Signal Detection Optimization For Underwater Acoustics

by

Mark D. Adams

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APPROVED:

F.J. Ricci, Chairman

D. Schaefer

A. Craig

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Committee Chairman: Fred J. Ricci
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(ABSTRACT)

The Ocean Environment was explored to determine the effects of temperature, absorption and leakage on acoustic propagation. Having modeled the acoustic region, a cylindrical sonar is evaluated to determine its array gain as a function of frequency. The Detection Threshold is analyzed and its impact on signal detectability discussed in detail. Utilizing the Passive Sonar Equation, the sonar's effectiveness is quantitatively measured as a function of frequency for a typical noise source. Correlation techniques, modeled on an IBM PC, are then introduced to improve the sonar's effectiveness.

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1.0 Introduction

In the following report, many of the considerations necessary and techniques used to detect and optimize an underwater signal are analyzed and discussed.

The Ocean Environment will be explored to determine the effects of temperature, absorption and leakage on acoustic propagation. From the temperature dependency, the sound velocity profile will be calculated and graphed. The effects of absorption due to ionic relaxation of Magnesium and Boron-borate molecules will be quantified and introduced into the propagation model. Finally, ductal leakage will be identified and used to further refine the propagation loss calculations.

Having modeled the acoustic region, a cylindrical sonar will be evaluated to determine its array gain as a function of frequency. The threat signal level as well as the identification and analysis of ownship noise will be quantified and discussed. The Detection Threshold will be analyzed and its impact on signal detectability discussed in detail.

Having identified all of the necessary sonar elements, the Passive Sonar Equation will be utilized to measure the sonar's effectiveness as a function of frequency for a typical noise source. Finally, cross-correlation and auto-correlation techniques will be introduced to improve the sonar's effectiveness. These correlation techniques will be modeled on an IBM PC, with the resulting graphs displayed and discussed.

A third world nation has sponsored a terrorist group to launch several stolen submarine launched Tomahawk cruise missiles against the United States. To accomplish this task, a diesel submarine, bought from the French, has been provided to the terrorist group. Through the IUSS network the U.S. has identified the general area and speed of the threat submarine. A U.S. submarine is in the area and is ordered to intercept and destroy this threat platform.

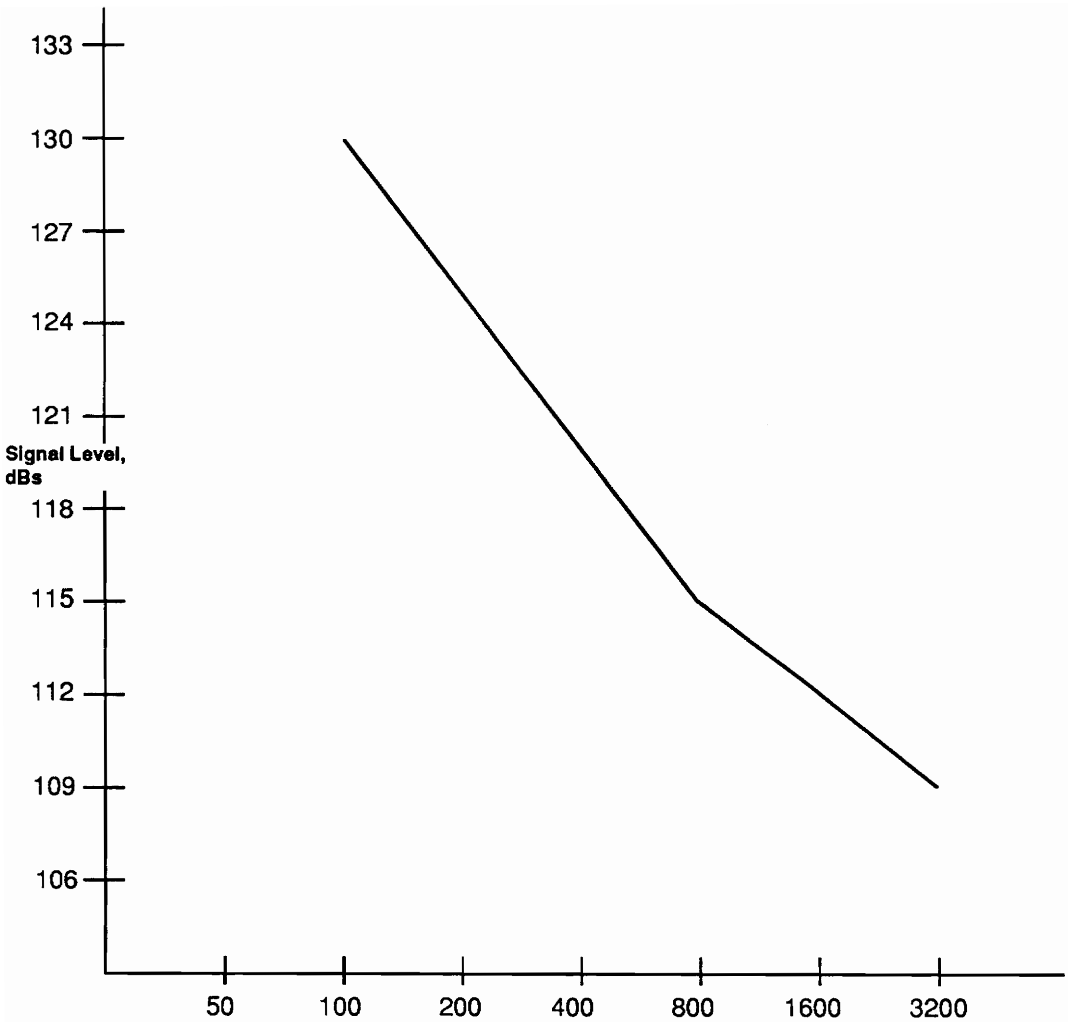
The U.S. platform is provided with the following data on the threat submarine:

* Structural condition of platform will constrain maximum operating depth to 500 feet.
* Platform generated noise (Signal Level, S_L, Figure 1, p. 3)
* Flank speed 25 knots

2.0 Acoustic Propagation

2.1 Sound Velocity Profile Expendable Bathythermographs (XBTs) are used to provide a detailed profile of the prevailing regional oceanic conditions. The XBTs provide information on the water temperature as a function of depth. Ocean salinity information, when added to this temperature vs. depth data, are used to provide an accurate sound velocity profile.

For water temperatures $28.4^{\circ}F<T<76.1^{\circ}F$, an accurate



Threat Noise "Signal" vs. Frequency

Figure 1

expression for the calculation of sound velocity (c) is the following: (Leroy, pp. 216)

$$c = 1492.9+3(T-10)-6*10^{-3}(T-10)^{2}-4*10^{-2}*(T-18)^{2}+1.2(S-35)-10^{-2}(T-18)(S-35)+D/200$$

where --D is in the depth in feet, S is salinity (in parts per thousand) and T is temperature in degrees Centigrade.

With the above expression, the sound velocity profile is calculated in 100 foot increments for the entire range of possible threat depths (Table 1, p. 5). These results are particularly significant when one analyzes the velocity profile and extracts the resulting ray tracings. A brief discussion on ray theory follows below.

Ray theory is an approximate solution to the fundamental wave equation. Although it neither provides a complete solution to the wave equation, nor handles diffraction, it is easier to use and to visualize. It illustrates, visually, the propagation path of a mono-phased acoustic wavefront. Several useful properties of ray acoustics will be exploited in the upcoming analysis.

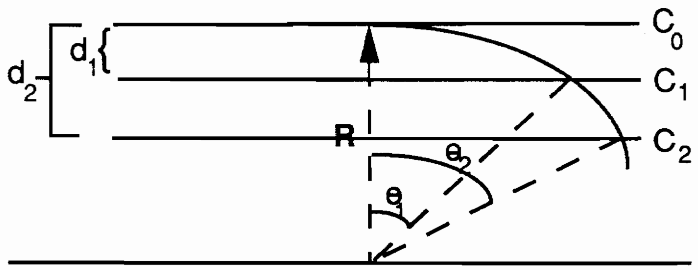
Snells law, which states that the cosine of the grazing angle at a boundary divided by the speed of the ray in the medium is a constant $[\frac{\cos\theta_0}{c_0} = \frac{\cos\theta_1}{c_1} = \frac{\cos\theta_2}{c_2}]$, is one such property.

From Table 1, it is apparent that the sound velocity changes linearly with the change in depth. As demonstrated

Table 1

Ocean Depth	Temperature, °F	Sound Velocity, ft/s
Surface	51.0	4903.0
100 ft	53.4	4912.4
200 ft	54.5	4921.8
300 ft	55.7	4931.2
400 ft	56.8	4940.6
500 ft	58.0	4950.0

Sound velocity profile for prevailing water conditions below, this allows the ray to be represented as an arc with a constant radius of curvature (Urick, pp. 124).



Constant ROC for sound velocity varying linearly with depth

From the above illustration, we note

$$d_1 - d_2 = R(\cos\theta_1 - \cos\theta_2)$$

since gradient ($\Delta c/\Delta z$) is linear

$$c_1 = c_o + gd_1$$
$$c_2 = c_o + gd_2$$

subtracting these we obtain

$$d_2 - d_1 = \frac{c_2 - c_1}{g}$$

Using snells law,

$$\cos\theta_1 = (c_1/c_o)\cos\theta_o = c_1/c_o, \text{ since } \cos\theta_o=1$$
$$\cos\theta_2 = c_2/c_o, \text{ where again } \cos\theta_o=1$$

Substituting above for $d_2 - d_1$, $\cos\theta_1$, and $\cos\theta_2$ we obtain

$$R = -c_o/g$$

Hence the radius of curvature is a constant for each specified source depth. With these tools, and the relationships below, the ray traces can be generated for the sound velocity profile in Table 1.

$$\Delta c/\Delta z = g = -.094 \text{ ft/s/ft}$$
$$R = -c_o/g$$

For distance traversed in the horizontal plane

$$Y = R\sin\theta_2 - R\sin\theta_1 = R(\theta_2 - \theta_1) \text{ for small } \theta.$$

For small θ, $\cos\theta = 1 + \theta^2/2$ which allows the distance traversed in the vertical plane to be expressed as

$$z = \frac{R(\theta_2^2 - \theta_1^2)}{2}$$

For the limiting ray $\theta_1 = 0$, therefore

$$z = R(\theta_2^2)/2$$

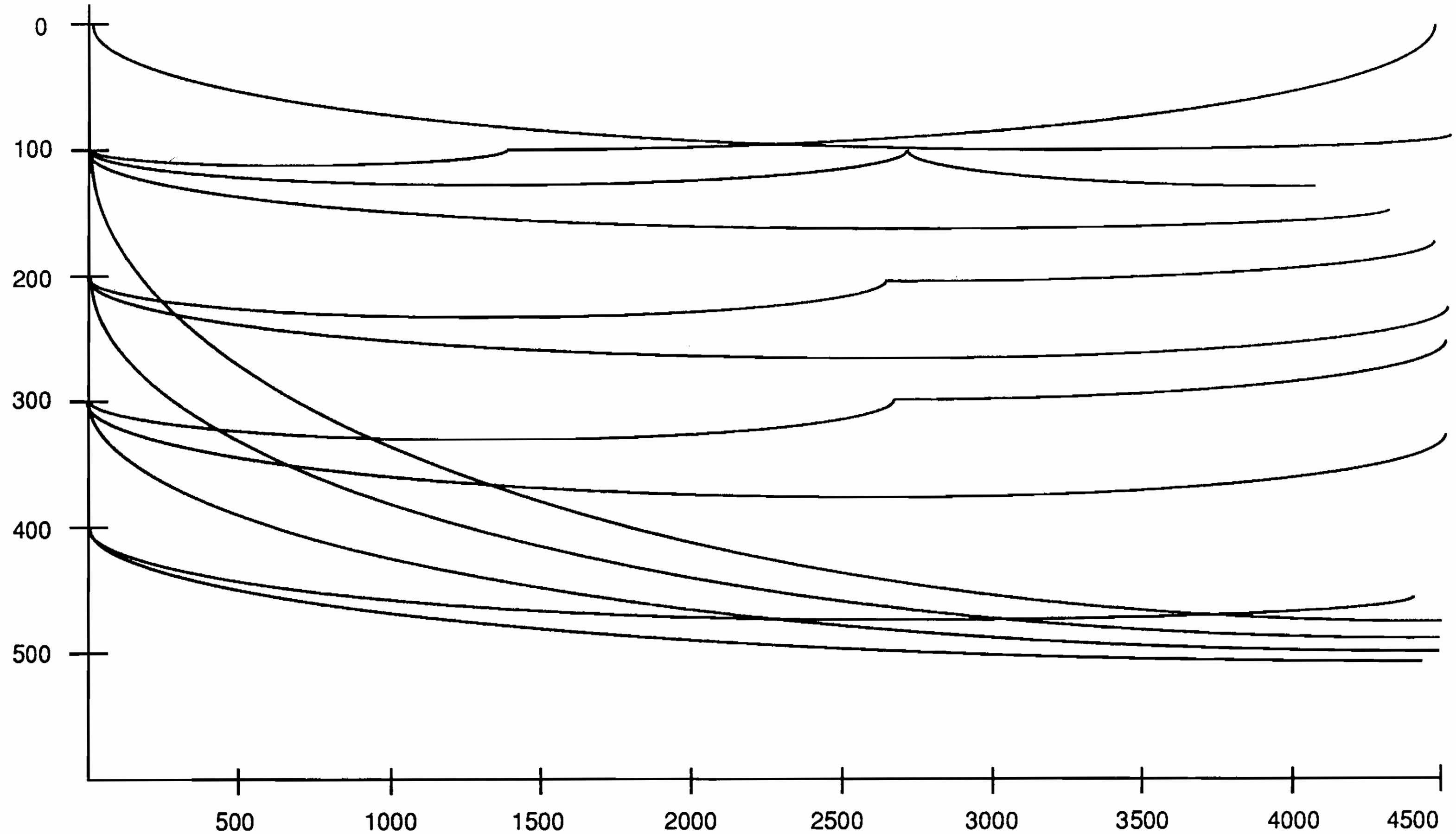
Table 2, p. 8, displays the various ray data which is plotted on Figure 2, p. 9, From Figure 2, it is apparent that a rather well defined ductal region exists from the surface to a depth of 500ft. This is of particular significance since the threat platform is constrained to a maximum depth of 500ft. Using the historical noise data on the threat platform combined with a fairly good picture of the region's acoustical profile, transmission loss can be calculated.

