expenditures (1989) showed no significant differences between Black households and non-Black households with respect to clothing expenditures.

Economic Foundations of Demand Theory

The present research basically focuses on demand analysis. The fundamental framework of demand analysis is required to achieve the goals of this empirical work in applied economics. Thus, the following sections briefly sketch the concepts of neoclassical consumer demand theory and describe theoretical topics for empirical demand estimation.

Utility Maximization and Consumer Demand

Neoclassical economic demand theory assumes that consumer demand is derived from constrained utility maximization. The basic axiom of the utility maximization process is that a rational consumer will always choose a most preferred bundle of goods from the feasible set of consumption bundles allowed by his budget. The utility function is denoted by

$$u = u(\mathbf{q}) \qquad 2.1$$

where **q** is a vector of the n goods demanded, subject to a linear budget constraint,

$$\sum_{k=1}^{n} p_k q_k = x \qquad k = 1, \ldots, n \qquad 2.2$$

where q_k is the quantity demanded of the k good, p_k is the price of the k good, and x is income or total expenditure. Mathematically, the consumer demand for a good derived from utility maximization is found by the Lagrangian method:

Primal problem

$$\text{Maximizing } u = u(\mathbf{q}) \qquad 2.3$$

$$\text{subject to } \sum_{k=1}^{n} p_k q_k = x,$$

$$L(\mathbf{q}, \lambda) = u(\mathbf{q}) + \lambda(x - \Sigma p_k q_k) \qquad 2.4$$

where λ is the Lagrangian multiplier interpreted as the marginal utility of income. Then, the first-order conditions of $\frac{\partial L}{\partial q_i}$, for i = 1, . . . ,n, and $\frac{\partial L}{\partial \lambda}$ yield

$$\frac{\partial u(\mathbf{q})}{\partial q_i} = \lambda p_i \qquad 2.5$$

$$x - \Sigma p_k q_k = 0. \qquad 2.6$$

Solving equations 2.5 and 2.6 with respect to q_i,

$$q_i = f_i(\mathbf{p}, x). \qquad 2.7$$

Equation 2.7 is a Marshallian or uncompensated demand function of good i. If equation 2.7 is substituted back into the problem 2.3, it yields the indirect utility function, which expresses the maximum attainable utility given prices p and outlay x, that is

$$u^* = v(\mathbf{p}, x). \qquad 2.8$$

<u>Cost Minimization and Consumer Demand</u>

The dual to utility maximization is cost (expenditure) minimization. In the primal problem, the objective function given by u =u(**q**) is maximized subject to the budget constraint, $\Sigma p_k q_k = x$, and the optimal solution is u^*. In the dual problem setting, the objective of the problem is cost minimization. Thus, the objective function is given by

<u>Dual problem</u>

Minimizing $\Sigma p_k q_k = x$ 2.9

subject to the constraint $u = u(\mathbf{q})$. 2.10

Utilizing the same procedure as in the primal problem solving, optimal values of q are obtained. In the dual problem, however, the determining variables are u and p, not x and p as in Marshallian demands. The dual problem setting provides the same solution of q, but is denoted by

$$q_i = h_i(u, \mathbf{p}). \qquad 2.11$$

This is the income-compensated demand, or Hicksian demand function; it implies how q_i is influenced by prices with utility held constant. If equation 2.11 is plugged into the dual problem, $\sum_{k=1}^{n} p_k q_k = x$, then it provides a cost function which is the minimum cost of obtaining the given level of utility at prices **p**:

$$x = c(u, \mathbf{p}). \quad 2.12$$

The cost function can be inverted to the indirect utility function that is a function of x:

$$x = c(u, \mathbf{p}) \text{ <---------- inversion -----------> } u^* = v(\mathbf{p}, x)$$

Properties of Demand

Deaton and Muellbauer (1993) reviewed the properties of consumer demand which provide reasonable restrictions to demand models. In many empirical works, these restrictions have been tested to confirm the theoretical plausibility of estimated demand functions. One of the most important, though trivial, properties is adding up, that is,

$$\Sigma p_k h_k (u, \mathbf{p}) = \Sigma p_x f_k (\mathbf{p}, x) = x. \quad 2.13$$

