A Game-Theoretic Approach to Incentivize Landowners to Mitigate an Emerald Ash Borer Outbreak

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Abstract

This paper addresses one of the greatest challenges facing our forests, the invasion of the emerald ash borer (EAB), a non-native, wood-boring insect that threatens to kill most ash trees, one of North America's most vital and widely-distributed tree genera. Current strategies to slow ash mortality due to the EAB infestation include surveillance of ash tree health coupled with insecticide treatment and/or removal of infested trees. Most ash trees grow on private land, and the growing spread of the EAB infestation is largely due to the lack of a private-public partnership. Local governments need programs to induce landowners to take action in order to slow ash mortality. We present a principal-agent modeling framework to design two cost-sharing programs in which a local government offers reimbursements to landowners to cover a portion of their management costs. Our principal-agent approach appears to be the first that keeps track of a biological invader's dynamic growth in two consecutive periods based on different treatment decisions in the first period. We develop two mathematical models: one in which the reimbursement is based on the number of infested trees and another in which reimbursement is based on the number of treated trees. For the infestation-based reimbursement model, we derive the analytical solution for the optimal treatment decision in the first period and the reimbursement. For both the infestation-based reimbursement and the treatment-based reimbursement models, we characterize the conditions under which each treatment decision is optimal and show that they depend on both the treatment effectiveness and the likelihood of new infestations in the second period. Compared to the infestation-based reimbursement program, the treatment-based reimbursement program induces the landowner to treat more trees through a higher reimbursement and provides a higher overall benefit. The results further indicate that continuing the treatment of EAB infestations beyond the two-year cost-sharing program is anticipated to yield superior long-term benefits. Our approach is expected to inspire other private-public partnerships to solve various environmental and societal spatio-temporal problems through better resource sharing, such as the management of water, land, and wildfire.

Keywords: principal-agent; information asymmetry; cost-sharing; emerald ash borer

1 INTRODUCTION

Invasive species are non-native plants, animals, or pests that cause significant economic and environmental damage by harming biodiversity and degrading the environment (Levine et al. 2003,

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Wilcove et al. 1998). The cost of invasive species to the United States' economy is estimated to be more than 20 billion dollars each year, and the associated costs continue to increase (Fantle-Lepczyk et al. 2022). Because invasive species threaten biodiversity, controlling the growing spread of invasive alien species has been one of the United Nations (U.N.) Sustainable Development Goals (Mohieldin and Caballero 2015, United Nations 2021). Furthermore, various national and international communities, such as the United Nations' Global Invasive Species Program (GISP) and National Invasive Species Council (NISC), have called for rapid management and control of invaders to minimize their harmful impacts on sustainability and human well-being (Büyüktahtakın and Haight 2018).

One prime example of an invasive species is the emerald ash borer (EAB) (*Agrilus planipennis*), a non-native forest insect that attacks and kills North American ash trees (*Fraxinus* spp.) (see Herms and McCullough (2014) for review). Ash trees are one of North America's most widely distributed tree genera (Atha and Boom 2017), growing as components of more than 25 forest community types throughout the eastern U.S. (MacFarlane and Meyer 2005) and providing food and habitat for numerous organisms from birds and mammals to insects and microorganisms (Atha and Boom 2017). Among ash species, white ash (*Fraxinus americana*) is the most abundant, and white ash wood is known for its strength and resiliency and is used for products such as tool handles, flooring, and furniture (MacFarlane and Meyer 2005). Following the EAB invasion of a forest stand, mature ash dies within ten years (Klooster et al. 2014). If ash seedlings and saplings become infested and die prior to reaching reproductive maturity, then ash will be extirpated from the stand, with cascading direct and indirect effects on forest community composition and ecosystem processes (Gandhi and Herms 2010).

Ash trees are also an important part of urban forests. Landscape ash trees have been planted along streets, in parks, and in people's yards for decades because of their tolerance to ecological stresses such as drought and soil compaction (MacFarlane and Meyer 2005). As part of the urban forest, ash trees provide ecological benefits such as wildlife habitat, stormwater absorption, and local cooling, social benefits such as pleasing environments for people to live and work, and human health benefits such as reduced stress (Dwyer et al. 1992, Pataki et al. 2021). Since EAB was discovered near Detroit, Michigan, and Windsor, Ontario, in 2002, the insect has spread to 36 states in the U.S. and five Canadian provinces (Emerald Ash Borer Information Network 2022), killed millions of ash trees (Herms and McCullough 2014), and cost homeowners and cities hundreds of millions of dollars in losses of property value and damage mitigation (Kovacs et al. 2010).

In response to the threat of EAB, cities and communities have developed EAB management plans with the goal of slowing ash mortality. The management plans include surveillance of ash tree health, application of systemic insecticides to protect trees by killing any EAB adults or larvae, and preemptive removal of highly infested trees before larvae can complete development (Herms and McCullough 2014). An obstacle to implementing a city-wide EAB management plan arises where ash trees are owned by both public and private entities. The city forester usually does not have the authority to manage ash trees growing on private land, where landowners manage their trees using their own preferences and budgets without consideration of city-wide objectives. Because many landowners do not have knowledge or budgets for tree care activities, levels of management, tree survival, and net benefits across ownerships may be much less than those attainable with a centralized cross-ownership management plan and actions (Kovacs et al. 2014).

To overcome this obstacle, local governments may use cost-share programs to induce landowners to undertake management actions. To design such programs, we present a principal-agent framework where a city forester (i.e., the principal) offers to reimburse a landowner (i.e., the agent) for a portion of its costs of EAB management. Within this framework, we use a two-period horizon to capture the dynamics of tree management, infestation, and survival. If the landowner does not accept the city forester's offer, then they do not inspect or treat any ash trees in the first period and bear the costs of removing and foregoing benefits of any infested trees that perish in the second period. If the landowner accepts the offer, they identify infested ash trees through surveillance and determine how many trees to treat and/or remove in the first period to maximize their expected net benefits of surviving trees in the second period. The city forester sets its reimbursement offer to ensure that the landowner participates and to maximize the expected benefits of surviving trees net a penalty for trees that perish and the cost of reimbursement.

We develop and analyze two program designs: one in which reimbursement is based on the number of infested trees and another in which reimbursement is based on the number of treated trees. In the infestation-based reimbursement (IBR) design, the city forester offers a payment based on the landowner's reported infestation level, and if accepted, the landowner decides how many trees to treat to maximize their net benefits. We find three possible treatment decisions that can be optimal for the landowner - treating all (infested and healthy) trees, treating only the infested trees, and treating none of them - depending on treatment effectiveness and the likelihood of new infestations. We provide the analytical solution for optimal reimbursement and characterize the conditions under which each of the treatment decisions can be optimal.

In the treatment-based reimbursement (TBR) design, the city forester announces a reimbursement schedule in which the level of reimbursement increases with the number of trees that are treated. Following surveillance of the infestation, the landowner decides how many trees to treat to maximize their net benefit, accounting for the level of reimbursement. We find eight optimal reimbursement schedules and identify the conditions under which each can be optimal.

We compare the efficacy of the two cost-sharing models through a numerical analysis over some infestation and treatment effectiveness scenarios and conclude that the treatment-based reimbursement program is superior. Despite the higher reimbursement required, the treatment-based reimbursement program induces the landowner to treat more trees, and as a result, the city forester achieves higher expected net benefits.

2 LITERATURE REVIEW

2.1 Management of Invasive Species and the EAB

Several optimization models have been presented to study the surveillance (Horie et al. 2013, Epanchin-Niell et al. 2012, Kıbış et al. 2021, Onal et al. 2020, Bushaj et al. 2021) and control (Büyüktahtakın et al. 2011, Kıbış and Büyüktahtakın 2017, Kovacs et al. 2014, Büyüktahtakın et al. 2015) of invasive species. For a detailed background in invasive species management and a review of optimization models to detect and control biological invasions, see, e.g., Büyüktahtakın and Haight (2018).

Some of the optimization models have focused on the surveillance and control of the EAB (Kovacs et al. 2014, Kıbış et al. 2021, Bushaj et al. 2021, 2022). A central assumption of these EAB models is that a central planner makes allocation decisions across time and space to maximize public benefits without consideration of ownership boundaries. For example, Kıbış et al. (2021) present a multi-stage stochastic mixed-integer programming model for optimizing surveillance, treatment, and removal of ash trees across neighborhoods of a city. However, bio-invasions usually take place in landscapes encompassing multiple landowners - a management mosaic - where each landowner's control decisions affect invasion spread and damages across ownership boundaries (Wilen 2007, Epanchin-Niell et al. 2012). When landowners make control decisions based on their own preferences, costs, and expected damages, they may be under control from a cross-ownership perspective, leading to increased invasion of the landscape (Wilen 2007). A central planner who determines the optimal control of a bio-invasion can internalize this diffusion externality and increase total net benefits across ownerships (e.g., Feder and Regev (1975); Bhat and Huffaker (2007); Richards et al. (2010); Sims-Chilton et al. (2010)). However, there are practical obstacles to implementing a centralized plan across ownerships. For example, the central planner may not have the authority to manage infestations on private land or may not have concurrence from municipal planners to provide funding for management activities outside their municipal boundaries (Kovacs et al. 2014). As a result, the potential welfare gains from centralized planning in a landscape with multiple stakeholders may not be attainable.

Game-theoretical methods are increasingly being employed to tackle challenges in invasive species management by incorporating the actions of multiple stakeholders. For example, Bhat and Huffaker (2007) develop a two-person differential game to model a dynamic contract that allows renegotiation and variable transfer payments between landowners for the control of a mammal population that moves across boundaries and damages resources on both ownerships. Liu and Sims (2016) develop an optimization model for spatially-connected landowners to determine the timing and level of side-payments in which un-invaded landowners compensate their invaded neighbors to engage in control actions, thereby slowing the spread of bio-invasion. Atallah et al. (2017) use non-cooperative and cooperative games to determine aggregate payoffs between two managers whose independent production processes are spatially connected through a network. Cobourn et al. (2019) employ a dynamic, cooperative Nash bargaining model to determine the sequence of transfer payments from an un-infested city to an infested city to engage in controls that slow the spread of the invader. Siriwardena et al. (2018) extend the cooperative Nash bargaining model to examine how the mix of land ownership within each municipality affects the path of a negotiated transfer payment from the un-infested to the infested jurisdiction. Büyüktahtakin et al. (2013) develop a long-term dynamic model for controlling invasive species using cooperative games. While these examples demonstrate a growing interest in using game theory for determining transfer or subsidy payments between landowners for invasive species management (ISM), to the best of our knowledge, the design of incentive programs to strengthen public-private partnerships in ISM has not been addressed.

2.2 Incentive Design using the Principal-Agent Framework

The principal-agent framework was developed to better align the conflicting interests of two parties and achieve a better outcome via incentive structuring when there is information asymmetry. It has been widely adopted in many studies, mostly in the coordination of decentralized players in supply chain management (Taylor and Xiao 2009, Yang et al. 2009, 2012, Kim and Netessine 2013, Chen et al. 2016) and in production (Iyer et al. 2005, Chick et al. 2017). There is also a growing body of literature on public-private partnerships or authority-led initiatives, such as funding and auditing of non-profit organizations (Privett and Erhun 2011), information sharing among farmers (Xiao et al. 2020), emissions mitigation through carbon-capturing (Cai and Singham 2018), infrastructure development (Paez-Perez and Sanchez-Silva 2016), container-inspection policy (Bakshi and Gans 2010), and performance-based contracts in health services (Fuloria and Zenios 2001, Jiang et al. 2012, Andritsos and Tang 2014, Aswani et al. 2019). While the principal-agent framework has been studied in various applications, as stated above, we are unaware of any principal-agent framework that tackles a spatio-temporally growing environmental problem.

Applications of principal-agent models may be divided into two types: those dealing with a moral hazard or hidden actions and those dealing with adverse selection or hidden information. We adapt both types of models to design incentive programs for public-private partnerships in invasive species management. In both cases, a city forester (the principal) offers to reimburse the landowner (the agent) for a portion of its costs of EAB management. In the first case, the city forester sets the reimbursement based on the observed level of infestation, which is assumed to be known by both the city forester and the landowner. This is a moral hazard problem because the landowner decides how many trees to treat and remove after receiving the reimbursement (i.e., the agent's action is hidden when the principal sets the incentive). The city forester sets the level of reimbursement according to the infestation level so that the landowner's management decision will align with the city forester's objective. In the second case, the city forester offers a schedule of reimbursements based on how many trees the landowner treats. This is an adverse selection problem because the landowner knows the number of infested trees, but the city forester does not (hidden information). Therefore, the city forester must design the reimbursement schedule to induce the landowner to treat the desirable number of trees without knowing the initial level of infestation.

2.3 Key Contributions

To the best of our knowledge, we are the first to adopt a principal-agent framework to design incentive programs for a local government to induce a private landowner to manage a dynamically evolving environmental problem. Specifically, we focus on controlling the harmful impacts of an invasive beetle, namely the emerald ash borer. We hypothesize that government subsidies can be used to motivate the participation of landowners and, as a result, improve the outcome of reducing the harmful impacts of an invasive forest beetle. Our game-theoretic models could also be adapted to a variety of other public-private relationships that require joint resource contributions over space and time, such as the management of water, land, and wildfire. Thus, our study is expected to motivate other partnerships among local governments and private owners to deal with important social and environmental problems through better resource sharing.

Our principal-agent models are new in that they incorporate the population dynamics of an environmentally-harming species. Specifically, in both the infestation-based and treatment-based reimbursement models, we consider a two-period principal-agent framework to capture the progression of the ash tree infestation by EAB. Both models are dynamic since they keep track of the growth of infestation in two consecutive periods based on different treatment decisions in the first period. The length of the first period is one year, while that of the second period can be up to two years because the protection from pesticide treatment lasts 2-3 years. In addition, we assume that the city forester offers the reimbursement program only in the first period since the city forester wants to evaluate the program every 2-3 years.

Further, our models differ from an assumption in the classic principal-agent framework in that the agent's utility function does not satisfy the single-crossing property. Specifically, when the infestation level is high in the first period, it may be desirable not to treat any trees, especially when the treatment is ineffective, as it leads to fewer trees being successfully treated. On the other hand, when the infestation level is low initially, it may be beneficial to treat only infested trees (but not healthy) because the treatment is not very effective. It may also be advantageous to treat all (infested and healthy) trees in order to prevent any new infestations. Consequently, the optimal treatment decision in our models differs from the seminal result of Maskin and Riley (1984). Recall that in the optimal solution of their work, the quantity offered to the agent is non-decreasing in his type. Further, the highest type receives the efficient amount, which is computed without the presence of information asymmetry. Lower types, however, receive less than efficient amounts. In the context of invasive species management, this would mean that a landowner with a higher infestation level will be induced to treat more trees. However, we show that the optimal number of trees to be treated is, in general, non-monotonic in the infestation level in either model.

In order to assess the strengths and weaknesses of the two-period framework as a representation of a multi-period decision problem, we conduct a comparative analysis focusing on two actions following the conclusion of the cost-sharing program. The actions examined are: 1) The landowner continues inspecting and treating EAB-infested ash trees over multiple periods. 2) The landowner discontinues the treatment of EAB infestations. Our findings indicate that, in most instances, the long-term discounted utility of action 1 surpasses that of action 2, except for a specific condition where the long-term discounted utility in action 2 is marginally better than action 1. This result suggests that, on the whole, the ongoing treatment of EAB infestations is expected to provide superior long-term benefits.

The rest of the paper is organized as follows. In Section 3, we introduce the infestation-based reimbursement model, where the infestation level is verifiable by the city forester. The optimal solutions are presented analytically. In Section 4, we present the treatment-based reimbursement where the number of trees treated is verifiable instead and several sets of optimal treatment decisions. The two cost-sharing models are compared against the case where no programs are offered in Section 5. We summarize the key takeaways in Section 7 and conclude the paper in Section 8.

3 THE INFESTATION-BASED REIMBURSEMENT MODEL

We set up a two-period model of the decisions made by a city forester (she henceforth) and a private landowner (he henceforth). Table 1 summarizes the notation used in the model. At the beginning of the first period, the city forester verifies the initial infestation level (number of infested trees) of a private landowner. This can be achieved by requiring the landowner to submit a surveillance report issued by a professional tree service. Based on the reported infestation level, she selects a reimbursement level to maximize her expected utility. The reimbursement helps the landowner pay for insecticide treatments to protect his ash trees from EAB infestation and death. The city forester's utility depends on the number of surviving trees at the end of the second period, where s is the marginal value the city forester has for a healthy ash tree. Because treatments protect trees for up to three years (Herms and McCullough 2014), the length of the management horizon is three years. The city forester selects the reimbursement based on what she expects the landowner to do after receiving the payment. The landowner is assumed to be rational, i.e., his treatment decisions maximize his utility, which depends on the number of surviving trees at the end of the horizon, where θ is the marginal value the landowner has for a surviving ash tree. The landowner's treatment decisions also depend on the reimbursement, the evolution of the number of infested trees in the two periods, and the consequences of each possible treatment decision.

Characterization of the initial infestation level of a private property. Let n denote the number of ash trees a landowner has on his private land at the beginning of the first period. This information is considered common knowledge because trees can be counted along the curb without accessing the landowner's property. Let π denote the likelihood of an ash tree being infested by EAB in the first period. We refer to π as the initial (or first-period) attack rate. It can be computed based on the proportion of infested trees on the neighboring properties or other infestation information provided by local agencies. We characterize the landowner's ash trees as either healthy-looking or infested-looking, based on the ash canopy condition rating scale described by Knight et al. (2014). Healthy-looking trees have no visible canopy damage, while infested trees have visible canopy damage. Professional tree services easily identify these canopy condition classes and corresponding degree of EAB infestation (Flower et al. 2013). See Section A.1 of the Electronic Companion (EC henceforth) for a more detailed description of our ash tree classification. The initial infestation level of a private property is denoted by i, the number of trees in the infested tree class. Since each tree can be attacked independently by EAB, we assume that the infestation level i follows a binomial distribution with the rate π, i.e., $i \sim B(n, \pi)$. Let α denote the visual surveillance cost per tree. Its value depends on the hourly cost of tree-care professionals and the amount of time spent inspecting each tree.