2.2 Propagation Loss Transmission loss is normally given as a ratio of the intensity of the transmitted signal at the source (often 1 meter from the source) and the intensity at the observer. Simple spherical spreading, for example, will consider the total power over the surface of a sphere surrounding the source of radius A_1 (over a region of area $4\pi A_1^2$) and compare this with the total power over a second larger sphere of radius A_2. Excluding absorption, which will be addressed momentarily, the total power passing

Table 2

Source Depth	θ_2(Rads)	ROC	Limiting Ray depth	Range Y
100ft	0.0125	52,255ft	104.1ft	653ft
	0.025	" "	116.3ft	1306ft
	0.050	" "	165.3ft	2613ft
	0.123	" "	495.3ft	6427ft
200ft	0.0125	52,351ft	204.9ft	654ft
	0.025	" "	216.4ft	1308ft
	0.050	" "	265.4ft	2618ft
	0.107	" "	499.7ft	5601ft
300ft	0.0125	52,457ft	304.1ft	655ft
	0.025	" "	316.4ft	1311ft
	0.050	" "	365.6ft	2623ft
	0.087	" "	498.5ft	4564ft
400ft	0.0125	52,553ft	404.1ft	657ft
	0.025	" "	416.4ft	1313ft
	0.050	" "	465.7ft	2628ft
	0.062	" "	501.0ft	3258ft

Ray Propagation Data as a function of source Depth



Ray Propagation vs. Source Depth

Figure 2

through these two spheres should be the same, therefore,

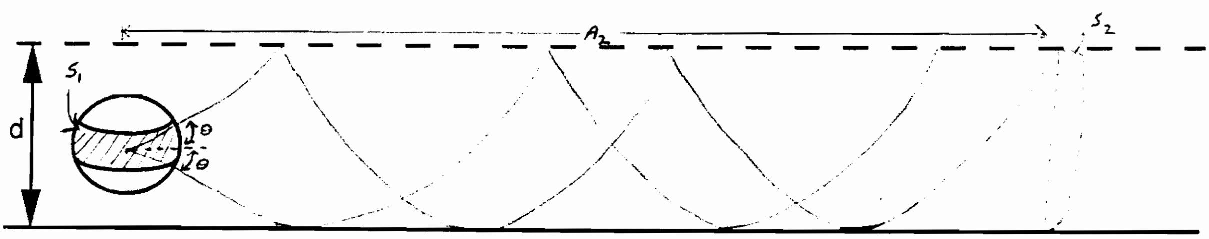
$I_1*4\pi A_1^2 = I_2*4\pi A_2^2$, where I is the intensity of the signal at the spheres surface. Hence

$$\text{Transmission Loss (TL)} = 10\log I_1/I_2 = 10\log 4\pi A_2^2/4\pi A_1^2 = 10\log A_2^2/A_1^2$$

Since $A_1 = 1$ meter, the TL expression for spherical spreading can be more simply expressed as

$$TL = 10\log A_2^2 = 20\log A_2$$

From Figure 2 however, it is evident that in this region (between the surfaces and 500ft), spherical spreading does not accurately model the signal propagation. Since spherical spreading occurs only out to the surface of the duct (the ocean surface and a depth of 500ft), the spreading occurs within a ductal region of thickness 500ft. This situation is illustrated below.



Transmission Loss in a ductal region

As is evident above, only those rays which emanate at angles less than the critical (limiting) angles remain in the ductal region. Again using the power over this spherical surface strip 1 meter from the source, we have the total radiated source power $=I_1*S_1$. Where S_1 is the surface area of the strip. At an appreciable distance form the source (>500ft), the signal would be modeled as distributed over a cylinder. Therefore, the power at a second radius A_2 would be given by I_2*S_2, where S_2 is the surface area of the cylinder of radius A_2. Recall that $S_2 = 2\pi A_2*D$, where D is the depth or thickness of the duct.

Again $TL = 10\log S_2/S_1$

Since $S_2 = 2\pi A_2*D$ and $S_1 = 2\pi \int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta = 4\pi \sin\theta$

$TL = 10\log A_2*D/2\sin\theta$

where θ is the angle of the limiting ray emanating from the source.

This TL value is a good first approximation. Two modifications however will greatly improve this TL value. These refinements will take absorption and ductal leakage into account.

2.3 <u>Absorption Effects</u> Absorption in the ocean is caused by ionic relaxation of the magnesium sulfate ($MgSO_4$) molecules present in seawater (Leonard, Combs, Skidmore, pp. 63), shear viscosity (Rayleigh, pp. 316), and a boron-borate relaxation process (Yeager, Fisher, et al., pp. 1705). Perhaps the most researched and most widely accepted expression for this absorption process has been determined by (Fisher and Simmons, pp.558) to take the form of

$$= A_1P_1\left(\frac{f^2}{f_1^2 + f^2}\right)f_1 + A_2P_2\left(\frac{f^2}{f_2^2 + f^2}\right)f_2 + A_3P_3f^2$$

where A_1, A_2, A_3, f_1, and f_2 are functions of temperature and P_1, P_2, and P_3 are functions of pressure.

Emperical data suggests the following form for these terms.

$$A_1 = (1.03*10^{-8} + 2.36*10^{-10} * T - 5.22*10^{-12} * T^2)$$
$$f_1 = (1.32*10^3 * (T + 273.1)\exp[-1700/(T + 273.1)]$$
$$A_2 = (5.62*10^{-8} + 7.52*10^{-10} * T)$$
$$f_2 = 1.55*10^7 * (T + 273.1)\exp[-3052/(T + 273.1)]$$
$$P_2 = 1 - 10.3*10^{-4} * P + 3.7*10^{-7} * P^2$$
$$A_3 = (55.9 - 2.37 * T + 4.77*10^{-2} * T^2 - 3.48*10^{-4} * T^3)*10^{-15}$$
$$P_3 = 1 - 3.84*10^{-4} * P + 7.57*10^{-8} * P^2$$

where f is in Hz, T is in degrees centigrade, and P is in atm.

Absorption coefficients (α_a) are displayed as a function of frequency and temperature in Table 3, p. 13. Note that these Coefficients are in dB/kyd and are valid for a source depth of 300 feet.

2.4 <u>Ductal Leakage</u> Ductal Leakage (α_L) can be expressed more easily than that of absorption. An accurate expression for this leakage in the desired frequency range was introduced by M. Schulkin (pp. 1152) as

$$\alpha_L = 2S*(f/D)^{.5}$$

where α_L is in dB/Kyd, S is the Sea State, f is the frequency in KHz, and D is the Depth or duct thickness.

Table 4, page 14, presents the leakage coefficient as a function of frequency and sea state. Note that the sea state was chosen for the current regional oceanic conditions --Sea State 3.

Table 3

Frequency	Absrp. (α_2) Temp. 39°F	Absorp (α_1) Temp. 56°F	Absrp (α_1) Temp. 68°F	Absrp (α_3) Temp.86°F
100	0.002	0.001	0.001	0.000
200	0.006	0.004	0.003	0.002
300	0.009	0.008	0.006	0.004
400	0.020	0.014	0.012	0.010
600	0.031	0.027	0.025	0.024
800	0.040	0.040	0.040	0.041
1000	0.051	0.055	0.058	0.059
1200	0.060	0.062	0.080	0.085
1600	0.075	0.077	0.100	0.110
2000	0.089	0.087	0.110	0.120
3200	0.120	0.112	0.140	0.180

Absorption as a function of Frequency/Temperature

Table 4

Frequency (Hz)	Leakage Coefficient (α_L)	Sea State
100	0.085	3
200	0.120	3
300	0.147	3
400	0.170	3
600	0.208	3
800	0.240	3
1000	0.268	3
1200	0.294	3
1600	0.339	3
2000	0.379	3
3200	0.480	3
4000	0.537	3

Leakage Coefficients as a function of Frequency/Sea State.

Combining all of the transmission losses discussed above results in the following expression.

$$TL = 10\log A_2 * D/2\sin\theta + \alpha_a * A_2 * 10^{-3} + \alpha_L * A_2 * 10^{-3}$$

where A_2 is the distance form the source to the observer, and D is the Duct thickness.

Table 5, p. 16, provides TL data as a function of frequency, range and temperature.

3.0 Sonar Analysis

Having completed the acoustical analysis of the region, a quick evaluation of the submarine's passive array, which will be used in the search, is now necessary. The array is circular with a radius of 2.5 meters.

3.1 Array Gain Array Gain is a measure of the increase of signal to noise obtained when using multiple hydrophones in an array as opposed to a single hydrophone. The array gain expression takes the form of

$$AG = 10\log \frac{(S/N)_1}{(S/N)_2}$$

where $(S/N)_1$ is the signal to noise ratio for the array and $(S/N)_2$ is the signal to noise ratio of one hydrophone.

Introducing the corresponding signal and noise directional patterns and their corresponding beam pattern weighting factors results in the following (Urick, pp. 35)

$$AG = 10\log \frac{\int_{4\pi} S(\theta,\psi)b(\theta,\psi)d\Omega / \int_{4\pi} N(\theta,\psi)b(\theta,\psi)d\Omega}{\int_{4\pi} S(\theta,\psi)d\Omega / \int_{4\pi} N(\theta,\psi)d\Omega}$$

where $S(\theta,\psi)$ and $N(\theta,\psi)$ are the signal and noise patterns and the $b(\theta,\psi)$ term is the corresponding beam pattern weighting.

Table 5

TL (dB) 10 Kyd	TL (dB) 20 Kyd	Frequency (Hz)	Temperature (°F)
71.5	74.5	100	56
72.3	75.3	200	56
72.9	75.9	300	56
73.5	76.5	400	56
74.5	77.5	600	56
75.4	78.4	800	56
76.3	79.3	1000	56
76.9	79.9	1200	56
78.1	81.1	1600	56
79.1	82.1	2000	56
81.7	84.7	3200	56

Propagation Loss as a function of Range,
Frequency, and Temperature

Note that for the single hydrophone, the $b(\theta,\Psi)$ term is simply equal to 1.

An array structure makes use of the fact that many signals are coherent across several hydrophones in the array while the noise is often uniform in all directions near the hydrophone (isotropic noise). If these two conditions are present, the Array Gain can be expressed as the Directivity Index (DI) which can be written as

$AG = DI = 10\log \int_{4\pi} d\Omega / \int_{4\pi} b(\theta,\Psi)\, d\Omega$ which simplifies to

$AG = DI = 10\log 2/ \int b(\theta) \cos\theta\, d\theta$ if $b(\theta,\Psi)$ has rotational symmetry about the Ψ axis.

Note that this expression also assumes that the signal is coherent and the noise is isotropic.

For a cylindrical array of radius r, the beam pattern function is given by the following expression (Urick, pp.59)

$$b(O) = \frac{2J_1[(2\pi r/\lambda)\sin\theta]}{(2\pi r/\lambda)\sin\theta}$$

where J_1 is a first order bessel function.

Substituting $b(\theta)$ into the array gain integral above results in

$$AG = DI = 10\log \frac{2}{\frac{2J_1[(2\pi r/\lambda)\sin\theta]}{(2\pi r/\lambda)\sin\theta} \cos\theta\, d\theta}$$

The resulting simplified form is

$AG = DI = 10\log (2\pi r/\lambda)^2$ for $2r \gg \lambda$

Table 6, p. 18, displays AG data as a function of Frequency.

Table 6

Frequency (Hz)	Wavelength (Meters)	Array Gain (dB)
100	15.03	0.38
200	7.52	6.40
300	5.01	9.93
400	3.76	12.42
600	2.51	15.93
800	1.88	18.44
1000	1.51	20.34
1200	1.25	21.98
1600	0.94	24.46
3200	0.47	30.48

Array Gain (in dBs) as a function of Frequency

3.2 Flow Noise (effective) The noise at the SSN hydrophones caused by ownship structural noises, flow noise, and ambient oceanic conditions is shown in Figure 3, p. 20, as a function of ownship velocity. If this noise is combined with the Array Gain data in table 6, pp. 18, the effective noise at the hydophone level can be computed for several SSN platform speeds. The resulting effective noise data (N_a) are presented in tables 7a, 7b, 7c, p. 21, as a function of frequency and are plotted in Figure 4, pp. 22 for three platform speeds.

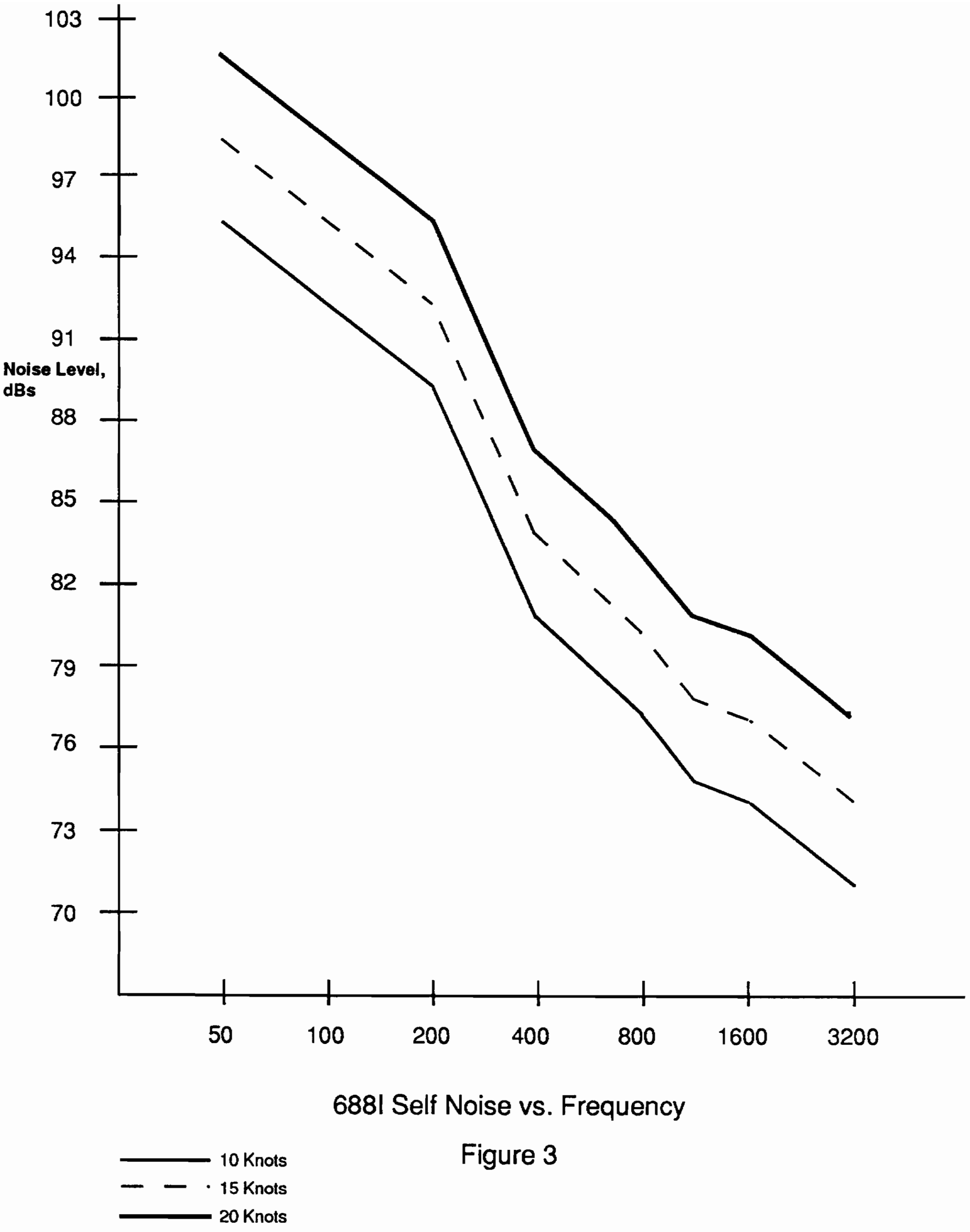
From Figure 4, it is readily seen that the effective noise decreases as the frequency increases. Unfortunately, as the frequency increases the signal strength radiated from the target decreases and propagation loss increases. Therefore, the total noise in each frequency band needs to be quantified and a trade-off analysis relating all of these factors needs to be performed to adequately evaluate its impact upon threat acquisition.

