Three Essays in Economic Growth

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(ABSTRACT)

This dissertation is comprised of four chapters. Chapter 1 provides an introduction to economic growth and discusses the topics covered in each of the following chapters along with some main results therein.

In Chapter 2, I develop a dynamic general equilibrium of innovation and imitation in which once a higher quality good is developed, there is an exogenously given rate at which the good is targeted for imitation. However, the innovator can undertake expenditure to protect the good from imitation and thereby lower the effective probability of imitation. It is shown that the total expenditure toward property right protection is inversely related to the cost of property right protection and the effectiveness of the property right system. Moreover, a subsidy that reduces the per unit cost of property right protection leads to an increase in the intensity of innovation. In the long run, the economy exhibits a constant steady state growth. I further show that an improvement in the efficiency of the property right system has an ambiguous effect on overall consumer welfare.

Chapter 3 develops a two-good, closed economy model, that provides a possible explanation for the existence of misallocation of resources and examines the long-run consequences. In the model, inefficiencies arise as a result of lobbying by firms to establish or prevent barriers to the competitive allocation of factors of production (labor). First, I show that the extent of the inefficiency is determined by the relative lobbying power of the firms. The inefficiencies lead to a static welfare loss, which increase in the relative lobbying power of firms seeking to establish barriers. I further show that if the relative lobbying power of firms seeking the barriers is large, the economy will end up producing a "wrong" mix of goods in the long-run, relative to the perfectly competitive equilibrium. The resulting welfare loss depends on the elasticity of substitution between the two goods, and in the case when the two goods are poor substitutes, the total utility may go to zero in the long-run.

In Chapter 4, I apply the model of lobbying developed in Chapter 3 to understand the link between misallocation of resources, international trade and economic growth. Misallocation leads to the possibility that the benchmark competitive free trade equilibrium is not achieved. This leads to a reduction in trade volume and consequently to welfare losses even for a country without domestic barriers. Further, domestic barriers cause a reduction in output growth in the short run. In the long run, however, there is a convergence to the competitive growth rates.

Dedication

Dedicated to the memory of the late Dr. Gaurav S. Chaudhary.

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Chapter 1

Introduction

One of the most fundamental questions in economics is: Why some countries are wealthy, while others are poor? According to latest World Bank data, the per capita income in the United States is nearly twenty eight times that of Bangladesh. This translates into vast differences in the standard of living for the citizens of these countries. It is imperative to understand the factors behind such vast differences, in both the level and growth rates, of per capita income across countries. This is best summarized in the following quote by Robert Lucas

> Is there some action a government of India could take that would lead the Indian economy to grow like Indonesias or Egypts? If so, *what*, exactly? If not, what is it about the nature of India that makes it so? The consequences for human welfare involved in questions like these are simply staggering: Once one starts to think about them, it is hard to think about anything else.
>
> (Lucas 1988, p. 5; italics in original)

The neoclassical growth models which began with the work of Solow (1956, 1957) and Swan (1957), provided an initial framework to address these cross-country differences. There are two main features of these and the later models of Cass (1965) and Koopman (1965). First, long-run economic growth is driven by exogenous technological progress while the steady state income level is determined by the saving rate and labor force growth. An implication of this feature is that economic policy can only have an effect on the income levels, without having any impact on growth rates. Second, the models predict a convergence in the per capita income growth and levels across countries, a natural consequence of diminishing returns. While the neoclassical growth models found some degree of empirical support, not enough evidence could be provided in support of the convergence result. Further, by relegating growth exclusively as a consequence of exogenous technological progress, the models left little scope for economic policy to have any impact on improving growth performance.

The endogenous growth models which began with the pioneering work of Romer(1986, 1990) arose to explain the lack of convergence among economies and provide an *economic explanation* for growth. In these models steady state growth is generated endogenously and may depend upon factors that can be influenced by economic policy, for example, tax policies, education policies, and efficiency of intellectual property rights. More importantly, in these models growth rates may differ between economies over the long-run in contrast to the convergence results of the neoclassical growth models. The endogenous growth models took two distinct approaches. In the first approach technological progress was explicitly modeled as an outcome of intentional investment by profit-seeking agents. Since knowledge by definition is a public good, these models dropped the assumption of perfect competition and developed models of monopolistic competition. In these models, sustained increase in research and development (R&D) activities lead to long run economic growth.

The second approach retains the assumption of competitive markets. Growth, in these models is not the outcome of R&D, but instead arises as a result of capital accumulation. The key assumptions of these models is that there are no diminishing returns in the accumulation of reproducible capital. Human capital accumulation, for example, satisfies this property. One of the implications of the models of endogenous growth is that cross country differences in policy or preferences lead to permanent differences in growth rate of per capita output.

The majority of the above mentioned models, however, assume pareto optimal allocation of resources. That is, they assume that all resources in the economy are efficiently employed. Recent literature on growth now places misallocation of resources, both at the firm and economy level, as a major source of productivity differences among countries. There is evidence, both at the macro and micro level, that there exist considerable misallocation of resources within economies. The central theme in this line of analysis is that what matters for productivity is how given stocks of physical capital, human capital and knowledge are allocated across firms and sectors. The best allocation maximizes output and welfare. This literature argues that differences in per capita income across countries can be explained in terms of such misallocation of resources.

This dissertation consists of three essays in endogenous growth models and contributes to the above mentioned strands of the theory. In the first essay, I develop an endogenous growth model wherein growth is driven by innovations arising from R&D activities by firms. Traditionally, such models have assumed either an infinitely-lived fully-enforced patent on the innovators' products or that imitation is prohibitively expensive. Therefore, in most of these models imitation does not take place. Even if the models account for imitation, it is assumed that the innovator is passive and does not fight to protect his/her property rights. However, in reality patents are are of a limited duration, are frequently imitated upon before the full extent of the patent period, and the innovators undertake substantial expenditure to protect their innovations. The model developed in the first chapter takes into account these possibilities. In the model, the expenditure undertaken to infringe or protect a patent jointly the determine the effective duration of a patent. I show that the total expenditure toward property right protection is inversely related to the cost of property right protection and the effectiveness of the property right system. Moreover, a subsidy that reduces the per unit cost of property right protection leads to an increase in the intensity of innovation. In the long run, the economy exhibits a constant steady state growth. I further show that an improvement in the efficiency of property right system has an ambiguous effect on overall consumer welfare.

In the second and third essays of my dissertation I consider a model of human capital accumulation with the possibility of misallocation of resources. I show that some of the earlier results derived in this area are completely reversed if we assume even a slight deviation from a competitive allocation of resources. In the second essay, I develop a learning-by-doing model of human capital accumulation in the presence of allocative inefficiencies. The allocative inefficiencies arise as a result of lobbying by firms to prevent or establish barriers to the competitive allocation of labor. First, I show that the extent of the inefficiency is determined by the relative lobbying power of the firms. The inefficiencies lead to a static welfare loss, which increase in the relative lobbying power of firms seeking to establish barriers. I further show that if the relative lobbying power of firms seeking the barriers is large, the economy will end up producing a "wrong" mix of goods in the long-run, relative to the perfectly competitive equilibrium. The resulting welfare loss depends on the elasticity of substitution between the two goods, and in the case when the two goods are poor substitutes, the total utility may go to zero in the long-run.

In the third essay, I apply this model of lobbying and misallocation to understand the relationship between international trade and economic growth. While the conventional wisdom on this relationship states that more open free trade policies lead to higher growth rates, the empirical evidence in support of this view is divided. There is growing evidence that open trade policies have a positive impact of growth only after taking into account several country specific factors. In the third essay, I develop a two-country two-good model of international trade and incorporate barriers to competitive allocation of labor in both countries. I show that barriers to the competitive allocation of labor in one country leads to a reduction in trade volumes, even in the absence of trade barriers. This in turn leads to a welfare loss for both the countries. Further, domestic barriers cause a reduction in growth rates as compared to the growth rate achieved in the case of competitive allocation. Finally, it is shown that with international trade the country with barriers does go on to converge to the competitive growth rates in the long run.

Chapter 2

Endogenous Patent Protection Expenditure and Economic Growth

2.1 Introduction

Innovation and imitation both constitute technological progress. While innovation creates new products and production processes, imitation leads to the spread of the new technologies. It is also true that both innovation and imitation are uncertain processes. In either case there is uncertainty about the success of a particular research and development (R&D) project. Innovative firms are also uncertain about how long they will be the sole producers of the state of the art product. They face the threat of other innovative firms who might discover a higher quality product and replace them at the top. This is the idea of creative destruction stressed by Joseph Schumpeter (1942). Moreover, the new knowledge created as a result of innovation by its very nature is non-rival and non-excludable. This creates a threat of imitation by other firms. To prevent imitation a system of property rights, usually in the form of patents (monopoly power), is granted to the innovator.

The growth theory models pioneered by the work of Romer (1986,1990) have shown that intentional investment by economic agents in R&D leads to technological progress and consequently drives economic growth. The underlying feature of these models is the idea that a system of property rights make the new knowledge a partially excludable good and therefore the innovators can appropriate the returns from their investment in R&D. This provides the innovators with the *ex ante* incentive to innovate. In the absence of a system of patents there will be no ex ante incentive to innovate and therefore no technological progress and growth. Empirical evidence also points to the fact that intellectual property rights play an important role in spurring innovations. Kanwar and Evenson (2003) using a cross country panel data show that stronger intellectual property rights lead to a faster rate of technological progress. In a more direct approach Favley, Foster and Greenaway (2006) show, using a threshold regression model, that stronger intellectual property rights lead to a higher growth in high and low income countries, but does not significantly affect growth in the middle income countries.

While much of the theoretical literature focuses on the link between innovation, imitation, and growth, little attention has been paid to the role that property rights play in these processes. The literature circumvents the problem of property right protection by either assuming that there is an infinitely long patent or that imitation is costly and therefore no imitation takes place. However, in practice, patents are awarded for a finite period of time and usually an innovation is imitated before the actual duration of the patent ends. For instance, Mansfield, Schwatrz, and Wagner (1981) found that 60 percent of the patented innovations they studied were imitated within 4 years. Some of the papers that model costly imitative activities assume that property rights are given exogenously and the innovator remains passive, expending no resources to protect the patent over the innovation. This also misses an important point, because, in reality patent holders undertake substantial expenditure to protect their patents from potential infringement. An article in *Fortune* magazine (May, 2008) noted that protecting intellectual assets was among the top legal challenges for business leaders. Intellectual property lawyers note that "*applicants need to deploy more legal muscle to counter each rejection and get their patents through....And fight harder to keep them.*"

Innovative firms undertake various forms of expenditure to protect their intellectual property rights from potential imitators, including, for instance, litigation expenditure and adding new features to their products so to render them more difficult to imitate. Lerner (1995) notes that the direct patent litigation costs for the year 1991 accounted for more than 25 per cent of R&D expenditure for that year. More recently Bessen and Meuer (2008) show that between 1985-2004 attorney fees for a patentee were on an average $1.04 million (in 1992 dollars) per case. They further show that the the total loss to the litigating parties (both the patentee and the infringer) is much greater than the cost of the lawyers. As Crampes and Langinier (2002) note,

> while patents are typically viewed in the economic literature as a perfect exogenously given protection against imitation, their efficiency depends on their idiosyncratic characteristics and on the effort of the patent owner in trial and settlement procedures.

My goal in this chapter is to incorporate this aspect of the intellectual property rights system by considering the expenditure undertaken by the innovators to protect their patents and by analyzing how the expenditure affects innovation, technological progress, growth, and welfare.

There is a growing literature that discusses agents' incentives to expend resources toward protection of their property rights and the resulting impact on efficiency.[1] Dinopolous and Syropolous (2007) have extended the discussion to the context of growth with innovation and rent protection. The present chapter adds to this literature. We analyze how the expenditures undertaken by innovative firms to protect their property rights impacts the innovative intensity and economic growth. We incorporate these expenditures in the standard quality ladder framework wherein, once a firm innovates and attains a leadership position in a specific industry, it starts expending resources to protect its monopoly power. As noted above, these resources may take the form of legal fees and expenditures undertaken to keep the blue print of a product secret.

This chapter is different from earlier work in this area in several ways. Most of the earlier work concentrates on incentives to innovate and imitate, taking intellectual property rights, if any, as exogenously given. Dinopolous and Syropolous (2007) incorporate rent protecting activities (RPA) in this framework. In their model, however, there is no threat of imitation and innovators protect their innovations from other potential innovators, making innovation progressively more difficult over time. This chapter considers the threat of imitation and allows innovators to expend resources to protect their property rights. Successful imitation shifts the good from a monopoly market to a perfectly competitive market. So at each instant there are goods that are produced and sold in monopolistic and competitive markets. We derive the steady state rate of innovative intensity and the intensity of property right protection. We show that the intensity of property right protection depends on the relative cost of innovation to property right protection, the difficulty of R&D and the effectiveness of

[1] See for instance Tornell 1997, Parente and Prescott 1999, 2000, Skaperdas and Syropoulos 2001,2002

the property right system. Finally, allowing for imitation in the model also has implications for policy and welfare. Since with imitation the good is provided competitively, leading to welfare gains, the model explicitly takes into account the trade off between these static welfare gains and the potential loss in growth resulting from reduced incentives to innovate.

The remainder of the chapter is organized as follows. Section 2.2 presents a brief literature review. In section 2.3 we set up the model and solve for the steady state equilibrium. In section 2.4 we present some comparative static results and determine the welfare properties of the model. Section 2.5 contains concluding remarks.

2.2 Related Literature

The role of technological progress and the interaction between innovation and imitation in economic growth has been analyzed in several papers. Endogenous growth models explain growth as arising from endogenous technological change. In these models both innovation and imitation occur as a result of purposive expenditure undertaken by entrepreneurs in the R&D sectors (see for instance, Romer (1991), Grossman and Helpman (1991 a,b)). Grossman and Helpman (1991a), develop a quality ladder model in which repeated product improvement takes place in a continuum of sectors. The equilibrium distribution of qualities evolve overtime, but the rate of aggregate growth is constant. They extend this model in a later paper, in which they introduce imitation. They study a two country model, where firms in the North innovate and firms in the South learn to imitate the products developed in the North. They assume that firms in the South are inept at innovation, while they have a comparative advantage in manufacturing the good, once they learn to imitate. The equilibrium rate of innovation and imitation is constant in the steady state and they show that subsidies to innovation in the North prevents imitation by the South under certain cases; while an imitation subsidy in South hinders innovation in the North. In both the papers R&D decisions are made by forward looking, profit maximizing entrepreneurs.

Segerstorm (1991), extends the model to allow goods in the North to be copied in the North itself. The paper develops a dynamic general equilibrium (DGE) model of growth with quality ladders in which both innovation and imitation occurs in the North. Innovation and imitation are stochastic processes, whereby the probability of success in either case is increasing in the resources devoted to same. They also assume that the R&D technology is linear. The paper determines a steady state equilibrium in which some firms devote resources to innovation and other firms devote resources toward imitation. The possibility of collusion between the innovators and the imitators gives the imitators an ex-ante incentive to imitate. The paper goes on to analyze the impact of subsidies to innovation, and shows that such a subsidy decreases the rate of innovation and increases the rate of imitation. Further, it leads to a higher growth rate in the economy and only increases welfare if innovative intensity is above a critical level.

Much of the work following Segerstorm, has been devoted to explaining the seemingly counterintuitive result that an increase in innovative R&D subsidy leads to an increase in imitative R&D and a decrease in innovative R&D. Cheng and Tao (1999) show that this result is caused by the assumption of a linear R&D technology. They show that by replacing this linear technology with a sufficiently convex R&D technology, the counterintuitive results are reversed. They assume that the R&D technology is linear at the firm level, but convex at the industry wide level. Davidson and Segerstorm (1998), assume that R&D is subject to diminishing returns at the firm level as well. They introduce specialized labor in both innovation and imitation, where there are constant returns in both the generalized and specialized labor but diminishing returns to each, individually. They show that in such a model an innovative (imitative) subsidy leads to higher innovation (imitation) and a lower imitation (innovation).

In all of the papers mentioned above little or no importance has been given to the role of property rights. Helpman (1993) addresses this issue in a two country DGE framework in which North innovates goods and South imitates. The paper shows that stronger intellectual property rights hurt the South. Kwan and Lai (2003) work out the optimal level of intellectual property right protection, by looking at the trade off between the loss in current consumption and gain in consumption growth as a result of stronger property right protection. They calibrate the model to US data and conclude that under-protection of property rights is more likely than over-protection. Further, the welfare losses in case of under-protection is substantially greater than in the case of over-protection. These papers, however, ignore the expenditure undertaken by the imitators to infringe on the intellectual property rights of the innovator and the resources devoted by the innovator to protect the intellectual property rights. In a sense, they take the security of property rights as an exogenous variable.

A recent strand of literature on fully endogenous growth models has incorporated rent protection into their models. Dinopoulos and Syropoulos (2007) build a quality ladder model where incumbent firms devote resources to safeguard monopoly rents from their past innovations. They term these expenses as rent protecting activities (*RPA's*). In this model discovery of higher quality products is a result of sequential stochastic innovation contest between the incumbent industry leaders and followers. RPA's by the incumbents make innovation progressively more difficult. They show that in this case, long run growth depends positively on proportional innovative R&D subsidies and negatively on the effectiveness of rent protecting activities. Sener (2008), develops a quality ladder model without scale effects in which incumbent innovators undertake *RPA's* against potential innovators and additionally innovation gets more difficult as goods move up in the quality ladder. Sener concludes that in such a scenario, it is optimal to tax R&D when innovations are of a very small and very large magnitudes. It is however optimal to subsidize them when innovations are of medium size. Neither of these papers take into account imitation and the contest between innovators and imitators. The present work is closely related to Segerstorm (2007), where there is both innovation and imitation. Innovation gets progressively more difficult and is carried out by both incumbents and followers. Imitation leads to elimination of monopoly power. Segerstorm, however, does not consider property right protection on the part of the incumbent monopolist. My work in essence incorporates expenditure toward property right protection from potential imitators, when innovation becomes difficult over time.

2.3 Model

We follow the basic quality ladders framework of Grossman and Helpman (1991a) adapted by Segerstorm (2007). In Segerstorm (2007), once innovation takes place there is a constant threat of imitation, and once imitation occurs the product is supplied competitively. The rate of imitation is exogenously given. We modify this set-up by allowing the innovator to undertake expenditure to protect his innovation from imitation. The effective rate of imitation is determined endogenously within the model.

2.3.1 *Industry Structure*

There is an economy with a continuum of industries indexed $\omega \in [0,1]$. The products in each industry can be supplied in a countable number of qualities $j = 0, 1, 2, 3...$ Firms are distinguished on the basis of the quality of product they produce. Quality improvement takes place as a result of uncertain and costly innovative R&D by firms. Every time a firm is successful in innovating, the quality of a good jumps from j to $j+1$. At any given point of time there are two types of industries. First, there are industries where there is a single quality leader as a result of successful innovation. The quality leader in these industries earns monopoly profits. Second, there are industries in which successful imitation has taken place. In these industries there are multiple producers of the highest quality good in a competitive market.

2.3.2 *Consumer Behavior*

There are a fixed number of identical households that provide labor services for manufacturing, research and development, and protection of property rights. The number of members of each household grows over time at an exogenous rate $n> 0$. The supply of labor at each point in time is therefore given by $L(t) = L_0 e^{nt}$. The wage earned by labor is assumed to be the numeraire and is constant through time. Given this, each household maximizes the inter-temporal utility function given by

$$U = \int_0^\infty e^{-(\theta - n)t} lnu(t)dt, \tag{2.1}$$

where θ is the subjective discount rate, n is the population growth rate and $u(t)$ is the consumer's instantaneous utility at time t. Instantaneous utility is given by

$$u(t) = \left[\int_0^1 \left(\sum_{j=0}^{\infty} \lambda^j d(j, \omega, t) \right)^{\kappa} d\omega \right]^{1/\kappa}, \tag{2.2}$$

where $d(j, \omega, t)$ denotes the quantity consumed of a product of quality j produced by industry ω at time t, and $\lambda > 1$ represents the extent to which higher quality products improve on the lower quality products and $\kappa \in (0, 1)$ determines the elasticity of substitution between consumer goods given by $\sigma = 1/(1 - \kappa)$. The consumer maximizes the discounted utility subject to the inter-temporal budget constraint

$$\int_0^{\infty} e^{-R(t)} E(t) dt = A(0), \tag{2.3}$$

where $R(t)$ is the cumulative interest factor up to time t, $A(0)$ is the summed value of asset holdings at time $t = 0$ and the present value of future factor income, and $E(t)$ is the consumer's per capita expenditure flow at time t. The consumer's time t expenditure flow is given by

$$E(t) = \int_0^1 \sum_{j=0}^{\infty} p(j, \omega, t) d(j, \omega, t) d\omega, \tag{2.4}$$

where $p(j, \omega, t)$ is the price of a product of quality j produced by industry ω at time t. As stated earlier, we normalize a worker's wage equal to one and analyze the steady state equilibrium where the consumer's expenditure $E(t)$ becomes constant over time.

Utility maximization involves two steps. First is a static problem where the consumer chooses, for a given expenditure $E(t)$, and price vector $p(j, \omega, t)$ the quantity of each good of quality j from industry ω. This involves maximizing the instantaneous utility function given in equation (2.2) subject to the budget constraint imposed in equation (2.4). In this set up the consumer choice will depend upon the quality adjusted price of a good. We assume that goods of different quality within the same industry are perfect substitutes; therefore, consumers only buy goods with the lowest quality adjusted price. We further assume that in cases where goods of different qualities have the same quality adjusted price, consumers buy goods with the highest quality. As will be shown later, these assumptions imply that in equilibrium the consumer will consume only the highest quality goods. The demand for lower quality goods will be zero. The Lagrangian function for the static maximization problem is given by [2]

$$L = u(t)^{\kappa} + \mu \left[E(t) - \int_0^1 \sum_{j=0}^{\infty} p(j,\omega,t) d(j,\omega,t) d\omega \right]. \tag{2.5}$$

The first order condition for this problem is [3]

$$d(j,\omega,t) = \left[\frac{\mu p(j,\omega,t)}{\kappa \lambda^{j\kappa}} \right]^{\frac{1}{(\kappa-1)}}. \tag{2.6}$$

λ^j denotes the quality of the product on step j of the quality ladder in industry ω at time t. Using the fact that $\sigma = 1/(1-\kappa)$, we define

$$\delta = \lambda^{\sigma-1}, \tag{2.7}$$

where δ is interpreted as the demand adjusted effective measure of quality improvements. Therefore, the demand function can be re-written as

$$d(j,\omega,t) = \left[\frac{\mu p(j,\omega,t)}{\kappa} \right]^{\frac{1}{(\kappa-1)}} \delta^j.$$

[2] $u(t)^{\kappa}$ is a strictly increasing monotone transformation of the instantaneous utility function and therefore yields the same optimization solution. We use this form, since it simplifies the algebra

[3] The consumer chooses $d(j,\omega,t)$ for each industry ω, so there is a continuum of first order conditions. Equation (6) describes each of them.

