Essays on Factor Models

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ABSTRACT

This dissertation consists of three chapters describing the applications of factor models in different fields of asset pricing. The first chapter addresses the following issue: Prominent volatility-based factor pricing models focus exclusively on the second moment of asset returns, and hence, tend to identify volatile factors but with little risk premia. This chapter demonstrates that a simple asset return transform can arbitrarily upset the ranking of volatility-based factors, but not their prices of risks. Accordingly, we propose a new framework to identify factors based on their prices of risks, or the so-called *principally priced risk factors* (PPRFs). We construct these factors by generalizing the standard Sharpe ratio for a single asset to a set of assets, incorporating information from both the first and second moments of asset returns. The PPRF framework improves out-of-sample pricing performance in both equity and currency markets.

The second chapter identifies the origins of covariance in institutional trading. Conceptually, we introduce two perspectives: the asset perspective, which prioritizes assets as the key market fundamentals, and the manager perspective, which prioritizes fund managers as the key market fundamentals that drive institutional trading covariance. Empirically, we establish that the asset perspective is the primary driver of covariance in institutional trading. Our analysis documents two further empirical patterns. First, returns stemming from the covariance in institutional trading from the asset perspective have higher volatility, offering valuable insights into the demand-based asset pricing literature. Second, the persistence in trading often breaks down during economic downturns, suggesting potential connections to the uncertainty-based business cycle literature.

Finally, the third chapter examines the impact of changes in monetary policy rules on the asset valuations of firms with different profitability. I have the following two empirical findings. First, during periods of hawkish monetary policies, the 'profitability premium'— the expected extra return on investments in more profitable firms — tends to increase. Second, when analyzing the factors mediating this effect, changes in inflation expectations play a more significant role in influencing the profitability premium during transitions to a hawkish monetary regime, compared to the effects of real interest rate adjustments on production costs. These observations suggest a possible mechanism by which monetary policy may have different long-term effects on firms with different characteristics.

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GENERAL AUDIENCE ABSTRACT

This dissertation explores factor models in asset pricing across three chapters. The first chapter critiques volatility-based models that focus on asset return variance and introduces a new framework for identifying factors based on risk prices, enhancing pricing performance in equity and currency markets. The second chapter investigates the origins of covariance in institutional trading, emphasizing the asset perspective as the dominant influence and documenting higher volatility and breakdowns in trading persistence during economic downturns. The third chapter examines the effects of monetary policy changes on firm asset valuations, finding that hawkish policies increase the profitability premium, significantly influenced by shifts in inflation expectations rather than changes in real interest rates. These insights highlight the nuanced impacts of market fundamentals and monetary policy on asset pricing and firm profitability.

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Chapter 1

Principally Priced Risk Factors

A fundamental tenet of the rational asset pricing framework is the trade-off between risk exposures and compensated returns in the financial asset market. But risks vary in their natures, with some requiring higher compensated returns (risk premia) while other subjecting financial assets to more widespread exposures. Pricing factors constructed to encapsulate risks inherit these diverse natures and therefore also exhibit various levels of statistical properties and pricing performances. The quest of identifying relevant and stable pricing factors in asset returns has long fascinated researchers and practitioners because these factors inform both conceptual and practical understanding of the financial market. The current paper proposes a novel but well-founded framework to incorporate both risk and return characteristics of traded assets to identify and prioritize important pricing factors, namely the principally priced risk factors, of the financial market. The proposed framework to construct these factors are simple and flexible, allowing it to be integrated and implemented in other factor constructions to deliver pricing improvements.

The framework starts with a known leverage-based illustration of the risk-return trade-off in a frictionless financial market. That is, financial leveraging (i.e., borrowing more or less from a money market account to finance a risky asset position) scales the position's return and return volatility proportionally, leaving the position's Sharpe ratio intact and equal to the Sharpe ratio of the underlying risky asset. When more risky assets are included in the consideration, the leverage transforms that scale individual risky assets' returns and volatilities now can also change the covariance structure of asset returns to identify prominent systematic risks (those explaining large shares of covariations). Importantly, these transforms can be informed by the risk premia of assets to further identify prominent priced risks (those offering significant risk premia). As a result, the framework delivers principally priced risk factors (PPRFs) whose prominent first and second moments price a cross-section of asset returns in-sample, and are subject to rigorous out-of-sample tests.

Intuitively, the PPRF framework is based on the idea of first changing the set of original assets into an equivalent set of transformed assets of desired characteristics, for which pricing factors can be identified more robustly. The construction and employment of equivalent asset returns are instrumental to all three main results of the paper. First, the volatility-based factor construction and ranking (as in the standard principal component analysis) are highly amenable to returns' transforms because the return covariance structure varies with these transforms. This variability indicates significant pricing improvements. Second and related to the first point, we transform asset returns to control for (i.e., homogenize) their first moments. A standard PCA implemented on these transformed asset returns then delivers PCs as pricing factors that are infused with information about both the first and second moments of the original asset returns. Third, on top of the U.S. equity market, we also examine the foreign exchange (FX) market where currencies are traded. Conceptually, an integrated FX market offers a natural setting to implement and test PPRFs. It is where investors in all base (denomination) currencies face an identical set of FX risks, while FX covariance structure and risk premia vary strongly with the base currencies. The variability of these return characteristics with base currencies helps to demonstrate the flexibility and robustness of PPRF framework to identify important risk factors adaptable to changes in base currencies, while the FX risk space stays fixed.

Our empirical analysis employs three equity data sets, namely Fama-French 25 size-B/M available from Ken French's website sorted portfolios (denoted as FF25), 74 extreme-decile anomaly portfolios (denoted as LP74) and 370 anomaly portfolios (denoted as LP370) from Lettau and Pelger [33]. We also employ daily exchange rate data of 11 developed countries from Thompson Reuters (WM/Reuters benchmark rates). For each equity data set, we examine the performance of five different factor pricing models based on four different standard measures of the pricing power, both in and out of samples, while retaining 3 or 5 model-implied leading factors. The five pricing models are standard PCA (PCA), principally priced risk factor model (PPRF), risk-premium PCA (RP-PCA) Lettau and Pelger [33], combined PPRF with RP-PCA (RP-PPRF), and Sharpe Ratio Matrix (SRM). Given a factor pricing model, four pricing measures are the maximum SR attainable (Max SR), the square root of the average pricing error (RMSE-α), percentage of unexplained return variation ($\overline{\sigma}_e$), and Gibbons-Ross-Shanken statistic (GRS). Out-of-sample, and therefore the most relevant pricing power measure of Max SR for the PPRF framework, the empirical analysis shows that PPRF outperforms PCA, and RP-PPRF outperforms RP-PCA, whether we retain 3 or 5 model-implied leading factors. Because the PPRF framework is designed to improve over a given model by re-prioritizing factors based on the prices of risks they represent, these improvements between PPRF-versions (PPRF and RP-PPRF) and the respective original factor models (PCA and RP-PCA) present OOS empirical evidence to support the working mechanism of principally priced risk factors. On the other hand, PPRF-versions have a significantly higher percentage of unexplained return variation ($\overline{\sigma}_e$) than the respective original factor models. However, this measure of pricing power is scalable by asset return transforms.

For FX data, in addition to five-factor pricing models considered in the equity data tests, we also examine a model based on two prominent FX factors, namely the Dollar factor (RX, which borrows the base currency and lends equally in foreign currencies) and the carry trade factor (HML) [36]. We also employ the four different standard measures of the pricing power as for the equity data, both in and out of samples, in four different base currencies, namely, USD, AUD, JPY and GBP. Out-of-sample, and therefore the most relevant pricing power measure of Max SR for the PPRF framework, the empirical analysis shows that PPRF and RP-PPRF outperform other factor pricing models in all four base currencies. These OOS empirical results demonstrate the adaptability and robustness of the framework of principally priced risk factors for different investors associated with different base currencies in FX market.

The paper is organized as follows. Section 1.1 introduces the transforms of asset returns and illustrates the relevance of these transforms in asset pricing in three settings, namely, the standard PCA, the rare disaster risks, and the returns in different base currencies. Section 1.2 formalizes the framework of principally priced risk factors and introduces the Sharpe Ratio Matrix. Section 1.3 presents the empirical analysis for equity data and Section 1.4 for FX data.

1.1 Principally Priced Risk Factors: Setup and Illustrations

This section specifies the asset market setup and consider a transformation of asset returns to characterize and motivate the principal pricing factors in the asset market (Section 1.1.1). Because this transformation preserves the asset return space, it enables the identification of important factors of the same asset pricing model underlying the original asset returns. The section then illustrates the relevance of the principal pricing factors in the context the risk-premium principal component analysis (RP-PCA), rare disaster risks, and foreign exchange (FX) risks (Section 1.1.2).

1.1.1 Market Setup and Risk Factor Transformation

We model a financial market in a discrete-time setting with time index $t \in \{0, \ldots, T\}$. Let the market consist of a money market account (i.e., risk-free bond) Y_0 and N non-redundant risky assets $\{Y_n\}_{n=1}^N$. For the holding period from t to $t+1$, the bond offers the risk-free rate return r_t and the risky assets offer the following respective excess returns

$$Y_{nt+1} = \mu_n + \sigma_n \varepsilon_{nt+1}, \qquad n \in \{1, \dots, N\}, \;\; t \in \{0, \dots, T-1\}, \tag{1.1}$$

where $\{\varepsilon_{nt+1}\}_{n=1}^N$ are standard normal (possibly correlated) shocks. To set up the notation, we adopt the excess return convention throughout. Namely, a generic risky return X denotes the return in excess of the contemporaneous risk-free rate (i.e., excess returns), and $\mu[X]$ and $\sigma[X]$ denote respectively the conditional mean (i.e., risk premium) and volatility of the excess return X. In general, these moments can be time-varying in the setup. For notational simplicity, we omit their time index unless ambiguities arise. The Sharpe ratio of the excess return X then is $SR[X] = \frac{\mu[X]}{\sigma[X]}$. We also use a tilde notation $\widetilde{X}$ to denote a generic demeaned return.

We assume no arbitrage opportunities and no frictions in the financial market. Given the return space spanned by $N+1$ original assets $\{Y_0, Y_n\}$, there exist multiple equivalent sets of other basis assets that span the same return space. The alternative basis assets are non-redundant traded portfolios of the original assets. As alternative return bases leave the asset return space invariant, they are subject to and priced by same underlying (possibly unobserved) asset pricing model.[1] Our analysis examines a transform between equivalent asset return bases that is indicative of the underlying pricing structure and its important pricing factors. We first recall two simple and known properties concerning a linear transform of excess returns.

Remark 1.1 (Excess return transform)**.** Assuming an arbitrage-free and frictionless financial market,

1. any linear combination $R_{t+1} = \sum_{n=1}^N \theta_n Y_{nt+1}$ of excess returns $\{Y_n\}$ on original traded assets, where $\{\theta_n\}$ are arbitrary real weights (possibly, $\sum_{n=1}^N \theta_n \neq 1$), is also a traded excess return

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[1] That is, given a frictionless and arbitrage-free financial market, when every constituent asset of a traded portfolio is priced by an asset pricing model, the portfolio is also priced by the same model.

(i.e., the excess return of some traded portfolio),

2. given a traded excess return R_{t+1}, there exist a family of infinitely many traded excess returns parametrized as $\{R_{t+1}(\theta) \equiv \theta R_{t+1}\}$, $\forall \theta \neq 0$, of identical Sharpe ratio but different means and volatilities,

$$\mu\left[R_{t+1}(\theta)\right] = \theta\mu\left[R_{t+1}\right], \quad \sigma\left[R_{t+1}(\theta)\right] = \theta\sigma\left[R_{t+1}\right], \quad SR\left[R_{t+1}(\theta)\right] = SR\left[R_{t+1}\right].$$

The generality and simplicity of relations between traded excess returns underlie the adoption of the excess return convention in the current paper. The first point of Remark 1.1 quantifies a result that a linear combination of excess returns is always an excess return.[2] The second point of Remark 1.1 arises from the fact that $R_{t+1}(\theta) = \theta R_{t+1}$ is the excess return of a traded portfolio of weight θ in the risky asset and $(1-\theta)$ in the risk-free bond Y_{0t+1}, whose full return is

$$\underbrace{\theta\left(R_{t+1} + Y_{0t+1}\right) + (1-\theta)Y_{0t+1}}_{\text{portfolio's full return}} = \underbrace{\theta R_{t+1}}_{\text{portfolio's excess return}} + \underbrace{Y_{0t+1}}_{\text{bond return}}. \tag{1.2}$$

The magnitude of portfolio weight θ (possibly higher than one) also characterizes the leverage level of the portfolio. A higher θ scales a traded portfolio's mean return and volatility proportionally in an arbitrage-free and frictionless financial market, leaving the SR of the portfolio invariant with its leverage level θ. Furthermore, returns on portfolios $\{R_{t+1}(\theta)\}$ perfectly correlate for all $\theta \neq 0$. This perfect correlation implies that these portfolios represent an identical risk, whose price equals the portfolios' common SR, even though these portfolios load on this risk differently.

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[2] To see this, consider the following construction of a traded portfolio,

$$\left(1 - \sum_{n=1}^{N} \theta_n\right) Y_{0t+1} + \sum_{n=1}^{N} \theta_n \left(Y_{nt+1} + Y_{0t+1}\right) = Y_{0t+1} + \sum_{n=1}^{N} \theta_n Y_{nt+1}.$$

First, the tradability of this portfolio is assured as sum of portfolio's weights (associated with both the bond and risky assets) equals one, $\left(1 - \sum_{n=1}^{N} \theta_n\right) + \sum_{n=1}^{N} \theta_n = 1$. Second, the excess return of this traded portfolio is $\sum_{n=1}^{N} \theta_n Y_{nt+1}$, affirming the first point of Remark 1.1.

A well-known special case of this transform result is that co-linear excess returns, $X_{t+1}(\theta) \equiv \theta X_{t+1}$ and X_{t+1}, have identical SR but (arbitrarily) different volatility levels.[3] The robustness of SR under Remark 1.1's transform suggests an approach to construct, rank and retain the pricing factors. The approach centers on the prices of risks, rather than mean excess returns (i.e., risk premia) or volatility levels, to formulate the associated principally priced risk factors (PPRFs). We first motivate and illustrate this approach in several prominent settings, before formalizing the construction of the PPRFs in Section (1.2).

1.1.2 Illustrations

A primary implication of Remark 1.1 is that risk premia and volatilities of asset returns can be arbitrarily transformed without altering the return space and its underlying pricing model. The construction and ranking of pricing factors based separately on their risk premia or volatilities then are subject to ambiguities because these quantities can be arbitrarily changed. We illustrate these ambiguities in settings of (i) the standard principal component analysis, (ii) rare disaster risks, and (iii) foreign exchange risks. The illustrations make the case for the principally priced risk factors, which are prioritized on their prices of risks.

Principal Component Analysis

Given a set of asset returns, the principal component analysis (PCA) is a standard statistical approach that identifies and ranks principal components (PCs) based on their shares in explaining common variations (covariations) in the returns. We first construct standard PCs and employ a transform (Remark 1.1) to illustrate the ambiguity in their ranking. Such a transform exploits the fact that PCA relies solely on the second moment of returns, hence also indicates pricing

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[3] In general, excess returns are vectors in the return space spanned by $\{Y_n\}$, $n \in \{1, \ldots, N\}$. Two co-linear excess return vectors $Y_{bt} = \theta Y_{at}$ also have co-linear volatility vectors, whose magnitudes (i.e., volatility levels) are scaled by θ.

improvement when it incorporates the first moment (i.e., risk premia) into the transformed pricing factors.

Let V denote the $N \times N$ covariance matrix of the N original returns

$$V = \frac{1}{T}\begin{bmatrix} \widetilde{Y}_{11} & \dots & \widetilde{Y}_{1T} \\ \vdots & \ddots & \vdots \\ \widetilde{Y}_{N1} & \dots & \widetilde{Y}_{NT} \end{bmatrix}\begin{bmatrix} \widetilde{Y}_{11} & \dots & \widetilde{Y}_{N1} \\ \vdots & \ddots & \vdots \\ \widetilde{Y}_{1T} & \dots & \widetilde{Y}_{NT} \end{bmatrix} \equiv \frac{1}{T}\widetilde{Y}'\widetilde{Y}, \tag{1.3}$$

columns of $T \times N$ matrix $\widetilde{Y}$ represent N time series of the (demeaned) asset returns $\{\widetilde{Y}_n\}_{n=1}^{N}$, and $\widetilde{Y}'$ is the transposed matrix of $\widetilde{Y}$. For the consistency in notation, we use X_k to denote the k-th column of a generic matrix X_k hereafter.

The PCA starts with the diagonalization of convariance matrix V by a $N \times N$ orthogonal matrix O,

$$O'VO = \frac{1}{T}O'\widetilde{Y}'\widetilde{Y}O = \begin{bmatrix} \lambda_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \lambda_N \end{bmatrix} \equiv Diag\left(\lambda\right). \tag{1.4}$$

Principal components $\{\widetilde{P}_n\}_{n=1}^{N}$ associated with the asset return set $\{\widetilde{Y}_n\}$ are defined as N columns of the following $T \times N$ matrix,

$$\widetilde{P} = \begin{bmatrix} \widetilde{P}_{11} & \dots & \widetilde{P}_{N1} \\ \vdots & \ddots & \vdots \\ \widetilde{P}_{1T} & \dots & \widetilde{P}_{NT} \end{bmatrix} \equiv \begin{bmatrix} \widetilde{Y}_{11} & \dots & \widetilde{Y}_{N1} \\ \vdots & \ddots & \vdots \\ \widetilde{Y}_{1T} & \dots & \widetilde{Y}_{NT} \end{bmatrix}\begin{bmatrix} O_{11} & \dots & O_{1N} \\ \vdots & \ddots & \vdots \\ O_{N1} & \dots & O_{NN} \end{bmatrix} = \widetilde{Y}O. \tag{1.5}$$

By construction, PCs have variances equal to eigenvalues of the covariance matrix V and are pairwise uncorrelated, $\frac{1}{T}P_n'P_n = \lambda_n$ and $\frac{1}{T}P_n'P_k = 0$ for all $n \neq k \in \{1, \dots, N\}$ (1.4), (1.5). Note that PCs are linear combinations of demeaned asset returns so are also demeaned quantities.[4] Their traded

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[4] The demeaned PCs $\{\widetilde{P}_n\}$, $n \in \{1, \dots, N\}$, are associated with the tilde in accordance with the notation convention

versions, $\{P_{nt}\}$, $n \in \{1, \dots, N\}$, or the traded PCs, are constructed as portfolios whose returns perfectly correlate with the respective PCs (i.e., PC-mimicking assets). Remark 1.1's observation that any linear combination of excess returns is an excess return implies a construction of the traded PCs by simply replacing the demeaned returns by excess returns in the expressions of PCs (1.5),

$$P_n = \sum_{k=1}^{N} Y_k O_{kn}, \quad \forall n \in \{1, \dots, N\}, \quad \text{or in matrix form,} \quad P_{T\times N} = Y_{T\times N} O_{N\times N}, \tag{1.6}$$

where the columns of $T \times N$ matrices P and Y respectively represent N traded PCs $\{P_n\}$ and N original excess returns, and $O_{N\times N}$ the orthogonal matrix (1.4). Evidently, $\{P_n\}$, $n \in \{1, \dots, N\}$, perfectly correlate with the respective original (demeaned) PCs $\widetilde{P}_n$ in (1.5).

We observe that traded PCs can only be determined up to a multiplicative scalar parameter. This is because $P_n(\theta_n) \equiv \theta_n P_n$, $\forall \theta_n \neq 0$, has the same key pricing-factor properties as P_n. Namely, $P_n(\theta_n)$ (i) is an excess return (hence traded, Remark 1.1), and (ii) perfectly correlates with the original PC $\widetilde{P}_n$. Property (i) implies that the excess returns $P_n(\theta_n)$, $\forall \theta_n \neq 0$, and original traded PC P_n have an identical SR, and proportional expected excess returns and volatilities. Property (ii) implies that the perfectly correlating quantities $\widetilde{P}_n$, P_n, and $P_n(\theta_n)$, $\forall \theta_n \neq 0$, all represent an unique normalized risk $\frac{\widetilde{P}_n}{\sqrt{\lambda_n}} \in \mathcal{N}(0,1)$. Note that for a set of non-zero scaling parameters $\{\theta_n\}$, traded PCs $\{P_n(\theta_n)\}$ are non-redundant. Therefore, for any set $\{\theta_n \neq 0\}$, the original assets $\{Y_0, Y_n\}$ (1.1) and traded PCs $\{Y_0, P_n(\theta_n)\}$ (1.6) are two equivalent bases that span the same original return space. We summarize these observations on the invariance of the SRs, risks, and return space represented by the traded PCs before discussing their implications on the choice of PCs as pricing factors.

Remark 1.2 (SR-preserving transform of PCs)**.** Given the original asset returns $\{Y_0, Y_n\}_{n=1}^{N}$ (1.1), for every PC $\widetilde{P}_n$ there are infinitely many traded PCs parametrized as

$$P_n(\theta_n) = \theta_n P_n, \quad \theta_n \neq 0, \quad n \in \{1 \dots, N\}, \tag{1.7}$$

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specified below Equation (1.1).

that (i) represent an identical normalized risk $\frac{\widetilde{P}_n}{\sqrt{\lambda_n}}$, (ii) have an identical Sharpe ratio $SR\left[P_n(\theta_n)\right] = SR\left[P_n\right]$, $\forall\theta_n \neq 0$, (iii) have proportional excess returns $\mu\left[P_n(\theta_n)\right] = \theta_n\mu\left[P_n\right]$ and volatilities $\sigma\left[P_n(\theta_n)\right] = \theta_n\sigma\left[P_n\right]$, and (iv) form an equivalent basis of the same return space as the original assets $\{Y_0, P_n(\theta_n)\} = \{Y_0, Y_n\}$.

These properties help to deliver a simple illustration of the ambiguities in employing volatilities (or risk premia) to prioritize PCs as pricing factors.

First, because the transform (1.7) preserves the return space, the equivalent basis of the traded PCs $\{Y_0, P_n(\theta_n)\}$, for a set of non-zero parameters $\{\theta_n\}$, is subject to the same underlying asset pricing model that governs the dynamics of the given original assets $\{Y_0, Y_n\}$.[5] Second, since the volatilities (or risk premia) of the traded PCs vary with scaling parameters, the flexibility in the choice of parameters $\{\theta_n\}$ can change the ranking of these PCs arbitrarily when this ranking is based on PCs' volatilities (as in the standard PCA). Specifically, when one starts out with an equivalent basis returns $\{Y_0, P_n(\theta_n)\}$ (as opposed to the original and equivalent basis $\{Y_0, Y_n\}$) and perform a PCA, one obtains the N PCs $\widetilde{P}_n(\theta_n) = \theta_n\widetilde{P}_n$, $n \in \{1, \dots, N\}$ (as opposed to N original PCs $\widetilde{P}_n$, $n \in \{1, \dots, N\}$ obtained from a PCA on the original basis $\{Y_0, Y_n\}$).[6] While the return space (and hence, the underlying pricing model) remains unchanged, PCs $\{\widetilde{P}_n(\theta_n)\}$ and original PCs $\{\widetilde{P}_n\}$ can have (arbitrarily) different volatilities and volatility-based ranking. Finally, respective n-th PCs $\widetilde{P}_n(\theta_n)$ and $\widetilde{P}_n$ represent identical (normalized) risk $\frac{\widetilde{P}_n}{\sqrt{\lambda_n}}$ for all $n \in \{1, \dots, N\}$ (Remark 1.2). The price of this risk equals to the SR of the n-th traded PC $P_n(\theta_n)$, for all values of the scaling

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[5] There exit other transforms of equivalent return bases that leave unchanged the asset return space. Such transforms generally produce different sets of PCs and thus a rich but complex comparative analysis of these PC sets. We consider such a transform in the construction of the inverse SR matrix (Section 1.2.2 below). For an explicit illustration and analysis of ambiguities associated with PCs representing identical risks and return space, it suffices to consider the simple transform (1.7).

[6] Since the traded PCs $\{P_n(\theta_n)\}$ are pairwise uncorrelated, a PCA on equivalent basis returns $\{Y_0, P_n(\theta_n)\}$ results in $\{P_n(\theta_n)\}$ as PCs.

parameter $\theta_n \neq 0$,

$$SR\left[P_n(\theta_n)\right] = \frac{\mu\left[P_n(\theta_n)\right]}{\sigma\left[P_n(\theta_n)\right]} = \frac{\mu\left[P_n\right]}{\sigma\left[P_n\right]} = \frac{\sum_{k=1}^{N} \mu\left[Y_k\right] O_{kn}}{\sqrt{\lambda_n}}, \quad \forall \theta_n \neq 0. \tag{1.8}$$

Accordingly, we use interchangeably the SR of PCs and the price of PC factors (i.e., the price of risk represented by PCs) hereafter.

The above illustration not only indicates the ambiguities with the volatility-based (or risk-premium-based) ranking of pricing factors, but also hints at a more robust price-based ranking of factors studied in Section 1.2 below.

Rare Disaster Risk Factors

Facing with high dimensional data, the importance and popularity of the standard PCA also reflects in its application towards the dimensionality reduction of data. In this procedure, only a limited number of PCs are retained based on their second moments (i.e., volatilities), which quantify the shares of data covariations explained by the retained factors. Because asset pricing models aim to quantify the tradeoff between risks and returns, important pricing factors should be quantified by both first and second moments. Rare disaster risks present a notable example for this point. Intuitively, factors representing rare disaster risks would rank lower due to infrequent incidence, limiting their shares to the return covariations. Yet these factors carry significant prices due to market's aversion to these risks, making these factors important in pricing models. These intuitive arguments appear to indicate that the volatility-based ranking and dimensionality reduction are biased against risk factors of a rare disaster nature. This section presents an analytical setting to formalize and examine the above intuitions. The analysis demonstrates rare disaster risks' non-perturbative (significant) impact on the risk premia, and perturbative (small) impact on the volatilities, of PC factors. These patterns illustrate how the volatility-based dimensionality reduction overlooks rare disaster risk factors and also hint at a more robust price-based retention criterion of factors given high dimensional data.

Consider a financial market $\{Y_0, Y_1, Y_2\}$ of a risk free-bond and two risky assets. The excess returns on risky assets load on two types of risks, namely, normal (ε_{t+1}) and disaster ($\mathcal{J}_{t+1}$) risks,

$$Y_{nt+1} = \mu_n + \sigma'_{n\varepsilon}\varepsilon_{t+1} + \Delta_{nJ}\underbrace{(\mathcal{J}_{t+1} - \kappa_J)}_{\equiv \widetilde{\mathcal{J}}_{t+1}} = \mu_n + \widetilde{Y}_{nt+1}, \qquad n \in \{1, 2\}, \tag{1.9}$$

where $\varepsilon_{t+1} \in$ denotes shocks of standard normal distributions. The discrete random variable $\mathcal{J}_{t+1} \in \mathcal{P}(\kappa_J)$ counts the number of disasters taking place in the next period, which has a Poisson distribution with disaster arrival rate (i.e., intensity) κ_J, or

$$Prob(\mathcal{J}_{t+1} = m) = \frac{\exp(-\kappa_J)\kappa_J^m}{m!}, \qquad E_t\left[\mathcal{J}_{t+1}\right] = \kappa_J, \qquad Var_t\left[\mathcal{J}_{t+1}\right] = \kappa_J. \tag{1.10}$$

Therefore, $\widetilde{\mathcal{J}}_{t+1} \equiv \mathcal{J}_{t+1} - \kappa_J$ is a compensated (demeaned) Poisson random variable. The intensity κ_J quantifies occurrence's frequency of disasters and coefficient Δ_{nJ} quantifies risky return Y_{nt+1}'s loading on the disaster risk (i.e., Y_{nt+1}'s disaster size). There are possibly multiple (uncorrelated) risks of the normal type to allow for an incomplete financial market, in which case ε_{t+1} denotes a random vector of multivariate normal distribution. For simplicity, we assume that there is only one risk of the disaster type, i.e., $\mathcal{J}_{t+1}$ has a univariate Poisson distribution. We also assume that normal shocks ε_{t+1} (and Poisson counter $\mathcal{J}_{t+1}$) are independently and identically distributed (iid) over different time periods.

We analyze how the ranking and retention of risk factors are impacted by various values of the rare disaster risk's intensity (i.e., likelihood) in the asset market. To this end, the analysis varies κ_J in asset return $\{Y_1, Y_2\}$ while fixing the underlying pricing model.[7] The latter is implemented

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[7] If we vary both (i) asset return set $\{Y_1, Y_2\}$ and (ii) the underlying model M that prices these returns, then it is ambiguous whether the changes in pricing factors' ranking and retention are due to changes in disaster risk's intensity in the asset market or in the underlying pricing model. Note that a change (scaling) in the intensity $\kappa_J \to \theta\kappa_J$ amounts equivalently to a change (scaling) in the the disaster risk's exposure uniformly in all asset returns, $\Delta_{nJ} \to \theta\Delta_{nJ}$, $\forall n$. Therefore, our comparative analysis in varying disaster risk's intensity can also be

by specifying an underlying and exogenous stochastic discount factor (SDF) M_{t+1}. This SDF determines a corresponding mean excess return μ_n for every asset Y_n (and Y_n's exposure to risks) considered in the analysis and under the same underlying asset pricing model. Note the knowledge about the underlying SDF M_{t+1} is not needed for the ranking and retention of standard PC factors because they are based solely on return volatilities. We consider the following SDF to fix the pricing model,

$$M_{t+1} = 1 - r_f - \eta_\varepsilon' \varepsilon_{t+1} - \Delta_{MJ} \left(\mathcal{J}_{t+1} - \kappa_J \right) = 1 - r_f + \widetilde{M}_{t+1}, \tag{1.11}$$

where (possibly, vector) η_ε and Δ_{MJ} denotes respectively the underlying prices of normal risks ε_{t+1} and disaster risk $\mathcal{J}_{t+1}$. Given this underlying SDF, the mean excess return $\{\mu_n\}$ in (1.9) of risky assets are obtained from Euler pricing equations,

$$\mu_n = -E_t \left[\widetilde{M}_{t+1} \widetilde{Y}_{nt+1} \right] = \underbrace{\eta_\varepsilon' \sigma_{n\varepsilon}}_{\equiv \mu_{n\varepsilon}} + \underbrace{\kappa_J \Delta_{MJ} \Delta_{nJ}}_{\equiv \mu_{nJ}}, \quad n \in \{1, 2\}. \tag{1.12}$$

A severe rare disaster risk for asset pricing is quantified by a small intensity κ_J, a large price Δ_{MJ}, and significant risk premia $\mu_{nJ} \equiv \kappa_J \Delta_{MJ} \Delta_{nJ}$, $n \in \{1, 2\}$. Therefore, even though the risk premia μ_{nJ} (and hence μ_n) are functions of (linear in) the intensity κ_J, they represent the non-perturbative (leading order) impacts of the rare disaster risk on asset returns in the perturbative analysis below. The covariance matrix (1.3) of the risky asset returns $\{Y_1, Y_2\}$ (1.9) can be determined using (1.10),

$$V = \underbrace{\begin{bmatrix} \sigma_{1\varepsilon}^2 & \sigma_{1\varepsilon}' \sigma_{2\varepsilon} \\ \sigma_{2\varepsilon}' \sigma_{1\varepsilon} & \sigma_{2\varepsilon}^2 \end{bmatrix}}_{\equiv V_\varepsilon} + \kappa_J \underbrace{\begin{bmatrix} \Delta_{1J}^2 & \Delta_{1J} \Delta_{2J} \\ \Delta_{2J} \Delta_{1J} & \Delta_{2J}^2 \end{bmatrix}}_{\equiv V_J} = V_\varepsilon + \kappa_J V_J, \tag{1.13}$$

where V_ε and V_J are components of the asset return variance arising respectively from the normal and disaster risks.

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interpreted as the one in varying asset market's uniform exposure to the disaster risk.

Given that disasters are rare (i.e., small κ_J), we perform a perturbative analysis by expanding in the power of κ_J. Starting with the eigen-problem and diagonalization of the (unperturbed) 2×2 covariance matrix V_ε (1.13) in the zero order of κ_J,

$$V_\varepsilon O_\varepsilon = O_\varepsilon Diag\left(\lambda_{1\varepsilon}, \lambda_{2\varepsilon}\right), \quad O'_\varepsilon V_\varepsilon O_\varepsilon = Diag\left(\lambda_{1\varepsilon}, \lambda_{2\varepsilon}\right), \quad O'_\varepsilon O_\varepsilon = \mathbb{1}_{2\times 2} \tag{1.14}$$

we look for the eigenvalues and eigenvectors of the (perturbed) covariance matrix $V = V_\varepsilon + \kappa_J V_J$ (1.13) in the first order of a small intensity κ_J as follows,

$$\begin{gathered}
\left[V_\varepsilon + \kappa_J V_J\right]\left[O_\varepsilon + \kappa_J O_J\right] = \left[O_\varepsilon + \kappa_J O_J\right]\; Diag\left(\lambda_{1\varepsilon} + \kappa_J \lambda_{1J}, \lambda_{2\varepsilon} + \kappa_J \lambda_{2J}\right), \\
\left[O'_\varepsilon + \kappa_J O'_J\right]\left[V_\varepsilon + \kappa_J V_J\right]\left[O_\varepsilon + \kappa_J O_J\right] = Diag\left(\lambda_{1\varepsilon} + \kappa_J \lambda_{1J}, \lambda_{2\varepsilon} + \kappa_J \lambda_{2J}\right), \\
\left[O'_\varepsilon + \kappa_J O'_J\right]\left[O_\varepsilon + \kappa_J O_J\right] = \mathbb{1}_{2\times 2}.
\end{gathered} \tag{1.15}$$

By matching terms of the same order of κ_J on both sides of (1.15) and using the unperturbed eigenproblem (1.14), the perturbative components are

$$\begin{gathered}
\lambda_{1J} = O'_{1\varepsilon} V_J O_{1\varepsilon}, \qquad \lambda_{2J} = O'_{2\varepsilon} V_J O_{2\varepsilon}, \\
O_{1J} = \frac{O'_{1\varepsilon} V_J O_{2\varepsilon}}{\lambda_{1\varepsilon} - \lambda_{2\varepsilon}}\, O_{2\varepsilon}, \qquad O_{2J} = \frac{O'_{2\varepsilon} V_J O_{1\varepsilon}}{\lambda_{2\varepsilon} - \lambda_{1\varepsilon}}\, O_{1\varepsilon},
\end{gathered} \tag{1.16}$$

where O_{kJ} and $O_{k\varepsilon}$ denote respectively the k-th column of matrices O_J and O_ε per the notation convention specified below (1.3),

$$O_\varepsilon = \left[O_{1\varepsilon}\;\; O_{2\varepsilon}\right] = \begin{bmatrix} O_{11\varepsilon} & O_{12\varepsilon} \\ O_{21\varepsilon} & O_{22\varepsilon} \end{bmatrix}, \qquad O_J = \left[O_{1J}\;\; O_{2J}\right] = \begin{bmatrix} O_{11J} & O_{12J} \\ O_{21J} & O_{22J} \end{bmatrix}. \tag{1.17}$$

In the standard PCA (1.5), the diagonalization of the covariance matrix results in the $2 \times T$ PC matrix $\widetilde{P} = \widetilde{Y}O$, where $2 \times T$ matrix $\widetilde{Y}$ contains (as columns) the demeaned returns of the two risk assets in (1.9) and $O \equiv [O_\varepsilon + \kappa_J O_J]$ is the 2×2 rotation matrix diagonalizing the (perturbed) covariance matrix V (1.13). The two PCs $\{\widetilde{P}_1, \widetilde{P}_2\}$ are columns of $\widetilde{P}$, and the corresponding traded PCs $\{P_1, P_2\}$ (constructed similarly to (1.6) based on Remark 1.1) have the following excess returns

$$P_1 = [O_{11\varepsilon} + \kappa_J O_{11J}]\, Y_1 + [O_{21\varepsilon} + \kappa_J O_{21J}]\, Y_2,$$

$$P_2 = [O_{12\varepsilon} + \kappa_J O_{12J}]\, Y_1 + [O_{22\varepsilon} + \kappa_J O_{22J}]\, Y_2. \tag{1.18}$$

In the first order of κ_J, the expected excess returns of the traded PCs are

$$\mu[P_1] = \underbrace{[O_{11\varepsilon}\mu_1 + O_{21\varepsilon}\mu_2]}_{\equiv \mu[P_{1\varepsilon}]} + \kappa_J \underbrace{[O_{11J}\mu_1 + O_{21J}\mu_2]}_{\equiv \mu[P_{1J}]} = \mu[P_{1\varepsilon}] + \kappa_J\, \mu[P_{1J}],$$

$$\mu[P_2] = \underbrace{[O_{12\varepsilon}\mu_1 + O_{22\varepsilon}\mu_2]}_{\equiv \mu[P_{2\varepsilon}]} + \kappa_J \underbrace{[O_{12J}\mu_1 + O_{22J}\mu_2]}_{\equiv \mu[P_{2J}]} = \mu[P_{2\varepsilon}] + \kappa_J\, \mu[P_{2J}], \tag{1.19}$$

where assets' risk premia $\{\mu_1, \mu_2\}$ are given in (1.12). The return volatilities of the traded PCs (1.18) are

$$\sigma[P_1] = \sqrt{\lambda_{1\varepsilon} + \kappa_J \lambda_{1J}} = \sqrt{\lambda_{1\varepsilon}} + \frac{1}{2}\kappa_J \frac{O'_{1\varepsilon} V_J O_{1\varepsilon}}{\sqrt{\lambda_{1\varepsilon}}},$$

$$\sigma[P_2] = \sqrt{\lambda_{2\varepsilon} + \kappa_J \lambda_{2J}} = \sqrt{\lambda_{2\varepsilon}} + \frac{1}{2}\kappa_J \frac{O'_{2\varepsilon} V_J O_{2\varepsilon}}{\sqrt{\lambda_{2\varepsilon}}}. \tag{1.20}$$

The risk premia (1.19) and return volatilities (1.20) of the traded PCs then imply the prices of risks represented by the two PCs in the leading order

$$SR[P_1] = \frac{\mu[P_1]}{\sigma[P_1]} \approx \frac{O_{11\varepsilon}\mu_{1\varepsilon} + O_{21\varepsilon}\mu_{2\varepsilon}}{\sqrt{\lambda_{1\varepsilon}}} + \frac{O_{11\varepsilon}\mu_{1J} + O_{21\varepsilon}\mu_{2J}}{\sqrt{\lambda_{1\varepsilon}}}$$

$$= SR[P_{1\varepsilon}] + \left(\frac{\mu_{1J}}{\sqrt{\lambda_{1\varepsilon}}} O_{11\varepsilon} + \frac{\mu_{2J}}{\sqrt{\lambda_{1\varepsilon}}} O_{21\varepsilon}\right) \tag{1.21}$$

and similarly,

$$SR\left[P_2\right]=\frac{\mu\left[P_2\right]}{\sigma\left[P_2\right]}\approx SR\left[P_{2\varepsilon}\right]+\left(\frac{\mu_{1J}}{\sqrt{\lambda_{2\varepsilon}}}O_{12\varepsilon}+\frac{\mu_{2J}}{\sqrt{\lambda_{2\varepsilon}}}O_{22\varepsilon}\right). \tag{1.22}$$

To streamline the discussion, we make two innocuous assumptions.

$$\text{Assumption 1: } \Delta_{1J}>0,\ \Delta_{2J}>0, \qquad \text{Assumption 2: } \mu_1>0,\ \mu_2>0. \tag{1.23}$$

The first assumption postulates that the rare disaster is an adverse event for asset returns, i.e., $\Delta_{1J},\Delta_{2J}>0$ in (1.9), and therefore, the disaster risk covariance matrix component V_J (1.13) is a strictly positive matrix. The second assumption postulates that both assets $\{Y_1,Y_2\}$ are risk (as opposed to hedge) assets, i.e., their risk premia $\mu_1,\mu_2>0$.[8]

We consider two exhaustive and mutually exclusive configurations concerning the asset return correlation with respect to normal risks

$$\begin{array}{cc}\text{Configuration } I\text{:}\quad \sigma_{1\varepsilon}'\sigma_{2\varepsilon}<0 & \text{Configuration } II\text{:}\quad \sigma_{1\varepsilon}'\sigma_{2\varepsilon}>0 \\ \Longrightarrow\begin{cases}O_{11\varepsilon}O_{21\varepsilon}<0,\\ O_{12\varepsilon}O_{22\varepsilon}>0,\end{cases} & \Longrightarrow\begin{cases}O_{11\varepsilon}O_{21\varepsilon}>0,\\ O_{12\varepsilon}O_{22\varepsilon}<0,\end{cases}\end{array}$$

Configuration I takes place when the normal risk components of two assets negatively correlate, implying that the first PC $P_{1\varepsilon}$ (defined in (1.19), explains the most covariations due to normal risks and arises in the diagonalization of normal risk covariance matrix V_ε (1.13)) loads oppositely on the two assets, hence, $O_{11\varepsilon}O_{21\varepsilon}<0$. The second PC $P_{2\varepsilon}$ (defined in (1.19)) is uncorrelated to the first $P_{1\varepsilon}$ by construction, implying that that it loads similarly on the two assets, hence, $O_{12\varepsilon}O_{22\varepsilon}>0$. The opposite holds for the configuration II, implying $O_{11\varepsilon}O_{21\varepsilon}>0$, and $O_{12\varepsilon}O_{22\varepsilon}<0$.

