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Pati-Salam Unification from Non-commutative Geometry and the TeV-scale W_R boson

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We analyze the compatibility of the unified left-right symmetric Pati-Salam models motivated by non-commutative geometry and the TeV scale right-handed W boson suggested by recent LHC data. We find that the unification/matching conditions place conflicting demands on the symmetry breaking scales and that generating the required W_R mass and coupling is non-trivial.

Keywords: Non-commutative Geometry, Left-Right Symmetric Model, Pati-Salam

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1. Introduction and Overview

Canonical discussions on possible new physics beyond the Standard Model (SM) have been centered around the hierarchy problem and the unifications of couplings. The current favorites among various approaches to stabilizing the low Higgs mass (126 GeV as found at the LHC^{1,2}) are supersymmetry, technicolor, and extra dimensions. These approaches also incorporate the philosophy of coupling unification in Grand Unified Theories (GUT's).

To this list, we seek to add another contender, namely models based on the non-commutative geometry (NCG) of Connes.^{3,4} In a series of papers starting from 1990,⁵⁻¹⁴ Connes and collaborators have argued that the SM action could be derived from a particular NCG via the construction of what they call the “spectral” action,⁷ in essence geometrizing the SM and placing it on a similar footing to gravity. Several of the predictions that result from the approach, according to our current understanding, are quite remarkable:^a

^aWe are currently working on a review article explaining how these predictions come about.¹⁵

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- The $SU(2)_L$ gauge bosons and the Higgs doublet are unified into a single “superconnection,” one of the consequences being that the $SU(2)_L$ gauge coupling g_L and the Yukawa couplings are related in a particular way.^{5,8–10}
- The $SU(2)_L \times U(1)_Y \times SU(3)_C$ gauge couplings satisfy an $SO(10)$ GUT-like relation, even though the particle content of the model is that of the SM.^{7,12}
- Anomaly cancellation requires the presence of both electroweak and QCD sectors, another GUT-like feature.⁸
- The smallness of the Higgs boson mass can potentially be explained via an extra-dimension-like mechanism involving a ‘warp’-factor.¹²

The approach, of course, is not without its problems:

- The GUT-like relations on the gauge couplings can only be imposed at a single scale, so one must interpret the NCG spectral action as that which ‘emerges’ from an underlying NCG theory at the ‘unification’ scale.
- Quantization of the model within the NCG framework (in the sense of path integrals) is yet to be fully explored,^{3,4} so one usually treats the NCG spectral action as an effective QFT action at the unification scale, and evolve down to lower energies using the usual Renormalization Group (RG) equations to work out the infrared consequences.
- The minimal version of NCG model which describes the SM predicts a Higgs mass of ~ 170 GeV, in clear contradiction with experiment.¹² This issue could be remedied by turning one of the off-diagonal entries of the Dirac operator, which is responsible for the neutrino Majorana mass, into a field. With this singlet field coupled to the SM Higgs field, the model accommodates a 126 GeV Higgs boson.¹³ This could also be accomplished by extending the NCG to that which leads to a left-right symmetric Pati-Salam type action with coupling unification which automatically involves this singlet field.¹⁴

In addition, the NCG spectral action approach to particle physics is under continued development by Connes and collaborators, and sorting out the various versions can be difficult.

Despite these caveats, however, or any other reservation one may have about the entire approach, it is not without its merits, as explained above, and we feel that it may have the potential to develop into a full-fledged paradigm. In particular, from the phenomenological standpoint, the necessity to enlarge the gauge symmetry (via an enlargement of the underlying NCG) to accommodate the Higgs mass can be considered a strength rather than a weakness. It tells us that the approach is restrictive enough for the models to be confronted by experiment, and point us in new directions to explore.

Indeed, in a recent paper,¹⁶ Chamseddine, Connes, and van Suijlekom have proposed a new formulation of an NCG based unified left-right symmetric Pati-

Salam model, which comes in three different versions differing in Higgs content. In all three, the gauge theory which emerges from the underlying NCG at the unification scale, which we will call M_U , is that with gauge symmetry $G_{224} = SU(2)_L \times SU(2)_R \times SU(4)_C$ with unified couplings:

$$g_L(M_U) = g_R(M_U) = g_4(M_U). \quad (1)$$

In one version, the symmetry is actually $G_{224D} = G_{224} \times D$, where D denotes parity which maintains left-right symmetry.^b

G_{224} or G_{224D} is assumed to break down to $G_{213} = SU(2)_L \times U(1)_Y \times SU(3)_C$ of the SM at scale M_R with matching conditions

$$\begin{aligned} \frac{1}{[g_1(M_R)]^2} &= \frac{2}{3} \frac{1}{[g_4(M_R)]^2} + \frac{1}{[g_R(M_R)]^2}, \\ \frac{1}{[g_2(M_R)]^2} &= \frac{1}{[g_L(M_R)]^2}, \\ \frac{1}{[g_3(M_R)]^2} &= \frac{1}{[g_4(M_R)]^2}. \end{aligned} \quad (2)$$

For all three versions, which differ in particle content, Ref. 16 argues that both boundary conditions can be satisfied if $M_U \sim 10^{15}$ GeV and $M_R \sim 10^{13}$ GeV.

In this paper, we will not attempt to review or justify the derivation of these models, but look at their consequences purely phenomenologically. From that viewpoint, the high value of M_R is problematic in light of recent hints of a W_R with a mass of around 2 TeV at the LHC.^{17–22} If the LHC signal is indeed the gauge boson of the $SU(2)_R$ group, then M_R on the order of a few TeV would be more compatible with that possibility. For instance, in Refs. 23, 24 we proposed an $su(2/2)$ superconnection-based left-right symmetric model for which $M_R = 4$ TeV, placing the mass of W_R in the correct range. We address the question whether M_R for Chamseddine et al.'s NCG models can be lowered by the addition of intermediate breaking scales between M_U and M_R at which the symmetry breaks down from G_{224D}/G_{224} to G_{213} via several intermediate steps. In other words, is any symmetry breaking pattern compatible with a unified left-right symmetric Pati-Salam model at M_U , and the SM below $M_R \sim \text{few TeV}$? We will demand that M_U stay below the Planck mass at 10^{19} GeV. Similar analyses have been carried out in the context of non-supersymmetric $SO(10)$ GUT models in Refs. 25–32 for a variety of symmetry breaking chains.^c Our analysis differs from these due to the NCG models considered here differing in Higgs content since NCG does not require the Higgs fields to fall into $SO(10)$ multiplets.

^b D -parity is slightly different from the usual Lorentz (P) parity in that the former interchanges the $SU(2)_L$ and $SU(2)_R$ sectors completely (including the scalars), while the latter does not transform the scalars. For example, the D -parity interchanges the $SU(2)_L$ Higgs fields and their $SU(2)_R$ counterparts, and transforms the bidoublet ϕ into ϕ^\dagger (and vice versa), while the P -parity leaves them unchanged.

^cFor analyses of supersymmetric $SO(10)$ GUT, see Refs. 33–36.

While originally motivated by the desire to confront the viability of NCG derived unified left-right symmetric Pati-Salam models, we note in passing that similar models may emerge in a large class of string compactifications as discovered by Dienes.³⁷ So the results presented here may have a wider range of applicability.

This paper is organized as follows. In section 2, we briefly review the current status of the W_R like signal seen at the LHC, and what the phenomenological constraints are. In section 3, we cover various symmetry breaking chains from G_{224D}/G_{224} down to G_{213} , and solve the renormalization group evolution equations for breaking scales which would satisfy the boundary/matching conditions for the given particle content. The list includes those that were considered by Chamseddine, Connes, and van Suijlekom in Ref. 16. We conclude in section 4 with a discussion of what was discovered.

2. Status of the W_R signal at the LHC

Recently, ATLAS reported on a search for narrow resonances hadronically decaying into a pair of SM gauge bosons WW , WZ , or ZZ .¹⁷ The largest excess occurs in the WZ channel at around 2 TeV with a local significance of 3.4σ and a global 2.5σ . Moreover, both CMS¹⁸ and ATLAS¹⁹ notice an excess at around 1.8 TeV in the dijet distributions albeit with low significance (2.2σ and 1σ). In addition, CMS observes an excess, again at around 2 TeV, both in their search for massive WH production in the $\ell\nu b\bar{b}$ final state²⁰ and in massive resonance production decaying into two SM vector bosons (one of which is leptonically tagged²¹), both of which have lower significance than 2σ .

It is discussed in Refs. 38–41 that these results may be interpreted in the context of the left-right model with the gauge group $SU(2)_L \times SU(2)_R \times U(1)'$ and it is shown that a heavy right-handed gauge boson W_R with a single coupling $g_R(M_R) \simeq 0.4$ can explain the current measurements. Note that this coupling is different from the SM left-handed W_L coupling $g_L(5\text{TeV}) \simeq 0.63$.^{42,43} Many other authors have also discussed possible phenomenological consequences of the W_R interpretation, e.g. Refs. 44–55 to list just a few, but we refrain from reviewing them here.

3. TeV-scale left-right model in the light of latest LHC searches

3.1. Setup of the Problem

We would like to see whether such a W_R can be accommodated within an NCG induced unified left-right symmetric Pati-Salam model. The left-right symmetric model naturally has $g_R = g_L$. However, one can have an asymmetry between g_R and g_L if one separates the D -parity⁵⁶ breaking scale M_D from the the scale M_R where $SU(2)_R$ is broken.^{57,58}

As an intermediate symmetry between G_{224D}/G_{224} and G_{213} of the SM, we introduce

$$G_{2213} = SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C, \quad (3)$$

with gauge couplings g_L , g_R , g_{BL} , and g_3 . The most general breaking sequence will then be

$$\text{NCG} \xrightarrow{M_U} G_{224D} \xrightarrow{M_D} G_{224} \xrightarrow{M_C} G_{2213} \xrightarrow{M_R} G_{213} \xrightarrow{M_Z} G_{13}, \quad (4)$$

where the double-line arrow indicates the emergence of the G_{224D} theory from the underlying NCG, and $G_{13} = U(1)_{\text{EM}} \times SU(3)_C$ is the unbroken group which remains below the electroweak scale with couplings e and g_3 .

We label the energy intervals in between symmetry breaking scales starting from $[M_Z, M_R]$ up to $[M_D, M_U]$ with Roman numerals as:

$$\begin{aligned} \text{I} &: [M_Z, M_R], & G_{213} \text{ (SM)}, \\ \text{II} &: [M_R, M_C], & G_{2213}, \\ \text{III} &: [M_C, M_D], & G_{224}, \\ \text{IV} &: [M_D, M_U], & G_{224D}. \end{aligned} \quad (5)$$

The ordering of the breaking scales must be strictly maintained, that is

$$M_Z \leq M_R \leq M_C \leq M_D \leq M_U. \quad (6)$$

However, adjacent scales can be set equal which collapses the corresponding energy interval and skips the intermediate step in between. For instance, if $M_R = M_C$, then G_{224} breaks directly to G_{213} , and interval III will be followed by interval I, skipping interval II.

In the following, we will investigate whether it is possible to set $M_R \sim 5$ TeV, while maintaining M_U below the Planck scale. The IR data which we will keep fixed as boundary conditions to the RG running are^{42,43}

$$\begin{aligned} \alpha(M_Z) &= 1/127.9, \\ \alpha_s(M_Z) &= 0.118, \\ \sin^2 \theta_W(M_Z) &= 0.2312, \end{aligned} \quad (7)$$

at $M_Z = 91.1876$ GeV, which translates to

$$g_1(M_Z) = 0.36, \quad g_2(M_Z) = 0.65, \quad g_3(M_Z) = 1.22. \quad (8)$$

The coupling constants are all required to remain in the perturbative regime during the evolution from M_U down to M_Z .

