PRELIMINARY INVESTIGATION OF THE VIBRATION CHARACTERISTICS OF A SLIP TABLE

by

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Thesis submitted to the Graduate Faculty of the
Virginia Polytechnic Institute
in candidacy for the degree of

MASTER OF SCIENCE

in

ENGINEERING MECHANICS

January, 1964

Blacksburg, Virginia

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C. LIST OF SYMBOLS

Symbol	Description	Units
A	cross sectional area of bar	in^2
a_{22}	influence coefficient	-
c	$\frac{Eq}{T} = 20.2 \times 10^4$(steel)	in/sec.
D	$\frac{AE\omega}{c}$	-
E	Modulus of Elasticity = 30×10^6(steel)	psi
F	Magnitude of force on "concentrated weight", $F \sin \omega t$	pounds
f	frequency	cycles per sec.
f_{11}	fundamental frequency of structure only	cps
f_{n_1}	fundamental frequency of structure and W_1	cps
f_{n_2}	fundamental frequency of structure and W_2	cps
g	gravitational constant = 386	$in/sec.^2$
H	$\frac{EA\omega\ \beta}{c\,J}$	-
J	Determinant of coefficients of C_1, C_2, C_3 and C_4 in equations (12)	-
k	constant denoting "concentrated weight" location	-
L	length of bar	inches
M	mass of "concentrated weight" attached to bar $= \frac{W}{g}$	inch pounds/sec^2
m	mass of the bar $= \frac{ATL}{g}$	inch pounds/sec^2
P	force in bar	pounds

R	weight ratio of W to AT L	-
t	time	seconds
TR	transmissibility = $\frac{\text{force on "concentrated weight"}}{\text{Input force}}$	-
u	displacement of any cross section in bar between free end and weight location	inches
$\bar{u}$	displacement of any cross section in bar between weight location and shaker	inches
W	weight of block attached to bar	pounds
w	weight per unit length of bar	pounds
W_1	weight of shaker and coupling	pounds
W_2	W_1 plus weight attached to coupling	pounds
x	coordinate of system	inches
X	an unknown function of x	inches
α	frequency parameter = $\frac{\omega L}{c}$	radians
β	magnitude of forcing function $\beta \sin \omega t$	pounds
Δ	deflection	inches
ε	strain in bar = $\frac{\partial u}{\partial x}$ or $\frac{\partial \bar{u}}{\partial x}$	in/in
π	3.1416	-
T	weight per unit volume of bar	pounds/in^3
ω	frequency	radians per second

III. INTRODUCTION

Magnetic shakers, or other types of shaking units, have long been used in vibration testing. Normally, the item to be vibrated is supported by suspension wires or mounted directly on the shaking unit. Recently it has been found that savings of up to 70 per cent in the time required to suspend the object to be vibrated can be realized by the use of a shaker table[6]. This is essentially a flat plate of light weight metal mounted in a horizontal position on an oil film and excited in a longitudinal direction by a magnetic shaker attached at one end of the plate. The item to be vibrated is mounted on this plate. A standard bolt spacing arrangement allows for easy adaptation of the test object to the plate. For small slip tables, approximately one feet square, the assumption that the plate behaves as a rigid body is probably quite realistic, but as the size of the slip table increases and frequency increases, this assumption becomes increasingly erroneous. Some tables of four feet by six feet are now in use.

There are two primary points of interest that will be investigated in this study: (1) The driving force will excite the vibration modes of both the table and the test object. Therefore the investigation of the fundamental modes of the combined test package and plate seems worthy of further study. (2) The transmissibility (ratio of the force on the test package to the input force of the shaker) may vary and therefore some means of predicting the force on the test package is of major practical importance. Both of the above would be influenced, among other things, by the frequency of the driving force, the material of which the plate is fabricated, the vertical and longitudinal position of attachment of the test package, the weight of the test package and the weight of the plate.

The exact analysis of the slip table is quite complex and no attempt will be made in this thesis to present such an analysis[8]. Normally, the shaker is attached along part of one end of the table with the other three edges free of load. The bottom of the table is supported on a pressurized oil film which may be assumed to be uniform. This may or may not have some significant damping effect. Since the test package is attached to the top of the slab, its inertia will produce a bending as well as an in-plane loading. The plate is a continuous body with an infinite number of degrees of freedom and the study of its vibration and of the propagation of elastic waves within it would be quite complex.

For this investigation, the previously described problem will be simplified to that of a bar of constant cross section, free at one end and along its longitudinal surface and subjected to a sinusoidal force at the other end. This bar represents the plate or table. On this bar, at any arbitrary point along its length, will be located a weight (representing the test package) whose centroid coincides with the longitudinal axis of the bar. No damping will be considered.

IV. REVIEW OF LITERATURE

Volumes could be written concerning the literature available in this field. Engineering Vibrations by Ayre and Jacobsen[3] is an excellent source for listings of specific literature on numerous types of vibration problems. Several solutions similar to those included in this investigation are included in reference 3 and referred to in this thesis. This book was the primary source of information for the analytical part of the thesis.

For the experimental work, the major portion of useful information was from company publications. While there are many other sources, the primary sources of information for this thesis were those publications of the Endevco Corporation and the MB Manufacturing Company listed in the Bibliography[1, 2, 4, 5, 6].

V. OBJECT

The simplified structure to be considered was described at the end of the INTRODUCTION. The objects of this investigation are (1) to determine analytically the fundamental frequencies of the structure, (2) to determine analytically the transmissibility of the system, and (3) to verify with reasonable accuracy the analytical expressions of (1) and (2) with experimental data.

VI. METHOD OF PROCEDURE

A. Analytical

This particular problem could be approached in various ways; a Lagrangian equation solution, for instance. The author has chosen to solve it as a boundary value problem, since this is the most direct approach.

B. Experimental

1. **Test Specimen** - The bar used was $\frac{1}{2}$ inch in diameter, 24 inches long, made of mild steel, and weighing 1.32 pounds. The "concentrated" weight was a mild steel or aluminum block (depending on the weight ratio.) Three blocks of varying weights were used. These blocks were first drilled to the diameter of the bar, then sawed in two through the center. The blocks were then attached to the bar by clamping pressure applied by two bolts passing through them. Provisions for mounting two accelerometers equally spaced from the bar centerline were made on each block. Although the thickness of the blocks varied (depending on the weight ratio) the thickest of the three was one inch. The approximate dimensions of a typical block were 2" x 2" x 1" thick. The center of the accelerometer mounting stud hole was 3/4" from the bar centerline.

2. **Test Equipment** - To give a known frequency of vibration, a MB Model SD vibration exciter was used. This shaker provides a force

of up to ten pounds at frequencies of up to 1000 cycles per second. Frequency is controlled by varying the voltage input to the shaker. A Hewlett-Packard audio-oscillator with a range of from 20 to 20,000 cps was used for this purpose. In order to accurately read the impressed frequency, a Hewlett-Packard model 522B electronic counter was used.

Accelerations were measured by Endevco accelerometers[2] and associated amplifiers with the readings taken from a Tektronix type 555 dual beam oscilloscope. Accelerometer #5614 had an output of 14.4 millivolts/g, peak to peak ; #306 had an output of 7.2 millivolts/g, peak to peak.

An Endevco model 2103-100 force gage was used to determine the force applied to the end of the bar. This gage measures a dynamic force of up to 100 pounds over a frequency range of 2 to 6000 cps[5]. It has an output of 0.47 volts/pound, peak to peak.

3. Test Procedure - Two different test setups were used for the experimental work. The first test setup, shown in Figure 1a, was used to determine the fundamental frequency. The bar was connected directly to a coupling mounted on the shaker. By noting the frequency at which the displacement of the end of the bar attached to the shaker became zero, then adding a known weight to the coupling and repeating the frequency determination, the fundamental frequency of the structure itself was determined using Dunkerley's method[9]. This determination is explained in more detail in EXPERIMENTAL RESULTS.

