Modeling the Dynamic Behavior of Rain Attenuation

by

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(ABSTRACT)

This thesis addresses the problem of predicting satellite path rain fade duration statistics for an arbitrary location, frequency, elevation angle and polarization. It summarizes the development of a dynamic stochastic model. From this model a technique is derived for predicting fade duration statistics for one site using measured attenuation data at another site. This technique is evaluated by comparing predicted and experimental results for several locations, frequencies, elevation angles and polarizations.

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I would especially like to thank my advisor, Dr. C.W. Bostian, for giving me guidance, encouragement and advice, and for allowing me to combine my interests with his research goals. Also, I appreciate the financial support of NASA and the Jet Propulsion Laboratory. Finally, I would like to thank my wife, Laura Bottomley, and our families for their continuing support and encouragement.

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CHAPTER 1 - INTRODUCTION

As satellite communication moves to frequency bands above 6 GHz, the use of a fixed design margin for rain attenuation becomes prohibitive [1]. Adaptive power control is being used as a means of overcoming fades due to rain attenuation [1-5]. Also, adaptive forward-error-correcting codes provide a means of maintaining communications through a fading channel. To design a satellite communications system with such adaptive controls, one must understand the dynamic behavior of rain attenuation on a satellite link.

In this thesis we study the problem of predicting the dynamics of rain attenuation for a given location, frequency, elevation angle and polarization. In Chapter 2, we investigate how the dynamic behavior of rain attenuation on satellite links is characterized and how it has been modeled. In Chapters 3 and 4 we develop a dynamic stochastic model and adapt it to global use. In Chapter 5, we show how rain attenuation data at one site can be used to predict the dynamics of rain attenuation at another site. In Chapter 6 we investigate the model's shortcomings.

We made several important discoveries. We found that an existing stochastic model can be improved by driving it with a non-Gaussian input process. There is also evidence that the actual rain attenuation process is nonstationary, with model parameters varying with the level of attenuation. Most importantly we discovered a way to relate the dynamics of two attenuation data sets so that any two sets can be compared, and one set can be used to predict the dynamics of attenuation for another location.

CHAPTER 2 - REVIEW OF THE LITERATURE

The dynamics of rain attenuation have been observed and reported since the mid 1960's [6], and many results have been published [2,3,5-27]. In this chapter we review how the dynamics of rain attenuation have been characterized and modeled.

2.1 FADE AND INTERFADE DURATION DEFINITION

The dynamics of rain attenuation are most commonly reported in terms of fade duration and interfade duration. Fade duration is the amount of time attenuation exceeds a given threshold. Interfade duration is the amount of time between fades of a given duration. A graphical example is given in Figure 2.1 [8]. Here we are primarily concerned with fade durations.

There are three common ways to display fade and interfade duration results for a specified time period. One way is the fade duration table shown in Table 2.1 [22], which records the number of events which fall in each threshold and duration bin. A second way is with a histogram showing the number of events for each threshold. An example is given in Figure 2.2 [22]. Finally, fade and interfade duration results can be displayed as cumulative distributions, one for each threshold. In this case the ordinate indicates the number of events with a duration greater than or equal to the abscissa. An example is given in Figure 2.3 [19].

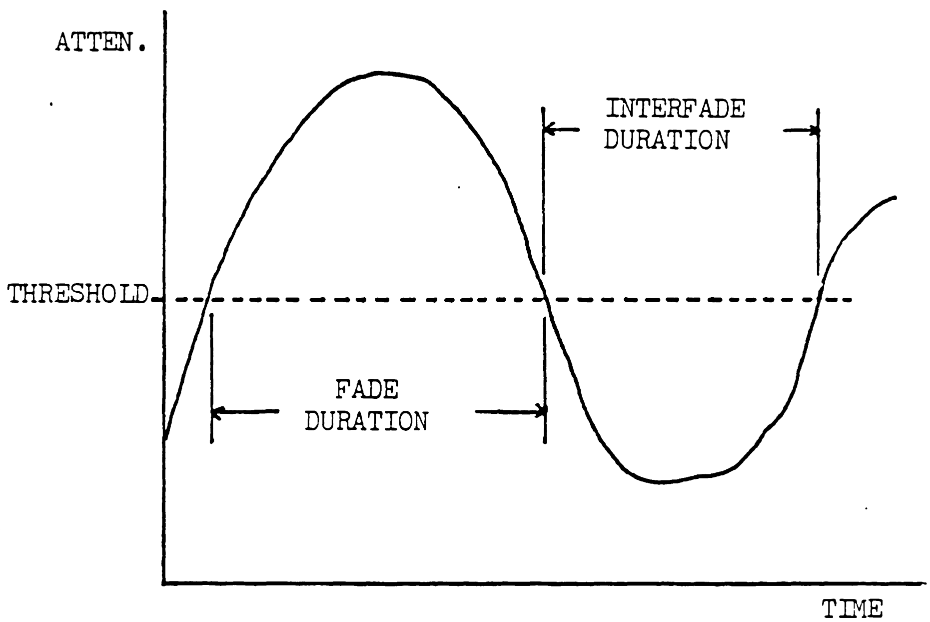

Figure 2.1 Definition of fade and interfade duration.

Table 2.1 Fade duration table for Vogel's 1978 CTS data at 11.7 GHz, 50.0° elevation angle, circular polarization [22].

Duration (min.)	Threshold (dB) 3	6	10	20	25
0 - 1	32	31	11	16	2
1 - 2	9	7	4	1	0
2 - 4	11	12	6	0	1
4 - 8	13	9	5	1	
8 - 16	11	7	2		
16 - 32	9	1			
32+	2				

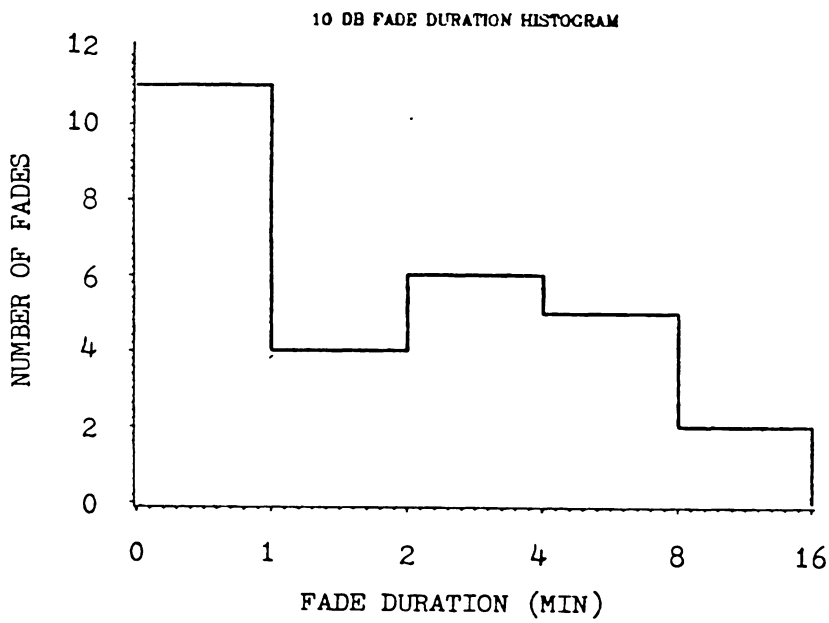

Figure 2.2 Fade duration histogram for 10 dB fades from Vogel's February 1978 - January 1979 CTS data at 11.7 GHz, 50.0° elevation angle, circular polarization from [22].

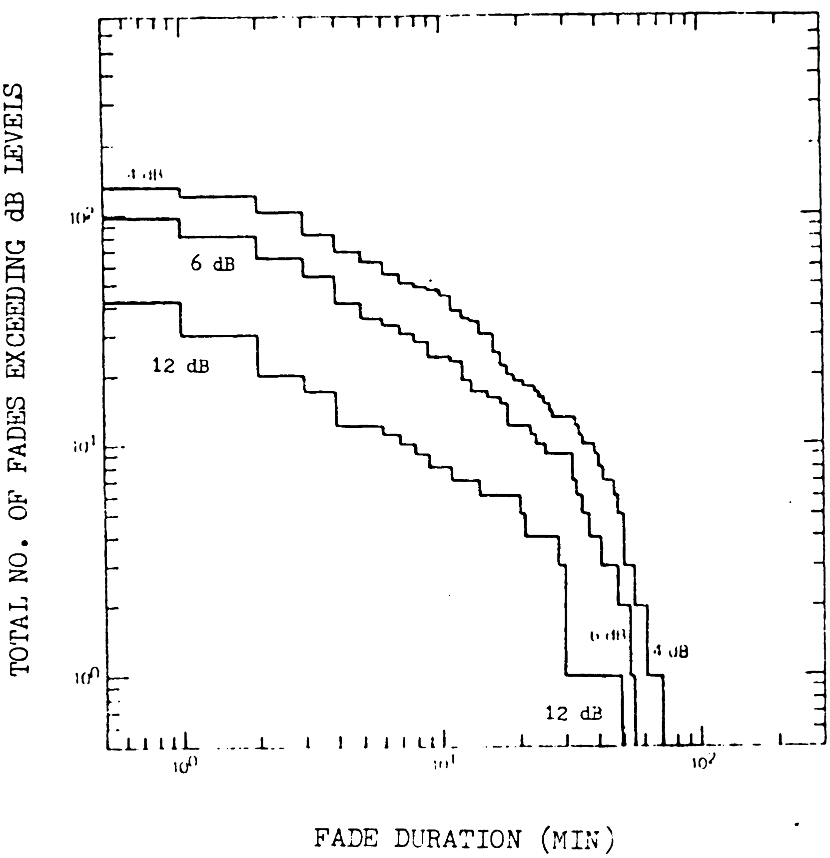

Figure 2.3 Cumulative fade duration distributions for Kumar's 7/76 - 8/77 COMSTAR 19 GHz data, 21° elevation angle, vertical polarization. Reproduced with permission from P.N. Kumar, "Precipitation fade statistics for 19/29-GHz COMSTAR beacon signals and 12-GHz radiometric measurements," <u>COMSAT Technical Review</u>, Vol. 12, No. 1, Spring 1982.

In this thesis we are interested in finding a way to predict fade duration results for a location where attenuation data have been collected. Consequently, we are interested in models which either generate time sequences of attenuation or develop analytical results which can be used to provide fade duration tables.

2.2 STATISTICAL MODELS WITH MOVING RAIN CELLS

Very little has been done to model or predict the dynamic behavior of rain attenuation. One approach that was been taken by several Japanese scientists is to do statistical modeling with moving rain cells.

In 1980, K. Morita referred to a method for calculating fade duration statistics from rain rate duration statistics using a lognormal distribution for 10 minute rain rate duration time distributions [28]. (The original paper is not available in English.) He used a statistical model to relate rain rate dynamics to attenuation dynamics. In it the lognormal distribution for rain rate duration was expressed by two parameters, the mean and standard deviation of the logarithm of duration. Both these parameters varied with rain rate threshold according to a power law relationship which was derived from rain rate data. Morita assumed that the rain cell moved 5 to 7 km in any 10 minute period so that the average rain rate over the path was almost the same as the 10 minute rain rate. The resulting fade duration distribution is also lognormal.

This approach has two shortcomings. First, it relies heavily on the assumption that fade duration distributions are lognormal. Second, using 10 minute rain rate values severely limits the model's accuracy in describing propagation effects less than 10 minutes long.

In 1982, S. Ito [18] referred to a method by K. Murakami [29] for predicting fade duration statistics. Only an abstract of Murakami's paper is available in English. Murakami assumed a lognormal distribution for fade duration distributions, and a constant velocity rain cell with a certain spatial structure and direction. He compared his predictions with measurements, but the results are plotted on a scale which severely compresses the data.

Like Morita's work, this model relied heavily on the assumption that fade duration distributions are all lognormal. We believe this is not necessarily true. Also, the model requires, as input, a rain cell velocity and direction which may be difficult to determine for a given site.

2.3 THE MASENG-BAKKEN MODEL

In 1978, T. Maseng and P. M. Bakken presented a stochastic dynamic model for rain attenuation at an AGARD Conference [30]. They published a paper on the model in 1981 [4].

To derive their model, Maseng and Bakken assumed that rain attenuation was based on a first-order Markov process. For discrete samples of attenuation, this implies that each attenuation sample is a function of the previous value and a random variable generated by a stochastic input process. Maseng and Bakken mathmatically derived their model by starting with the first-order differential equation for a Markov process. To find the necessary parameters of the model, they assumed that attenuation is lognormal and that the rate of change of attenuation increases with attenuation level. This led to a known solution of the Fokker-Planck equation which provided formulas for the model parameters.

Because Maseng and Bakken assumed that attenuation is lognormally distributed, they decided to work with a process proportional to the log of attenuation, called X. The resulting model is given by the following equations.

$$X_{(k)} = A\, X_{(k-1)} + B\, N_{(k)} \qquad (2.3\text{-}1)$$

$$A_{(k)} = \Gamma \exp\,(\sigma_L\, X_{(k)}) \qquad (2.3\text{-}2)$$

where

$A_{(k)}$ = discrete values of attenuation occuring at time t_k where $t_k - t_{k-1} = \Delta t$.

$N_{(k)}$ = white, Gaussian, zero-mean, unity variance input process.

$A = \exp\,(-\,\beta\,\Delta t)$

$B = (1 - A^2)^{(1/2)}$

Δt = sampling period in minutes.

$$\beta = E\{(A(t + \Delta t) - A(t))^2/A(t)\}/(2\,\sigma_L^{\,2}\,\Delta t) \qquad (2.3\text{-}3)$$

where E{} indicates expected value.

σ_L = standard deviation of the log of attenuation not including clear weather.

Γ = median value of attenuation during periods of rain, i.e. not including clear weather.

The process $X_{(k)}$ is a normalized version of the log of attenuation ($L_{(k)}$) that has zero mean and unity variance. This is shown in the following equations:

$$X_{(k)} = \log(A_{(k)}/\Gamma)/\sigma_L \qquad (2.3\text{-}3)$$

$$L_{(k)} = \log(A_{(k)}) \qquad (2.3\text{-}4)$$

$$X_{(k)} = (L_{(k)} - \log(\Gamma))/\sigma_L \qquad (2.3\text{-}5)$$

The $\log(\Gamma)$ in equation 2.3-5 acts as the mean of L since Maseng and Bakken assumed L is Gaussian.

Since the input process is Gaussian, Maseng and Bakken were able to derive analytical formulas for the fade duration distribution of $X_{(k)}$. They used these formulas to predict fade duration statistics for ATS-6 links at 20 and 30 GHz. However, the displayed results were normalized with respect to average fade duration. This can hide an incorrect average fade duration and distribution. Also the results were plotted with a log-probability scale. This severely compressed the apparent spread of the data.

2.4 SUMMARY

The dynamics of rain attenuation are commonly described in terms of fade and interfade durations. Results for a certain time period are specified in three forms: a fade duration table, a fade duration histogram, or a fade duration distribution. In modeling rain attenuation dynamics, we are interested in ways of predicting fade duration results for any location, frequency, elevation angle, and polarization.

So far, two approaches have been taken. The first is to assume that fade duration distributions are lognormal. Physical models including rain cell velocity are used to derive the necessary parameters from rain rate dynamics. This approach is limited since fade duration distributions are only approximately lognormal over certain duration values.

The second approach is a stochastic model, which assumes rain attenuation is a first-order Markov process. This approach uses the statistics of attenuation plus a data-derived dynamic parameter to model the dynamic behavior.

We felt that the second approach provided the greatest flexibility for modeling. It was easy to implement and could be adapted as assumptions changed.

CHAPTER 3 - IMPLEMENTATION AND EVALUATION OF THE MASENG-BAKKEN MODEL

The Maseng-Bakken model (M & B model) was implemented and used to model one of our attenuation data bases. Then, we used parameter estimation and residual analysis techniques to evaluate the model's structure, parameters, and assumptions.

3.1 MODEL IMPLEMENTATION

The task of implementing the model in software was broken down into several parts. First we had to prepare a data base of attenuation measurements to compare with the model. Second, the model inputs had to be calculated from the attenuation measurements. Then, we needed to implement the model equations to generate a time sequence of attenuation samples. Finally the measured data and model data had to be analyzed and compared with each other to determine model performance. All software was developed on a Harris 800 computer in FORTRAN 77. A software system flowchart is given in Figure 3.1.

For our data base we used 1980 and 1981 SIRIO attenuation measurements at 11.6 GHz, 10.7° elevation angle, circularly polarized. These data were collected at Blacksburg, Virginia by Virginia Tech's Satellite Communications Group under INTELSAT contract number IS-901 and JPL contract 955954. The data were either recorded on strip charts and

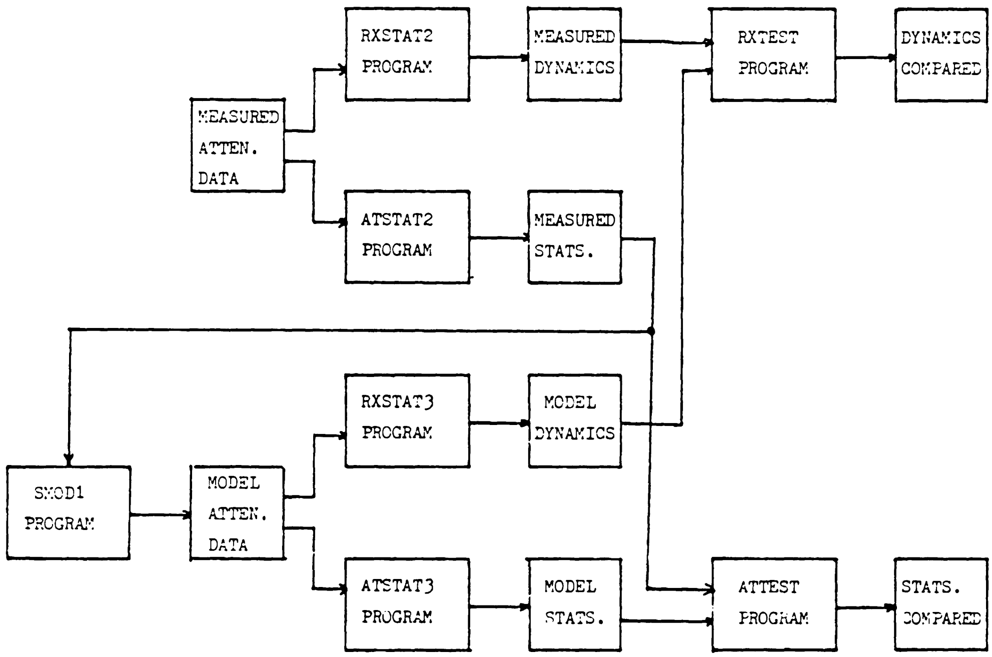

Figure 3.1 Software system diagram for model implementation.

digitized, or directly sampled by a computer. Samples were recorded every 30 seconds [31].

