CALCULATION OF THE WAVE DRAG DUE TO LIFT FOR AN ARBITRARY RECTILINEAR-PLANFORM WING-BODY COMBINATION

by

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TABLE OF CONTENTS

		PAGE
I.	LIST OF FIGURES	3
II.	INTRODUCTION	4
III.	LIST OF SYMBOLS	5
IV.	METHOD OF ANALYSIS	7
V.	ANALYSIS	15
	A. Body of Revolution	15
	B. Wing With Subsonic Leading Edges	16
	C. Wing With Supersonic Leading Edges	20
	D. Twisted Wing	25
	E. Body Induced Upwash	27
VI.	SAMPLE CALCULATIONS	28
VII.	DISCUSSION	36
VIII.	CONCLUSIONS	39
IX.	ACKNOWLEDGEMENTS	40
X.	BIBLIOGRAPHY	41
XI.	VITA	42

I. LIST OF FIGURES

FIGURE		PAGE
1.	General System of Coordinates.	8
2.	Pressures acting on the Oblique Sections Lying in the Mach Planes	9
3.	Scheme of Integration for Determining the Oblique Section Lift	12
4.	Interference Model of Lagerstrom and Van Dyke	13
5.	Interference Model Used in Analysis	13
6.	Model Used in Sample Calculations	29
7.	Distribution of $\beta l/2q$ for a Typical Wing Body Configuration. $\theta = 0^{o}$	30
8.	Distribution of $\beta l/2q$ for a Typical Wing-Body Combination. $\theta = 45^{o}$	31
9.	Distribution of $\beta l/2q$ for a Typical Wing-Body Combination.	35
10.	Comparison of the Distributions of $\beta l/2q$ for a Sears-Haach Body of Fineness Ratio 12.5 and a Symmetrically Indented Body	37

II. INTRODUCTION

A paper by Lomax (ref. 1) shows that the wave drag of an object in a steady supersonic flow is identical at a fixed Mach number to the wave drag of a series of equivalent bodies of revolution. The streamwise gradient of cross-sectional area of these equivalent bodies of revolution is given by the sum of two quantities, namely:

1. The streamwise gradient of area, measured in oblique planes tangent to the Mach cones, along the given object.

2. A term proportional to the resultant aerodynamic force on the object measured in the same oblique planes.

If the body is considered to be planar (which is the case for most slender wing-body combinations) then the two quantities above may be treated separately and will yield the wave drag due to thickness and the wave drag due to lift, respectively. The wave drag due to thickness can be computed solely from a knowledge of the geometry of the body as is indicated by the transonic area rule (ref. 2) and the supersonic area rule (refs. 3 and 4). The wave drag due to lift, however, can be determined only if both the geometry and the pressure distribution on the body are known. This knowledge always, of course, fixes the drag of the body.

It will be the purpose of this thesis, then, to apply linearized supersonic theory to rectilinear plan form wing-body combinations in an effort to derive expressions which will predict the shape of the equivalent bodies of revolution without a prior knowledge of the pressure distribution. Then it will be possible to determine both the wave drag due to thickness and the wave drag due to lift solely from the geometry of the wing-body combination.

III. LIST OF SYMBOLS

a	$\frac{\text{Slope of ray from origin}}{\text{Slope of Mach lines}}$, $\beta \frac{y}{x}$
b	wing span
C_p	pressure coefficient, $\frac{\text{Local pressure - Free-stream pressure}}{q}$
ΔC_p	differential pressure coefficient, $C_{p_{lower}} - C_{p_{upper}}$
c_R	root chord
D	wave drag
$E'(m)$	complete elliptic integral of second kind of modulus $\sqrt{1 - m^2}$
$L(\theta)$	downstream limit of distribution of $\beta l/2q$ for constant θ
$-L_1(\theta)$	upstream limit of distribution of $\beta l/2q$ for constant θ
l or $l(\xi,\theta)$	oblique section lift
M	free-stream Mach number
m	$\frac{\text{Slope of leading edge}}{\text{Slope of Mach lines}}$, $\beta \cot \Lambda$
m_t	$\frac{\text{Slope of trailing edge}}{\text{Slope of Mach lines}}$
q	free-stream dynamic pressure
r	body radius

r_o	average body radius in vicinity of wing-body juncture
$S(x,\theta)$	oblique section area
t	$\frac{\text{Slope of ray from leading-edge tip}}{\text{Slope of Mach lines}}$, $\beta \frac{y_a}{x_a}$
x,y,z	Cartesian coordinates in body-axis system
x_a,y_a	Cartesian coordinates with origin at leading-edge tip
x_w	axial coordinate with origin at wing apex
α	angle of attack
$\beta = \sqrt{M^2 - 1}$	
θ	roll angle of Mach cutting plane
Λ	angle of sweep of leading edge measured from y-axis
$\Lambda_{t.e.}$	angle of sweep of trailing edge measured from y-axis
ξ	intersection of Mach cutting plane with body axis measured from nose of configuration
ξ_w	intersection of Mach cutting plane with body axis measured from wing apex
φ	angular displacement of body radius r from z-axis
Ψ	angle between intersection of Mach cutting plane with wing-chord plane and body axis, $\cot^{-1}(\beta \sin \theta)$
$d\sigma_\theta$	incremental element of length along intersection of Mach cutting plane with wing-chord plane

IV. METHOD OF ANALYSIS

The wave drag of any lifting or nonlifting object in a supersonic stream is given by (see ref. 1)

$$\frac{D}{q} = -\frac{1}{4\pi^2}\int_0^{2\pi} d\theta \int_{-L_1(\theta)}^{L(\theta)} dx_1 \int_{-L_1(\theta)}^{L(\theta)} dx_2 \left[S''(x_1,\theta) - \frac{\beta}{2q} l'(x_1,\theta)\right] \left[S''(x_2,\theta) - \frac{\beta}{2q} l'(x_2,\theta)\right] \ln|x_1 - x_2| \tag{1}$$

Where $S''(x,\theta)$ is the second derivative with respect to x of the oblique section area and $l'(x,\theta)$ is the rate of change of the oblique section lift with respect to x. The coordinate system is shown in Figure 1.

Equation (1) was derived from the linearized potential equation for supersonic flow by expressing the wave drag in terms of the perturbation velocities induced by the object on an enclosing cylindrical control surface of infinite radius. This control surface is parallel to the free-stream direction. The only approximations in equation (1) are those basic to linearized supersonic flow.

In the case of a planar system, that is, a system in which the surfaces are everywhere close to a given plane, Lomax has shown that equation (1) reduces to

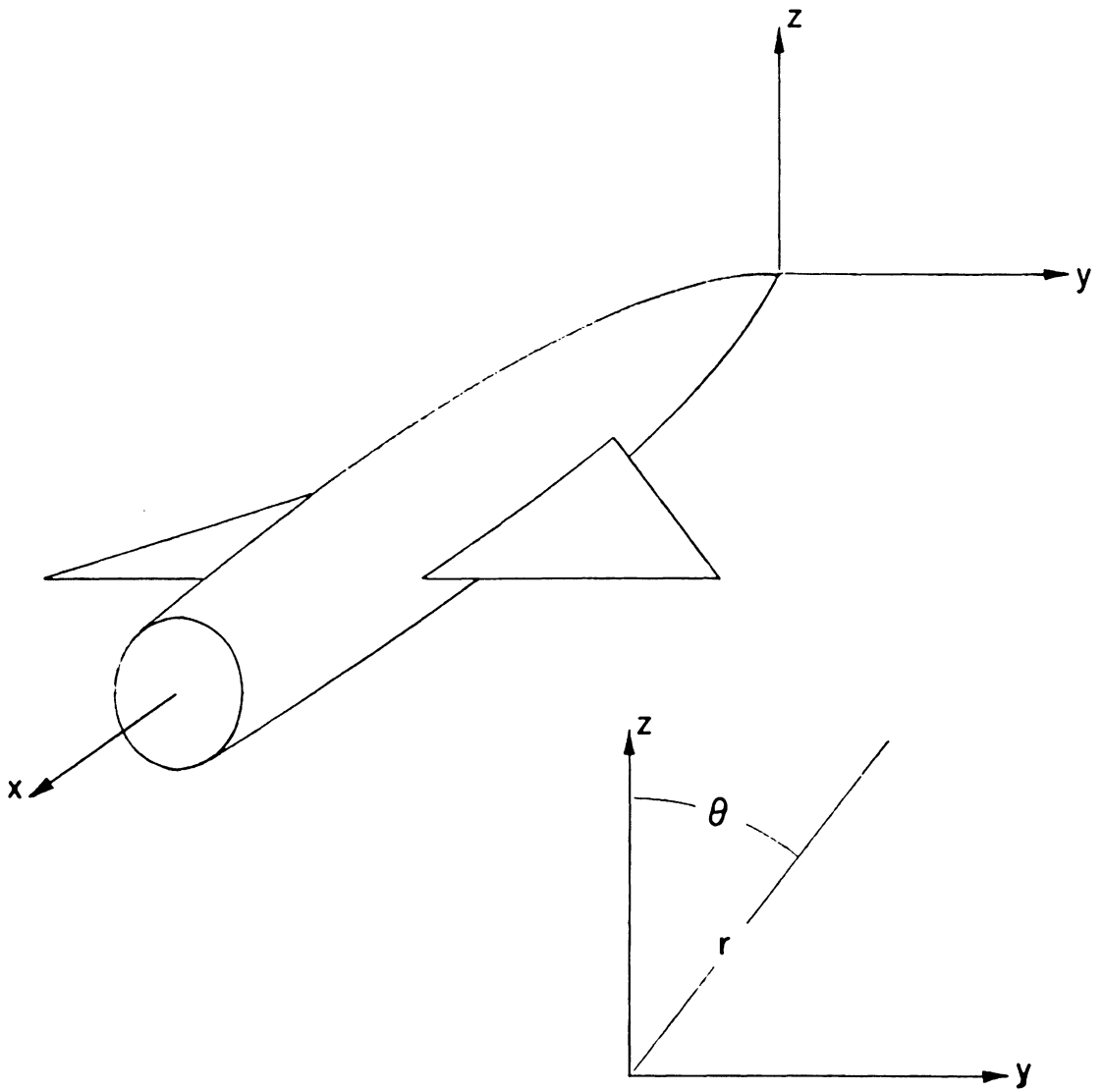


Figure 1.- General system of coordinates.