The second test setup, shown in Figure 1b, was used to determine the transmissibility. The bar was suspended with wires at each end from a beam approximately 3-1/2 feet above the bar. The force gage was mounted on the bar and then connected to the shaker through a connector. This connector was necessary to protect the force gage from any loading other than longitudinal. This was accomplished by the connector's "necked down" section. (Sketches and suggested dimensions are shown in most shaker instruction manuals.) The voltage output of the force gage was then read directly and converted to the input force applied to the bar. The force acting on the concentrated weight was then computed by multiplying its mass times the acceleration. The ratio of the force on the concentrated weight to the input force is defined as the transmissibility.

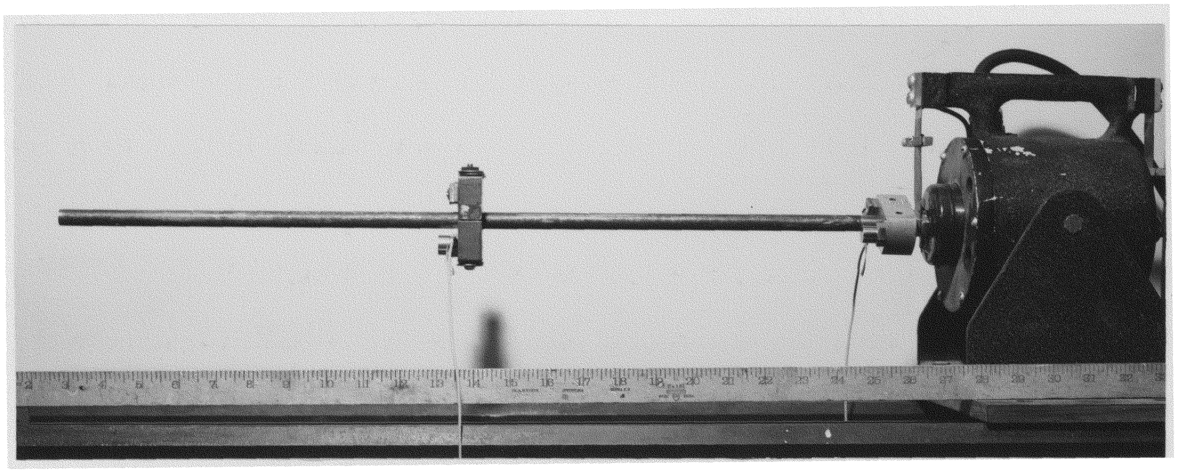


Figure 1a - Test Specimen - Fundamental Frequency Experiment

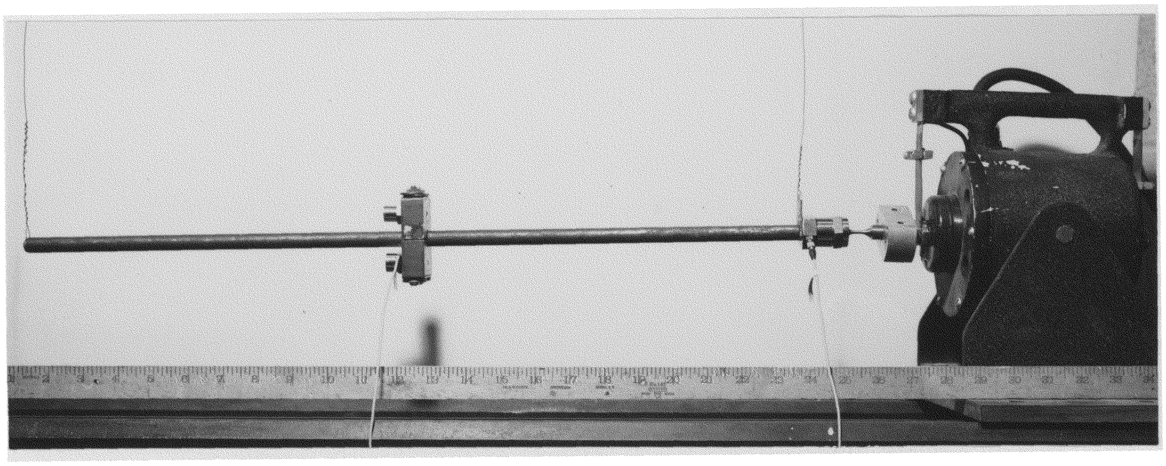


Figure 1b - Test Specimen - Transmissibility Experiment

VII. ANALYTICAL RESULTS

A diagram of the system with boundary conditions is shown in Figure 2a. For an infinitesimal length of the bar between $kL < x < L$, the free body diagram shown in Figure 2b can be drawn. In the latter referenced figure, the term $\frac{w}{g} \frac{\partial^2 u}{\partial t^2} dx$ is the inertia force acting on the element.

The differential force equilibrium equation of the element is then as follows:

$$\frac{w}{g} \frac{\partial^2 u}{\partial t^2} dx = -P + P + \frac{\partial P}{\partial x} dx ,$$

but since $\frac{\partial u}{\partial x} = \epsilon = \frac{\Delta}{L}$ and $\Delta = \frac{PL}{AE}$ for an axially loaded bar,

$$P = \frac{\Delta}{L} AE = \frac{\partial u}{\partial x} AE$$

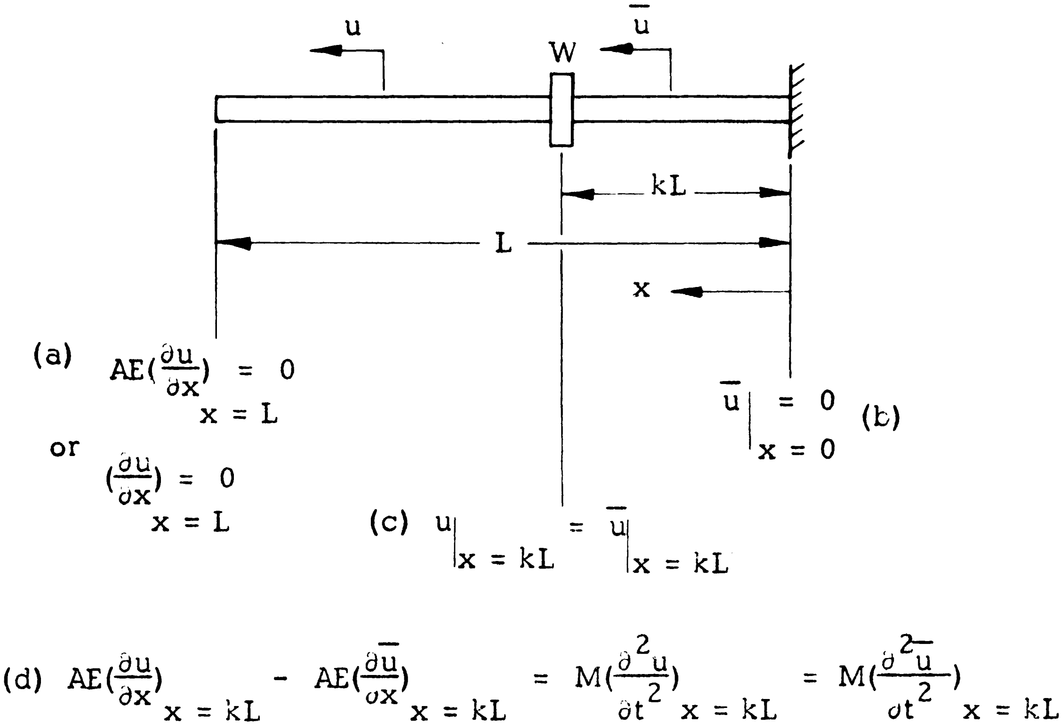
and the equilibrium equation on substituting for P becomes

$$\frac{\partial^2 u}{\partial t^2} = AE \frac{g}{w} \frac{\partial^2 u}{\partial x^2} .$$