The M & B model requires four inputs:

1. The median value of attenuation not including clear weather: Γ.

2. The standard deviation of the natural log of attenuation during fades: σ_L.

3. A dynamic parameter given by equation 2.3-3: β.

4. The number of nonzero 30 second attenuation samples to generate for the model data set: NPTS.

We developed a program called ATSTAT2 to determine the four inputs from a data set. ATSTAT2 takes the measured attenuation data and sorts it into 0.2 dB bins. From these bins the attenuation cumulative distribution, mean, median and standard deviation are calculated. Binning is also done with L, the natural log of attenuation, so that the standard deviation of L, σ_L, can be calculated. β is calculated as an average according to equation 2.3-3. NPTS is the number of nonzero attenuation points found in the data set.

To generate model data, we developed a program called SMOD1. The program receives input parameters interactively, and generates model data using equations 2.3-1 and 2.3-2.

Dynamic analysis of measured and model data is done by the programs RXSTAT2 and RXSTAT3. These programs calculate fade and interfade duration tables, cumulative distributions, means and medians. They had been developed by Robert Porter, a member of the Satellite Communications Group at Virginia Tech. Statistical analysis of measured and model data is done by the programs ATSTAT2 and ATSTAT3.

Finally, two programs, RXTEST and ATTEST, were developed to compare the dynamic and statistical behavior of real and model data. RXTEST displays fade duration tables showing the difference between measured results and model predicted results. ATTEST displays differences in measured and model attenuation distributions, means, medians and standard deviations. Both programs also calculate root-mean-square error quantities to ease evaluation of model performance.

3.2 MODEL RESULTS

We used the M & B model on the 1980 and 1981 SIRIO data sets. The results are presented in Tables 3.1 and 3.2. In Table 3.1, for a threshold of 10 dB and a duration of 0.5 - 1.0 minutes the model predicted 23 events when only 9 such events occured in 1980.

The M & B model overpredicted short duration fades at low thresholds, and underpredicted long duration fades at high thresholds. For example, for a threshold of 5 dB and duration of 1-3 minutes the model predicted 72 events whereas only 30 occurred. We felt that the model performance could be improved significantly.

Table 3.1 Comparison of measured and model fade duration table for 1980 SIRIO data set at 11.6 GHz, 10.7° elevation angle, circular polarization.

Duration	Threshold (dB) 3		5		10		15		20	
(min.)	MR	MB	MR	MB	MR	MB	MR	MB	MR	MB
0.5	55	326	24	216	7	60	6	18	1	19
0.5 - 1.0	38	132	24	88	9	23	1	6	1	5
1.0 - 3.0	66	238	34	135	10	33	8	11	2	2
3.0 - 10.0	45	176	36	83	15	12	8	5	2	1
10.0 - 30.0	36	72	18	24	3	4	1	0		
30.0 +	18	8	4	0	1	0				

MR - Measured Results
MB - M & B Model Results

Table 3.2 Comparison of measured and model fade duration tables for 1981 SIRIO data set at 11.6 GHz, 10.7° elevation angle, circular polarization.

Duration	Threshold (dB) 3		5		10		15		20	
(min.)	MR	MB	MR	MB	MR	MB	MR	MB	MR	MB
0.5	21	176	5	118	4	25	4	8	4	1
0.5 - 1.0	11	82	9	55	6	11	3	3	2	0
1.0 - 3.0	15	125	30	72	12	20	8	6	3	1
3.0 - 10.0	38	102	26	51	20	5	7	1	3	0
10.0 - 30.0	32	52	15	20	5	2	2	0		
30.0 +	15	8	5	0	1	0				

MR - Measured Results
MB - M & B Model Results

3.3 MODEL EVALUATION

To improve the M & B model, we needed to investigate the order of the model, the autoregressive coefficient (A), and the input process. To do this, we reformulated the model in terms of the natural log of attenuation, L, and the natural log of attenuation with its mean removed, Λ. The model equations in terms of L and Λ are given below.

$$\Lambda_{(k)} = A\ \Lambda_{(k-1)} + W_{(k)} \qquad (3.3\text{-}1)$$

$$L_{(k)} = \Lambda_{(k)} + \mu_L \qquad (3.3\text{-}2)$$

$$A_{(k)} = \exp\ (L_{(k)}) \qquad (3.3\text{-}3)$$

where

$W_{(k)}$ = input process with standard deviation B2.

μ_L = mean of the natural log of attenuation, L.

Relating the new model formulation to equations 2.3-1 through 2.3-5, we find that:

$$\mu_L = \log(\Gamma) \qquad (3.3\text{-}4)$$

$$B2 = \sigma_L\ B \qquad (3.3\text{-}5)$$

$$L_{(k)} = \sigma_L X_{(k)} + \log(\Gamma) \qquad (3.3\text{-}6)$$

$$\Lambda_{(k)} = \sigma_L X_{(k)} \qquad (3.3\text{-}7)$$

Thus, according to equation 3.3-4, the M & B model assumes that the median attenuation value maps into the mean of L. Also, $X_{(k)}$ is a normalized version of $L_{(k)}$ as shown in equation 3.3-6. Consequently, the A parameter is the same in both formulations, and B is related to the standard deviation B2 of $W_{(k)}$ by a multiplicative constant.

To study the new model formulation, we considered a general model given below.

$$\Lambda_{(k)} = A(1)\ \Lambda_{(k-1)} + \ldots + A(N)\ \Lambda_{(k-N)} + W_{(k)} \qquad (3.3\text{-}8)$$

$$W_{(k)} = C(1)\ W_{(k-1)} + C(2)\ W_{(k-2)} + \ldots + C(M)\ W_{(k-M)} \qquad (3.3\text{-}9)$$

This is the general form of an autoregressive moving average model, commonly used in parameter estimation problems [32].

We chose a least squares algorithm to determine the order (N) and the autoregressive coefficients (A's) from the data. The input process was derived from the measured data and the determined A values by working backwards using equation 3.3-8. We decided to apply the same least squares algorithm to the input process to determine its whiteness by determining the C parameters in equation 3.3-9. We also evaluated the input process mean, standard deviation and distribution to test the zero mean assumption, the B parameter calculation, and the Gaussian assumption. Finally, we looked at implementation of the new model formulation with both a Gaussian input process and a data-derived input process.

To determine the autoregressive order and coefficients, we developed a program called PAREST. We implemented a recursive least squares algorithm given by Ljung [32]. The input process analysis, or residual analysis, was done in a program called RESAN.

The general model was implemented in a program called SMOD2. The program had two options to allow for a Gaussian input process and a data-derived input process. To generate the data-derived input process, a uniform random variable was used with the data-derived input process distribution [33]. A program listing of SMOD2 is given in Appendix A.

The model data was analyzed and compared to measured data using the programs discussed in Section 3.1. The entire software system diagram is given in Figure 3.2.

3.4 MODEL EVALUATION RESULTS

First of all, we examined a first-order autoregressive model. The results are summarized in Table 3.3. We found that the M & B model correctly estimated the A and B parameters. Also, we verified that the input process was zero mean. We felt that the C parameters indicated a fairly white input process.

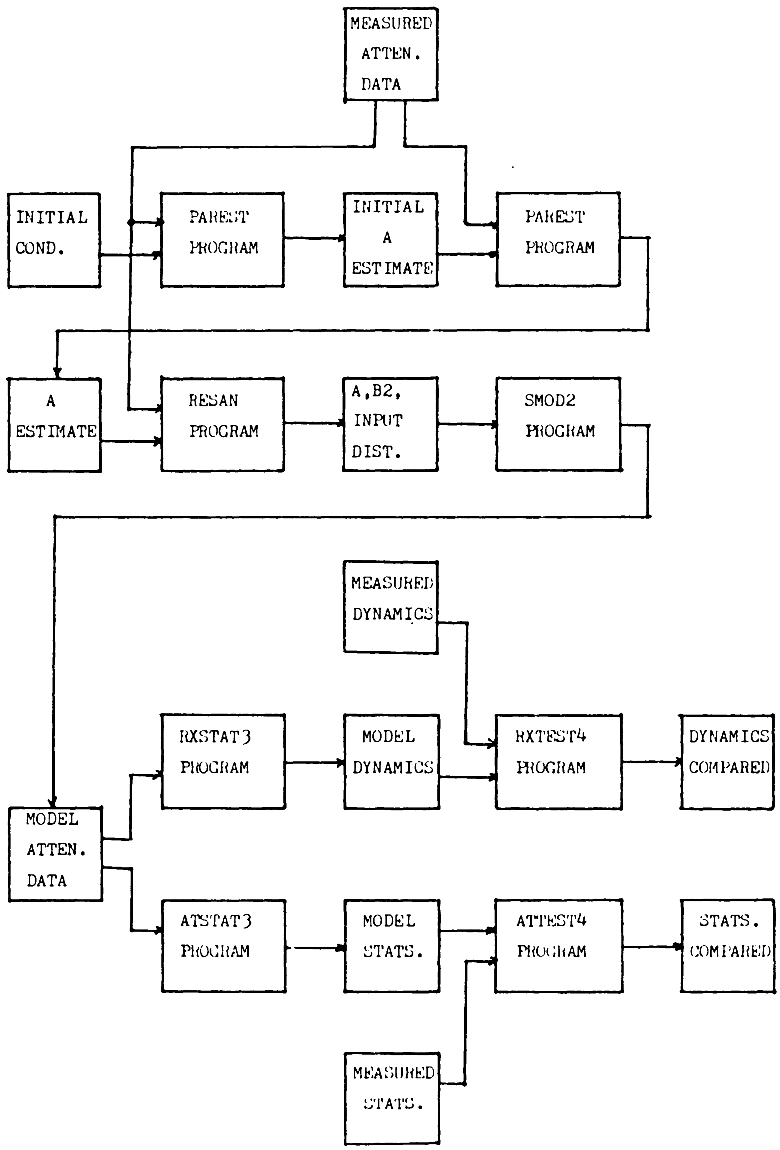

Figure 3.2 The software system for model evaluation.

Table 3.3 First-order model evaluation results.

Item	Year 1980	Year 1981
1. A from parameter estimation	.97119	.97495
A from M & B model	.97346	.97578
2. B2 from residual analysis	.24727	.21830
3. B from residual analysis	.23109	.22276
B from M & B model	.22884	.21874
4. W(k) mean from residual analysis	0.00	0.00
5. C(1) from residual analysis	-.034	-.054
C(2) from residual analysis	-.103	-.118

We implemented the first-order model with both a Gaussian and a data-derived input process. The results are given in Tables 3.4 and 3.5. For most threshold and duration bin combinations, the data-derived input model better predicted the number of events. For example, in Table 3.5 for a threshold of 5 dB and duration of 1.0 - 3.0 minutes, the Gaussian input model predicted 87 events and the data-derived input model predicted 52 events when the measured result was 30 events. The data-derived input process distribution was definitely not Gaussian. Both Gaussian and data-derived input process distributions for 1980 SIRIO are given to one decimal place in Table 3.6 and plotted in Figure 3.3. The actual data-derived input process distribution used was specified to two decimal places over the range (-1.0, 1.0).

We had difficulty developing a standard for comparing the fade duration table predictions of two models to determine which model performed better. We tried root-mean-square error indicators for each threshold and duration bin, but found them to be highly sensitive to threshold and duration bin.

Investigation of a second-order autoregressive model yielded the results in Table 3.7. We implemented both the Gaussian input model and the data-derived input model. Both showed marginal improvement over their first order counterparts, but not enough to warrant a second order model.

Third-order and sixth-order models were investigated to determine how the least squares algorithm would specify insignificant parameters. The results are tabulated in Table 3.8.

Table 3.4 Comparison of Gaussian Input Model and Data-derived Input Model with measured fade duration results for 1980 SIRIO data at 11.6 GHz, 10.7° elevation angle, circular polarization.

	Threshold (dB)														
Duration	3			5			10			15			20		
(min.)	MR	GI	DI	MR	GI	DI	MR	GI	DI	MR	GI	DI	MR	GI	DI
0.5	55	319	120	24	165	64	7	50	42	6	13	21	1	4	11
0.5-1.0	38	140	69	24	81	39	9	9	20	1	6	11	1	0	3
1.0-3.0	66	216	156	34	114	106	10	23	43	8	6	17	2	2	9
3.0-10.0	45	152	170	36	74	115	15	7	28	8	2	11	2	0	4
10.0-30.0	36	55	89	18	10	25	3	2	1	1	0	0			
30.0 +	18	4	3	4	0	0	1	0	0						

MR - Measured Results
GI - Gaussian Input Model Results
DI - Data-Derived Input Model Results

Table 3.5 Comparison of Gaussian Input Model and Data-derived Input Model with measured fade duration results for our 1981 SIRIO data at 11.6 GHz, 10.7° elevation angle, circular polarization.

Duration	Threshold (dB)														
	3			5			10			15			20		
(min.)	MR	GI	DI	MR	GI	DI	MR	GI	DI	MR	GI	DI	MR	GI	DI
0.5	21	171	44	5	120	34	4	40	10	4	16	6	4	8	2
0.5-1.0	11	83	32	9	52	21	6	22	9	3	0	3	2	1	2
1.0-3.0	15	131	83	30	87	52	12	19	25	8	8	6	3	1	4
3.0-10.0	38	103	103	26	64	66	20	10	9	7	2	5	3	0	1
10.0-30.0	32	57	58	15	20	15	5	2	1	2	0	0			
30.0 +	15	9	6	5	0	1	1	0	0						

MR - Measured Results
GI - Gaussian Input Model Results
DI - Data-Derived Input Model Results

Table 3.6 Data-derived input process cumulative distribution and Gaussian distribution for SIRIO 1980 data set.

x	Data F(x)	Gaussian F(x)	x	Data F(x)	Gaussian F(x)
-3.0	100.000	100.00	0.0	50.801	50.00
-2.9	99.996	100.00	0.1	15.078	34.30
-2.8	99.996	100.00	0.2	8.011	20.93
-2.7	99.996	100.00	0.3	4.529	11.25
-2.6	99.994	100.00	0.4	2.755	5.28
-2.5	99.992	100.00	0.5	2.235	2.16
-2.4	99.992	100.00	0.6	1.996	0.76
-2.3	99.881	100.00	0.7	0.903	0.23
-2.2	99.881	100.00	0.8	0.839	0.06
-2.1	99.880	100.00	0.9	0.746	0.01
-2.0	99.872	100.00	1.0	0.662	0.00
-1.9	99.830	100.00	1.1	0.398	0.00
-1.8	99.828	100.00	1.2	0.161	0.00
-1.7	99.809	100.00	1.3	0.312	0.00
-1.6	99.792	100.00	1.4	0.272	0.00
-1.5	99.792	100.00	1.5	0.216	0.00
-1.4	99.690	100.00	1.6	0.187	0.00
-1.3	99.677	100.00	1.7	0.185	0.00
-1.2	99.621	100.00	1.8	0.161	0.00
-1.1	99.434	100.00	1.9	0.126	0.00
-1.0	99.390	100.00	2.0	0.124	0.00
-0.9	99.275	99.99	2.1	0.122	0.00
-0.8	99.224	99.94	2.2	0.119	0.00
-0.7	98.293	99.77	2.3	0.117	0.00
-0.6	98.163	99.24	2.4	0.006	0.00
-0.5	97.828	97.84	2.5	0.006	0.00
-0.4	96.543	94.72	2.6	0.006	0.00
-0.3	95.113	88.75	2.7	0.006	0.00
-0.2	93.050	79.07	2.8	0.006	0.00
-0.1	88.348	65.70	2.9	0.002	0.00
			3.0	0.000	0.00

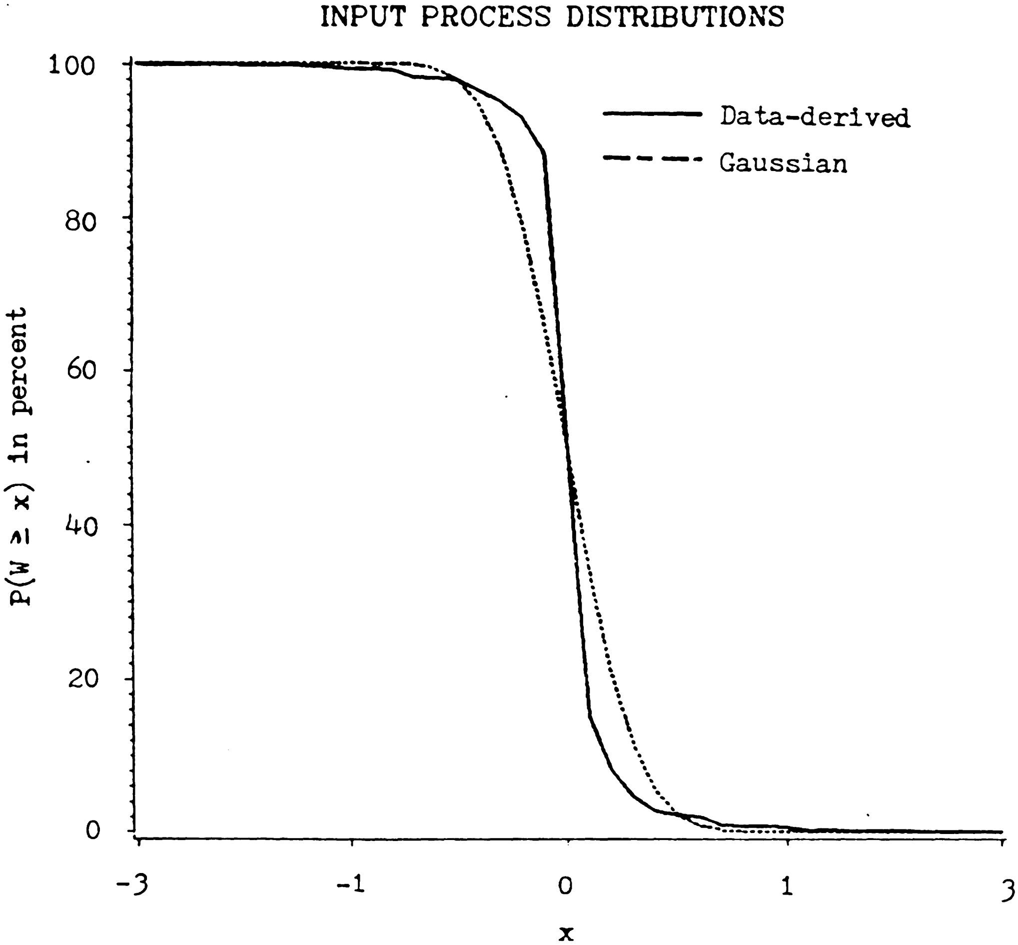

Figure 3.3 Plot of Data-derived input process cumulative distribution (solid line) and Gaussian distribution (dashed line) for SIRIO 1980 data set. Both have zero mean and same standard deviation.

Table 3.7 Second-order model evaluation results.