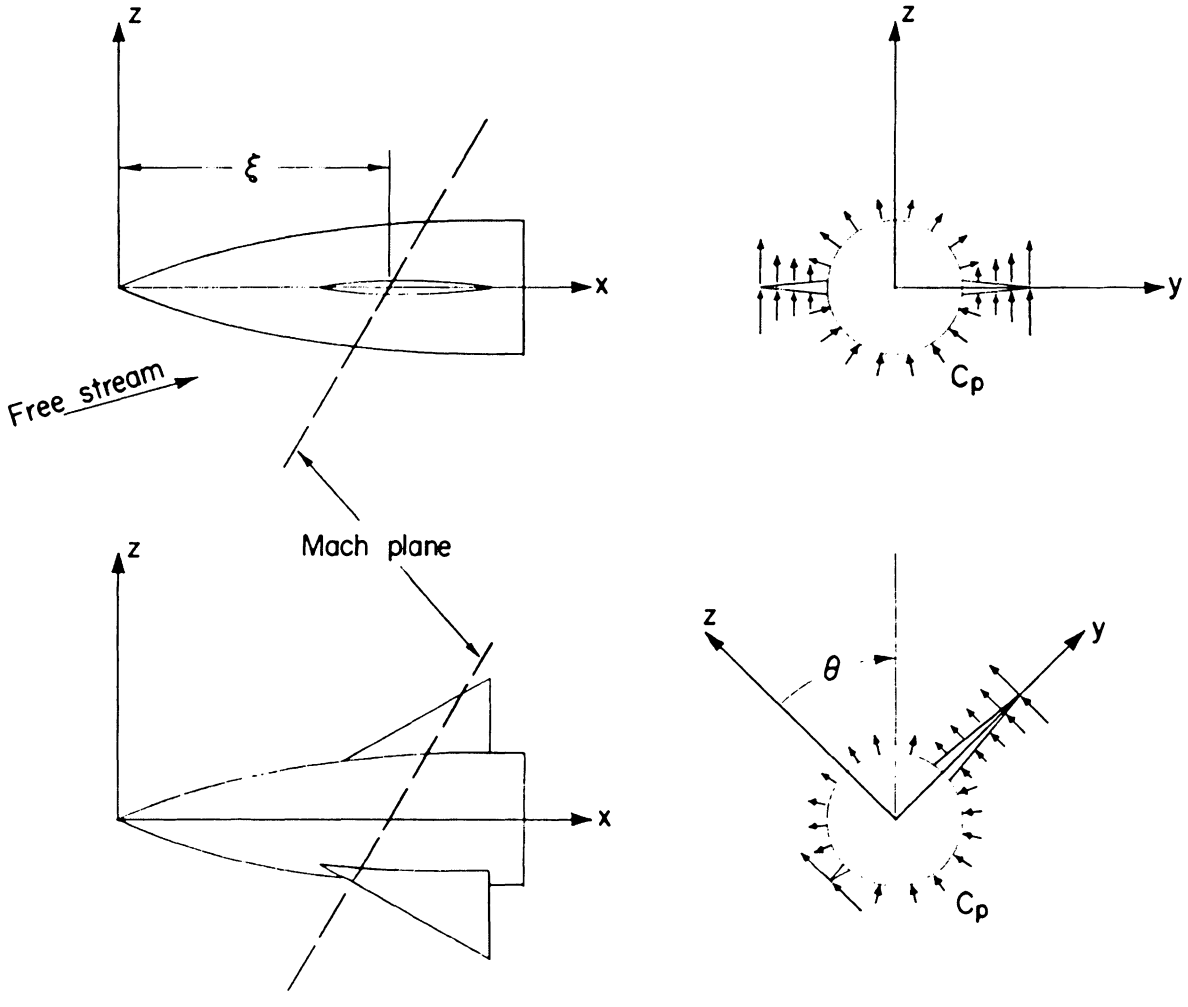


Figure 2.- Pressures acting on the oblique sections lying in the Mach planes

$$\frac{D}{q} = -\frac{1}{4\pi^2}\int_0^{2\pi} d\theta \int_{-L_1(\theta)}^{L(\theta)} dx_1 \int_{-L_1(\theta)}^{L(\theta)} dx_2\, S''(x_1,\theta)\, \ln|x_1 - x_2| - \tag{2}$$

$$\frac{1}{4\pi^2}\int_0^{2\pi} d\theta \int_{-L_1(\theta)}^{L(\theta)} dx_1 \int_{-L_1(\theta)}^{L(\theta)} dx_2 \left(\frac{\beta}{2q}\right)^2 l'(x_1,\theta)\, l'(x_2,\theta)\, \ln|x_1 - x_2|$$

The first part of equation (2) yields the wave drag due to thickness. This component of the total wave drag can be determined by the method of reference 5 from the streamwise gradient of the normally projected, obliquely cut areas, $S'(\xi,\theta)$. The second component of the total wave drag, the wave drag due to lift, can also be obtained by the method of reference 5 if the quantity $S'(\xi,\theta)$ is replaced by $-\beta l(\xi,\theta)/2q$. In reference 1 $l(\xi,\theta)$ is defined as "... the lift (the component of net resultant force parallel to the θ = constant plane and normal to the free stream) on a given section formed by the intersection of a Mach plane with the airplane surface..." If the airplane is fixed and the Mach planes are rotated about the x-axis, $l(\xi,\theta)$ represents the resultant obliquely cut section force normal to the free stream and parallel to the plane θ = constant (see Figure 2).

Two simplifying assumptions which will be used in the following analysis are:

1. The wind axis specified in the theory of reference 1 will be replaced by body axis since the angle of attack is small.

2. All pressures will be considered as acting on a projection of the wing-body combination on the x,y-plane.

Utilising the definition of $l(\xi,\theta)$ and the simplifying assumptions listed above a general expression for the axial distribution of $\beta l/2q$ can be developed. With the aid of the graphical representation of Figure 3 it is seen that

$$-\frac{\beta l(\xi,\theta)}{2q} = -\frac{\beta\cos\theta}{2\sqrt{1+\beta^2\sin^2\theta}}\int \Delta C_p d\sigma_\theta \qquad (3)$$

where $d\sigma_\theta$ indicates integration along the intersection of the Mach cutting plane with the horizontal (or x,y) plane of the object. The local lift coefficient ΔC_p is a function of both x and y. Along the line of integration, however, $x = \beta y\sin\theta + \xi$ (see Figure 3) and equation (3) can thus be rewritten in terms of y as follows,

$$-\frac{\beta l(\xi,\theta)}{2q} = -\frac{\beta\cos\theta}{2}\int \Delta C_p\, dy \qquad (4)$$

since $dy = d\sigma_\theta/\sqrt{1+\beta^2\sin^2\theta}$

It now becomes necessary to consider the manner in which the load is distributed over the surface of a typical wing-body combination. Reference 6 indicates that the lift of a complete configuration, with the wing at an angle of attack α_w and the body at an angle of attack α_B, is equal to the lift of the isolated body plus the lift of the two configurations on the right hand side of Figure 4. In each of these two configurations the body is at an angle of attack of zero and extends to infinity ahead of the wing; in the one case the wing is at an angle of

