CHAPTER 3

VERIFICATION OF PROPOSED EQUATIONS FOR THE EFFECTIVE MOMENT OF INERTIA OF STEEL JOIST - CONCRETE SLAB SYSTEMS

3.1 Overview

The proposed equations in the AISC Guide (Murray et al. 1997) for the effective moment of inertia of a steel joist-concrete slab system, Equations (1.7) and (1.8), are based on research from Kitterman (1994) and Band (1996), and a theoretical derivation. The modeling techniques developed in Chapter 2 were used to verify the validity of Equation (1.7) as described in the following sections. Equation (1.7) is used to estimate the effective moment of inertia, I_{eff}, when computing the fundamental frequency of a composite joist or joist girder.

3.2 Description of Procedure to Verify I_{eff}

3.2.1 Development of Test Cases

The joists used in this study to verify Equation (1.7) are from a study Kitterman (1994) did to develop Equation (1.3). The joists are a set of 25 joist and joist girder designs. A detailed description of the joists is presented in Appendix B. The concrete slabs developed for the joists were based on the width and thickness of a slab which would reasonably be used in a building. Table 3.1 is the test matrix of slab thickness and widths used. In Table 3.1, the Joist Group refers to the type of joist being modeled, the term RB stands for joists with round bar webs. The round bar web joists have depths which range from 10 in. to 24 in. The numbers listed across the top of the table refer to the range of depths for angle web joists associated with the slab configurations listed in that column. The letters A, B, C, and D represent different slab configurations, defined in a footnote to the table. The numbers in the spacing section of the table are the various widths that are used for the slabs. Within each column of the test matrix, every combination of slab thickness and slab width is modeled on each joist in the joist group.

Table 3.1 Test Matrix for Joist-Slab Cases

Joist Group	RB	20-30	32	36-40	48-72
Slab Type	I II III	II III	III IV	III IV	III IV
Spacing (in.)	24	24 36	24 36	36 48	48 60 120

Notes: The slab types in Table 3.1 are defined as follows:

I: 0.6 in. deck, 1.9 in. slab, 2.5 in. total thickness
II: 1.0 in. deck, 2.5 in. slab, 3.5 in. total thickness
III: 2.0 in. deck, 3.25 in. slab, 5.25 in. total thickness
IV: 3.0 in. deck, 3.5 in. slab, 6.5 in. total thickness

In addition to the above set of cases, a maximum slab case was developed for each of the 25 joists. This slab width is defined as four-tenths of the length of the beam, 0.4L. To size the steel deck, the minimum height of the deck ribs is defined as the width of the slab divided by 120. Once the steel deck is chosen, 3.5 in. of concrete is modeled as the concrete depth above the ribs.

3.2.2 Description of the Computer Model Used

The program SAP2000 (Wilson and Habibullah 1996) was used to model each joist, using Model D as defined in Section 2.5.1. The details of this model are given in Sections 2.3.5 and 2.3.6. Every model used pinned-roller support conditions for the joist. The dynamic modulus of elasticity of the concrete was used, which is 1.35 E_c. For all cases a concrete unit weight of 145 pcf and a f_c’ of 3000 psi was used, which resulted in a dynamic modulus of elasticity of 4270 ksi.

3.2.3 Method of Analysis

For each of the 130 cases, the value for the effective moment of inertia, I_{eff}, was calculated from Equation (1.7). This value was then used to calculate the first natural frequency, f_n, from Equation (1.1). Appendix D shows a sample calculation for I_{eff} and f_n. SAP2000 was then used to develop finite element model results for each case as described in Section 3.3.2. The values for I_{eff} and f_n were obtained from the analysis of the models as follows. To determine I_{eff}, a uniform load acting as a floor load was applied to the frame elements representing the slab. Once the model was analyzed, the centerline deflection due to this uniform load was obtained and used to back calculate the moment of inertia of the finite element model, using Equation (3.1):

$$I_{eff} = \frac{5wL^4}{384E\Delta_{cl}} \tag{3.1}$$

where w is the uniform load, kips/in.; L is the joist span, in.; E is the modulus of elasticity, ksi; Δ_{cl} is the centerline deflection of the joist, in.; and I_{eff} is the effective moment of inertia of the joist-slab system, in^4. The frequency of the model was determined by SAP2000 using an eigenvector analysis. Only the first mode of vibration was calculated, giving the first natural frequency of the model.