Item	Year 1980	Year 1981
1. A(1) from parameter est.	.93792	.91359
A(2) from parameter est.	.03846	.06944
2. B2 from residual analysis	.24168	.20748
3. W(K) mean from res. analysis	0.00	0.00
4. C(1) mean from res. analysis	-.00037	.0211
C(2) mean from res. analysis	-.11007	-.1224

Table 3.8 Third and sixth-order model evaluation results.

Item	Year 1981
Third-order Model	
1. A(1)	.91775
A(2)	-.06289
A(3)	.13077
2. B2	.199244
3. W(K) mean	0.00
4. C(1)	.01653
C(2)	-.00761
Sixth-order Model	
1. A(1)	.93046
A(2)	-.07137
A(3)	.08350
A(4)	-.02087
A(5)	.05063
A(6)	.01535
2. B2	.18989
3. W(K) mean	0.00
4. C(1)	-.00031
C(2)	.01264

3.5 CONCLUSION

The M & B model accurately estimates the first order model parameters A and B. A second order model appears to be unnecessary. The data-derived input process provides significant improvement over the Gaussian input process used in the M & B model in predicting fade duration statistics. In Chapter 4 we adapt the improved model for global use.

CHAPTER 4 - OPTIMIZED FIXED PARAMETER MODEL

In Chapter 3 we found that the Maseng-Bakken model worked better when driven by the data derived input process. Consequently, the improved M & B model is run with the same inputs as listed in Section 3.1. The data derived input process is scaled to have the desired standard deviation.

Unfortunately, not all of these inputs are easily available, even for sites with existing earth stations. In fact, the dynamic parameter β is not available for sites where no attenuation data currently exist. This severely limits global application of the improved model. In this chapter we investigate what simplifications and techniques can be employed to adapt the model for global use.

4.1 UNIVERSAL ATTENUATION DATA SET

One way to simplify the improved model is to assume β, given by equation 2.3-3, is the same for all sites. This is equivalent to fixing the A and B parameters of equation 2.3-1, but not the standard deviation of the input process B2. The term site refers to frequency, elevation angle and polarization as well as geographical location. If β is fixed, only the mean μ_L, standard deviation σ_L and NPTS vary from site to site. We called this the Fixed Parameter Model.

Since statistically only the mean and standard deviation of L change, any two model data sets can be related both statistically and on a point to point basis by

$$L2 = (\sigma_{L2}/\sigma_{L1})\ (L1 - \mu_{L1}) + \mu_{L2} \qquad (4.1\text{-}1)$$

The numbers 1 and 2 refer to the two different model data sets. The statistical relationship means that any pair L2 and L1 having a common cumulative distribution value also satisfy equation 4.1-1. In terms of attenuation, equation 4.1-1 implies:

$$A2 = K(A1)^C \qquad (4.1\text{-}2)$$

where

$$K = \exp(\mu_{L2} - (\sigma_{L2}/\sigma_{L1})\ \mu_{L1}) \qquad (4.1\text{-}3)$$

$$C = \sigma_{L2}/\sigma_{L1} \qquad (4.1\text{-}4)$$

Thus, we can relate any two model data sets with two scaling factors, assuming we used the same random number sequence in both. This implies that we need only generate model data once. Instead of rerunning the model for a new site with new μ_L and σ_L, we can scale an existing model data set with K and C given by equations 4.1-3 and 4.1-4. The NPTS input determines how many points of the universal data set to use, as discussed in Section 3.1. The NPTS can be estimated from the percent of time it rains during a year.

4.2 ESTIMATING K AND C FROM PARTIAL ATTENUATION DISTRIBUTIONS

In Section 4.1 we developed a fixed parameter model which could be applied globally. However, to determine the inputs μ_L and σ_L we need a complete cumulative attenuation distribution for periods of rain, i.e. not including clear weather. Common global statistical models for rain attenuation only provide several distribution values. Consequently, we are faced with the problem of estimating μ_L and σ_L from a partial attenuation distribution.

Our first step in solving this problem was to realize that we did not need to estimate μ_L and σ_L if we could estimate K and C directly. We developed a simple technique for estimating K and C using the statistical relationship given in equation 4.1-2. We assume all cumulative distributions exclude clear weather and define the following terms:

Au	universal attenuation data set previously generated by model.
Am	model attenuation for site of interest.
A	measured attenuation for site of interest.

We define Lu, Lm and L to be the natural log of the above quantities. In Section 4.1 we used the measured data's μ_L and σ_L as inputs for the model. This was equivalent to aligning the model's L process mean and standard deviation with the measured L process mean and standard deviation. We can take this a step further and align the model's L process cumulative distribution with the measured L process cumulative distribution. This is equivalent to aligning the attenuation distributions of the measured and model data sets A and Am. Thus, for several attenuation levels ym_i, we can write:

$$P(Am \geq ym_i) = P(A \geq ym_i) \qquad (4.2\text{-}1)$$

where we are given $P(A \geq ym_i)$ for several values of ym_i. For example, suppose we are given that attenuation exceeds 5 dB for 3 % of the time, 7 dB for 1% of the time, and 10 dB for 0.5 % of the time, all excluding clear weather. This means that we have:

i	ym_i	$P(Am \geq ym_i)$
1	5 dB	3 %
2	7 dB	1 %
3	10 dB	0.5 %

From equation 4.1-2 we know that:

$$Am = K\ Au^C \qquad (4.2\text{-}2)$$

this implies that:

$$P(Am \geq ym) = P(K\ Au^C \geq ym) = P(Au \geq (ym/K)^{(1/C)}) \qquad (4.2\text{-}3)$$

In terms of the L process:

$$P(Lm \geq xm)=P(C\ Lu+\log(K) \geq xm)=P(Lu \geq (xm-\log(K))/C) \qquad (4.2\text{-}4)$$

where

$$xm = \log(ym)$$

We know $P(Lu \geq xu)$ at all possible xu values since we have the universal data set.

Thus to estimate K and C, we start with a partial cumulative distribution of attenuation, A, specified as:

$$(y_i, p_i) \quad i = 1, S \qquad \text{where} \quad p_i = P(A \geq y_i) \qquad (4.2\text{-}5)$$

From equation 4.1-5 and the facts that Lm is the natural log of Am and xm_i is the natural log of ym_i we have:

$$(xm_i, p_i) \quad \text{where} \quad p_i = P(Lm \geq xm_i) \qquad (4.2\text{-}6)$$

For each p_i, we can determine xu_i such that:

$$P(Lu \geq xu_i) = p_i \qquad (4.2\text{-}7)$$

With equation 4.2-4 we can relate xm_i and xu_i by:

$$xu_i = (xm_i - \log(K))/C$$

or

$$xm_i = C\ xu_i + \log(K) \qquad i = 1,\ S \qquad (4.2\text{-}8)$$

This set of linear equations can be solved using a least squares fit to estimate K and C. Sample calculations are given in Section 4.5.

We did not determine how many equations should be used to obtain the optimal estimates for K and C. We found that as we used more equations, our estimates for K and C improved.

4.3 IMPLEMENTATION OF FIXED PARAMETER MODEL

To implement this model, we developed software to estimate K and C and to scale and analyze model data. Estimating K and C was done in a program called PRERUN. The program took measured cumulative attenuation distribution values and scaled them to exclude clear weather. The universal data cumulative distribution was also used by the program. The xm_i were calculated directly from the y_i given, and the corresponding xu_i were found by taking the inverse universal distribution function of the p_i. The least squares fit was performed with a Gaussian elimination subroutine solving $A^T Ax = A^T b$ [34]. A program listing is given in Appendix B.

To scale and analyze model data, programs called RXSTAT5 and ATSTAT5 were used. This allowed us to minimize storage requirements by not storing scaled data. A software system diagram is given in Figure 4.1.

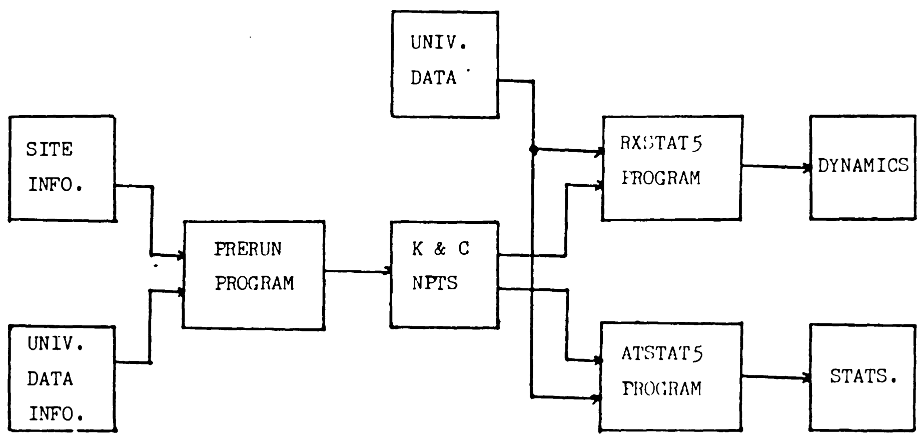

Figure 4.1 Software diagram of Fixed Parameter Model implementation.

4.4 FIXED PARAMETER MODEL RESULTS AND DISCUSSION

We implemented the fixed parameter model on our 1980 SIRIO data set and Vogel's 1978 CTS data set [22]. We had to assume a probability of rain of 4 % for Vogel's data. The results are presented in Tables 4.1 and 4.2.

The fixed parameter model produced slightly better results than the improved M & B model. We believe this was due to the way K and C were estimated. In the improved M & B model, μ_L and σ_L are determined from an entire distribution where every 30 second sample point has equal weight. Consequently, the model is greatly influenced by the range of attenuation where most attenuation sample points occur, the 0-1 dB range. This is not good from a modeling perspective since the higher levels of attenuation are of more interest. The calculation of K and C from a partial distribution using a least squares fit overcomes this problem by giving each attenuation level equal weight.

Table 4.1 Comparison of measured and Fixed Parameter Model fade duration tables for 1981 SIRIO data at 11.6 GHz, 10.7° elevation angle, circular polarization.

Duration	Threshold (dB) 3		5		10		15		20	
(min.)	MR	FP	MR	FP	MR	FP	MR	FP	MR	FP
0.5	21	48	5	25	4	16	4	6	4	2
0.5 - 1.0	11	36	9	25	6	6	3	2	2	2
1.0 - 3.0	15	62	30	55	12	29	8	4	3	4
3.0 - 10.0	38	117	26	68	20	8	7	5	3	1
10.0 - 30.0	32	56	15	22	5	3	2	0		
30.0 +	15	17	5	1	1	0				

MR - Measured Results
FP - Fixed Parameter Results

Table 4.2 Comparison of measured and Fixed Parameter Model fade duration tables for Vogel's 1978 CTS data [22] at 11.7 GHz, 50.0° elevation angle, circular polarization.

Duration	Threshold (dB) 3		6		10		20		25	
(min.)	MR	FP	MR	FP	MR	FP	MR	FP	MR	FP
0-1	32	71	31	50	11	24	16	6	2	4
1-2	9	45	7	20	4	9	1	1	0	2
2-4	11	49	12	14	6	9	0	4	1	
4-8	13	36	9	14	5	6	1			
8-16	11	19	7	4	2					
16-32	9	1	1							
32+	2									

MR - Measured Results
FP - Fixed Parameter Results

4.5 ADVANCED COMMUNICATIONS TECHNOLOGY SATELLITE EXAMPLE

The Advanced Communications Technology Satellite (ACTS) will be capable of controlling the transmit power as well as implementing forward error correction coding (FEC) to overcome the effects of rain attenuation. However, power can only be boosted for limited amounts of time based on battery capabilities, and forward error correction creates a backlog of traffic. Consequently, in determining system availability, one must examine fade duration statistics as well as cumulative attenuation distributions and traffic specifications. In this section we present an example of how the Fixed Parameter Model can be used to predict downlink fade duration statistics for an ACTS earth station [35].

The earth station, assumed to be in Blacksburg, Virginia, receives a 20 GHz linearly polarized signal at an elevation angle of 33°. We assume an arbitrary tilt angle of 45°. First, we use the CCIR method for predicting rain attenuation to obtain a cumulative attenuation distribution for the following percentages of time: 0.1, 0.03, 0.01 and 0.005 [9, 36, 37]. The intermediate results are presented below.

1. h_R = 3.91246 km
2. L_S = 6.08285 km
3. L_G = 5.10151 km
4. r_p = 0.81517 km
5. R_p = 42 mm/hr for p = 0.01%

6. $k_H = 0.0751$ $\alpha_H = 1.099$

$k_V = 0.0691$ $\alpha_V = 1.065$

$k = 0.0721$ $\alpha = 1.0827$

7. $A_{0.01\%} = 20.45$ dB

8. $A_{0.1\%} = 7.96$ dB

$A_{0.03\%} = 13.04$ dB

$A_{0.005\%} = 25.71$ dB

The cumulative attenuation distribution can be summarized as:

x	P(A≥x)
7.96 dB	0.1 %
13.04 dB	0.03 %
20.45 dB	0.01 %
25.71 dB	0.005 %

To obtain the cumulative attenuation distribution excluding clear weather (equation 4.2-5), we use the following:

$$P(A \geq x) = P(A \geq x \mid A>0)\ P(A>0)$$

or

$$P(A \geq x \mid A>0) = P(A \geq x)/P(A>0) \qquad (4.5\text{-}1)$$

where

$P(A \geq x)$ = cumulative attenuation distribution.

$P(A \geq x \mid A>0)$ = cumulative attenuation distribution excluding clear weather.

$P(A>0)$ = probability of nonzero attenuation which is approximated by the probability of rain.

Using equation 4.5-1 and assuming a probability of rain of 4% we obtain the cumulative attenuation distribution excluding clear weather given below.

i	y_i	p_i
1	7.96 dB	2.5 %
2	13.04 dB	0.75 %
3	20.45 dB	0.25 %
4	25.71 dB	0.005 %

This is also the desired model cumulative attenuation distribution (equation 4.2-7). The desired model cumulative L process distribution is found by taking the natural log of the y_i to obtain the xm_i.

i	xm_i	p_i
1	2.0744	2.5 %
2	2.5680	0.75 %
3	3.0180	0.25 %
4	3.2470	0.005 %

The next step is to determine the xu_i that correspond to the p_i given above (equation 4.2-7). To do this, we need the cumulative attenuation and L process distribution for the universal data set. The necessary parts of those distributions are given below.

yu	xu	P(Lu≥xu | Au>0)
...		
7.0 dB	1.9459	2.59413 %
7.2 dB	1.9741	2.44917 %
...		
11.8 dB	2.4681	0.76242 %
12.0 dB	2.4849	0.72854 %
...		
17.4 dB	2.8565	0.25040 %
17.6 dB	2.8679	0.24473 %
...		
21.2 dB	3.0540	0.12801 %
21.4 dB	3.0634	0.12425 %

We can interpolate to obtain the following:

i	p_i	xu_i
1	2.5 %	1.9642
2	0.75 %	2.4743
3	0.25 %	2.8573
4	0.125 %	3.0615

From equation 4.2-8 we can write the following equations:

$$2.0744 = 1.9642\ C + \log(K)$$
$$2.5680 = 2.4743\ C + \log(K)$$
$$3.0180 = 2.8573\ C + \log(K)$$
$$3.2470 = 3.0615\ C + \log(K)$$

In matrix form, $Ax = b$, we have:

$$\begin{bmatrix} 1 & 1.9642 \\ 1 & 2.4743 \\ 1 & 2.8573 \\ 1 & 3.0615 \end{bmatrix} \begin{bmatrix} \log(K) \\ C \end{bmatrix} = \begin{bmatrix} 2.0744 \\ 2.5680 \\ 3.0180 \\ 3.2470 \end{bmatrix}$$

To obtain a least-squares solution for K and C, we solve the equation $A^TAx = A^Tb$:

$$\begin{bmatrix} 4.0000 & 10.3573 \\ 10.3573 & 27.5172 \end{bmatrix} \begin{bmatrix} \log(K) \\ C \end{bmatrix} = \begin{bmatrix} 10.9074 \\ 28.9926 \end{bmatrix}$$

The solution is:

$K = 0.9498$

$C = 1.0730$

These constants were used to scale our universal attenuation data set to obtain the fade duration table given in Table 4.3.

The ACTS satellite is capable of boosting its transmitting power as much as 8 dB to overcome rain attenuation. Thus it could overcome the 5 dB fades predicted in Table 4.3. However, some of the 5 dB fades also exceed 8 dB. The satellite can overcome only part of these fades. The

Table 4.3 Fade duration table using Fixed Parameter Model for ACTS example at 20 GHz, 33° elevation angle, linear polarization.

Duration	Threshold (dB)		
(min.)	5	10	15
0-1	92	57	28
1-2	55	23	13
2-4	62	29	11
4-8	55	19	8
8-16	34	7	2
16-30	9		
30+			

duration of the fade is another constraint. Suppose the satellite can draw excess battery power for no more than 16 minutes. In that case, the satellite could only overcome part of the nine predicted 5 dB fades which last over 16 minutes.

CHAPTER 5 - SCALED ATTENUATION METHOD

In Chapter 4 we derived a way to relate any two model data sets both statistically and instantaneously. In this chapter we use this result to relate any two measured attenuation data sets. This allows us to use one measured data set to predict the fade duration table of another.

5.1 DERIVATION FROM MODEL

The Fixed Parameter Model structure implied that any two model data sets can be related by:

$$A2 = K(A1)^{C} \qquad (4.1\text{-}2)$$

If the model is accurate in characterizing the actual process, then we can assume equation 4.1-2 holds for any two measured attenuation data sets, both statistically and instantaneously.

This assumption has some background in the literature. In 1975, Ippolito related attenuation on an ATS-6 link at 20 and 30 GHz by [17]

$$A_{30}/A_{20} = 1.8755\ A_{20}^{-0.0306}$$

where attenuation is in dB. This can be rewritten in the form of equation (4.1-2). As recently as 1985 Matricciani and Paraboni [38] related attenuation at two different frequencies with the same power law form as

Ippolito. They found that the long term scaling ratio relating the two attenuation levels with the same probability value was a very good estimate of the instantaneous ratio.

5.2 DERIVATION FROM SIMPLIFIED STATISTICAL MODEL

The statistical relationship given by equation (4.1-2) can be alternatively derived from a simplified statistical model relating rain rate probabilities to attenuation probability levels. The main purpose of this derivation is to show that the relationship given by equation (4.1-2) is reasonable.

We start with the Lin model [39] in which attenuation A for rain rate R is given by:

$$A(R) = \alpha(R)\ L\ (1 + 1/LBAR(R))^{(-1)} \quad dB \qquad (5.2\text{-}2)$$

$$\alpha(R) = a\ R^{b} \quad dB/km \qquad (5.2\text{-}1)$$

where

$\alpha(R)$ is the specific attenuation.

$$L = (H - Hg)/\sin\Theta \quad km \qquad (5.2\text{-}4)$$

$$LBAR(R) \simeq 2636/(R - 6.2) \quad km \qquad (5.2\text{-}3)$$

and

a,b = coefficients which vary with frequency, polarization,and elevation angle.