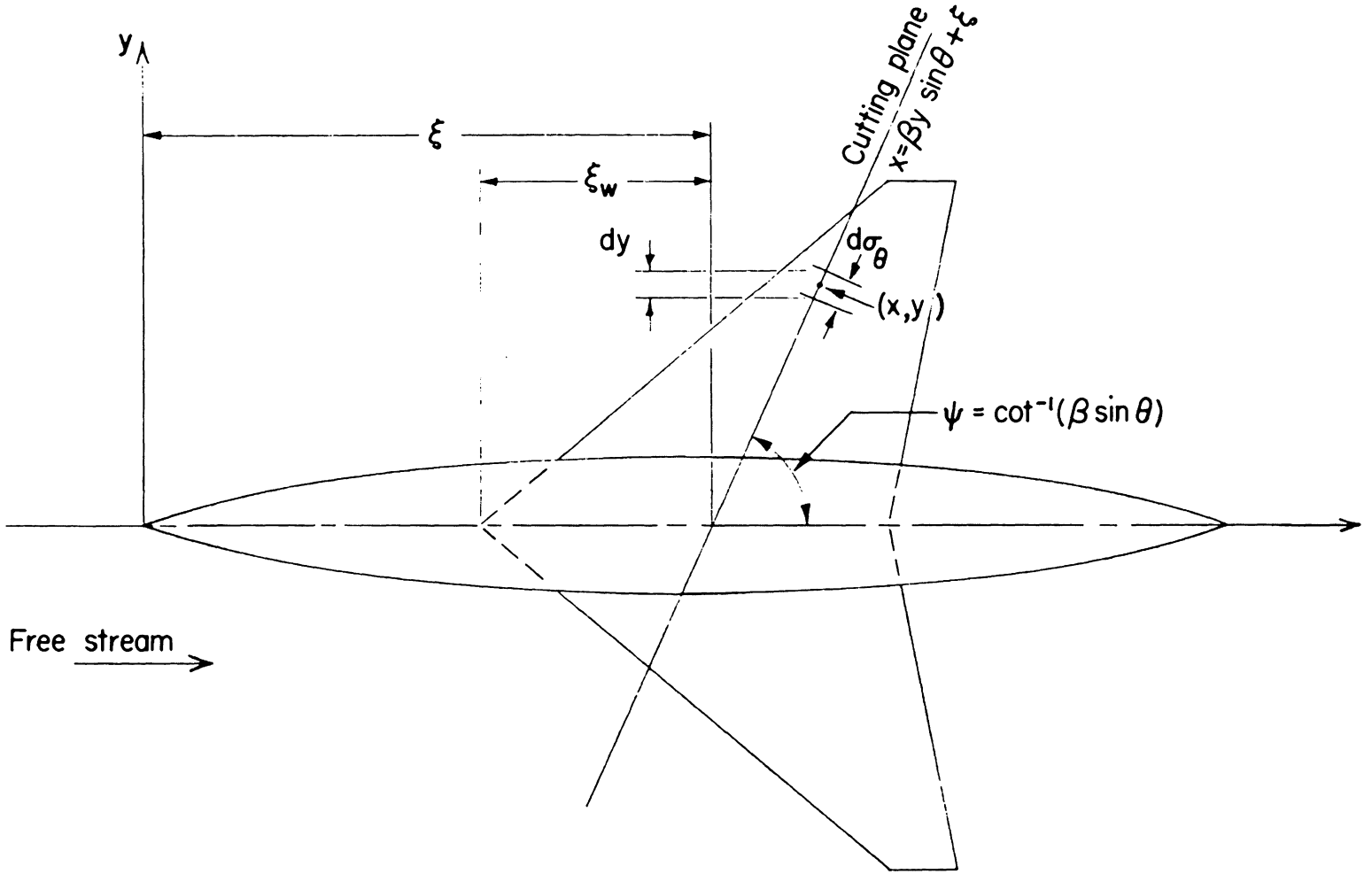


Figure 3.- Scheme of integration for determining the oblique section lift.

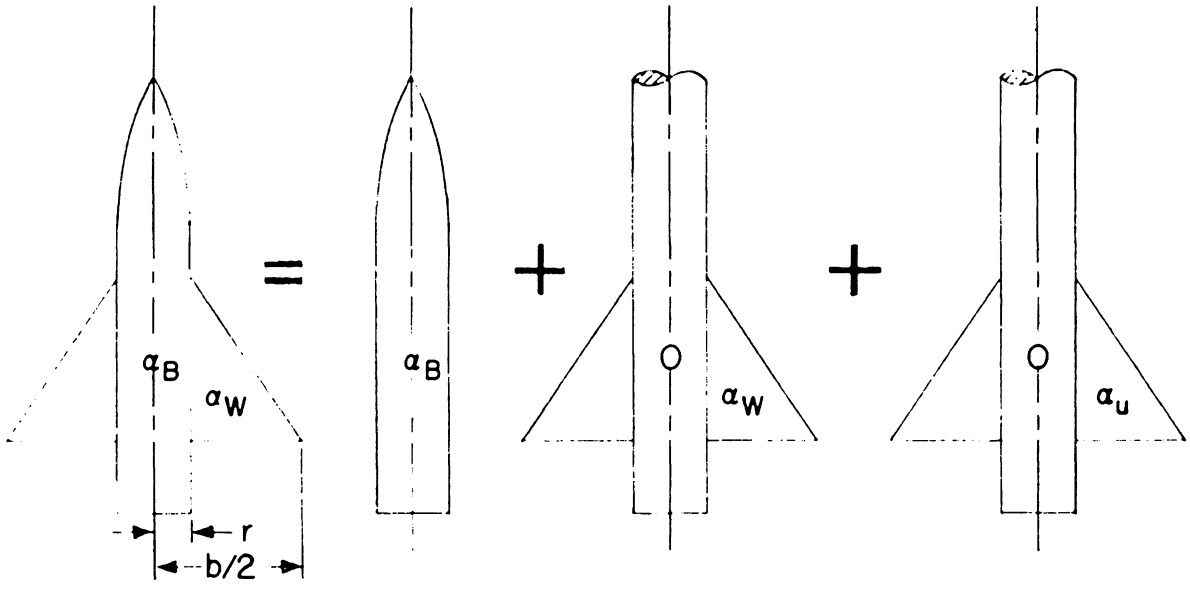


Figure 4.- Interference model of Lagerstrom and Van Dyke.

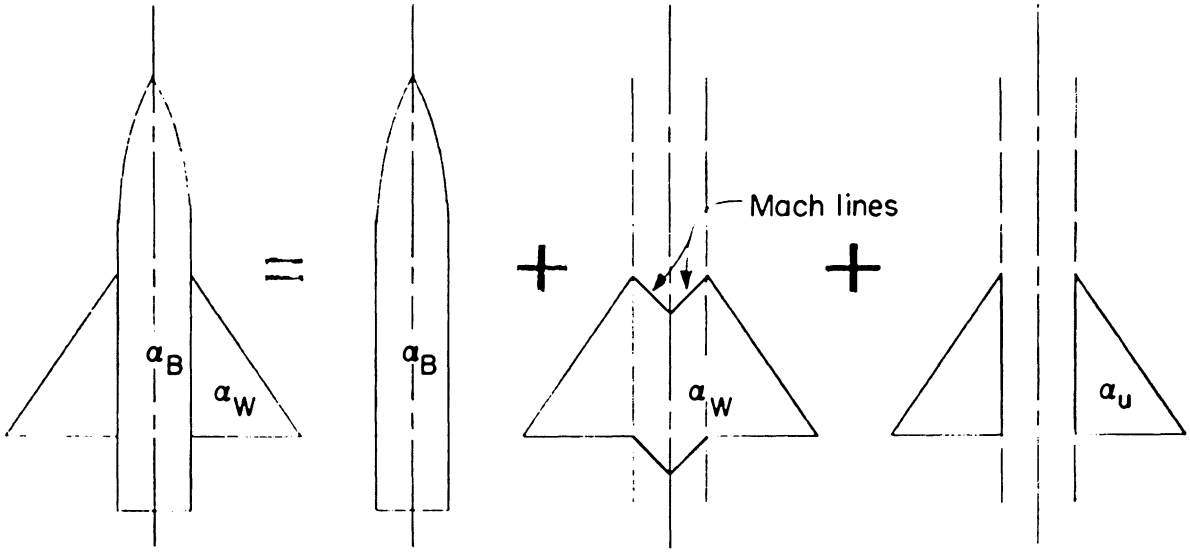


Figure 5.- Interference model used in analysis.

attack α_w, and in the other case the angle of attack, α_u, which varies with the spanwise position along the wing. The angle α_u is the upwash-angle distribution around the isolated body at an angle of attack α_B, and is given by (see ref. 7)

$$\alpha_u = \alpha_B \left(\frac{r}{y}\right)^2 \tag{5}$$

When determining the lineal distribution of $pl/2q$ due to angle of attack the following procedure will be used. The pressures on the body alone will be determined from slender body theory. The pressures on a wing alone, in the region shown in Figure 5, will be computed, and the pressures on the exposed wing due to body upwash (wing will be treated as if it were twisted with the angle of attack distribution α_u) will be found. The total distribution of $pl/2q$ for a wing-body combination at an angle of attack will be the sum of the three components discussed above. It should be noted that the two wing configurations on the right hand side of Figure 5 are not equivalent to the corresponding wing configurations in Figure 4. A rigorous solution of the flow about the wing configuration in Figure 4 has not been published. Therefore, in order to obtain useful approximate values for the local lift coefficient on the wing and on the body, two assumptions are made. The first of these assumptions is that the cylindrical body can be replaced by a flat plate in the plane of the wing (but upwash is still taken to be that around the cylindrical body). The ~~section~~ second assumption is that the influence of the wing on the body is confined to the area bounded by the Mach lines extending from the leading and trailing edge wing-body juncture.

V. ANALYSIS

A. Body of Revolution

The pressure distribution on a slender body of revolution due to an angle of attack α_B is given in reference 8 as

$$\Delta C_p = 8\alpha_B \frac{dr}{d\xi} \cos \varphi \tag{6}$$

where φ is the angular displacement from the positive z axis.

If the body is slender the integrated load on a section formed by the intersection of an oblique cutting plane with the body surface is approximately the same as the integrated load on a cross-section normal to the body axis. Thus the distribution of $\beta l/2q$ may be written as

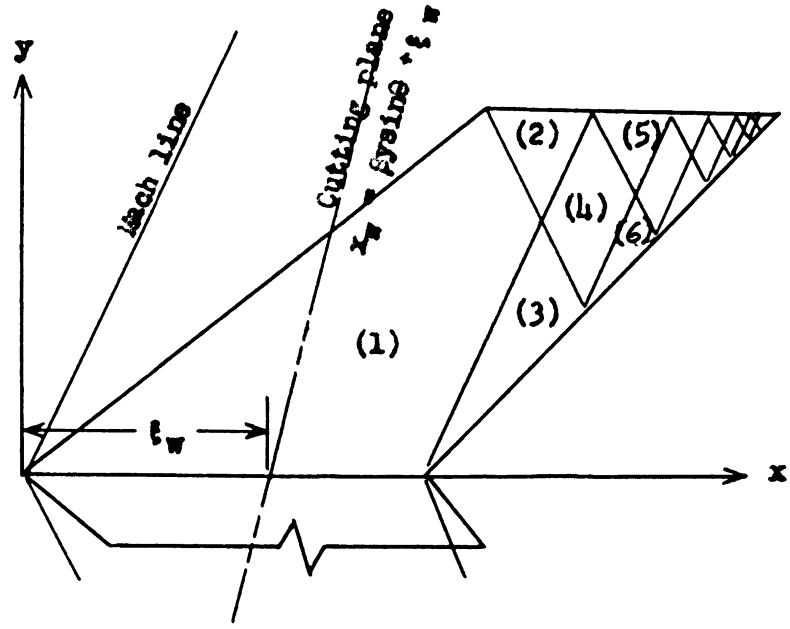
$$-\frac{\beta l}{2q} = -\beta \cos \theta \int_0^{\pi/2} \Delta C_p \, r \sin \varphi \, d\varphi \tag{7}$$

Upon substitution of equation (6) and completion of the required integration equation (7) becomes

$$\left.\begin{aligned} -\frac{\beta l}{2q} &= -4\beta\alpha_B r \cos\theta \frac{dr}{d\xi} \\ \text{or} \\ -\frac{\beta l}{2q} &= -\frac{2}{\pi} \beta\alpha_B \cos\theta \frac{dS_{Body}}{d\xi} \end{aligned}\right\} \tag{8}$$

where S_{Body} is the cross-sectional area distribution of the body normal to the free stream.