Taking the ratio of demands for two industries ω and $\tilde{\omega}$, yields the relative demand as

$$\frac{d(j,\omega,t)}{d(j,\tilde{\omega},t)} = \left[\frac{p(j,\omega,t)}{p(j,\tilde{\omega},t)}\right]^{-\sigma} \frac{\delta^{j(\omega,t)}}{\delta^{j(\tilde{\omega},t)}}.$$

Further, define

$$q_j(\omega,t) \equiv \delta^{j(\omega,t)}, \tag{2.8}$$

as the measure of quality of a good on step j of the quality ladder in industry ω, that is, the quality of the state of the art good. In what follows we will use $j(\omega)$ to denote the state of the art quality level in industry ω instead of $j(\omega,t)$ and leave the dependence on t implicit. The demand and price of a good of quality j in industry ω are denoted as $d_j(\omega)$ and $p_j(\omega)$ respectively. Making these substitutions in the above equation we get

$$\frac{d_j(\omega)}{d_j(\tilde{\omega})} = \left[\frac{p_j(\omega)}{p_j(\tilde{\omega})}\right]^{-\sigma} \frac{q_j(\omega)}{q_j(\tilde{\omega})}.$$

Rearranging terms and integrating over ω, we get

$$\int_0^1 p_j(\omega)d_j(\omega)d\omega = \frac{p_j(\tilde{\omega})^\sigma}{q_j(\tilde{\omega})} d_j(\tilde{\omega}) \int_0^1 p_j(\omega)^{1-\sigma} q_j(\omega)\; d\omega.$$

Note that the left side of the equation above is the expenditure $E(t)$. Re-arranging terms and using the notations above we can write down the demand function for good of quality j in industry $\tilde{\omega}$ as

$$d_j(\tilde{\omega}) = \frac{q_j(\tilde{\omega})p_j(\tilde{\omega})^{-\sigma}E(t)}{\int_0^1 q_j(\omega)p_j(\omega)^{1-\sigma}d\omega}.$$

Thus the demand for the good of highest quality j in any industry ω is given by

$$d_j(\omega) = \frac{q_j(\omega)p_j(\omega)^{-\sigma}E(t)}{\int_0^1 q_j(\omega)p_j(\omega)^{1-\sigma}d\omega}. \tag{2.9}$$

As noted earlier, the demand for all goods below quality j is zero.

This completes the first step of the optimization problem. The second step of optimization is a dynamic problem in which the households decide on the allocation of expenditure $E(t)$ overtime. We assume the existence of capital markets where the households save by investing either in a portfolio of risky securities or in risk free bonds which bears a risk free interest rate. The equilibrium interest rate $r(t) \equiv dR(t)/dt$ clears the capital market at each moment in time. Further, the no arbitrage condition in the asset market requires that, in equilibrium, the return from investing in a portfolio of securities should equal the risk free rate of return.

Substituting the demand function from equation (2.9) into equation (2.2) and substituting the resulting expression into equation (2.1), we get the inter-temporal indirect utility function of the households. The dynamic optimization problem of the household then reduces to maximizing this indirect utility function, subject to the budget constraint in equation (2.3) given the rate of interest $r(t)$. That is,

$$max \int_0^\infty e^{-(\theta-n)t} \left[lnE(t) + \left(\left(ln \int_0^1 \frac{\lambda^j q_j(\omega) p_j(\omega)^{-\sigma}}{\int_0^1 q_j(\omega) p_j(\omega)^{1-\sigma} d\omega} \right)^{1/\alpha} d\omega \right)^{1/\alpha} \right] dt,$$

subject to $\int_0^\infty e^{-R(t)} E(t) dt = A(0)$. This problem yields the following Euler equation

$$\frac{\dot{E}}{E} = r(t) - \theta. \tag{2.10}$$

Equations (2.9) and (2.10) describe optimal consumer behavior.

2.3.3 *Producer Behavior*

For each industry and each quality product constant returns to scale prevails, with one unit labor producing one unit of the output. Labor is the only factor of production. By assumption, labor is homogeneous and the economy wide endowment of labor grows at a constant rate n. Further, labor markets are assumed to be perfectly competitive and all workers earn the same equilibrium wage rate. At any given point of time there are two types of industries. First, industries where imitation has occurred. In these industries there are multiple producers of the highest quality good. Second, there are industries where imitation has not taken place, and we have only one quality leader. We analyze producer behavior in each type of industry. We assume that once imitation takes place there is Bertrand competition between the incumbent and the imitators, driving down the price of the good to its marginal cost and consequently the firms earn zero economic profit.

Once a higher quality good is innovated in a particular industry the innovator is awarded a property right in the form of a patent and becomes the sole producer of the good in the industry. Recall that the demand for all lower quality products is zero and the demand for the innovators good is given by equation (2.9). We assume that the system of patents is not perfect and the good is targeted for imitation. The effective duration of the patent is determined by the resources that the leader expends to protect the property right. Thus the industry leaders' problem is two fold. The leader acts as a monopolist and therefore chooses the price (p_L) for his product given the demand. The leader also has to decide on the amount of resources to be devoted toward property right protection.

Let $a_p\delta^j$ be the unit cost of property right protection for a good of quality j in any ω industry per unit of time. The cost of property right protection does not depend on the industry. However, the unit cost is increasing in the product quality. Thus protection of property rights becomes progressively more expensive as product quality increases. As quality improves, protecting the property right over a good can become more costly for a variety of reasons. For instance, more money needs to be spend to keep the blue print of several components of the product a secret. Alternatively, one might need to apply for several patents for different components of the same product. For example, *Apple* filed 200 patents for the technology relating to the *iPhone* alone.[4] The parameter a_p may be interpreted as a fixed cost such as the hourly rate of a patent attorney.

If we let P be the total intensity of property right protection (for example the total number of hours for which a patent attorney is hired), then the total cost of property right protection at each time would be $Pa_p\delta^j$. Given this, the instantaneous profit function of the single quality leader is given by

$$\pi_L = \begin{cases} (p_L - 1)\frac{q_j(\omega,t)p_L^{-\sigma}E(t)L(t)}{\int_0^1 q_j(\omega,t)p(j,\omega,t)^{1-\sigma}d\omega} - Pa_p\delta^j & p_L \leq \lambda \\ 0 & p_L > \lambda \end{cases} \tag{2.11}$$

It is easy to see that if the single quality leader charges a price greater than λ, then the quality adjusted price of the good will be greater than the price of the good one step below in the ladder. Therefore, the demand for the highest quality good is zero and the resulting profit is zero. If on the other hand the quality leader charges a price less than or equal to λ, the profit depends upon the difference between the price p_L and the marginal cost, the market demand, and the expenditure toward property right protection. Given the per capita demand from equation (2.9), the total market demand is derived by multiplying (2.9) by the

[4] According to World Intellectual Property Organization, the number of claims on a patent is one the major factors determining the cost of patent application and prosecution

size of the population, $L(t)$. It is obvious that in this case, as long as $\lambda > 1/\kappa$ the single quality leader would charge the limit price. Therefore assuming that $\lambda > 1/\kappa$, the single quality leader would charge the limit price $p_L = \lambda$.

Denote

$$Q(t) \equiv \int_0^1 q_j(\omega) d\omega \tag{2.12}$$

as the average quality level across industries at time t and

$$z(t) = \frac{Q(t) p_L^{-\sigma} E(t)}{\int_0^1 q_j(\omega) p_j(\omega)^{1-\sigma} d\omega} \tag{2.13}$$

as the per capita quantity demanded for a single quality leader's product when the product is of average quality. This representation is useful because, as will be shown, the dynamic behavior of the model is determined by how $Q(t)$ evolves over time as a result of innovative activity. Using this notation the profit function for the quality leader can be rewritten as

$$\pi_L = (\lambda - 1) \frac{q_j(\omega)}{Q(t)} z(t) L(t) - P a_p \delta^j. \tag{2.14}$$

The term $q_j(\omega)/Q(t)$ can be interpreted as the quality of the good in industry ω relative to the average quality of goods across all industries. Further, since $z(t)$ is a measure of demand, the term $z(t)L(t)$ captures the market size. Therefore the profit flow is an increasing function of the relative quality of the firm's product and the market size. Note also that while instantaneous profit falls as a result of property right protection expenditure (P), undertaking this expenditure increases the monopoly duration which increases the present discounted value of expected profits. Therefore, the quality leader chooses P to maximize the present discounted value of expected profits given by

$$V_I[j(\omega)] = \mathbb{E}_t\left[\int_0^\infty \pi_L e^{-\int_0^t r(s)ds} dt\right], \tag{2.15}$$

where $\mathbb{E}_t$ in the above equation denotes the expectation operator.

2.3.4 R&D Races

We assume that innovation is uncertain and costly and only takes place in industries with multiple quality leaders, i.e, only goods that have been imitated are targeted for innovation. This will certainly be the case if we consider a situation in which further innovation upon a good can only take place if the blue print of the current highest quality good is available to other potential innovators. Until the time a successful imitation takes place the blue print remains a secret and no other firms can innovate further. While this blue print is available to the property right holder, it can be shown that for some reasonable parameter restrictions the leader has no incentive to go two steps above in the quality ladder.

We further assume that innovation becomes progressively more difficult as a good moves up the quality ladder. The amount of labor required for each unit of innovative R&D in industry ω is given by $a_I\delta^j$, so as j goes up, the per unit cost of innovation in industry ω goes up. If a firm invests $Ia_I\delta^j$ units of labor, then it is successful in discovering the next highest quality product $j(\omega)+1$ with instantaneous probability Idt. Since labor is the numeraire good, the total cost of innovation is also $Ia_I\delta^j$. Innovations therefore follow a Poisson process with arrival rate I.

Imitation is modeled in the simplest possible way. There is an exogenous imitative intensity

$\overline{C}$ with which all goods are targeted for imitation. In case of a successful imitation, Bertrand competition ensues between the incumbent monopolist and the imitator and both of them end up making zero economic profit. This intensity of imitation does not vary across industries or overtime. However, as mentioned earlier the single quality leader expends resources (P) to protect his property right from the imitators. Therefore the effective probability of imitation is a function of both $\overline{C}$ and P. The effective probability of imitation is given by

$$\rho = \frac{\overline{C}}{\varphi P}, \tag{2.16}$$

where φ can be interpreted as the effectiveness of the property right system. Note that this probability is increasing in $\overline{C}$ and decreasing in φ and P. If the quality leader expends more resources or the system of property rights become more effective, then the instantaneous probability of imitation goes down.

2.3.5 R&D Optimization

In this section we analyze the firms' choice of the optimal innovative intensity and property right protection. All firms engaged in R&D are assumed to maximize their expected discounted profits (market value). Once a firm innovates, its market value is given by $V_I[j(\omega)]$. If the product is imitated, it becomes available competitively and the market value of the firm becomes zero. Therefore, the firms problem is to choose the intensity of property right protection so as to maximize (2.15). The Hamilton-Jacobi-Bellman equation for the leader is given by [5]

$$r(t)V_I[j(\omega)] = \max_P \left\{\pi_L - \rho V_I[j(\omega)]\right\}, \tag{2.17}$$

[5] See Malliaris and Brock (1982,pp.123-24) for the application of stochastic dynamic programming to Poisson Jump Process. The same technique has been used in Segerstorm(2007), Dinopolous and Syropolous(2007)

where $\rho \equiv \overline{C}/\varphi P$ is the effective rate of imitation. The right side of equation (2.17) states that the monopolist makes a profit of π_L each time period and with instantaneous probability ρ loses the market value as a result of imitation. Thus it is the maximized expected return on the stock of the monopolist. The left side is the return on an equal sized investment in a risk free bond. In equilibrium the returns must be equal. Solving the optimization problem in the right side of equation (2.17) we get

$$a_p \delta^{j_\omega} = \frac{\overline{C}}{\varphi P^2} V_I[j(\omega)]. \tag{2.18}$$

For a potential innovator in an industry where imitation has already taken place, the Hamilton-Jacobi-Bellman equation is given by

$$r(t)V_{PI} = \max_I \left\{ -I a_I \delta^{j(\omega)} + I[V_I[j(\omega)+1] - V_{PI}] \right\}, \tag{2.19}$$

where V_{PI} is the market value of the potential innovator. The potential innovator incurs a cost of $-I a_I \delta^{j_\omega}$ in each time period and with probability I succeeds in innovating the next higher quality good $j(\omega)+1$ and becomes the quality leader in the industry. The firm raises the resources for R&D in the capital market by issuing securities to households. Since innovation is an uncertain process these securities are risky. However, the risk is idiosyncratic and the households can earn a sure rate of return by investing in a diversified portfolio.

Note that the optimization problem of the potential innovator is linear in the innovative intensity I. If $a_I \delta^{j(\omega)} > V_I[j(\omega)+1] - V_{PI}$, then given free entry into innovation an infinite amount of resources will be devoted to innovative activity or there will be an infinite demand for loans (supply of securities). This will cause interest rate to rise to infinity and would drive down the present expected market value of the firm to zero. Therefore, the above inequality cannot be sustained in equilibrium. Similar argument rules out $a_I\delta^{j(\omega)} < V_I[j(\omega)+1] - V_{PI}$ as an equilibrium condition. In this model the interest rate adjusts at each point in time to ensure that at equilibrium

$$a_I\delta^{j(\omega)} = [V_I[j(\omega)+1] - V_{PI}]. \tag{2.20}$$

In the steady state, however, the interest is constant and is equal to the household discount factor θ. This is shown in next two sections. As in standard models of free entry and linear technology, the optimal scale of innovative intensity I is indeterminate. In the steady state the innovative intensity is determined by the rate of growth of quality, as will be shown in the next section. Multiplying both sides of the equation (2.20) with I we see that the total costs of innovation exactly equals the expected benefits from innovation, thus the present market value of the potential innovator, $V_{PI} = 0$. Therefore we get

$$a_I\delta^{j(\omega)} = V_I[(j(\omega)+1)],$$

or

$$V_I[j(\omega)] = \frac{a_I\delta^{j(\omega)}}{\delta}. \tag{2.21}$$

Substituting for $V_I[j(\omega)]$ from equation (2.21) into equation (2.18) we get

$$P = \left(\frac{\overline{C}a_I}{\varphi\delta a_p}\right)^{1/2}. \tag{2.22}$$

Equation (2.22) determines the intensity of property right protection completely in terms of the parameters of the model. As expected, P is negatively related to a_p and φ. As the per unit fixed cost of property right protection goes up, the total intensity of property right protection goes down. φ is the effectiveness or efficiency of the property right system, if that increases, firms need not devote as many resources toward property right protection.

Additionally, an increase in imitative intensity, $\overline{C}$, would prompt firms to devote additional resources toward property right protection and therefore P goes up. An increase in $\delta \equiv \lambda^{\sigma-1}$, implies that the monopoly price that can be charged goes up and the firm can make a higher profit at each instant of time. So everything else remaining same, the firm can generate the same expected discounted profit in a shorter duration, and will reduce the expenditure on property right protection to prolong the monopoly power. Substituting the value P from equation (2.22) into (2.16), we get the effective probability of imitation as

$$\rho = \left(\frac{\overline{C}\delta a_P}{\varphi a_I}\right)^{1/2} \tag{2.23}$$

Further, substituting equation (2.14), (2.18) and (2.21) into equation (2.17) we get

$$r(t) + 2\rho = \frac{\delta(\lambda - 1)z(t)}{a_I x(t)}, \tag{2.24}$$

where

$$x(t) \equiv \frac{Q(t)}{L(t)}. \tag{2.25}$$

Since R&D becomes progressively more difficult as we move up the quality ladder, more labor is required to come up with each successive innovation. An increase in $Q(t)$ means that the average quality is increasing and more labor is required to maintain the rate of innovation. An increase in $L(t)$ implies that more labor is available for R&D. Thus $x(t)$ can be interpreted as a measure of the relative difficulty of R&D. This interpretation follows from Segerstorm (2007). Equation (2.25) is one of the key relationships of the model, linking the state variables $x(t)$ and the demand measure $z(t)$ as given in equation (2.13).

Before turning to the dynamic behavior of the model it is worthwhile to summarize the results thus far. It has been shown that the monopoly producers of the highest quality good charge a limit price λ. The demand for their good at this price depends upon the relative quality of the good and the market size. The intensity of property right protection at a point in time is determined by maximizing the expected discounted value of future profits arising from the monopoly. The optimal intensity of property right protection P, given in equation (2.22), turns out to be a constant and this in turn determines a constant effective rate of imitation ρ as given in equation (2.23). While the intensity of property right protection is constant, the total expenditure to achieve this intensity, $Pa_P\delta^{j(\omega)}$, increases as the good moves up the quality ladder. This essentially means that as a good moves up the quality ladder, a greater amount resources need to be devoted to maintain the effective imitation rate ρ.

To proceed with the dynamic behavior, note that there are two state variables in the model; the relative difficulty of R&D $x(t) \equiv Q(t)/L(t)$, and the demand measure $z(t)$ whose values we need to determine at each point in time. We are interested in a steady where $x(t)$ is constant over time. This is because, in a steady state that exhibits a positive but finite rate of innovation $x(t)$ must be constant. An increasing $x(t)$ means that innovation becomes progressively more difficult and eventually no resources will be devoted to innovative activities. On the other hand, if $x(t)$ falls over time then an infinite amount of resources will be devoted to R&D. Thus for a positive but finite rate of innovation the state variable $x(t)$ must grow at a constant rate. In the following section we analyze a steady state where this property holds. It will be shown that in the steady state characterized by a constant $x(t)$, the state variable $z(t)$ and the per capita expenditure $E(t)$ are also constant.

2.3.6 *Dynamic Behavior*

In this section we will analyze the behavior of the variable $Q(t)$, the average quality of goods across industries over time. Let l be the measure of industries with a single quality leader and c the measure of industries with multiple producers of the highest quality good. We will also refer to industries with a single quality leader as 'l-industries' and those with multiple producers of highest quality good as the 'c-industries'. Using this notation, innovation in our model only takes place in the c industries. We can now determine the evolution of the average quality over time. Note that

$$Q(t) = \int_0^1 \delta^{j(\omega)} d\omega,$$

or

$$Q(t) = \int_l \delta^{j(\omega)} d\omega + \int_c \delta^{j(\omega)} d\omega,$$

or

$$Q(t) = Q_L(t) + Q_C(t).$$

Q_L and Q_C are the measure of qualities in l and c-industries. The time derivative of $Q(t)$ can therefore be written as

$$\dot{Q}(t) = \dot{Q}_L(t) + \dot{Q}_C(t). \tag{2.26}$$

To see the time derivative of Q_L note that quality in a particular l-industry jumps from $j(\omega)$ to $j(\omega) + 1$ when innovation takes place in a c-industry. Similarly, quality in a particular l-industry drops when imitation occurs in that industry and the good of quality $j(\omega)$ leaves that industry for a c-industry. Thus we have

$$\dot{Q}_L(t) = \int_c \delta^{j(\omega)+1} I d\omega - \int_l \delta^{j(\omega)} \rho d\omega,$$

or

$$\dot{Q}_L(t) = \delta I Q_C - \rho Q_I. \tag{2.27}$$

Using similar reasoning we can establish

$$\dot{Q}_C(t) = \rho Q_L - I Q_C. \tag{2.28}$$

Substituting equations (2.27) and (2.28) into (2.26) we get

$$\dot{Q}(t) = (\delta - 1) I Q_C.$$

Denoting

$$q_c \equiv Q_C / Q(t), \tag{2.29}$$

we get

$$\frac{\dot{Q}(t)}{Q(t)} = (\delta - 1) I q_c. \tag{2.30}$$

q_c can be interpreted as a measure of quality in the c-industries relative to the overall quality. Equation (2.30) gives us the evolution of average qualities across all industries through time. Note that the growth rate is proportional to the innovation rate over the relative quality in the c-industries. This makes sense because innovation in our model occurs only in the c-industries.

Recall that the level of difficulty of R&D is given by the variable $x(t) \equiv Q(t)/L(t)$. Differentiating $x(t)$ over time we get

$$\frac{\dot{x}(t)}{x(t)} = \frac{\dot{Q}(t)}{Q(t)} - \frac{\dot{L}(t)}{L(t)},$$

or

$$\frac{\dot{x}(t)}{x(t)} = (\delta - 1)Iq_c - n. \tag{2.31}$$

In a steady state determined by a constant $x(t)$ we must have

$$I = \frac{n}{(\delta - 1)q_c}. \tag{2.32}$$

The steady state level of innovation intensity depends on population growth rate and the relative quality of goods in the c-industries. This is driven by the fact that innovation becomes progressively more difficult as goods climb up the quality ladder. An increase in q_c implies that quality in the c-industries goes up relative to the average quality of goods. Since innovation only takes place in the c-industries a higher relative quality in these industries implies innovation becomes more difficult and therefore innovative intensity will fall. A higher population growth on the other hand, enables the economy to devote more resources to innovation and therefore innovative intensity is positively related to the rate of population growth.

We are now in a position to determine q_c and solve for the steady state innovative intensity completely in terms of the parameters of the model. We know from equation (2.28) that

$$\frac{\dot{Q}_C(t)}{Q_C} = \rho \frac{Q_L}{Q_C} - I,$$

or

$$\frac{\dot{q}_c}{q_c} = \frac{\dot{Q}_C}{Q_C} - \frac{\dot{Q}}{Q}.$$

So it follows that in the steady state when $\dot{q}_c = 0$,

$$q_c = \frac{\rho(\delta - 1) - n}{(n + \rho)(\delta - 1)}, \tag{2.33}$$

and

$$q_l \equiv Q_L/Q = 1 - q_c = \frac{n\delta}{(n + \rho)(\delta - 1)}, \tag{2.34}$$

where ρ is given in equation (2.23). Substituting equation (2.33) into (2.32) we get

$$I = \frac{n(n + \rho)}{\rho(\delta - 1) - n}. \tag{2.35}$$

Note that a positive rate of innovation requires $\rho > n/(\delta - 1)$. That is, imitation is required to sustain innovative activity. This again follows from the fact that innovation takes place only in the c-industries. The condition states that rate of imitation should be greater than the relative ease of innovation. This is satisfied if we assume that a_p/a_I is sufficiently large. That is, the fixed cost of protecting property is sufficiently large.

2.3.7 *Labor Market*

In this model labor is used for manufacturing the final good in the l and c-industries, for innovative R&D and for property right protection. The sum of labor used for each type of activity should be equal to the labor of the economy at each point of time. We will begin by first solving for the total labor requirement for manufacturing.

