Predicting bark thickness with one- and two-stage regression models for three hardwood species in the southeastern US

[Author Information]

First and corresponding author: Dr. Sheng-I Yang. Department of Forestry, Wildlife and Fisheries, University of Tennessee. 274 Ellington Plant Sciences Building, Knoxville, TN 37996 USA. Email: syang47@utk.edu.

Second author: Dr. Philip J. Radtke. Department of Forest Resources & Environmental Conservation, Virginia Tech. 319E Cheatham Hall, Blacksburg, VA 24061 USA. Email: pradtke@vt.edu

Predicting bark thickness with one- and two-stage regression models for three hardwood species in the southeastern US

Abstract

Tree bark, as the outermost protective layer of tree stems, is an important indicator to evaluate the fire resistance properties of trees and to assess the tree mortality induced by fire. Despite its importance, many existing bark thickness models were not primarily developed for predicting bark thickness directly, i.e. with bark thickness as a response variable, and most past studies were focused on modeling bark thickness in conifers. Thus, the objective of this study was to compare the efficacy of various bark thickness models/methods for three common hardwood species in the southeastern US. A total number of 47,281 measurements from 2,070 trees were used in analysis.

Results showed that bark thickness at breast height (1.37 m or 4.5 ft above ground) varies by tree size and species, which can be predicted by a species-specific linear regression model with DBH as a single predictor. To predict bark thickness profile, a combination of stem taper function and bark thickness model, a two-stage method, is suggested, which generally performs better than a single bark thickness function (one-stage method) in terms of bias and precision. For a given model form, the two-stage method produced more reliable prediction of bark thickness at upper and lower portions of tree stem than the one-stage method. With the three species examined, the segmented stem taper functions provided more accurate predictions than the variable-exponent function. The results of this study can provide guidance for ecologists and forest managers when selecting appropriate approaches to predict bark thickness.

[**Keywords**] white oak, yellow popular, red maple, stem taper function, cluster bootstrap

1 Introduction

Tree bark is an outermost protective layer of tree stem, which insulates the cambium and living tissues from fire damage, insect attack or infection from disease. In general, the thickness of the bark increases with the increase of tree size and age. Bark thickness varies by genetics, species, environmental and climatic conditions (Kohnle et al., 2012; Laasasenaho et al., 2005; Nolan et al., 2020; Sonmez et al., 2007; Stängle et al., 2017). In forest fire ecology, bark thickness at breast height (BT_{bh}; 1.37 m or 4.5 ft above ground) is an important quantity to evaluate fire resistance properties of trees and to assess tree mortality induced by fire (Nolan et al., 2020; Roula et al., 2020). Diameter at breast height over bark (DBH) is the most widely-used tree variable to predict BT_{bh} because DBH is highly correlated with bark thickness and easy to measure in the field. By examining eight conifer species in Klamath and Sierra Nevada Mountains of California, Zeibig-Kichas et al. (2016) pointed out that the relationships between DBH and BT_{bh} vary among species. A simple linear regression (SLR) model with the square root of DBH as a single predictor provided reliable predictions of BT_{bh} for eight species they examined (Zeibig-Kichas et al., 2016). By comparing sixteen tree species in the central hardwood region of North America, Hengst and Dawson (1993) found that a SLR model can reasonably well depict the relationships between DBH and BT_{bh} for most species.

In addition to a single bark thickness measure, accurate information on bark thickness at any given height along tree stem is also of interest, especially for calculating bark biomass and volume (e.g., cork industry in the Mediterranean region) (Costa et al., 2020; Sánchez-González et al., 2021). The thickness of the bark generally decreases with increasing height from the bottom to the tree top (Costa et al., 2020; Hammond et al., 2015). In forest modeling, bark thickness at a given point of the stem is often calculated by subtracting inside bark diameter (dib) from outside bark diameter (dob). Stem taper functions are often implemented to predict dob and dib in standing trees, but the compatibility between the two quantities must be considered when the method is applied (i.e., dob $\geq$ dib) (Burkhart and Tomé, 2012). An alternative approach is to predict dob by stem taper functions, and then to predict the dib using predicted dob as the independent variable in an equation often referred to as a "bark thickness model" (e.g., Li and Weiskittel, 2011). Li and Weiskittel (2011) compared the performance of eight bark thickness models for seven conifer species in the Acadian Region of North America. Their results showed that the model proposed by Cao and Pepper (1986) performed better than other models (Li and Weiskittel, 2011). Stängle et al. (2017) found that prediction accuracy was improved when models with more variables were used for Norway spruce in southwest Germany; however, in the past the accuracy of dob predicted from taper equations was not always considered when comparing bark thickness models. The effect of taper equation selection on bark thickness prediction accuracy remains unclear.

A reason many existing bark thickness models predict dib rather than bark thickness is because dib is used to calculate under-bark, wood volume in timber production models (e.g., Cao and Pepper, 1986). As a result, bark thickness is not directly predicted in some bark thickness models (i.e., bark thickness is not the response variable in the model). With increasing concerns over drought and wildfire under climate change, bark thickness becomes an important indicator to quantify and track changes in stand structure and production of ecosystem services following fire (Nolan et al., 2020). In the southeastern US, forest sustainability in mixed hardwood forests has received increased attention because of decreasing abundance of certain valuable species, such as oaks, in the region (e.g., Oak et al., 2015). Meanwhile, the abundance of red maple has rapidly increased in formerly oak-dominant forests due to fire suppression and extensive logging (Fei and Steiner, 2007). Prescribed fire has proven useful in control of red maple to promote oak regeneration because red maple invests the least in bark thickness, making it particularly susceptible to fire. (e.g., Brose et al., 1999; Hammond et al., 2015; Thomas-Van Gundy et al., 2014). Thus, developing reliable models for bark thickness is necessary to inform forest management and environmental conservation efforts. In addition, developing tree bark profiles for coniferous species has been widely investigated (e.g., Li and Weiskittel, 2011; Maguire and Hann, 1990; Stängle et al., 2017), but fewer efforts have been aimed at modeling bark thickness for hardwoods.

The objectives of this study were 1) to examine relationships between BT_{bh} and DBH among species and crown classes, 2) to compare the efficacy of various models and methods for predicting BT_{bh}, and 3) to compare methods and model forms for predicting bark thickness profiles. As a part of the third objective, a combination stem taper function and bark thickness model was compared to a single bark thickness profile function. According to the objectives, the hypotheses were developed as 1) BT_{bh} and DBH follow a linear relationship regardless of species, 2) adding crown class can enhance the predictability of the model, and 3) bark thickness profiles can be more accurately predicted when increasing number of parameters in the taper equations. Cluster bootstrap, as a nonparametric approach, was used to account for the within-tree correlation. Red maple (*Acer rubrum*), white oak (*Quercus alba*), and yellow poplar (*Liriodendron tulipifera*) as three common species in the mixed-hardwood forests in the southeastern US (Burns and Honkala, 1990), were examined. In pursuing these research objectives, overall goals of the work were to present models that could be used directly or provide guidance for ecologists and forest managers when selecting other models to use when predicting bark thickness.

2 Materials and Methodology

2.1 Tree bark Data

Tree data for the three species were collected from the LegacyTree Database (`http://www.legacytreedata.org`), a large collection of individual tree data compiled from North American trees sampled in the past century (Radtke et al., 2015). Sample trees used in this study were collected from 13 states in the southeastern US, including Alabama, Arkansas, Florida, Georgia, Kentucky, Louisiana, Mississippi, North Carolina, Oklahoma, South Carolina, Tennessee, Texas and Virginia. Felled trees with dob, dib, DBH and total tree height measured were used in analysis. For a given tree, the observed single bark thickness (BT, mm) was computed as:

$$BT = 10 * [0.5 * (dob - dib)] \quad (1)$$

where 10 is the number of mm in 1 cm, and dob and dib are diameters (cm) measured at points along the main stem outside and inside bark, respectively. Observed $\mathrm{BT_{bh}}$ was calculated in the same way on trees having outside and inside bark measurements at breast height. Irregular observed bark thickness (e.g., negative values) were manually examined and removed. A total number of 47,281 observations from 2,070 trees were used in analysis. A summary of tree characteristics for each species is given in Table 1.

2.2 Prediction of bark thickness at breast height ($\mathrm{BT_{bh}}$)

As suggested by Hengst and Dawson (1993), SLR was used to describe relationships between $\mathrm{BT_{bh}}$ and DBH for each species. The model can be written as:

$$BT_{bh} = \beta_0 + \beta_1 * DBH \quad (2)$$

where $\beta_0 - \beta_1$ are model coefficients.