B. Wing With Subsonic Leading Edges



Sketch 1.

The flow field over the wing shown in sketch 1 can be determined from conical flow relations. When integrating the loading along the cutting plane in order to determine $\beta l/2q$ the problem is simplified if only half of the wing panel is considered at one time. Consider first the right wing panel. The loading in region (1) (see ref. 9) is given by the relation

$$\Delta C_{p(1)} = \frac{4m^2 a}{\beta E'(m)\sqrt{m^2 - a^2}} \tag{9}$$

Combining equation (9) and equation (4) yields

$$-\frac{\beta l}{2q}\bigg|^{r.v.}_{(1)} = -\frac{2m^2 a \cos\theta}{\sqrt{1 + \beta^2 \sin^2\theta}\, E'(m)} \int \frac{d\sigma\theta}{\sqrt{m^2 - a^2}} \tag{10}$$

Making the substitutions $a = \beta y/x_w$ and $x_w = \beta y \sin\theta + \xi_w$ and performing the specified integration equation (10) becomes

$$-\frac{\beta l}{2q}\Bigg|_{(1)}^{r.w.} = -\frac{2m^2 a \xi_w \cos\theta}{\beta(1 - m^2\sin^2\theta)E'(m)} \left\{ \frac{-\sin\theta\sqrt{m^2 - a^2}}{1 - a\sin\theta} + \frac{\sin^{-1}\dfrac{a - m^2\sin\theta}{m - am\sin\theta}}{\sqrt{1 - m^2\sin^2\theta}} \right\}_{a_1}^{a_0} \tag{11}$$

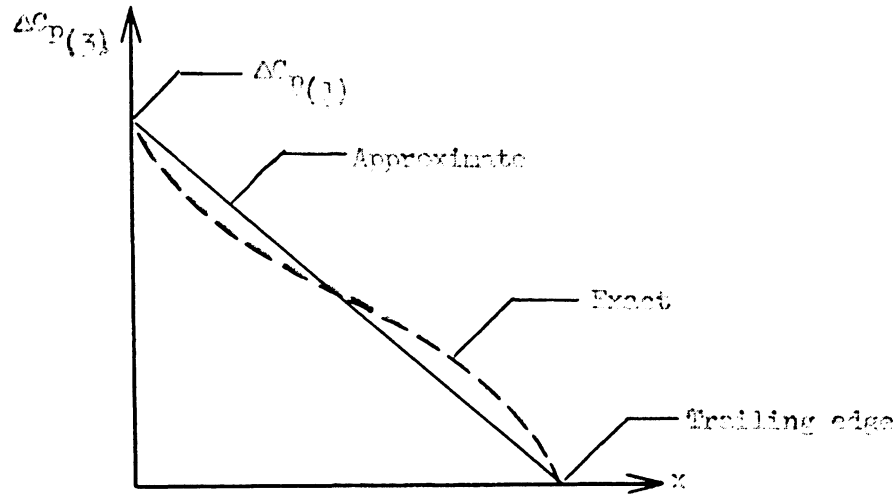
The limits of integration a_1 and a_0 are the inboard and outboard intersections, respectively, of the Mach cutting plane with the boundaries of region (1) for the right wing panel. These limits are in terms of the conical coordinate a. The subscript w indicates that the coordinates x_w and ξ_w are measured from the wing apex and not from the nose of the wing-body combination.

Equation (11) is also valid for the left wing panel. However, the values for a and m will be negative.

In region (2) the integration of the loading along the intersection of the Mach cutting plane and the wing involves the integration of a combination of complete and incomplete elliptic integrals. This integration is not straight forward and will not be attempted here. However, the lack of an expression for $\beta l/2q$ in this region generally will have little effect upon the accuracy of the distribution for the total wing, since the lift in this region is small in comparison to the lift of the total wing (see ref. 9).

When the wing has a subsonic trailing edge the pressure coefficient in region (3) is given by an expression not suitable for direct integration.

Therefore, the chordwise pressure distribution in this region will be approximated by a triangular distribution. A comparison of the two loadings is shown in sketch 2.



Sketch 2.

It should be noted that the area under the two curves is the same so that the total lift is unaffected by the use of the approximate pressure distribution.

The approximate pressure coefficient in region (3) is

$$\Delta C_{p(3)} = \Delta C_{p(1)} \frac{\beta(1/m_t - \sin\theta)y + (c_R - t_w)}{\beta(1/m_t - 1)y} \tag{12}$$

where

$$\Delta C_{p(1)} = \frac{4m^2 a}{\beta E'(m)\sqrt{m^2 - a^2}}$$

and

$$a = \frac{\beta y}{\beta y + c_R}$$

Therefore,

$$\Delta C_{p(3)} = \frac{4m^2 a}{\beta^2 E'(m)(1/m_t - \beta)} \left\{ \frac{(\beta y + c_R)\left[(1/m_t - \sin\theta)\beta y + (c_R - \epsilon_w)\right]}{y\sqrt{(m^2-1)\beta^2 y^2 + 2\beta m^2 c_R y + c_R{}^2 m^2}} \right\} \quad (13)$$

Performing the integration of $\Delta C_{p(3)}$ along the cutting plane in the manner prescribed by equation (4) yields

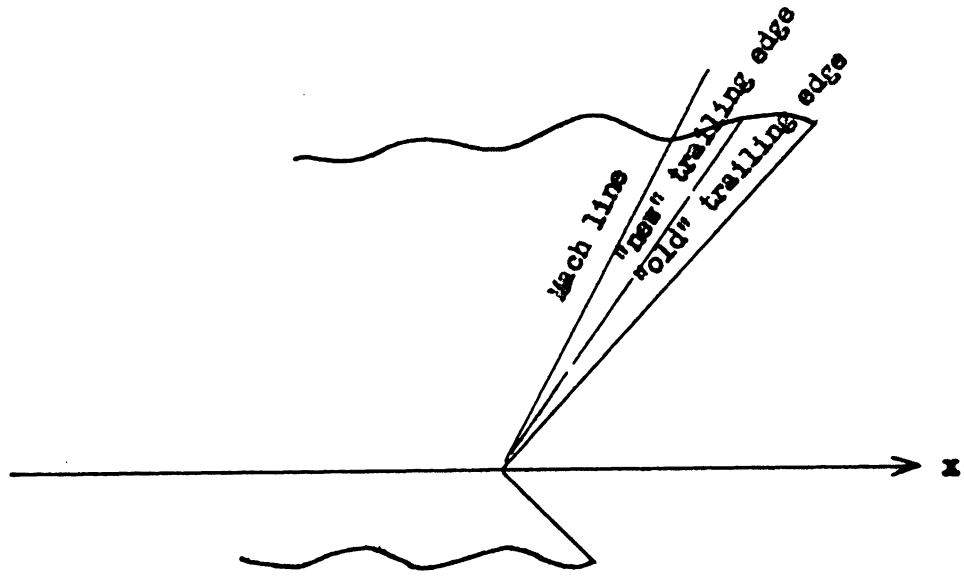
$$-\frac{\beta l}{2q}\bigg|_{(3)} = -\frac{2m^2 a \cos\theta}{\beta E'(m)(1/m_t - 1)} \left\{ \frac{(1/m_t - \sin\theta)}{(m^2-1)} \sqrt{(m^2-1)\beta^2 y^2 + 2\beta m^2 c_R y + m^2 c_R{}^2} + \right.$$

$$\beta \left(\frac{-(m^2-1)(c_R - \epsilon_w) + c_R(1/m_t - \sin\theta)}{m^2 - 1} \right) \sin^{-1}\left(\frac{\beta(m^2-1)y + m^2 c_R}{m c_R} \right) -$$

$$\left. \frac{(c_R - \epsilon_w)}{m} \log\left[\frac{\sqrt{\beta^2(m^2-1)y^2 + 2\beta m^2 c_R y + m^2 c_R{}^2} + m c_R}{y} + \beta m \right] \right\}_{y_1}^{y_o}$$

(13a)

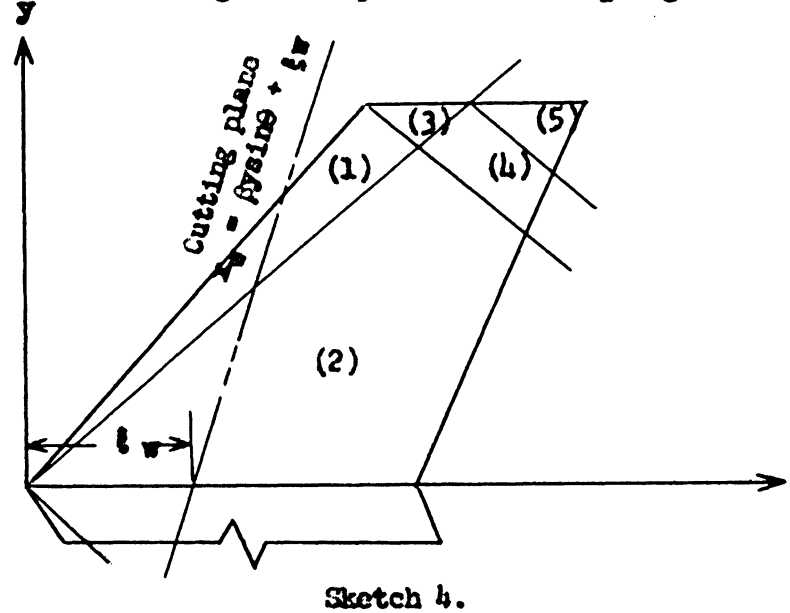
If the area of region (3) is small compared to the total wing area a further approximation of the loading in region (3) can be used. Assume that the wing has a new trailing edge given by the line which bisects the angle formed by the old trailing edge and the Mach line emanating from the trailing edge vertex (see sketch 3). Next assume that all of the wing ahead of the new trailing edge and outside the influence of the tip has a region (1) loading and compute the distribution of $\beta l/2q$ on this basis.