We first assume and discuss configuration I. Regarding the PCs' risk premia $\mu\left[P_1\right]$ and $\mu\left[P_2\right]$

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[8] In principle, if a traded asset Y_n is a hedge asset, we can substitute it by $-Y_n$, which is also a traded and (now) risk asset. However, such a change also flips the sign of the Y_n's disaster size Δ_{nJ} and precludes the first assumption on the adverse nature of the disaster risk on asset returns. Therefore, we need both assumptions.

(1.19), recall that the severity of and aversion to the disaster risk make both assets' risk premia μ_1, μ_2 depend and vary non-perturbatively with intensity κ_J (as explained below (1.12), via the assets' disaster risk premium components μ_{1J}, and μ_{2J}). As a result, the non-perturbative terms $\mu\left[P_{1\varepsilon}\right]$ and $\mu\left[P_{2\varepsilon}\right]$ respectively dominate the first-order perturbalive terms $\kappa_J\mu\left[P_{1J}\right]$ and $\kappa_J\mu\left[P_{2J}\right]$ in the risk premia (1.19) of the two PCs. However, in configuration I, the first PC's loadings on the two assets have opposite signs, $O_{11\varepsilon}O_{21\varepsilon} < 0$, while $\mu_1, \mu_2 > 0$ (Assumption 2 in (1.23)), generating off-setting effects of the disaster risk on the first PC's risk premium via its dominant (non-perturbative) term $\mu\left[P_{1\varepsilon}\right] = O_{11\varepsilon}\mu_1 + O_{21\varepsilon}\mu_2$. In contrast, the second PC's loadings on the two assets have same signs, $O_{12\varepsilon}O_{22\varepsilon} > 0$, generating enhancing effects of the disaster risk on the second PC's risk premium via its dominant (non-perturbative) term $\mu\left[P_{2\varepsilon}\right] = O_{12\varepsilon}\mu_1 + O_{22\varepsilon}\mu_2$. Therefore, in configuration I, the disaster risk increases the second PC's risk premium $\mu\left[P_2\right]$ relatively more than the first PC's risk premium $\mu\left[P_1\right]$ in the leading (non-perturbative) order.

Regarding the PCs' volatilities $\sigma\left[P_1\right]$ and $\sigma\left[P_2\right]$ (1.20), the leading terms $\sqrt{\lambda_{1\varepsilon}}$ and $\sqrt{\lambda_{2\varepsilon}}$ are not affected by the disaster risk. In the first order of the disaster risk intensity, the first PC's pertubative term $\frac{1}{2}\kappa_J\frac{O_{1\varepsilon}'V_JO_{1\varepsilon}}{\sqrt{\lambda_{1\varepsilon}}}$ has a smaller magnitude than the second PC's pertubative term $\frac{1}{2}\kappa_J\frac{O_{2\varepsilon}'V_JO_{2\varepsilon}}{\sqrt{\lambda_{2\varepsilon}}}$ because (i) the matrix V_J is strictly positive (Assumption 1) and (ii) column vector $O_{1\varepsilon}$'s components have opposite signs and column vector $O_{1\varepsilon}$'s components have same signs, and (iii) $\lambda_{1\varepsilon} > \lambda_{2\varepsilon}$ in PCA ranking convention. Intuitively, when the first PC is long in one asset and short in the other (configuration I) while disaster risk moves asset returns in the same direction (Assumption 1), then the presence of disaster risk weakens the share of return covariations explained by the first PC (and enforces that by the second PC). Therefore, in configuration I, the disaster risk boosts the second PC's volatility $\sigma\left[P_2\right]$ relatively more than the first PC's volatility's $\sigma\left[P_1\right]$, but such a relative boost only takes place in the first perturbative order.

Regarding the prices of risks represented by PCs $SR\left[P_1\right]$ (1.21) and $SR\left[P_2\right]$ (1.22), as they are ratios of PCs' risk premia and volatilities, in which the former are affected by the disaster risk in the leading order (non-perturbatively) while the latter in the first order. As a result, these prices of risks vary with the disaster risk non-perturbatively. Quantitatively, in $SR\left[P_1\right]$ (1.21), the first term $SR\left[P_{1\varepsilon}\right]$ does not vary with κ_J, while the second term $\left(\frac{\mu_{1J}}{\sqrt{\lambda_{1\varepsilon}}}O_{11\varepsilon}+\frac{\mu_{2J}}{\sqrt{\lambda_{1\varepsilon}}}O_{21\varepsilon}\right)$ is the leading (non-perturbative) in the disaster intensity. Under Assumptions 1 and 2 in (1.23), the two components of this leading term have opposite signs, generating offsetting effects of the disaster risk on the first PC's price of risk $SR\left[P_1\right]$. Similarly, the second term $\left(\frac{\mu_{1J}}{\sqrt{\lambda_{1\varepsilon}}}O_{12\varepsilon}+\frac{\mu_{2J}}{\sqrt{\lambda_{1\varepsilon}}}O_{22\varepsilon}\right)$ of $SR\left[P_2\right]$ (1.22) is the leading (non-perturbative) in the disaster intensity. The two components of this leading term have same signs under Assumptions 1 and 2 (1.23), generating enhancing effects of the disaster risk on the second PC's price of risk $SR\left[P_2\right]$. Therefore, in configuration I, the disaster risk increases the price of risk $SR\left[P_2\right]$ represented by the second PC relatively more than $SR\left[P_1\right]$ represented by the first PC in the leading (non-perturbative) order.

Altogether, when the disaster risk is sufficiently severe,[9] its non-perturbative and enhancing contribution to the risk price of the less volatile (second) PC means that a dimensionality reduction scheme overlooking the second PC omits a key pricing factor for asset returns. We summarize these findings in the following remark.

Remark 1.3 (Rare disaster risk upsetting standard dimensionality reduction).

1. In configuration I, the disaster risk increases non-perturbatively the risk premium $\mu\left[P_2\right]$ and price of risk $SR\left[P_2\right]$, and perturbatively (in the first perturbative order of κ_J) the volatility, of the second PC, all relative to the corresponding quantities of the first PC.

2. When rare disaster is severe, it is possible that the first PC remains the most volatile PC, hence is retained by the standard PCA's volatility-based dimensionality reduction, whereas the omitted second PC has a significant risk premium and represents a risk of prominent price.

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[9] The severity of the rare disaster risk is characterized by a significant disaster risk premium component $\kappa_J\Delta_{MJ}\Delta_{nJ}$, $n\in\{1,2\}$, in asset returns as discussed below (1.12).

Intuitively, given two assumptions in (1.23), configuration I features a clash between the covariance structure of normal risks vs. that of the disaster risk in asset returns. The volatility-based ranking based on the normal risk covariance favors opposite (long and short) positions of the two assets in the most volatile PC, whereas such opposite positions weaken the PC's risk premium by offsetting the risk premia of the two assets in this PC. As a result, the first PC is volatile but does not necessarily represent a risk of high price. The less volatile (second) PC features similar (long and long, or short and short) positions of the two assets, which enhance the second PC's risk premium, making it an important pricing factor albeit not volatile. The dimensionality reduction procedure tends to omit important pricing factors in configuration I.

In configuration II, by similar arguments, comparative results reverse. That is, in configuration II, the disaster risk increases non-perturbatively the risk premium $\mu\left[P_1\right]$ and price of risk $SR\left[P_1\right]$, and perturbatively (in the first perturbative order of κ_J) the volatility, of the first PC, all relative to the corresponding quantities of the second PC. Intuitively, given two assumptions in (1.23), configuration II features a harmony between the covariance structure of normal risks vs. that of the disaster risk in asset returns. The volatility-based ranking based on the normal risk covariance favors similar positions of the two assets in the most volatile PC. These similar positions enhance the PC's risk premium by summing the risk premia of the two assets in this PC. As a result, the first PC both is volatile and represents a risk of high price. The dimensionality reduction procedure tends to retain important pricing factors in configuration II.

Foreign Exchange Risks

An integrated currency (FX) market enables investors based in different countries to trade an identical set of currencies and their derivatives albeit their returns depend on the denomination (base) currencies of investors. As currency return risks are exchange rate risks,[10] the set of currency re-

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[10] To an investor based in country A, trading the risk-free bond of another country B exposes the investor exclusively to the exchange risk between the two currencies. Therefore, currency strategies (e.g., currency carry trades) can

turns expose investors to a common risk space but different (denomination-dependent) covariance structures and returns. FX market offers a rich setting to conceptually discuss (this section) and empirically verify (Section 1.3) different sets of volatility-based pricing factors associated with various base currencies (i.e., in the perspectives of different countries' investors) while maintaining the same space of underlying (exchange rate) risks.

Let us consider a set of a base country (indexed by 0) and K foreign countries (indexed by $k \in \{1, \ldots, I\}$), with $I+1$ associated different currencies. The pricing in currency k's denomination is characterized by the respective country k's SDF,

$$M_{kt+1} = 1 - r_{kf} - \eta'_{k\varepsilon}\varepsilon_{t+1}, \qquad k \in \{0, 1, \ldots, I\}, \tag{1.24}$$

where $\eta_{k\varepsilon}$ represents a $I \times 1$ vector of prices in currency k of I risks in ε_{t+1}, which have a multivariate standard normal distribution of I dimensions and are common to all investors (FX risks). We assume that the FX market is arbitrage-free, complete, frictionless and fully integrated. We adopt the exchange rate convention that S_{kt} units of the base currency buys one unit of currency k in period t. Given a complete and arbitrage-free FX market, the exchange rate growth between currency k and the base currency is given by the ratio of the two countries' SDFs

$$\frac{S_{k/0t+1}}{S_{k/0t}} = \frac{M_{kt+1}}{M_{0t+1}} = 1 - (r_{kf} - r_{0f}) - (\eta_{k\varepsilon} - \eta_{0\varepsilon})' \eta_{0\varepsilon} - (\eta_{k\varepsilon} - \eta_{0\varepsilon})' \varepsilon_{t+1}, \quad \forall k \in \{1, \ldots, K\}, \tag{1.25}$$

Consider a typical currency trade in the FX market from the perspective of an investor in the base country. At time t, the trade takes a long position (lending) in currency k and a short position (borrowing) in the base currency 0, in which both positions have an equal notional value of one unit of the base currency. At time $t+1$, the trade closes all positions by converting proceeds to the base currency. The realized excess return (i.e., in excess of the base country's risk-free rate r_{0f}) of

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be visualized as trading (e.g., taking long and short positions in) the risk-free bonds of different countries.

this trade at $t+1$ is

$$CT_{k/0,t+1} = \frac{S_{kt+1}}{S_{kt}}\left(1+r_{kf}\right) - \left(1+r_{0f}\right) = \left(\eta_{0\varepsilon} - \eta_{k\varepsilon}\right)' \eta_{0\varepsilon} + \left(\eta_{0\varepsilon} - \eta_{k\varepsilon}\right)' \varepsilon_{t+1}, \tag{1.26}$$

where the second equality is obtained by employing the exchange rate growth's expression (1.25). Evidently, the currency excess return (1.26) and exchange rate growth (1.25) have identical volatility components, i.e., $CT_{k/0,t+1}$ is the traded version of $\frac{S_{k/0t+1}}{S_{k/0t}}$. An investor in the base country can obtain FX PC factors and their traded version by implementing PCA on currency returns $\{CT_{k/0,t+1}\}$, $k \in \{1,\ldots,K\}$. The first and second moments of these returns hence carry information about both the covariance structure of exchange rate risks and their pricing seen specifically from the base currency perspective.

Regarding the exchange rate covariance structure, it is important to observe that the loadings $\{\eta_{0\varepsilon} - \eta_{k\varepsilon}\}$ of K exchange rate growths $\{\frac{S_{k/0t+1}}{S_{k/0t}}\}$, $k \in \{1,\ldots,K\}$, on the exchange rate risks in ε_{t+1} vary with the denomination.[11] As a result, while all investors are subject to the same set of exchange rate risks ε_{t+1}, the exchange rate covariance structure and the associated PCs depend on the denomination currency. Regarding the pricing of exchange rate risks, the specificity of the denomination currency is characterized by the by the price of risk vector $\eta_{0\varepsilon}$. While this vector is exogenous (given) in the current reduced-form setting, it is inherent in the exchange growth dynamics via the no-arbitrage complete-market relationship (1.25) between SDFs and exchange rates. These observations indicate that, given an integrated and complete FX market in which all investors are subject to a common set of I risks in ε_{t+1}, how risk factors are construed and their pricing performances may vary significantly with the currency base. This variability hinders the consistence and robustness of the pricing factors, those are constructed solely on means or volatilities of asset returns, in different base currencies. We qualitatively illustrate this variability

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[11] The dependence of exchange rate growth's risk loadings on the currency denomination is reflected reflected by the presence of the price of risk $\eta_{0\varepsilon}$ associated with the base (denomination) currency in all loading vectors $\{\eta_{0\varepsilon} - \eta_{k\varepsilon}\}$, $\forall k \in \{1,\ldots,K\}$ (1.25).

and ambiguity for the standard PCA next.

In the base currency, the exchange rate covariance matrix is

$$V_0 = \begin{bmatrix} (\eta_{1\varepsilon} - \eta_{0\varepsilon})' (\eta_{1\varepsilon} - \eta_{0\varepsilon}) & \dots & (\eta_{1\varepsilon} - \eta_{0\varepsilon})' (\eta_{K\varepsilon} - \eta_{0\varepsilon}) \\ \vdots & \ddots & \vdots \\ (\eta_{K\varepsilon} - \eta_{0\varepsilon})' (\eta_{1\varepsilon} - \eta_{0\varepsilon}) & \dots & (\eta_{K\varepsilon} - \eta_{0\varepsilon})' (\eta_{I\varepsilon} - \eta_{K\varepsilon}) \end{bmatrix}. \tag{1.27}$$

The diagonalization of the above covariance matrix, $O_0' V_0 O_0 = Diag\left(\lambda_{0,1}, \dots, \lambda_{0,K}\right)$, yields the FX PC factors and their traded versions associated with the base country, [12]

$$\underbrace{\left[\widetilde{P}_{1/0} \;\dots\; \widetilde{P}_{K/0}\right]}_{T\times K} = \underbrace{\left[\widetilde{CT}_{1/0} \;\dots\; \widetilde{CT}_{K/0}\right]}_{T\times K} \underbrace{O_0}_{K\times K}, \tag{1.28}$$

$$\left[P_{1/0} \;\dots\; P_{K/0}\right] = \left[CT_{1/0} \;\dots\; CT_{K/0}\right] O_0$$

where $T \times 1$ columns $\widetilde{P}_{k/0}$ and $P_{k/0}$ denote respectively the k-th PC and its traded version, and $T \times 1$ column $\widetilde{CT}_{k/0}$ the demeaned version of the currency return $CT_{k/0,t}$. By substituting expression (1.26) into the right-hand side of the above equation, we can express FX PCs, their risk loading vectors $\{\eta_{k/0}\}$, risk premia $\{\mu\left[P_{k/0}\right]\}$, and volatilities $\{\sigma\left[P_{k/0}\right]\}$ more explicitly

$$\widetilde{P}_{k/0} = \eta_{k/0}' \varepsilon_{t+1}, \quad k \in \{1, \dots, K\}, \quad \text{with:} \quad \underbrace{\left[\eta_{1/0} \;\dots\; \eta_{K/0}\right]}_{I\times K} = \underbrace{\left[\eta_1 - \eta_0 \;\dots\; \eta_k - \eta_0\right]}_{I\times K} \underbrace{O_0}_{K\times K}, \tag{1.29}$$

$$\mu\left[P_{k/0}\right] = \eta_{k/0}' \eta_0, \qquad \sigma\left[P_{k/0}\right] = \sqrt{\eta_{k/0}' \eta_{k/0}} = |\eta_{k/0}| = \sqrt{\lambda_{0,k}}, \tag{1.30}$$

The similarity between PCs' construction (1.28) and their risk loadings (1.29) allows us to visualize PCs $\{\widetilde{P}_{k/0}\}$ (which are $T \times 1$ vectors in the return space) and their variability with the base currency

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[12] Based on Remark 1.1 and similarly to (1.6), the traded PCs are constructed from the original (demeaned) PCs, $\left[P_{1/0} \;\dots\; P_{K/0}\right] = \left[CT_{1/0} \;\dots\; CT_{K/0}\right] O_0$.

in terms of the variability of their risk loadings $\{\eta_{k/0}\}$ (which are $I \times 1$ vectors in the risk space).[13] Figure 1.1 illustrates the variability of the standard volatility-based PCs with the base currency in the risk space. In a base currency A, the standard PCA identifies PCs as orthogonalized factors

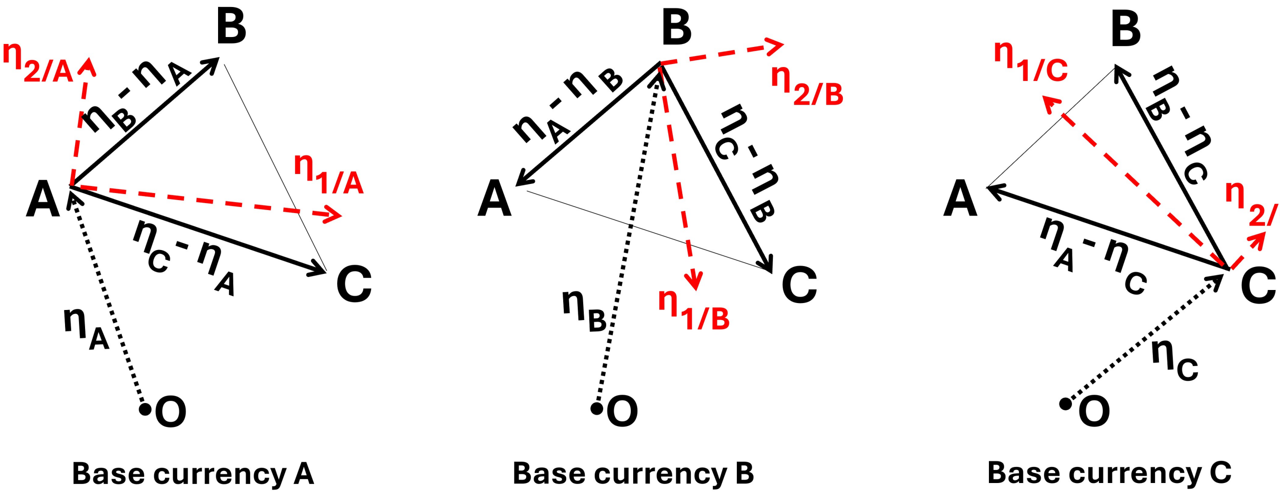

Figure 1.1: **Illustrating principal components associated with different base currencies in the risk space.**

The k-th principal component in the base currency j is associated with and uniquely characterized by its risk loading vector $\eta_{k/j}$ (red arrow) in the risk space (Equation (1.29)). The base currency respectively is A (left panel), B (middle panel), and C (right panel). In a (any) base currency j: (i) per Equation (1.29), the determination of PCs involves only the relative prices of risks $\{\eta_k - \eta_j\}$, $k \neq j$ (i.e., triangle ABC), and (ii) per Equation (1.30), the pricing of the PCs' risks further involves the price of risk η_j in the base currency (i.e., quadrilateral $ABCO$).

explaining the covariance structure of risk loading vectors $\{\eta_k - \eta_A\}_k$ (1.29), all of which clearly are characterized relative to the base country's price of risk vector η_A. A change from the base currency A to another base currency B therefore amounts to changing the reference point from A to B (the triangular ABC in Figure 1.1). First, such a change of the reference point can dramatically change the PC system, both in PCs' directions (i.e., risks represented by PCs) and their magnitudes (i.e.,

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[13] Equations (1.28) and (1.29) show that the orthogonalization of $T \times 1$ currency returns $\{\widetilde{CT}_{k/0}\}$ into principal components $\{\widetilde{P}_{k/0}\}$ is identical to the orthogonalization of $I \times 1$ risk loading vectors $\{\eta_k - \eta_0\}$ (of K currency returns) into K risk loading vectors $\{\eta_{k/0}\}$ (of K principal components).

shares of covariations explained by PCs). This indicates a significant variability of PCs' statistical properties with the base currency. Second, PCs' mean excess returns $\{\eta'_{k/A}\eta_A\}_k$ (1.30), which quantify how the risks represented by PCs are priced in the base currency, further involves the price of risk η_A of the base currency (the quadrilateral $ABCO$). The exogeneity of O from ABC is the current illustration further indicates a significant variability of PCs' pricing properties with the base currency. That is, the volatility-based construction may identify superior factors (featuring significant shares of exchange rate covariations or high risk premia) in a base currency and inferior factors in others. Our empirical analysis of Section 1.3 evaluate the performance and robustness of various pricing factor constructions in different currency bases.

1.2 PPRF: Implementation Approaches

1.2.1 PPRF: Brute-force Implementation

From an asset pricing perspective, Remark 1.2 shows that the characterization of PCs based on their volatilities is ambiguous. PCs $\widetilde{P}_n$ and $\widetilde{P}_n(\theta_n)$ represent an identical risk of the same return space but have arbitrarily different volatilities. Therefore, these PCs can be arbitrarily ranked based on their volatilities while being the same risk factor. In contrast, SRs are invariant under transformations (1.7), indicating a pricing characterization of PCs based on their SRs. Intuitively, such a characterization takes into account the information from both first and second moments of asset returns, hence incorporates the central tradeoff between risks and expected excess returns of asset pricing. Remark 1.2 motivates a price-based ranking of PCs as pricing factors, which we referred to as principally priced risk factors.

Definition 1.4 (Principally priced risk factors (PCA case))**.** Given the original asset returns $\{Y_0, Y_n\}$ (1.1), the principally priced risk factors are PCs of these asset returns which are ranked based on the prices of the risks they represent,

$$\widetilde{P}_n \succ \widetilde{P}_k \quad \text{if} \quad SR\left[P_n\right] > SR\left[P_k\right], \quad k, n \in \{1, \ldots, N\}. \tag{1.31}$$

In general, a ranking criterion of pricing factors is important when one wishes to retain only a limited number of most relevant factors that suffice to capture the essence of the underlying pricing model while the full data processing burden is nontrivial. E.g., standard PCA retains a limited number of most volatile PCs to capture the essence of the covariation structure of the concerned random variables. Such a criterion, however

By construction, the ranking (1.31) is invariant under the risk-preserving transformations (1.7). The principally priced risk factors therefore are not subject to the ambiguity of the standard PCA ranking (based on volatilities) under these transformations. It is important to observe that the ranking based on SRs is a general organizational principle to order pricing factors. Definition 1.4 is a specific setting in which this ranking principle is applied on PC factors (and Definition 1.5 below applies the ranking on other specific factors). More generally, given a set of factors, the price-based ranking can be employed as a simple and versatile add-on procedure to prioritize factors and resulting models along the pricing-relevant risk dimensions of higher prices. Compared to the ranking based solely on the second moment of asset returns, the ranking based on Sharpe ratios also requires the estimates of expected returns. The current paper employs expansive windows, historical means and simple shrinkage to determine these estimates.

It is also important to note that the differences in the ranking based on volatilities and SRs can be mitigated because SR-preserving transformations (Remark 1.2) can change and align PCs' volatilities with their SRs. The mitigation is performed by identifying equivalent bases of the given asset return space such that more volatile PCs in these bases also represent risks of higher prices. Specifically, starting with the original asset returns $\{Y_0, Y_n\}_{n=1}^{N}$ (1.1), a standard PCA procedure produces N PCs $\{P_n\}_{n=1}^{N}$ (1.6) with associated variances $\{\lambda_n\}_{n=1}^{N}$ (1.4) and Sharpe ratios $\{SR\left[P_n\right]\}_{n=1}^{N}$ (1.8).

Using the following N specific parameters, $\theta_n = \frac{SR[P_n]}{\sqrt{\lambda_n}}$, $n \in \{1, \ldots, N\}$, we then construct a set of N specific traded PCs $\{P_n(\theta_n)\}_{n=1}^N$ (1.7). These specific traded PCs $\{P_n(\theta_n)\}_{n=1}^N$ have three properties, namely, (i) span the same return space as the original assets $\{Y_n\}_{n=1}^N$, (ii) are mutually uncorrelated, and (iii) have volatilities $\left\{\theta_n \sqrt{\lambda_n} = SR\left[P_n\right]\right\}$ clearly aligned with (equal to) their SRs $\{SR\left[P_n\right]\}$. These properties assure that when we start with the equivalent return basis $\{Y_0, P_n(\theta_n)\}_{n=1}^N$, the implementation of PCA produces PCs $\{P_n(\theta_n)\}_{n=1}^N$ of unambiguous and desirable features that more volatile factors also present risks of higher prices. Finally, starting with any other equivalent basis that is related to $\{P_n(\theta_n)\}_{n=1}^N$ by a (any) orthogonal matrix $Q_{N\times N}$ yields the same features (producing the same PCs $\{P_n(\theta_n)\}_{n=1}^N$) because PCA is invariant to a rotation transformation. We recap the steps of constructing and aligning PPRFs with the standard PCA below.

Definition 1.5 (Principally priced risk factors (rotation formulation))**.** Given the original asset returns $\{Y_0, Y_n\}_{n=1}^N$, the associated PCs $\{P_n\}_{n=1}^N$ and eigenvalues $\{\lambda_n\}_{n=1}^N$, we construct a new equivalent basis $\{Y_0, X_n\}_{n=1}^N$ of the given asset return space (with X_n being the n-th column of matrix $X_{T\times N}$): $X_{T\times N} = P(\theta)_{T\times N} Q_{N\times N}$, where $Q_{N\times N}$ is a (any) $N \times N$ orthogonal matrix, and n-th column of matrix $P(\theta)_{T\times N}$ is the scaled n-th PC $P_n(\theta_n) = \theta_n P_n$ with $\theta_n = \frac{SR[P_n]}{\sqrt{\lambda_n}}$, $n \in \{1, \ldots, N\}$. The principally priced risk factors then are the standard PCs (i.e., ranked on their volatilities) of the equivalent asset return basis $\{Y_0, X_n\}_{n=1}^N$.

While this rotation formulation of PPRFs is equivalent to the direct re-ranking of PCs based on their SRs (Definition 1.4), it illustrates the flexibility of rotating, aligning and integrating the volatilitiy-based and price-based insights in constructing pricing factors in sample for OOS pricing tests (Section 1.3). The rotation formulation also motivates another approach to price-based factors named Sharpe ratio matrix (SRM) introduced below.

1.2.2 PPRFs: Inverse Sharpe Ratio Matrix

Sharpe ratio matrix: SRM is an alternative approach to incorporate information about the first moments of asset returns into the standard PCA. SRM consists of two steps. In the first step, we adjust and normalize the expected excess return of every original risky asset to a common notional value using the SR-preserving transformation (1.2). In the second step, we define SRM as the inverse of the covariance matrix of the normalized asset returns, whose diagonalization generates PCs that are informative about the mean returns of original assets. The following definition formalizes the construction of the SRM.

Definition 1.6 (Sharpe ratio matrix)**.** Given the original asset returns $\{Y_0, Y_n\}_{n=1}^N$, we employ the transformation (1.2) to construct an equivalent return basis $\{Y_0, Y_{SRM,n}\}_{n=1}^N$ in which every risky asset has a same expected excess return of a common notional value μ_{SRM},[14]

$$Y_{SRM,nt+1} = \theta_{SRM,n} Y_{nt+1}, \quad \text{with} \quad \theta_{SRM,n} = \frac{\mu_{SRM}}{\mu\left[Y_n\right]} \;\; \forall n \in \{1, \ldots, N\}. \tag{1.32}$$

The $N \times N$ Sharpe ratio matrix $[SRM]$ then is defined as the inverse of the covariance matrix V_{SRM} of the normalized returns $Y_{SRM} \equiv \{Y_{SRM,n}\}_{n=1}^N$,

$$[SRM] \equiv \left[\frac{1}{T}\tilde{Y}'_{SRM}\tilde{Y}_{SRM}\right]^{-1} = \left[V_{SRM}\right]^{-1}. \tag{1.33}$$

Several observations concerning the SRM are in order. First, observe that once the expected excess returns of $\{Y_{SRM,n}\}_{n=1}^N$ have been equalized to μ_{SRM}, these normalized assets' SRs equal to to the inverse of their volatilities (up to the same proportionality constant μ_{SRM}). This observation then presents an intuition and rationale to define the Sharpe ratio matrix to be the inverse of

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[14] Indeed, with the weight $\theta_{SRM,n}$ given in (1.32), the expected excess return of $Y_{SRM,n}$ is $mu\left[Y_{SRM,n}\right] = \theta_{SRM,n}\mu\left[Y_n\right] = \mu_{SRM}$, $\forall n \in \{1, \ldots, N\}$. Note that the notional value μ_{SRM} is canceled out in all observable quantities implied by the SRM. As a result, the specific choice of the notional value μ_{SRM} is immaterial in the construction of the SRM.

the covariance matrix of equal-mean-return assets (1.33). Second, the construction (1.32) of the normalized asset returns employs as inputs the original assets' expected returns, which then affects the normalized asset returns' covariances. The construction of the SRM therefore incorporates the information from both first and second moments of asset returns, and so does the PCA implemented on the normalized return basis $\{Y_{SRM,n}\}_{n=1}^{N}$. Specifically, by equalizing the expected excess returns of the normalized asset returns, we control for and absorb differences in the first moments of asset returns before inputting them into the PCA process. As a result, the resulting traded PCs $\{P_{SRM,n}\}$ and their volatilities reflect both first and second moments of the original asset expected returns. The diagonalization of $[SRM]$ produces these PCs $\{P_{SRM,n}\}$,

$$O'_{SRM}[SRM]O_{SRM} = Diag\left(S_n\right), \quad P_{SRM} = Y_{SRM}O_{SRM}, \quad \text{with} \quad \sigma\left[P_{SRM,n}\right] = \frac{1}{\sqrt{S_n}}, \tag{1.34}$$

where $\{S_n\}$ and $\{\sigma\left[P_{SRM,n}\right]\}$, $n \in \{1, \ldots, N\}$, respectively are eigenvalues of $[SRM]$ and volatilities of PCs associated with the normalized returns.[15] Third, adapting Definition 1.4 for the SRM formulation, the principally priced risk factors (PPRFs) in the SRM approach are PCs ranked base on the eigenvalues of the Shapre ratio matrix,

$$\widetilde{P}_{SRM,n} \succ \widetilde{P}_{SRM,k} \quad \text{if} \quad S_n > S_k, \quad k, n \in \{1, \ldots, N\}. \tag{1.35}$$

Note that this ranking is inverse to the ranking based on PCs' volatilities because of an inverse relationship (1.34) between eigenvalues of SRM and volatilities of PCs.[16] Fourth, in comparison

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[15] Because both a (symmetric) matrix and its inverse are diagonalized by the same orthogonal matrix, the relationship (1.33) implies that the traded PCs $\{P_{SRM,n}\}$ can also be obtained from the diagonalization of $[SRM]$,

$$O'_{SRM}V_{SRM}O_{SRM} = O'_{SRM}[SRM]^{-1}O_{SRM} = \left[O'_{SRM}[SRM]O_{SRM}\right]^{-1} = \left[Diag\left(S_n\right)\right]^{-1} = Diag\left(\frac{1}{S_n}\right).$$

Therefore the variance of n-th PC is $\frac{1}{\sqrt{S_n}}$ as in (1.34).

[16] Intuitively, once we control for the mean returns in the SRM approach, SRs and volatilities are inversely related. However, note that traded PCs $\{P_{SRM,n}\}$ (1.34) do not necessarily have same expected excess returns. As a result, the ranking of PCs based on their Sharpe ratio does not exactly coincide with the ranking (1.35) based on SRM's

with Remark 1.2's SR-preserving transformation, the transformation underlying the SRM approach is more elaborate because the latter (former) concerns the scaling of asset returns before (after) implementing the PCA factor analysis. As a result, SRM approach results in a set of PCs $\{P_{SRM,n}\}$ that are different from the PCs $\{P_n\}$ associated with original assets (whereas the set of factors obtained in the PPRF approach of Definition 1.4 remains identical to the set of original PCs $\{P_n\}$). Both approaches preserve the given asset return space, hence address the same underlying pricing model. It is an empirical question how these two approaches price asset returns in various data sets (Sections 1.3 and 1.4). Finally, the formulation of the SRM is also related to the Risk-premium PCA (RP-PCA) approach of the literature as we explain below.

Risk-premium PCA and SRM: We first summarize the key features of the Risk-premium PCA (RP-PCA) approach of Lettau and Pelger [33].

1.3 Empirical Analysis: Equity Markets

This section discusses empirical analysis and evidence obtained from equity data.

1.3.1 Data Sources

This section describes data sources and treatments. We use 74 extreme-decile anomaly portfolios (denoted as LP74), 370 anomaly portfolios (denoted as LP370) from Lettau and Pelger [33], [17]and the Fama-French 25 size-B/M sorted portfolios (denoted as FF25) available from Ken French's website. [18] The sample period is from November 1963 to December 2021.

Besides using the original dataset, we have also constructed artificially rotated asset spaces from

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eigenvalues.

[17] https://www.serhiykozak.com/data, under the heading "Portfolio Sorts". Lettau and Pelger [33] select 37 out of 50 characteristics from the website that are available as of November 1963. While the data on the website is only available up to December 2019, we follow their instruction on the website, "Anomaly Definitions", to replicate the anomaly portfolios for observations up to December 2021.

[18] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data-library.html

LP74 and LP370. We applied our algorithms to these rotated spaces to support our argument that scaling asset returns is crucial for optimizing the Sharpe ratio of factor portfolios. We create the following two rotated asset spaces,

1. The SRM space: We enforce the mean return of all portfolios to be 5%.
2. The PCA space: We set the principal components as the test assets, matching their number to the number of portfolios in the original space. For instance, in the LP74 dataset, the corresponding PCA space also consists of 74 portfolios.

1.3.2 Empirical Tests

This section describes empirical tests, and defines tests statistics, and quantities involved in the tests. We compare the in-sample and out-of-sample performances of the following algorithms:

- PCA
- RP-PCA in Lettau and Pelger [33] with $\gamma = 10$, in which γ is the regularization term on the first moment of returns.
- PPRF: Run PCA first, then sort the corresponding factors based on their Sharpe ratios.
- RP-PPRF: Run RP-PCA first, then sort by the corresponding Sharpe ratios.
- SRM: We combine each asset return with the bond to form a new asset return of a fixed mean return of 5%. Next, we perform PCA on the resulting new asset returns and rank and retain PCs based on their Sharpe ratios.

And we report the results of the three- and five-factor models for each algorithm. To prevent arbitrage opportunities resulting from excessively high Sharpe ratios of idiosyncratic risks, we only retain the first 20 factors for PPRF, RP-PPRF, and SRM with the highest volatility before re-ranking them based on their Sharpe ratios. We obtain the out-of-sample results using the expanding window approach.

For each data set, we run algorithms on the covariance matrix that we:

1. Estimate from the historical mean return.

2. Estimate from the historical mean return, but further adjust the eigenvalues of the covariance matrix:
$$\tilde{\lambda}_k = \frac{\hat{\lambda}_k}{1 + \bar{c}c_k}, \quad c_k = N/(T\hat{\lambda}_k) \tag{1.36}$$
   $\hat{\lambda}_k$ is the k-th originally estimated eigenvalue, and $\tilde{\lambda}_k$ is the adjusted eigenvalue. We take the number of factors that explain 95% of the variation, then $\bar{c}$ is the average of eigenvalue for the rest. We divide the matrix of eigenvectors into two halves, $\mathbf{w}^{(1)}, \mathbf{w}^{(2)}$, write
$$b_k = \max\{|\mathbf{w}_k^{(1)\prime}\mathbf{w}_k^{(2)}|, 0.75\} \tag{1.37}$$
   And estimate the Sharpe ratio by
$$\frac{b_k^{-1/2}\mu'\mathbf{w}_k}{\sqrt{\tilde{\lambda}_k}} \equiv b_k^{-1/2}\mathrm{SR} \tag{1.38}$$

With approach 2, we obtain a more accurate estimate in the Sharpe ratio, but the pricing error is unchanged. Finally, given the high-dimensionality of the data, the covariance matrix may not be invertible due to its sparse structure. To address this issue, we adjust for the potentially ill-conditioned covariance matrix of the residuals before computing the GRS statistics by applying the following procedure:
$$\tilde{\Sigma}_r = \widehat{\Sigma}_r + 0.1 \times \mathbf{I} \tag{1.39}$$

In which $\tilde{\Sigma}_r$ is the adjusted covariance matrix of the residual and Σ_r is the original covariance matrix of the residual. We then compute the GRS statistics, a measure of the pricing error, in the following formula:

$$\text{GRS} = \frac{T}{N} \times \frac{T-N-k}{T-k-1} \times \frac{\boldsymbol{\alpha}' \tilde{\Sigma}_r^{-1} \boldsymbol{\alpha}}{1 + \boldsymbol{\mu}_f' \Sigma_f^{-1} \boldsymbol{\mu}_f} \tag{1.40}$$

In which $\boldsymbol{\alpha}$ is the vector of alpha, $\boldsymbol{\mu}_f$ is the vector of mean factor returns, and Σ_f is the covariance matrix of the factor returns. We report the maximum Sharpe ratio attainable, root-mean-squared alpha, percentage of unexplained idiosyncratic return variation, and the GRS statistics for each algorithm.

1.3.3 Empirical Analysis: Original Space

This section discusses empirical results and why they support our principal factors (i.e., factors of higher SR but not high volatilities) in pricing models. We first report the results on the original dataset in Tables 1.1, 1.2, 1.3, 1.4, 1.5, and 1.6 in this subsection, then report the results on the rotated test assets from LP74 and LP370 in Tables 1.7 and 1.8 in subsection 1.3.4.

In each table, column 1 presents the maximum Sharpe ratio achieved. Table 1.1 displays the in-sample results, while Table 1.2 shows the out-of-sample results for LP74. According to Table 1.1, the in-sample PPRF achieves a higher Sharpe ratio than PCA, and RP-PPRF outperforms RP-PCA. Significantly, sorting factors based on their risk premium alone yields a higher Sharpe ratio than PCA, yet it does not perform as well as PPRF. This supports our claim that prioritizing factors based on Sharpe ratios is more effective than sorting by mean excess return or volatility alone. For all algorithms, adjusting the estimated eigenvalues of the covariance matrix enhances the Sharpe ratio without altering the pricing error. Table 1.2 presents out-of-sample results using an expanding window approach, where PPRF again surpasses PCA, with the three-factor Sharpe ratio for PPRF nearly doubling that of PCA. While RP-PPRF also achieves a higher Sharpe ratio than RP-PCA, the improvement is modest, suggesting that RP-PCA may have already selected factors with a high

Sharpe ratio.

Table 1.3 presents the in-sample results, and Table 1.4 shows the out-of-sample results for LP370. The findings in Table 1.3 are consistent with those in Table 1.1, where PPRF outperforms PCA, and RP-PPRF surpasses RP-PCA in terms of Sharpe ratios. Notably, in the LP370 dataset, the SRM algorithm achieves the highest Sharpe ratio among all tested algorithms. Table 1.4 details the out-of-sample results for LP370, confirming that PPRF continues to provide a higher Sharpe ratio than PCA. In the five-factor model, the out-of-sample Sharpe ratio for RP-PPRF is approximately equivalent to that of RP-PCA.

Table 1.5 displays the in-sample results, while Table 1.6 details the out-of-sample results for FF25. The patterns observed for LP74 and LP370 are also evident in FF25. Furthermore, as indicated in Table 1.6, PPRF exceeds RP-PCA in achieving higher out-of-sample Sharpe ratios for both the three-factor and five-factor models. Additionally, both PPRF and SRM surpass RP-PCA when using the adjusted covariance matrix.

However, when comparing PPRF and RP-PCA, both of which incorporate the first and second moments of returns to construct factors, it is important to note two primary differences. Firstly, PPRF aligns both the mean and volatility of returns, while RP-PCA does not. Secondly, the effectiveness of RP-PCA is contingent upon the regularization parameter γ applied to the first moment of asset returns. Intriguingly, PCA can be considered a specific instance of RP-PCA where $\gamma = -1$. Conversely, PPRF's performance is independent of any regularization, complicating direct comparisons between the performances of PPRF and RP-PCA.