L = effective path length.

LBAR = nonlinear factor to account for non-uniform rain rate on path.

Θ = elevation angle.

H = average height of the freezing layer.

Hg = ground elevation above sea level.

For the purpose of this derivation, we must simplify the model and assume that the auxiliary nonlinear factor is not significant. This is equivalent to assuming:

$$(1 + 1/LBAR(R)) \approx 1.0 \qquad (5.2\text{-}5)$$

This provides a simpler model which can be rewritten as:

$$\alpha = a\ R^{b} \quad \text{dB/km} \qquad (5.2\text{-}6)$$

$$A = \alpha\ h\ \csc\Theta \quad \text{dB} \qquad (5.2\text{-}7)$$

where

$$h = H - Hg \quad km$$

With this simple statistical model we first derive a relationship for two attenuation data sets with the same rain rate distribution. We use numbers to denote the two data sets or sites.

First of all, we combine equations 5.2-6 and 5.2-7 to obtain:

$$A = a\ h\ \csc\Theta\ R^{b} \qquad (5.2\text{-}8)$$

We can write this equation for each site giving:

$$A1 = a1\ h1\ \csc\Theta1\ R^{b1} \qquad (5.2\text{-}9)$$

$$A2 = a2\ h2\ \csc\Theta2\ R^{b2} \qquad (5.2\text{-}10)$$

We do not need a subscript on R since both sites have the same rain rate distribution. Equations 5.2-9 and 5.2-10 can both be solved for R and equated to give:

$$(A2/(a2\ h2\ \csc\Theta2))^{(1/b2)} = (A1/(a1\ h1\ \csc\Theta1))^{(1/b1)}$$

This equation can be simplified to:

$$A2 = \frac{[a2\ h2\ csc\Theta2\ ]}{(a1\ h1\ csc\Theta1)^{(b2/b1)}}\ A1^{(b2/b1)} \qquad (5.2\text{-}11)$$

This equation is of the form:

$$A2 = K\ A1^{C}$$

where

$$K = \frac{[a2\ h2\ csc\Theta2\ ]}{(a1\ h1\ csc\Theta1)^{(b2/b1)}} \qquad (5.2\text{-}12)$$

$$C = b2/b1 \qquad (5.2\text{-}13)$$

Equation 5.2-8 was previously derived in Japan to relate attenuation data at two different frequencies and elevation angles [23]. This method worked quite well.

To relate two sites with different rain rate distributions we assume that the rain rate distributions are approximately lognormal. This assumption is common in the literature [23,28,40,41] and allows us to relate any two rain rate distributions statistically by:

$$R2 = p1\ R1^{q1} \qquad (5.2\text{-}14)$$

Rewriting equations 5.2-9 and 5.2-10 we obtain:

$$A1 = a1\ h1\ \csc\Theta 1\ R1^{b1} \qquad (5.2\text{-}15)$$

$$A2 = a2\ h2\ \csc\Theta 2\ R2^{b2} \qquad (5.2\text{-}16)$$

Solving for R we obtain:

$$R1 = (A1/(a1\ h1\ \csc\Theta 1))^{(1/b1)} \qquad (5.2\text{-}17)$$

$$R2 = (A2/(a2\ h2\ \csc\Theta 2))^{(1/b2)} \qquad (5.2\text{-}18)$$

Substituting in equations 5.2-14 and 5.2-15 into 5.2-11:

$$(A2/(a2\ h2\ \csc\Theta 2))^{(1/b2)} = p1\ [(A1/(a1\ h1\ \csc\Theta 1))^{(1/b1)}]^{q1}$$

which can be simplified to:

$$A2 = \frac{[a2\ h2\ \csc\Theta 2\qquad]}{(a1\ h1\ \csc\Theta 1)^{(q1\ b2/b1)}}\ p1^{b2}\ \ A1^{(q1\ b2/b1)} \qquad (5.2\text{-}19)$$

Again we have developed an equation of the form:

$$A2 = K\ A1^{C}$$

where

$$K = \frac{[a2\ h2\ \csc\Theta 2\qquad\qquad]}{(a1\ h1\ \csc\Theta 1)^{(q1\ b2/b1)}}\ p1^{b2} \qquad (5.2\text{-}20)$$

$$C = q1\ b2/b1 \qquad (5.2\text{-}21)$$

These derivations show that a power law relationship as simple as equation 4.1-2 can adequately take into account such factors as geographical region, frequency, elevation angle and polarization.

5.3 IMPLEMENTATION OF THE SCALED ATTENUATION METHOD

Implementation of this method required no new software development. The Fixed Parameter Model software was used with our SIRIO 1980 data set as the universal attenuation data set.

5.4 RESULTS AND CONCLUSIONS

We implemented the Scaled Attenuation Method to predict fade duration statistics for data from Vogel at the University of Texas [22], Cox and Arnold at Bell Labs [10], and Kumar at COMSAT Labs [19]. The measured and predicted results are given in Table 5.1 and Figures 5.1 - 5.5. We must point out that the one predicted 3 dB fade lasting over 32 minutes in Table 5.1 was actually predicted to last over 30 minutes.

Table 5.1 Comparison of experimental and Scaled Attenuation Method predicted results for Vogel's 1978 CTS data at 11.7 GHz, 50.0° elevation angle, circular polarization [22].

Duration (min.)	Threshold (dB) 3		6		10		20		25	
	MR	SA	MR	SA	MR	SA	MR	SA	MR	SA
0-1	32	27	31	13	11	8	16	1	2	1
1-2	9	9	7	9	4	3	1	1	0	1
2-4	11	13	12	2	6	5	0	2	1	
4-8	13	18	9	8	5	3	1			
8-16	11	9	7	7	2	2				
16-32	9	8	1							
32+	2	1								

MR - Measured Results
SA - Scaled Attenuation Results

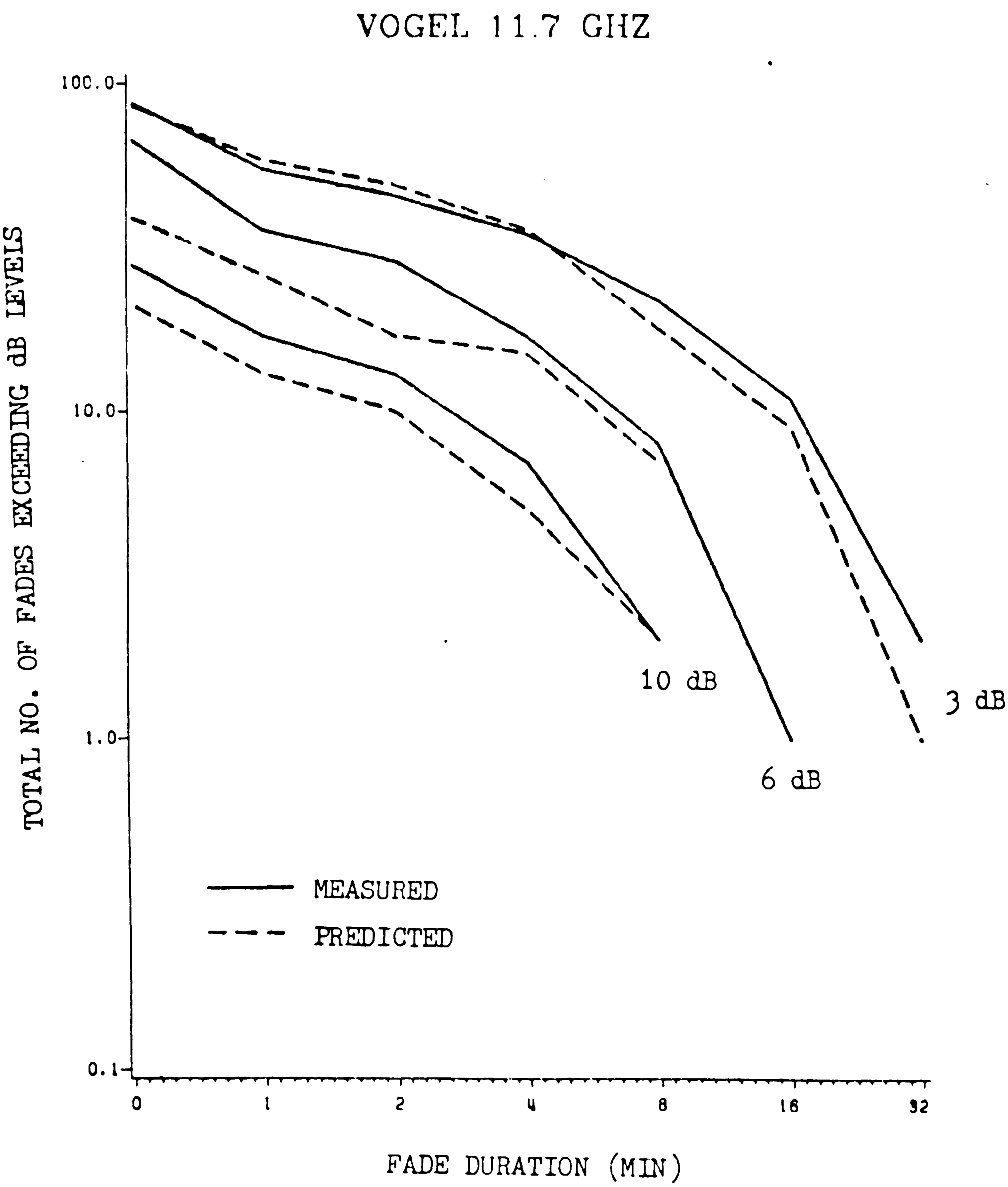

Figure 5.1 Comparison of experimental and Scaled Attenuation Method predicted fade duration distributions for Vogel's 1978 CTS data at 11.7 GHz, 50.0° elevation angle, circular polarization [22].

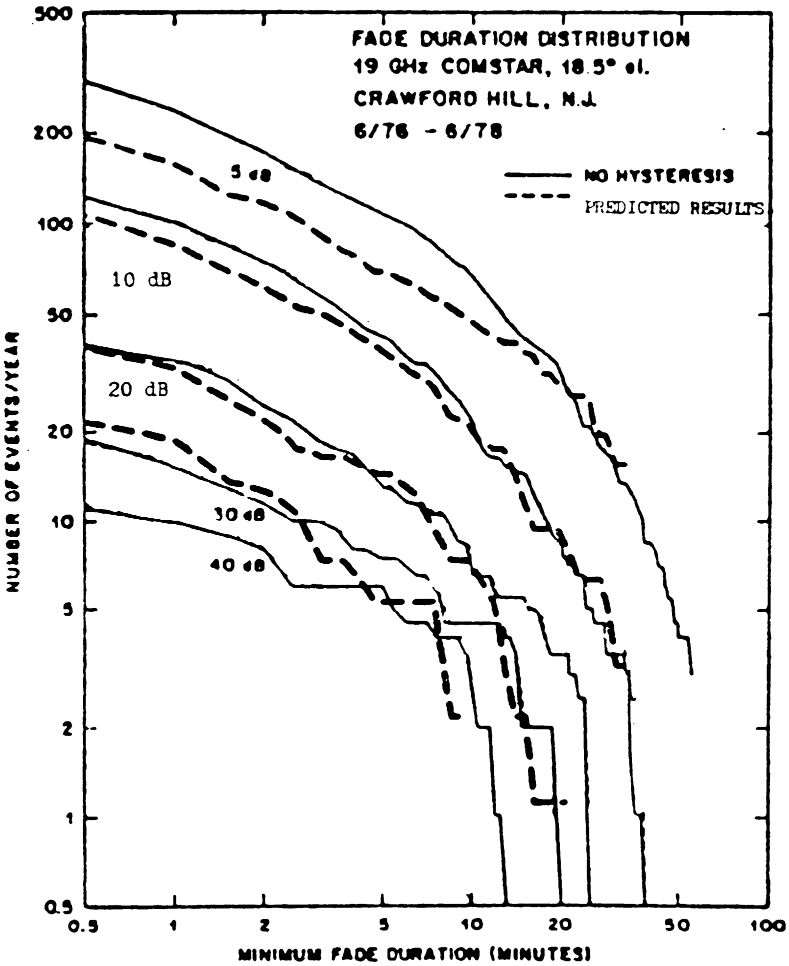

Figure 5.2 Comparison of experimental and Scaled Attenuation Method predicted results for thresholds of 5, 10, 20 and 30 dB for Cox and Arnold's data at 19 GHz for 6/76 - 6/78 normalized to one year. The measured results reproduced with permission of D.C. Cox, "Results from the 19- and 28- GHz COMSTAR Satellite Propagation Experiments at Crawford Hill," Proceedings of the IEEE, Vol 70, No. 5, May 1982. (© 1982 IEEE)

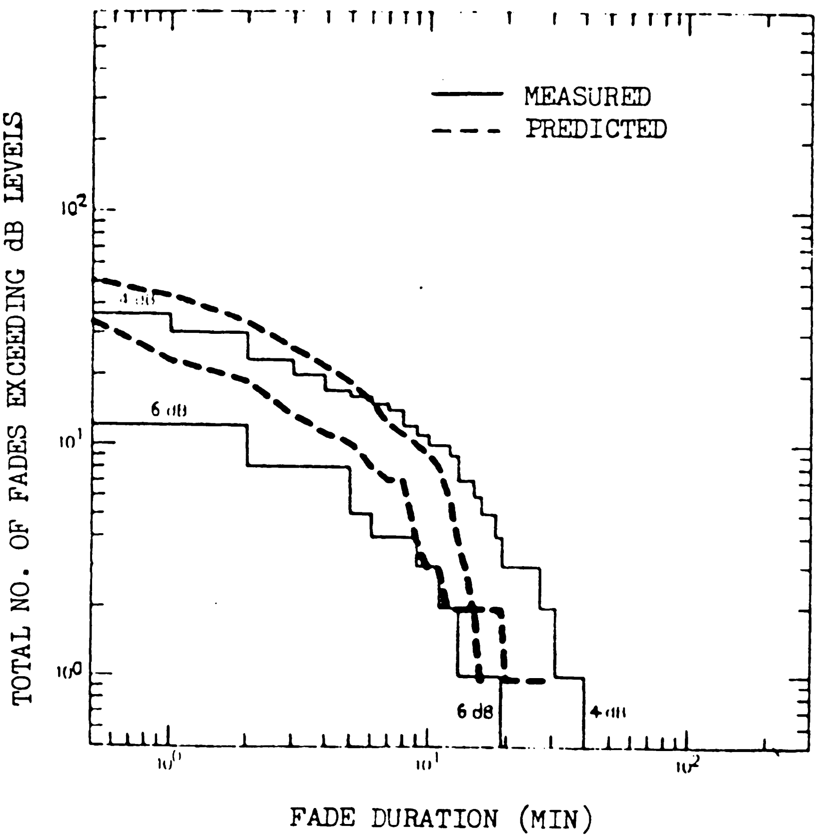

Figure 5.3 Comparison of experimental and Scaled Attenuation Method predicted results for Kumar's 12 GHz radiometer data for 7/76 - 8/77.
Reproduced with permission from P.N. Kumar, "Precipitation Fade Statistics for 19/29-GHz COMSTAR beacon signals and 12-GHz radiometric measurements," COMSAT Technical Review, Vol. 12, No. 1, Spring 1982.

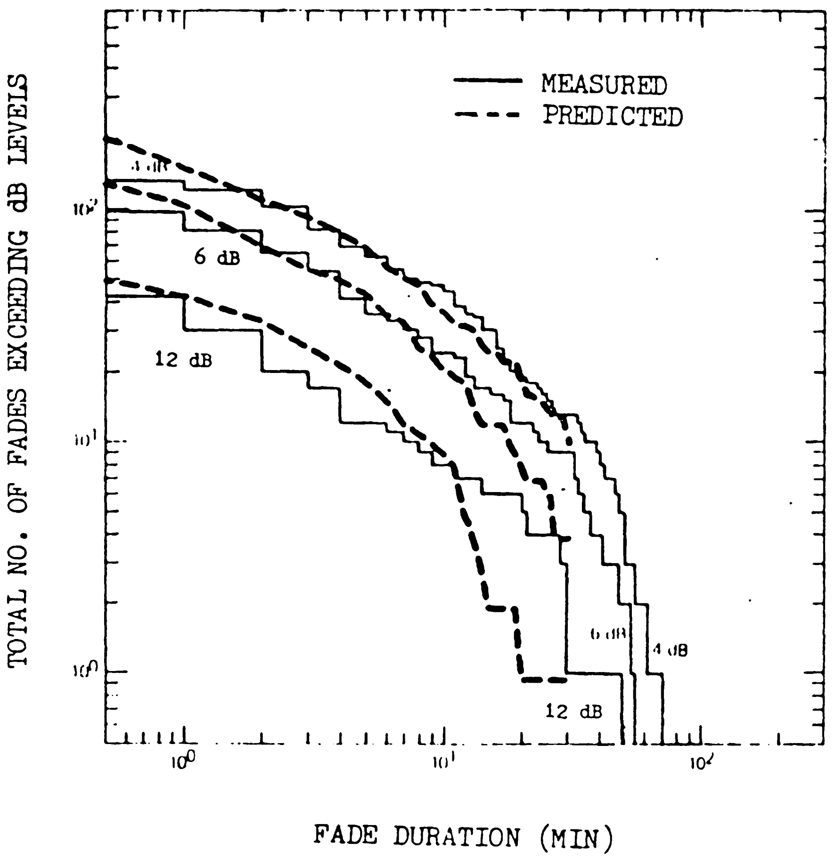

Figure 5.4 Comparison of experimental and Scaled Attenuation Method predicted results for Kumar's COMSTAR 19 GHz data at 21° elevation angle, vertical polarization.
Reproduced with permission from P.N. Kumar, "Precipitation Fade Statistics for 19/29-GHz COMSTAR beacon signals and 12-GHz radiometric measurements," COMSAT Technical Review, Vol. 12, No. 1, Spring 1982.

