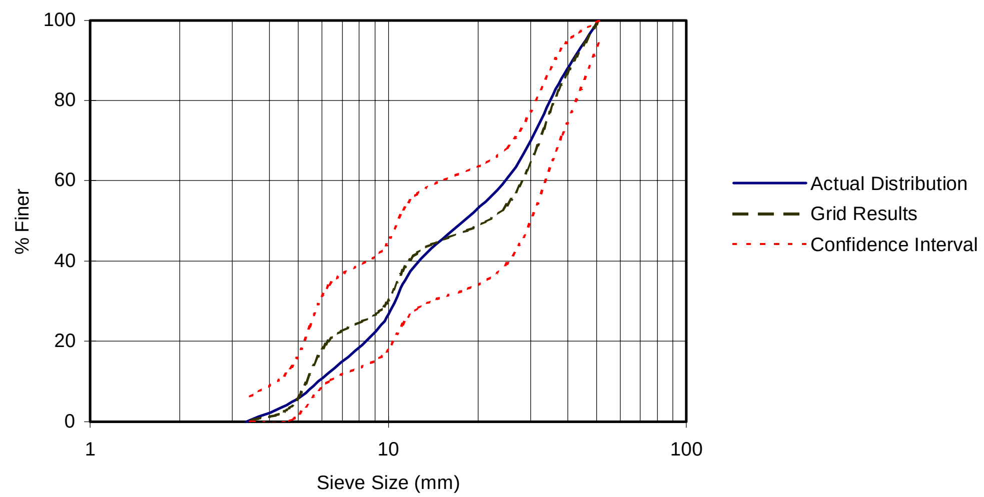
Third 50 Stone Grid Sample with 95%Binomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
50.8	100	100	94.18	100
38.1	85.48	82	68.56	91.42
25	60.50	54	39.32	68.19
12.5	39.53	26	14.63	40.34
9.5	24.20	12	4.53	24.31
6.3	12.10	4	0.49	13.71
4.75	4.84	2	0.05	10.65
3.35	0	0	0	5.82



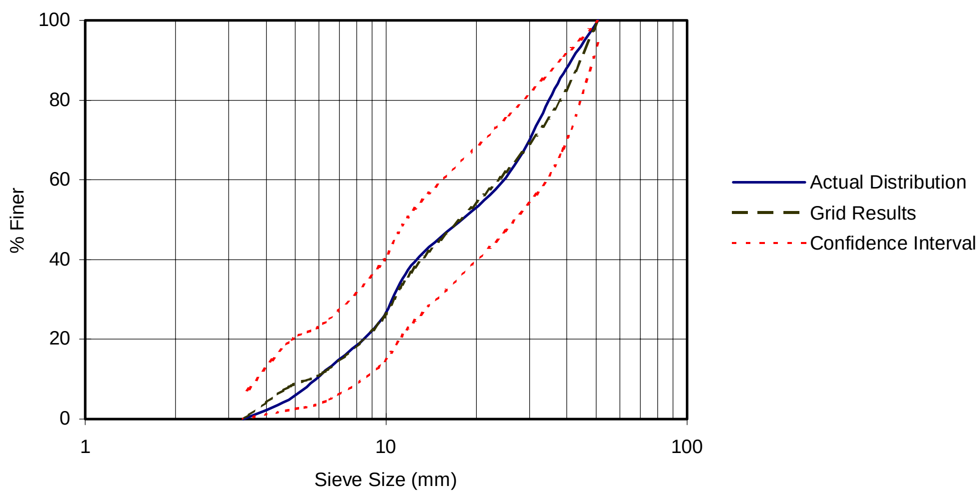
Fourth 50 Stone Grid Sample with 95% Binomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
50.8	100	100	94.18	100
38.1	85.48	84	70.89	92.83
25	60.50	54	39.32	68.19
12.5	39.53	42	28.19	56.79
9.5	24.20	28	16.23	42.49
6.3	12.10	20	10.03	33.72
4.75	4.84	4	0.49	13.71
3.35	0	0	0	5.82



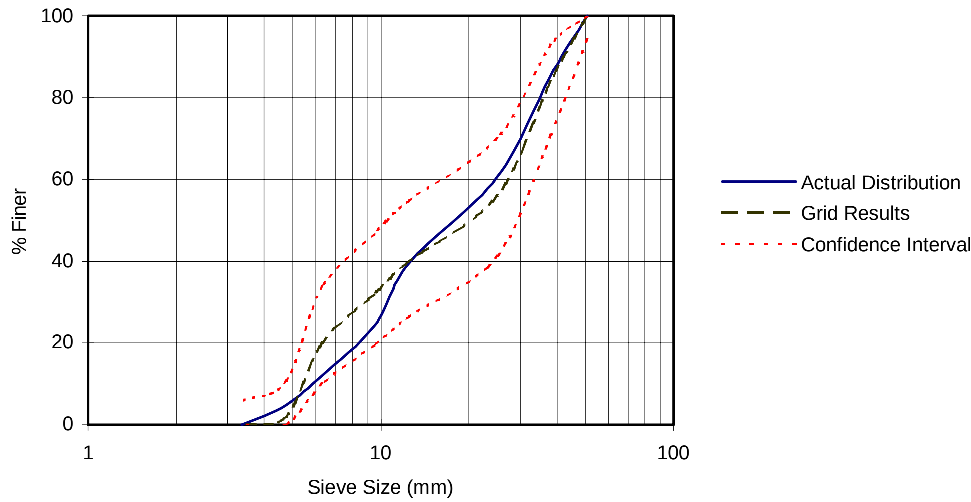
Fifth 50 Stone Grid Sample with 95% Binomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
50.8	100	100	94.18	100
38.1	85.48	80	66.28	89.97
25	60.50	62	47.17	75.35
12.5	39.53	38	24.65	52.83
9.5	24.20	24	13.06	38.17
6.3	12.10	12	4.53	24.31
4.75	4.84	8	2.22	19.23
3.35	0	0	0	5.82



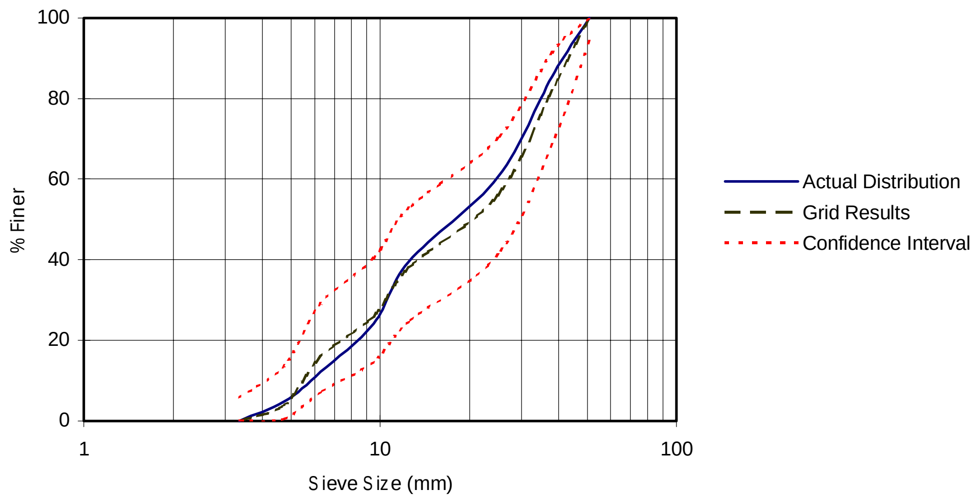
Sixth 50 Stone Grid Sample with 95% Binomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
50.8	100	100	94.18	100
38.1	85.48	84	70.89	92.83
25	60.50	56	41.25	70.01
12.5	39.53	40	26.41	54.82
9.5	24.20	32	19.52	46.70
6.3	12.10	20	10.03	33.72
4.75	4.84	2	0.05	10.65
3.35	0	0	0	5.82