If $c^2 = AE \frac{g}{w} = \frac{Eg}{T}$, the above equation becomes

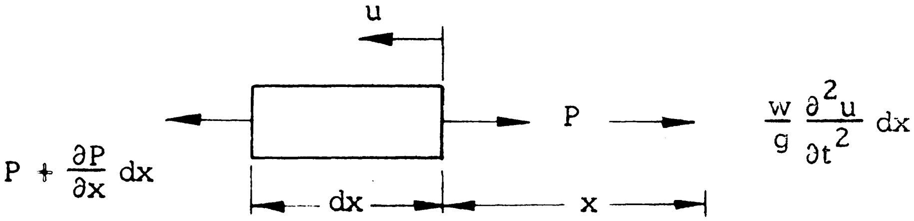
$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \tag{1}$$

which is the familiar wave equation in one dimension. A similar



Boundary Conditions for Deriving Frequency Equation

Figure 2a



Free Body Diagram for an Infinitesimal Length of Bar

Figure 2b

analysis of the portion of the bar from $0 < x < kL$ yields

$$\frac{\partial^2 \bar{u}}{\partial t^2} = c^2 \frac{\partial^2 \bar{u}}{\partial x^2} \qquad (2)$$

It is assumed that the displacement of any cross section of the bar between $kL < x < L$ is given by $u = X \sin \omega t$ where X is an unknown function of x. Substituting the assumed displacement into the partial differential equation (1), we obtain $\frac{\partial^2 X}{\partial x^2} + \frac{\omega^2}{c^2} X = 0$, the solution for which may be written as $X = C_1 \cos \frac{\omega}{c} x + C_2 \sin \frac{\omega}{c} x$. We therefore have the following expression for u:

$$u = (C_1 \cos \frac{\omega}{c} x + C_2 \sin \frac{\omega}{c} x) \sin \omega t \qquad (3)$$

Similarly, since equation (2) is identical in form to (1), we have the following expression for $\bar{u}$:

$$\bar{u} = (C_3 \cos \frac{\omega}{c} x + C_4 \sin \frac{\omega}{c} x) \sin \omega t \qquad (4)$$

When the end of the bar attached to the shaker suffers no displacement, the modes of vibration will be those of a fixed-free bar.

Applying boundary condition (a) from Figure 2a, at $x = 0$

$$(C_3 \cos 0 + C_4 \sin 0) \sin \omega t = 0 \quad \therefore \quad C_3 = 0$$

Therefore equation (4) reduces to $\bar{u} = C_4 \sin \frac{\omega}{c} x \sin \omega t$. Applying boundary conditions (b), (c), and (d) we have a set of 3 simultaneous equations:

$$\left.\begin{aligned}
&C_1\left(-\frac{\omega}{c}\sin\alpha\right) + C_2\left(\frac{\omega}{c}\cos\alpha\right) && = 0\\
&C_1(\cos k\alpha) + C_2(\sin k\alpha) + C_4(-\sin k\alpha) && = 0\\
&C_1(-D\sin k\alpha) + C_2(D\cos k\alpha) + C_4(M\omega^2\sin k\alpha - D\cos k\alpha) && = 0
\end{aligned}\right\} \quad (5)$$

where $\alpha = \frac{\omega L}{c}$ and $D = \frac{AE\omega}{c}$.

For any non-trivial solution, the determinant of coefficients of (5) must equal zero, or

$$-\frac{\omega}{c}M\omega^2\sin\alpha\sin^2 k\alpha + \frac{\omega}{c}D\sin\alpha\sin k\alpha\cos k\alpha + \frac{\omega}{c}D\cos\alpha\sin^2 k\alpha$$

$$-\frac{\omega}{c}M\omega^2\cos\alpha\cos k\alpha\sin k\alpha + \frac{\omega}{c}D\cos\alpha\cos^2 k\alpha$$

$$-\frac{\omega}{c}D\sin\alpha\sin k\alpha\cos k\alpha = 0$$

Noting that the second and last terms cancel as well as an $\frac{\omega}{c}$ in each term, we now have

$$M\omega^2\sin k\alpha\,(\sin\alpha\sin k\alpha + \cos\alpha\cos k\alpha) - D\cos\alpha = 0 \qquad (6)$$

Now dividing by $D\cos\alpha$ and factoring out $\sin k\alpha$ from the term in brackets,

$$\frac{M\omega^2 c}{AE\omega}\sin^2 k\alpha\left(\frac{\sin\alpha}{\cos\alpha} + \frac{\cos k\alpha}{\sin k\alpha}\right) - 1 = 0 ,$$

which simplifies to

$$\frac{W}{ATL}\,\alpha\sin^2 k\alpha\,(\tan\alpha + \operatorname{ctn} k\alpha) - 1 = 0 \qquad (7)$$

upon multiplying by $\frac{Lc}{Lc}$ and noting that $\frac{W}{g} = M$.

It should be noted here that if $k = 1$ (i.e. concentrated weight located at free end of bar), (6) reduces to

$\frac{A L T}{W} = \alpha \tan \alpha$ which is derived on page 475 of reference 3.

The term $\frac{W}{AT\,L}$ represents the ratio of the weight of the "concentrated" mass to the weight of the bar.

Equation (7) has been solved by computer for various weight ratios and k values; the computer results are shown in Table 1 .

In order to derive an expression for the transmissibility, it is necessary to solve for u and $\bar{u}$. This can be done by making use of the boundary conditions as shown in Figure 3.

Note, that for simplification, the origin of x is changed to the free end of the bar. Applying boundary condition (e) :

$$\left.\frac{\partial u}{\partial x}\right|_{x=0} = 0 , \qquad \frac{\partial u}{\partial x} = \left(-C_1 \frac{\omega}{c} \sin \frac{\omega}{c} x + C_2 \frac{\omega}{c} \cos \frac{\omega}{c} x\right) \sin \omega t$$

Therefore, $C_2 = 0$ and u becomes $C_1 \cos \frac{\omega}{c} x \sin \omega t$ (8)

The boundary condition (f) at $x = L$ yields the following relationship:

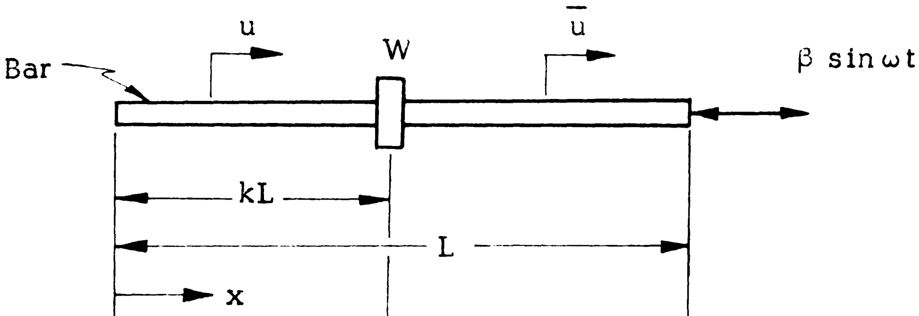
$$EA\left(\frac{\partial \bar{u}}{\partial x}\right)_{x=L} = \beta \sin \omega t = EA\left(-C_3 \frac{\omega}{c} \sin \alpha + C_4 \frac{\omega}{c} \cos \alpha\right) \sin \omega t \qquad (9)$$

TABLE 1

COMPUTER SOLUTIONS TO FREQUENCY EQUATION

R=WT.RATIO, MASS TO BAR. A = ALPHA. K LOCATES MASS

R	K	A
1.00	1.00	.865
1.00	.75	.940
1.00	.50	1.080
1.00	.25	1.335
.75	1.00	.955
.75	.75	1.030
.75	.50	1.165
.75	.25	1.395
.50	1.00	1.080
.50	.75	1.145
.50	.50	1.265
.50	.25	1.455
.25	1.00	1.265
.25	.75	1.310
.25	.50	1.400
.25	.25	1.515



