Suggestions for Deontic Logicians

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Thesis submitted to the Faculty of the
Virginia Polytechnic Institute and State University
in partial fulfillment of the requirements for the degree of

Master of Arts
in
Philosophy

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December 28, 2012
Blacksburg, Virginia

Keywords: Deontic Logic, Hume's Law, Is-Ought Problem, Many-Valued Logics

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(ABSTRACT)

The purpose of this paper is to make a suggestion to deontic logic: Respect Hume's Law, the answer to the is-ought problem that says that all ought-talk is completely cut off from is-talk. Most deontic logicians have sought another solution: Namely, the solution that says that we can bridge the is-ought gap. Thus, a century's worth of research into these normative systems of logic has lead to many attempts at doing just that. At the same time, the field of deontic logic has come to be plagued with paradox. My argument essentially depends upon there being a substantive relation between this betrayal of Hume and the plethora of paradoxes that have appeared in two-adic (binary normative operator), one-adic (unary normative operator), and zero-adic (constant normative operator) deontic systems, expressed in the traditions of von Wright, Kripke, and Anderson, respectively. My suggestion has two motivations: First, to rid the philosophical literature of its puzzles and second, to give Hume's Law a proper formalization. Exploring the issues related to this project also points to the idea that maybe we should re-engineer (e.g., further generalize) our classical calculus, which might involve the adoption of many-valued logics somewhere down the line.

Dedication

"The *a priori* is independent of experience not because
It prescribes a form which the data of sense must fit...
[It is] because it prescribes **nothing** to experience."

CLARENCE IRVING LEWIS (1923)

Acknowledgments

I would like to thank Joseph C. Pitt for advising me throughout the creation of this thesis, as well as the other readers, Kevin Coffey and David Faraci. This work would not have seen its completion if it were not for their pragmatic, syntactic, and semantic support, respectively. More than anything, though, is the endless support and inspiration from my wife, Li, that which gives these few ideas wings to soar free.

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Chapter 1

Introduction

Ought-talk is the term for talking about: obligations, prescriptions, imperatives, permissions, sanctions, liberties, forbiddings, proscriptions, taboos, omissions, supererogations, and waivers.[1] General statements about what is good or bad, right or wrong, or beautiful or ugly are examples of ought-talk as well. 'Ought-talk' can be used interchangeably with 'normative discourse'.

Is-talk is the term for talking about: necessities, descriptions, indicatives, possibilities, capacities, probabilities, impossibilities, incapacities, miracles, unnecessities, accidents, and actualities. [2] 'Is-talk' can be used interchangeably with 'abnormative discourse'.[3]

As Ingvar Johansson succinctly puts it, no one knows how many times David Hume's famous last paragraph in Book III of *A Treatise of Human Nature* has been referenced [9]. Nonetheless, we must look at his words in order to see exactly what is at issue. Hume writes,

[1] Supererogatory things are those that are 'beyond the call of duty', so to speak. Also, things that are optional or non-optional are included under the umbrella of ought-talk.

[2] Unnecessary things are those that are 'beyond the reach of truisms', so to speak. Also, things that are contingent or non-contingent are included under the umbrella of is-talk.

[3] These two domains of language are probably not exhaustive of 'all that can be said'. The same disclaimer will be made in regards to their corresponding logics up ahead. Nonetheless, these 'ways of talking' are the ones that enter into our logical systems most frequently.

> I cannot forbear adding to these reasonings an observation, which may, perhaps, be found of some importance. In every system of morality which I have hitherto met with, I have always [remarked], that the author proceeds for some time in the ordinary way of reasoning, and establishes the being of a God, or makes observations concerning human affairs; when of a sudden I am [surprised] to find, that instead of the usual copulations of propositions, *is*, and *is not*, I meet with no proposition that is not connected with an *ought*, or an *ought not*. This change is imperceptible; but is, however, of the last consequence. For as this *ought*, or *ought not*, expresses some new relation or affirmation, [it is] necessary that it should be [observed] and [explained]; and at the same time that a reason should be given, for what seems altogether inconceivable, how this new relation can be a deduction from others, which are entirely different from it... [I] am [persuaded], that this small attention would subvert all the vulgar systems of morality, and let us see that the distinction of vice and virtue is not [founded] merely on the relations of objects, nor is [perceived] by reason. [8]

This excerpt gives us the 'is-ought problem'. The is-ought problem in my terms is:[4]

- (Deduction Clause) We believe that we can deduce ought-talk from is-talk and;
- (Justification Clause) We have never produced a justification for such a deduction.

But why do we want to justify the deducing of ought-talk from is-talk? The short answer is: because ought-talk is important.[5] People who are in the business of constructing various 'deontic logics' – logics with ought-talk – are usually inclined to pursue such justifications. The converse, though, is definitely not true. This is summarized in Karl Pettersen's observations:

> It is not easy to find moral philosophers who actually make use of deontic logic, e.g., in the formulation of different ethical theories. The lack of consensus in deontic logic in comparison with ordinary [alethic] modal logic, and, of course, ordinary first-order [alethic] logic, has prevented its use as a tool. As a moral philosopher, one can hardly know which, if any, interpretation of [OUGHT] it is that captures what one wants to say, and one cannot suppose one's audience to understand formulae with such expressions without detailed explanation. [13]

[4]Historically there have been at least two separate is-ought problems, a metalogical version and a metaphysical version. Gerard Schurz, in his indispensable *The Is-Ought Problem*, makes this distinction, albeit with different terminology. For this project we will only directly address the metalogical side. The metaphysical repercussions, if there are any, are also of the utmost concern, but they will have to be postponed to a future project. Also, notice that we concentrate only on *deduction* – OUGHT may very well fit in better with inductive or abductive inferential patterns, but that would be to merely sidestep Hume's concerns.

[5]But why is ought-talk important? Because ought-talk is key to giving us *identities*, which in turn makes it crucial to how we go about interacting in the World. Thanks to Kevin Coffey for the second half of this concise capturing of the significance of normative notions.