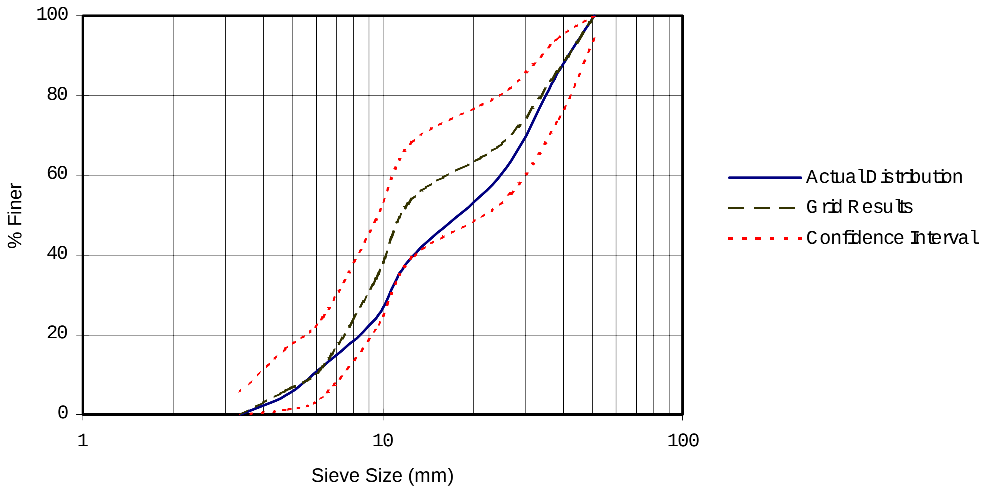
Seventh 50 Stone Grid Sample with 95% Binomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
50.8	100	100	94.18	100
38.1	85.48	82	68.56	91.42
25	60.50	56	41.25	70.01
12.5	39.53	38	24.65	52.83
9.5	24.20	26	14.63	40.34
6.3	12.10	16	7.17	29.11
4.75	4.84	4	0.49	13.71
3.35	0	0	0	5.82



Eighth 50 Stone Grid Sample with 95% Binomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
50.8	100	100	94.18	100
38.1	85.48	86	73.26	94.18
25	60.50	68	53.30	80.48
12.5	39.53	54	39.32	68.19
9.5	24.20	34	21.21	48.77
6.3	12.10	12	4.53	24.31
4.75	4.84	6	1.25	16.55
3.35	0	0	0	5.82

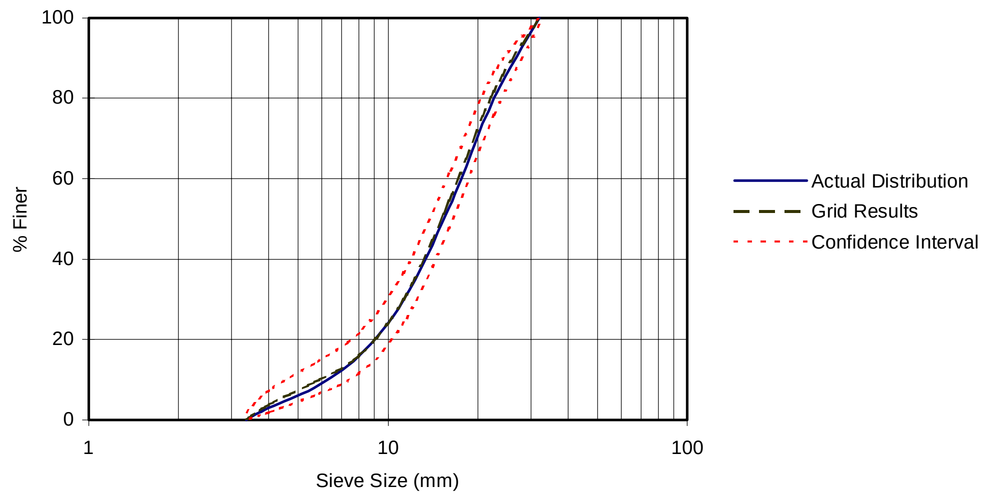


Appendix K. Multinomial Confidence Interval Results for Glass Bead Grid Samples

Multinomial confidence intervals are applied to the grid sampling results for the glass bead distribution using the Goodman (1965) technique described in section 2.4.2. These sampling results for the different sample sizes are shown in Appendix D. The sample grain size curve and the 95% multinomial confidence intervals are compared with the actual grain size distribution

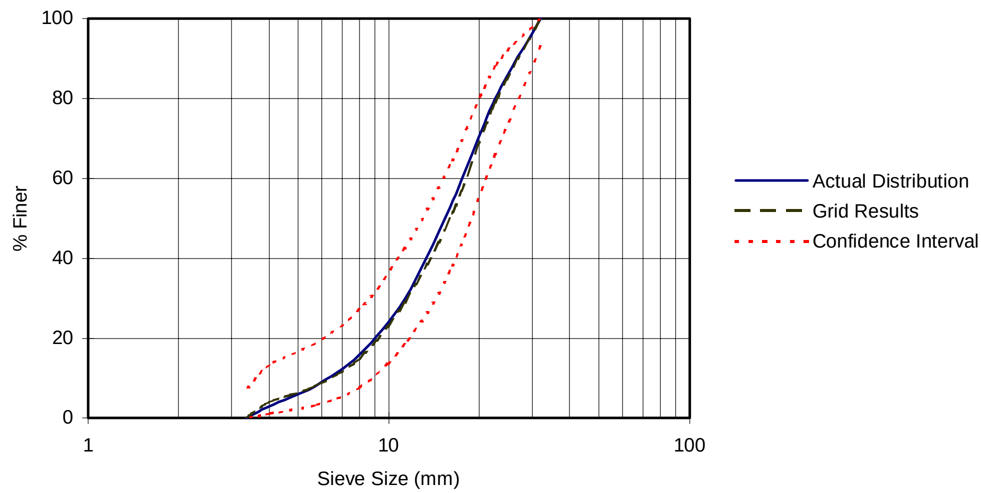
400 Stone Grid Sample with 95% Multinomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
32	100	100	98.17	100.00
22.6	80.039	81.75	75.90	86.43
16	52.833	54.5	47.67	61.16
11.3	29.913	30	24.15	36.59
8	15.952	16	11.62	21.63
5.66	7.982	9.25	6.00	13.99
4	2.992	3.75	1.89	7.31
3.36	0	0	0.00	1.83



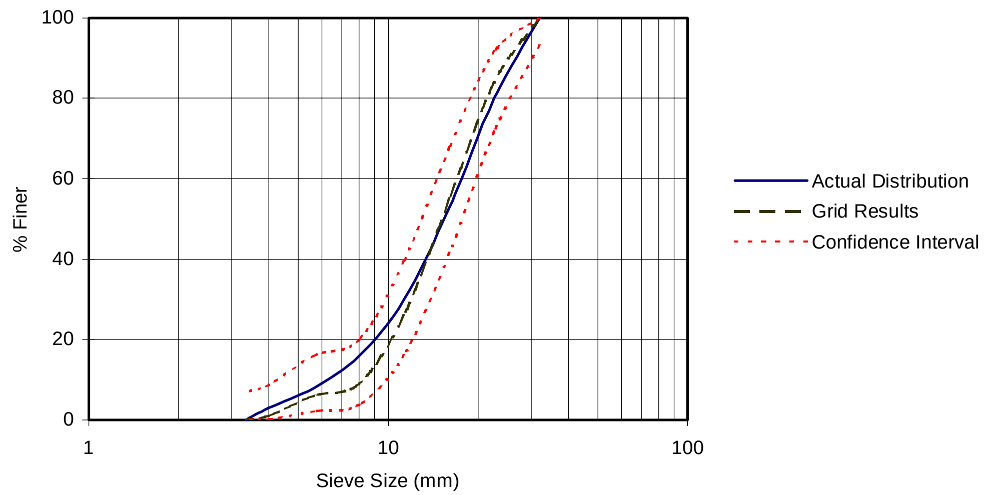
First 100 Stone Grid Sample with 95% Multinomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
32	100	100	93.04	100.00
22.6	80.039	79	66.05	87.91
16	52.833	50	36.81	63.19
11.3	29.913	29	18.40	42.52
8	15.952	15	7.71	27.16
5.66	7.982	8	3.19	18.65
4	2.992	4	1.12	13.28
3.36	0	0	0.00	6.96