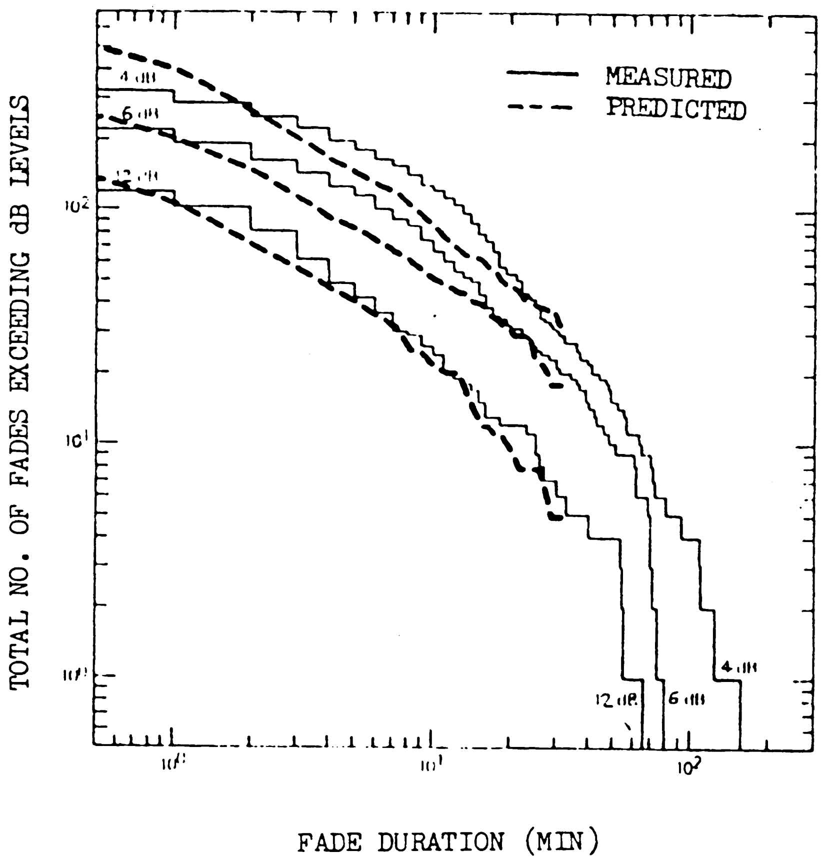

Figure 5.5 Comparison of experimental and Scaled Attenuation Method predicted results for Kumar's COMSTAR 29 GHz data at 21° elevation angle, linear polarization.
Reproduced with permission from P.N. Kumar, "Precipitation Fade Statistics for 19/29-GHz COMSTAR beacon signals and 12-GHz radiometric measurements," COMSAT Technical Review, Vol. 12, No. 1, Spring 1982.

The Scaled Attenuation Method worked extremely well in predicting the fade duration statistics of other sites. For example, for most thresholds and duration bins the Scaled Attenation Method better predicted the number of events than the Fixed Parameter Model (See Table 4.2). The plots in Figures 5.1 - 5.5 show excellent agreement between measured and predicted fade duration distributions. The predicted fade duration distributions are only plotted up to a duration of 30.5 minutes due to resolution limitations of the software.

5.5 ADVANCED COMMUNICATIONS TECHNOLOGY SATELLITE EXAMPLE

We applied the Scaled Attenuation Method to the ACTS example in Section 4.5. Using Virginia Tech's 1980 SIRIO data as the universal attenuation data set, we found that:

$$K = 1.3103$$

$$C = 1.0237$$

These constants were used to scale the universal attenuation data set to obtain the fade duration table given in Table 5.2.

The ACTS satellite is capable of boosting its transmitting power up to 8 dB to overcome rain attenuation. However, the satellite does not have an unlimited supply of power. Consequently, the fade duration becomes a constraint. For example, suppose the satellite can draw excess battery power for no more than 30 minutes. In that case, the satellite

Table 5.2 Fade duration table using Scaled Attenuation Method for ACTS example at 20 GHz, 33° elevation angle, linear polarization.

Duration	Threshold (dB)		
(min.)	5	10	15
0-1	58	22	11
1-2	20	9	3
2-4	21	6	6
4-8	23	7	2
8-16	20	9	5
16-30	11	2	
30+	11		

could only overcome part of the 11 predicted 5 dB fades which last over 30 minutes.

CHAPTER 6 - MODEL IMPROVEMENT

In Chapter 5 we found that the Scaled Attenuation Method was superior to the Fixed Parameter Model in predicting fade duration statistics. To explain why the model does not perform as well, we re-examined the assumptions of the model.

6.1 PARAMETER VARIATION

We initially assumed the parameters A and B2 were constant for a given site. Further analysis revealed that these parameters are actually functions of the level of attenuation. In other words, rain attenuation is a nonstationary process. It is important to note that variation in B2 may imply variation in the shape of the input process distribution as well. We did analyze the mean of the input process and found that it was zero regardless of attenuation range. The parameter variation is shown in Tables 6.1 and 6.2. Also shown are results from an analysis of the Fixed Parameter Model data; this should show no parameter variation. This was done to show what variation would be observed from quantization and roundoff error. To show the significance of the parameter variation, Tables 6.1 and 6.2 were plotted in Figures 6.1 through 6.4. Also plotted were the constant parameter values used by the Fixed Parameter Model.

Table 6.1 A and B2 as a function of attenuation level for Virginia Tech's 1980 SIRIO data at 11.6 GHz, 10.7° elevation angle, circular polarization.

Range	SIRIO data		Fixed Parameter Model data	
(dB)	A	B2	A	B2
0.0-1.0	.9669	.2695	.9545	.2553
1.0-1.5	.9590	.2403	.9715	.2558
1.5-2.0	.9873	.1234	.9678	.2556
2.0-2.5	.9892	.1039	.9666	.2684
2.5-3.0	.9854	.1448	.9680	.2586
3.0-5.0	.9937	.1053	.9729	.2375
5.0+	.9878	.1976	.9688	.2228

Table 6.2 A and B2 as a function of attenuation level for Virginia Tech's 1981 SIRIO data at 11.6 GHz, 10.7° elevation angle, circular polarization.

Range	SIRIO data		Fixed Parameter Model data	
(dB)	A	B2	A	B2
0.0-1.0	.9705	.2648	.9680	.2316
1.0-1.5	.8843	.1353	.9215	.2221
1.5-2.0	.9793	.1259	.9523	.2291
2.0-2.5	.9895	.1091	.9553	.2253
2.5-3.0	.9882	.0856	.9878	.2206
3.0-5.0	.9937	.0973	.9705	.2138
5.0+	.9927	.1126	.9723	.1987

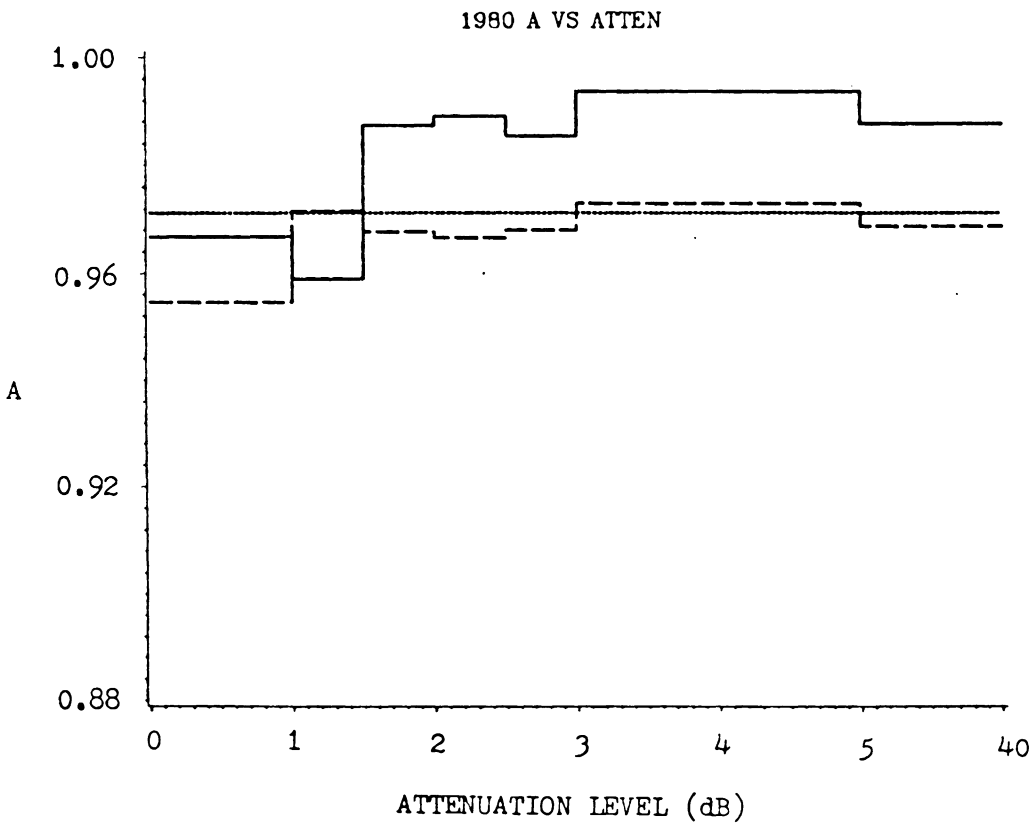

Figure 6.1 Plot of A as a function of attenuation level for Virginia Tech's 1980 SIRIO data at 11.6 GHz, 10.7° elevation angle, circular polarization (solid line), the Fixed Parameter Model data (long dashed line), and the constant value (short dashed line).

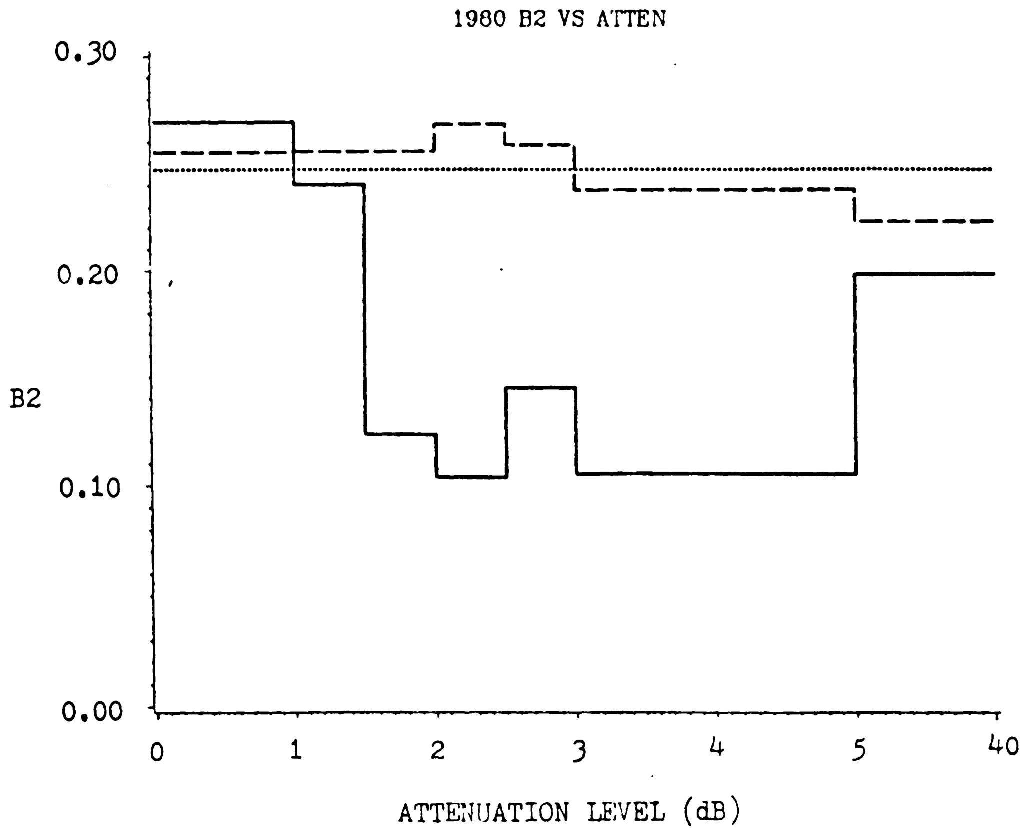

Figure 6.2 Plot of B2 as a function of attenuation level for Virginia Tech's 1980 SIRIO data at 11.6 GHz, 10.7° elevation angle, circular polarization (solid line), the Fixed Parameter Model data (long dashed line), and the constant value (short dashed line).

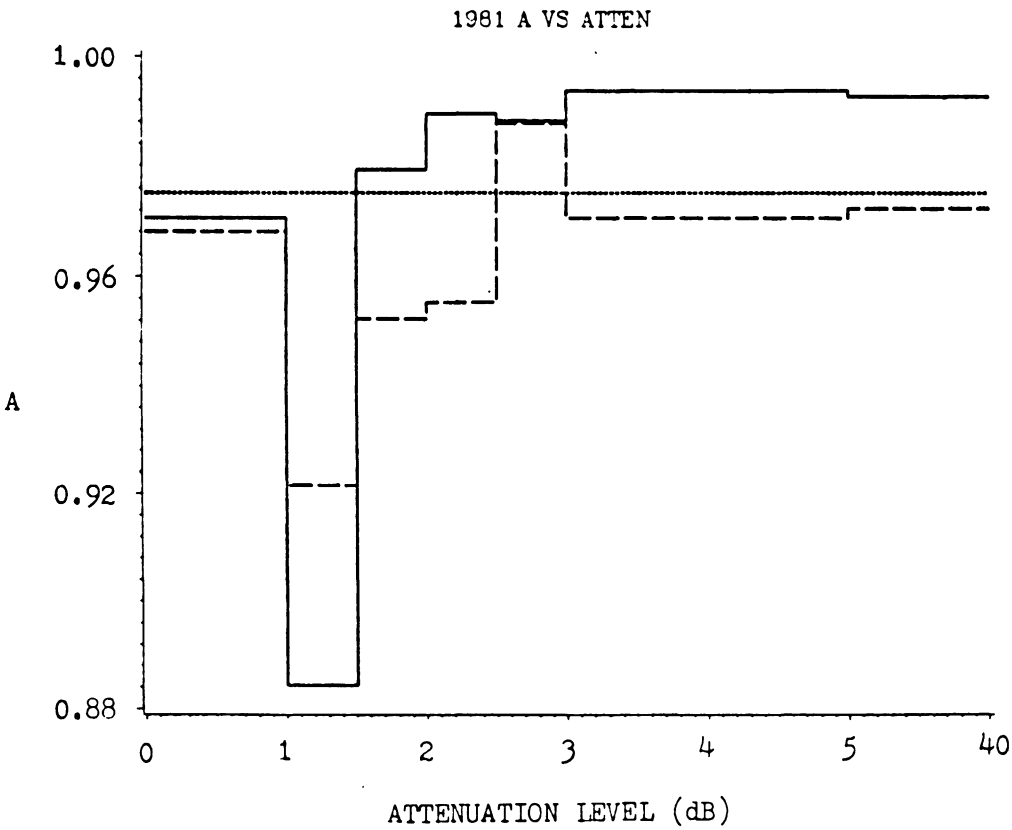

Figure 6.3 Plot of A as a function of attenuation level for Virginia Tech's 1981 SIRIO data at 11.6 GHz, 10.7° elevation angle, circular polarization (solid line), the Fixed Parameter Model data (long dashed line), and the constant value (short dashed line).

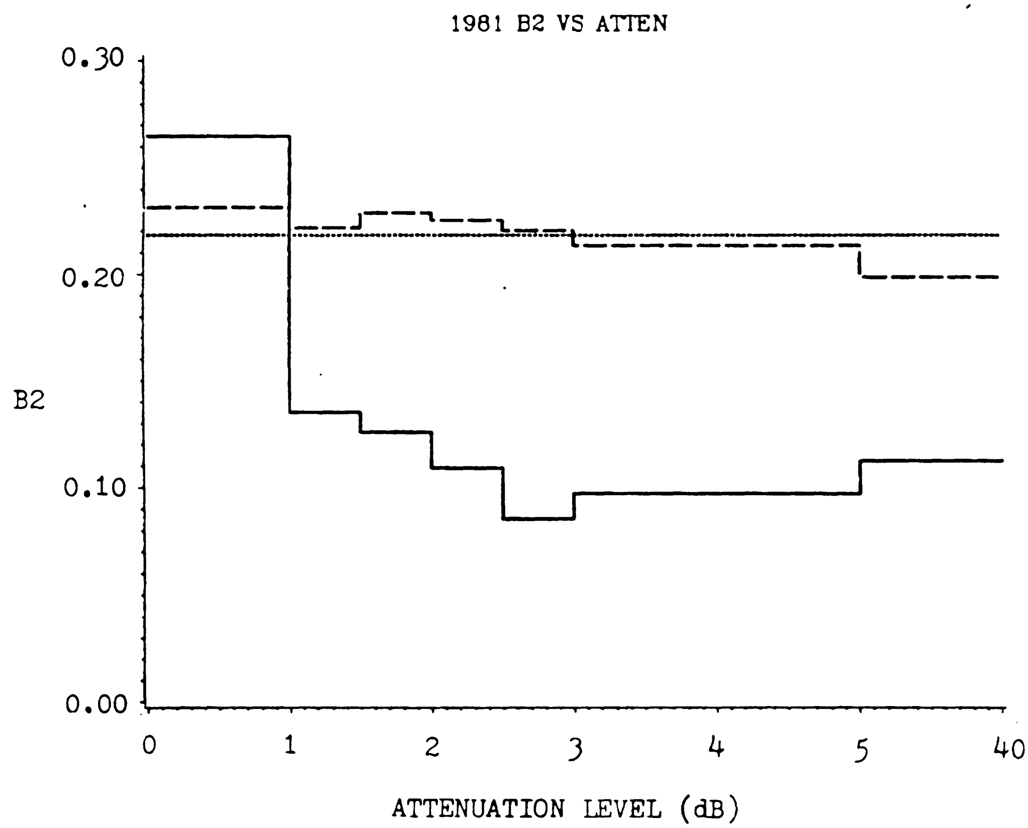

Figure 6.4 Plot of B2 as a function of attenuation level for Virginia Tech's 1981 SIRIO data at 11.6 GHz, 10.7° elevation angle, circular polarization (solid line), the Fixed Parameter Model data (long dashed line), and the constant value (short dashed line).

The slight variation in the A parameter is very significant because its value is so close to one. The model can be thought of as a first order autoregressive filter followed by a nonlinear device. As A approaches one, the filter transfer characteristic H(z) changes dramatically. For example, a variation in A of 0.96 to 0.99 results in a change in the magnitude of H(z) at z = 0 from 28 to 40 dB. This large variation is due to the proximity of the filter's pole to the unit circle in the z domain.

The variation in B2 depends partly on the variation of A and partly on the range of attenuation. To analyze this dependence, we start with equation 3.3-1:

$$\Lambda_{(k)} = A\ \Lambda_{(k-1)} + W_{(k)} \qquad (3.3\text{-}1)$$

Taking the expected value of Λ^2, we obtain:

$$E\{\Lambda^2_{(k)}\} = A^2\ E\{\Lambda^2_{(k-1)}\} + B2^2 \qquad (6.1\text{-}1)$$

Since we found A as a function of $\Lambda_{(k-1)}$, $\Lambda_{(k)}$ is not necessarily in the same range as $\Lambda_{(k-1)}$. However, if it is not in the same range, it is nearby so that:

$$E\{\Lambda^2_{(k)}\} \approx E\{\Lambda^2_{(k-1)}\} = E\{\Lambda^2\} \qquad (6.1\text{-}2)$$

We can now rewrite 6.1-1 in the following forms:

$$1 = A^2 + B2^2/E\{\Lambda^2\} \qquad (6.1\text{-}3)$$

$$B2^2 = (1 - A^2)\ E\{\Lambda^2\} \qquad (6.1\text{-}4)$$

Equation 6.1-4 shows how B2 varies with A and range. If A gets larger, B2 gets smaller. As range goes above 1 dB, $E\{\Lambda^2\}$ gets larger, causing B2 to get larger. Both of these effects cause the variation of B2 shown in Figures 6.2 and 6.4.