3.3 Comparison of Results

The results for the 130 cases analyzed are found in Appendix C. This data shows the results for I_{eff} and f_n determined from both the SAP2000 analysis and the values determined using Equations (1.7) and (1.1). The purpose for this data is to determine if the values calculated by Equations (1.7) and (1.1) can accurately predict the values determined from SAP2000, which are treated as the true behavior of the steel joist-concrete slab tee-beam system in this study. A statistical analysis was done on similar groups of data. These groups of data were separated based on the web type, round bar or angle, and on the type of value determined, I_{eff} or f_n. In all, four separate sets of data were created so that reasonable comparisons of the results within each group could be made.

3.3.1 Analysis of Results for I_{eff}

Once the data for I_{eff} was separated into round bar web cases and angle web cases, a statistical analysis was done on each set. First, the ratio of the SAP2000 generated value for I_{eff} was divided by the value obtained from Equation (1.7). These ratios are listed for each case in Appendix C. For each web group, the average, standard deviation, and coefficient of variation of the ratios was calculated. These values are listed in Table 3.2. The coefficient of variation, which is the standard deviation divided by the average, is listed in the form of a percent.

Table 3.2 Statistical Calculations for I_{eff} Results

	Round Bar Web	Angle Web
Average of Ratios	1.117	0.956
Standard Deviation	0.14	0.054
Coefficient of Variation (%)	12.5	5.64

From the coefficient of variation, the variability of the data set about its average can be quantified. The smaller the coefficient of variation, the more related the data points are to each other. In this study, a value of less than 10% for the coefficient of variation indicates well related data.

In addition to the statistical analysis, a graph was developed for the round bar and angle web data sets. These graphs plot the value for I_{eff} obtained from Equation (1.7) against the value obtained from SAP2000. The solid line on these graphs represents the line of exact correlation between the values from Equation (1.7). and the values from SAP2000. Values above this line indicate that Equation (1.7) values are smaller than those obtained by SAP2000, and values below this line indicate the reverse.

From the values contained in Table 3.2, and from the trends in the data seen in Figures 3.1 and 3.2, some observations can be made. First, from Table 3.2, it can be seen that the average of the ratios for round bar web joists is 1.117. This average is almost 12% above the ratio for an exact match of the values, or 1.0. This value indicates that, on average, the value for I_{eff} obtained from Equation (1.7) for round bar web joists is about 12% lower than the value obtained from SAP2000. Also, the coefficient of variation for this data set is 12.5%, which shows that the data is more variable than the less than 10% variation which is desired. Second, Figure 3.1 shows the round bar web data plotted against each other. As can be seen from the graph, most of the data points are above the line and are not in a tight band, as quantified by the coefficient of variation.

For the angle web cases, the following observations can be made. The average of the ratios for this set of data is 0.956. This average indicates that Equation (1.7) is calculating values very close to those obtained by SAP2000 for angle web joists. Also, the coefficient of variation is only 5.6%, which is well within the less than 10% range desired. This fact indicates a tight grouping of the data. Both of the above mentioned observations can be seen graphically in Figure 3.2. Almost all of the data points are close to the line representing an exact match of values in I_{eff}.