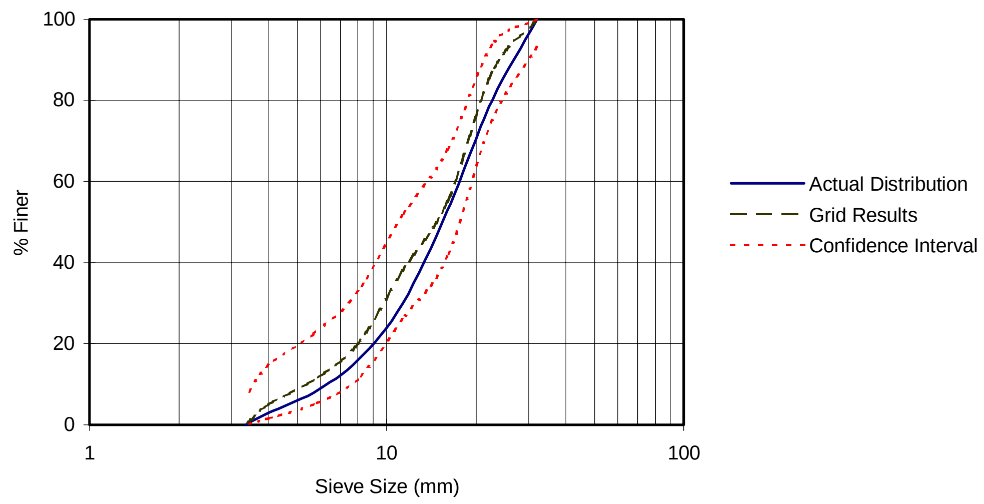
Second 100 Stone Grid Sample with 95% Multinomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
32	100	100	93.04	100.00
22.6	80.039	84	71.68	91.59
16	52.833	55	41.53	67.78
11.3	29.913	26	15.98	39.36
8	15.952	9	3.78	19.92
5.66	7.982	6	2.09	16.03
4	2.992	1	0.11	8.71
3.36	0	0	0.00	6.96



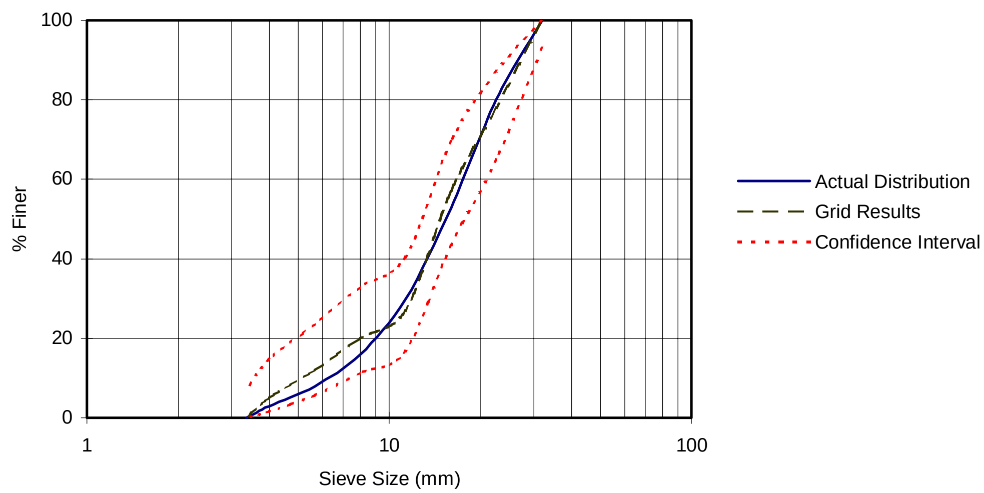
Third 100 Stone Grid Sample with 95% Multinomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
32	100	100	93.04	100.00
22.6	80.039	87	75.19	93.66
16	52.833	55	41.53	67.78
11.3	29.913	38	26.01	51.66
8	15.952	20	11.33	32.84
5.66	7.982	11	5.03	22.40
4	2.992	5	1.58	14.68
3.36	0	0	0.00	6.96



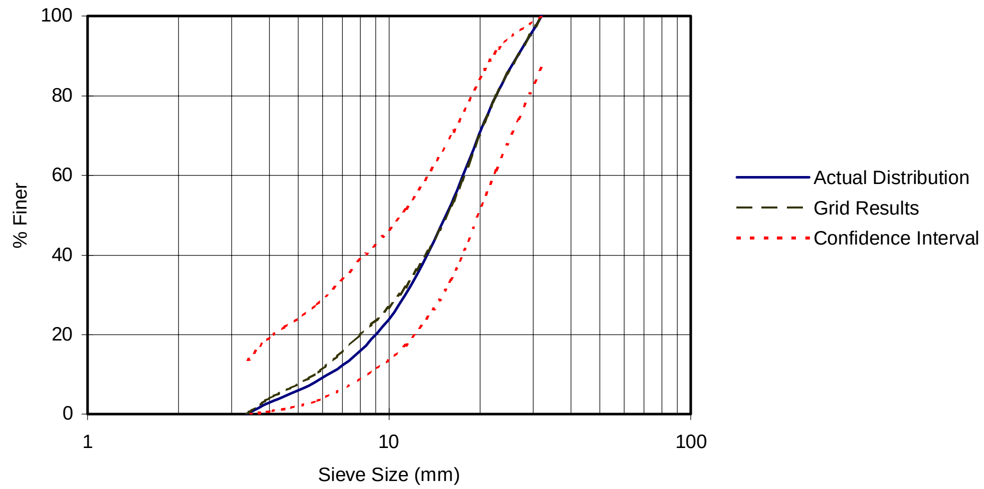
Fourth 100 Stone Grid Sample with 95% Multinomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
32	100	100	93.04	100.00
22.6	80.039	78	64.95	87.15
16	52.833	57	43.45	69.58
11.3	29.913	27	16.78	40.42
8	15.952	20	11.33	32.84
5.66	7.982	12	5.67	23.61
4	2.992	5	1.58	14.68
3.36	0	0	0.00	6.96