To further verify the significance of the parameter variation, we implemented a model which varied A and B2 with attenuation range according to our 1980 SIRIO analysis. To do this, we used the input process distribution derived for the Fixed Parameter Model and scaled the generated input process values to have the desired standard deviation B2. Consequently , we assumed that the nonstationarity in the input process was only reflected in the standard deviation. We used this varying parameter model to predict Vogel's fade duration table, and the results are given in Table 6.3. We must point out that the 3 dB fade lasting over 32 minutes predicted by the Varying Parameter Model was actually predicted to last over 30 minutes.

The Varying Parameter Model performed better than the Fixed Parameter Model at the low thresholds of 3, 6 and 10 dB, and worse at the high thresholds of 20 and 25 dB. While this does not conclusively show that the Varying Parameter Model is an improvement over the Fixed Parameter Model, it does indicate that the variation in parameters observed in Tables 6.1 and 6.2 has a significant impact on the model results.

Table 6.3 Comparison of experimental, Fixed Parameter Model and Varying Parameter Model results for Vogel's 1978 CTS data at 11.7 GHz, 50.0° elevation angle, circular polarization [22].

Duration	Threshold (dB)														
	3			6			10			20			25		
(min.)	MR	FP	VP	MR	FP	VP	MR	FP	VP	MR	FP	VP	MR	FP	VP
0-1	32	71	53	31	50	18	11	24	19	16	6	10	2	4	7
1-2	9	45	25	7	20	9	4	9	8	1	1	2	0	2	3
2-4	11	49	27	12	14	14	6	9	9	0	4	4	1		11
4-8	13	36	23	9	14	8	5	6	9	1		7			
8-16	11	19	13	7	4	8	2		4						
16-32	9	1	8	1		3									
32+	2		1												

MR - Measured Results
FP - Fixed Parameter Results
VP - Varying Parameter Results

6.2 QUANTIZATION ERROR EFFECTS

Another problem with the Fixed Parameter Model is the technique used to obtain the input process distribution. This distribution was derived from an experimental data set with values resolved to within a tenth of a dB. Consequently, the measured attenuation process can be thought of as a discrete random process. Since there are a finite number of transitions from one attenuation sample to another, the input process derived from measured data is also discrete. The distribution of this input process is stepped instead of smooth. This introduces quantization error into the model implementation. In Tables 6.4 and 6.5 we made up an input process to show how the derived input process can be in error from the actual input process, and how this effect is more pronounced at lower attenuation levels.

One way to overcome this quantization problem is to determine the shape of the input process distribution from high attenuation values only. However, this technique will only work if the input process distribution shape is stationary, not a function of attenuation range. This technique was implemented along with the Varying Parameter Model to yield the results given in Table 6.6. We must point out that the nine 3 dB fades lasting over 32 minutes predicted by the Varying Parameter with 4-40 dB Input Model were actually predicted to last over 30 minutes.

Table 6.4 Example of quantization effects in deriving input process from measured data at low attenuation levels.

Item	k values 0	1	2	3
1. ACTUAL $W_{(k)}$	-----	-0.11	0.02	0.04
2. Actual $\Lambda_{(k)}$	0.000	-0.107	-0.084	-0.041
3. Actual $L_{(k)}$	-0.250	-0.357	-0.334	-0.291
4. Actual $A_{(k)}$	0.779	0.700	0.716	0.747
5. Measured $A_{(k)}$	0.8	0.7	0.7	0.7
6. Measured $L_{(k)}$	-0.223	-0.357	-0.357	-0.357
7. Measured $\Lambda_{(k)}$	0.027	-0.107	-0.107	-0.107
8. MEASURED $W_{(k)}$	-----	-0.133	-0.003	-0.003

Table 6.5 Example of quantization effects in deriving input process from measured data at high attenuation levels.

Item	k values 0	1	2	3
1. ACTUAL $W_{(k)}$	-----	-0.11	0.02	0.04
2. Actual $\Lambda_{(k)}$	3.246	3.042	2.974	2.927
3. Actual $L_{(k)}$	2.996	2.792	2.724	2.667
4. Actual $A_{(k)}$	20.005	16.311	15.236	14.547
5. Measured $A_{(k)}$	20.0	16.3	15.2	14.5
6. Measured $L_{(k)}$	2.996	2.790	2.721	2.674
7. Measured $\Lambda_{(k)}$	3.246	3.041	2.971	2.924
8. MEASURED $W_{(k)}$	-----	-0.110	0.018	0.039

Table 6.6 Comparison of experimental, Fixed Parameter Model and Varying Parameter Model with input process derived from 4-40 dB 1980 SIRIO data results for Vogel's 1978 CTS data [22] at 11.7 GHz, 50.0° elevation angle, circular polarization.

Duration	Threshold (dB) 3			6			10			20			25		
(min.)	MR	FP	RP	MR	FP	RP	MR	FP	RP	MR	FP	RP	MR	FP	RP
0-1	32	71	51	31	50	21	11	24	12	16	6	2	2	4	3
1-2	9	45	19	7	20	6	4	9	5	1	1	0	0	2	0
2-4	11	49	14	12	14	9	6	9	3	0	4	0	1		1
4-8	13	36	16	9	14	9	5	6	5	1		1			
8-16	11	19	25	7	4	3	2		2						
16-32	9	1	4	1		4									
32+	2		9			2									

MR - Measured Results
FP - Fixed Parameter Results
RP - Varying Parameter With 4-40 dB Input Results

Table 6.6 shows that the Varying Parameter Model with the less quantized input process performed much better than the Fixed Parameter Model at the higher thresholds of 10, 20 and 25 dB, and did worse at the lower thresholds of 3 and 6 dB for large duration bins. Thus, the results indicate that quantization error has a significant effect on the model's results.

6.3 SUMMARY

In this chapter we found two reasons why the Fixed Parameter Model does not predict fade duration statistics as well as the Scaled Attenuation Method. First of all, the parameters A and B2 are actually functions of attenuation level, implying that the attenuation process is nonstationary, even during rain. Second, we showed that quantization error introduced in deriving the input process is significant when using low level attenuation data. Finally, the shape of the input process distribution may also be a function of attenuation level.

CHAPTER 7: CONCLUSIONS AND RECOMMENDATIONS

In this thesis, we investigated ways of predicting fade duration statistics for a given location, frequency, elevation angle and polarization. We started with the existing Maseng-Bakken dynamic stochastic model, and improved it by driving it with a non-Gaussian input process, using one universal attenuation data set, and developing a new way of calculating its inputs. From the structure of the resulting model, called the Fixed Parameter Model, we derived the Scaled Attenuation Method, a way of relating two measured attenuation data sets. We found that this method was the best way to predict the fade duration statistics of other locations, frequencies, elevation angles and polarizations. Finally, we found that the Fixed Parameter Model parameters were not constant , but varied with the level of attenuation. We also found that quantization error influenced the input process being used.

We have several recommendations for further investigation. First of all, we would like to see the Scaled Attenuation Method evaluated using other attenuation data sets. Also, we would like to see this method applied to the Land Mobile Satellite System fading problem. Criteria are needed for evaluating methods that predict fade duration statistics. Finally, dialogue with satellite system designers is needed to determine the most useful way of presenting the dynamics of rain attenuation.

APPENDIX A. SMOD2 PROGRAM WHICH GENERATES MODEL ATTENUATION DATA

```
C------------------------------------------------------------------------
C PROGRAM:      SMOD2.F
C DATE:         19 NOV 84
C PGMMER:       GREG BOTTOMLEY
C PURPOSE:   THIS PROGRAM IS AN IMPLEMENTATION OF A GENERAL STOCHASTIC DYNAMIC
C     MODEL OF RAIN ATTENUATION
C
C------------------------------------------------------------------------
C VARIABLE DECLARATIONS
C
      DOUBLE PRECISION          OLDX(20)      !PREVIOUS LK VALUES
      DOUBLE PRECISION          OLDN(20)      !PREVIOUS NK VALUES
      DOUBLE PRECISION          XK            !CURRENT L OF MODEL
      DOUBLE PRECISION          XKS           ! L + LMEAN
      DOUBLE PRECISION          XMEAN .       !LMEAN
      DOUBLE PRECISION          NK            !NORMAL (0,1) VARIABLE
      DOUBLE PRECISION          AK            !ATTENUATION
      DOUBLE PRECISION          A(20),C(20)   !AR AND MA PARAMETERS
      DOUBLE PRECISION          WK            !NORMALIZED INPUT PROCESS AS WHOLE
      DOUBLE PRECISION          B             !STDDEV OF INPUT NOISE PROCESS
      INTEGER                   N,M           !AR AND MA ORDERS
      INTEGER                   I,J           !COUNTERS
      INTEGER                   K             !COUNTER FROM 1 TO XCOUNT
      INTEGER                   TIME          !FOR DATA OUTPUT
      INTEGER                   XCOUNT        !MAX K,MAX NUMBER OF POINTS
      INTEGER                   YR,DAYS       !YEAR AND DAYS IN YEAR
      INTEGER                   OPT           !OPTION FOR RUNNING MODEL
      INTEGER                   LASWDIS       !END OF WDIS ARRAY (FIRST 0)
      CHARACTER *3              COFLAG        !CO LOCK
      CHARACTER *80             RECOUT        !RECORD MASK
C
C OPTION B SPLINE VARIABLE DECLARATIONS (FOR SPLINING DATA DERIVED INPUT DIST.
C
      DOUBLE PRECISION        DATA(0:260,3)   !(XJ,F(XJ))
      DOUBLE PRECISION        DDATA(0:260,3)  !TEMP
      DOUBLE PRECISION        ALPHA           !X TO BE INTERPOLATED
      DOUBLE PRECISION        SA              !S(A)
      DOUBLE PRECISION        MM(260)         !SLOPE PER INTERVAL
      DOUBLE PRECISION        PB(260)         !INTERCEPT PER INTERVAL
      INTEGER       NN                        !NUMBER OF DATA POINTS MINUS ONE
      INTEGER       L                         !COUNTERS
      INTEGER       JINT                      !SPLINE INTERVAL
      COMMON/JUNK/DDATA                       !SAVE ON STORAGE SPACE
C
C---------------------------------------------------------------------
C TWO MODEL OPTIONS:
C       OPT      DESCRIPTION
C        1       WHITE GAUSSIAN INPUT NOISE
C        2       WHITE DATA-DERIVED DISTRIBUTION INPUT NOISE
C
```

```
C-----------------------------------------------------------------
C INPUT DATA FROM DD FILE FROM RESAN
C
      READ(23,*)N
      DO 484 I = 1,N
        READ(23,*)A(I)
  484   CONTINUE
      READ(23,*)B
      READ(23,*)M
      DO 485 I = 1,M
        READ(23,*)C(I)
  485   CONTINUE
      READ(23,*)XCOUNT
      READ(23,*) YR
      READ(23,*) XMEAN
      DO 486 I = 0,260
        READ(23,*,END=487) DATA(I,1),DATA(I,2)
        IF(DATA(I,2).NE.100..AND.DATA(I-1,2).EQ.100.) FIRWDIS = I-1
        IF(DATA(I,2).EQ.0.) GO TO 487
  486   CONTINUE
  487 CONTINUE
      LASWDIS = I
      WRITE(10,FMT='(" SMOD2 PROGRAM")')
      DAYS = 365
      IF (YR.EQ.30.OR.YR.EQ.34) DAYS = DAYS +1
C
C-----------------------------------------------------------------
C DEBUG OUTPUT
C
D     WRITE(6,FMT='(" N= ",I4,/," A(N) ARRAY",/,10F8.4)')N,(A(I),I=1,N)
D     WRITE(6,FMT='(" B= ",F20.10)')B
D     WRITE(6,FMT='(" M= ",I4,/," C(M) ARRAY",/,10F8.4)')M,(C(I),I=1,M)
D     WRITE(6,FMT='(" XCOUNT AND YEAR",I3,I4)')XCOUNT,YR
C
C--------------------------------------------------------------------
C READ IN OPTION
C
      CALL OPTION(OPT)
D     WRITE(6,FMT='(" OPT= ",I4)')OPT
C
C--------------------------------------------------------------------
C CONSTANTS
C
      K = 0
      DO 130 I = 1,N
        OLDX(I) = 0.D0
        OLDN(I) = 0.D0
  130   CONTINUE
C----------------------------------------------------------------------
C HEADER FOR MODEL DATA FILE
C
```

```
      WRITE(20,FMT='("YER",I2,"    1    1",I4)')YR,DAYS
C
C-----------------------------------------------------------------
C IF OPTION 2, WE NEED TO SPLINE INVERSE OF WK DISTRIBUTION
C
      IF (OPT.EQ.2) THEN
C
C PREPROCESS DATA
C
      INDEX = FIRWDIS
      DDATA(0,1) = DATA(FIRWDIS,2)
      DDATA(0,2) = DATA(FIRWDIS,1)
      DO 1100 I = 1,LASWDIS
        INDEX = INDEX + 1
        CNT = 0
        IF(DATA(INDEX,2).EQ.0.) GO TO 1060
        DO 1050 J = 1,LASWDIS
          IF(DATA(INDEX,2).NE.DATA((INDEX+J),2)) GO TO 1060
          CNT = CNT + 1
 1050     CONTINUE
 1060 DDATA(I,1) = DATA(INDEX,2)
      DDATA(I,2) = DATA(INDEX,1)
      DDATA(I,3) = DATA((INDEX+CNT),1)
      IF(DATA(INDEX,2).EQ.0.) GO TO 1200
      INDEX = INDEX + CNT
 1100 CONTINUE
 1200 NN = I
C
C COPY IN REVERSE ORDER
C
D     WRITE(6,FMT='(" I",7X,"DATA(I,1) DATA(I,2) DATA(I,3)")')
      DO 1300 I = 0,NN
        DATA(I,1) = DDATA((NN-I),1)
        DATA(I,2) = DDATA((NN-I),3)
        DATA(I,3) = DDATA((NN-I),2)
D       WRITE(6,FMT='(I8,3F10.5)')I,DATA(I,1),DATA(I,2),DATA(I,3)
 1300   CONTINUE
C
C INITIAL VALUES
C
      DO 93 I = 1,NN
        MM(I) = 0.D0
        BB(I) = 0.D0
   93   CONTINUE
C
C FIND SPLINE
C
      CALL SPLINE(DATA,MM,BB,NN)
C
      END IF
C
```

```
C-----------------------------------------------------------------------
C MAIN SYSTEM CALCULATIONS
C
  500 K = K+1                                      !INCREMENT K
C
  600 IF (OPT.EQ.1) THEN
        NK = RANN(0.,1.)
        WK = B*NK
        XK = WK
        DO 610 I = 1,N
          XK = XK + A(I)*OLDX(I)
  610     CONTINUE
        XKS = XK + XMEAN
        AK = EXP(XKS)
        COFLAG = " LK"
C       IF(AK.GE.40.) THEN
C         AK = 40.0
C         COFLAG = "UNL"
C         END IF
        CALL CIRC(OLDX,N,XK)
        END IF
C
  800 IF (OPT.EQ.2) THEN
        NK = RANU(0.,100.)
        CALL LOCATE(NK,DATA,NN,JINT)                  !RUN NK OVER INPUT DIST
        IF (JINT.LT.0.OR.JINT.GT.NN) THEN
          WRITE(10,FMT='(" POINT NOT IN INTERVAL, TRY AGAIN")')
          WRITE(6,FMT='(" POINT ",F10.2," NOT IN INTERVAL")')NK
          GO TO 9900
          END IF
        CALL SALPHA(NK,MM,3B,DATA,JINT,WK)             !CALC WK
D     WRITE(6,FMT='(" NK=",F10.5," JINT=",I4," WK=",F10.6)')NK,JINT,WK
        XK = WK
        DO 810 I = 1,N
          XK = XK + A(I)*OLDX(I)
  810     CONTINUE
        XKS = XK + XMEAN
        AK = EXP(XKS)
        COFLAG = " LK"
        CALL CIRC(OLDX,N,XK)
        END IF
C
C-----------------------------------------------------------------------
C DEBUG OUTPUT
C
D     WRITE(6,FMT='(" K,AK,XK,WK,NK",I6,3F20.10)')K,AK,XK,WK,NK
C
C GENERATE OUTPUT TO EXAMINE ON TERMINAL
C
      WRITE(10,1) K,WK,AK
    1 FORMAT(" K,NK AND AK: ",2X,I8,2X,2G15.5,/)
```

```
C
C FOR ANALYSIS
C
      TIME = K
      WRITE(20,38) TIME,AK,XK,NK,WK,COFLAG
   38 FORMAT("   1",I8,F6.1,2F6.2,F7.3,1X,A3," LK  LK  DF  DF   1.1")
C
C----------------------------------------------------
 9700 CONTINUE
      IF(K.GE.XCOUNT) GO TO 9900
      GO TO 500
C------------------------------------------------------------------
C END OF DATA LOOP
C
 9900 CONTINUE
C
      WRITE(20,39)(TIME+1),AK,XK,NK,WK,COFLAG
   39 FORMAT("   0",I8,F6.1,2F6.2,F7.3,1X,A3," LK  LK  DF  DF   1.1")
      STOP
      END
C----------------------------------------------------------------------
C SUBROUTINE: CIRC
C PGMMER:     GREG BOTTOMLEY
C DATE:       17NOV84
C PURPOSE: TO CIRCULATE PROCESS VECTOR
C------------------------------------------------------------------
C DECLARATIONS
C
      SUBROUTINE CIRC(VEC,N,VAL)
C
      DOUBLE PRECISION  VAL                  !NEW PROCESS VALUE
      DOUBLE PRECISION  VEC(20)              !PROCESS VECTOR
      INTEGER           N                    !ORDER OF AR PROCESS
C
      DO 10 I = N,2,-1
        VEC(I) = VEC(I-1)
   10   CONTINUE
      VEC(1) = VAL
      RETURN
      END
C
C------------------------------------------------------------------
C SUBROUTINE:  SPLINE(DATA,M,B,N)
C INPUTS: DATA      DATA
C         N         NUMBER OF DATA POINTS
C OUTPUT: M,B       SLOPE AND INTERCEPTS
C PURPOSE:  DETERMINE THE LINEAR SPLINE FOR THE DATA GIVEN.
C------------------------------------------------------------------
C
C DECLARATIONS
C
```

```
      SUBROUTINE SPLINE(DATA,M,B,N)
C
      DOUBLE PRECISION      DATA(0:260,3)
      DOUBLE PRECISION      M(260)
      DOUBLE PRECISION      B(260)
      INTEGER               N                      !NUMBER OF POINTS MINUS ONE
      INTEGER               J                      !COUNTER
C
C-----------------------------------------------------------------
C MAIN EXECUTION
C
C CALCULATE SLOPE AND INTERCEPT BY THE FOLLOWING FORMULAS
C
C     M = (Y2 - Y1)/(X2 - X1)
C
C     B = (Y1*X2 - Y2*X1)/(X2 - X1)
C
C NOTE: THE I'TH INTERVAL IS FORMED FROM THE (I-1)TH AND ITH POINT
C
      DO 100 I = 1,N
        M(I) = (DATA(I,2)-DATA(I-1,3))/(DATA(I,1)-DATA(I-1,1))
        B(I) = (DATA(I-1,3)*DATA(I,1) - DATA(I,2)*DATA(I-1,1))/
     *          (DATA(I,1) - DATA(I-1,1))
  100   CONTINUE
C
   90 RETURN
      END

C
C----------------------------------------------------------------------
C SUBROUTINE:  LOCATE(X,DATA,N,JINT)
C INPUTS: X                 X TO BE INTERPOLATED
C         DATA              DATA POINTS
C         N                 NUMBER OF DATA POINTS MINUS ONE
C OUTPUT: JINT              INTERVAL WHERE X LIES
C PURPOSE: DETERMINE THE INTERVAL WHERE X LIES WITHIN THE DATA
C---------------------------------------------------------------------
C DECLARATIONS
C
      SUBROUTINE LOCATE(X,DATA,N,JINT)
C
      DOUBLE PRECISION      X,DATA(0:260,3)
      INTEGER               JINT,N
C
C---------------------------------------------------------------------
C
C MAIN EXECUTION
C
      DO 10 JINT = 0,N
        IF(X.GE.DATA(JINT,1).AND.X.LE.DATA(JINT+1,1)) GO TO 20
   10   CONTINUE
```

```
      JINT = -2                                         !ERROR
   20 JINT = JINT + 1
      RETURN
      END
C-----------------------------------------------------------------------
C SUBROUTINE:  SALPHA(X,M,B,DATA,JINT,SX)
C INPUTS: X       POINT OF INTERPOLATION
C         M,B     SPLINE PARAMETERS
C         DATA    DATA POINTS
C         JINT    INTERVAL WHERE X CAN BE FOUND
C OUTPUT: SX      S(X)
C PURPOSE:  EVALUATE SPLINE AT X
C-----------------------------------------------------------------------
C DECLARATIONS
C
      SUBROUTINE SALPHA(X,M,B,DATA,JINT,SX)
C
      DOUBLE PRECISION      X
      DOUBLE PRECISION      M(260),B(260)
      DOUBLE PRECISION      DATA(0:260,3)
      DOUBLE PRECISION      SX
      INTEGER JINT
C
C-----------------------------------------------------------------------
C MAIN EXECUTION
C
      SX = M(JINT)*X + B(JINT)
C
      RETURN
      END
C
```