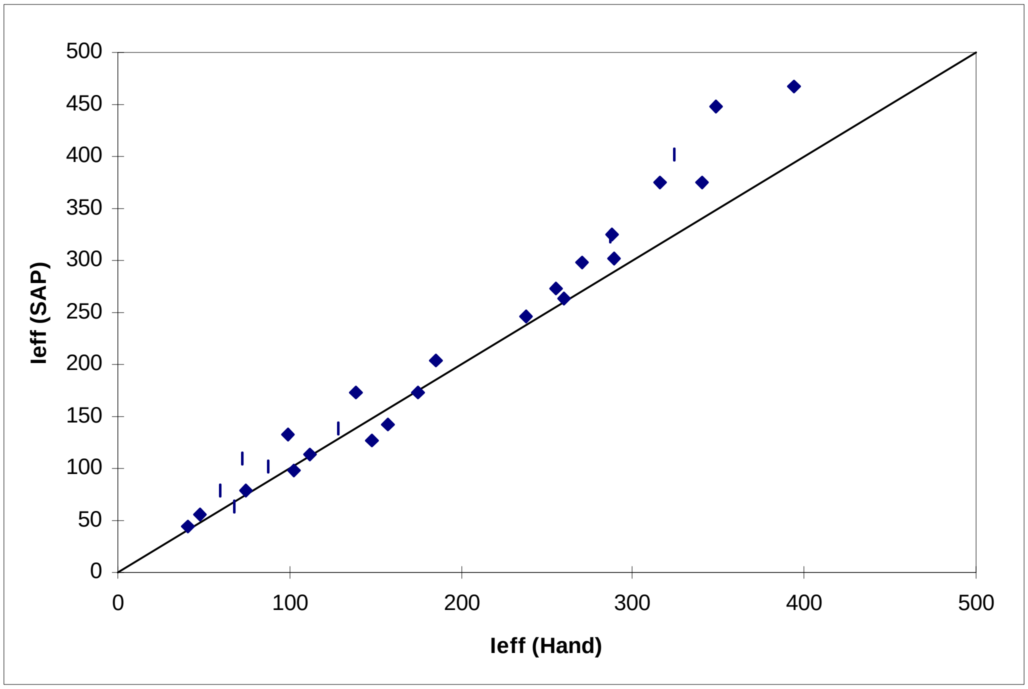

Figure 3.1 SAP2000 vs. Hand Results for I_{eff} of Round Bar Web Joists

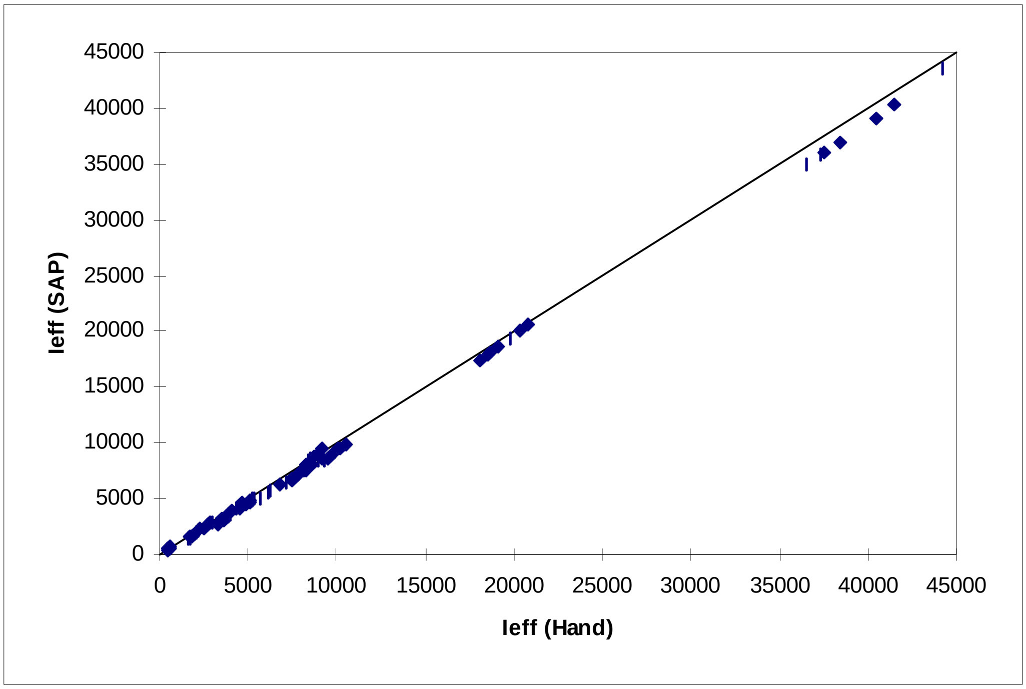

Figure 3.2 SAP2000 vs. Hand Results for I_{eff} of Angle Web Joists

3.3.2 Trend Lines for I_{eff}

To further investigate the ability of Equation (1.7) to predict the results obtained by SAP2000, another set of graphs was developed. These graphs plot the I_{eff} determined from SAP2000 divided by the fully composite moment of inertia against the span-to-depth ratio of the joist. To obtain the required relationship, both sides of Equation (1.7) were divided by I_{comp} to obtain

$$\frac{I_{eff}}{I_{comp}} = \frac{1}{\frac{\gamma}{\frac{I_{chords}}{I_{comp}}} + 1} \tag{3.2}$$

The non-dimensional term I_{chords}/I_{comp} was calculated for every case. A maximum, minimum, and average value was found for both the round bar web cases and the angle web cases. These values were used to generate three trend lines shown in Figures 3.3 and 3.4. The data was then plotted on top of these trend lines. If the relationships for determining the reduction factors for round bar webs and angle webs give results similar to that of SAP2000, then the data points should generally be within the maximum and minimum trend lines.