The total manufacturing employment is equal to total employment in l-industries plus the total employment in c-industries. The total employment in both industries is determined by the demand for the products in each industry. Recall that the demand for the product in each industry depends on the price of the good, the relative quality and the market size. Consequently the total labor requirement for manufacturing in a l-industry is given by

$$d_j(\omega)L(t) = \frac{q_j(\omega)}{Q(t)} z(t)L(t),$$

where $d_j(\omega)$ is the per capita demand for a good produced in an l-industry as given in equation (2.9). Similarly, the manufacturing employment in a c-industry is given by

$$d_j(\omega)L(t) = \frac{q_j(\omega)}{Q(t)} \lambda^\sigma z(t)L(t).$$

The total manufacturing employment is therefore given by

$$\int_l d_j(\omega)z(t)L(t)d\omega + \int_c d_j(\omega)z(t)L(t)d\omega.$$

Using $q_c \equiv Q_C/Q(t)$, where $Q_C = \int_c q_j(\omega)d\omega$ yields the total manufacturing employment L_m as

$$L_m = z(t)L(t)\left[(1 - q_c) + \lambda^\sigma q_c\right]. \tag{2.36}$$

Note that the manufacturing employment is an increasing function of q_c. This is because we have competitive pricing in the c-industries, and therefore demand for the goods is higher in these industries so more labor has to be utilized in production.

We now consider the labor devoted to innovation and property right protection. Innovative R&D only takes place in c-industries, therefore the total labor used for this purpose is given by

$$L_I = \int_c I a_I \delta^{j(\omega)} d\omega,$$

or

$$L_I = Ia_I Q q_c. \tag{2.37}$$

Once again, employment for R&D is an increasing function of q_c, because as quality in c-industry increases, innovation gets more difficult and more labor is required for innovative R&D, to maintain the innovative intensity. Property right protection is only undertaken by the single quality leaders in the l-industries. Therefore

$$L_P = \int_l P a_p \delta^{j_\omega} d\omega,$$

or

$$L_P = P a_p Q q_l. \tag{2.38}$$

The employment for property right protection is an increasing function of q_l. This is because the per unit labor requirement on property rights is increasing in quality of goods. A rise in q_l implies that quality of goods in the l-industries goes up, therefore total employment for property right protection also goes up. The total labor demand is given by $L_m + L_I + L_P$ which must be equal to the labor supply at each time. Thus we get the following

$$z(t)L(t)\left[q_l + \lambda^\sigma q_c\right] + Ia_I Q q_c + P a_p Q q_l = L(t).$$

Dividing both sides of the equation by L(t) we get

$$z(t)\left[q_l + \lambda^\sigma q_c\right] + Ia_I x(t) q_c + P a_p x(t) q_l = 1. \tag{2.39}$$

In a steady state defined by constant $x(t)$ and q_c,

$$I = \frac{n(n+\rho)}{\rho(\delta - 1)} \textit{ and } P = \left(\frac{\overline{C}a_I}{\varphi\delta a_P}\right)^{1/2}$$

are constant. Equation (2.39) therefore implies that in the steady state

$$z(t) = \frac{Q(t)p_L^{-\sigma}E(t)}{\int_0^1 q_j(\omega)p_j(\omega)^{1-\sigma}d\omega}$$

is also a constant, its value is given by

$$z = \frac{\lambda^{-\sigma}E(t)}{q_l\lambda^{1-\sigma} + q_c}. \tag{2.40}$$

A constant z implies that in steady state $E(t)$ is also a constant and therefore the interest rate, $r(t)$, is equal to the discount factor θ. This result is summarized in the following proposition

Proposition 2.1 *In a steady state defined by constant $x(t)$ and q_c, the innovative intensity I and the per capita expenditure E(t) are constant. Further, the interest rate in the steady state is also constant and is equal to the household discount rate θ.*

Now, using (2.40) in equation (2.24) we get

$$z = \frac{(\theta + 2\rho)a_I x}{\delta(\lambda - 1)}. \tag{2.41}$$

Equation (2.41) describes the relationship between the variables z and x to maintain zero profits in R&D. Note that z and x are positively related. This is because, as x goes up, the cost of maintaining the innovative intensity goes up. This requires that demand z go up in order to earn a higher revenue to justify the additional resources into R&D.

The labor condition in equation (2.39) can be rewritten as

$$z = \frac{1}{q_l + \lambda^\sigma q_c} - x \left[\frac{I a_I q_c + P a_p q_l}{q_l + \lambda^\sigma q_c} \right]. \tag{2.42}$$

Again note that labor condition implies a negative relationship between z and x. As the level of difficulty, x, of R&D rises more resources are needed to maintain the steady state value of innovation leaving fewer resources for the production of goods thus z falls. Equation (2.41) and (2.42) can be solved simultaneously to pin down the steady state values of z and x. Figure 2.1 below plots these two equations in the (z, x) space. The intersection of the two lines determines the equilibrium values. The downward sloping line is the labor market constraint while the upward sloping line is the zero profit condition.

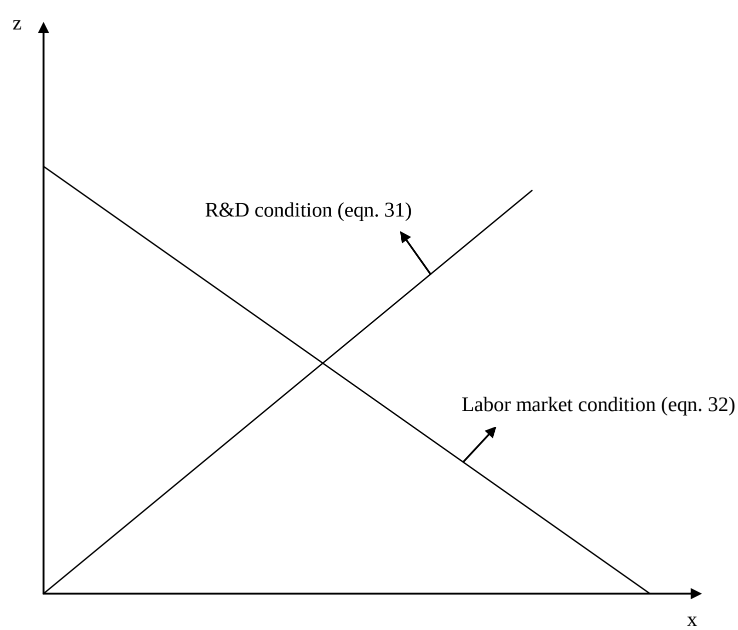

Figure 2.1: steady state z and x

2.4 Comparative statics and Welfare

In this section we derive the comparative static results from the model and analyze the welfare properties.

2.4.1 Comparative Statics

Change in φ

We first consider the effect of a change in the parameter φ on the intensity of innovation, demand (z) and relative difficulty of R&D (x). It can be easily verified that $\partial I/\partial\varphi > 0$. That is an increase in the efficiency of the property right system, leads to increase in the Innovative intensity. This is because an increase in φ, everything else same, leads to a fall in ρ or the probability of successful imitation. Thus the monopoly of the quality leaders lasts for a longer period, hence the innovative intensity goes up.

We now look at the impact of an increase in φ on z and x. An increase in φ, decreases ρ, so for each level of x, the firms can maintain a zero profit with lower demand, therefore z decreases. Thus the R&D condition from figure 2.1, shifts rightward. Also an increase in φ, increases both the z and the x intercept of the resource condition. Therefore we have the following proposition

Proposition 2.2 *An increase in the efficiency of the property rights, leads to an increase in the innovative intensity, an increase in the relative difficulty of R&D and has an ambiguous effect on the demand.*

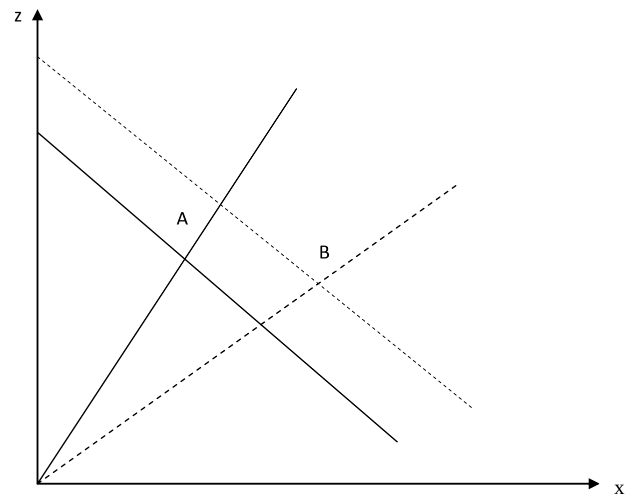

Figure 2.2: Impact of increase in φ

In figure 2.2, A represents the original equilibrium in the economy. The new equilibrium is given by a point such as B. It is clear from the figure that depending on the way the two lines shift, z can either go up or down. But it is clear that x goes up. An increase in x implies that the relative difficulty of R&D has increased. This can only happen if during the transition period, rate of growth of Q exceeds the rate of growth of L. Since the rate of growth of Q depends positively on both I and q_c, this means that I goes up proportionately more than the decline in q_c.

Impact of an R&D subsidy

Next, we look at the impact of an R&D subsidy to innovative firms. Denote by s_I the per unit subsidy given for innovative R&D. Thus the per unit cost of R&D in industry ω is now given by $(1 - s_I)a_I\delta^j$. Incorporating this subsidy into the optimization problem of firms, the zero profit condition in equation (2.24) is modified to

$$z(t) = \frac{(r + 2\rho)(1 - s_I)a_I x(t)}{\delta(\lambda - 1)}. \tag{2.43}$$

Additionally, note that the steady state values of P, ρ, q_c and q_l also change. These are given as follows

$$P = \left(\frac{\overline{C}(1-s_I)a_I}{\varphi\delta a_p}\right)^{1/2} \;;\; \rho = \left(\frac{\overline{C}a_p\delta}{\varphi(1-s_I)a_I}\right)^{1/2}. \tag{2.44}$$

Substituting the value of ρ in equations (2.33) and (2.34) we get the new steady state values of q_c and q_l. Note that P is now a decreasing function of the R&D subsidies. This is because as s_I increases, the cost of R&D to the innovative firm decreases. The zero profit condition in R&D implies that the expected present value of profits as a result of successful innovation must also decrease in equilibrium. This implies that at the margin, the expected benefit from a unit increase in property right protection expenditure must also have decreased. Therefore, the firms decrease the intensity of property right protection. Further, a decrease in P with a constant $\overline{C}$ and φ implies that ρ must increase. This is clear from the second equation. An increase in ρ leads to an increase in q_c and a decrease in q_l.

It can be seen easily that $\partial I/\partial s_I < 0$. Therefore an increase in R&D subsidy leads to a decrease in innovative intensity. The intuition behind this result is as follows. An increase in q_c implies that the quality in the c industries has risen relative to the overall quality. This would mean that innovation becomes more difficult in these industries and as innovation only takes place in the c industries, overall intensity of innovation decreases. This result is in direct contrast to the result in Segerstorm (2007), where it is shown that an increase in R&D subsidy leads to a temporary increase in I, but does not lead to a long run change in it. In his model the R&D subsidy does not affect the probability of successful imitation and therefore does not affect the relative quality in the c industries. However, in our model an increase in s_I changes the incentives for expenditure on property right protection and therefore the probability of successful imitation. To analyze the impact of an increase in s_I on z and x we again look at the shifts in the labor market resource condition and the zero profit condition. This is shown in figure 2.3 on the next page.

The zero profit condition pivots about the origin to the right. The labor market condition also shifts, the z intercept falls and the x intercept increases. It is clear from the figure that depending on how the two curves shift both z and x may increase or decrease. Although if z increases then x also increases. However, a fall in z may be accompanied by either an increase or decrease in x.

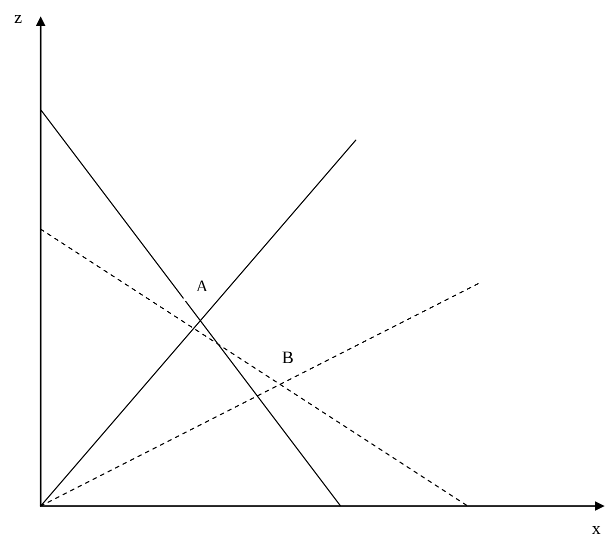

Figure 2.3: Impact of increase in s

Subsidy to property right protection expenditure

The next change we consider is a subsidy on property right protection expenditure for firms. Let us assume there is a subsidy rate s_P on property rights expenditure that is undertaken by industry leaders in the l industries. An increase in s_P leads to an increase in P, increase in q_l and a decrease in q_c. An increase in P leads to a decrease in ρ and this in turn causes I to increase. Thus, a subsidy to property right protection expenditure has the same effect as an improvement in the effectiveness of the property rights system. The impact on z and x will also be similar to the case of an improvement in the efficiency of the property rights system. Thus, qualitatively a subsidy on property right protection expenditure has the same effect as an improvement in the system of property rights.

2.4.2 Welfare and Growth

To derive the welfare implications of the model we first compute the instantaneous utility of the household. This is given by:

$$u(t) = \left(\int_0^1 [\lambda^j d_{jt}(\omega)]^\kappa d\omega \right)^{1/\kappa} \tag{2.45}$$

Since the only products that are consumed in equilibrium are the highest quality goods, demand for all other goods is zero and they do not enter the utility function. Now, substituting the value of $d_j(\omega)$ from equation (2.9) into the above equation we get

$$u(t) = EQ(t)^{\frac{1-\kappa}{\kappa}} [q_l \lambda^{1-\sigma} + q_c]^{\frac{1-\kappa}{\kappa}}. \tag{2.46}$$

The instantaneous household utility is increasing in per capita expenditure E, the average quality of products $Q(t)$ and the relative quality of goods in the c industries,i.e q_c and decreasing in q_l. An interesting question is the impact of an improvement in the efficiency of property right system (φ) on the overall utility of the consumers. To see this, note that in the steady state quality $Q(t)$ grows at the rate n. Therefore, $Q(t) = Q_0 e^{nt}$, where Q_0 is the initial average quality across all industries. Assuming that initial quality in each industry is one, Q_0 is also one. Further, substituting for E in (2.46) from equation (2.40), we can re-write the instantaneous utility in the steady state as

$$u(t) = z\lambda^{\sigma}(e^{nt})^{(1-\kappa)/\kappa}\left[1 + q_c(1 - \lambda^{1-\sigma})\right]^{1/\kappa}$$

Now, substituting for z from equation (2.41) we can re-write the instantaneous utility in terms of $x(t) \equiv Q(t)/L(t)$, the relative difficulty of R&D. The resulting expression is

$$u(t) = \frac{(\theta + 2\rho)a_I x}{\delta(\lambda - 1)}\lambda^{\sigma}(e^{nt})^{(1-\kappa)/\kappa}\left[1 + q_c(1 - \lambda^{1-\sigma})\right]^{1/\kappa}$$

Denoting

$$\Delta \equiv \frac{a_I\lambda^{\sigma}(e^{nt})^{(1-\kappa)/\kappa}}{\delta(\lambda - 1)},$$

and substituting in the expression for $u(t)$ we get

$$u(t) = \Delta(\theta + 2\rho)x\left[1 + q_c(1 - \lambda^{1-\sigma})\right]^{1/\kappa}. \tag{2.47}$$

In the above expression the term Δ is a constant, while ρ, x and q_c are all functions of φ. We know from equation (2.23) that $\partial\rho/\partial\varphi < 0$. Further, from proposition 2.1, we know that $\partial x/\partial\varphi > 0$ and $\partial q_c/\partial\varphi < 0$. Therefore, the impact of an increase in φ on the instantaneous utility $u(t)$ is ambiguous. This is expected because the improvement in the efficiency of property right protection has two effects. First, by increasing the rate of innovation it increases the overall quality of goods available for consumption. This leads to a rise in the instantaneous utility. Second, an improvement in the efficiency of property right protection reduces the effective rate of imitation (ρ), thereby decreasing the number of goods that are available competitively. This leads to a decrease in the instantaneous utility of the consumer. These two effects counteract each other and therefore the overall impact on the consumers utility is ambiguous. This particular result of the model is different from previous papers, as they do not model imitation.

We now turn to the growth implication of the model. Taking natural logs on both sides of equation (2.46) and differentiating with respect to time we can derive the expression for the rate of growth of the economy.

$$\gamma = \left(\frac{1-\kappa}{\kappa}\right)\frac{\dot{Q}}{Q} \tag{2.48}$$

In the steady state, $\frac{\dot{Q}}{Q} = n$. Therefore the steady state rate of growth for the economy is given by

$$\gamma = \left(\frac{1-\kappa}{\kappa}\right)n \tag{2.49}$$

The steady state rate of growth is constant and is positively related to the rate of population growth. This is because in the long run growth in driven entirely by quality improvements as a result of innovative activity. Innovative activity however depends on population growth as innovation in our model becomes progressively more difficult and positive population growth is required to sustain innovation.

2.5 Conclusion

In this chapter we analyzed the role of expenditures toward property right protection with a standard quality ladders framework where innovation becomes progressively difficult overtime. Innovator undertakes purposive investment into R&D and innovates a higher quality good and becomes a monopoly producer of the good. As soon as the good is innovated it is also targeted for imitation at an exogenously given rate. By expending resources toward thwarting imitation, the innovator attempts to reduce the effective probability of imitation. While this reduces the instantaneous profit of the innovator, it also increases the expected duration of the monopoly profits. The innovator decides on the optimal contribution toward property right protection by balancing these two effects. We find that the optimal contribution is negatively related to the per unit cost of property right protection and the efficiency of the property rights system. At the same time an increase in the exogenous imitative intensity or per unit cost of innovation decreases the optimal property right expenditure. We also derive the optimal innovative intensity in the steady state and show that it is negatively related to the effective probability of imitation and positively related to population growth. Since, innovation becomes difficult overtime a positive population growth is required to sustain innovations in the long run.

In terms of welfare, it is shown that the steady state utility of consumers depends on both the overall quality of goods and the fraction of goods supplies competitively. An improvement in the efficiency of property rights system increases the overall quality of goods, at the same time reducing the fraction of goods supplied competitively. This leads to an ambiguous change in steady state utility for the consumer. This result is different from previous work in this area. Most of the earlier literature does not incorporate imitation and expenditure toward property right protection. Since in these models all goods are supplied in monopolistic markets, there is no scope for utility gain resulting from goods becoming available competitively. Therefore, in these models any policy that leads to higher overall quality of goods is unambiguously welfare enhancing. Finally, we turn to the growth implications of the model and show that in the steady state the rate of growth of the economy is constant and is proportional to the rate of population growth.

This work leaves various avenues for further research. First, in this model we have assumed that imitative intensity is given exogenously. A richer model would endogenize the rate of imitation by modeling the behavior of the imitator. One can then analyze the impact of changes in cost of imitation and optimal property right expenditure. Second, this model analyzes the steady state growth and innovation. It is worthwhile to investigate what happens to innovation, growth and welfare during the transition to the steady state.

Chapter 3

Learning by Doing in a Model of Allocative Inefficiency

3.1 Introduction

The central task of growth and development economics is to explain the vast observed differences in output growth and per capita income across countries. The neoclassical growth models which began with the work of Solow(1956, 1957) and Swan(1957), provided an initial framework to address these cross-country differences. The models assume two factors of production (labor and physical capital) with diminishing returns to each factor in the production process. There are two main features of these and the later models of Cass (1965) and Koopman (1965). First, long-run economic growth is driven by exogenous technological progress while the steady state income level is determined by the saving rate and labor force growth. Second, the models predict a convergence in the per capita income growth and levels across countries, a natural consequence of diminishing returns.[1] The availability of detailed

[1]See McCallum (1996) for a survey of neoclassical growth models and its predictions.

cross-country data since the beginning of 1990's also gave rise to a slew of empirical research, testing if these predictions fit the actual performance of countries.

Mankiw, Romer and Weil (1992) employ a simple cross country linear regression to show that the Solow framework modified appropriately to account for human capital accumulation does a good job of explaining cross country differences in per capita income. That is, differences in saving, education and population growth account for the differences in income per capita. The main drawback of the MRW paper however is its assumption of no differences in the aggregate production function across countries. In particular, they assume that all countries have the same level of productivity. Islam (1995) looks at the same questions raised by MRW in a panel data framework accounting for the unobservable 'country effects' in the production function. The paper argues that ignoring these differences in the production function creates an omitted variable bias. The results show that differences in the unobservable productivity terms across countries are a significant source of per capita income differences across countries. In the absence of these differences, convergence would have occurred as the predicted by the neoclassical models.

The endogenous growth models initiated by the pioneering work of Romer (1986, 1987) and Lucas 1988) arose as a critique to the predictions of the neoclassical growth models. In these models steady state growth is generated endogenously and may depend upon factors that can be influenced by economic policy, for example, tax policies, education policies, and efficiency of intellectual property rights. More importantly, in these models growth rates may differ between economies over the long-run in contrast to the convergence results of the neoclassical growth models. These models incorporate externalities into the production process that generate increasing returns to factors at the economy wide level, while maintaining the assumption of diminishing returns at the firm level.[2] In contrast to the neoclassical growth models which attribute the differences in per capita income to just the differences in savings rate and labor force growth, the endogenous growth models enabled researchers to include several policy variables in an econometric regression to explain the behavior of per capita income levels and growth. In some sense, therefore, the endogenous growth models provide an *economic explanation* for the cross country differentials.

There has been considerable research following Islam (1995) and the endogenous growth literature that corroborates the finding of productivity differences amongst countries. For example, Hall and Jones (1999), using a simple growth accounting exercise, show that it is productivity differences and not physical or human capital differences that explain differences in income levels across countries. Parente and Prescott (1994) and Cole et al. (2005) also reach a similar conclusion. A natural question, then, is why does productivity differ so widely among countries? Hall and Jones (1999) attribute the differences in productivity to differences in what they term 'social infrastructure' across countries. By social infrastructure they mean "institutions and government policies that determine the economic environment within which individuals accumulate skills, and firms accumulate capital and produce output". They argue that good 'social infrastructure' prevents output of individual productive units from diversion and that they are essential for a high level of output per worker. They test their hypothesis for a cross section of countries using instrumental variable estimation. They find that differences in social infrastructure across countries lead to differences in not only the productivity but also the on the rate of capital accumulation and educational attainment.

[2] See Romer (1994)for an excellent survey of the endogenous growth literature

These papers reinforce the point raised by Parente and Prescott (1994), who argue that differences in technology adoption rates are a primary cause of difference in productivity. Their paper considers the decision of a firm to upgrade the technology of its plant. Unlike the original neoclassical models, the paper argues that technology adoption is a costly decision which depends on country specific factors and the existing stock of world knowledge. They derive an equilibrium wherein countries with significant policy and institutional barriers use less of the available knowledge in the world. The size of these barriers is given exogenously. This work is extended in Parente and Prescott (1999) to analyze why some of the policies that prevent adoption of latest technologies are in place. They conclude that most of these policies exist to protect special interest groups within the economy. Specifically, they develop a model in which a labor coalition monopolizes the supply of its services to all firms using a particular production process. The monopoly rights of the coalition is protected via regulation. Given this, they derive an equilibrium wherein the labor coalition can prevent the adoption of newer technologies and results in firms employing inferior technologies. They find that eliminating such arrangements could increase output by a factor of 3 without increasing inputs.