It is obvious that there are many deontic logics out there. As a small taste, dynamic logic, the computer scientist's favorite that is unarguably a logic of ought-talk, originates from von Wright's very first system of 1951 which features action-types instead of propositions as input into OUGHT.[6] Nevertheless, most ethicists ignore its success rate, as its particular interpretations do not fit the kind of theorizing that they want to do. In terms of logics that actually have come to equilibria of agreement amongst philosophers, Pettersen cites only those logics that purely involve is-talk.[7] The important thing to note here is that the metalogical route is only one way to try to get at the is-ought problem – the metaphysical route (that which many meta-ethicists take) is also perfectly allowable in terms at trying to ameliorate these two conceptual domains (see footnote 4 of this chapter). Thus noted, I will restrict the range of this paper to just the metalogical – i.e., deontic logic – side of the philosophical project, but always keeping in mind the fact that there are many metaphysical worries to be had.

The purpose of this paper is to make a suggestion for deontic logicians. To get to my suggestion I will organize the paper as follows:

First, §2 will present an overview of the is-ought problem and defend its philosophical significance. Next, in §3 I will discuss how currently prominent deontic logics face countless paradoxes. §3 will focus on the major deontic logics. (There are, indeed, an uncountable number of systems out there in the literature; for my purposes, though, I will focus only on the most widely discussed positions.) Then, in §4 I will argue for the plausibility of this claim:

(Claim) Currently prominent deontic logics disrespect – i.e., violate – Hume's Law.

[6]This first effort of von Wright undoubtably helped to make deontic logic its own self-contained area of research. Nevertheless, we will be ignoring this system in this paper, focusing more on systems akin to his later dyadic formulations.

[7]His list of is-talk logics might not be exhaustive, but it at least has these, plus some variations like temporal logic, *etc.*

This claim may make it seem like I can read the minds of deontic logicians. Alas, I cannot. But a century's worth of accumulated research strongly suggests that there is a trend. That trend, as will be shown, is that any robust sense of Hume's Law – namely, the one I defend which will be precisely defined in §2 – is slowly becoming antiquated.

Finally, in §5 I will argue abductively that this suggestion follows from my offered argument, especially from acknowledging the above claim:

(Thesis) Deontic logics should respect Hume's Law.

I will conclude the paper with some last remarks about the potential for progress in the field of deontic logic. Specifically, I will suggest the possibility of a promising, novel approach to deontic logic – one that fully respects Hume's Law. Programming Hume's Law into the very syntactic structure of a logic might, at the end of the day, prove to be very helpful. I will end by discussing possible repercussions were deontic logicians to dismiss my suggestions.

Chapter 2

The Is-Ought Problem

Hume's answer to the is-ought problem resonates throughout the *Treatise*. Simply put, Hume takes the Justification Clause as fact while he questions the very possibility of the Deduction Clause. Hume questions the very possibility of such a deduction by claiming that no one *could ever* deduce ought-talk from is-talk. It is only philosophers who make it formally appear so. Ought-talk is a disjoint domain of discourse.

Hume's Law is the name for this above claim hereafter; to fully appreciate the authority of the claim, I precisely formulate it as such:

> HUME'S LAW: One cannot deduce ought-talk from a premise set that is completely lacking in ought-talk.

One such philosopher who makes it formally appear like we can accomplish such a deduction is A. N. Prior. In 1960, Prior produces a particularly troublesome counterexample to Hume's Law [15]. It involves two entailments of propositional logic.

The first is an instance of $\vee$-Introduction whereby any accepted premised automatically entails a disjunction:

$$\frac{\text{Tea drinking is common in England.}}{\text{Tea drinking is common in England } \textit{or} \text{ all Kiwis ought to be shot.}}$$

The second is an instance of disjunctive syllogism, whereby the negation of a disjunct 'detaches' the other:

$$\frac{\text{Tea drinking is common in England } \textit{or} \text{ all Kiwis ought to be shot.} \qquad \text{Tea drinking is } \textit{not} \text{ common in England.}}{\text{All Kiwis ought to be shot.}}$$

Prior's argument is that our intuition is to count the conclusion of the first derivation as ought-talk. There, right away, he says, we have a direct violation of Hume's Law: We have validly derived ought-talk from is-talk. But wait: what if we allow ourselves to discount that mixed is-ought conclusion from being genuinely ought-talk? Then one must look no further than the second derivation. Assuming now that the first premise, the disjunction of the first derivation, is purely is-talk, it seems as if again we are deriving ought-talk from is-talk, for no one will deny that *that* conclusion is authentically normative in nature.

In taking Hume seriously, whilst simultaneously appreciating the force of Prior's dichotomous trap, we must begin to wonder: Is Hume's answer – i.e., Hume's Law – the right answer? This is the gateway into a non-Humean answer: By remaining skeptical (i.e., 'agnostic') about the Deduction Clause, non-Humeans are free to question the future impossibility of the Justification Clause. And how do they exactly go about doing this? Well, by actively searching for a possible justification for deducing ought-talk from is-talk. In doing so they of course ignore the potency of Hume's Law.[1] This is a mistake, Hume thinks. I concur.

[1] Non-Humeans are not necessarily coextensive with any particular group, e.g., deontic logicians.

No matter how one would answer the is-ought problem, one thing is set in stone: The is-ought problem has to be seen as important because of the deep ramifications *any* of its answers may have. Moreover, it is our philosophical responsibility to come up with a solid solution. Inaction in this situation is the worst kind of action – we must commit ourselves to one path in order to see where it leads. If we do not, then we have no way of reflecting upon the actual semantic import of our normative discourse.[2]

One objection to this motivation runs as follows. The task on the table – for those choosing the metalogical route – is to design a logic that accounts for how we use normative discourse. There is no pre-theoretical need to be a Humean or non-Humean on this matter ahead of time. That decision is just irrelevant to what stuff we end up building into our logical systems.

I respond to this general criticism in two ways. First, by pointing to the agnosticism that non-Humeans already subscribe to, one can see that not much of a decision has been made anyways. By loose analogy, it is just categorically more of a scientific approach, as it embraces a kind of open-mindedness that is essential to experimental research in general. Thus, the decision is relevant, but more so in how one will methodologically approach the problem, and not so much in the resultant content of the constructed logics.