To calculate the total noise within each frequency band, the following general formula can be used to calculate the bands mid-point.

$$f_o = \left[\frac{(f_2 - f_1)(1-n) f_2^{(n-1)} f_1^{(n-1)}}{f_1^{(n-1)} - f_2^{(n-1)}} \right]^{\frac{1}{n}}$$

where n is a function of the slope of the log plot (-3n) dB/octive.

Adjusting the effective noise dB level at the midpoint for



6881 Self Noise vs. Frequency

Figure 3

Table 7A

Frequency	Noise (element)	Array Gain	Effective Noise (N_a)
100 Hz	92.0	0.38	91.6
200 Hz	89.0	6.40	82.6
300 Hz	84.5	9.93	74.6
400 Hz	80.0	12.42	67.6
600 Hz	78.5	15.93	62.6
800 Hz	77.0	18.44	58.6
1000 Hz	76.0	20.34	55.7
1200 Hz	75.0	21.98	53.0
1600 Hz	74.0	24.46	49.5
3200 Hz	71.0	30.48	40.5

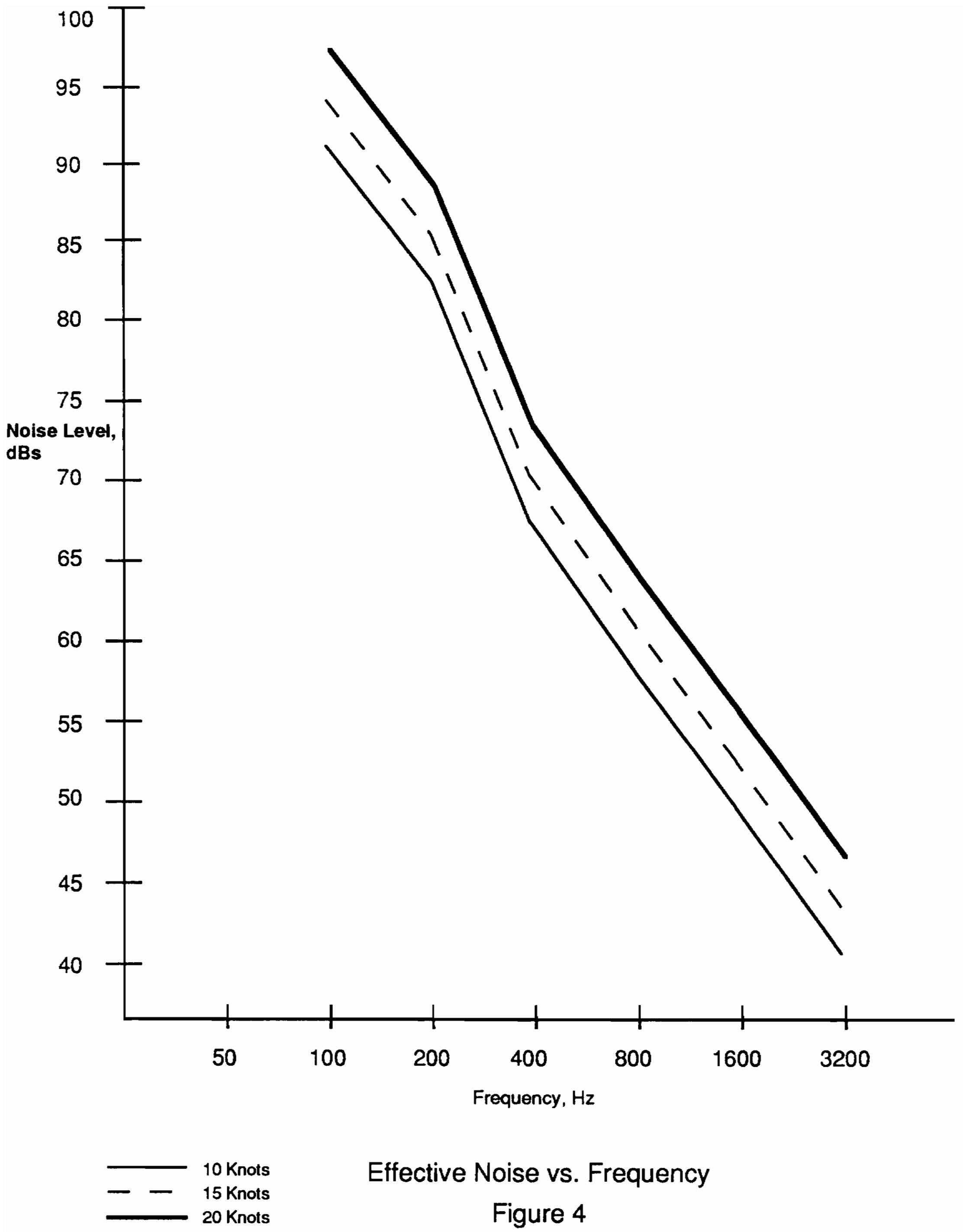
Table 7B

Frequency	Noise (element)	Array Gain	Effective Noise (N_a)
100 Hz	95.0	0.38	94.6
200 Hz	92.0	6.40	85.6
300 Hz	87.5	9.93	77.6
400 Hz	83.0	12.42	70.6
600 Hz	81.5	15.93	65.6
800 Hz	80.0	18.44	61.6
1000 Hz	79.0	20.34	58.7
1200 Hz	78.0	21.98	56.0
1600 Hz	77.0	24.46	52.5
3200 Hz	74.0	30.48	43.5

Table 7C

Frequency	Noise (element)	Array Gain	Effective Noise (N_a)
100 Hz	98.0	0.38	97.6
200 Hz	95.0	6.40	88.6
300 Hz	90.5	9.93	80.6
400 Hz	86.0	12.42	73.6
600 Hz	84.5	15.93	68.6
800 Hz	83.0	18.44	64.6
1000 Hz	82.0	20.34	61.7
1200 Hz	81.0	21.98	59.0
1600 Hz	80.0	24.46	55.5
3200 Hz	77.0	30.48	46.5

Effective Noise Data as a function of Frequency



Effective Noise vs. Frequency

Figure 4

the width of the frequency band provides the total effective noise present in the band. The appropriate expression for total bandwidth noise is shown below.

Total effective bandwidth noise (N_{BL}) is

$N_{BL} = N_{afo} + 10\log BW$

where N_{afo} is the effective noise level at the band mid-point frequency, f_o, and BW is the bandwidth in Hz.

Tables 8a, 8b, and 8c; p. 24, display the resulting frequency band mid-points, the adjustment terms, and the total frequency band noise for the three SSN speeds. Tables 8d, 8e, and 8f; p. 25-27, show all of the resulting self noise, N_{BL}, level possibilities for the three ownship speeds.

4.0 **Threat Signal Characteristics**

The expected intensity of the threat's radiated signal reaching the U.S. SSN can be calculated by adjusting the intelligience data on the threat platform noise, Figure 1 p. 3, by the associated propagation loss data (TL, table 5, p. 15). Although counterdetection concerns will necessitate the SSN remain a distance of at least 15 Kyd from the threat platform, early detection of the enemy platform (a distance of at least 20Kyd) is desired for target solution and confirmation. A best and worst case scenario (attempted detection at 20 Kyds and 10 Kyds for ownship platform speeds of 10, 15, and 20 knots) will be analyzed.

Table 8A

Frequency Band (Hz)	f_o (Hz)	N_{fo}	10logBW	N_{BL}
100 - 200	141.0	87.9	20.0	107.9
200 - 400	267.3	77.6	23.0	100.6
400 - 800	558.3	64.0	26.0	90.0
800 - 1600	1113.0	55.0	29.0	84.0
1600 - 3200	2255.3	45.8	32.0	77.8

Self Noise as a function of Frequency Band
(Platform Speed 10 Knots)

Table 8B

Frequency Band (Hz)	f_o (Hz)	N_{fo}	10logBW	N_{BL}
100 - 200	141.0	90.9	20.0	110.9
200 - 400	267.3	80.6	23.0	103.6
400 - 800	558.3	67.0	26.0	93.0
800 - 1600	1113.0	58.0	29.0	87.0
1600 - 3200	2255.3	48.8	32.0	80.8

Self Noise as a function of Frequency Band
(Platform Speed 15 Knots)

Table 8C

Frequency Band (Hz)	f_o (Hz)	N_{fo}	10logBW	N_{BL}
100 - 200	141.0	93.9	20.0	113.9
200 - 400	267.3	83.6	23.0	106.6
400 - 800	558.3	70.0	26.0	96.0
800 - 1600	1113.0	61.0	29.0	90.0
1600 - 3200	2255.3	51.8	32.0	83.8

Self Noise as a function of Frequency Band
(Platform Speed 20 Knots)

Table 8D

Frequency Band	Total noise in Frequency Band (N_{BL})	
100 - 3200	100-1600 range +6.0E07	= 108.7 dB
100 - 1600	6.2E10 +...	= 108.7 dB
100 - 800	6.2E10 +1.1E10 +1.0E09	= 108.7 dB
100 - 400	6.2E10 +1.1E10	= 108.6 dB
100 - 200	6.2E10	= 107.9 dB
200 - 3200	1.1E10 +...	= 101.1 dB
200 - 1600	1.1E10 +1.0E09 +2.5E08	= 101.0 dB
200 - 800	1.1E10 +1.0E09	= 101.0 dB
200 - 400	1.1E10	= 100.6 dB
400 - 3200	1.0E09 +2.5E08 +6.0E07	= 91.2 dB
400 - 1600	1.0E09 +...	= 91.0 dB
400 - 800	1.0E09	= 90.0 dB
800 - 1600	2.5E08	= 84.0 dB
800 - 3200	2.5E08 +6.0E07	= 84.9 dB
1600 - 3200	6.0E07	= 77.8 dB

Total Noise as a Function of Frequency Band
(10 Knot search speed)

Table 8E

Frequency Band	Total noise in Frequency Band (N_{BL})
100 - 3200	111.7 dB
100 - 1600	111.7 dB
100 - 800	111.7 dB
100 - 400	111.6 dB
100 - 200	110.9 dB
200 - 3200	104.1 dB
200 - 1600	104.0 dB
200 - 800	104.0 dB
200 - 400	103.6 dB
400 - 3200	94.2 dB
400 - 1600	94.0 dB
400 - 800	93.0 dB
800 - 1600	87.0 dB
800 - 3200	87.9 dB
1600 - 3200	80.8 dB

Total Noise as a Function of Frequency Band
(15 Knot search speed)

Table 8F

Frequency Band	Total noise in Frequency Band (N_{BL})
100 - 3200	114.7 dB
100 - 1600	114.7 dB
100 - 800	114.7 dB
100 - 400	114.6 dB
100 - 200	113.9 dB
200 - 3200	107.1 dB
200 - 1600	107.0 dB
200 - 800	107.0 dB
200 - 400	106.6 dB
400 - 3200	97.2 dB
400 - 1600	97.0 dB
400 - 800	96.0 dB
800 - 1600	90.0 dB
800 - 3200	90.9 dB
1600 - 3200	83.8 dB

Total Noise as a Function of Frequency Band
(20 Knot search speed)

Table 9, p. 29 displays the resulting threat signal intensity for the 10 Kyd and 20 Kyd ranges as a function of frequency. Using the same techniques that were used above to compute the total self noise, the total resulting signal level at the hydrophone input, Table 9, in each frequency band needs to be quantified to adequately evaluate its threat acquisition impact.

Tables 10a and 10c, p. 30, display the resulting frequency band mid-points, the adjustment terms, and the total frequency band signals for ranges of 10 Kyds and 20 Kyds. Tables 10b and 10d, p. 31, show all of the resulting frequency band signal level possibilities for the 10 and 20 Kyd ranges.

5.0 Detection Threshold (Uncorrelated)

The last remaining quantity to be discussed is the detection threshold. The strict definition of the detection threshold is the ratio of the total signal (across the entire bandwidth, S_B) at the detector divided by the total noise at the detector (across the entire bandwidth, N_B). The resulting expression is

$$DT = 10\log S_B/N_B$$

Since this DT value may vary widely depending upon input signal to noise ratio, a threshold needs to be established. It is important to note, however, that a threshold which is established too high would exclude all but the extremely high signals and a threshold which is too low would often.

Table 9

Frequency (Hz)	Signal Level (SL)	S_L - TL (10Kyd)	(20Kyd)
100	130.0 dB	58.5 dB	55.5 dB
200	125.0 dB	52.7 dB	49.7 dB
300	122.5 dB	49.6 dB	46.6 dB
400	120.0 dB	46.5 dB	43.5 dB
600	117.5 dB	43.0 dB	40.0 dB
800	115.0 dB	39.6 dB	36.6 dB
1000	114.3 dB	38.0 dB	35.0 dB
1600	112.0 dB	33.9 dB	30.9 dB
2000	111.3 dB	32.2 dB	29.2 dB
3200	109.0 dB	27.3 dB	24.3 dB

Signal Level minus Propagation Loss as a function of Frequency

Tables 10 A/C

Frequency Band (Hz)	f_o (Hz)	10logBW	S_{Lfo} 10Kyd	SL_{BL} 10Kyd
100 - 200	141.6	20.0	56.1	76.1
200 - 400	282.5	23.0	50.1	73.1
400 - 800	562.3	26.0	43.7	69.7
800 - 1600	1133.6	29.0	37.2	66.2
1600 - 3200	2253.8	32.0	31.2	63.2

Table 10 A
Signal minus propagation loss at 10kyd as a function of Bandwidth

Frequency Band (Hz)	f_o (Hz)	10logBW	S_{Lfo} 20Kyd	S_{LBL} 20Kyd
100 - 200	141.6	20.0	53.1	73.1
200 - 400	282.5	23.0	47.1	70.1
400 - 800	558.3	26.0	40.7	66.7
800 - 1600	1113.0	29.0	34.2	63.2
1600 - 3200	2255.3	32.0	28.2	60.2

Table 10C
Signal minus propagation loss at 20kyd as a function of Bandwidth

Tables B/D

Frequency Band (Hz)	SL_{BL} 10Kyd
100 - 200	76.1
100 - 400	77.9
100 - 800	78.5
100 - 1600	78.7
100 - 3200	78.9
200 - 400	73.1
200 - 800	74.7
200 - 1600	75.3
200 - 3200	75.6
400 - 800	69.7
400 - 1600	71.3
400 - 3200	71.9
800 - 1600	66.2
800 - 3200	68.0
1600 - 3200	63.2

Table 10 B
Signal minus Propagaton Loss in all frequency bands at 10 Kyds

Frequency Band (Hz)	SL_{BL} 20Kyd
100 - 200	73.1
100 - 400	74.9
100 - 800	75.5
100 - 1600	75.7
100 - 3200	75.9
200 - 400	70.1
200 - 800	71.7
200 - 1600	72.3
200 - 3200	72.6
400 - 800	66.7
400 - 1600	68.3
400 - 3200	68.9
800 - 1600	63.2
800 - 3200	65.0
1600 - 3200	60.2

Table 10D
Signal minus Propagaton Loss in all frequency bands at 20 Kyds

falsely indicate the presence of a threat. Hence a trade-off must be made between the probability of detection and the probability of a false alarm. The ralationship between these two factors for a given sonar set is termed the Receiver-Operating-Characteristics. Figure 5, p. 33, shows these characteristics for a broad range of detection and false alarm probabilities (Urick, 382). Note that this plot includes the corresponding detection index values for each curve.