3.2. One-Loop Running and the Extended Survival Hypothesis

For a given particle content, the gauge couplings are evolved according to the 1-loop RG relation

$$\frac{1}{g_i^2(M_A)} - \frac{1}{g_i^2(M_B)} = \frac{a_i}{8\pi^2} \ln \frac{M_B}{M_A}, \quad (9)$$

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where the RG coefficients a_i are given by^{59,60}

$$\begin{aligned}
 a_i = & -\frac{11}{3}C_2(G_i) + \frac{2}{3} \sum_{R_f} T_i(R_f) \cdot d_1(R_f) \cdots d_n(R_f) \\
 & + \frac{1}{3} \sum_{R_s} T_i(R_s) \cdot d_1(R_s) \cdots d_n(R_s) .
 \end{aligned} \tag{10}$$

Here, the summation is over irreducible chiral representations of fermions (R_f) in the second term and those of scalars (R_s) in the third. $C_2(G_i)$ is the quadratic Casimir for the adjoint representation of the group G_i , and T_i is the Dynkin index of each (complex) representation.^d For $SU(2)$, $C_2(G) = 2$, $T = 1/2$ for doublet representations and $T = 2$ for triplets. See Table 1 for the Dynkin indexes of other representations. For $U(1)$, $C_2(G) = 0$ and

$$\sum_{f,s} T = \sum_{f,s} \left(\frac{Y}{2} \right)^2 , \tag{11}$$

where $Y/2$ is the $U(1)$ charge, the factor of $1/2$ coming from the traditional normalizations of the hypercharge Y and $B - L$ charges. The a_i 's will differ depending on the particle content, which changes every time symmetry breaking occurs. We will distinguish the a_i 's in different energy intervals with the corresponding roman numeral superscript, cf. Eq. (5).

For the particle content in each energy interval we impose the *Extended Survival Hypothesis* (ESH).⁶² ESH states that at every step of the symmetry breaking chain,

^dIf the representation is real a factor of $\frac{1}{2}$ comes about in the third term.

Table 1. Dynkin index T_i for several irreducible representations of $SU(2)$, $SU(3)$, and $SU(4)$. Different normalization conventions are used in the literature. For example, there is a factor of 2 difference between those given in Ref. 60 and those in Ref. 61. Our convention follows the former. For $SU(3)$, there exist two inequivalent 15 dimensional irreducible representations.

Representation	$SU(2)$	$SU(3)$	$SU(4)$
2	$\frac{1}{2}$	—	—
3	2	$\frac{1}{2}$	—
4	5	—	$\frac{1}{2}$
6	$\frac{35}{2}$	$\frac{5}{2}$	1
8	42	3	—
10	$\frac{165}{2}$	$\frac{15}{2}$	3
15	280	10, $\frac{35}{2}$	4

the only scalars which survive below the symmetry breaking scale are the ones which acquire vacuum expectation values (VEV's) at the subsequent levels of the symmetry breaking. For instance, the only scalar assumed to survive below M_R would be the SM Higgs doublet which acquires a VEV to break G_{213} further down to G_{13} under the ESH.

3.3. Non-Unified Left-Right Symmetric Pati-Salam

We begin by looking at the Pati-Salam model^{63–66} without the unification of all three couplings as demanded in the NCG approach. We impose left-right symmetry $g_L = g_R$ at scale M_D , which we identify as the scale at which G_{224D} breaks to G_{224} , and evolve our couplings down from M_D :

$$G_{224D} \xrightarrow{M_D} G_{224} \xrightarrow{M_C} G_{2213} \xrightarrow{M_R} G_{213} \xrightarrow{M_Z} G_{13}. \quad (12)$$

Note that energy interval IV is absent. In addition to Eq. (8), the boundary/matching conditions we impose on the couplings at the symmetry breaking scales are:

$$M_D : g_L(M_D) = g_R(M_D), \quad (13)$$

$$M_C : \sqrt{\frac{2}{3}} g_{BL}(M_C) = g_3(M_C) = g_4(M_C), \quad (14)$$

$$M_R : \frac{1}{g_1^2(M_R)} = \frac{1}{g_R^2(M_R)} + \frac{1}{g_{BL}^2(M_R)}, \quad g_2(M_R) = g_L(M_R), \quad (15)$$

$$M_Z : \frac{1}{e^2(M_Z)} = \frac{1}{g_1^2(M_Z)} + \frac{1}{g_2^2(M_Z)}. \quad (16)$$

Note that if $M_C = M_R$, then the conditions at those scales reduce to those given in Eq. (2).

We assume that the above breaking sequence is accomplished by a Higgs sector consisting of scalars which transform under G_{224} as

$$\phi(2, 2, 1), \quad \Delta_R(1, 3, 10), \quad \Sigma(1, 1, 15). \quad (17)$$

These fields decompose into irreducible representations of G_{2213} as:

$$\begin{aligned} \Sigma(1, 1, 15) &= \Sigma_1(1, 1, 0, 1) \oplus \Sigma_3 \left(1, 1, \frac{4}{3}, 3\right) \oplus \Sigma_{\bar{3}} \left(1, 1, \frac{-4}{3}, \bar{3}\right) \oplus \Sigma_8(1, 1, 0, 8), \\ \Delta_R(1, 3, 10) &= \Delta_{R1}(1, 3, 2, 1) \oplus \Delta_{R3} \left(1, 3, \frac{2}{3}, 3\right) \oplus \Delta_{R6} \left(1, 3, \frac{-2}{3}, 6\right), \\ \phi(2, 2, 1) &= \phi(2, 2, 0, 1). \end{aligned} \quad (18)$$

The breaking of G_{224} down to G_{2213} would be accomplished by the field Σ_1 acquiring a VEV. $\Sigma_3, \Sigma_{\bar{3}}, \Sigma_8, \Delta_{R3}, \Delta_{R6}$ are all colored, so they will not be acquiring VEV's in the subsequent steps. Thus, under the ESH, all these fields will become heavy at M_C

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Table 2. The Higgs content and RG coefficients in the three energy intervals for the non-unified left-right symmetric Pati-Salam model under the Extended Survival Hypothesis (ESH).

Interval	Higgs content	RG coefficients
III	$\phi(2, 2, 1), \Delta_R(1, 3, 10), \Sigma(1, 1, 15)$	$(a_L, a_R, a_4)^{\text{III}} = \left(-3, \frac{11}{3}, -7\right)$
II	$\phi(2, 2, 0, 1), \Delta_{R1}(1, 3, 2, 1)$	$(a_L, a_R, a_{BL}, a_3)^{\text{II}} = \left(-3, \frac{-7}{3}, \frac{11}{3}, -7\right)$
I	$\phi_2(2, 1, 1)$	$(a_1, a_2, a_3)^{\text{I}} = \left(\frac{41}{6}, \frac{-19}{6}, -7\right)$

and decouple from the RG equations below M_C . The remaining fields decompose into irreducible representations of G_{213} as:

$$\begin{aligned} \Delta_{R1}(1, 3, 2, 1) &= \Delta_{R1}^0(1, 0, 1) \oplus \Delta_{R1}^+(1, 2, 1) \oplus \Delta_{R1}^{++}(1, 4, 1), \\ \phi(2, 2, 0, 1) &= \phi_2(2, 1, 1) \oplus \phi'_2(2, -1, 1). \end{aligned} \quad (19)$$

The breaking of G_{2213} down to G_{213} would be accomplished by the field Δ_{R1}^0 , while that of G_{213} down to G_{13} would be realised by the neutral (diagonal) components of $\phi_2(2, 2, 0, 1)$, acquiring VEVs. The fields Δ_{R1}^+ and Δ_{R1}^{++} would be both charged under electromagnetism, so they will not be acquiring VEV's in the subsequent steps. Thus, under the ESH, these fields will become heavy at M_R . In addition, only one of the two physical states (which are linear combinations of ϕ_2 and ϕ'_2) remains light while the other picks a mass at M_R , unless we apply fine-tuning.⁶⁷ The left-over field, the SM Higgs (which can be identified without loss of generality as $\phi_2(2, 1, 1)$) is left to be the only field in the Higgs spectrum below M_R . Thus, under the ESH, the particle content (other than the fermions and gauge bosons) of our model in the three energy intervals I through III are:

$$\begin{aligned} \text{III} &: \phi(2, 2, 1), \Delta_R(1, 3, 10), \Sigma(1, 1, 15), \\ \text{II} &: \phi(2, 2, 0, 1), \Delta_{R1}(1, 3, 2, 1), \\ \text{I} &: \phi_2(2, 1, 1). \end{aligned} \quad (20)$$

The values of the RG coefficients for this Higgs content are listed in Table 2.

Taking advantage of the boundary/matching conditions, the following relations can be derived between the boundary values $\alpha(M_Z)$, $\alpha_s(M_Z)$, $\sin^2 \theta_W(M_Z)$, $g_R(M_R)$, and the ratios of the successive symmetry breaking scales:

$$\begin{aligned} &2\pi \left[\frac{3 - 6 \sin^2 \theta_W(M_Z)}{\alpha(M_Z)} - \frac{2}{\alpha_s(M_Z)} \right] \\ &= \underbrace{(3a_1 - 3a_2 - 2a_3)^{\text{I}}}_{44} \ln \frac{M_R}{M_Z} + \underbrace{(-3a_L + 3a_R + 3a_{BL} - 2a_3)^{\text{II}}}_{27} \ln \frac{M_C}{M_R} \\ &\quad + \underbrace{(-3a_L + 3a_R)^{\text{III}}}_{20} \ln \frac{M_D}{M_C}, \end{aligned}$$

$$\begin{aligned}
 & 2\pi \left[\frac{4\pi}{g_R^2(M_R)} - \frac{\sin^2 \theta_W(M_Z)}{\alpha(M_Z)} \right] \\
 &= \underbrace{(-a_2)^I}_{19/6} \ln \frac{M_R}{M_Z} + \underbrace{(a_R - a_L)^{II}}_{2/3} \ln \frac{M_C}{M_R} + \underbrace{(a_R - a_L)^{III}}_{20/3} \ln \frac{M_D}{M_C}. \quad (21)
 \end{aligned}$$

The derivation is shown in the Appendix. To maintain the ordering of the mass scales, all logarithms in these expressions must be non-negative. Numerically, we have

$$\begin{aligned}
 517 &= 44x + 27y + 20z, \\
 \frac{206}{g_R^2(M_R)} - 484 &= 19x + 4y + 40z, \quad (22)
 \end{aligned}$$

where

$$x = \log_{10} \frac{M_R}{M_Z}, \quad y = \log_{10} \frac{M_C}{M_R}, \quad z = \log_{10} \frac{M_D}{M_C}. \quad (23)$$

If we fix $M_R = 5 \text{ TeV}$, then $x = \log_{10}(M_R/M_Z) = 1.74$, and the above system of linear equations yields

$$y = 27.9 - \frac{4.11}{g_R^2(M_R)}, \quad z = -15.7 + \frac{5.56}{g_R^2(M_R)}. \quad (24)$$

Since both y and z must be positive, we must have

$$0.38 < g_R(M_R) < 0.59. \quad (25)$$

We would also like to impose the condition

$$x + y + z = 14.0 + \frac{1.44}{g_R^2(M_R)} = \log_{10} \frac{M_D}{M_Z} < \log_{10} \frac{10^{19} \text{ GeV}}{M_Z} = 17.0, \quad (26)$$

which constrains $g_R(M_R)$ to

$$g_R(M_R) > 0.69, \quad (27)$$

which is incompatible with Eq. (25). Thus, the system does not allow for a parity breaking scale M_D lower than the Planck mass.