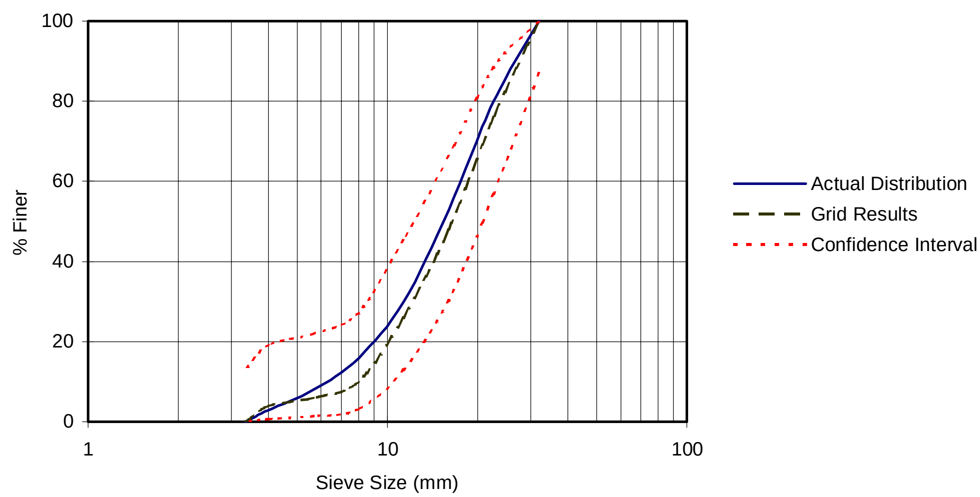
First 50 Stone Grid Sample with 95% Multinomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
32	100	100	86.99	100.00
22.6	80.039	80	61.15	91.04
16	52.833	52	33.72	69.76
11.3	29.913	32	17.35	51.33
8	15.952	20	8.96	38.85
5.66	7.982	10	3.20	27.21
4	2.992	4	0.72	19.24
3.36	0	0	0.00	13.01



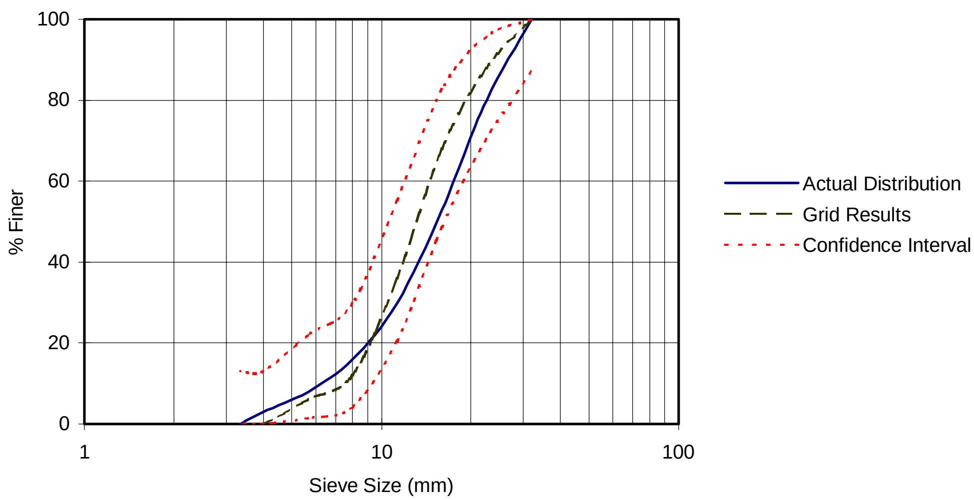
Second 50 Stone Grid Sample with 95% Multinomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
32	100	100	86.99	100.00
22.6	80.039	76	56.85	88.39
16	52.833	48	30.24	66.28
11.3	29.913	26	13.00	45.25
8	15.952	10	3.20	27.21
5.66	7.982	6	1.42	22.03
4	2.992	4	0.72	19.24
3.36	0	0	0.00	13.01



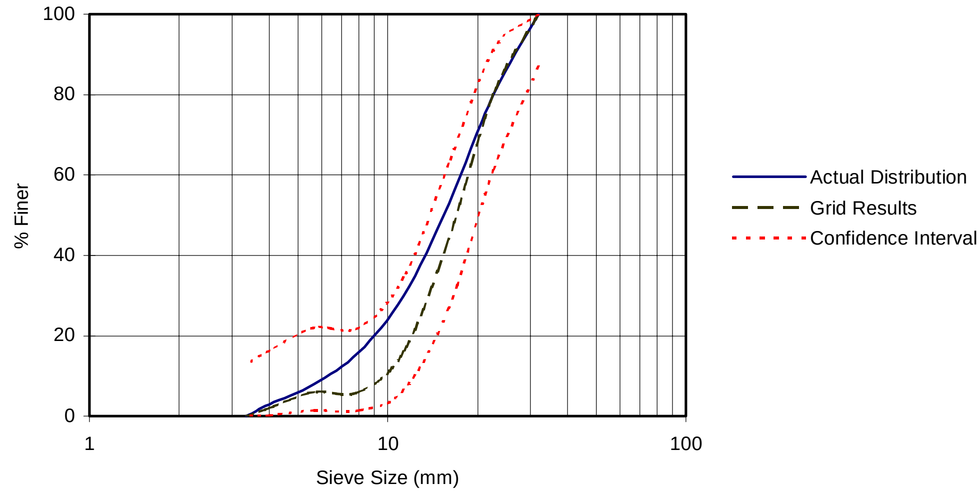
Third 50 Stone Grid Sample with 95% Multinomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
32	100	100	86.99	100.00
22.6	80.039	88	70.34	95.78
16	52.833	68	48.67	82.65
11.3	29.913	36	20.41	55.23
8	15.952	12	4.22	29.66
5.66	7.982	6	1.42	22.03
4	2.992	0	0.00	13.01
3.36	0	0	0.00	13.01



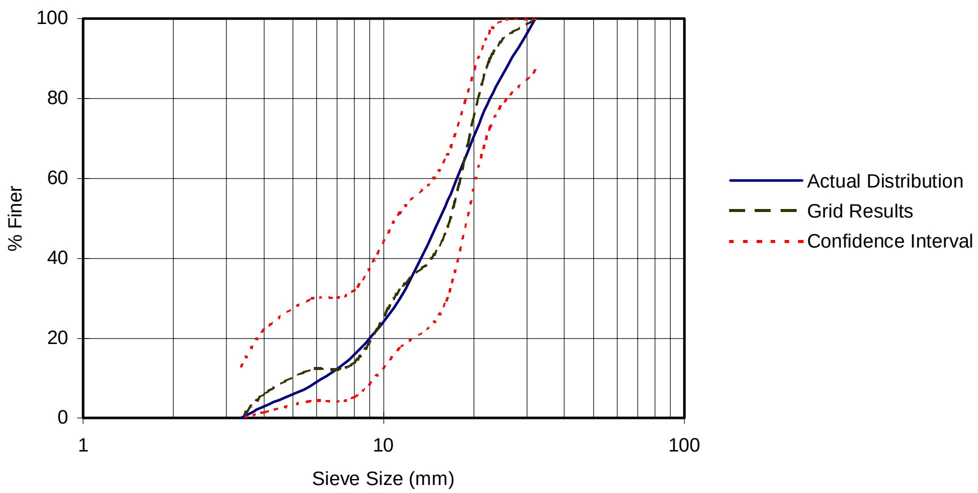
Fourth 50 Stone Grid Sample with 95% Multinomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
32	100	100	86.99	100.00
22.6	80.039	80	61.15	91.04
16	52.833	44	26.86	62.70
11.3	29.913	16	6.48	34.37
8	15.952	6	1.42	22.03
5.66	7.982	6	1.42	22.03
4	2.992	2	0.21	16.27
3.36	0	0	0.00	13.01



