

Assessment of a Leading Edge Fillet for Decreasing Vane Endwall Temperatures in a Gas Turbine Engine

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Abstract

The objective of this investigation was to improve the thermal environment for a turbine vane through reduction of passage secondary flows. This was accomplished by modifying the vane/endwall junction to include a leading edge fillet. The problem approach was to integrate optimization methods with computational fluid dynamics to optimize the fillet design. The resulting leading edge fillet was then tested in a large-scale, low speed cascade to verify thermal performance. A combustor simulator located upstream of the cascade was used to generate realistic inlet conditions for the turbine vane. Both computational and experimental results underscore the importance of properly modeling the inlet conditions to the turbine.

Results of the computational optimization process indicate that significant reductions in adiabatic wall temperature can be achieved with a leading edge fillet. While the intent of the initial fillet design was to improve the thermal environment for the vane endwall, computational results also indicate thermal benefit to the vane surfaces. Flow and thermal field results show that a fillet can enhance coolant effectiveness, prevent formation of the leading edge horseshoe vortex, and preclude full development of a passage vortex.

In experimental testing, four cascade inlet conditions were investigated to evaluate the effectiveness of the fillet in reducing endwall temperature levels. Two tested conditions featured a flush combustor/cascade interface, while the remaining two included coolant injection through a backward-facing slot. With the flush interface, fillet thermal performance was evaluated for two inlet total pressure profiles. For the design profile, the fillet had a positive impact on the endwall temperature distribution as well as on the passage thermal field. For the off-design profile, the fillet was observed to have a slightly detrimental impact on the endwall adiabatic temperature distribution; however, passage thermal field results indicate a thermal benefit for the vane suction surface. With the backward-facing slot, thermal tests were conducted for two slot coolant flow rates. For both slot flow rates, the fillet improved endwall thermal protection and prevented coolant lift-off. While increasing the flow rate of slot coolant enhanced endwall effectiveness, fillet thermal performance was similar for the two slot flow rates.

To my family.

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Nomenclature

A	= surface area as specified by limits of integration
A_p	= airfoil projected area perpendicular to the lift vector
C	= true chord of turbine vane
C_{axial}	= axial chord of turbine vane
C_d	= discharge coefficient
C_L	= coefficient of lift, $C_L = \frac{F_L}{\frac{1}{2}\rho V^2 A_p}$
c_p	= specific heat at constant pressure
C_p	= pressure coefficient (Euler number), $C_p = \frac{p_s - p_{s,inlet}}{\frac{1}{2}\rho U_{inlet}^2}$
d	= slot feed hole diameter
D	= fillet extent normal to the vane surface = dilution hole diameter = film cooling hole diameter
D_{manuf}	= manufacturing fillet height and extent
D_{max}	= maximum fillet extent normal to the vane surface
$D_{ps}(s)$	= fillet extent as a function of distance along the vane pressure surface
$D_{ss}(s)$	= fillet extent as a function of distance along the vane suction surface
F_L	= lift force normal to the flow direction
$F(\mathbf{X})$	= optimization objective function
$F_{norm}(\mathbf{X})$	= baseline normalized objective function
h	= convective heat transfer coefficient
H	= fillet height = height of backward-facing cooling slot

H_{\max}	= maximum fillet height
$H_{ps}(s)$	= fillet height as a function of distance along the vane pressure surface
$H_{ss}(s)$	= fillet height as a function of distance along the vane suction surface
I	= momentum flux ratio, $I = \frac{\rho_c V_{\text{jet}}^2}{\rho_\infty V_\infty^2}$
k_{endwall}	= thermal conductivity of the experimental measurement endwall
k_{vane}	= thermal conductivity of the experimental vane
k	= thermal conductivity of air = turbulent kinetic energy, $k = 0.5(u_{\text{rms}}^2 + v_{\text{rms}}^2 + w_{\text{rms}}^2)$
l	= cross-passage length
\dot{m}	= mass flow rate
Ma	= Mach number
n	= coordinate normal to inviscid streamline
N	= number of film or dilution holes
p_s	= static pressure
p_o	= total pressure
p_{oper}	= operating static pressure used in computations
P	= vane pitch = pitchwise film cooling hole spacing
q	= heat rate
R	= electrical resistance = gas constant for air
Re_{exit}	= turbine cascade exit Reynolds number, $U_{\text{exit}}C/\nu$
Re_{inlet}	= turbine cascade inlet Reynolds number, $U_{\text{inlet}}C/\nu$