Indeed, if we set $g_R(M_R) = 0.4$ as preferred by experiment,^{38–41} we obtain $y = 2.2$, $z = 19.0$, which translates to

$$M_R = 5 \text{ TeV}, \quad M_C = 8 \times 10^5 \text{ GeV}, \quad M_D = 8 \times 10^{24} \text{ GeV}, \quad (28)$$

with

$$g_L(M_D) = g_R(M_D) = 0.48, \quad g_4(M_D) = 0.43. \quad (29)$$

If we allow $g_R(M_R)$ to be as large as 0.59, we obtain $y = 16.3$, $z = 0$, which translates to

$$M_R = 5 \text{ TeV}, \quad M_C = M_D = 1 \times 10^{20} \text{ GeV}, \quad (30)$$

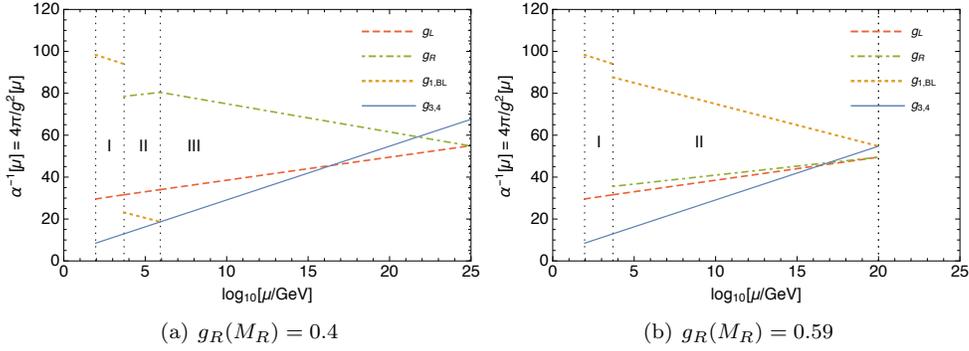


Fig. 1. Running of the gauge couplings for the left-right symmetric Pati-Salam model. The vertical dotted lines from left to right correspond to the symmetry breaking scales M_Z , M_R , and M_C . M_R is fixed at 5 TeV. For the $U(1)_{B-L}$ coupling between M_R and M_C , we plot $\frac{3}{2}\alpha_{BL}^{-1}(\mu) = \frac{6\pi}{g_{BL}^2(\mu)}$ so that it agrees with $\alpha_4^{-1}(\mu)$ at $\mu = M_C$. The two cases shown are (a) $g_R(M_R) = 0.4$ is imposed, and (b) M_D is minimized by collapsing the energy interval III.

with

$$g_L(M_D) = g_R(M_D) = 0.50, \quad g_4(M_D) = 0.48. \quad (31)$$

The evolution of the couplings for these choices of scales are shown in Fig. 1. For each choice of $g_R(M_R)$, the value of $g_{BL}(M_R)$ is determined from the known value of the hypercharge coupling $g_1(M_R)$ and the matching condition Eq. (15). Larger values of $g_R(M_R)$ closer to $g_L(M_R)$ will lower the scale M_D at which the RG flow of the two couplings separate. At the same time, larger values of $g_R(M_R)$ demand smaller values of $g_{BL}(M_R)$, which pushes up the scale M_C where the RG flow of g_{BL} bifurcates from that of g_3 . Since the order $M_C \leq M_D$ cannot be violated, M_D cannot be lowered further by increasing $g_R(M_R)$ once the two scales meet.

Looking at Fig. 1(b), however, we notice that in energy interval II g_L and g_R do flow apart, but not as much as in energy interval III. A larger difference between g_L and g_R could be generated in interval II if $(a_L - a_R)^{\text{II}}$ could be enhanced. To this end, let us relax the ESH and allow some of the colored Δ_R fields to survive into interval II. The RG coefficients for three Higgs-content scenarios in interval II different from the ESH case are listed in Table 3. Clearly, the addition of extra Δ_R fields enhances $(a_L - a_R)^{\text{II}}$.

We perform the same analyses as above for the three ESH-breaking cases, namely, the calculation of the symmetry breaking scales to reproduce $g_R(M_R) = 0.4$, and then by allowing the value of $g_R(M_R)$ to float in order to find the lowest value of M_D :

- (1) Δ_{R1} and Δ_{R3} survive:

To reproduce $g_R(M_R) = 0.4$, we find

$$M_R = 5 \text{ TeV}, \quad M_C = 8 \times 10^5 \text{ GeV}, \quad M_D = 2 \times 10^{24} \text{ GeV}, \quad (32)$$

Table 3. The dependence of the RG coefficients on the Higgs content in energy interval II where the symmetry is G_{2213} . Relaxing the ESH will lead to different Higgs content and different RG coefficients.

Interval	Higgs content	$(a_L, a_R, a_{BL}, a_3)^{\text{II}}$
II	$\phi(2, 2, 0, 1), \Delta_{R1}(1, 3, 2, 1)$	$\left(-3, \frac{-7}{3}, \frac{11}{3}, -7\right)$
	$\phi(2, 2, 0, 1), \Delta_{R1}(1, 3, 2, 1), \Delta_{R3}\left(1, 3, \frac{2}{3}, 3\right)$	$\left(-3, \frac{-1}{3}, 4, \frac{-13}{2}\right)$
	$\phi(2, 2, 0, 1), \Delta_{R1}(1, 3, 2, 1), \Delta_{R6}\left(1, 3, \frac{-2}{3}, 6\right)$	$\left(-3, \frac{5}{3}, \frac{13}{3}, \frac{-9}{2}\right)$
	$\phi(2, 2, 0, 1), \Delta_{R1}(1, 3, 2, 1), \Delta_{R3}\left(1, 3, \frac{2}{3}, 3\right), \Delta_{R6}\left(1, 3, \frac{-2}{3}, 6\right)$	$\left(-3, \frac{11}{3}, \frac{14}{3}, -4\right)$

with

$$g_L(M_D) = g_R(M_D) = 0.48, \quad g_4(M_D) = 0.44. \quad (33)$$

If $g_R(M_R)$ is allowed to float, the minimum of M_D is achieved when $g_R(M_R) = 0.53$ with

$$M_R = 5 \text{ TeV}, \quad M_C = M_D = 1 \times 10^{17} \text{ GeV}, \quad (34)$$

with

$$g_L(M_D) = g_R(M_D) = 0.52, \quad g_4(M_D) = 0.53. \quad (35)$$

The runnings of the couplings for these cases are shown in Fig. 2(a) and (b).

(2) Δ_{R1} and Δ_{R6} survive:

To reproduce $g_R(M_R) = 0.4$, we find

$$M_R = 5 \text{ TeV}, \quad M_C = 2 \times 10^6 \text{ GeV}, \quad M_D = 5 \times 10^{23} \text{ GeV}, \quad (36)$$

with

$$g_L(M_D) = g_R(M_D) = 0.48, \quad g_4(M_D) = 0.45. \quad (37)$$

If $g_R(M_R)$ is allowed to float, the minimum of M_D is achieved when $g_R(M_R) = 0.49$ with

$$M_R = 5 \text{ TeV}, \quad M_C = M_D = 8 \times 10^{15} \text{ GeV}, \quad (38)$$

with

$$g_L(M_D) = g_R(M_D) = 0.53, \quad g_4(M_D) = 0.62. \quad (39)$$

The runnings of the couplings for these cases are shown in Fig. 2(c) and (d).

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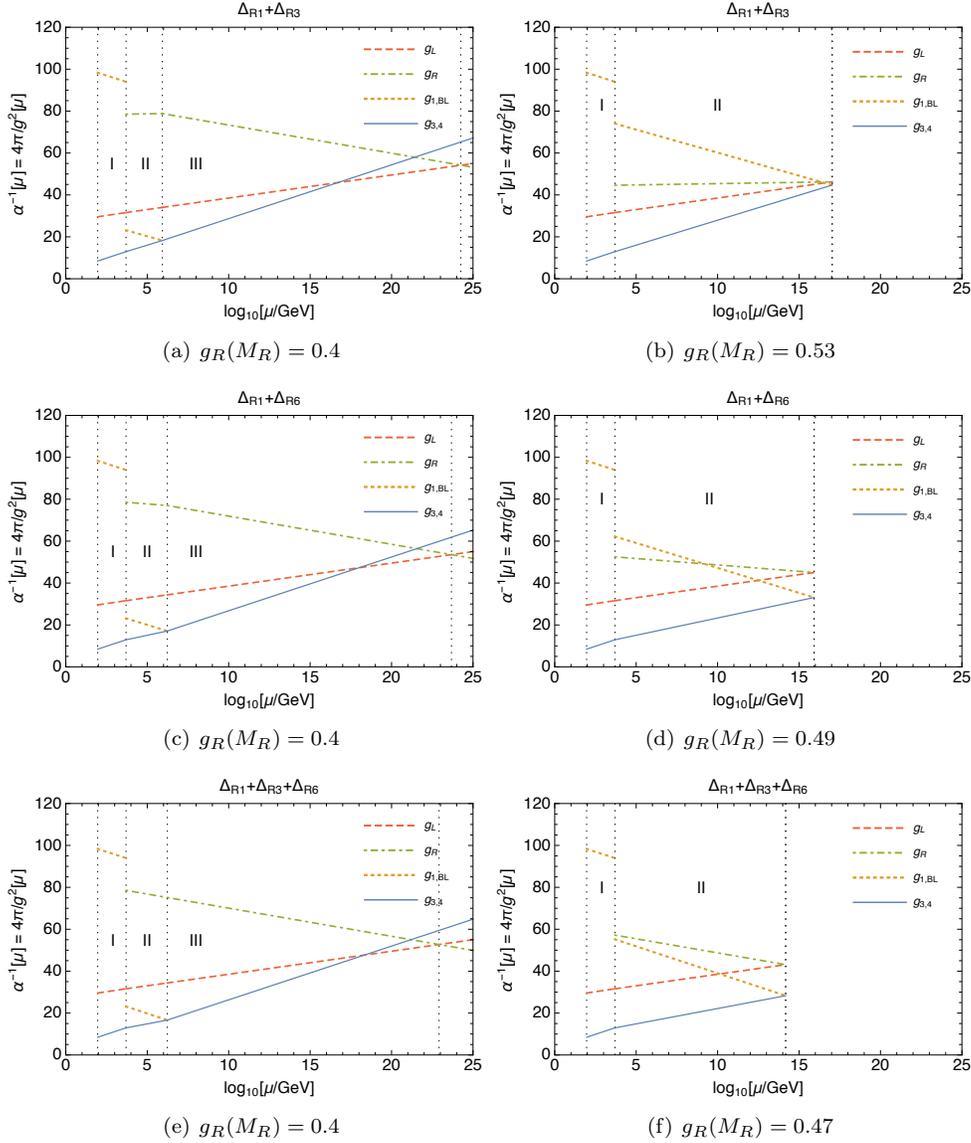


Fig. 2. Running of the gauge couplings for the left-right symmetric Pati-Salam model with more than Δ_{R1} surviving into energy interval II. Vertical dotted lines indicate symmetry breaking scales. M_R is fixed at 5 TeV. In (a), (c), and (e) $g_R(M_R) = 0.4$ is imposed, while in (b), (d), and (f) M_D is minimized by collapsing the energy interval III.

(3) All three multiplets Δ_{R1} , Δ_{R3} , and Δ_{R6} survive:

To reproduce $g_R(M_R) = 0.4$, we find

$$M_R = 5 \text{ TeV}, \quad M_C = 2 \times 10^6 \text{ GeV}, \quad M_D = 8 \times 10^{22} \text{ GeV}, \quad (40)$$

with

$$g_L(M_D) = g_R(M_D) = 0.49, \quad g_4(M_D) = 0.46. \quad (41)$$

If $g_R(M_R)$ is allowed to float, the minimum of M_D is achieved when $g_R(M_R) = 0.47$ with

$$M_R = 5 \text{ TeV}, \quad M_C = M_D = 2 \times 10^{14} \text{ GeV}, \quad (42)$$

with

$$g_L(M_D) = g_R(M_D) = 0.54, \quad g_4(M_D) = 0.67. \quad (43)$$

The runnings of the couplings for these cases are shown in Fig. 2(e) and (f).