Sketch 3.

Other regions of interacting flow fields, such as regions (4), (5), (6), etc., (see sketch 1) will be considered to have no lifting pressures and will be omitted from these calculations. This assumption should add little error to the total axial distribution of $\beta l/2q$ if these regions are small. Reference 9 indicates that the contribution of these regions to the total lift is negligible.

C. Wing With Supersonic Leading Edge



Sketch 4.

In region (1) the flow is the same as the flow over a swept wing of infinite aspect ratio. The local lift coefficient is given by

$$\Delta C_{p(1)} = \frac{4m\alpha}{\beta\sqrt{m^2 - 1}} \tag{14}$$

Substituting the value of the local lift coefficient (eq. (14)) into equation (4) and performing the specified integration yields

$$-\frac{\beta l}{2q}\bigg|_{(1)} = -\frac{2m\alpha \cos\theta}{\sqrt{m^2 - 1}}\, y\bigg|_{y_i}^{y_o} \tag{15}$$

In region (2) the expression for the local lift coefficient is (see ref. 10)

$$\Delta C_{p(2)} = \frac{4m\alpha}{n\beta\sqrt{m^2 - 1}}\left[\cos^{-1}\left(\frac{1 - am}{m - a}\right) + \cos^{-1}\left(\frac{1 + am}{m + a}\right)\right] \tag{16}$$

Combining equations (16) and (4) results in the expression for the distribution of $\beta l/2q$.

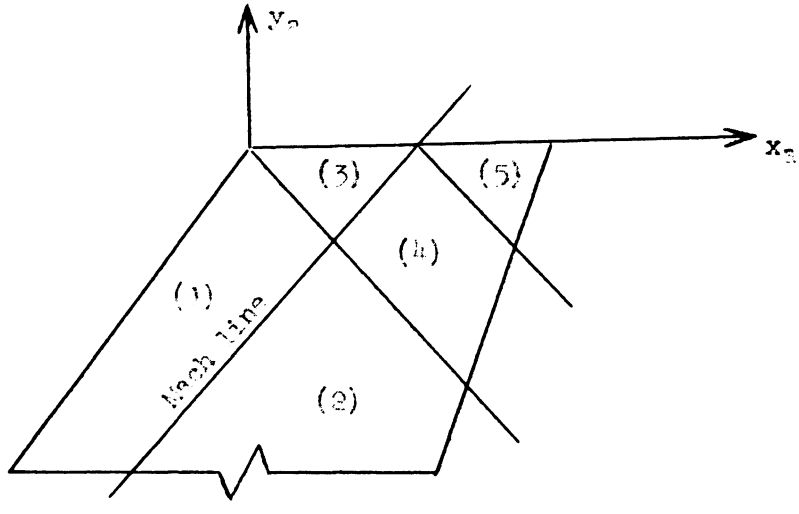
$$-\frac{\beta l}{2q}\bigg|_{(2)} = -\frac{2m\epsilon_w\alpha \cos\theta}{\beta n(1 - m^2\sin^2\theta)\sqrt{m^2 - 1}}\left\{(a - m^2\sin\theta)\left[\cos^{-1}\left(\frac{1 - am}{m - a}\right) + \cos^{-1}\left(\frac{1 + am}{m + a}\right)\right] - m\left[\cos^{-1}\left(\frac{1 - am}{m - a}\right) - \cos^{-1}\left(\frac{1 + am}{m + a}\right)\right] + 2\sqrt{\frac{m^2 - 1}{1 - \sin^2\theta}}\sin^{-1}\left(\frac{a - \sin\theta}{1 - a\sin\theta}\right)\right\}_{a_i}^{a_o} \tag{17}$$

In reference 11 the differential pressure coefficient in region (3) is shown to be

$$\Delta C_{p_{(3)}} = \Delta C_{p_{(1)}} + \Delta C_{p_t} \tag{18}$$

where

$$\Delta C_{p_t} = -\frac{4m_1}{\beta\pi\sqrt{m^2 - 1}} \cos^{-1}\left[\frac{(2m + 1)t + m}{m - t}\right] \tag{19}$$



Sketch 5.

and $t = \beta y_a / x_a$.

In region (4) the differential pressure coefficient is

$$\Delta C_{p_{(4)}} = \Delta C_{p_{(2)}} + \Delta C_{p_t} \tag{20}$$

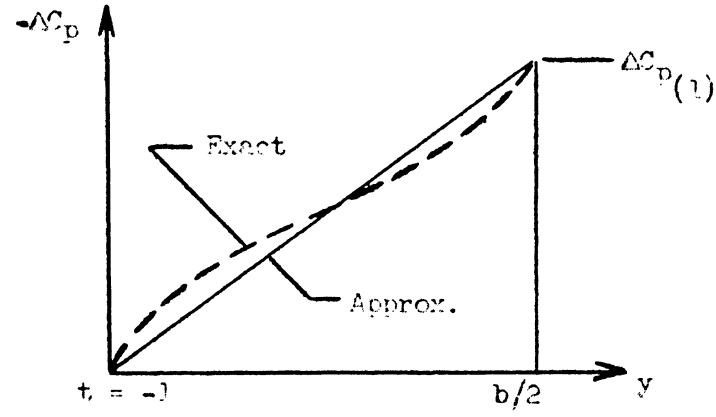
The distribution of $\beta l/2q$ for region (3) can be determined by first calculating the distribution as if it were region (1) and adding to it the distribution derived from equation (19). Similarly the

distribution of $\beta l/2q$ for region (4) can be determined by calculating the distribution as if it were region (2) and adding to it the distribution obtained from equation (19).

In order to simplify the integration equation (19) has been approximated by the following expression,

$$\Delta C_{p_t} \approx -\frac{4m_x}{\beta\sqrt{m^2-1}}\left[1-\frac{\beta(b/2-y)}{t_w-\beta b/2m+\beta y \sin\theta}\right] \tag{21}$$

A comparison of the two curves defined by equations (19) and (21) is shown in sketch 6.



Sketch 6.

The lift forces on a typical lateral section such as that shown in sketch 6 are equal for the two load distributions illustrated. They are equal since the areas under the two curves are equal. The error in $\beta l/2q$ introduced by the approximate form of the loading will be small since the load on the tip is only a small part of the load on the entire wing. Also when $\theta = 0^\circ$ the error will be zero if the

integration proceeds from the Mach line emanating from the tip leading edge to $b/2$, which is the case everywhere except at the trailing edge and at the inner boundary of region (5). The error will increase slightly as θ increases.

Performing the integration of equation (21) along the cutting plane as indicated by equation (4) results in the following expression

$$-\frac{\beta l(\xi_w,\theta)}{2q}\bigg|_t = \frac{2m\alpha\cos\theta}{\sqrt{m^2-1}}\left\{\left(\frac{\sin\theta+1}{\sin\theta}\right)y - \frac{\beta b/2\left(\sin\theta-\frac{1}{m}\right)+\xi_w}{\beta\sin^2\theta}\log_e(\xi_w-\beta b/2m+\beta y\sin\theta)\right\}_{y_i}^{y_o} \tag{22}$$

When $\theta = 0^o$ it is more convenient to use the form

$$-\frac{\beta l(\xi_w,0^o)}{2q}\bigg|_t = \frac{2m\alpha}{\sqrt{m^2-1}}\left\{\left(1+\frac{\beta b/2}{\xi_w-\beta b/2m}\right)y+\frac{\beta y^2}{2(\xi_w-\beta b/2m)}\right\}_{y_i}^{y_o} \tag{23}$$

The lift in region (5) is negligible (see ref. 11) in comparison to the lift of the whole wing and will be treated here as if it added no lift at all.

D. Twisted Wing

For the case of an arbitrarily twisted wing the pressure distribution will be determined by supersonic strip theory, due to the lack of a better method. In supersonic strip theory each chordwise section of the wing is treated as if it was a section of a two-dimensional wing. That is, the loading on a section is proportional to the angle of attack of that section. If the wing is twisted or lies partially or wholly within the influence of some disturbing element (such as the wing apex) strip theory will not yield a good approximation of the pressure distribution. However, if the wing has supersonic edges the total lift of the wing will be correctly predicted by strip theory. This result is a consequence of a theorem and a corollary about the preservation of lift which are given in reference 6, and are given below.

Theorem I: For a wing with supersonic edges whose trailing edge is perpendicular to the flow direction, the lift coefficient due to a deflected element has the two-dimensional value $\frac{4\alpha}{\beta}$, when based on the area of the deflected element. In general this area carries only part of the generated lift. However, the center of lift is at the centroid of the element.

Corollary 1: For a wing with a planform as described in Theorem I but with an arbitrary distribution of local angle of attack α the lift coefficient is

$$C_L = \frac{4\alpha_{av}}{\beta} = \frac{4}{\beta S}\iint_{wing} \alpha dxdy$$

where C_L = total lift coefficient, S = wing area and α_{av} = average angle of attack. The integration is extended over the total area of the wing.

Proofs of the above theorem can be found in reference 6. A further consequence of the above theorem for wings with all supersonic edges is that an integration with respect to the conical variable a from 0 to m yields the section lift coefficient Δc_{a_v}. Thus an integration of the pressures on such a wing along a section formed by the intersection of the Mach cutting plane with the wing chord plane, which proceeds from the body axis (a = 0) to the wing leading edge (a = m), will equal to $\frac{\Delta c_{a_v}}{\beta}$ times the length of the section. If strip theory is used instead of a more exact determination of the pressure distribution, the above integration will yield the same result. It appears, therefore, that strip theory is capable of providing a reasonable approximation of the distribution of $\beta l/2q$ on a wing with supersonic edges. The largest errors will probably occur for wings of high aspect ratio and highly swept trailing edges.