The landowner's available options. In addition to estimates of the numbers of healthy-looking and infested trees, the landowner has information about the effectiveness of insecticide treatments, such as soil or trunk injections. With treatment, infested trees with visible canopy damage may recover depending on the extent of the EAB damage and which insecticide treatment is used (Herms et al. 2019). For example, while treated trees with less than 50 percent canopy dieback will likely survive and recover after 2-3 years, treated trees with greater than 50 percent dieback may not recover (Herms et al. 2019). Therefore, we assume the treatment's success rate represents the likelihood that an infested tree with visible canopy damage that is treated in the first period survives to the end of the second period. Without treatment, infested trees with visible canopy damage die in 2-4 years (Herms and McCullough 2014). Healthy-looking trees (without visible canopy damage) that are treated in the first period survive until the end of the two-period horizon, which is consistent with the management guidance described by Herms et al. (2019). Dead trees at the end of the horizon are assumed to be hazardous and must be removed with cost c. The cost of treatment is β per tree, where β is much lower than c.

Table 1: Notation.

Input parameters:	
n	The number of ash tree a landowner has.
π	The initial (or first-period) attack rate. $\bar{\pi} = 1 - \pi$.
i	The initial infestation level, which follows a binomial distribution with rate π.
c	Cost of removing a dead tree due to the EAB infestation.
α	Cost of inspecting an ash tree.
β	Cost of treating an ash tree.
γ	Penalty cost for a dying tree.
ρ	The treatment success rate. $\rho < 1$ and $\bar{\rho} = 1 - \rho$.
θ	The marginal value of a landowner having a surviving ash tree.
s	The marginal value the city forester has for a healthy ash tree.
π^l	The low second-period attack rate. $\bar{\pi}^l = 1 - \pi^l$. $\pi^l < \pi$.
π^h	The high second-period attack rate. $\bar{\pi}^h = 1 - \pi^h$. $\pi < \pi^h$.
Value function:	
$v(w, d)$	The value function of the city forester of having w surviving ash trees while losing d trees. $v(w, d) = s \cdot w - \gamma \cdot d$.
$V(w)$	The value function of the landowner having w surviving ash trees at the end of the second period. $V(w) = \theta \cdot w$.
Decision variables only applicable to the infestation-based reimbursement model:	
$q(i)$	The number of trees a landowner will treat given i out of n ash trees are infested.
$r(i)$	Reimbursement offered to the landowner for having i infested ash trees.
Decision variables only applicable to the treatment-based reimbursement model:	
$\mathbf{q}$	$\mathbf{q} = [q(0), q(1), \cdots, q(n)]$ is a $(n+1)$-tuple that prescribes the number of trees to be treated based on infestation level i, where $q(i) \in [0, n]$.
$\mathbf{r}$	$\mathbf{r} = [r(0), r(1), \cdots, r(n)]$ is a $(n+1)$-tuple that prescribes the reimbursement scheme based on the treatment decision $q(i)$.

Progression of the infestation level. Let $q(i)$ denote the number of treated trees in the first period. If no trees are infested ($i = 0$) or if all of the infested trees are treated ($q(i) = i$) in the first period, the chance of a healthy, untreated tree becoming newly infested in the next period is assumed to decrease from π to π^l. On the other hand, if any of the infested trees are left untreated

$(q(i) < i)$, the attack rate in the second period increases from π to π^h. Further, any healthy tree that is treated in period one becomes EAB-resistant for both periods; that is, it will not be infested in period two. If all trees are infested $(i = n)$, there will be no new infestation in the second period.

Landowner's expected utility if he does *not* participate in the cost-sharing program. The landowner is assumed to neither inspect nor treat any trees in either period. Otherwise, the landowner should participate to offset the cost with the reimbursement. Let θ denote the landowner's marginal value of a healthy ash tree; his value of having w surviving trees at the end of the planning horizon, denoted by $V(w)$, is thus $\theta \cdot w$. Let a_0 denote the decision of not participating in the cost-sharing program, then his expected utility given the initial infestation level (i) is

$$\phi(a_0|n,i) = \begin{cases} \theta\bar{\pi}^l n - c\pi^l n & \text{if } i = 0 \\ \theta\bar{\pi}^h(n-i) - c(i + \pi^h(n-i)) & \text{if } 1 \leq i \leq n. \end{cases} \tag{1}$$

If there are no infested trees $(i = 0)$ initially, the attack rate in the second period is π^l. Therefore, $\pi^l n$ trees are expected to be infested in the second period and die without treatment. The term $\theta\bar{\pi}^l n$ in Eq. (1) is the landowner's value of having $\bar{\pi}^l n$ surviving trees, while $c\pi^l n$ is the cost of removing $\pi^l n$ dead trees. If at least one tree is infested $(i \geq 1)$ initially, inaction in the first period would lead to a higher attack rate (π^h) in the second period. The expected number of trees to be infested in the next period is thus $\pi^h(n - i)$. The landowner's expected utility is the difference between the value of healthy trees, $\theta \cdot \bar{\pi}^h(n - i)$, and the removal cost, $c \cdot (i + \pi^h(n - i))$.

Landowner's expected utility if he participates in the cost-sharing program. Here, the landowner will first pay for an inspection to identify the infested trees. The inspection cost of all trees is αn, where α is the unit cost. Upon learning the infestation level i, the landowner decides on how many trees to treat, denoted by $q(i)$, by considering the progression of the infestation level. Figures 1 and 2 depict the two possible outcomes, which depend on the relationship of $q(i)$ and i. In both figures, I_t denotes the number of newly infested trees in period t, where $t = 1, 2$. H_t represents the number of healthy trees in period t, T_t the number of treated trees in period t, IST_t the number of infested trees that are successfully treated in period t, IUT_t the number of infested trees that are unsuccessfully treated in period t, NT_t the number of trees that are not treated in period t, and D_t the number of dead trees in period t.

The consequence of not treating all infested trees in the first period, i.e., $q(i) < i$ is illustrated in Figure 1. When $q(i)$ out of the i infested trees are treated, the $i - q(i)$ untreated ones will die and

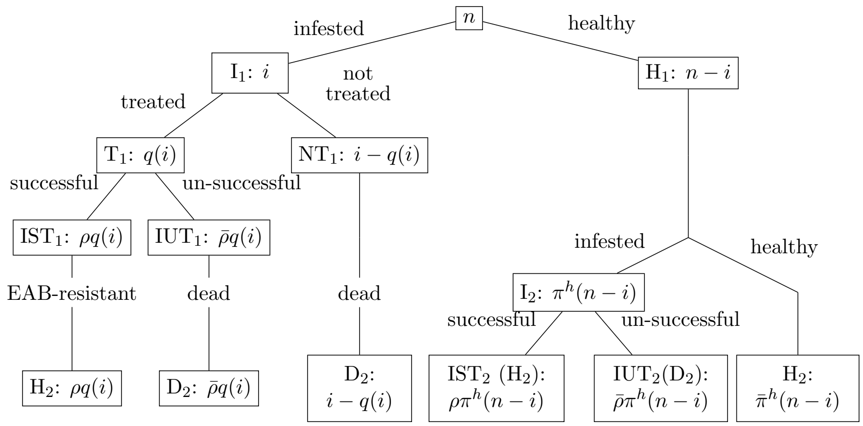

Figure 1: Progression of infestation in two consecutive periods when some infested trees are *not* treated in the first period ($q(i) < i$).

must be removed. Further, the $\bar{\rho}q(i)$ unsuccessfully-treated trees will also not survive, whereas the $\rho q(i)$ successfully-treated ones will recover and stay healthy in the second period. Because not all infested trees are treated, the attack rate among the $n-i$ healthy trees increases to π^h in the second period. As a result, $\pi^h(n-i)$ trees may become infested in period two while $\bar{\pi}^h(n-i)$ ones remain healthy. For simplicity, we assume that all newly infested trees will be treated, then $\rho\pi^h(n-i)$ trees will become healthy, but $\bar{\rho}\pi^h(n-i)$ trees may die because the treatment is unsuccessful. Therefore, the overall number of treated trees is $t_1 = q(i) + \pi^h(n-i)$, the total number of trees survived by the end of period two is $w_1 = \rho q(i) + \bar{\pi}^h(n-i) + \rho\pi^h(n-i)$, and the aggregated number of dead trees is $d_1 = i - q(i) + \bar{\rho}q(i) + \bar{\rho}\pi^h(n-i)$. The landowner's expected utility when $q(i) < i$ is

$$\phi(q(i), r(i)\big| n, i, q(i) < i) = \quad \theta \cdot w_1 + r(i) - \alpha \cdot n - \beta \cdot t_1 - c \cdot d_1. \tag{2}$$

The first two terms in Eq. (2) are the landowner's value of having w surviving trees and the reimbursement received from the city forester. The next three terms represent the inspection cost, the treatment cost, and the removal cost, respectively.

Similarly, Figure 2 shows the outcome of treating not only infested trees but some healthy trees in the first period, i.e., $q(i) \geq i$. Because all infested trees (i) are treated, the successfully-treated trees (ρi) will survive and stay healthy in the second period, while the unsuccessfully-treated ones ($\bar{\rho}i$) would die. The healthy trees that are treated ($q(i) - i$) in the first period become resistant to EAB. Further, the attack rate among the healthy, untreated trees ($n - q(i)$) decreases to π^l in the second period. Therefore, $\pi^l(n - q(i))$ trees are expected to be infested in period two, while $\bar{\pi}^l(n-q(i))$ trees are expected to remain healthy. Since all newly infested trees in the second period are assumed to be treated, $\rho\pi^l(n-q(i))$ of them are expected to be successfully treated and survive, while the rest are expected to die. The landowner's expected utility when $q(i) \geq i$ is

$$\phi(q(i), r(i)\big|n, i, q(i) \geq i) = \quad \theta \cdot w_0 + r(i) - \alpha \cdot n - \beta \cdot t_0 - c \cdot d_0. \tag{3}$$

The terms in Eq. (3) are similar to those in Eq. (2), except that the total number of treated trees is $t_0 = i + (q(i) - i) + \pi^l(n - q(i))$, the aggregated number of trees surviving by the end of the second period is $w_0 = (q(i) - i) + \rho i + \bar{\pi}^l(n - q(i)) + \rho\pi^l(n - q(i))$, and the overall number of dead trees is $d_0 = \bar{\rho} i + \bar{\rho}\pi^l(n - q(i))$. Let μ_i be an indicator such that $\mu_i = 1$ when $q(i) < i$ and $\mu_i = 0$ otherwise. $\bar{\mu}_i = 1 - \mu_i$. The landowner's expected utility from participating in the cost-sharing program is

$$\phi(q(i), r(i)\big|n, i) = \mu_i \cdot \phi(q(i), r(i)|n, i, q(i) < i) + \bar{\mu}_i \cdot \phi(q(i), r(i)|n, i, q(i) \geq i). \tag{4}$$

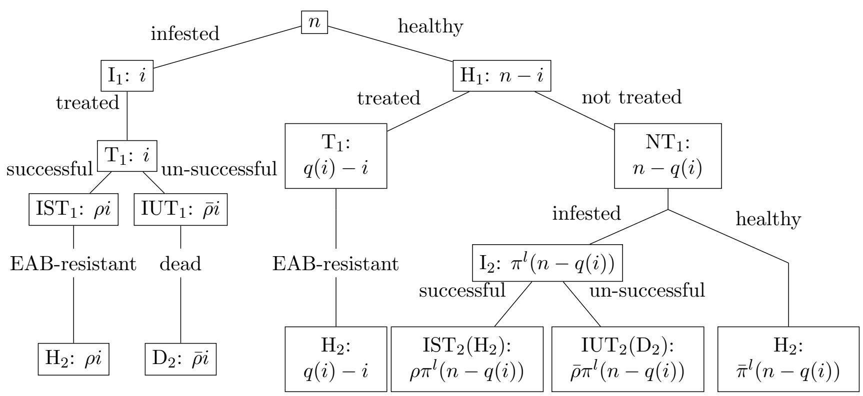

Figure 2: Progression of infestation in two consecutive periods when *all* of the infested trees are treated in the first period ($q(i) \geq i$).

Sequence of events. Figure 3 illustrates the interactions between the city forester and the landowner over two consecutive periods if he participates in the cost-sharing program. At the beginning of the first period, both parties have the same knowledge about the ash trees on the landowner's property: there are n ash trees, and the attack rate is π. The landowner pays for a tree care professional to inspect ash trees and finds out the number of trees (i) that are already infested. He reports the information to the city forester, who then offers a financial assistance ($r(i)$) that depends on the infestation level (i). Upon receiving the reimbursement $r(i)$, the landowner decides how many trees ($q(i)$) to treat. If he does not treat all of the infested trees ($q(i) < i$) in the first period, then the outcome of his decision over the next two periods is depicted in Figure 1. Otherwise, the outcome follows Figure 2.

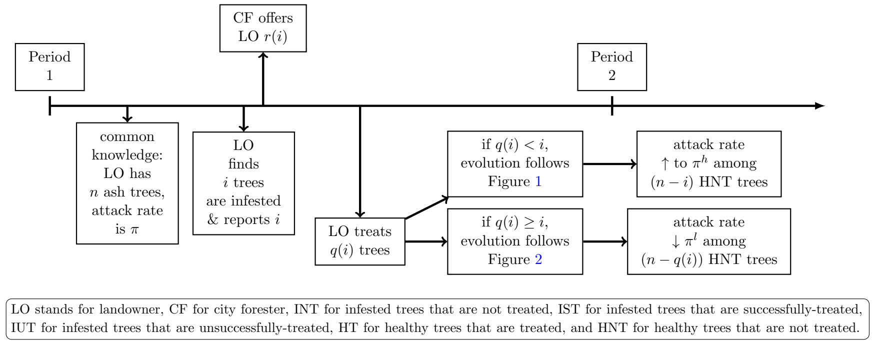

Figure 3: Sequence of events for the infestation-based reimbursement model.

City forester's optimization problem. We assume that both the city forester and the landowner are rational decision makers who maximize their utilities over a two-period planning horizon. Let $v(w,d) = s \cdot w - \gamma \cdot d$ denotes the city forester's value function. The first term is the city forester's value of having w surviving trees, where s is the marginal value of a tree. The second term, $\gamma \cdot d$, is the city forester's penalty of having d infested trees either untreated or unsuccessfully-treated. This penalty is included because if an infested tree on the landowner's parcel is not treated or not successfully treated, then ash trees in nearby parcels would be more likely to be infested in the following period. The penalty also represents the cost of enforcing a city ordinance to remove dead trees. The city forester's goal is to maximize her utility function, which is the difference between her value function and the reimbursement. In the case where $q(i) < i$, as shown in Figure 1, the number of surviving trees at the end of the period is $w_1 = \rho q(i) + \bar{\pi}^h(n-i) + \rho\pi^h(n-i)$, while the number of untreated or unsuccessfully-treated trees is $d_1 = i - \rho q(i) + \bar{\rho}\pi^h(n-i)$. Similarly, in the case where $q(i) \geq i$, $w_0 = q(i) - \bar{\rho} i + \bar{\pi}^l(n-q(i)) + \rho\pi^l(n-q(i))$ and $d_0 = \bar{\rho} i + \bar{\rho}\pi^l(n-q(i))$, as illustrated in Figure 2. The city forester's problem is as follows:

$$\begin{aligned}
\max_{q(i),r(i)} \ \Psi(q(i),r(i)|n,i) = \quad & \mu_i \cdot \Big(s \cdot w_1 - \gamma \cdot d_1\Big) + \bar{\mu}_i \Big(s \cdot w_0 - \gamma \cdot d_0\Big) - r(i) \\
\text{s.t.} \quad & \phi(q(i),r(i)|n,i) \geq \phi(a_0|n,i) && \text{(IR)} \\
& \phi(q(i),r(i)|n,i) \geq \phi(j,r(i)|n,i) \qquad \forall\ 0 \leq j \leq n && (\text{IC}_{ij}) \\
\text{and} \quad & q(i), r(i) \geq 0 && (\text{NN}_i)
\end{aligned} \tag{5}$$

The objective function in Eq. (5) is the net expected utility of the city forester, considering two scenarios: (1) when $q(i) < i$ (or $\mu_i = 1$) and (2) when $q(i) \geq i$ (or $\mu_i = 0$). The first term is the value of having w_1 surviving trees at the end of the second period minus the penalty for losing d_1 trees when $q(i) < i$. Similarly, the second term is the difference between the value of having w_0 surviving trees and the penalty for losing d_0 trees when $q(i) \geq i$. The last term is the cost of providing the financial award to the landowner. The individual rationality (IR) constraint ensures the landowner is incentivized to participate in the cost-sharing program. The incentive compatibility (IC_j) constraints guarantee that the landowner will prefer to treat the number of trees desired by the city forester $q(i)$ based on the infestation level (i) rather than another treatment decision (j). The non-negativity constraints (NN_i) ensure that neither $q(i)$ nor $r(i)$ is negative.

3.1 Analytical Solutions and Optimal Solution Characteristics for the IBR Model

In this section, we first present the optimal treatment decision (OTD) in the first period visually and then discuss the optimal solution characteristics based on the initial infestation level. There are three types of OTD: treating no trees (N as an abbreviated notation), treating only infested trees (I), and treating all trees (A). As shown in the tree diagram of Figure 4, the OTD depends on the infestation level (i) in the first period, the treatment effectiveness (ρ), and the second-period attack rates (π^l and π^h). In this figure, LE, SE, and VE stand for less effective, somewhat effective, and very effective, respectively. They correspond to the categorizations of ρ, which are clarified in Definition 1 in Section A.2 of the EC. Similarly, L (resp. M and H) stand for low (resp. medium and high) levels of π^l and π^h. See Definitions 2 and 3 in Section A.2 for details.