(e) $AE\left(\frac{\partial u}{\partial x}\right)_{x=0} = 0$

or $\left(\frac{\partial u}{\partial x}\right)_{x=0} = 0$

$AE\left(\frac{\partial \bar{u}}{\partial x}\right)_{x=L} = \beta \sin \omega t$ (f)

(g) $u\Big|_{x=kL} = \bar{u}\Big|_{x=kL}$

(h) $AE\left(\frac{\partial \bar{u}}{\partial x}\right)_{x=kL} - AE\left(\frac{\partial u}{\partial x}\right)_{x=kL} = M\left(\frac{\partial^2 u}{\partial t^2}\right)_{x=kL} = M\left(\frac{\partial^2 \bar{u}}{\partial t^2}\right)_{x=kL}$

Boundary Conditions For Deriving Transmissibility Equation

Figure 3

Boundary condition (g) from Figure 3 and the use of (8) above state that

$$(C_1 \cos ka) \sin\omega t = (C_3 \cos ka + C_4 \sin ka) \sin\omega t \tag{10}$$

Applying boundary condition (h) from Figure 3, we have

$$(C_1 D \sin ka) \sin\omega t - (C_3 D \sin ka - C_4 D \cos ka) \sin\omega t$$

$$= (-M\omega^2 C_1 \cos ka) \sin\omega t \tag{11}$$

Noting that the term $\sin\omega t$ is common to all terms in relationships (9), (10), and (11) above, the relationships can be rewritten as a set of 3 simultaneous equations:

$$\left.\begin{array}{rrrl} & -C_3 D \sin a & + C_4 D \cos a & = \beta \\ -C_1 \cos ka & +C_3 \cos ka & + C_4 \sin ka & = 0 \\ C_1(D \sin ka + M\omega^2 \cos ka) & - C_3 D \sin ka & + C_4 D \cos ka & = 0 \end{array}\right\} \tag{12}$$

The constants C_1, C_3, and C_4 are found by solving (12) by determinants as follows:

$$C_1 = \frac{\begin{vmatrix} \beta & -D \sin a & D \cos a \\ 0 & \cos ka & \sin ka \\ 0 & -D \sin ka & D \cos ka \end{vmatrix}}{\begin{vmatrix} 0 & -D \sin a & D \cos a \\ -\cos ka & \cos ka & \sin ka \\ D \sin ka + M\omega^2 \cos ka & -D \sin ka & D \cos ka \end{vmatrix}}$$

$$= \frac{\beta D \cos^2 k\alpha \; + \beta D \sin^2 k\alpha}{-D^2 \sin^2 k\alpha \sin\alpha - DM\omega^2 \sin k\alpha \cos k\alpha \sin\alpha - DM\omega^2 \cos^2 k\alpha \cos\alpha - D^2 \cos^2 k\alpha \sin\alpha}$$

$$= \frac{\beta D (\cos^2 k\alpha \; + \sin^2 k\alpha)}{-D^2 \sin\alpha(\sin^2 k\alpha + \cos^2 k\alpha) - DM\omega^2 \cos k\alpha(\sin k\alpha \sin\alpha + \cos k\alpha \cos\alpha)}$$

Making use of the identity $\cos^2\alpha + \sin^2\alpha = 1$ and expanding the last term in brackets of the denominator by the identities:

$$\sin\alpha \sin\beta = \frac{1}{2}\cos(\alpha - \beta) - \frac{1}{2}\cos(\alpha + \beta)$$

and

$$\cos\alpha \cos\beta = \frac{1}{2}\cos(\alpha - \beta) + \frac{1}{2}\cos(\alpha + \beta)$$

$$C_1 = \frac{\beta D}{-D(D\sin\alpha + M\omega^2 \cos k\alpha \cos(\alpha - k\alpha))} \qquad (13)$$

Referring to the denominator of (13) as J:

$$C_3 = \frac{\begin{vmatrix} 0 & \beta & D\cos\alpha \\ -\cos k\alpha & 0 & \sin k\alpha \\ D\sin k\alpha + M\omega^2 \cos k\alpha & 0 & D\cos k\alpha \end{vmatrix}}{J}$$

$$= \frac{\beta \sin k\alpha\,(D\sin k\alpha \; + M\omega^2 \cos k\alpha) + \beta D \cos^2 k\alpha}{J}$$

$$= \frac{\beta D(\sin^2 k\alpha + \cos^2 k\alpha) \; + \beta M\omega^2 \sin k\alpha \cos k\alpha}{J}$$

or, by making use of the identity, $\sin 2a = 2\sin a \cos a$,

$$C_3 = \frac{\beta D + \frac{\beta}{2} M\omega^2 \sin 2ka}{J} \tag{14}$$

$$C_4 = \frac{\begin{vmatrix} 0 & -D\sin a & \beta \\ -\cos ka & \cos ka & 0 \\ D\sin ka + M\omega^2\cos ka & -D\sin ka & 0 \end{vmatrix}}{J}$$

or

$$C_4 = \frac{\beta M\omega^2 \cos^2 ka}{J} \tag{15}$$

Defining $H = \frac{\beta D}{J} = -\frac{\beta}{D\sin a + M\omega^2 \cos ka \cos(a - ka)}$,

and substituting the constants C_1, C_3, and C_4 evaluated in (13), (14), and (15) above, equation (8) now becomes for $0 < x < kL$:

$$u = H \cos \frac{\omega}{c} x \sin \omega t \tag{16}$$

And equation (4) now becomes, for $kL < x < L$:

$$\bar{u} = \left[H\left(1 + \frac{M\omega c}{2EA} \sin 2ka\right)\cos\frac{\omega}{c}x - H\frac{M\omega c}{EA}\cos^2 ka \sin\frac{\omega}{c}x \right] \sin \omega t \tag{17}$$

If $M = 0$ (i.e. no concentrated mass), equations (16) and (17) reduce to $u = \bar{u} = -\frac{\beta L}{AE}\,\frac{c}{\omega L} \csc\frac{\omega L}{c} \cos\frac{\omega}{c} x \sin \omega t$, which is the solution for a bar with no concentrated mass and a forcing function at the end

$x = L$. Reference 3 presents a similar solution for a bar with no concentrated mass and a forcing function at the end $x = 0$.

To arrive at an expression for the transmissibility, the ratio of the force acting on the concentrated mass to that applied by the shaker, we solve for the right hand side of boundary condition (h) (see Table 3) which yields the force $F \sin \omega t$ acting on the concentrated mass:

$$M \left(\frac{\partial^2 u}{\partial t^2} \right)_{kL} = F \sin \omega t$$

or

$$M(-H \omega^2 \cos k\alpha) \sin \omega t = F \sin \omega t$$

Substituting for H and multiplying by $\frac{m\omega^2}{m\omega^2}$ where m is the mass of the bar, the transmissibility becomes:

$$TR = \frac{F \sin \omega t}{\beta \sin \omega t} = \frac{F}{\beta} = \frac{\frac{M}{m} \cos k\alpha}{\frac{1}{\alpha} \sin \alpha + \frac{M}{m} \cos k\alpha \cos(\alpha - k\alpha)} \qquad (18)$$

VIII. EXPERIMENTAL RESULTS

A. Fundamental Frequencies

Verification of equation (7) with experimental results is shown in Figure 4. While the basic method is described in METHOD OF PROCEDURE, a few more details are included here.

Table 2 is a tabulation of the actual test data as recorded using the test specimen as previously described.