APPENDIX B. PRERUN PROGRAM USED IN SCALED ATTENUATION METHOD

```
C-----------------------------------------------------------------------
C PROGRAM:     PRERUN.F
C DATE:        07 FEB 85
C PGMMER:      GREG BOTTOMLEY
C PURPOSE:  THIS PROGRAM CALCULATES K AND C FROM A PARTIAL ATTENUATION DIST.
C
C-----------------------------------------------------------------------
C VARIABLE DECLARATIONS
C
      DOUBLE PRECISION         UDIS(0:260,3)  !UNIVERSAL ATTEN DIST
      DOUBLE PRECISION         ADIS(0:201,2)  !SITE ATTEN DIST
      DOUBLE PRECISION         ADISP(0:201,2) !MATCHED AM DISTRIBUTION
      DOUBLE PRECISION         SCALE          !USED TO SCALE DIST TO APPROPRIATE
                                              !CONDITIONAL DISTRIBUTION
                                              !(EXCLUDING CLEAR WEATHER)
      DOUBLE PRECISION         SCALE2         !FOR UNIVERSAL DIST.
      DOUBLE PRECISION         AMAT(201,2)   !MATRIX FOR LEAST SQUARES REDUCTION
      DOUBLE PRECISION         BVEC(201)     !VECTOR FOR LEAST SQUARES REDUCTION
      DOUBLE PRECISION         ATA(2,2)      !REDUCED A MATRIX
      DOUBLE PRECISION         ATB(2)        !REDUCED BVEC
      DOUBLE PRECISION         A1,A2,L1,L2   !FOR LEAST SQUARES REDUCTION
      DOUBLE PRECISION         PER           !PERCENT (PI'S)
      DOUBLE PRECISION         X(2)          !GAUSS ELIM ANSWER
      DOUBLE PRECISION         THRESH        !THRESHOLD FOR COND DISTRIBUTION
      REAL                     PEAK          !PEAK ATTEN FOR RXSTATS AND ATSTATS
      INTEGER                  I,J           !COUNTERS
      DOUBLE PRECISION         K             !PARAMETER
      INTEGER                  XCOUNT        !MAX K
      INTEGER                  YR,DAYS       !YEAR AND DAYS IN YEAR
      INTEGER                  NDIS          !NPTS USED FOR ADIS DISTRIBUTION
      INTEGER                  NINT          !NUMBER OF POINTS IN YEAR
      INTEGER                  SIZE          !SIZE (#ROWS) OF AMAT AND BVEC
      INTEGER                  TINDA,TINDU   !INDEX FOR THRESH
      INTEGER                  FIRUDIS       !FIRST SIGNIFICANT 100% IN DIST.
      INTEGER                  LASUDIS       !FIRST SIGNIFICANT 0% IN DIST.
      INTEGER                  FIRADIS       !FIRST SIGNIFICANT 100% IN DIST.
      INTEGER                  LASADIS       !FIRST SIGNIFICANT 0% IN ATTEN DIST
      CHARACTER *80            RECOUT        !RECORD MASK
C
C SPLINE VARIABLE DECLARATIONS (FOR CALCULATING THE INVERSE UNIVERSAL DIST.
C
      DOUBLE PRECISION     DDATA(0:260,3)     !INTERMEDIATE
      DOUBLE PRECISION     DATA(0:260,3)      !UDIS FOR SPLINE
      DOUBLE PRECISION     ALPHA             !X TO BE INTERPOLATED
      DOUBLE PRECISION     SA                !S(A)
      DOUBLE PRECISION     MM(260)           !SLOPE PER INTERVAL
      DOUBLE PRECISION     BB(260)           !INTERCEPT PER INTERVAL
```

```
      INTEGER       NN                            !NUMBER OF DATA POINTS MINUS ONE
      INTEGER       L                             !COUNTERS
      INTEGER       JINT                          !SPLINE INTERVAL
C
C-------------------------------------------------------------------
C INPUT UNIVERSAL DATA
C
      DO 10 I = 0,201
        READ(20,*,END=20)UDIS(I,1),UDIS(I,2)
        UDIS(I,3) = UDIS(I,2)                      !SPLINE PURPOSES
        IF(UDIS(I,2).NE.100..AND.UDIS(I-1,2).EQ.100.) FIRUDIS = I-1
   10   CONTINUE
   20 LASUDIS = I-1
C
C INPUT SITE DATA
C
      READ(23,*)YR                                 !YEAR OF DATA TO BE MODELED
      READ(23,*)THRESH                             !THRESHOLD FOR COND DIST TO USE
      READ(23,*)NPTS                               !NPTS TO GENERATE >= THRESH
      READ(23,*)NDIS                               !NPTS USED FOR INPUT DIST.
      READ(23,*)NINT                               !NO OF INTERVALS IN PERIOD
      READ(23,*)PEAK                               !PEAK ATTEN FOR RXSTAT & ATSTAT
      FIRADIS = 0
      DO 30 I = 0,201                              !SITE ATTEN DIST VALUES
        READ(23,*,END=40)ADIS(I,1),ADIS(I,2)
        IF(ADIS(I,2).NE.100..AND.ADIS(I-1,2).EQ.100.) FIRADIS = I-1
   30   CONTINUE
   40 CONTINUE
      LASADIS = I-1
      IF(ADIS(I,2).NE.0) LASADIS = I + 1
C
C-------------------------------------------------------------------
C DEBUG OUTPUT
C
D     WRITE(21,FMT='(" FIRUDIS,LASUDIS",2I5)')FIRUDIS,LASUDIS
D     WRITE(21,FMT='("    I        UDIS(I,1)        UDIS(I,2)")')
D     DO 121 I = 0,LASUDIS
D       WRITE(21,FMT='(I4,5X,F10.5,5X,F10.5)')I,UDIS(I,1),UDIS(I,2)
D 121   CONTINUE
D     WRITE(21,FMT='(//," YR = ",I2,5X," NPTS = ",I10)')YR,NPTS
D     WRITE(21,FMT='(/," NDIS = ",I8)')NDIS
D     WRITE(21,FMT='(" FIRADIS,LASADIS",2I5)')FIRADIS,LASADIS
D     WRITE(21,FMT='(/,"    I        ADIS(I,1)        ADIS(I,2)")')
D     DO 122 I = 0,LASADIS
D       WRITE(21,FMT='(I4,5X,F10.5,5X,F10.5)')I,ADIS(I,1),ADIS(I,2)
D 122   CONTINUE
D     WRITE(21,FMT='(//," NINT = ",I10)')NINT
C
C-------------------------------------------------------------------
C SCALING AND INITIAL CONDITIONS
C
```

```
      DO 125 I = 0,201
        IF(UDIS(I,1).GT.THRESH) GO TO 126
 125    CONTINUE
 126  TINDU = I-1
      DO 127 I = 0,201
        IF(ADIS(I,1).GT.THRESH) GO TO 128
 127    CONTINUE
 128  TINDA = I-1
C
      SCALE = REAL(NDIS)/REAL(NPTS)
      SCALE2 = 100./UDIS(TINDU,2)
      DO 130 I = 0,201
        ADIS(I,2) = SCALE*ADIS(I,2)
        UDIS(I,2) = SCALE2*UDIS(I,2)
        UDIS(I,3) = UDIS(I,2)
 130    CONTINUE
C
      FIRUDIS = TINDU
      FIRADIS = TINDA
C
C-------------------------------------------------------------------
C DEBUG OUTPUT
C
D     WRITE(21,FMT='(" TINDA,TINDU",2I8)')TINDA,TINDU
D     WRITE(21,FMT='(" SCALE,SCALE2",2F15.3)')SCALE,SCALE2
D     WRITE(21,FMT='(" SCALED UNIV. DISTRUBTION",/,"ATTEN",5X,"DIST")')
D     DO 912 I = 0,LASUDIS
D       WRITE(21,FMT='(F5.2,5X,F15.8)')UDIS(I,1),UDIS(I,2)
D 912   CONTINUE
D     WRITE(21,FMT='(" SCALED ATTEN DISTRUBTION",/,"ATTEN",5X,"DIST")')
D     DO 913 I = 0,LASADIS
D       WRITE(21,FMT='(F5.2,5X,F15.8)')ADIS(I,1),ADIS(I,2)
D 913   CONTINUE
C-----------------------------------------------------------------------
C WE NEED TO SPLINE THE INVERSE OF UDIS
C
      INDEX = FIRUDIS
      DDATA(0,1) = UDIS(FIRUDIS,2)
      DDATA(0,2) = UDIS(FIRUDIS,1)
      DO 1100 I = 1,LASUDIS
        INDEX = INDEX + 1
        CNT = 0
        IF(UDIS(INDEX,2).EQ.0.) GO TO 1060
        DO 1050 J = 1,LASUDIS
          IF((INDEX+J).GT.LASUDIS) GO TO 1060
          IF(UDIS(INDEX,2).NE.UDIS((INDEX+J),2)) GO TO 1060
          CNT = CNT + 1
 1050     CONTINUE
 1060 DDATA(I,1) = UDIS(INDEX,2)
      DDATA(I,2) = UDIS(INDEX,1)
      DDATA(I,3) = UDIS((INDEX+CNT),1)
```

```
      IF(UDIS(INDEX,2).EQ.0.) GO TO 1200
      INDEX = INDEX + CNT
      IF(INDEX.EQ.LASUDIS) GO TO 1150
 1100 CONTINUE
 1150 DDATA(I+1,1) = 0.D0
      DDATA(I+1,2) = UDIS(INDEX,1)
      DDATA(I+1,3) = UDIS(INDEX,1)
      I = I+1
 1200 NN = I
C
C COPY IN REVERSE ORDER (LOW PERCENT TO HIGH PERCENT)
C
D     WRITE(21,FMT='(" I",7X,"DATA(I,1) DATA(I,2) DATA(I,3)")')
      DO 1300 I = 0,NN
        DATA(I,1) = DDATA((NN-I),1)
        DATA(I,2) = DDATA((NN-I),3)
        DATA(I,3) = DDATA((NN-I),2)
D       WRITE(21,FMT='(I3,3F10.5)')I,DATA(I,1),DATA(I,2),DATA(I,3)
 1300   CONTINUE
C
C INITIAL VALUES FOR SPLINE CONSTANTS
C
      DO 93 I = 1,NN
        MM(I) = 0.D0
        BB(I) = 0.D0
   93   CONTINUE
C
C FIND SPLINE
C
      CALL SPLINE(DATA,MM,BB,NN)
C
C---------------------------------------------------------------------
C MAIN SYSTEM CALCULATIONS
C
C SET UP AMAT SET OF MANY EQUATIONS (S) IN 2 UNKNOWNS (LN(K) AND C)
C
      INDEX = 0
      DO 600 I = (FIRADIS+1),LASADIS
        INDEX = INDEX + 1
        A2 = ADIS(I,1)
        PER = ADIS(I,2)
        CALL LOCATE(PER,DATA,NN,JINT)
        IF (JINT.LT.0.OR.JINT.GT.NN) THEN
          WRITE(10,FMT='(" POINT NOT IN INTERVAL, TRY AGAIN")')
          WRITE(21,FMT='(" POINT ",F10.2," NOT IN INTERVAL")')PER
          GO TO 9900
          END IF
        CALL SALPHA(PER,MM,BB,DATA,JINT,A1)              !CALC A1
C
C DEBUG OUTPUT
D     WRITE(21,FMT='(" A2=",F10.5," JINT=",I4," A1=",F10.6)')A2,JINT,A1
```

```
C
        L1 = LOG(A1)
        L2 = LOG(A2)
        AMAT(INDEX,1) = 1.D0
        AMAT(INDEX,2) = L1
        BVEC(INDEX) = L2
  600   CONTINUE
        SIZE = INDEX
C
C DEBUG TEST OF LEAST-SQUARE FIT
C
D     AMAT(1,1) = 2
D     AMAT(1,2) = 1
D     AMAT(2,1) = 1
D     AMAT(2,2) = -1
D     AMAT(3,1) = 3
D     AMAT(3,2) = -2
D     SIZE = 3
D     BVEC(1) = 2
D     BVEC(2) = 1
D     BVEC(3) = 1
C
C CALCULATE (AMAT TRANSPOSE) TIMES AMAT
C
      DO 310 I = 1,2
        DO 210 J = 1,2
          ATB(J) = 0
          ATA(I,J) = 0
          DO 110 KK = 1,SIZE
            ATB(J) = ATB(J) + AMAT(KK,J)*BVEC(KK)
            ATA(I,J) = ATA(I,J) + AMAT(KK,I)*AMAT(KK,J)
  110       CONTINUE
  210     CONTINUE
  310   CONTINUE
C
C DEBUG OUTPUT
C
D     WRITE(21,FMT='(" ATA 11,12,21,22",4F10.4)')ATA(1,1),ATA(1,2),
D    *                  ATA(2,1),ATA(2,2)
D     WRITE(21,FMT='(" ATB 1,2",2F10.4)')ATB(1),ATB(2)
C
C SOLVE FOR X BY GAUSSIAN ELIMINATION
C
      CALL GAUSS(ATA,ATB,X,2,2,II,R)
      IF (II.NE.1) GO TO 9801
C
C CALCULATE K AND C FROM SOLUTION
C
      IF (X(1).GT.0.) K = EXP(X(1))
      IF (X(1).LT.0.) K = 1/EXP(ABS(X(1)))
      IF (X(1).EQ.0.) K = 1.D0
```

```
      C = X(2)
C
C OUTPUT RESULTS
C
      WRITE(21,*) K
      WRITE(21,*) C
      WRITE(21,*) NPTS
      WRITE(21,*) NINT
      WRITE(21,*) YR
      WRITE(21,*) THRESH
      WRITE(21,*) PEAK
      WRITE(21,FMT='(//," VALUE OF B: ",F20.8)') X(1)
      WRITE(21,FMT='(/," VALUE OF M: ",F20.8)') X(2)
      WRITE(21,FMT='(/," VALUE OF K: ",F20.8)') K
      WRITE(21,FMT='(/," VALUE OF C: ",F20.8)') C
C
C CALCULATE ADISP (THE RESULTING ATTEN DIST FOR MODEL DATA
C
      CALL SPLINE(UDIS,MM,BB,LASUDIS)
      DO 700 I = 0,LASADIS
        ADISP(I,1) = ADIS(I,1)
        A2 = ADIS(I,1)
        A1 = (A2/K)**(1./C)
        CALL LOCATE(A1,UDIS,LASUDIS,JINT)
        IF (JINT.LT.0) THEN
          PER = 100.
          GO TO 691
          END IF
        IF (JINT.GT.LASUDIS) THEN
          PER = 0.
          GO TO 691
C         WRITE(10,FMT='(" POINT NOT IN INTERVAL, TRY AGAIN")')
C         WRITE(21,FMT='(" POINT ",F10.2," NOT IN INTERVAL")')A1
C         GO TO 9900
          END IF
        CALL SALPHA(A1,MM,BB,UDIS,JINT,PER)            !CALC PER
D       WRITE(21,FMT='(" A2,A1,PER ",3F10.6)')A2,A1,PER
  691   ADISP(I,2) = PER
  700   CONTINUE
C
C OUTPUT ADIS (SITE DIST) VS ADISP (MODEL DIST)
C
      WRITE(21,FMT='(//," ATTENUATION DISTRIBUTION GIVEN ",F4.1)')THRESH
      WRITE(21,FMT='(/," ATTEN",5X,"ACTUAL DIS",5X,"MATCHED DIS")')
      DO 750 I = FIRADIS,LASADIS
        WRITE(21,FMT='(F6.1,5X,F10.6,6X,F10.6)')ADIS(I,1),ADIS(I,2),
     *           ADISP(I,2)
  750   CONTINUE
C
      WRITE(21,FMT='(//," ATTENUATION DISTRIBUTION ENTIRE PERIOD")')
      WRITE(21,FMT='(" ASSUMING ALL MISSING POINTS ARE CLEAR WEATHER")')
```

```
      WRITE(21,FMT='(/," ATTEN",5X,"ACTUAL DIS",5X,"MATCHED DIS")')
      SCALE = REAL(NPTS)/REAL(NINT)
      DO 760 I = 0,LASADIS
        WRITE(21,FMT='(F5.1,5X,F10.6,6X,F10.6)')ADIS(I,1),
     *           (ADIS(I,2)*SCALE),(ADISP(I,2)*SCALE)
  760   CONTINUE
C
      GO TO 9900
C-----------------------------------------------------
C ERROR MESSAGES
C
 9801 WRITE(21,FMT='(" GAUSS ELIM NOT POSSIBLE")')
      WRITE(10,FMT='(" GAUSS ELIM NOT POSSIBLE")')
C-----------------------------------------------------------------------
 9900 CONTINUE
C
      STOP
      END
C
C-----------------------------------------------------------------------
C SUBROUTINE:  SPLINE(DATA,M,B,N)
C INPUTS: DATA       DATA
C         N          NUMBER OF DATA POINTS
C OUTPUT: M,B        SLOPE AND INTERCEPTS
C PURPOSE: DETERMINE THE LINEAR SPLINE FOR THE DATA GIVEN.
C-----------------------------------------------------------------------
C
C DECLARATIONS
C
      SUBROUTINE SPLINE(DATA,M,B,N)
C
      DOUBLE PRECISION      DATA(0:260,3)
      DOUBLE PRECISION      M(260)
      DOUBLE PRECISION      B(260)
      INTEGER               N                  !NUMBER OF POINTS MINUS ONE
      INTEGER               J                  !COUNTER
C
C---------------------------------------------------------------------
C MAIN EXECUTION
C
C CALCULATE SLOPE AND INTERCEPT BY THE FOLLOWING FORMULAS
C
C     M = (Y2 - Y1)/(X2 - X1)
C
C     B = (Y1*X2 - Y2*X1)/(X2 - X1)
C
C NOTE: THE I'TH INTERVAL IS FORMED FROM THE (I-1)TH AND ITH POINT
C
      DO 100 I = 1,N
        M(I) = (DATA(I,2)-DATA(I-1,3))/(DATA(I,1)-DATA(I-1,1))
        B(I) = (DATA(I-1,3)*DATA(I,1) - DATA(I,2)*DATA(I-1,1))/
```

```
     *          (DATA(I,1) - DATA(I-1,1))
  100   CONTINUE
C
   90 RETURN
      END

C
C-----------------------------------------------------------------------
C SUBROUTINE:  LOCATE(X,DATA,N,JINT)
C INPUTS: X                  X TO BE INTERPOLATED
C         DATA               DATA POINTS
C         N                  NUMBER OF DATA POINTS MINUS ONE
C OUTPUT: JINT               INTERVAL WHERE X LIES
C PURPOSE: DETERMINE THE INTERVAL WHERE X LIES WITHIN THE DATA
C----------------------------------------------------------------------
C DECLARATIONS
C
      SUBROUTINE LOCATE(X,DATA,N,JINT)
C
      DOUBLE PRECISION       X,DATA(0:260,3)
      INTEGER                JINT,N
C
C----------------------------------------------------------------------
C
C MAIN EXECUTION
C
      DO 10 JINT = 0,(N-1)
        IF(X.GE.DATA(JINT,1).AND.X.LE.DATA(JINT+1,1)) GO TO 20
   10   CONTINUE
      JINT = -2
      IF(X.GT.DATA(N,1)) JINT=N
   20 JINT = JINT + 1
      RETURN
      END
C---------------------------------------------------------------------
C SUBROUTINE:  SALPHA(X,M,B,DATA,JINT,SX)
C INPUTS: X       POINT OF INTERPOLATION
C         M,B     SPLINE PARAMETERS
C         DATA    DATA POINTS
C         JINT    INTERVAL WHERE X CAN BE FOUND
C OUTPUT: SX      S(X)
C PURPOSE:  EVALUATE SPLINE AT X
C---------------------------------------------------------------------
C DECLARATIONS
C
      SUBROUTINE SALPHA(X,M,B,DATA,JINT,SX)
C
      DOUBLE PRECISION       X
      DOUBLE PRECISION       M(260),B(260)
      DOUBLE PRECISION       DATA(0:260,3)
      DOUBLE PRECISION       SX
```

```
      INTEGER JINT
C
C---------------------------------------------------------------------
C MAIN EXECUTION
C
      SX = M(JINT)*X + B(JINT)
C
      RETURN
      END
C
C---------------------------------------------------------------------
C SUBROUTINE:  GAUSS(A,B,X,N,MAINDM,IERROR,RNORN)
C INPUTS: A             A MATRIX
C         B             B VECTOR
C         N             SIZE OF ACTUAL MATRIX AND VECTORS
C         MAINDM        DIMENSION OF A(MAINDM X MAINDM),B,AND X
C OUTPUT: X             X VECTOR
C         IERROR        ERROR INDICATOR
C         RNORM         MEASURE OF RELATIVE CALCULATION ERROR
C PURPOSE:  THIS ROUTINE APPLIES GAUSSIAN ELIMINATION WITH PARTIAL PIVOTING
C     TO SOLVE THE MATRIX EQUATION AX = B.
C
C ACKNOWLEDGEMENT:  THIS ROUTINE WAS TAKEN FROM LEE W. JOHNSON AND R. DEAN
C     RIESS, NUMERICAL ANALYSIS 2ND ED., ADDISON-WESLEY PUBLISHING CO., 1982,
C     PP. 29-30.
C---------------------------------------------------------------------
C
C DECLARATIONS
C
      SUBROUTINE GAUSS(A,B,X,N,MAINDM,IERROR,RNORM)

      DOUBLE PRECISION A(MAINDM,MAINDM)
      DOUBLE PRECISION B(MAINDM)
      DOUBLE PRECISION X(MAINDM)
      DOUBLE PRECISION AUG(50,51)     !AUGMENTED MATRIX A:B
      DOUBLE PRECISION TEMP           !TEMP STORAGE
      DOUBLE PRECISION PIVOT
      DOUBLE PRECISION Q
      REAL             RSQ
      REAL             RMAG           !MAGNITUDE OF R
      REAL             RESI
      REAL             RNORM
      INTEGER                   I,J,K          !COUNTERS
      INTEGER                   N              !SIZE OF DATA
      INTEGER                   MAINDM         !DIMENSIONS OF INPUT ARRAYS
      INTEGER                   IERROR         !ERR RETURN CODE
      INTEGER                   IPIVOT
C
C---------------------------------------------------------------------
C
C MAIN EXECUTION
```

```
C
      NM1 = N-1
      NP1 = N+1
C
C SET UP THE AUGMENTED MATRIX FOR AX=B
C
      DO 2 I=1,N
        DO 1 J=1,N
          AUG(I,J) = A(I,J)
    1     CONTINUE
        AUG(I,NP1) = B(I)
    2   CONTINUE
C
C THE OUTER LOOP USES ELEMENTARY ROW OEPRATIONS TO TRANSFORM
C THE AUGMENTED MATRIX TO ECHELON FORM.
C
      DO 8 I=1,NM1
C
C SEARCH FOR THE LARGEST ENTRY IN COLUMN I, ROWS I THROUGH N.
C IPIVOT IS THE ROW INDEX OF THE LARGEST ENTRY.
C
      PIVOT = 0.
      DO 3 J=I,N
        TEMP = ABS(AUG(J,I))
        IF (PIVOT.GE.TEMP) GO TO 3
        PIVOT = TEMP
        IPIVOT = J
    3   CONTINUE
      IF (PIVOT.EQ.0.) GO TO 13
      IF (IPIVOT.EQ.I) GO TO 5
C
C INTERCHANGE ROW I AND ROW IPIVOT.
C
      DO 4 K=I,NP1
        TEMP = AUG(I,K)
        AUG(I,K) = AUG(IPIVOT,K)
        AUG(IPIVOT,K) = TEMP
    4   CONTINUE
C
C ZERO ENTRIES (I+1,I),(I+2,I),....,(N,I) IN THE AUGMENTED MATRIX.
C
    5 IP1 = I + 1
      DO 7 K=IP1,N
        Q = -AUG(K,I)/AUG(I,I)
        AUG(K,I) = 0.
        DO 6 J = IP1,NP1
          AUG(K,J) = Q*AUG(I,J) + AUG(K,J)
    6     CONTINUE
    7   CONTINUE
    8 CONTINUE
      IF (AUG(N,N).EQ.0.) GO TO 13
```

```
C
C BACKSOLVE TO OBTAIN A SOLUTION TO AX=B.
C
      X(N) = AUG(N,NP1)/AUG(N,N)
      DO 10 K=1,NM1
        Q = 0.
        DO 9 J = 1,K
          Q = Q + AUG(N-K,NP1-J)*X(NP1-J)
    9     CONTINUE
        X(N-K) = (AUG(N-K,NP1) - Q)/AUG(N-K,N-K)
   10   CONTINUE
C
C CALCULATE THE NORM OF THE RESIDUAL VECOTR, B-AX.
C SET IERROR = 1 AND RETURN
C
      RSQ = 0.
      DO 12 I = 1,N
        Q = 0.
        DO 11 J = 1,N
          Q = Q + A(I,J)*X(J)
   11     CONTINUE
        RESI = B(I) - Q
        RMAG = ABS(RESI)
        RSQ = RSQ + RMAG**2
   12   CONTINUE
      RNORM = SQRT(RSQ)
      IERROR = 1
      RETURN
C
C ABNORMAL RETURN --- REDUCTION TO ECHELON FORM PRODUCES A ZERO
C ENTRY ON THE DIAGONAL.  THE MATRIX A MAY BE SINGULAR.
C
   13 IERROR = 2
      RETURN
      END
```