These results indicate that achieving a value of $g_R(M_R) = 0.4$ at $M_R = 5 \text{ TeV}$ is not trivial in this model, requiring a very high value of the parity breaking scale M_D . Lowering this scale below the Planck mass cannot be achieved with the minimal Higgs content in energy interval II as required by the ESH even if the value of $g_R(M_R)$ were allowed to float. If one relaxes the ESH, then M_D lower than the Planck mass is possible provided $g_R(M_R)$ is allowed to be as large as ~ 0.5 . It is also preferable for M_C and M_D to be degenerate, that is, for G_{224D} to break directly to G_{2213} .

3.4. Unified Left-Right Symmetric Pati-Salam from NCG

With the above results in mind, let us now look at the unified left-right symmetric Pati-Salam model which we expect to emerge from an underlying NCG. The breaking pattern now includes an emergence/unification scale as in Eq. (4), and all four energy intervals listed in Eq. (5) must be taken into account with an extra boundary condition at M_U :

$$M_U : g_L(M_U) = g_R(M_U) = g_4(M_U). \quad (44)$$

This leads to the relations

$$\begin{aligned} & 2\pi \left[\frac{3 - 8 \sin^2 \theta_W(M_Z)}{\alpha(M_Z)} \right] \\ &= (3a_1 - 5a_2)^{\text{I}} \ln \frac{M_R}{M_Z} + (-5a_L + 3a_R + 3a_{BL})^{\text{II}} \ln \frac{M_C}{M_R} \\ & \quad + (-5a_L + 3a_R + 2a_4)^{\text{III}} \ln \frac{M_D}{M_C} + (-5a_L + 3a_R + 2a_4)^{\text{IV}} \ln \frac{M_U}{M_D}, \quad (45) \end{aligned}$$

$$\begin{aligned} & 2\pi \left[\frac{3}{\alpha(M_Z)} - \frac{8}{\alpha_s(M_Z)} \right] \\ &= (3a_1 + 3a_2 - 8a_3)^{\text{I}} \ln \frac{M_R}{M_Z} + (3a_L + 3a_R + 3a_{BL} - 8a_3)^{\text{II}} \ln \frac{M_C}{M_R} \\ & \quad + (3a_L + 3a_R - 6a_4)^{\text{III}} \ln \frac{M_D}{M_C} + (3a_L + 3a_R - 6a_4)^{\text{IV}} \ln \frac{M_U}{M_D}, \quad (46) \end{aligned}$$

$$2\pi \left[\frac{4\pi}{g_R^2(M_R)} - \frac{\sin^2 \theta_W(M_Z)}{\alpha(M_Z)} \right]$$

Table 4. Higgs content of Model A of Ref. 16. In Ref. 16, the model emerges with symmetry G_{224} at $M_U = M_D$. This breaks directly to G_{213} of the SM at $M_C = M_R$. We modify this process by allowing $M_C \neq M_R$, inserting energy interval II with symmetry G_{2213} between intervals III and I. The Higgs content in interval II is based on the ESH.

Interval	Higgs content	RG coefficients
III	$\phi(2, 2, 1), \Delta_R(1, 2, 4), \Sigma(1, 1, 15)$	$(a_L, a_R, a_4)^{\text{III}} = \left(-3, \frac{-7}{3}, \frac{-29}{3}\right)$
II	$\phi(2, 2, 0, 1), \Delta_{R1}(1, 2, 1, 1)$	$(a_L, a_R, a_{BL}, a_3)^{\text{II}} = \left(-3, \frac{-17}{6}, \frac{17}{6}, -7\right)$
I	$\phi_2(2, 1, 1)$	$(a_1, a_2, a_3)^{\text{I}} = \left(\frac{41}{6}, \frac{-19}{6}, -7\right)$

$$= (-a_2^{\text{I}}) \ln \frac{M_R}{M_Z} + (a_R - a_L)^{\text{II}} \ln \frac{M_C}{M_R} + (a_R - a_L)^{\text{III}} \ln \frac{M_D}{M_C}. \quad (47)$$

The derivation is given in the Appendix. Note that there is no $\ln M_U/M_D$ term in the last line since parity is not broken in energy interval IV and $a_L^{\text{IV}} = a_R^{\text{IV}}$. We will now look at the three models of Chamseddine, Connes, and van Suijlekom in Ref. 16 one by one.

(A) Pati-Salam with “composite” Higgs fields^e

The first model of Ref. 16 emerges with symmetry G_{224} at $M_U = M_D$, which breaks directly to G_{213} of the SM at $M_C = M_R$. So only energy intervals I and III are present. The Higgs content of this model in interval III, as specified in Ref. 16, is in shown in Table 4. We make a slight modification by taking the $\Sigma(1, 1, 15)$ field to be real, conforming to standard Pati-Salam literature, whereas Ref. 16 assumes it to be complex.

In this case, Eqs. (45) through (47) simply reduce to those with the II and IV terms missing. Then, the system of three equations has three unknowns, namely $\ln \frac{M_D}{M_C}$, $\ln \frac{M_R}{M_Z}$, and $g_R(M_R)$, which allows us to determine all three. We find:

$$\begin{aligned} g_R(M_R) &= 0.54, \\ M_R &= M_C = 4.1 \times 10^{13} \text{ GeV}, \\ M_U &= M_D = 3.5 \times 10^{15} \text{ GeV}, \end{aligned} \quad (48)$$

in agreement with Ref. 16. The unified coupling in this case is $g_L(M_U) = g_R(M_U) = g_4(M_U) = 0.53$. The running of the couplings for this case is shown in Fig. 3(a).

^eThe terminology of Ref. 16 may cause confusion with the standard concept of compositeness that is found in the literature. What the authors of Ref. 16 call a “composite” field seems to be a field which does not transform under a single irreducible representation.

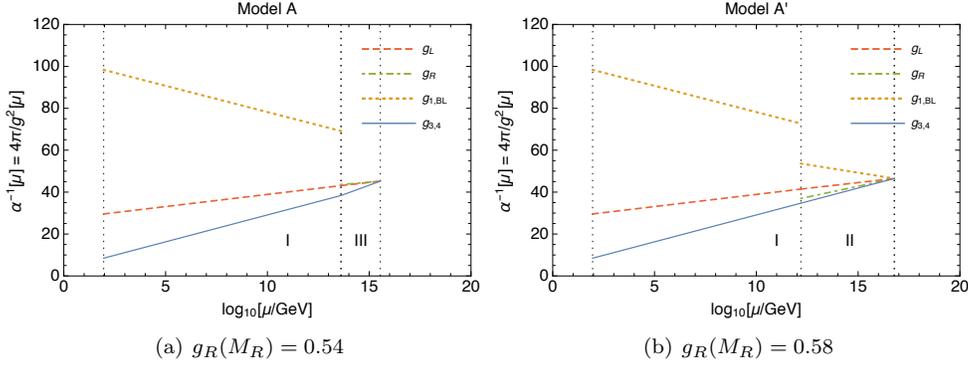


Fig. 3. Running of the gauge couplings for Model A of Ref. 16, with (a) G_{224} breaking directly into G_{213} , and (b) G_{224} breaks immediately to G_{2213} as it emerges. In (a), the dashed line indicating g_L and the dot-dashed line indicating g_R are almost overlapping in interval III.

We now allow for $M_C \neq M_R$ and insert the energy interval II with symmetry G_{2213} . To determine the Higgs content in this interval, we again invoke the ESH. The decomposition of $\Sigma(1, 1, 15)$ into irreducible representations of G_{2213} was given in Eq. (18) and it was concluded that all the components of $\Sigma(1, 1, 15)$ become heavy and decouple from the RG equations at M_C . The decomposition of $\Delta_R(1, 2, 4)$ into irreducible representations of G_{2213} is given by

$$\Delta_R(1, 2, 4) = \Delta_{R1}(1, 2, 1, 1) \oplus \Delta_{R3}\left(1, 2, \frac{-1}{3}, 3\right). \quad (49)$$

Δ_{R3} is colored so again by ESH it will become heavy and only Δ_{R1} will survive into II. The decomposition of Δ_{R1} into irreducible representations of G_{213} is given by

$$\Delta_{R1}(1, 2, 1, 1) = \Delta_{R1}^0(1, 0, 1) \oplus \Delta_{R1}^+(1, 1, 1). \quad (50)$$

The breaking of G_{2213} down to G_{213} would be accomplished by the field Δ_{R1}^0 acquiring a VEV, while Δ_{R1}^+ has electromagnetic charge so it must become heavy. The survival of $\phi_2(2, 1, 1)$ into I is as before. Thus, the Higgs content of the model is as shown in Table 4.

Eqs. (45) through (47) now has four unknowns instead of three. Numerically, they are given by

$$\begin{aligned} 401 &= \frac{109}{3}x + \frac{13}{2}y - \frac{34}{3}z, \\ 862 &= 67x + \frac{77}{2}y + 42z, \\ \frac{206}{g_R^2(M_R)} - 484 &= 19x - 16y + 4z, \end{aligned} \quad (51)$$

where x , y , and z are defined as in Eq. (23). Solving this system for x , y , and

Table 5. Higgs content of Model B of Ref. 16. In Ref. 16, the model emerges with symmetry G_{224} at $M_U = M_D$. This breaks directly to G_{213} of the SM at $M_C = M_R$. We modify this process by allowing $M_C \neq M_R$, inserting energy interval II with symmetry G_{2213} between intervals III and I. The Higgs content in interval II is based on the ESH. The particle content and RG coefficients in intervals I and II are the same as those listed in Table 2.

Interval	Higgs content	RG coefficients
III	$\phi(2, 2, 1), H(1, 1, 6), \Delta_R(1, 3, 10),$ $\tilde{\Sigma}(2, 2, 15)$	$(a_L, a_R, a_4)^{\text{III}} = \left(2, \frac{26}{3}, -2\right)$
II	$\phi(2, 2, 0, 1), \Delta_{R1}(1, 3, 2, 1)$	$(a_L, a_R, a_{BL}, a_3)^{\text{II}} = \left(-3, \frac{-7}{3}, \frac{11}{3}, -7\right)$
I	$\phi_2(2, 1, 1)$	$(a_1, a_2, a_3)^{\text{I}} = \left(\frac{41}{6}, \frac{-19}{6}, -7\right)$

z we find

$$\begin{aligned}
 x &= 2.3 + \frac{2.71}{g_R^2(M_R)}, \\
 y &= 30.2 - \frac{8.72}{g_R^2(M_R)}, \\
 z &= -10.8 + \frac{3.68}{g_R^2(M_R)}.
 \end{aligned} \tag{52}$$

Demanding that both y and z be positive restricts $g_R(M_R)$ to the range

$$0.54 < g_R(M_R) < 0.58. \tag{53}$$

The lower bound corresponds to the case considered in Ref. 16 at which $y = 0$. Since we would like to minimize x , and thereby M_R , we set $g_R(M_R)$ to the upper bound of this range where $x = 10.2$, $y = 4.6$, and $z = 0$. This corresponds to

$$M_R = 1.5 \times 10^{12} \text{ GeV}, \quad M_C = M_D = M_U = 6 \times 10^{14} \text{ GeV}. \tag{54}$$

The unified coupling in this case is $g_L(M_U) = g_R(M_U) = g_4(M_U) = 0.52$. The running of the couplings for this case is shown in Fig. 3(b).