The computation of f_{11} in Table 2 is accomplished by using a modified form of Dunkerley's equation [9] as follows:

$$\left.\begin{aligned} \frac{1}{(2\pi f_{n_1})^2} &= \frac{1}{(2\pi f_{11})^2} + \frac{W_1}{386} a_{22} \\ \frac{1}{(2\pi f_{n_2})^2} &= \frac{1}{(2\pi f_{11})^2} + \frac{W_2}{386} a_{22} \end{aligned}\right\} \qquad (19)$$

where f_{n_1} = fundamental frequency of W_1 and structure.

f_{n_2} = fundamental frequency of W_2 and structure.

W_1 = weight of shaker and coupling.

W_2 = W_1 + weight attached to coupling.

a_{22} = influence coefficient.

f_{11} = fundamental frequency of structure only.

Elimination of a_{22} from equations (19) results in the following:

$$f_{11} = \sqrt{\frac{f_{n_1}^{\;2}\, f_{n_2}^{\;2}}{2.658\, f_{n_2}^{\;2} - 1.658\, f_{n_1}^{\;2}}} \qquad (20)$$

TABLE 2

EXPERIMENTAL DATA FOR DETERMINATION OF FUNDAMENTAL FREQUENCIES

$\frac{W}{ATL}$	k	W_1 (Pounds)	W_2 (Pounds)	f_{n_1} cps	f_{n_2} cps	f_{11} cps	α
$\frac{1}{4}$	1	0.8288	1.3288	1574	1540	1638	1.23
	$\frac{3}{4}$	0.8288	1.3288	1630	1592	1710	1.278
	$\frac{1}{2}$	0.8288	1.3288	1727	1653	1880	1.402
	$\frac{1}{4}$	0.8288	1.3288	1720	1618	1935	1.45
$\frac{1}{2}$	1	0.8288	1.3288	1400	1390	1420	1.06
	$\frac{3}{4}$	0.8288	1.3288	1500	1490	1519	1.133
	$\frac{1}{2}$	0.8288	1.3288	1614	1563	1710	1.278
	$\frac{1}{4}$	0.8288	1.3288	1688	1597	1885	1.40

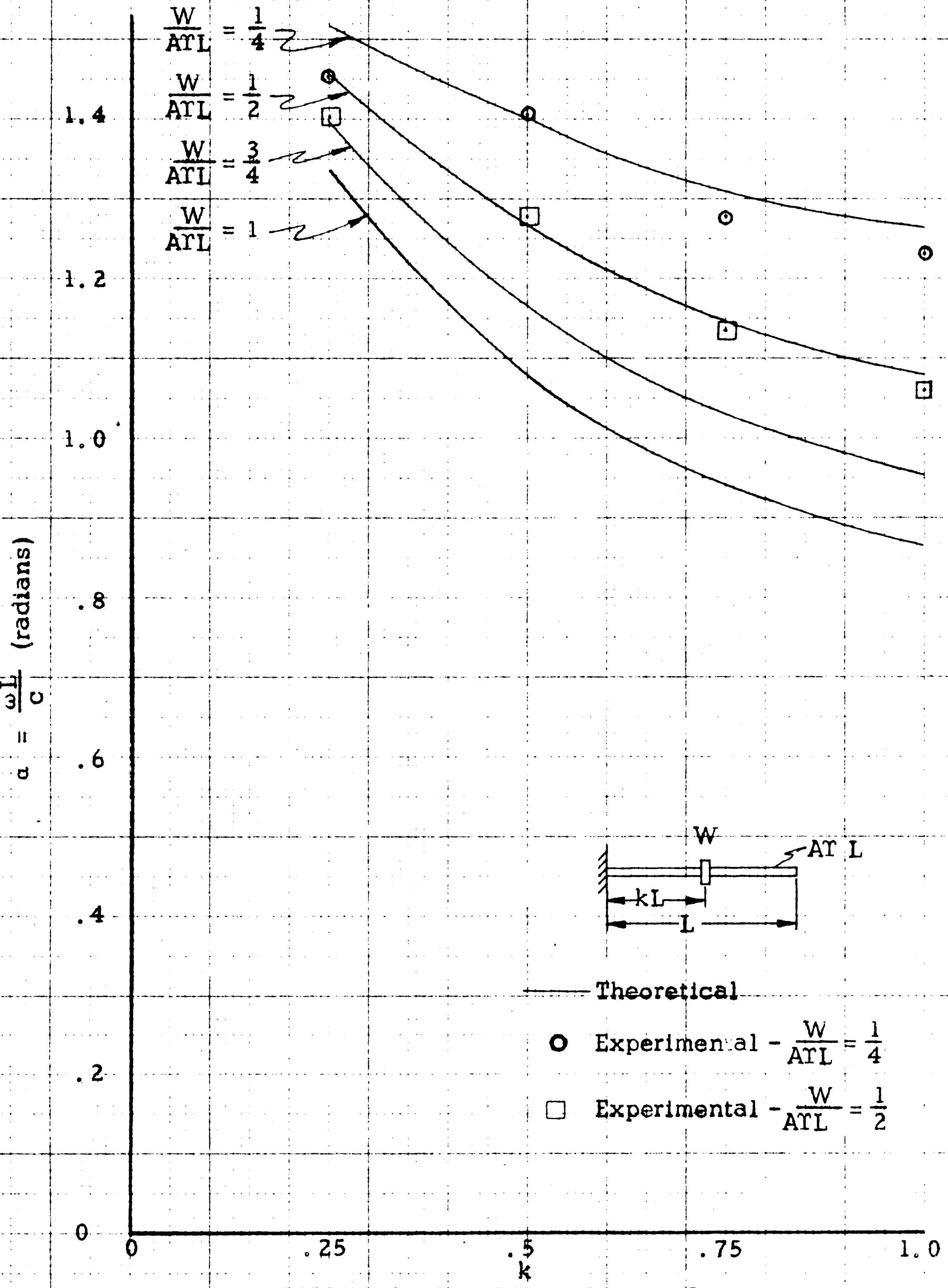


FIGURE 4 - FUNDAMENTAL FREQUENCIES

B. TRANSMISSIBILITY

Some difficulty was experienced in first attempting to determine the input force applied by the shaker. It was first thought that there was a linear relationship between the current in the audio oscillator and the force in the shaker[7]. Results of experimentation and reference 4 indicate that this is not true except at frequencies of less than 500 cps. The force gage was then used to provide the input force reading. Plots of the data for the transmissibility experimentation are shown in two Figures, 5 and 6. Data tables from which these curves were plotted are Tables 3 through 8.