The estimated total value of both the Hicksian and Marshallian demands is total expenditures. In other words, the sum of the estimated expenditures on the different goods equals the consumer's total expenditures at any given time period. This property of demand provides another reasonable restriction, the so-called adding-up restriction. The adding-up restriction implies that (see Deaton & Muellbauer, 1993)

$$\sum_k p_k \frac{\partial q_k}{\partial x} = 1 \quad \text{and} \quad 2.14$$

$$\sum_k w_k e_k = 1 \quad 2.15$$

where w_k is the budget share of good k and e_k is total expenditure elasticity. This implies that the marginal propensities to consume should sum to one. The second property of demand is homogeneity of degree zero in prices for Hicksian and uncompensated demand and in total expenditures for uncompensated demand. If all prices and total expenditures are changed by an equal proportion, the quantity demanded must remain unchanged. This property is sometimes called "absence of money illusion". The homogeneity property provides the homogeneity restriction which implies that (see Deaton & Muellbauer, 1993), for i=1, . . ., n,

$$\sum_k p_k \frac{\partial q_i}{\partial p_k} + x \frac{\partial q_i}{\partial x} = 0 \quad \text{and} \quad 2.16$$

$$\sum_k e_{ik} + e_i = 0 \qquad 2.17$$

where $\sum_k e_{ik}$ is the sum of the own price elasticity and cross price elasticities of good i, and e_i is total expenditure elasticity of good i. The third property of demand is symmetry of the cross price derivatives of the Hicksian demands, that is,

$$\partial h_k(u, \mathbf{p})/\partial p_j = \partial h_j(u, \mathbf{p})/\partial p_k \qquad \text{for all } i \neq j. \qquad 2.18$$

The symmetry expressed in equation 2.18 can be proved through Shephard's Lemma and Young's theorem: by Shephard's Lemma

$$h_k(u, \mathbf{p}) = \partial c(u, \mathbf{p})/\partial p_k \qquad h_j = \partial c(u, \mathbf{p})/\partial p_j \qquad 2.19$$

$$\partial h_k(u, \mathbf{p})/\partial p_j = \partial^2 c/\partial p_j p_k \qquad \partial h_j(u, \mathbf{p})/\partial p_k = \partial^2 c/p_k p_j \qquad 2.20$$

and by Young's theorem, $\partial^2 c/\partial p_j p_k$ equals $\partial^2 c/p_k p_j$. The last property of demand is negativity, which implies downward sloping compensated demand functions.

Two-Stage Budgeting and Separability

Multi-stage budgeting occurs when the consumer or household allocates its total expenditures in sequential stages, and is represented as a utility tree. For example, in a simple case of two-stage budgeting, the consumer can allocate his total current expenditures to broad groups of products, such as durables, nondurables and service goods at the first stage. At this first stage, the only information needed is total expenditures and appropriately defined prices for each product class. At the second stage, total expenditures on, for example, nondurables determined in the first stage are allocated among the various classes of nondurable products, such as food, clothing and shoes, gasoline and oil, fuel oil and coal, and other nondurables. The expenditures for individual product classes are expressed as functions of the group expenditure on all nondurables, in the example, allocated in the first stage and of the prices within the group only. In such a manner, the consumer can allocate the expenditures to the subgroups in sequential stages. Because expenditure allocation to any good within a group can be written as a function only of the total group expenditure and the prices of goods within that group, the demand for any good belonging to the group must also be expressed as a function only of total expenditures on the group and the prices of goods within the group. That is,

$$q_i = f_G(\mathbf{p}_G, x_G) \quad 2.21$$

where $\mathbf{p}_G$ is a vector of prices of goods within the group, and x_G is total expenditures on the group. In order to satisfy such a condition, certain assumptions must be made. A necessary and sufficient condition for the second stage of two-stage budgeting is "weak separability". The primary ideas of separability of preferences and two-stage budgeting are tightly related to one another (Deaton & Muellbauer, 1993).