Comparing the two cases, allowing $M_R \neq M_C$ has lowered M_R from 10^{13} GeV to 10^{12} GeV. This is due to the bifurcation of g_4 into g_3 and g_{BL} at M_C . The hypercharge coupling at M_R must be matched to g_R and g_4 if $M_R = M_C$, but it will be matched to g_R and g_{BL} if $M_R \neq M_C$. Since g_{BL} decreases in II, one can allow g_R to increase further to generate the numerically correct value for g_1 . This lowers the scale M_R . However, 10^{12} GeV is still too large compared to the TeV scale. This lowering is also at the expense of G_{224} breaking immediately to G_{2213} as the model emerges from the underlying NCG theory.

(B) Pati-Salam with fundamental Higgs fields

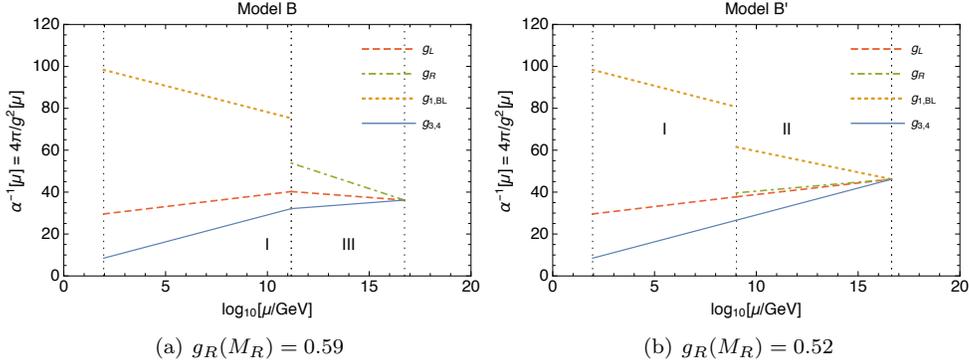


Fig. 4. Running of the gauge couplings for Model B of Ref. 16, with (a) G_{224} breaking directly into G_{213} , and (b) G_{224} breaks immediately to G_{2213} as it emerges.

The Higgs content of Model B of Ref. 16 is shown in Table 5, together with what the Higgs content in interval II would be under the ESH if the condition $M_C = M_R$ were relaxed. As in Model A, it is assumed that $M_U = M_D$. We first follow Ref. 16 and also assume $M_C = M_R$ and find

$$\begin{aligned} g_R(M_R) &= 0.48, \\ M_R &= M_C = 1.5 \times 10^{11} \text{ GeV}, \\ M_U &= M_D = 5.4 \times 10^{16} \text{ GeV}. \end{aligned} \quad (55)$$

The unified coupling is $g_L(M_U) = g_R(M_U) = g_4(M_U) = 0.59$. The running of the couplings for this case is shown in Fig. 4(a).

Let us now relax the condition $M_C = M_R$ and insert the energy interval II with symmetry G_{2213} between intervals I and III. Without going into detail, we list the Higgs fields that survive via the ESH into II from III in Table 5. Note that the Higgs content in I and II are exactly the same as the non-unified Pati-Salam model we considered earlier. In the exact repeat of our analysis of Model A, it can be shown that for the ordering of the symmetry breaking scale to be maintained, $g_R(M_R)$ is restricted to the range

$$0.48 < g_R(M_R) < 0.56, \quad (56)$$

with the higher bound giving the smallest possible M_R . This is found to be

$$M_R = 1.1 \times 10^9 \text{ GeV}, \quad M_C = M_D = M_U = 4.4 \times 10^{16} \text{ GeV}. \quad (57)$$

with the unified coupling $g_L(M_U) = g_R(M_U) = g_4(M_U) = 0.52$. The running of the couplings for this case is shown in Fig. 4(b).

While this result is somewhat more promising than Model A, M_R is still too large, as is the value of $g_R(M_R)$ necessary for M_R to be pushed down to this scale. Let us see if the situation may be improved by relaxing the ESH as we

did for the non-unified Pati-Salam case. We will allow some or all of the colored Δ_R fields to survive into interval II to enhance the difference between g_L and g_R . We consider the same three cases listed in Table 3.

(i) Δ_{R1} and Δ_{R3} survive:

To maintain the ordering of the symmetry breaking scales, it is found that $g_R(M_R)$ is restricted to the narrow range

$$0.48 < g_R(M_R) < 0.51 . \quad (58)$$

As $g_R(M_R)$ is increased, M_R/M_Z and M_D/M_C decrease while M_C/M_R increases. In terms of scale, M_R decreases while both M_C and M_D increase. The upper bound of this range is when $M_R/M_Z = 1$, so this case actually allows for $M_R = 5$ TeV. The other parameters in this case is

$$\begin{aligned} g_R(M_R) &= 0.51 , \\ M_R &= 5 \times 10^3 \text{ GeV} , \\ M_C &= 5 \times 10^{15} \text{ GeV} , \\ M_U = M_D &= 8 \times 10^{17} \text{ GeV} , \end{aligned} \quad (59)$$

with the unified coupling $g_L(M_U) = g_R(M_U) = g_4(M_U) = 0.54$. The running of the couplings for this case is shown in Fig. 5(a).

(ii) Δ_{R1} and Δ_{R6} survive:

To maintain the ordering of the symmetry breaking scales, it is found that $g_R(M_R)$ is restricted to the range

$$0.42 < g_R(M_R) < 0.48 , \quad (60)$$

with smaller $g_R(M_R)$ associated with smaller M_R , which drops down to M_Z at the lower bound. Imposing $M_R = 5$ TeV we obtain:

$$\begin{aligned} g_R(M_R) &= 0.43 , \\ M_R &= 5 \times 10^3 \text{ GeV} , \\ M_C &= 2 \times 10^{10} \text{ GeV} , \\ M_U = M_D &= 3 \times 10^{20} \text{ GeV} , \end{aligned} \quad (61)$$

with the unified coupling $g_L(M_U) = g_R(M_U) = g_4(M_U) = 0.63$. The running of the couplings for this case is shown in Fig. 5(b). Maintaining M_D below 10^{19} requires

$$0.45 < g_R(M_R) . \quad (62)$$

Selecting this boundary value for $g_R(M_R)$, we find

$$\begin{aligned} g_R(M_R) &= 0.45 , \\ M_R &= 5 \times 10^6 \text{ GeV} , \\ M_C &= 4 \times 10^{10} \text{ GeV} , \\ M_U = M_D &= 1 \times 10^{19} \text{ GeV} , \end{aligned} \quad (63)$$

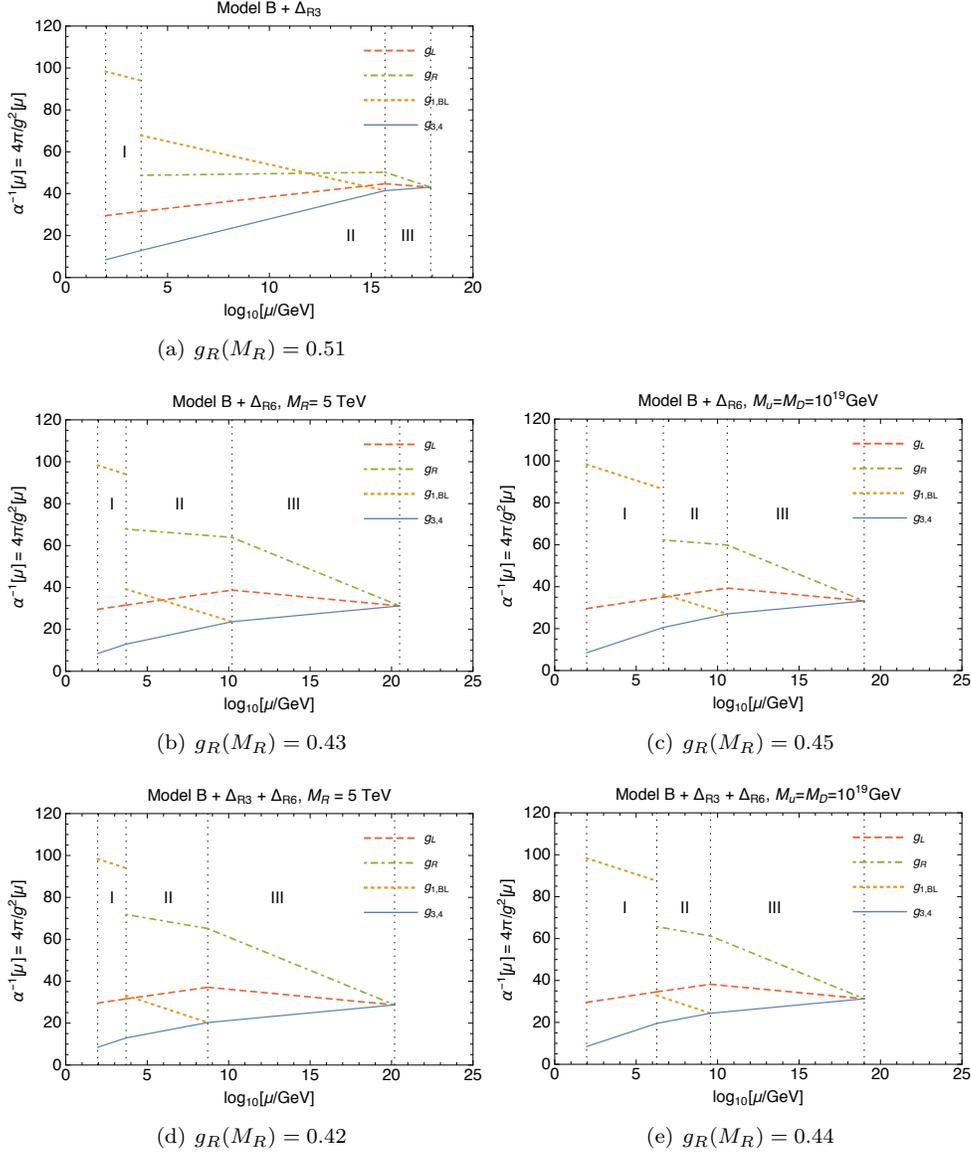


Fig. 5. Running of the gauge couplings for Model B of Ref. 16 with extended Higgs content in energy interval II. In addition to Δ_{R1} , the field Δ_{R3} survives into II in (a), (d), and (e), while the field Δ_{R6} also survives into II in (b), (c), (d) and (e). In (a), (b), and (d) we impose $M_R = 5$ TeV. In (c) and (e) we impose $M_U = M_D = 10^{19}$ GeV.

with the unified coupling $g_L(M_U) = g_R(M_U) = g_4(M_U) = 0.62$. The running of the couplings for this case is shown in Fig. 5(c).

(iii) Δ_{R1} , Δ_{R3} , and Δ_{R6} all survive:

Table 6. Higgs content of Model C of Ref. 16. In Ref. 16, the model emerges with symmetry G_{224D} at M_U . This breaks directly to G_{213} of the SM at $M_D = M_C = M_R$. We modify this process by allowing $M_D \neq M_C \neq M_R$, inserting energy intervals II and III with symmetries G_{2213} and G_{224} , respectively, between intervals I and IV. The Higgs content in intervals I, II, and III are based on the ESH. An extra D-parity singlet field $\sigma(1, 1, 1)$ is introduced in interval IV to break parity spontaneously. The particle content and RG coefficients in intervals I and II are the same as those listed in Table 2.