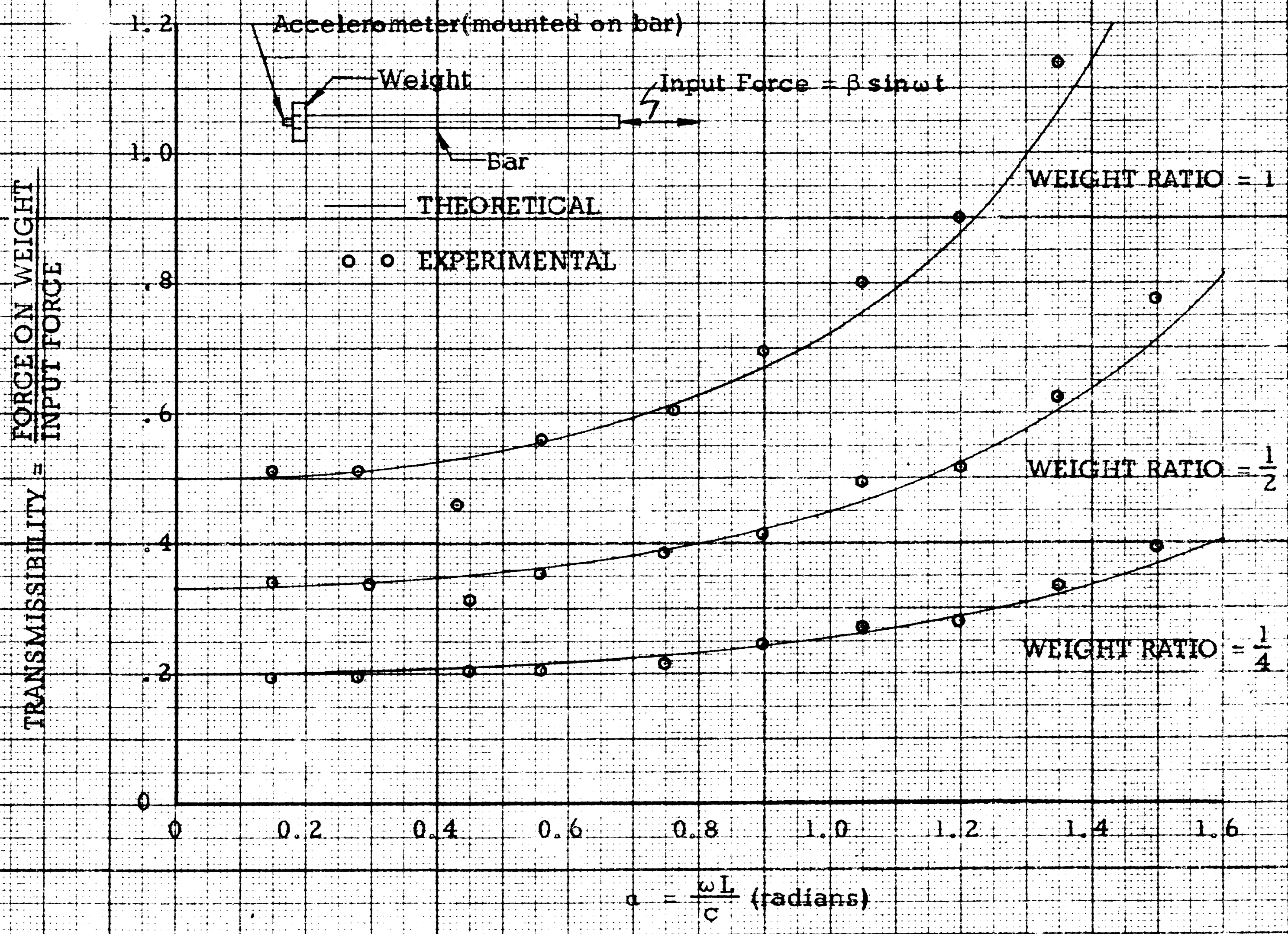


FIGURE 5 - TRANSMISSIBILITY VS. α FOR WEIGHT AT FREE END

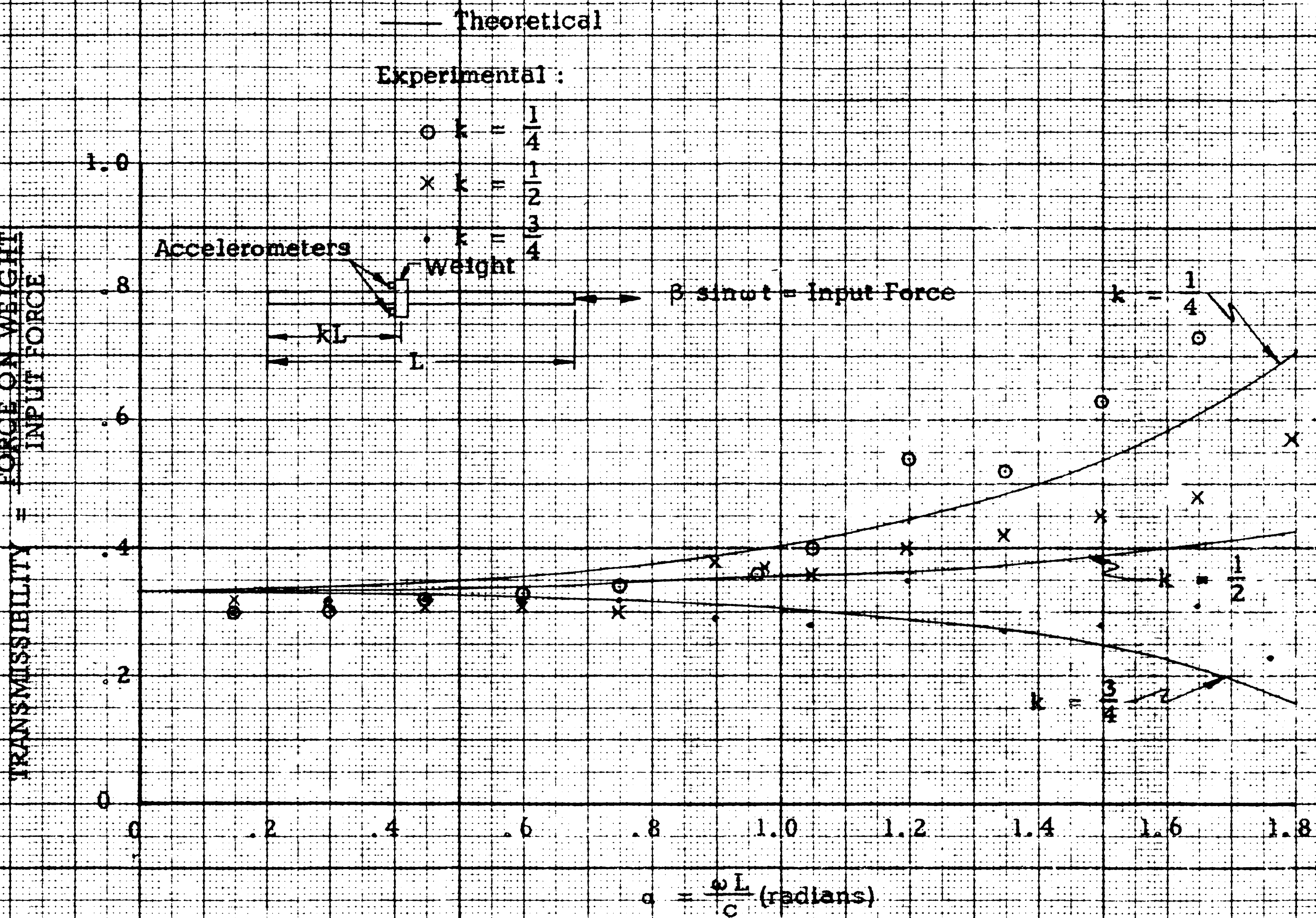


FIGURE 6 – TRANSMISSIBILITY VERSUS FREQUENCY PARAMETER α – WEIGHT RATIO = $\frac{1}{2}$

TABLE 3
DATA FOR INPUT FORCE AND FORCE ON "CONCENTRATED MASS"

Weight Ratio = 1   k = 0   (kL = distance from free end of test specimen)

Freq. cps	Force Gage Reading			Accelerometer Reading ⓒ			Transmissibility = Column 7 / Column 4
	Oscilloscope reading ⓐ	Volts	Pounds	Oscilloscope ⓐ reading ⓑ	Millivolts	Pounds (F = Ma)	
200	1/2.0	2.0	4.25	0.2/1.2	24	2.17	0.51
375	1/2.0	2.0	4.25	0.2/1.2	24	2.17	0.51
575	1/2.4	2.4	5.15	0.2/1.3	26	2.37	0.46
800	1/3.5	3.5	7.50	0.2/2.3	46	4.20	0.56
1025	1/2.7	2.7	5.75	0.2/1.9	38	3.47	0.60
1200	1/3.6	3.6	7.62	0.2/2.9	58	5.30	0.70
1400	1/3.0	3.0	6.40	0.2/2.8	56	5.12	0.80
1600	1/2.0	2.0	4.25	0.2/2.1	42	3.82	0.90
1800	1/2.0	2.0	4.25	0.2/2.7	54	4.84	1.14

ⓐ volts per cm/cms.   ⓑ gain of 10   ⓒ #5614

TABLE 4
DATA FOR INPUT FORCE AND FORCE ON "CONCENTRATED MASS"