Second, and much more importantly, is that the name of the game is just wrong in the above objection. The task *is not* to design a logic that *describes* how we use normative discourse (insofar as our normative *utterances* qua *phonetical strings* just amount to being contingent – i.e., abnormative – things in the World). The task is to design a logic that *prescribes* how we *ought* to use normative discourse. The former task would be indeed the project of the mathematical logician, but insofar as we are concerned with a project whose province is that

[2]As a slogan: 'ought-talk collapses into naught-talk'. That is, even if it is internally consistent, without anything in our *ideology*, let alone the World, to latch onto, it becomes vacuous and mere game-playing.

of *philosophical* logic, we are best to couch it in its own ought-talk.

One might continue: If our best logical system for telling us how we ought to reason violates Hume's Law, then that should count as a good reason to think that Hume's Law is wrong. *But that would just be yet another instance of breaking Hume's Law.*[3] The only way to avoid these circular paths is to be clear about our stance from the beginning.

Without a doubt, a lot of deontic logicians come across instrumentally as non-Humeans. That fact leaves us with a question: Is the project of constructing deontic logics even worthwhile? Is the project missing the point? Having discussed already the two main answers to the is-ought problem – Humean and non-Humean – while now realizing that the first approach – following Hume's Law – is largely ignored (metalogically), we can begin to ask: is there an underlying problem in how we are dealing with this problem (metalogically)? These questions and more will be investigated in the chapter up ahead.

[3] Any ought-talk in the antecedent of the preceding sentence is merely being mentioned, whereas ought-talk is being used in the consequent. Importantly, the topic for us is always the use of ought-talk and never mere instances of the word 'ought' or 'should' and so on. Thanks to David Faraci for important discussions pertaining to this objection.

Chapter 3

Deontic Logic

The idea of a 'deontic logic' does *not* necessarily miss the point: That is, the project of constructing deontic logics does not necessarily make its researchers non-Humeans (although this turns out to be the case in the current scene – this premise is argued for in §4).

Thus, insofar as *the idea of* a 'deontic logic' leaves open the possibilty of formally representing Hume's Law, the project of constructing a deontic logic is worthwhile to explore.[1] Well, except perhaps for the reasons given by the Dane Jørgen Jørgensen in his famous dilemma. It seems, Jørgensen writes, that philosophy has this pair of facts:

(Fact 1) Norms cannot be true or false; and

(Fact 2) Truth and falsity are essential to Logic.

Nowadays Jørgensen's dilemma thankfully seems fairly surmountable.[2] Unfortunately, cur-

[1] Amongst other reasons for exploring it, of course. All that is meant is that there exist no *prima facie* reasons for *not* exploring deontic logics, i.e., reasons for why the project itself is incoherent (besides Jørgensen's worries below).

[2] Common solutions to this otherwise pressing predicament are "deontic logic takes normative propositions, not norms" or "deontic logic uses a different conception of truth". This dilemma is still a problem, though, for command-oriented logics (whose basic terms are like 'Do X!' or 'Make Y!'), which is part of the reason why philosophers have steered away from a lot of those varieties. Dynamic logic would count as one of these.

rently prominent deontic logics face countless other paradoxes. To see where these paradoxes spring up from we will explore the varieties of systems currently being discussed in the contemporary literature. Generally, there are three main kinds of (philosophical) systems of deontic logic.[3]

3.1 von Wright-style systems (DDL)

In 1956, the Finnish philosopher Henrik von Wright produces the first system of *dyadic* deontic logic [20]. As footnoted earlier, this attempt is seen as a big step forward from his original monadic, action-type theory that is riddled with complications. This, then, will be taken as the mold for what I am calling collectively von Wright-style systems, or **DDL**.

The general axiomatic system I will use to represent **DDL** is depicted below. It uses a primitive dyadic operator '$\mathsf{O}(\chi \,|\, \phi)$' to represent 'Given ϕ, it is obligatory that χ' where 'χ' and 'ϕ' are variables that range over propositions. The symbol $\mathsf{P}(\_ \,|\, \_)$ is read the same as the obligation operator, just with 'permissible' in place of 'obligatory'. It is equivalent to '$\sim\mathsf{O}(\sim\_ \,|\, \_)$'. Lastly, $\top$ stands for any tautology (i.e., any recursively iterated form of any logical truth, theorem, or Axiom) of the language.[4]

- **CPL**-Theorem: $\{\top\} \vdash \chi \in \{\top\}$
- *Modus Ponens*: $\chi \wedge (\chi \supset \psi) \vdash \psi$
- 'Necessitation': $\chi \equiv \psi \vdash \mathsf{O}(\chi \,|\, \phi) \equiv \mathsf{O}(\psi \,|\, \phi)$

[3]Some linguists like Kratzer (1977) offer up semantic models that have quite a complex and rich structure. Nonetheless, I would still place Kratzer's work in the syntactic category of von Wright-style systems.

[4]Some conventions: **CPL** will be my shorthand for Classical Propositional Logic which has, as primitives, both $\sim$ ('negation') and $\supset$ ('implication') plus all of the sentence letters ($p, q, r, ...$). Also, 'necessitation' is scare-quoted so as not to imply anything substantive; it serves as a dummy name that is sometimes a very appropriate label and sometimes the worst. The $\vdash$ symbol will be used in standard Fregean fashion to stand for the 'consequence relation' respective to each system.

- ◦ **DDL** Axiom 1: $\mathsf{O}(\chi \wedge \psi \,|\, \phi) \supset \mathsf{O}(\chi \,|\, \phi) \wedge \mathsf{O}(\psi \,|\, \phi)$
- ◦ **DDL** Axiom 2: $\mathsf{O}(\chi \,|\, \phi) \supset \mathsf{P}(\chi \,|\, \phi)$
- ◦ **DDL** Axiom 3: $\phi \supset \mathsf{O}(\top \,|\, \phi)$

In English, the first three schemata represent the rules of inference for **DDL**. The first just says that we are allowed to use classical logical-truths ($p \supset p$, or $p \vee q \equiv q \vee p$, *etc.*) The second is the standard form of *modus ponens* 'detachment' in classical logic. The third serves as the introduction rule for our $\mathsf{O}(\,|\,)$ operator wherein an equivalence can be operated upon on both sides as long as the given condition (ϕ in the schema) is the same.