Interval	Higgs content	RG coefficients
IV	$\phi(2, 2, 1), H(1, 1, 6) \times 2, \tilde{\Sigma}(2, 2, 15)$ $\Delta_R(1, 3, 10), \Delta_L(3, 1, 10), \sigma(1, 1, 1)$	$(a_L, a_R, a_4)^{\text{IV}} = \left(\frac{26}{3}, \frac{26}{3}, \frac{4}{3}\right)$
III	$\phi(2, 2, 1), H(1, 1, 6), \Delta_R(1, 3, 10)$	$(a_L, a_R, a_4)^{\text{III}} = \left(-3, \frac{11}{3}, \frac{-22}{3}\right)$
II	$\phi(2, 2, 0, 1), \Delta_{R1}(1, 3, 2, 1)$	$(a_L, a_R, a_{BL}, a_3)^{\text{II}} = \left(-3, \frac{-7}{3}, \frac{11}{3}, -7\right)$
I	$\phi_2(2, 1, 1)$	$(a_1, a_2, a_3) = \left(\frac{41}{6}, \frac{-19}{6}, -7\right)$

To maintain the ordering of the symmetry breaking scales, it is found that $g_R(M_R)$ is restricted to the range

$$0.41 < g_R(M_R) < 0.48, \quad (64)$$

while to maintain $M_U = M_D$ below 10^{19} GeV we must have

$$0.44 < g_R(M_R). \quad (65)$$

If we demand $M_R = 5$ TeV, we find

$$\begin{aligned} g_R(M_R) &= 0.42, \\ M_R &= 5 \times 10^3 \text{ GeV}, \\ M_C &= 5 \times 10^8 \text{ GeV}, \\ M_U = M_D &= 2 \times 10^{20} \text{ GeV}, \end{aligned} \quad (66)$$

with the unified coupling $g_L(M_U) = g_R(M_U) = g_4(M_U) = 0.66$. The running of the couplings for this case is shown in Fig. 5(d). If we demand $M_U = M_D = 10^{19}$ GeV, we find

$$\begin{aligned} g_R(M_R) &= 0.44, \\ M_R &= 2 \times 10^6 \text{ GeV}, \\ M_C &= 4 \times 10^9 \text{ GeV}, \\ M_U = M_D &= 1 \times 10^{19} \text{ GeV}, \end{aligned} \quad (67)$$

with the unified coupling $g_L(M_U) = g_R(M_U) = g_4(M_U) = 0.63$. The running of the couplings for this case is shown in Fig. 5(e).

(C) Left-right symmetric Pati-Salam with fundamental Higgs fields

Finally, the last and most general scenario of Ref. 16 is where G_{224D} instead of G_{224} is the emergent symmetry of the spectral action. The assumed Higgs content of the model is shown in Table 6.

First, assuming $M_D = M_C = M_R$ as in Ref. 16, we solve Eqs. (45) through (47) and find

$$\begin{aligned} g_R(M_R) &= 0.54, \\ M_D = M_C = M_R &= 5 \times 10^{13} \text{ GeV}, \\ M_U &= 3 \times 10^{15} \text{ GeV}, \end{aligned} \quad (68)$$

with the unified coupling $g_L(M_U) = g_R(M_U) = g_4(M_U) = 0.58$. The running of the couplings for this case is shown in Fig. 6(a).

We next relax the relation $M_D = M_C = M_R$ and insert energy intervals II and III in between intervals I and IV with the Higgs content listed in Table 6. Eqs. (45) through (47) now read

$$\begin{aligned} 401 &= \frac{109}{3}x + 19y + \frac{34}{3}z - \frac{44}{3}w, \\ 862 &= 67x + 51y + 46z + 44w, \\ \frac{206}{g_R^2(M_R)} - 484 &= 19x + 4y + 40z, \end{aligned} \quad (69)$$

where x , y , and z are defined as in Eq. (23), and $w = \log_{10} M_U/M_D$. Solving this system for y , z , and w we find:

$$\begin{aligned} y &= 30.3 - 1.38x - \frac{4.11}{g_R^2(M_R)}, \\ z &= 15.1 + 0.34x - \frac{5.56}{g_R^2(M_R)}, \\ w &= 0.24 + 0.43x - \frac{1.04}{g_R^2(M_R)}. \end{aligned} \quad (70)$$

Demanding that y , z , and w are all positive restricts $x = \log_{10} M_R/M_Z$ and $g_R^{-2}(M_R)$ to the triangular region shown in Fig. 7(a). It is clear from the figure that M_R is minimized when $M_U = M_D = M_C$, that is, energy regions III and IV are collapsed and only I and II remain. On the other hand, $g_R(M_R)$ is minimized when $M_U = M_D$ and $M_C = M_R$, that is, energy regions II and IV are collapsed and only I and III remain.

For the $M_U = M_D = M_C$ case, we find

$$\begin{aligned} g_R(M_R) &= 0.56, \\ M_R &= 1 \times 10^9 \text{ GeV}, \\ M_U = M_D = M_C &= 4 \times 10^{16} \text{ GeV}, \end{aligned} \quad (71)$$

with the unified coupling $g_L(M_U) = g_R(M_U) = g_4(M_U) = 0.52$. The running of the couplings for this case is shown in Fig. 6(b).

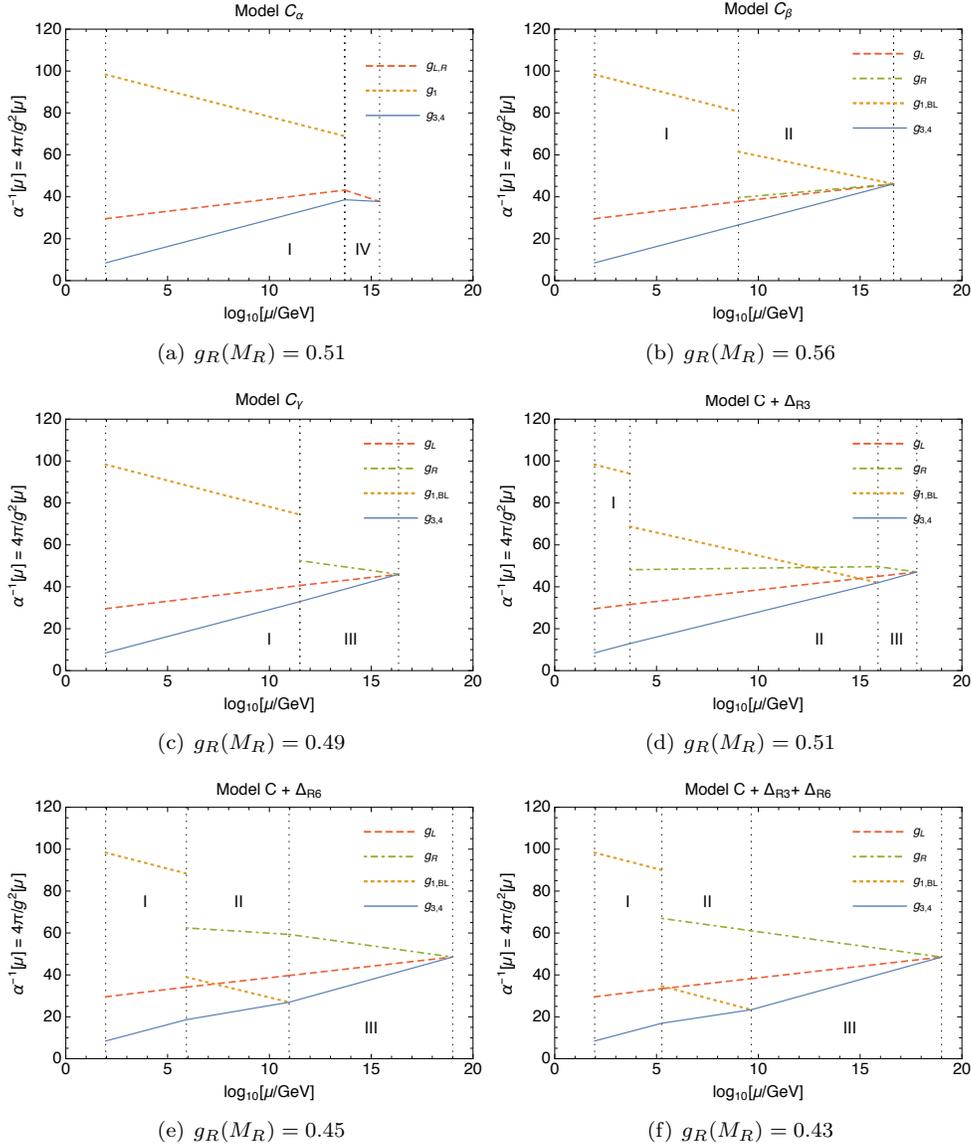


Fig. 6. (a) Running of the gauge couplings for Model C of Ref. 16 where $M_D = M_C = M_R$, (b) $M_U = M_D = M_C$, (c) $M_U = M_D$, $M_C = M_R$, (d) with Δ_{R3} surviving in II, (e) with Δ_{R6} surviving in II, and (f) with Δ_{R3} and Δ_{R6} surviving in II.

For the $M_U = M_D$, $M_C = M_R$ case, we find

$$\begin{aligned}
 g_R(M_R) &= 0.49, \\
 M_C &= M_R = 3 \times 10^{11} \text{ GeV}, \\
 M_U &= M_D = 2 \times 10^{16} \text{ GeV},
 \end{aligned}
 \tag{72}$$

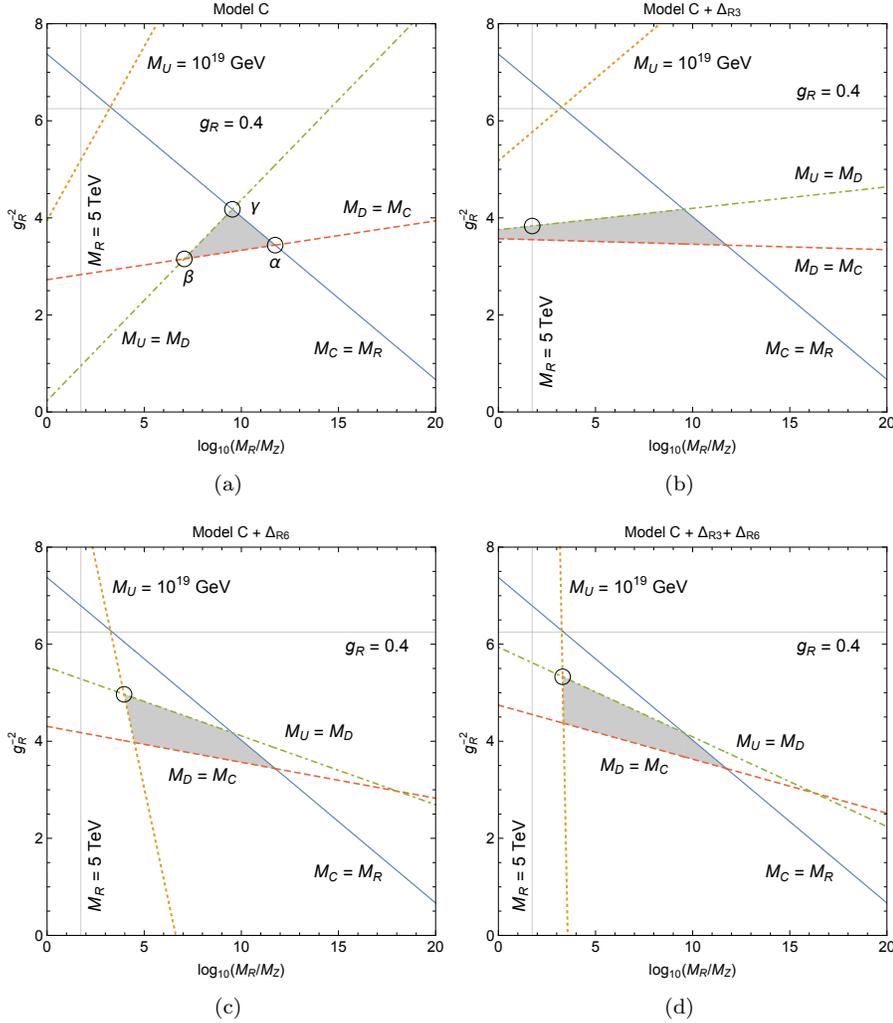


Fig. 7. (a) For Model C, the values of $x = \log_{10} M_R/M_Z$ and $g_R^{-2}(M_R)$ must lie inside the shaded triangle shown to maintain the ordering of the symmetry breaking scales. Ref. 16 selects the values at point α , where $M_D = M_C = M_R$. M_R is minimized at point β where $M_U = M_D = M_C$, while $g_R(M_R)$ is minimized at point γ where $M_U = M_D$ and $M_C = M_R$. (b), (c), and (d) show how the allowed region changes with the addition of extra colored Δ_R fields in energy interval II. The requirement that $M_U \leq 10^{19} \text{ GeV}$ demands that one stay to the right of the dotted line, and this restricts us to the interiors of the shaded quadrangles shown. Consequently, only case (b) allows for $M_R = 5 \text{ GeV}$. In all three cases, $g_R(M_R)$ is minimized for a given choice of M_R when $M_U = M_D$. The optimum points for each case discussed in the text are indicated by circles.

with the unified coupling $g_L(M_U) = g_R(M_U) = g_4(M_U) = 0.52$. The running of the couplings for this case is shown in Fig. 6(c).