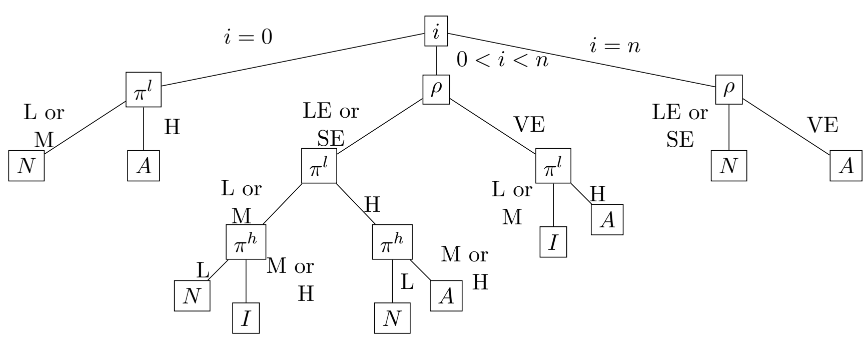

Figure 4: The optimal treatment decision (OTD) vs. key parameters.

At the top level, the tree diagram is split into three branches based on the initial infestation level (i). The left branch corresponds to when there are no infested trees ($i = 0$). The OTD is governed by the value of the low second-period attack rate (π^l). Recall that if a healthy tree is not treated in the first period, it may become infested in the second period with the probability π^l. When π^l is high, treating all trees (A) in the first period will prevent all healthy trees from becoming infested in the second period. On the other hand, when π^l is low or medium, healthy trees are unlikely to be infested in the second period, and thus, there is no need to treat any of them (N) in the first period. Similarly, the right branch represents when all trees are infested ($i = n$), the OTD is solely driven by whether or not the treatment is very effective. If yes, then the landowner prefers to treat all trees (A). Otherwise, the landowner is better off not treating them (N) and simply removing all in the first period.

The middle branch shows when some (but not all) trees are infested, the OTD is jointly determined by ρ, π^l, and π^h. This branch is thus further split into multiple sub-branches. Rather than discussing each branch separately, we summarize the results based on the OTD. First, the landowner is induced to treat all trees (A) if π^l is high. Treating all trees will not only improve the chance of survival for the infested trees but also make the healthy trees EAB-resistant. The only exception is when π^h is low. Because the risk of having new infestations is low, treating all trees is unnecessary. Second, treating only infested trees (I) is optimal if π^l is low or medium, and either the treatment is very effective, or π^h is medium or high. Because healthy trees in the first period are less likely to get infested in the second period when the infested trees are treated, they do not require preventative treatment. In the first case, where the treatment is very effective, infested trees that are treated would likely survive. In the second case, where π^h is medium or high, treating infested trees would prevent the infestation level from increasing significantly in the second period. Last, it is optimal not to treat any trees (N) if π^h is low and the treatment is less or somewhat effective. Because π^h is low, the infestation level will not increase significantly in the second period. Further, since the benefit of treating infested trees does not justify the cost of treatment, the landowner is better off removing the infested trees.

In the next sections, we present both the OTD and the optimal reimbursement under three settings: when no trees are infested, when all trees are infested, and when some trees are infested.

3.1.1 Optimal solution characteristics when no trees are infested

Because there are no infested trees ($i = 0$) in the first period, the second-period attack rate is π^l no matter what the landowner chooses to do in the previous period. Let $q^*(i)$ and $r^*(i)$ denote the optimal treatment option selected by the landowner and the reimbursement from the city forester, respectively. The characteristics of the optimal solution when no trees are infested are summarized in Proposition 1. See Section A.3.1 of the EC for the proof of Proposition 1.

Proposition 1. *When no trees are infested in the first period, both the OTD and the optimal reimbursement depend on the low second-period attack rate (π^l), as shown in Table 2. If π^l is low or medium, the landowner is induced not to treat any trees (N) in the first period. The reimbursement decreases in the treatment effectiveness (ρ). Further, it is less (resp. greater) than or equal to the inspection cost, αn, if the treatment is very effective (resp. less effective or somewhat effective). If π^l is high, the city forester encourages the landowner to treat all trees (A). The reimbursement decreases in π^l. Further, it is less than the inspection cost when π^l is very high $\left(\pi^l > \max\{\frac{\beta}{\theta+c}, \ddot{\pi}^l\}\right)$. Otherwise, the reimbursement is greater than the inspection cost.*

Table 2: The optimal solution when $i = 0$.

Conditions	Opt. Treatment	Opt. Reimbursement
$\pi^l \in [0, \ddot{\pi}^l)$	$q^*(0) = 0$ (N)	$r^*(0) = \max\left\{0, \alpha n + a_1 \pi^l n\right\}$
$\pi^l \in [\ddot{\pi}^l, 1]$	$q^*(0) = n$ (A)	$r^*(0) = \max\left\{0, \alpha n + \left(\beta - (\theta + c)\pi^l\right) n\right\}$

Insights from Proposition 1. There are two interesting results. First, even though the city forester does not want the landowner to treat any trees (N) in the first period when π^l is low or medium, the reimbursement must include the future cost of (or benefit from) treating newly infested trees in the second period. If the treatment is not very effective ($\rho \in [0, \ddot{\rho})$), the net loss for the landowner is $a_1 = \beta - \rho(\theta + c)$ per infested tree. Anticipating $\pi^l n$ trees to be infested in the second period, the city forester reimburses the landowner the net expected loss from unsuccessful treatment, $a_1 \pi^l n$, to induce the landowner's participation. Therefore, the reimbursement is greater than the inspection cost ($r^*(0) > \alpha n$). As the treatment effectiveness (ρ) increases, a_1 decreases. When ρ becomes very effective, a_1 turns negative, and its absolute value represents the landowner's net gain per infested tree. Consequently, the reimbursement becomes less than or equal to the inspection cost ($r^*(0) \leq \alpha n$) and thus the $\max\{0, \cdot\}$ operator ensures a non-negative reimbursement. Second, the treatment strategy must change when the likelihood of new infestation in the second period is high ($\pi^l \in [\ddot{\pi}^l, 1]$). The city forester prefers the landowner to treat all trees (A) in the first period so that all healthy trees are resistant to EAB in the second period. The reimbursement is smaller than the inspection cost because $\beta - (\theta + c)\pi^l$ is negative when π^l is greater than $\frac{\beta}{\theta+c}$.

3.1.2 Optimal solution characteristics when all trees are infested

In the case where all trees are already infested in the first period, any untreated trees will die, while the successfully-treated ones will be EAB-resistant in the next period. We summarize the results in Proposition 2, and its proof can be found in Section A.3.2 of the EC.

Proposition 2. *When all trees are infested in the first period, both the OTD and the optimal reimbursement are determined by the treatment effectiveness (ρ), as presented in Table 3. If the treatment is less or somewhat effective, the city forester reimburses the landowner's inspection cost, and consequently, the landowner does not treat any trees (N). If, on the other hand, the treatment is very effective, the city forester induces the landowner to treat all trees (A) with a reimbursement less than the inspection cost.*

Table 3: The optimal solution when $i = n$.

Condition	Opt. Treatment	Opt. Reimbursement
$\rho \in [0, \ddot{\rho})$	$q^*(n) = 0$ (N)	$r^*(n) = \alpha n$
$\rho \in [\ddot{\rho}, 1]$	$q^*(n) = n$ (A)	$r^*(n) = \max\{0, \alpha n + a_1 n\} < \alpha n$

Insights from Proposition 2. The reimbursement is always less than or equal to the inspection cost. When the treatment is less effective or somewhat effective, all trees will die because of the absence of treatment. Since the landowner is responsible for removing the dead ash trees, the city forester only needs to compensate the landowner with the inspection cost. On the other hand, when the treatment is very effective, the landowner treats all trees. Therefore, the cost of the inspection is offset by the landowner's gain in utility from successfully-treated trees. Consequently, the city forester only needs to cover a portion of the inspection cost to induce the landowner to participate. The reimbursement monotonically decreases as ρ increases, and it can be reduced to zero when ρ is really high (or $\rho \geq \frac{\alpha+\beta}{\theta+c}$).

3.1.3 Optimal solution characteristics when some trees are infested

We discuss two scenarios based on the treatment effectiveness. First, we examine the case where the treatment is very effective, and the key results are summarized in Proposition 3. See EC A.3.3 for proof of Proposition 3.

Proposition 3. *If the treatment is very effective, the OTD depends on the low second-period attack rate (π^l), while the reimbursement depends on both second-period attack rates (π^l & π^h), as shown in Table 4. The landowner is induced to treat all infested trees (I) when π^l is low or medium and to treat all trees (A) when π^l is high. The reimbursement is only positive when π^h is low or medium.*

Insights from Proposition 3. Because the landowner will, at a minimum, treat the infested trees in the first period, π^h would not be realized in the second period. However, the reimbursement

Table 4: The optimal solution when $0 < i < n$ and the treatment is very effective ($\rho \geq \ddot{\rho}$).

π^l is low or medium: $\pi^l \in [0, \ddot{\pi}^l)$		
Condition	Opt. Treatment	Opt. Reimbursement
$\pi^h \in [0, \ddot{\pi}^h(i))$	$q^*(i) = i$ (I)	$r^*(i) = \alpha n + a_1 n - [a_1 \bar{\pi}^l + (\theta + c)(\pi^h - \pi^l)](n - i) > 0$
$\pi^h \in [\ddot{\pi}^h(i), 1]$	$q^*(i) = i$ (I)	$r^*(i) = 0$
π^l is high: $\pi^l \in [\ddot{\pi}^l, 1]$		
Condition	Opt. Treatment	Opt. Reimbursement
$\pi^h \in [0, \ddot{\pi}^h(i))$	$q^*(i) = n$ (A)	$r^*(i) = \alpha n + a_1 n - [a_1 - \beta + (\theta + c)\pi^h](n - i) > 0$
$\pi^h \in [\ddot{\pi}^h(i), 1]$	$q^*(i) = n$ (A)	$r^*(i) = 0$

decreases as π^h increases. When π^h is high, the reimbursement is zero. This is because as the risk of healthy trees getting infested in the second period increases, the landowner would have to treat the trees anyhow. Therefore, the city forester only needs to offer a small award to induce the landowner to participate.

Next, we present the optimal solution when the treatment is not very effective in Proposition 4. The proof of Proposition 4 is relegated in Section A.3.4 of the EC.

Proposition 4. *If the treatment is not very effective, the OTD and the optimal reimbursement are determined by both of the second-period attack rates (π^l & π^h), as illustrated in Table 5. When π^h is low, the landowner is induced not to treat any trees (N). The city forester offers a reimbursement that is higher than the inspection cost. When π^h is medium, the value of π^l determines the optimal solution. For a low or medium π^l, the landowner is induced to treat all infested trees (I) with a positive reimbursement. For a high π^l, the city forester prefers the landowner to treat all trees (A) and offers a positive reimbursement. When π^h is high, the reimbursement is always zero. The landowner will treat all trees when π^l is high and treat only the infested trees (I) otherwise.*

Table 5: The optimal solution when $0 < i < n$, and the treatment is not very effective ($\rho < \ddot{\rho}$).

π^h is low: $\pi^h \in [0, \dot{\pi}^h(i))$		
Condition	Opt. Treatment	Opt. Reimbursement
$\pi^l \in [0, 1]$	$q^*(i) = 0$ (N)	$r^*(i) = \alpha n + a_1 \pi^h (n - i) > \alpha n$
π^h is medium: $\pi^h \in [\dot{\pi}^h(i), \ddot{\pi}^h(i))$		
Condition	Opt. Treatment	Opt. Reimbursement
$\pi^l \in [0, \ddot{\pi}^l)$	$q^*(i) = i$ (I)	$r^*(i) = \alpha n + a_1 n - [a_1 \bar{\pi}^l + (\theta + c)(\pi^h - \pi^l)](n - i) > 0$
$\pi^l \in [\ddot{\pi}^l, 1]$	$q^*(i) = n$ (A)	$r^*(i) = \alpha n + a_1 n - [a_1 - \beta + (\theta + c)\pi^h](n - i) > 0$
π^h is high: $\pi^h \in [\ddot{\pi}^h(i), 1]$		
Condition	Opt. Treatment	Opt. Reimbursement
$\pi^l \in [0, \ddot{\pi}^l)$	$q^*(i) = i$ (I)	$r^*(i) = 0$
$\pi^l \in [\ddot{\pi}^l, 1]$	$q^*(i) = n$ (A)	$r^*(i) = 0$

Insights from Proposition 4. When the treatment is not very effective, the benefit from treatment does not justify the cost of inducing the landowner to treat any of the infested trees, especially when π^h is low. Therefore, N is optimal in this case. However, the city forester still needs to offer a reimbursement that covers not only the inspection cost but also part of the treatment cost in the second period, as there will be newly infested trees, and the landowner is assumed to treat them.

4 THE TREATMENT-BASED REIMBURSEMENT MODEL

In the treatment-based reimbursement (TBR) model, we assume that the number of treated trees (q) is verifiable. This can be achieved if the city forester requires the submission of a service receipt issued by a professional tree care service who provided the EAB treatment. Differently from the previous model, the infestation level (i) is assumed to be not verifiable. Moreover, the reimbursement is given to the landowner after treatment, not prior to it.

Sequence of events. The interactions between the city forester and the landowner over two consecutive periods if a landowner participates in the cost-sharing program are illustrated in Figure 5. At the start of the first period, both parties have the same knowledge about the ash trees: the landowner has n ash trees, and the attack rate is π. The city forester announces a reimbursement schedule, $\mathbf{r} = [r(0), r(1), \cdots, r(n)]$, that prescribes the reimbursement corresponding to the number of treated trees (q). As an example, $r(0)$ is the reimbursement if no trees are treated ($q = 0$). After inspection, the landowner realizes the infestation level (i) but does not report this information to the city forester. Instead, he decides the number of trees to be treated ($q(i)$) based on the infestation level (i) and the reimbursement schedule ($\mathbf{r}$). After treatment, the landowner submits the service receipt to the city forester and receives the reimbursement according to the schedule. The progression of the infestation level follows the depiction in Figure 1 if some infested trees are not treated and in Figure 2 otherwise. Similar to the IBR model, reimbursement is only available in the first period. Therefore, a landowner who wishes to participate must sign up before the infestation level is revealed. Further, for simplicity, we assume that the landowner will treat all newly infested trees in the second period if he participates in the program.

Landowner's expected utility when he does *not* participate in the program. Because the first-period attack rate is π, the chance of having i infested trees is $\binom{n}{i} \cdot \pi^i \bar{\pi}^{n-i}$. If no trees are infested ($i = 0$) in the first period, the attack rate in the second period decreases to π^l. The expected number of trees infested in period two is $\pi^l n$, while the expected number of surviving trees is $\bar{\pi}^l n$. The landowner's utility is the difference between the value of surviving trees and the cost of removal: $\theta\bar{\pi}^l n - c\pi^l n$. Conversely, if the initial infestation level is at least one, the second-period attack rate is increased to π^h in the absence of any treatment in the first period. The death toll is the sum of the infested trees (i) in the first period and the expected number of trees that will be infested ($\pi^h(n-i)$) in the second period. The number of surviving trees at the end of period two is $\bar{\pi}^h(n-i)$. The landowner's expected utility is, therefore $\binom{n}{i}\pi^i\bar{\pi}^{n-i} \cdot \Big(\theta\bar{\pi}^h(n-i) - c(\pi^h(n-i)+i)\Big)$. His expected utility over all possible infestation levels, $\Phi(a_0|n)$, can be computed as follows:

$$\Phi(a_0|n) = \bar{\pi}^n \cdot \Big(\theta\bar{\pi}^l n - c\pi^l n\Big) + \sum_{i=1}^{n}\binom{n}{i}\pi^i\bar{\pi}^{n-i} \cdot \Big(\theta\bar{\pi}^h(n-i) - c(\pi^h(n-i)+i)\Big). \tag{6}$$

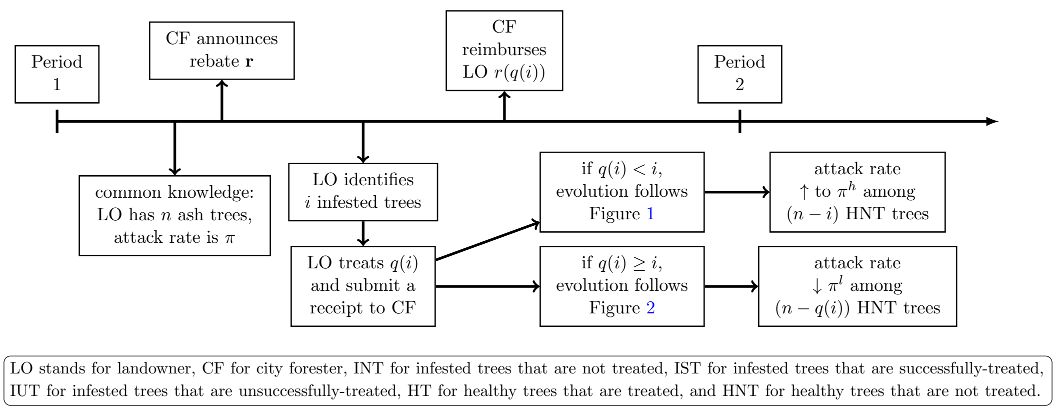

Figure 5: Sequence of events for the treatment-based reimbursement model.