Weight Ratio = $\frac{1}{2}$ k = 0   (kL = distance from free end of test specimen)

Freq. cps	Force Gage Reading			Accelerometer Reading ⓒ			Transmissibility = $\frac{\text{Column 7}}{\text{Column 4}}$
	Oscilloscope reading ⓐ	Volts	Pounds	Oscilloscope ⓐ reading ⓑ	Millivolts	Pounds (F = Ma)	
200	1/2.0	2.0	4.25	0.2/1.6	32	1.45	0.34
400	1/2.0	2.0	4.25	0.2/1.6	32	1.45	0.34
600	1/3.0	3.0	6.40	0.2/2.2	44	2.00	0.31
800	1/4.0	4.0	8.50	0.2/3.3	66	3.00	0.35
1000	1/2.0	2.0	4.25	0.2/1.8	36	1.65	0.39
1200	1/2.0	2.0	4.25	0.2/1.9	38	1.75	0.41
1400	1/2.0	2.0	4.25	0.2/2.3	46	2.10	0.49
1600	1/2.0	2.0	4.25	0.2/2.4	48	2.20	0.52
1800	1/2.0	2.0	4.25	0.2/2.9	58	2.65	0.62
2000	1/2.6	2.6	5.55	0.2/4.7	94	4.30	0.77
2200	1/2.0	2.0	4.25	0.2/4.8	96	4.40	1.03

ⓐ volts per cm/cms.   ⓑ gain of 10   ⓒ # 5614

TABLE 5
DATA FOR INPUT FORCE AND FORCE ON "CONCENTRATED MASS"

Weight Ratio = $\frac{1}{4}$ k = 0 (kL = distance from free end of test specimen)

Freq. cps	Force Gage Reading			Accelerometer Reading ⓒ			Transmissibility = $\frac{\text{Column 7}}{\text{Column 4}}$
	Oscilloscope reading ⓐ	Volts	Pounds	Oscilloscope ⓐ reading ⓑ	Millivolts	Pounds (F = Ma)	
200	1/2.0	2.0	4.25	0.2/1.8	36	0.82	0.19
400	1/2.0	2.0	4.25	0.2/1.8	36	0.82	0.19
600	1/2.0	2.0	4.25	0.2/1.9	38	0.87	0.21
800	1/2.0	2.0	4.25	0.2/1.9	38	0.87	0.21
1000	1/3.0	3.0	6.40	0.2/3.0	60	1.38	0.22
1200	1/2.0	2.0	4.25	0.2/2.3	46	1.06	0.25
1400	1/2.0	2.0	4.25	0.2/2.5	50	1.15	0.27
1600	1/2.0	2.0	4.25	0.2/2.6	52	1.19	0.28
1800	1/2.0	2.0	4.25	0.2/3.1	62	1.42	0.33
2000	1/2.6	2.6	5.55	0.2/4.8	96	2.20	0.40
2200	1/2.0	2.0	4.25	0.2/4.8	96	2.20	0.52
2400	1/1.0	1.0	2.10	0.2/3.5	70	1.60	0.76

ⓐ volts per cm/cms. ⓑ gain of 10 ⓒ #5614

TABLE 6

DATA FOR INPUT FORCE AND FORCE ON "CONCENTRATED MASS"

Weight Ratio = $\frac{1}{2}$ k = $\frac{1}{4}$ (kL = distance from free end of test specimen)

Freq. cps	Force Gage Reading			Accelerometer Reading				Transmissibility = Column 8 / Column 4
	Oscilloscope reading ⓐ	Volts	Pounds	Oscilloscope ⓐ reading ⓑ	Millivolts	Pounds (F = Ma)	Average Pounds	
200	1/2.0	2.0	4.25	0.2/1.4	28	1.28	1.28	0.30
				0.2/0.7	14	1.28		
400	1/2.0	2.0	4.25	0.2/1.4	28	1.28	1.28	0.30
				0.2/0.7	14	1.28		
600	1/2.0	2.0	4.25	0.2/1.4	28	1.28	1.37	0.32
				0.2/0.8	16	1.46		
800	1/2.0	2.0	4.25	0.2/1.5	30	1.36	1.41	0.33
				0.2/0.8	16	1.46		
1000	1/3.0	3.0	6.40	0.2/2.4	48	2.20	2.20	0.34
				0.2/1.2	24	2.20		
1288	10/1.5	15.0	35.00	1.0/2.5	250	11.50	12.60	0.36
				1.0/1.5	150	13.70		
1400	1/4.0	4.0	8.50	0.2/3.2	64	2.92	3.38	0.40
				0.2/2.1	42	3.85		
1600	1/2.0	2.0	4.25	0.2/2.6	52	2.37	2.28	0.54
				0.2/1.2	24	2.20		
1800	1/2.0	2.0	4.25	0.2/3.2	64	2.92	2.19	0.52
				0.2/0.8	16	1.46		
2000	1/2.2	2.2	4.70	0.2/3.1	62	2.83	2.98	0.63
				0.2/1.7	34	3.12		
2200	1/2.0	2.0	4.25	0.2/3.4	68	3.10	3.10	0.73
				0.2/1.7	34	3.12		

ⓐ volts per cm/cms. ⓑ gain of 10

TABLE 7

DATA FOR INPUT FORCE AND FORCE ON "CONCENTRATED MASS"

Weight Ratio = $\frac{1}{2}$ k = $\frac{1}{2}$ (kL = distance from free end of test specimen)

Freq. cps	Force Gage Reading			Accelerometer Reading				Transmissibility = $\frac{\text{Column 8}}{\text{Column 4}}$
	Oscilloscope reading ⓐ	Volts	Pounds	Oscilloscope ⓐ reading ⓑ	Millivolts	Pounds (F = Ma)	Average Pounds	
200	1/2.0	2.0	4.25	0.2/1.6	32	1.46	1.38	0.32
				0.2/1.4	28	1.30		
400	1/2.0	2.0	4.25	0.2/1.4	28	1.30	1.30	0.31
				0.2/1.4	28	1.30		
600	1/2.0	2.0	4.25	0.2/1.4	28	1.30	1.30	0.31
				0.2/1.4	28	1.30		
800	1/2.6	2.6	5.56	0.2/1.8	36	1.65	1.74	0.31
				0.2/2.0	40	1.83		
1000	1/4.3	4.3	9.15	0.2/2.6	52	2.37	2.73	0.30
				0.2/3.4	68	3.10		
1200	1/3.0	3.0	6.40	0.2/3.7	74	3.40	2.43	0.38
				0.2/1.6	32	1.46		
1300	10/2.0	20.0	43.50	2.0/1.7	340	15.6	16.10	0.37
				2.0/1.8	360	16.10		
1400	1/2.0	2.0	4.25	0.2/1.8	36	1.65	1.51	0.36
				0.2/1.5	30	1.38		
1600	1/2.0	2.0	4.25	0.2/1.8	36	1.65	1.70	0.40
				0.2/1.9	38	1.75		
1800	1/2.0	2.0	4.25	0.2/1.8	36	1.65	1.78	0.42
				0.2/2.1	42	1.92		
2000	1/2.0	2.0	4.25	0.2/1.4	28	1.30	1.93	0.45
				0.2/2.8	56	2.56		
2200	1/2.0	2.0	4.25	0.2/1.8	36	1.65	2.05	0.48
				0.2/2.7	54	2.46		

ⓐ volts per cm/cms. ⓑ gain of 10

TABLE 8

DATA FOR INPUT FORCE AND FORCE ON "CONCENTRATED MASS"

Weight Ratio = $\frac{1}{2}$ k = $\frac{3}{4}$ (kL = distance from free end of test specimen)