The basic concept of separability is that commodities can be partitioned into groups so that preferences within the same group can be described independently of the quantities in the other groups. Thus, the utility function can be expressed

$$u = u(q_1, q_2, q_3, q_4, q_5, q_6) = g[u_1(q_1, q_2), u_2(q_3, q_4), u_3(q_5, q_6)]. \quad 2.22$$

If the marginal rate of substitution between any two goods belonging to the same group is independent of the consumption of goods within the other groups, we consider this as weak separability of preferences. By contrast, if the marginal rate of substitution between any two goods belonging to two different groups is independent of the consumption of any good in any third group, this separability is called strong separability or block additivity. A strongly separable utility function is written

$$u = f[g_1(q_1) + g_2(q_2) + g_3(q_3) + \ldots\ldots + g_m(q_m)]. \quad 2.23$$

Each group contains only one good. The above exposition of weak and strong separability does not mean that the price changes of goods belonging to other groups do not influence the consumption of goods in a group. The change in the demand for q_{mk} (quantity demanded of good k belonging to m commodity group), for example, induced by a change in the price of good j in commodity group n (p_{nj}) is proportional to the change in demand for good mk caused by the change in total expenditures, that is, where $m \neq n$,

$q_{mk} = g(p_m, x_m(\mathbf{p}, x))$	p_m = all the prices in group m
$\partial q_{mk}/\partial p_{nj} = \partial g/\partial x_m \cdot \partial x_m/\partial p_{nj}$	p_{nj} = price of good j in group n
$\partial q_{mk}/\partial x = \partial g/\partial x_m \cdot \partial x_m/\partial x$	x_m = total expenditures on group m
	x = total expenditures.

Thus,

$$\partial q_{mk}/\partial p_{nj} = \frac{\partial x_m / \partial p_{nj}}{\partial x_m / \partial x} \cdot \frac{\partial q_{mk}}{\partial x} = \mu_m \cdot \frac{\partial q_{mk}}{\partial x} \qquad 2.24$$

where μ_m is the factor of proportionality. As seen in the function 2.21, weak separability allows us to express quantity demanded of a good as a function of its own price, the prices of goods within the group, and the total expenditures on the group if a good is “separable” from groups of other goods. The advantage of weak separability is in drastic reduction of the number of independent variables required in estimation.

Aggregation

Consumer demand theory refers to the individual consumer’s demand for individual goods. However, available time-series data tend to be aggregate in broad classifications such as food, clothing, and entertainment, and this type of data refers to large groups of consumers rather than individual consumers. Aggregate demand derived from macro or aggregate data creates a problem as to whether this demand is consistent with microeconomic theory under which demand estimations are based on individual consumer behavior. This problem is referred to as the “aggregation problem” which frequently presents obstacles to direct application of aggregate data to demand analysis. To overcome the aggregation problem, certain conditions under which we can treat aggregate demand estimations as resulting from the behavior of a single utility-maximizing consumer (exact aggregation) are necessary.

One possibility is exact linear aggregation. In exact linear aggregation, individual consumers or households are assumed to be only different in expenditures, but to meet the same price factors; thus, aggregate demand can be written as a function of only prices and aggregate or mean expenditure. That is,

$$q_{ih} = f_{ih}(x_h, \mathbf{p}) \qquad 2.25$$

where q_{ih} is the demand for good i of household h, x_h is the total expenditures of the household, and **p** is a price vector. If there are H households in the whole population, the mean demand will be

$$\bar{q}_i = q_i(x_1, \ldots\ldots, x_H, \mathbf{p}) = 1/H\ \Sigma_h\ f_{ih}(x_h, \mathbf{p}). \qquad 2.26$$