In addition, crown class was added to the regression model to examine the explanatory power of crown class as a predictor when DBH is already included as a predictor. That is,

$$BT_{bh} = \beta_0 + \beta_1 * DBH + \beta_2 * Cr \tag{3}$$

where $\beta_0 - \beta_2$ are model coefficients, and

$$\begin{aligned} Cr &= 0,\ for\ DC \\ &= 1,\ for\ IS \end{aligned}$$

where DC is dominant/codominant trees, and IS is intermediate/suppressed trees. The equation 3 was only fitted by the trees with crown class measured (i.e., 455 of 2070 trees). Estimates, the upper and lower bounds of 95% confidence intervals of model coefficients were computed by the median, 2.5% and 97.5% quantiles of 1,000 bootstrap samples.

Table 1: Sample characteristics for data used in the study from three hardwood species: number of sample trees (N); average number of measurement points per tree ($N_{obs/tree}$); total height (Ht); DBH; and single bark thickness at breast height (BT_{bh}) (standard deviations in parentheses).

Species	N	$N_{obs/tree}$	Total Ht (m)	DBH (cm)	BT_{bh} (mm)
Red maple	251	20(7.8)	20.6(5.4)	28.0(12.7)	8.7(4.1)
White oak	943	23(6.6)	22.4(4.5)	34.2(11.4)	13.6(4.8)
Yellow poplar	876	24(7.0)	27.0(6.9)	35.5(14.6)	16.7(6.3)

2.3 Prediction of bark thickness profile

In this study, two approaches for modeling bark thickness were evaluated. The first approach predicts bark thickness profiles directly from tree-level predictors DBH and Ht and the height (h, m) of prediction using a single functional form, namely $\mathrm{BT} = f(\mathrm{DBH}, \mathrm{h/Ht})$. The second procedure uses a two-step approach, first predicting dob at h using a taper equation fitted to the available data, followed by a prediction of BT from the predicted dob and relative height information, i.e. $\mathrm{BT} = f(\hat{\mathrm{dob}}, \mathrm{h/Ht})$. The first and second approaches are denoted in the following sections as one-stage and two-stage methods, respectively.

2.3.1 One-stage method

Four equations were selected for testing as bark thickness profile models. The first two were simple quadratic and polynomial functional forms, The second two were segmented and variable-exponent model forms adapted from taper functions previously developed for use in modeling tree stem diameter profiles.

2.3.1.1 Quadratic regression equation

The first model has the simplest form, predicting bark thickness as a fraction of DBH with only relative height as a predictor.

$$BT/DBH = a_0 + a_1 * (h/Ht) + a_2 * (h/Ht)^2 \tag{4}$$

where BT is bark thickness in mm at any given height h (m) above ground, DBH is diameter at breast height in cm, Ht is total tree height in m, and $a_0 - a_2$ are model coefficients.

2.3.1.2 Polynomial regression equation

The form of the second model is similar to the one proposed by Cao and Pepper (1986). Its form is similar to the first regression Eq. (4), but with total tree height included as an additional predictor.

$$BT/DBH = a_0 + a_1 * (h/Ht) + a_2 * (h/Ht)^2 + a_3 * Ht \quad (5)$$

where $a_0 - a_3$ are model coefficients. Other variables remain as defined above.

2.3.1.3 Segmented polynomial regression equation

The segmented polynomial regression model proposed by Max and Burkhart (1976) has proven to well describe the various shapes of sections along the tree bole. Max and Burkhart (1976) reported that the four-parameter model with the join points of 11% and 75% of total tree height performed better than the eight- or six-parameter ones. In the original model, the squared ratio as the response variable was incorporated (Max and Burkhart, 1976), but Yang and Burkhart (2020) found that the model with the plain ratio (i.e., first order ratio) provided more accurate predictions than the one using the squared ratio. To be comparable with other models, the model with the plain ratio was used in this study. That is,

$$BT/DBH = a_0(x - 1) + a_1(x^2 - 1) + a_2(0.11 - x)^2 I_1 + a_3(0.75 - x)^2 I_2 \quad (6)$$

where x is h/Ht, $a_0 - a_3$ are model coefficients,

$$I_1 = 1, \; if \; 0.11 \geq x$$
$$= 0, \; if \; 0.11 < x$$

, and

$$I_2 = 1,\ if\ 0.75 \geq x$$
$$= 0,\ if\ 0.75 < x.$$

2.3.1.4 Variable exponent equation

The fourth model was a variable exponent function adapted from Kozak (2004). The six-parameter model can be written as:

$$BT = a_0 \mathrm{DBH}^{a_1} K^z \tag{7}$$

where

$$K = [1 - (\frac{h}{Ht})^{\frac{1}{4}}]/[1 - (0.01)^{\frac{1}{4}}]$$
$$Z = a_2 + a_3(1/e^{(\frac{\mathrm{DBH}}{Ht})}) + a_4 \mathrm{DBH}^K + a_5 K^{(\frac{\mathrm{DBH}}{Ht})}$$

, and $a_0 - a_5$ are model coefficients. Other variables remain as defined above.

2.3.2 Two-stage method

Unlike the single equation forms developed in the one-stage method, this method developed equations for implementation in two stages. The first stage predicts dob using an equation adapted from existing stem taper functions. The second stage subsequently predicts bark thickness using the dob predicted in the first stage. To predict $\hat{dob}$ (stem taper) in the first stage, the same model forms as Eqs. (4)-(7) were considered, but BT on the left-hand-side of the equation was replaced with dob. In the second stage, the equation proposed by Cao and Pepper (1986) was modified as

$$BT = dob[b_0 + b_1 * (h/Ht) + b_2 * (h/Ht)^2 + b_3 * Ht] \quad (8)$$

where $b_0 - b_3$ are model coefficients and other variables as defined above. In the two-stage method, there were four different model combinations: four Eqs. (4)-(7) modified to predict dob $\times$ Eq.(8) for BT.

2.3.3 Parameter estimation

The cluster bootstrap technique was used in parameter estimation in order to account for the repeated measurements within trees. Cluster bootstrapping is a nonparametric resampling procedure commonly used for clustered or multi-level data when the assumption of independent observations is not appropriate (Davison and Hinkley, 1997; Field and Welsh, 2007; Ren et al., 2010). Unlike the basic bootstrap algorithm, observations in cluster bootstrapping are selected in groups or clusters that mimic the data generation mechanism of selecting subjects (trees) for taper model development; thus, the correlation structure of observations within each cluster is retained. In this study, individual trees served as clusters and were randomly selected with replacement for a given species. Then all measurements of the selected trees were used in model fitting. The process was repeated 1,000 times (i.e., 1,000 bootstrap samples). Model fitting was conducted using the nlsLM function in R (Elzhov et al., 2016). Estimates, the upper and lower bounds of 95% confidence intervals of model coefficients were computed by the median, 2.5% and 97.5% quantiles of 1,000 bootstrap samples.

2.4 Model evaluation

2.4.1 Bark thickness at breast height ($\mathrm{BT_{bh}}$)

To evaluate the model performance, the mean bias (MB) and root mean square error (RMSE) for $\mathrm{BT_{bh}}$ were calculated as:

$$\mathrm{MB} = \frac{\sum e}{N}, \tag{9}$$

$$\mathrm{RMSE} = [\frac{\sum e^2}{N}]^{\frac{1}{2}}, \tag{10}$$

where N is the total number of sample trees, and the residuals for $\mathrm{BT_{bh}}$ (e) were calculated as:

$$e = BT_{bh} - \hat{BT}_{bh}$$

where $\hat{BT}_{bh}$ is the predicted $\mathrm{BT_{bh}}$ (mm). In addition to examining the SLR models (Eqs. 2 and 3), we also compared $\hat{BT}_{bh}$ from the one- and two-stage methods as predicted from fitted Eqs. (4)-(8) at $h = 1.37$ (m).

2.4.2 Bark thickness profile

Similarly, to compare the model performance between one- and two-stage methods, the mean bias (MB) and root mean square error (RMSE) for bark thickness (BT) were calculated. Equations 9 and 10 were used where N is the total number of measurement points, and the residuals were calculated as:

$$e = BT - \hat{BT}$$

where $\hat{BT}$ is the predicted BT in mm. All measurements of a sample tree were divided into three sections by relative height (h/Ht): lower ($h/Ht \leq 1/3$), middle ($1/3 < h/Ht \leq 2/3$), and upper ($2/3 < h/Ht$) sections (from ground level to the tree top). The models were evaluated by summing rank statistics for bias and precision (Kozak and Smith, 1993). For each section, the models were ranked by MB and RMSE, respectively. Then, a sub-rank was determined based on the combination of the ranks of MB and RMSE. The overall rank of the model was calculated from the total rank of every section to evaluate the overall model performance.

For the two-stage method, the residuals for dobs (e_{dob}) were calculated to examine the performance of the stem taper models; that is,

$$e_{dob} = dob - \hat{dob}$$

where $\hat{dob}$ is the predicted dob in cm.