The second set of three schemata serve as the Axioms for **DDL**. Axiom 1 can be best described as a kind of distribution over conjunction law, wherein the obligation of a conjunction (given a condition) implies the obligation of both of its conjuncts individually. Axiom 2 parses as "if it is obligatory that χ, given some ϕ, then it is permissible that χ, given some ϕ. This is intuitive: How could it ever be that someone was obligated to do something that was forbidden? Axiom 3, however, is where things might turn counterintuitive. Roughly, Axiom 3 is saying that given some condition ϕ, it is obligatory that some tautology be true, given the same condition ϕ. What does it mean to say "it is obligatory that 'the sky is cloudy' implies 'the sky is cloudy'"? For now we will pocket that thought as we move on to the most reasonable semantic interpretation to give to **DDL**.

The best semantic interpretation for this general axiomatic system **DDL** is probably something like the one David Lewis sketches using possible worlds [10, 11]. To begin, one starts with Lewis' correctness conditions. To construct these we represent the actual world as w_i, any accessible (evaluable) world as w_e, and any 'best' world as w_b. Most importantly, there is the $\succ$ relation which establishes a preference ordering among possible worlds (that is ultimately based on some supervenient abstract ethical principles). We can now write:

Table 3.1: David Lewis' possible worlds semantics of **DDL**.

'$\mathsf{O}(\chi \mid \phi)$' is true in the actual world just in case *either*

(a) $\sim \exists w_e$ such that 'ϕ' is true in w_e, seen from w_i , or
(b) $\exists u \forall v$ such that $u = \{\phi \wedge \chi, ...\} \succ v = \{\phi \wedge \sim \chi, ...\}$, seen from w_i.

In English, the above conditions read as 'it is obligatory that χ given ϕ' iff there is no sufficiently morally similar world where ϕ is the case from our perspective or there is some world where both χ and ϕ are the case that is *morally better than* all other worlds in which ϕ is the case but χ is not, all from our perspective.

In criticizing Lewis' interpretation for **DDL**, Holly S. Goldman (1977) argues that Lewis, even with the $\succ$ definition of conditional obligation, fails to account for contingent features of the World [6]. Most notably, Goldman argues that any world in which 'Mr. Lingens does not break a promise' obtains is surely better than any other world in which 'he does, followed by his apology for doing so'. However, Goldman says, this former subset of worlds – the ones in which Mr. Lingens never breaks the promise to begin with – are automatically included in the contrast class, thus making statements like 'If Mr. Lingens breaks his promise, then he ought to apologize' false according to the correctness condition above. Nonetheless, Goldman concludes, the statement in the World could turn out to be true – it could be a contingent fact that Mr. Lingens is never released from his promise and he indeed makes one to begin with. Therefore the Lewisian analysis is inadequate.

Thus, although the Lewisian semantics is not enough, there is one particularly nice feature: One can translate the seemingly 'unconditional' obligations – i.e., terms that appear to have the more traditional monadic operator – into these generalized conditional obligations, symbolized with the $\mathsf{O}(\,\mid\,)$ notation. To do this one just plugs in a tautology into the second slot of the dyadic operator: Concretely, this looks like $\mathsf{O}(\chi \mid \top)$ for any seemingly unconditional obligation (e.g., 'it is obligatory not to kill'). This, once again, is very helpful formally, especially when comparing **DDL** to some of the monadic systems. However, things begin to go awry when we extend this further into the interpretation. Under Lewis' correctness conditions, it now appears that we will be morally evaluating things like $\mathsf{O}(\chi \supset \psi \mid \top)$ for any statement 'it is obligatory that, if χ then ψ' . But this is *precisely* the situation deontic logicians like von Wright originally found themselves in that inspired the need for a dyadic formulation. The case in point can be seen in the following pair of formulae:[5]

$$\text{Conditional Obligation} \begin{cases} O(\chi \supset \psi) \\ \chi \supset O\psi \end{cases}$$

Linguistically, it is often hard to tell which of the two above formulae should be used to symbolize a given sentence of ought-talk. As a result, **DDL** uses a kind of middle-ground between these two forms. However, when we have a sentence $S = O(\chi \supset \psi \mid \top)$ as expressed above, it seems as if we are left in the same sort of puzzle as we were before. It looks like the problems that first plagued monadic systems is resurrecting itself inside the new syntax. This fact, along with the limitations of its best possible semantics, puts **DDL** in a relatively weak position when we turn to look at its competitors.

3.2 Kripke-style systems (SDL)

The next family of systems I will associate with Kripke, as it is Kripke's models that apply. This family is usually called standard deontic logic, or **SDL**.

With the dominance of Kripke possible worlds semantics (especially with their use of the familiar systems that Petterssen mentions), the natural thing to do is to apply these well-

[5] I will hereafter make use of O as the generic monadic 'ought' operator, disentangled from any axiomatics (although I will sparingly use it to represent Ernst Mally's 'ought-to-be' operator as well).

known systems to the deontic concepts. The generalized system below uses a primitive monadic operator '$\mathcal{O}\chi$' to represent 'it is obligatory that χ'; '$\mathcal{O}$' takes the exact syntactic role of '$\Box$' ('neccesity') within a normal modal logic framework: That is, we have the relations $\mathcal{O}\chi \equiv {\sim}\mathcal{P}{\sim}\chi$, $\chi \supset \mathcal{P}\chi$, and so on. All notation from **DDL** is otherwise the same:

- **CPL**-Theorem: $\{\top\} \vdash \chi \in \{\top\}$
- *Modus Ponens*: $\chi \wedge (\chi \supset \psi) \vdash \psi$
- 'Necessitation': $\top \vdash \mathcal{O}\top$
- **SDL** Axiom 1: $\mathcal{O}(\chi \supset \psi) \supset (\mathcal{O}\chi \supset \mathcal{O}\psi)$
- **SDL** Axiom 2: $\mathcal{O}\chi \supset \mathcal{P}\chi$
- **SDL** Axiom 3: $\mathcal{O}\mathcal{O}\chi \supset \mathcal{O}\chi$

One often seen candidate for a desired deontic theorem is the Utopia sentence: $\mathcal{O}(\mathcal{O}\chi \supset \chi)$. If one assumes the Utopia sentence in **SDL**, then, in tandem with Axiom 1 (which is just distribution over implication like the K Axiom in modal logic), one gets precisely Axiom 3, a kind of iteration collapse formula. Further inspiration for Axiom 3 comes from Ruth Barcan's 1966 paper in which she argued for extreme skepticism regarding iterated deontic modalities: Thus, Axiom 3 makes sure that these multiply iterated 'ought' operators do not add up to anything beyond their single instance [2]. Axiom 2 may best be seen in regards to the fundamental idea behind a 'no conflicts law'. For example, if it is obligatory that you call your sister in emergencies then it is not the case that it is obligatory that you not call her. All of that consequent translates into the $\mathcal{P}$ terminology, thus giving us Axiom 2, of 'obligation entails permission'.

One resultant paradox from this straightforward monadic system is due to Chisholm (1963). Chisholm's paradox – often claimed to be the most damaging result around – not only

Table 3.2: Roderick Chisholm's contrary-to-duty quartet in **SDL**.

$\mathcal{O}\chi$	It is obligatory that Mr. Lingens leaves to help his neighbors.
$\mathcal{O}(\chi \supset \psi)$	It is obligatory that if Mr. Lingens leaves, he tells them first.
$\sim\chi \supset \mathcal{F}\psi$	If Mr. Lingens stays, then it is forbidden to tell his neighbors.
$\sim\chi$	Mr. Lingens stays.

derives conflicting obligations, but reminds us that there is absolutely no decision procedure available when it comes to 'ranking duties'.

Quickly sketched, Chisholm's argument relies on the incompatibility of the four above mutually *consistent* and yet logically *independent* statements, where the operator $\mathcal{F}\chi =: \mathcal{O}{\sim}\chi$ or $\mathcal{F}\chi =: {\sim}\mathcal{P}\chi$.

It is often said that this quartet brings two more problems to the table. The first is the question of how to reason in the face of violations. Here we are not talking about any kind of metalogical violations of the Humean sort, but direct violations of the prescriptions given by **SDL**. Furthermore, not only does Mr. Lingens go against the Moral Authority, but he also finds himself, even after his bad behavior, in a bit of a quandary. To see this, one simply applies Modus Ponens to the first pair of sentences to get $\mathcal{O}\psi$ (after applying Axiom 1 to distribute the $\mathcal{O}$), as well as to the second pair, to get $\mathcal{O}{\sim}\psi$, or, equivalently, $\mathcal{F}\psi$. The result is Mr. Lingens is obligated to both tell his neighbors *and* not tell them. The only way to cure this ailment, most philosophers say, is to rethink the structure of deontic conditionals (hence, the continued research into dyadic approaches seen in the last section).

3.3 Anderson-style systems ($\mathbf{MPL}^e$)

The third and final family of systems is credited to Alan Anderson (1958) [1]. What Anderson does for his operator is he uses a 'translation schema' '$\mathbb{O}\chi =: \Box(e \supset \chi)$' to represent 'It is obligatory that χ' where 'e' is a primitive constant – meaning "the excellent thing", making the translation schema read as, "It is obligatory that χ just means it is necessary that if the excellent thing has been achieved then χ has been done." Here 'χ' is a variable that ranges over all descriptive propositions while '$\Diamond$' denotes 'possibility'.[6]

- **CPL**-Theorem: $\{\top\} \vdash \chi \in \{\top\}$
- *Modus Ponens*: $\chi \wedge (\chi \supset \psi) \vdash \psi$
- 'Necessitation': $\top \vdash \Box\top$
- $\mathbf{MPL}^e$ Axiom 1: $\Box(\chi \supset \psi) \supset (\Box\chi \supset \Box\psi)$
- $\mathbf{MPL}^e$ Axiom 2: $\Diamond e$
- $\mathbf{MPL}^e$ Axiom 3: $\Box\chi \supset \chi$

As displayed, $\mathbf{MPL}^e$ looks like it can stand up to a lot of Humean tests. Furthermore, non-Humeans' generally lose interest with such a system as soon as they realize that the ought-talk is just encoded into 'e', thus making any talk of a 'reduction' highly misleading. To be honest, $\mathbf{MPL}^e$ presents itself as being the most friendly towards Humean approaches. The main roadblock, then, outside of the metalogical topic this paper is focused on, is in finding the proper interpretation of the primitive constant 'e'.

Looking back to Leibniz, we can note that this strategy is not unique to Anderson (see Table 3.3 on next page). The fact is *everyone* who has tried such a reduction, after noting

[6] Ernst Mally, the pioneer of deontic logic, also used a unary deontic operator. He built a system that nowadays we know can be transformed into Anderson's or Von Wright's system with just a few adjustments. An early-on theorem was $O\chi \equiv \chi$. This made his work seem useless. Lokhorst argues that this is unfortunate for we can learn a lot from Mally's efforts. We just have to put Mally's 'surprising' results to the side.

Table 3.3: Deontic logicians' constancy of constants in $\mathbf{MPL}^e$.

Constant	Creator	English Proposition Representation
b	Leibniz	"This action is done by a perfectly benevolent being"
u	Mally	"The unconditionally obligatory thing is achieved"
s	Anderson	"It is required for the sanction to soon be invoked"
d	Kanger	"All relevant normative demands have been fully met"

the superficially similar modal character of deontic notions with alethic ones, has ended up positing some arbitrary, highly *deontic* constant into the fundamental woodwork of their 'reduced' deontic logic. In other words, none of these attempts count as genuine reductions of the prescriptive to the descriptive.