5.1 <u>Detection Index Values</u> The detection index (Figure 5, p. 33) is defined as the square of the magnitude of the mean noise subtracted from the magnitude of the noise plus signal, all divided by the variance of the signal or noise. This can be expressed as

$$d = [M_{(S+N)} - M_N]^2/\sigma^2$$

This detection index was related to the detection threshold by Peterson and Birdshall (pp. 13) for an optimum receiver for an unknown signal. This relationship is

$$d = wt(S/N)^2$$

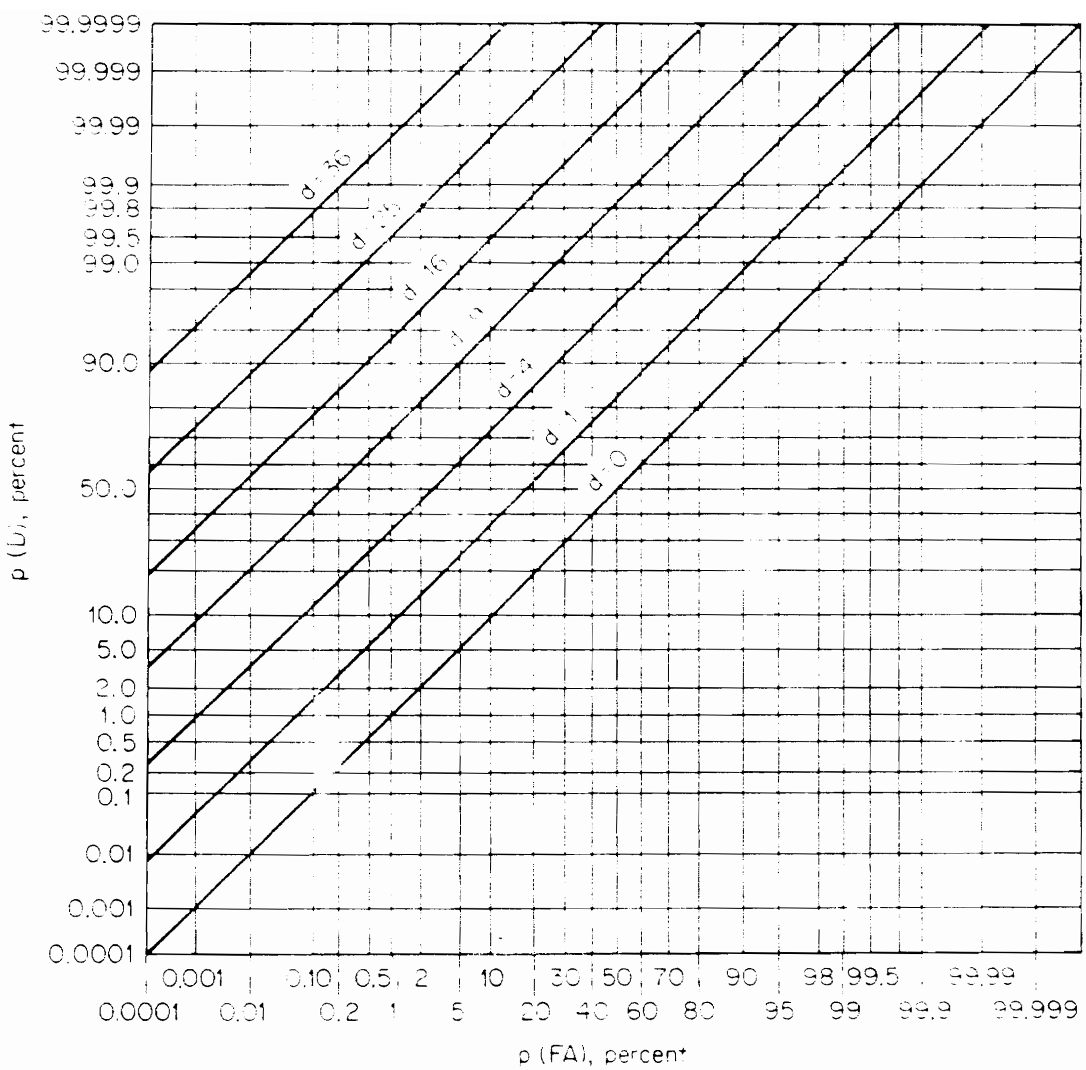
where w = the bandwidth, t = signal duration, and (S/N) is the signal to noise ratio.

Solving for (S/N) results in

$$(S/N) = (d/wt)^{.5}$$

Subtituting this (S/N) into the expression for DT, above, provides

$$DT = 10\log\ (d/wt)^{.5} = 5\log d - 5\log wt$$



Detection Index values as a function of Detection Probability and False Alarm Probability

Figure 5

Table 11, p. 35, presents the detection threshold values (in dBs) as a function of the frequency range for a probability of detection of 90% and a false alarm probability of 0.10%.

6.0 Passive Sonar Equation

All of the elements needed to find the SSN sonars' optimal frequency range for the regional environment, radiated threat platform noise and SSN search speed and the resulting sonar effectiveness have now been identified and calculated. To extract this information from the above terms: Signal level (S_L), Noise Level (N_L), Array Gain (AG), Propagation Loss (TL), and Detection Threshold (DT); it is necessary to use the Passive Sonar Equation (PSE). The PSE is given by

$$SE = S_L - TL - N_L + AG - DT$$

where SE is the signal level (dB) in excess of the minimum needed to detect the threat for the conditions and values chosen in the sonar equation terms.

The above Signal Excess equation is a very concise and useful measure of a sonars' effectiveness for given conditions. If, after substituting in for the various sonar parameters, the result is positive, then the threat will indeed be detected. If the resulting SE is negative, then the threat will go undetected. Tables (12a, 12b, 12c) p. 36-38, and (13a, 13b, 13c) p. 39-41, display the values of the sonar equation terms as a function of frequency for the three SSN search speeds (10, 15, and 20 Knots) for ranges of

Table 11

Frequency Band (Hz)	Signal duration, t	Detection index,d	DT
100 - 200	300 secs.	21	-15.8
100 - 400	300 secs.	21	-18.2
100 - 800	300 secs.	21	-20.0
100 - 1600	300 secs.	21	-21.7
100 - 3200	300 secs.	21	-23.2
200 - 400	300 secs.	21	-17.3
200 - 800	300 secs.	21	-19.7
200 - 1600	300 secs.	21	-21.5
200 - 3200	300 secs.	21	-23.2
400 - 800	300 secs.	21	-18.8
400 - 1600	300 secs.	21	-21.2
400 - 3200	300 secs.	21	-23.0
800 - 1600	300 secs.	21	-21.8
800 - 3200	300 secs.	21	-22.7
1600 - 3200	300 secs.	21	-21.8

Detection Index (Uncorrelated) as a function of Frequency Band

Table 12A

Frequency (Hz)	SL - TL (10Kyd)	NL - AG	DT
100 - 200	76.1	107.9	-15.8
100 - 400	77.9	108.6	-18.2
100 - 800	78.5	108.7	-20.0
100 - 1600	78.7	108.7	-21.7
100 - 3200	78.9	108.7	-23.2
200 - 400	73.1	100.6	-17.3
200 - 800	74.7	101.0	-19.7
200 - 1600	75.3	101.0	-21.5
200 - 3200	75.6	101.1	-23.2
400 - 800	69.7	90.0	-18.8
400 - 1600	71.3	91.0	-21.2
400 - 3200	71.9	91.2	-23.0
800 - 1600	66.2	84.0	-21.8
800 - 3200	68.0	84.9	-22.7
1600 -3200	63.2	77.8	-21.8

Sonar Equation Components as a function of Frequency Band
10 Kyd Range and 10 Knot search Speed

Table 12B

Frequency (Hz)	SL - TL (10Kyd)	NL - AG	DT
100 - 200	76.1	110.9	-15.8
100 - 400	77.9	111.6	-18.2
100 - 800	78.5	111.7	-20.0
100 - 1600	78.7	111.7	-21.7
100 - 3200	78.9	111.7	-23.2
200 - 400	73.1	103.6	-17.3
200 - 800	74.7	104.0	-19.7
200 - 1600	75.3	104.0	-21.5
200 - 3200	75.6	104.1	-23.2
400 - 800	69.7	93.0	-18.8
400 - 1600	71.3	94.0	-21.2
400 - 3200	71.9	94.2	-23.0
800 - 1600	66.2	87.0	-21.8
800 - 3200	68.0	87.9	-22.7
1600 -3200	63.2	80.8	-21.8

Sonar Equation Components as a function of Frequency Band
10 Kyd Range and 15 Knot search Speed

Table 12C

Frequency (Hz)	SL - TL (10Kyd)	NL - AG	DT
100 - 200	76.1	113.9	-15.8
100 - 400	77.9	114.6	-18.2
100 - 800	78.5	114.7	-20.0
100 - 1600	78.7	114.7	-21.7
100 - 3200	78.9	114.7	-23.2
200 - 400	73.1	106.6	-17.3
200 - 800	74.7	107.0	-19.7
200 - 1600	75.3	107.0	-21.5
200 - 3200	75.6	107.1	-23.2
400 - 800	69.7	96.0	-18.8
400 - 1600	71.3	97.0	-21.2
400 - 3200	71.9	97.2	-23.0
800 - 1600	66.2	90.0	-21.8
800 - 3200	68.0	90.9	-22.7
1600 -3200	63.2	83.8	-21.8

Sonar Equation Components as a function of Frequency Band
10 Kyd Range and 20 Knot search Speed

Table 13A

Frequency (Hz)	SL - TL (20Kyd)	NL - AG	DT
100 - 200	73.1	107.9	-15.8
100 - 400	74.9	108.6	-18.2
100 - 800	75.5	108.7	-20.0
100 - 1600	75.7	108.7	-21.7
100 - 3200	75.9	108.7	-23.2
200 - 400	70.1	100.6	-17.3
200 - 800	71.7	101.0	-19.7
200 - 1600	72.3	101.0	-21.5
200 - 3200	72.6	101.1	-23.2
400 - 800	66.7	90.0	-18.8
400 - 1600	68.3	91.0	-21.2
400 - 3200	68.9	91.2	-23.0
800 - 1600	63.2	84.0	-21.8
800 - 3200	65.0	84.9	-22.7
1600 -3200	60.2	77.8	-21.8

Sonar Equation Components as a function of Frequency Band
20 Kyd Range and 10 Knot search Speed

Table 13B

Frequency (Hz)	SL - TL (20Kyd)	NL - AG	DT
100 - 200	73.1	110.9	-15.8
100 - 400	74.9	111.6	-18.2
100 - 800	75.5	111.7	-20.0
100 - 1600	75.7	111.7	-21.7
100 - 3200	75.9	111.7	-23.2
200 - 400	70.1	103.6	-17.3
200 - 800	71.7	104.0	-19.7
200 - 1600	72.3	104.0	-21.5
200 - 3200	72.6	104.1	-23.2
400 - 800	66.7	93.0	-18.8
400 - 1600	68.3	94.0	-21.2
400 - 3200	68.9	94.2	-23.0
800 - 1600	63.2	87.0	-21.8
800 - 3200	65.0	87.9	-22.7
1600 -3200	60.2	80.8	-21.8

Sonar Equation Components as a function of Frequency Band
20 Kyd Range and 15 Knot search Speed

Table 13C

Frequency (Hz)	SL - TL (20Kyd)	NL - AG	DT
100 - 200	73.1	113.9	-15.8
100 - 400	74.9	114.6	-18.2
100 - 800	75.5	114.7	-20.0
100 - 1600	75.7	114.7	-21.7
100 - 3200	75.9	114.7	-23.2
200 - 400	70.1	106.6	-17.3
200 - 800	71.7	107.0	-19.7
200 - 1600	72.3	107.0	-21.5
200 - 3200	72.6	107.1	-23.2
400 - 800	66.7	96.0	-18.8
400 - 1600	68.3	97.0	-21.2
400 - 3200	68.9	97.2	-23.0
800 - 1600	63.2	90.0	-21.8
800 - 3200	65.0	90.9	-22.7
1600 -3200	60.2	83.8	-21.8

Sonar Equation Components as a function of Frequency Band
20 Kyd Range and 20 Knot search Speed

10 and 20 Kyds respectively.

7.0 Signal Excess Values (Uncorrelated)

The resulting Signal Excess, for each SSN speed, is displayed in Tables 14a, 14b, and 14c, p. 43-45, as a function of frequency for the ranges of 10 and 20 Kyds.

From the Signal Excess tables (14 -a,b,c) it is evident that largest Signal Excess is generated when the 100 - 1599 Hz signals are filtered out, resulting in the SE values below.

SSN Speed	SE 10 Kyd	SE 20 Kyd
10 Kts.	+7.2 dB	+4.2 dB
15 Kts.	+4.2 dB	+1.2 dB
20 Kts.	+1.2 dB	-1.8 dB

Therefore, the optimum frequency range resulting from the given oceanic conditions and the threat and ownship characteristics is the 1600 - 3200 Hz band for all three speeds and at both the 10 Kyd and 20 Kyd ranges. Note that the SSN sonar, even while operating in the optimum frequency band will not allow a search rate of 20 knots.

With a large search area, it is desirable to proceed at the fastest search rate possible. Ideally, this would be limited from the threats counterdetection capability, not ownship acquisistion range. Since the threat platform has very little detection capability, an increased 668I detection capability at greater speeds would be of great value.

Table 14A

Frequency (Hz)	SE 10 Kyds	SE 20 Kyds
100 - 200	-16.0	-19.0
100 - 400	-12.5	-15.5
100 - 800	-10.2	-13.2
100 - 1600	-8.3	-11.3
100 - 3200	-6.6	-9.6
200 - 400	-10.2	-13.2
200 - 800	-6.6	-9.6
200 - 1600	-4.2	-7.2
200 - 3200	-2.2	-5.2
400 - 800	-1.5	-4.5
400 - 1600	+1.5	-1.5
400 - 3200	+3.7	+0.7
800 - 1600	+4.0	+1.0
800 - 3200	+5.8	+2.8
1600 - 3200	+7.2	+4.2

Signal Excess as a function of Frequency Band (10 Knots)

Table 14B

Frequency (Hz)	SE 10 Kyds	SE 20 Kyds
100 - 200	-19.0	-22.0
100 - 400	-15.5	-18.5
100 - 800	-13.2	-16.2
100 - 1600	-11.3	-14.3
100 - 3200	-9.6	-12.6
200 - 400	-13.2	-16.2
200 - 800	-9.6	-12.6
200 - 1600	-7.2	-10.2
200 - 3200	-5.3	-8.3
400 - 800	-4.5	-7.5
400 - 1600	-1.5	-4.5
400 - 3200	+0.7	-2.3
800 - 1600	+1.0	-2.0
800 - 3200	+2.8	-0.2
1600 - 3200	+4.2	+1.2

Signal Excess as a function of Frequency Band (15 Knots)

Table 14C

Frequency (Hz)	SE 10 Kyds	SE 20 Kyds
100 - 200	-22.0	-25.0
100 - 400	-18.5	-21.5
100 - 800	-16.2	-19.2
100 - 1600	-14.3	-17.3
100 - 3200	-12.6	-15.6
200 - 400	-16.2	-19.2
200 - 800	-12.6	-15.6
200 - 1600	-10.2	-13.2
200 - 3200	-8.3	-11.3
400 - 800	-7.5	-10.5
400 - 1600	-4.5	-7.5
400 - 3200	-2.3	-5.3
800 - 1600	-2.0	-5.0
800 - 3200	-0.2	-3.2
1600 - 3200	+1.2	-1.8

Signal Excess as a function of Frequency Band (20 Knots)

8.0 Cross/Auto-Correlation

Since the threat signal is well known (from historical intelligence data) the increased capability can be provided by the use of Cross/Auto-Correlation techniques. An exactly known signal is related to the detection threshold, d, by the following expression (Peterson and Birdsall, pp. 13).

$$d = 2Ew/N_{BL}$$

where E is the Total energy in the receiver band, w is the bandwidth, and N_{BL} is the total noise in the band. With a Source Signal level of S_L with duration t, then E $=S_L*t$.