Landowner's expected utility when he participates in the program. If the landowner does not treat all infested trees ($q(i) < i$), his expected utility would be

$$\begin{aligned}\phi\Big(q(i), r(q(i))\big|n, i, q(i) < i\Big) = \;& \theta \cdot \big[\rho q(i) + \bar{\pi}^h(n-i) + \rho\pi^h(n-i)\big] + r(q(i)) - \alpha \cdot n \\ & -\beta \cdot \big[q(i) + \pi^h(n-i)\big] - c \cdot \big[i - \rho q(i) + \bar{\rho}\pi^h(n-i)\big].\end{aligned} \tag{7}$$

The terms in Eq. (7) are similar to those in Eq. (2), except that $r(i)$ has been replaced by $r(q(i))$. Similarly, the landowner's expected utility is

$$\begin{aligned}\phi\Big(q(i), r(q(i))\big|n, i, q(i) \geq i\Big) = \;& \theta \cdot \big[q(i) - \bar{\rho} i + \bar{\pi}^l(n-q(i)) + \rho\pi^l(n-q(i))\big] + r(q(i)) \\ & -\alpha \cdot n - \beta \cdot \big[q(i) + \pi^l(n-q(i))\big] - c \cdot \big[\bar{\rho} i + \bar{\rho}\pi^l(n-q(i))\big]\end{aligned} \tag{8}$$

if he treats not only all infested trees but also some healthy trees ($q(i) \geq i$). Recall that μ_i is defined as an indicator such that $\mu_i = 1$ when $q(i) < i$ and $\mu_i = 0$ otherwise. $\bar{\mu}_i = 1 - \mu_i$. The landowner's expected utility from participating in the cost-sharing program is thus

$$\Phi(\mathbf{q},\mathbf{r}\big|n) = \quad \textstyle\sum_0^n \binom{n}{i}\pi^i\bar{\pi}^{n-i}\cdot\phi\Big(q(i),r(q(i))|n,i\Big), \text{ where} \tag{9}$$

$$\phi\Big(q(i),r(q(i))\big|n,i\Big) = \mu_i\cdot\phi\Big(q(i),r(q(i))|n,i,q(i)<i\Big) + \bar{\mu}_i\cdot\phi\Big(q(i),r(q(i))\big|n,i,q(i)\geq i\Big). \tag{10}$$

City forester's optimization problem. The city forester's objective is to maximize her net expected utility, which is the difference between her value from the surviving trees and the reimbursement provided to the landowner. We can write the problem of the city forester as follows:

$$\begin{aligned}
&\max_{\mathbf{q},\mathbf{r}} \Psi(\mathbf{q},\mathbf{r}|n) = && \textstyle\sum_{i=0}^n \binom{n}{i}\pi^i\bar{\pi}^{n-i}\cdot\Big[\mu_i\cdot\big(s\cdot w_1-\gamma\cdot d_1\big)+\bar{\mu}_i\cdot\big(s\cdot w_0-\gamma\cdot d_0\big)-r(q(i))\Big] && \\
& && \Phi(\mathbf{q},\mathbf{r}|n)\geq\Phi(a_0|n) && && \text{(IR)}\\
&\text{s.t.} && \phi(q(i)|n,i)\geq\phi(j|n,i) && \forall\, 0\leq i\leq n,\quad 0\leq j\leq n && (\text{IC}_{ij})\\
& && r(q(i))\geq r(q(i)-1) && \forall\, 1\leq q(i)\leq n && (\text{MON}_i)\\
& && q(i),r(q(i))\geq 0 && \forall\, 0\leq i\leq n && (\text{NN}_i),\\
&\text{where} && w_1=\rho q(i)+(\bar{\pi}^h+\rho\pi^h)(n-i), && d_1=i-\rho q(i)+\bar{\rho}\pi^h(n-i), &&\\
& && w_0=q(i)-\bar{\rho}i+(\bar{\pi}^l+\rho\pi^l)(n-q(i)), && d_0=\bar{\rho}i+\bar{\rho}\pi^l(n-q(i)). &&
\end{aligned} \tag{11}$$

The objective function in Eq. (11) is the city forester's net expected utility given a set of treatment decisions ($\mathbf{q}$) and a reimbursement schedule ($\mathbf{r}$). The individual rationality (IR) constraint encourages the landowner to participate in the cost-sharing program by ensuring the landowner's expected utility is higher if he participates. The incentive compatibility (IC_{ij}) constraints induce the landowner to pick the treatment decision ($q(i)$) desired by the city forester. The monotonic (MON_i) constraints ensure the reimbursement is non-decreasing in the number of treated trees. This set of constraints is unnecessary in the infestation-based reimbursement model because the landowner must first report the infestation level and select the reimbursement accordingly. In fact, the optimal reimbursement is non-monotonic in the infestation level. Finally, both the number of treated trees and the reimbursement are restricted to be non-negative (NN_i).

4.1 Optimal Solution Characteristics for the TBR Model

We classify the optimal treatment decisions (OTDs) into eight sets, which are identified through an extensive numerical analysis. Each set contains $n+1$ treatment decisions, with each decision corresponding to the initial infestation level (i). Figure 8 in EC A.5 provides a graphical illustration. Different from the IBR model, there are four types of OTD: treating none (N), treating only infested trees (I), treating all trees (A), and treating some trees (S). To simplify the representation, we use a subscript and a superscript that follow an OTD to represent the starting and ending infestation levels, respectively, where the same OTD applies. As an example, $N_0^0 I_1^{n-1} A_n^n$ denotes the set of OTDs where it is optimal for the landowner to not treat any trees when no trees are infested initially ($i = 0$), treat only infested trees when the initial infestation level (i) is between 1 and $n-1$, and treat all trees when all are infested ($i = n$).

The OTDs are determined by the treatment effectiveness (ρ) and the second-period attack rates (π^l and π^h). We use the tree diagram in Figure 6 to facilitate our discussion. Similar to the IBR model, we categorize the treatment effectiveness (ρ) into three regions: less effective (LE), somewhat effective (SE), and very effective (VE). Further, we classify π^l to low (L), medium-low (ML), medium-high (MH), and high (H) levels and π^h to low (L), medium (M) and high (H) levels. See Definitions 4 and 5 in EC A.4 for detailed definitions.

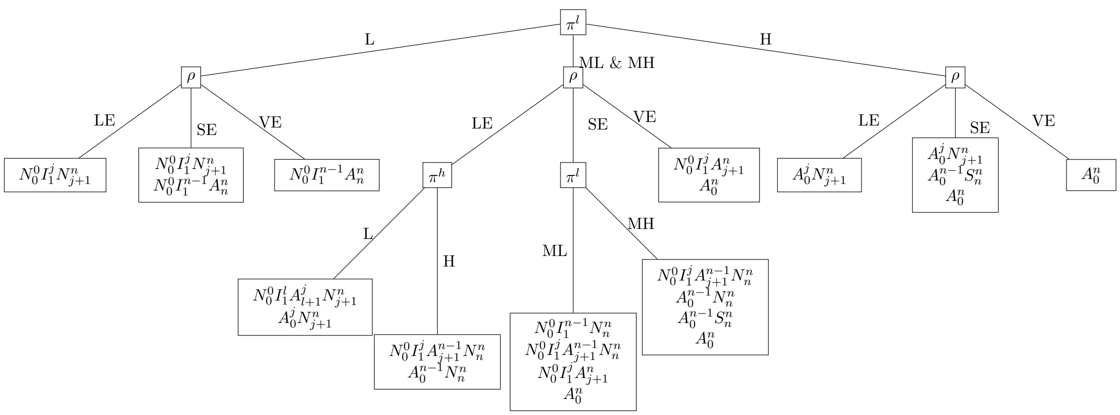

Figure 6: OTDs vs. key parameters.

Next, we examine the OTDs when π^l is high. As shown by the right branch of the tree diagram in Figure 6, either one of the following sets can be optimal: A_0^n, $A_0^{n-1} S_n^n$, $A_0^j N_{j+1}^n$. All three sets of decisions agree to treat all trees in the first period, so long as the infestation level is at or below a cutoff value (j) and j can be $n-1$, to avoid the realization of a high π^l in the second period. When most of the trees are already infested in the first period ($i > j$), however, the decision depends on the treatment effectiveness. If the treatment is very effective, then treating all trees is optimal because the chance of survival is high. On the contrary, if the treatment is not effective, then all trees should be removed. Interestingly, when the treatment is somewhat effective, it can be optimal to treat some (but not all) infested trees.

Last, the OTDs when π^l is medium-low[1] or medium-high[1] are driven by the treatment effectiveness, as shown in the middle branch of the tree diagram in Figure 6. If the treatment is very effective (VE), either A_0^n or $N_0^0 I_1^j A_{j+1}^n$ can be optimal. Both sets of OTDs agree to treat all trees if the infestation level is beyond some threshold j. Below that value, the landowner may either treat only infested trees or treat all. If, on the other hand, the treatment is less effective (LE), the value of the high second-period attack rate (π^h) determines the optimal treatment decisions. Specifically, when π^h is low[1], either $N_0^0 I_1^l A_{l+1}^j N_{j+1}^n$ or $A_0^j N_{j+1}^n$ is optimal. The landowner would treat either the infested trees or all trees if the infestation level is low, while not treating any trees if the infestation is high. This result is driven by the fact that the treatment is less effective, and therefore it should be used mainly for the prevention of trees becoming infested in the next period. When π^h is high[1], both OTDs ($N_0^0 I_1^j A_{j+1}^{n-1} N_n^n$ and $A_0^{n-1} N_n^n$) agree with treating all trees if the infestation level is sufficiently high ($j \leq i < n$) to prevent healthy trees in the first period from becoming infested in the second. If the treatment is somewhat effective (SE) and π^l is medium-low (ML), then $N_0^0 I_1^{n-1} N_n^n$, $N_0^0 I_1^j A_{j+1}^{n-1} N_n^n$, $N_0^0 I_1^j A_{j+1}^n$ or A_0^n can be optimal. Although the four sets of OTDs appear to be quite different, they all settle on treating at least the infested trees so long as not all trees are already infested. On the other hand, if the treatment is somewhat effective (SE) and π^l is medium-high (MH), then the set of optimal decisions includes $N_0^0 I_1^{n-1} N_n^n$, $A_0^{n-1} N_n^n$, $A_0^{n-1} S_n^n$, and A_0^n. Here, the common optimal decisions among the OTDs are to treat all trees when the infestation level is high, save for the case where all trees are already infested.

5 MODEL COMPARISONS

In this section, we compare the efficacy of the two cost-sharing models in several metrics. In addition to the IBR and TBR models, we also consider a third model in which the city forester does not offer a cost-sharing (or NCS) program. Four scenarios are analyzed by varying the treatment effectiveness (between very effective and not very effective) and the low second-period attack rate (between low/medium and high). Due to the page limit, the most interesting scenario is presented here, while the remaining scenarios are relegated to Section A.6 of the EC. Managerial insights from all scenarios are summarized in Section 7.

[1] See Section A.4 of the EC for a formal definition.

5.1 Scenario 1: The treatment is not very effective, and the low second-period attack rate is low or medium.

Scenario 1 represents a situation where infested trees have relatively high levels of canopy damage, causing low treatment efficacy. Further, EAB population pressure in the neighborhood is relatively low, causing the low second-period attack rate to be medium or low. Table 6 presents the expected number of surviving trees (E_{trees}), the expected reimbursement (E_r), the expected objective function value (E_{obj}) of the city forester, and the OTDs of the three models. Under NCS, the city forester does not offer any financial assistant, and thus, E_r is zero. E_{trees} is computed as the expected number of surviving trees, assuming that the landowner would not take any actions in mitigating the negative impact of EAB, i.e., the landowner's OTDs are N_0^5. The E_{obj} is computed as the expected value of $\Psi(q(i), r(i)|n, i)$, the objective function value in Eq. (5) where both $q(i)$ and $r(i)$ are set to zero, over i. Similarly, E_r in IBR (resp. TBR) is the expected value of $r^*(i)$ (resp. $r^*(q(i))$) over i, while E_{obj} is the expected value of $\Psi(q^*(i), r^*(i)|n, i)$ (resp. $\Psi(\mathbf{q}^*, \mathbf{r}^*|n)$) in Eq. (5) (resp. Eq. (11)) over i .

Table 6: Comparisons under Scenario 1 ($\pi^l < \ddot{\pi}^l$ and $\rho < \ddot{\rho}$). Other parameters used: $n = 5$, $\alpha = 40$, $\beta = 294$, $c = 738$, $\theta = 50$, $s = 100$, $\gamma = 100$, $\rho = 0.20$, $\pi = 0.30$, $\pi^l = 0.25$. Calculated cutoffs: $\dot{\rho} = 0.30$, $\ddot{\rho} = 0.37$, $\dot{\pi}^l = 0.27$, $\ddot{\pi}^l = 0.32$, $\hat{\pi}^l(\rho) = 0.27$, $\hat{\pi}^h = 0.57$.

	$\pi^h = 0.35$				$\pi^h = 0.50$				$\pi^h = 0.70$			
	E_{trees}	E_r	E_{obj}	OTDs	E_{trees}	E_r	E_{obj}	OTDs	E_{trees}	E_r	E_{obj}	OTDs
IBR	2.97	270	-176	$N_0^0 I_1^2 N_3^5$	3.07	108	6	$N_0^0 I_1^3 N_4^5$	3.06	71	42	$N_0^0 I_1^3 N_4^5$
TBR	2.97	270	-176	$N_0^0 I_1^2 N_3^5$	3.07	0	114	$N_0^0 I_1^3 N_4^5$	3.06	0	113	$N_0^0 I_1^3 N_4^5$
NCS	2.59	0	17	N_0^5	2.27	0	-46	N_0^5	1.84	0	-131	N_0^5

In this scenario, π^l is low ($\pi^l < \dot{\pi}^l$) , ρ is less effective ($\rho < \dot{\rho}$), and π^h varies from low ($\pi^h = 0.35$ or 0.50, $\pi^h < \hat{\pi}^h$) to high ($\pi^h = 0.70 > \hat{\pi}^h$). Under both IBR and TBR, the OTDs all follow the same structure: $N_0^0 I_1^j N_{j+1}^n$, which is non-monotonic in the initial infestation level. As π^h increases, the index j increases. This suggests that the landowner prefers to treat more infested trees as the likelihood of new infestation increases. As the value of π^h increases, the required expected reimbursement is lower under TBR and consequently a higher expected objective function value than that under IBR. Further, though the OTDs are different under TBR and IBR, neither is monotonic in the initial infestation level. Interestingly, not offering a cost-sharing program may be desirable to the city forester even though it always leads to the lowest number of surviving trees. Case in point: when π^h is 0.35, the expected objective function value is the highest under NCS, even though the expected number of surviving trees is lower. This is due to the high expected reimbursement required to run either cost-sharing program. Another intriguing result is that the expected number of surviving trees increases as π^h increases from 0.35 to 0.5 or 0.7 under both IBR and TBR. This is due to the landowner's higher treatment effort, which is incentivized by those two cost-sharing programs.

6 STRENGTHS AND WEAKNESSES OF THE TWO-PERIOD FRAMEWORK

To better address the multi-period nature of the landowner's decision-making process, we acknowledge that our two-period framework may oversimplify the actual scenario. In reality, the landowner may perceive their inspection and treatment decisions as part of a longer-term strategy. For instance, they might consider their commitment to treating EAB infestations for the entire duration of property ownership, or they may anticipate discontinuing treatment once a reimbursement program expires. These factors introduce complexities that extend beyond the two-period framework.

To assess the strengths and weaknesses of the two-period framework, we expand the time horizon to cover T periods ($T > 2$). While maintaining the analytical models for the first two periods, we conduct a comparative analysis on the landowner's long-term discounted utilities over periods 3 to T. We consider two actions in our analysis: 1) Continuing surveillance and treating any newly infested trees until the end of the T^{th} period or until all ash trees have died. Note that the reimbursement program ends after period two and the landowner pays the full cost of inspection and treatment in periods 3 through T. 2) Stopping the surveillance or treatment of any ash trees, while remaining responsible for removing dead ash trees until the end of period T. This analysis aims to provide a more comprehensive understanding of the impact of the landowner's potential strategies on the long-term outcomes.

Through simulations, we assess the impact of different actions on the infestation dynamics and the resulting outcomes for the landowner. We use the IBR model since it has a simpler optimal treatment decision (OTD) tree and thus is easier to implement. For each initial infestation level, we follow the OTD in the first period. In the second period, an untreated healthy tree from the first period is susceptible to becoming infested with either a high (π^h) or low (π^l) attack rate, depending on the OTD. In line with our assumption, the landowner is committed to treating any newly infested trees in the second period, leading to a low attack rate in the third period. From the third period onward, the landowner considers the two aforementioned actions. If the first action is chosen, a low attack rate is assumed for the remaining periods. Alternatively, if the second action is selected, the attack rate for the remaining periods is π^h. To capture the dynamics of infestation on private land following both actions, we generate 1000 instances through simulations by varying combinations of multiple key input parameters, such as the initial infestation levels, the treatment effectiveness, and the high second-period attack rate. These instances allow us to simulate the progression of infestation over time and evaluate the outcomes associated with each action.

Let $\phi_{1,s|i}$ and $\phi_{2,s|i}$ denote the landowner's discounted utility with initial infestation level i ($i \in \{0, 1, \cdots, n\}$) for choosing action 1 and action 2 at the beginning of period 3, respectively, in instance s ($s = \{1, 2, \cdots, 1000\}$). These can be computed using Eq. (12):

$$\begin{aligned} \phi_{1,s|i} &= \sum_{k=3}^{T} \delta^{k-3} \left(\frac{\theta}{2} \cdot w^k - \frac{\alpha}{2} \cdot s^k - \beta \cdot t^k - c \cdot d^k \right), \\ \phi_{2,s|i} &= \sum_{k=3}^{T} \delta^{k-3} \left(\frac{\theta}{2} \cdot w^k - c \cdot d^k \right). \end{aligned} \tag{12}$$

Here, w^k, s^k, t^k and d^k denote the number of surviving trees, the number of susceptible trees, the number of treated trees, and the number of dead trees in period k, respectively. The time discount factor is denoted as δ. Recall that θ (resp. α) represents the valuation (resp. surveillance cost) for both periods, the per-period valuation (resp. surveillance cost) is thus scaled to be $\frac{\theta}{2}$ (resp. $\frac{\alpha}{2}$). Note, there is no reimbursement for treatment in the computation of $\phi_{1,s|i}$. Let $\phi_1|i$ and $\phi_2|i$ represent the long-term discounted utilities under action 1 and action 2, respectively. These values are computed as the simple averages of $\phi_{1,s|i}$ and $\phi_{2,s|i}$ over the 1000 instances, respectively. Our goal is to compare these two values for various combinations of input parameters, such as the time horizon (T), time discount factor (δ), treatment effectiveness (ρ), and high second-period attack rate (π_h) over various initial infestation levels. To study the impact of the time horizon, we select three values: 10, 15, and 20, which are chosen to approximate the median homeowner tenure in the U.S. based on a recent Redfin study (Anderson 2022). Furthermore, we vary δ from 0.9 to 0.99, but we only report results for $\delta = 0.95$ and 0.98 as they are representative enough.