Exact linear aggregation is possible if 2.26 can be written as

$$\bar{q}_i = f_i(\bar{x}, \mathbf{p}) \qquad 2.27$$

where $\bar{x}$ = 1/H$\Sigma_h x_h$, average total expenditure. Unlike 2.26, 2.27 does not depend on the distribution of total expenditures x_h across households. Expression 2.27 can only occur if all individual households have identical marginal propensities to spend on each good; that is, a change in the total expenditure distribution would not affect mean demand, and clearly all individual demand equations must be linear in x_h and have the same slope with respect to x_h . Generally, for some functions $a_{ih}(\mathbf{p})$ and $b_i(\mathbf{p})$, the individual demand equations must take the form (see Gorman, 1953, 1961)

$$q_{ih} = a_{ih}(\mathbf{p}) + b_i(\mathbf{p})x_h. \qquad 2.28$$

If aggregated, then

$$\bar{q}_i = a_i(\mathbf{p}) + b_i(\mathbf{p})\bar{x} \qquad 2.29$$

where $a_i(\mathbf{p})$ is the average of the $a_{ih}(\mathbf{p})$. The above demand equations, 2.28 and 2.29, imply that q_{ih} and $\bar{q}_i$ are proportional to x_h and $\bar{x}$, respectively. Deaton and Muellbauer (1993) note that 2.29 is necessary and sufficient for 2.27 whether or not aggregate or individual utility maximization is assumed. If individual households or consumers are assumed to maximize utility, the individual cost (expenditure) functions have the form

$$c_h(u_h, \mathbf{p}) = a_h(\mathbf{p}) + u_h b(\mathbf{p}) \qquad 2.30$$

where $c_h(u_h, \mathbf{p})$ is the cost function of individual households or consumers h, with $u_h = v_h(x_h, \mathbf{p})$. This specification is known as the Gorman polar form. The Gorman polar form of a cost function implies linear Engel curves, for which quasi-homothetic preferences are necessary and sufficient. Thus, this cost form has extremely restrictive requirements for exact aggregation.

Muellbauer (1975, 1976) introduced exact aggregation with nonlinear Engel curves to relax the restrictions of using average total expenditures in aggregate demand. He defined exact aggregation: aggregation is over budget shares, rather than goods quantities, of different consumers, and the aggregate budget share for good i, $\bar{w}_i$, is defined to be a weighted average, rather than a simple average, of individual budget shares, w_{ih}, with weights equal to the share of each individual in total expenditure on the good I; that is,

$$\overline{w}_i = [p_i \Sigma_h q_{ih}(x_h, \mathbf{p})]/\Sigma_h x_h = \Sigma_h[w_{ih}(x_h/\Sigma_h x_h] \qquad 2.31$$

and

$$\overline{w}_i = w_i(u_0, \mathbf{p}) = \partial \ln c(u_0, \mathbf{p})/\partial \ln p_i = \Sigma_h(x_h/\Sigma x_h) \cdot \partial \ln c_h(u_h, \mathbf{p})/\partial \ln p_i \qquad 2.32$$

where u_0=v($\mathbf{p}$, x_0), the indirect utility function of a representative consumer. This representative budget share function shows that, although expenditure redistribution happens among the consumers, the representative consumer utility function, u_0, does not change. Therefore, this function can capture the change of u_h or different preferences among consumers while keeping u_0 constant (Deaton & Muellbauer, 1993). Muellbauer (1975, 1976) reported that individual cost functions should take a particular form to aggregate individual budget share equations into an aggregate equation which can be regarded as being derived from the cost function of some representative utility maximizing individual. It turns out that, for exact nonlinear aggregation, the representative individual should have a cost function of the form

$$c_h(\mathbf{p}, u_h) = \lambda_h[u_h, a(\mathbf{p}), b(\mathbf{p})] + \delta_h(\mathbf{p}) \qquad 2.33$$

where a($\mathbf{p}$), b($\mathbf{p}$), and $\delta_h(\mathbf{p})$ are linearly homogeneous functions of prices and λ_h is linearly homogeneous in a and b. This cost function is called generalized Gorman polar form (GL). In the special case where the representative expenditure depends only on the distribution of total expenditures, it takes the forms of price independent generalized linearity (PIGL) and price independent generalized log linearity (PIGLOG), that is,

$$\log c(u_0, \mathbf{p}) = (1-u_0)\log a(\mathbf{p}) + u_0 \log b(\mathbf{p}). \qquad 2.34$$

The Almost Ideal Demand System (AIDS) has a cost function that belongs to the PIGL family; this system can generate exact nonlinear aggregation over individual consumers or households.