Again, the values of M_R and $g_R(M_R)$ thus obtained are more promising that

what could be achieved in Model A, but nevertheless they are both still too large. So let us relax the ESH in energy interval II again to see whether things are improved. As we did for Model B, we consider the three cases listed in Table 3. The allowed regions in (x, g_R^{-2}) space is shown in Figs. 7(b) through 7(d). Taking M_R to be as close to 5 TeV as possible while minimizing $g_R(M_R)$ and maintaining $M_U \leq 10^{19}$ GeV leads to the following optimum solutions:

- (i) Δ_{1R} and Δ_{3R} survive:

$$\begin{aligned} g_R(M_R) &= 0.51 , \\ M_R &= 5 \times 10^3 \text{ GeV} , \\ M_C &= 8 \times 10^{15} \text{ GeV} , \\ M_U = M_D &= 6 \times 10^{17} \text{ GeV} , \end{aligned} \quad (73)$$

with the unified coupling $g_L(M_U) = g_R(M_U) = g_4(M_U) = 0.52$. The running of the couplings for this case is shown in Fig. 6(d).

- (ii) Δ_{1R} and Δ_{6R} survive:

$$\begin{aligned} g_R(M_R) &= 0.45 , \\ M_R &= 8 \times 10^5 \text{ GeV} , \\ M_C &= 9 \times 10^{10} \text{ GeV} , \\ M_U = M_D &= 1 \times 10^{19} \text{ GeV} , \end{aligned} \quad (74)$$

with the unified coupling $g_L(M_U) = g_R(M_U) = g_4(M_U) = 0.51$. The running of the couplings for this case is shown in Fig. 6(e).

- (iii) Δ_{1R} , Δ_{3R} , and Δ_{6R} survive:

$$\begin{aligned} g_R(M_R) &= 0.43 , \\ M_R &= 2 \times 10^5 \text{ GeV} , \\ M_C &= 4 \times 10^9 \text{ GeV} , \\ M_U = M_D &= 1 \times 10^{19} \text{ GeV} , \end{aligned} \quad (75)$$

with the unified coupling $g_L(M_U) = g_R(M_U) = g_4(M_U) = 0.51$. The running of the couplings for this case is shown in Fig. 6(f).

3.5. Summary of Results

In this section, we have looked at whether the IR conditions $M_R = 5$ TeV and $g_R(M_R) = 0.4$ could be realized within left-right symmetric, and unified left-right symmetric Pati-Salam models in which the unification/emergence scale is below the Planck mass. The left-right symmetric Pati-Salam demands the unification of g_L and g_R , while the unified left-right symmetric Pati-Salam demands further unification of g_L and g_R with g_4 . The requirements that these couplings unify at a single scale, and the matching conditions between g_1 , g_{BL} , and g_R at M_R , and that between g_{BL} , g_3 and g_4 at M_C , place conflicting demands on the various symmetry breaking

scales, and it is found that the target IR conditions cannot be realized so easily. In particular, if the Higgs content at various energy intervals is determined based on the Extended Survival Hypothesis (ESM), M_R and $g_R(M_R)$ tend to be much larger than our target values. Lowering these values requires the breaking of ESH. The most promising cases are Models B and C of Ref. 16 with the colored Δ_{3R} field surviving below M_C . We note that this may put the Δ_{3R} particles within reach of the LHC. But even for those cases $g_R(M_R)$ cannot be made as low as 0.4. In all cases, the optimum conditions for minimum M_R and/or minimum $g_R(M_R)$ requires degeneracies of some of the symmetry breaking scales.

4. Discussion and Conclusions

In this paper, we have initiated a purely phenomenological analysis of Connes' NCG approach to the SM and beyond, in the light of the latest experimental results from the LHC. In particular, we have concentrated on the remarkable left-right symmetric structure that is inherent in the NCG of the SM, embodied in the unified left-right symmetric Pati-Salam models of Ref. 16, and explored its phenomenological consequences by concentrating on the possible existence of a TeV scale W_R boson. We find that generating a TeV scale W_R boson with the small coupling of $g_R = 0.4$ within NCG motivated models is not trivial and places strong constraints on the particle content and symmetry breaking scales.

We note that we have also conducted a preliminary analysis of the constraints imposed by proton stability,⁶⁸ the $\Delta B = 2$ neutron-antineutron and hydrogen-antihydrogen oscillations⁶⁷ as well as the constraints coming from the inflationary cosmological models.²⁸ In principle, these constraints are not prohibitive of the phenomenological analysis carried out in this paper.

While our analysis could suggest that the NCG motivated unified left-right Pati-Salam model is not favored phenomenologically by the current LHC data, we note the possibility that the current approach of grafting the NCG spectral action to RG evolution of standard QFT at the GUT scale may not capture the true nature and predictions of NCG theories.

Finally, we address the closely related question of the hierarchy problem. One of the most interesting aspects of the NCG of the SM and its Pati-Salam-like completion is the existence of the GUT scale which can be found in the close proximity to the Planck scale, i.e. the scale of quantum gravity. Given this fact as well as the presence of a hidden fundamental non-commutative structure in this approach, this suggests that the hierarchy problem should get a quantum gravitational, and not an effective field theory treatment. The more convincing physical meaning of this GUT scale also comes after one realizes that Connes' approach also produces a gravity sector in parallel with the standard model (and its Pati-Salam completion) and thus the GUT scale should be viewed as being close to the natural scale of gravity, i.e. Planck scale, and indeed the two scales are not that far apart in the

non-commutative approach. In particular if one views quantum gravity as having origins in metastring theory^{69–71} then one finds the fundamental non-commutative structure, and also, the two-scale renormalization group, which sheds new light on such fundamental issues as the hierarchy problem: the two scales that feature in Refs. 69–71 are both the UV and the IR scales and thus the stability of the Higgs mass becomes two-fold, both with respect to the UV and to the IR. In other words, the question is now not only why the Higgs mass is not of the Planck scale (or the GUT scale) but also why the Higgs mass is not of the Hubble (vacuum energy) scale. It is well known that numerologically, the Higgs scale (~ 1 TeV) is the geometric mean between these two scales, at the point of a UV/IR invariant energy scale. The Higgs scale also naturally appears as a geometric scale in Connes' non-commutative geometry approach, in complete analogy with the geometric meaning of the Planck and the Hubble scales. Actually, because of the appearance of gravity and the standard model Lagrangians in the Connes's spectral action, and because of the discrete nature of the Higgs dimension, there is a natural Higgs-like degree of freedom on the gravity side – a Brans-Dicki-Jordan-like scalar – which can be argued to contribute to the geometric warping of the Higgs discrete dimension. This is similar to the infinite extra dimensional scenarios, however, without infinite extra dimensions.^{4, 12}

In our view the approach based on NCG (and its related proposal based on the superconnection approach^{23, 24}) offers a new and, phenomenologically, almost completely unexplored view on the rationale for the SM and also for its natural completion. This approach also offers a possibly exciting relation with the fundamental physics of quantum gravity, thus relating the infrared physics of the current exciting experimental searches conducted at the LHC to the hidden ultraviolet physics of quantum theory of space and time.

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Appendix A. Derivation of Relations Between Symmetry Breaking Scales

$$\begin{aligned}
 \frac{1}{g_1^2(M_Z)} &= \frac{1}{g_1^2(M_R)} + \frac{a_1^I}{8\pi^2} \ln \frac{M_R}{M_Z} \\
 &= \frac{1}{g_R^2(M_R)} + \frac{1}{g_{BL}^2(M_R)} + \frac{a_1^I}{8\pi^2} \ln \frac{M_R}{M_Z} \\
 &= \frac{1}{g_R^2(M_C)} + \frac{1}{g_{BL}^2(M_C)} + \frac{(a_R + a_{BL})^{II}}{8\pi^2} \ln \frac{M_C}{M_R} + \frac{a_1^I}{8\pi^2} \ln \frac{M_R}{M_Z}
 \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{g_R^2(M_C)} + \frac{2}{3} \frac{1}{g_4^2(M_C)} + \frac{(a_R + a_{BL})^{\text{II}}}{8\pi^2} \ln \frac{M_C}{M_R} + \frac{a_1^{\text{I}}}{8\pi^2} \ln \frac{M_R}{M_Z} \\
&= \frac{1}{g_R^2(M_D)} + \frac{2}{3} \frac{1}{g_4^2(M_D)} + \frac{1}{8\pi^2} \left(a_R + \frac{2}{3} a_4 \right)^{\text{III}} \ln \frac{M_D}{M_C} \\
&\quad + \frac{(a_R + a_{BL})^{\text{II}}}{8\pi^2} \ln \frac{M_C}{M_R} + \frac{a_1^{\text{I}}}{8\pi^2} \ln \frac{M_R}{M_Z} \\
&= \frac{1}{g_R^2(M_U)} + \frac{2}{3} \frac{1}{g_4^2(M_U)} + \frac{1}{8\pi^2} \left(a_R + \frac{2}{3} a_4 \right)^{\text{IV}} \ln \frac{M_U}{M_D} \\
&\quad + \frac{1}{8\pi^2} \left(a_R + \frac{2}{3} a_4 \right)^{\text{III}} \ln \frac{M_D}{M_C} + \frac{(a_R + a_{BL})^{\text{II}}}{8\pi^2} \ln \frac{M_C}{M_R} + \frac{a_1^{\text{I}}}{8\pi^2} \ln \frac{M_R}{M_Z}, \\
\frac{1}{g_2^2(M_Z)} &= \frac{1}{g_2^2(M_R)} + \frac{a_2^{\text{I}}}{8\pi^2} \ln \frac{M_R}{M_Z} \\
&= \frac{1}{g_L^2(M_R)} + \frac{a_2^{\text{I}}}{8\pi^2} \ln \frac{M_R}{M_Z} \\
&= \frac{1}{g_L^2(M_C)} + \frac{a_L^{\text{II}}}{8\pi^2} \ln \frac{M_C}{M_R} + \frac{a_2^{\text{I}}}{8\pi^2} \ln \frac{M_R}{M_Z} \\
&= \frac{1}{g_L^2(M_D)} + \frac{a_L^{\text{III}}}{8\pi^2} \ln \frac{M_D}{M_C} + \frac{a_L^{\text{II}}}{8\pi^2} \ln \frac{M_C}{M_R} + \frac{a_2^{\text{I}}}{8\pi^2} \ln \frac{M_R}{M_Z} \\
&= \frac{1}{g_L^2(M_U)} + \frac{a_L^{\text{IV}}}{8\pi^2} \ln \frac{M_U}{M_D} + \frac{a_L^{\text{III}}}{8\pi^2} \ln \frac{M_D}{M_C} + \frac{a_L^{\text{II}}}{8\pi^2} \ln \frac{M_C}{M_R} + \frac{a_2^{\text{I}}}{8\pi^2} \ln \frac{M_R}{M_Z}, \\
\frac{1}{e^2(M_Z)} &= \frac{1}{g_2^2(M_Z)} + \frac{1}{g_1^2(M_Z)} \\
&= \frac{1}{g_2^2(M_R)} + \frac{1}{g_1^2(M_R)} + \frac{1}{8\pi^2} (a_1 + a_2)^{\text{I}} \ln \frac{M_R}{M_Z} \\
&= \frac{1}{g_L^2(M_R)} + \frac{1}{g_R^2(M_R)} + \frac{1}{g_{BL}^2(M_R)} + \frac{1}{8\pi^2} (a_1 + a_2)^{\text{I}} \ln \frac{M_R}{M_Z} \\
&= \frac{1}{g_L^2(M_C)} + \frac{1}{g_R^2(M_C)} + \frac{1}{g_{BL}^2(M_C)} + \frac{1}{8\pi^2} (a_L + a_R + a_{BL})^{\text{II}} \ln \frac{M_C}{M_R} \\
&\quad + \frac{1}{8\pi^2} (a_1 + a_2)^{\text{I}} \ln \frac{M_R}{M_Z} \\
&= \frac{1}{g_L^2(M_C)} + \frac{1}{g_R^2(M_C)} + \frac{2}{3} \frac{1}{g_4^2(M_C)} \\
&\quad + \frac{1}{8\pi^2} (a_L + a_R + a_{BL})^{\text{II}} \ln \frac{M_C}{M_R} + \frac{1}{8\pi^2} (a_1 + a_2)^{\text{I}} \ln \frac{M_R}{M_Z} \\
&= \frac{1}{g_L^2(M_D)} + \frac{1}{g_R^2(M_D)} + \frac{2}{3} \frac{1}{g_4^2(M_D)} + \frac{1}{8\pi^2} \left(a_L + a_R + \frac{2}{3} a_4 \right)^{\text{III}} \ln \frac{M_D}{M_C} \\
&\quad + \frac{1}{8\pi^2} (a_L + a_R + a_{BL})^{\text{II}} \ln \frac{M_C}{M_R} + \frac{1}{8\pi^2} (a_1 + a_2)^{\text{I}} \ln \frac{M_R}{M_Z} \\
&= \frac{1}{g_L^2(M_U)} + \frac{1}{g_R^2(M_U)} + \frac{2}{3} \frac{1}{g_4^2(M_U)} \\
&\quad + \frac{1}{8\pi^2} \left(a_L + a_R + \frac{2}{3} a_4 \right)^{\text{IV}} \ln \frac{M_U}{M_D} + \frac{1}{8\pi^2} \left(a_L + a_R + \frac{2}{3} a_4 \right)^{\text{III}} \ln \frac{M_D}{M_C} \\
&\quad + \frac{1}{8\pi^2} (a_L + a_R + a_{BL})^{\text{II}} \ln \frac{M_C}{M_R} + \frac{1}{8\pi^2} (a_1 + a_2)^{\text{I}} \ln \frac{M_R}{M_Z},
\end{aligned}$$