We provide key takeaways here and defer detailed results to Section A.7 of the EC. The observed trend reveals that the long-term discounted utilities under both actions ($\phi_1|i$ and $\phi_2|i$) decrease as the time horizon (T) or the time discount factor (δ) increases. This outcome aligns with our intuition, as an extended ownership duration incurs higher costs associated with managing ash trees, including the removal of dead trees. Moreover, a higher discount factor amplifies the net present value of costs, further contributing to the decrease in long-term discounted utilities under both actions, with the impact being more pronounced under the first action.

We proceed by comparing the values of $\phi_1|i$ and $\phi_2|i$ to determine the preferable course of action for the landowner. Notably, $\phi_1|i$ dominates $\phi_2|i$ when the time horizon spans 10 or 15 years, irrespective of the specific values of other input parameters. This relationship holds true in the majority of cases, even when the time horizon extends to 20 years. Consequently, the optimal strategy for the landowner is to persist with surveillance and treatment efforts. However, when the time horizon is 20 years, the discount factor is at least 0.98, and the second-period attack rate is low, $\phi_2|i$ can surpass $\phi_1|i$ for some initial infestation levels. Under these conditions, the landowner would prefer to discontinue surveillance or treatment instead.

In conclusion, our two-period framework, while a simplification of a multi-period problem, demonstrates practical applicability, particularly when local foresters aim to encourage landowners to proactively manage EAB infestations by providing financial assistance. Landowners participating in such programs are likely to continue treating EAB infestations even when the reimbursement program ends after two periods (or 2-3 years). However, it is important to acknowledge a potential weakness of this framework, which is the possibility of undesirable dynamically inconsistent behavior, such as the discontinuation of surveillance or treatment by the landowner. Such behavior may arise in situations where the expected ownership tenure is long, the time discount factor is high, and the absence of treatment is not anticipated to result in significant damage over subsequent periods.

7 MANAGERIAL INSIGHTS

In this section, we highlight three key managerial insights for city foresters and relate them to current EAB management practices and research results. First, our results from the comparison of the performance of the cost-sharing models suggest that the TBR model is superior to the IBR model. The TBR model induces the landowner to treat more trees. While the expected reimbursement is higher, the city forester is likely to achieve a greater net benefit because more ash trees survive at the end of the planning period. Second, our results show that the optimal treatment decision in the first period depends on the number of infested trees and the second-period attack rate for the TBR model. When the number of infested trees is low, the city forester creates a subsidy that induces the landowner to treat, at a minimum, all infested trees. Further, the optimal number of treated trees increases as the likelihood of second-period attack increases and the city forester's reimbursement encourages the landowner to treat all (healthy and infested) trees in the first period. Third, results for the TBR model also show that the optimal treatment decision depends on the effectiveness of the insecticide treatment. When treatment is very effective (i.e., when trees are either healthy or have less than 50 percent canopy decline), treating all trees is optimal regardless of the number of trees infested. The reimbursement required to induce the landowner to treat all trees is likely to be small. When the treatment is not very effective (i.e., when infested trees have greater than 50 percent canopy damage) and the number of infested trees in the first period is high, not treating any trees and removing them within 1-3 years is optimal.

These three insights are consistent with EAB management practice and research results. In practice, most communities do not offer incentive programs to induce landowners to protect their ash trees. Instead, landowners are required to remove their ash trees when they die. Communities that do have incentive programs roughly follow the TBR model, where the city forester subsidizes the cost of insecticide treatment in proportion to the number of trees the landowner decides to treat. The EAB Management Plan of the city of Burnsville, Minnesota, offers such a program where the cost of insecticide treatment per tree is discounted by a fixed amount. Our TBR model could be used to show the benefits of employing a cost-sharing program where one does not already exist or to refine an existing program to increase net benefits. Our results also concur with broad EAB management guidance (Herms et al. 2019). These guidelines recommend applying insecticide treatments to healthy ash trees when EAB has a moderate to high risk of attack, for example, when EAB has been detected in the local community or neighborhood. Further, these guidelines recommend not treating trees with greater than 50 percent canopy decline because studies have shown that ash trees in the later stages of infestation are unlikely to be saved by insecticide treatment (Herms et al. 2019). The optimal treatment decisions, which are determined by the number of infested trees, treatment effectiveness, and second-period attack rates, provide additional detail and refinement to the broader management guidance in the literature.

Additionally, our results are in line with the results of simulation and optimization studies of urban ash populations, which show that EAB management strategies emphasizing insecticide treatment have higher net benefits than strategies that focus on tree removal (McCullough and Mercader 2012, Kovacs et al. 2014, Kıbış et al. 2021, Bushaj et al. 2021). For example, Kıbış et al. (2021) developed a multi-stage stochastic mixed integer programming formulation of surveillance and control decisions for EAB with the objective of maximizing the net benefits from ash trees surviving over a 5-year horizon. Their results show that it is critical to surveil and locate EAB immediately and then prioritize the treatment of minimally-infested trees followed by the removal of highly infested and dead trees. While the simulation and optimization results provide guidance for a city forester managing a large urban ash population in public ownership, our results apply to a private landowner who must decide how to manage the small number of ash trees on their property. Further, our results pertain to a city forester who is designing an incentive program for landowners to manage their ash.

The practicality of our models may be limited in two ways. First, we assume that the landowner is perfectly rational and takes the action that maximizes his utility. In reality, however, a landowner may not have the accurate information, such as the second-period attack rates, to select the best course of action. Further, he may not have the budget to treat trees even with the reimbursement from the city forester. Second, some parameter values, such as the marginal valuation of having a surviving ash tree, may be hard to estimate and can vary drastically among landowners. Dealing with heterogeneous landowners could complicate the city forester's design of a cost-sharing program.

8 CONCLUSIONS

In this work, we develop two cost-sharing models for a city forester to induce the participation of a private landowner in mitigating the negative impact of emerald ash borer on ash trees. Both types of information asymmetry are considered. In the infestation-based reimbursement model, the city forester encounters a moral hazard problem because the reimbursement, distributed to the landowner prior to treatment, must incentivize the landowner to treat the desired number of trees based on the infestation level post-reimbursement. The city forester, on the other hand, deals with an adverse selection problem in the treatment-based reimbursement model. She designs the reimbursement schedule so that the landowner would not treat more than the optimal number of trees in order to claim a higher reimbursement. We identify possible optimal treatment decisions and the conditions under which each treatment decision can be optimal for both models. Specifically, we analytically characterize the optimal treatment decisions and the reimbursement schedule for the IBR model. Further, we investigate the efficacy of the cost-sharing programs by comparing the IBR and TBR models to the one where the city forester does not offer any cost-sharing program. We conclude that in all scenarios but one, the treatment-based reimbursement model leads to the highest number of surviving trees while achieving the highest objective function value of the city forester, even though the reimbursement required to run the program is the highest. The only exception is when the treatment is not effective and both second-period attack rates are low.

Our future work would focus on two tasks. First, develop a heuristic to find the optimal solution for the treatment-based reimbursement model. Currently, finding the optimal treatment decisions and the reimbursement schedule requires a complete search with all possible combinations of treatment decisions. Because we have been able to narrow down the structures of the optimal treatment decisions, they can be used to reduce the search time. Second, integrate the game-theoretic models with an optimization model to maximize the number of surviving ash trees in both public and private lands under a limited budget through a public-private partnership. This integrated framework will provide optimal budget allocations for managing public trees and running cost-sharing programs with landowners that are heterogeneous in the number of ash trees and attack rates.

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A ELECTRONIC COMPANION

A.1 Classification of ash trees

Knight et al. (2014) classified ash trees into five levels based on their canopy health conditions. A level-1 ash tree has "a full, healthy canopy." A level-2 ash tree is minimally infested and has "a thinning canopy but no dead twigs or branches near the top of the tree." Level-3 ash trees have "dead twigs or branches near the top of the tree" while the level-4 ones have "less than 50% of a full canopy." A level-5 ash tree has "no foliage in the canopy portion of the tree" (Knight et al. 2014). As shown in Figure 7, we consider level-1 (healthy) and level-2 (minimally infested) ash trees as healthy-looking because visual inspections may not be able to differentiate them. Similarly, we group level-3 to level-5 trees as infested-looking ones since dead branches and canopy can be easily observed. If a level-1 tree is treated in the first period, it becomes EAB-resistant and remains healthy in period 2. Otherwise, it may become infested in the second period. If a level-2 tree is treated in the first period, it would recover and remain so in the second period because pesticide treatment for minimally infested trees is nearly 100% effective. However, if a level-2 tree is not treated in the first period, its health would deteriorate and it will become a level-3 or higher tree. For an infested-looking tree, applying pesticide treatment may not be effective for higher-level trees. Since unsuccessfully treated trees would die in 2-4 years, they should be removed by the end of period 2.

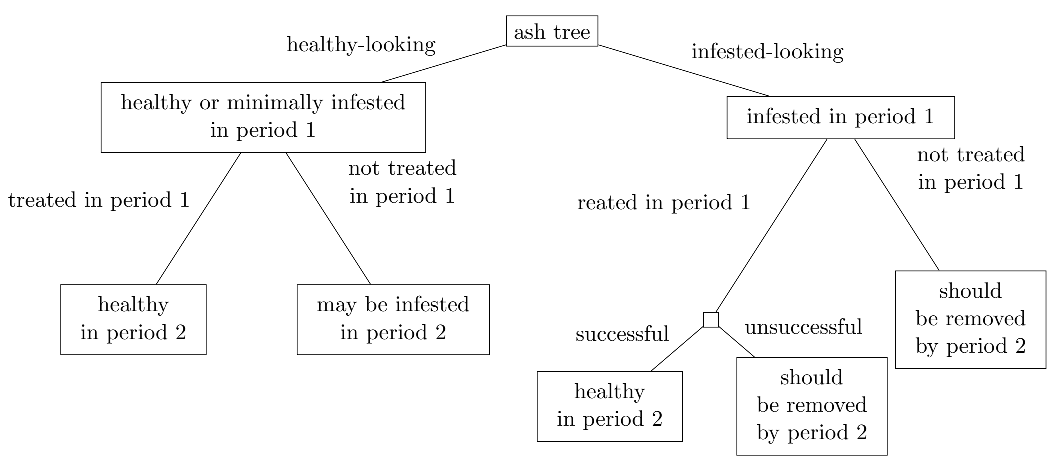

Figure 7: Classification of ash trees.

A.2 Categorization of key parameters for the IBR model

We define the following parameters and conditions in order to characterize the optimal solutions analytically.

First, let $a_1 := \beta - \rho(\theta + c)$ and $a_2 := \beta - \rho(s + \gamma + \theta + c)$. It is apparent that $a_1 > a_2$. Further, let $\dot{\rho} := \frac{\beta}{s+\gamma+\theta+c}$ and $\ddot{\rho} := \frac{\beta}{\theta+c}$, then $0 < \dot{\rho} < \ddot{\rho}$. Because the cost of treatment is generally much lower than the sum of the valuation of a live ash tree and the removal cost, $\ddot{\rho}$ is likely to be small. We can categorize the treatment effectiveness (ρ) into three levels.

Definition 1. As summarized in Table 7, the treatment is considered to be *less effective* if $\rho \in [0, \dot{\rho})$, *somewhat effective* if $\rho \in [\dot{\rho}, \ddot{\rho})$, and *very effective* if $\rho \in [\ddot{\rho}, 1]$.

Table 7: Categorization of treatment effectiveness (ρ).

Condition		ρ
$\rho \in [0, \dot{\rho})$	or $a_1 > a_2 > 0$	less effective
$\rho \in [\dot{\rho}, \ddot{\rho})$	or $a_1 > 0 \geq a_2$	somewhat effective
$\rho \in [\ddot{\rho}, 1]$	or $0 \geq a_1 > a_2$	very effective

Table 8: Classification of the low second-period attack rate (π^l).

Condition		π^l
$\pi^l \in [0, \dot{\pi}^l)$	or $b_1 > b_2 > 0$	low
$\pi^l \in [\dot{\pi}^l, \ddot{\pi}^l)$	or $b_1 > 0 \geq b_2$	medium
$\pi^l \in [\ddot{\pi}^l, 1]$	or $0 \geq b_1 > b_2$	high

Second, let $b_1 := \beta - \pi^l\big[\bar{\rho}(\theta + c) + \beta\big]$, $b_2 := \beta - \pi^l\big[\bar{\rho}(s + \gamma + \theta + c) + \beta\big]$, and $b_1 > b_2$. Further, let $\dot{\pi}^l := \frac{\beta}{\bar{\rho}(s+\gamma+\theta+c)+\beta}$ and $\ddot{\pi}^l := \frac{\beta}{\bar{\rho}(\theta+c)+\beta}$, which are used to classify the different levels of the low second-period attack rate (π^l). Since $s + \gamma + \theta + c > \theta + c$, $0 < \dot{\pi}^l < \ddot{\pi}^l < 1$.

Definition 2. As shown in Table 8, the low second-period attack rate (π^l) is considered *low* if $\pi^l \in [0, \dot{\pi}^l)$, *medium* if $\pi^l \in [\dot{\pi}^l, \ddot{\pi}^l)$, and *high* if $\pi^l \in [\ddot{\pi}^l, 1]$.

Third, let $\dot{\pi}^h(i) := \max\{0, \min\{1, \pi^l + \frac{a_1}{\bar{\rho}(\theta+c)+\beta} \cdot \frac{i}{n-i}\}\}$ and $\ddot{\pi}^h(i) := \max\{0, \min\{1, \frac{\pi^l(\bar{\rho}(\theta+c)+\beta)+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i}\}\}$ when π^l when π^l is low or medium ($\pi^l \in [0, \ddot{\pi}^l)$). Further, let $\dot{\pi}^h(i) := \max\{0, \min\{1, \frac{\beta}{\bar{\rho}(\theta+c)+\beta} + \frac{a_1}{\bar{\rho}(\theta+c)+\beta} \cdot \frac{i}{n-i}\}\}$ and $\ddot{\pi}^h(i) := \max\{0, \min\{1, \frac{\beta+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i}\}\}$ when π^l is high ($\ddot{\pi}^l \leq \pi^l \leq 1$). Both $\dot{\pi}^h(i)$ and $\ddot{\pi}^h(i)$ are valid for $0 < i < n$, and they describe the different levels of the high second-period attack rate (π^h). As shown in Section A.2.1, $\ddot{\pi}^h(i) \geq \dot{\pi}^h(i)$ when the treatment is either less effective or somewhat effective ($\rho \in [0, \ddot{\rho})$). $\dot{\pi}^h(i)$ increases (resp. decreases) in i when a_1 is positive (resp. negative), while $\ddot{\pi}^h(i)$ increases (resp. decreases) in i when $\alpha + a_1$ is positive (resp. negative).

Table 9: Categorization of the high second-period attack rate (π^h).

When $\rho \in [0, \ddot{\rho})$	
Condition	π^h
$\pi^h \in [0, \dot{\pi}^h(i))$	low
$\pi^h \in [\dot{\pi}^h(i), \ddot{\pi}^h(i))$	medium
$\pi^h \in [\ddot{\pi}^h(i), 1]$	high

When $\rho \in [\ddot{\rho}, 1]$	
Condition	π^h
$\pi^h \in [0, \dot{\pi}^h(i))$	low
$\pi^h \in [\ddot{\pi}^h(i), 1]$	high

Definition 3. We classify the high second-period attack rate (π^h) under two scenarios, as summarized in Table 9. In the first scenario, the treatment is less effective or somewhat effective ($\rho \in [0, \ddot{\rho})$). π^h is considered *low* if $\pi^h \in [0, \dot{\pi}^h(i))$, *medium* if $\pi^h \in [\dot{\pi}^h(i), \ddot{\pi}^h(i))$, and *high* if $\pi^h \in [\ddot{\pi}^h(i), 1]$. In the second scenario where the treatment is very effective. π^h is considered *low* when $\pi^h \in [0, \ddot{\pi}^h(i))$ and *high* otherwise.

A.2.1 Relationship between the two cutoff values of the high second-period attack rate

We show that $\ddot{\pi}^h(i) \geq \dot{\pi}^h(i)$ when $\rho < \ddot{\rho}$. The proof is separated into two cases: when π^l is low or medium and when π^h is high.

Case 1: When the low second-period attack rate is low or medium ($0 \leq \pi^l < \ddot{\pi}^l$). As discussed earlier, $\dot{\pi}^h$ and $\ddot{\pi}^h$ are defined as follows: $\dot{\pi}^h(i) := \max\{0, \min\{1, \pi^l + \frac{a_1}{\bar{\rho}(\theta+c)+\beta} \cdot \frac{i}{n-i}\}\}$ and $\ddot{\pi}^h(i) := \max\{0, \min\{1, \frac{\pi^l(\bar{\rho}(\theta+c)+\beta)+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i}\}\}$. Because $\ddot{\pi}^h$ is in between 0 and 1, we divide our discussion into two sub-cases.

Sub-case 1: If $\frac{\pi^l(\bar{\rho}(\theta+c)+\beta)+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i} \geq 1$, then $\ddot{\pi}^h(i) = 1$. Because $\dot{\pi}^h$ is also between 0 and 1, $\ddot{\pi}^h(i) \geq \dot{\pi}^h(i)$.