The previous discussions summarize the maximum attainable Sharpe ratios from each algorithm, which is the main focus of this paper. However, measuring the pricing error is also important and can depend on the specific choice of test assets. [8] argue that validation tests of asset price models should rely on measures less sensitive to the choice of test assets, such as the GRS test statistics. Therefore, we report two measures of the pricing error: the root-mean-squared alpha and the GRS statistic. Both measures are based on the alpha in the time-series regression, but the former takes the square root, while the latter takes the square. Given an asset pricing model, the GRS statistic gives an aggregate measure of individual asset pricing errors $\{\alpha_i\}$ weighted by the inverse of the covariance matrix Σ_ϵ^{-1} of asset return idiosyncratic innovations $\{R_{\epsilon it}\}$, $i \in \{1, \ldots, N\}$ in the pricing model. It is important to note that the GRS statistic equals the difference between the squared Sharpe ratio of the tangency portfolio of traded assets and traded factors when the underlying asset pricing model is linear in traded factors $\{F_k\}$, $k \in \{1, \ldots, K\}$. When considering traded factors, the returns of the assets can be partitioned into two subspaces of mutually uncorrelated returns. The first subspace is spanned by the factor-mimicking returns R_{Fkt}. The returns in the second subspace are uncorrelated with the first subspace and are spanned by the idiosyncratic innovations R_{it} of the asset returns in this factor model. The covariance matrix of the asset returns and its inverse can be decomposed into two blocks associated with the two return subspaces, which are unrelated. The Sharpe ratio of the tangency portfolio weights concerns the inverse of the asset return covariance matrix, and thus can be decomposed into two unrelated components associated with the two return subspaces. We demonstrate this argument in the following formula:

$$GRS \equiv \sum_{i,j} \alpha_i \Sigma_{\epsilon ij}^{-1} \alpha_j = SR^2_{\{R_i\}} - SR^2_{\{F_k\}}. \tag{1.41}$$

Hence, as equation (1.41) suggests, a large GRS test statistic indicates significant pricing errors in the model, which may justify a low Sharpe ratio. We further argue that the GRS statistic does not depend on the rotation of the asset return basis, which means that any two N-asset bases R_X and R_Y related by an orthogonal matrix, $R_{eY} = R_{eX} O$, have the same GRS statistic under the pricing model $m(K)$.

$$\begin{aligned} GRS(K) =& \alpha_R \left[\widetilde{R}'_{\epsilon t}\widetilde{R}_{\epsilon t}\right]^{-1}\alpha'_R = \alpha_\Pi W' \left[W\widetilde{R}'_{\epsilon\Pi t}\widetilde{R}_{\epsilon\Pi t}W'\right]^{-1} W\alpha'_\Pi \\ =& \alpha_\Pi \left[\widetilde{R}'_{\epsilon\Pi t}\widetilde{R}_{\epsilon\Pi t}\right]^{-1}\alpha'_\Pi = \sum_{k=K+1}^{N} \frac{\overline{R}^2_{\epsilon\Pi k}}{\lambda_k} = \sum_{k=K+1}^{N} \eta^2_{\Pi k} \end{aligned} \tag{1.42}$$

In which W is the $N \times N$ matrix that diagonalizes the covariance matrix, $\alpha_R = \alpha_\Pi W'$, $\widetilde{R}_{\epsilon t} = \widetilde{R}_{\epsilon\Pi t}W'$, and $\eta_{\Pi k}$ is the price of risk (also the SR of the mimicking asset) of the k-th PC. In contrast, the root-mean-squared alpha depends on the rotation, and thus, the relationship between the Sharpe ratio and the root-mean-squared alpha is unclear.

Column 2 in each table reports the root-mean-squared alpha, while column 4 reports the GRS statistics. We observe that PPRF generates lower GRS than PCA, and RP-PPRF generates lower GRS than RP-PCA. This result is consistent with the pattern we observed in column 1, where the maximum attainable Sharpe ratio from PPRF is higher than PCA, and the maximum attainable Sharpe ratio from RP-PPRF is higher than RP-PCA. For in-sample results, these findings hold for all test assets we use, as shown in Tables 1.1, 1.3, and 1.5. For out-of-sample results, this pattern emerges in LP74 in Table 1.2 and LP370 in Table 1.4.

Although we have argued in equation (1.41) that the GRS is inversely proportional to the maximum attainable Sharpe ratio in the algorithm, we compute the GRS using equation (1.40) to follow the F-distribution under the finite sample. As FF25 consists of only 25 assets, which is significantly fewer than the 74 assets in LP74 and 370 assets in LP370, the computation results for FF25 may not fully support the conclusion that the GRS statistic is inversely related to the maximum attainable Sharpe ratio. However, this conclusion holds pretty well when the number of assets is large.

Finally, in column 3 of each table, we present the unexplained idiosyncratic time-series variation resulting from each algorithm. This variation represents the residual of the time-series regression on factors. The conventional PCA aims to minimize the idiosyncratic variation only, while RP-

PCA aims to minimize the cross-sectional pricing error (as the root-mean-squared alpha reported in Column 2) and the time-series unexplained variation (as the idiosyncratic variation reported in Column 3). However, our approaches, PPRF, RP-PPRF, and SRM, aim to maximize the Sharpe ratio, not minimize the unexplained variation. As shown in Column 2, the root-mean-squared error resulting from each algorithm does not align with the Sharpe ratio. In this context, we discuss whether the unexplained idiosyncratic variation aligns with Sharpe ratios.

Based on the out-of-sample results in Tables 1.2, 1.4, and 1.6, we observe that the unexplained idiosyncratic variation from PPRF is higher than that from PCA, and the unexplained idiosyncratic variation from RP-PPRF is higher than that from RP-PCA. However, the Sharpe ratios for PPRF and RP-PPRF follow the opposite sorting. This finding suggests that a factor model that aims to maximize the Sharpe ratio does not necessarily need to minimize all the idiosyncratic variation. This argument can be further supported by comparing PCA and SRM: In the out-of-sample results, both algorithms tend to generate a similar level of unexplained idiosyncratic variation, yet the Sharpe ratios for SRM are higher than those for PCA in Tables 1.2 and 1.4. Thus, we can conclude that the unexplained idiosyncratic variation does not necessarily align with the Sharpe ratios when comparing different factor models.

In conclusion, based on the discussions of Sharpe ratios, pricing errors, and unexplained idiosyncratic variations, we suggest that statistical factors should be sorted based on high Sharpe ratios rather than high volatilities. Our results show that the Sharpe ratio is a useful metric for evaluating the performance of factor models in terms of risk-adjusted returns. Additionally, we find that minimizing the unexplained idiosyncratic variation may not be necessary for achieving high Sharpe ratios. Therefore, we recommend that investors focus on selecting factors with high Sharpe ratios rather than those that minimize all sources of variation.

1.3.4 Empirical Analysis: Rotated Space

This subsection reports the results on the rotated asset space, where we show that the maximum Sharpe ratios from SRM and PPRF can be higher even than the RP-PCA, thus the rotation (scaling) in the asset return space matters for the factor portfolios.

In the SRM space, we standardize all portfolio returns to be 5%. A notable difference emerges between PCA and SRM within this space: when PCA is applied, factors are ordered by volatility in descending order, resulting in Sharpe ratios being ranked in ascending order. Conversely, when SRM is applied, factors are ordered by their Sharpe ratios in descending order.

In the PCA space, we use the principal components of the original test asset, employing conventional methods as our test assets. We calculate the weights or eigenvectors of the covariance matrix using the in-sample data, then use these to predict portfolio returns in the out-of-sample period. The results of this out-of-sample analysis are then documented. In both the SRM and PCA spaces, we do not apply our algorithms to the adjusted covariance matrix as done in the original space. Therefore, the calculation of Sharpe ratios relies on the sample mean of portfolio returns and the unadjusted eigenvalues of the covariance matrix.

Panel A of Table 1.7 displays the out-of-sample results from the SRM space in LP74, while Panel B presents the out-of-sample results from the PCA space in LP74. Compared to the corresponding results for LP74 in Table 1.2, we find that the Sharpe ratios for PPRF, RP-PPRF, and SRM in the SRM space, for both three-factor and five-factor models, are twice as high as those for PCA and RP-PCA. This underscores the significant role that scaling of asset returns plays in achieving higher Sharpe ratios in factor portfolios. In the PCA space, although PPRF, RP-PPRF, and SRM still outperform PCA and RP-PCA, the differences in Sharpe ratios are less pronounced than in the SRM space.

Regarding the pricing errors of RMSE-α and the unexplained idiosyncratic variation $\overline{\sigma}_e$, our results indicate that the sorting of asset returns is crucial. For instance, the unexplained idiosyncratic variation for PPRF, RP-PPRF, and SRM is significantly lower than in the original space. This reduction is intuitive as the principal factors are designed to minimize idiosyncratic variation, and in the rotated space, minimizing this variation contributes to maximizing the attainable Sharpe ratios. Achieving this relationship is more challenging in the original space, where there appears to be a trade-off between minimizing idiosyncratic variation and maximizing Sharpe ratios.

Panel A of Table 1.8 presents the out-of-sample results from the SRM space in LP370, while Panel B provides the out-of-sample results from the PCA space in LP370. In the SRM space, RP-PPRF and SRM marginally outperform PCA and RP-PCA in terms of Sharpe ratios; however, the pricing errors are significantly lower in the SRM space compared to the original space of LP370, as shown in Table 1.4. In the PCA space of LP370, the Sharpe ratios for PPRF, RP-PPRF, and SRM are superior to those of PCA and RP-PCA. Notably, the Sharpe ratio for SRM in the three-factor models is approximately twice as high as that of RP-PCA.

1.4 Empirical Analysis: FX Markets

This section discusses empirical analysis and evidence obtained from FX data. We compare our approaches to Lustig et al. [36], an important paper in the field of international finance that discovers huge co-movement among exchange rates of currencies with similar interest rates. While Lustig et al. [36] work on the US dollar only, we argue that the result depends heavily on the currency denominations. Therefore, we extend the analysis to the bases of the Japanese Yen (JP), Australian dollar (AU), and British Pounds Sterling (UK). Among these currencies, JP has the lowest, AU has the highest, and the UK has a similar interest rate to the US among developed countries. We find that the result differs across currency denominations.

1.4.1 Data Sources

This section describes data sources and treatments. We calculate the returns of carry trades that involve shorting the US dollar and long positions in other currencies using exchange rate and interest rate differential data. The data comprises daily exchange rates among 11 developed countries obtained from WM/Reuters, namely the United States (US), Australia (AU), Canada (CA), Denmark (DK), the Eurozone (EU), Japan (JP), New Zealand (NZ), Norway (NO), Sweden (SE), Switzerland (CH), and the UK. For each currency base in our study, there are 10 exchange rates. The data covers the period from January 1, 1983, to December 31, 2020. Before the introduction of the Euro on January 1, 1999, we used the Deutschmark as a proxy for the Euro. We used the average of bid and ask rates for both the spot rate and the 1-month forward exchange rate.

The monthly interest rate differential data up to April 2020 is available on Adrien Verdelhan‘s website. [19] We also employ the forward discount (the 1-month forward rate minus the spot rate) to estimate the interest rate differential with respect to the US, consistent with the covered interest parity. [20] We denote the interest rate differential as Δr and the spot exchange rate return as Δs, and compute the carry trade return as $\Delta r - \Delta s + 0.5\Delta s^2$. Using the carry trade return data, we construct the PCA, PPRF, RP-PCA, RP-PPRF factors, and two factors proposed by Lustig et al. [36]: the Dollar factor (RX), which is the average of the carry trade return, and the HML factor, which involves borrowing the two currencies with the lowest interest rates and lending the two currencies with the highest interest rates. We analyze denominations based on the US dollar (US), Japanese yen (JP), Australian dollar (AU), and British pound sterling (UK).

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[19] http://web.mit.edu/adrienv/www/Data.html, under the heading “Monthly Changes in Exchange Rates and Global Risk Factors“.

[20] Some literature, such as [20], documents large and persistent deviations from the covered interest parity after the financial crisis in 2008 across the developed and emerging economies. Therefore, calculating the carry trade returns based on the covered interest parity may be inaccurate after 2008. However, we only use the covered interest parity to compute the carry trade returns after April 2020, as monthly interest rate differential data was unavailable during this period. This period is relatively short when compared to our entire sample.

1.4.2 Empirical Tests

This section describes empirical tests, and defines tests statistics, and quantities involved in the tests. We compare the in-sample and out-of-sample performances of the two-factor models of the following algorithms:

- PCA
- RP-PCA in Lettau and Pelger [33] with $\gamma = 10$, in which γ is the regularization term on the first moment of returns.
- PPRF: Run PCA first, then sort the corresponding factors based on their Sharpe ratios.
- RP-PPRF: Run RP-PCA first, then sort the corresponding factors on their Sharpe ratios.
- SRM.
- RX & HML: The two-factor model proposed by Lustig et al. [36].

The out-of-sample results come from the expanding window approach. We retain all factors for data on the carry trade returns before we re-rank them by their Sharpe ratios in PPRF, RP-PPRF, and SRM.

1.4.3 Empirical Analysis

This section discusses the empirical results, and why they support our principal factors (i.e., factors of higher SR but not high volatilities) in pricing models.

Table 1.9 presents the correlation between the first two principal components and the Lustig et al. [36] factors across currency denominations of the US, JP, AU, and the UK. Panels A, B, and C report the correlations for PC2 and the HML and RX factors, respectively.

In Panel A, we replicate the findings in Lustig et al. [36] that the correlation between the HML factor and PC2 is 0.94 in the US denomination. We argue that as the HML factor is more important in terms of the price of risk, it should have a non-trivial correlation with the PPRF factor. We find that the correlation between the HML factor and SPC1 is 0.29, while the correlation between the RX factor and PPRF is only 0.08. Additionally, Panel A suggests that the importance of the HML in pricing differs significantly across currency denominations, as no other currency denomination has a higher correlation between the HML and SPC1 than the US.

In Panels B and C, we report the correlation of the RX and HML factors across currency denominations. Panel B confirms that the RX factor, which proxies for the PC1 for each denomination, should not highly correlate. Furthermore, as the HML factor comes from borrowing the lowest-interest-rate currency and lending the highest-interest-rate currency regardless of the currency denominations, the HML factors across currency denominations should be highly similar, which is confirmed by Panel C.

Table 1.10 presents the annualized Sharpe ratio for the first ten PCs, RX, and HML factors for different currency denominations. We compute the annualized Sharpe ratio by multiplying the daily Sharpe ratio by $\sqrt{360}$. Our findings suggest that the sorting by volatility does not align with the sorting by the Sharpe ratio across all denominations. Additionally, the annualized Sharpe ratio for the HML factor is higher than the RX factor across all currency denominations, supporting our view that sorting the Sharpe ratio is more crucial than sorting by volatility. Panel B reports the annualized Sharpe ratio for the first factors generated by SRM. They generate a higher Sharpe ratio than the PCA in the carry trade return.

Table 1.11 reports the in-sample maximum Sharpe ratio, the RMSE-α, the unexplained idiosyncratic variation, and the GRS statistics on the carry trade returns for currency denominations US, JP, AU, and UK. As discussed in Section 1.3, we estimate the Sharpe ratio based on the covariance matrix from the mean return and the adjusted covariance matrix that leads to the Sharpe ratio in equation (1.38), and compute the GRS statistics based on equation (1.40). We annualize the Sharpe ratio by multiplying the daily Sharpe ratio by $\sqrt{360}$, the annualized RMSE-α by multiplying the daily RMSE-α by 360, and the annualized GRS by multiplying the daily GRS by 360^2. We report results with the covariance matrix estimated from the sample mean and a further adjustment in the eigenvalues of the estimated covariance matrix and run the expanding window to report the out-of-sample results.

Our analysis reveals that the Sharpe ratios generated by PCA and RP-PCA remain constant across all currency denominations, irrespective of eigenvalue corrections. Furthermore, PPRF and RP-PPRF consistently outperform RP-PCA for all denominations, while SRM delivers superior performance in the AU denomination. These findings hold for the expanding window approach, as presented in Table 1.12.

The in-sample results in Table 1.11 and the out-of-sample results in Table 1.12 confirm the underperformance of the carry trade strategies, as reported by [36], especially after the 2008 financial crisis. The decreasing profitability of currency trade strategies can be attributed to the increasing market volatility and risk aversion, which leads to a heightened flight to safe assets, as noted by [36].

In the US denomination, the results from PCA and RP-PCA are similar to those reported by [36], as the first two factors generated by these algorithms are largely consistent with the [36] factors. However, PPRF and SRM yield higher Sharpe ratios than the other algorithms, indicating that sorting the currency factors based on Sharpe ratios may be useful in creating a hedge against market volatility. This sorting method has been effective recently and holds for other currency denominations in our sample.

We further observe that the maximum attainable Sharpe ratios of PPRF, RP-PPRF, and SRM are inversely proportional to the interest rate of the denominated currency. This observation can be explained by the fact that countries with higher interest rates tend to have higher risks associated with investing in their currency. As a result, when the return is the same, the Sharpe ratio tends to be lower. This increased risk can be due to factors such as historical inflation or an economy heavily reliant on commodities. For instance, Australia and New Zealand have economies that depend heavily on commodity exports, leading to higher risks associated with their currencies. As a result, their central banks need to raise interest rates to compensate for these risks. This argument reflects in Tables 1.10, 1.11, and 1.12, which show that the Sharpe ratios for PPRF and SRM are highest among the JP denomination but lowest among the AU denomination.

The out-of-sample results in Table 1.12 reveal that the GRS statistics for the carry trade returns with only 10 exchange rates follow the Sharpe ratio pattern only in the US denomination. However, this pattern is not evident in other denominations. This inconsistency may be attributed to the computation of the GRS statistics based on 1.40, which considers the finite-sample correction. This correction can result in discrepancies between the patterns of the GRS statistics and the Sharpe ratios when the number of assets is small, as we discuss in Section 1.3.

Our findings in this section contribute to the existing literature on preference-free approaches in international finance research ([36]; [37]). We demonstrate that achieving a high Sharpe ratio from a model does not require specifying individual preferences. When analyzing exchange rate patterns, we can adapt to real-world complexity and high dimensionality. Moreover, our results contribute to the literature on the performance of carry trade returns and the breakdown of the covered interest parity after the 2008 financial crisis due to heightened market volatility ([36]; [20]). We highlight that sorting carry trade factors based on their Sharpe ratios can serve as a hedge against market volatility.

Table 1.1: **Lettau-Pelger 74 Portfolio In-Sample (IS) Results**

This table reports statistics for the performance of various factor models on the 74 extreme decline anomaly portfolios analyzed by Lettau and Pelger [33]. Results are 'in-sample' for the period 1963:11- 2021:12. PCA refers to factors extracted using conventional principal components analysis. PPRF refers to factors obtained by first applying PCA and then sorting the resulting PCs by Sharpe ratios rather than by variances. RP-PCA refers to the risk premium PCA approach of Lettau and Pelger [33] using a γ value of 10. RP-PPRF refers to applying RP-PCA on the covariance matrix and then sorting the resulting factors by Sharpe ratios. SRM refers to the 'Sharpe ratio matrix' approach in which we first rescale assets to have a common mean of 5%, and then apply PCA and sort by Sharpe ratio. Max SR is the maximum Sharpe ratio attained with the factors. RMSE-α is the square root of the average pricing error and $\overline{\sigma}_e$ is the percentage of unexplained return variation. GRS is the GRS statistics. p is the p-value of the GRS statistics. 'Sample mean' indicates that we estimate the covariance matrix from the sample mean. 'Adjust' indicates that we adjust the eigenvalues of the estimated covariance matrix.

Panel A: Three factors (Sample mean)					
	Max SR	RMSE-α	$\overline{\sigma}_e$	GRS	p
PCA	0.21	0.28	15.45	5.91	0
PPRF	0.40	0.69	98.35	4.43	0
RPPCA	0.36	0.25	15.66	4.86	0
RP-PPRF	0.44	0.26	18.08	4.14	0
SRM	0.33	0.70	99.81	5.14	0
Three factors (Adjust)					
PCA	0.24	0.28	15.45	5.91	0
PPRF	0.47	0.69	98.35	4.43	0
RPPCA	0.41	0.25	15.66	4.86	0
RP-PPRF	0.51	0.26	18.08	4.14	0
SRM	0.38	0.70	99.81	5.14	0
Panel B: Five factors (Sample mean)					
PCA	0.27	0.23	11.64	5.48	0
PPRF	0.46	0.65	92.91	3.94	0
RPPCA	0.47	0.22	11.81	3.89	0
RP-PPRF	0.60	0.29	16.65	2.63	0
SRM	0.38	0.70	99.76	4.72	0
Five factors (Adjust)					
PCA	0.32	0.23	11.64	5.48	0
PPRF	0.53	0.65	92.91	3.94	0
RPPCA	0.54	0.22	11.81	3.89	0
RP-PPRF	0.69	0.29	16.65	2.63	0
SRM	0.44	0.70	99.76	4.72	0

Table 1.2: **Lettau-Pelger 74 Portfolio Out-of-Sample (OOS) Results**

This table reports statistics for the performance of various factor models on the 74 extreme decline anomaly portfolios analyzed by Lettau and Pelger [33]. Results are 'out-of-sample' for the period 1963:11- 2021:12. PCA refers to factors extracted using conventional principal components analysis. PPRF refers to factors obtained by first applying PCA and then sorting the resulting PCs by Sharpe ratios rather than by variances. RP-PCA refers to the risk premium PCA approach of Lettau and Pelger [33] using a γ value of 10. RP-PPRF refers to applying RP-PCA on the covariance matrix and then sorting the resulting factors by Sharpe ratios. SRM refers to the 'Sharpe ratio matrix' approach in which we first rescale assets to have a common mean of 5%, and then apply PCA and sort by Sharpe ratio. Max SR is the maximum Sharpe ratio attained with the factors. RMSE-α is the square root of the average pricing error and $\overline{\sigma}_e$ is the percentage of unexplained return variation. GRS is the GRS statistics. p is the p-value of the GRS statistics. 'Sample mean' indicates that we estimate the covariance matrix from the sample mean. 'Adjust' indicates that we adjust the eigenvalues of the estimated covariance matrix.

Panel A: Three factors (Sample mean)					
	Max SR	RMSE-α	$\overline{\sigma}_e$	GRS	p
PCA	0.16	0.27	17.24	5.04	0
PPRF	0.32	0.79	96.96	4.48	0
RPPCA	0.40	0.22	17.55	4.14	0
RP-PPRF	0.44	0.23	19.65	3.52	0
SRM	0.36	0.27	20.06	4.23	0
Three factors (Adjust)					
PCA	0.18	0.28	17.42	4.99	0
PPRF	0.31	0.77	95.76	4.49	0
RPPCA	0.47	0.22	17.55	4.14	0
RP-PPRF	0.50	0.23	19.65	3.82	0
SRM	0.30	0.26	19.17	4.39	0
Panel B: Five factors (Sample mean)					
PCA	0.29	0.20	12.65	4.70	0
PPRF	0.37	0.53	80.42	4.51	0
RPPCA	0.56	0.17	13.13	3.48	0
RP-PPRF	0.60	0.18	17.15	3.14	0
SRM	0.38	0.23	16.40	4.18	0
Five factors (Adjust)					
PCA	0.34	0.20	12.65	4.70	0
PPRF	0.41	0.59	81.08	4.44	0
RPPCA	0.64	0.17	13.13	3.48	0
RP-PPRF	0.66	0.19	17.62	3.54	0
SRM	0.33	0.23	15.28	5.01	0

Table 1.3: **Lettau-Pelger 370 Portfolio IS Results**

This table reports statistics for the performance of various factor models on the 370 decline anomaly portfolios analyzed by Lettau and Pelger [33]. Results are 'in-sample' for the period 1963:11-2021:12. PCA refers to factors extracted using conventional principal components analysis. PPRF refers to factors obtained by first applying PCA and then sorting the resulting PCs by Sharpe ratios rather than by variances. RP-PCA refers to the risk premium PCA approach of Lettau and Pelger [33] using a γ value of 10. RP-PPRF refers to applying RP-PCA on the covariance matrix and then sorting the resulting factors by Sharpe ratios. SRM refers to the 'Sharpe ratio matrix' approach in which we first rescale assets to have a common mean of 5%, and then apply PCA and sort by Sharpe ratio. Max SR is the maximum Sharpe ratio attained with the factors. RMSE-α is the square root of the average pricing error and $\overline{\sigma}_e$ is the percentage of unexplained return variation. GRS is the GRS statistics. p is the p-value of the GRS statistics. 'Sample mean' indicates that we estimate the covariance matrix from the sample mean. 'Adjust' indicates that we adjust the eigenvalues of the estimated covariance matrix.

Panel A: Three factors (Sample mean)					
	Max SR	RMSE-α	$\overline{\sigma}_e$	GRS	p
PCA	0.17	0.17	13.91	2.00	0
PPRF	0.26	0.67	98.85	1.90	0
RPPCA	0.22	0.17	13.96	1.95	0
RP-PPRF	0.35	0.27	16.03	1.79	0
SRM	0.55	0.67	99.99	1.51	0
Three factors (Adjust)					
PCA	0.20	0.17	13.91	2.00	0
PPRF	0.30	0.67	98.85	1.90	0
RPPCA	0.26	0.17	13.96	1.95	0
RP-PPRF	0.40	0.27	16.03	1.79	0
SRM	0.63	0.67	99.99	1.51	0
Panel B: Five factors (Sample mean)					
PCA	0.23	0.15	11.66	1.93	0
PPRF	0.33	0.15	16.04	1.79	0
RPPCA	0.34	0.15	11.71	1.78	0
RP-PPRF	0.56	0.13	13.26	1.40	0
SRM	0.61	0.67	99.99	1.41	0
Five factors (Adjust)					
PCA	0.27	0.15	11.66	1.93	0
PPRF	0.38	0.15	16.04	1.79	0
RPPCA	0.40	0.15	11.71	1.78	0
RP-PPRF	0.64	0.13	13.26	1.40	0
SRM	0.71	0.67	99.99	1.41	0

Table 1.4: **Lettau-Pelger 370 Portfolio OOS Results**

This table reports statistics for the performance of various factor models on the 370 decline anomaly portfolios analyzed by Lettau and Pelger [33]. Results are 'out-of-sample' for the period 1963:11-2021:12. PCA refers to factors extracted using conventional principal components analysis. PPRF refers to factors obtained by first applying PCA and then sorting the resulting PCs by Sharpe ratios rather than by variances. RP-PCA refers to the risk premium PCA approach of Lettau and Pelger [33] using a γ value of 10. RP-PPRF refers to applying RP-PCA on the covariance matrix and then sorting the resulting factors by Sharpe ratios. SRM refers to the 'Sharpe ratio matrix' approach in which we first rescale assets to have a common mean of 5%, and then apply PCA and sort by Sharpe ratio. Max SR is the maximum Sharpe ratio attained with the factors. RMSE-α is the square root of the average pricing error and $\overline{\sigma}_e$ is the percentage of unexplained return variation. GRS is the GRS statistics. p is the p-value of the GRS statistics. 'Sample mean' indicates that we estimate the covariance matrix from the sample mean. 'Adjust' indicates that we adjust the eigenvalues of the estimated covariance matrix.

Panel A: Three factors (Sample mean)					
	Max SR	RMSE-α	$\overline{\sigma}_e$	GRS	p
PCA	0.21	0.16	15.54	2.97	0
PPRF	0.28	0.80	97.69	2.75	0
RPPCA	0.33	0.15	15.72	2.75	0
RP-PPRF	0.34	0.19	17.35	2.65	0
SRM	0.12	0.16	17.65	2.78	0
Three factors (Adjust)					
PCA	0.25	0.16	15.54	2.97	0
PPRF	0.32	0.80	97.69	2.75	0
RPPCA	0.38	0.15	15.73	2.75	0
RP-PPRF	0.40	0.19	17.35	2.65	0
SRM	0.20	0.17	17.07	2.55	0
Panel B: Five factors (Sample mean)					
PCA	0.25	0.13	12.53	2.91	0
PPRF	0.27	0.56	77.37	2.76	0
RPPCA	0.54	0.12	12.91	2.82	0
RP-PPRF	0.53	0.14	14.07	2.64	0
SRM	0.13	0.15	15.04	2.69	0
Five factors (Adjust)					
PCA	0.29	0.13	12.53	2.91	0
PPRF	0.31	0.56	77.37	2.76	0
RPPCA	0.63	0.12	12.91	2.82	0
RP-PPRF	0.62	0.14	14.04	2.61	0
SRM	0.23	0.15	14.61	2.61	0

Table 1.5: **Fama-French 25 Portfolio IS Results**

This table reports statistics for the performance of various factor models on the Fama-French 25 size-B/M sorted portfolios. Results are 'in-sample' for the period 1963:11- 2021:12. PCA refers to factors extracted using conventional principal components analysis. PPRF refers to factors obtained by first applying PCA and then sorting the resulting PCs by Sharpe ratios rather than by variances. RP-PCA refers to the risk premium PCA approach of Lettau and Pelger [33] using a γ value of 10. RP-PPRF refers to applying RP-PCA on the covariance matrix and then sorting the resulting factors by Sharpe ratios. SRM refers to the 'Sharpe ratio matrix' approach in which we first rescale assets to have a common mean of 5%, and then apply PCA and sort by Sharpe ratio. Max SR is the maximum Sharpe ratio attained with the factors. RMSE-α is the square root of the average pricing error and $\overline{\sigma}_e$ is the percentage of unexplained return variation. GRS is the GRS statistics. p is the p-value of the GRS statistics. 'Sample mean' indicates that we estimate the covariance matrix from the sample mean. 'Adjust' indicates that we adjust the eigenvalues of the estimated covariance matrix.

Panel A: Three factors (Sample mean)					
	Max SR	RMSE-α	$\overline{\sigma}_e$	GRS	p
PCA	0.26	0.15	6.72	4.02	0
PPRF	0.35	0.23	15.98	2.67	0
RPPCA	0.27	0.15	6.73	3.89	0
RP-PPRF	0.31	0.17	9.47	3.20	0
SRM	0.30	1.13	98.88	3.42	0
Three factors (Adjust)					
PCA	0.30	0.15	6.72	4.02	0
PPRF	0.40	0.23	15.98	2.67	0
RPPCA	0.31	0.15	6.73	3.89	0
RP-PPRF	0.36	0.17	9.47	3.20	0
SRM	0.40	0.15	9.99	2.60	0
Panel B: Five factors (Sample mean)					
PCA	0.30	0.12	4.72	3.48	0
PPRF	0.39	0.13	9.18	1.82	0.01
RPPCA	0.37	0.10	4.78	2.21	0
RP-PPRF	0.41	0.14	7.66	1.40	0.09
SRM	0.33	1.13	98.35	2.94	0
Five factors (Adjust)					
PCA	0.34	0.12	4.72	3.48	0
PPRF	0.45	0.13	9.18	1.82	0.01
RPPCA	0.43	0.10	4.78	2.21	0
RP-PPRF	0.48	0.14	7.66	1.40	0.09
SRM	0.47	0.10	8.20	1.57	0.04

Table 1.6: **Fama-French 25 Portfolio OOS Results**

This table reports statistics for the performance of various factor models on the Fama-French 25 size-B/M sorted portfolios. Results are 'out-of-sample' for the period 1963:11- 2021:12. PCA refers to factors extracted using conventional principal components analysis. PPRF refers to factors obtained by first applying PCA and then sorting the resulting PCs by Sharpe ratios rather than by variances. RP-PCA refers to the risk premium PCA approach of Lettau and Pelger [33] using a γ value of 10. RP-PPRF refers to applying RP-PCA on the covariance matrix and then sorting the resulting factors by Sharpe ratios. SRM refers to the 'Sharpe ratio matrix' approach in which we first rescale assets to have a common mean of 5%, and then apply PCA and sort by Sharpe ratio. Max SR is the maximum Sharpe ratio attained with the factors. RMSE-α is the square root of the average pricing error and $\overline{\sigma}_e$ is the percentage of unexplained return variation. GRS is the GRS statistics. p is the p-value of the GRS statistics. 'Sample mean' indicates that we estimate the covariance matrix from the sample mean. 'Adjust' indicates that we adjust the eigenvalues of the estimated covariance matrix.

Panel A: Three factors (Sample mean)					
	Max SR	RMSE-α	$\overline{\sigma}_e$	GRS	p
PCA	0.26	0.17	7.40	6.24	0
PPRF	0.38	0.26	16.65	7.04	0
RPPCA	0.27	0.17	7.42	5.83	0
RP-PPRF	0.35	0.20	10.05	6.41	0
SRM	0.31	0.26	15.70	5.13	0
Three factors (Adjust)					
PCA	0.30	0.17	7.40	6.24	0
PPRF	0.41	0.26	17.14	6.79	0
RPPCA	0.32	0.17	7.42	5.83	0
RP-PPRF	0.40	0.20	10.04	6.45	0
SRM	0.40	0.26	14.60	5.69	0
Panel B: Five factors (Sample mean)					
PCA	0.29	0.16	5.03	5.98	0
PPRF	0.40	0.19	8.94	5.62	0
RPPCA	0.38	0.13	5.08	4.58	0
RP-PPRF	0.38	0.17	6.86	6.16	0
SRM	0.35	0.22	12.63	5.03	0
Five factors (Adjust)					
PCA	0.34	0.16	5.03	5.98	0
PPRF	0.48	0.20	9.24	6.06	0
RPPCA	0.44	0.13	5.07	4.58	0
RP-PPRF	0.44	0.17	6.96	6.21	0
SRM	0.43	0.24	10.11	6.75	0

Table 1.7: **Rotated Lettau-Pelger 74 Portfolio OOS Results**

This table reports statistics for the performance of various factor models on the rotated space of the extreme-decile portfolios in Lettau and Pelger [33]. Results are 'out-of-sample' for the period 1963:11- 2021:12. PCA refers to factors extracted using conventional principal components analysis. PPRF refers to factors obtained by first applying PCA and then sorting the resulting PCs by Sharpe ratios rather than by variances. RP-PCA refers to the risk premium PCA approach of Lettau and Pelger [33] using a γ value of 10. RP-PPRF refers to applying RP-PCA on the covariance matrix and then sorting the resulting factors by Sharpe ratios. SRM refers to the 'Sharpe ratio matrix' approach in which we first rescale assets to have a common mean of 5%, and then apply PCA and sort by Sharpe ratio. Max SR is the maximum Sharpe ratio attained with the factors. RMSE-α is the square root of the average pricing error and $\overline{\sigma}_e$ is the percentage of unexplained return variation. GRS is the GRS statistics. p is the p-value of the GRS statistics. We estimate the covariance matrix from the sample mean.

Panel A: SRM Space: Three Factors					
	Max SR	RMSE-α	$\overline{\sigma}_e$	GRS	p
PCA	0.13	0.26	18.93	0.66	0
PPRF	0.31	0.23	19.45	0.58	0
RPPCA	0.12	0.26	19.12	0.65	0
RP-PPRF	0.31	0.24	19.48	0.56	0
SRM	0.36	0.27	20.06	0.58	0
SRM Space: Five Factors					
PCA	0.16	0.25	14.72	0.66	0
PPRF	0.34	0.21	15.73	0.59	0
RPPCA	0.15	0.21	14.55	0.67	0
RP-PPRF	0.35	0.21	15.35	0.58	0
SRM	0.38	0.23	16.40	0.58	0
Panel B: PCA Space: Three Factors					
PCA	0.25	0.24	16.76	0.64	0
PPRF	0.38	0.51	51.84	0.59	0
RPPCA	0.35	0.22	17.30	0.70	0
RP-PPRF	0.40	0.30	22.28	0.60	0
SRM	0.45	0.37	58.71	0.54	0
PCA Space: Five Factors					
PCA	0.29	0.21	12.17	0.65	0
PPRF	0.42	0.26	32.21	0.55	0
RPPCA	0.45	0.17	12.66	0.60	0
RP-PPRF	0.47	0.20	16.29	0.56	0
SRM	0.47	0.28	36.72	0.46	0

Table 1.8: **Rotated Lettau-Pelger 370 Portfolio OOS Results**

This table reports statistics for the performance of various factor models on the rotated space of the portfolios in Lettau and Pelger [33]. Results are 'out-of-sample' for the period 1963:11- 2021:12. PCA refers to factors extracted using conventional principal components analysis. PPRF refers to factors obtained by first applying PCA and then sorting the resulting PCs by Sharpe ratios rather than by variances. RP-PCA refers to the risk premium PCA approach of Lettau and Pelger [33] using a γ value of 10. RP-PPRF refers to applying RP-PCA on the covariance matrix and then sorting the resulting factors by Sharpe ratios. SRM refers to the 'Sharpe ratio matrix' approach in which we first rescale assets to have a common mean of 5%, and then apply PCA and sort by Sharpe ratio. Max SR is the maximum Sharpe ratio attained with the factors. RMSE-α is the square root of the average pricing error and $\overline{\sigma}_e$ is the percentage of unexplained return variation. GRS is the GRS statistics. p is the p-value of the GRS statistics. We estimate the covariance matrix from the sample mean.

Panel A: SRM Space: Three Factors					
	Max SR	RMSE-α	$\overline{\sigma}_e$	GRS	p
PCA	0.20	0.16	16.11	7.67	0
PPRF	0.18	0.16	16.86	3.42	0
RPPCA	0.21	0.15	16.25	8.57	0
RP-PPRF	0.23	0.16	16.90	3.26	0
SRM	0.31	0.15	17.60	3.40	0
SRM Space: Five Factors					
PCA	0.22	0.14	13.47	7.89	0
PPRF	0.22	0.15	14.25	3.50	0
RPPCA	0.26	0.13	13.70	7.49	0
RP-PPRF	0.26	0.15	14.57	3.21	0
SRM	0.32	0.14	14.97	3.36	0
Panel B: PCA Space: Three Factors					
PCA	0.23	0.16	15.49	8.09	0
PPRF	0.28	0.23	26.20	3.31	0
RPPCA	0.26	0.15	15.48	8.21	0
RP-PPRF	0.34	0.16	18.44	3.36	0
SRM	0.53	0.30	33.80	2.89	0
PCA Space: Five Factors					
PCA	0.25	0.14	12.15	8.04	0
PPRF	0.32	0.17	20.12	3.33	0
RPPCA	0.37	0.13	12.26	8.01	0
RP-PPRF	0.40	0.13	14.25	3.39	0
SRM	0.56	0.22	25.46	2.69	0

Table 1.9: **Correlation of Foreign Exchange Factors**

This table reports the correlation coefficient for each factor across four different currency bases: US, JP, AU, and UK. The data consists of 11 currencies in the developed countries and spans the period of 1983:1:1 – 2020:12:31. RX is the dollar factor in Lustig et al. [36], which is the average of all carry trade returns. HML is the carry trade factor in Lustig et al. [36], which is the difference in carry trade portfolio returns on the highest and lowest interest rate differentials.

Panel A

	RX-US	HML-US		RX-JP	HML-JP
PC1-US	0.98	0.04	PC1-JP	0.99	−0.36
PC2-US	−0.05	0.94	PC2-JP	−0.03	0.12
SPC1-US	0.08	0.29	SPC1-JP	0.00	0.17
SPC2-US	0.13	−0.01	SPC2-JP	0.01	0.17
SRM1-US	0.16	0.13	SRM1-JP	0.04	0.22
SRM2-US	0.08	0.22	SRM2-JP	−0.08	0.35
	RX-AU	HML-AU		RX-UK	HML-UK
PC1-AU	0.99	−0.57	PC1-UK	0.99	0.28
PC2-AU	−0.02	0.24	PC2-UK	−0.01	0.87
SPC1-AU	0.00	0.09	SPC1-UK	−0.02	0.12
SPC2-AU	0.06	0.31	SPC2-UK	0.00	0.01
SRM1-AU	0.08	0.46	SRM1-UK	0.05	0.25
SRM2-AU	0.01	0.02	SRM2-UK	0.13	0.58

Panel B

	RX-US	RX-JP	RX-AU	RX-UK
RX-US	1			
RX-JP	−0.11	1		
RX-AU	−0.10	0.32	1	
RX-UK	0.52	−0.16	−0.32	1

Panel C

	HML-US	HML-JP	HML-AU	HML-UK
HML-US	1			
HML-JP	0.91	1		
HML-AU	0.89	0.68	1	
HML-UK	0.99	0.91	0.89	1

Table 1.10: **Sharpe Ratios of Principal Components for Currency Bases**

This table reports the annualized Sharpe ratio for each factor across four different currency bases: US, JP, AU, and UK. The data consists of 11 currencies in the developed countries and spans the period of 1983:1:1 – 2020:12:31. The annualized Sharpe ratio is from multiplying the daily Sharpe ratio by $\sqrt{360}$. PC is the PC-mimicking carry trade portfolios, which the loading matrix is the portfolio weight. SRM is the 'Sharpe ratio matrix' approach in which we first rescale assets to have a common mean of 5%, and then apply PCA and sort by Sharpe ratio. RX is the dollar factor in Lustig et al. [36], which is the average of all carry trade returns. HML is the carry trade factor in Lustig et al. [36], which is the difference in carry trade portfolio returns on the highest and lowest interest rate differentials.

Panel A: PC	US	JP	AU	UK
PC1	0.34	0.00	0.31	0.13
PC2	0.12	0.00	0.08	0.20
PC3	0.43	0.19	0.35	0.12
PC4	0.45	0.61	0.36	0.41
PC5	0.23	0.31	0.48	0.17
PC6	0.12	0.15	0.03	0.06
PC7	0.06	0.19	0.08	0.08
PC8	0.08	0.06	0.11	0.11
PC9	0.10	0.95	0.25	0.11
PC10	0.25	0.20	0.29	0.29
Panel B: SRM	US	JP	AU	UK
PC1	0.52	1.07	0.69	0.38
PC2	0.43	0.41	0.28	0.33
PC3	0.25	0.35	0.24	0.30
PC4	0.23	0.24	0.22	0.14
PC5	0.20	0.16	0.22	0.11
PC6	0.18	0.07	0.19	0.09
PC7	0.16	0.06	0.11	0.01
PC8	0.07	0.04	0.11	0.00
PC9	0.03	0.02	0.02	0.00
PC10	0.02	0.01	0.01	0.00
Panel C: FX Factors	US	JP	AU	UK
RX	0.20	0.19	0.18	−0.04
HML	0.27	0.50	0.29	0.30

Table 1.11: **Carry Trade Portfolios IS Results**

This table reports statistics for the in-sample performance of various two-factor models on the daily carry trade return data for 11 developed countries. Results are 'in-sample' for the period 1983:1:1 - 2020:12:31. PCA refers to factors extracted using conventional principal components analysis. PPRF refers to factors obtained by first applying PCA and then sorting the resulting PCs by Sharpe ratios rather than by variances. RP-PCA refers to the risk premium PCA approach of Lettau and Pelger [33] using a γ value of 10. RP-PPRF refers to applying RP-PCA on the covariance matrix and then sorting the resulting factors by Sharpe ratios. SRM refers to the 'Sharpe ratio matrix' approach in which we first rescale assets to have a common mean of 5%, and then apply PCA and sort by Sharpe ratio. RX is the dollar factor in Lustig et al. [36]. HML is the carry trade factor in Lustig et al. [36]. Max SR is the annualized maximum Sharpe ratio attained with the factors. RMSE-α is the annualized square root of the average pricing error and $\overline{\sigma}_e$ is the percentage of unexplained return variation. GRS is the annualized GRS statistics. p is the p-value of the GRS statistics.