Parente and Prescott papers highlight the importance of differential technology adoption rates in explaining differential growth rates. However, once a technology is adopted there is a Pareto optimal allocation of resources. A growing strand of literature now places misallocation of resources, both at the firm and economy level, as a major source of productivity differences among countries. There is evidence, both at the macro and micro level, that there exist considerable misallocation of resources within economies. The central theme in this line of analysis is that what matters for productivity is how given stocks of physical capital, human capital and knowledge are allocated across firms and sectors. The best allocation maximizes output and welfare. For example, Banerjee and Duflo (2005) argue that the low overall output of India is essentially due to differing marginal productivities of capital across firms arising from misallocation. Hsieh and Klenow (2009) cite empirical evidence which demonstrates the misallocation across plants within 4-digit industries potentially reduces the TFP in manufacturing by a factor of two to three in India and China. In a recent working paper Jones (2010) develops a model wherein the misallocation in the intermediate goods at the firm level is amplified through the input-output structure of an economy.

Cole et al. (2005) examines the reason Latin American countries have not caught up to their western peer economies and successful east Asian economies. The paper presents two key findings. First, a stagnant relative TFP is the key determinant of relative income and labor productivity stagnation. Second, human capital differences are not the key determinant of Latin American TFP gap. The main factor explaining the TFP gap is barriers to competition. The barriers are both international (tariffs, quotas) and domestic (entry barriers, inefficient financial system and large subsidized state owned enterprises). The authors present evidence which shows that Latin American countries have erected international and domestic barriers that have closed them off from domestic and international competition. The paper further shows that productivity and output increases significantly when these barriers are dropped.

In a recent working paper Bergoeing et al. (2010) analyze the role of barriers resulting from underdevelopment and policy distortions that alter the entry and exit decisions of firms. They develop a general equilibrium model with heterogeneous firms subject to idiosyncratic shocks to their productivity. They show that as the cost of entry imposed by the barriers increases, the firm distribution is altered in such a way that too many inefficient firms remain in the market. This impedes the allocation of resources to more efficient firms and delays technology adoption. On calibrating the model to the leading and developing countries, the authors find that the barriers account for more than 50 percent of the income gap between the U.S and the developing countries.

This chapter adds to the literature on the effects of misallocation of resources on the long-run performance of an economy. We incorporate resource misallocation into the learning-by-doing framework of Lucas (1988). Lucas develops a two-good model in which growth is driven by the accumulation of human capital. Learning by doing implies that labor becomes more productive when more of it is used in the production of a good. In the model labor is allocated competitively across the two goods and these factor shares evolve over time. The evolution of the factor shares depends on the elasticity of substitution between the two goods. If the two goods are good substitutes, then as labor becomes more productive in the production of one good, an increasing amount of labor is devoted to its production. This eventually leads to specialization in the production of that good. Alternatively, if the two goods are poor substitutes, then the gains realized as a result of becoming more productive in one good enables resources to be released from that good to the production of the other good. Therefore, in the long-run both the goods are produced. Given this and assuming there are no diminishing returns in the accumulation of human capital, the model exhibits positive growth in the long-run even in the absence of exogenous technological progress.

As mentioned, considerable empirical evidence suggests resource misallocation at the micro and macro level. In this chapter we address the impact of allocative inefficiencies on the growth and welfare of economies. We also outline a possible mechanism which may lead to the said allocative inefficiencies. To this end, we build on the Lucas model by incorporating a simple model of lobbying by firms to establish barriers to the competitive allocation of labor or alternatively prevent the establishment of such barriers. We first show that the extent of these barriers and the resulting allocative inefficiency is determined by the relative strength of lobbying of the two types of firms. If the relative lobbying strength of firms seeking to establish barriers is sufficiently high, then the equilibrium allocation of labor across the two types of goods is not competitive. There is a loss in static welfare as a result of this, and the loss in welfare is greater the lower the elasticity of substitution and greater the relative lobbying strength of firms seeking barriers.

We then consider the dynamic properties of the model and its implications for the long-run allocation of labor across the two goods and growth. We show that a deviation from the competitive allocation of labor as a result of lobbying can completely alter the results of the Lucas model. For instance, when the two goods are good substitutes there exists a range of parameter values for which the economy may end up specializing in the "wrong" good. Further, if the two goods are not good substitutes then contrary to the Lucas model the economy may not produce both the goods. This leads to a substantial welfare loss in the long-run.

The present work also fits in with the literature that emphasizes the role of institutional arrangements like the extent of special interest lobbying on the economic performance of a country, such as Olson (1982, 1996). This literature argues that economic policies and institutions determine the extent to which nations attain their potential. That is, poor policies and institutions create a set of incentives such that nations are not operating on their production frontiers, contrary to the assumption of Pareto optimal allocation of conventional growth models. These policies prevent the economies from from fully realizing the gains from specialization and trade. Therefore, the literature argues that large gains in economic performance can be achieved as a result of adopting the right mix of policies. This is best summarized in Olson (1996, p. 20),

> *Countries that adopt relatively good economic policies and institutions enjoy rapid catch up growth: since they are far short of their potential, their per capita income can increase not only because of the technological and other advances that simultaneously bring growth to the richest countries, but also by narrowing the huge gap between their actual and potential income.*

This chapter is organized as follows. In section 3.2 we briefly summarize the Lucas (1988) model and outline its main results in propositions 3.1 and 3.2. In section Section 3.3 we build on the Lucas model by incorporating a lobbying contest between firms and derive the subgame perfect Nash equilibrium of the lobbying contest and account for the possibility of a uncompetitive allocation of labor. Section 3.4 analyzes the static welfare implications and section 3.5 looks at the dynamic behavior of the complete model. In section 6 we offer some concluding comments.

3.2 Model

3.2.1 The Competitive Equilibrium

We first outline the Lucas (1988) model of human capital accumulation. We briefly present his model and its main conclusions. In this set up, the economy produces two goods - red widgets and blue widgets. We modify the set up from the original Lucas formulation by assuming that there are two types of firms in the economy - Type B and Type R. Type B firms can only produce blue widgets and Type R firms only red. While this does not change the results of the Lucas model with free movement of labor between the two types of firms, the assumption is helpful later on when we introduce rigidities in the economy by constraining the movement of labor between the two types of firms. The two goods are produced using the Ricardian technology:

$$B = h_B \lambda L, \tag{3.1}$$

and

$$R = h_R(1-\lambda)L, \tag{3.2}$$

where h_B and h_R are the marginal product of labor in the production of blue and red widgets. The marginal products are constant at a point in time but they evolve over time as a result of learning by doing. Thus one can interpret each h_i as the human capital specialized to the production of good i, which capital evolves as more effort is expended to the production process. Further, λ and $(1-\lambda)$ are the fraction of labor devoted to the production of blue and red widgets respectively. In what follows we will normalize the total labor supply, L, to one. To formally incorporate learning by doing into the model it is assumed that growth of the human capital terms increases with the share of labor devoted to the production of the two goods. That is,

$$\dot{h}_B = h_B \delta_B \lambda \tag{3.3}$$

and

$$\dot{h}_R = h_R \delta_R (1-\lambda). \tag{3.4}$$

The individual consumers' utility function is given by

$$U = \left[\alpha_B b^{-\gamma} + \alpha_R r^{-\gamma}\right]^{-\frac{1}{\gamma}}, \tag{3.5}$$

where b and r are the quantities consumed of blue and red widgets respectively. Further, $\gamma > -1$, $\alpha_B, \alpha_R \geq 0$ and $\alpha_B + \alpha_R = 1$. The constant elasticity of substitution is given by $\sigma = 1/(1+\gamma)$. If P is the blue price of the red widgets (blue widgets are assumed to be the numeraire), then utility maximization by an individual requires that

$$\frac{r}{b} = \left(\frac{\alpha_R}{\alpha_B}\right)^{\sigma} P^{-\sigma}. \tag{3.6}$$

In the original Lucas formulation labor is free to move between the two types of firms and there is free entry into the production process. All firms in the economy earn zero economic profits and all proceeds from production flow to labor. Under these assumptions profit maximization by firms implies that the relative price under perfect competition, P_c, must be equal to the ratio of productivities,

$$P = \frac{h_B}{h_R} \equiv P_c. \tag{3.7}$$

So by combining (3.1), (3.2), (3.6) and (3.7), we can derive the equilibrium work force allocation from

$$\frac{1-\lambda}{\lambda} = \left(\frac{\alpha_R}{\alpha_B}\right)^{\sigma} \left(\frac{h_R}{h_B}\right)^{\sigma-1}. \tag{3.8}$$

Solving the above equation for λ and substituting the value of P_c from equation (3.7) gives us the optimal static labor force allocation under perfect competition as

$$\lambda = \frac{1}{1+\left(\frac{\alpha_R}{\alpha_B}\right)^{\sigma}\left(\frac{h_R}{h_B}\right)^{\sigma-1}} \equiv \lambda_c = \frac{1}{1+\left(\frac{\alpha_R}{\alpha_B}\right)^{\sigma} P_c^{1-\sigma}} \tag{3.9}$$

Equations (3.7) and (3.9) are the static equilibrium variables of the Lucas model. These results are summarized in the following proposition

Proposition 3.1 *The total share of labor devoted to the production of blue widgets under perfect competition is given by* $\lambda_c = [1 + (\alpha_R/\alpha_B)^\sigma (h_R/h_B)^{\sigma-1}]^{-1}$ *and the equilibrium price is given by* $P_c = h_B/h_R$.

Proceeding with Lucas' formulation we can analyze the evolution of this closed economy overtime. Combining the above results with equations (3.3) and (3.4), the autarky price in the Lucas model evolves as follows

$$\frac{\dot{P}_c}{P_c} = \frac{\dot{h}_B}{h_B} - \frac{\dot{h}_R}{h_R} = \delta_B \lambda_c - \delta_R (1 - \lambda_c),$$

or, as follows from equations (3.7) and (3.9)

$$\frac{\dot{P}_c}{P_c} = (\delta_B + \delta_R) \left[1 + \left(\frac{\alpha_R}{\alpha_B} \right)^\sigma P_c^{1-\sigma} \right]^{-1} - \delta_R. \tag{3.10}$$

The solution for the above differential equation determines the labor force allocation at each date and therefore determines the time paths of h_B and h_R. The solution to equation (3.10) depends on the elasticity of substitution σ. To see this let $\lambda_c(P_c) \equiv [1 + (\alpha_R/\alpha_B)^\sigma P_c^{1-\sigma}]^{-1}$. We now have three possible cases

Case 1: $\sigma > 1$. When $\sigma > 1$, the two goods are good substitutes. Note that as P_c approaches 0 , $\lambda_c(P_c)$ approaches 0 and as P_c approaches ∞, $\lambda_c(P_c)$ approaches 1. Also note from equation (3.10) that when $\lambda_c(P_c) = \delta_R/(\delta_R + \delta_B)$, $\dot{P}_c = 0$. This is depicted in figure 3.1 below. It is easy to see that if initial P_c is less than P_c^*, $\dot{P}_c$ is negative and P_c falls overtime leading to a specialization in red widgets. Similarly if initial P_c is greater than P_c^* then P_c increases overtime leading to specialization in blue widgets. Thus the system under autarky converges to specialization in one of the two goods. If the economy is initially relatively better at producing blue widgets (i.e $h_B/h_R > P_c^*$), then the economy will eventually only produce blue widgets. On the other hand if initially the economy is relatively better at producing red widgets, then the economy will eventually produce only red widgets.

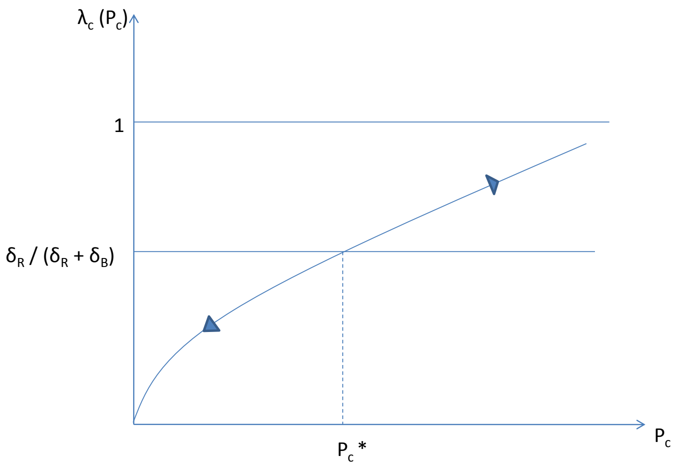

Figure 3.1: $\sigma > 1$

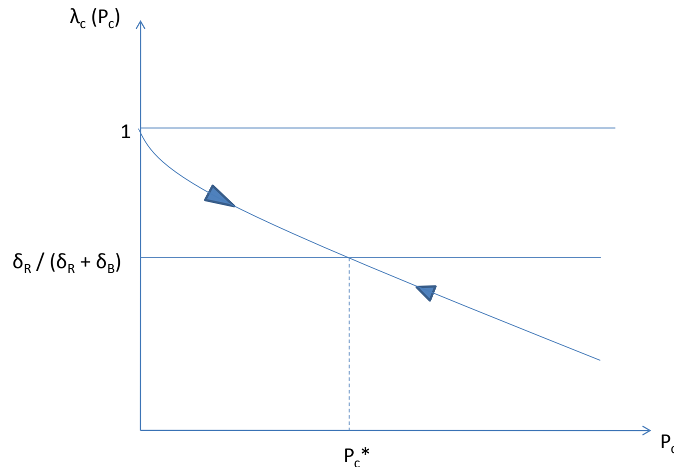

Figure 3.2: $\sigma < 1$

Case 2: $\sigma < 1$. In this case the two goods are poor substitutes, $\lambda_c(P_c)$ is downward sloping and the point P_c^* is a stable stationary point. At this point the labor is allocated at each time period so as to equate $\delta_B\lambda_c$ and $\delta_R(1-\lambda_c)$. That is, in the long-run both the goods will be produced. This makes sense as the goods are poor substitutes.

Case 3: $\sigma = 1$. In this borderline case, $\lambda_c(P_c)$ is a flat line. The workforce force is allocated according to the demand weights α_B and α_R and this allocation is maintained forever. Since h_B/h_R increases or decreases forever, the autarky price P_c also increases or decreases forever at a constant rate. The dynamic properties of the Lucas model are summarized in the proposition below

Proposition 3.2 *The long run equilibrium of the economy depends on the elasticity of substitution σ between red and blue widgets as follows*

1. *If $\sigma > 1$ then the economy goes on to completely specialize in the production of either red of blue widgets depending on the initial value of $P_c \equiv h_B/h_R$. If initial $P_c < P_c^*$, then P_c goes to zero and the economy specializes in the production of red widgets. If initial $P > P_c^*$, then P_c goes to ∞ and the economy specializes in the production of red widgets.*

2. *if $\sigma < 1$, then irrespective of the initial value of h_B/h_R, P_c goes to a stable stationary value P_c^* and the economy produces both goods in the long-run. Factor shares are allocated such that $\delta_B \alpha_B = \delta_R \alpha_R$.*

3. *if $\sigma = 1$, then irrespective of the initial value of h_B/h_R, P_c either increases of decreases indefinitely and the factor shares are allocated according to the demand weights α_B, α_R and these shares are maintained forever.*

3.2.2 The Labor Allocation Model

We modify the above set up by asking what happens if the labor is not allocated competitively across the two type of firms. As noted earlier, firms in this economy earn zero economic profits and all proceeds from production flow to labor. We assume that each firm is controlled by an entrepreneur whose services are provided at zero cost. The entrepreneur, however, does derive satisfaction from producing goods and therefore wishes to maximize the firm's output. The allocation of labor itself is the outcome of a contest between red and blue widget producing firms. One can envision this contest as a political one whereby firms lobby the government to establish barriers to the allocation of labor or alternatively to prevent the establishment of barriers. We assume that the set of blue widget producing firms act collectively and allocate resources to determine optimally their share of the labor force, which share is allocated equally among the type 1 firms. The red widget producing firms behave in similar fashion. (As implied, we assume the "free-rider" problem in each industry has been overcome.)

We assume that the perfectly competitive allocation of labor is the benchmark or default allocation of labor in the economy. Given this allocation, there is an effort on the part of the producers to either move away from or to defend this benchmark allocation. Specifically, the type R firms desire a greater share of labor than is allocated to them under the benchmark and type B firms defend the benchmark allocation. Both type R and type B firms expend resources to obtain their desired outcomes. As noted in the previous section, λ_c is the perfectly competitive benchmark allocation of labor to the production of blue widgets as determined by equation (3.9). We have a scenario wherein the actual labor allocation achieved is $\lambda_n = z\lambda_c$, where $z \in [0, 1]$. The fraction z is determined by the resources expended by the type B and type R firms. If we let β be the resources devoted by the type B firms and ρ be the resources of type R firms, then the fraction z is given as

$$z = min\left[1, \frac{\theta\beta}{\rho}\right], \tag{3.11}$$

where θ is an exogenously given parameter, which can be interpreted as the lobbying power of type B firms. Alternatively, $1/\theta$ is the lobbying power of type R firms. As can be seen, z is non-decreasing in β and non-increasing in ρ.

We model the contest as a leader-follower game where the type B firms are the leaders and type R are the followers. That is, type B firms devote β in the first stage of the game and given β, type R firms select ρ. The choice of the leader is motivated by our assumption that the competitive allocation is the default or benchmark allocation in the economy and type B firms are defending this allocation. Once this defense is set up, the red widget producers expend resources to break this defensive position and achieve an outcome which is not competitive. As mentioned, the payoff to each type of firm is the output that can be produced with the labor share achieved. The respective payoffs for the type B and type R firms are

$$\pi_B = \begin{cases} h_B \lambda_c - \beta & \text{if } \theta\beta \geq \rho \\ (\theta\beta/\rho)\lambda_c h_B - \beta & \text{if } \theta\beta < \rho \end{cases} \tag{3.12}$$

$$\pi_R = \begin{cases} [1 - (\theta\beta/\rho)\lambda_c]h_R - \rho & \text{if } \theta\beta < \rho \\ (1 - \lambda_B)h_R - \rho & \text{if } \theta\beta \geq \rho \end{cases} \tag{3.13}$$

We solve this problem using backward induction wherein type R firms decide on the optimal ρ for a given β. Type B firms use this information to decide their optimal contribution of resources. As is clear from (3.13), in the interval $[0, \theta\beta]$ the output of type R firms is constant and therefore the payoff decreases in ρ. Therefore, over this interval the optimal contribution of type R firms is $\rho^* = 0$. Thus for an interior solution with $\rho > 0$, we must have $\rho \in (\theta\beta, \infty)$. The necessary condition for type R firms to expend an amount $\rho > \theta\beta$ is that the payoff must be rising at the the cutoff point $\theta\beta$. That is, the first derivative of the payoff function with respect to ρ evaluated at $\theta\beta$ must be positive. This requires that

$$\beta < (\lambda_c h_R)/\theta \equiv \tilde{\beta}. \tag{3.14}$$

Assuming this condition holds, one finds that the payoff function of type R firms in the interval $(\theta\beta, \infty)$ is $[1 - (\theta\beta/\rho)\lambda_c]h_R - \rho$, which is maximized at

$$\rho = (\theta\beta\lambda_c h_R)^{1/2}. \tag{3.15}$$

However, this value of ρ is optimal if and only if the payoff at $\rho = (\theta\beta\lambda_c h_R)^{1/2}$ is greater than the payoff obtained when $\rho = 0$. This requires

$$\beta < (\lambda_c h_R)/4\theta \equiv \hat{\beta}. \tag{3.16}$$

Note that if (3.16) is satisfied then (3.14) also holds, since $\tilde{\beta} < \hat{\beta}$. This establishes that the optimal value for ρ, ρ^*, is

$$\rho^* = 0, if \beta \geq \hat{\beta} \equiv (\lambda_c h_R)/4\theta, \rho^* = (\theta\beta\lambda_c h_R)^{1/2}, if \beta < \hat{\beta}. \tag{3.17}$$

Given the values for ρ^* in equation (3.17), we can now determine the optimal allocation for the type B leader. As apparent from equation (3.17), the optimal value for β lies in the interval $[0, \hat{\beta} \equiv (\lambda_c h_R)/4\theta]$. Given equation (3.17), the payoff function for the type B firms is

$$\pi_B = \begin{cases} (\theta\beta\lambda_c)^{1/2} h_B / h_R^{1/2} - \beta & \text{if } \beta < \hat{\beta} \\ \lambda_c h_B - \hat{\beta} & \text{if } \beta = \hat{\beta} \end{cases}$$

Let $f(\beta) \equiv (\theta\beta\lambda_c)^{1/2} h_B / h_R^{1/2} - \beta$ and note that $f'(\beta) >, <$, or $= 0$, as $\beta <, >$, or $= \beta^* \equiv (\theta\lambda_c h_B^2)/4h_R$. From these facts it follows that π_B is maximized at $\hat{\beta}$, if $\hat{\beta} \equiv (\lambda_c h_R)/4\theta \leq \beta^* \equiv (\theta\lambda_c h_B^2)/4h_R$, that is, if $\theta \geq (h_R/h_B)$.

Suppose $\theta < (h_R/h_B)$ and thus β^* is less than $\hat{\beta}$. Then in this case $f'(\beta) = 0$ at $\beta = \beta^* \in [0, \hat{\beta})$ and therefore β^* is a *candidate* optimal value for β. We say *candidate* because it still may pay the type B firms to generate a discrete jump in π_B by increasing β from β^* to $\hat{\beta}$.

Note that if $\beta = \beta^* \equiv (\theta\lambda_c h_B^2)/4h_R$ then

$$z^* \equiv (\theta/2)(h_B/h_R), \tag{3.18}$$

and the payoff to the type B firms is $\pi_B(\beta^*) = (\theta\lambda_c h_B^2)/2h_R - (\theta\lambda_c h_B^2)/4h_R = (\theta\lambda_c h_B^2)/4h_R$. However, this must be compared to $\pi_B(\hat{\beta}) = h_B\lambda_c - (\lambda_c h_R)/4\theta$. One finds that when $\beta^* < \hat{\beta}$,

$$\pi_1(\beta^*) - \pi_1(\hat{\beta}) >, < or = 0 \; as \; g(\theta) \equiv \theta^2 h_B^2 - 4h_R h_B \theta + h_R^2 >, <, or = 0.$$

The quadratic function $g(\theta)$ has zeros at $\theta = (h_R/h_B)(2-\sqrt{3})$ and $\theta = (h_R/h_B)(2+\sqrt{3})$ and is negative on the interval $(h_R/h_B)(2-\sqrt{3}), (h_R/h_B)(2+\sqrt{3}))$. Thus on this interval the optimal value for β is $\hat{\beta}$. When $\theta < \theta_C \equiv (h_R/h_B)(2-\sqrt{3})$, the optimal value is β^* and we have an interior solution with $z^* < 1$. When $\theta > (h_R/h_B)(2+\sqrt{3})$, $\theta > (h_R/h_B)$, and our previous analysis has established that the optimal value is $\hat{\beta}$. As expected, the equilibrium that the economy attains depends on the value of θ, the relative lobbying strength of the type B firms. If θ is big enough, then type B firms have the advantage in lobbying and since they are defending the competitive allocation they are able to generate the default competitive allocation of labor. If on the other hand θ is low, which implies that the type R firms have the lobbying power, then they can impose an allocation of labor which is not competitive.