Sub-case 2: If $\frac{\pi^l(\bar{\rho}(\theta+c)+\beta)+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i} < 1$, then $\ddot{\pi}^h(i) = \max\{0, \frac{\pi^l(\bar{\rho}(\theta+c)+\beta)+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i}\}$. $\dot{\pi}^h(i)$ is either zero, or $\min\{1, \pi^l + \frac{a_1}{\bar{\rho}(\theta+c)+\beta} \cdot \frac{i}{n-i}\}$. If $\dot{\pi}^h(i) = 0$, $\ddot{\pi}^h(i) = \max\{0, \frac{\pi^l(\bar{\rho}(\theta+c)+\beta)+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i}\} \geq 0 = \dot{\pi}^h(i)$, i.e., $\ddot{\pi}^h(i) \geq \dot{\pi}^h(i)$. Otherwise, $\dot{\pi}^h(i) = \min\{1, \pi^l + \frac{a_1}{\bar{\rho}(\theta+c)+\beta} \cdot \frac{i}{n-i}\}$. Equivalently, $-\dot{\pi}^h(i) = -\min\{1, \pi^l + \frac{a_1}{\bar{\rho}(\theta+c)+\beta} \cdot \frac{i}{n-i}\} \geq -\left(\pi^l + \frac{a_1}{\bar{\rho}(\theta+c)+\beta} \cdot \frac{i}{n-i}\right)$. Since $\ddot{\pi}^h(i) = \max\{0, \frac{\pi^l(\bar{\rho}(\theta+c)+\beta)+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i}\} \geq \frac{\pi^l(\bar{\rho}(\theta+c)+\beta)+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i}$, the difference of the two cutoffs is thus

$$\begin{aligned}\ddot{\pi}^h(i) - \dot{\pi}^h(i) &\geq \frac{\pi^l(\bar{\rho}(\theta+c)+\beta)+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i} - \left(\pi^l + \frac{a_1}{\bar{\rho}(\theta+c)+\beta} \cdot \frac{i}{n-i}\right) \\ &= \frac{\alpha+\pi^l a_1}{\theta+c} + \frac{\alpha(\bar{\rho}(\theta+c)+\beta)+a_1^2}{(\theta+c)(\bar{\rho}(\theta+c)+\beta)} \cdot \frac{i}{n-i}.\end{aligned}$$

Because $\frac{\alpha(\bar{\rho}(\theta+c)+\beta)+a_1^2}{(\theta+c)(\bar{\rho}(\theta+c)+\beta)} \cdot \frac{i}{n-i} \geq 0$ for $0 \leq i < n$, $\ddot{\pi}^h(i) - \dot{\pi}^h(i) \geq \frac{\alpha+\pi^l a_1}{\theta+c}$. When $0 \leq \rho < \ddot{\rho}$, $a_1 > 0$, and thus, $\ddot{\pi}^h(i) - \dot{\pi}^h(i) \geq \frac{\alpha+\pi^l a_1}{\theta+c} > 0$.

Case 2: When the low second-period attack rate is high ($\ddot{\pi}^l \leq \pi^l \leq 1$). Recall $\dot{\pi}^h$ and $\ddot{\pi}^h$ are defined as follows: $\dot{\pi}^h(i) := \max\{0, \min\{1, \frac{\beta}{\bar{\rho}(\theta+c)+\beta} + \frac{a_1}{\bar{\rho}(\theta+c)+\beta} \cdot \frac{i}{n-i}\}\}$ and $\ddot{\pi}^h(i) := \max\{0, \min\{1, \frac{\beta+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i}\}\}$. Similar to the previous case, our discussion is separated into two sub-cases.

Sub-case 1: If $\frac{\beta+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i} \geq 1$, then $\ddot{\pi}^h(i) = 1$. Because $\dot{\pi}^h$ is also between 0 and 1, $\ddot{\pi}^h(i) \geq \dot{\pi}^h(i)$.

Sub-case 2: If $\frac{\beta+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i} < 1$, $\ddot{\pi}^h(i) = \max\{0, \frac{\beta+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i}\}$. $\dot{\pi}^h(i)$ is either zero or $\min\{1, \frac{\beta}{\bar{\rho}(\theta+c)+\beta} + \frac{a_1}{\bar{\rho}(\theta+c)+\beta} \cdot \frac{i}{n-i}\}$. If $\dot{\pi}^h(i) = 0$, $\ddot{\pi}^h(i) = \max\{0, \frac{\beta+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c} \cdot \frac{i}{n-i}\} \geq 0 = \dot{\pi}^h(i)$. Therefore, $\ddot{\pi}^h(i) \geq \dot{\pi}^h(i)$. Otherwise,

$$\begin{aligned}
\ddot{\pi}^h(i) - \dot{\pi}^h(i) &= \max\{0, \tfrac{\beta+\alpha}{\theta+c} + \tfrac{\alpha+a_1}{\theta+c} \cdot \tfrac{i}{n-i}\} - \min\{1, \tfrac{\beta}{\bar{\rho}(\theta+c)+\beta} + \tfrac{a_1}{\bar{\rho}(\theta+c)+\beta} \cdot \tfrac{i}{n-i}\} \\
&\geq \tfrac{\beta+\alpha}{\theta+c} + \tfrac{\alpha+a_1}{\theta+c} \cdot \tfrac{i}{n-i} - \left(\tfrac{\beta}{\bar{\rho}(\theta+c)+\beta} + \tfrac{a_1}{\bar{\rho}(\theta+c)+\beta} \cdot \tfrac{i}{n-i}\right) \\
&= \tfrac{\beta a_1+\alpha(\bar{\rho}(\theta+c)+\beta)}{\theta+c} + \tfrac{\alpha(\bar{\rho}(\theta+c)+\beta)+a_1^2}{(\theta+c)(\bar{\rho}(\theta+c)+\beta)} \cdot \tfrac{i}{n-i}.
\end{aligned}$$

Since $\frac{\alpha(\bar{\rho}(\theta+c)+\beta)+a_1^2}{(\theta+c)(\bar{\rho}(\theta+c)+\beta)} \cdot \frac{i}{n-i} \geq 0$ for $0 \leq i < n$, $\ddot{\pi}^h(i) - \dot{\pi}^h(i) \geq \frac{\beta a_1+\alpha(\bar{\rho}(\theta+c)+\beta)}{\theta+c}$. When $0 \leq \rho < \ddot{\rho}$, $a_1 > 0$, and thus, $\ddot{\pi}^h(i) - \dot{\pi}^h(i) \geq \frac{\beta a_1+\alpha(\bar{\rho}(\theta+c)+\beta)}{\theta+c} > 0$.

We can thus conclude that $\ddot{\pi}^h(i) \geq \dot{\pi}^h(i)$ when $0 \leq \rho < \ddot{\rho}$. □

A.3 Proof of Propositions

A.3.1 Proof of Proposition 1

When no trees are infested, the objective function value in Eq. (5) is $\Psi(q(0), r(0)|n, 0) = (s\bar{\rho} + \beta)\pi^l q(0) + (s - (s\bar{\rho} + \beta)\pi^l)n - r(0)$. Because $(s\bar{\rho} + \beta)\pi^l > 0$, Ψ increases in $q(0)$ and decreases in $r(0)$. From the *IR* and *NN* constraints, we get the lower bound of the reimbursement as $\max\{0, b_1 q(0) + \alpha n + a_1 \pi^l n\}$, where $a_1 := \beta - \rho(\theta + c)$ and $b_1 := \beta - \pi^l\left[\bar{\rho}(\theta + c) + \beta\right]$. Further, the *IC* constraints require $b_1(q(0) - j) \leq 0$ for all j. The remainder of the proof is divided into two cases: when π^l is low/medium and when π^l is high.

Case 1: When the low second-period attack rate is low or medium ($0 \le \pi^l < \ddot{\pi}^l$). In this case, $b_1 > 0$. Since $b_1(q(0) - j) \le 0$ must hold for all j, $q(0)$ must be less than j $\forall$ $0 \le j \le n$. $q(0) = 0$ is hence the only feasible and, consequently the optimal treatment decision. The optimal reimbursement is thus $r^*(0) = \max\{0, \alpha n + a_1 \pi^l n\}$. If the treatment is very effective ($\ddot{\rho} \le \rho \le 1$), $a_1 \le 0$ and hence $r^*(0) \le \alpha n$. Otherwise, $r^*(0) = \alpha n + a_1 \pi^l n > \alpha n$.

Case 2: When the low second-period attack rate is high ($\ddot{\pi}^l \le \pi^l < 1$). Here, $b_1 \le 0$. If $b_1 < 0$, $q(0)$ must equal to n to satisfy $q(0) - j \ge 0$ for all j or equivalently, $b_1(q(0) - j) \le 0$. Therefore, $q(0) = n$ is the only feasible solution. If $b_1 = 0$, all IC constraints are satisfied because $b_1(q(0) - j) = 0 \le 0$ for all j. Because Ψ increases in $q(0)$ and IC constraints are all satisfied, it is optimal to set $q^*(0) = n$ and $r^*(0) = \max\{0, \alpha n + (\beta - (\theta + c)\pi^l)n\}$. Further, when π^l is greater than both $\frac{\alpha+\beta}{\theta+c}$ and $\ddot{\pi}^l$, $\alpha n + (\beta - (\theta + c)\pi^l)n < 0$, and thus, $r^*(0) = 0$. □

A.3.2 Proof of Proposition 2

When all trees are infested, the objective function value in Eq. (5) is $\Psi(q(n), r(n)|n, n) = \rho(s + \gamma)q(n) - \gamma n - r(n)$. Because $\rho(s + \gamma) > 0$, Ψ increases in $q(n)$. We proceed with the proof by discussing the optimal solution for each category of treatment effectiveness.

Case 1: When the treatment is very effective ($\ddot{\rho} \le \rho \le 1$). In this case, $a_1 \le 0$. If we set $q(n) = n$, then $\mu_n = 0$ by definition. The IC constraints require $a_1(n - j) \le 0$ to hold for all j. Because $a_1 \le 0$, $n - j$ must be non-negative, and thus, all IC constraints are satisfied, making $q(n) = n$ a feasible solution. The lower bound of the reimbursement obtained from the IR and NN constraints is $\max\{0, \alpha n + a_1 n\}$. Since Ψ decreases in $r(n)$, setting it to the lower bound maximizes Ψ.

If $q(n)$ is less than n, then $\mu_n = 1$ by definition. If $a_1 < 0$, the IC constraint $-a_1(n - q(n)) \le 0$ is violated because $n - q(n) > 0$. Therefore, a feasible solution exists only when $a_1 = 0$. Since Ψ increases in $q(n)$, the best solution when $q(n) < n$ is $q(n) = n - 1$. and $r(n) = \alpha n$. The lower bound of $r(n)$ is $\max\{0, a_1 q(n) + \alpha n\}$ instead. However, when compared to $\Psi(n, \max\{0, \alpha n + a_1 n\}|n, n)$, $\Psi(n - 1, \alpha n|n, n)$ is lower when $a_1 = 0$: $\Psi(n - 1, \alpha n|n, n) - \Psi(n, \max\{0, \alpha n + a_1 n\}|n, n) = \big(\rho(s + \gamma)(n - 1) - \gamma n - \alpha n\big) - \big(\rho(s + \gamma)n - \gamma n - \alpha n\big) = -\rho(s + \gamma) < 0$. Therefore, $q(n) = n - 1$ and $r(n) = \alpha n$ cannot be an optimal solution. In conclusion, the optimal solution is $q^*(n) = n$ and

$r^*(n) = \max\{0, \alpha n + a_1 n\}$ when the treatment ρ is very effective.

Case 2: When the treatment is not very effective ($0 \leq \rho < \ddot{\rho}$). Here, $a_1 > 0$. If we assume $q(n) = n$, then $\mu_n = 0$ by definition. The lower bound of the reimbursement is $\max\{0, \alpha n + a_1 n\}$. IC constraints are satisfied if $a_1(n-j) \leq 0$ for all j. Because $a_1 > 0$ and $n - j \geq 0$ for all j, IC constraints are violated. Therefore, $q(n)$ can not be n.

If, on the other hand, $q(n) < n$, $\mu_n = 1$. The lower bound of $r(n)$ is $\max\{0, a_1 q(n) + \alpha n\}$. To satisfy IC constraints, i.e., $a_1(n-j) \leq 0$ for all j, $q(n) - j$ must be non-positive for all j. Consequently, $q(n) = 0$ is the only feasible solution, and thus, optimal. Since Ψ decreases in $r(n)$, $r^*(n) = \alpha n$. □

A.3.3 Proof of Proposition 3

Recall that Proposition 3 pertains to the case when some trees are infested, and the treatment is very effective. In this case, $a_1 \leq 0$. First, we argue that the number of treated trees must be no less than the number of infested trees ($q(i) \geq i$) by showing that IC constraints are violated if $q(i) < i$. The IC constraints can be re-written as $\big(\bar{\rho}(\theta + c) + \beta\big)(\pi^h - \pi^l)(n-i) - a_1(i - q(i)) - b_1(j - i) \leq 0$. Equivalently, $LHS = \pi^l + \frac{a_1(i-q(i))}{\big(\bar{\rho}(\theta+c)+\beta\big)(n-i)} + \frac{b_1(j-i)}{\big(\bar{\rho}(\theta+c)+\beta\big)(n-i)} \geq \pi^h$ must hold for all $j \in [i, n]$. If π^l is low or medium, $b_1 > 0$. At least one of the IC constraints is violated. Specifically, when $j = i$, the third term in LHS is zero. The second term is non-positive because $a_1 \leq 0$ and $q(i) < i$. Hence, $LHS \leq \pi^l$, which is less than π^h by assumption. Therefore, the IC constraint is violated. Similarly, if π^l is high, $b_1 \leq 0$. Here, all IC constraints are violated since $LHS \leq \pi^l < \pi^h$. As a result, $q(i) \geq i$.

The objective function value in Eq. (5) is thus $\Psi = (s+\gamma)\bar{\rho}\pi^l q(i) - \bar{\rho}(s+\gamma)i + \big(s - (s+\gamma)\bar{\rho}\pi^l\big)n - r(i)$. Because $(s+\gamma)\bar{\rho}\pi^l > 0$, the objective value increases in $q(i)$ and decreases in $r(i)$. A lower bound of $r(i)$ obtained from the IR and NN constraints is $\max\{0, b_1 q(i) - \big(\theta - \alpha - (\bar{\rho}(\theta + c) + \beta)\pi^l\big)n + \bar{\rho}(\theta + c)i + \phi(a_0|i)\}$. Further, to satisfy the IC constraints, the following inequalities must hold simultaneously: $\big(\bar{\rho}(\theta+c)+\beta\big)\big(\pi^l(n-q(i)) - \pi^h(n-i)\big) + a_1(i-j) + \beta(q(i)-i) \leq 0$ for all $j \in [0, i)$ and $b_1(q(i) - j) \leq 0$ for all $j \in [i, n]$. Next, we divide the proof into two cases: when π^l is low/medium and when it is high.

Case 1: When the low second-period attack rate is low or medium ($0 \leq \pi^l < \ddot{\pi}^l$). Here, $b_1 > 0$.

In order to satisfy the IC constraints or $b_1(q(i)-j) \leq 0$ for all $j \in [i,n]$, $q(i)-j$ needs to be non-positive. We can get $q(i)=i$ is the only feasible solution, and thus, optimal. To minimize the objective function value, $r^*(i) = \max\{0, \alpha n + a_1 n - [a_1\bar{\pi}^l + (\theta+c)(\pi^h - \pi^l)](n-i)\}$. $r^*(i)$ decreases in π^h. As per its definition, $\ddot{\pi}^h(i) = \max\{0, \min\{1, \frac{\pi^l(\bar{\rho}(\theta+c)+\beta)+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c}\cdot\frac{i}{n-i}\}\}$, which is $\leq \frac{\pi^l(\bar{\rho}(\theta+c)+\beta)+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c}\cdot\frac{i}{n-i}$. Since $\alpha n + a_1 n - \big[a_1\bar{\pi}^l + (\theta+c)\big(\frac{\pi^l(\bar{\rho}(\theta+c)+\beta)+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c}\cdot\frac{i}{n-i} - \pi^l\big)\big](n-i) = 0$, $\alpha n + a_1 n - [a_1\bar{\pi}^l + (\theta+c)(\pi^h-\pi^l)](n-i) > 0$ for all $\pi^h > \ddot{\pi}^h(i)$. Thus, $r^*(i) = \alpha n + a_1 n - [a_1\bar{\pi}^l + (\theta+c)(\pi^h-\pi^l)](n-i) > 0$. Similarly, when $\pi^h \leq \ddot{\pi}^h(i)$, $\alpha n + a_1 n - [a_1\bar{\pi}^l + (\theta+c)(\pi^h-\pi^l)](n-i) \leq 0$. Therefore, $r^*(i) = 0$.

Case 2: When the low second-period attack rate is high ($\ddot{\pi}^l \leq \pi^l \leq 1$). In this case, $b_1 \leq 0$. If $b_1 = 0$, the IC constraints are satisfied automatically. Since Ψ increases in $q(i)$, setting $q(i) = n$ is optimal. If, on the other hand, $b_1 < 0$, $q(i)-j$ needs to be non-negative for all $j \in [i,n]$ to satisfy the second set of IC constraints. Therefore, $q(i) = n$ is the only feasible solution, and thus, optimal. To minimize the objective function value, $r(i)$ should take its lower bound, which is simplified to $r^*(i) = \max\{0, \alpha n + a_1 n - (a_1 - \beta + (\theta+c)\pi^h)(n-i)\}$. $r^*(i)$ decreases in π^h.