<u>Demographic Translating and Scaling</u>

Demographic variables are major determinants of consumer consumption patterns, and changes in demographic variables can cause shifts in demand structure.

Although these shifts may be caused by changes in such non-economic factors as psychological needs, attitudes, and sociological influences, demographic variables are useful proxies when investigating the underlying shifts in demand and the differences in consumption patterns among individuals.

Demographic translating and demographic scaling are generally used to investigate the influences of demographic variables precisely. Demographic translating replaces the original demand function by

$$q_i(\mathbf{p}, x) = d_i + f_i(\mathbf{p}, x - \Sigma p_k d_k) \qquad 2.35$$

where the d's are translating parameters that depend on the demographic variables and are commonly taken to be expressed by a linear function,

$$d_i = f(A_1, A_2, \ldots\ldots, A_k) = \Sigma_k R_{ki} A_k \qquad 2.36$$

or an exponential function,

$$d_i = \Pi_k A_k^{Rki} \qquad 2.37$$

where A_k are demographic variables such as age, family size, and residence locations. Translating, in the case of the Linear Expenditure System (LES), is sometimes interpreted as allowing "necessary" or "subsistence" parameters of a demand system to depend on demographic variables. If the original demand system is theoretically plausible, then the translated one is also plausible (Pollak & Wales, 1992): the indirect utility function $v(\mathbf{p}, x) = v(\mathbf{p}, x - \Sigma_k p_k d_k)$ or equivalently the direct utility function, $u(q) = u(q_1 - d_1, \ldots\ldots, q_n - d_n)$. Alternatively, demographic scaling replaces the original demand function by

$$q_i(\mathbf{p}, x) = m_i f_i(p_1 m_1, \ldots\ldots, p_n m_n, x) \qquad 2.38$$

where the m's are scaling parameters that depend on the demographic variables, that is

$$m_i = \Sigma_k R_{ki} A_k \text{ or } m_i = \Pi_k A_k^{Rki}. \qquad 2.39$$

Since total expenditure remains unchanged, any change in the m's implies a reallocation of expenditure among the consumption categories. Thus, another interpretation is that the change in demographic variables is equivalent to a change in the prices of goods consumed. If the original demand system is theoretically plausible, so is the scaled system (Pollak & Wales, 1992): the indirect utility function $u(\mathbf{p}, x) = v(p_1 m_1, \ldots\ldots, p_n m_n, x)$ or equivalently the direct utility function, $u(\mathbf{q}) = u(q_1/m_1, \ldots\ldots, q_n/m_n)$. There are other alternatives in the treatment of demographic effects: the Gorman Specification which includes first scaling and then translating the original demand function; the Reverse Gorman Specification; and the Modified Prais-Houthakker procedure. The first two are theoretically plausible, but the third one yields a theoretically plausible demand system only if the original demand system corresponds to an additive direct utility function (Pollak & Wales, 1992).

Functional Forms of Demand

Many efforts have been made to model the functional forms which satisfy theoretical plausibility when a researcher ensures that a derived demand exactly came from maximizing a utility function. For example, Stone's Linear Expenditure System (LES) was exactly derived from maximizing consumer's utility by using the Stone-Geary utility function. Imposing general restrictions on the functional forms has been one typical approach to test demand theory. This approach really results in a reduction of the number of parameters in the system of demand functions, and easily tests whether the resulting functional forms satisfy basic properties of demand functions or not. If the complete system of demand equations is considered, the degrees of freedom problem can be reduced by use of the restrictions on the parameters in an equation which are implied by consumer theory, as mentioned in earlier sections. This section reviews four major demand functional forms: the LES, Rotterdam Model, Indirect Translog Model, and Almost Ideal Demand System (AIDS). The first two models are classified as linear functional forms. The LES is assumed to take a particular utility function, but the Rotterdam model is not. The latter two models are called "flexible functional forms" which do not require particular functional forms of utility functions.