All in all, the project of constructing deontic logics is definitely worthwhile: Through all the trial and error so far, the research discussed in this section supports this claim. Progress is slow, but the very idea of deontic logic, contrary to Jørgensen's first worries, is very much in the positive. Each system that is built has been put to the test of seeing whether it can capture some of our deepest intuitions about what deontic theorems should look like.

Chapter 4

Disrespecting Hume's Law

Do these aforementioned systems of deontic logic obey Hume's Law? No. They are non-Humean deontic logics.

4.1 Von Wright-style systems (DDL)

Going back to **DDL** axiomatics, we see that **DDL** Axiom Schema 3 directly breaks Humean Law: That is, the main culprit is $\phi \supset \mathsf{O}(\top \mid \phi)$. It is a foundational fact of **DDL** that from pure is-talk – e.g., anything abnormative plugged into ϕ such as 'it is raining on Titan' – one can derive an obligation – e.g., 'it is obligatory that if she can fly then she can fly, given the condition that it is raining on Titan'. This seems innocuous *prima facie*, yet it is the direct cause of multiple paradoxes. I will discuss the most curious of these.

One resultant paradox stemming from this Humean law-breaking Axiom is attributed to von Wright himself (1951). The theorem is derived like this:

P.1	$(\phi \supset \phi) \supset \mathsf{O}(\top \mid \phi \supset \phi)$	Instance of **DDL** Axiom Schema 3
P.2	$\vdash \phi \supset \phi$	**CPL**-Theorem (Self-Implication)
$\therefore$	$\mathsf{O}(\top \mid \phi \supset \phi)$	P.1, P.2 *Modus Ponens* Rule

In general, we get $\vdash \mathsf{O}(\top \mid \top)$. One way to look at it is the denial of possibly empty normative systems – i.e., systems completely devoid of obligations. This is immediately counterintuitive because we can imagine worlds with no obligations (e.g., worlds with no agents, the Hobbesian 'state of nature', *etc.*). Even worse, the above result under Lewis' interpretation would be stating that worlds where a tautology is the case is always better than worlds where it is not. But what would a world that is filled with contradictions (precisely 'tautologies not being the case') even look like? Are they not *impossible* worlds? And what intrinsic moral worth do tautologies really have? It seems as if we would need to go outside the logical system in order to entertain answers to a lot of these questions, for something being obligatory never implies that it then simply is the case, which in turn means we can always imagine something that is obligatory just not being the case. With this paradox, though, which puts the blame squarely on Humean-Law-breaking Axiom 3, we are left with few resources, syntactic or semantic, to try to to justify its status as a basic theorem of **DDL**.

Now I want to move on to the even more disrespectful family of systems, the Kripke-style **SDL** family.

4.2 Kripke-style systems (SDL)

The biggest culprit in **SDL** is $(\vdash \chi) \supset \mathcal{O}\chi$, which is just a rewriting of the 'necessitation' rule. To reiterate, this rule allows for the derivation of ought-talk – $\mathcal{O}\chi$ – just from theorems of the system. The paradoxes that result from these 'necessitation instances' are plentiful.[1] One of the first paradoxes to result from **SDL** systems is due to the Dane, Alf Ross (1941). Typically it goes like this:

$$\frac{\text{It is obligatory that the letter is mailed.}}{\text{It is obligatory that the letter is mailed } \textit{or} \text{ the letter is burned.}}$$

Of course, this is symptomatic of all non-relevance logics.[2] However, though, this problem manifests itself in the opposite sense of the one that traditionally prompts the relevance logician (e.g., Anderson, who we will see next) – the new disjunct ('the letter is burned') *is* relevant to the first ('the letter is mailed'), whereas the worry for the relevance logician stems from deductions like "The Earth is round, so the Earth is round or Li Shen was a Tang Dynasty poet" (i.e., utter irrelevance). Even worse, it seems like the additional disjunct has the power to prevent or interfere with the realization of the first, possibly the most harmful kind of disjunct one can imagine. Filling in the details, the full derivation in **SDL** is this:

P.3	$\mathcal{O}\chi$	Premise Assumed for Argument
P.4	$\vdash \chi \supset (\chi \vee \psi)$	**CPL**-Theorem ($\vee$-Introduction)
P.5	$\mathcal{O}\big[\chi \supset (\chi \vee \psi)\big]$	P.4 **SDL** 'Necessitation' Rule
P.6	$\mathcal{O}\big[\chi \supset (\chi \vee \psi)\big] \supset \big[\mathcal{O}\chi \supset \mathcal{O}(\chi \vee \psi)\big]$	Instance of **SDL** Axiom Schema 1
P.7	$\mathcal{O}\chi \supset \mathcal{O}(\chi \vee \psi)$	P.5, P.6 *Modus Ponens* Rule
$\therefore$	$\mathcal{O}(\chi \vee \psi)$	P.3, P.7 *Modus Ponens* Rule

This causes huge problems due to the second disjunct falling under the scope of the obligation

[1] This result was known from the beginning of the adoption of **SDL**. Although it had strong counterintuitive implications, the 'good' algebraic behavior of Lewis systems convinced people to stick it out and see where it leads.

[2] In other words: this is a big reason why people become relevantists, i.e., use some kind of 'relevant implication' connective.

operator. This result essentially, when taken to its limits, trivializes all of our obligations, for it says that we can always choose any other arbitrary disjunct as our real obligation.

One thing to note is that the proof *as a whole* does not violate Hume's Law. The premise set has ought-talk, so that suffices for having ought-talk in the conclusion. Nevertheless, moving from P.2 to P.3 is a direct violation, for it employs the culprit **SDL** 'necessitation' rule. Without that key ingredient Ross' paradox would be impossible to formulate.

4.3 Anderson-style systems ($\mathbf{MPL}^e$)

Last but not least, we have the most Humean of the system families: $\mathbf{MPL}^e$. But as sympathetic to Hume's view as $\mathbf{MPL}^e$ is, there still remains a big culprit, this time in the form of a provable theorem: $\Box\chi \supset \mathbb{O}\chi$. In English, this Law-breaking statement says, 'if it is necessary that water is H_2O then it is obligatory that water is H_2O'. Once again, at first this seems rather harmless, but beyond the surface it becomes increasingly strange. And all of its strangeness goes right back to its violations of Hume's Law.