We now look at what happens to the relative price level. To differentiate this case from the perfectly competitive benchmark, we denote the relative price level in the presence of barriers as P_n (the non-competitive outcome). As is clear from the above discussion, the relative price depends on the value of θ. To see this let

$$\tau \equiv (2-\sqrt{3}) \; and \; \theta_C \equiv (h_R/h_B)\tau. \tag{3.19}$$

If $\theta \geq \theta_C$, then we have a perfectly competitive outcome with $P_c = h_B/h_R$. On the other hand if $\theta < \theta_C$, then we know that $z^* < 1$ and there is an over-production of red widgets. We know from equation 6 that the relative price is determined by the relative supply of the two goods. An over-production of red widgets implies that the relative price should be less than that obtained under perfect competition. Substituting the relative output of red and blue widgets in equation 6, we get

$$P_n = \left(\frac{\alpha_R}{\alpha_B}\right)\left(\frac{1 - z^*\lambda_c}{z^*\lambda_c}\right)^{-1/\sigma}\left(\frac{h_R}{h_B}\right)^{-1/\sigma} < \frac{h_B}{h_R} \equiv P_c,$$

where z^* is given from equation (3.18). Let $\lambda_n \equiv z^*\lambda_c$ denote the non competitive allocation of labor. We can then rewrite the price under the non-competitive scenario as

$$P_n = \left(\frac{\alpha_R}{\alpha_B}\right)\left(\frac{1 - \lambda_n}{\lambda_n}\right)^{-1/\sigma}\left(\frac{h_B}{h_R}\right)^{1/\sigma} \tag{3.20}$$

We are now in a position to characterize the equilibrium of the economy in terms of $P_c \equiv h_B/h_R$. Given the value of τ from equation (3.19) we can see that competitive equilibrium arises if $P_c \geq \tau/\theta$ and the non competitive equilibrium is reached if $P_c < \tau/\theta$. Let

$$P_s \equiv \frac{\tau}{\theta} \tag{3.21}$$

be the critical price below which the economy attains a non-competitive equilibrium. Note that this critical value is negatively related to θ. As θ increases P_s decreases, increasing the possibility of a competitive equilibrium. This is expected because an increasing θ implies that the lobbying power of type B firms increases. Since type B firms defend the default competitive allocation, higher values of θ makes this outcome more likely. The following proposition summarizes the results obtained in this section

Proposition 3.3 *The total share of labor allocated to type B and type R firms depend on the critical price $P_s \equiv \tau/\theta$ as follows*

1. *If $P_c < P_s$, then $\beta^* = \theta\lambda_c(h_B)^2/4h_R$, $\rho = \theta\lambda_c/2h_B$ and $z^* = \theta h_B/2h_R$. The total labor appropriated by type 1 firms is $\lambda = z^*\lambda_c \equiv \lambda_n$ and that by type 2 firms is $(1-\lambda_n) = 1 - z^*\lambda_c$. The relative price in this case is less than what would exist under perfectly competitive allocation of labor, i.e. $P_n < P_c \equiv h_B/h_R$, as shown in equation (3.20).*

2. *If $P_c \geq P_s$, then $\beta^* = \hat{\beta} \equiv \lambda_c h_R/4\theta$, $\rho^* = 0$ and $z^* = 1$. In this case, we have a perfectly competitive allocation with $\lambda = \lambda_c$. The relative price level is equal to the price under perfect competition, P_c.*

3.3 Static Welfare Analysis

In this section we determine the static welfare loses from barriers to the perfectly competitive allocation of labor. With perfectly competitive allocation of labor the total consumption of blue and red widgets is given by

$$b = \lambda_c h_B \text{ and } r = (1-\lambda_c)h_R,$$

where λ_c is given from equation (3.9). Substituting for λ_c, the utility level attained under perfect competition is given by

$$U_c = \left[\alpha_B(\lambda_c h_B)^{-\gamma} + \alpha_R((1-\lambda_c)h_R)^{-\gamma}\right]^{-\frac{1}{\gamma}}.$$

Similarly, when there are barriers to the allocation of labor, the non competitive consumption levels are given by

$$b = z^*\lambda_c h_B \ and \ r = (1 - z^*\lambda_c)h_R,$$

which yields an instantaneous utility of

$$U_n = \left[\alpha_B(z^*\lambda_c h_B)^{-\gamma} + \alpha_R((1 - z^*\lambda_c)h_R)^{-\gamma}\right]^{-\frac{1}{\gamma}}$$

Taking log differences of the two utility levels we get

$$lnU_c - lnU_b \equiv \Delta U = -\frac{1}{\gamma}ln\left[\frac{\alpha_B(\lambda_c h_B)^{-\gamma} + \alpha_R((1 - \lambda_c)h_R)^{-\gamma}}{\alpha_B(z^*\lambda_c h_B)^{-\gamma} + \alpha_R((1 - z^*\lambda_c)h_R)^{-\gamma}}\right].$$

We note that the first derivative of ΔU with respect to z^* is negative, i.e. $\partial\Delta U/\partial z^* < 0$. Therefore as z^* increases the welfare loss decreases. Since z^* is an increasing function of θ , it implies that as θ increases the welfare loss decreases.

One would also expect that utility losses should be negatively related to the elasticity of substitution $\sigma \equiv 1/(1 + \gamma)$. If the two goods are good substitutes, then producing more reds relative to the competitive solution should lead to smaller losses in utility than if the goods are not good substitutes. To see this we note that the partial derivative of ΔU with respect to γ is positive. Thus for a given θ, as γ increases, σ decreases and the welfare losses increase. Since σ is inversely related to γ, this implies that as σ decreases, the welfare losses increase. The following proposition summarizes theses results

Proposition 3.4 *The utility losses arising from barriers is negatively related to both the elasticity of substitution σ and the parameter θ.*

3.4 Dynamics

In this section we determine the dynamics of the closed economy. As discussed previously, in the original Lucas formulation the dynamics of the system under perfect competition depend on the elasticity of substitution, σ. If the two goods are good substitutes ($\sigma > 1$) then in the long-run we have specialization in one of the two goods. On the other hand, if the two goods are poor substitutes ($\sigma < 1$), then in the long-run the economy ends up producing both goods. We now analyze the dynamics when there are barriers to the allocation of labor.

As follows from proposition 3.3, P_s is the crucial price that determines if the economy has a competitive or non-competitive allocation of labor. Given this, the factor share λ is a function of P_c given by

$$\lambda(P_c) = \begin{cases} \lambda_n \equiv (\theta/2)P_c\lambda_c & \text{if } \ P_c < P_s \equiv \tau/\theta \\ \lambda_c(P_c) & \text{if } \ P_c \geq P_s \equiv \tau/\theta \end{cases} \tag{3.22}$$

where λ_c is given from equation (3.9) as

$$\lambda_c = \frac{1}{1 + \left(\frac{\alpha_R}{\alpha_B}\right)^{\sigma} P_c^{1-\sigma}}$$

The dynamics of $\lambda(P_c)$ depend on the nature of the $\lambda(P_c)$ function. Whether $\lambda(P_c)$ is increasing or decreasing in P_c depends upon the value of σ, the elasticity of substitution between the two goods. Moreover, the dynamics of $\lambda(P_c)$ also depend upon σ. The various possibilities will be considered shortly.

In addition, as implied by (3.22), the dynamics of $\lambda(P_c)$ depend upon the dynamic properties of P_c. As follows from equations (3.3) and (3.4), $\dot{P}_c/P_c$ is given by

$$\frac{\dot{P}_c}{P_c} = \frac{\dot{h}_B}{h_B} - \frac{\dot{h}_R}{h_R} = (\delta_B + \delta_R)\lambda - \delta_R.$$

From this fact it follows that

$$\dot{P}_c \ >,\ <\ or\ =\ 0\ as\ \lambda(P_c)\ >,\ <\ or\ = \delta_R/(\delta_R + \delta_B). \tag{3.23}$$

Let $\hat{P}_c$ be that value of P_c at which $\lambda(P_c) = \delta_R/(\delta_R + \delta_B)$. Of crucial importance to determining the dynamic behavior of $\lambda(P_c)$ is a comparison of $\hat{P}_c$ to P_s. We are now prepared to consider the possibilities.

Case 1: $\sigma > 1$. When $\sigma > 1$, as can be seen from equation (3.22), $\partial\lambda/\partial P_c > 0$ for all P_c. Further, as P_c approaches P_s, $\lambda_n(P_c)$ approaches $\lambda_c(P_c)$ and as P_c approaches infinity, $\lambda(P_c)$ approaches 1. Similarly as P_c approaches 0, $\lambda(P_c)$ also approaches 0. Figure 3 plots $\lambda(P_c)$ as a function of P_c. Recall that for all $P_c < P_s$, $\lambda(P_c) = \lambda_n(P_c)$. This is the solid curve in Figure 3.3. When $P_c \geq P_s$, $\lambda(P_c) = \lambda_c(P_c)$, as given in equation(3.9). This is the dotted line in Figure 3.3. As mentioned previously , the dynamics of the model will depend upon on how the stationary value $\hat{P}_c$ compares with the crucial value P_s. This in turn depend on whether $\lambda(\hat{P}_c) \equiv \delta_R/(\delta_R+\delta_B)$ is greater or lesser than $\lambda(P_s)$. We discuss each sub-case below.

Case 1a: $\hat{P}_c < P_s$. When this is true, $\lambda(\hat{P}_c) < \lambda(P_s)$. This is illustrated in figure 3.4 below. To provide a clear comparison to the case without barriers, the dotted curve traces out the entire $\lambda_c(P_c)$ function. It is clear that to the left of $\hat{P}_c$, $\dot{P}_c$ is less than zero and P_c tends to zero. To the right, $\dot{P}_c$ is greater than zero and P_c tends to infinity. So the economy converges to specialization in one of the two goods. As can be seen if the initial

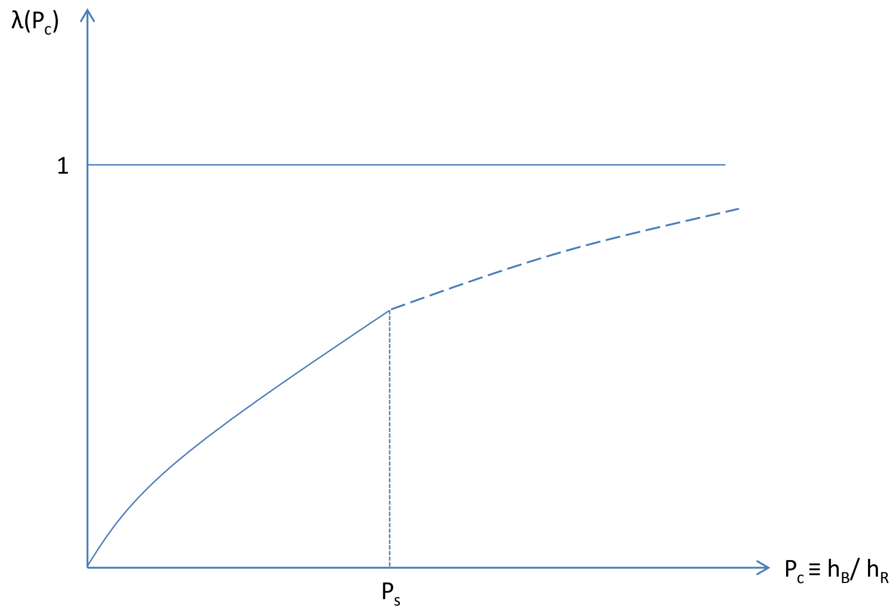

Figure 3.3: $\lambda(P_c)$ when $\sigma > 1$

$P_c \in (\hat{P}_c, P_S)$, that is, if initially there are barriers, the economy does go on to specialize in the production of blue widgets. However, we note that the stationary point $\hat{P}_c$ obtained in this case is greater than the one obtained in the perfectly competitive benchmark model of Lucas (P_c^*). This has important implications for the long-run dynamics.

To see this, consider an initial value of P_c in the interval $(P_c^*, \hat{P}_c)$. In Lucas' formulation the economy should end up specializing in blue widgets (see proposition 3.2), however in our model the economy specializes in the production of red widgets in the long-run. Therefore, in this sense, for a range of initial values of $P_c \equiv h_B/h_R$, the economy ends up specializing in the production of the "wrong" good as compared to the competitive outcome. However, since in this case the two goods are good substitutes, there will be no long-run welfare loss.

Another interesting aspect in this case relates to the speed of convergence. If the economy starts off with barriers then the speed of convergence to specialization in blue widgets is

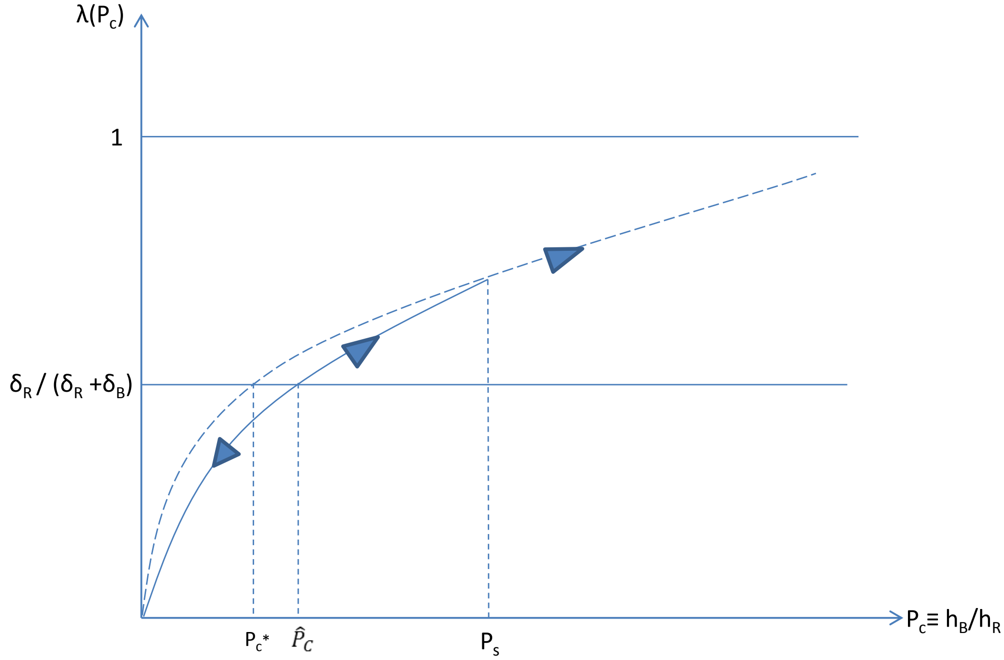

Figure 3.4: $\sigma > 1$ and $\hat{P}_c < P_s$.

lower than would be attained without barriers. This can be seen by noting that the $\lambda_n(P_c)$ curve lies below the $\lambda_c(P_c)$ curve, when $P_c < P_s$. Therefore, in the interval $(0, P_s)$, for any given value of P_c, the type B firms have a lower share of labor than would be obtained under perfect competition. This implies that if the initial value of P_c lies in the interval $(P_c^*, \hat{P}_c)$, the economy would take a longer time to converge to a specialization in blue widgets. Alternatively, if the initial value of P_c lies in $(0, \hat{P}_c)$, then the convergence to specialization in the production of red widgets would be more rapid than would be obtained under perfect competition.

Case 1b: $\hat{P}_c \geq P_s$. When this is true, $\lambda(\hat{P}_c) > \lambda(P_s)$. In this case, $\lambda(\hat{P}_c) = \lambda_c(P_c^*) = \delta_R/(\delta_R + \delta_B)$, so that $\hat{P}_c = P_c^*$, and the stationary point obtained is the same as we would obtain in the perfectly competitive case. If the initial value of P_c is greater than P_c^*, then P_c goes to ∞ and the economy ends up specializing in blue widgets. On the other hand if $P_c < P_c^*$, then P_c approaches zero and in the long-run the economy specializes in the production of red widgets. Therefore, the economy will specialize in the same good as would be the case under perfect competition. However, as discussed previously, the speed of convergence to specialization in red widgets would be faster than that obtained in the perfectly competitive set up. This is illustrated in figure 3.5 below

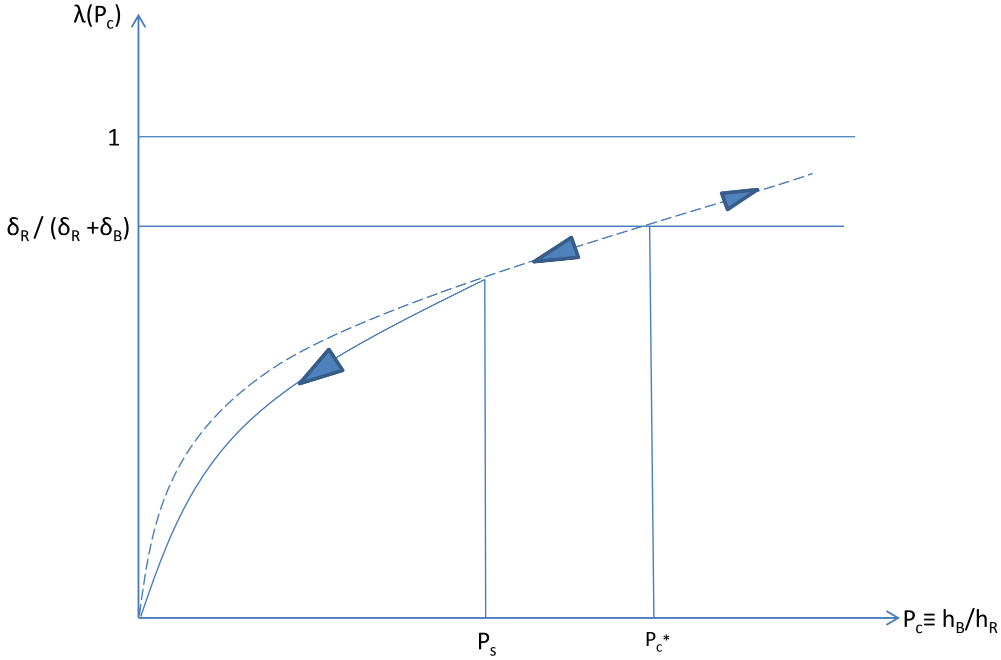

Figure 3.5: $\sigma > 1$ and $\hat{P}_c > P_s$.

Case 2: $\sigma < 1$. We again start with the characterization of the $\lambda(P_c)$ function. As was the case previously, when P_c approaches P_s, $\lambda_n(P_c)$ approaches $\lambda_c(P_c)$. The crucial distinction in this case, however, is the slope of the $\lambda(P_c)$ function. We can seen from the equation (3.22) when $P_c < P_s$, $\lambda(P_c) = \lambda_n(P_c)$. When $\sigma < 1$, it can be shown that $\partial\lambda_n(P_c)/\partial P_c > 0$. However, when $P_c \geq P_s$, $\lambda(P_c) = \lambda_c(P_c)$ and we know that $\partial\lambda_c/\partial P_c < 0$. Therefore, as P_c approaches zero or ∞, $\lambda(P_c)$ approaches zero. Figure 3.6 plots $\lambda(P_c)$ as a function of P_c when $\sigma < 1$.

On the solid curve solid curve $\lambda(P_c) = \lambda_n(P_c)$ for all $P_c < P_s$, on the dotted curve

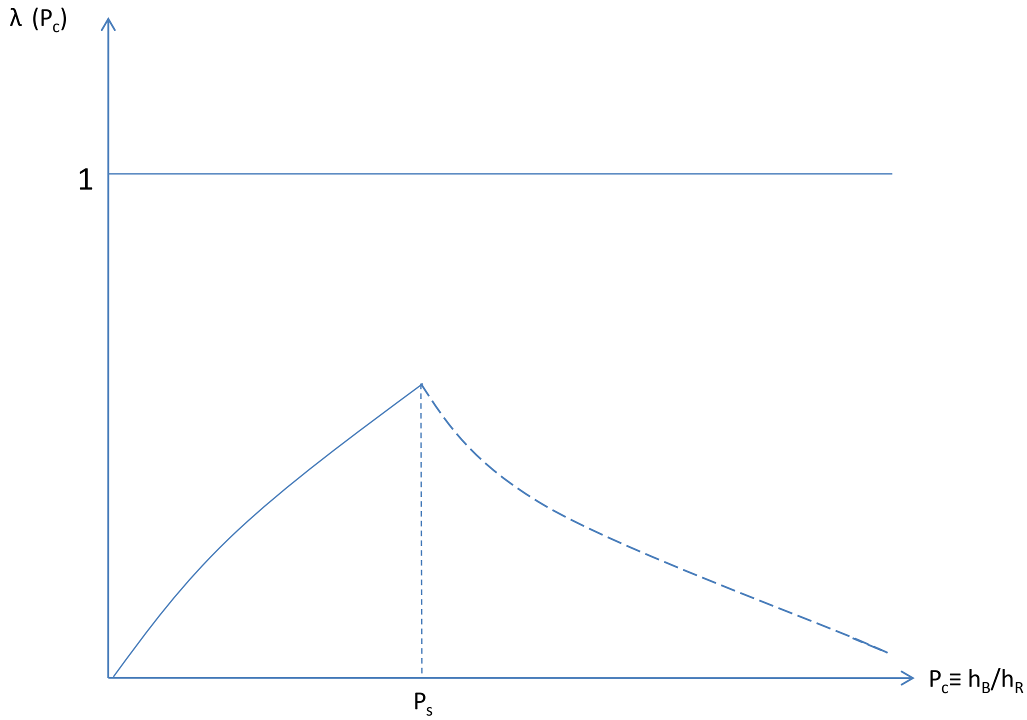

Figure 3.6: $\lambda(P_c)$ when $\sigma < 1$

$\lambda(P_c) = \lambda_c(P_c)$ for all $P_c \geq P_s$. We can see that in this case the maximum value $\lambda(P_c)$ takes is $\lambda(P_s)$. This raises the possibility that $\hat{P}_c$ is not defined, that is $\dot{P}_c$ is never zero. It is clear that this would be the case if $\delta_R/(\delta_R + \delta_B)$ is greater than $\lambda(P_s)$. Therefore we have the following two sub-cases:

Case 2a: $\hat{P}_c$ *exists and* $\hat{P}_c < P_s$. In this case we get very different results as compared to the Lucas formulation. Figure 3.7 illustrates this case. In the figure the dotted curve represents $\lambda_c(P_c)$ as would be obtained under perfect competition.