Recall that $\ddot{\pi}^h(i)$ is defined as $\ddot{\pi}^h(i) = \max\{0, \min\{1, \frac{\beta+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c}\cdot\frac{i}{n-i}\}\}$, which is $\leq \frac{\beta+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c}\cdot\frac{i}{n-i}$. Because $\alpha n + a_1 n - \big(a_1 - \beta + (\theta+c)\big(\frac{\beta+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c}\cdot\frac{i}{n-i}\big)\big)(n-i) = 0$, $\alpha n + a_1 n - (a_1 - \beta + (\theta+c)\pi^h)(n-i) > 0$ when $\pi^h > \ddot{\pi}^h$. Hence, $r^*(i) = \alpha n + a_1 n - (a_1 - \beta + (\theta+c)\pi^h)(n-i) > 0$. On the other hand, when $\pi^h \leq \ddot{\pi}^h$, $\alpha n + a_1 n - (a_1 - \beta + (\theta+c)\pi^h)(n-i) \leq 0$, and thus, $r^*(i) = 0$. □

A.3.4 Proof of Proposition 4

Proposition 4 discusses the optimal solution when some trees are infested and the treatment is not very effective. The parameter a_1 is positive in this case. The proof is separated into two cases: when π^l is low/medium and when it is high. Each case is then further divided into two sub-cases: when π^h is low and when it is medium/high.

Case 1: When the low second-period attack rate is low or medium ($0 \leq \pi^l < \ddot{\pi}^l$). In this case, $b_1 > 0$.

Sub-case 1: When the high second-period attack rate is low ($0 \leq \pi^h < \dot{\pi}^h$). First, we show that no feasible solutions exist when $q(i) \geq i$. The IC constraints require $\big(\bar{\rho}(\theta+c)+\beta\big)\big[\pi^l(n-q(i)) - \pi^h(n-i)\big] + a_1(i-j) + \beta(q(i)-i) \leq 0$ for all $j \in [0,i)$ and $b_1(q(i)-j) \leq 0$

for all $j \in [i,n]$ to hold jointly. Because $b_1 > 0$, to satisfy the second set of IC constraints, $q(i) = i$ is the only solution that can be feasible. The first set of IC constraints is simplified to $\big(\bar{\rho}(\theta+c)+\beta\big)(\pi^l - \pi^h)(n-i) + a_1(i-j) \leq 0$ for all $j \in [0,i)$. Recall that $\dot{\pi}^h$ is defined as $\dot{\pi}^h(i) = \max\{0, \min\{1, \pi^l + \frac{a_1}{\bar{\rho}(\theta+c)+\beta} \cdot \frac{i}{n-i}\}\}$. Since $a_1 > 0$, $\pi^l + \frac{a_1}{\bar{\rho}(\theta+c)+\beta} \cdot \frac{i}{n-i} > 0$, and thus, $\dot{\pi}^h(i) \leq \pi^l + \frac{a_1}{\bar{\rho}(\theta+c)+\beta} \cdot \frac{i}{n-i}$. Moreover, $\big(\bar{\rho}(\theta+c)+\beta\big)\big[\pi^l - \big(\pi^l + \frac{a_1}{\bar{\rho}(\theta+c)+\beta} \cdot \frac{i}{n-i}\big)\big](n-i) + a_1(i-j) = 0$ when $j = 0$. Because the expression $\big(\bar{\rho}(\theta+c)+\beta\big)\big(\pi^l - \pi^h\big)(n-i) + a_1(i-j)$ decreases in π^h, $\big(\bar{\rho}(\theta+c)+\beta\big)\big(\pi^l - \pi^h\big)(n-i) + a_1(i-j) > 0$ when $j = 0$ for any $\pi^h < \dot{\pi}^h$. $q(i) = i$ is not a feasible solution since at least one of the IC constraints is violated.

Next, we examine whether a feasible solution exists when $q(i) < i$. In this case, $\mu_i = 1$. The objective function value in Eq. (5) is $\Psi = \rho(s+\gamma)q(i) + (\bar{\pi}^h + \rho\pi^h)s(n-i) - \bar{\rho}\gamma i - \bar{\rho}\pi^l\gamma n - r(i)$, which increases in $q(i)$ and decreases in $r(i)$. To satisfy the IC constraints, both $a_1(q(i)-j) \leq 0$ for all $j \in [0,i)$ and $\big(\bar{\rho}(\theta+c)+\beta\big)\big(\pi^h(n-i) - \pi^l(n-j)\big) - a_1(i-q(i)) - \beta(j-i) \leq 0$ for all $j \in [i,n]$ must hold. Because $a_1 > 0$, $q(i)$ must be zero to satisfy the first set of the IC constraints and $q(i) < i$. These constraints are simplified to $\big(\bar{\rho}(\theta+c)+\beta\big)\big(\pi^h(n-i) - \pi^l(n-j)\big) - a_1 i - \beta(j-i) \leq 0$ for all $j \in [i,n]$. As shown earlier, $\dot{\pi}^h(i) \leq \pi^l + \frac{a_1}{\bar{\rho}(\theta+c)+\beta} \cdot \frac{i}{n-i}$. Therefore, for any $\pi^h < \ddot{\pi}^h$, $\big(\bar{\rho}(\theta+c)+\beta\big)\big(\pi^h(n-i) - \pi^l(n-j)\big) - a_1(i-q(i)) - \beta(j-i) = (\bar{\rho}(\theta+c)+\beta)(\pi^h - \pi^l)(n-i) - a_1 i - b_1(j-i)$ $< (\bar{\rho}(\theta+c)+\beta)(\pi^l + \frac{a_1}{\bar{\rho}(\theta+c)+\beta} \cdot \frac{i}{n-i} - \pi^l)(n-i) - a_1 i - b_1(j-i) = -b_1(j-i) \leq 0$ since $b_1 > 0$ and $j \in [i,n]$. As a result, the second set of IC constraints are satisfied.

The lower bound of $r(i)$ obtained from the IR and NN constraints is $\max\{0, a_1 q(i) - \big(\theta - \alpha - \big(\bar{\rho}(\theta+c)+\beta\big)\pi^h\big)n + \big(\theta + c - \big(\bar{\rho}(\theta+c)+\beta\big)\pi^h\big)i + \phi(a_0|i)\}$. This lower bound when $q^*(i) = 0$ is simplified to $r^*(i) = \alpha n + a_1\pi^h(n-i)$, which is greater than αn since $a_1 > 0$.

Sub-case 2: When the high second-period attack rate is medium or high ($\dot{\pi}^h \leq \pi^h \leq 1$). Similar to the previous sub-case, we argue that $q(i) \geq i$ when $\pi^h \geq \dot{\pi}^h$. Here, $\mu_i = 0$ and the objective function value is $\Psi = (s+\gamma)\bar{\rho}\pi^l q(i) - \bar{\rho}(s+\gamma)i + \big(s - (s+\gamma)\bar{\rho}\pi^l\big)n - r(i)$, which increases in $q(i)$ and decreases in $r(i)$. The IC constraints require $\big(\bar{\rho}(\theta+c)+\beta\big)\big(\pi^l(n-q(i)) - \pi^h(n-i)\big) + a_1(i-j) + \beta(q(i)-i) \leq 0$ for all $j \in [0,i)$ and $b_1(q(i)-j) \leq 0$ for all $j \in [i,n]$. Since $b_1 > 0$ and $q(i) \geq i$, $q(i)$ must be i to satisfy the second set of IC constraints. Similar to the previous sub-case, we can show that the first set of constraints become $\big(\bar{\rho}(\theta+c)+\beta\big)(\pi^l - \pi^h)(n-i) + a_1(i-j) \leq 0$ for all $j \in [0,i)$, which are satisfied when $\pi^h \geq \dot{\pi}^h$.

The lower bound of $r(i)$ obtained from the IR and NN constraints is $\max\{0, b_1q(i) - \big(\theta - \alpha - (\bar{\rho}(\theta+c)+\beta)\pi^l\big)n + \bar{\rho}(\theta+c)i + \phi(a_0|i)\}$. Since Ψ decreases in $r(i)$, the lower bound maximizes Ψ. Further, the lower bound can be simplified to $\max\{0, \alpha n + a_1 n - [a_1\bar{\pi}^l + (\theta+c)(\pi^h - \pi^l)](n-i)\}$, which decreases in π^h. Recall that $\pi^h := \max\{0, \min\{1, \frac{\alpha+\pi^l a_1}{\theta+c} + \frac{\alpha+a_1}{\theta+c}\cdot\frac{i}{n-i}\}\}$. Because $a_1 > 0$, $\pi^h \leq \frac{\alpha+\pi^l a_1}{\theta+c} + \frac{\alpha+a_1}{\theta+c}\cdot\frac{i}{n-i}$. For any $\pi^h < \ddot{\pi}^h$, $\alpha n + a_1 n - [a_1\bar{\pi}^l + (\theta+c)(\pi^h - \pi^l)](n-i) > \alpha n + a_1 n - \left[a_1\bar{\pi}^l + (\theta+c)\left(\frac{\alpha+\pi^l a_1}{\theta+c} + \frac{\alpha+a_1}{\theta+c}\cdot\frac{i}{n-i}\right)\right](n-i) = 0$. On the other hand, when $\pi^h \geq \ddot{\pi}^h$, $\alpha n + a_1 n - [a_1\bar{\pi}^l + (\theta+c)(\pi^h - \pi^l)](n-i) \leq 0$. Therefore, $r^*(i) = 0$.

Case 2: When the low second-period attack rate is high ($\ddot{\pi}^l \leq \pi^l < 1$). In this case, $b_1 \leq 0$.
Sub-case 1: When the high second-period attack rate is low ($0 \leq \pi^h < \dot{\pi}^h$). We first show that no feasible solutions exist when $q(i) \geq i$. The IC constraints are simplified as follows: $\big(\bar{\rho}(\theta+c)+\beta\big)\big(\pi^l(n-q(i)) - \pi^h(n-i)\big) + a_1(i-j) + \beta(q(i)-i) \leq 0$ for all $j \in [0, i)$ and $b_1(q(i)-j) \leq 0$ for all $j \in [i, n]$. Because $b_1 \leq 0$, $q(i) = n$. At least one of the first sets of the IC constraints is violated when $\pi^h < \dot{\pi}^h$. Therefore, $q(i) = n$ is not a feasible solution. Next, using the same logic as Sub-case 1 of case 1, we can conclude that it is optimal to set $q^*(i) = 0$ and, consequently $r^*(i) = \alpha n + a_1\pi^h(n-i)$.
Sub-case 2: When the high second-period attack rate is medium or high ($\dot{\pi}^h \leq \pi^h \leq 1$). As established earlier, when $\pi^h > \dot{\pi}^h$, $q(i) \geq i$. Since $\mu_i = 0$, the objective function value is $\Psi = (s+\gamma)\bar{\rho}\pi^l q(i) - \bar{\rho}(s+\gamma)i + \big(s - (s+\gamma)\bar{\rho}\pi^l\big)n - r(i)$, which increases in $q(i)$ and decreases in $r(i)$. To satisfy the IC constraints, two sets of conditions need to hold: $\big(\bar{\rho}(\theta+c)+\beta\big)\big(\pi^l(n - q(i)) - \pi^h(n-i)\big) + a_1(i-j) + \beta(q(i)-i) \leq 0$ and $b_1(q(i)-j) \leq 0$ for all $j \in [i, n)$. Because $b_1 \leq 0$, $q(i) = n$ is the only feasible solution that meets the second set of the IC constraints.

A lower bound obtained from the IR and NN constraints when $q^*(i) = n$ is simplified to $\max\{0, \alpha n + a_1 i + \beta(n-i) - (\theta+c)\pi^h(n-i)\}$, which decreases in π^h. For any value any $\pi^h < \ddot{\pi}^h$, $\alpha n + a_1 i + \beta(n-i) - (\theta+c)\pi^h(n-i) > \alpha n + a_1 i + \beta(n-i) - (\theta+c)\ddot{\pi}^h(n-i) \geq \alpha n + \beta n - \rho(\theta+c)i - (\theta+c)\left(\frac{\beta+\alpha}{\theta+c} + \frac{\alpha+a_1}{\theta+c}\cdot\frac{i}{n-i}\right)(n-i) = 0$. On the other hand, when $\pi^h \geq \ddot{\pi}^h$, $\alpha n + a_1 i + \beta(n-i) - (\theta+c)\pi^h(n-i) \leq 0$, and thus, $r^*(i) = 0$. □

A.4 Categorization of key parameters for the TBR model

The classifications for the treatment effectiveness (ρ) in the TBR model are the same as those in the TBR model. See Definition 1. We introduce a few new parameters to characterize the optimal solutions in addition to the former notation. All of them are used to characterize the optimal solutions.

First, let $\hat{\pi}^l(\rho) := \max\{\dot{\pi}^l, \frac{\rho(\theta+c)}{\bar{\rho}(\theta+c)+\beta}\}$ when the low second-period attack rate (π^l) is medium ($\dot{\pi}^l \leq \pi^l < \ddot{\pi}^l$), and the treatment ($\rho$) is somewhat effective ($\dot{\rho} \leq \rho < \ddot{\rho}$). $\hat{\pi}^l(\rho)$ is less than $\ddot{\pi}^l$ because $\ddot{\pi}^l - \frac{\rho(\theta+c)}{\bar{\rho}(\theta+c)+\beta} = \frac{\beta-\rho(\theta+c)}{\bar{\rho}(\theta+c)+\beta} > 0$ and $\ddot{\pi}^l > \dot{\pi}^l$. We use $\hat{\pi}^l(\rho)$ to further classify π^l.

Definition 4. As illustrated in Table 10, the low second-period attack rate (π^l) is considered *low* if $\pi^l \in [0, \dot{\pi}^l)$, *medium-low* if $\dot{\pi}^l \leq \pi^l < \hat{\pi}^l(\rho)$, *medium-high* if $\hat{\pi}^l(\rho) \leq \pi^l < \ddot{\pi}^l$, and *high* if $\pi^l \in [\ddot{\pi}^l, 1]$.

Table 10: Categorization of the low second-period attack rate (π^l).

Condition	π^l
$0 \leq \pi^l < \dot{\pi}^l$	low
$\dot{\pi}^l \leq \pi^l < \hat{\pi}^l(\rho)$	medium-low
$\hat{\pi}^l(\rho) \leq \pi^l < \ddot{\pi}^l$	medium-high
$\ddot{\pi}^l \leq \pi^l \leq 1$	high

Next, let $\hat{\pi}^h := \max\{0, \min\{1, \pi^l + \frac{a_1-b_1}{\bar{\rho}(\theta+c)+\beta} \cdot (n-1)\}\}$, which represents a cutoff value that classifies π^h.

Definition 5. As shown in Table 11, the high second-period attack rate (π^h) is considered *low* if $0 \leq \pi^h < \hat{\pi}^h$ and *high* if $\hat{\pi}^h \leq \pi^h \leq 1$.

Table 11: Categorization of the high second-period attack rate (π^h).

Condition	π^h
$0 \leq \pi^h < \hat{\pi}^h$	low
$\hat{\pi}^h \leq \pi^h \leq 1$	high

A.5 Structures of the Optimal Treatment Decisions for the TBR Model

Figure 8 illustrates the eight sets of OTDs. Firstly, $N_0^0 I_1^{n-1} A_n^n$ denotes the set of OTDs where it is optimal for the landowner to not treat any trees when no trees are infested initially, treat only infested trees when the initial infestation level (i) is between 1 and $n-1$, and treat all trees when all are infested. Secondly, $N_0^0 I_1^j N_{j+1}^n$ represents the set where not treating any trees when none are infested, treating only infested trees when the infestation level (i) is between 1 and j, and not treating any trees when the infestation level is between $j+1$ and n is optimal. Similarly, $N_0^0 I_1^j A_{j+1}^n$ depicts not treating any trees when none are infested, treating all infested trees when the infestation level is at or below j, and treating all trees (A) when more than j trees are infested as the set of OTDs. $N_0^0 I_1^l A_{l+1}^j N_{j+1}^n$ represents the most complex set where only infested trees are treated if the number of trees being infested is between 1 and l, all trees are treated if the infestation level is lower than j, but none is treated if the infestation level is either zero or greater than j. $N_0^0 A_1^j N_{j+1}^n$ denotes the set where treating all trees is optimal when the infestation level is between 1 and j while not treating any trees otherwise is optimal. $A_0^j N_{j+1}^n$ deviates slightly from $N_0^0 A_1^j N_{j+1}^n$ in that all trees are treated even when zero trees are infested. $A_0^{n-1} S_n^n$ depicts the set where it is optimal to treat all trees so long as not of them are already infested ($i < n$), and only treat some trees when all of them are already infested. Lastly, A_0^n represents treating all trees regardless of the initial infestation level is optimal.

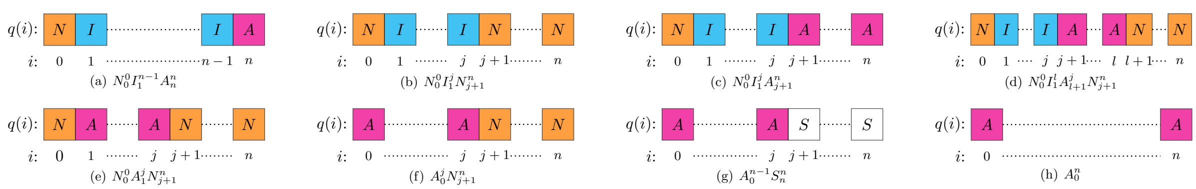

Figure 8: Eight sets of optimal treatment decisions (OTDs).

A.6 Model Comparisons - Remaining Scenarios

A.6.1 Scenario 2: The treatment is not very effective and the low second-period attack rate is high.

We present the numerical results in Table 12 when π^l is high ($\pi^l = 0.40 > \ddot{\pi}^l = 0.35$), ρ is somewhat effective ($\dot{\rho} = 0.30 \leq \rho = 0.30 < \ddot{\rho} = 0.37$), and π^h is low ($\pi^h = 0.42$ or 0.55 or $0.70 < \hat{\pi}^h = 0.78$). Under IBR, the landowner treats all trees when the infestation level is below a certain cutoff (j) and not treat any trees beyond the cutoff ($A_0^j N_{j+1}^n$). As π^h increases, the landowner treats all trees up to a higher infestation level to prevent the high risk of getting newly infested trees in the next period. Consequently, j increases.