3 Results and Discussion

3.1 Predicting bark thickness at breast height ($\mathrm{BT_{bh}}$)

3.1.1 $\mathrm{BT_{bh}}$ models - simple linear regression (SLR)

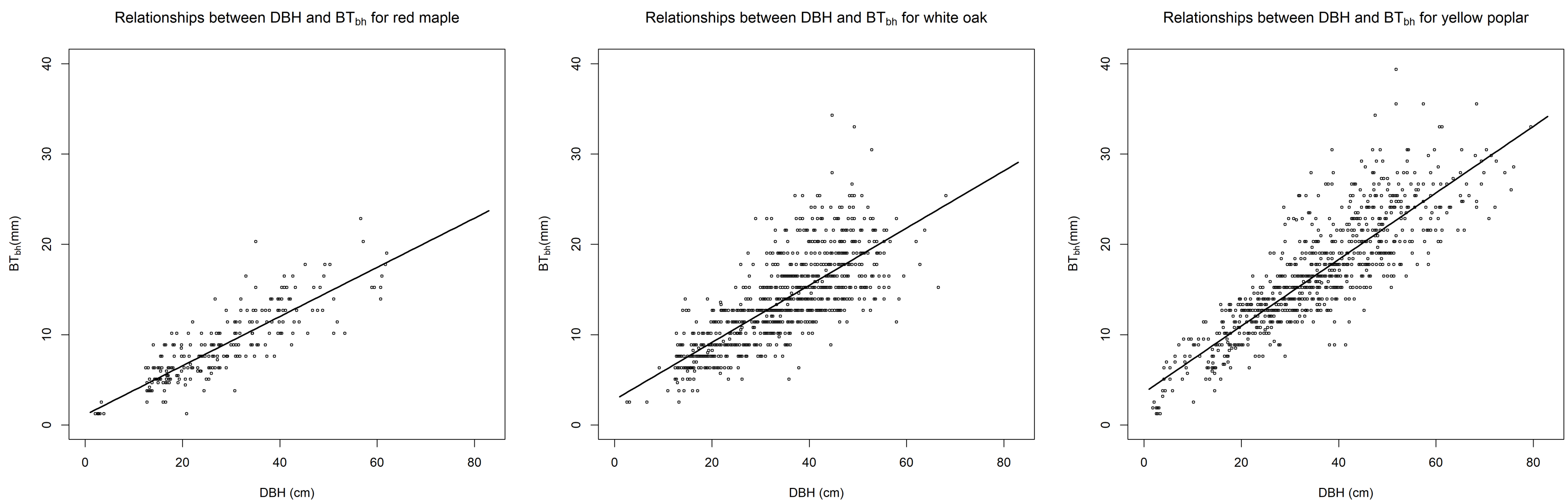
Figure 1 illustrates that $\mathrm{BT_{bh}}$ and DBH generally exhibit a linear and positive relationship regardless of species, which confirms our first hypothesis. Similar findings were reported from Hengst and Dawson (1993), and Hammond et al. (2015). With a unit change in DBH, yellow poplar yields the greatest change in $\mathrm{BT_{bh}}$ (i.e., steepest slope) compared to white oak and red maple. On average, for a given tree size, red maple has the thinnest tree bark, while yellow poplar bark is thickest. Differences in BT_{bh} between species increases with increasing tree DBH (see Fig. 1). For the full data set of 2,070 trees DBH as a single predictor explained considerably more variation in BT_{bh} for yellow poplar and red maple than white oak (Table 2). Based on 95% confidence intervals shown in Table 2 (top section, all trees), all coefficients were statistically different from zero and statistical evidence was clear ($\alpha = .05$) for each species requiring a distinct slope coefficient. Evidence was not as strong for different intercepts between white oak and yellow poplar (Table 2).

A subset of 455 trees with crown class recorded were used to examine the explanatory power of crown class in the bark thickness model. The results showed that adding crown class as an additional predictor did not improve predictive abilities of the models, which indicates our second hypothesis should be rejected. As shown in Table 2, the adjusted R^2 values were identical with and without crown class included for yellow poplar, while they only increased 1% for red maple and white oak with crown class included as a predictor. None of the crown class coefficients were significantly different from zero, meaning the data used here do not support the need for separate intercepts for different crown classes in our prediction models for BT_{bh}.

Fit results for SLR models of BT_{bh} on DBH in the three species examined here were comparable to Sonmez et al. (2007), who reported that DBH alone explained 50-68% of variation in BT_{bh} of oriental spruce (*Picea orientalis*). Other than crown class, tree age is another important tree variable associated with bark thickness (Mosaffaei and Jahani, 2020). However, as indicated by Sonmez et al. (2007), adding tree age as an additional predictor in their model did not substantially improve model performance (R^2 value only increased 1-2%). Unlike plantations, tree age is not usually recorded in inventories for natural mixed-hardwood forests. In our dataset, only three trees have record of age. Measuring tree age in natural forests requires tree ring analysis using increment cores or disks cut from breast height, procedures typically beyond the scope of volumetric studies. Predicting BT_{bh} from DBH is a cost-effective and commonly-used approach in inventory and other management applications.

3.1.2 BT_{bh} from bark thickness profile models

We further compared the accuracy of BT_{bh} predictions from SLR models to BT_{bh} predicted using BT profile models setting $h = 1.37$ m. The additional predictor Ht in profile models (Eq. 5 - 8) was seen as a potential source of information for increasing predictive ability. The magnitudes of red maple BT_{bh} prediction errors were smaller than white oak and yellow poplar, a result consistent with red maple having the thinnest bark of the three species studied (Figure 2). Of the profile models, the two-stage method produced more accurate predictions of BT_{bh} than the one-stage method regardless of the model form used. Two-stage regression models showed similar predictive ability as SLR despite very slight biases; however, not all model forms predicted equally well in two-stage framework for all three species. Specifically, the segmented model (Eq. 6) in the two-stage approach showed the largest absolute bias in red maple, and smallest biases in white oak and yellow poplar (Figure 2). Considering the comparable RMSE of SLR and two-stage models when used to predict BT_{bh}, along with the miniscule mean biases (≤ 0.1 mm) for some BT profile model forms, a case could be made to adopt Eqs. (7) and (6), for red maple and white oak, respectively. Either Eq. (6) or (7) could be justified in place of the SLR BT_{bh} model for yellow poplar. In either case, the main justification for using a two-stage approach for predicting BT_{bh} likely would hinge on a desire to maintain consistency and smoothness of BT predictions slightly above or below breast height when a single profile modeling approach was adopted.



(a) Red maple

(b) White oak

(c) Yellow poplar

Figure 1: Relationships between diameter at breast height (DBH) and bark thickness at breast height (BT_{bh}) for red maple (a), white oak (b) and yellow poplar (c). The observations are shown in black circles, and the solid black line represents the fitted regression (Eq. 2).

Table 2: Summary of the estimated coefficients in Equations 2 and 3 for three species, which were calculated from 1,000 bootstrap samples.

All trees (2,070 trees)						
	Intercept(β_0)		Slope(β_1)		$Adj.R^2$	
Species	Estimate	95% CI	Estimate	95% CI	Estimate	95% CI
Red maple	1.10	(0.53,1.65)	0.27	(0.25,0.30)	0.72	(0.65,0.78)
White oak	2.83	(2.33,3.42)	0.32	(0.30,0.33)	0.56	(0.52,0.60)
Yellow poplar	3.58	(3.13,4.09)	0.37	(0.35,0.38)	0.73	(0.70,0.76)