The local lift coefficient on an arbitrarily twisted wing is given by strip theory by

$$\Delta C_p = \frac{\Delta \alpha}{\beta} \tag{24}$$

The distribution of $\beta l/2q$ on a twisted wing can be obtained in the same manner as for an untwisted wing. Namely, by substitution of the pressure coefficient into equation (4) and by performing the required integration. This has been done, and the result is given below as

$$-\frac{\beta l}{2q} = -2 \cos\theta \int_{y_i}^{y_o} \alpha \, dy \tag{25}$$

If the function a is not easily integrated, the distribution of $\beta l/2q$ can be obtained by plotting the variation of c with y and graphically integrating between the desired limits.

E. Body Induced Upwash

The spanwise distribution of angle of attack due to body induced upwash is given by the expression (equation (6))

$$\alpha_u = \alpha_B \left(\frac{r}{y}\right)^2$$

where α_B is the angle of attack of the body and r is the body radius. Substituting equation (6) into the expression for the distribution of $\beta l/2q$ for a twisted wing, derived in the preceeding section, and performing the specified integration yields

$$-\frac{\beta l}{2q} = 2\alpha_B r^2 \cos\theta \left(\frac{1}{y_o} - \frac{1}{y_i}\right) \tag{26}$$

or in terms of the conical coordinate a

$$-\frac{\beta l}{2q} = -\frac{2\beta\alpha_B r^2 \cos\theta}{\xi} \left(\frac{a_o - a_i}{a_o a_i}\right) \tag{27}$$

VI. SAMPLE CALCULATIONS

In order to facilitate the use of the expressions derived in the preceeding sections of this thesis sample calculations have been made for the distribution of $\beta l/2q$ for a typical configuration. A sketch of this configuration with the pertinent dimensions is presented in Figure 6. The calculations are for a Mach number of $M = \sqrt{2}$ and for an angle of attack $\alpha = 4^{\circ}$. The calculations are presented below.

As was pointed out in the "Methods of Analysis" section, the distribution of $\beta l/2q$ for a wing-body combination will be determined from the wing-body interference model depicted in Figure 5.

1. Body contribution.- The contribution of the body to the distribution of $\beta l/2q$ is given by equation (8)

$$-\left.\frac{\beta l}{2q}\right|_{body} = -\frac{2}{\pi}\beta\alpha_B\cos\theta\,\frac{dS_{body}}{d\xi} \tag{8}$$

The body used in this example is the Sears-Haack minimum drag body for a given length and volume (see ref. 12). The cross-sectional area distribution of this body is

$$S = \frac{\pi r_{max}^2}{8}\left(\sqrt{3 + 4\xi/L - 4\xi^2/L^2}\right)^3$$

and the rate of change of the area in the axial direction is

$$\frac{dS}{d\xi} = \frac{3\pi r_{max}^2}{2L}\left(1 - 2\xi/L\right)\sqrt{3 + 4\xi/L - 4\xi^2/L^2}$$

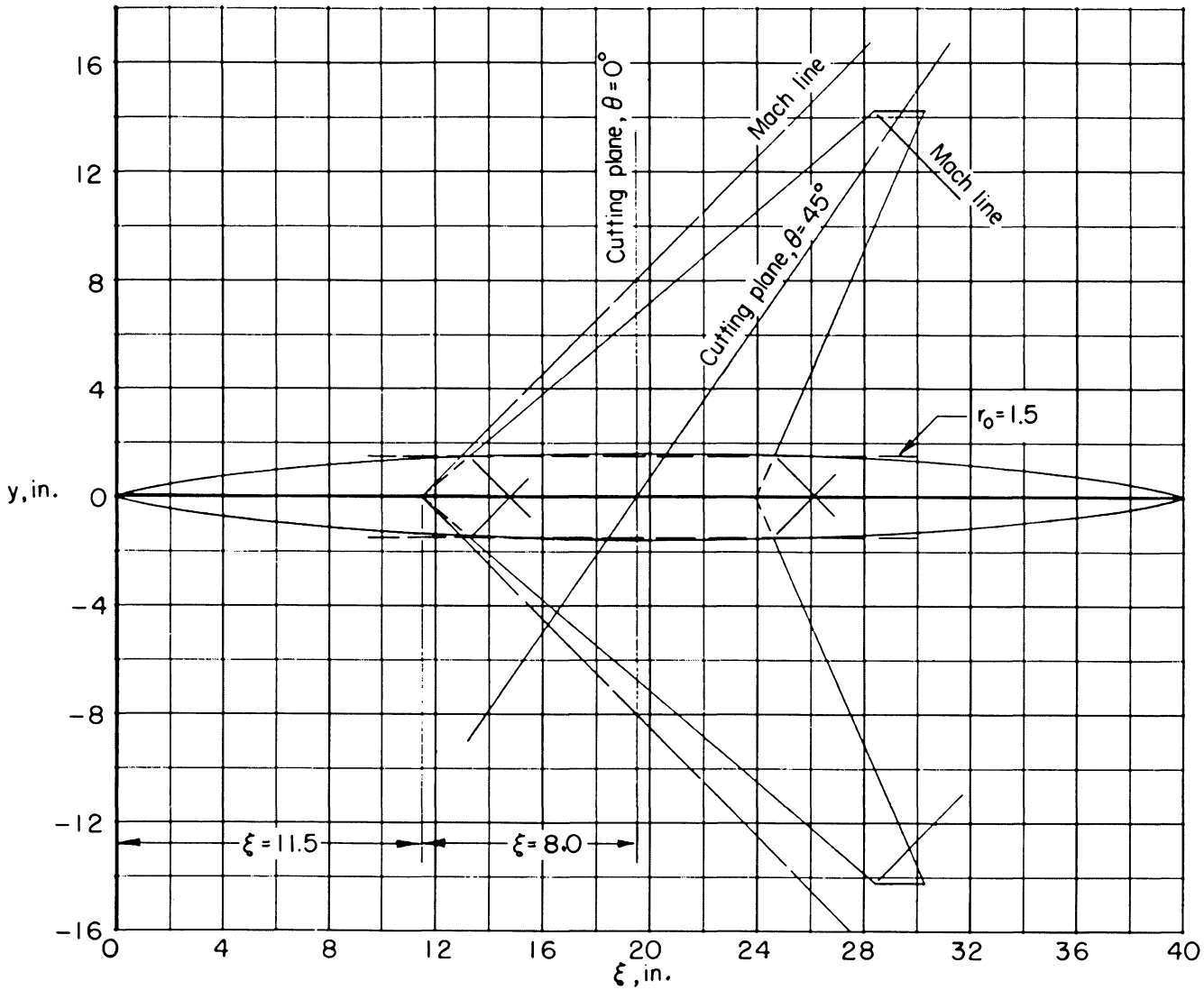


Figure 6.- Model used in sample calculations.

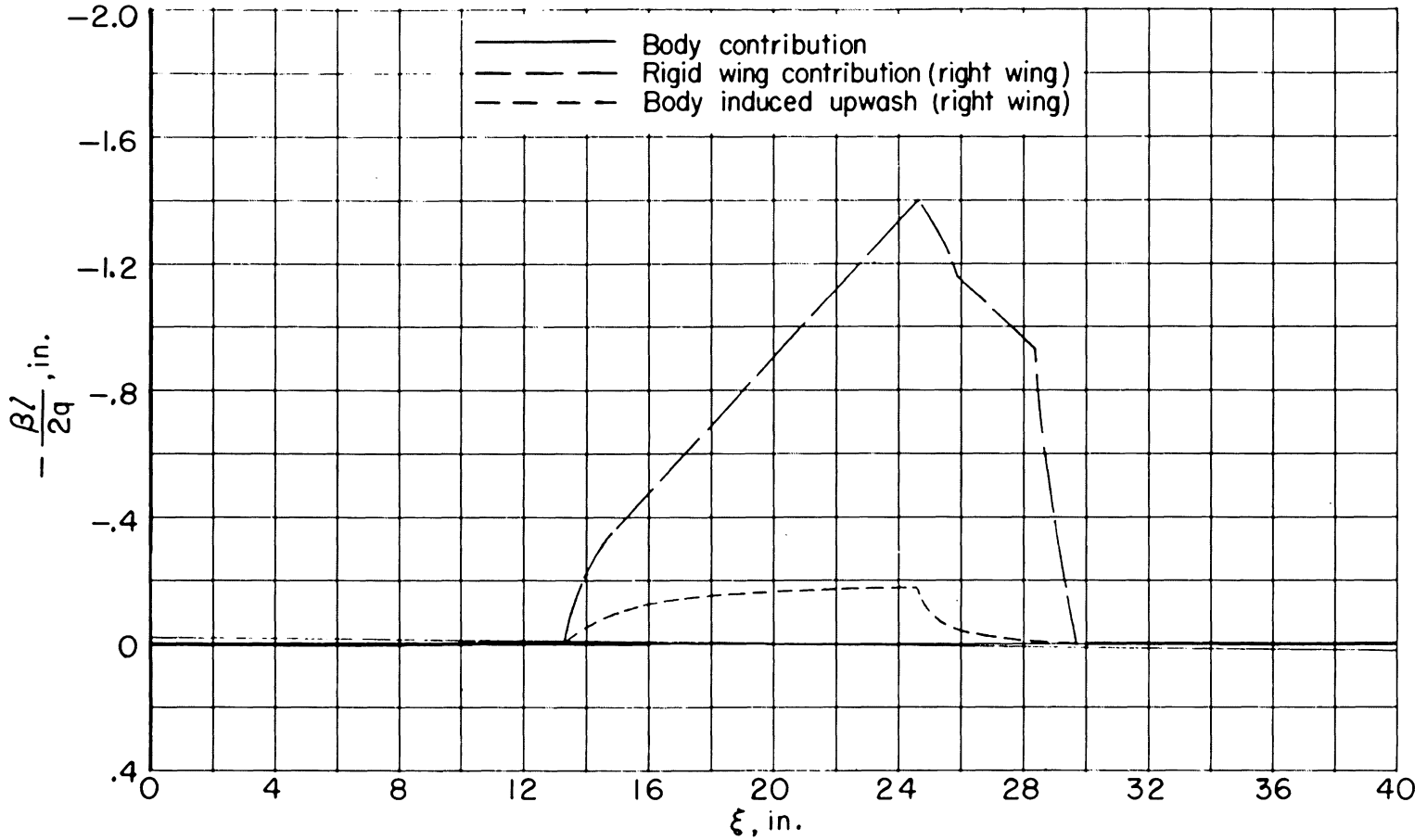


Figure 7.- Distribution of $\beta l/2q$ for a typical wing body configuration.
$\theta = 0^{o}$