The landowner takes a more aggressive approach in TBR when π^h is 0.42. He treats all trees except when all of them are already infested in the first period. Both the expected number of surviving trees and the expected objective function value are highest under TBR. This shows that offering reimbursement based on the number of treated trees is superior under this scenario.

Table 12: Comparisons under Scenario 2 ($\pi^l \geq \ddot{\pi}^l$ and $\rho < \ddot{\rho}$). Other parameters used: $n = 5$, $\alpha = 40$, $\beta = 294$, $c = 738$, $\theta = 50$, $s = 100$, $\gamma = 100$, $\rho = 0.30$, $\pi = 0.40$, $\pi^l = 0.38$. Calculated cutoffs: $\dot{\rho} = 0.30$, $\ddot{\rho} = 0.37$, $\dot{\pi}^l = 0.30$, $\ddot{\pi}^l = 0.35$, $\hat{\pi}^l(\rho) = 0.30$, $\hat{\pi}^h = 0.78$.

	$\pi^h = 0.42$				$\pi^h = 0.55$				$\pi^h = 0.70$			
	E_{trees}	E_r	E_{obj}	OTD	E_{trees}	E_r	E_{obj}	OTD	E_{trees}	E_r	E_{obj}	OTD
IBR	3.13	189	-64	$A_0^2N_3^5$	3.46	55	138	$A_0^3N_4^5$	3.58	29	188	$A_0^4N_5^5$
TBR	3.59	215	3	$A_0^4S_5^5$ (S=3)	3.46	0	193	$A_0^3N_4^5$	3.58	0	217	$A_0^4N_5^5$
NCS	2.13	0	-74	N_0^5	1.89	0	-122	N_0^5	1.62	0	-177	N_0^5

A.6.2 Scenario 3: The treatment is very effective and the low second-period attack rate is low or medium.

Table 13 summarizes the numerical results when π^l is low ($\pi^l = 0.29 \leq \dot{\pi}^l = 0.32$), the treatment is very effective ($\rho = 0.38 > \ddot{\rho} = 0.37$), and π^h is high ($\pi^h = 0.73$ or 0.53 or $0.33 > \hat{\pi}^h = 0.32$). The OTDs for the landowner are to treat the infested trees ($N_0^0 I_1^4 A_5^5$) under both IBR and TBR for all values of π^h, which leads to the same number of the expected number of surviving trees. However, the expected reimbursement is lower under TBR than IBR when π^h is higher. As a result, the objective function value is higher under TBR. Both models outperform NCS as a result of having a high success rate in treating infested trees.

Table 13: Comparisons under Scenario 3 ($\pi^l < \ddot{\pi}^l$ and $\rho \geq \ddot{\rho}$). Other parameters used: $n = 5$, $\alpha = 40$, $\beta = 294$, $c = 738$, $\theta = 50$, $s = 100$, $\gamma = 100$, $\rho = 0.38$, $\pi = 0.30$, $\pi^l = 0.29$. Calculated cutoffs: $\dot{\rho} = 0.30$, $\ddot{\rho} = 0.37$, $\dot{\pi}^l = 0.32$, $\ddot{\pi}^l = 0.38$, $\hat{\pi}^l(\rho) = 0.38$, $\hat{\pi}^h = 0.32$.

	$\pi^h = 0.33$				$\pi^h = 0.53$				$\pi^h = 0.73$			
	E_{trees}	E_r	E_{obj}	OTD	E_{trees}	E_r	E_{obj}	OTD	E_{trees}	E_r	E_{obj}	OTD
IBR	3.44	102	86	$N_0^0I_1^4A_5^5$	3.44	33	155	$N_0^0I_1^4A_5^5$	3.44	33	155	$N_0^0I_1^4A_5^5$
TBR	3.44	102	86	$N_0^0I_1^4A_5^5$	3.44	0	188	$N_0^0I_1^4A_5^5$	3.44	0	188	$N_0^0I_1^4A_5^5$
NCS	2.80	0	61	N_0^5	2.47	0	-5	N_0^5	2.15	0	-71	N_0^5

As π^h gets higher (0.53 or 0.73), the landowner only treats the infested trees ($N_0^0 I_1^4 A_5^5$) under both IBR and TBR. Because the expected reimbursement required under TBR is zero, the objective function value is higher.

A.6.3 *Scenario 4: The treatment is very effective and the low second-period attack rate is high.*

We present the numerical results in Table 14 when π^l is high ($\pi^l = 0.25 > \ddot{\pi}^l = 0.23$), the treatment is very effective ($\rho = 0.30 > \ddot{\rho} = 0.21$), and π^h is high ($\pi^h = 0.70$ or 0.50 or $0.33 > \hat{\pi}^h = 0.2$). The OTDs under both IBR and TBR are to treat all trees (A_0^5) regardless of the value of π^h. Similar to the results in the previous scenario, the expected number of surviving trees and the expected objective function value are highest under TBR, which attests to the superiority of providing reimbursement based on the number of treated trees over the infestation level.

Table 14: Comparisons under Scenario 4 ($\pi^l \geq \ddot{\pi}^l$ and $\rho \geq \ddot{\rho}$). Other parameters used: $n = 5$, $\alpha = 100$, $\beta = 200$, $c = 900$, $\theta = 50$, $s = 100$, $\gamma = 100$, $\rho = 0.30$, $\pi = 0.30$, $\pi^l = 0.25$. Calculated cutoffs: $\dot{\rho} = 0.17$, $\ddot{\rho} = 0.21$, $\dot{\pi}^l = 0.20$, $\ddot{\pi}^l = 0.23$, $\hat{\pi}^l(\rho) = 0.33$, $\hat{\pi}^h = 0.2$.

	$\pi^h = 0.33$				$\pi^h = 0.50$				$\pi^h = 0.70$			
	E_{trees}	E_r	E_{obj}	OTD	E_{trees}	E_r	E_{obj}	OTD	E_{trees}	E_r	E_{obj}	OTD
IBR	3.95	56	234	A_0^5	3.95	53	237	A_0^5	3.95	53	237	A_0^5
TBR	3.95	39	251	A_0^5	3.95	0	290	A_0^5	3.95	0	290	A_0^5
NCS	2.74	0	48	N_0^5	2.42	0	-16	N_0^5	2.05	0	-90	N_0^5

A.7 Long-term Discounted Utility Comparisons

The simulation results are presented in Tables 15, 16, and 17, corresponding to time horizons of 10, 15, and 20 periods, respectively. For all simulations, we use the following set of parameters to generate the dynamic infestation evolution of the private land: $n = 5$, $\alpha = 40$, $\beta = 294$, $c = 738$, $\theta = 50$, $s = 100$, $\gamma = 100$, $\pi = 0.3$, and $\pi_l = 0.2$. The treatment effectiveness (ρ) varies among 0.2 to 0.35 and 0.5, which corresponds to less effective (LE), somewhat effective (SE) and very effective (VE), respectively. Furthermore, the high second-period attack rate (π_h) ranges from 0.35 to 0.7, encompassing both low (L) and high (H) levels. Finally, the time discount factor δ is set at either 0.95 or 0.98.

First, both $\phi_1|i$ and $\phi_2|i$ decrease as either the time horizon (T) or the time discount factor (δ) increases. This outcome is intuitive, as a longer duration of landowner responsibility results in higher costs associated with managing the ash trees, including the removal of dead trees. Moreover, a higher discount factor leads to a greater net present value of costs. As expected, both $\phi_1|i$ and $\phi_2|i$ experience an increase in treatment effectiveness (ρ). However, it is important to note that

Table 15: Long-term discounted-utility comparison: $T = 10$

δ	ρ	i	$\pi_h = 0.35$			$\pi_h = 0.5$			$\pi_h = 0.7$		
			OTD	$\phi_1\|i$	$\phi_2\|i$	OTD	$\phi_1\|i$	$\phi_2\|i$	OTD	$\phi_1\|i$	$\phi_2\|i$
0.95	0.2	0	I	-1726	-2763	I	-1726	-2977	I	-1722	-3096
		1	I	-1441	-2321	I	-1469	-2517	I	-1475	-2614
		2	I	-1161	-1886	I	-1137	-2023	I	-1124	-2081
		3	N	-817	-1170	I	-860	-1532	I	-862	-1598
		4	N	-391	-569	N	-441	-635	N	-505	-666
		5	N	0	0	N	0	0	N	0	0
0.95	0.35	0	I	-1648	-2726	I	-1629	-2964	I	-1689	-3083
		1	I	-1428	-2369	I	-1402	-2544	I	-1413	-2674
		2	I	-1172	-2014	I	-1182	-2157	I	-1151	-2288
		3	I	-961	-1637	I	-964	-1792	I	-945	-1849
		4	I	-704	-1266	I	-722	-1408	I	-722	-1466
		5	N	0	0	N	0	0	N	0	0
0.95	0.5	0	I	-1549	-2689	I	-1589	-2933	I	-1599	-3053
		1	I	-1395	-2420	I	-1426	-2612	I	-1403	-2741
		2	I	-1212	-2122	I	-1208	-2311	I	-1230	-2447
		3	I	-1057	-1850	I	-1028	-2026	I	-1033	-2120
		4	I	-879	-1576	I	-888	-1731	I	-859	-1808
		5	A	-694	-1293	A	-719	-1460	A	-695	-1527
0.98	0.2	0	I	-1938	-3084	I	-1921	-3299	I	-1924	-3385
		1	I	-1590	-2528	I	-1581	-2741	I	-1587	-2839
		2	I	-1281	-2083	I	-1284	-2231	I	-1259	-2287
		3	N	-860	-1258	I	-935	-1691	I	-932	-1731
		4	N	-418	-637	N	-490	-682	N	-571	-712
		5	N	0	0	N	0	0	N	0	0
0.98	0.35	0	I	-1830	-3056	I	-1867	-3249	I	-1810	-3369
		1	I	-1541	-2632	I	-1561	-2823	I	-1580	-2909
		2	I	-1327	-2229	I	-1333	-2410	I	-1321	-2483
		3	I	-1065	-1832	I	-1076	-1962	I	-1054	-2032
		4	I	-814	-1441	I	-804	-1538	I	-813	-1603
		5	N	0	0	N	0	0	N	0	0
0.98	0.5	0	I	-1740	-2996	I	-1733	-3253	I	-1779	-3357
		1	I	-1575	-2706	I	-1571	-2914	I	-1566	-3021
		2	I	-1368	-2413	I	-1410	-2604	I	-1392	-2664
		3	I	-1177	-2080	I	-1188	-2279	I	-1173	-2343
		4	I	-1004	-1805	I	-959	-1922	I	-973	-1980
		5	A	-776	-1438	A	-788	-1595	A	-766	-1595

$\phi_1|i$ may not display a monotonic relationship with respect to the high second-period attack rate (π_h) due to potential variations in the infestation realizations and the first period's OTDs. $\phi_2|i$, on the other hand, decreases in (π_h).

Table 16: Long-term discounted-utility comparison: $T = 15$

δ	ρ	i	$\pi_h = 0.35$			$\pi_h = 0.5$			$\pi_h = 0.7$		
			OTD	$\phi_1\|i$	$\phi_2\|i$	OTD	$\phi_1\|i$	$\phi_2\|i$	OTD	$\phi_1\|i$	$\phi_2\|i$
0.95	0.2	0	I	-2230	-2873	I	-2183	-3004	I	-2219	-3093
		1	I	-1865	-2412	I	-1875	-2523	I	-1858	-2611
		2	I	-1491	-1919	I	-1464	-2030	I	-1500	-2076
		3	N	-964	-1179	I	-1126	-1548	I	-1128	-1599
		4	N	-488	-592	N	-517	-630	N	-572	-667
		5	N	0	0	N	0	0	N	0	0
0.95	0.35	0	I	-2146	-2815	I	-2150	-2979	I	-2166	-3086
		1	I	-1867	-2465	I	-1861	-2578	I	-1889	-2666
		2	I	-1563	-2056	I	-1591	-2178	I	-1572	-2263
		3	I	-1288	-1714	I	-1288	-1801	I	-1257	-1846
		4	I	-1002	-1321	I	-983	-1401	I	-989	-1458
		5	N	0	0	N	0	0	N	0	0
0.95	0.5	0	I	-2086	-2787	I	-2129	-2954	I	-2070	-3057
		1	I	-1909	-2516	I	-1886	-2670	I	-1892	-2750
		2	I	-1681	-2232	I	-1677	-2341	I	-1702	-2442
		3	I	-1431	-1913	I	-1432	-2052	I	-1398	-2112
		4	I	-1201	-1638	I	-1216	-1749	I	-1238	-1827
		5	A	-960	-1321	A	-1001	-1486	A	-999	-1504
0.98	0.2	0	I	-2602	-3197	I	-2621	-3303	I	-2575	-3385
		1	I	-2181	-2698	I	-2175	-2770	I	-2186	-2835
		2	I	-1804	-2185	I	-1757	-2241	I	-1751	-2310
		3	N	-1092	-1291	I	-1324	-1731	I	-1330	-1746
		4	N	-559	-651	N	-576	-689	N	-627	-709
		5	N	0	0	N	0	0	N	0	0
0.98	0.35	0	I	-2555	-3167	I	-2585	-3286	I	-2567	-3368
		1	I	-2212	-2759	I	-2240	-2870	I	-2221	-2932
		2	I	-1908	-2367	I	-1842	-2398	I	-1878	-2478
		3	I	-1503	-1896	I	-1547	-2014	I	-1529	-2031
		4	I	-1186	-1481	I	-1209	-1600	I	-1162	-1608
		5	N	0	0	N	0	0	N	0	0
0.98	0.5	0	I	-2516	-3150	I	-2485	-3278	I	-2483	-3355
		1	I	-2257	-2830	I	-2215	-2944	I	-2220	-3009
		2	I	-1981	-2540	I	-1966	-2627	I	-1962	-2675
		3	I	-1700	-2167	I	-1746	-2290	I	-1669	-2297
		4	I	-1444	-1868	I	-1492	-1961	I	-1494	-2026
		5	A	-1171	-1536	A	-1198	-1622	A	-1198	-1689

Next, $\phi_1|i \geq \phi_2|i$ for all parameter combinations when $T = 10$ or 15. This relationship also holds when $T = 20$ and either 1) $\delta = 0.95$ or 2) $\delta = 0.98$ and $\pi^h \geq 0.5$. Under these cases, it is more advantageous for the landowner to continue inspection and treatment until the end of period T. When $T = 20$, $\delta = 0.98$, and $\pi_h = 0.35$, however, $\phi_2|i$ is slightly higher than $\phi_1|i$ for some initial infestation levels (see cells highlighted in purple in Table 17). In these cases, the landowner is better off stop inspection and treatment starting in the third period.

Table 17: Long-term discounted-utility comparison: $T = 20$

δ	ρ	i	$\pi_h = 0.35$			$\pi_h = 0.5$			$\pi_h = 0.7$		
			OTD	$\phi_1\|i$	$\phi_2\|i$	OTD	$\phi_1\|i$	$\phi_2\|i$	OTD	$\phi_1\|i$	$\phi_2\|i$
0.95	0.2	0	I	-2591	-2865	I	-2577	-2993	I	-2605	-3102
		1	I	-2203	-2405	I	-2187	-2506	I	-2188	-2597
		2	I	-1787	-1968	I	-1713	-2036	I	-1769	-2106
		3	N	-1105	-1187	I	-1331	-1540	I	-1324	-1588
		4	N	-538	-597	N	-573	-630	N	-601	-667
		5	N	0	0	N	0	0	N	0	0
0.95	0.35	0	I	-2543	-2820	I	-2575	-2978	I	-2537	-3076
		1	I	-2213	-2470	I	-2226	-2582	I	-2226	-2678
		2	I	-1869	-2084	I	-1879	-2200	I	-1843	-2265
		3	I	-1538	-1705	I	-1534	-1801	I	-1536	-1861
		4	I	-1176	-1324	I	-1202	-1407	I	-1184	-1453
		5	N	0	0	N	0	0	N	0	0
0.95	0.5	0	I	-2532	-2797	I	-2523	-2951	I	-2503	-3059
		1	I	-2225	-2487	I	-2245	-2655	I	-2241	-2741
		2	I	-1992	-2220	I	-2019	-2372	I	-2026	-2450
		3	I	-1739	-1941	I	-1703	-2041	I	-1770	-2139
		4	I	-1448	-1627	I	-1494	-1748	I	-1441	-1800
		5	A	-1221	-1367	A	-1188	-1424	A	-1208	-1482
0.98	0.2	0	I	-3240	-3219	I	-3225	-3318	I	-3228	-3386
		1	I	-2684	2691	I	-2711	-2778	I	-2734	-2843
		2	I	-2181	-2180	I	-2184	-2251	I	-2213	-2301
		3	N	-1319	-1312	I	-1670	-1734	I	-1683	-1761
		4	N	-652	-657	N	-667	-684	N	-686	-711
		5	N	0	0	N	0	0	N	0	0
0.98	0.35	0	I	-3221	-3185	I	-3177	-3295	I	-3191	-3368
		1	I	-2752	-2768	I	-2769	-2838	I	-2785	-2935
		2	I	-2386	-2354	I	-2370	-2454	I	-2386	-2502
		3	I	-1939	-1916	I	-1938	-1989	I	-1919	-2045
		4	I	-1527	-1518	I	-1503	-1572	I	-1563	-1639
		5	N	0	0	N	0	0	N	0	0
0.98	0.5	0	I	-3200	-3173	I	-3169	-3279	I	-3192	-3357
		1	I	-2859	-2841	I	-2816	-2921	I	-2844	-3013
		2	I	-2538	-2545	I	-2495	-2619	I	-2534	-2703
		3	I	-2198	-2190	I	-2194	-2268	I	-2213	-2330
		4	I	-1914	-1914	I	-1875	-1950	I	-1893	-2005
		5	A	-1525	-1517	A	-1576	-1622	A	-1555	-1640