Freq. cps	Force Gage Reading			Accelerometer Reading				Transmissibility = Column 8 / Column 4
	Oscilloscope reading ⓐ	Volts	Pounds	Oscilloscope ⓐ reading ⓑ	Millivolts	Pounds (F = Ma)	Average Pounds	
200	1/2.0	2.0	4.25	0.2/1.3	26	1.20	1.28	0.30
				0.2/0.8	16	1.46		
400	1/2.0	2.0	4.25	0.2/1.4	28	1.28	1.37	0.32
				0.2/0.8	16	1.46		
600	1/2.0	2.0	4.25	0.2/1.4	28	1.28	1.37	0.32
				0.2/0.8	16	1.46		
800	1/2.0	2.0	4.25	0.2/1.4	28	1.28	1.37	0.32
				0.2/0.8	16	1.46		
1000	1/3.0	3.0	6.40	0.2/2.1	42	1.92	2.06	0.32
				0.2/1.2	24	2.20		
1200	2/3.0	6.0	12.80	0.2/2.9	58	2.65	3.71	0.29
				0.2/2.6	52	4.76		
1400	2/3.0	6.0	12.80	0.2/3.5	70	3.20	3.62	0.28
				0.2/2.2	44	4.03		
1600	1/2.0	2.0	4.25	0.2/1.0	20	0.92	1.47	0.35
				0.2/1.1	22	2.02		
1800	1/2.0	2.0	4.25	0.2/2.1	42	1.92	1.14	0.27
				0.1/0.4	4	0.36		
2000	1/2.0	2.0	4.25	0.2/1.4	28	1.28	1.19	0.28
				0.2/0.6	12	1.10		
2200	1/2.0	2.0	4.25	0.2/0.5	10	0.45	1.33	0.31
				0.2/1.2	24	2.20		
2350	1/3.3	3.3	7.00	0.2/1.9	38	1.75	1.61	0.23
				0.2/0.8	16	1.46		

ⓐ volts per cm/cms. ⓑ gain of 10

TABLE 9
DATA FOR INPUT FORCE AND FORCE ON "CONCENTRATED MASS"

Weight Ratio = 1 Bar Length = Thickness of Block $M = \frac{0.85\#}{g}$

Freq. cps	Force Gage Readings (converted to pounds)	Accelerometer Readings						
		2 Symmetrical Accelerometers Mounted on Block				One Accelerometer Mounted at Bar Centerline		
		Oscilloscope ⓐ reading ⓑ	Millivolts	Pounds (F = Ma)	Average Pounds	Oscilloscope ⓐ reading ⓑ	Millivolts	Pounds (F = Ma)
200	4.25	0.2/3.3	66	3.90	3.84	0.2/3.6	72	4.25
		0.2/1.6	32	3.78				
400	4.25	0.2/3.4	68	4.00	3.89	0.2/3.5	70	4.14
		0.2/1.6	32	3.78				
600	4.25	0.2/3.8	76	4.50	4.02	0.2/3.6	72	4.25
		0.2/1.5	30	3.54				
800	4.25	0.2/3.6	72	4.25	4.01	0.2/3.5	70	4.14
		0.2/1.6	32	3.78				
1000	6.40	0.5/2.3	115	6.80	5.76	0.5/2.2	110	6.49
		0.2/2.0	40	4.72				
1200	9.80	0.5/3.1	155	9.15	9.18	0.5/3.6	180	10.60
		0.2/3.9	78	9.20				
1400	6.40	0.5/2.2	110	6.50	6.08	0.5/2.1	100	5.90
		0.2/2.4	48	5.66				
1600	4.25	0.2/3.2	64	3.78	4.25	0.2/3.8	76	4.49
		0.2/2.0	40	4.72				
1800	4.25	0.2/3.6	72	4.25	4.37	0.2/3.8	76	4.49
		0.2/1.9	38	4.50				
2000	4.25	0.2/3.7	74	4.37	4.43	0.2/3.4	68	4.00
		0.2/1.9	38	4.50				
2200	4.25	0.2/3.8	76	4.50	4.37	0.2/3.4	68	4.00
		0.2/1.8	36	4.25				
2400	4.25	0.2/4.2	84	4.96	4.84	0.2/3.4	68	4.00
		0.2/2.0	40	4.72				

ⓐ volts per cm/cms.
ⓑ gain of 10

IX. DISCUSSION OF RESULTS

The fundamental frequencies determined experimentally and shown in Figure 4 are in good agreement with equation (7). The larger error at $k = 0.25$ is attributed to the fact that as the position of the weight was moved closer to the shaker, the natural frequency of the system was less clearly defined resulting in erroneous readings.

The transmissibility curves shown in Figure 5 reflect very good accuracy. However, these three curves were for the weight mounted at the free end of the bar with the accelerometer mounted at the center of the bar. An additional test was run with the bar length changed to be equal to the thickness of the block. With the accelerometer mounted at the center of the bar, results of comparing the force gage reading with the product of mass times acceleration were reasonably accurate. But upon switching the accelerometer from this position to a position on the block and adding a second accelerometer symmetrical with the first, significant errors appeared, especially at the higher frequencies. The average reading of these two accelerometers gave consistently higher readings at frequencies above 1800 cps. Data is included in Table 9. This effect is thought to be attributable to the non-rigid body vibration of the block representing the concentrated weight. This vibration could be described as a combination of shear and bending actions.

Results comparable with the accuracy of Figure 5 were not attainable with the accelerometers mounted on the block itself. Low frequency results were good, but at the higher frequencies large discrepancies were noted with the accelerometers giving the same results at one frequency, then one accelerometer reading higher than the other at another frequency, or vice versa at still another frequency. The average of the two accelerometers did give curves which verified the trend of the analytical solution. These curves are shown for a weight ratio of one-half in Figure 6. The points that are below the theoretical curves can be explained by noting that the weight of the bar clamped by the block was not included in the weight of the concentrated weight, hence the mass times acceleration term is somewhat on the low side and the experimental points fall below the theoretical curve. As the frequency increases, the non-rigid body vibration of the block begins to predominate resulting in higher acceleration readings than actually exist at the bar centerline. Hence, the experimental points are above the theoretical curve.

X. CONCLUSIONS

The expression given in equation (15) was verified by experimentation although measurement of the acceleration at the bar centerline would have reduced the error considerably.

For low frequencies ($a < 0.75$), the change in transmissibility is small and can probably be ignored.

The curves shown in Figure 5 and 6 do not exhibit any discontinuities at the natural frequencies as evidenced by these curves. There is, however, much less shaker input current required to produce a large input force at and near the system fundamental frequency.

As noted in Test Equipment, the shaker is rated only up to a frequency of 1000 cps. While this is probably a conservative rating, this limitation has restricted the range of the results presented in this thesis. A greater force rating than the 10 pounds available would have helped considerably, as the acceleration and force gage readings would have been larger, resulting in lower reading errors. However, with the idea in mind that the problem investigated is not the practical problem, but only a simplification of it, the author feels that with the data presented herein as a foundation, far more practical and useful data could be obtained from experiments on an actual shaker table. This investigation has pointed to some of the difficulties, and their solution, that such experiments would entail.

XI. ACKNOWLEDGEMENTS

The author wishes to express his gratitude to Profs. Pletta and Maher for their help in the early stages of this thesis formulation, to Mr. Kenneth McCauley and Mr. Thomas Ewers for their help in the laboratory, and especially to Dr. Ricardo Chicurel for his advice and guidance and contribution of many hours of time and thought; many during the school vacation periods during which much of this thesis work was accomplished.

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ABSTRACT

The problem of analyzing a shake table is simplified to that of a bar of constant cross section with a weight attached at any point along the bar, all subjected to a sinusoidal force at one end of the bar. The bar, suspended by wires, represents the table. The weight used in the experiment represents any piece of equipment which might be subjected to vibration testing. The fundamental frequencies of the system and the ratio of the force acting on the weight to the input force for various weight ratios are determined analytically and experimentally. This ratio is called the transmissibility. Experimental results were good under certain conditions, but tended to have increasingly larger errors as frequencies increased.