Panel A: US						**Panel A: US (Adjust)**					
	Max SR	RMSE-α	$\overline{\sigma}_e$	GRS	p		Max SR	RMSE-α	$\overline{\sigma}_e$	GRS	p
PCA	0.36	0.02	30.66	48.3	0	PCA	0.41	0.02	30.66	48.3	0
PPRF	0.62	0.03	84.64	93.5	0	PPRF	0.72	0.03	84.64	93.5	0
RPPCA	0.36	0.02	30.66	48.3	0	RPPCA	0.41	0.02	30.66	48.3	0
RP-PPRF	0.62	0.03	84.64	95.9	0	RP-PPRF	0.72	0.03	84.64	95.9	0
SRM	0.67	0.03	88.03	69.0	0	SRM	0.78	0.03	88.03	69.0	0
RX & HML	0.33	0.02	33.23	54.8	0	RX & HML	0.33	0.02	33.23	54.8	0
Panel B: JP						**Panel B: JP (Adjust)**					
PCA	0.00	0.03	23.77	62.0	0	PCA	0.00	0.03	23.77	62.0	0
PPRF	1.13	0.01	95.49	15.9	0	PPRF	1.13	0.01	95.49	15.9	0
RPPCA	0.00	0.03	23.77	62.0	0	RPPCA	0.00	0.03	23.77	62.0	0
RP-PPRF	1.13	0.01	95.49	17.0	0	RP-PPRF	1.13	0.01	95.49	17.0	0
SRM	1.14	0.01	94.43	9.7	0	SRM	1.32	0.01	94.44	9.7	0
RX & HML	0.62	0.03	25.96	99.3	0	RX & HML	0.62	0.03	25.96	99.3	0
Panel C: AU						**Panel C: AU (Adjust)**					
PCA	0.32	0.02	20.63	46.8	0	PCA	0.37	0.02	20.63	46.8	0
PPRF	0.61	0.04	91.60	154.1	0	PPRF	0.70	0.04	91.60	154.1	0
RPPCA	0.32	0.02	20.63	46.7	0	RPPCA	0.37	0.02	20.63	46.7	0
RP-PPRF	0.60	0.04	91.24	156.4	0	RP-PPRF	0.70	0.04	91.24	156.4	0
SRM	0.74	0.04	95.03	127.8	0	SRM	0.86	0.04	95.04	127.8	0
RX & HML	0.48	0.02	27.71	56.1	0	RX & HML	0.48	0.02	27.71	56.1	0
Panel D: UK						**Panel D: UK (Adjust)**					
PCA	0.24	0.01	14.37	18.6	0	PCA	0.27	0.01	14.37	18.6	0
PPRF	0.50	0.02	96.78	48.8	0	PPRF	0.58	0.02	96.78	48.8	0
RPPCA	0.24	0.01	14.37	18.5	0	RPPCA	0.27	0.01	14.37	18.5	0
RP-PPRF	0.50	0.02	96.78	49.6	0	RP-PPRF	0.58	0.02	96.78	49.6	0
SRM	0.51	0.01	91.89	11.7	0	SRM	0.59	0.01	91.89	11.7	0
RX & HML	0.33	0.03	15.31	75.4	0	RX & HML	0.33	0.03	15.31	75.4	0

Table 1.12: **Carry Trade Portfolios OOS Results**

This table reports statistics for the out-of-sample performance of various two-factor models on the daily carry trade return data for 11 developed countries. Results are ‘out-of-sample’ for the period 1983:1:1 - 2020:12:31. PCA refers to factors extracted using conventional principal components analysis. PPRF refers to factors obtained by first applying PCA and then sorting the resulting PCs by Sharpe ratios rather than by variances. RP-PCA refers to the risk premium PCA approach of Lettau and Pelger [33] using a γ value of 10. RP-PPRF refers to applying RP-PCA on the covariance matrix and then sorting the resulting factors by Sharpe ratios. SRM refers to the ‘Sharpe ratio matrix’ approach in which we first rescale assets to have a common mean of 5%, and then apply PCA and sort by Sharpe ratio. RX is the dollar factor in Lustig et al. [36]. HML is the carry trade factor in Lustig et al. [36]. Max SR is the annualized maximum Sharpe ratio attained with the factors. RMSE-α is the annualized square root of the average pricing error and $\overline{\sigma}_e$ is the percentage of unexplained return variation. GRS is the annualized GRS statistics. p is the p-value of the GRS statistics.

Panel A: US						**Panel A: US (Adjust)**					
	Max SR	RMSE-α	$\overline{\sigma}_e$	GRS	p		Max SR	RMSE-α	$\overline{\sigma}_e$	GRS	p
PCA	0.18	0.02	34.63	46.0	0	PCA	0.20	0.02	34.63	46.0	0
PPRF	0.63	0.02	73.19	31.7	0	PPRF	0.72	0.02	73.19	31.7	0
RPPCA	0.18	0.02	34.64	45.9	0	RPPCA	0.21	0.02	34.64	45.9	0
RP-PPRF	0.75	0.02	73.61	37.0	0	RP-PPRF	0.87	0.02	73.61	37.0	0
SRM	0.15	0.02	49.64	46.8	0	SRM	0.11	0.03	49.69	53.0	0
RX & HML	0.37	0.02	36.77	45.9	0	RX & HML	0.37	0.02	36.77	45.9	0
Panel B: JP						**Panel B: JP (Adjust)**					
PCA	0.78	0.02	28.48	28.3	0	PCA	0.90	0.02	28.48	28.3	0
PPRF	0.82	0.04	95.93	118.5	0	PPRF	0.94	0.04	95.93	118.5	0
RPPCA	0.78	0.02	28.51	28.3	0	RPPCA	0.90	0.02	28.51	28.3	0
RP-PPRF	0.81	0.04	95.92	119.5	0	RP-PPRF	0.93	0.04	95.92	119.5	0
SRM	0.44	0.02	28.40	55.9	0	SRM	0.34	0.02	27.39	47.6	0
RX & HML	0.58	0.02	29.76	27.0	0	RX & HML	0.58	0.02	29.76	27.0	0
Panel C: AU						**Panel C: AU (Adjust)**					
PCA	0.03	0.02	23.35	27.9	0	PCA	0.04	0.02	23.35	27.9	0
PPRF	0.28	0.03	91.12	59.3	0	PPRF	0.32	0.03	91.12	59.3	0
RPPCA	0.03	0.02	23.35	27.8	0	RPPCA	0.04	0.02	23.35	27.8	0
RP-PPRF	0.17	0.03	91.24	59.0	0	RP-PPRF	0.19	0.03	91.24	59.0	0
SRM	0.26	0.02	46.42	38.5	0	SRM	0.44	0.02	29.69	58.3	0
RX & HML	0.01	0.02	36.97	46.3	0	RX & HML	0.01	0.02	36.97	46.3	0
Panel D: UK						**Panel D: UK (Adjust)**					
PCA	0.37	0.02	8.61	34.7	0	PCA	0.43	0.02	8.61	34.7	0
PPRF	0.63	0.02	98.54	41.0	0	PPRF	0.50	0.02	98.54	41.0	0
RPPCA	0.37	0.02	8.61	34.7	0	RPPCA	0.43	0.02	8.61	34.7	0
RP-PPRF	0.63	0.02	98.49	41.0	0	RP-PPRF	0.71	0.02	98.49	41.0	0
SRM	−0.10	0.02	10.37	52.3	0	SRM	−0.10	0.03	12.63	77.6	0
RX & HML	0.38	0.02	8.39	38.7	0	RX & HML	0.38	0.02	8.39	38.7	0

Chapter 2

Origins and Implications of Covariance in Institutional Trading

2.1 Introduction

Many studies suggest an asset-pricing role for institutional trading. Their perspectives differ, but in most cases, the core metric involves an aggregation of trading aligned with specific stock characteristics. For example, [23] examine the evolution of the small-cap premium. [21] examine how institutional trading relates to a variety of anomalies. [29] incorporate characteristic-based demands more generally, examining returns at a quarterly horizon to analyze the impact of latent correlated demands on asset prices. This 'demand-based' view of asset pricing also appears in many other forms in the literature, including [14], [9], and [43].

The connection between latent characteristics and institutional trading suggests a need for understanding the source of the covariance in institutional trading and how they impact asset prices. This study aims to achieve this goal by exploring the algebraic structure within the institutional trading space. In particular, we develop a framework within the institutional trading space to identify two potential sources of covariance in institutional trading: (1) a network of fund managers with similar traits that may induce them to trade in similar patterns (the manager perspective), and (2) a basket of assets traded together by multiple fund managers (the asset perspective), which might affect the asset return through different channels and at different magnitudes.

To illustrate the differences between the manager and asset perspectives regarding their potential impact on asset returns, we propose relevant examples. The asset perspective is exemplified by fund managers' mandate to "beat the market," as documented in studies such as [9] and [43]. Many fund managers trade the benchmark index such as the S&P 500, as well as their constituent stocks. As a result, a plethora of managers possess strikingly similar asset collections, leading to correlated demand shocks, even though they may employ diverse strategies to outperform the index. Contrarily, the manager perspective indicates the role of the traits in the fund managers. The finding in [16] shows a related example: The educational background of the mutual fund managers may play an important role in their common trading patterns and the performance of their portfolios, highlighting the roles of information advantage through the education networks. Our framework aids in disentangling these two distinct patterns of covariance in institutional trading.

Our primary approach involves constructing institutional trading matrices for each quarter and applying Principal Component Analysis (PCA) to identify dimensions of covariance in trading. We analyze the covariance in the asset and manager perspectives. In the asset perspective, we identify the baskets (i.e., a linear combination of original assets) that are responsible for principal dimensions of covariation in how assets are traded in relation to one another. We refer to the vector of weights in this linear combination as the principal basket composition, and the vector of implied trades in this basket as the principal basket (much like we would refer to a column of trades in Tesla stock as 'Tesla'). In the manager perspective, we identify the networks (i.e., a linear combination of original managers) that are responsible for principal dimensions of covariation in how managers trade in relation to one another. We refer to the vector of weights in this linear combination as the principal network composition, and the vector of implied trades by the network as the principal network (much like we would refer to a column of trades by Vanguard as 'Vanguard.')

We first develop an intra-quarter static setting, and then extend the analysis to an intertemporal dynamic setting. By employing Singular Value Decomposition (SVD) on the institutional trading matrix in the static setting, we demonstrate a duality between these two perspectives. This means that both perspectives have identical information sparsity and that principal networks trade only the corresponding principal baskets. In the dynamic setting, we consider the evolution of principal baskets and networks across quarters of data.

Our empirical analysis uses 13-F institutional holdings data, which confirms the intra-period duality asset and manager perspectives on trading. That is, the same number of principal baskets are needed to explain a given degree of covariation in how assets are traded as the number of principal networks required to explain covariation in how managers trade. We document that this number typically rises during economic downturns, indicating investors trade in a less synchronized manner. This result sheds light on a potential driving force of the time-varying covariance in institutional trading, and provides some valuable direction of research on the connection between uncertainty-based business cycle literature (e.g., [26], [42], and [12]) and institutional trading.

However, these perspectives in the trading space become distinct in the dynamic setting. We apply Canonical Correlation Analysis (CCA) to principal network and principal basket composition matrices to separate persistent sources of common variation from quarter-specific noise. We then compare persistence in the asset and manager perspectives (i.e., basket and network composition, respectively). A persistent basket composition means that a particular dimension of covariance in how assets are traded is preserved over time – irrespective of who does the trading. Conversely, a persistent network composition means that a particular dimension of covariance in how managers trade is preserved over time – irrespective of what they trade.

Note that intra-quarter duality implies a linkage between the two perspectives. Thus, consider a principal basket in the asset space whose trading persists across quarters. The quarter q trades in that principal basket are proportional to the corresponding principal network's quarter q composition. However, that composition may or may not persist across quarters – irrespective of the persistent covariation in the asset space. Lack of persistence just means that different managers trade a relatively fixed basket over time. And of course, the converse may alternatively apply – the network composition persists but what that network trades (i.e., the corresponding principal basket composition under duality) may vary quarter by quarter.

We take the premise that a persistent covariance structure reveals the underlying economic drivers of that covariance. More specifically, the locus of persistence reveals the locus of economic origin. In our empirical analysis using the 13-F data, we find that the asset perspective is the main driver of covariance in trading. That is, what is traded matters much more than who is trading in explaining the dynamics of trading. This result has important implications for asset returns. Portfolios constructed via principal baskets are likely to exhibit higher volatility compared to portfolios derived from principal networks because the principal basket incorporates a higher level of predictable covariance in institutional trading, generating excess volatility in the asset return space, as suggested by the demand-based asset pricing theory.

Our study expands upon the demand-based asset pricing literature, including earlier empirical literature on how groups of investors taking similar actions simultaneously impact asset prices, such as 'herding' in the behavioral sense or the 'correlated demand shocks' in the rational or fundamental sense. However, our generalization is based on the PCA of the 'institutional trading matrix' for each quarter. Through this framework, we provide a more complete picture of the role of institutional trading in asset pricing, at a resolution that is generally disregarded in direct factor analyses of returns. In a sense, we hope to tie much of the noise in deviations from factor returns to the price impact of principal components of institutional trading.

Our study also relates to the recent literature that employs high-dimensional statistical methods to explore the common variation in institutional and retail investing. We highlight two papers that are particularly relevant in this regard.

First, [32] invokes the Tucker decomposition to analyze a three-dimensional tensor comprising 1,342 mutual funds with 25 characteristics across 34 quarters. The Tucker decomposition generalizes the concept of PCA to tensors, which incorporates the time dimension. While our approaches have commonalities, there are significant differences. [32] focuses on fund characteristics rather than trading. Moreover, we analyze persistence in the factor structure of trading explicitly. Also, [32] demonstrates the application of the Tucker decomposition to finance research, but does not discuss the economic mechanisms in detail. Our approach, on the other hand, seeks to frame systematic covariation in trading through the asset and manager perspectives, thereby identifying the economic origins of aggregate trading pressure.

Second, [5] apply PCA to a dataset of approximately 9.7 million Indian household investors and 3,000 stocks in August 2011. They aim to dichotomize the asset demand space, which aligns with our objective. However, their analysis primarily focuses on the cross-sectional dimension of stock ownership, assuming that the asset demand space remains constant over time. In contrast, our study emphasizes the time-series dimension and explores how the asset demand space varies over time through stock trading.

The structure of this chapter is outlined as follows. Section 2.2 provides a comprehensive definition of the framework for the institutional trading matrix and delineates the distinction between the two perspectives. Section 2.3 proposes a series of empirical tests to validate our framework using data from the 13-F institutional holding database. Finally, Section 2.4 concludes this chapter.

2.2 A Quantitative Framework of Trading

This section presents a framework to analyze the economic and statistical properties of the trading in financial assets (i.e., stocks) by managers (i.e., funds under their management). The framework is designed to lay out economic intuitions and formulate testable implications of our institutional trading data. We first introduce the asset and manager perspectives on trading and their dual nature in a static (intra-period) setting in Section 2.2.1. We then analyze these perspectives and demonstrate their divergence in a dynamic (intertemporal) setting in Section 2.2.2.

2.2.1 Intra-Period Setting

To introduce the notations and basic economic perspectives, we consider first the trading setup within one quarter, i.e., the smallest time period determined by data availability. For each quarter $q \in \{1, \dots Q_q\}$, the institutional trading data concerns A_q assets traded by M_q managers and is recorded in a $A_q \times M_q$ matrix T_q. Specifically, element T_{qij} of matrix T_q denotes the trading of asset $i \in \{1, \dots A_q\}$ by fund $j \in \{1, \dots M_q\}$ during period $q \in \{1, \dots Q_q\}$,

$$T_{qij} = \frac{\text{net change in asset } i\text{'s shares held by manager } j \text{ in quarter } q}{\text{asset } i\text{'s total shares outstanding at the end of quarter } q-1}. \tag{2.1}$$

We note that our notation and analysis allow for the numbers of actively traded assets (A_q) and active fund managers (M_q) to vary from one quarter to the next. The same trading data can be analyzed from either the asset or manager perspective.

The Asset Perspective on Trading

In the asset perspective on trading, we take every asset i as a fundamental variable of the market. For an asset i, we take i's trading data $\{T_{qij}\}$, $j \in \{1, \dots M_q\}$, by M_q managers as the M_q different realizations of the fundamental variable associated with asset i in quarter q. Adopting the notation convention in which columns represent the variables and rows represent the realizations, the asset perspective's trading matrix in quarter q is

$$\textit{Asset perspective:} \qquad T'_q = \overbrace{\left.\begin{bmatrix} T_{q11} & \dots & T_{qi1} & \dots & T_{qA_q1} \\ \vdots & \dots & \vdots & \dots & \vdots \\ T_{q1j} & \dots & T_{qij} & \dots & T_{qA_qj} \\ \vdots & \dots & \vdots & \dots & \vdots \\ T_{q1M_q} & \dots & T_{qiM_q} & \dots & T_{qA_qM_q} \end{bmatrix}\right\}}^{A_q \text{ asset variables (columns)}} \begin{array}{l} M_q \text{ manager} \\ \text{realizations (rows).} \end{array} \tag{2.2}$$

Hereafter, we refer to T'_q as the asset trading matrix. In the asset perspective, the trade covariations in quarter q are measured by the covariances of A_q fundamental assets' trading by managers and are quantified by the (uncentered) second moment

$$V_q^{(A)} = \underbrace{T_q}_{A_q \times M_q} \underbrace{T'_q}_{M_q \times A_q}, \tag{2.3}$$

where X' denotes the transpose of matrix X. To identify the principal components that drive the trade covariations from the asset perspective, we employ the singular value decomposition (SVD) for the trading matrix T_q (2.1)

$$\underbrace{T_q}_{A_q \times M_q} = \underbrace{B_q}_{A_q \times A_q} \underbrace{\sqrt{\Lambda_q}}_{A_q \times M_q} \underbrace{N'_q}_{M_q \times M_q}, \qquad \underbrace{T'_q}_{M_q \times A_q} = \underbrace{N_q}_{M_q \times M_q} \underbrace{\sqrt{\Lambda_q}'}_{M_q \times A_q} \underbrace{B'_q}_{A_q \times A_q}, \tag{2.4}$$

where the $A_q \times A_q$ matrix B_q and the $M_q \times M_q$ matrix N_q are orthogonal (rotation) matrices satisfying respectively $B_q B'_q = B'_q B_q = \mathbb{1}_{A_q \times A_q}$ and $N_q N'_q = N'_q N_q = \mathbb{1}_{M_q \times M_q}$. The $A_q \times M_q$ matrix $\sqrt{\Lambda_q}$ and its transposed $M_q \times A_q$ matrix $\sqrt{\Lambda_q}'$ are diagonal (possibly rectangular) matrices containing entries $\{\sqrt{\lambda_{qi}}\}$ on their diagonals. Since $\{\lambda_{qi}\}$ are eigenvalues of the symmetric matrix $T_q T'_q$ (explained in Equations (2.5) and (2.11) below), $\{\sqrt{\lambda_{qi}}\}$ are loosely referred to as eigenvalues of the trading matrix T_q.

The diagonalization of the covariance matrix $V_q^{(A)}$ (2.3) follows from the SVD (2.4)

$$B_q' V^{(A)} B_q = \underbrace{B_q' T_q}_{\mathcal{B}_q'} \underbrace{T_q' B_q}_{\mathcal{B}_q} = \sqrt{\Lambda_q}\sqrt{\Lambda_q}' = \begin{bmatrix} \lambda_{q1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \lambda_{qA_q} \end{bmatrix} \equiv \Lambda_q^{(A)}. \tag{2.5}$$

Note that because the rank of the covariance matrix $V_q^{(A)} = T_q T_q'$ (2.3) does not exceed T_q's lower dimension $\min\{A_q, M_q\}$, when $A_q > M_q$, the $(A_q - M_q)$ smallest eigenvalues of $V_q^{(A)}$ are identically zeros, $\lambda_{qM_q+1} = \ldots = \lambda_{qA_q} = 0$.

In the asset perspective , the diagonalization (2.5) identifies the principal baskets, i.e., baskets of A_q original assets that are responsible for principal covariation dimensions in asset trading. Specifically, the principal baskets are represented by the columns of the $M_q \times A_q$ matrix $\mathcal{B}_q \equiv T_q' B_q$ (2.5) and are as below

$$\underbrace{\begin{bmatrix} \mathcal{B}_{q11} & \dots & \mathcal{B}_{q1A_q} \\ \vdots & \ddots & \vdots \\ \mathcal{B}_{qM_q1} & \dots & \mathcal{B}_{qM_qA_q} \end{bmatrix}}_{\mathcal{B}_q\text{: Basket trading matrix}} = \underbrace{\begin{bmatrix} T_{q11} & \dots & T_{qA_q1} \\ \vdots & \ddots & \vdots \\ T_{q1M_q} & \dots & T_{qA_qM_q} \end{bmatrix}}_{T_q'\text{: Asset trading matrix}} \underbrace{\begin{bmatrix} B_{q11} & \dots & B_{q1A_q} \\ \vdots & \ddots & \vdots \\ B_{qA_q1} & \dots & B_{qA_qA_q} \end{bmatrix}}_{B_q\text{: Basket composition matrix}}, \tag{2.6}$$

where B_{qki} presents the composition (i.e., weight) of the k-th original asset in the i-th principal basket, i.e., the i-th column of the rotation matrix B_q presents the compositions of A_q original assets in the i-th principal basket in quarter q. As a result, the trading $\mathcal{B}_{qji}$ of the i-th principal basket by the original manager j equals the tradings $\{T_{qkj}\}$ of original assets $k \in \{1, \ldots, A_q\}$ by the manager j weighted by the respective compositions $\{B_{qki}\}$, or $\mathcal{B}_{qji} = \sum_k T_{qkj} B_{qki}$ (2.6). In a more tangible sense of trading T_{qij} in (2.1), we have,

$$\mathcal{B}_{qji} = \frac{\text{net change in } i\text{-th principal basket's shares held by manager } j \text{ in quarter } q}{i\text{-th principal basket's total shares outstanding at the end of quarter } q-1}. \tag{2.7}$$

Hereafter, we refer to B_q as the basket composition matrix, and $\mathcal{B}_q$ the basket trading matrix.

In the asset perspective, principal baskets play the role of fundamental (principal) variables. Fixing a i-th principal basket, the set of $\{\mathcal{B}_{qji}\}$, $j \in \{1, \ldots, M_q\}$, presents M_q realizations of i-th principal basket trading by M_q original managers in quarter q. In the notation convention in which columns represent fundamental variables, rows represent the realizations (see (2.2)), $\mathcal{B}_q$ (2.6) represents the trading matrix associated with the principal baskets in the asset perspective

$$\textit{Asset perspective:} \qquad \mathcal{B}_q = \overbrace{\left.\begin{bmatrix} \mathcal{B}_{q11} & \ldots & \mathcal{B}_{q1i} & \ldots & \mathcal{B}_{q1A_q} \\ \vdots & \ldots & \vdots & \ldots & \vdots \\ \mathcal{B}_{qj1} & \ldots & \mathcal{B}_{qji} & \ldots & \mathcal{B}_{qjA_q} \\ \vdots & \ldots & \vdots & \ldots & \vdots \\ \mathcal{B}_{qM_q1} & \ldots & \mathcal{B}_{qM_qi} & \ldots & \mathcal{B}_{qM_qA_q} \end{bmatrix}\right\}}^{A_q \text{ principal basket variables (columns)}} \begin{array}{l} M_q \text{ manager} \\ \text{realizations (rows).} \end{array} \tag{2.8}$$

The tradings of different principal baskets are unrelated, $\mathcal{B}'_{qi}\mathcal{B}_{qk} \equiv \sum_j \mathcal{B}_{qji}\mathcal{B}_{qjk} = 0$. Furthermore, the i-th principal basket's (uncentered) variance and its contribution to the total (uncentered) variance in asset trading are respectively λ_{qi} and $\frac{\lambda_{qi}}{\sum_k \lambda_{qk}}$ (2.5).

The Manager Perspective on Trading

In the manager perspective on trading, we take every manager j as a fundamental variable of the market. For a manager j, we take j's trading data $\{T_{qij}\}$, $i \in \{1, \ldots A_q\}$ in A_q assets as the A_q different realizations of the fundamental variable associated with manager j in quarter q. Adopting the notation convention in which columns represent the variables and rows represent the realizations, the manager perspective's trading matrix in quarter q is

$$\textit{Manager perspective:} \qquad T_q = \overbrace{\left.\begin{bmatrix} T_{q11} & \dots & T_{q1j} & \dots & T_{q1M_q} \\ \vdots & \dots & \vdots & \dots & \vdots \\ T_{qi1} & \dots & T_{qij} & \dots & T_{qiM_q} \\ \vdots & \dots & \vdots & \dots & \vdots \\ T_{qA_q1} & \dots & T_{qA_qj} & \dots & T_{qA_qM_q} \end{bmatrix}\right\}}^{M_q \text{ manager variables (columns)}} \begin{array}{l} A_q \text{ asset} \\ \text{realizations (rows).} \end{array} \tag{2.9}$$

We refer to T_q as the managers' trading matrix (as opposed to the asset trading matrix T_q' in (2.2)). In the manager perspective, the covariations of trading in quarter q are measured by the covariances of M_q fundamental managers' trading and are quantified by the (uncentered) second moment

$$V_q^{(M)} = \underbrace{T_q'}_{M_q \times A_q} \underbrace{T_q}_{A_q \times M_q} . \tag{2.10}$$

The diagonalization of the covariance matrix $V_q^{(M)}$ (2.10) also follows from the SVD (2.4)

$$N_q' V^{(M)} N_q = \underbrace{N_q' T_q'}_{\mathcal{N}_q'} \underbrace{T_q N_q}_{\mathcal{N}_q} = \sqrt{\Lambda}' \sqrt{\Lambda} = \begin{bmatrix} \lambda_{q1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \lambda_{qM_q} \end{bmatrix} \equiv \Lambda_q^{(M)}. \tag{2.11}$$

As discussed below (2.11), as the rank of the covariance matrix $V_q^{(M)} = T_q' T_q$ (2.10) does not exceed T_q's lower dimension $\min\{A_q, M_q\}$, when $M_q > A_q$, the $(M_q - A_q)$ smallest eigenvalues of $V_q^{(M)}$ are identically zeros, $\lambda_{qA_q+1} = \ldots = \lambda_{qM_q} = 0$.

In the manager perspective, the diagonalization (2.11) identifies the principal networks, i.e., networks of M_q original managers that are responsible for principal covariation dimensions in managers' trading. Specifically, the principal networks are columns of the $A_q \times M_q$ matrix $\mathcal{N}_q \equiv T_q N_q$ and are as below

$$\underbrace{\begin{bmatrix} \mathcal{N}_{q11} & \dots & \mathcal{N}_{q1M_q} \\ \vdots & \ddots & \vdots \\ \mathcal{N}_{qA_q1} & \dots & \mathcal{N}_{qA_qM_q} \end{bmatrix}}_{\mathcal{N}_q\text{: Network trading matrix}} = \underbrace{\begin{bmatrix} T_{q11} & \dots & T_{q1M_q} \\ \vdots & \ddots & \vdots \\ T_{qA_q1} & \dots & T_{qA_qM_q} \end{bmatrix}}_{T_q\text{: Managers' trading matrix}} \underbrace{\begin{bmatrix} N_{q11} & \dots & N_{q1M_q} \\ \vdots & \ddots & \vdots \\ N_{qM_q1} & \dots & N_{qM_qM_q} \end{bmatrix}}_{N_q\text{: Network composition matrix}}, \tag{2.12}$$

where N_{qkj} presents the composition (i.e., weight) of the k-th original manager in the j-th principal network, i.e., the j-th column of the rotation matrix N_q presents the compositions of M_q original managers in the j-th principal network in quarter q. As a result, the j-th principal network's trading $\mathcal{N}_{qij}$ in the original asset i equals the tradings $\{T_{qik}\}$ in the original asset i by original managers $k \in \{1, \dots, M_q\}$ weighted by the respective compositions $\{N_{qkj}\}$, or $\mathcal{N}_{qij} = \sum_k T_{qik} N_{qkj}$ (2.12). In a more tangible sense of trading T_{qij} in (2.1), we have,

$$\mathcal{N}_{qij} = \frac{\text{net change in asset } i\text{'s shares held by } j\text{-th principal network in quarter } q}{\text{asset } i\text{'s total shares outstanding at the end of quarter } q-1}. \tag{2.13}$$

Hereafter, we refer to N_q as the network composition matrix, and $\mathcal{N}_q$ the network trading matrix.

In the manager perspective, principal networks play the role of fundamental (principal) variables. Fixing a j-th principal network, the set of $\{\mathcal{N}_{qij}\}$, $i \in \{1, \dots, A_q\}$, presents A_q realizations of j-th principal network's trading in A_q original assets in quarter q. In the notation convention in which columns represent fundamental variables, rows represent the realizations (see (2.8)), $\mathcal{N}_q$ (2.12) represents the trading matrix associated with the principal networks in the manager perspective

$$\textit{Manager perspective:}\qquad \mathcal{N}_q = \overbrace{\begin{bmatrix} \mathcal{N}_{q11} & \ldots & \mathcal{N}_{q1j} & \ldots & \mathcal{N}_{q1M_q} \\ \vdots & \ldots & \vdots & \ldots & \vdots \\ \mathcal{N}_{qi1} & \ldots & \mathcal{N}_{qij} & \ldots & \mathcal{N}_{qiM_q} \\ \vdots & \ldots & \vdots & \ldots & \vdots \\ \mathcal{N}_{qA_q1} & \ldots & \mathcal{N}_{qA_qj} & \ldots & \mathcal{N}_{qA_qM_q} \end{bmatrix}}^{M_q \text{ principal network variables (columns)}} \left.\vphantom{\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}}\right\} \begin{array}{l} A_q \text{ asset} \\ \text{realizations (rows).} \end{array} \tag{2.14}$$

As a result, the trading by different principal networks are unrelated, $\mathcal{N}'_{qj}\mathcal{N}_{ql} \equiv \sum_i \mathcal{N}_{qij}\mathcal{N}_{qil} = 0$. Furthermore, the j-th principal network's (uncentered) variance and its contribution to the total (uncentered) variance in managers' trading are respectively λ_{qj} and $\frac{\lambda_{qj}}{\sum_k \lambda_{qk}}$ (2.5).

Duality of Perspectives

In the intra-period setting, a strong duality between asset and manager perspectives exists, allowing for a unified and equivalent view on the trading covariations among managers and the trading covariations among assets. Insights from this duality also help to derive implications and formulate tests specific to each perspective in the intertemporal setting.

First, as implied by the singular value decomposition and the discussions following Equations (2.7) and (2.13), covariance matrices $V_q^{(A)}$ (2.3) and $V_q^{(M)}$ (2.10) have an identical set of non-trivial eigenvalues. As a result, the sparsity structure of covariations, i.e., the distribution and dominance of principal factors driving the trading covariations, is identical in the asset and manager perspectives. In particular, the covariations' portion in asset trading explained by k-th principal basket (i.e., in the asset perspective) is equal to that in managers' trading (i.e., in the manager perspective) explained by k-th principal network for all $k \leq \min\{A_q, M_q\}$. The remaining principal baskets and principal networks (indexed by $k > \min\{A_q, M_q\}$) do not explain any of the covariations in trading, i.e., representing noises.

Second, the singular value decomposition implies a simple mapping between the basket trading matrix $\mathcal{B}_q$ and the network composition matrix N_q, and vice versa, between the network trading matrix $\mathcal{N}_q$ and the basket composition matrix B_a. The combination of the basket trading matrix's construction $\mathcal{B}_q = T_q' B_q$ (2.6) with the SVD (2.4) produces

$$\underbrace{\mathcal{B}_q}_{M_q \times A_q} = \underbrace{N_q}_{M_q \times M_q} \underbrace{\sqrt{\Lambda_q}'}_{M_q \times A_q}. \tag{2.15}$$

Note that because $\sqrt{\Lambda_q}'$ is a diagonal (rectangular) matrix, the k-th column of matrix $\mathcal{B}_q$ is proportional to the k-th column of matrix N_q, with the proportional constant being $\sqrt{\lambda_{qk}}$ for all $k \leq \min\{A_q, M_q\}$. When $A_q > M_q$ (i.e., $\mathcal{B}_q$ has more rows than N_q), the last $A_q - M_q$ columns of matrix $\sqrt{\Lambda_q}'$ are zeros, implying that the last (surplus) $A_q - M_q$ columns of $\mathcal{B}_q$ are also zeros. Given that columns of $\mathcal{B}_q$ represent principal baskets (2.6), this result means that none of M_q original managers trade the last (surplus) $A_q - M_q$ principal baskets when $A_q > M_q$, i.e., these surplus principal baskets are irrelevant. The first M_q principal baskets (first M_q first columns of $\mathcal{B}_q$) remain relevant, and are proportional respectively to the M_q columns of matrix N_q as mentioned above. Similar to (2.15), we have

$$\underbrace{\mathcal{N}_q}_{A_q \times M_q} = \underbrace{B_q}_{A_q \times A_q} \underbrace{\sqrt{\Lambda_q}}_{A_q \times M_q}, \tag{2.16}$$

or the k-th column of matrix $\mathcal{N}_q$ is proportional to the k-th column of matrix B_q, with the proportional constant being $\sqrt{\lambda_{qk}}$ for all $k \leq \min\{A_q, M_q\}$. When $M_q > A_q$, the last (surplus) $M_q - A_q$ columns of $\mathcal{N}_q$ are zeros, implying these last (surplus) $M_q - A_q$ principal networks are irrelevant (or noise-trading principal networks). These results show that principal baskets in the asset perspective inform the compositions of principal networks in the manager perspective, and vice versa.

Third, the above mapping between principal baskets and principal networks leads to a concrete trade result, namely, principal networks only trade the corresponding principal baskets. To understand this result, in the asset perspective, we change the basis from A_q original assets (i.e., columns of T'_q (2.2)) to A_q principal baskets (i.e., columns of $\mathcal{B}_q$ (2.8)) and examine the trading of principal networks in the new basis. Two observations are in order; (i) the principal networks' trading in the original assets is given in $\mathcal{N}_q$ (2.14), and (ii) the change of basis is performed via a multiplication by the rotation matrix B'_q.[1] Combining these two observations, the principal networks' trading in the new basis of principal baskets is $B'_q\mathcal{N}_q$, or

$$B'_q\mathcal{N}_q = B'_q T_q N_q = B'_q(B_q\sqrt{\Lambda_q}N'_q)N_q$$

$$= \sqrt{\Lambda_q} = \overbrace{\left.\begin{bmatrix} \sqrt{\lambda_{q1}} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \sqrt{\lambda_{qM_q}} \\ 0 & \dots & 0 \\ \vdots & \dots & \vdots \\ 0 & \dots & 0 \end{bmatrix}\right\}}^{M_q \text{ principal network variables}} \begin{array}{l} A_q \text{ principal} \\ \text{basket realizations,} \end{array} \tag{2.17}$$

where we have used the definition of $\mathcal{N}_q = T_q N_q$ (2.12), the SVD (2.4) and the notation convention (2.14) that columns of $B'_q\mathcal{N}_q$ are fundamental variables (M_q principle networks) and rows $B'_q\mathcal{N}_q$ are realizations (trades in the new basis of A_q principal baskets). In this notation convention, the diagonal form of matrix $\sqrt{\Lambda_q}$ in (2.17) demonstrates that the k-th principle network trades exactly

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[1] Equation $\mathcal{B}_q = T'_q B_q$ (2.6) implies that the new basis of principal baskets (i.e., columns of $\mathcal{B}_q$) is obtained from the original basis of original assets (i.e., columns of T'_q) by a rotation B_t. As a result, the same quantities but expressed in these two bases are related by the inverse rotation B'_q.

an amount of $\sqrt{\lambda_{qk}}$ in the k-th principal basket in period q and nothing else,[2]

$$\frac{\text{net change in } k\text{-th principal basket's shares held by } k\text{-th principal network in quarter } q}{k\text{-th principal basket's total shares outstanding at the end of quarter } q-1} = \sqrt{\lambda_{qk}}. \tag{2.18}$$

Similarly, in the manager perspective, we can also perform a change of basis from M_q original managers (i.e., columns of T_q) to M_q principal networks (i.e., columns of $\mathcal{N}_q$ (2.14)) to demonstrate that principal baskets are traded only by the respective principal networks and no one else. The following proposition recapitulates these duality results between the asset and manager perspectives.

Proposition 2.1 (Duality of perspectives). *In any single quarter q, the asset and manager perspectives are dual to each other in that*

1. *trading covariations in the two perspective have an identical sparsity structure,*
2. *trades in principal baskets in the asset perspective are proportional to the decomposition of the respective principal networks in the manager perspective (and vice versa),*
3. *principal networks trade only the corresponding principle baskets with the amounts being the corresponding eigenvalues of the trade matrix.*

The singular value decomposition helps to delineate the general trading configuration in any single quarter into principal baskets and principal networks that are responsible for the most prominent comovement dimensions respectively in asset and managers' tradings. The insight of the duality between asset and manager perspectives is that these trading perspectives are perfectly coherent intra period. In any single quarter, principal networks trading only the respective principal baskets and the associated trade amounts are completely characterized by a simple diagonal trading matrix

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[2] Note that Equation (2.17) depicts explicitly the diagonal matrix $\sqrt{\Lambda_q}$ for the case of $A_q > M_q$. In the opposite case, $A_q < M_q$, the last $(M_q - A_q)$ columns of the diagonal matrix $\sqrt{\Lambda_q}$ are zeros. This implies that the last (surplus) $(M_q - A_q)$ principal networks do not trade any of the A_q principal baskets as they are irrelevant (noise) principal networks.

$\sqrt{\Lambda_q}$ (2.17). Surplus principal networks (in case $M_q > A_q$) and principal baskets (in case $A_q > M_q$) are irrelevant noises as they do not contribute to trading covariations.

2.2.2 Intertemporal Setting

While asset and manager perspectives are coherent and the general trade configuration can be neatly delineated into the trades of principal baskets by principal networks in each quarter, both principal baskets and principal networks vary from one quarter to the next. To the extent that principal factors that drive the trade comovements depend on the underlying economic and market conditions, time variations in baskets' and networks' compositions and their persistence reflect economic dynamics and persistence of trading. For a simple exposition, we consider a conceptual two-quarter setting with quarter index $q \in \{1, 2\}$. Including additional quarters follows in a similar qualitative manner and is adopted in our empirical analysis.

To characterize the (cross-quarter) persistence in the asset perspective, in which assets are fundamental economic variables, we examine the similarity in the compositions of leading principal baskets across the two quarters.[3] In each quarter q, per Equation (2.16), the quarter-specific principal baskets' compositions (columns of matrix B_q (2.6)) are proportional to respective principal networks' trades (columns of matrix $\mathcal{N}_q$ (2.14)). Therefore, in the intertemporal setting, Proposition 2.1's intra-quarter duality between the asset and manager perspectives translates into the asset-perspective duality between the persistence in the principal baskets' compositions and the persistence in the principal networks' trades across the two quarters $q \in \{1, 2\}$. Similarly, we characterize the (cross-quarter) persistence in the manager perspective, in which managers are fundamental economic variables, as the similarity in the compositions of leading principal networks (columns of matrices N_q (2.12)) across $q \in \{1, 2\}$. Per Proposition 2.1 and Equation (2.15), this manager-perspective persistence is equivalently characterized by the similarity in trades of the prin-

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[3] The employment of principal baskets (as opposed to individual original assets) helps to focus on important quarter-specific factors while alleviating computational burdens and statistical noises.

cipal baskets (columns of matrices $\mathcal{B}_q$ (2.8)) across $q \in \{1, 2\}$.

The time dimension not only gives rise to the notion of trade persistence but also enables differentiation of the persistence and the implied economic contents intrinsic to the asset and manager perspectives. To connect and contrast with the intra-period setting in the previous subsection, we first approach each of the quarters separately, before integrating them into an intertemporal framework.

Time-Separable Approach

The input data to the two-quarter setting are the tradings of available managers in available assets each period. In the asset perspective, the time-separable approach organizes the input trading data into matrices T_1' and T_2' (2.2) for the two respective quarters (columns representing assets as fundamental variables, rows representing the asset tradings by different managers as the realizations). In the manager perspective, the time-separable approach organizes the trading data into matrices T_1 and T_2 (2.9) for $q \in \{1, 2\}$ (columns representing managers as fundamental variables, rows representing the managers' tradings of different assets as the realizations).

In the asset perspective and in a time-separable approach, We quantify the trading persistence as the similarity between leading principal baskets' compositions (or columns of matrix B_q) across the two quarters in the following steps. First, we retain only P_q leading principal baskets (i.e., P_q first columns $\{B_{q1}, \ldots, B_{qP_q}\}$ of matrix B_q) which together explain 25% of the trade covariation in period q. To compute the fraction of variation explained by a principal basket or principal network, we divide the variance of a PC (not the eigenvalue) by the sum of all variances of PCs. Second, we perform a canonical correlation analysis (CCA) to measure the common variation between the space spanned by P_{q+1} leading principal baskets in quarter q and P_{q+1} leading principal baskets in quarter $q + 1$.