One way to see the strangeness of the culprit theorem is in this idea: If every world has it, then the *ideal world* has it, too. Thus, unlike the semantics for **DDL**, another possible worlds semantics for these systems might have the 'best worlds' as being 'outside' the realm of the merely 'possible'. This idea is nothing new, for all one needs to imagine is some notion of the 'ultimate achievement' – surely some people want their ultimate goal in their life to not be actually achievable, for what if they achieve it the age of 16? The Platonic notion of perfection captures this idea well, for to reach the realm of the Forms is really to go beyond the sphere of merely possible worlds.

DDL and **SDL** are both guilty of breaking Hume's Law as the culprit violators are taken as *Axioms*. $\mathbf{MPL}^e$, then, is the least guilty, as its violations turn on a *derivable theorem*. However, $\mathbf{MPL}^e$ may in fact be worse off. Here is how.

The $\mathbf{MPL}^e$ theorem ($\text{¤}\chi \supset \mathbb{O}\chi$) is nothing more than the Weakening Axiom in disguise, i.e., the very first Axiom of Łukasiewicz's original system for propositional logic, i.e., **CPL**:

P.8	$\vdash \chi \supset (e \supset \chi)$	**CPL**-Theorem (Weakening)
P.9	$\text{¤}\big[\chi \supset (e \supset \chi)\big]$	P.8 $\mathbf{MPL}^e$ 'Necessitation' Rule
P.10	$\text{¤}\big[\chi \supset (e \supset \chi)\big] \supset \text{¤}\chi \supset \text{¤}(e \supset \chi)$	Instance of $\mathbf{MPL}^e$Axiom Schema 1
P.11	$\text{¤}\chi \supset \text{¤}(e \supset \chi)$	P.9, P.10 *Modus Ponens* Rule
$\therefore$	$\text{¤}\chi \supset \mathbb{O}\chi$	P.11 Translation Schema for $\mathbb{O}$

This last observation *strongly* suggests that we should look beyond classical foundations for building a deontic logic: The rock bottom itself is seen to be the direct source of Hume's Law violations for the $\mathbf{MPL}^e$ family (but otherwise these Anderson-style systems present themselves as being the most Humean in design.) Most surprisingly, though, is the fact that all of the systems I have looked at – **DDL**, **SDL**, and $\mathbf{MPL}^e$ – have problems with *Modus Ponens*: This is just another classical issue. Thus, we have come full circle, back to Prior's original counterexample. Prior uses disjunctive syllogism, but that is really nothing more than standard *Modus Ponens* in disguise. For with any deontic system that has *Modus Ponens* as a rule, when one has the following

$$\chi \wedge (\chi \supset O\psi) \vdash O\psi,$$

one is then licensed to infer any ought-talk whatsoever from pure is-talk, exactly like Prior's second derivation. We might want to take a pause here: In Charles Pigden's famous response to Prior, he concludes by pointing out that this supposed 'autonomy of ethics' found in the is-ought problem is not unique to normative discourse [14]. Pigden explains that the conservativeness of classical logic itself prevents us from inferring even hedgehog-talk from non-hedgehog-talk: That is, no conclusions which *non-vacuously* (so ignoring the dummy disjuncts that Prior picks on) contain the predicate 'is a hedgehog' can be validly derived from premises which do not have such hedgehog predicates. Thus, Pigden tells us, the is-ought problem is simply a very special case of a much more general theorem.

Pigden is quite wrong, however. The autonomy of normative discourse is not like the autonomy of hedgehog-talk. Yes, it is definitely true that we must add in explicitly some premise like 'if that animal has this trait and that trait and... then it is a hedgehog' in order to later deduce that 'some animal is indeed a hedghog', but that is not to say that this newly added premise is controversial in itself. For ought-talk, the analogue of the above explicit premise ('if this is the case and that is the case and... then this ought to be the case') is unacceptable under Hume's Law. To assume that it is true is to give no more respect to Humean Law than it would be to derive it later on. Either way, autonomy of ought-talk goes well beyond the level of the autonomy of hedgehog-talk.

Thus, we have seen that even beyond the augmented deontic axiomatics of these systems that violate Hume's Law, their very classical base is allowing for even larger violations. One might now wonder: But do all of the paradoxes of deontic logics solely stem from the denial of Hume's Law?

Chapter 5

Conclusion

Yes, the majority of deontic logic paradoxes do in fact come from a foundational denial of Hume's Law. Thus, deontic logics – if they employ the $\mathsf{O}(\,|\,)$, $\mathcal{O}_{-}$, $\mathbb{O}_{-}$, or O_{-} operator – should respect Hume's Law because that will most likely allow for the extermination of many of the most persistent paradoxes. A conscious acceptance of Hume's Law would immediately allow for less controversial deontic Axioms and theorems, but it is also observed that classical laws themselves should be modified appropriately. Anderson-style systems, most notably, do not intentionally mean any violence against Hume's Law – on the surface, they actually are rather respectful. Nevertheless, the classical frameworks that they build upon already have violations of Hume's Law within them.

Accordingly, Hume's Law ought to be seen as a satisfactory answer to the is-ought problem, but only in conjunction with the modest recognition that we might in fact have to look beyond our classical Logic. The reasons that support my thesis are pragmatic in origin – namely, irrespective of the 'Truth' of Hume's Law, a deontic logician ought to respect it so as to minimize the number of paradoxes found in deontic logics.

All deontic logicians have to go on is the success of the past: Logics of is-talk are familiar. Therefore we have the desire to use logics of is-talk, thereby making use of their ('supposedly purely') descriptive terms (e.g., Anderson-style systems) or, in the least, employing their familiar formalisms and inference rules (e.g., Kripke-style systems).