s	= coordinate aligned with inviscid streamline; surface distance along vane measured from flow stagnation
S	= span of turbine vane = streamwise film cooling hole spacing
S_{Dmax}	= location of maximum fillet extent along the vane surface
S_{Hmax}	= location of maximum fillet height along the vane surface
$S_{max,ps}$	= extent of fillet wrap around the pressure surface of the vane
$S_{max,ss}$	= extent of fillet wrap around the suction surface of the vane
$S_{total,ps}$	= total vane pressure surface length
$S_{total,ss}$	= total vane suction surface length
S_x	= sample standard deviation
T	= static temperature
T_{aw}	= adiabatic wall temperature
\bar{T}_{aw}	= area-weighted average adiabatic wall temperature
TB	= temperature based thermal benefit, $TB[\%] = \frac{\bar{T}_{aw} _{baseline} - \bar{T}_{aw} _{fillet}}{T_{ms} - T_c} \times 100$
TB_η	= effectiveness based thermal benefit, $TB_\eta[\%] = \frac{\eta_{fillet} - \eta_{baseline}}{\eta_{baseline}} \times 100$
T_c	= coolant flow temperature
T_{ms}	= midspan flow temperature
T_{sur}	= temperature of the surroundings
Tu	= turbulence intensity based on inlet velocity
U	= overall heat transfer coefficient of a heat exchanger
U_{exit}	= vane cascade exit freestream velocity

- U_{inlet} = vane cascade inlet freestream velocity
 U, V, W = absolute velocity components
 u, v, w = local flow plane, transformed velocity components
 u_x = uncertainty in measured quantity x
 V = velocity magnitude
 V_L = line voltage
 V_p = phase voltage
 V_s = streamwise velocity, $V_s = u \cos \Psi_{ms} + v \sin \Psi_{ms}$
 V_n = normal velocity, $V_n = -u \sin \Psi_{ms} + v \cos \Psi_{ms}$
 V_z = spanwise velocity, $V_z = w$
 W_{Delta} = maximum power output of each duct heater zone
 \mathbf{X} = fillet design variable vector
 X, Y, Z = global, stationary, coordinate system
 x, y, z = local coordinate system

y^+ = inner coordinate transverse distance (2-D), $y^+ = \frac{y \sqrt{\tau_w / \rho}}{\nu}$

Greek

- α = angle of attack
 β = transformation angle between the global and local coordinate systems
 γ = ratio of specific heats
 δ = boundary layer thickness
 δ^* = displacement thickness
 Δ = denotes a difference in value

- Δp = total to static pressure differential
- Δp_o = normalized total pressure, $\Delta p_o = \frac{(p_o - p_{o,ms,ave})}{\frac{1}{2}\rho U_{ave}^2}$
= total pressure loss through the vane passage
- ε = turbulence dissipation rate
= surface emissivity
- η = turbine engine cycle efficiency
= adiabatic effectiveness, $\eta = \frac{T_{ms} - T_{aw}}{T_{ms} - T_c}$
- $\bar{\eta}$ = lateral average adiabatic effectiveness, $\bar{\eta} = \frac{\int_{ss}^{ps} \eta(\xi) d\xi}{\int_{ss}^{ps} d\xi}$
- $\bar{\eta}$ = area-weighted average adiabatic effectiveness, $\bar{\eta} = \frac{\sum_{i=1}^n \eta_i l_i}{\sum_{i=1}^n l_i}$
- $\eta_{\infty c}$ = compressor polytropic efficiency
- $\eta_{\infty t}$ = turbine polytropic efficiency
- θ = nondimensional flow temperature, $\theta = \frac{T_{ms} - T}{T_{ms} - T_c}$
- θ' = complementary nondimensional flow temperature, $\theta' = 1 - \theta = \frac{T - T_c}{T_{ms} - T_c}$
- μ = dynamic viscosity
- μ_t = turbulent viscosity
- ν = kinematic viscosity
- π_c = compressor pressure ratio
- ρ = density

- σ = Stefan-Boltzmann constant, $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$
- ϕ = pitch angle, $\phi = \tan^{-1}(w/u)$
- ψ = yaw angle, $\psi = \tan^{-1}(v/u)$
- ψ_{ms} = midspan yaw angle, $\psi_{ms} = \tan^{-1}(v_{ms}/u_{ms})$

Subscripts

- a = ambient
- ave = mass average value
- aw = adiabatic wall
- c = coolant condition
- cond = conduction
- conv = convection
- exit = exit value at midspan
- hx = heat exchanger
- inlet = inlet value at mid-span
- jet = dilution or film-cooling jet value
- o = stagnation value
- max = maximum
- min = minimum
- ms = mid-span value
- o = stagnation value
- primary = primary flow
- ps = pressure surface
- rad = radiation
- rms = root mean square
- secondary = secondary flow
- ss = suction surface
- ∞ = mainstream