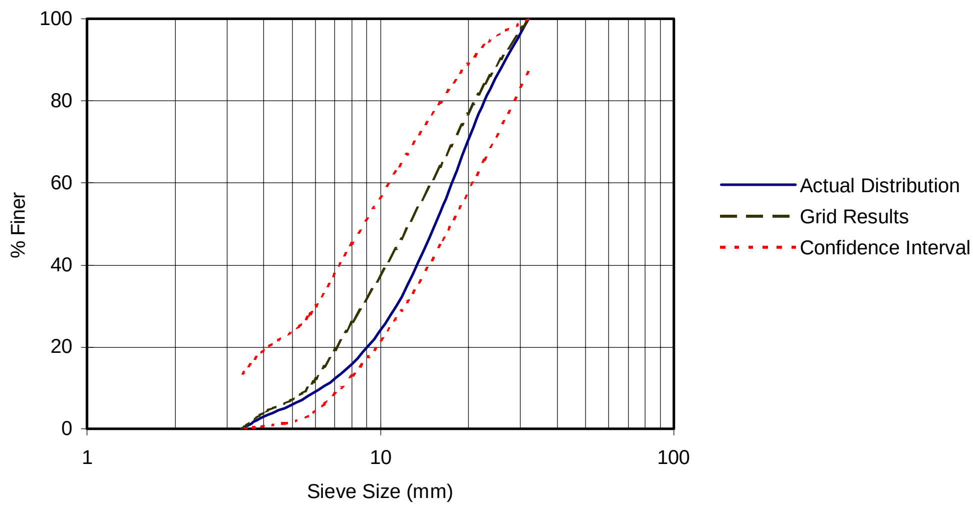
Fifth 50 Stone Grid Sample with 95% Multinomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
32	100	100	86.99	100.00
22.6	80.039	90	72.79	96.80
16	52.833	46	28.54	64.50
11.3	29.913	32	17.35	51.33
8	15.952	14	5.32	32.05
5.66	7.982	12	4.22	29.66
4	2.992	6	1.42	22.03
3.36	0	0	0.00	13.01



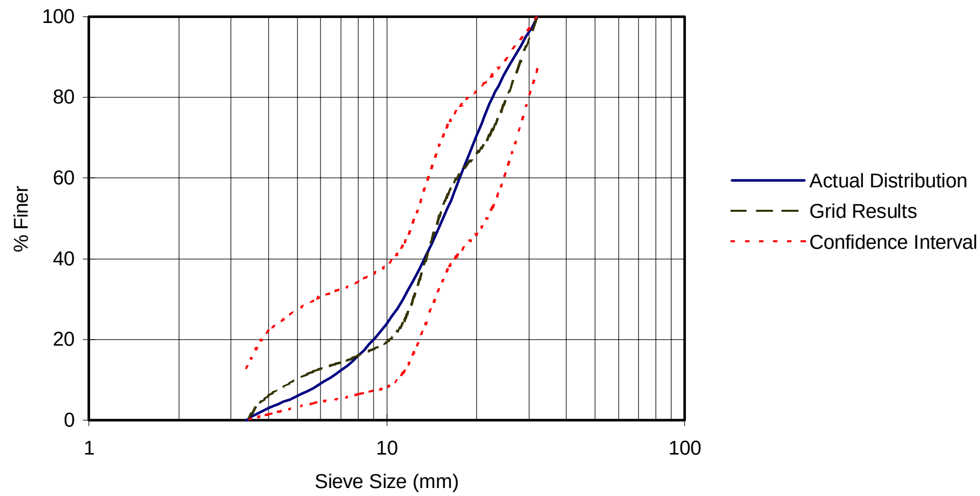
Sixth 50 Stone Grid Sample with 95% Multinomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
32	100	100	86.99	100.00
22.6	80.039	84	65.63	93.52
16	52.833	64	44.77	79.59
11.3	29.913	44	26.86	62.70
8	15.952	26	13.00	45.25
5.66	7.982	10	3.20	27.21
4	2.992	4	0.72	19.24
3.36	0	0	0.00	13.01



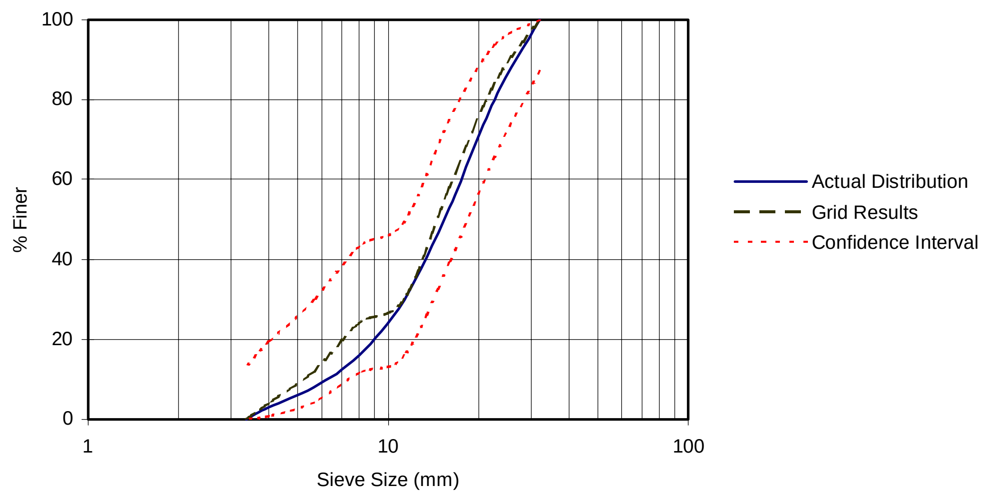
Seventh 50 Stone Grid Sample with 95% Multinomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
32	100	100	86.99	100.00
22.6	80.039	72	52.69	85.58
16	52.833	56	37.30	73.14
11.3	29.913	24	11.61	43.15
8	15.952	16	6.48	34.37
5.66	7.982	12	4.22	29.66
4	2.992	6	1.42	22.03
3.36	0	0	0.00	13.01



Eighth 50 Stone Grid Sample with 95% Multinomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
32	100	100	86.99	100.00
22.6	80.039	84	65.63	93.52
16	52.833	58	39.13	74.79
11.3	29.913	30	15.87	49.33
8	15.952	24	11.61	43.15
5.66	7.982	12	4.22	29.66
4	2.992	4	0.72	19.24
3.36	0	0	0.00	13.01

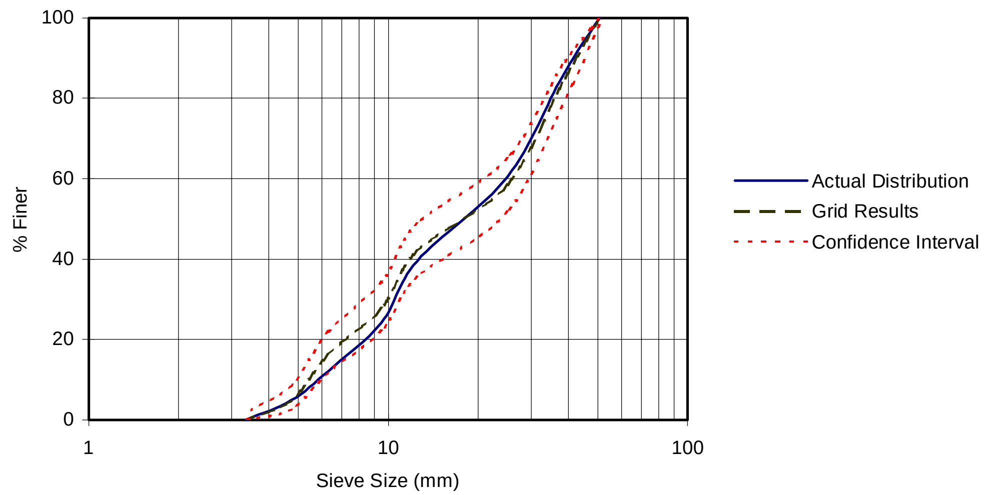


Appendix L. Multinomial Confidence Interval Results for Natural Sediment Grid Samples