For the round bar web cases, the data points are generally above the average trend line with most of the data points above the maximum trend line. This observation indicates that the expression for the reduction factor of round bar web joists given in Equation (1.5) may need further study. For the angle web case, the data points are generally within the trend lines.

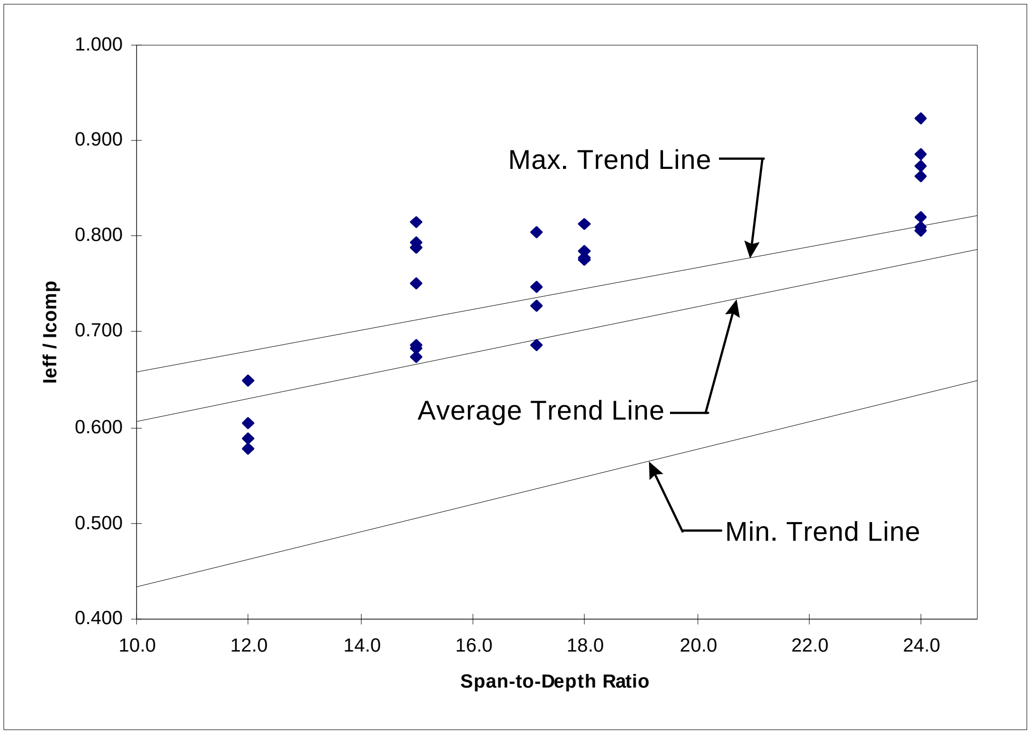

Figure 3.3 I_{eff}/I_{comp} vs. L/D for Round Bar Web Joists

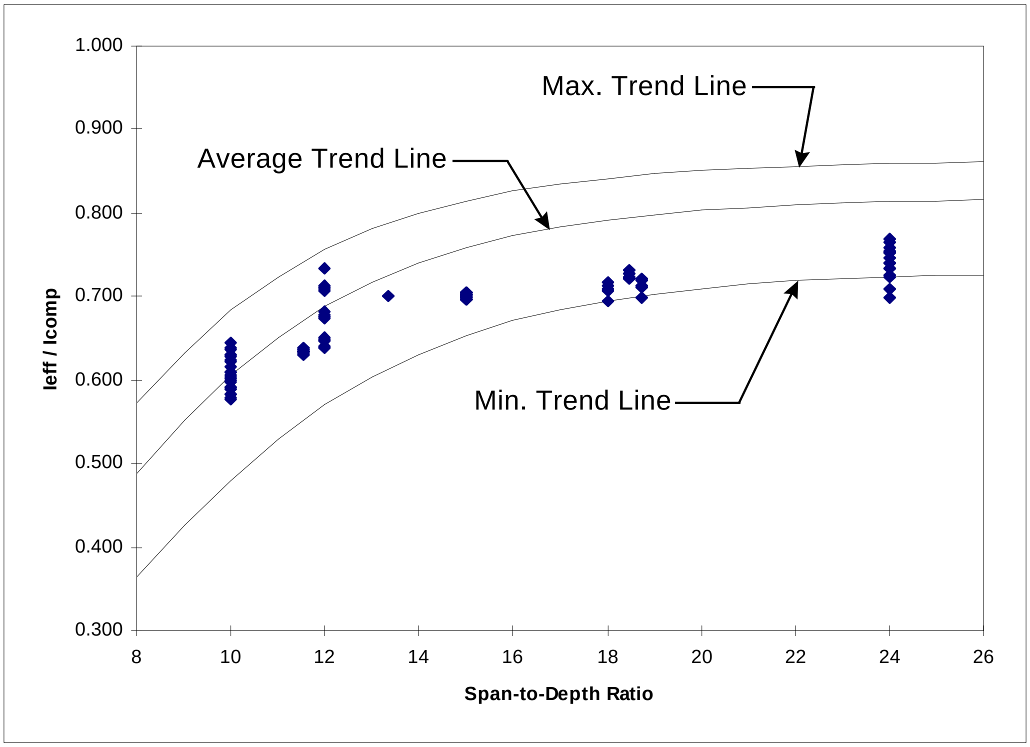

Figure 3.4 I_{eff}/I_{comp} vs. L/D for Angle Web Joists

3.3.3 Analysis of Results for Frequency

The analysis of the results for the first natural frequency, f_n, are similar to those done in Section 3.4.1. Analyzing these results can determine if Equation (1.1), using the value for moment of inertia determined by Equation (1.7), can predict a value of f_n that is similar to the value obtained from SAP2000. The values for f_n were separated based on the web type of the joist, round bar or angle. As in Section 3.4.1, the ratio of the SAP2000 value divided by the value calculated from Equation (1.1) was determined for each case. A statistical analysis similar to that in Section 3.4.1 was done on both sets of ratios. Table 3.3 contains the average, standard deviation, and the coefficient of variation, in percent, of the ratios.

Table 3.3 Statistical Calculations for Frequency Results

	Round Bar Web	Angle Web
Average of Ratios	1.051	0.976
Standard Deviation	0.065	0.030
Coefficient of Variation (%)	6.23	3.04

Similar to Section 3.4.1, a graph was developed for both the round bar and angle web data sets. These graphs plot the value for f_n obtained from Equation (1.1) against the value obtained from SAP2000. The line on these graphs represents the line of exact correlation between the values from Equation (1.1). and the values from SAP2000. Values above this line indicate that the value from Equation (1.1) is predicting a value smaller than the one obtained by SAP2000, and values below this line indicate the reverse.

From the data shown in Table 3.3 and the graphs in Figures 3.5 and 3.6, several observations can be made. The average of the ratios for the round bar web cases is 1.051. This value is only 5% above the ratio for an exact match of values. The coefficient of variation, which is 6.23%, is well within the range deemed acceptable for this study. The statistical values for the angle web cases are even better, with an average of 0.976 and a coefficient of variation of 3.04%. Figure 3.5 shows that while a majority of the data points are above the line, they are closer to the line than the values in Figure 3.3, which dealt with I_{eff} of round bar web joists. Figure 3.6 shows almost all of the data points for angle web joists grouped right on the line.