The CCA works in the following manner. Suppose we have B_q, the principal basket composition matrix in quarter q, and B_{q+1}, the principal basket composition matrix in quarter $q+1$. Without loss of generality, we assume that N_q has m variables (columns), N_{q+1} has n variables (columns), and both matrices have p rows. The basic idea of the CCA is to find the linear combination of variables in B_{q+1} that has the maximum correlation with the linear combination of B_q; that is,

$$\begin{aligned}
Y_i =& a_1 B_{1,q+1} + a_2 B_{2,q+1} + \cdots + a_n B_{n,q+1} \\
X_i =& b_1 B_{1,q} + b_2 B_{2,q} + \cdots + b_m B_{m,q} \\
(a', b') =& \underset{a,b}{argmax} \quad \rho \equiv \text{Corr}(Y, X)
\end{aligned} \tag{2.19}$$

Where we denote (Y_i, X_i) as a canonical pair, which are pairwise uncorrelated (i.e. $(Y_i, X_i) \perp (Y_j, X_j), \forall i \neq j$). We denote Y_i as the rotated B_{q+1} and X_i as the rotated B_q In equation (2.19), the first canonical pair (Y_1, X_1) gives the linear combination of the variables in B_q and B_{q+1} giving rise to the highest correlation coefficient. The second canonical pair (Y_2, X_2) gives the linear combination of the variables in B_q and B_{q+1} giving rise to the second highest correlation coefficient, and so on. We denote ρ as the correlation coefficient in the canonical pair, which we refer to as the canonical correlation coefficient, and there are up to $\min\{m, n\}$ canonical correlation coefficients. If we identify a higher canonical correlation coefficient between B_q and B_{q+1}, then we identify a higher level of common variation between B_q and N_{q+1}, indicating that the principal basket composition matrix is more persistent.

Specifically, we denote S_A as the number of columns in the principal basket composition of quarter Y_i required to explain 70% of total cross variation with X_i, as identified by CCA; that is,

$$\begin{aligned}
&\frac{\sum_{k=1}^{S_A-1} \text{Cancorr}_{k,q}}{\sum_{k=1}^{P_{q+1}} \text{Cancorr}_{k,q}} < 0.7 \leq \frac{\sum_{k=1}^{S_A} \text{Cancorr}_{k,q}}{\sum_{k=1}^{P_{q+1}} \text{Cancorr}_{k,q}} \\
&\text{CanCorr}_{k,q} = \rho_k \sigma_{k,q} \sigma_{k,q+1}
\end{aligned} \tag{2.20}$$

Where ρ_k is the k-th canonical correlation coefficient between the canonical pairs, $\sigma_{k,q}$ is the standard deviation for the k-th column in X and $\sigma_{k,q+1}$ is the standard deviation for the k-th column in Y. We take the format in $\text{CanCorr}_{k,q}$ to simultaneously consider the persistence in the matrix and how the component can explain the fraction of intra-quarter variation of a matrix. In this way, we can make sure the components that explain a non-trivial fraction of intra-quarter variation of the trading can also predict future trading, rather than the noise that we might have accidentally captured.

Similarly, the trading persistence in the manager perspective and in a time-separable approach is characterized by the similarity between leading principal networks' compositions (or columns of matrix N_q) across the two quarters, which we denote as S_M: The number of columns in the principal network composition matrix in quarter $q+1$ we retain to explain 70% of the total cross variation with the principal network composition matrix in quarter q. We employ the steps similar to those underlying equation (2.20),

$$\frac{\sum_{k=1}^{S_M-1}\text{Cancorr}_{k,q}}{\sum_{k=1}^{P_{q+1}}\text{Cancorr}_{k,q}} < 0.7 \leq \frac{\sum_{k=1}^{S_M}\text{Cancorr}_{k,q}}{\sum_{k=1}^{P_{q+1}}\text{Cancorr}_{k,q}} \tag{2.21}$$

Due to the intra-period duality (Proposition 2.1), the sparsity structure is identical in the asset and manager perspectives, implying the same number P_q of the retained principal baskets and principal networks in equations (2.20) and (2.21). However, the intra-period duality does not preclude a difference in the persistence in the asset and manager perspectives, $S_A \neq S_M$.[4].

While it is an empirical matter whether the persistence in the asset perspective is higher, equal, or lower than that in the manager perspective, this empirical matter offers important insights into the trading dynamics. Since idiosyncratic components and other noises specific to a single quarter tend to be absent in other quarters, persistent components reflect underlying persistent economic and

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[4] To illustrate that the algebra does not rule out $S_A \neq S_M$, we identify a data set in which the persistence differs in the two perspectives. We take the rotation matrices B_q (2.6), N_q (2.12), and a diagonal matrix $\sqrt{\Lambda}$ as exogenous inputs (and trading data matrix T_q (2.2) as implied) in each quarter q. Clearly, the exogeneity of B_q and N_q allows for a difference in the spaces spanned by their columns across quarters, i.e., $S_A \neq S_M$.

market factors. When $S_A > S_M$, the leading principal network compositions together span a more persistent space and represent a more stable structure over time than the leading principal basket compositions. In the alternative scenario, $S_M > S_A$, the leading principal basket compositions together span a more persistent space and represent a more stable structure over time than the leading principal network compositions.

Time-Integrated Approach

We can further differentiate the persistence in the asset and manager perspectives by integrating the time series data across different quarters into a unified analysis as opposed to considering each quarter separately. First, in the asset perspective, the time-integrated approach organizes the input trading data into a single matrix T'_A by stacking the single-quarter trading matrices T'_1 and T'_2 (2.2) one on top of the other,[5]

$$T'_A = \begin{bmatrix} T'_1 \\ \\ \\ T'_2 \end{bmatrix} = \left.\overbrace{\begin{bmatrix} T_{111} & \dots & T_{1i1} & \dots & T_{1A1} \\ \vdots & \dots & \vdots & \dots & \vdots \\ T_{11M_1} & \dots & T_{1iM_1} & \dots & T_{1AM_1} \\ \\ T_{211} & \dots & T_{2i1} & \dots & T_{2A1} \\ \vdots & \dots & \vdots & \dots & \vdots \\ T_{21M_2} & \dots & T_{2iM_2} & \dots & T_{2AM_2} \end{bmatrix}}^{A \text{ asset variables (columns)}}\right\} \begin{array}{l} (M_1 + M_2) \text{ manager} \\ \text{realizations (rows),} \end{array} \tag{2.22}$$

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[5] The trading matrix T'_1 in quarter $q = 1$ has dimension $M_1 \times A_1$, T'_2 has dimension $M_2 \times A_2$. The stacking of T'_1 on top of T'_2 requires that those matrices have the same number of columns (representing the numbers A_1 and A_2 of assets traded in $q = 1$ and $q = 2$). We address the possibility that $A_1 \neq A_2$ by finding the union of assets that are traded at least in one of the two quarters, $A = A_1 \cup A_2$. We then augment T'_1 and T'_2 so that both of them have the same A columns by assigning a column of zeros to T'_q for assets that are not traded in quarter q. This augmentation makes the stacking feasible.

where $A = A_1 \cup A_2$ is the number of union assets traded in the two quarters $q = 1$ and $q = 2$ (see Footnote 5). In the asset perspective, the representation (2.22) adopts the notation convention in which columns represent assets as fundamental variables. The time-integrated trading matrix T'_A generalizes the intra-period trading matrix T'_q (2.2) by organizing managers trading these assets in both quarters into $(M_1 + M_2)$ extended rows of matrix T'_A. We take an asset i's trading data $\{T_{qij}\}$, $q \in \{1, 2\}$, $j \in \{1, \ldots M_q\}$ by $M_1 + M_2$ managers in the two quarters as $M_1 + M_2$ different realizations of the fundamental variable associated with asset i. That is, the time-integrated approach to trading data in the asset perspective treats a manager j's trades of an asset i in two quarters as two different realizations $\{T_{1ij}, T_{2ij}\}$ of the fundamental variable i.

The $(M_1 + M_2) \times A$ time-integrated trading data matrix T'_A (2.22) now replaces the previous intra-period trading matrix T'_q (2.2) in the asset perspective's analysis. Accordingly, the trade covariations are measured by the covariances of A fundamental assets' trading by managers in both quarters and are characterized by the $A \times A$ (uncentered) second moment matrix (using (2.22))

$$V^{(A)} = T_A T'_A = \sum_{q \in \{1,2\}} T_q T'_q = \sum_{q \in \{1,2\}} V_q^{(A)}. \tag{2.23}$$

This time-integrated covariance structure generalizes the intra-period covariance $V_q^{(A)}$ (2.3) in the asset perspective. The SVD of the time-integrated trading matrix reads $T'_A = N_A \sqrt{\Lambda_A}' B'_A$, where N_A and B_A are rotation matrices, $N_A N'_A = N'_A N_A = \mathbb{1}_{(M_1+M_2)\times(M_1+M_2)}$, $B_A B'_A = B'_A B_A = \mathbb{1}_{A \times A}$, and $\sqrt{\Lambda_A}'$ is a $(M_1 + M_2) \times A$ diagonal (rectangular) matrix containing the trading matrix T_A's eigenvalues $\left\{\sqrt{\lambda_{Ai}}\right\}$ on its diagonal. Similar to (2.5), the diagonalization of the covariance matrix $V^{(A)}$ reads

$$B'_A V^{(A)} B_A = \underbrace{B'_A T_A}_{\mathcal{B}'_A} \underbrace{T'_A B_A}_{\mathcal{B}_A} = \sqrt{\Lambda_A} \sqrt{\Lambda_A}' = \begin{bmatrix} \lambda_{A1} & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & \lambda_{AA} \end{bmatrix}. \tag{2.24}$$

Because the rank of the covariance matrix $V^{(A)} = T_A T'_A$ does not exceed T_A's lower dimension

$\min\{A,(M_1+M_2)\}$, when $A>(M_1+M_2)$, the $A_q-(M_1+M_2)$ smallest eigenvalues of $V^{(A)}$ are identically zeros, $\lambda_{AM_1+M_2+1}=\ldots=\lambda_{AA}=0$. These zero eigenvalues are associated with the noise trading dimensions which do not explain any of the trading comovements in the asset perspective. The diagonalization (2.24) identifies the principal baskets (i.e., baskets of original assets) responsible for the most prominent comovement dimensions in asset tradings in two quarters. Similar to the intra-quarter decomposition (2.6), we have the following time-integrated decomposition for the principal baskets

$$\underbrace{\begin{bmatrix} \mathcal{B}_{111} & \dots & \mathcal{B}_{11A} \\ \vdots & \dots & \vdots \\ \mathcal{B}_{1M_11} & \dots & \mathcal{B}_{1M_1A} \\ & & \\ \mathcal{B}_{211} & \dots & \mathcal{B}_{21A} \\ \vdots & \dots & \vdots \\ \mathcal{B}_{2M_21} & \dots & \mathcal{B}_{2M_2A} \end{bmatrix}}_{\mathcal{B}_A\text{: Basket trading matrix}} = \underbrace{\begin{bmatrix} T_{111} & \dots & T_{1A1} \\ \vdots & \dots & \vdots \\ T_{11M_1} & \dots & T_{1AM_1} \\ & & \\ T_{211} & \dots & T_{2A1} \\ \vdots & \dots & \vdots \\ T_{21M_2} & \dots & T_{2AM_2} \end{bmatrix}}_{T'_A\text{: Asset trading matrix}} \underbrace{\begin{bmatrix} B_{A11} & \dots & B_{A1A} \\ \vdots & \ddots & \vdots \\ B_{AA1} & \dots & B_{AAA} \end{bmatrix}}_{B_A\text{: Basket composition matrix}}, \tag{2.25}$$

where the i-th basket's compositions in terms of A original union assets (Footnote union) are represented by the i-th column of the rotation matrix B_A (referred to as the basket composition matrix), and the i-th basket's tradings by (M_1+M_2) managers are represented by the i-th column of the matrix $\mathcal{B}_A$ (referred to as the basket trading matrix). Specifically, in the asset-perspective notation convention in which columns represent principal baskets as fundamental variables, i-th column of the $(M_1+M_2)\times A$ basket trading matrix $\mathcal{B}_A$ represents the i-th principal basket's time-integrated tradings (i.e., realizations) by (M_1+M_2) managers in two quarters $q=1$ and $q=2$.

Second, in the manager perspective, the time-integrated approach organizes the input trading data into a single matrix T_M by stacking the single-quarter trading matrices T_1 and T_2 (2.9) one on top of the other,[6]

$$T_M = \begin{bmatrix} T_1 \\ \\ \\ T_2 \end{bmatrix} = \left.\overbrace{\begin{bmatrix} T_{111} & \dots & T_{112} & \dots & T_{11M} \\ \vdots & \dots & \vdots & \dots & \vdots \\ T_{1A_11} & \dots & T_{1A_1j} & \dots & T_{1A_1M} \\ & & & & \\ T_{211} & \dots & T_{212} & \dots & T_{21M} \\ \vdots & \dots & \vdots & \dots & \vdots \\ T_{2A_21} & \dots & T_{2A_2j} & \dots & T_{2A_2M} \end{bmatrix}}^{M \text{ manager variables (columns)}}\right\} \begin{array}{l} (A_1 + A_2) \text{ asset} \\ \text{realizations (rows)}, \end{array} \tag{2.26}$$

where $M = M_1 \cup M_2$ is the number of union managers trading in the two quarters $q = 1$ and $q = 2$ (see Footnote 6). In the manager perspective, the representation (2.26) adopts the notation convention in which columns represent managers as fundamental variables. The time-integrated trading matrix T_M generalizes the intra-period trading matrix T_q (2.9) by organizing assets traded by managers in both quarters into $(A_1 + A_2)$ extended rows of matrix T_M. We take a manager j's trading data $\{T_{qij}\}$, $q \in \{1, 2\}$, $i \in \{1, \dots, A_q\}$ in $A_1 + A_2$ assets in the two quarters as $A_1 + A_2$ different realizations of the fundamental variable associated with manager j. That is, the time-integrated approach to trading data in the manager perspective treats an asset i traded by a manager j in two quarters as two different realizations $\{T_{1ij}, T_{2ij}\}$ of the fundamental variable j.

The $(A_1 + A_2) \times M$ time-integrated trading data matrix T_M (2.26) now replaces the previous intra-period trading matrix T_q (2.9) in the asset perspective's analysis. Accordingly, the trade covariations are measured by the covariances of M fundamental managers' trading in assets in both quarters

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[6] The trading matrix T_1 in quarter $q = 1$ has dimension $A_1 \times M_1$, T_2 has dimension $A_2 \times M_2$. The stacking of T_1 on top of T_2 requires that those matrices have the same number of columns (representing the numbers M_1 and M_2 of trading managers in $q = 1$ and $q = 2$). We address the possibility that $M_1 \neq M_2$ by finding the union of managers that trade at least in one of the two quarters, $M = M_1 \cup M_2$. We then augment T_1 and T_2 so that both of them have the same M columns by assigning a column of zeros to T_q for managers that do not trade in quarter q. This augmentation makes the stacking feasible.

and are characterized by the $M \times M$ (uncentered) second-moment matrix (using (2.26))

$$V^{(M)} = T_M' T_M = \sum_{q \in \{1,2\}} T_q' T_q = \sum_{q \in \{1,2\}} V_q^{(M)}. \tag{2.27}$$

This time-integrated covariance structure generalizes the intra-period covariance $V_q^{(M)}$ (2.10) in the manager perspective. The SVD of the time-integrated trading matrix reads $T_M = B_M \sqrt{\Lambda_M} N_M'$, where N_M and B_M are rotation matrices, $N_M N_M' = N_M' N_M = \mathbb{1}_{M \times M}$, $B_M B_M' = B_M' B_M = \mathbb{1}_{(A_1+A_2)\times(A_1+A_2)}$, and $\sqrt{\Lambda_M}$ is a $(A_1 + A_2) \times M$ diagonal (rectangular) matrix containing the trading matrix T_M's eigenvalues $\left\{\sqrt{\lambda_{Mi}}\right\}$ on its diagonal. Similar to (2.11), the diagonalization of the covariance matrix $V^{(M)}$ reads

$$N_M' V^{(M)} N_M = \underbrace{N_M' T_M'}_{\mathcal{N}_M'} \underbrace{T_M N_M}_{\mathcal{N}_M} = \sqrt{\Lambda_M}' \sqrt{\Lambda_M} = \begin{bmatrix} \lambda_{M1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \lambda_{MM} \end{bmatrix}. \tag{2.28}$$

Because the rank of the covariance matrix $V^{(M)} = T_M' T_M$ does not exceed T_M's lower dimension $\min\{M, (A_1 + A_2)\}$, when $M > (A_1 + A_2)$, the $M - (A_1 + A_2)$ smallest eigenvalues of $V^{(M)}$ are identically zeros, $\lambda_{MA_1+A_2+1} = \ldots = \lambda_{MM} = 0$. These zero eigenvalues are associated with the noise trading dimensions which do not explain any of the trading comovements in the manager perspective.

The diagonalization (2.28) identifies the principal networks (i.e., networks of original managers) responsible for the most prominent comovement dimensions in managers' tradings in two quarters. Similar to the intra-quarter decomposition (2.12), we have the following time-integrated decomposition for the principal networks

$$\underbrace{\begin{bmatrix} \mathcal{N}_{111} & \dots & \mathcal{N}_{11M} \\ \vdots & \dots & \vdots \\ \mathcal{N}_{1A_11} & \dots & \mathcal{N}_{1A_1M} \\ \\ \mathcal{N}_{211} & \dots & \mathcal{N}_{21M} \\ \vdots & \dots & \vdots \\ \mathcal{N}_{2A_21} & \dots & \mathcal{N}_{2A_2M} \end{bmatrix}}_{\mathcal{N}_M\text{: Network trading matrix}} = \underbrace{\begin{bmatrix} T_{111} & \dots & T_{11M} \\ \vdots & \dots & \vdots \\ T_{1A_11} & \dots & T_{1A_1M} \\ \\ T_{211} & \dots & T_{21M} \\ \vdots & \dots & \vdots \\ T_{2A_21} & \dots & T_{2A_2M} \end{bmatrix}}_{T_M\text{: Manager trading matrix}} \underbrace{\begin{bmatrix} N_{M11} & \dots & N_{M1M} \\ \vdots & \ddots & \vdots \\ N_{MM1} & \dots & N_{MMM} \end{bmatrix}}_{N_M\text{: Network composition matrix}}, \tag{2.29}$$

where the j-th network's compositions in terms of M original union managers (Footnote 6) are represented by the j-th column of the rotation matrix N_M (referred to as the manager composition matrix), and the j-th network's tradings in $(A_1 + A_2)$ assets are represented by the j-th column of the matrix $\mathcal{N}_M$ (referred to as the network trading matrix). Specifically, in the manager-perspective notation convention in which columns represent principal networks as fundamental variables, j-th column of the $(A_1 + A_2) \times M$ network trading matrix $\mathcal{N}_M$ represents the j-th principal network's time-integrated tradings (i.e., realizations) in $(A_1 + A_2)$ assets in two quarters $q = 1$ and $q = 2$.

Third, it is important to observe that in the time-integrated approach to trading data, the duality does not hold between the asset and manager perspectives. This is because the data are organized differently in the trading matrices T'_A and T_M, leading to different covariance structures. The second moments $V^{(A)}$ (2.23) and $V^{(M)}$ (2.27) show that even though the duality is preserved in every single quarter $q \in \{1, 2\}$, it is broken in the integration of the two periods.[7] Absence of a duality in the current time-integrated approach, the trading persistence in the asset and manager perspectives would diverge.

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[7] Mechanically, the duality would be preserved if the covariances were characterized by the matrix pair $T'_A T_A$ and $T_A T'_A$ (or by the pair $T'_M T_M$ and $T_M T'_M$). However, these pairs do not consistently represent the trading covariances across different quarters either in the asset or the manager perspective.

To verify and quantify this divergence, we examine the sparsity structure of trading covariances in the two perspectives. If the distribution of $V^{(A)}$'s eigenvalues (2.24) concentrates on a few dominant eigenvalues, while the distribution of $V^{(M)}$'s eigenvalues (2.28) is more flat, the time-integrated trading covariation exhibits a stronger structure in the asset perspective. To relate with the time-separable approach above, this empirical pattern in the time-integrated approach indicates that trading tends to be more persistent across quarters in the asset perspective than in the manager perspective. Alternatively, if the distribution of $V^{(M)}$'s eigenvalues concentrates on a few dominant eigenvalues, while the distribution of $V^{(A)}$'s eigenvalues is flatter, the time-integrated trading covariation exhibits a stronger structure in the manager perspective. This empirical pattern intuitively indicates that tradings tend to be more persistent across quarters in the manager perspective than in the asset perspective.

In summary, the time-separable approach to trading data first works in the cross-section of assets and managers to identify quarter-specific principal factors, then integrates (compares and retains) the components of these principal factors that are persistent over quarters. In contrast, the time-integrated approach first integrates the trading of assets or managers over quarters, then works in the cross-section of assets and managers to identify the cross-quarter principal factors.

2.3 Empirical Analysis

This section outlines the procedures involved in testing our framework on the covariance of institutional trading. Section 2.3.1 introduces the data source and the trading measure employed for the analysis. Section 2.3.2 presents the results of preliminary empirical tests conducted using both the time-separable and time-integrated approaches, as outlined at the end of Section 2.2.2. These tests aim to highlight the differences between the asset and manager perspectives of covariance in trading across different periods. Section 2.3.3 is dedicated to the formal testing of the intra-quarter duality. Section 2.3.4 delves into the formal testing of the intertemporal analysis outlined in our framework. Finally, Section 2.3.5 establishes the connection between the covariance in institutional trading and asset returns.

2.3.1 Data Sources and the Institutional Trading Measure

Our primary source for institutional portfolio holdings is the Thomson Reuters 13F dataset. We merge these data with the CRSP databases to incorporate stock prices and shares outstanding. The merged dataset is quarterly from 1983 through 2020.[8] We exclude non-U.S. institutions and only keep domestic equity listed on the NYSE, AMEX, or NASDAQ with a share code of 10 or 11. In this way, our analysis can align with other literature on institutional ownership that focuses on the impact on the large U.S. equity market, such as [23], [34], and [9].

We define our trading measures as follows,

$$\text{RelPTrade}_{ijq} = 100 \times \frac{\text{shares}_{ijq} - \text{shares}_{ij,q-1}}{\text{TSO}_{i,q-1}} \tag{2.30}$$

where shares_{ijq} is the split-adjusted number of shares of stock i held by fund manager j in quarter q and $\text{TSO}_{i,q-1}$ is the number of the split-adjusted shares outstanding in quarter $q-1$. For example, $\text{RelPTrade} = 1$ means the fund manager buys 1% of shares outstanding in quarter $q-1$.[9]

One potential issue with using 13F data to measure institutional trading by equation (2.30) is that the data only reflects institutional portfolio holdings and changes in holdings may not necessarily indicate actual trading activity. To address this concern, we have taken several steps to refine our measurement. First, we adjust for stock splits by using split-adjusted share numbers. Second, we only consider data from fund managers who have a long history of existence to avoid potential

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[8] Thomson Reuters 13F data are available from 1980. However, daily trading volume for stocks listed on the NASDAQ stock exchange is not available prior to 1983.

[9] Details on our data-cleaning steps for inputs to equation (2.30) can be found in Appendix A.

distortions from mergers and acquisitions. Finally, we take into account that dividend payments can also impact portfolio holdings, but not all funds necessarily pay dividends.

Figure 2.1 shows their values of the number of assets and fund managers in time series after all the filtering processes. On average, there are 1,350 assets and 1,352 managers each quarter. The number of managers increases substantially over time, which is consistent with the evidence of the increasing institutional ownership in the US equity market. However, the number of assets traded by the fund managers is relatively stable. The discrepancy in the number of assets and fund managers in each quarter also motivates the singular value decomposition to build the duality in the covariances in the institutional trading matrix between the two perspectives that we show in Section 2.2. We will test this theoretical prediction in the following subsection.

2.3.2 A Preliminary Test for the Theoretical Prediction

This chapter summarizes how we invoke the 13-F data to test the relationship between the degree of covariance in institutional trading and the persistence in the composition matrix, as outlined in chapter 2.2.2.

We invoke the following steps to test our hypotheses:

Step 1: (Determine and compare the trading persistence in the asset and manage perspectives using the time-separable approach). For a given two consecutive quarters $\{q, q+1\}$, compute S_A in equation (2.20) and S_M in equation (2.21). Because the number of rows between B_q and B_{q+1} is not identical, we identify the manager corresponding to each row, and if a manager in a matrix is missing in another, we fill those rows in that matrix with 0. A similar way works for N_q and N_{q+1}, where we identify if the stock in a matrix is missing in another and fill the corresponding rows with missing stocks with 0. After this process, we ensure the identical number of rows between the two matrices so that we can run the CCA.

Step 2: (Determine and compare the trading sparsity in the asset and manage perspectives using the time-integrated approach). For the same two consecutive quarters $\{q, q+1\}$ above, we compute

(i) The set of eigenvalues $\{\lambda_{A1}, \ldots, \lambda_{AA}\}$ of the time-integrated covariance matrix $V^{(A)}$ from equation (2.24) in the asset perspective.

(ii) the set of eigenvalues $\{\lambda_{M1}, \ldots, \lambda_{MM}\}$ of the time-integrated covariance matrix $V^{(M)}$ from equation (2.28) in the manager perspective.

Then, we compute the fraction of the variation explained by the first S_A eigenvalues in the asset perspective, and the first S_M eigenvalues in the manager perspective. [10] We repeat this set of hypothesis testing across 148 quarters, from 1983Q2 to 2020Q1, where we refer to quarter $q+1$. If $S_A > S_M$, then the principal network composition matrix is more persistent than the principal basket composition matrix. However, the first S_A eigenvalues in the asset perspective explain a higher fraction of variation than the first S_M eigenvalues in the manager perspective. This result arises from the duality established in Proposition 2.1. Equation (2.15) implies that the principal basket trading matrix is nested in the principal network trading composition matrix, and equation (2.16) implies that the principal network trading matrix is nested in the principal basket trading composition. Hence, when the principal network composition matrix is more persistent, the principal basket trading matrix should explain a higher fraction of variation in the institutional trading space and the principal basket trading matrix is more persistent across time. Intuitively, this result indicates that a similar set of fund managers will trade different assets across quarters, and hence, would generate the covariance in trading and induce fluctuations in the asset market: What is traded matters more than who is trading in explaining the dynamics of trading.

Figure 2.2 shows the time series of S_A and S_M. The figure shows that $S_A > S_M$, indicating that the principal network composition matrix is more persistent than the principal basket composition

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[10] In a strict sense, both $V^{(A)}$ and $V^{(M)}$ are un-centered second-moment matrices. Consequently, the eigenvalues for $V^{(A)}$ and $V^{(M)}$ do not equal the variance of the principal components (or in this context, the principal trading matrix). Yet, when calculating the proportion of variation elucidated in the institutional trading matrix, using both the eigenvalues of the second-moment matrix and the variance of the principal components, the difference we observe is negligible.

matrix. On the other hand, Figure 2.3 shows the fraction of variation in the time-integrated trading spaces explained by the first S_A principal basket trading matrix or the first S_M principal manager trading matrix. The results indicate that the fraction of variation explained in the asset perspective is higher than that in the manager perspective. This result is consistent with our expectations.

Moreover, to mitigate any potential influence of the unequal dimensions ($S_A \neq S_M$) on the outcomes, we perform an additional test. This test involves calculating the proportion of variation in the time-integrated covariance matrix that is accounted for by an equal number of columns from the principal basket composition or principal network composition. The corresponding results refer to Figure 2.4, and they align with the findings in Figure 2.3.

2.3.3 Principal Networks and Principal Baskets: The Intra-Period Analysis

We formally test our framework on the intra-quarter duality in this subsection. The duality is important in explaining the result of the more persistent principal network composition matrix will lead to the more persistent principal basket trading matrix as shown in the previous subsection.

For each quarter, we diagonalize $T_q'T_q$, yielding the $A_q \times M_q$ matrix $\mathcal{N}_q \equiv T_q N_q$ as the principal component, which we refer to as a principal network trading matrix. We also diagonalize $T_q T_q'$ and generate the $M_q \times A_q$ matrix $\mathcal{B}_q' \equiv T_q' B_q$ as the principal component, which we refer to as a principal basket trading matrix. We do not demean T_q; as a result, the eigenvalues do not equal the variance of the principal components. To compute the fraction of variation explained by a PC, we divide the variance of a PC (not the eigenvalue) by the sum of all variances of PCs. This process is repeated for 149 quarters from 1983Q1 to 2020Q1.

Table 2.1 presents information about the number of columns of the principal network trading matrix and principal basket trading matrix required to explain 70% or 90% of the total variation in each quarter. Whenever we require more columns to explain the same fraction of variation in the institutional trading space, it indicates a less structured trading pattern within a quarter. The top panel shows the summary statistics, while the bottom panel shows the slope coefficient of the following AR(1) model:

$$\begin{aligned} P_{N,q} =& \alpha_N + \beta_N P_{N,q-1} + \epsilon_N \\ P_{B,q} =& \alpha_B + \beta_B P_{B,q-1} + \epsilon_B \end{aligned} \tag{2.31}$$

Where $P_{N,q}$ is the number of columns of the principal network trading matrix we retain in quarter q, and $P_{B,q}$ is the number of columns of the principal basket trading matrix we retain in quarter q. Our test results reveal that a similar number (while non-exact) of columns in the principal network trading matrix and principal basket trading matrix are needed to explain 70% of the total variation within each quarter. However, the test result does not reject the null that the numbers of columns in principal networks and principal baskets to explain 90% of the total variation are different. Hence, our results support the prediction of the intra-quarter duality from the singular value decomposition, which is important in the mapping of the principal basket composition and the principal network trading. The time series follows a normal distribution, as the skewness coefficient is close to 0 and the excess kurtosis is close to 3. Finally, the first autocorrelation coefficient for both perspectives is high, indicating that the information structure of T_q is persistent.

Figure 2.5 presents the time series of $P_{N,q}$ and $P_{B,q}$, offering valuable insights into trading activity. The top panel plots the time series of the number of principal components required to explain 70% of the total variation, while the bottom panel is for explaining 90% of the total variation. Notably, during periods of economic turmoil, such as recessions or financial crises, we often need more principal components to explain a given fraction of the total variation in trading activity, meaning a weaker factor structure in the institutional demand space. We observe a higher level of idiosyncratic trading and reduced adherence to a structured trading pattern. This indicates a greater degree of variability in trading decisions among investors.

Whenever we observe a weaker factor structure in institutional trading, it often indicates increased ”disagreement” among investors, which often reflects conflicting opinions on the prospects of asset prices. For example, the trade cannot be completed if both investors wish to sell the same stock. However, if one investor desires to sell and the other wishes to buy the same stock, the trade can occur if one believes the asset price will rise while the other thinks it will fall.

Literature often suggests that during economic turmoil, increased trading activity but in a less structured manner may be driven by a higher level of disagreement among investors (as suggested by [30]). This can result in more autocorrelated patterns in asset price dynamics, as seen in financial crises or recessions (as suggested by [4]). The business cycle or other systematic events naturally generate fluctuations in the number of columns in the principal network trading matrix or the principal basket trading matrix.

Results in Figure 2.5 could be analogous to the ”macro uncertainty” and ”firm-level common uncertainty” constructed in [26], which is brought by our discussions above. [26] take the conditional volatility of the residuals from the large predictive regressions of macro variables and firm profits on principal components (a proxy for the full information set). As noted by [26], there is typically higher uncertainty in the financial market or economy during economic downturns or financial crises. Compared to traditional measures of economic and financial uncertainty, such as the market volatility and dispersion in firm profits or earnings forecasts, the index in [26] better captures the business cycle and is sufficiently persistent in explaining the prolonged decline and recoveries of economic activities around recessions. Existing literature, such as [42] and [12], has also argued that conventional proxies of uncertainty lack the necessary persistence to fully explain the prolonged decline in output during and after recessions.

Since we construct the principal network trading matrix and principal basket trading matrix from high-dimensional institutional trading data, which typically reflects a faster learning rate in market or macroeconomic information than other types of agents such as retail investors, firm owners, or households, the time series variation shown in Figure 2.5 could motivate the construction of an uncertainty index with similar characteristics to that of [26]. An index based on institutional trading could offer a more precise characterization of the business cycle compared to the index in [26]. Since financial institutions play a major role in the financial markets, fluctuations in institutional trading can serve as a natural indicator of economic uncertainty or may be reflective of it.

Furthermore, Panel B of Table 2.1 reveals high persistence in the information structure of the trading matrix, making it a promising candidate for an economic uncertainty index. In contrast, our measure based on the covariance in institutional trading exhibits greater persistence than other uncertainty proxies, offering potential improvements in understanding uncertainty-based business cycles. Overall, these results boost further exploration into the linkages between the financial market and real economic activities, shedding light on a channel through which they interact.

2.3.4 Analyzing Persistence: The Intertemporal Analysis

We confirm the prediction of the intra-period duality of the two perspectives in the previous subsection. Next, we present the empirical results of the intertemporal analysis, which is central to this paper. We will demonstrate the true source of correlated institutional trading by showing that either the principal basket trading matrix or the principal network trading matrix is more persistent, which we show in Section 2.2.2 and the preliminary empirical tests in Section 2.3.2.

Appendix B describes how we construct the measure of the cross variation of the principal basket composition matrix and principal network composition matrix across time, as defined in equation (B1). Then we take the first S columns in the principal basket composition matrix B_{q+1} or the principal network composition matrix N_{q+1} that explains up to 70% of the total cross variation to construct the out-of-sample principal basket trading matrix in quarter $q+2$, denoted as $\mathcal{B}^{oos*}_{q+2} = T'_{q+2}\mathbf{B}_{q+1}$ and the out-of-sample principal network trading matrix in quarter $q+2$, denoted as $\mathcal{N}^{oos*}_{q+2} = T_{q+2}\mathbf{N}_{q+1}$. In this way, we make sure the components that explain a non-trivial fraction of intra-quarter variation of the trading can also predict future trading, rather than the noise that we might have accidentally captured.

Our out-of-sample regression in the manager perspective is as follows,

$$T_{q+2} = \alpha_{M,q+2} + \beta_{M,q+2}\mathcal{N}^{oos*}_{q+2} + \epsilon_{M,q+2} \tag{2.32}$$

The dependent variables in the regression are columns of the $A_{q+2} \times M_{q+2}$ matrix T_{q+2}, and the independent variables are the columns of the $A_{q+2} \times S_M$ matrix $T_{q+2} \times N_{q+1}$. On the other hand, the regression in the asset perspective is as follows,

$$T'_{q+2} = \alpha_{A,q+2} + \beta_{A,q+2}\mathcal{B}^{oos*}_{q+2} + \epsilon_{A,q+2} \tag{2.33}$$

The dependent variables in the regression are the columns of the $M_{q+2} \times S_A$ matrix T'_{q+2}, while the independent variable is the $M_{q+2} \times S$ matrix $T'_{q+2} \times B'_{q+1}$. Hence, in either perspective, the independent variables are the columns of the principal trading matrix, and we report the equally weighted R-squared as the measure of persistence in the institutional trading space. According to the duality, if the principal network composition matrix is more persistent than the principal basket composition matrix, then the principal basket trading matrix is more persistent, which should be reflected in the higher R-squared in the regression in equation (2.33) than in the regression in equation (2.32).

In Panel A of Table 2.2, we present the summary statistics for S. Our results reveal that we need to retain more columns of the principal basket composition matrix than the principal network composition matrix to explain the same proportion of cross-covariation. This suggests that the principal network composition matrix is more persistent than the principal basket composition matrix. We run the t-test to test if the average S from the asset perspective is higher than the average S from the manager perspective. The result confirms that we need a higher S in the asset perspective.

Figure 2.6 illustrates how the number of columns in the principal network composition and principal basket composition matrices varies over time. The pattern of the number of columns in the composition matrix in the asset perspective follows a countercyclical trend. In contrast, the trend in the number of canonical compositions in the manager perspective consistently decreases over time. This observation is largely consistent with the result in Figure 2.5 in that the fluctuation in the degree of institutional trading covariance might reflect economic uncertainty, the time series of the number of columns in the principal basket composition matrix follows the countercyclical trend more than the number of columns in the principal network composition matrix.

In Panel B of Table 2.2, we report the summary statistics of the out-of-sample R-squared for future trading, which corresponds to the regressions in equations (2.32) and (2.33). The out-of-sample R-squared on future trading from equation (2.33) is higher than the out-of-sample R-squared from equation (2.32), indicating that the principal basket trading matrix is more persistent than the principal network trading matrix. This result is consistent with the duality that the principal basket trading matrix is nested in the principal network composition matrix as well as the preliminary empirical result in Section 2.3.2. This result also indicates that the asset is the primary source of covariance in institutional trading, with a higher fraction of the variation explained in the time-integrated institutional trading matrix.

Figure 2.7 shows the time series of the out-of-sample R-squared. We observe that the time series of the out-of-sample R-squared is persistent for both perspectives. We have two additional observations about the time trend in the out-of-sample R-squared. First, the decreasing out-of-sample R-squared in the manager perspective is closely related to the decreasing S in the principal network composition matrix, shown in Figure 2.6. This observation can be explained by the fact that when the number of independent variables in a regression decreases, the R-squared will also decrease. For the time trend of the R-squared to align with the time trend of S, A_{q+2} should be almost constant or not exhibit a clear time trend across quarters. Figure 2.1 confirms this observation.

Second, the out-of-sample R-squared for the asset perspective does not exhibit a clear time trend. Since S in the asset view is countercyclical and M_{q+2} is increasing, we cannot determine whether including more independent variables will increase the R-squared in this regression setup.

2.3.5 Asset Pricing Implication: Connection to the Return Space

In this subsection, we delve into the relationship between institutional trading and the returns of stocks traded by fund managers. We present two sets of empirical analyses to shed light on this connection.

First, we will construct portfolio returns based on the principal basket trading matrix and the principal network trading matrix. As we have shown that the principal basket trading matrix is more persistent than the principal network trading matrix, we aim to examine whether such discrepancy leads to the difference in the predictability of the asset returns incurred in the trading. We conjecture that the return predictability from the trading in the asset perspective will be worse than the manager view because the asset perspective is the main source of covariance in institutional trading, and a more synchronized trading pattern would generate more noise in the return, making the return less predictable.

Second, considering that trading can lead to increased volatility in asset returns, we will conduct a series of tests to investigate whether the volatility of returns differs between the two perspectives. In these tests, we construct quarter-q portfolio returns based on the information from principal assets or principal managers in quarter q. However, it is important to note that we may not necessarily have the intra-period duality established in Section 2.2, as the duality from the singular value decomposition requires the same matrix to hold, and we link the institutional trading matrix to the asset return matrix in this section. By examining whether the volatility of portfolio returns, constructed from the principal assets and principal managers, differs, we ascertain the true source of covariance in institutional trading based on our framework.

We define the returns incurred to the fund managers and the asset returns in the following steps. We focus on common stocks listed on AMEX/NYSE/NASDAQ with observations that fall in March, June, September, and December of each year from 1983 to 2020. We merge CRSP with our S34 sample, which we have constructed based on PERMNO, and only retain observations for PERMNOs that exist in both datasets. We note that each PERMNO might match with multiple fund managers as multiple fund managers can trade the same stock within a quarter. We denote the vector of stocks in each quarter q as r_q. We winsorize the return r at the 1st and 99th percentiles across all the observations across time. For each quarter, we have the data in the long format with four columns: the fund manager (mgrno), the stock (PERMNO), the stock return, and the trading measure RelPTrade.

Given M_q fund managers and A_q stocks in quarter q, we calculate the average return of the traded stocks i (where $i = 1, 2, ..., A_q$) for each fund manager j in quarter q. The weight used for the calculation will be $\text{RelPTrade}_{i,j,q}$. We denote the weighted average return for fund manager j in quarter q as $R_{j,q}$ and the return on each stock as $r_{i,j}$, taking the following format:

$$r_q \equiv \begin{bmatrix} r_{1,1} & r_{2,1} & r_{3,1} & \dots & r_{A,1} & r_{1,2} & r_{2,2} & \dots & r_{A,M} \end{bmatrix}' \tag{2.34}$$

And the weighted average return will be as follows:

$$R_{j,q} = \frac{1}{\sum_{j=1}^{M_q} \text{RelPTrade}_{i,j,q}} \sum_{j=1}^{M_q} \text{RelPTrade}_{i,j,q} \times r_{i,j,q} \tag{2.35}$$

Hence, for each quarter, we form a vector of fund managers and the weighted return as a vector, which we denote as R_q with the following format:

$$R_q \equiv \begin{bmatrix} R_1 & R_2 & R_3 & \dots & R_M \end{bmatrix}' \tag{2.36}$$

We first discuss the predictability of past trading from the principal network trading matrix and the principal basket trading matrix on future asset returns, which is our second set of empirical analyses in this section. Specifically, we examine how the past trading activities of the principal network and principal basket can be used to forecast the asset returns of these fund managers and asset returns.