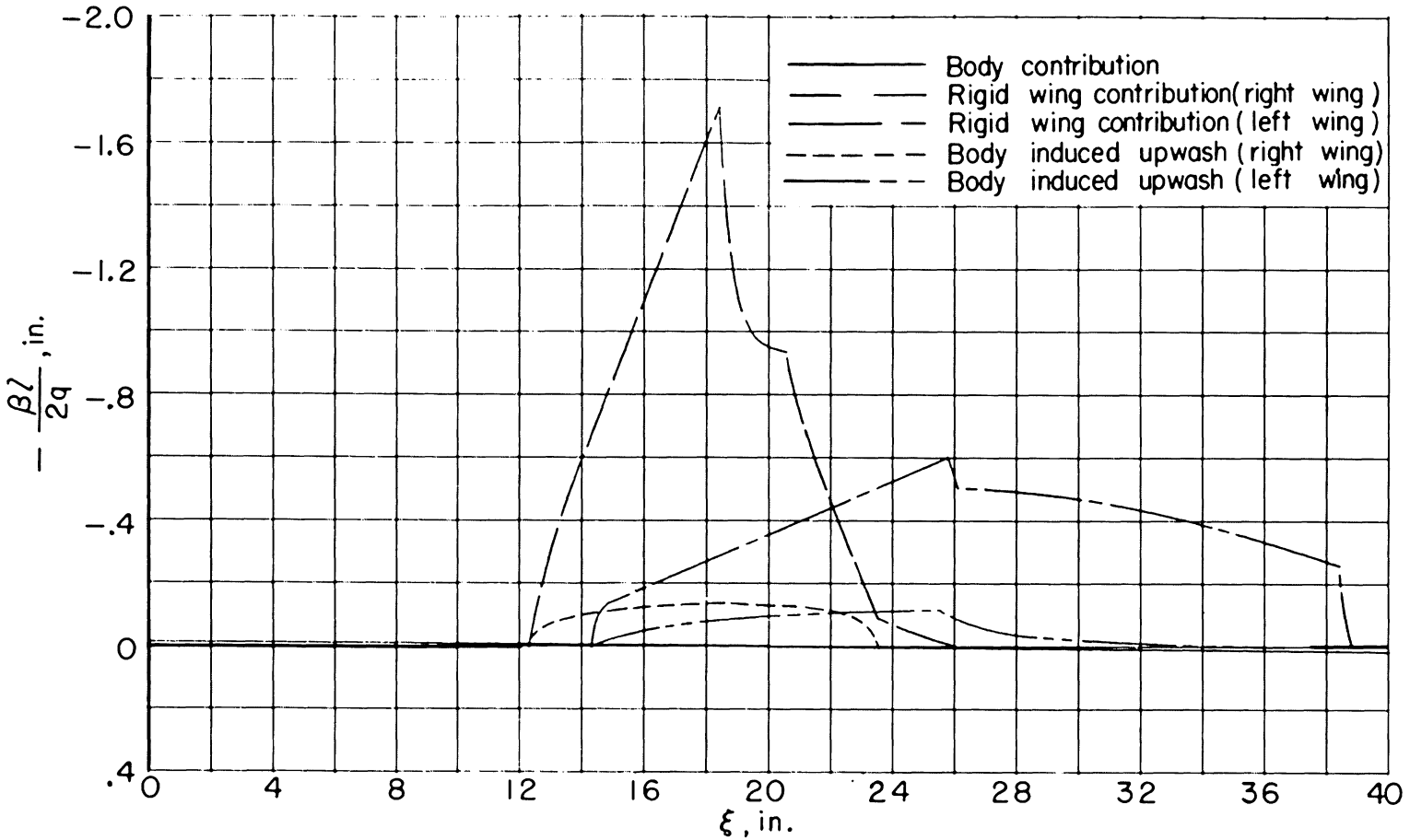


Figure 8.- Distribution of $\beta l/2q$ for a typical wing-body combination.
$\theta = 45^{\circ}$

Thus $$-\frac{\beta l}{2q} = -\frac{3\beta c_p r_{max}^2 \cos\theta}{L}(1 - 2\ell/L)\sqrt{3 + 4\ell/L - 4\ell^2/L^2}$$

Substituting the conditions of the problem into the above equation yields the contribution of the body to the distribution of $\beta l/2q$ which is presented in Figures 7 and 8.

II. Contribution of a rigid wing alone.- Figure 6 indicates that at a Mach number $M = \sqrt{2}$ the wing has subsonic leading edges while the trailing edges are supersonic. Thus only region (1) and region (2) type loadings exist on the wing surface. The contribution of the rigid wing alone to the distribution of $\beta l/2q$ is then given by equation (11)

$$-\left.\frac{\beta l}{2q}\right|_{(1)}^{r.w.} = -\frac{2m^2 a \xi_w \cos\theta}{\beta(1 - m^2\sin^2\theta)E'(m)}\left\{\frac{-\sin\theta\sqrt{m^2 - a^2}}{1 - a\sin\theta} + \frac{\sin^{-1}\left(\frac{a - m^2\sin\theta}{m - a\sin\theta}\right)}{\sqrt{1 - m^2\sin^2\theta}}\right\}_{a_1}^{a_0} \qquad (11)$$

When $\theta = 0^\circ$ the intersection of the Mach cutting plane with the x,y-plane is perpendicular to the stream direction and the distribution of $\beta l/2q$, as determined from the equation above, will be the same for both the right and left wing panels. This is obvious from the symmetry of the configuration. The limits of integration a_0 and a_1 are determined from the intersection of the cutting plane with the boundaries of the region of interest. The cutting plane for $\theta = 0^\circ$ and $\xi = 8$ is

shown in Figure 6. The outboard limit for this cutting plane, in terms of the conical coordinate $a = \beta y/x$, is 0.842. This, of course, is the value of the sweepback parameter m since the outboard limit lies along the wing leading edge. The inboard limit lies along the body axis and is equal to zero. The contribution of the right half of the rigid wing to the distribution of $\beta l/2q$ has been computed for the conditions of the problem and is presented in Figure 7.

When θ is different from zero the contributions (to the distribution of $\beta l/2q$) of the right half and the left half of the rigid wing alone are not equal. The cutting plane for $\theta = 45^\circ$ and $\xi = 8$ is shown on Figure 6. The outboard limit of integration for the right half of the wing lies along the Mach line emanating from the wing tip and is given by $a_o = 0.774$. For the left half of the wing the outboard limit lies along the wing leading edge and is equal to -0.842. The inboard limits for both wing halves lie along the body axis and are equal to zero.

The contributions to the distribution of $\beta l/2q$ of the right and left halves of the rigid wing alone have been computed from equation (12) using the limits defined above and are presented in Figure 8.

III. Contribution of body induced upwash.- The contribution of body induced upwash to the distribution of $\beta l/2q$ can be determined from equation (27);

$$-\frac{\beta l}{2q}\bigg|_{\substack{\text{body}\\ \text{upwash}}} = -\frac{2\beta a r_o^2 \cos\theta}{\xi_w}\left(\frac{a_o - a_1}{a_o a_1}\right) \qquad (27)$$

The values of a_o and a_i are determined from the intersection of the Mach cutting plane with the boundaries of the region which is composed of all the wing surface outboard of the average body radius r_o. When $\theta = 0^o$ the contribution of the right and left exposed wing panels to the distribution of $\beta l/2q$ are identical due to the symmetry of the configuration. For the cutting plane defined by $\theta = 0^o$ and $\xi_w = 8$, the limits of integration are $a_o = m = 0.842$ and $a_i = \beta r_o/\xi_w = 0.1875$ (see Figure 6). The contribution of the right exposed wing panel has been computed for the conditions of the problem and is presented in Figure 7.

When θ is different from zero the contributions of the two exposed wing panels are no longer equal and must be determined separately. For $\theta = 45^o$ and $\xi_w = 8$, the limits of integration for the right exposed wing panel are $a_o = 0.790$ and $a_i = 0.166$. For the left exposed wing panel these values are $a_o = -0.842$ and $a_i = -0.216$ (see Figure 6). The contributions of the right and left exposed wing panels have been computed and are presented in Figure 8.

IV. Total distribution of $\beta l/2q$.- The total distribution of $\beta l/2q$ is obtained simply by summing the contributions of the various parts of the wing-body interference model for like values of θ and ξ. The total distributions of $\beta l/2q$ for the conditions of this problem with θ equal to 0^o and 45^o are presented in Figure 9.