It can be seen that to any point to the left of $\hat{P}_c$ and to the right of P_c^*, $\dot{P}_c$ is negative and P_c falls over time. In the interval $(\hat{P}_c, P_c^*)$, $\dot{P}_c$ is greater than zero and P_c increases over time. Therefore, in this case there are two possible states the economy can converge to in

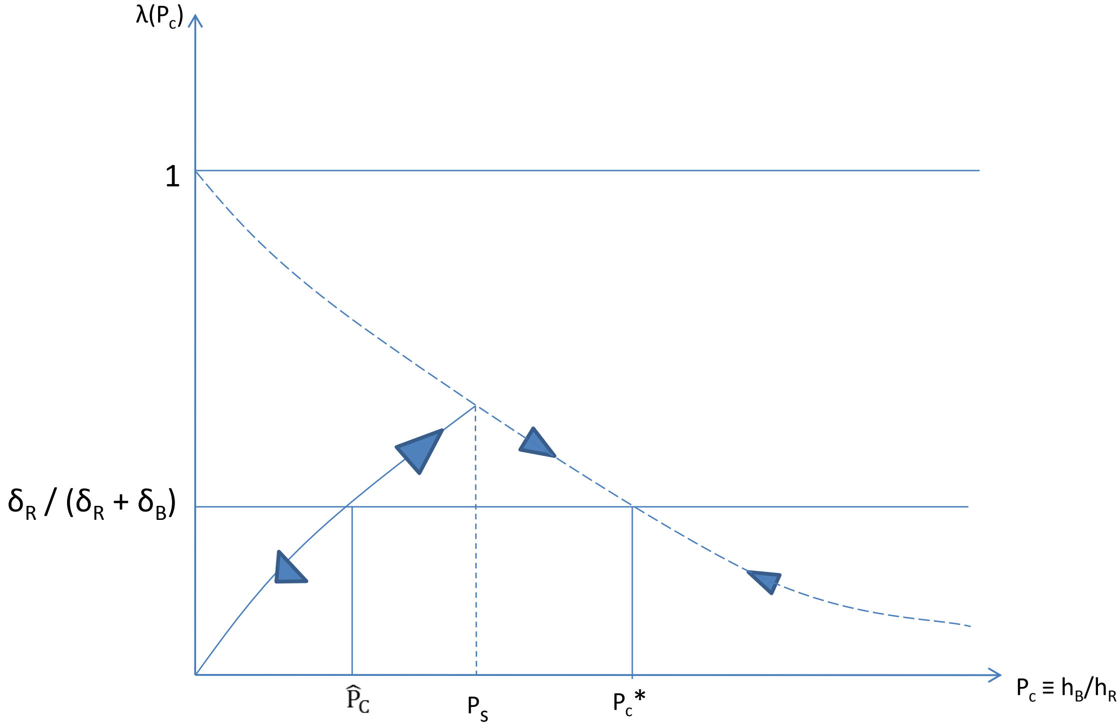

Figure 3.7: $\sigma < 1$ and $\hat{P}_c < P_s$.

the long-run. If initially $P_c < \hat{P}_c$, P_c falls over time and the economy converges to specialization in the production of red widgets. On the other hand if initially $P_c > \hat{P}_c$, then the economy converges to the stable stationary point P_c^*, which is the equilibrium that would arise if the labor were allocated competitively. Thus with $\sigma < 1$, depending on the initial value of P_c, the economy may or may not converge to a perfectly competitive solution. This is in contrast to the result derived by Lucas, where irrespective of the initial P_c the economy always converges to the stationary point P_c^*.

It is interesting to note the welfare implications of this case if the economy starts off with an initial value of $P_c \in (0, \hat{P}_c)$. As mentioned above, in this case P_c goes to 0 and converges to specialization in red widgets. Since the goods are not good substitutes this implies there will be a large welfare loss. In fact, since h_R goes to infinity, it is easy to see that the utility of consumers goes to zero. If the economy starts off with an initial value of $P_c > \hat{P}_c$, then the economy does converge to a perfectly competitive solution and there are no long-run welfare losses.

Case 2b: $\hat{P}_c$ does not exist and $\dot{P}_c < 0$. This is illustrated in the graph below. As can be seen, $\dot{P}_c$ is always negative. Irrespective of the initial value of P_c, the economy always converges to specialization in red widgets. This is again in contrast to the result in the Lucas formulation where in the long-run both goods are produced. As discussed in the previous paragraph in this case the household utility goes to zero.

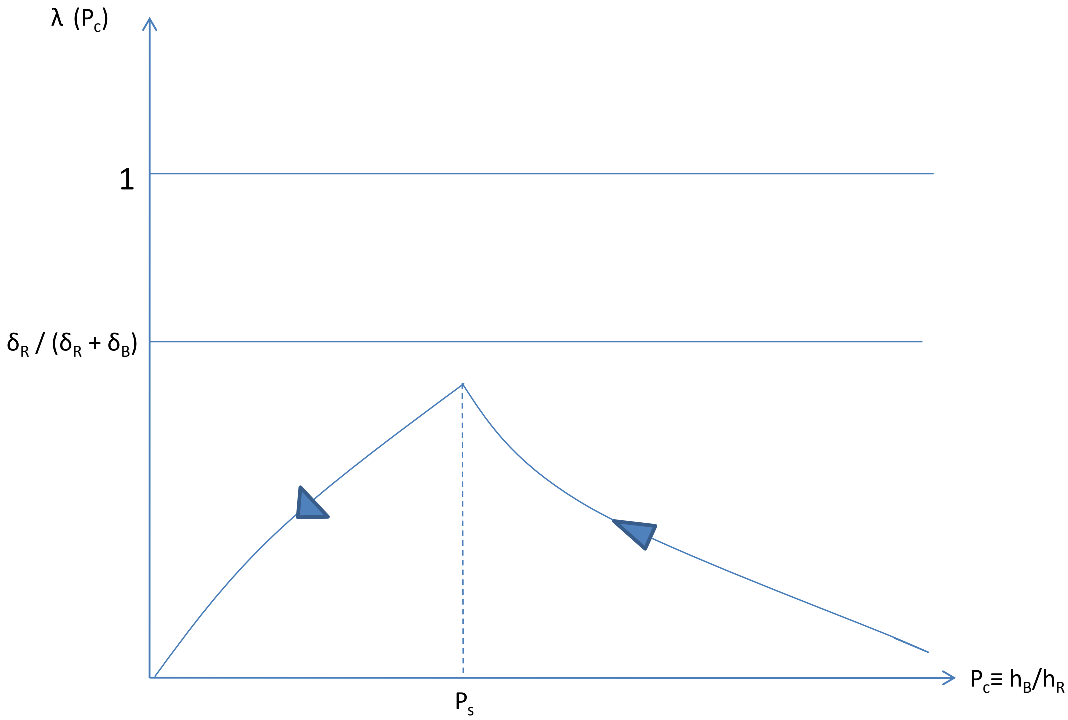

Figure 3.8: $\sigma < 1$, $\hat{P}_c$ does not exist and $\dot{P}_c < 0$.

Case 3: $\sigma = 1$. As can be seen from equation (3.22), when $\sigma = 1$, $\lambda(P_c) = \alpha_B$, for all $P_c \geq P_s$. When $P_c < P_s$, $\lambda(P_c) = \lambda_n(P_c)$ is upward sloping. As in the previous case, we have the following two sub-cases:

Case 3a: $\hat{P}_c < P_s$. Figure 3.9 illustrate this case.

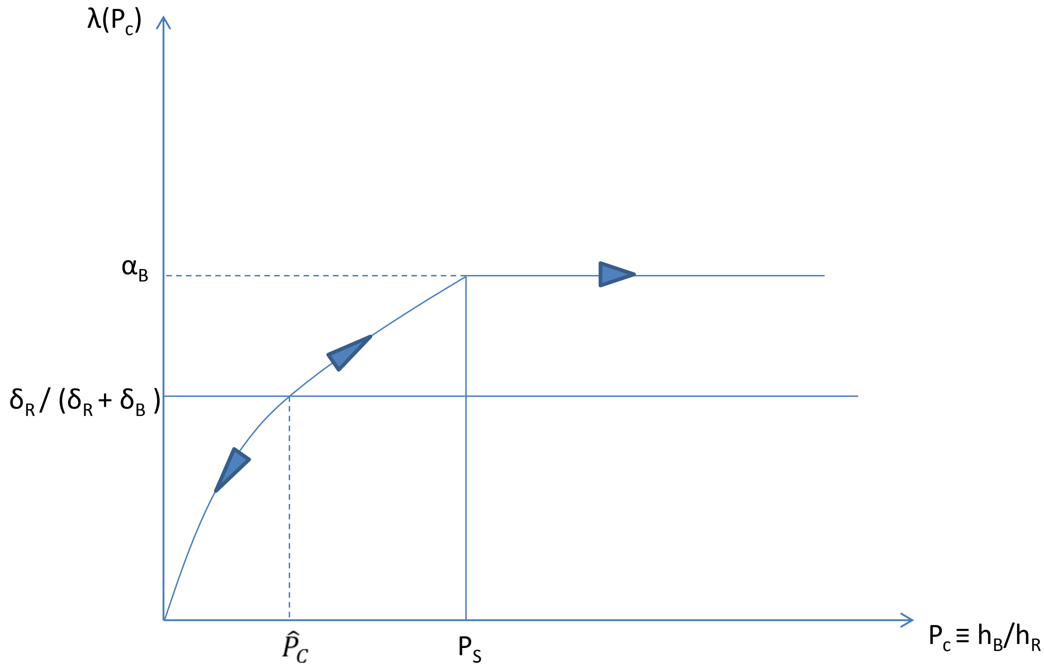

Figure 3.9: $\sigma = 1$, $\hat{P}_c$ exists and $\hat{P}_c < P_s$.

If the initial P_c is less than $\hat{P}_c$, then $\dot{P}_c$ is negative and P_c falls overtime leading to specialization in red widgets. This is again in contrast to the result derived by Lucas wherein P_c increases overtime and the economy produces both red and blue widgets indefinitely. Again as discussed above, in this case there will be welfare losses in the long-run because the economy fails to produce both goods. If the initial P_c is greater than $\hat{P}_c$, then P_c rises overtime and the economy eventually converges to a perfectly competitive equilibrium as obtained in the Lucas framework.

Case 3b: $\hat{P}_c$ *does not exist and* $\dot{P}_c < 0$. Figure 3.10 above depicts this. In this case, $\dot{P}_c$ is always negative and P_c falls overtime. However, in our formulation the factor share λ

eventually goes to zero and the economy ends up specializing in red widgets. Thus the economy does not converge to a perfectly competitive solution where both red and blue widgets are produced in the long-run leading to a loss in welfare.

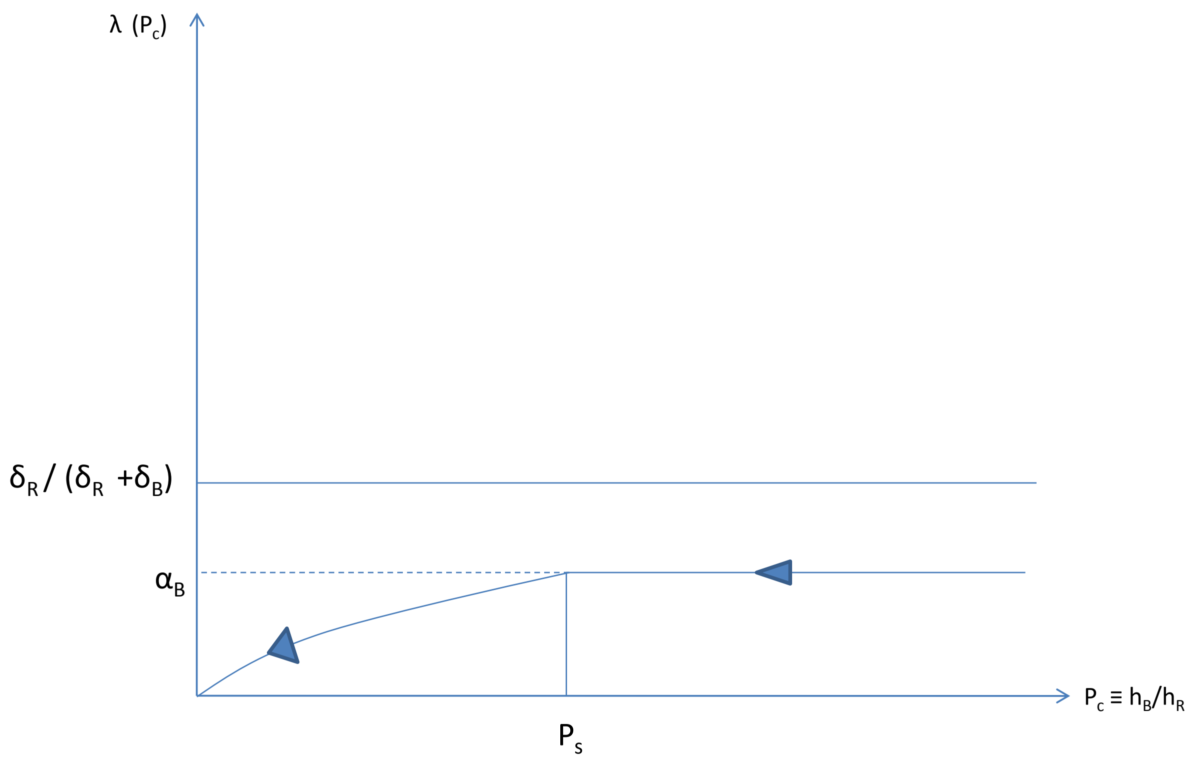

Figure 3.10: $\sigma = 1$, $\hat{P}_c$ does not exist and $\dot{P}_c < 0$

The results of the dynamic behavior of the economy are summarized in the following proposition

Proposition 3.5 *The long-run equilibrium of the economy depends on the elasticity of substitution between and red and blue widgets as follows:*

1. *If $\sigma > 1$, then depending on the initial value of $P_c \equiv h_B/h_R$, the economy specializes in the production of either red or blue widgets. There is a range of initial values of P_c for*

which specialization occurs in the production of red widgets contrary to the prediction of the Lucas model.

2. *If $\sigma < 1$, then depending on the initial value of P_c either the economy converges to a perfectly competitive solution or if initial P_c is low enough, to a specialization in the production of red widgets. The latter case involves a loss in welfare compared to the perfectly competitive solution of the Lucas model.*

3. *If $\sigma = 1$, then again the economy either converges to a perfectly competitive solution producing both goods as determined by the demand weights, or if demand weight α_B is low enough, to a specialization in red widgets. The latter case involves a loss in welfare as compared to the perfectly competitive solution of the Lucas model.*

It is also of interest to note that the range of initial values which may lead to long-run factor allocation contrary to the competitive outcome is determined by the parameter θ. A large value of θ implies that the crucial price P_s is low which means that the range of possible values for which we have non-competitive allocation decreases. The reverse is true when θ is small. In this case, P_s would be large and therefore the range of initial values from which the economy converges to a non-competitive long-run allocation is large. Therefore, the relative lobbying power of the two industries in some sense determines whether the economy converges in the long-run to a perfectly competitive allocation and the resulting speed of convergence.

3.5 Conclusion

There is a growing strand of literature that places misallocation of resources as a major factor explaining low level of productivity and per capita income of economies. This chapter adds to this strand of work. We build on the Lucas(1988) model of human capital accumulation via learning by doing, by incorporating a model of lobbying by firms to establish or prevent barriers to the competitive allocation of labor. In the original Lucas model with two goods (red and blue widgets), labor is allocated competitively across the two goods at each point in time and the factor shares evolve over time based on the elasticity of substitution between the two goods. Learning by doing implies that as more labor is devoted to the production of a good, the productivity of labor increases. Assuming that there are no diminishing returns to human capital, long-run growth is driven by the accumulation of human capital. We modify this set up by allowing for lobbying by firms producing the two different goods to establish barriers to the competitive allocation of labor or alternatively prevent the establishment of barriers.

We show that if the relative lobbying power of firms seeking barriers is high enough then there is a possibility of uncompetitive allocation of labor. This in turn leads to an inefficiency in terms of an over-production of the good produced by these firms (the red widgets) and a loss in welfare of the consumers. We find that the loss in welfare is greater, the lower the elasticity of substitution between the goods and higher the relative lobbying power of firms seeking the barriers.

We then proceed to analyze the dynamic behavior of the model with lobbying. As in the Lucas model, the long-run factor allocation and welfare depend upon the elasticity of substitution between the two goods. However, we find that the results change substantially in the presence of barriers to the competitive allocation of labor. When the two goods are good substitutes, there is a range of initial values of price level for which the economy may go on to specialize in the “wrong” good relative to the one implied by competitive allocation.

Further, we show that if the economy starts off from such a point, then the speed of convergence to specialization in blue widgets is slower than would be achieved without barriers. On the other hand, the speed of convergence to specialization in the production of red widgets would be faster. However, the goods being good substitutes there is no long-run welfare loss. There are substantial welfare losses though when the two goods are not good substitutes. In this case, the Lucas model predicts that the economy would produce both the goods in the long-run. However, we show that there is a range of initial values of prices for which the economy may end up specializing in the production of red widgets. We also show that this range is larger, the greater the relative lobbying power of the firms establishing barriers to the competitive allocation. With the two goods being poor substitutes, this implies that in the long-run the total welfare of the economy would fall to zero.

An interesting extension of this model would be to open up the economy to trade with other countries which may or may not have barriers to the allocation of labor. If there are no barriers to allocation in countries, then the domestic prices in each country would reflect the relative productivities and the nations would specialize in the good in which they have a comparative advantage. However, if there are barriers present in the countries then, as our model predicts, the domestic prices may not reflect the relative productivities. Therefore there is a possibility that trade may take place contrary to strict technological comparative advantage. These inefficiencies in trade may lead to an overall loss in welfare. Another interesting aspect would be to see would be if significant barriers in one country can lead to adverse outcome in other countries which may not have any barriers. We develop this theme in the next chapter.

Chapter 4

Allocative Inefficiencies, International Trade and Economic Growth

4.1 Introduction

There is a vast literature that explores the link between international trade and economic growth. The widespread view among economists and policy makers is that more trade openness leads to higher growth rates. For instance, Fischer (2000) notes that "integration into the world economy is the best way for countries to grow." An implication of this view is that one of the most important strategies to achieve economic growth is to dismantle trade barriers. There is, however, considerable disagreement on the empirical evidence supporting this view. While several influential papers, like Sachs and Warner (1995), Edwards (1998), and Dollar (1992) have presented evidence in favor of open trade policies leading to better economic performance, others have questioned the validity of these results. Rodriguez and Rodrik (2000), for example, state that most of the aforementioned studies suffer from either problematic measures of trade barriers, or the measures being correlated with other sources of economic performance. They conclude, “the nature of the relationship between trade policy and economic growth remains very much an open question,” going on to state that the relationship between open trade policies and growth is “a contingent one, dependent on a host of country and external characteristics.”

Recent literature on growth and development has also highlighted the role of misallocation of resources within an economy as being one of the primary factors explaining the long run economic performance of economies. The central theme in this line of analysis is that what matters for productivity and growth is how given stocks of physical capital, human capital and knowledge are allocated across firms and sectors. The best allocation maximizes output and welfare. For example, Banerjee and Duflo (2005) argue that the low overall output of India is essentially due to differing marginal productivities of capital across firms arising from misallocation. Hsieh and Klenow (2009) cite empirical evidence which demonstrates the misallocation across plants within 4-digit industries potentially reduces the TFP in manufacturing by a factor of two to three in India and China.

Our goal in this paper is to explore the relationship between international trade and economic growth, allowing for the possibility of misallocation of resources within the economies. We develop a model of international trade where growth is driven as a result of human capital accumulation via learning-by-doing. We introduce barriers to competitive allocation of inputs (labor) within the economies and analyze its implications for international trade and economic growth. Misallocation of resources arises as a result of lobbying by firms to prevent or establish barriers to the competitive allocation of labor. The relative lobbying power of firms is a crucial parameter that determines the presence and extent of misallocation. If the relative lobbying power of firms seeking to establish barriers is large enough, then they achieve a labor share greater than the competitive labor share; the resulting inefficiencies decrease the volume of trade between countries even in the absence of trade barriers. The reduction in the volume of trade leads to welfare losses (both for the country with barriers and for the one without) and may adversely impact the growth rate of output.

We find these results provide some justification for the ambiguous relationship between trade barriers and economic growth observed in the data. The paper advocates more broad based reforms as opposed to removing trade barriers to achieve higher growth rates. Our results seem to be in line with Rodriguez and Rodrik (2000); that the relationship between international trade and growth is in fact contingent upon various country specific characteristics.

The rest of the paper is organized as follows. Section 4.2 provides a brief literature review and section 4.3 outlines a simple two-country, two-good model of international trade and derives the benchmark competitive equilibrium. In section 4.4 we develop a model of lobbying and the resulting labor allocation and domestic prices for both countries. Section 4.5 we look at the direction and volume of trade in the presence of barriers and section 4.6 derives the equilibrium world price. Section 4.7 looks at the growth implications of the model and section 4.9 offers some concluding comments.

4.2 Literature Review

There are several papers that look at the relationship between international trade and economic growth. The endogenous growth models have provided mixed conclusions as far as the relationship between trade openness and growth is concerned. For example Romer (1990) develops a model of endogenous growth where growth is driven by the technological change arising from from investment in research and development (R&D) by profit maximizing agents. One of the determinants of growth rate in the model is the stock of human capital. The paper predicts that opening up to trade can increase the stock of human capital available to a country and therefore leads to higher growth rates. Grossman and Helpman (1990) build on Romer's model to outline specifically the impact of trade policies on growth. They develop a two country model with international trade where growth is driven by R&D in the production of intermediate goods in both countries. The final good in both countries is produced using the intermediate goods. R&D increases the measure of intermediate goods available. There are differences in the productivity of R&D in both countries, with one country having the comparative advantage in R&D. In this set up they show that any policy of export promotion in the final good, leads to an overall increase in the long run growth rate of world economy if and only if the policy is enacted in the economy with comparative disadvantage in R&D.

Lucas (1998) develops a two-good, multi-country model of international trade and growth wherein growth is driven by the accumulation of human capital embodied in the production of the two goods. There is a differential rate to the accumulation of human capital in the two goods. The paper shows that if the two goods are good substitutes then international trade leads some countries to specialize in the good with lower rate of human capital accumulation and therefore forever grow at lower growth rates. Other papers in which countries with different endowments and technologies may end up with lower growth as a result of economic integration include Rivera-Batiz and Xie (1993) and Young (1991).

The empirical literature on the link between international trade and growth has also been largely inconclusive. For example, while there are several prominent papers which have used cross-country regressions to show that trade openness leads to more growth (Fischer (2000), Edwards (1998), Dollar (1992), Sachs and Warner (1995)), other papers have shown that the conclusions in these papers are incorrect. For example, Rodrik et al. (2004) showed that trade shares are not a significant determinant of growth rates when entered into cross country regressions along with institutional quality variables measured by the rule of law and property rights. More recently Yanikkaya (2003) has demonstrated that the relationship between trade liberalization and growth is not straightforward. The paper employs different measures of trade openness, representing both trade volumes and trade barriers, for a cross section of countries and estimates its relationship with growth rates. The paper shows that while there is a positive association between trade volumes and growth; there is also a positive and significant relationship between measure of trade barriers and growth, especially for developing countries.