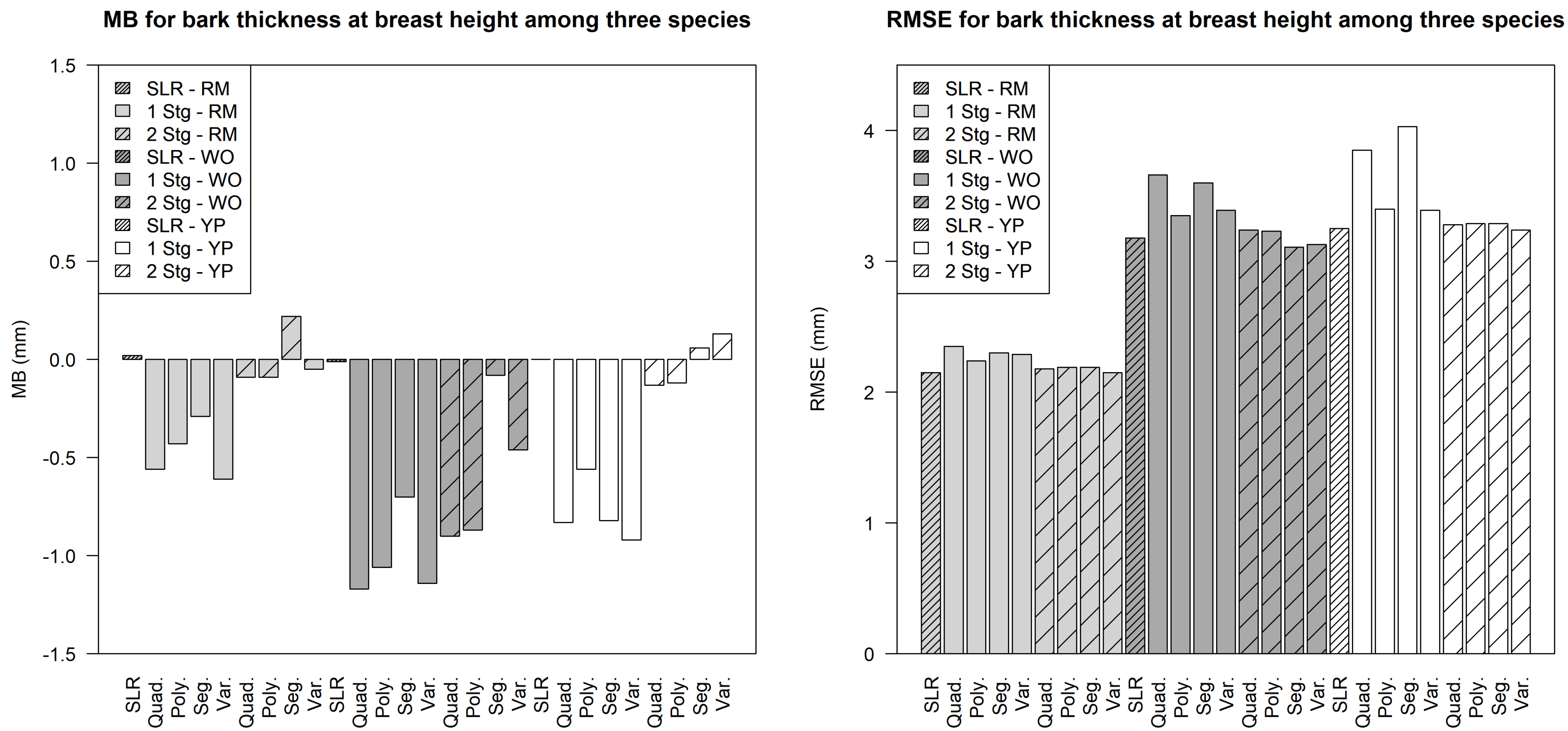
Trees with crown class measured (455 trees)

Without crown class added

	Intercept(β_0)		Slope(β_1)		$Adj.R^2$	
Species	Estimate	95% CI	Estimate	95% CI	Estimate	95% CI
Red maple	0.97	(0.22,1.72)	0.25	(0.21,0.30)	0.58	(0.45,0.68)
White oak	2.03	(1.28,2.75)	0.32	(0.29,0.35)	0.75	(0.71,0.82)
Yellow poplar	1.49	(0.99,2.05)	0.37	(0.35,0.39)	0.87	(0.79,0.93)

With crown class added

	Intercept(β_0)		Slope(β_1)		Crown(β_2)		$Adj.R^2$	
Species	Estimate	95% CI	Estimate	95% CI	Estimate	95% CI	Estimate	95% CI
Red maple	0.22	(-1.16,1.51)	0.27	(0.22,0.33)	0.65	(-0.29,1.63)	0.59	(0.46,0.71)
White oak	1.53	(0.63,2.47)	0.33	(0.30,0.37)	0.56	(-0.07,1.21)	0.76	(0.68,0.83)
Yellow poplar	1.61	(0.80,2.42)	0.36	(0.34,0.39)	-0.13	(-0.77,0.52)	0.87	(0.79,0.93)



(a) Mean bias (MB)

(b) Root mean square error (RMSE)

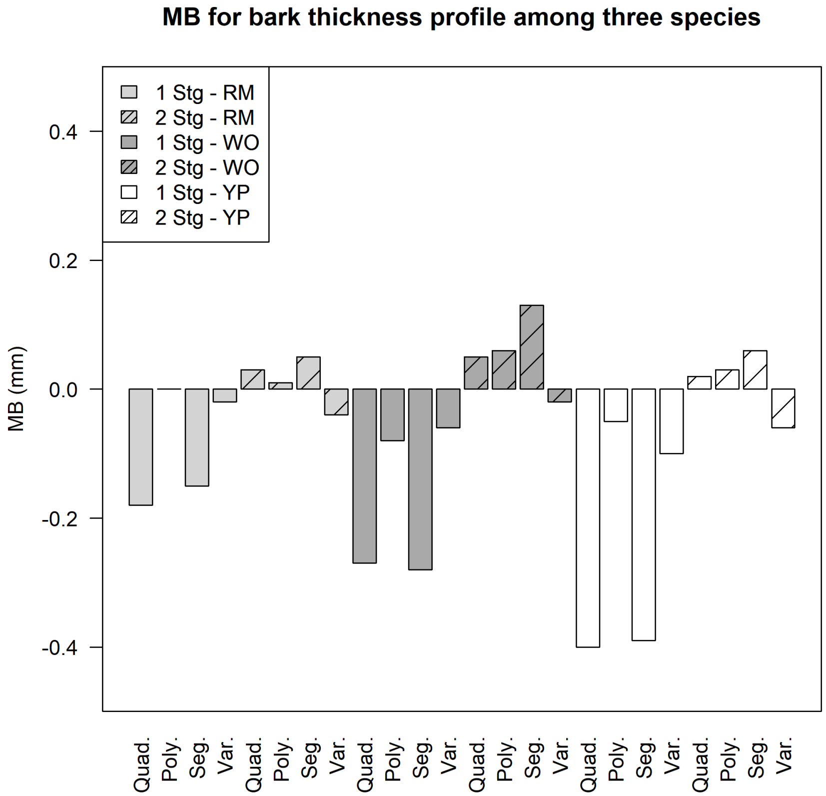
Figure 2: Comparison of mean bias (MB) and root mean square error (RMSE) for bark thickness at breast height (BT_{bh}) among species (red maple, RM; white oak, WO; yellow poplar, YP) and models. SLR, 1 stg. and 2 stg. stand for the simple linear regression model (Eq. 2), one-stage and two-stage methods, respectively. Quad., Poly., Seg., and Var. represent quadratic regression, polynomial regression, segmented regression and variable-exponent models, respectively.

3.2 Predicting bark thickness at any point along tree bole

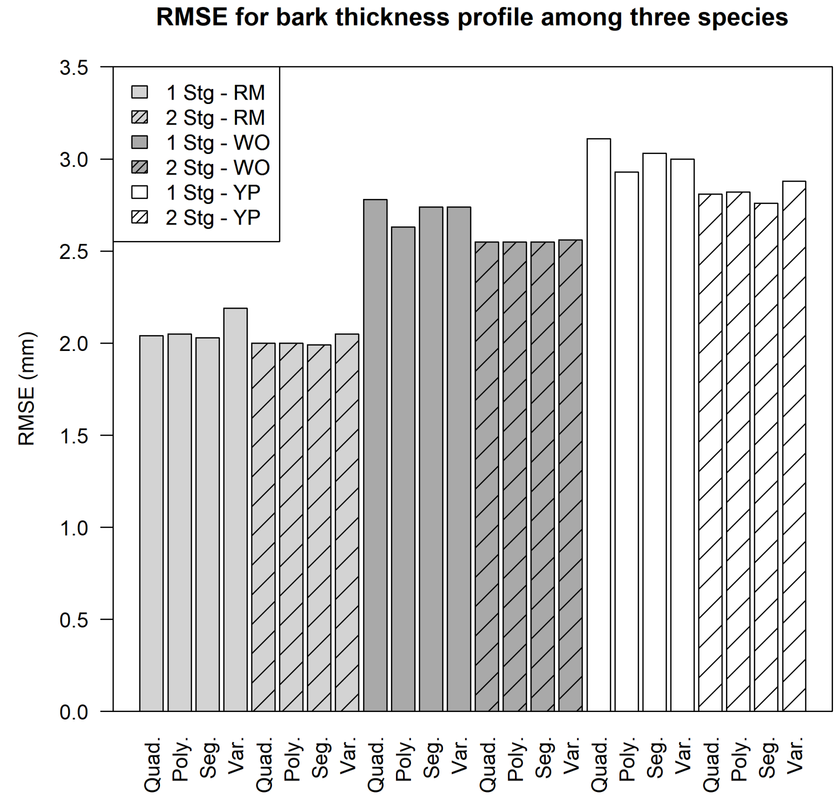
For a given model form, a combination of stem taper function and bark thickness model (two-stage method) generally produced more reliable prediction of bark thickness profile than a single bark thickness equation (one-stage method) (Figure 3 and Table 3). The two-stage method yielded less biased predictions than the one-stage method, except for the polynomial (Eq. 5) and variable-exponent (Eq. 7) equations fitted for red maple. Regardless of model form and species, the precision of prediction was improved when the two-stage approach was implemented (Figure 3). To further examine the predictability of the models/methods, the measurement points of each sample tree were divided into three sections by relative height (h/Ht): lower ($h/Ht \leq 1/3$), middle ($1/3 < h/Ht \leq 2/3$), and upper ($2/3 < h/Ht$) sections. For a given model, bark thickness at each section was generally more accurately predicted by the two-stage method than the one-stage approach (see subrank in Table 3); however, for some models, the one-stage method performed slightly better than the two-stage approach in the middle section (e.g., segmented regression model for red maple). It was found that using the two-stage approach can improve the prediction accuracy of bark thickness near the two ends of the stem. For example, the one-stage, variable exponent model overpredicted the bark thickness of white oak at the lower and upper sections, but the apparent biases were reduced when the two-stage method was used (Fig. 4), which makes the rank of the model jump from the 8th to the 4th (see overall rank in Table 3). Another example was found for the segmented regression model. As shown in Figure 4, the apparent biases at the lower end of yellow poplar relative height were reduced when changing to the two-stage approach.