This leads to the expression

$$d = 2wtSL/N \text{ or } SL/N = d/2wt$$

Therefore, $DT = 10Log\ d/2wt$

$$= 10Logd - 10Log2wt$$

8.1 Detection Threshold (Correlated) These correlated DT values are displayed on Table 15, p. 48, as a function of Frequency, Time and detection threshold index. Note the considerable increase in the correlated DT values over the uncorrelated ones. Table 16, p. 49, displays the difference between the uncorrelated and correlated DT values. Cross-correlation and Auto-correlation techniques have been implemented on a Personal Computer to analyze a sinusoidal waveform with additive white (random) noise. The results of these correlation techniques, including data and graphs is located in appendix A.

8.2 Signal Excess Values (Correlated)

Using the same procedures utilized for the previous uncorrelated SE values, the correlated Signal Excess is computed as a function of Frequency for 10 and 20 Kyd ranges at the three SSN search speeds. The results are shown on tables 17a, 17b, and 17c, p. 50-52.

Table 15

Frequency Band (Hz)	Signal duration, t	Detection index,d	DT
100 - 200	300 secs.	21	-34.6
100 - 400	300 secs.	21	-39.3
100 - 800	300 secs.	21	-43.0
100 - 1600	300 secs.	21	-46.3
100 - 3200	300 secs.	21	-49.5
200 - 400	300 secs.	21	-37.6
200 - 800	300 secs.	21	-42.3
200 - 1600	300 secs.	21	-46.0
200 - 3200	300 secs.	21	-49.3
400 - 800	300 secs.	21	-40.6
400 - 1600	300 secs.	21	-45.4
400 - 3200	300 secs.	21	-49.0
800 - 1600	300 secs.	21	-43.6
800 - 3200	300 secs.	21	-48.4
1600 - 3200	300 secs.	21	-46.6

Correlated Detection Threshold Values for 90% Probability of detection and .10% probability of false alarm.

Table 16

Frequency Band (Hz)	Signal duration, t	d	$DT_c - DT_u$
100 - 200	300 secs.	21	-18.8
100 - 400	300 secs.	21	-21.1
100 - 800	300 secs.	21	-19.8
100 - 1600	300 secs.	21	-24.6
100 - 3200	300 secs.	21	-26.3
200 - 400	300 secs.	21	-20.3
200 - 800	300 secs.	21	-22.6
200 - 1600	300 secs.	21	-24.5
200 - 3200	300 secs.	21	-26.1
400 - 800	300 secs.	21	-21.8
400 - 1600	300 secs.	21	-24.2
400 - 3200	300 secs.	21	-26.0
800 - 1600	300 secs.	21	-21.8
800 - 3200	300 secs.	21	-25.7
1600 - 3200	300 secs.	21	-24.8

Correlated DT values minus Uncorrelated DT Values as a function of Frequency Band.

Table 17A

Frequency (Hz)	SE 10 Kyds	SE 20 Kyds
100 - 200	+2.8	-0.2
100 - 400	+8.6	+5.6
100 - 800	+12.8	+9.8
100 - 1600	+16.3	+13.3
100 - 3200	+19.7	+16.7
200 - 400	+10.1	+7.1
200 - 800	+16.0	+13.0
200 - 1600	+20.3	+17.3
200 - 3200	+29.7	+26.7
400 - 800	+20.3	+17.3
400 - 1600	+25.7	+22.7
400 - 3200	+29.7	+26.7
800 - 1600	+25.8	+22.8
800 - 3200	+31.5	+28.5
1600 - 3200	+32.0	+29.0

Signal Excess with Correlated DT Values (10 Knots)

Table 17B

Frequency (Hz)	SE 10 Kyds	SE 20 Kyds
100 - 200	-0.2	-3.2
100 - 400	+5.6	+2.6
100 - 800	+9.8	+6.8
100 - 1600	+13.3	+10.3
100 - 3200	+16.7	+13.7
200 - 400	+7.1	+10.1
200 - 800	+13.0	+10.0
200 - 1600	+17.3	+14.3
200 - 3200	+20.8	+17.8
400 - 800	+17.3	+14.3
400 - 1600	+22.7	+19.7
400 - 3200	+26.7	+23.7
800 - 1600	+22.8	+19.8
800 - 3200	+28.5	+25.5
1600 - 3200	+29.0	+26.0

Signal Excess with Correlated DT Values (15 Knots)

Table 17C

Frequency (Hz)	SE 10 Kyds	SE 20 Kyds
100 - 200	-3.2	-6.2
100 - 400	+2.6	-0.4
100 - 800	+6.8	+3.8
100 - 1600	+10.3	+7.3
100 - 3200	+13.7	+10.7
200 - 400	+4.1	+1.1
200 - 800	+10.0	+7.0
200 - 1600	+14.3	+11.3
200 - 3200	+17.8	+14.8
400 - 800	+14.3	+11.3
400 - 1600	+19.7	+16.7
400 - 3200	+23.7	+20.7
800 - 1600	+19.8	+16.8
800 - 3200	+25.5	+22.5
1600 - 3200	+26.0	+23.0

Signal Excess with Correlated DT Values (20Knots)

Again, from the Signal Excess tables (17 -a,b,c) it is evident that the largest Signal Excess is generated when the 100 - 1599 Hz signals are filtered out, resulting in the SE values below.

SSN Speed	SE 10 Kyd	SE 20 Kyd
10 Kts.	+32.0 dB	+29.0 dB
15 Kts	+29.0 dB	+26.0 dB
20 Kts.	+26.0 dB	+23.0 dB

From the above Signal Excess values, it is evident that a search speed of at least 20 Knots can be supported as long as cross correlation processing is used on the input signal.

9.0 Conclusions

Summarizing the analyses, data and discussions herein, the following conclusions are presented.

The acoustic propagation is strongly affected by water temperature and pressure. The thermocline presented in this analysis was found to generate a well defined ductal region between the surface and a depth of 500 feet. The effects of absorption due to the ionic relaxation of Magnesium Sulfate and Boron-borate molecules was calculated and introduced into the propagation model. Finally, the effects of ductal leakage were identified and added to the model. The sum of these terms provided an accurate expression for the propagation loss as a function of frequency and range (TL = 10 Log $A_2*D/2\sin\Theta + \alpha_a*A_2*10^{-3} + \alpha_L*A_2*10^{-3}$); where A2 is the distance from the source to the observer, D is the ductal thickness, and α_a, α_L are the absorption and ductal leakage coefficients respectively.

The Array Gain of a cylindrical sonar was computed as a function of frequency. Flow noise induced in the hydrophones was analyzed for several platform speeds and calculated as a function of frequency.

The Detection Index was computed for a probability of detection of 90% and a probability of false alarm of 0.1%. This index was then related to the detection threshold by a detection optimization algorithm and its impact discussed.

The Passive Sonar Equation was then used to relate these sonar elements and to provide a quantitative measure of the sonar's effectiveness. By analyzing the sonar effectiveness as a function of frequency, the sonar's optimum operating frequency range for the given conditions was identified. For the oceanic and threat characteristics provided at the outset, this optimum frequency range was found to be the 1600 - 3200 Hz range.

Cross-Correlation and Auto-Correlation were then introduced to improve the sonar's effectiveness. An increase of 24.8 dB was introduced for the optimum operating frequency band by the addition of these correlation techniques. These techniques were then modeled on an IBM PC to simulate the effects of cross-correlation and auto-correlation on a noise corrupted periodic signal. Signal to Noise Ratios for these corrupted signals ranged from 3dB to -9dB. To quantify the improvement resulting from correlation use, the correlated and uncorrelated plots of the corrupted signals were evaluated for periodicity. For the -9dB noise corrupted plot, the uncorrelated signal period was typically in error by more than 60%. The correlated data for the -9dB noise corrupted plot provided errors typically less than 8%.

Works Cited

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Appendix A

Cross-Correlation and Auto-Correlation Data, Graphs and Discussion

The computer program, pages A4-A5, was generated to illustrate the benefits of correlation techniques in the detection of a periodic signal. The details of the computer simulation and a discussion of the resulting data follow below.

To implement this computer analysis, it was first necessary to generate a set of random numbers over a specified range. The range is chosen to provide the proper magnitude of the "white noise" (Proakis and Manolakis, pp. 124). The expression

$$\text{dB level} = 10 \text{ Log } Px/Pw$$

where dB level is the Signal to Noise Ratio (SNR) in dBs, Px is the signal strength, and Pw is the noise level;

provides the relationship necessary to establish the random number range. For "white noise" the noise level can be expressed by

$$Pw = d^2/12$$

where d is a parameter of the distribution.

When simulating the noise with random numbers, the range is related to the distribution parameter, d, by

$$-d/2 < r < d/2$$

where r is the random number range.

The Signal to Noise Ratios analyzed and the corresponding random number frequency ranges are provided below.

SNR	Random Number Range (r)
3dB	-0.86705...0.86705
1dB	-1.09109...1.09109
-1dB	-1.37419...1.37419
-3dB	-1.73000...1.73000
-9dB	-3.45180...3.45180

As can be seen by a cursory review of the program code, this noise is used to corrupt a sinusoidal signal of period n=10. Two subroutines are then used to cross-correlate the signal to noise, and noise to signal; and auto-correlate the signal to signal and noise to noise. Obviously, the auto-correlation of the random noise is extremely small due to the randomness of the noise. The two cross-correlation terms are also small since there is very little correlation between the signal and the random noise.

To quantitatively analyze the graphs, pp. A6-A15, (generated by the data on pp. A16-A35), the distance between the negative axis crossings has been measured. For a perfectly periodic signal of period n=10, these negative sloped axis crossings should be spaced by a distance of n=10. The percentage deviation from this period for each dB value is provided on the following page.

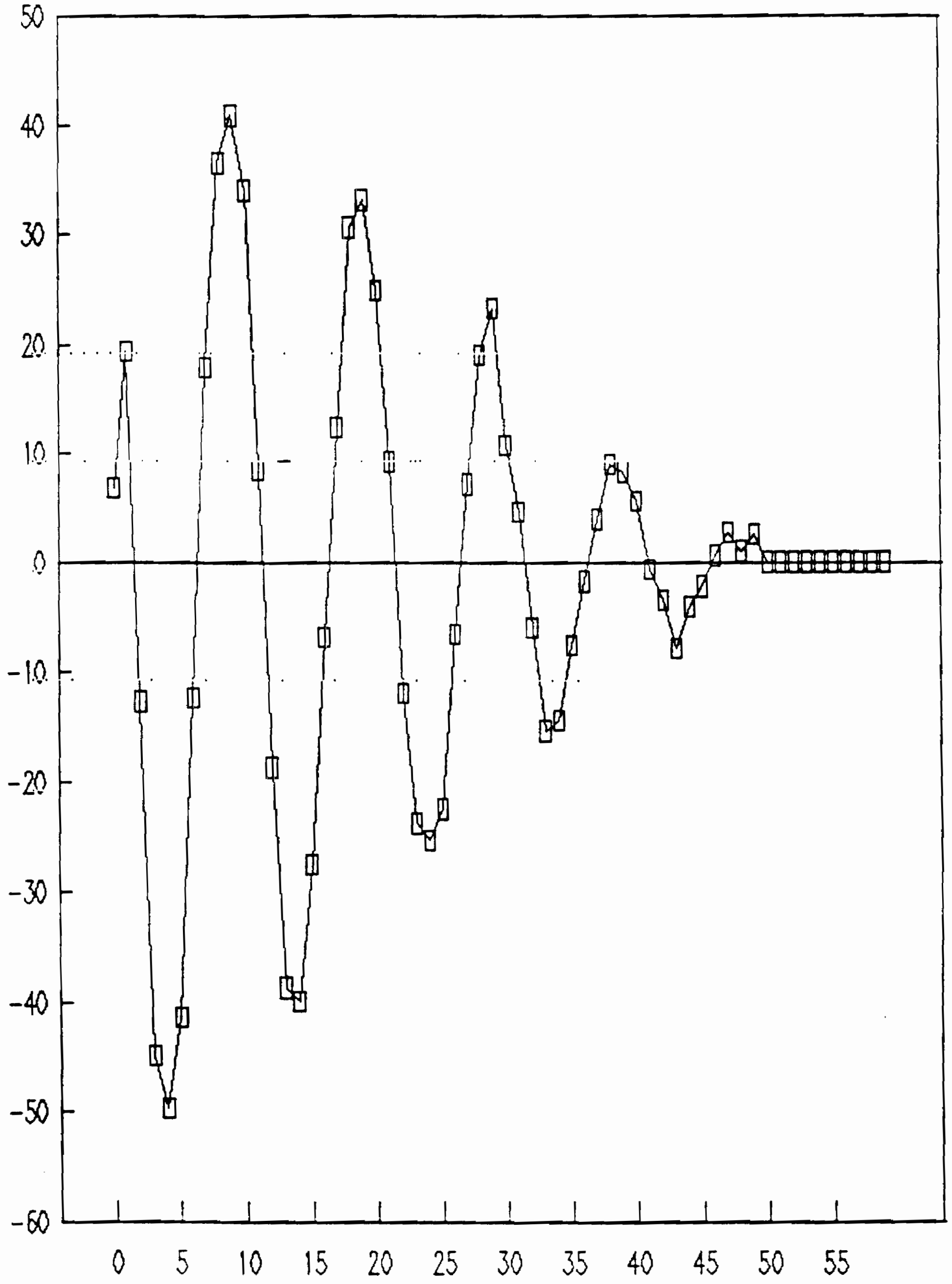
dB Level	% error for period 1	2	3	4
3dB (CORR)	8%	0%	0%	8%
3dB (UNCORR)	8%	0%	8%	8%
1dB (CORR)	0%	6%	6%	0%
1dB (UNCORR)	11%	5%	17%	11%
-1dB (CORR)	0%	0%	8%	8%
-1dB (UNCORR)	62%	15%	0%	23%
-3dB (CORR)	8%	0%	8%	8%
-3dB (UNCORR)	8%	77%	77%	77%
-9dB (CORR)	8%	0%	8%	62%
-9dB (UNCORR)	62%	77%	77%	8%

From a visual inspection of the SNR graphs or the above error values, the benefits of correlation are evident. The uncorrelated 9dB periodicity data, for example, is in error by more than 50%, while three of the four correlated data points are within 8% of the uncorrupted signal period.