The Linear Expenditure System (LES)

The LES is derived from the Klein-Rubin utility function, which is also referred to as the Stone-Geary utility function written as

$$u = \ln u = \Sigma_i \beta_i \ln(q_i - \gamma_i) \qquad 2.40$$

where q_i is the quantity of good i, $0<\beta_i<1$, $\Sigma_i\beta_i = 1$, $\gamma_i>0$, and $q_i - \gamma_I>0$. Maximizing the utility function expressed in 2.40 subject to the budget constraint $\Sigma_i p_i q_i=x$, yields the demand function

$$q_i=\gamma_i+\beta_i[(x-\Sigma_j p_j\gamma_j)/p_i] \qquad (i, j = 1, \ldots, n) \qquad 2.41$$

where γ_i is referred to as the subsistence level of good i. The Engel expenditure function is obtained by multiplying equation 2.41 by price p_i . It is written as follows:

$$p_iq_i=p_i\gamma_i+\beta_i(x-\Sigma_j p_j\gamma_j) \qquad (i, j = 1, \ldots, n) \qquad 2.42$$

where $0<\beta_i<1$, $\Sigma_i\beta_i=1$, $q_i>\gamma_i$, and x is total expenditures; $p_i\gamma_i$ is the minimum expenditure to attain a minimal subsistence level, and $x-\Sigma_j p_j\gamma_j$ is 'supernumerary expenditure' which is allocated between the goods in the fixed proportions β_i. Equation 2.42 is known as the Linear Expenditure System (Stone, 1954). It represents the transformation of the original equation 2.41 into a theoretically acceptable form without losing its linearity: adding-up, homogeneity, symmetry, and negativity hold in LES (Phlips, 1983). If adding-up, homogeneity, and symmetry are imposed, the $n^2 + n$ original parameters being estimated reduce to $(n-1)(n/2 +1)$.

The Rotterdam Model

The Rotterdam model of Theil (1965) and Barten (1966) is expressed by a double-logarithmic system of infinitesimal changes, but it does not use any explicit utility functional form. The Rotterdam model is derived by totally differentiating a double logarithmic demand function, $\ln q_i = \alpha_i + \Sigma_k e_{ik} \ln p_k + e_i \ln x$, so that

$$d\ln q_i = \Sigma_j \partial\ln q_i/\partial\ln p_j \cdot d\ln p_j + \partial\ln q_i/\partial\ln x \cdot d\ln x = \Sigma_j e_{ij}\, d\ln p_j + e_i\, d\ln x \qquad 2.43$$

where e_{ij} is uncompensated cross price elasticity, and e_i is total expenditure elasticity. The Slutsky equation can be written as

$$e_{ij} = e^*_{ij} - e_i w_j \qquad 2.44$$

where e^*_{ij} is compensated cross price elasticity and w_j is the budget share of good j. Substituting 2.44 into 2.43 yields

$$d\ln q_i = e_i\,(d\ln x - \Sigma_j w_j\, d\ln p_j) + e^*_{ij}\, d\ln p_j. \qquad 2.45$$

For imposition of symmetry, multiply equation 2.45 by the budget share w_i, so that the final equation is derived as

$$w_i \, d\ln q_i = e_i w_i(d\ln x - \Sigma_j w_j d\ln p_j) + \Sigma_j e^*_{ij} w_i d\ln p_j \qquad 2.46$$

$$w_i \, d\ln q_i = b_i(d\ln x - \Sigma_j w_j d\ln p_j) + \Sigma_j c_{ij} d\ln p_j \qquad (i, j = 1, \ldots, n). \qquad 2.47$$

In practice, w_i is estimated by the mean of w_i, $\overline{w}_i = (w_{it}-w_{it-1})/2$. Equation 2.47 shows that $b_i=w_ie_i$ is the marginal propensity to spend on the good i, and c_{ij} estimates the net effect of a price change. Adding-up requires $\Sigma_k b_k = 1$, and $\Sigma_k c_{kj}= 0$. The Rotterdam model will be homogeneous if $\Sigma_k c_{jk}= 0$. Symmetry is simply $c_{ij}=c_{ji}$.