In the two-stage method, dobs were predicted by different stem taper functions, but the same bark thickness model was applied, which can be used to examine how the choice of stem taper function affects bark thickness prediction accuracy. When the two-stage method was applied, the overall performance of the quadratic, polynomial and segmented regression functions is better than the variable exponent model among three species (see overall rank in Table 3). The quadratic, polynomial, and segmented models won the top three for all species. The variable exponent model ranked 6th for red maple and 4th for white oak, while it dropped to rank 7th for yellow poplar. Although the performance of the variable-exponent model was improved in the two-stage method, the predictive ability was still lower than some models in the one-stage method (e.g., polynomial function for red maple). The inaccurate bark thickness profile generated by the variable-exponent function could be due to poor predictions of diameter outside bark. Notably, Figure 5 illustrates that the variable-exponent model yields highly biased predictions of outside-bark diameters among all species, especially for white oak and yellow poplar. The differences between the observed and predicted dobs are around 2 cm (Fig. 5). The variable exponent model used in this study (Eq. 7) includes six parameters, which has two more parameters than other models, but the prediction accuracy did not improve with increasing number of parameters (i.e., the third hypothesis was rejected). In addition, when fitting the variable exponent models, we found it was challenging to choose the appropriate initial values. In fact, Kozak (2004) reported that the nine-parameter variable exponent model provided the most reliable stem taper estimates than the six-parameter one. However, in our preliminary analysis, the convergence criterion was not always met in parameter estimation for the nine-parameter model, especially when fitting dob. Thus, the six-parameter model was used instead.

In addition, as noted in Tables 4 and 5, a few coefficients were not significantly different from zero, which implies that a model with fewer parameters could predict BT profiles equally well as some of the models examined here. However, to be comparable with the results from other studies, we did not modify the model form. Investigation on a simplified form of bark thickness model for a variety of hardwoods is suggested for the future studies.



(a) Mean bias (MB)

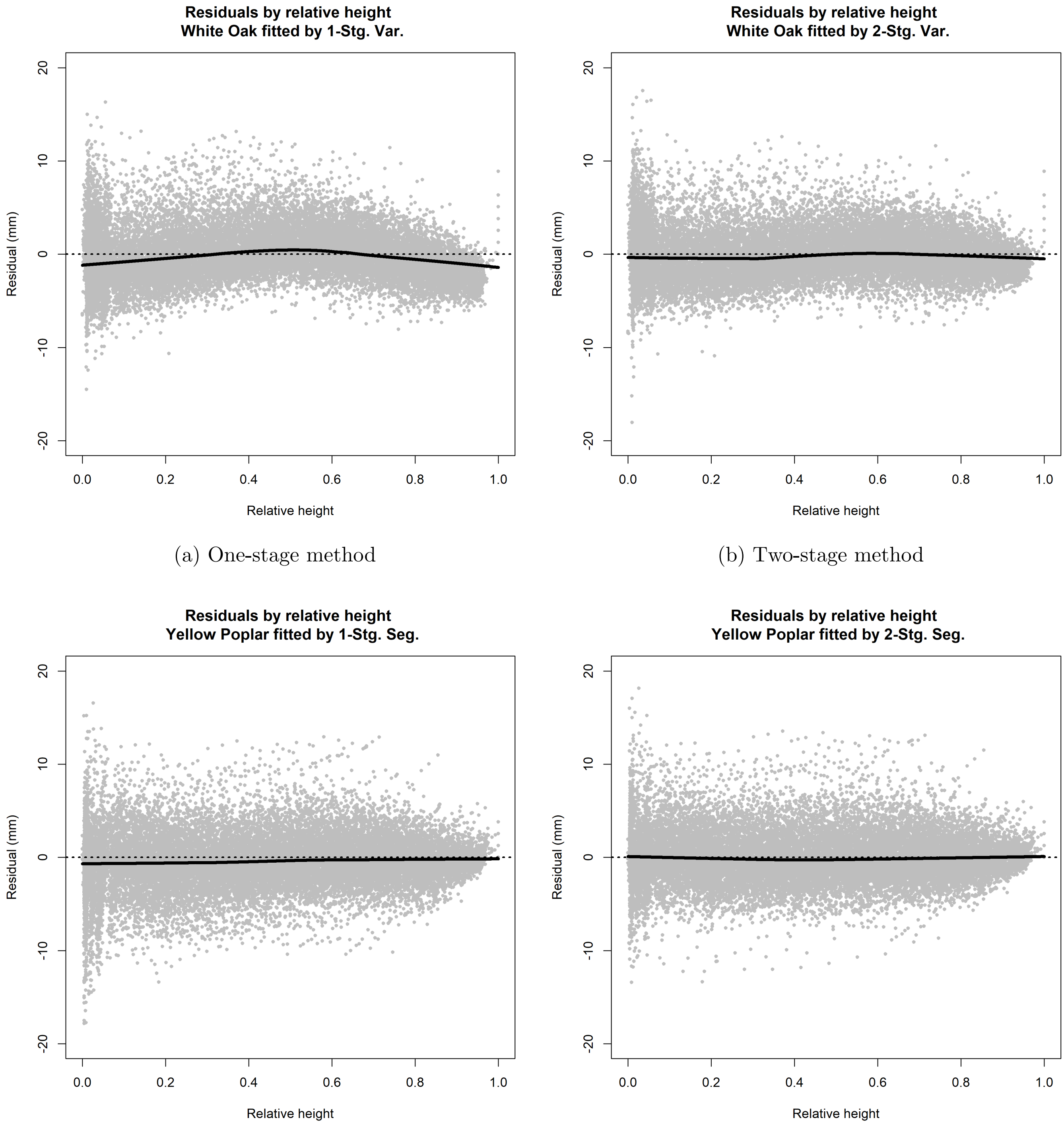


(b) Root mean square error (RMSE)

Figure 3: Comparison of mean bias (MB) and root mean square error (RMSE) for bark thickness profile among species (red maple, RM; white oak, WO; yellow poplar, YP) and models. 1 stg. and 2 stg. stand for the one-stage and two-stage methods, respectively. Quad., Poly., Seg., and Var. represent quadratic regression, polynomial regression, segmented regression and variable-exponent models, respectively.

Table 3: Prediction accuracy of bark thickness profile by species, methods and models. A total number of 2,070 trees was used in analysis. All measurements of a sample tree were divided into three sections by relative height (h/Ht): lower ($h/Ht \leq 1/3$), middle ($1/3 < h/Ht \leq 2/3$), and upper ($2/3 < h/Ht$) sections (from ground level to the tree top). RM, WO and YP represent red maple, white oak and yellow poplar, respectively. Rk., Subrk., and OA Rk. stand for rank, subrank and overall rank, respectively. Quad., Poly., Seg., and Var. represent quadratic regression, polynomial regression, segmented regression and variable-exponent models, respectively.