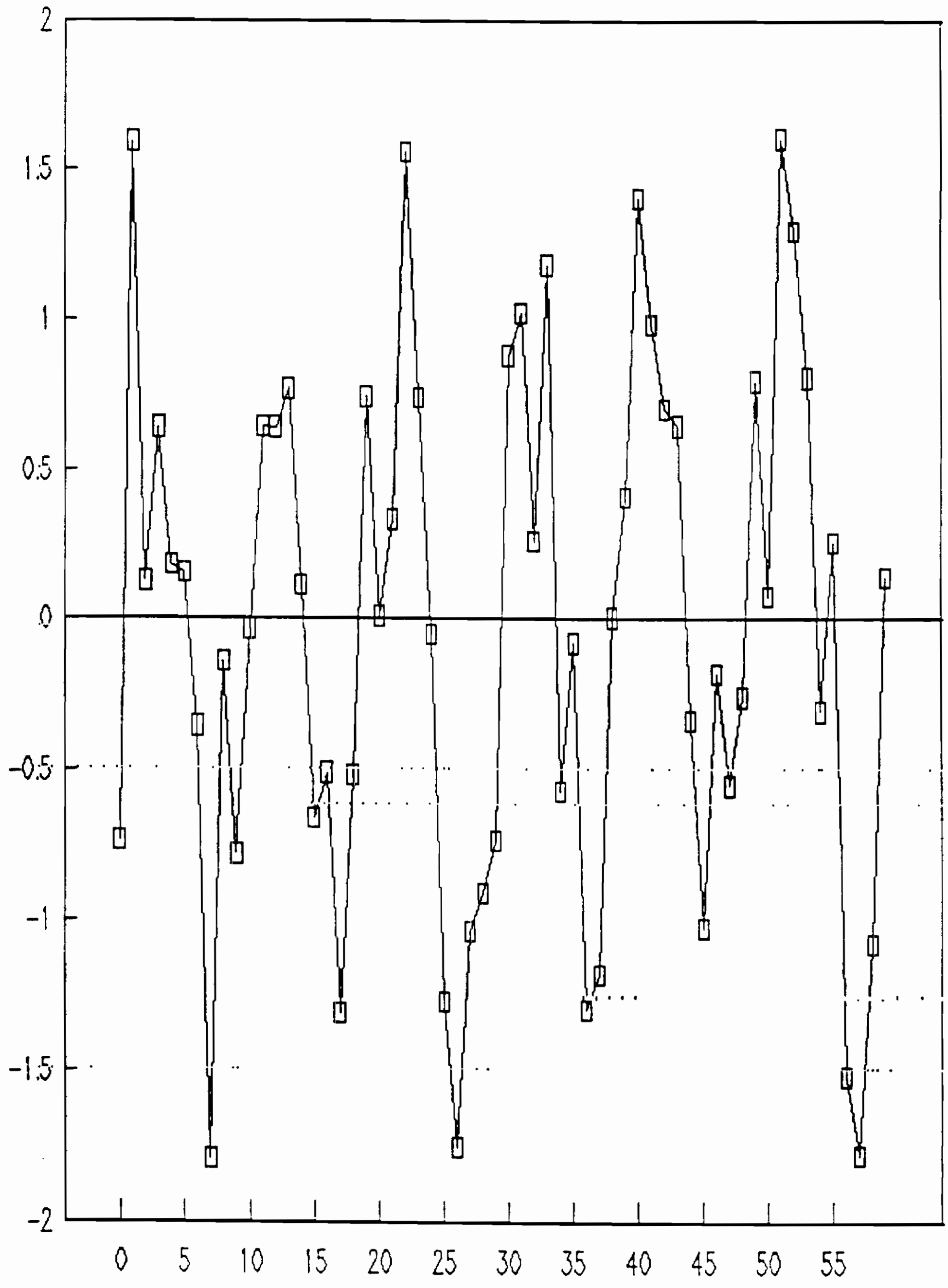
CROSS-CORRELATION/ AUTO-CORRELATION PROGRAM

```
      real signal(1:100), w(1:100), ryy(1:100), rndm
      real autosig(1:100), crossig(1:100)
      real crosnois(1:100), autonois(1:100)
      integer n
      external rndm, irand
      intrinsic sin
      open(unit=1,file='dumpfile')
      do 3 n = 1, 100
     signal(n) = sin (.62831854 * n)
     w(n) = rndm()
c    write(*,*) signal(n)
3     continue
      call autoc(signal,n,autosig,n)
      call cros(signal,n,w,n,crossig,n)
      call cros(w,n,signal,n,crosnois,n)
      call autoc(w,n,autonois,n)
      do 4 n = 1, 100
c    ryy(n) = autonois(n) + autosig(n) + crosnois(n) + crossig(n)
     ryy(n) = signal(n)
     write(*, *) ryy(n)
     write(1, *) ryy(n)
4     continue
      close(unit=1)
      end

      real function rndm()
      logical first
      integer irand, iseed
      intrinsic abs
      save first, iseed
      data first/ .true. /
      if (first) then
     write(*,*)'Give me a positive whole-number seed:'
     read*, iseed
     iseed = abs(iseed)
     first = .false.
      endif
      iseed = irand(iseed)
      rndm = iseed / 10000000.00
      rndm = rndm - 5.00
      rndm = rndm / 5.766680122
      end

      integer function irand(iseed)
      integer iseed
      integer iseed1, iseed2, m1, m2, mult1, mult2, i
      parameter( mult1 = 3141, mult2 = 5821 )
      parameter( m1 = 100000000, m2 = 10000 )
      intrinsic mod
      iseed1 = iseed / m2
      iseed2 = mod( iseed, m2 )
      i = mod( ( ( mod( (iseed2*mult1 + iseed1*mult2), m2)
     +           * m2 ) + (iseed2*mult2) ), m1)
      irand = mod((i+1),m1)
```

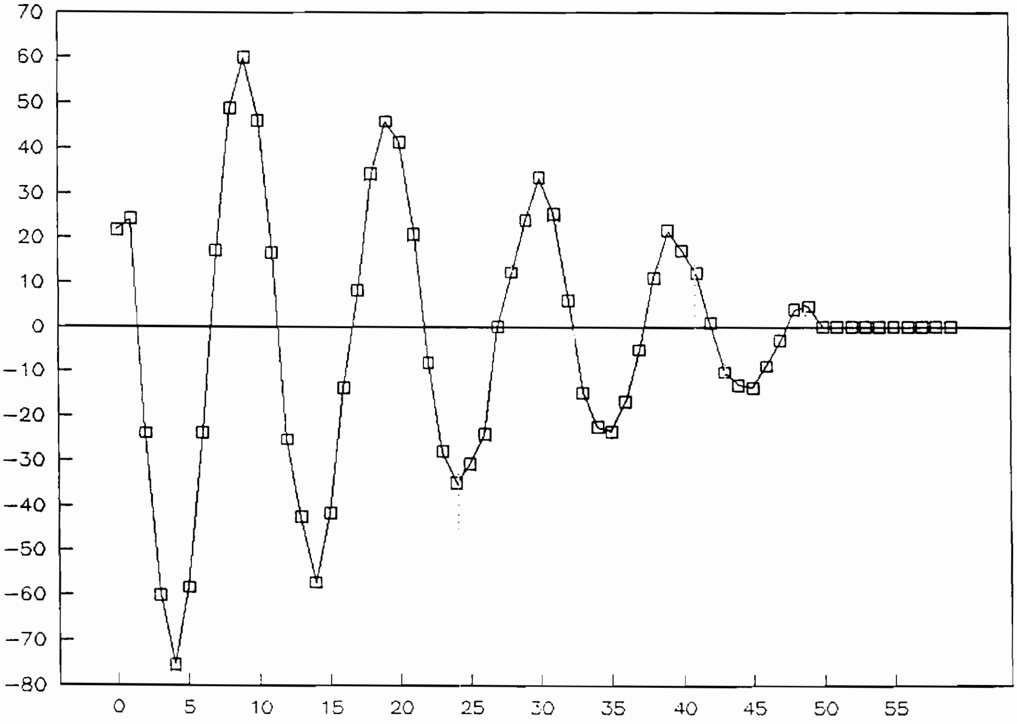
```
      end

      subroutine cros (x,n,y,m,r,lmax)
c
c     subroutine cros computes the crosscorrelation
c     sequence between x and y
c     parameters
c    x: array containing seq x
c    n: the length of seq x
c    y: array containing seq y
c    m: the length of seq y
c    r: array containing the correlation
c      lmax: the length of the correlation
c
      dimension x(1),y(1),r(1)
      do 10 l=1,lmax
     nl=m+1-l
     if (nl.ge.n-l) nl=n-l
       r(l)=0.0
       do 10 k=1,nl
         r(l)=r(l)+x(k)*y(k-l)
10    continue
      return
      end
c
c
c     computation of autocorrelation function
c
      subroutine autoc (x,n,r,l)
      dimension x(1),r(1)
      do 20 k=1,l
     r(k)=0.0
     nk=n-k+1
     do 20 nd=1,nk
       ndk=nd+k-1
       r(k)=r(k)+x(nd)*x(nd-k)
20    continue
      return
      end
c
c
```