Multinomial confidence intervals are applied to the grid results for the natural sediment distribution using the Goodman (1965) technique described in section 2.4.2. These sampling results for the different sample sizes are shown in Appendix E. The sample grain size curve and the 95% multinomial confidence intervals are compared with the actual grain size distribution

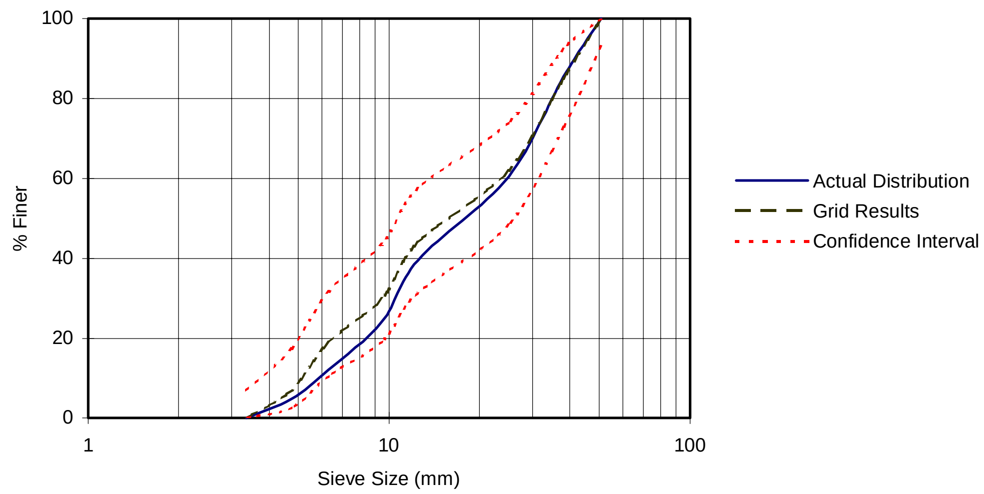
400 Stone Grid Sample with 95% Multinomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
50.8	100	100	98.17	100
38.1	85.48	83.75	78.10	88.17
25	60.50	58.50	51.67	65.02
12.5	39.53	42.00	35.46	48.83
9.5	24.20	27.75	22.08	34.24
6.3	12.10	16.25	11.83	21.90
4.75	4.84	4.75	2.58	8.58
3.35	0	0	0	1.83



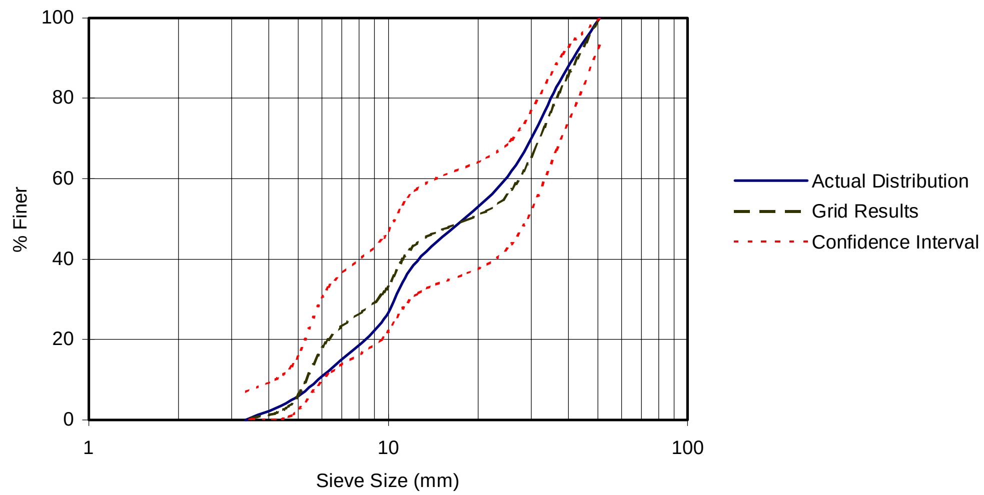
First 100 Stone Grid Sample with 95% Multinomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
50.8	100	100	93.04	100
38.1	85.48	85	72.84	92.29
25	60.50	62	48.34	73.99
12.5	39.53	44	31.32	57.52
9.5	24.20	30	19.22	43.56
6.3	12.10	19	10.59	31.73
4.75	4.84	7	2.63	17.36
3.35	0	0	0	6.96



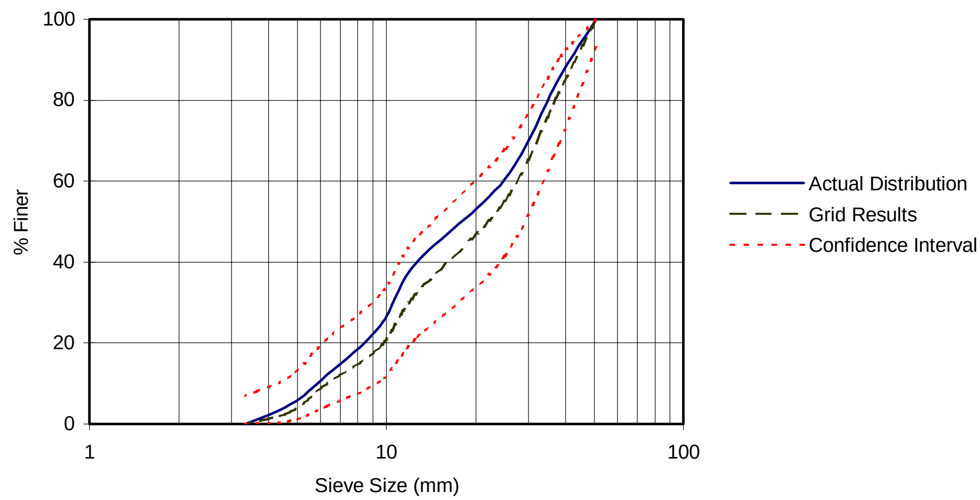
Second 100 Stone Grid Sample with 95% Multinomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
50.8	100	100	93.04	100
38.1	85.48	83	70.53	90.87
25	60.50	56	42.48	68.68
12.5	39.53	44	31.32	57.52
9.5	24.20	31	20.05	44.59
6.3	12.10	20	11.33	32.84
4.75	4.84	4	1.12	13.28
3.35	0	0	0	6.96



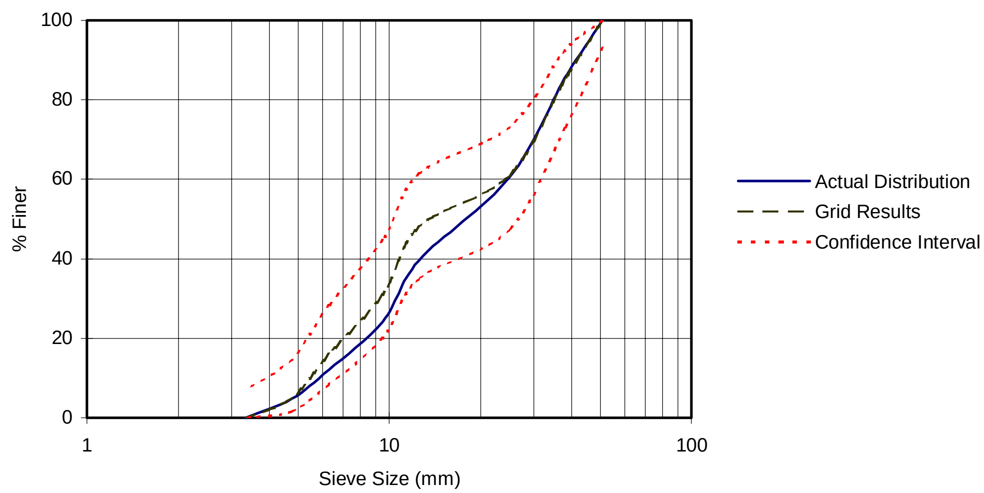
Third 100 Stone Grid Sample with 95% Multinomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
50.8	100	100	93.04	100
38.1	85.48	82	69.40	90.15
25	60.50	55	41.53	67.78
12.5	39.53	32	20.89	45.62
9.5	24.20	19	10.59	31.73
6.3	12.10	10	4.39	21.17
4.75	4.84	3	0.71	11.83
3.35	0	0	0.00	6.96