<u>The Indirect Translog Model</u>

In order to derive the indirect translog demand model, first, an indirect utility function should be approximated by the translog second-order Taylor approximation. Approximating the indirect utility function log u = f(log p_1, ..., log p_n, log x) in second order results in the following utility function:

$$\log u = \alpha_0 + \Sigma_i \alpha_i \log(p_i/x) + ½\Sigma_i\Sigma_j\beta_{ij}\log(p_i/x)\log(p_j/x) \qquad 2.48$$

where α_0, α, and β are parameters. The equation 2.48 is a second-order Taylor approximation to any arbitrary utility function, which was developed by Christensen, Jorgenson, and Lau (1975). By applying Roy's identity to equation 2.48, the equation provides the system of demand equations

$$w_i = \frac{\alpha_i + \sum_j \beta_{ij} \log(\frac{p_j}{x})}{\sum_j \alpha_j + \sum_j \sum_i \beta_{ij} \log \frac{p_i}{x}} \qquad (i, j =1, \ldots, n). \qquad 2.49$$

The additivity, homogeneity, and symmetry restrictions for the indirect translog model can be found. Major limitations of this model for estimating a demand system are the number of structural parameters required and the accuracy of the approximation only in the locality of some point, that is, at a particular value of x or p, not over an entire sampling period nor over an entire sample (Deaton & Muellbauer, 1993; Phlips, 1983).

<u>The Almost Ideal Demand System (AIDS) Model</u>

Deaton and Muellbauer (1980) proposed a demand system that they call the "almost ideal demand system (AIDS)". This system allows exact nonlinear aggregation in demand estimations, as discussed in the previous section on aggregation. The merit of the representation of market demands as if they were the outcome of decisions by a rational representative consumer has made for extensive application of the AIDS model to many demand system estimations. The AIDS can be derived from the PIGLOG class of cost functions. Deaton and Muellbauer (1980) defined a cost function (see Deaton and Muellbauer for details),

$$\log c(u, \mathbf{p}) = (1-u) \log\{a(\mathbf{p})\} + u \log\{b(\mathbf{p})\} \qquad 2.50$$

where a(**p**) and b(**p**) are functions of prices, **p** is a vector of prices, and u denotes utility. Utility lies between 0 (subsistence) and 1 (bliss) so that the positive linearly homogeneous functions a(**p**) and b(**p**) can be regarded as the costs of subsistence and bliss, respectively. Second-order approximation of Taylor series in equation 2.50 results in a cost function of a flexible functional form, so that a(**p**) and b(**p**) are defined by

$$\log a(p) = \alpha_0 + \Sigma_k\alpha_k \log p_k + ½\Sigma_k\Sigma_j\gamma_{kj}\log p_k \log p_j \qquad 2.51$$

$$\log b(p) = \log a\,(p) + \beta_0\Pi_k p_k^{\beta k} \qquad (j, k = 1, \ldots, n) \qquad 2.52$$

where α, β, and γ are parameters, and $\beta_0\Pi_k p_k^{\beta k}$ is a geometric price index. β_k is the weighted parameter of the price of good k. Thus, at any single point, derivatives of β_k can be set equal to the derivatives of an arbitrary cost function. Substituting equation 2.51 and 2.52 into 2.50 yields the AIDS cost function

$$\log c(u, p) = \alpha_0 + \Sigma_k\alpha_k\log p_k + 1/2\Sigma_k\Sigma_j\gamma_{kj}\log p_k\log p_j + u\beta_0\Pi_k P_k^{\beta k}. \qquad 2.53$$