In the manager perspective, we aim to examine if the stock return can be predicted by the return on the stocks traded by some aggregated measure of fund managers (i.e., the trading strategies adopted by some connections of managers). We perform regressions using Equation (1), where we regress r_{q+1} on $\mathcal{N}_q \equiv T_q N_q$. Each row in $\mathcal{N}_q$ corresponds to a stock, and each column corresponds to a network of fund managers. We consider the first three columns of $\mathcal{N}_q$ for our analysis. The regressions are as follows:

$$\begin{aligned}
r_{q+1} =& \alpha_{1,q} + \beta_{1,q}\mathcal{N}_{1,q} + \epsilon_{1,q} \\
r_{q+1} =& \alpha_{2,q} + \beta_{2,q}\mathcal{N}_{2,q} + \epsilon_{2,q} \\
r_{q+1} =& \alpha_{3,q} + \beta_{3,q}\mathcal{N}_{3,q} + \epsilon_{3,q} \\
r_{q+1} =& \alpha_{1,2,3,q} + \beta_{1,2,3,q}\mathcal{N}_{1,2,3,q} + \epsilon_{1,2,3,q}
\end{aligned} \tag{2.37}$$

In these equations, the first line represents the regression of the return on the first column of the principal network trading matrix, the second line represents the regression on the second column, the third line represents the regression on the third column, and the last line represents the regression on all three columns. We report the coefficient of determination (R^2) from each regression across quarters. We expect the R^2 value from the first regression to be higher than the second regression and higher than the third regression.

In the asset perspective, we aim to examine if the average fund manager's return can be predicted by the return on the fund managers who trade an aggregated measure of stocks. We perform regressions using Equation (3), where we regress R_{q+1} on $\mathcal{B}_q \equiv B_q T_q'$. Each row in $\mathcal{B}_q$ corresponds to a fund manager, and each column corresponds to the aggregated trading activities of the fund managers (i.e., the sense of what they trade). We consider the first three columns of $\mathcal{B}_q$ for our analysis. The regressions are as follows:

$$\begin{aligned}
R_{q+1} =& \alpha_{1,q} + \beta_{1,q}\mathcal{B}_{1,q} + \epsilon_{1,q} \\
R_{q+1} =& \alpha_{2,q} + \beta_{2,q}\mathcal{B}_{2,q} + \epsilon_{2,q} \\
R_{q+1} =& \alpha_{3,q} + \beta_{3,q}\mathcal{B}_{3,q} + \epsilon_{3,q} \\
R_{q+1} =& \alpha_{1,2,3,q} + \beta_{1,2,3,q}\mathcal{B}_{1,2,3,q} + +\epsilon_{1,2,3,q}
\end{aligned} \tag{2.38}$$

The first line represents the regression of the return on the first column of the principal basket trading matrix, the second line represents the regression on the second column, the third line represents the regression on the third column, and the last line represents the regression on all three columns.

We present the summary statistics of the R^2 across 149 quarters in Table 2.3. When we regress the return on the first three principal components, we find that the average (median) R^2 in the manager view is 0.11 (0.07), whereas it is only 0.003 (0.001) in the asset view.

These results suggest that the covariance pattern in institutional trading largely arises from the asset view, as the return space in the asset view is more noise-prone, making the return less predictable. Conversely, although the manager view is not the primary source of covariance in institutional trading, it generates less noise in the asset return space, leading to more predictable returns, as indicated by the higher R^2.

In the second analysis, we analyze the performance of portfolio returns in the principal network trading and principal basket trading. Each quarter, we construct the portfolio return from the principal network using the following equation:

$$\underbrace{r_{N,q}}_{1\times 10} = \underbrace{r'_q}_{1\times A} \underbrace{\mathcal{N}_q}_{A\times 10} \tag{2.39}$$

Similarly, we construct the portfolio return from the principal basket using the following equation:

$$\underbrace{r_{B,q}}_{1\times 10} = \underbrace{R'_q}_{1\times M} \underbrace{\mathcal{B}_q}_{M\times 10} \tag{2.40}$$

We stack the constructed returns over 149 quarters from 1983Q1 to 2020Q1.

The summary statistics of the resulting standard deviation of the quarterly returns refer to Table 2.4. The findings support the idea put forth by [9] that increased institutional trading contributes to higher fluctuations in the stock market. According to their perspective, if the majority of investors in the economy are institutional rather than retail investors, these institutions, driven by their objective to outperform the market, tend to trade heavily in the stock market index and its constituent stocks. As a consequence, institutional trading generates excess demand for the stock market index. Given the fixed supply of the stock market index, this increased demand causes the index to rise, resulting in higher stock market volatility compared to a scenario without institutional investors.

Based on our result that the covariance in institutional trading primarily comes from the asset perspective, we have a related hypothesis: Portfolio return volatility is higher in the asset perspective compared to the manager perspective. We invoke the variance ratio test to test this hypothesis: If we reject the null hypothesis that the variance of the portfolio returns in the asset perspective is equal to the manager perspective, then we confirm that the portfolio return from the asset perspective is higher. We show the relevant results in the last two rows of Table 2.4, which confirms our hypothesis.

2.4 Conclusions

We present a novel framework that disentangles the covariance of institutional trading into two distinct perspectives: the asset perspective and the manager perspective. By invoking singular value decomposition within each quarter, we ensure that each fund manager trades the corresponding assets, and each basket of assets is traded by multiple fund managers. This duality allows us to characterize the covariance in trading using a single matrix. Importantly, our intertemporal analysis reveals that these two perspectives exhibit different levels of persistence over time, enabling us to identify the primary force behind the covariance in institutional trading, with the basket of assets emerging as the main driver.

Our findings contribute significantly to the demand-based asset pricing literature by shedding light on the implications of covariance structure in institutional trading for asset returns. In particular, we find that the volatility of the portfolio return from the asset perspective is significantly higher. Moreover, we explore how this covariance structure may connect to uncertainty-based business cycle theories, providing insights into how systematic risks drive the covariance of institutional trading over time. We also offer a means to measure the degree of economic uncertainty through institutional trading data. Overall, we offer valuable insights into the dynamics of institutional trading and its impact on asset returns and the business cycle.

Table 2.1: **The Summary Statistics of the Intra-Period Analysis**

This table shows the summary statistics of the number of columns in the principal network trading matrix ($P_{N,q}$) and principal basket trading matrix ($P_{B,q}$) required to explain 70% or 90% of the total variation in the institutional trading matrix in quarter q. Panel A shows the summary statistics of $P_{N,q}$ and $P_{B,q}$, including the mean, standard deviation (std), skewness (skew), excess kurtosis (ex.kurt), minimum (min), 10th percentile (p10), median, 90th percentile (p90), and the maximum. For each case of reporting the number of columns in principal network (basket) tradings to explain 70% or 90% of total variation, we also report the t-statistics to compare the mean numbers between the principal basket and principal network. Panel B reports the slope coefficient and t-statistics from the AR(1) models in equation (2.31). The standard errors are Newey-West with 3 lags of autocorrelation. The sample spans from 1983Q1 to 2020Q1.

Panel A: Summary Statistics	Mean	std	skew	ex.kurt	min	p10	median	p90	max
70%									
$P_{N,q}$	107.94	9.69	-1.37	4.59	76	92	111	117	122
$P_{B,q}$	104.68	9.79	-1.26	4.26	73	89	107	114	120
t($P_{N,q} - P_{B,q}$)	2.89								
90%									
$P_{N,q}$	271.02	38.84	-0.88	3.29	157	217	276	315	333
$P_{B,q}$	266.89	38.16	-0.88	3.29	155	212	271	311	326
t($P_{N,q} - P_{B,q}$)	0.93								
Panel B: Coefficients	β	t							
70%									
β_N	0.85	19.02							
β_B	0.84	18.33							
90%									
β_N	0.94	49.82							
β_B	0.94	48.79							

Table 2.2: **The Summary Statistics of the Intertemporal Analysis**

Panel A shows the summary statistics of S, the number of columns in the principal composition matrix of quarter $q+1$ to retain. A smaller S indicates that the principal composition matrix is more persistent. We report the t-statistics on the third row in Panel B to test if the average number of columns in the asset perspective is higher than from the manager perspective. Panel B summarizes the R-squared obtained from the regressions in equations (2.32) and (2.33). For each quarter and fund manager, we regress the actual trading matrix on the predicted trading matrix and report the uniformly weighted average of the R-squared, denoted as the out-of-sample (OOS) R-squared. The detail of the construction steps refers to Appendix B. We report the t-statistics on the third row in Panel B to test if the average OOS R-squared from the asset perspective is higher than from the manager perspective. We note that if the principal network composition matrix is more persistent, then the principal basket composition matrix has better predictability in trading. The sample in Panel A spans from 1984Q2 to 2019Q4. The sample in Panel B spans from 1984Q3 to 2020Q1.

Panel A: Number of Columns, S	Mean	std	skew	ex.kurt	min	p10	median	p90	max
Network Composition	23.82	7.03	0.23	1.95	10	16	22	34	37
Basket Composition	60.88	5.00	−1.28	4.44	46	55	62	66	68
t(Asset − Manager)	51.19								
Panel B: The OOS R-Squared	Mean	std	skew	ex.kurt	min	p10	median	p90	max
Predicted Principal Network Trading	0.06	0.02	0.92	2.52	0.03	0.04	0.05	0.10	0.12
Predicted Principal Basket Trading	0.33	0.02	−0.04	4.90	0.26	0.31	0.33	0.35	0.41
t(Basket Trading − Network Trading)	101.35								

Table 2.3: **The Summary Statistics of the R-Squared of Regressing the Quarter $q+1$ Return on the Quarter q Principal Components**

This table shows the summary statistics of the R-squared by regressing the quarter $q+1$ return on the first three columns of the principal trading matrix in quarter q. The top panel shows the result of the regression in equation (2.37), in which r_{q+1} is the vector of stocks in quarter $q+1$, and $\mathcal{N}_{q,k}, k=1,2,3$ is the first k columns of the principal network trading matrix in quarter q. The fourth row in the top panel indicates that we regress r_{q+1} on the first three columns of the principal network trading matrix together. Likewise, the bottom panel shows the result of the regression in equation (2.38), in which R_{q+1} is the vector of weighted-average return incurred to the managers who trade the stocks in quarter $q+1$, and $\mathcal{B}_{q,k}, k=1,2,3$ is the first k columns of the principal basket trading matrix in quarter q. The fourth row in the top panel indicates that we regress R_{q+1} on the first three columns of the principal basket trading matrix together. The sample spans from 1983Q1 to 2020Q1.

Manager Perspective	Mean	Min	p10	Median	p90	Max
Reg r_{q+1} on $\mathcal{N}_{q,1}$	0.104	0.000	0.004	0.058	0.284	0.459
Reg r_{q+1} on $\mathcal{N}_{q,2}$	0.012	0.000	0.000	0.004	0.028	0.178
Reg r_{q+1} on $\mathcal{N}_{q,3}$	0.006	0.000	0.000	0.002	0.017	0.080
Reg r_{q+1} on $\mathcal{N}_{q,1,2,3}$	0.113	0.000	0.008	0.072	0.289	0.477
Asset Perspective	Mean	Min	p10	Median	p90	Max
Reg R_{q+1} on $\mathcal{B}_{q,1}$	0.001	0.000	0.000	0.000	0.003	0.023
Reg R_{q+1} on $\mathcal{B}_{q,2}$	0.002	0.000	0.000	0.000	0.003	0.055
Reg R_{q+1} on $\mathcal{B}_{q,3}$	0.003	0.000	0.000	0.000	0.003	0.055
Reg R_{q+1} on $\mathcal{B}_{q,1,2,3}$	0.003	0.000	0.000	0.001	0.008	0.058

Table 2.4: **The Standard Deviations of the Portfolio Returns for Each Perspective**

This table shows the standard deviations of the portfolio returns embedded from the first ten columns of the principal trading matrix in each perspective. Each quarter, we construct the portfolio return from the principal network trading matrix using $r_{N,q} = r_q' \mathcal{N}_q$, where we denote the weighted average return for fund manager j in quarter q as $R_{j,q}$. Similarly, we construct the portfolio return from the principal basket trading matrix using $r_{B,q} = R_q' \mathcal{B}_q$, where we denote the return on each stock as $r_{i,j}$. We stack the constructed returns over 149 quarters from 1983Q1 to 2020Q1. The row of VR Test Stats reports the test statistics for the variance ratio test of the variance from the asset perspective divided by the variance from the manager perspective. The p-value corresponds to the null hypothesis of equal variance, while the alternative hypothesis is unequal variance.

Manager	$\mathcal{N}_1$	$\mathcal{N}_2$	$\mathcal{N}_3$	$\mathcal{N}_4$	$\mathcal{N}_5$	$\mathcal{N}_6$	$\mathcal{N}_7$	$\mathcal{N}_8$	$\mathcal{N}_9$	$\mathcal{N}_{10}$
Standard deviation (%)	4.685	1.114	0.643	0.492	0.362	0.416	0.392	0.366	0.384	0.366
Asset	$\mathcal{B}_1$	$\mathcal{B}_2$	$\mathcal{B}_3$	$\mathcal{B}_4$	$\mathcal{B}_5$	$\mathcal{B}_6$	$\mathcal{B}_7$	$\mathcal{B}_8$	$\mathcal{B}_9$	$\mathcal{B}_{10}$
Standard deviation (%)	4.806	7.575	5.846	6.736	4.658	6.713	7.122	5.307	6.501	6.357
VR Test Stats	1.05	14.97	44.39	187.30	165.12	260.52	329.26	210.42	286.00	301.50
p-value	0.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

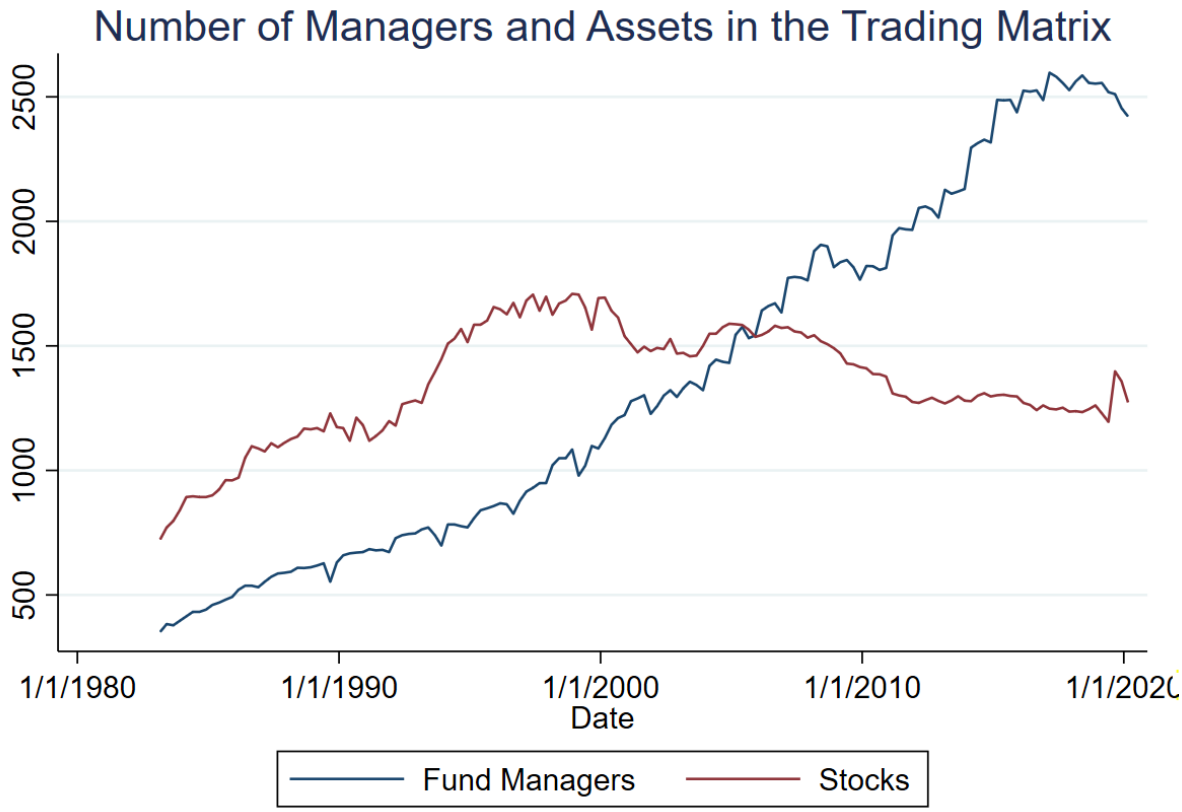

Figure 2.1: **The Number of Stocks and Fund Managers in Each Quarter.**

This figure shows the number of stocks (red line) and fund managers (blue line) in the trading matrix T_q for each quarter. The sample spans from 1983Q1 to 2020Q1.

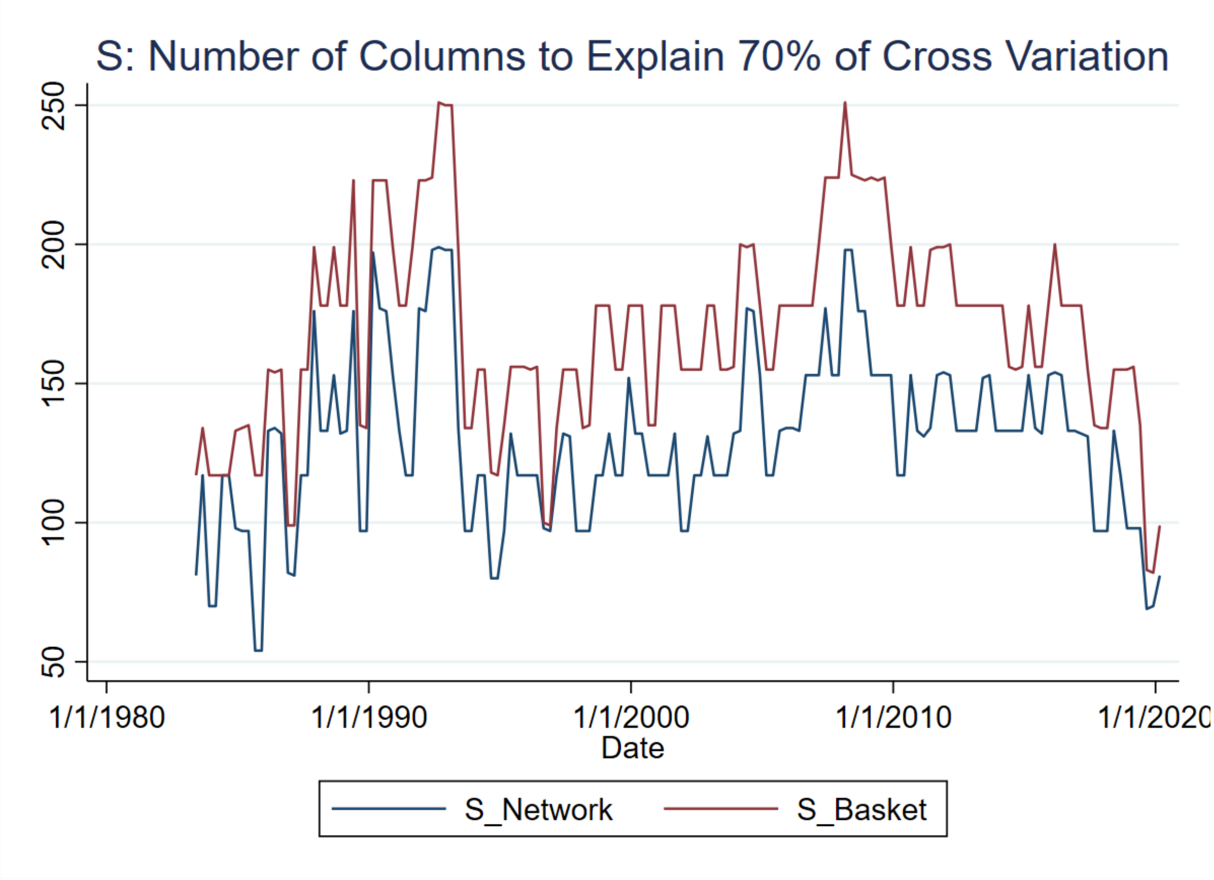

Figure 2.2: **The Number of Columns in the Principal Composition Matrix to Explain 70% of Cross Variation.**

The blue line shows the number of columns in the principal network composition matrix in quarter $q+1$, denoted as N_{q+1}, to explain up to 70% of the total cross variation with N_q. We denote this number of columns as S_N as in the figure, and the construction step is in equation (2.21). The red line shows the number of columns in the principal basket composition in quarter $q+1$, denoted as B_{q+1}, to explain up to 70% of the total cross variation with B_q. We denote this number of columns as S_B as in the figure, and the construction step is in equation (2.20). A smaller S_N or S_B indicates that the principal composition matrix is more persistent. Before we compute the cross variation, we retain the number of columns in the principal components that explain 25% of the total variation within each quarter. The sample is from 1983Q2 to 2020Q1.

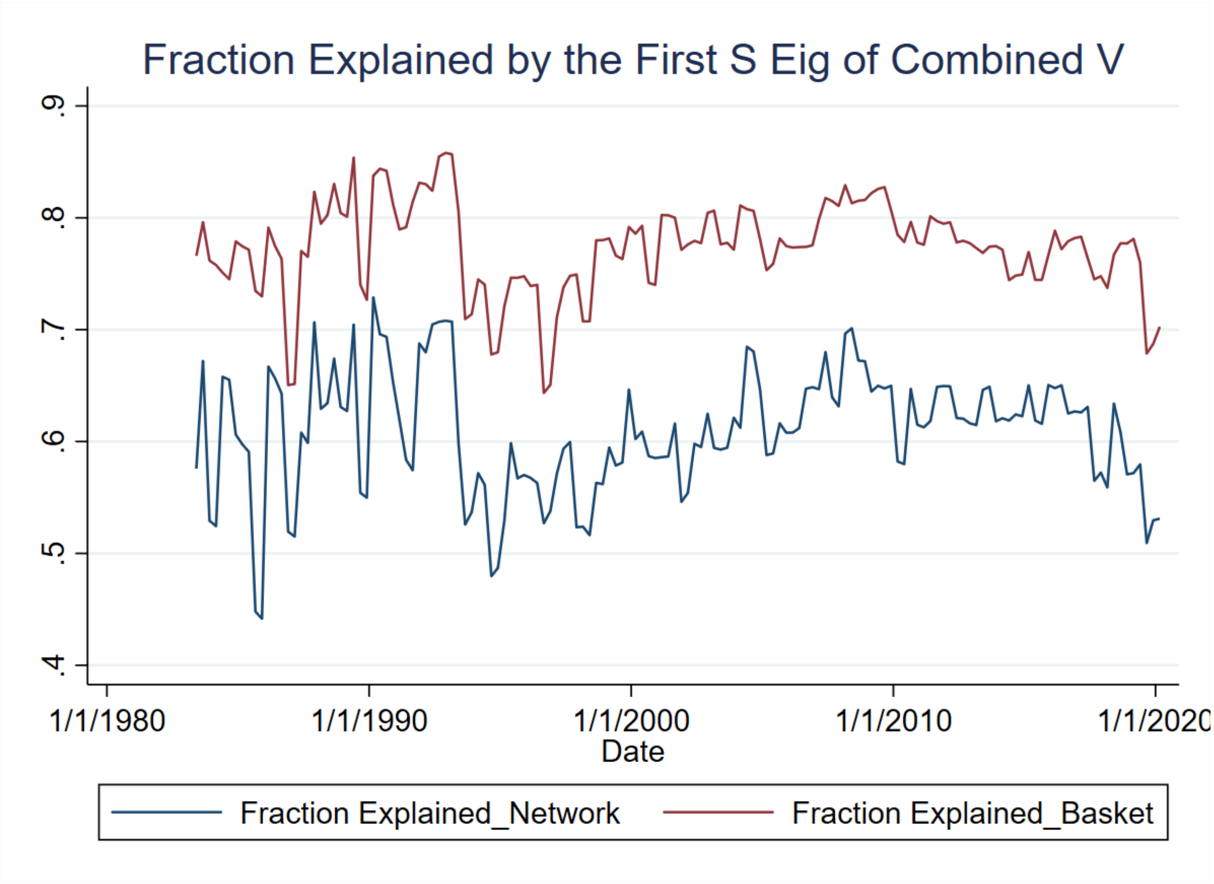

Figure 2.3: **Fraction of Variation Explained.**

The blue line shows the fraction of the summation of the first S_M eigenvalues of the time-integrated covariance matrix $V^{(M)}$ in equation (2.28) over the summation of M_{q+1} eigenvalues in the manager perspective. The red line shows the fraction of the summation of the first S_A eigenvalues of the time-integrated covariance matrix $V^{(A)}$ in equation (2.24) over the summation of A_{q+1} eigenvalues in the asset perspective. The sample is from 1983Q2 to 2020Q1.

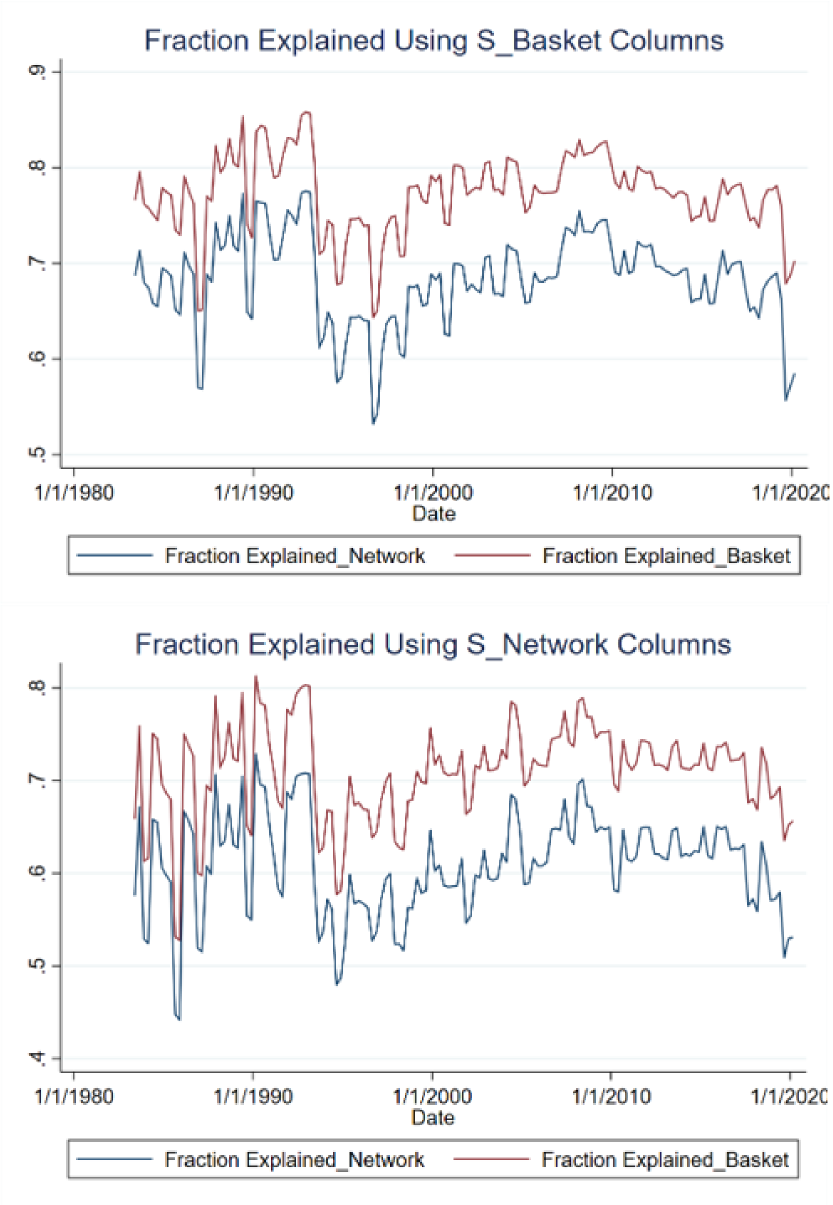

Figure 2.4: **Fraction of Variation Explained Using the Fixed Number of Columns.**

The blue line shows the fraction of the summation of the first fixed number of eigenvalues of the time-integrated covariance matrix $V^{(M)}$ in equation (2.28) over the summation of M_{q+1} eigenvalues in the manager perspective. The red line shows the fraction of the summation of the first fixed number of eigenvalues of the time-integrated covariance matrix $V^{(A)}$ in equation (2.24) over the summation of A_{q+1} eigenvalues in the asset perspective. In the top panel, we fix the number to be S_A, while in the bottom panel, we fix the number to be S_M. The sample is from 1983Q2 to 2020Q1.

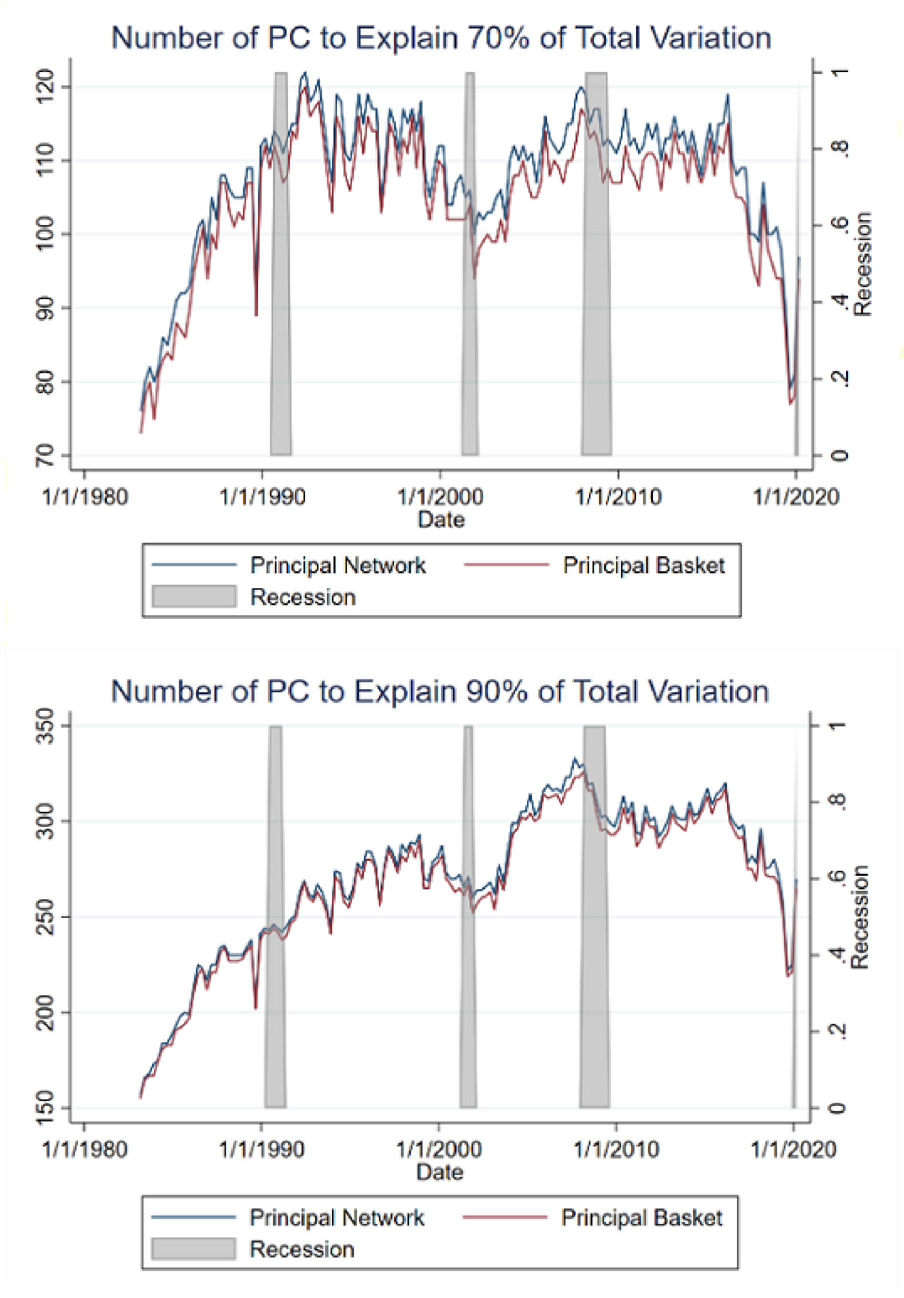

Figure 2.5: **Time Series of the Number of Columns in Principal Network Trading Matrix and Principal Basket Trading Matrix: Intra-Period Analysis.**

The top panel shows the number of columns in the principal network trading matrix (red line) and principal basket trading matrix (blue line) required to explain 70% of the total variation in the institutional trading matrix T_q in each quarter. The bottom panel shows the number to explain 90%. The gray area indicates the recessionary periods designated by the NBER. The sample spans from 1983Q1 to 2020Q1.

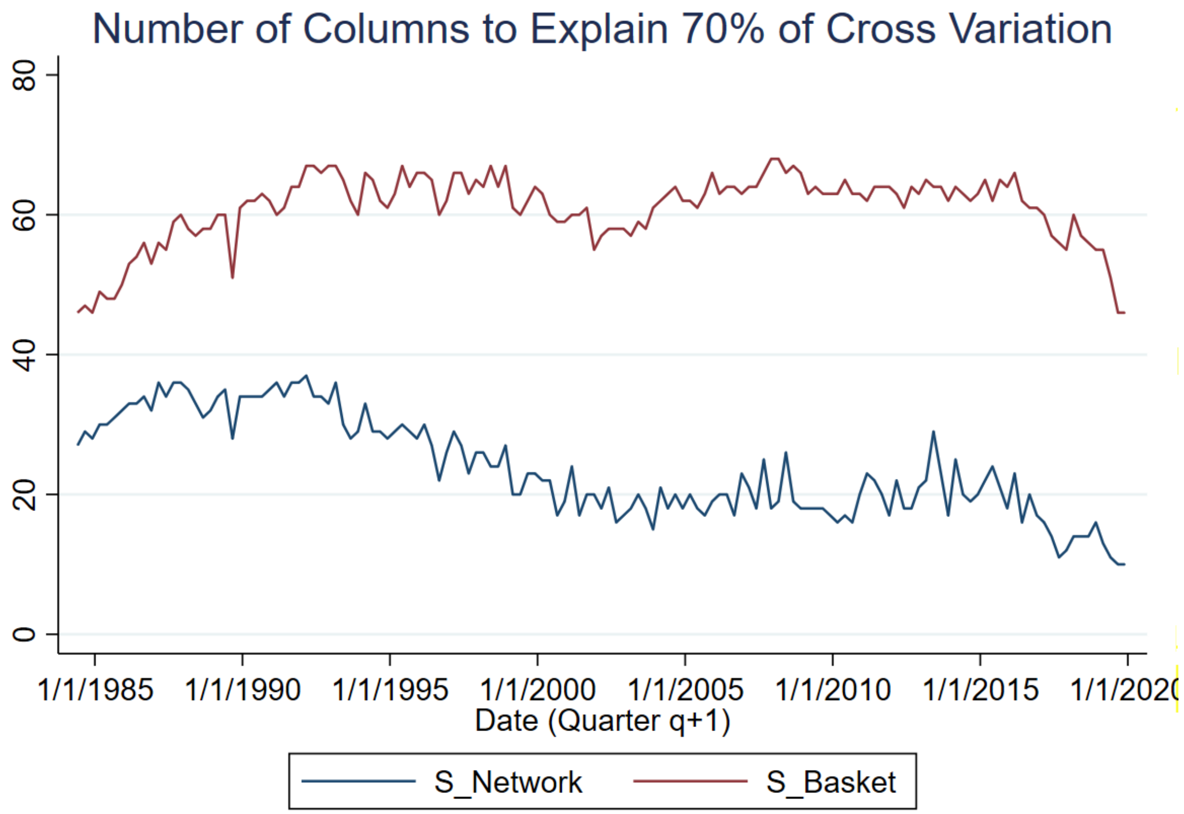

Figure 2.6: **Time Series of the Number of Columns in the Principal Composition Matrix in Quarter q+1: Intertemporal Analysis.**

This figure shows the time series of S, the number of columns in the principal composition matrix of quarter $q+1$ to retain to explain 70% of the cross variation with the time-integrated principal composition matrix from quarter $q-4$ to quarter q. The blue line shows the number of columns in the principal network composition matrix. The red line shows the number of columns in the principal basket composition matrix. A smaller S indicates that the principal composition matrix is more persistent. The construction steps refer to Appendix B. The sample spans from 1984Q2 to 2019Q4.

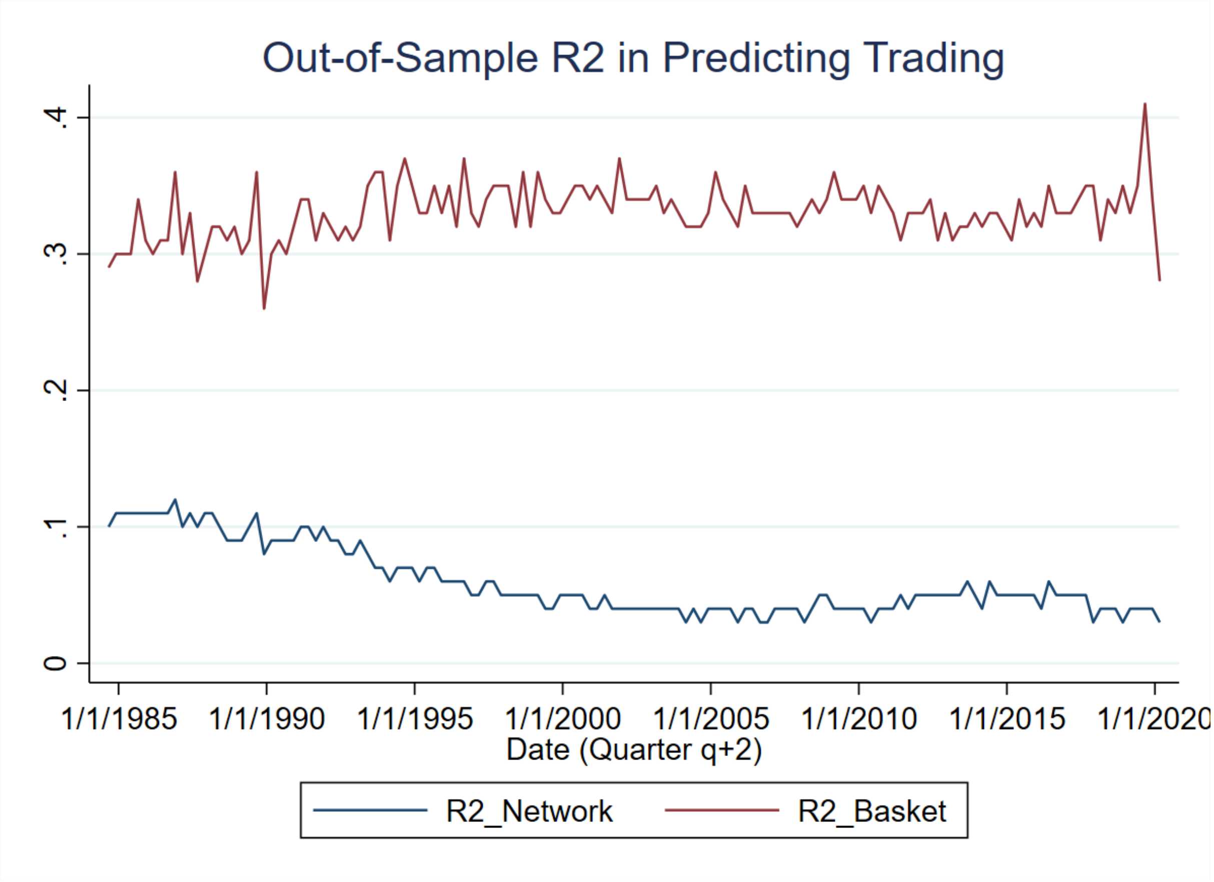

Figure 2.7: **Time Series of the Out-of-Sample R-Squared in Quarter q+2.**

This figure shows the time series of the R-squared obtained from the regressions in equations (2.32) and (2.33). The red line shows the R-squared from the asset perspective, and the blue line shows the R-squared from the manager perspective. In each perspective, we regress the actual trading on the predicted principal trading matrix and report the uniformly weighted average of the R-squared. The construction steps refer to Appendix B. The sample spans from 1984Q3 to 2020Q1.

Chapter 3

Monetary Policy and the Profitability Premium

3.1 Introduction

Recent studies have uncovered evidence suggesting a potentially long-lasting impact of monetary policy on asset valuation. This challenges the traditional belief that the effects of monetary policy on asset valuation and other economic activities are only short-lived. These findings highlight the growing significance of monetary policy in the financial market. However, the question arises: what could be the underlying explanations for this phenomenon?

To answer this question, [11] investigate the joint switching process between asset valuations and the conduct of monetary policy. They argue that the conduct of monetary policy can affect asset valuations through the channel of sticky expectations from the household. They find that the primary reason why announcements lead to lasting effects on the real interest rate and the equity premium is due to households underreacting to central bank announcements regarding inflation targets. Conventional monetary policy shocks, which are changes in the nominal interest rate not associated with inflation or output, fail to account for the enduring influence of monetary policy on asset valuation. Additionally, the aspects of households' underreaction to adjustments in the inflation target align with the regime-switching rules of monetary policy. [11] explain the existence of long-term monetary non-neutrality that is absent in the prior theoretical framework.