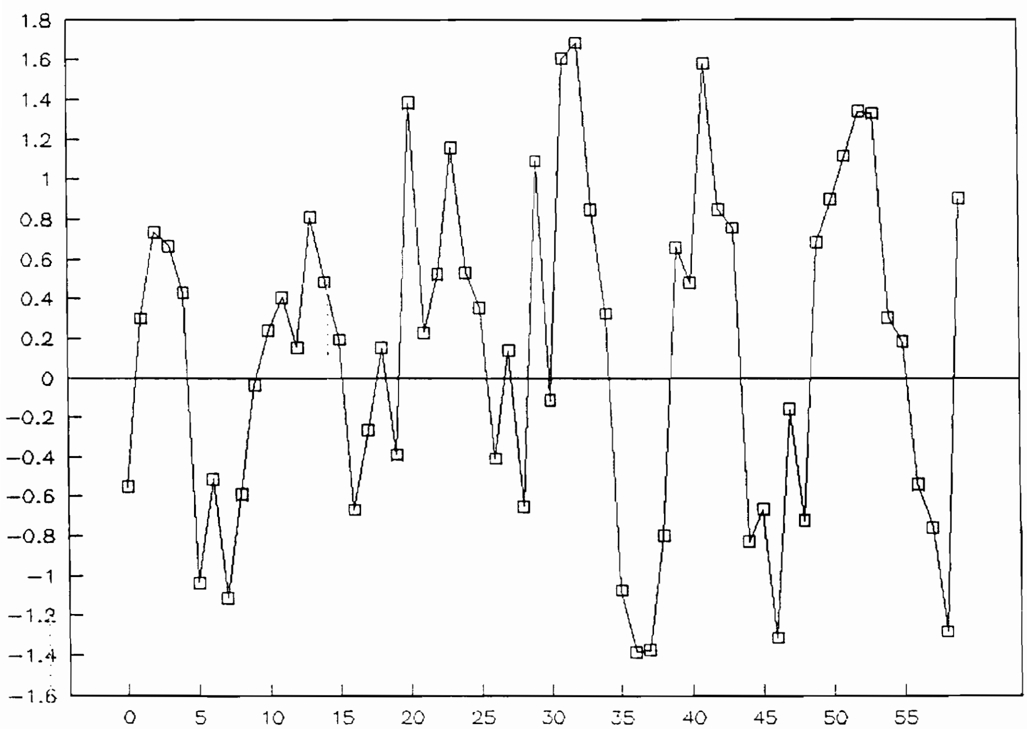
SNR = 3 dB, Correlated



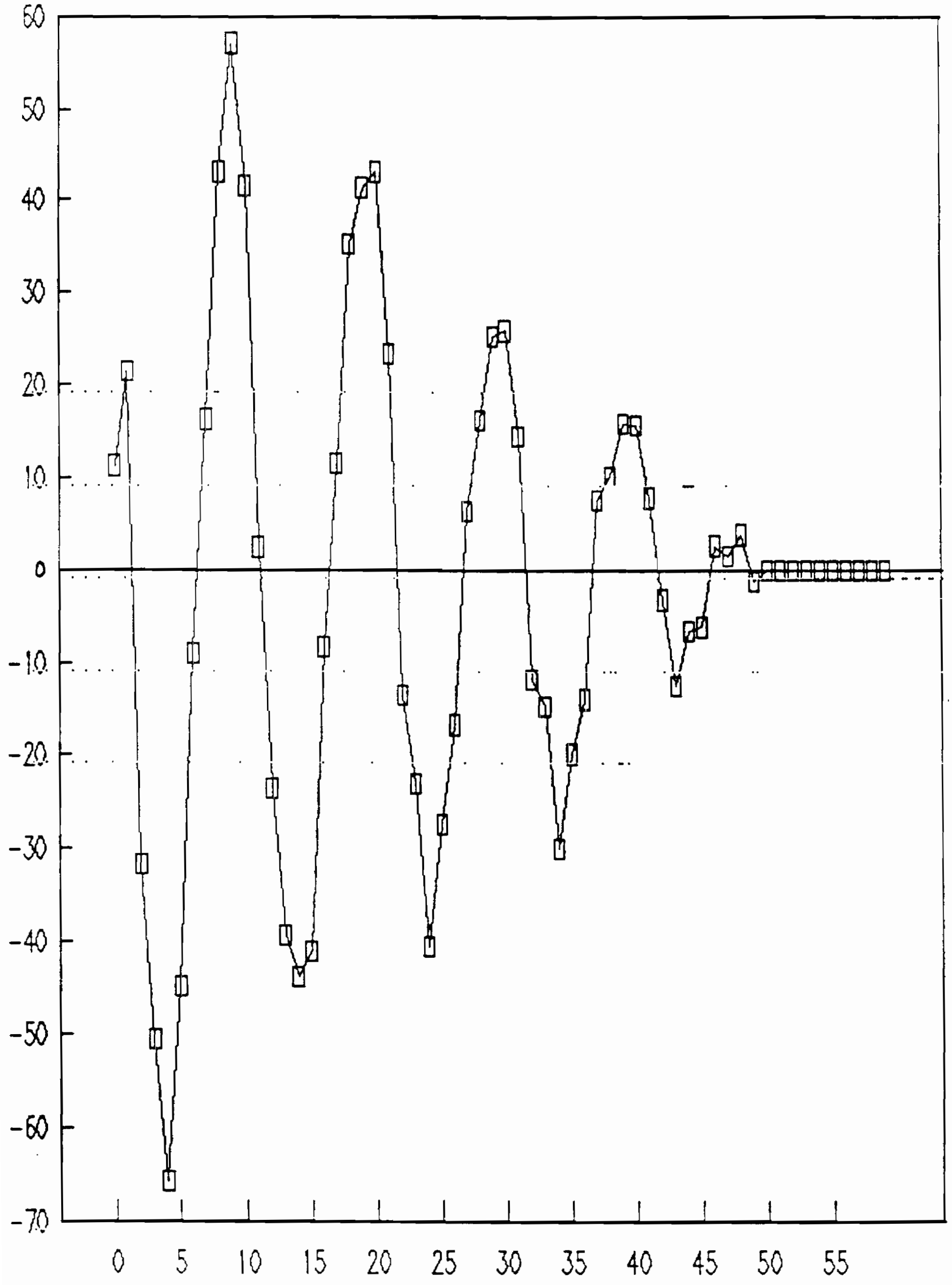
SNR = 3 dB, Uncorrelated



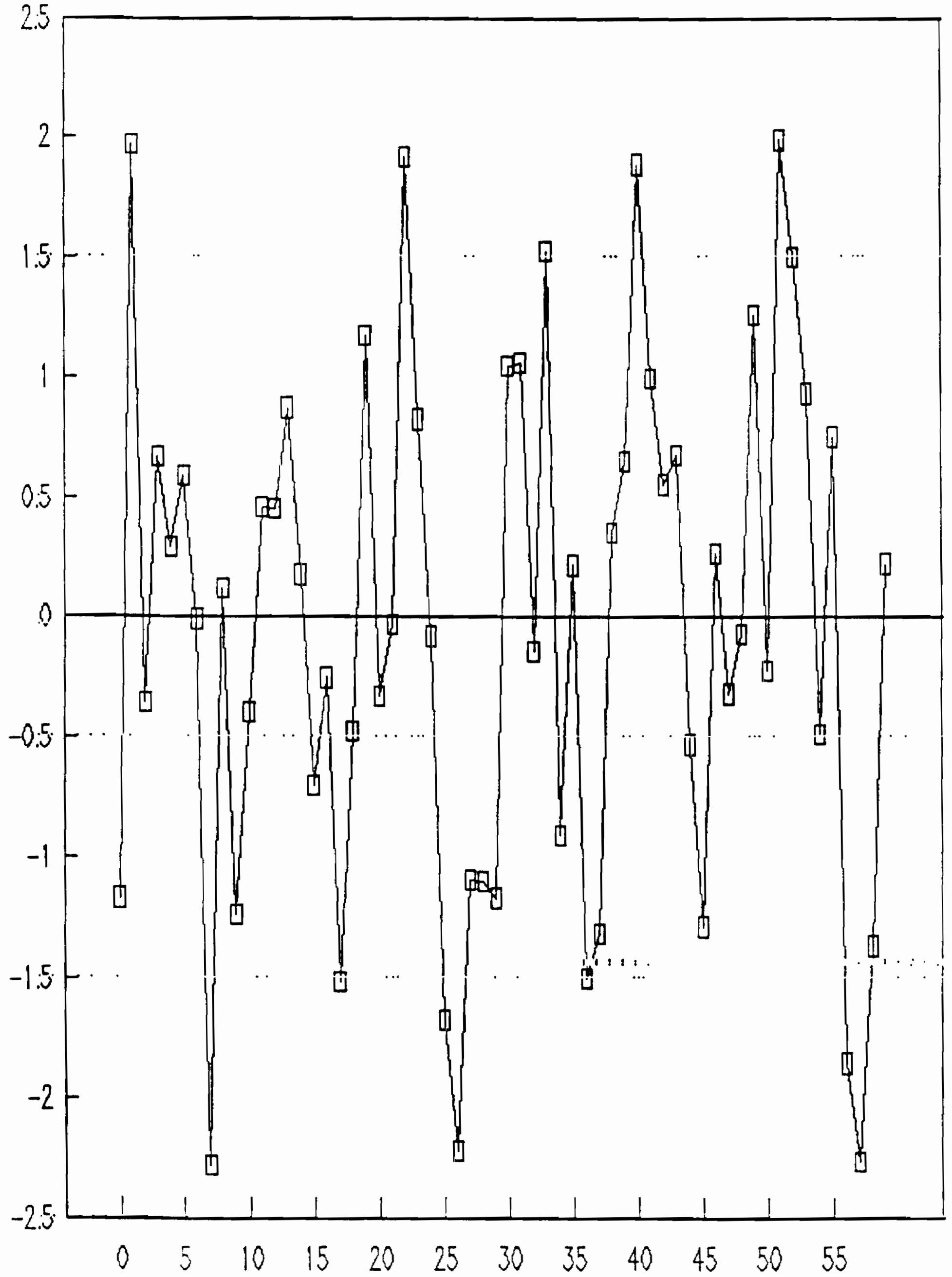
SNR = 1 dB, Correlated



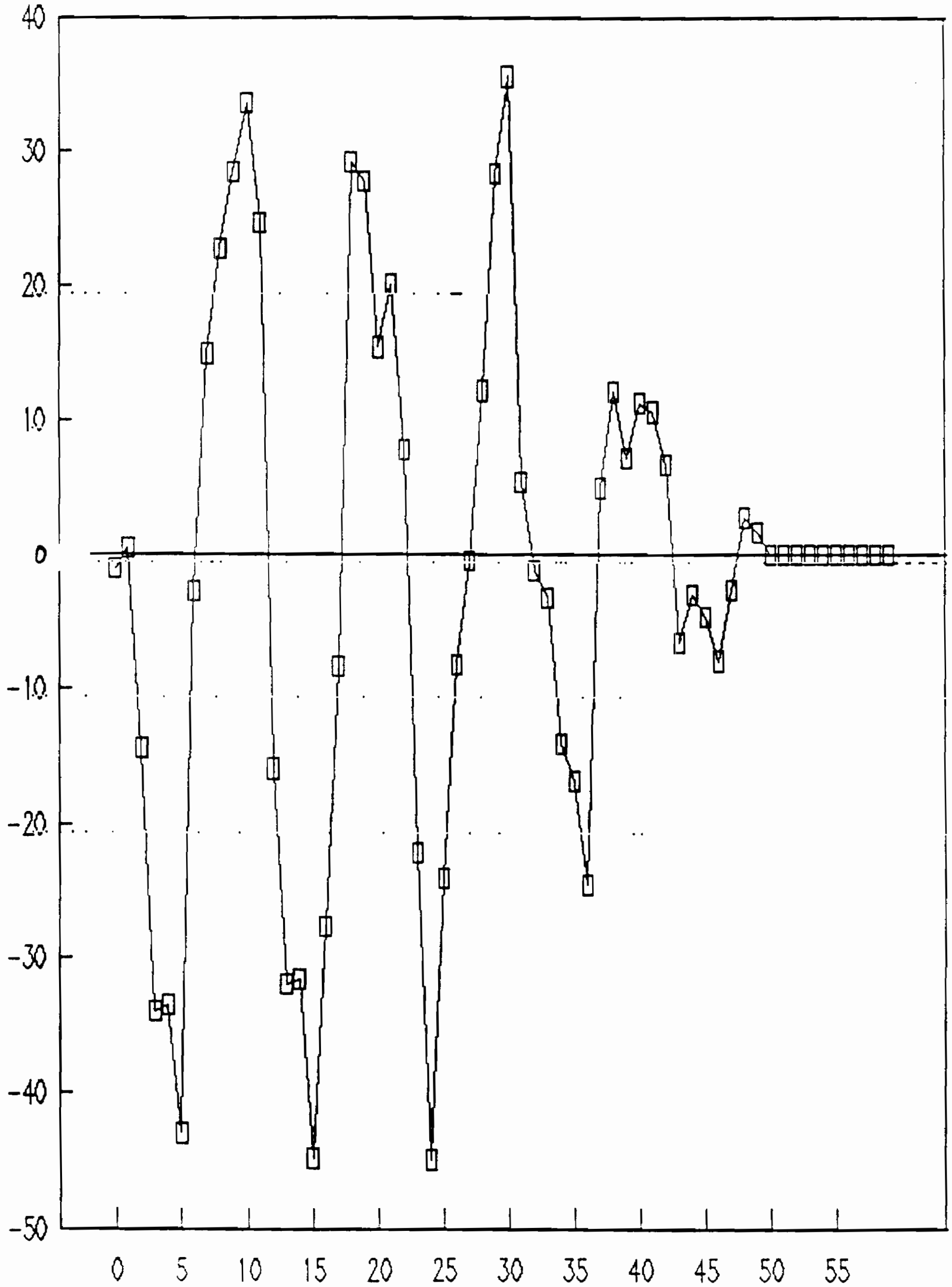
SNR = 1 dB, Uncorrelated



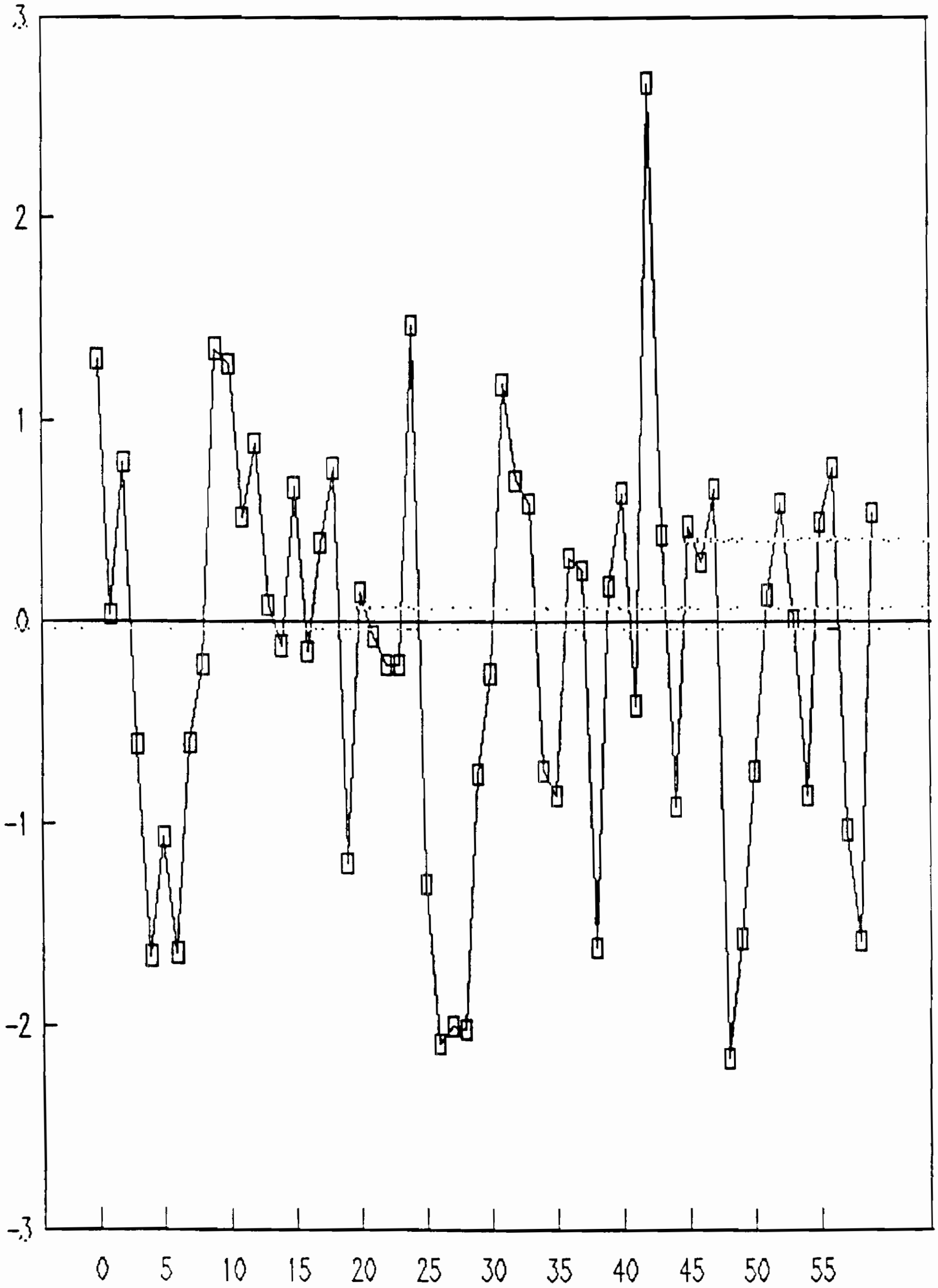
SNR = -1 dB, Correlated



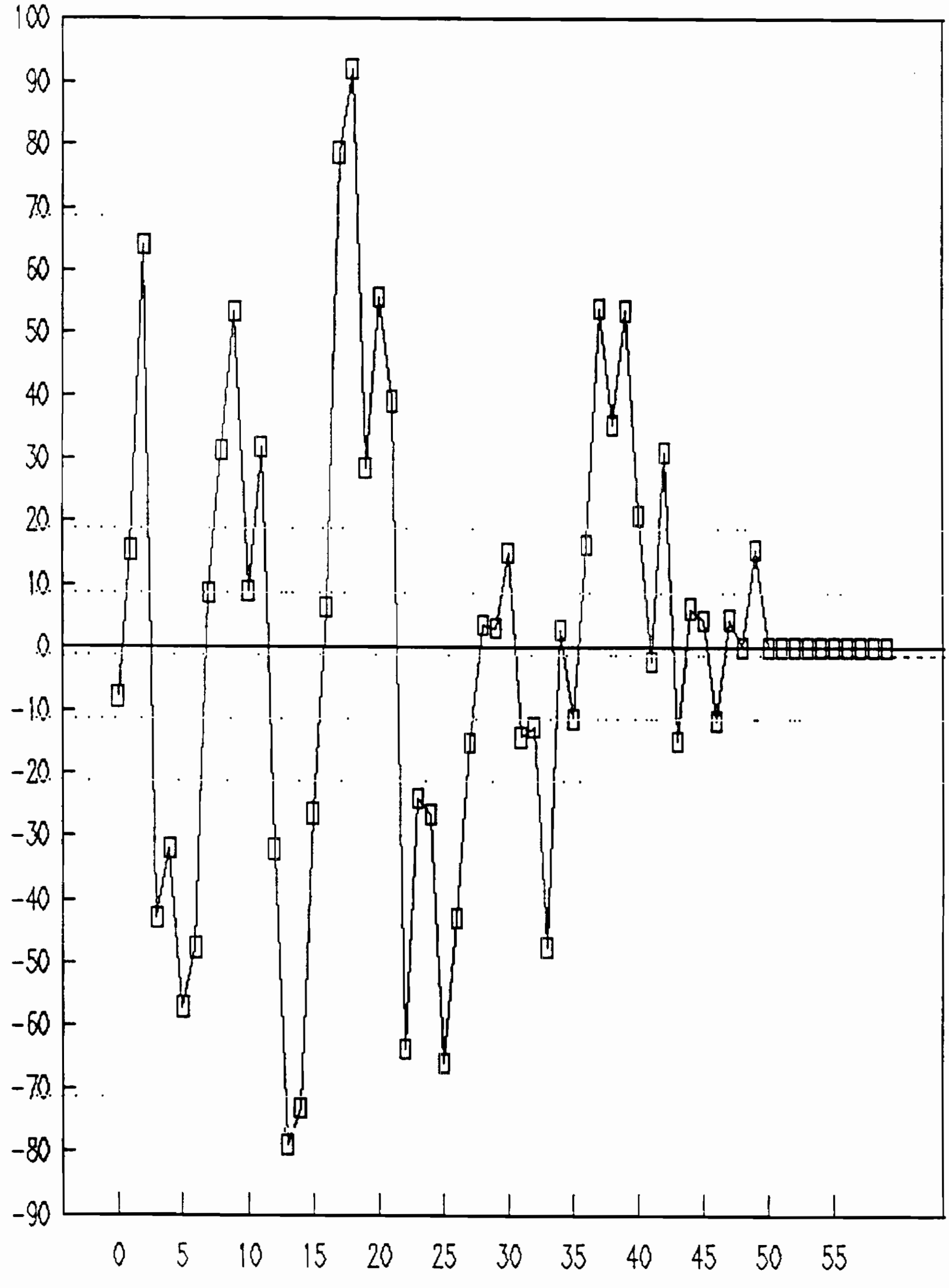
SNR = -1 dB, Uncorrelated



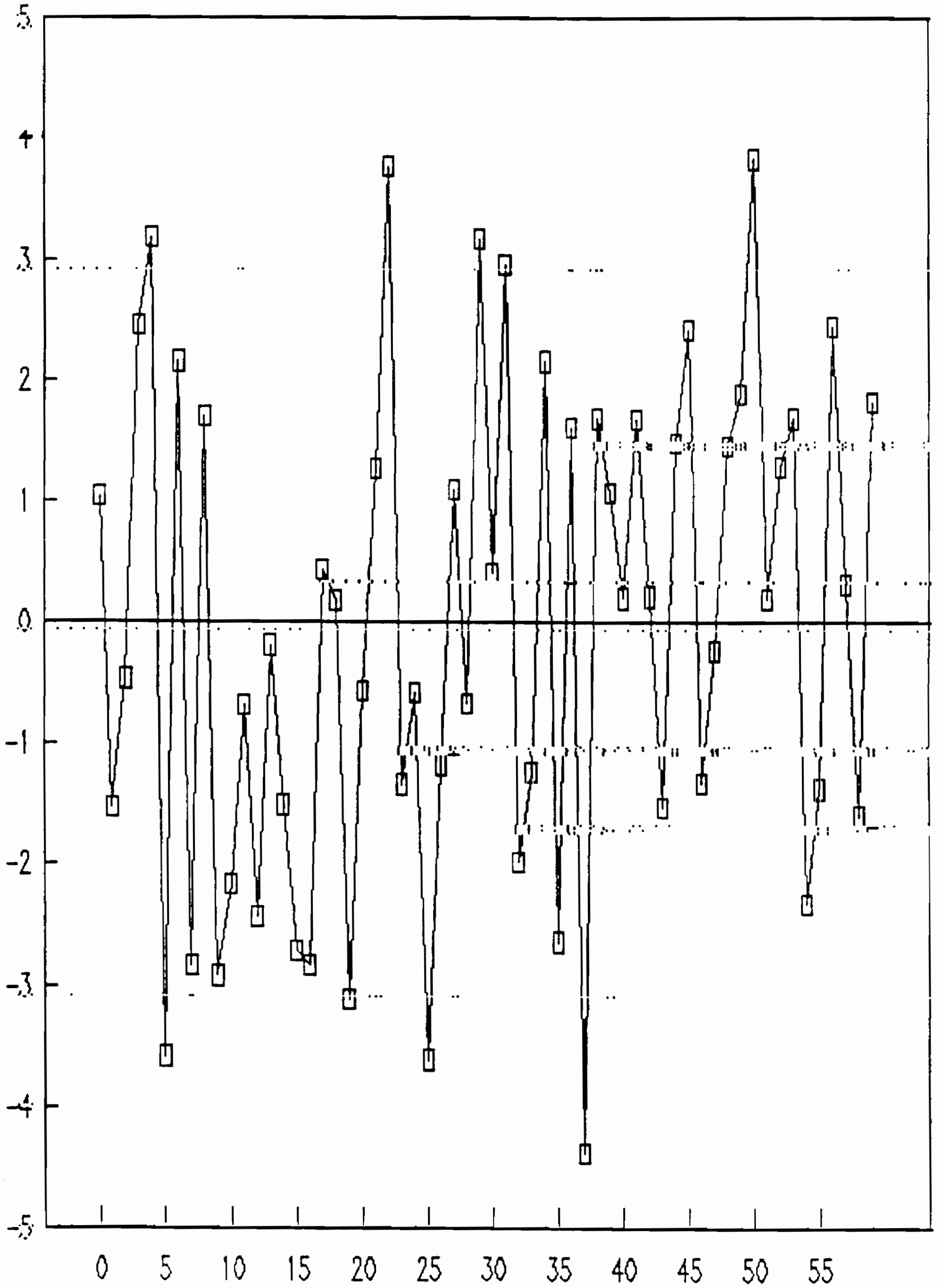
SNR = -3 dB, Correlated



SNR = -3 dB, Uncorrelated



SNR = -9 dB, Correlated



SNR = -9 dB, Uncorrelated

6.836707	0	0	55
19.37468	1	0	56
-12.7058	2	0	57
-44.9150	3	0	58
-49.6712	4	0	59
-41.4005	5		
-12.2709	6		
17.93306	7		
36.50703	8		
40.87254	9		
33.99157	10		
8.416156	11		
-18.6760	12		
-38.6703	13		
-39.8527	14		
-27.4593	15		
-6.82041	16		
12.3262	17		
30.67123	18		
33.20076	19		
24.8821	20		
9.247148	21		
-11.9110	22		
-23.7515	23		
-25.3147	24		
-22.4259	25		
-6.44254	26		
7.195776	27		
19.09197	28		
23.29208	29		
10.78435	30		
4.637113	31		
-5.88854	32		
-15.3509	33		
-14.3514	34		
-7.48015	35		
-1.72220	36		
3.935392	37		
8.99982	38		
8.296336	39		
5.614062	40		
-0.63236	41		
-3.38767	42		
-7.76275	43		
-3.96537	44		
-2.17561	45		
0.573798	46		
2.699674	47		
1.029805	48		
2.546162	49		
9.8E-45	50		
0	51		
0	52		
0	53		
0	54		

SNR = 3 dB, Correlated

-0.73676	0
1.593562	1
0.126157	2
0.638352	3
0.183460	4
0.155480	5
-0.35572	6
-1.78960	7
-0.14205	8
-0.77991	9
-0.03153	10
0.640585	11
0.638083	12
0.768085	13
0.110723	14
-0.66000	15
-0.51065	16
-1.30818	17
-0.51761	18
0.740653	19
0.007912	20
0.331688	21
1.558944	22
0.738663	23
-0.05111	24
-1.27256	25
-1.75212	26
-1.03863	27
-0.91014	28
-0.73769	29
0.878026	30
1.020628	31
0.259845	32
1.181297	33
-0.57256	34
-0.07988	35
-1.29866	36
-1.18211	37
0.005223	38
0.410464	39
1.406558	40
0.980239	41
0.701470	42
0.646280	43
-0.33687	44
-1.02879	45
-0.18377	46
-0.55249	47
-0.26168	48
0.793223	49
0.074435	50
1.604739	51
1.295384	52
0.805285	53
-0.30820	54
0.257592	55
-1.52191	56
-1.77867	57
-1.07965	58
0.140066	59

SNR = 3 dB, Uncorrelated

-0.55161	0
0.299487	1
0.735010	2
0.665574	3
0.427561	4
-1.03535	5
-0.51098	6
-1.11027	7
-0.58538	8
-0.03749	9
0.242197	10
0.407844	11
0.153104	12
0.810163	13
0.482598	14
0.193261	15
-0.66709	16
-0.26719	17
0.151032	18
-0.38963	19
1.382677	20
0.229506	21
0.524854	22
1.158612	23
0.532878	24
0.353091	25
-0.40722	26
0.137307	27
-0.65006	28
1.090869	29
-0.11243	30
1.606233	31
1.684415	32
0.846775	33
0.325267	34
-1.07086	35
-1.38112	36
-1.37191	37
-0.79778	38
0.659389	39
0.479093	40
1.578988	41
0.847324	42
0.756257	43
-0.82980	44
-0.66566	45
-1.31058	46
-0.15991	47
-0.71981	48
0.681410	49
0.896529	50
1.116661	51
1.339225	52
1.332219	53
0.306025	54
0.180806	55
-0.53602	56
-0.75690	57
-1.28031	58
0.903798	59

SNR = 1 dB, Uncorrelated

21.70579	0	1.7E-43	50
24.06465	1	0	51
-23.8571	2	0	52
-59.9994	3	0	53
-75.4439	4	0	54
-58.2775	5	0	55
-23.7009	6	0	56
17.0869	7	0	57
48.8127	8	0	58
59.84694	9	0	59
45.97027	10		
16.63373	11		
-25.2253	12		
-42.5564	13		
-57.1050	14		
-41.6760	15		
-13.6958	16		
8.12583	17		
34.24283	18		
45.85324	19		
41.23975	20		
20.74064	21		
-8.03601	22		
-27.9120	23		
-34.9032	24		
-30.6383	25		
-23.976	26		
-0.04027	27		
12.21246	28		
23.84666	29		
33.39807	30		
25.3037	31		
5.904254	32		
-14.6478	33		
-22.3693	34		
-23.3489	35		
-16.6014	36		
-5.05640	37		
10.96942	38		
21.52169	39		
17.08281	40		
12.06876	41		
0.869518	42		
-10.1886	43		
-13.0387	44		
-13.6369	45		
-8.72962	46		
-3.01654	47		
3.908934	48		
4.65426	49		

SNR = 1 dB, Correlated

-1.1677	0
1.969365	1
-0.35632	2
0.667929	3
0.290767	4
0.590218	5
-0.00751	6
-2.28007	7
0.118657	8
-1.23609	9
-0.39377	10
0.458990	11
0.455024	12
0.873544	13
0.175486	14
-0.70224	15
-0.25305	16
-1.51706	17
-0.47657	18
1.173864	19
-0.33125	20
-0.03058	21
1.9145	22
0.826912	23
-0.08100	24
-1.67308	25
-2.22067	26
-1.08985	27
-1.09869	28