			Lower					Middle					Upper					
Species	Method	Model	MB	Rk.	RMSE	Rk.	Subrk.	MB	Rk.	RMSE	Rk.	Subrk.	MB	Rk.	RMSE	Rk.	Subrk.	OA Rk.
RM	One-Stg.	Quad.	-0.36	8	2.26	6	7	0.14	5	2.12	2	2	-0.27	7	1.58	7	7	7
		Poly.	-0.18	5	2.19	3	4	0.29	7	2.24	7	7	-0.09	1	1.55	4	2	5
		Seg.	-0.27	7	2.26	6	6	0.00	1	2.11	1	1	-0.14	5	1.55	4	5	4
		Var.	-0.22	6	2.28	8	7	0.70	8	2.32	8	8	-0.53	8	1.89	8	8	8
	Two-Stg.	Quad.	0.03	1	2.18	1	1	0.15	6	2.14	4	6	-0.11	2	1.53	3	2	2
		Poly.	0.03	1	2.19	3	2	0.10	2	2.15	5	2	-0.11	2	1.52	2	1	1
		Seg.	0.10	3	2.18	1	2	-0.13	4	2.13	3	2	0.17	6	1.50	1	4	3
		Var.	-0.11	4	2.25	5	5	0.12	3	2.19	6	5	-0.13	4	1.56	6	6	6
WO	One-Stg.	Quad.	-0.59	8	3.53	8	8	0.03	1	2.53	6	2	-0.19	5	1.75	2	4	6
		Poly.	-0.38	6	3.24	5	6	0.19	2	2.44	5	2	0.01	1	1.85	7	5	5
		Seg.	-0.44	7	3.44	7	7	-0.21	4	2.55	7	7	-0.16	4	1.75	2	2	7
		Var.	-0.32	5	3.24	5	5	0.95	8	2.61	8	8	-0.78	8	2.11	8	8	8
	Two-Stg.	Quad.	-0.02	2	3.11	3	2	0.31	6	2.43	2	5	-0.12	3	1.74	1	1	1
		Poly.	0.00	1	3.11	3	1	0.30	5	2.43	2	2	-0.11	2	1.76	4	2	1
		Seg.	0.19	4	3.09	1	2	-0.19	2	2.43	2	1	0.39	7	1.79	5	6	3
		Var.	-0.15	3	3.10	2	2	0.34	7	2.42	1	5	-0.21	6	1.81	6	6	4
YP	One-Stg.	Quad.	-0.97	8	3.88	8	8	0.06	2	2.92	4	2	-0.22	4	2.03	4	4	5
		Poly.	-0.59	6	3.43	6	6	0.37	6	2.92	4	6	0.15	2	2.15	7	5	6
		Seg.	-0.73	7	3.73	7	7	-0.17	3	2.92	4	3	-0.20	3	2.02	2	2	4
		Var.	-0.45	5	3.29	4	4	1.01	8	3.09	8	8	-0.90	8	2.49	8	8	8
	Two-Stg.	Quad.	-0.02	2	3.28	2	2	0.34	5	2.85	2	3	-0.27	5	2.02	2	3	2
		Poly.	0.00	1	3.28	2	1	0.32	4	2.86	3	3	-0.27	5	2.04	5	6	3
		Seg.	0.13	3	3.20	1	2	0.00	1	2.83	1	1	0.05	1	1.96	1	1	1
		Var.	-0.20	4	3.31	5	4	0.41	7	2.93	7	7	-0.43	7	2.12	6	7	7



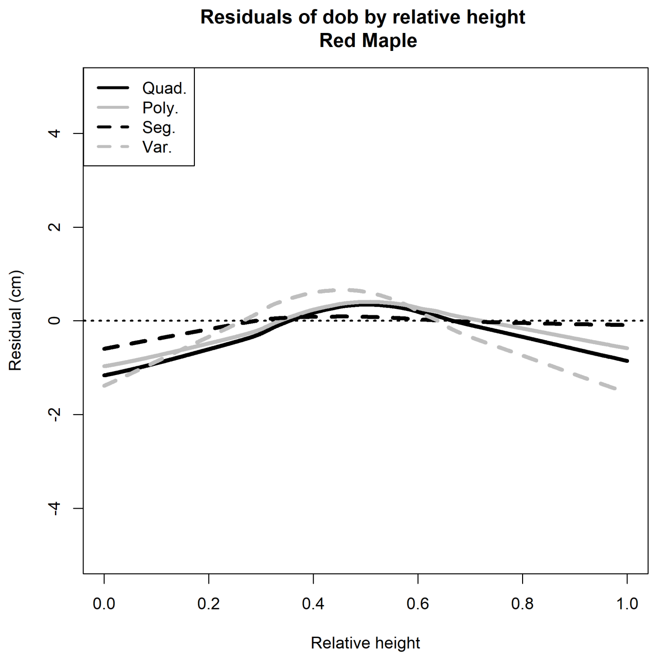
(a) One-stage method

(b) Two-stage method

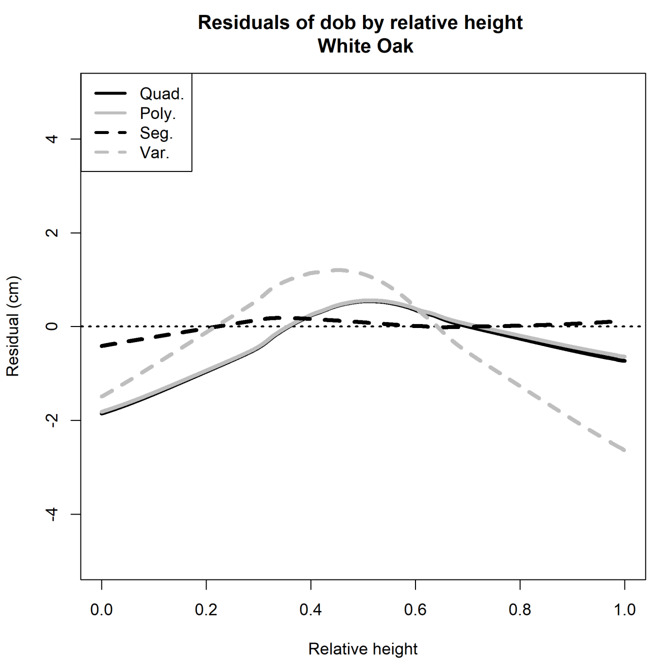
(c) One-stage method

(d) Two-stage method

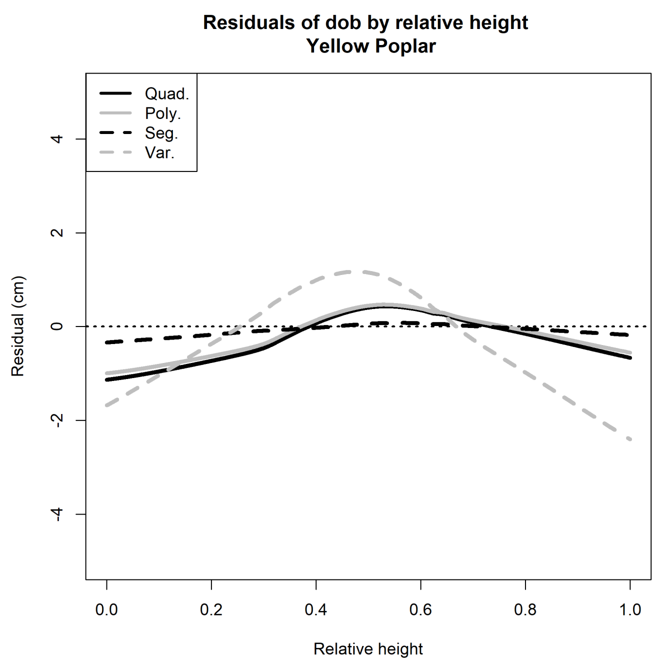
Figure 4: Residuals of bark thickness (e_{BT}, mm) by relative height (h/Ht). Bark thickness profile (BT) predicted by the variable-exponent function (Var.) for white oak (a-b) and by the segmented regression for yellow poplar (c-d). Black solid lines are Lowess smoothing curves.



(a) Red maple



(b) White oak



(c) Yellow poplar

Figure 5: Lowess smoothing curves for residuals of diameter outside bark (dob) (observed dob - predicted dob, cm) by relative height (h/Ht). Dob predicted by the quadratic function (Quad.), polynomial regression (Poly.), segmented regression (Seg.) and variable-exponent function (Var.) for three hardwood species.