The impact of monetary policy is also likely to vary among firms with different characteristics, an area that has relatively limited research coverage. Building on these strands of research, especially from Bianchi et al. [11], I investigate the impact of monetary policy announcements on stock returns associated with firms with different profitability and how these effects differ across hawkish and dovish monetary policy regimes. According to [19], firm owners with different levels of profitability may have different levels of responsiveness to monetary policy announcements, which may create a joint switching process between the profitability premium and the conduct of monetary policy. In particular, more profitable firms may be more sensitive to central bank information shocks, as they can update their expectations about future economic fundamentals more quickly, allowing them to make better production or employment decisions and earn higher profits. As a result, the stock returns may be more responsive to the aggregate risk. This difference in responsiveness could create a profitability premium between more and less profitable firms, which may vary depending on the stance of monetary policy.

Specifically, in hawkish regimes when the central bank focuses more on controlling inflation than promoting economic growth, firm owners may pay closer attention to monetary policy announcements when there is high inflation, leading to higher expected returns for more profitable firms. The Federal Reserve adjusts the nominal interest rate in response to inflation, but the natural rate of interest is beyond its control. Consequently, the difference between the real interest rate and the natural rate of interest serves as a gauge of the monetary policy stance. A hawkish regime arises when the real interest rate surpasses the natural rate of interest, prompting the Fed to raise interest rates to lower inflation. Conversely, a dovish regime arises when the opposite scenario plays out.

This research helps to shed light on the effect of monetary policy on different firms, a field that is still in its early stages.

I uncover a persistent relationship between the profitability premium, the valuation of the long-short profitability portfolio, and monetary policy regimes. This discovery supports my belief that monetary policy impacts the discount rate that influences the profitability premium. In my study, the first hawkish regime emerged at the end of 1980 when Paul Volcker took over as Chair of the Federal Reserve. Following 1980, the real interest rate declined, and people became less concerned with inflation-related news. Additionally, I observe that the declining profitability premium after 1980 is tied to movements in the real interest rate and people's sticky expectations toward the macroeconomic fundamentals.

The primary empirical analysis in this version of the study comprises three key segments. The first part employs a Markov-switching model to illustrate the correlation between the profitability premium, valuation, and monetary policy regimes. Monetary policy is regime-switching since each Chair of the Federal Reserve has a distinct stance on the conduct of monetary policy that seldom changes. To further validate this approach, the break test proposed by Bai and Perron [3] reveals that the breakpoints in the valuation for the long-short profitability portfolio overlap with regime-switching dates. My findings indicate that hawkish regimes with high monetary policy correspond to a low valuation of the long-short profitability portfolio and a high profitability premium, while the opposite scenario plays out in dovish regimes. These results lend credence to my belief that the conduct of monetary policy has a bearing on the valuation at the cross-sectional level.

In the second segment of the study, I aim to establish a link between the profitability premium and the concept of sticky expectations, drawing inspiration from the works of Bouchaud et al. [13] and Bianchi et al. [11]. If agents' expectations are rational, they will adjust their expectations immediately following the announcement of a new inflation target by the Federal Reserve. Consequently, inflation and the real interest rate may not be persistent and may not align with the dynamics of the profitability premium. To test the sticky expectation hypothesis, I gauge the degree of information stickiness and inflation expectation volatility. I find that if people have stickier inflation expectations, the profitability premium tends to be lower. Additionally, I observe a decline in the real interest rate and profitability premium since 1980, which is consistent with the emerging literature indicating that low real interest rates reduce the risk premium. In a related study, Kekre and Lenel [28] established that expansionary monetary policy reduces the equity premium since more risk-tolerant individuals hold wealth. My research reveals that the profitability premium has also remained low in the last decade, aligning with the downward trend in the real interest rate.

In the final part of this study, I employ a mediation analysis to investigate the causal effects of Paul Volcker's appointment as Chair of the Federal Reserve on the valuation ratio of the long-short operating profitability portfolio and the profitability premium. Following his appointment, Volcker implemented several policies aimed at increasing interest rates to reduce inflation. I have selected this event as a prominent example of hawkish monetary policy. The two different channels are compared, i.e., the impact on agents' inflation expectations and changes in the real interest rate. The findings indicate that both channels have a significant causal impact on the valuation ratio and the profitability premium. However, the causal effect of affecting agents' inflation expectations is more robust.

The rest of this chapter is outlined as follows. Section 3.2 discusses the data sources, including the firm characteristics, monetary policy instruments, and inflation expectations. Section 3.3 presents empirical results on the monetary policy and profitability premium. Section 3.4 concludes this chapter.

3.2 Data

This section describes the data source. They are the profitability premium, expectations of inflation, and monetary policy instruments. The sample is from 1963:7 to 2020:12.

- **Profitability premium and valuation:**

The data of firm characteristics and portfolio returns are available from the 'Open Source Asset Pricing' data repository maintained by Andrew Chen and Tom Zimmermann. [1] The P/E ratio is the 6-month lagged market value divided by income before extraordinary items (IB). The P/E (B/M) ratio for each leg is the value-weighted P/E (B/M) of firms within that portfolio, and the P/E of the long-short portfolio is the log of long-leg P/E (B/M) minus the log of short-leg P/E. I winsorize the P/E at the 1st and 99th percentiles before computing the portfolio P/E, and the P/E series is available from the 'Open Source Asset Pricing' data repository. I construct the annual book value with the technique of Lochstoer and Tetlock [35], then I follow Asness and Frazzini [1] to merge it with the monthly market value to construct the monthly firm-level B/M.

The Ball et al. [7] operating profitability is (REVT – COGS – XSGA + XRD) / AT. REVT is the total revenue. COGS is the cost of goods sold. XSGA is the selling and general administrative expenses. XRD is the R&D expenses, and AT is total assets. I exclude negative observations during the construction. Among the measures of profitability, I select operating profitability for two reasons. First, according to Ball et al. [6], if we have the consistent denominator to compute the profitability, then the gross profitability and net income have identical predictive power for the cross-section of stock return. Further, the operating profitability explains the cross-section of stock returns better than the other two measures. Second, Morlacco and Zeke [39] find that monetary policy shocks significantly affect XSGA, which is an important part of operating profitability and can explain the cross-sections of stock returns (see Eisfeldt and Papanikolaou [22]).

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[1] https://www.openassetpricing.com/data/. The relevant paper refers to [15].

- **Expectations of inflation:**

The quarterly data are available from Survey of Professional Forecasters (SPF) in the website of the Federal Reserves Bank of Philadelphia. [2] I obtain the mean responses of CPI annualized inflation that starts in 1981Q3, and the specification of expectation updating follows that in Coibion and Gorodnichenko [17],

$$F_t \pi_{t+h,t} = (1 - \lambda_\pi) E_t \pi_{t+h,t} + \lambda_\pi F_{t-1} \pi_{t+h,t} \tag{3.1}$$

π_t is the inflation rate in quarter t. $F_t \pi_{t+h,t}$ is the subjective expectation towards the h-quarter ahead inflation. E is the objective expectation. $\lambda_\pi \in [0, 1]$ is the probability of not acquiring new information in each period, interpreted as the degree of information stickiness. Coibion and Gorodnichenko [17] provides the following handy specification to map the forecast error to forecast revisions, [3]

$$\pi_{t+1,t} - F_t \pi_{t+1,t} = c + \beta (F_t \pi_{t+1,t} - F_{t-1} \pi_{t+1,t}) + \epsilon_t \tag{3.2}$$

$\pi_{t+1,t}$ is the inflation rate in quarter t. $F_t \pi_{t+1,t}$ is the quarter-t one-quarter ahead survey expectations. $F_{t-1} \pi_{t+1,t}$ is the survey done in quarter $t-1$, and $\lambda_\pi = \frac{\beta}{1+\beta}$ is the degree of information stickiness.

I run OLS to estimate equation (3.2) from 1981Q3 to 2020Q4 to obtain $\hat{\beta} \approx 1.11$ (s.e. $= 0.09$), which is close to the estimate from Coibion and Gorodnichenko [17]. $\hat{\lambda}_\pi = \frac{1.11}{2.11} = 0.53$ suggests that agents update their information sets every one or two months on average for the quarterly forecasts, which is a significant magnitude in terms of the forecasting horizon.

From equation (3.2), I construct the quarterly time series of λ_π with the rolling regression. After I

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[2] https://www.philadelphiafed.org/surveys-and-data/cpi-spf

[3] The full-information rational expectations is

$$E_t \pi_{t+1,t} = \pi_{t+1,t} + \nu_{t+1,t}$$

in which $\nu_{t+1,t}$ is the full-information rational expectations error and is uncorrelated with information dated t or earlier. By plugging $E_t \pi_{t+1,t}$ in equation (3.1) and make arrangements, we have equation (3.2).

merge λ_π with the monthly data, I fill in missing values with the previous observation.

- **Monetary policy instruments:**

I use the real federal funds effective rate and the natural rate of interest (r^*) to compute the monetary policy spread. The data is from the Federal Reserve Bank of New York. [4] The monetary policy spread is

$$\text{mps}_t = \text{FFR}_t - \pi_t - r_t^*$$

FFR_t is the federal funds effective rate in month t, π_t is the 12-month rolling average annualized inflation in month t, and r^* is available at the quarterly frequency. I compute π_t according to Bianchi et al. [11] to proxy for the inflation expectation. Similar to the quarterly time-series of λ_π, I fill in missing values of the previous r^* after merging with the monthly observations.

The monetary policy spread measures the monetary policy stance based on monetary policy rules (Laubach and Williams [31]). r^* is the level of long-run real interest rate at which the output reaches its maximum potential level and unemployment reaches its minimum potential level. If the real interest rate is above its natural level, then inflation is above the central bank's desired target. Hence, the central bank needs to raise the nominal interest rate to contract the money supply and dampen economic activity, and the monetary policy becomes hawkish. The opposite scenario happens in a dovish regime.

I also estimate the volatility of the inflation expectation by the GARCH(1,1) specification. By Reis [41] and Coibion and Gorodnichenko [17], people pay more attention to inflation news if inflation is more volatile. Also, inflation highly correlates with inflation expectations. Therefore, the inflation expectation volatility is an inverse proxy for the degree of information stickiness toward inflation.

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[4] https://www.newyorkfed.org/research/policy/rstar, I take the one-sided, filtered estimate.

3.3 Empirical Results

3.3.1 Monetary Policy and Valuation of the Long-Short Portfolio

Table 3.1 displays the returns, CAPM-α, and beta of the monetary policy spread. Table 3.1 indicates the existence of the profitability premium that loads on the monetary policy spread. The results in Panel A indicate the presence of the operating profitability premium, with the long-short portfolio generating an average annualized return of 4.6% and a standard error of 1.99%. In Panel B, the portfolio's outperformance over the CAPM is highlighted, with an annualized CAPM-α of 7.13% and a standard error of 1.87%. Finally, Panel C demonstrates that portfolios with higher profitability have a higher beta of the monetary policy spread, with the long-short portfolio exhibiting a beta of 2.37. These findings suggest that market risk alone cannot account for the profitability premium and that monetary policy may play a role in explaining it.

To further demonstrate my thought, Figure 3.1 documents a robust relationship between the long-short operating profitability portfolio valuation and the monetary policy spread. The figure finds two patterns. First, the monetary policy spread and valuation ratio seem to co-move in a regime-switching pattern. When the monetary policy spread is high, the valuation is low. Second, the valuation ratio starts to co-move in the opposite direction after 1980: There is a downward trend in the interest rate during an upward trend in the valuation.

Table 3.1 and Figure 3.1 indicate that monetary policy spread and valuation move together. Motivated by these results, I invoke the Markov-switching model to estimate the mean of mps with the following specification,

$$\text{mps}_t = \mu_{s,t} + \epsilon_{s,t} \tag{3.3}$$

In equation (3.3), $s = \{H, D\}$ are two states that stand for hawkish/dovish regimes, $\mu_{s,t}$ is the mean of mps, and $\epsilon \sim \mathcal{N}(0, \sigma_{s,t}^2)$ is a zero-mean error term. I assume that both the mean and variance of the residual are regime-switching.

Figure 3.2 depicts the smoothed probability of the hawkish regime, while Table 3.2 explicitly shows the periods of hawkish regimes. A regime is classified as hawkish if the smoothed probability for a hawkish regime reaches 0.5. The first estimated hawkish regime begins in October 1980, two months after Paul Volcker takes over as Chair of the Federal Reserve. During this regime, Volcker actively raised the federal funds rate to combat the 1970s Great Inflation. From 1980 to 2000, the only dovish monetary policy occurred from September 1990 to May 1994. This regime started about six months after the early-1990 recession, caused by the late-1980 interest rate hikes and the 1990 oil-price shock, when Alan Greenspan became the Chair. The policy then returned to a hawkish stance in mid-1994 when the Fed decided to increase the interest rate. After the onset of the eight-month recession in March 2001, the estimated monetary policy regime is dovish, except for the following four one-year periods:

a) September 2006 – September 2007, when Ben Bernanke, the Chair of the Federal Reserve at that time, finds sizable inflationary pressure and decides to keep the target for the federal funds rate at 5.25%. [5]

b) December 2008 – October 2009, the financial crisis and the zero lower bound (ZLB) period begins. I interpret this period as a ”weakly” dovish regime for three reasons. First, the Fed initiates the large-scale asset purchase program in agency debts, mortgage-backed securities, and long-term treasury bonds in November 2008. They aim to put downward pressure on the long-term interest rate to boost the recovery from the Great Recession. [6] Second, the monetary policy spread in my data is positive between April 2009 and September 2009, peaking at 1.05% in July 2009. Besides, the average monetary policy spread during this period is 0.06%. Finally, Figure 3.1 indicates that the natural rate of interest drops significantly from

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[5] The March 2007 FOMC statement mentions that ”In these circumstances, the Committee’s predominant policy concern remains the risk that inflation will fail to moderate as expected. Future policy adjustments will depend on the evolution of the outlook for both inflation and economic growth, as implied by incoming information.”. See the following link for details, https://www.federalreserve.gov/newsevents/pressreleases/monetary20070321a.htm.

[6] https://www.federalreserve.gov/newsevents/pressreleases/monetary20081125b.htm.

2.07% in April 2008 to 0.64% in March 2009 and recovers to 1.05% in July. However, the short-term real interest rate increases from -1.56% to 2.11% in the same period, which increases faster than the natural rate of interest. This phenomenon may lead to the seemingly estimated hawkish regime, and it arises because of the speedy recovery from the Great Recession that stimulates economic activities.

c) December 2014 – December 2015. As in period b), this period is also more precisely the "weakly" dovish regime than a hawkish regime. The monetary policy spread in my sample is 0.031% in January 2015 and 0.047% in February 2015, while negative for the other months. The monetary policy stance has been accommodative since late 2009, especially when Janet Yellen becomes the Chair in 2014.

d) November 2018 – October 2019. The monetary policy spread in this period is negative, but the estimated regime is hawkish. The Fed raises the interest rate 4 times after Jerome Powell becomes the Chair in February 2018, and he maintains in the December 2018 FOMC statement that future increases in the interest rate are appropriate. Thus, the market believes that the monetary policy stance will no longer be accommodative.[7]

In Panel A of Table 3.2, I replicate the regime sequence estimated by Bianchi et al. [11], but my sample period extends until December 2020, which leads to some differences in estimation results. For example, I capture the hawkish monetary policy statement made by Jerome Powell in late 2018 and early 2019. Additionally, I estimate a dovish monetary policy regime in the early 1990s when there was a mild recession and the monetary policy spread was lower than in the 1980s.

Panel B in Table 3.2 suggests that both hawkish and dovish monetary policy regimes are highly persistent. The probability of staying in either regime is close to unity, reflecting the fixed stance of

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[7] The December 2018 FOMC statement mentions that "The Committee judges that some further gradual increases in the target range for the federal funds rate will be consistent with sustained expansion of economic activity, strong labor market conditions, and inflation near the Committee's symmetric 2 percent objective over the medium term.". See the following link for details, https://www.federalreserve.gov/newsevents/pressreleases/monetary20181219a.htm.

each Chair of the Federal Reserve. To more rigorously model a common regime-switching pattern between monetary policy spread and valuation, as shown in Figure 3.1, I estimate the breakpoints of the monetary policy spread and the P/E of the long-short operating profitability portfolio using the Bai and Perron [3] method, and I present the results in Appendix C.

After I show the joint regime-switching pattern, I examine the evolution of the valuation ratios across monetary policy regimes, and the results are in Table 3.3. I assume that P/E and M/B follow the Markov-switching process that I estimate by equation (3.3). The results show that the hawkish (dovish) regimes are associated with a low (high) valuation in the long-short operating profitability portfolio across all percentiles. To examine if the mean of the monetary policy spread and valuation significantly differ across two regimes, I invoke the t-test, and the results of the t-test in Panel C strengthen my findings in Panels A and B.

I also plot the relationship between the monetary policy regimes and the P/E ratio of the long-short operating profitability portfolio in Figure 3.3. The figure shows that hawkish regimes correspond to low valuation for the long-short profitability portfolio. This pattern resembles the market return documented in Bianchi et al. [11], but I show that monetary policy also has long-lived effects at the cross-sectional level. Finally, I examine the long and short sides of the operating profitability portfolio in Appendix C, and the results largely emerge from the short side (i.e., low-profitability stocks) in dovish regimes.

Figure 3.3 also shows a notable trend of decreasing monetary policy spread and increasing valuation ratio since the 1980s, which is the early period of the first hawkish regime. This trend is consistent with recent studies such as Hanson and Stein [25] and Nakamura and Steinsson [40], which argue that monetary policy can also affect long-term interest rates, contradicting earlier comments made by Ben Bernanke. [8] The declining trend in the monetary policy spread and the increasing valuation

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[8] In the speech ”Why are interest rates so low? ”Ben Bernanke argues that ”The Fed’s ability to affect real rates of return, especially longer-term real rates, is transitory and limited. Except in the short run, real interest rates are determined by a wide range of economic factors, including prospects for economic growth—not by the Fed. ” See https://www.brookings.edu/blog/ben-bernanke/2015/03/30/why-are-interest-rates-so-low/.

ratio since the 1980s can be attributed to multiple factors. First, the declining inflation expectations and the natural rate of interest have reduced the short-term interest rate (Bauer and Rudebusch [10]). Second, the tightening stance on monetary policy since the 1980s has led to a prolonged decline in the real interest rate, contributing to an increase in the valuation ratio. These findings are consistent with recent literature that has argued that monetary policy can impact the long-term interest rate, contrary to earlier beliefs (Hanson and Stein [25]; Nakamura and Steinsson [40]). Furthermore, while Bianchi et al. [11] have discovered that monetary policy regimes lead to a persistent increasing trend in the consumption-wealth ratio since the 1980s, my results suggest that a similar pattern also emerges for the P/E ratio at the cross-sectional level.

3.3.2 The Profitability Premium

So far, I have provided evidence that the conduct of monetary policy affects the profitability and discount rate of the long-short operating profitability portfolio. This subsection further examines how the profitability premium evolves across the monetary policy regime. The formula of Vuolteenaho [46] implies that the value spread of the long-short portfolio is a proxy for the risk premium:

$$\sum_{j=0}^{\infty} \rho^j E\left[r_{t+1+j}\right] = \theta_L - \theta_S + \sum_{j=0}^{\infty} \rho^j E\left[e_{L,t+1+j} - e_{S,t+1+j}\right] \tag{3.4}$$

In Equation (3.4), r_{t+1+j} is the log return on the long-short operating profitability portfolio, $\theta_L - \theta_S$ is the value spread between the long and short legs of the operating profitability portfolio, and $e_{L,t+1+j} - e_{S,t+1+j}$ is the difference between the log E/P ratio of the long and short legs of the operating profitability portfolio. I choose $\rho = 0.95$ to compute the discounted profitability premium by summing the value spread and the discounted sum of the future expected earning difference. I then take the exponential on the calculated annual profitability premium.

In Figure 3.4, the evolution of the profitability premium in the data is presented, and the summary statistics of the profitability premium are shown in Table 3.4. The average annual profitability premium over the full sample period is 5.1%, which is close to the annual average return of 4.6% on the long-short portfolio reported in Table 3.1. There are three noteworthy time-series patterns in the present discounted profitability premium that are associated with monetary policy. First, the profitability premium is higher in hawkish regimes than in dovish regimes, with a difference in the mean of 6.8% annually. This finding supports the similar pattern observed in the market level documented in Bianchi et al. [11], indicating that the conduct of monetary policy has a long-lasting effect not just on the valuation or discount rate, but also on the profitability premium. Second, a secular downward trend in the profitability premium is observed after 1980, the first hawkish monetary policy regime, as shown in Panel B. The decline in the profitability premium is caused by the prolonged decline in both the short-term and long-term interest rates due to monetary policy. This decline leads to an increase in the valuation ratio and a decrease in the discount rate, as depicted in Figure 3.3 and Table 3.3. Finally, the profitability premium is persistent for both hawkish and dovish regimes.[9]

In conclusion, the regime-switching monetary policy stance has significant impacts on the operating profitability, valuation of the long-short operating profitability portfolio, and the operating profitability premium. Additionally, a counterfactual analysis in Appendix C separates the sample by expansionary and recessionary periods and shows that the results presented in this section do not hold during these periods. This finding suggests that the observed effects do not come from fundamental fluctuations.

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[9] There may be some concerns about spurious regressions due to the unit root. To address this problem, I have run the Dickey-Fuller test on the monetary policy spread, the P/E ratio, and the profitability premium, and the p-values are 0.0085, 0.0374, and 0.0002, respectively. The results indicate that the unit root is not a problem in this paper.

3.3.3 Sticky Expectations Toward Inflation

In this subsection, I argue that the findings of Bianchi et al. [11] regarding the role of sticky expectations in the equity premium also apply to the long-short operating profitability portfolio. Persistent departures from full-information rational expectations are necessary for monetary policy regimes to have a lasting impact on asset valuation. If agents are slow to update their beliefs about inflation, their expectations will persist, and this will lead to persistently high or low actual inflation rates [38]. The sluggishness in inflation expectations means that inflation will not immediately adjust to the new target set by the central bank to stabilize the economy. As a result, with persistent movements in inflation expectations and changes in the short-term nominal interest rate by the central bank, the real interest rate will also persist. Hawkish monetary policy has a persistent positive (negative) effect on the profitability premium (P/E) due to the persistent movement in the real interest rate.

However, how does the argument of sticky expectations apply to the cross-sectional level? An answer is the rational inattention, as proposed by Sims [44]. As inflation is not a significant driver of individual stock returns, investors may not frequently update their discount rate based on inflation. Consequently, stock returns may be sluggishly reactive to changes in inflation (Katz et al. [27]). Moreover, from the firm owners' perspective, even firms that are more informed about inflation forecasts may earn only slightly higher profits than uninformed firms. Thus, they may rationally choose not to pay much attention to inflation information (Coibion et al. [19]). As a result, when inflation or interest rates impact profit, the cash flow or cost of firms that fail to immediately incorporate the information generates persistent profits or earnings momentum, as argued by Bouchaud et al. [13].

With the rational inattention from the households or firms, the central bank's communications also often fail to let the inflation reach the target level. Particularly, past successful monetary policies, such as the Volcker disinflation in the 1980s, have resulted in stable, low, and tranquil inflation, leading to widespread inattention to monetary policy. According to a survey conducted in April 2018, 55% of US manufacturing and services firms reported that they did not know their point forecasts for inflation over the next twelve months, as reported by Coibion et al. [18].

Based on the two pieces of evidence presented, it appears that sticky information about inflation may weaken the profitability premium. When inflation is low and tranquil, firms may not pay close attention to news on inflation, resulting in an increased degree of information stickiness towards inflation among firm owners. As Coibion et al. [19] found that firms with more information on inflation can gain a higher profit, there is an information asymmetry between the two groups of firms. Profitable firms may demand lower compensation over time, resulting in a decrease in the profitability premium.

In order to investigate the relationship between information stickiness towards inflation and the profitability premium, I conducted a regression analysis. Specifically, I regressed the P/E ratio of the long-short operating profitability portfolio and the profitability premium on the degree of information stickiness towards inflation. The results are presented in Table 3.5.

In Column 1, I found that the P/E of the long-short operating profitability slightly increases as firm owners become more sticky toward inflation. However, when adding the monetary policy spread as a control variable in Column 2, the relationship between the degree of information stickiness and P/E becomes insignificant. As in Section 3.3, Column 2 also shows a negative relationship between the monetary policy spread and P/E.

To further investigate the impact of information stickiness on the profitability premium, Columns 3 and 4 confirm that a higher degree of information stickiness towards inflation drives down the profitability premium.

I also examined the relationship between inflation volatility and information stickiness in Columns 5 and 6. As expected, Column 5 shows that if the standard deviation of inflation expectation increases by 1%, then the probability that a person fails to receive inflation information immediately reduces by 1.2%. Column 6 compares the information stickiness toward inflation between the hawkish and dovish regimes. It reveals that people have less sticky expectations of inflation in a hawkish regime because hawkish regimes indicate a higher inflationary pressure. I confirm this thought by showing that the probability that a firm owner fails to receive inflation news immediately is 3% lower in a hawkish regime than in a dovish regime.

The result of this study is related to Bouchaud et al. [13], who propose that the profitability premium exists because of analysts' sticky expectations of future profits. However, there are two key differences between their work and mine. First, I find that the degree of information stickiness towards variables that differ in firm dynamics also affects the profitability premium. Second, I find a negative time-series relationship between the degree of information stickiness towards inflation and the profitability premium, whereas Bouchaud et al. [13] find a positive cross-sectional relationship between the profitability premium and the degree of information stickiness towards profitability.

In addition to persistent signals about future profits, such as those discussed by Bouchaud et al. [13], I argue that monetary policy regimes are also a persistent signal. Suppose the Fed raises the short-term interest rate to meet its inflation target in a hawkish regime, but firm owners' expectations towards inflation are sticky. In that case, they will not immediately adjust their expectations. This means that the Fed can persistently affect the real interest rate, as sticky expectations towards inflation generate persistent inflation and lead to a persistent real interest rate. As Bauer and Rudebusch [10] have discussed, the equilibrium dynamics between long-term trends and the short-term interest rate, such a persistent relationship should hold in equilibrium. The persistent path of the real interest rate, therefore, triggers the persistent movement of output and costs.

3.3.4 Relations to the Declining Real Interest Rate

Figure 3.1 depicts a clear downward trend of the real interest rate after 1980, which may also indicate the impact of monetary policy regimes on the profitability premium. This finding is consistent with recent research highlighting the significant influence of monetary policy on long-term real interest rates, as evidenced in studies by Hanson and Stein [25], Nakamura and Steinsson [40], and Bianchi et al. [11]. However, the declining real interest rate could also result from secular stagnation (Summers [45]) or decreasing productivity growth (Gordon [24]), which may lead to a lack of investment opportunities and lower demand for capital. It is worth considering whether monetary policy affects productivity growth by altering the real interest rate.

3.3.5 Mediation Analysis: Which Effect is Stronger?

In this subsection, I aim to examine the causal effect of monetary policy regimes on the valuation ratio and the profitability premium by considering the mediation of agents' inflation expectations and fluctuations in real interest rates, which were discussed in the previous two subsections. The results presented so far have only focused on the relationship between the valuation ratio and monetary policy regimes. Therefore, in Table 3.6, I present the results of the mediation analysis to shed light on this causal relationship.

In the first two columns of the table, I investigate the effects of the appointment of Paul Volcker as the Chair of the Federal Reserve on the valuation ratio and the profitability premium by considering the effect of altering agents' inflation expectations. Consistent with the previous findings, the results suggest that a switch from a dovish to a hawkish monetary policy regime causes a drop in the P/E ratio of the long-short operating profitability portfolio and an increase in the profitability premium. Similarly, in the last two columns, I examine the effects of hawkish regimes on the valuation ratio and the profitability premium through the effect of altering the real interest rate, and I observe a similar pattern.

Comparing the effects of switching to hawkish regimes on the P/E ratio through two different channels (columns 1 and 3 in Table 3.6), I argue that the channel through affecting agents' inflation expectations is stronger than through the real interest rate. Similarly, by comparing the effects of hawkish regimes on the profitability premium through different channels (columns 2 and 4), I find that the channel through agents' inflation expectations is also stronger.

Overall, the mediation analysis provides further evidence to support the idea that monetary policy regimes affect the valuation ratio and the profitability premium through the mediation of agents' inflation expectations. The results also support the important role that inflation expectations play in conducting monetary policy.

3.4 Conclusion

I find a connection between the valuation of the long-short profitability portfolio, the profitability premium, and monetary policy stances. This finding supports recent evidence of long-lived monetary non-neutrality and reveals a potential source of the time-varying premium. This paper also contributes to the literature on expectation-based business cycles by arguing the roles of different levels of responsiveness to monetary policy announcements can play in generating the premium. In particular, the profitability premium negatively relates to the degree of information stickiness toward inflation. Finally, I explore how monetary policy relates to the declining real interest rate associated with an overall profitability decline across various industries, affecting the profitability premium.

Table 3.1: **Summary Statistics of the Operating Profitability Portfolios**

Panel A shows the annualized time-series average of the portfolios sorted on operating profitability proposed by [7]. Panel B displays the annualized CAPM-alpha for each portfolio. Panel C presents the monetary policy beta of each portfolio. The monetary policy spread is the nominal interest rate minus the 12-month rolling average of the inflation rate and the natural rate of interest. A higher monetary policy spread indicates a more hawkish monetary policy stance. I multiply the monthly return by 12 to obtain the annualized return. In Panel C, the dependent variable of the regression is the return on profitability portfolios, and the independent variable is the monetary policy spread. The sample spans the period of 1963:7 - 2020:12.

Panel A: Return	Q1 (low)	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10 (high)	10-1
	7.94**	9.47***	10.78***	11.83***	11.33***	10.95***	11.53***	13.05***	12.33***	12.54***	4.60**
	(3.16)	(2.70)	(2.44)	(2.38)	(2.19)	(2.10)	(2.15)	(2.19)	(2.12)	(2.19)	(1.99)
Panel B: CAPM-α	Q1 (low)	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10 (high)	10-1
	-1.30	1.22	3.36***	4.59***	4.63***	4.56***	4.81***	6.16***	5.63***	5.83***	7.13***
	(1.56)	(1.11)	(1.00)	(1.00)	(0.87)	(0.88)	(0.78)	(0.73)	(0.70)	(0.89)	(1.87)
Panel C: Beta	Q1 (low)	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10 (high)	10-1
mps	-1.36**	-0.81*	-0.40	0.65*	-0.27	0.052	0.94***	0.015	0.18	1.01***	2.37***
	(0.62)	(0.44)	(0.41)	(0.38)	(0.35)	(0.35)	(0.33)	(0.31)	(0.30)	(0.36)	(0.79)
Mkt-RF	1.36***	1.21***	1.09***	1.06***	0.98***	0.94***	0.98***	1.01***	0.98***	0.98***	-0.37***
	(0.032)	(0.025)	(0.025)	(0.028)	(0.019)	(0.021)	(0.017)	(0.016)	(0.017)	(0.019)	(0.039)
cons	-3.48*	-0.079	2.71**	5.63***	4.20***	4.64***	6.33***	6.18***	5.92***	7.45***	10.9***
	(1.88)	(1.31)	(1.18)	(1.10)	(1.02)	(1.12)	(0.93)	(0.87)	(0.89)	(1.06)	(2.19)
Observations	690	690	690	690	690	690	690	690	690	690	690
R-squared	0.77	0.84	0.83	0.83	0.84	0.83	0.87	0.89	0.89	0.84	0.16

Standard errors in parentheses. * p<.10, ** p<.05, *** p<.01

Table 3.2: **Estimated Sequence of Monetary Policy Regimes**

This table reports the estimated monetary policy regimes by estimating the regime-switching model of equation (3.1) on the monetary policy spread. I assume that the mean and variance of the monetary policy spread are regime-switching. The sample spans the period of 1963:7 – 2020:12.

Panel A: Period and Most Likely Regime	
1963.7 - 1980.10	Dovish
1980.11 - 1990.8	Hawkish
1990.9 - 1994.4	Dovish
1994.5 - 2001.2	Hawkish
2001.3 - 2006.8	Dovish
2006.9 - 2007.9	Hawkish
2007.10 - 2008.11	Dovish
2008.12 - 2009.10	Hawkish
2009.11 - 2014.11	Dovish
2014.12 - 2015.12	Hawkish
2016.1 - 2018.10	Dovish
2018.11 - 2019.10	Hawkish
2019.11 - 2020.12	Dovish
Panel B: Regime Transition Probabilities	
p_{HH}	0.99
p_{DD}	0.97

Table 3.3: **Monetary Policy and Valuation across Estimated Regimes**

This table reports the summary statistics of the monetary policy spread and the valuation ratio of the long-short operating profitability portfolio. mps is the monetary policy spread, which is the real federal funds rate minus the natural rate of interest. Positive (negative) monetary policy spread refers to the hawkish (dovish) regime. P/E is the logarithm of the P/E for the long-leg minus that for the short-leg of the operating profitability portfolio. M/B is the logarithm of the M/B of the operating profitability portfolio. Panel A reports the summary statistics of the hawkish regime estimated by the Markov-switching model on mps. Panel B reports the summary statistics for the dovish regime. I assume that the valuation ratio is subject to the same regime as mps. Finally, Panel C tests if the average of each variable is higher or lower in the hawkish regime than in the dovish regime. The sample spans the period of 1963:7 – 2020:12.

Panel A: Hawkish	N	Mean	Std	Min	P10	P50	P90	Max
mps	249	0.82	1.32	−1.13	−0.59	0.57	2.57	5.90
P/E	249	−1.27	0.41	−2.09	−1.72	−1.32	−0.79	0.08
M/B	249	0.50	0.13	0.12	0.34	0.48	0.66	0.82
Panel B: Dovish	N	Mean	Std	Min	P10	P50	P90	Max
mps	441	−2.98	1.41	−9.27	−4.72	−2.68	−1.51	0.14
P/E	441	−0.76	0.36	−1.70	−1.14	−0.81	−0.27	0.39
M/B	441	0.78	0.29	0.25	0.47	0.73	1.17	1.72
Panel C: t-stats								
$\text{mps}_H - \text{mps}_D$	35.41							
$\text{P/E}_H - \text{P/E}_D$	−16.30							
$\text{M/B}_H - \text{M/B}_D$	−17.71							

Table 3.4: **The Profitability Premium**

Panel A shows the summary statistics of the profitability premium in the hawkish and dovish regimes. t(Hawk - Dove) tests if the mean profitability premium in the hawkish regime is higher than in the dovish regime. Panel B shows a comparison before and after 1980 by regressing the profitability premium on the linear time trend. Coef is the slope coefficient. t-stat is the t-statistics. R^2 is the R-squared, and 1st ACF is the first-order autocorrelation coefficient. The profitability premium is the sum of the value spread and the discounted sum of the E/P ratio of the long-short operating profitability portfolio, following the formula in [46], and taking the exponential. The sample spans the period of 1963:7 - 2020:12.

Panel A	N	Mean	Std	Min	P10	P50	P90	Max	1st ACF
Full	690	5.11	5.53	0.21	0.89	3.27	11.40	41.39	0.97
Hawkish	249	9.46	6.87	0.66	3.35	7.84	18.31	41.39	0.95
Dovish	441	2.65	2.12	0.21	0.63	2.23	4.83	14.86	0.97
t(Hawk - Dove)	15.23								
Panel B	N	Coef	t-stat	R^2					
Post-1980	492	-16.11	-12.30	0.30					
Pre-1980	198	9.14	22.54	0.61					

Table 3.5: **Profitability Premium and the Degree of Information Stickiness**

This table presents the regression taking the degree of information stickiness towards inflation estimated with the [17] method as the main explanatory variable, which I denote as λ_π. The dependent variable in Columns 1 and 2 is the P/E of the long-short operating profitability portfolio. The dependent variable from Columns 3 to 4 is the profitability premium. The dependent variable from Columns 5 to 6 is λ_π. Voleinf in Column 5 is inflation expectation volatility estimated by GARCH (1,1), and hawk in Column 6 is the indicator that equals 1 if the estimated monetary policy regime is hawkish. The sample spans the period of 1982:4 - 2020:12.

	P/E	P/E	premium	premium	λ_π	λ_π
λ_π	0.31*	0.21	-9.26***	-7.91***		
	(0.17)	(0.14)	(2.55)	(2.15)		
mps		-0.13***		1.74***		
		(0.01)		(0.13)		
Voleinf					-0.012*	
					(0.007)	
hawk						-0.03***
						(0.011)
cons	-1.19***	-1.26***	10.96***	11.83***	0.49***	0.49***
	(0.09)	(0.07)	(1.38)	(1.21)	(0.01)	(0.01)
Observations	465	465	465	465	465	465
R-squared	0.01	0.28	0.03	0.32	0.01	0.01

Standard errors in parentheses. * p<.10, ** p<.05, *** p<.01

Table 3.6: **Mediation Analysis: The Effect of Switching to Hawkish Regimes**

This table presents the results of the mediation analysis on the impact of transitioning to hawkish monetary policy regimes on the P/E ratio of the long-short operating profitability portfolio and the profitability premium. The table reports the estimates of two mediation variables: inflation expectations (einf) in the first two columns and the real interest rate (r_r) in the last two columns. The analysis covers the period from July 1963 to December 2020.

Panel A: Estimated Coefficients	P/E	premia	P/E	premia
Hawk	-0.57***	1.39***	-0.18***	0.73***
	(0.05)	(0.10)	(0.05)	(0.10)
einf	-0.04***	0.06		
	(0.006)	(0.01)		
Hawk × einf	0.008	-0.02		
	(0.01)	(0.02)		
r_r			0.03***	-0.13***
			(0.01)	(0.02)
Hawk × r_r			-0.12***	0.29***
			(0.02)	(0.03)
cons	-0.60***	0.45***	-0.77***	0.71***
	(0.03)	(0.06)	(0.02)	(0.03)
obs	690	690	690	690
R^2	0.35	0.42	0.36	0.46
Panel B: Estimated Effects				
Controlled Direct Effect	-0.56***	1.38***	-0.31***	1.02***
	(0.04)	(0.09)	(0.04)	(0.09)
Natural Direct Effect	-0.53***	1.33***	-0.19***	0.75***
	(0.03)	(0.06)	(0.05)	(0.10)
Natural Indirect Effect	0.02	-0.04	-0.32***	0.54***
	(0.01)	(0.02)	(0.04)	(0.09)
Marginal Total Effect	-0.51***	1.30***	-0.51***	1.30***
	(0.03)	(0.06)	(0.03)	(0.06)

Standard errors in parentheses. * p<.10, ** p<.05, *** p<.01

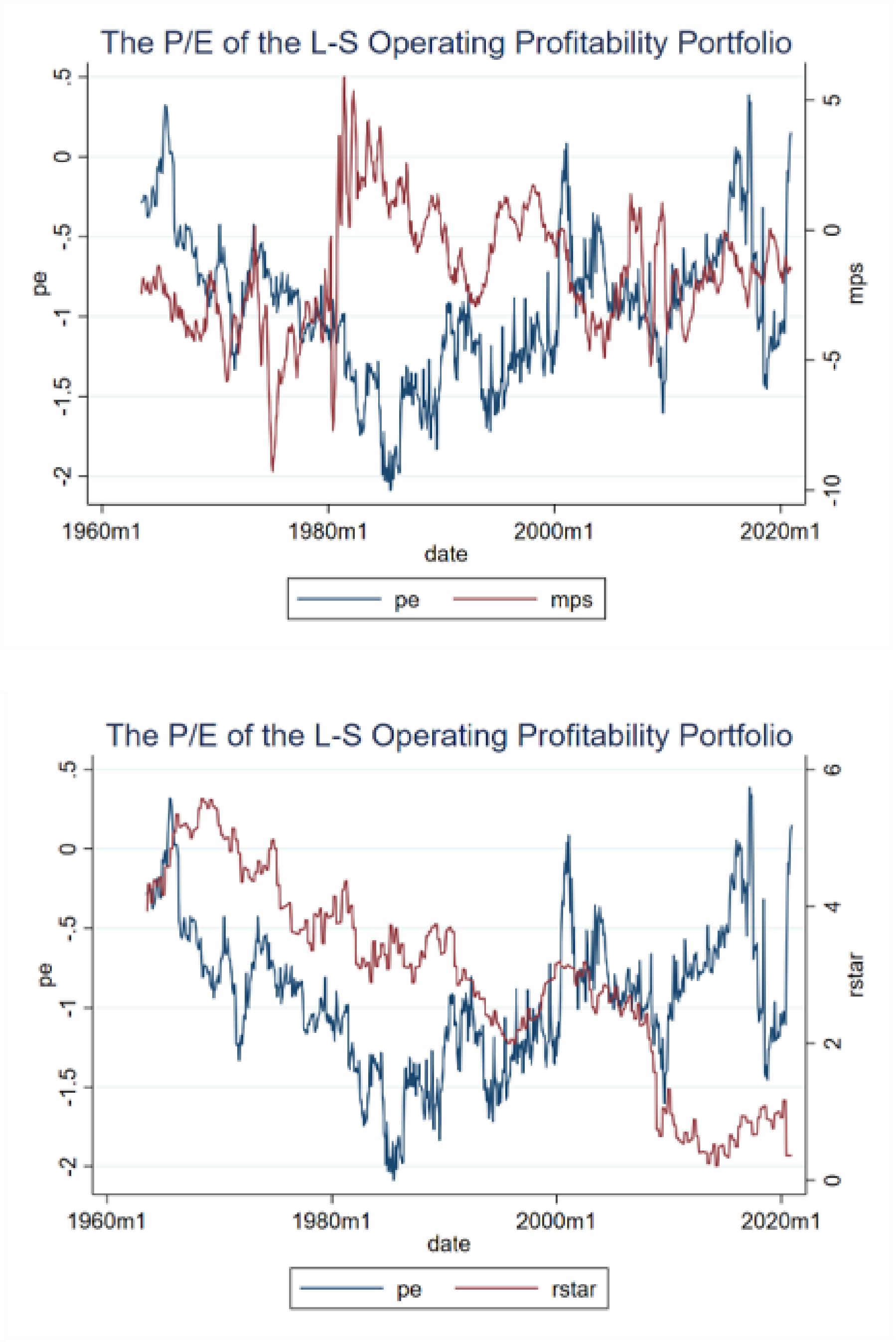

Figure 3.1: **Monetary Policy and Valuation**

This figure shows the P/E ratio of the long-short portfolio constructed from the Ball et al. [7] operating profitability, monetary policy spread, and the natural rate of interest. The sample spans the period of 1963:7 – 2020:12.