Most of the paradoxes arise out of this implicit denial – i.e., active agnosticism about acceptance – of Hume's Law, which has done us the great deed of showing us just how unfit the classical foundations we assume they all have really are. In ultimately arriving at a conclusion that argues for a certain 'autonomy of ought-talk', what are we to make of so-called 'bridge laws'? What is their 'status'? Philosophers of mind, for instance, entertain the idea of psycho-physical bridge laws. Would accepting the is-ought analogue (i.e., something like 'value-fact bridge laws') of this strategy be compatible with loyalty to Hume's Law?[1]

- Example I: $O\chi \supset \Diamond\chi$
- Example II: $\{\chi \;,\; \sim\!\chi\} \not\models O\bot$

For example I, one need not accept such a Kantian law since it is conceivable that 'what ought to be the case' – i.e., 'what is the case' in some *ideal world* – does not obtain in any real *possible world* (this is identical reasoning used with the Anderson-style culprit theorem). This just (once again) invokes the notion of an asymptotic limit such that what really 'ought to be' is aimed for, yet it is never fully achieved.

Nevertheless, the existence of example II makes one give the idea of value-fact bridge laws a second chance. The very form of example II (note: it is a non-entailment) is representative of *paraconsistent logics* (and the kind of 'trivialism' that they are designed to get around).

[1] This goes back to the beginning of the paper where I pointed out the two versions of the is-ought problem. Here the metaphysical route surfaces, so the consequences of these laws may or may not have an effect on the metalogical issues. For example, someone could believe in a value-fact bridge law without subscribing to the idea that sentences involving pure is-talk can capture the meaning of ought-talk. Thanks to David Faraci for comments on this point.

These logics are heavily studied in philosophy nowadays [18]. The most common paraconsistent logic is Graham Priest's Logic of Paradox. This logic takes example II – technically called 'No Explosion' – as foundational and builds a three-valued logic out of what remains. Its expressive power is weaker than that of classical two-valued logics (such as the systems of this paper), but recent work by Beall and others has shown how this weakness can be overcome [3].[2] Furthermore, a weaker version of Example I (i.e., using permissibility instead of obligation) *does* in fact violate Hume's Law when one takes the contrapositive, sometimes called Hintikka's Theorem: $\sim\Diamond\chi \supset O{\sim}\chi$ [16, 12]. Thus, this serves as more reason (for those who accept Example I) to 'go paraconsistent', as contraposition can be reworked in such systems (through notions like 'paraconsistent negation' and so forth).[3]

In sum, deontic logicians ought to respect Hume's Law. By doing so, the paradoxes that plague the various approaches should become much easier to handle. Furthermore, I suggest that exploration into many-valued deontic logics is likely to be a highly productive avenue of research within the still growing field of deontic logic. I hope to personally pursue this research in the future, as do I hope that the important field of deontic logic continues to generate new insights into how the fundamental logical structure of our normative discourse could and *should* look. My last suggestion for deontic logicians is this: Interdisciplinary collaboration amongst philosophers of logic, metaethicists, computer scientists, and sociolinguists cannot be overemphasized. We need all the help we can get.

[2] Additionally, I believe that encoding Hume's Law into the syntax of a logic – perhaps involving the machinery of many-valued logics – can have profound consequences. If a logic which produces Hume's Law as a theorem *also* lines up with all of our ordinary, intuitive conceptions of ought-talk, will we not have fully vindicated our previously provisional answer to the is-ought problem, i.e., Hume's Law?

[3] The contrapositive of the stronger version – i.e., $\sim\Diamond\chi \supset \sim O\chi$, where $\sim O\chi \equiv P{\sim}\chi$ – may be in violation as well, depending on the normative status of the 'P' operator (that is: might 'P' merely be a certain kind of is-talk capacity as opposed to an ought-talk sanction? This leads to important questions pertaining to the effects of '$\sim$', too). Also: the 'weaker version' of Kant's Law I refer to is $P\chi \supset \Diamond\chi$, which falls out when one takes $\Box(\chi \supset \psi) \vdash O\chi \supset O\psi$ as an Axiom (as both Prior and Hintikka in in fact do – interestingly, this particular Axiom supplements the Anderson-style systems very well if it used to generalize the translation schema); that is, along with some other assumptions.

If deontic logicians are to ignore my suggestions entirely, then what is to come of the field of deontic logic? I think that this question can be answered now, without invoking clairvoyance: Deontic logic will become less and less relevant to its practical importance. The biggest fear might be that one day, in the not-so-distant future, artificial intelligences will consult their 'ethical software' in order to make proper decisions. However, if those decisions ultimately depend upon the brute gathering of input from their *sensory* fields, we may have a problem much larger than we could ever imagine.

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Appendix A

Four-valued Logic

Campbell Brown, in his *Minding the is-ought gap*, suggests a semantic classification of sentence types in order to vindicate Hume's Law [4]. This strikes me as the right move; however, it needs to be taken all the way. The least *ad hoc* way to go about semantic classification is separating out sentences based on their semantic *value*. In two-valued systems, this just amounts to putting truths with the truths and falsehoods with the falsehoods. But Brown has a fourfold division. Thus, a four-valued logic would be required for a full semantic classification. A slight revision of Łukasiewicz's four-valued logic would look like the following:

χ	$\sim\chi$	$\Box\chi$	$\Diamond\chi$
$\top$	$\bot$	$\#$	$\top$
$\bot$	$\top$	$\bot$	$\bigstar$
$\#$	$\bigstar$	$\#$	$\top$
$\bigstar$	$\#$	$\bot$	$\bigstar$

$\supset$	$\top$	$\bot$	$\#$	$\bigstar$
$\top$	$\top$	$\bot$	$\#$	$\bigstar$
$\bot$	$\top$	$\top$	$\top$	$\top$
$\#$	$\top$	$\#$	$\#$	$\#$
$\bigstar$	$\top$	$\bigstar$	$\bigstar$	$\#$

Instead of Brown's ethical, non-ethical, wholly ethical, and wholly non-ethical, one would now have: logical ($\top$), tactical ($\bot$), ethical ($\#$), and physical ($\bigstar$).