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$$\begin{aligned}
\frac{1}{g_3^2(M_Z)} &= \frac{1}{g_3^2(M_R)} + \frac{a_3^I}{8\pi^2} \ln \frac{M_R}{M_Z} \\
&= \frac{1}{g_3^2(M_C)} + \frac{a_3^{II}}{8\pi^2} \ln \frac{M_R}{M_Z} + \frac{a_3^I}{8\pi^2} \ln \frac{M_R}{M_Z} \\
&= \frac{1}{g_4^2(M_C)} + \frac{a_3^{II}}{8\pi^2} \ln \frac{M_C}{M_R} + \frac{a_3^I}{8\pi^2} \ln \frac{M_R}{M_Z} \\
&= \frac{1}{g_4^2(M_D)} + \frac{a_4^{III}}{8\pi^2} \ln \frac{M_D}{M_C} + \frac{a_3^{II}}{8\pi^2} \ln \frac{M_C}{M_R} + \frac{a_3^I}{8\pi^2} \ln \frac{M_R}{M_Z} \\
&= \frac{1}{g_4^2(M_U)} + \frac{a_4^{IV}}{8\pi^2} \ln \frac{M_U}{M_D} + \frac{a_4^{III}}{8\pi^2} \ln \frac{M_D}{M_C} + \frac{a_3^{II}}{8\pi^2} \ln \frac{M_C}{M_R} + \frac{a_3^I}{8\pi^2} \ln \frac{M_R}{M_Z}, \\
\frac{1}{g_R^2(M_R)} &= \frac{1}{g_R^2(M_C)} + \frac{a_R^{II}}{8\pi^2} \ln \frac{M_C}{M_R} \\
&= \frac{1}{g_R^2(M_D)} + \frac{a_R^{III}}{8\pi^2} \ln \frac{M_D}{M_C} + \frac{a_R^{II}}{8\pi^2} \ln \frac{M_C}{M_R} \\
&= \frac{1}{g_R^2(M_U)} + \frac{a_R^{IV}}{8\pi^2} \ln \frac{M_U}{M_D} + \frac{a_R^{III}}{8\pi^2} \ln \frac{M_D}{M_C} + \frac{a_R^{II}}{8\pi^2} \ln \frac{M_C}{M_R}. \tag{A.1}
\end{aligned}$$

If we impose the condition

$$g_L(M_U) = g_R(M_U) = g_4(M_U) \equiv g_U, \tag{A.2}$$

then it is straightforward to show that

$$\begin{aligned}
&2\pi \left[\frac{3 - 8 \sin^2 \theta_W(M_Z)}{\alpha(M_Z)} \right] \\
&= 8\pi^2 \left[\frac{3}{e^2(M_Z)} - \frac{8}{g_2^2(M_Z)} \right] \\
&= \left[(-5a_L + 3a_R + 2a_4)^{IV} \ln \frac{M_U}{M_D} + (-5a_L + 3a_R + 2a_4)^{III} \ln \frac{M_D}{M_C} \right. \\
&\quad \left. + (-5a_L + 3a_R + 3a_{BL})^{II} \ln \frac{M_C}{M_R} + (3a_1 - 5a_2)^I \ln \frac{M_R}{M_Z} \right], \\
&2\pi \left[\frac{3}{\alpha(M_Z)} - \frac{8}{\alpha_s(M_Z)} \right] \\
&= 8\pi^2 \left[\frac{3}{e^2(M_Z)} - \frac{8}{g_3^2(M_Z)} \right] \\
&= \left[(3a_L + 3a_R - 6a_4)^{IV} \ln \frac{M_U}{M_D} + (3a_L + 3a_R - 6a_4)^{III} \ln \frac{M_D}{M_C} \right. \\
&\quad \left. + (3a_L + 3a_R + 3a_{BL} - 8a_3)^{II} \ln \frac{M_C}{M_R} + (3a_1 + 3a_2 - 8a_3)^I \ln \frac{M_R}{M_Z} \right], \\
&2\pi \left[\frac{4\pi}{g_R^2(M_R)} - \frac{\sin^2 \theta_W(M_Z)}{\alpha(M_Z)} \right] \\
&= 8\pi^2 \left[\frac{1}{g_R^2(M_R)} - \frac{1}{g_2^2(M_Z)} \right]
\end{aligned}$$

$$\begin{aligned}
 &= 8\pi^2 \left[\frac{1}{g_R^2(M_R)} - \frac{1}{g_L^2(M_R)} \right] + 8\pi^2 \left[\frac{1}{g_2^2(M_R)} - \frac{1}{g_2^2(M_Z)} \right] \\
 &= \left[(a_R - a_L)^{\text{III}} \ln \frac{M_D}{M_C} + (a_R - a_L)^{\text{II}} \ln \frac{M_C}{M_R} - a_2^{\text{I}} \ln \frac{M_R}{M_Z} \right], \\
 \frac{8\pi^2}{g_U^2} &= \frac{3}{8} \left[\frac{2\pi}{\alpha(M_Z)} - \left\{ \left(a_L + a_R + \frac{2}{3} a_4 \right)^{\text{IV}} \ln \frac{M_U}{M_D} + \left(a_L + a_R + \frac{2}{3} a_4 \right)^{\text{III}} \ln \frac{M_D}{M_C} \right. \right. \\
 &\quad \left. \left. + (a_L + a_R + a_{BL})^{\text{II}} \ln \frac{M_C}{M_R} + (a_1 + a_2)^{\text{I}} \ln \frac{M_R}{M_Z} \right\} \right] \\
 &= \frac{2\pi}{\alpha_s(M_Z)} - \left(a_4^{\text{IV}} \ln \frac{M_U}{M_D} + a_4^{\text{III}} \ln \frac{M_D}{M_C} + a_3^{\text{II}} \ln \frac{M_C}{M_R} + a_3^{\text{I}} \ln \frac{M_R}{M_Z} \right). \quad (\text{A.3})
 \end{aligned}$$

Note that $a_L^{\text{IV}} = a_R^{\text{IV}}$ since parity is not broken in energy interval IV.

If instead, we impose the conditions

$$g_L(M_D) = g_R(M_D) \equiv g_{LR}, \quad g_4(M_D) \equiv g_{4D}, \quad (\text{A.4})$$

where g_{LR} and g_{4D} are not necessarily equal, then the relations will be

$$\begin{aligned}
 &2\pi \left[\frac{3 - 6 \sin^2 \theta_W(M_Z)}{\alpha(M_Z)} - \frac{2}{\alpha_s(M_Z)} \right] \\
 &= 8\pi^2 \left[\frac{3}{e^2(M_Z)} - \frac{6}{g_2^2(M_Z)} - \frac{2}{g_3^2(M_Z)} \right] \\
 &= \left[(-3a_L + 3a_R)^{\text{III}} \ln \frac{M_D}{M_C} + (-3a_L + 3a_R + 3a_{BL} - 2a_3)^{\text{II}} \ln \frac{M_C}{M_R} \right. \\
 &\quad \left. + (3a_1 - 3a_2 - 2a_3)^{\text{I}} \ln \frac{M_R}{M_Z} \right], \\
 &2\pi \left[\frac{4\pi}{g_R^2(M_R)} - \frac{\sin^2 \theta_W(M_Z)}{\alpha(M_Z)} \right] \\
 &= 8\pi^2 \left[\frac{1}{g_R^2(M_R)} - \frac{1}{g_2^2(M_Z)} \right] \\
 &= \left[\frac{1}{g_R^2(M_R)} - \frac{1}{g_L^2(M_R)} \right] + \left[\frac{1}{g_2^2(M_R)} - \frac{1}{g_2^2(M_Z)} \right] \\
 &= \frac{1}{8\pi^2} \left[(a_R - a_L)^{\text{III}} \ln \frac{M_D}{M_C} + (a_R - a_L)^{\text{II}} \ln \frac{M_C}{M_R} - a_2^{\text{I}} \ln \frac{M_R}{M_Z} \right], \\
 \frac{8\pi^2}{g_{LR}^2} &= \frac{\pi}{\alpha(M_Z)} - \frac{1}{3} \frac{2\pi}{\alpha_s(M_Z)} - \frac{1}{6} \left[\{3(a_L + a_R)\}^{\text{III}} \ln \frac{M_D}{M_C} \right. \\
 &\quad \left. + \{3(a_L + a_R + a_{BL}) - 2a_3\}^{\text{II}} \ln \frac{M_C}{M_R} + \{3(a_1 + a_2) - 2a_3\}^{\text{I}} \ln \frac{M_R}{M_Z} \right], \\
 \frac{8\pi^2}{g_{4D}^2} &= \frac{2\pi}{\alpha_s(M_Z)} - \left(a_4^{\text{III}} \ln \frac{M_D}{M_C} + a_3^{\text{II}} \ln \frac{M_C}{M_R} + a_3^{\text{I}} \ln \frac{M_R}{M_Z} \right). \quad (\text{A.5})
 \end{aligned}$$

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