The demand functions can be derived directly from equation 2.53. Taking the price derivative of 2.53 provides the quantities demanded, $\partial c(u, p)/\partial p_i=q_i$. Multiplying both sides by $p_i/c(u, p)$, the budget share of good i, w_i, is obtained. Logarithmic differentiation of 2.53 gives the budget shares as a function of price and utility:

$$w_i = \alpha_i + \Sigma_j\gamma_{ij}\log p_j + \beta_i u\beta_0\Pi_k P_k^{\beta k} \qquad 2.54$$

where u is indirect utility which can be derived by inversion of the cost function of equation 2.53. Substituting the result of the inversion into 2.54, we finally obtain the AIDS:

$$w_i = \alpha_i + \Sigma_j\gamma_{ij}\log p_j + \beta_i\log (x/P) \qquad 2.55$$

where α, β, and γ are parameters, and P is a price index defined by

$$\log P = \alpha_0 + \Sigma_k\alpha_k\log p_k + 1/2\Sigma_j\Sigma_k\gamma_{kj}\log p_k\log p_j. \qquad 2.56$$

Equations 2.55 and 2.56 define a system of demand equations. The theoretical restrictions are defined by

$$\text{Adding-up: } \Sigma_i\alpha_i = 1;\ \Sigma_i\gamma_{ij} = \Sigma_i\beta_i = 0 \qquad 2.57$$

$$\text{Homogeneity: } \Sigma_i\gamma_{ij} = 0 \qquad 2.58$$

$$\text{Symmetry: } \gamma_{ij} = \gamma_{ji}. \qquad 2.59$$

Deaton and Muellbauer (1980) suggest approximating log P by using Stone's price index:

$$\log P = \Sigma_i w_i \log p_i. \qquad 2.60$$

Thus, equation 2.55 becomes

$$w_i = \alpha_i + \Sigma_j\gamma_{ij}\log p_j + \beta_i\log (x/P^*) \qquad 2.61$$

where P^* is Stone's price index. Equation 2.61 is referred to as the Linearized AIDS (LAIDS) model.

Summary

Besides changes in income, prices and expenditures for clothing, shoes, and other commodities, the demographic structure of the population of the United States is clearly changing. These changes are likely to have major impacts on aggregate demands for clothing and shoes. Some elements of the demographic structure are changing at a faster rate than others. The increase in the proportion of women in the labor force has been particularly rapid over the past decades. Other factors such as changes in the age distribution of the U.S. population occur at a slower pace, but are nonetheless of major significance for any long term forecasting. The aging of the population, increased female labor force participation rates, and changes in racial composition have all been identified as important aspects of the demographic structure in the U.S. which are likely to affect the composition of demand for clothing and shoes. These demographic factors are therefore incorporated in the model which is developed in Chapter 4. During World War II, U.S. government regulations and clothing shortages tremendously restricted the consumption of clothing items and shoes. Thus, World War II might have had a major impact on clothing consumption and demand during the wartime.

Many studies have investigated factors affecting the consumption of clothing products in the United States. The majority of these studies have focused on clothing expenditure analyses with single equation models. The literature contains relatively few time-series studies of the demand for clothing categories and shoes in the U.S. within the framework of demand systems. Fewer still are the time-series studies which also take into account the effects of demographic variables.

Economists have developed many functional forms for demand estimation. Among those which are "flexible functional forms" are the Almost Ideal Demand Systems (AIDS) developed by Deaton and Muellbauer (1980) and the Transcendental Logarithmic (Translog) model developed by Christensen, Jorgenson and Lau (1975). A functional form is considered flexible if it can reasonably approximate a true unknown direct utility function, indirect utility function or cost function, no matter what form that unknown function might take. Exact aggregation and the two-stage budgeting procedure are possible with the AIDS model, and the model satisfies the microeconomic theory of demand. Specifications of the AIDS for the present study are discussed in more detail in Chapter 4.