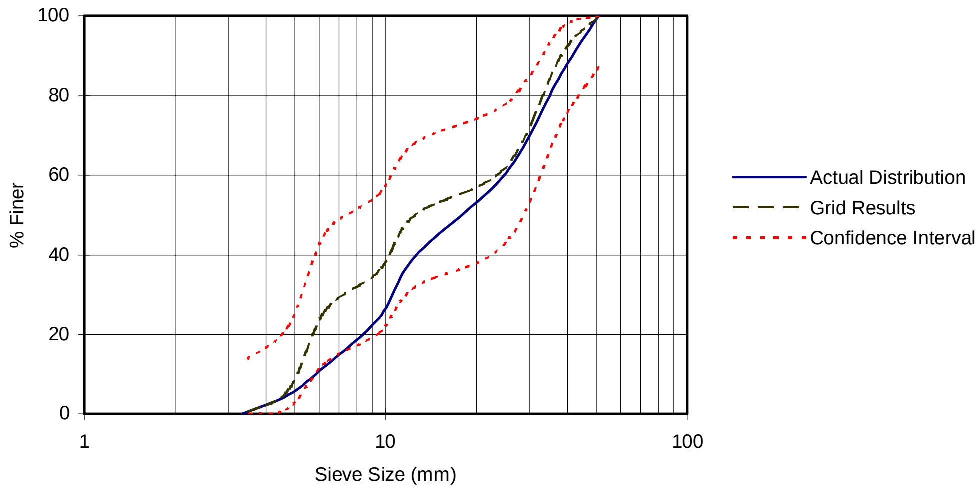
Fourth 100 Stone Grid Sample with 95% Multinomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
50.8	100	100	93.04	100
38.1	85.48	85	72.84	92.29
25	60.50	61	47.35	73.12
12.5	39.53	48	34.96	61.32
9.5	24.20	31	20.05	44.59
6.3	12.10	16	8.41	28.32
4.75	4.84	5	1.58	14.68
3.35	0	0	0.00	6.96



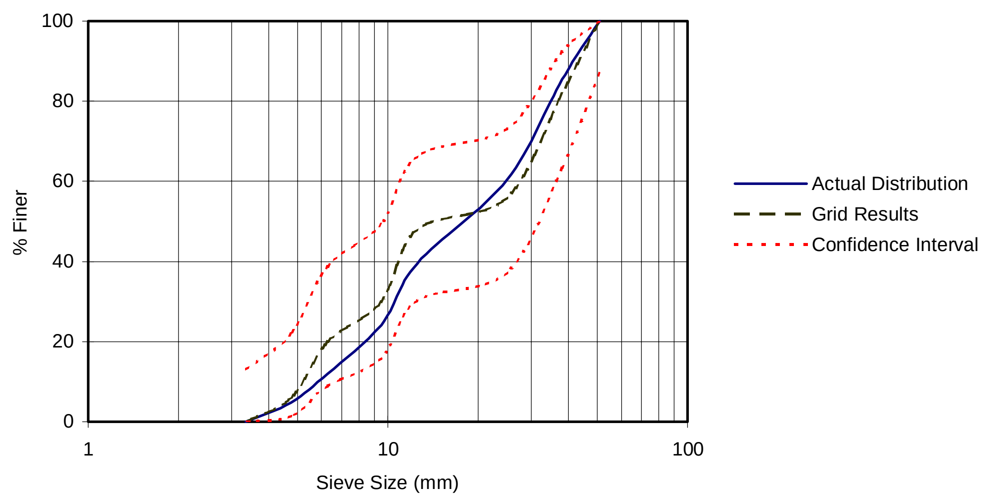
First 50 Stone Grid Sample with 95% Multinomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
50.8	100	100	86.99	100
38.1	85.48	90	72.79	96.80
25	60.50	62	42.86	78.01
12.5	39.53	50	31.97	68.03
9.5	24.20	36	20.41	55.23
6.3	12.10	26	13.00	45.25
4.75	4.84	6	1.42	22.03
3.35	0	0	0.00	13.01



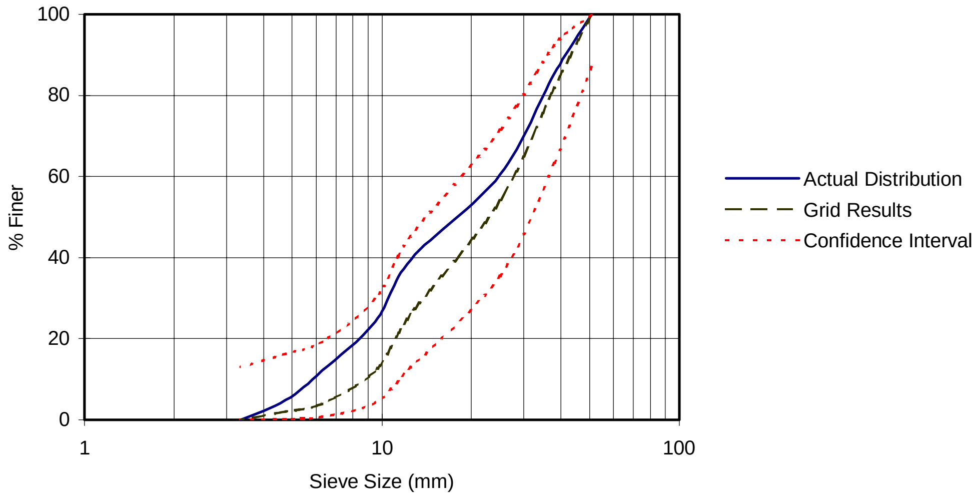
Second 50 Stone Grid Sample with 95% Multinomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
50.8	100	100	86.99	100
38.1	85.48	82	63.37	92.31
25	60.50	56	37.30	73.14
12.5	39.53	48	30.24	66.28
9.5	24.20	30	15.87	49.33
6.3	12.10	20	8.96	38.85
4.75	4.84	6	1.42	22.03
3.35	0	0	0.00	13.01



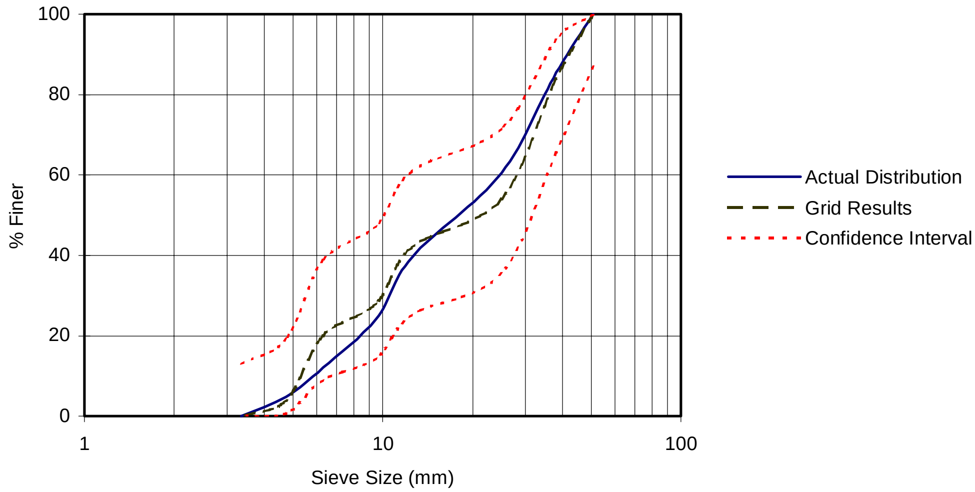
Third 50 Stone Grid Sample with 95% Multinomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
50.8	100	100	86.99	100
38.1	85.48	82	63.37	92.31
25	60.50	54	35.50	71.46
12.5	39.53	26	13.00	45.25
9.5	24.20	12	4.22	29.66
6.3	12.10	4	0.72	19.24
4.75	4.84	2	0.21	16.27
3.35	0	0	0.00	13.01