Our task in this paper is to re-examine the relationship between international trade and growth in the the presence of domestic barriers to the allocation of resources. A growing strand of literature places misallocation of resources, both at the firm and economy level, as a major source of productivity differences among countries. There is evidence, both at the macro and micro level, that there exist considerable misallocation of resources within economies. The central theme in this line of analysis is that what matters for productivity is how a given stocks of physical capital, human capital and knowledge are allocated across firms and sectors. The best allocation maximizes output and welfare. For example, Banerjee and Duflo (2005) argue that the low overall output of India is essentially due to differing marginal productivities of capital across firms arising from misallocation. Hsieh and Klenow (2009) cite empirical evidence which demonstrates the misallocation across plants within 4-digit industries potentially reduces the TFP in manufacturing by a factor of two to three in

India and China. In a recent working paper Jones (2010) develops a model wherein the misallocation in the intermediate goods at the firm level is amplified through the input-output structure of an economy.

Cole et al. (2005) examines the reason Latin American countries have not caught up to their western peer economies and successful east Asian economies. The paper presents two key findings. First, a stagnant relative TFP is the key determinant of relative income and labor productivity stagnation. Second, human capital differences are not the key determinant of Latin American TFP gap. The main factor explaining the TFP gap is barriers to competition. The paper further shows that productivity and output increases significantly when these barriers are dropped.

Bergoeing et al. (2010) analyze the role of barriers resulting from underdevelopment and policy distortions that alter the entry and exit decisions of firms. They develop a general equilibrium model with heterogeneous firms subject to idiosyncratic shocks to their productivity. They show that as the cost of entry imposed by the barriers increases, the firm distribution is altered in such a way that too many inefficient firms remain in the market. This impedes the allocation of resources to more efficient firms and delays technology adoption. On calibrating the model to the leading and developing countries, the authors and that the barriers account for more than 50 percent of the income gap between the U.S and the developing countries.

This paper adds to the literature by exploring the link between international trade and economic growth, by adding the possibility of misallocation of resources within the domestic economies. We incorporate resource misallocation into the learning-by-doing framework of

Lucas (1988). Lucas develops a two-good model in which growth is driven by the accumulation of human capital. Learning by doing implies that labor becomes more productive when more of it is used in the production of a good. In the model labor is allocated competitively across the two goods and these factor shares evolve over time.

4.3 Model

We consider a two-country, two-good model of international trade. Each country is endowed with a unit of labor and produces red and blue widgets using labor as the sole input. At a point in time, the marginal product of labor in each industry is constant. In the home country (country 1) the marginal product of labor in the production of blue widgets is h_B and in the production of red widgets, h_R. For the foreign country (country 2) the variables are $\tilde{h}_B$ and $\tilde{h}_R$. (In what follows a variable x in the foreign country is denoted as $\tilde{x}$).

In country 1 the marginal rate of transformation is $\tau \equiv h_B/h_R$. For the foreign country the marginal rate of transformation is $\tilde{\tau} \equiv \tilde{h}_B/\tilde{h}_R$. To motivate trade between the two countries, we assume that $\tau \equiv h_B/h_R > \tilde{\tau} \equiv \tilde{h}_B/\tilde{h}_R$ and $h_B > \tilde{h}_B$, so country 1 has a comparative advantage and absolute advantage in the production of blue widgets. Of course in the absence of barriers to foreign trade and in the absence of domestic barriers to the allocation of labor, country 1 would specialize in the production of blue widgets, country 2 would specialize in the production of red widgets and the world price of red widgets would be determined by the competitive relative supply and demand of red and blue widgets.

We modify this set up by developing a model in which there are barriers to the free allocation of labor in each country, but no trade barriers. These barriers raise the possibility that there are inefficiencies in the allocation of labor and the benchmark, free trade equilibrium will not be achieved. In addition to their impact on trade, these potential inefficiencies also may impact long-run economic growth, a topic developed in later sections.

4.3.1 Consumer Behavior

The consumers in each country have the following utility function

$$U = \left[\alpha_B b^{-\gamma} + \alpha_R r^{-\gamma}\right]^{-\frac{1}{\gamma}}, \tag{4.1}$$

where b and r are the quantities consumed of blue and red widgets respectively. Further, $\gamma > -1$, $\alpha_B, \alpha_R \geq 0$ and $\alpha_B + \alpha_R = 1$. The constant elasticity of substitution is given by $\sigma = 1/(1+\gamma)$. If P is the blue price of the red widgets (blue widgets are assumed to numeraire), then utility maximization by an individual requires that that

$$\frac{r}{b} = \left(\frac{\alpha_R}{\alpha_B}\right)^{\sigma} P^{-\sigma} \tag{4.2}$$

4.3.2 Producer Behavior and Trade in Country 1

There are two types of firms in country 1. Type 1 produce only blue widgets, while the type 2 firms can only produce red widgets. We assume free entry into the production process. Both types of firms earn zero economic profit and since labor is the sole input to production, all the proceeds from production flow to labor. The respective production levels of levels of blue and red widgets are

$$B = \lambda_B h_B L, \tag{4.3}$$

$$R = \lambda_R h_R L, \tag{4.4}$$

where L is the total labor endowment. Assuming $L = 1$, λ_B and λ_R are the fractions of the unit endowment of labor devoted to the production of blue and red widgets respectively. If labor is free to move between type 1 and type 2 firms then under perfect competition the equilibrium price would be

$$P = \frac{h_B}{h_R},$$

combining the above equation with (4.2)-(4.4), we have the following

$$\frac{1-\lambda_B}{\lambda_B} = \left(\frac{\alpha_R}{\alpha_B}\right)^{\sigma} \left(\frac{h_R}{h_B}\right)^{\sigma-1}.$$

Solving the above expression for σ_B, we get

$$\lambda_B^* = \frac{1}{1+\left(\frac{\alpha_R}{\alpha_B}\right)^{\sigma} \left(\frac{h_R}{h_B}\right)^{\sigma-1}}. \tag{4.5}$$

Now suppose we open up country 1 to international trade and that country 1 takes the world price as given. If P_W is the world blue price of a red widget then there are three possibilities. If $P > P_W$, then then country 1 will specialize in the production of blue widgets and export blue widgets for red widgets. On the other hand if $P < P_W$, then country 1 will specialize in the production of red widgets and exports red widgets for blue widgets. Finally, if $P = P_W$, then there is no incentive for trade and country 1 will produce both goods according to labor share given by equation (4.5). Without loss of generality we assume that $P > P_W$. In what follows we will determine exports as a function of P_W.

Since the home price is greater than the world price, country 1 will specialize in the production of blue widgets and trade them for reds. Let χ be the fraction of blues exported. Then we know that χh_B blues are traded for $\chi h_B/P_W$ red widgets. Trade will lead to an equilibrium where the marginal rate of substitution in country 1 is equal to the world price P_W. That is,

$$\frac{\frac{\chi h_B}{P_W}}{(1-\chi)h_B} = \left(\frac{\alpha_R}{\alpha_B}\right)^{\sigma} P_W^{-\sigma}.$$

Solving the above expression for χ gives the fraction of blues exported when there is perfectly competitive allocation of labor as

$$\chi^* = \frac{\left(\frac{\alpha_R}{\alpha_B}\right)^{\sigma} P_W^{1-\sigma}}{1 + \left(\frac{\alpha_R}{\alpha_B}\right)^{\sigma} P_W^{1-\sigma}}. \tag{4.6}$$

Given this, the total exports of blue widgets and imports of reds by the country 1 are given by

$$X_B^* = \left[\frac{\left(\frac{\alpha_R}{\alpha_B}\right)^{\sigma} P_W^{1-\sigma}}{1 + \left(\frac{\alpha_R}{\alpha_B}\right)^{\sigma} P_W^{1-\sigma}}\right] h_B,$$

and

$$M_R^* = \left[\frac{\left(\frac{\alpha_R}{\alpha_B}\right)^{\sigma} P_W^{-\sigma}}{1 + \left(\frac{\alpha_R}{\alpha_B}\right)^{\sigma} P_W^{1-\sigma}}\right] h_B.$$

Where X_B^* and M_R^* denote the exports of blues and imports of reds respectively. In the next section we introduce country 2 and determine its pattern of trade as a function of the world price P_W

4.3.3 Producer behavior and Trade in Country 2

We assume that the foreign country is a mirror image of country 2. Type 1 firms can only produce red widgets, while type 2 firms only produce blue widgets. The production functions are similar to country 1 given by

$$\tilde{R} = \tilde{\lambda}_R \tilde{h}_R,$$

and

$$\tilde{B} = \tilde{\lambda}_B \tilde{h}_B.$$

Under perfect competition and in the absence of trade, domestic price level is given by

$$\tilde{P} = \frac{\tilde{h}_B}{\tilde{h}_R}.$$

The fraction of labor devoted to the production of red widgets in country 2 is then given by

$$\tilde{\lambda}_R^* = \frac{\left(\frac{\alpha_R}{\alpha_B}\right)^\sigma \left(\frac{\tilde{h}_R}{\tilde{h}_B}\right)^{\sigma-1}}{1 + \left(\frac{\alpha_R}{\alpha_B}\right)^\sigma \left(\frac{\tilde{h}_R}{\tilde{h}_B}\right)^{\sigma-1}}. \tag{4.7}$$

Now suppose country 2 opens up to trade and takes the world price P_W as given. Then if $\tilde{P} < P_W$, country 2 will specialize in the production of red widgets and trade them for blues at the world price. Let $\tilde{\chi}$ be the fraction of reds that are exported. Then country 2 will export $\tilde{\chi}\tilde{h}_R$ red widgets for $\tilde{\chi}\tilde{h}_R P_W$ blue widgets. Trade will lead to an equilibrium where the marginal rate of substitution in country 1 is equal to the world price P_W. That is,

$$\frac{(1-\tilde{\chi})\tilde{h}_R}{\tilde{\chi}\tilde{h}_R P_W} = \left(\frac{\alpha_R}{\alpha_B}\right)^\sigma P_W^{-\sigma}.$$

Solving for $\tilde{\chi}$ we get the fraction of reds exported as

$$\tilde{\chi}^* = \frac{1}{\left(\frac{\alpha_R}{\alpha_B}\right)^\sigma P_W^{1-\sigma}}. \tag{4.8}$$

Given this we can derive the amount of red widgets exported and the amount blues imported as

$$\tilde{X}_R = \left[\frac{1}{1 + \left(\frac{\alpha_R}{\alpha_B}\right)^{\sigma} P_W^{1-\sigma}} \right] \tilde{h}_R,$$

and

$$\tilde{M}_B = \left[\frac{1}{1 + \left(\frac{\alpha_R}{\alpha_B}\right)^{\sigma} P_W^{1-\sigma}} \right] \tilde{h}_R P_W.$$

In the next section we will establish the world price P_W and the pattern of trade between country 1 and country 2 under perfect competition.

4.3.4 The World Price and Pattern of Trade

The final pattern of trade depends on how the autarky prices in the two countries P and $\tilde{P}$ compare against each other. Assuming without loss of generality that $h_B/h_R > \tilde{h}_B/\tilde{h}_R$, then $P_R > \tilde{P}_R$ and country 1 will export blue widgets and import red widgets in line with their technological comparative advantage. Similarly country 2 will specialize in reds and trade reds for blues. The world price P_W is such that the total export of red widgets by country 2 is equal to the total imports of red widgets by country 1. That is,

$$\left[\frac{1}{1 + \left(\frac{\alpha_R}{\alpha_B}\right)^{\sigma} P_W^{1-\sigma}} \right] \tilde{h}_R = \left[\frac{\left(\frac{\alpha_R}{\alpha_B}\right)^{\sigma} P_W^{-\sigma}}{1 + \left(\frac{\alpha_R}{\alpha_B}\right)^{\sigma} P_W^{1-\sigma}} \right] h_B.$$

Solving for P_W from the above equation we get the world price under perfect competition as

$$P_W = \left(\frac{\alpha_R}{\alpha_B}\right) \left(\frac{h_B}{\tilde{h}_R}\right)^{1/\sigma}. \tag{4.9}$$

The results of this section are summarized in the following proposition

Proposition 4.1 *Under perfect competition and free movement of labor between type 1 and type 2 firms in both countries, the volume of exports and imports for country 1 and country 2 are as follows*

Country 1

$$X_B^* = \left[\frac{\left(\frac{\alpha_R}{\alpha_B}\right)^\sigma P_W^{1-\sigma}}{1 + \left(\frac{\alpha_R}{\alpha_B}\right)^\sigma P_W^{1-\sigma}} \right] h_B \ , \ M_R^* = \left[\frac{\left(\frac{\alpha_R}{\alpha_B}\right)^\sigma P_W^{-\sigma}}{1 + \left(\frac{\alpha_R}{\alpha_B}\right)^\sigma P_W^{1-\sigma}} \right] h_B.$$

Country 2

$$\tilde{X}_R = \left[\frac{1}{1 + \left(\frac{\alpha_R}{\alpha_B}\right)^\sigma P_W^{1-\sigma}} \right] \tilde{h}_R \ , \ \tilde{M}_B = \left[\frac{1}{1 + \left(\frac{\alpha_R}{\alpha_B}\right)^\sigma P_W^{1-\sigma}} \right] \tilde{h}_R P_W.$$

The resulting world price is

$$P_W = \left(\frac{\alpha_R}{\alpha_B}\right) \left(\frac{h_B}{\tilde{h}_R}\right)^{1/\sigma}.$$

We refer to this perfectly competitive solution as the benchmark case. We compare the benchmark case to the case when labor is not free to move between type 1 and type 2 firms in both the countries. The allocation of labor is the outcome of a lobbying contest between type 1 and type 2 firms.

4.4 The Labor Allocation Model

4.4.1 Country 1

As noted earlier, firms in this economy earn zero economic profits and all proceeds from production flow to labor. We assume that each firm is controlled by an entrepreneur whose services are provided at zero cost. The entrepreneur, however, does derive satisfaction from producing goods and therefore wishes to maximize the firm's output of red and blue widgets. The allocation of labor itself is the outcome of a contest between red and blue widget producing firms. One can envision this contest as a political one whereby firms lobby the government to establish barriers to the allocation of labor or alternatively to prevent the establishment of barriers. We assume that the set of blue widget producing firms act collectively to allocate resources to determine optimally their share of the labor force, which share is allocated equally among the type 1 firms. The red widget producing firms behave in similar fashion. (As implied, we assume the "free-rider" problem in each industry has been overcome.)

Specifically, let the competitive labor allocation under free trade be the benchmark allocation of labor in the economy. Then one can envision a scenario wherein, the type 2 firms desire a positive share of labor for production of red widgets. Therefore, the type 2 firms are interested in establishing barriers to the competitive allocation of labor. So the actual labor allocation achieved is $\lambda_B = z\lambda_B^* = z$, where $z \in [0, 1]$. The fraction z is determined by a contest between the type 1 and type 2 industries. Let β be the resources devoted by the type 1 firms and ρ be the resources of type 2 firms, then the fraction z is given as

$$z = min\left[1, \frac{\theta\beta}{\rho}\right], \tag{4.10}$$

where θ is an exogenously given parameter. As can be seen z is non-decreasing in β and non-increasing in ρ.

We model the contest as a leader-follower game where the type 1 firms are the leaders and type 2 are the followers. That is, type 1 firms devote β in the first stage of the game and given β, type 2 firms select ρ. The payoff to each type of firm is once again the output that can be produced with the labor share achieved. The respective payoffs for the type 1 and 2 firms are

$$\pi_1 = \begin{cases} h_B - \beta & \text{if } \theta\beta \geq \rho \\ (\theta\beta/\rho)h_B - \beta & \text{if } \theta\beta < \rho \end{cases} \tag{4.11}$$

$$\pi_2 = \begin{cases} [1 - (\theta\beta/\rho)]h_R - \rho & \text{if } \theta\beta < \rho \\ -\rho & \text{if } \theta\beta \geq \rho \end{cases} \tag{4.12}$$

We solve this problem using backward induction wherein type 2 firms decide on the optimal ρ for a given β. Type 1 firms use this information to decide their optimal contribution of resources. As is clear from (4.12), the optimal contribution for type 2 firms in the interval $[0, \theta\beta]$ is $\rho^* = 0$. In the interval $(\theta\beta, \infty)$, the payoff function is $[1 - (\theta\beta/\rho)]h_R - \rho$. The necessary condition for the optimal ρ to be in this interval is that the first derivative of the payoff function with respect to ρ evaluated at the cut off point $\theta\beta$ must be positive, that is

$$\beta < h_R/\theta$$

Given this the payoff is maximized at

$$\rho = (\theta\beta h_R)^{1/2}. \tag{4.13}$$

However, this value of ρ is optimal if and only if the payoff at $\rho = (\theta\beta h_R)^{1/2}$ is greater than the payoff obtained when $\rho = 0$. This requires

$$\beta < (h_R)/4\theta \equiv \hat{\beta}. \tag{4.14}$$

This establishes that the optimal value for ρ, ρ^*, is

$$\rho^* = 0, if \beta \geq \hat{\beta} \equiv h_R/4\theta, \rho^* = (\theta\beta h_R)^{1/2}, if \beta < \hat{\beta}. \tag{4.15}$$

Given the values for ρ^* in equation (4.15), we can now determine the optimal allocation for the type 1 leader. As apparent from equation (4.15), the optimal value for β lies in the interval $[0, \hat{\beta} \equiv h_R/4\theta]$. Given equation (4.15), the payoff function for the type 1 firms is

$$\pi_1 = (\theta\beta)^{1/2} h_B/h_R^{1/2} - \beta, if \beta < \hat{\beta}; \pi_1 = h_B - \hat{\beta}, if \beta = \hat{\beta}.$$

Let $f(\beta) \equiv (\theta\beta)^{1/2} h_B/h_R^{1/2} - \beta$ and note that $f'(\beta) >, <$, or $= 0$, as $\beta <, >$, or $= \beta^* \equiv (\theta h_B^2)/4h_R$. From these facts it follows that π_1 is maximized at $\hat{\beta}$, if $\hat{\beta} \equiv h_R/4\theta \leq \beta^* \equiv (\theta h_B^2)/4h_R$, that is, if $\theta \geq (h_R/h_B)$.

Suppose $\theta < (h_R/h_B)$ and thus β^* is less than $\hat{\beta}$. Then in this case $f'(\beta) = 0$ at $\beta = \beta^* \in [0, \hat{\beta})$ and therefore β^* is a *candidate* optimal value for β. We say *candidate* because it still may pay the type 1 firms to generate a discrete jump in π_1 by increasing β from β^* to $\hat{\beta}$.

Note that when $\beta = \beta^* \equiv (\theta h_B^2)/4h_R$ then $z^* \equiv (\theta/2)(h_B/h_R)$ and the payoff to the type 1's is $\pi_1(\beta^*) = (\theta h_B^2)/2h_R - (\theta h_B^2)/4h_R = (\theta h_B^2)/4h_R$. However, this must be compared to $\pi_1(\hat{\beta}) = h_B - h_R/4\theta$. One finds that when $\beta^* < \hat{\beta}$,

$$\pi_1(\beta^*) >, < or = \pi_1(\hat{\beta}) \; as \; g(\theta) \equiv \theta^2 h_B^2 - 4h_R h_B \theta + h_R^2 >, <, or = 0.$$

The quadratic function $g(\theta)$ has zeros at $\theta = (h_R/h_B)(2 - \sqrt{3})$ and $\theta = (h_R/h_B)(2 + \sqrt{3})$ and is negative on the interval $(h_R/h_B)(2 - \sqrt{3}), (h_R/h_B)(2 + \sqrt{3}))$. Thus on this interval the optimal value for β is $\hat{\beta}$. When $\theta < \theta_C \equiv (h_R/h_B)(2-\sqrt{3})$, the optimal value is β^*. When $\theta > (h_R/h_B)(2+\sqrt{3})$, $\theta > (h_R/h_B)$, and our previous analysis has established that the optimal value is $\hat{\beta}$.

The results from the labor allocation model are summarized in the following proposition.

Proposition 4.2 *The total share of labor allocated to type 1 and type 2 firms depend on the parameter θ as follows*
if $\theta < (h_R/h_B)(2-\sqrt{3}) \equiv \theta_C$, then $\beta^* = (\theta h_B^2/4h_R)$, $\rho = (\theta h_B/2)$ and $z^* = (\theta h_B/2h_R)$. The total labor appropriated by type 1 firms is $\lambda_B = z^*$ and that by type 2 firms is $\lambda_R = 1 - z^*$.
if $\theta \geq (h_R/h_B)(2-\sqrt{3}) \equiv \theta_C$, then $\beta^* = \hat{\beta} \equiv h_R/4\theta$, $\rho^* = 0$ and $z^* = 1 \equiv \lambda_B^*$. In this case, we have a perfectly competitive equilibrium with $\lambda_B = \lambda_B^*$ and $\lambda_R = 0$.

4.4.2 Country 2

Since we have assumed country 2 to be a mirror image of country 1, the analysis parallels that in the previous section. Let $\tilde{\lambda}_R^* = 1$ be the perfectly competitive benchmark allocation of labor to the production of red widgets. Then $\tilde{\lambda}_R = \tilde{z}\tilde{\lambda}_R^* = \tilde{z}$, where $\tilde{z} \in [0,1]$. Let $\tilde{\rho}$ be the resources devoted by the type 1 firms and $\tilde{\beta}$ be the resources of type 2 firms, then the fraction $\tilde{z}$ is given as

$$\tilde{z} = min\left[1, \frac{\tilde{\theta}\tilde{\rho}}{\tilde{\beta}}\right], \tag{4.16}$$

where, $\tilde{\theta}$ is an exogenously given parameter, the counterpart of θ for country 2. The solution to this contest is similar to the one analyzed previously for country 1. The following proposition summarizes the results of the labor allocation contest in country 2

Proposition 4.3 *The total share of labor allocated to type 1 and type 2 firms depend on the parameter $\tilde{\theta}$ as follows*

if $\tilde{\theta} < (\tilde{h}_B/\tilde{h}_R)(2-\sqrt{3}) \equiv \tilde{\theta}_C$, *then* $\tilde{\rho}^* = (\tilde{\theta}\tilde{h}_R^2/4\tilde{h}_B)$, $\tilde{\beta}^* = (\tilde{\theta}\tilde{h}_R/2)$ *and* $\tilde{z}^* = (\tilde{\theta}\tilde{h}_R/2\tilde{h}_B)$. *The total labor appropriated by type 1 firms is* $\tilde{\lambda}_R = \tilde{z}^*$ *and that by type 2 firms is* $\tilde{\lambda}_B = 1 - \tilde{z}^*$.

if $\tilde{\theta} \geq (\tilde{h}_B/\tilde{h}_R)(2-\sqrt{3}) \equiv \tilde{\theta}_C$, *then* $z^* = 1$. *In this case, we have a perfectly competitive equilibrium with* $\tilde{\lambda}_R = \tilde{\lambda}_R^* = 1$ *and* $\tilde{\lambda}_B = 0$.