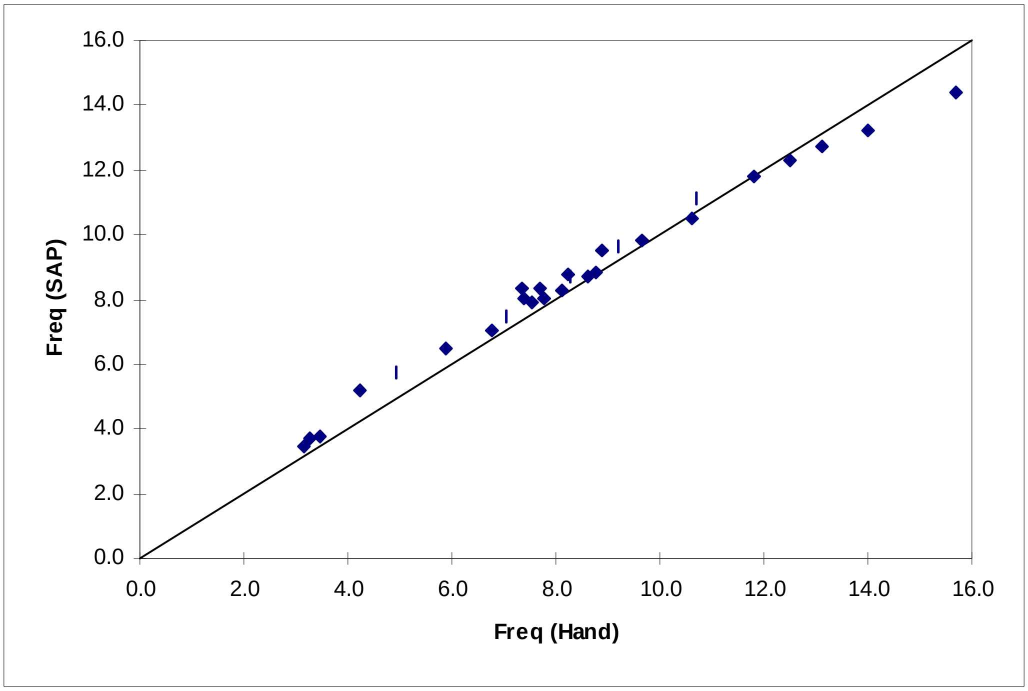

Figure 3.5 SAP2000 vs. Hand Results for Frequency of Round Bar Web Joists

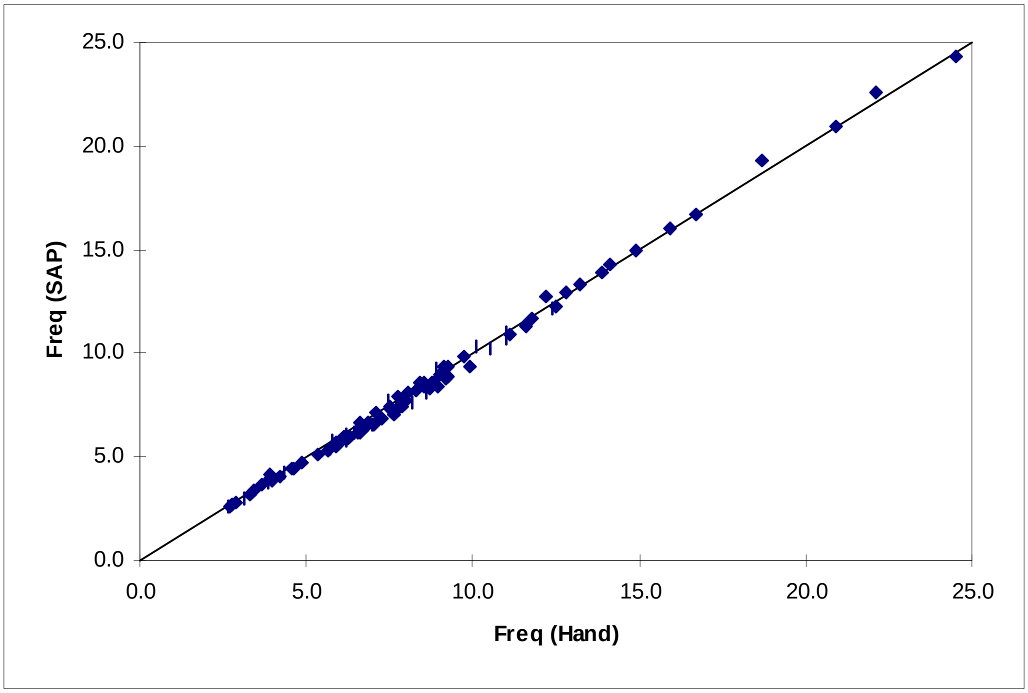

Figure 3.6 SAP2000 vs. Hand Results for Frequency of Angle Web Joists

3.4 Conclusions

From the data presented and the analysis done in Section 3.4, several c onclusions can be drawn. First, Equation (1.7) predicts values for I_{eff} very close to the values predicted by SAP2000. For angle web joists, Equation (1.7) is very good, predicting values within 5% of the value obtained from SAP2000, on average. For round bar web joists, Equation (1.7) is not as good at predicting the values obtained from SAP2000. Though the values predicted by Equation (1.7) for round bar web joists are generally within 12% of those predicted from SAP2000, it would be desirable to have Equation (1.7) predict values within 10%. A second conclusion is that the values of f_n calculated using Equation (1.1) are very good at predicting the value obtained from SAP2000 for both round bar web and angle web joists. This equation predicts values for f_n within 5% of the values predicted from SAP2000, on average.