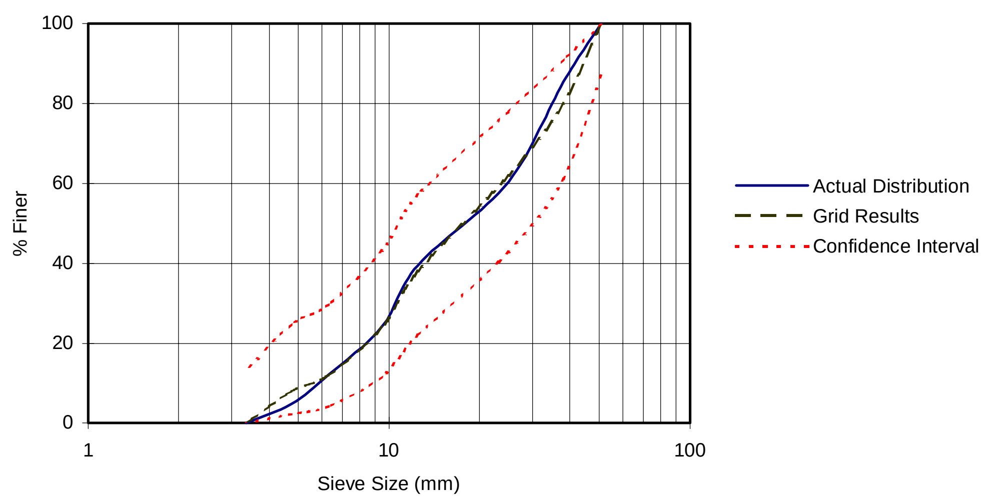
Fourth 50 Stone Grid Sample with 95% Multinomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
50.8	100	100	86.99	100
38.1	85.48	84	65.63	93.52
25	60.50	54	35.50	71.46
12.5	39.53	42	25.21	60.87
9.5	24.20	28	14.42	47.31
6.3	12.10	20	8.96	38.85
4.75	4.84	4	0.72	19.24
3.35	0	0	0.00	13.01



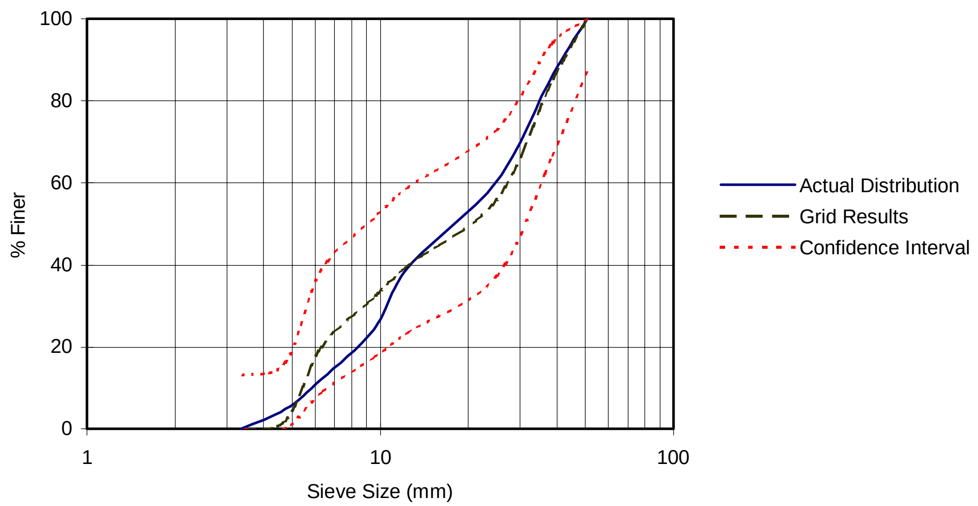
Fifth 50 Stone Grid Sample with 95% Multinomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
50.8	100	100	86.99	100
38.1	85.48	80	61.15	91.04
25	60.50	62	42.86	78.01
12.5	39.53	38	21.99	57.14
9.5	24.20	24	11.61	43.15
6.3	12.10	12	4.22	29.66
4.75	4.84	8	2.26	24.67
3.35	0	0	0.00	13.01



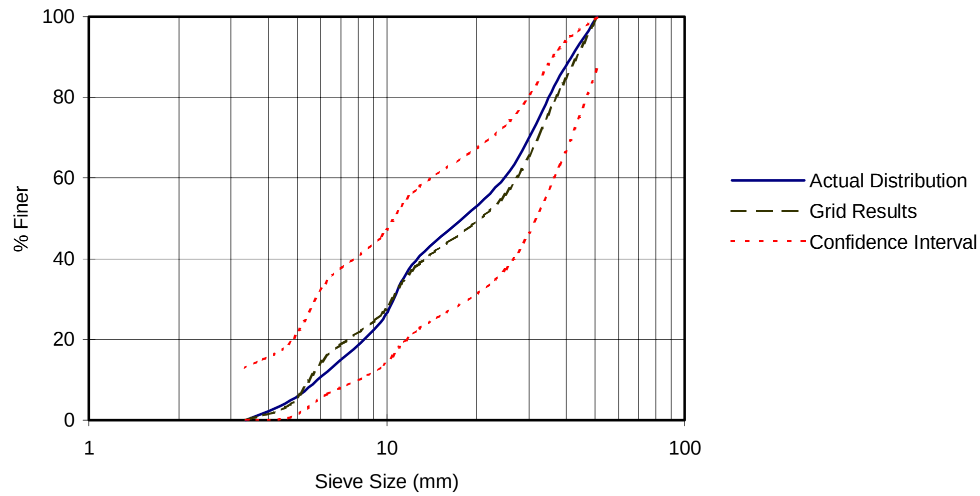
Sixth 50 Stone Grid Sample with 95% Multinomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
50.8	100	100	86.99	100
38.1	85.48	84	65.63	93.52
25	60.50	56	37.30	73.14
12.5	39.53	40	23.58	59.02
9.5	24.20	32	17.35	51.33
6.3	12.10	20	8.96	38.85
4.75	4.84	2	0.21	16.27
3.35	0	0	0.00	13.01



Seventh 50 Stone Grid Sample with 95% Multinomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
50.8	100	100	86.99	100
38.1	85.48	82	63.37	92.31
25	60.50	56	37.30	73.14
12.5	39.53	38	21.99	57.14
9.5	24.20	26	13.00	45.25
6.3	12.10	16	6.48	34.37
4.75	4.84	4	0.72	19.24
3.35	0	0	0.00	13.01



Eighth 50 Stone Grid Sample with 95% Multinomial Confidence Intervals

Sieve Size (mm)	Actual Distribution	Grid Results	Confidence Interval	
50.8	100	100	86.99	100
38.1	85.48	86	67.95	94.68
25	60.50	68	48.67	82.65
12.5	39.53	54	35.50	71.46
9.5	24.20	34	18.87	53.29
6.3	12.10	12	4.22	29.66
4.75	4.84	6	1.42	22.03
3.35	0	0	0.00	13.01