REFERENCES

1. Bakken, P.M. and T. Maseng, "Adaptive Control of Satellite EIRP to Reduce Outage Caused by Fading," IEEE Transactions on Communications, Vol. COM-31, No. 5, May 1983, pp. 726-734.

2. Dintelmann, F., "Analysis of 11 GHz Slant Path Fade Duration and Fade Slope," Electronics Letters, 2nd April 1981, Vol. 17, No. 7, pp. 267-268.

3. Jarett, D. and L.D. Spilman, "Impact of Rain Attenuation on 18/30 - GHz Satellite Systems," Progress in Astronautics and Aeronautics: Communications Satellite Developments: Technology, Vol. 42, 1976 (to be published), pp. 198-207.

4. Maseng, T. and P.M. Bakken, "A Stochastic Dynamic Model of Rain Attenuation," IEEE Transactions on Communications, Vol. COM-29, No. 5, May 1981, pp. 660-668.

5. Thirlwell, J. and R.G. Howell, "Slant-path Attenuation Measurements in the Range 12-30 GHz Using OTS and Passive Radiometers at Martlesham Heath," International Conference on Antennas and Propagation Part 2, IEEE Conference Publication Number 195, April 1981, pp. 29-35.

6. Lin, S.H., "Statistical Behavior of Rain Attenuation," Bell System Technical Journal, Vol. 52, No. 4, April 1973, pp. 557-581.

7. Andrzejewski, S.J., R.J. Brockway, H.S. Fitzhugh, and L.K. Harman, ATS-5 Millimeter Wave Propagation Experiment, ATS Technical Data Report, Final Report, Westinghouse Defense and Electronics Systems Center, Contract NAS5-21598, December 31, 1971.

8. Arnold, H.W. and D.C. Cox, "Fade Duration and Interfade Interval Statistics Measured on a 19-GHz Earth-Space Path," IEEE Transactions on Communications, Vol. COM-30, No. 1, January 1982, pp. 265-269.

9. "Propagation Data Required for Space Telecommunication Systems," (Report 564-2), Recommendations and Reports of the CCIR, 1982, Volume V, Propagation in Non-ionized Media, International Telecommunications Union, Geneva, Switzerland, 1982, pp. 331-373.

10. Cox, D.C. and H.W. Arnold, "Results from the 19- and 28-GHz COMSTAR Satellite Propagation Experiments at Crawford Hill," Proceedings of the IEEE, Vol. 70, No. 5, May 1982, pp. 458-488.

11. Craft , H.D., Jr., "CTR Notes: Attenuation Statistics at 15.3 GHz for Clarksburg, Maryland," COMSAT Technical Review, Vol. 1, No. 1, Fall 1971, pp. 221-225.

12. Davies, P.G., M.J. Courthold, and E.C. Mackenzie, "Measurements of Circularly-Polarized Transmissions from the OTS and SIRIO Satellites in the 11 GHz Band," International Conference on Antennas and Propagation Part 2, IEE Conference Publication Number 195, April 1981, pp. 76-80.

13. Davies, P.G. and E.C. Mackenzie, "Review of SHF and EHF Slant Path Propagation Measurements Made Near Slough (UK)," IEE Proceedings Part H, Vol. 128, No. 1, February 1981, pp. 53-65.

14. Davies, P.G. and E.C. Mackenzie, "Review of Slant Path Propagation Measurements Made at the Appleton Laboratory, Slough (UK)," AGARD Conference Preprints #284: Propagation Effects in Space/Earth Paths, London, May 1980, pp. 9-1 - 9-22.

15. Vogel, W.J., "CTS Attenuation and Cross Polarization Measurements at 11.7 GHz," Final Report Covering Period 16 June 1976 to 30 June 1979, The University of Texas at Austin, Electrical Engineering Research Lab, prepared for NASA Headquarters under contract NAS5-22576, June 1980.

16. Hyde, G., "ATS-6 Preliminary Results from the 13/18-GHz COMSAT Propagation Experiment," IEEE Transactions on Aerospace and Electronic Systems Vol. AES-11, No. 6, November 1975, pp. 1084-1094.

17. Ippolito, L.J., "ATS-6 Millimeter Wave Propagation and Communications Experiments at 20 and 30 GHz," IEEE Transactions on Aerospace and Electronic Systems, Vol. AES-11, No. 6, November 1975, pp. 1067-1083.

18. Ito, S., H. Fukuchi, C. Ohuchi, H. Hirano and I. Ono, "Statistics of Rain Attenuation and Other Environmental Effects Associated With the BSE Satellite Down-link at 12 GHz in Japan," IEEE Transactions on Broadcasting, Vol. BC-28, No. 4, December 1982, pp. 131-138.

19. Kumar, P.N., "Precipitation Fade Statistics for 19/29-GHz COMSTAR Beacon Signals and 12-GHz Radiometric Measurements," COMSAT Technical Review, Vol. 12, No. 1, Spring 1982, pp. 1-27.

20. Lin, S.H., H.J. Bergmann, and M.V. Pursley, "Rain Attenuation on Earth-Satellite Paths: Summary of 10-Year Experiments and Studies," The Bell System Technical Journal, Vol. 59, No. 2, February 1980, pp. 183-228.

21. Matricciani, E., "Duration of Rain-Induced Fades of Signal from SIRIO at 11.6 GHz," Electronics Letters, Vol. 17, No. 1, 1981, pp. 29-30.

22. Vogel, W.J., "CTS Attenuation and Cross Polarization Measurements at 11.7 GHz," Final Report Covering the Period 1 February 1978 to 31 January 1979, The University of Texas at Austin, Electrical Engineering Research Lab, Prepared for Goddard Space Flight Center under contract NAS5-22576.

23. "Centimeter Wave Propagation Experiments by Using Japan's Geostationary Satellite BSE," Radio Research Laboratories, Ministry of Posts and Telecommunications, March 1983.

24. Tang, D.D. and D. Davidson, COMSTAR 19/29-GHz Propagation Experiment II. 1979-1980, GTE Laboratories Incorporated, Communications Products Technology Center, Waltham, MA, June 22, 1981, pp. ii-v, 1-9, 35-44, 69-73.

25. Tang, D.D., D. Davidson and S.C. Bloch, "Diversity Reception of COMSTAR Satellite 10/29-GHz Beacons with the Tampa Triad, 1978-1981," Radio Science, Vol. 17, No. 6, November-December 1982, pp. 1477-1488.

26. Vogel, W.J., A.W. Straiton, B.M. Fannin and N.K. Wagner, "Attenuation Diversity Measurements at 20 and 30 GHz," Radio Science, Vol. 11, No. 3, March 1976, pp. 167-174.

27. Nackoney, O.G. and D. Davidson, "Results of 11.7-GHz CTS Rain Attenuation Measurements at Waltham, Massachusetts," Radio Science, Vol. 17, No. 6, November-December 1982, pp. 1435-1442.

28. Morita, K., "Estimation Methods for Propagation Characteristics on Earth-Satellite Links in Microwave and Millimeter Wavebands," Review of the Electrical Communication Laboratory, Vol. 28, Nos. 5-6, May-June 1980, pp. 459-471.

29. Murakami, K., "A Method for Calculating the Statistics of Rain Fade Duration on Earth-Space Paths," Transactions of the IECE of Japan, Vol. E65, No. 6, Abstracts, pg. 371.

30. Maseng, T. and P.M. Bakken, "A Stochastic Dynamic Model of Rain Attenuation," AGARD #238: Operational Modeling of the Aerospace Propagation Environment, Session X - EHF Propagation, Paper 50, Vol. II, 1978.

31. Bostian, C.W., T. Pratt and W.L. Stutzman, "Operation of the SIRIO Transportable Receiving Terminal: Final Report Covering January 1979 - December 1981," Report VPI&SU/EE/SATCM-83/5, August 1983, for INTELSAT Contract IS-901.

32. Ljung, L. and T. Soderstrom, Theory and Practice of Recursive Identification, Cambridge, Mass.: MIT Press, 1983, pp. 1-135.

33. Papoulis, A., Probability, Random Variables, and Stochastic Processes, New York: McGraw-Hill Book Company, 1965, p. 162.

34. Johnson, L.W. and R.D. Riess, Numerical Analysis, 2nd edition, Reading, Mass.: Addison-Wesley, 1982, pp. 29-30, 71-73.

35. NASA ACTS Experiments Meeting, March 26, 1985, Crystal City, VA..

36. "Attenuation by Hydrometeors in Particular Precipitation and Other Atmospheric Particles," (Report 721-1), Recommendations and Reports of the CCIR, 1982, Volume V, Propagation in Non-ionized Media, International Telecommunications Union, Geneva, Switzerland, 1982, pp. 167-181.

37. "Radiometeorological Data," (Report 563-1), Recommendations and Reports of the CCIR, 1982, Volume V, Propagation in Non-ionized Media, International Telecommunications Union, Geneva, Switzerland, 1982, pp. 96-123.

38. Matricciani, E. and A Paraboni, "Instantaneous Frequency Scaling of Rain Attenuation at 11.6-17.8 GHz with SIRIO Data," IEEE Transactions on Antennas and Propagation, Vo. AP-33, No. 3, March 1985, pp. 335-337.

39. Ippolito, L.J., R.D. Kaul and R.G. Wallace, Propagation Effects Handbook for Satellite Systems Design, NASA Reference Publication 1083(03), third edition, June 1983.

40. D'Alessio, T. and U. Merlo, "Synthesis of Models for Rain Induced Attenuation," IEE International Conference, Antennas and Propagation Part 2, Publication Number 169, November 1978, pp. 172-176.

41. Goldhirsh,J., "Slant Path Fade and Rain-Rate Statistics Associated with the COMSTAR Beacon at 28.56 GHz for Wallops Island, Virginia Over a Three-Year Period," IEEE Transactions on Antennas and Propagation, Vol. AP-30, No. 2, March 1982, pp. 191-198.

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