It is clear from the above results that barriers to the allocation of labor in a country can have an impact on the trade pattern as well. This arises from the fact that for a range of values of θ and $\tilde{\theta}$, it is not possible for the countries to specialize in the good in which they have a technological comparative advantage. This is because type 2 firms in each country can appropriate part of the labor and continue producing a positive amount of the good in which there is no comparative advantage. In the following section we make this point clear by exploring the pattern of trade when there are barriers to the allocation of labor.

4.5 Pattern of trade with barriers

4.5.1 Country 1

As mentioned above, the pattern of trade will depend on the value of θ. We have two separate cases depending on whether the value of θ is greater or less than the critical value (θ_C). We analyze each case below

Case 1: $\theta \geq \theta_C$. Since this case mimics the competitive outcome, the results would be the same as obtained under the competitive benchmark case discussed in section 1.2. Assuming that the world price P_W is less than the domestic price $P \equiv h_B/h_R$, country 1 specializes completely in the production of blue widgets and trades blue widgets for red widgets. The total exports of blue widgets and import of red widgets are respectively given by

$$X_B^* = \left[\frac{\left(\frac{\alpha_R}{\alpha_B}\right)^\sigma P_W^{1-\sigma}}{1 + \left(\frac{\alpha_R}{\alpha_B}\right)^\sigma P_W^{1-\sigma}} \right] h_B,$$

and

$$M_R^* = \left[\frac{\left(\frac{\alpha_R}{\alpha_B}\right)^\sigma P_W^{-\sigma}}{1 + \left(\frac{\alpha_R}{\alpha_B}\right)^\sigma P_W^{1-\sigma}} \right] h_B.$$

Case 2: $\theta < \theta_C$. In this case there are barriers to the competitive allocation of labor. With barriers, the allocation of labor to type 1 firms is $\lambda_B = z^* \equiv (\theta h_B / 2h_R)$ and the total production of blue widgets is $\lambda_B h_B = (\theta h_B^2 / 2h_R)$. The direction of trade would depend on the comparison of the marginal rate of substitution of blue widgets for red widgets with the world price P_W. Country 1 would trade blue widgets for reds if the following condition holds

$$\left(\frac{\alpha_R}{\alpha_B}\right)\left(\frac{b}{r}\right)^{1/\sigma} > P_W,$$

or

$$\frac{b}{r} > P_W^\sigma \left(\frac{\alpha_B}{\alpha_R}\right)^\sigma.$$

Substituting the value of b and r in the above expression we get

$$\frac{\theta h_B^2 / 2h_R}{(1 - \theta h_B / 2h_R) h_R} > P_W^\sigma \left(\frac{\alpha_B}{\alpha_R}\right)^\sigma.$$

Therefore, country 1 will trade blue widgets for reds if the following condition holds

$$\theta > \frac{2 P_W^\sigma (\alpha_B / \alpha_R)^\sigma h_R}{h_B [h_B / h_R + P_W^\sigma (\alpha_B / \alpha_R)^\sigma]} \equiv \theta_B \tag{4.17}$$

If $\theta_B < \theta < \theta_C \equiv (2-\sqrt{3})h_R/h_B$ then country 1 trades blue widgets for reds. Note that this condition is satisfied only if

$$P_W^\sigma < \left(\frac{2-\sqrt{3}}{\sqrt{3}}\right)\frac{h_B}{h_R}\left(\frac{\alpha_R}{\alpha_B}\right)^\sigma$$

It will be shown later that in this case the resulting world price is such that this condition is satisfied. Assuming for the moment that the condition holds, and country 1 exports $\chi(\theta h_B^2/2h_R)$ blue widgets for $\chi(\theta h_B^2/2h_R)P_W$ red widgets, the equilibrium occurs at the point where the marginal rate of substitution between the two goods in equal to the price ratio. That is,

$$\frac{\chi(\theta/2)h_B^2/h_R P_W + [1-(\theta/2)h_B/h_R]h_R}{(1-\chi)(\theta/2)h_B^2/h_R} = \left(\frac{\alpha_R}{\alpha_B}\right)^\sigma P_W^{-\sigma}.$$

Solving the above expression for χ we get

$$\chi = \frac{\left(\frac{\alpha_R}{\alpha_B}\right)^\sigma P_W^{1-\sigma}}{1+\left(\frac{\alpha_R}{\alpha_B}\right)^\sigma P_W^{1-\sigma}} - \frac{(2/\theta)(h_R^2/h_B^2)[1-(\theta/2)h_B/h_R]P_W}{1+\left(\frac{\alpha_R}{\alpha_B}\right)^\sigma P_W^{1-\sigma}}$$

Note that the first expression on the right side of the equation is the fraction of exports without any barriers, i.e. χ^* and let

$$\eta \equiv \frac{(2/\theta)(h_R^2/h_B^2)[1-(\theta/2)h_B/h_R]P_W}{1+\left(\frac{\alpha_R}{\alpha_B}\right)^\sigma P_W^{1-\sigma}}. \tag{4.18}$$

Then we can rewrite χ as

$$\chi = \chi^* - \eta \tag{4.19}$$

It can be seen that the fraction of trade in the presence of domestic barriers is lower than the fraction of trade without any barriers. This is because with barriers country 1 cannot fully specialize in the production of blue widgets. Using equation (4.19) we can derive the volume of trade for country 1 as

$$X_B = (\chi^* - \eta)\frac{\theta}{2}\frac{h_B^2}{h_R},$$

and

$$M_R = (\chi^* - \eta)\frac{\theta}{2}\frac{h_B^2}{h_R P_W}$$

The following proposition summarizes the results

Proposition 4.4 *If $\theta \geq \theta_C$ then the exports and imports of country 1 are given by*

$$X_B^* = \chi^* h_B \ , M_R^* = \chi^* h_B / P_W.$$

if $\theta_B < \theta < \theta_C$, then the exports and imports of country 1 are given by

$$X_B = (\chi^* - \eta)\frac{\theta}{2}\frac{h_B^2}{h_R} \ , M_R = (\chi^* - \eta)\frac{\theta}{2}\frac{h_B^2}{h_R P_W}$$

where χ^* is given in equation (4.6) and η is given in equation (4.18).

4.5.2 Country 2

Using analysis similar to the previous section can establish the pattern of trade in country 2. First, denoting

$$\tilde{\theta}_R \equiv \frac{2\tilde{h}_B/\tilde{h}_R}{1+\tilde{h}_R/h_B(\alpha_B/\alpha_R)^\sigma P_W^\sigma}, \tag{4.20}$$

as the crucial value for $\tilde{\theta}$ above which country 2 exports red widgets for blues, it can be shown that the condition $\tilde{\theta}_R < \tilde{\theta} < \tilde{\theta}_C$ is satisfied only if

$$P_W^\sigma > \left(\frac{2-\sqrt{3}}{\sqrt{3}}\right)\frac{\tilde{h}_B}{\tilde{h}_R}\left(\frac{\alpha_R}{\alpha_B}\right)^\sigma \tag{4.21}$$

Further, denoting

$$\tilde{\eta} \equiv \frac{(\alpha_R/\alpha_B)^\sigma P_W^{-\sigma}(2/\theta)(\tilde{h}_B^2/\tilde{h}_R^2)[1-\tilde{(}\theta/2)\tilde{h}_R/\tilde{h}_B]}{1+\left(\frac{\alpha_R}{\alpha_B}\right)^\sigma P_W^{1-\sigma}},$$

we can summarize the trade pattern for country 2 in the following proposition

Proposition 4.5 *If $\tilde{\theta} \geq \tilde{\theta}_C$ then the exports and imports of country 2 are given by*

$$\tilde{X}_R = \tilde{\chi}^*\tilde{h}_R\ , \tilde{M}_B = \tilde{\chi}^*\tilde{h}_R P_W.$$

if $\tilde{\theta}_R < \tilde{\theta} < \tilde{\theta_C}$, then exports and imports of country 2 are given by

$$\tilde{X}_R = (\tilde{\chi}^* - \tilde{\eta})\frac{\tilde{\theta}}{2}\frac{\tilde{h}_R^2}{\tilde{h}_B}, \tilde{M}_R = (\tilde{\chi}^* - \tilde{\eta})\frac{\tilde{\theta}}{2}\frac{\tilde{h}_R^2 P_W}{\tilde{h}_B},$$

where $\tilde{\chi}^*$ is given in equation (4.8).

In the following section we use the results derived above to solve for the equilibrium world price and outline the final pattern of trade based on the values of θ and $\tilde{\theta}$.

4.6 The World Price and Pattern of Trade

The final pattern of trade and the world price depends on the comparison of the autarky prices in the two countries. Since the autarky price in country 1 is greater than the autarky price in country 2, country 1 will export blue widgets and import red widgets. However, the volume of trade depends upon the value of θ and $\tilde{\theta}$. Therefore, the final world price based on the relative supply and demand will also depend on these values. We break down the analysis into the following four sub-cases.

1. $\theta \geq \theta_C$ and $\tilde{\theta} \geq \tilde{\theta}_C$
2. $\theta \geq \theta_C$ and $\tilde{\theta}_R < \tilde{\theta} < \tilde{\theta}_C$
3. $\theta_B < \theta < \theta_C$ and $\tilde{\theta}_R < \tilde{\theta} < \tilde{\theta}_C$
4. $\theta_B < \theta < \theta_C$ and $\tilde{\theta} \geq \tilde{\theta}_C$

We analyze each case individually to determine the world price P_W.

Case 1: $\theta \geq \theta_C$ and $\tilde{\theta} \geq \tilde{\theta}_C$. In this case neither country has barriers to the competitive allocation of labor. Therefore, this case mimics the results derived in section 4.3. Given that $P \equiv h_B/h_R > \tilde{P} \equiv \tilde{h}_B/\tilde{h}_R$, country 1 can completely specialize in the production of blue widgets and country 2 can completely specialize in reds. The equilibrium world price is such that the amount reds exported by country 2 is equal to amount of reds imported by country 1. That is, as given in proposition 1, the world price is

$$P_W = \left(\frac{\alpha_R}{\alpha_B}\right)\left(\frac{h_B}{\tilde{h}_R}\right)^{1/\sigma}$$

Case 2: $\theta \geq \theta_C$ and $\tilde{\theta}_R < \tilde{\theta} < \tilde{\theta}_C$. Country 1 has no barriers to the competitive allocation of labor, while country 2 has barriers. Since country 2 cannot specialize completely in the production of red widgets, there is a decrease in the volume of exports of red widgets. World price would be such that the total exports of red widgets by country 2 with barriers is equal to the total import of red widgets by country 1 in the absence of barriers. That is,

$$(\tilde{\chi}^* - \tilde{\eta})\frac{\tilde{\theta}}{2}\frac{\tilde{h}_R^2}{\tilde{h}_B} = \chi^*$$

Substituting the values for χ^*, $\tilde{\chi}^*$, and $\tilde{\eta}$ and solving the resulting equation for P_W we get,

$$P_W = \left(\frac{\alpha_R}{\alpha_B}\right)\left[\frac{h_B + \left(1 - \frac{\tilde{\theta}}{2}\frac{\tilde{h}_R}{\tilde{h}_B}\right)\tilde{h}_B}{\frac{\tilde{\theta}}{2}\frac{\tilde{h}_R^2}{\tilde{h}_B}}\right]^{1/\sigma} > \left(\frac{\alpha_R}{\alpha_B}\right)\left(\frac{h_B}{\tilde{h}_R}\right)^{1/\sigma} \tag{4.22}$$

Therefore the world price of red widgets is greater in this case than in the case without barriers. This is because barriers in country 2 reduce the supply of red widgets compared to the benchmark case. Thus a shortage of red widgets causes the relative price of red widgets to go up. This in turn decreases the overall welfare of consumers in both economies. Therefore, the presence of barriers in one country leads to adverse welfare consequences for the country without barriers. Before we proceed we ensure that the world price given by equation (4.22) satisfies the condition given in equation (4.21). This requires

$$\left(\frac{\alpha_R}{\alpha_B}\right)^\sigma\left[\frac{h_B + \left(1 - \frac{\tilde{\theta}}{2}\frac{\tilde{h}_R}{\tilde{h}_B}\right)\tilde{h}_B}{\frac{\tilde{\theta}}{2}\frac{\tilde{h}_R^2}{\tilde{h}_B}}\right] > \left(\frac{2-\sqrt{3}}{\sqrt{3}}\right)\frac{\tilde{h}_B}{\tilde{h}_R}\left(\frac{\alpha_R}{\alpha_B}\right)^\sigma.$$

Canceling out the common terms and simplifying, the above expression is satisfied if

$$\tilde{\theta} < (2 - \sqrt{3}) \left(\frac{\tilde{h}_B}{\tilde{h}_R} + \frac{h_B}{\tilde{h}_R} \right)$$

Since in this case $\tilde{\theta} < \tilde{\theta}_C \equiv (2 - \sqrt{3})\tilde{h}_B/\tilde{h}_R$, the above inequality is always satisfied. Therefore, the resulting world price ensures that country 2 does trade red widgets for blue.

Case 3: $\theta_B < \theta < \theta_C$ and $\tilde{\theta}_R < \tilde{\theta} < \tilde{\theta}_C$. There are barriers to the competitive allocation of labor in both the countries. In this case, there is a reduction in quantity supplied of both red and blue widgets. Again the world price will be such that the the total exports of red widgets by country 2 is equal to the total import of red widgets by country 1, when barriers are present in both countries. That is,

$$(\tilde{\chi}^* - \tilde{\eta}) \frac{\tilde{\theta}}{2} \frac{\tilde{h}_R^2}{\tilde{h}_B} = (\chi^* - \eta) \frac{\theta}{2} \frac{h_B^2}{h_R P_W}.$$

Again solving for P_W, we get

$$P_W = \left(\frac{\alpha_R}{\alpha_B} \right) \left[\frac{\frac{\theta}{2} \frac{h_B^2}{h_R} + \left(1 - \frac{\tilde{\theta}}{2} \frac{\tilde{h}_R}{\tilde{h}_B}\right) \tilde{h}_B}{\frac{\tilde{\theta}}{2} \frac{\tilde{h}_R^2}{\tilde{h}_B} + \left(1 - \frac{\theta}{2} \frac{h_B}{h_R}\right) h_R} \right]^{1/\sigma} \tag{4.23}$$

As in case 2, it can be shown that the world price given in equation (4.23) is such that θ and $\tilde{\theta}$, line up in way that both countries trade according to their technological comparative advantage.

Case 4: $\theta_B < \theta < \theta_C$ and $\tilde{\theta} \geq \tilde{\theta}_C$. Given the symmetry in our model, this case would be identical to Case 2.

4.7 Allocative Barriers and Output Growth

In this section we analyze the growth rate of output for the two countries under the cases described in the previous section. We use a learning-by-doing framework wherein there is accumulation of human capital as a result of devoting more labor to the production of a good. To simplify matters we initially assume that there is learning only in one good via type 1 firms. For country 1 there is learning in the production of blue widgets, while in country 2 there is learning in the production of red widgets. Therefore we have the following for country 1

$$\dot{h}_B = \delta_B \lambda_B h_B \ , \ \dot{h}_R = 0, \tag{4.24}$$

while for country 2 we have

$$\dot{\tilde{h}}_R = \tilde{\delta}_R \tilde{\lambda}_R \tilde{h}_R \ , \ \dot{\tilde{h}}_B = 0. \tag{4.25}$$

The above equations state for type 1 firms in both countries, the marginal product of labor (human capital) increases over time as more labor is devoted to the production of type 1 goods. The parameters δ_B and $\tilde{\delta}_R$ are the constant rates at which the productivities evolve. To make the analysis simpler we assume that $\delta_B = \tilde{\delta}_R = \delta$. In the following analysis we meausre output as valued in blue widgets at the world price and compute the rate of growth of output for different values of θ and $\tilde{\theta}$

4.7.1 $\theta \geq \theta_C$ and $\tilde{\theta} \geq \tilde{\theta}_C$

In this case neither country faces barriers to the competitive allocation of labor. Therfore, country 1 specializes in the production of blue widgets and country 2 specializes in the production of red widgets. The resulting labor shares in both the countries is given by

$$\lambda_B = 1\ , \lambda_R = 0\ and\ \tilde{\lambda}_B = 0\ , \tilde{\lambda}_R = 1.$$

Given these labor shares and the production functions given in equation (4.3) and 4.4) we know that the total value of output in country 1 is

$$Y = B = h_B.$$

The rate of growth of output is therefore given by

$$\frac{\dot{Y}}{Y} = \gamma = \delta. \tag{4.26}$$

Similarly, the total value of output for country 2 is given by

$$\tilde{Y} = \tilde{R} = P_W \tilde{h}_R,$$

where

$$P_W = \left(\frac{\alpha_R}{\alpha_B}\right)\left(\frac{h_B}{\tilde{h}_R}\right)^{1/\sigma}$$

Since $\dot{P}_W/P_W = 0$, the world price P_W is constant over time. Therefore growth of output in country 2 is driven entirely by growth in the production of red widgets. That is,

$$\frac{\dot{\tilde{Y}}}{\tilde{Y}} \equiv \tilde{\gamma} = \delta \tag{4.27}$$

4.7.2 $\theta \geq \theta_C$ and $\tilde{\theta}_R < \tilde{\theta} < \tilde{\theta}_C$

This is the case where country 1 has no barriers, while country 2 has barriers to the competitive allocation of labor. Recall that in this case there is complete specialization in the production of blue widgets in country 1. Country 2 produces both red widgets and blue widgets as a result of barriers. Therefore, as in the previous case the rate of growth of output in country 1 is $\gamma = \delta$. For country 2, the factor shares are given by

$$\tilde{\lambda}_R = \tilde{z}^* \ , \ \tilde{\lambda}_B = 1 - \tilde{z}^*,$$

where $\tilde{z}^* = \tilde{\theta}\tilde{h}_R/2\tilde{h}_B$. The total value output of country 2 is therefore given by

$$\tilde{Y} = (1 - \tilde{z}^*)\tilde{h}_B + P_W \tilde{z}^* \tilde{h}_R,$$

where P_W in this case is given in equation (4.22). The growth rate of output in this case is given by

$$\tilde{\gamma} = \Delta\delta, \tag{4.28}$$

where $\Delta < 1$. Therefore, the growth rate in country 2 as a result of barriers is less than that obtained in the perfectly competitive case.

4.7.3 $\theta_B < \theta < \theta_C$ and $\tilde{\theta}_R < \tilde{\theta} < \tilde{\theta}_C$

In this case both the countries have barriers to the competitive allocation of labor and there will be incomplete specialization in the production of blue widgets in country 1 and incomplete specialization in the production of red widgets in country 2. The growth rates achieved by both the countries will be less than that achieved under the competitive allocation.

4.7.4 $\theta_B < \theta < \theta_C$ and $\tilde{\theta} \geq \tilde{\theta}_C$

Given the symmetry in our model this case would be similar to the case discussed in section 5.2.

4.8 Long Run Growth

In this section we explore whether countries maintain their initial growth rates, or is there is a possibility of transitioning to a higher or lower growth path in the long run. We break down the analysis again into four sub-cases depending on the initial values θ and $\tilde{\theta}$. The transition would depend on whether the critical values $\theta_C \equiv (2-\sqrt{3})h_R/h_B$ and $\tilde{\theta}_C \equiv (2-\sqrt{3})\tilde{h}_B/\tilde{h}_R$, increase or decrease over time.

4.8.1 $\theta \geq \theta_C \equiv (2-\sqrt{3})h_R/h_B$ and $\tilde{\theta} \geq \tilde{\theta}_C \equiv (2-\sqrt{3})\tilde{h}_B/\tilde{h}_R$

In this case, neither countries have barriers and they specialize completely in the production of the good in which they have a comparative advantage. This means, for country 1, h_B grows over time and h_R remains constant. Similarly for country 2, $\tilde{h}_R$ grows over time and $\tilde{h}_B$ remains constant. Given this it is easy to see that for both countries, the critical values fall over time. Therefore given the initial values of θ and θ_C, both countries would maintain the growth rate δ in the long run.

4.8.2 $\theta \geq \theta_C \equiv (2-\sqrt{3})h_R/h_B$ and $\tilde{\theta}_R < \tilde{\theta} < \tilde{\theta}_C \equiv (2-\sqrt{3})\tilde{h}_B/\tilde{h}_R$

Since country 1 has no barriers to the allocation of labor, it specializes completely in the production of blue widgets and the analysis is similar to that of the previous case. Country 1 starts off initially with a growth rate of δ and stays on that path in the long run. Country 2, however starts off with barriers and as shown in the previous section grows at the rate $\Delta\delta$, where $\Delta < 1$. However, since country 2 still produces a positive amount of red widgets and there is learning in the production of red widgets, $\tilde{h}_R$ grows over time. On the other hand, since we have assumed there is no learning in the production of blue widgets, $\tilde{h}_B$, remains constant. This implies that in the long run, for country 2, the ratio $\tilde{h}_B/\tilde{h}_R$ falls over time. Therefore, the critical value $\tilde{\theta}_C$ also falls in the long run. So even though country 2 starts off initially with $\tilde{\theta} < \tilde{\theta}_C$, in the long run this will be reversed, leading to the removal of barriers. Country 2 will therefore achieve growth rate δ in the long run as well. Free trade therefore enables country 2 to move to a higher growth path eventually.

We can make similar arguments to show that in the remaining two cases as well irrespective of the initial staring point, the two countries will eventually converge to the competitive growth rate δ.

4.9 Conclusion

In this paper we developed a two-country two-good model of international trade with the possibility of barriers to the competitive allocation of labor in each country. Barriers arise as a result of lobbying by firms to establish or prevent competitive allocation of labor. We first of all show that in each country the extent of the barriers is determined by the relative lobbying power of the firms. If the lobbying power of firms defending the competitive outcome is high enough in the two countries, then competitive allocation follows. In this case the autarky prices in the two countries reflects the marginal productivities and therefore trade takes place in accordance with technological comparative advantage. On the other hand, if the lobbying power of firms that prevent competitive allocation of labor is high enough then a non-competitive labor allocation follows, with barriers preventing complete specialization in the production of the good in which the country has a comparative advantage. This leads to a reduction in the volume of trade, even in the absence of trade barriers, and therefore leads to a distortion in world price relative to competitive prices. This has negative welfare consequences for not only the country with barriers, but also for the country which may not have any barriers

We then turn to the growth implications of the model. Growth in the model is driven by the accumulation of human capital as a result of learning-by-doing. We assume that in each country learning only takes place in one good. We show that growth rates are maximized when countries achieve a competitive allocation of labor. In cases where there are barriers to competitive allocation growth rate falls. However, with free trade and countries exporting the good in which there is learning, the economies eventually converge to the competitive growth rates.

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