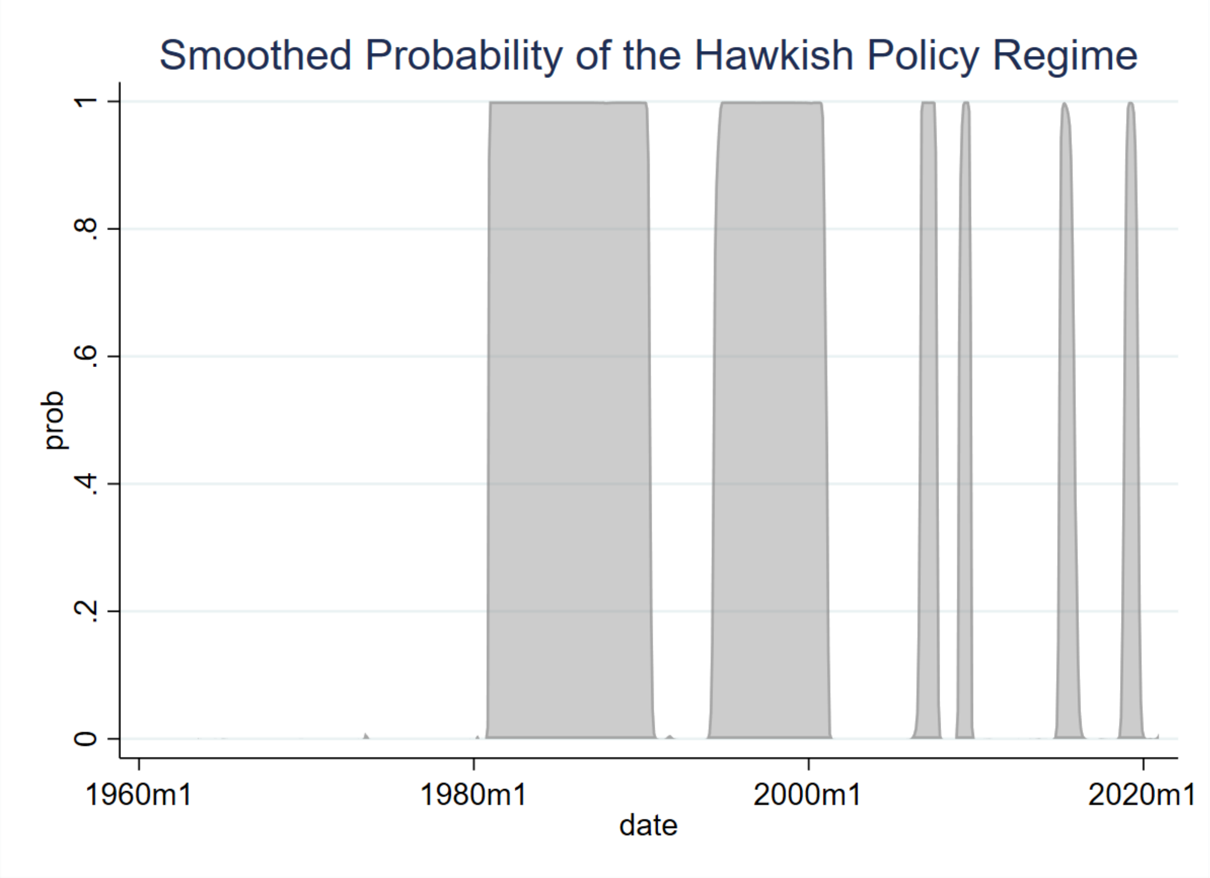

Figure 3.2: **The Probability of the Hawkish Monetary Policy Regime.**

This figure shows the smoothed probability for the hawkish monetary policy regime. The sample spans the period of 1963:7 – 2020:12.

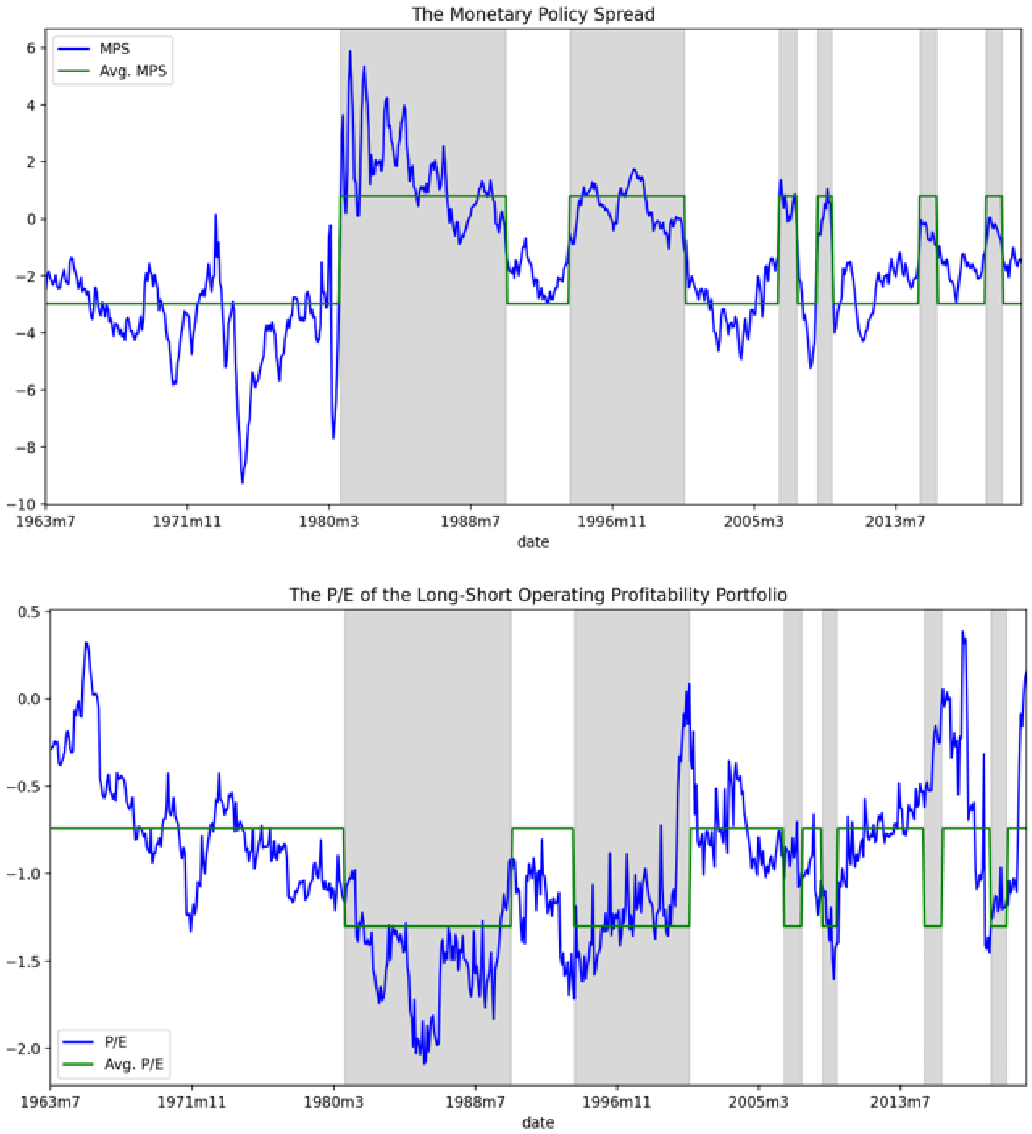

Figure 3.3: **Regime-Average Monetary Policy Spread and P/E.**

The blue lines are the monetary policy spread and the P/E of the long-short operating portfolio. The green lines are the regime-average monetary policy spread and the P/E. The monetary policy regime is hawkish if the smoothed probability of a hawkish regime exceeds 0.5, which I indicate as the gray shaded area. The P/E of the long-short operating portfolio is the logarithm of the P/E for the long-leg minus that for the short-leg. The profitability premium is the return on the long-short operating profitability portfolio. The sample spans the period of 1963:7 – 2020:12.

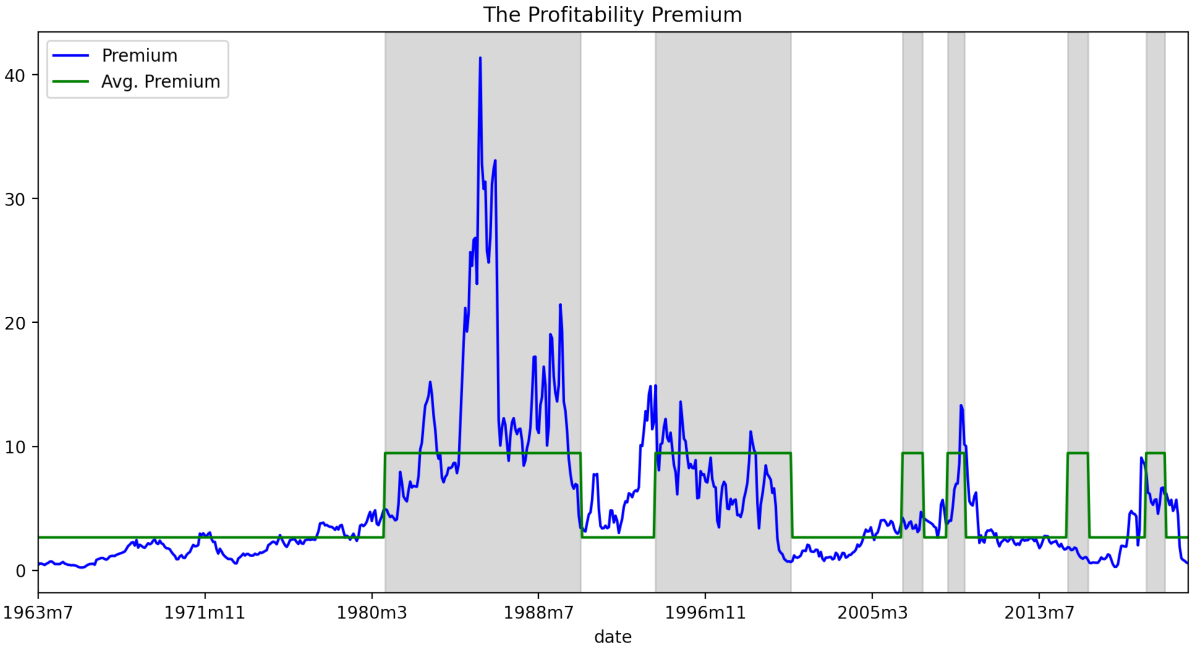

Figure 3.4: **Evolution of the Profitability Premium.**

This figure shows the evolution of the profitability premium in the data. I take Vuolteenaho [46] to proxy by the sum of the value spread and the E/P ratio of the long-short operating profitability portfolio, then take the exponential. The gray shaded area indicated the estimated hawkish monetary policy regimes. The sample spans the period of 1963:7 – 2020:12.

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Appendices

Appendix A

Data Cleaning Process for Chapter 2

We generate RelPTrade_{ijq} in the following steps.

1. To calculate the split-adjusted shares traded by manager j, we first remove observations with missing or zero beginning-of-period prices from CRSP and observations with missing or zero beginning-of-period shares outstanding. Note that the first quarter of trade data for fund manager j occurs with the second quarter of holdings data.

2. We determine RelPTrade_{ijq} by dividing the number of split-adjusted shares traded by the beginning-of-period total shares outstanding in quarter q. To make it easier to interpret, we express this as a percentage by multiplying it by 100.

3. We winzorize trading at the 1st and 99th percentiles of RelPTrade_{ijq}, ranked across all stocks, managers, and quarters.

4. For each quarter q, we rank managers based on their average dollar trading volume over the preceding 12 quarters. Thus, a manager must have been in the 13F dataset for at least three years. Dollar trading volume is shares traded times the average of the end-of-quarter and beginning-of-quarter split-adjusted stock prices. We keep manager j in quarter q if they rank

above the 25th percentile.

5. Likewise, we rank stocks based on their beginning-of-quarter q market capitalization and keep only those above the 25th percentile.

6. We eliminate any trading observation with absolute values greater than 0.2.

Table A1 presents the summary statistics of RelPTrade_{ikq}, where i, k, and q denote a stock, a fund manager, and a quarter, respectively. The table shows that after the seven steps of data cleaning, the unadjusted measure of RelPTrade_{ikq} exhibits fat tails and high kurtosis, indicating the presence of outliers. However, the tails become thinner after truncation or winsorization of the measure. Additionally, the table suggests that the distribution of RelPTrade_{ijq} is almost symmetric, indicating that buyers and sellers are well-matched in the sample. Finally, the summary statistics reveal that institutional trading is highly sparse, with institutions trading only 0.0003% of the total number of shares outstanding.

We also note that the growth rate of institutional trading appears to be relatively low, as indicated by the summary statistics of $(\text{RelPTrade}_{ijq} + 1)$. On average, the growth rate of RelPTrade_{ijq} from quarter $q - 1$ to quarter q is approximately -0.02% in the unadjusted sample and close to 0 in the adjusted sample. Furthermore, the growth rate does not exceed 2.6% in the unadjusted sample or 0.22% in the adjusted sample. These findings suggest that the structure of institutional trading does not change significantly over time, as the growth rate in institutional trading is relatively low.

Figure A1 displays two histograms of the truncated RelPTrade_{ijq} for two periods of 74 quarters, illustrating our argument that the distribution of institutional trading does not undergo huge changes over time. As we show in the figure, approximately 60% of the RelPTrade_{ijq} observations fall within ± 0.008, indicating a highly sparse institutional trading structure.

Appendix B

CCA Construction Details for Chapter 2

This appendix describes how we use CCA to conduct the intertemporal analysis in Section 2.3.4. We describe the process to conduct the CCA in the manager perspective, then briefly repeat the process for the asset perspective.

1. Let $\breve{M}_q$ denote an augmented number of managers assigned to quarter q equal to the union of those appearing in quarters $s = q - 4$ to q. We augment the columns of each T_s matrix to include all $\breve{M}_q$ managers, with zeros replacing missing trading data,[1] then stack vertically to create the manager-history trading matrix $H_q^{(\mathrm{m})}$ with dimensions $(A_{q-4} + \cdots + A_q) \times \breve{M}_q$. Construct the $\breve{M}_q \times \breve{M}_q$ covariance matrix $H_q^{(\mathrm{m})'} H_q^{(\mathrm{m})}$ and diagonalize to obtain the $\breve{M}_q \times \breve{M}_q$ matrix of historical principal network compositions denoted $N_q^{(\mathrm{h})}$ (superscript indicates 'historical'). Let P_q denote the number of columns of $N_q^{(\mathrm{h})}$ needed to explain 70% of the uncentered variance in $H_q^{(\mathrm{m})'} H_q^{(\mathrm{m})}$. Trim to just these columns so that 'trimmed' $\hat{N}_q^{(\mathrm{h})}$ is now dimension $\breve{M}_q \times P_q$.

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[1] By 'augment' we mean the following. Suppose that there are 5 managers appearing in the union of quarter q and $q+1$, managers A, B, C, D, and E. T_q has 3 columns; manager A, D, and E. T_{q+1} has four columns: manager B, C, D, and E. The augmented matrix for BOTH T_q and T_{q+1} must have five columns, A, B, C, D, and E. That is, we insert two columns of zeros in T_q, between the existing first and second columns; and one column of zeros before the existing first column of T_{q+1}. Thus, column k in each quarter always corresponds to the same manager.

2. There are M_{q+1} rows in N_{q+1} (the matrix of quarter $q+1$ principal network compositions. To ensure compatibility for CCA, augment the rows of both $\hat{N}_q^{(\mathrm{h})}$ and N_{q+1} to accommodate the union of managers appearing in both matrices so that $\breve{M}_{q+1}$ denotes the resulting number of rows after this second augmentation.

3. Perform CCA between $\hat{N}_q^{(\mathrm{h})}$ and N_{q+1}. For each canonical variate (CV) pair k, retain the canonical correlation ρ_k and the two corresponding canonical variates; denoted U_k (constructed from columns of $\hat{N}_q^{(\mathrm{h})}$) and V_k (constructed from columns of N_{q+1}). Let $\sigma_{k,q}$ and $\sigma_{k,q+1}$ denote the Euclidean norms (uncentered standard deviations) of U_k and V_k, respectively.

4. Define the canonical covariance of the k'th CV-pair as

$$CanCov_{k,q} = \rho_k \sigma_{k,q} \sigma_{k,q+1}. \tag{B1}$$

   This measure takes into account both persistence and explained variance, both historical and in the forecast quarter. Rank CV pairs from large to small according to $CanCov$ and choose the first $S^{(m)}$ such that we capture 70% of $\sum_{k=1}^{P_q} CanCov_{k,q}$. Now use $\{V_k\}$, $k=1$ to $S^{(m)}$ to form an $\breve{M}_{q+1} \times S^{(m)}$ 'out-of-sample principal network' composition matrix, N_{q+1}^{oos}. Augment the rows for a third time to conform to managers in $q+2$ – denote the result $\breve{M}_{q+2}$.

5. Use the $\breve{M}_{q+2} \times S^{(m)}$ matrix N_{q+1}^{oos} to predict trading in quarter $q+2$. More specifically, regress each column of T_{q+2} on each of the first $S^{(m)}$ columns of $T_{q+2}N_{q+1}^{oos}$, and report the equal-weighted R-squared as our measure of out-of-sample predictability.

We briefly outline parallel construction steps taken under the asset perspective.

1. Let $\breve{A}_q$ be the dimension of the union of all assets appearing in quarter q. Augment the rows of each $\mathbf{T}_s$, $s = q-4$ to q to include the union of $\breve{A}_q$ relevant assets. Stack horizontally to

create the historical trading matrix $H_q^{(\mathrm{a})}$ with dimensions $\breve{A}_q \times (M_{q-4} + \cdots + M_q)$ and the $\breve{A}_q \times \breve{A}_q$ covariance matrix $H_q^{(\mathrm{a})} H_q^{(\mathrm{a})'}$. Denote the historical principal basket compositions from this covariance matrix $\mathbf{B}_q^{(\mathrm{h})}$, and trim to 70% explanatory power.

2. Then perform CCA on $\mathbf{B}_q^{(\mathrm{h})}$ and $\mathbf{B}_{q+1}$ (the matrix of quarter $q+1$ principal basket compositions) and keep the largest $S^{(a)}$ pairs to yield 70% of the total canonical covariance. Form the out-of-sample principal basket trading matrix $\mathbf{B}_{q+1}^{oos}$ from the rotated basket composition.

3. Finally, we regress T'_{q+2} on $T'_{q+2} N_{q+1}^{\mathrm{oos}}$ and report the equal-weighted average R-squared.

Table A1: **The Summary Statistics of the Trading Measure**

This table presents various measures of trading activity in our data-cleaning process. Each row corresponds to a different measure applied to a particular combination of stock, fund manager, and quarter denoted by i, j, and q, respectively. The RelPTrade measure represents the proportion of shares traded by a fund manager relative to the total number of shares outstanding, with a value of 1 indicating that the fund manager trades 1% of shares outstanding. Row 1 shows the measure obtained from the first 5 steps described in A. In Row 2, we further adjust Row 1 by excluding observations with absolute values greater than 0.2, the version used in our main analysis. Row 3 is an additional adjustment that applies a winsorization procedure to Row 1, capping values at ±0.2. Rows 4, 5, and 6 apply a natural logarithm transformation to Rows 1, 2, and 3, respectively, and add 1 to the values to accommodate cases with zero trading activity. The sample spans from 1983Q1 to 2020Q1.

	N	Mean	std	skew	ex.kurt	min	p1	median	p99	max
RelPTrade_{ijq}	35,183,328	0.002	0.17	0.59	22.8	−0.93	−0.72	0	0.77	1.04
RelPTrade_{ijq}, Truncated at ±0.2	32,452,921	0.0003	0.04	0.05	11.3	−0.20	−0.15	0	0.15	0.20
RelPTrade_{ijq}, Winsorized at ±0.2	35,183,328	0.001	0.07	0.04	7.2	−0.20	−0.20	0	0.20	0.20
$\ln(\text{RelPTrade}_{ijq}+1)$	35,183,328	−0.021	0.27	−6.80	63.3	−2.60	−1.28	0	0.57	0.71
$\ln(\text{RelPTrade}_{ijq}+1)$, Truncate RelPTrade_{ikq} at ±0.2	32,452,921	−0.001	0.04	−0.58	11.7	−0.22	−0.16	0	0.14	0.18
$\ln(\text{RelPTrade}_{ijq}+1)$, Winsorize RelPTrade_{ikq} at ±0.2	35,183,328	−0.002	0.07	−0.59	7.4	−0.22	−0.22	0	0.18	0.18

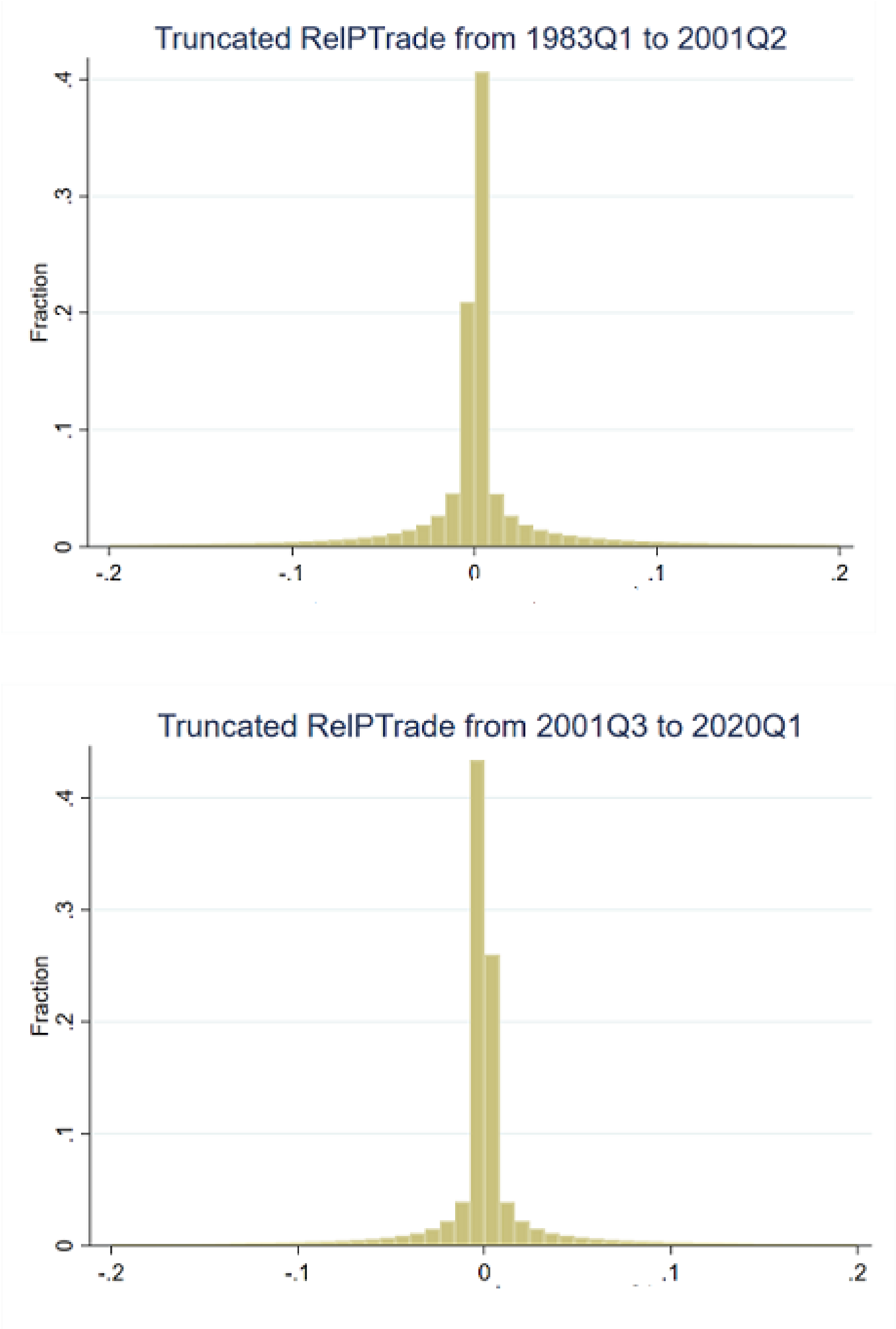

Figure A1: **Histogram of the Trading Measure in Our Analysis.**

This histogram displays the distribution of the truncated RelPTrade shown in Row 2 of Table A1. The histogram is divided into 50 bins that range from -0.2 to 0.2, with each bin having a width of 0.008. The vertical axis represents the fraction of the data within the range indicated by each bin. The histogram shows the results for two periods of 74 quarters.

Appendix C

Robustness Checks for Chapter 3

In the first robustness check, I demonstrate a joint regime-switching pattern between the valuation ratio and the monetary policy spread.

Panels A and B of Table C1 estimate the unknown breakpoints and assume that the maximum number of breakpoints is 4. Panel A shows that the breakpoints largely coincide with the dates in Table 3.2 that the monetary policy regime switches from hawkish to dovish or vice versa. This result confirms that a change in the conduct of monetary policy causes a structural shift in the monetary policy spread. I further test the breakpoints of the P/E ratio in Panel B and examine if the confidence interval overlaps with the confidence interval of the breakpoints of the monetary policy spread. It turns out that two estimated break points in the monetary policy spread are consistent with a breakpoint of the P/E ratio. One breakpoint is in November 1989, which largely corresponds to the end of the first hawkish regime, and another one is in March 2003, which is roughly the end of the second hawkish regime.

Panel C of Table C1 further tests if a monetary policy regime-switching date in Table 3.2 corresponds to a breakpoint in the P/E ratio with the Bai and Perron [3] technique. The null hypothesis is that the regime-switching date is not a breakpoint in the P/E ratio. The result shows that the only two estimated regime-switching dates that fail to reject the null at the 5% significance level are September 1990 and November 2019. However, the difference between the former estimated regime-switching date and the closest estimated breakpoint in the P/E ratio is only two months. The result confirms the argument that there is a joint regime-switching pattern between the monetary policy spread and the valuation of the long-short operating profitability portfolio.

The second robustness check focuses on the long and short sides of the Ball et al. [7] operating profitability portfolios in this section. I examine if the effect of monetary policy regimes on the valuation and profitability premium mainly comes from the long side or the short side.

The top panel in Figure C1 examines how the P/E ratio of the long side relates to the monetary policy spread and the short-term real interest rate. On the contrary, the bottom panel examines the P/E ratio of the short side. Loosely speaking, there are close relationships between the P/E of both sides and the monetary policy spread and short-term real interest rate before 1980. After 1980, such a relationship is stronger on the short side. Therefore, the results of the P/E ratio of the long-short portfolio are mainly from the short side.

I explicitly show the relationship between the valuation for each side of the operating profitability portfolio and the monetary policy in Table C2. Panel A shows that across the entire sample period, the P/E ratio for the short side is more sensitive to the conduct of monetary policy than that for the long side. Similar results hold for the real interest rate. This result may not be surprising, as the short side with lower profitability firms have a longer duration than the long side with higher profitability, and longer-duration stocks are more sensitive to interest rate movements.

However, if we analyze the results from the hawkish and dovish regimes, then the conclusion will be different. Panel B shows that in hawkish regimes, only the P/E for the long side is sensitive to the interest rate movements. On the other hand, Panel C shows that in dovish regimes, the P/E for the short side becomes sensitive to the real interest rate movements.

The final robustness check confirms that the fluctuations in the fundamentals do not drive my empirical results in Section 3.3. Figure C2 plots the hawkish and recessionary periods, which do not overlap very much. In addition, hawkish periods are longer than recessionary periods. Table C3 examines the monetary policy spread, valuation, and profitability premium across expansionary and recessionary periods. I show that my empirical results in Section 3.3 do not hold if I separate my sample based on expansionary and recessionary periods. I have three results in Table C3. First, the average monetary policy spread in expansionary and recessionary periods is negative. However, the average monetary policy spread in hawkish regimes is positive. Second, there is no significant difference in the P/E ratio between expansionary and recessionary periods, while the P/E ratio in hawkish regimes is significantly lower than in dovish regimes. Finally, although the average profitability premium is higher in expansionary periods according to the t-test, this result emerges from the maximum value. The profitability premium across most percentiles is, in fact, lower than in recessionary periods.

Table C1: **Estimated Break Points and Monetary Policy Regimes**

Panels A and B report the estimated breakpoints for the level estimate of the monetary policy spread and the P/E of the long-short operating profitability portfolio by the [3] technique. The trimming is 0.15, and I assume that the maximum number of breaks is 4. Overlap: PE in Panel A reports if the 95% confidence intervals of the break estimates for the monetary policy spread and the P/E ratio in Panel B overlap. Overlap: Regime in Panel A reports if the 95% confidence interval of the break estimate for the monetary policy spread overlaps for a regime-switching date estimated in Table 3.2. Finally, Panel C tests if the regime-switching dates estimated in Table 3.2 correspond to break dates of the P/E with the [2] technique, and the null hypothesis is no break. The sample spans the period of 1963:7 – 2020:12.

Panel A: MPS					
Break Point #	Estimate	95% CI		Overlap: PE	Overlap: Regime
1	1980.10	1980.9	1980.11	No	Yes
2	1989.11	1989.9	1990.1	Yes	No
3	2001.3	2001.1	2001.5	Yes	Yes
4	2012.5	2010.7	2014.3	No	No
Panel B: P/E					
Break Point #	Estimate	95% CI			
1	1972.1	1968.8	1975.6		
2	1981.6	1980.12	1981.12		
3	1990.1	1983.10	1996.4		
4	2000.4	1998.11	2001.9		
Panel C: P/E and Regimes					
Regime Dates	p-value				
1980.11	0.00				
1990.9	0.07				
1994.5	0.00				
2001.3	0.00				
2006.9	0.00				
2007.10	0.00				
2008.12	0.00				
2009.11	0.00				
2014.12	0.00				
2016.1	0.00				
2018.11	0.03				
2019.11	0.27				

Table C2: **The Long and Short Operating Profitability Portfolios**

This table presents the regression results. The dependent variable is the P/E for the long and short sides, the independent variables are the monetary policy spread (mps) and the short-term real interest rate (r_r). P/E Long is the P/E ratio for the long side. The sample spans the period of 1963:7 - 2020:12.

Panel A: Full Sample	P/E Long	P/E Long	P/E Short	P/E Short
mps	0.04***		0.13***	
	(0.008)		(0.007)	
r_r		-0.01**		0.06***
		(0.007)		(0.007)
cons	3.21***	3.17***	4.31***	4.01***
	(0.018)	(0.020)	(0.020)	(0.022)
Observations	690	690	690	690
R-squared	0.06	0.01	0.32	0.09
Panel B: Hawkish	P/E Long	P/E Long	P/E Short	P/E Short
mps	-0.13***		-0.03	
	(0.012)		(0.018)	
r_r		-0.11***		-0.01
		(0.008)		(0.014)
cons	3.32***	3.58***	4.51***	4.53***
	(0.020)	(0.033)	(0.029)	(0.058)
Observations	249	249	249	249
R-squared	0.27	0.38	0.01	0.01
Panel C: Dovish	P/E Long	P/E Long	P/E Short	P/E Short
mps	0.16***		0.13***	
	(0.01)		(0.01)	
r_r		-0.01		-0.05***
		(0.012)		(0.014)
cons	3.59***	3.11***	4.26***	3.88***
	(0.033)	(0.020)	(0.048)	(0.023)
Observations	441	441	441	441
R-squared	0.31	0.003	0.14	0.02

Standard errors in parentheses. * p<.10, ** p<.05, *** p<.01

Table C3: **Valuation across Expansionary and Recessionary Periods**

This table reports the summary statistics of. mps is the monetary policy spread, which is the real federal funds rate minus the natural rate of interest. Positive (negative) monetary policy spread refers to the hawkish (dovish) regime. P/E is the logarithm of the P/E for the long-leg minus that for the short-leg of the operating profitability portfolio. M/B is the logarithm of the M/B of the operating profitability portfolio. The profitability premium is the sum of the value spread and the discounted sum of the E/P ratio of the long-short operating profitability portfolio, following the formula in [46], and taking the exponential. The sample spans the period of 1963:7 – 2020:12.

Panel A: Expansions	N	Mean	Std	Min	P10	P50	P90	Max
mps	605	−1.52	2.12	−8.57	−3.99	−1.76	1.18	5.90
P/E	605	−0.94	0.47	−2.09	−1.52	−0.96	−0.29	0.39
M/B	605	0.66	0.29	0.12	0.38	0.58	1.08	1.72
premium	605	5.29	5.80	0.21	0.84	3.19	11.73	41.39
Panel B: Recessions	N	Mean	Std	Min	P10	P50	P90	Max
mps	85	−2.22	3.14	−9.27	−6.16	−2.41	1.58	5.35
P/E	85	−0.97	0.32	−1.74	−1.40	−0.93	−0.59	−0.19
M/B	85	0.81	0.21	0.41	0.56	0.76	1.12	1.22
premium	85	3.78	2.68	0.89	1.16	3.51	7.00	13.30
Panel C: t-stats								
$\text{mps}_E - \text{mps}_R$	1.99							
$\text{P/E}_E - \text{P/E}_R$	0.79							
$\text{M/B}_E - \text{M/B}_R$	−5.60							
$\text{premia}_E - \text{premia}_R$	4.04							
Panel D: Correlation	rec	hawk						
rec	1							
hawk	−0.06	1						

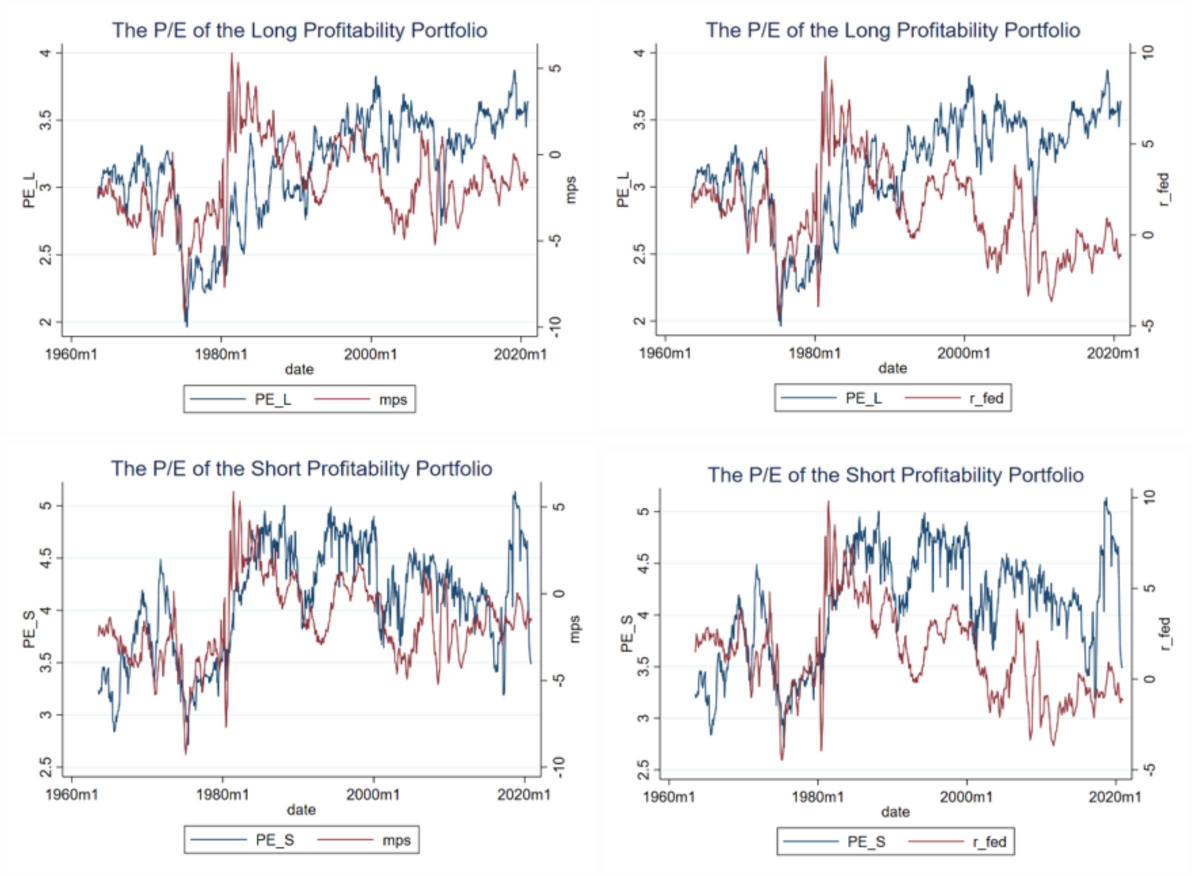

Figure C1: **The Valuation of the Long and Short Operating Profitability Portfolios.**

The top panel shows the relationship between the P/E of the long side of the operating profitability portfolio, the monetary policy spread, and the real interest rate. The bottom panel shows the short side. The P/E ratio of each portfolio is the natural log of the market-value-weighted average of the P/E ratio for each firm. The sample spans the period of 1963:7 – 2020:12.

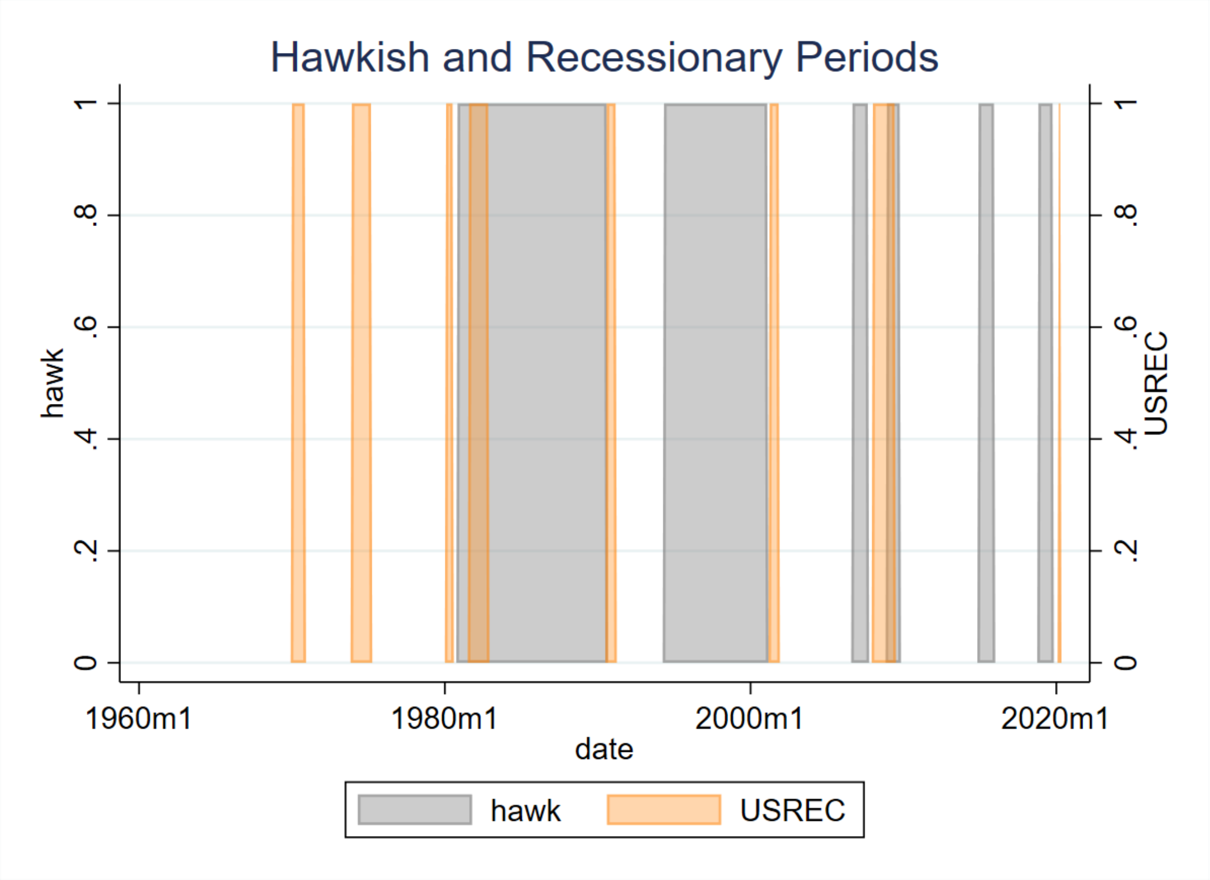

Figure C2: **Hawkish and Recessionary Periods.**

The figure shows the hawkish and recessionary periods. The sample spans the period of 1963:7 – 2020:12.