Table 4: Summary of the estimated coefficients for eight models among three species. The estimates and 95% confidence interval were computed from the median, 2.5% and 97.5% quantiles of 1,000 bootstrap samples, respectively. RM, WO and YP represent red maple, white oak and yellow poplar, respectively. Insignificant estimates were shown in bold.

			a0		a1		a2		a3		a4		a5	
Species	Method	Model	Est.	95%CI	Est.	95%CI	Est.	95%CI	Est.	95%CI	Est.	95%CI	Est.	95%CI
RM	One-Stg.	Quad.	0.335	(0.325,0.346)	**-0.013**	(-0.054,0.026)	-0.302	(-0.339,-0.259)	-	-	-	-	-	-
		Poly.	0.431	(0.394,0.468)	**0.002**	(-0.037,0.039)	-0.313	(-0.349,-0.274)	-0.005	(-0.006,-0.003)	-	-	-	-
		Seg.	**0.197**	(-0.106,0.474)	-0.472	(-0.640,-0.292)	**0.044**	(-0.173,0.247)	6.242	(4.760,7.743)	-	-	-	-
		Var.	0.280	(0.178,0.422)	1.055	(0.938,1.188)	0.234	(0.174,0.293)	0.424	(0.204,0.683)	0.034	(0.016,0.054)	-1.16	(-1.459,-0.875)
	Two-Stg.	Quad.	1.125	(1.117,1.134)	-1.009	(-1.060,-0.961)	-0.103	(-0.150,-0.055)	-	-	-	-	-	-
		Poly.	1.253	(1.221,1.286)	-0.989	(-1.038,-0.943)	-0.119	(-0.163,-0.074)	-0.006	(-0.007,-0.005)	-	-	-	-
		Seg.	-2.914	(-3.326,-2.502)	0.973	(0.725,1.226)	-1.699	(-1.997,-1.404)	23.924	(22.121,25.815)	-	-	-	-
		Var.	0.913	(0.832,1.008)	1.075	(1.047,1.102)	0.536	(0.501,0.573)	0.129	(0.005,0.264)	0.028	(0.020,0.036)	-0.898	(-1.031,-0.768)
WO	One-Stg.	Quad.	0.437	(0.431,0.444)	-0.053	(-0.074,-0.033)	-0.369	(-0.386,-0.351)	-	-	-	-	-	-
		Poly.	0.584	(0.562,0.609)	-0.040	(-0.061,-0.022)	**-0.377**	(-0.395,0.359)	-0.006	(-0.007,-0.006)	-	-	-	-
		Seg.	**0.081**	(-0.055,0.221)	-0.495	(-0.578,-0.412)	**-0.061**	(-0.165,0.044)	10.900	(9.843,11.991)	-	-	-	-
		Var.	0.873	(0.727,1.036)	0.834	(0.788,0.884)	0.250	(0.224,0.275)	0.463	(0.351,0.581)	0.045	(0.039,0.052)	-1.013	(-1.137,-0.899)
	Two-Stg.	Quad.	1.142	(1.138,1.146)	-0.979	(-1.002,-0.958)	-0.167	(-0.187,-0.145)	-	-	-	-	-	-
		Poly.	1.200	(1.185,1.218)	-0.974	(-0.998,-0.953)	-0.170	(-0.190,-0.149)	-0.003	(-0.003,-0.002)	-	-	-	-
		Seg.	-4.144	(-4.301,-3.990)	1.673	(1.580,1.770)	-2.733	(-2.848,-2.622)	35.618	(34.724,36.630)	-	-	-	-
		Var.	0.821	(0.754,0.883)	1.126	(1.104,1.151)	0.493	(0.473,0.512)	0.227	(0.150,0.316)	0.054	(0.047,0.062)	-1.094	(-1.198,-0.982)
YP	One-Stg.	Quad.	0.500	(0.492,0.508)	-0.131	(-0.154,-0.109)	-0.350	(-0.372,-0.330)	-	-	-	-	-	-
		Poly.	0.650	(0.627,0.677)	-0.112	(-0.134,-0.091)	-0.367	(-0.386,-0.348)	-0.005	(-0.006,-0.005)	-	-	-	-
		Seg.	**0.102**	(-0.054,0.260)	-0.541	(-0.638,-0.447)	**0.001**	(-0.112,0.118)	11.870	(11.133,12.699)	-	-	-	-
		Var.	1.072	(0.924,1.242)	0.799	(0.759,0.840)	0.243	(0.210,0.276)	0.552	(0.418,0.681)	0.015	(0.014,0.023)	-0.708	(-0.844,-0.634)
	Two-Stg.	Quad.	1.067	(1.063,1.071)	-0.689	(-0.709,-0.669)	**-0.377**	(-0.394,0.359)	-	-	-	-	-	-
		Poly.	1.143	(1.125,1.160)	-0.679	(-0.698,-0.661)	-0.385	(-0.402,-0.368)	-0.003	(-0.003,-0.002)	-	-	-	-
		Seg.	-2.326	(-2.520,-2.135)	0.566	(0.447,0.683)	-1.444	(-1.580,-1.309)	21.185	(20.331,22.206)	-	-	-	-
		Var.	0.853	(0.800,0.903)	1.070	(1.056,1.088)	0.477	(0.452,0.500)	0.135	(0.051,0.227)	0.018	(0.016,0.024)	-0.875	(-0.997,-0.830)

Table 5: (*Continued*) Summary of the estimated coefficients for eight models among three species. The estimates and 95% confidence interval were computed from the median, 2.5% and 97.5% quantiles of 1,000 bootstrap samples, respectively. RM, WO and YP represent red maple, white oak and yellow poplar, respectively. Insignificant estimates were shown in bold.

			b0		b1		b2		b3	
Species	Method	Model	Est.	95%CI	Est.	95%CI	Est.	95%CI	Est.	95%CI
RM	One-Stg.	Quad.	-	-	-	-	-	-	-	-
		Poly.	-	-	-	-	-	-	-	-
		Seg.	-	-	-	-	-	-	-	-
		Var.	-	-	-	-	-	-	-	-
	Two-Stg.	Quad.	0.325	(0.256,0.394)	0.318	(0.251,0.390)	**-0.039**	(-0.133,0.050)	**-0.002**	(-0.005,0.001)
		Poly.	0.267	(0.200,0.332)	0.296	(0.230,0.366)	**0.015**	(-0.075,0.101)	**0.001**	(-0.002,0.003)
		Seg.	0.324	(0.253,0.394)	0.337	(0.275,0.404)	**-0.076**	(-0.160,0.002)	**-0.002**	(-0.005,0.001)
		Var.	0.295	(0.226,0.361)	0.591	(0.513,0.671)	-0.424	(-0.518,-0.334)	**-0.001**	(-0.004,0.001)
WO	One-Stg.	Quad.	-	-	-	-	-	-	-	-
		Poly.	-	-	-	-	-	-	-	-
		Seg.	-	-	-	-	-	-	-	-
		Var.	-	-	-	-	-	-	-	-
	Two-Stg.	Quad.	0.620	(0.585,0.656)	0.135	(0.105,0.167)	0.257	(0.215,0.297)	-0.010	(-0.011,-0.009)
		Poly.	0.584	(0.553,0.624)	0.129	(0.099,0.161)	0.272	(0.230,0.311)	-0.009	(-0.010,-0.007)
		Seg.	0.616	(0.581,0.653)	0.221	(0.189,0.253)	0.111	(0.068,0.151)	-0.010	(-0.012,-0.009)
		Var.	0.600	(0.567,0.634)	0.541	(0.509,0.573)	-0.352	(-0.394,-0.312)	-0.010	(-0.012,-0.009)
YP	One-Stg.	Quad.	-	-	-	-	-	-	-	-
		Poly.	-	-	-	-	-	-	-	-
		Seg.	-	-	-	-	-	-	-	-
		Var.	-	-	-	-	-	-	-	-
	Two-Stg.	Quad.	0.658	(0.615,0.694)	0.077	(0.045,0.109)	0.219	(0.179,0.259)	-0.007	(-0.008,-0.006)
		Poly.	0.607	(0.566,0.644)	0.062	(0.032,0.095)	0.251	(0.211,0.290)	-0.005	(-0.006,-0.004)
		Seg.	0.651	(0.608,0.688)	0.135	(0.105,0.166)	0.122	(0.084,0.162)	-0.007	(-0.008,-0.006)
		Var.	0.630	(0.590,0.667)	0.462	(0.424,0.499)	-0.314	(-0.359,-0.268)	-0.007	(-0.008,-0.006)

3.3 Further discussion

Mathematical and statistical models of bark thickness have played a secondary role in forest modeling in the past, where primary interest was on wood volumes mainly in species grown for their timber value (McTague and Weiskittel, 2021). However, the need for robust and accurate quantitative models of bark thickness has increased substantially in recent years as changing climate regimes have brought greater attention to future implications of wildfire on forest ecology and management (Moritz et al., 2012). As mentioned, BT_{bh} has been known as a useful indicator to assess post-fire mortality of trees in forest fire ecology. Using a SLR model with DBH as a single independent variable has proven to provide the most reliable predictions of BT_{bh} among all models examined. Although BT_{bh} model provides a quick and cost-efficient quantity in forestry practice, variation of bark thickness along the stem can not be well characterised with a single measure of bark thickness. Instead, bark thickness models depict the whole profile of bark thickness, which may enhance the prediction of tree mortality from fire, and provide a better quantification of tree health. Recently, Scheiter et al. (2013) reviewed the need for species and community-specific bark allocation information to account for fire effects on forest ecosystems in dynamic global vegetation models. Pellegrini et al. (2017) found that the use of mean-values of bark thickness for species functional groups was problematic in global projections, favoring instead a more robust approach accounting for variation in bark thickness across ecological gradients, and preserving local variation by the use of distributions instead of means. The results of this study demonstrate several regression modeling techniques that can be employed in such work, making better use of large existing bark thickness data sets like the one used here.