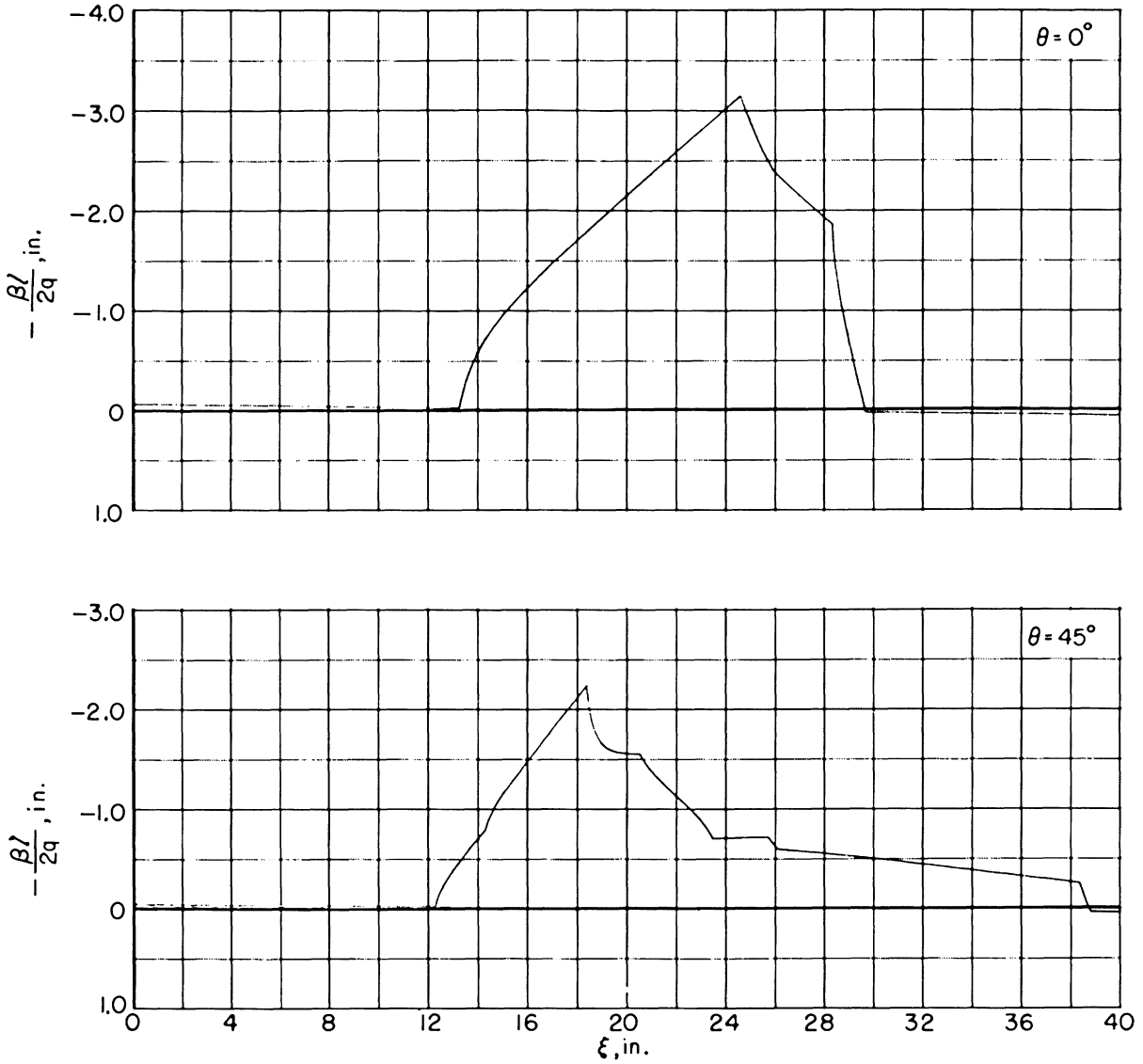


Figure 9.- Distribution of $\beta l/2q$ for a typical wing-body combination.

VII. DISCUSSION

In this thesis a method has been developed whereby the wave drag due to lift of a wing-body combination can be determined solely from the geometry of the configuration. The various expressions for $\beta l/2q$ (from which the wave drag due to lift is calculated) were derived from linearized supersonic flow relations. Thus this method can be expected to apply to any slender rectilinear planform wing-body combination to which linearized supersonic theory is applicable.

At Mach numbers approaching unity linearized supersonic theory loses its validity and consequently this method would be expected to lose its validity also. However, the term $\beta = \sqrt{M^2-1}$ becomes small as the Mach number nears one and the wave drag due to lift, which is proportional to β^2, becomes only a small part of the total drag. Therefore, it is possible to use this method for Mach numbers close to one without incurring excessive errors in the total wave drag.

The results of the sample calculations (see Figures 7, 8, and 9) indicate that the contribution of the body to the distribution of $\beta l/2q$ is small. The body used in these calculations was a Sears-Haack body of fineness ratio 12.5. In order to illustrate the effect of a change in body shape on the distribution of $\beta l/2q$, calculations of $\beta l/2q$ were made for an indented Sears-Haack body (Fig. 10) and compared with the results for the Sears-Haack body. The change in body shape had a considerable effect upon the contribution of the body to the distribution of $\beta l/2q$. It should be mentioned that the effect on the wing of the change in body pressures has not been considered. This induced

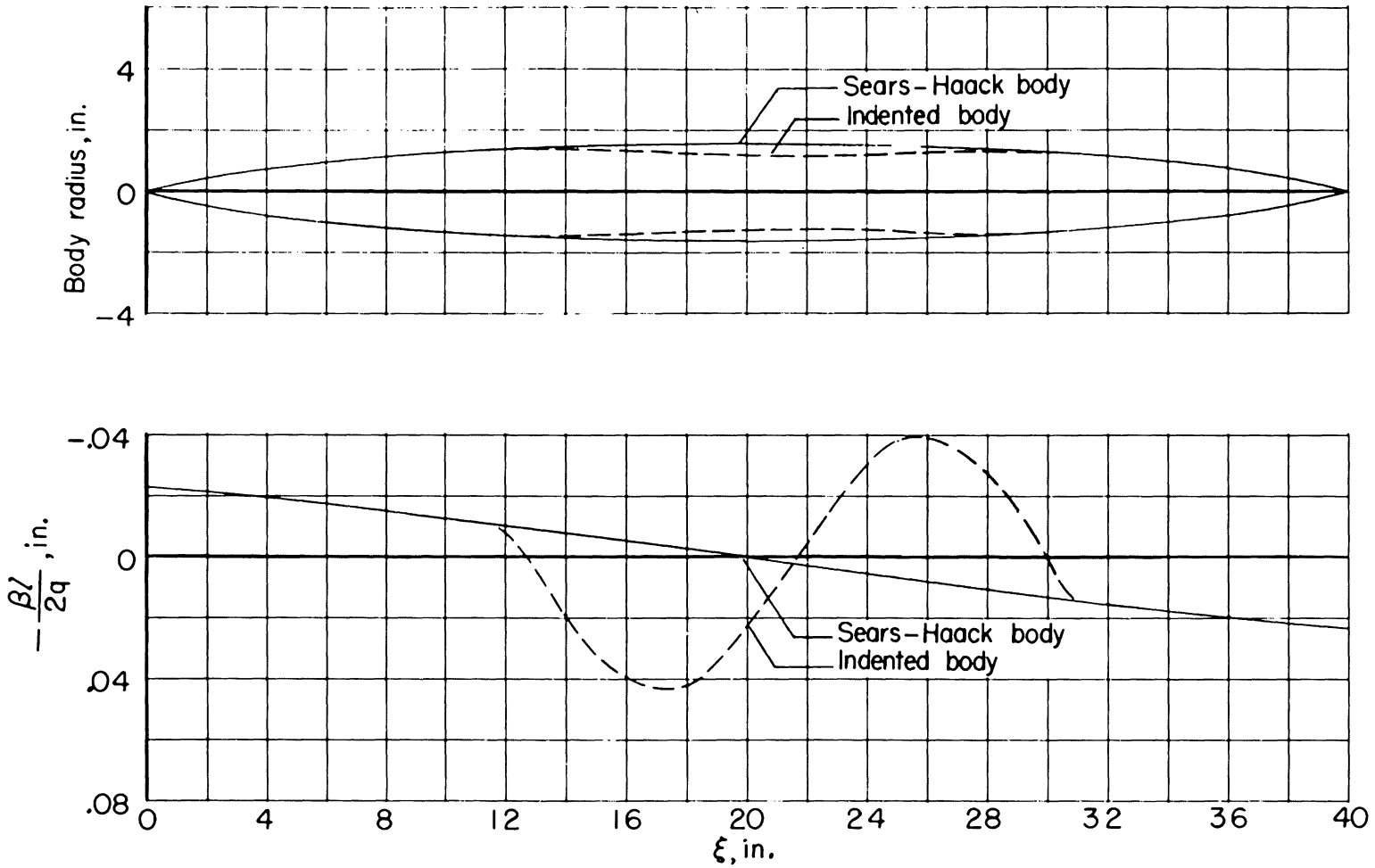


Figure 10.- Comparison of the distributions of $\beta l/2q$ for a Sears-Haack body of fineness ratio 12.5 and a symmetrically indented body.

effect will probably be as large, if not larger, than the effect of the change in body shape on the body. Thus it seems likely that the total effect of a change in body shape is actually several times larger than that calculated and presented in Figure 10. However, the contribution of the body change to the distribution of $\mu l/2q$, including induced effects, is small in comparison to the total distribution of $\mu l/2q$ for the complete configuration. Small enough, in fact, to be neglected if the body is smooth and slender. It appears, then, that a symmetrical change in body shape, such as indentation, which has a considerable effect upon the wave drag due to thickness, will have only a slight effect upon the wave drag due to lift.

Since the effects of changes in body shape are small the wave drag due to lift, for the type of configuration studied in this thesis, depends almost entirely upon the wing planform. The method developed herein can be used to determine the variation of the wave drag due to lift with the various planform parameters such as sweepback, aspect ratio, and taper ratio.

VIII. CONCLUSIONS

A method has been developed in this thesis whereby the wave drag due to lift of a wing-body combination can be determined solely from the geometry of the configuration. The cases of supersonic and subsonic leading edges have both been treated. Sample calculations have been made for a typical wing-body configuration and the results indicate that symmetrical changes in body shape, such as indentation, which may have a considerable effect upon the wave drag due to thickness, will have only a slight effect upon the wave drag due to lift. The wave drag due to lift of a wing-body combination, for which the body is fairly smooth and slender, is shown to depend almost entirely upon the wing planform.

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