-1.16917	29
1.04779	30
1.061321	31
-0.14444	32
1.528445	33
-0.90745	34
0.217183	35
-1.50197	36
-1.31725	37
0.352075	38
0.650545	39
1.88546	40
0.997308	41
0.555486	42
0.680495	43
-0.53391	44
-1.28673	45
0.265007	46
-0.31936	47
-0.07094	48
1.257182	49
-0.22582	50
1.98708	51
1.496782	52
0.932502	53
-0.48846	54
0.752057	55
-1.85581	56
-2.26274	57
-1.36734	58
0.221991	59

SNR = -1 dB, Uncorrelated

11.30172	0
21.45116	1
-31.6690	2
-50.4877	3
-65.7200	4
-44.8042	5
-9.08149	6
16.28338	7
43.13945	8
57.10151	9
41.53896	10
2.499433	11
-23.5189	12
-39.3201	13
-43.8137	14
-41.0635	15
-8.29382	16
11.5814	17
35.17485	18
41.34438	19
43.11062	20
23.35101	21
-13.5614	22
-23.0477	23
-40.5306	24
-27.349	25
-16.7454	26
6.385636	27
16.15451	28
25.19314	29
25.7914	30
14.48764	31
-11.8513	32
-14.7736	33
-29.9354	34
-19.8817	35
-13.9768	36
7.483633	37
10.24038	38
15.81841	39
15.65625	40
7.770146	41
-3.27354	42
-12.4683	43
-6.62681	44
-6.19303	45
2.648227	46
1.458084	47
3.802929	48
-1.20836	49
-7.8E-44	50
0	51
0	52
0	53
0	54
0	55
0	56
0	57
0	58
0	59

SNR = -1 dB, Correlated

1.301043	0
0.028030	1
0.791969	2
-0.61600	3
-1.65395	4
-1.06958	5
-1.63776	6
-0.60812	7
-0.21554	8
1.349095	9
1.278184	10
0.515042	11
0.883441	12
0.080694	13
-0.12352	14
0.667765	15
-0.15146	16
0.393358	17
0.762147	18
-1.19147	19
0.142380	20
-0.07531	21
-0.21179	22
-0.21417	23
1.468565	24
-1.29632	25
-2.08842	26
-1.99934	27
-2.01281	28
-0.75706	29
-0.25914	30
1.177184	31
0.703293	32
0.589238	33
-0.73904	34
-0.85841	35
0.315596	36
0.254799	37
-1.60657	38
0.174956	39
0.641057	40
-0.41920	41
2.673753	42
0.428760	43
-0.91972	44
0.472930	45
0.292847	46
0.664834	47
-2.15730	48
-1.56476	49
-0.74158	50
0.131745	51
0.592622	52
0.010235	53
-0.85882	54
0.497161	55
0.763563	56
-1.03220	57
-1.57516	58
0.542416	59

SNR = -3 dB, Uncorrelated

-1.02231	0
0.558254	1
-14.4532	2
-33.9047	3
-33.4318	4
-42.9546	5
-2.70510	6
14.8927	7
22.72163	8
28.41927	9
33.61829	10
24.64916	11
-15.9187	12
-31.9203	13
-31.5528	14
-44.8037	15
-27.5419	16
-8.35393	17
29.24006	18
27.75653	19
15.41201	20
20.12152	21
7.798084	22
-22.0474	23
-44.8945	24
-23.9668	25
-8.21048	26
-0.46281	27
12.14737	28
28.37618	29
35.58132	30
5.44963	31
-1.17697	32
-3.23262	33
-14.1114	34
-16.8858	35
-24.5266	36
5.022751	37
12.03435	38
7.133132	39
11.26507	40
10.59132	41
6.706447	42
-6.53471	43
-2.99017	44
-4.65481	45
-7.92951	46
-2.58586	47
2.748745	48
1.704967	49
1.5E-43	50
0	51
0	52
0	53
0	54
0	55
0	56
0	57
0	58
0	59

SNR = -3 dB, Correlated

1.038199	0
-1.53907	1
-0.46757	2
2.453526	3
3.181632	4
-3.58159	5
2.155336	6
-2.82947	7
1.695962	8
-2.91756	9
-2.17329	10
-0.68862	11
-2.43681	12
-0.18949	13
-1.51837	14
-2.71396	15
-2.83321	16
0.423230	17
0.175534	18
-3.11082	19
-0.56691	20
1.262396	21
3.76656	22
-1.33982	23
-0.58076	24
-3.60662	25
-1.1845	26
1.092749	27
-0.66682	28
3.175766	29
0.397566	30
2.960883	31
-1.98834	32
-1.24598	33
2.156069	34
-2.64207	35
1.606205	36
-4.38317	37
1.681805	38
1.058053	39
0.195478	40
1.67301	41
0.208459	42
-1.53722	43
1.472993	44
2.416451	45
-1.33708	46
-0.23673	47
1.450004	48
1.885554	49
3.825027	50
0.187832	51
1.271828	52
1.680084	53
-2.32623	54
-1.38220	55
2.439733	56
0.304795	57
-1.61075	58
1.819867	59

SNR = -9 dB, Uncorrelated

-7.88366	0
15.39614	1
64.09659	2
-42.7631	3
-31.8983	4
-56.9680	5
-47.6061	6
8.754358	7
31.34637	8
53.39524	9
8.839727	10
31.85546	11
-32.0793	12
-78.8232	13
-72.9914	14
-26.5015	15
6.483152	16
78.69638	17
92.0265	18
28.51464	19
55.63842	20
39.04184	21
-63.691	22
-24.1397	23
-26.5110	24
-65.9844	25
-42.932	26
-15.1490	27
3.600243	28
3.109817	29
15.1111	30
-14.3146	31
-12.5598	32
-47.4574	33
2.873367	34
-11.4119	35
16.23656	36
53.8498	37
35.38948	38
53.57365	39
20.84566	40
-2.17004	41
31.05666	42
-15.0255	43
6.295218	44
4.415704	45
-11.5268	46
4.476462	47
0.132567	48
15.62216	49
1.9E-43	50
0	51
0	52
0	53
0	54
0	55
0	56
0	57
0	58
0	59

SNR = -9 dB, Correlated