Lastly, the models/methods examined in this study can be applied to predict not only bark thickness profile, but also dibs along tree bole. As indicated by McTague and Weiskittel (2021), newly emerging technologies (e.g., terrestrial laser scanning, TLS) have been widely used to efficiently measure dobs along the stem of standing trees in forest management. With the bark thickness model, the TLS data can be easily converted to estimate inside-bark diameters, and then calculate under-bark wood volume.

4 Conclusion

In short, we examined the model predictability for bark thickness at breast height (BT_{bh}) and bark thickness profile. This finding notwithstanding, the predictive accuracy of BT profile equations in predicting BT_{bh} compared favorably to the simple linear regression (SLR) models with DBH as a single predictor. Using a BT profile model to predict BT_{bh} ensures that BT predictions above or below breast height will be continuous in the range of heights that includes $h = 1.37$ m.

To predict bark thickness profile, a combination of stem taper function and bark thickness model (two-stage method) is suggested, which generally performs better than a single bark thickness function (one-stage method) in terms of prediction accuracy. For a given model form, the two-stage method produced more reliable prediction of bark thickness at upper and lower portions of tree stem than the one-stage method. Suitable two-stage BT profile equations were identified for the three species examined. The variable-exponent function (Eq. 7) provided the poorest predictions among all models. A possible simplification could be realized in the red maple, bark thickness model by omitting the final term in Eq. 5, as total height alone as a predictor does not seem to improve the predictive ability of the best BT profile model we identified for the species. As the results show, model/method selection greatly impacts the prediction accuracy of bark thickness, so choosing an appropriate model is essential to provide reliable predictions of bark thickness for different management objectives. In addition to the three species examined in this study, the methodology can be applied to build and evaluate bark thickness models for other species in different regions when quantifying bark thickness is needed.

References

Brose, P., D. Van Lear, and R. Cooper (1999). Using shelterwood harvests and prescribed fire to regenerate oak stands on productive upland sites. *Forest Ecology and Management 113*, 125–141.

Burkhart, H. E. and M. Tomé (2012). *Modeling forest trees and stands*. Springer.

Burns, R. M. and B. H. Honkala (1990). Silvics manual volume 2: Hardwoods. *United States Department of Agriculture (USDA), Forest Service, Agriculture Handbook 654*, –undefined.

Cao, Q. V. and W. D. Pepper (1986). Predicting inside bark diameter for shortleaf, loblolly and longleaf pine. *Southern Journal of Applied Forestry 10*(4), 220–224.

Costa, A., I. Barbosa, M. Pestana, and C. Miguel (2020, 8). Modelling bark thickness variation in stems of cork oak in south-western Portugal. *European Journal of Forest Research 139*(4), 611–625.

Davison, A. C. and D. V. Hinkley (1997, 10). *Bootstrap Methods and their Application.* Cambridge University Press.

Elzhov, V., K. M. Mullen, A.-N. Spiess, and B. B. Maintainer (2016). Package 'minpack.lm' - R Interface to the Levenberg-Marquardt Nonlinear Least-Squares Algorithm Found in MINPACK, Plus Support for Bounds.

Fei, S. and K. C. Steiner (2007). Evidence for increasing red maple abundance in the eastern United States. *Forest Science 53*(4), 473–477.

Field, C. A. and A. H. Welsh (2007). Bootstrapping clustered data. Technical report.

Hammond, D. H., J. M. Varner, J. S. Kush, and Z. Fan (2015, 7). Contrasting sapling bark allocation of five southeastern USA hardwood tree species in a fire prone ecosystem. *Ecosphere 6*(7).

Hengst, G. E. and J. Dawson (1993). Bark properties and fire resistance of selected tree species from the central hardwood region of North America. *Canadian Journal of Forest Research 24*, 688–696.

Kohnle, U., S. Hein, F. C. Sorensen, and A. R. Weiskittel (2012, 2). Effects of seed source origin on bark thickness of douglas-fir (pseudotsuga menziesii) growing in southwestern Germany. *Canadian Journal of Forest Research 42*(2), 382–399.

Kozak, A. (2004). My last words on taper equations. *Forestry Chronicle 80*(4), 507–515.

Kozak, A. and J. Smith (1993). Standards for evaluating taper estimating systems. *Forestry chronicle 69*(4), 438–444.

Laasasenaho, J., T. Melkas, and S. Aldén (2005, 2). Modelling bark thickness of Picea abies with taper curves. *Forest Ecology and Management 206*(1-3), 35–47.

Li, R. and A. R. Weiskittel (2011, 3). Estimating and predicting bark thickness for seven conifer species in the Acadian Region of North America using a mixed-effects modeling approach: Comparison of model forms and subsampling strategies. *European Journal of Forest Research 130*(2), 219–233.

Maguire, D. A. and D. W. Hann (1990). Bark thickness and bark volume in southwestern Oregon Douglas-fir. *Western Journal of Applied Forestry 5*(1), 5–8.

Max, T. and H. Burkhart (1976). Segmented Polynomial Regression Applied to Taper Equations. *Forest Science 22*(3), 283–289.

McTague, J. P. and A. Weiskittel (2021, 2). Evolution, history, and use of stem taper equations: a review of their development, application, and implementation. *Canadian Journal of Forest Research 51*(2), 252–265.

Moritz, M. A., M.-A. Parisien, E. Batllori, M. A. Krawchuk, J. Van Dorn, D. J. Ganz, and K. Hayhoe (2012, 6). Climate change and disruptions to global fire activity. *Ecosphere 3*(6).

Mosaffaei, Z. and A. Jahani (2020). Modeling of ash (Fraxinus excelsior) bark thickness in urban forests using artificial neural network (ANN) and regression models.

Nolan, R. H., S. Rahmani, S. A. Samson, H. M. Simpson-Southward, M. M. Boer, and R. A. Bradstock (2020, 10). Bark attributes determine variation in fire resistance in resprouting tree species. *Forest Ecology and Management 474*.

Oak, S. W., M. A. Spetich, and R. S. Morin (2015). Oak decline in central hardwood forests: frequency, spatial extent, and scale. *In: Greenberg, Cathryn H.; Collins, Beverly S.(eds.). Natural disturbances and historic range of variation: Type, frequency, severity, and post-disturbance structure in central hardwood forests USA. Managing Forest Ecosystems. Vol. 32. 400pp.*.

Pellegrini, A. F. A., W. R. L. Anderegg, C. E. T. Paine, W. A. Hoffmann, T. Kartzinel, S. S. Rabin, D. Sheil, A. C. Franco, and S. W. Pacala (2017, 3). Convergence of bark investment according to fire and climate structures ecosystem vulnerability to future change. *Ecology Letters 20*(3).

Radtke, P. J., D. M. Walker, A. R. Weiskittel, J. Frank, J. W. Coulston, and J. A. Westfall (2015). Legacy tree data: A national database of detailed tree measurements for volume, weight, and physical properties. *Gen. Tech. Rep. PNW-GTR-931*, 25–30.

Ren, S., H. Lai, W. Tong, M. Aminzadeh, X. Hou, and S. Lai (2010). Nonparametric bootstrapping for hierarchical data. *Journal of Applied Statistics 37*(9), 1487–1498.

Roula, S. E., R. T. Bouhraoua, and F. X. Catry (2020). Factors affecting post-fire regeneration after coppicing of cork oak (Quercus suber) trees in northeastern Algeria. *Canadian Journal of Forest Research 50*(4), 371–379.

Sánchez-González, M., M. F. Sánchez, and C. Prades (2021, 10). Fitting and calibrating a three-level mixed effects cork growth model. *Forest Ecology and Management 497*, 119510.

Scheiter, S., L. Langan, and S. I. Higgins (2013, 5). Next-generation dynamic global vegetation models: learning from community ecology. *New Phytologist 198*(3).

Sonmez, T., S. Keles, and F. Tilki (2007). Effect of aspect, tree age and tree diameter on bark thickness of Picea orientalis. *Scandinavian Journal of Forest Research 22*(3), 193–197.

Stängle, S. M., U. H. Sauter, and C. F. Dormann (2017, 3). Comparison of models for estimating bark thickness of Picea abies in southwest Germany: the role of tree, stand, and environmental factors. *Annals of Forest Science 74*(1).

Thomas-Van Gundy, M., J. Rentch, M. B. Adams, and W. Carson (2014, 2). Reversing legacy effects in the understory of an oak-dominated forest. *Canadian Journal of Forest Research 44*(4), 350–364.

Yang, S.-I. and H. E. Burkhart (2020, 11). Robustness of parametric and nonparametric fitting procedures of tree-stem taper with alternative definitions for validation data. *Journal of Forestry 118*(6).

Zeibig-Kichas, N. E., C. W. Ardis, J. P. Berrill, and J. P. King (2016). Bark Thickness Equations for Mixed-Conifer Forest Type in Klamath and Sierra Nevada Mountains of California. *International Journal of Forestry Research 2016*.